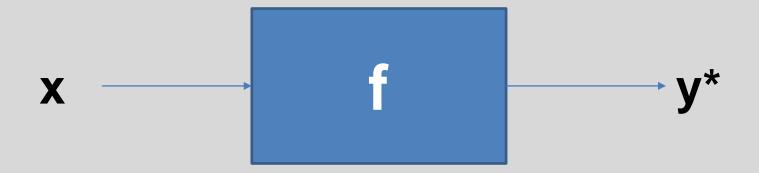


Computer Vision

Lecture 03: Machine Learning Basics-2

Machine learning

• x: data, y*: prediction, f: function (ie. machine)



Find f from data=learning a machine f from (x, y).
 ie. machine learning



Regression

- Regression
 - Data x, ground-truth y.
 - The ground-truth y is continuous.
 - It is trained in the supervised way.



Regression vs. Classification

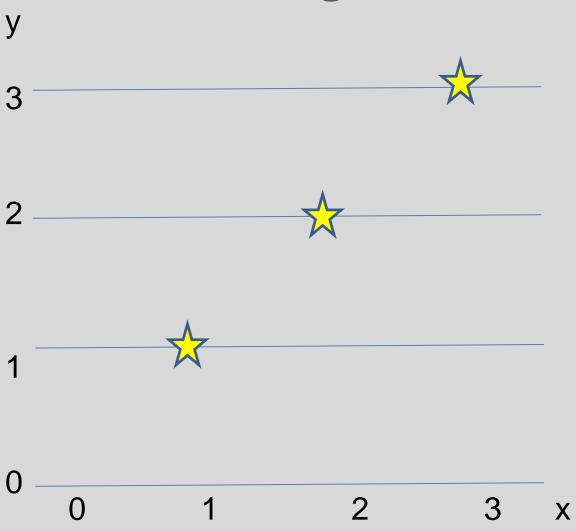
- In classification, y label does not have meaning by itself.
- E.g. class 1, 2, 3, ... → Class index can be interchanged.
- In regression, y label itself has some meaning.
- E.g. mid-term exam score, weight/height, ...

