CMPEN 454 PROJECT-3 Writeup

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Part 1 Theory Questions

Q1.1: Calculating the Jacobian

Assuming the affine warp model defined in Equation 3, derive the expression for the Jacobian Matrix J in terms of the warp parameters $p = [p_1, p_2, p_3, p_4, p_5, p_6]'$.

$$\text{equation 2 and 3}$$

$$\mathbf{W}(\mathbf{x};\mathbf{p}) = \begin{bmatrix} \mathbf{W}_x \\ \mathbf{W}_y \\ \mathbf{W}_1 \end{bmatrix} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} 1+p_1 & p_3 & p_5 \\ p_2 & 1+p_4 & p_6 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \text{ and,}$$

$$\mathbf{p} = \begin{bmatrix} p_1 & p_2 & p_3 & p_4 & p_5 & p_6 \end{bmatrix}^T$$

$$\frac{\partial \mathbf{W}}{\partial \mathbf{p}} = \begin{bmatrix} \frac{\partial \mathbf{W}_u}{\partial \mathbf{p}_1} & \cdots & \frac{\partial \mathbf{W}_u}{\partial \mathbf{p}_6} \\ \frac{\partial \mathbf{W}_v}{\partial \mathbf{p}_1} & \cdots & \frac{\partial \mathbf{W}_v}{\partial \mathbf{p}_6} \end{bmatrix} = \begin{bmatrix} u & 0 & v & 0 & 1 & 0 \\ 0 & u & 0 & v & 0 & 1 \end{bmatrix}$$

$$L = \sum_{\mathbf{x}} [\mathbf{T}(x) - \mathbf{I}(\mathbf{W})]^2$$

$$\begin{aligned} \mathbf{J} = \frac{\partial L}{\partial \mathbf{p}} &= \frac{\partial L}{\partial \mathbf{I}(\mathbf{W})} \frac{\partial \mathbf{I}(\mathbf{W})}{\partial \mathbf{W}} \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \\ &= -2 \sum_{\mathbf{x}} [\mathbf{T}(x) - \mathbf{I}(\mathbf{W})] \begin{bmatrix} \frac{\partial \mathbf{I}}{\partial u} & \frac{\partial \mathbf{I}}{\partial v} \end{bmatrix} \begin{bmatrix} u & 0 & v & 0 & 1 & 0 \\ 0 & u & 0 & v & 0 & 1 \end{bmatrix} \end{aligned}$$

Q1.2: Computational complexity

Initialization step

First calculate $\nabla \mathbf{T}$, which the runtime is $O(n)$, then calculate $\mathbf{J} = \frac{\partial \mathbf{W}}{\partial \mathbf{p}}$ at $\mathbf{p} = \mathbf{0}$. To calculate the $\nabla \mathbf{T} \cdot \frac{\partial \mathbf{W}}{\partial \mathbf{p}}$, since $\nabla \mathbf{T}$ is of dimension $(m \times 2)$ and $\frac{\partial \mathbf{W}}{\partial \mathbf{p}}$ is of dimension (2×6), this multiplication should be $O(n)$. To calculate the Hessian matrix $\mathbf{H} = \mathbf{J^T J}$, since $\mathbf{J}$ is of dimension $(m \times 6)$, this multiplication should be $O(n^2)$.

Thus the initialization cost will be $O(n + n^2) = O(n^2)$.

Incremental step

First, calculate the $\mathbf{I(W(x;p))}$cost should be $O(n)$. Then calculate the cost of $[\mathbf{I}(\mathbf{W(x;p)}) - \mathbf{T}]$= $O(n)$.

Then calculate the cost of $\sum_\mathbf{x} \mathbf{(\nabla T \frac{\partial W}{\partial p})^T} [\mathbf{I}(\mathbf{W(x;p)}) - \mathbf{T(x)}]$= $O(n)$, multiply the previous result with the inverse of Hessian to get $\mathbf{\Delta p}$= $O(n^3)$. Then update the warp function, which is $O(n^2)$

The total time complexity is $O(n + n^3) = O(n^3)$

Part 2 Lucas-Kanade Tracker

Best The best comes from the Lucas Kanade tracker.