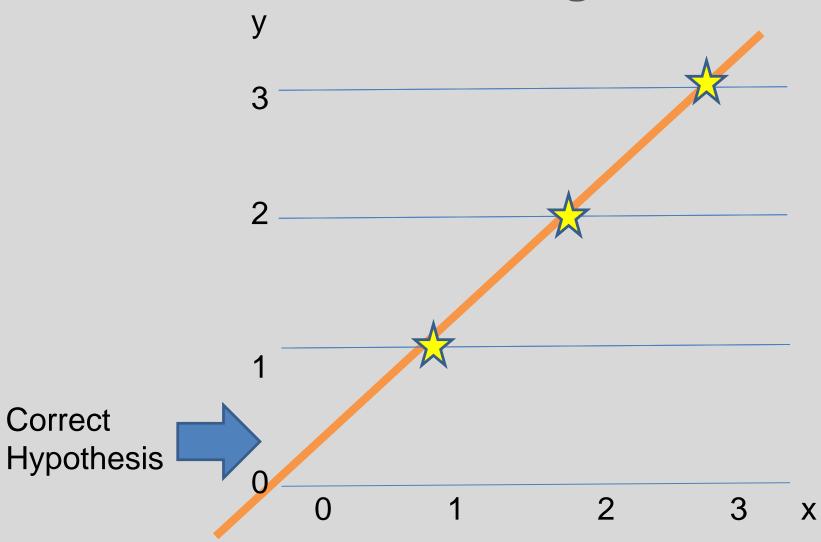


x (attendance score)	y (mid-term score)	
10	90	
9	70	
1	20	
4	50	
8	60	

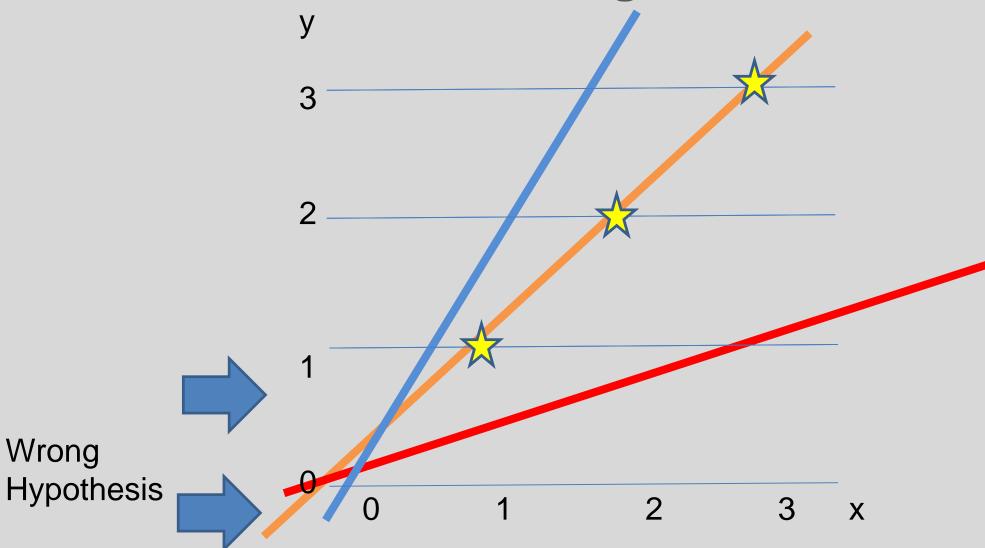


X	y
1	1
2	2
3	3

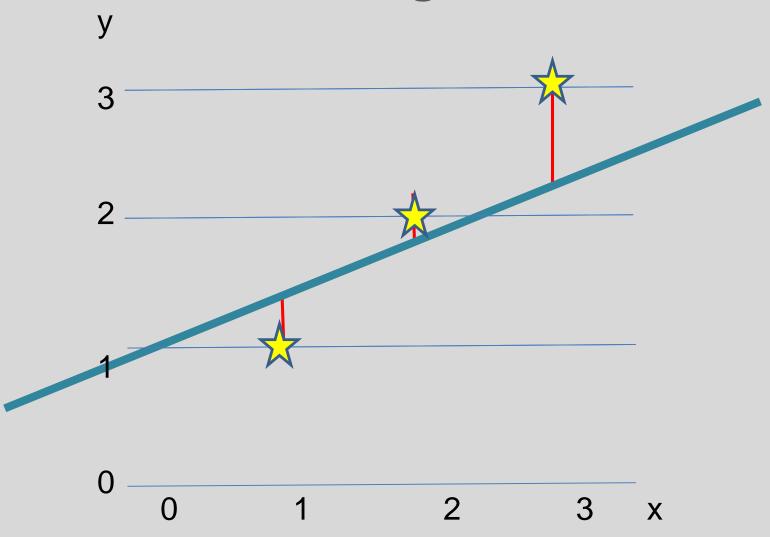


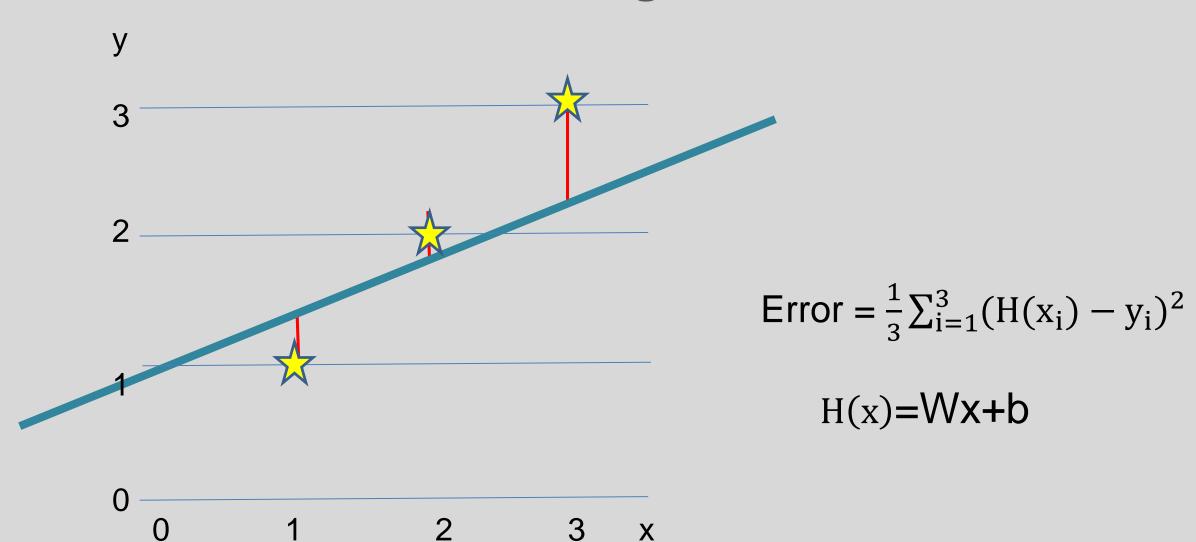






Hypothesis:
$$H(x) = y = Wx + b$$

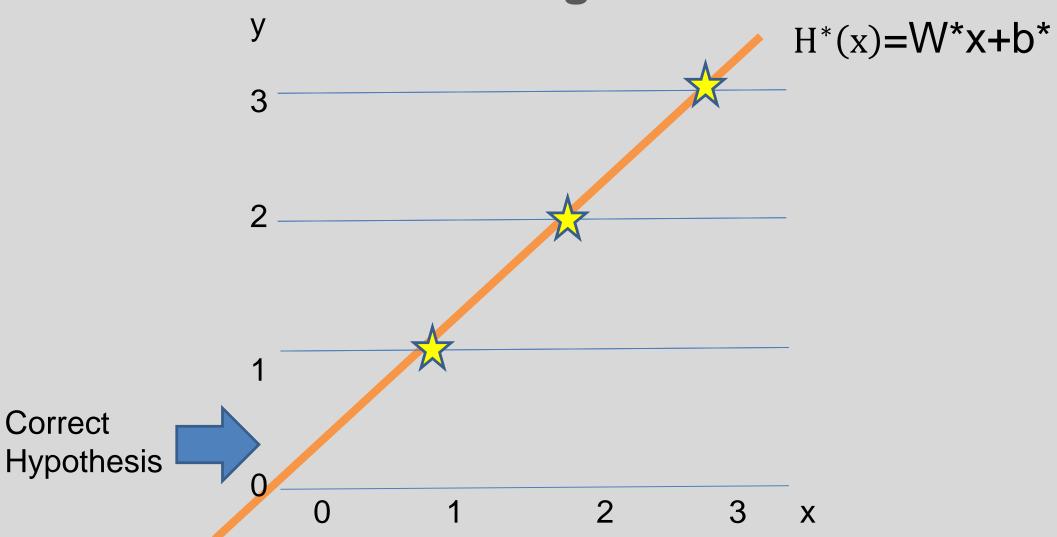




Error(W,b) =
$$\frac{1}{3}\sum_{i=1}^{3}(H(x_i) - y_i)^2$$
, $H(x) = Wx + b$

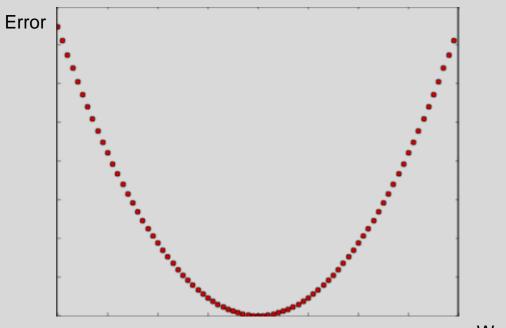
$$W^*$$
, $b^* = minimize_{W,b}Error(W, b)$





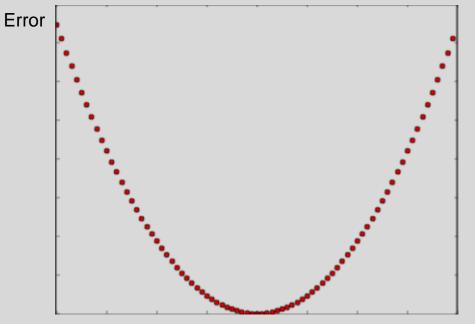
How to optimize it?

Error(W,b) =
$$\frac{1}{3}\sum_{i=1}^{3}(Wx_i + b - y_i)^2$$
,



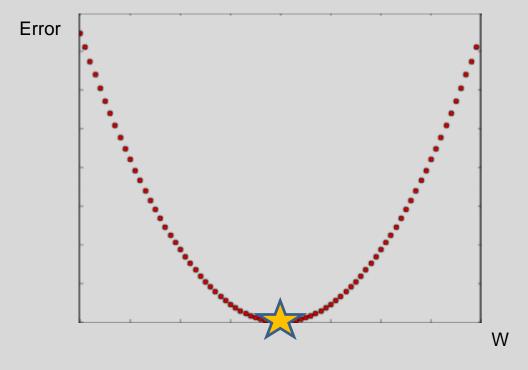
How to optimize it? → Gradient descent algorithm

Error(W,b) =
$$\frac{1}{3}\sum_{i=1}^{3}(Wx_i + b - y_i)^2$$
,



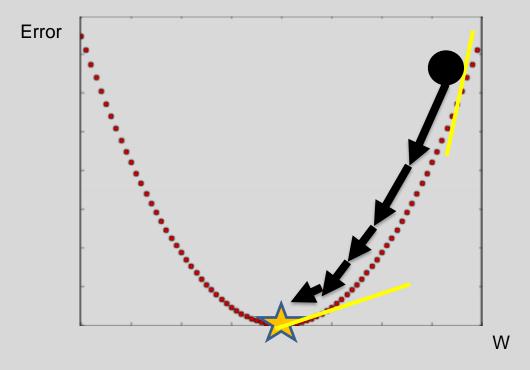
Convex function

For the convex function, we manually can find the global minima by differentiating it.



Convex function

We can also iteratively reach the global minima.



Local minima = Global minima

Gradient descent

Error(W, b) =
$$\frac{1}{3}\sum_{i=1}^{3}(Wx_i + b - y_i)^2$$
,

$$W^{(t+1)} = W^{(t)} - \epsilon \frac{\partial}{\partial W} \text{Error(W, b)}$$
$$b^{(t+1)} = b^{(t)} - \epsilon \frac{\partial}{\partial b} \text{Error(W, b)}$$

 ϵ : Learning rate (small value e.g. 0.001)



Multi-variate linear regression

x1 (attendance score)	x2 (quiz score)	x3 (assignment 1 score)	y (mid-term score)
10	8	5	90
9	7	7	70
1	4	3	20
4	6	6	50
8	3	7	60

Multi-variate linear regression

$$H(x)=W^Tx+b$$

$$H(x_1, x_2, x_3) = w_1 x_1 + w_2 x_2 + w_3 x_3 + b$$

W and x become vectors.



PyTorch library

https://numpy.org/



Python library for multi-dimensional array.

https://pytorch.org/



Python library for multi-dimensional array.

+

Simple gradient computation. (loss.backward()) Simple GPU usage.

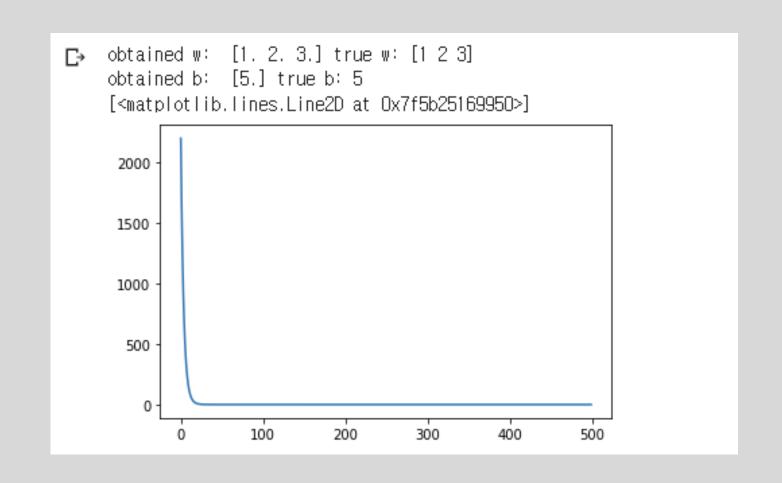
Implementing linear regression using Numpy

```
import numpy as np
w true = np.array([1, 2, 3])
b true = 5
w = np.random.rand(3, 1).squeeze(1)
b = np.random.rand(1)
X = np.random.rand(100, 3)
y = np.matmul(X, w true) + b true
qamma = 0.1
losses = []
for i in range(100):
  errors = y - (np.dot(X, w) + b)
  dEdw = np.dot(X.T, errors)
  dEdb = errors.sum()
  loss = (errors**2).sum()/2.0
  w += gamma * dEdw
  b += gamma * dEdb
  losses.append(loss)
print('obtained w: ', w, 'true w:', w true)
print('obtained b: ', b, 'true b:', b true)
from matplotlib import pyplot as plt
plt.plot(losses)
```

Numpy does not support backward()!

"By ourselves, we have to find the closed form for the gradient."

Implementing linear regression using Numpy



```
import torch
w true = torch. Tensor([1, 2, 3])
b true = 5
w = torch.randn(3, requires grad = True)
b = torch.randn(1, requires grad = True)
X = torch.randn(100, 3)
y = torch.mv(X, w true) + b true
qamma = 0.1
losses = []
for i in range (100):
  w.grad = None
  b.grad = None
  y \text{ pred} = \text{torch.mv}(X, w) + b
  loss = torch.mean((y- y pred)**2)
  loss.backward()
  w.data = w.data - gamma * w.grad.data
  b.data = b.data - gamma * b.grad.data
  losses.append(loss.item())
```

Ground-truth linear regression parameter (W, b) we decided.

Implementing linear regression using PyTorch

```
import torch
w true = torch. Tensor ([1, 2, 3])
b true = 5
w = torch.randn(3, requires grad = True)
b = torch.randn(1, requires grad = True)
X = torch.randn(100, 3)
y = torch.mv(X, w true) + b true
qamma = 0.1
losses = []
for i in range (100):
  w.grad = None
  b.grad = None
  y \text{ pred} = \text{torch.mv}(X, w) + b
  loss = torch.mean((y- y pred)**2)
  loss.backward()
  w.data = w.data - gamma * w.grad.data
  b.data = b.data - gamma * b.grad.data
  losses.append(loss.item())
```



Ground-truth linear regression parameter (W, b) we decided.

We will find the solution (W, b) in this random initialized variable.

```
import torch
w true = torch. Tensor ([1, 2, 3])
b true = 5
w = torch.randn(3, requires grad = True)
b = torch.randn(1, requires grad = True)
X = torch.randn(100, 3)
y = torch.mv(X, w true) + b true
qamma = 0.1
losses = []
for i in range (100):
  w.grad = None
  b.grad = None
  y \text{ pred} = \text{torch.mv}(X, w) + b
  loss = torch.mean((y- y pred)**2)
  loss.backward()
  w.data = w.data - gamma * w.grad.data
  b.data = b.data - gamma * b.grad.data
  losses.append(loss.item())
```



Ground-truth linear regression parameter (W, b) we decided.

We will find the solution (W, b) in this random initialized variable.

Data (X, y) are generated using ground-truth (W, b).

Implementing linear regression using PyTorch

```
import torch
w true = torch. Tensor([1, 2, 3])
b true = 5
w = torch.randn(3, requires grad = True)
b = torch.randn(1, requires grad = True)
X = torch.randn(100, 3)
y = torch.mv(X, w true) + b true
qamma = 0.1
losses = []
for i in range(100):
  w.grad = None
  b.grad = None
  y \text{ pred} = \text{torch.mv}(X, w) + b
  loss = torch.mean((y- y pred)**2)
  loss.backward()
  w.data = w.data - gamma * w.grad.data
  b.data = b.data - gamma * b.grad.data
  losses.append(loss.item())
```



Ground-truth linear regression parameter (W, b) we decided.

We will find the solution (W, b) in this random initialized variable.

Data (X, y) are generated using ground-truth (W, b).

Learning rate and the variable we will accumulate our loss.

Implementing linear regression using PyTorch

```
import torch
w true = torch. Tensor([1, 2, 3])
b true = 5
w = torch.randn(3, requires grad = True)
b = torch.randn(1, requires grad = True)
X = torch.randn(100, 3)
y = torch.mv(X, w true) + b true
qamma = 0.1
losses = []
for i in range(100):
  w.grad = None
  b.grad = None
  y \text{ pred} = \text{torch.mv}(X, w) + b
  loss = torch.mean((y- y pred)**2)
  loss.backward()
  w.data = w.data - gamma * w.grad.data
  b.data = b.data - gamma * b.grad.data
  losses.append(loss.item())
```



Ground-truth linear regression parameter (W, b) we decided.

We will find the solution (W, b) in this random initialized variable.

Data (X, y) are generated using ground-truth (W, b).

Learning rate and the variable we will accumulate our loss.

Loop until 100 iteration to change (W, b) solution using gradient descent.

Implementing linear regression using PyTorch

```
import torch
w true = torch. Tensor([1, 2, 3])
b true = 5
w = torch.randn(3, requires grad = True)
b = torch.randn(1, requires grad = True)
X = torch.randn(100, 3)
y = torch.mv(X, w true) + b true
qamma = 0.1
losses = []
for i in range(100):
  w.grad = None
  b.grad = None
  y \text{ pred} = \text{torch.mv}(X, w) + b
  loss = torch.mean((y- y pred)**2)
  loss.backward()
  w.data = w.data - gamma * w.grad.data
  b.data = b.data - gamma * b.grad.data
  losses.append(loss.item())
```



Ground-truth linear regression parameter (W, b) we decided.

We will find the solution (W, b) in this random initialized variable.

Data (X, y) are generated using ground-truth (W, b).

Learning rate and the variable we will accumulate our loss.

Loop until 100 iteration to change (W, b) solution using gradient descent.

Regression loss.

Implementing linear regression using PyTorch

```
import torch
w true = torch. Tensor ([1, 2, 3])
b true = 5
w = torch.randn(3, requires grad = True)
b = torch.randn(1, requires grad = True)
X = torch.randn(100, 3)
y = torch.mv(X, w true) + b true
qamma = 0.1
losses = []
for i in range(100):
  w.grad = None
  b.grad = None
  y \text{ pred} = \text{torch.mv}(X, w) + b
  loss = torch.mean((y- y pred)**2)
  loss.backward()
  w.data = w.data - gamma * w.grad.data
  b.data = b.data - gamma * b.grad.data
  losses.append(loss.item())
```



Ground-truth linear regression parameter (W, b) we decided.

We will find the solution (W, b) in this random initialized variable.

Data (X, y) are generated using ground-truth (W, b).

Learning rate and the variable we will accumulate our loss.

Loop until 100 iteration to change (W, b) solution using gradient descent.

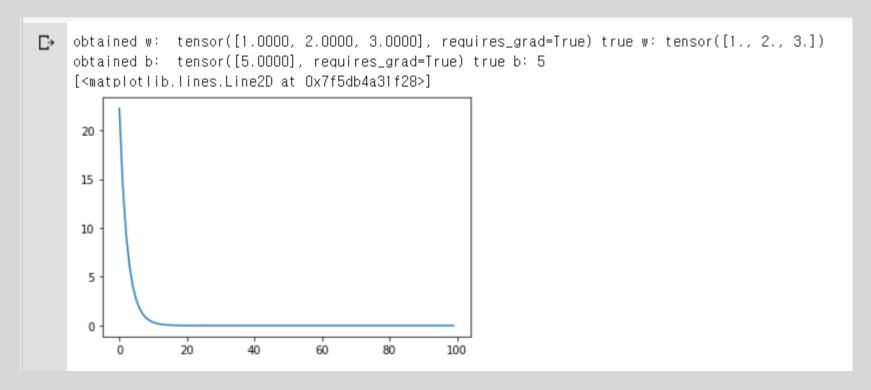
Regression loss.

Gradient descent formula.

```
print('obtained w: ', w, 'true w:', w_true)
print('obtained b: ', b, 'true b:', b_true)

from matplotlib import pyplot as plt
plt.plot(losses)
```

Compare obtained (W, b) with their ground-truth. It's same!



Implementing linear regression using PyTorch

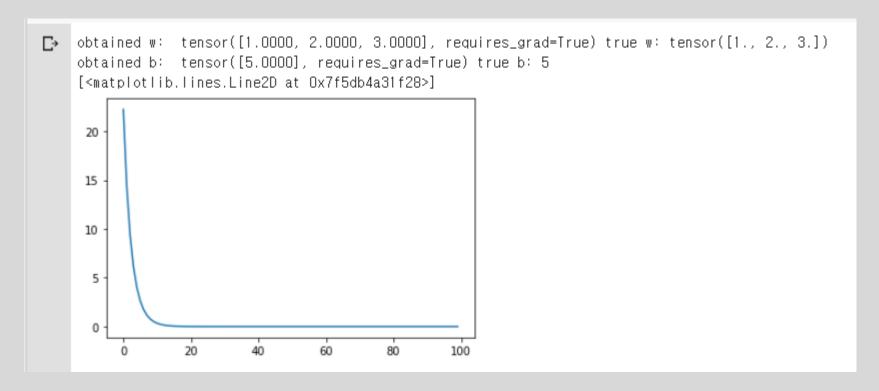
```
print('obtained w: ', w, 'true w:', w_true)
print('obtained b: ', b, 'true b:', b_true)

from matplotlib import pyplot as plt
plt.plot(losses)
```



Compare obtained (W, b) with their ground-truth. It's same!

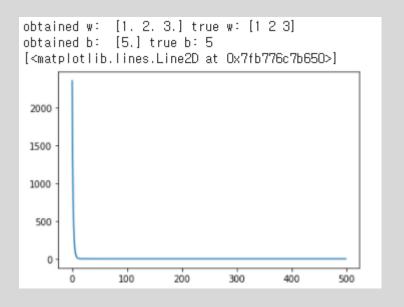
The loss plotted for each iteration. It is gradually reduced!

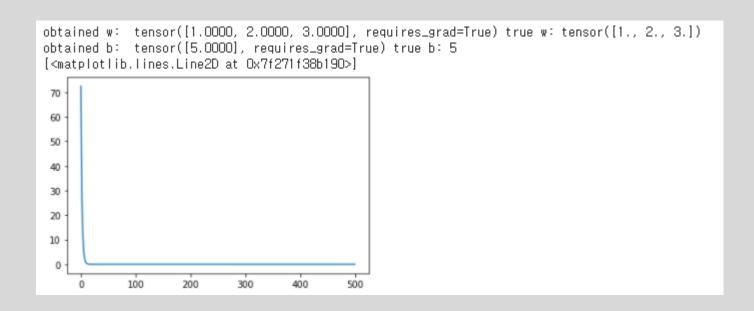




```
import numpy as np
w_{true} = np.array([1, 2, 3])
b_{true} = 5
w = np.random.rand(3, 1).squeeze(1)
b = np.random.rand(1)
X = np.random.rand(100.3)
y = np.dot(X, w_true) + b_true
gamma = 0.01
losses = []
for i in range(500):
  errors = y - (np.dot(X, w) + b)
  dEdw = np.dot(X.T, errors)
  dEdb = errors.sum()
  loss = (errors**2).sum() / 2.0
  w += gamma * dEdw
  b += gamma * dEdb
  losses.append(loss)
print('obtained w: ', w, 'true w:', w_true)
print('obtained b: ', b, 'true b:', b_true)
from matplotlib import pyplot as plt
plt.plot(losses)
```

```
import torch
w_{true} = torch.Tensor([1, 2, 3])
b_{true} = 5
w = torch.randn(3, requires_grad=True)
b = torch.randn(1, requires_grad=True)
X = torch.randn(100, 3)
y = torch.mv(X, w_true) + b_true
gamma = 0.01
losses = []
for i in range(500):
  w.grad = None
  b.grad = None
 y_pred = torch.mv(X, w) + b
  loss = torch.mean((y-y\_pred)**2)
  loss.backward()
  w.data -= gamma * w.grad.data
  b.data -= gamma * b.grad.data
  losses.append(loss.item())
print('obtained w: ', w, 'true w:', w_true)
print('obtained b: ', b, 'true b:', b_true)
from matplotlib import pyplot as plt
plt.plot(losses)
```





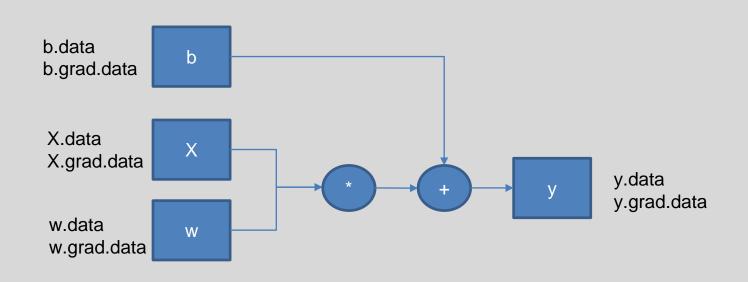
[Numpy result]

[PyTorch result]

Result is same; however we don't need to explicitly specify the differentiation form.



```
import torch
w_{true} = torch.Tensor([1, 2, 3])
b_{true} = 5
w = torch.randn(3, requires_grad=True)
b = torch.randn(1, requires_grad=True)
X = torch.randn(100, 3)
y = torch.mv(X, w_true) + b_true
gamma = 0.01
losses = []
for i in range(500):
  w.grad = None
 b.grad = None
 y_pred = torch.mv(X, w) + b
  loss = torch.mean((y-y\_pred)**2)
  loss.backward()
  w.data -= gamma * w.grad.data
 b.data -= gamma * b.grad.data
  losses.append(loss.item())
print('obtained w: ', w, 'true w:', w_true)
print('obtained b: ', b, 'true b:', b_true)
from matplotlib import pyplot as plt
plt.plot(losses)
```



After calling loss.backward(), .grad values are calculated.

```
import torch
w true = torch. Tensor([1, 2, 3])
b true = 5
w = torch.randn(3, requires grad = True)
b = torch.randn(1, requires grad = True)
X = torch.randn(100, 3)
y = torch.mv(X, w true) + b true
gamma = 0.1
losses = []
for i in range(100):
 w.grad = None
 b.grad = None
 y pred = torch.mv(X, w) + b
 loss = torch.mean((y- y pred) **2)
 loss.backward()
 w.data = w.data - gamma * w.grad.data
 b.data = b.data - gamma * b.grad.data
 losses.append(loss.item())
```



```
import torch
w true = torch. Tensor([1, 2, 3])
b true = 5
net = torch.nn.Linear(in features = 3, out features = 1, bias = True)
X = torch.randn(100, 3)
y = torch.mv(X, w true) + b true
gamma = 0.1
losses = []
for i in range(100):
  w.grad = None
  b.grad = None
 y pred = net(X)
  loss = torch.mean((y- y pred.squeeze(1))**2)
  loss.backward()
  w.data = w.data - gamma * w.grad.data
  b.data = b.data - gamma * b.grad.data
  losses.append(loss.item())
```

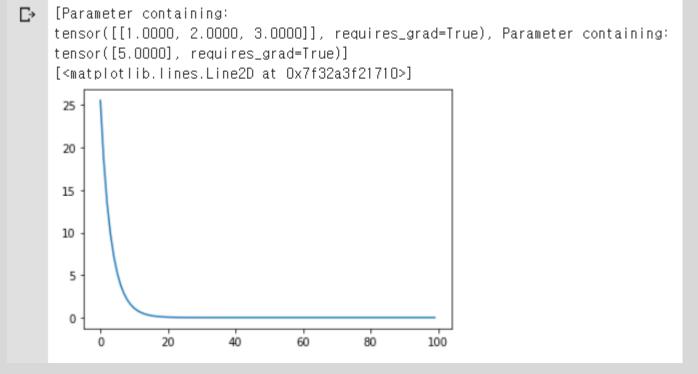
```
import torch
w true = torch. Tensor([1, 2, 3])
b true = 5
X = torch.randn(100, 3)
y = torch.mv(X, w true) + b true
gamma = 0.1
losses = []
for i in range(100):
 w.grad = None
 b.grad = None
 y pred = net(X)
 loss = torch.mean((y- y pred)**2)
 loss.backward()
  w.data = w.data - gamma * w.grad.data
 b.data = b.data - gamma * b.grad.data
 losses.append(loss.item())
```

```
import torch
                                                                        w true = torch. Tensor([1, 2, 3])
                                                                        b true = 5
net = torch.nn.Linear(in features = 3, out features = 1, bias = True) net = torch.nn.Linear(in features = 3, out features = 1, bias = True)
                                                                        X = torch.randn(100, 3)
                                                                        y = torch.mv(X, w true) + b true
                                                                        gamma = 0.1
                                                                        losses = []
                                                                        optimizer = torch.optim.SGD(net.parameters(), lr=gamma)
                                                                         for i in range(100):
                                                                          optimizer.zero grad()
                                                                          y pred = net(X)
                                                                          loss = torch.mean((y- y pred.squeeze(1))**2)
                                                                          loss.backward()
                                                                          optimizer.step()
                                                                          losses.append(loss.item())
```

```
import torch
                                                                          import torch
w true = torch. Tensor([1, 2, 3])
                                                                          w true = torch. Tensor([1, 2, 3])
b true = 5
                                                                          b true = 5
net = torch.nn.Linear(in features = 3, out features = 1, bias = True)
                                                                         net = torch.nn.Linear(in features = 3, out features = 1, bias = True)
X = torch.randn(100, 3)
                                                                          X = torch.randn(100, 3)
y = torch.mv(X, w true) + b true
                                                                          y = torch.mv(X, w true) + b true
gamma = 0.1
                                                                          gamma = 0.1
losses = []
                                                                          losses = []
optimizer = torch.optim.SGD(net.parameters(), lr=gamma)
                                                                          optimizer = torch.optim.SGD(net.parameters(), lr=gamma)
                                                                          loss fn = torch.nn.MSELoss()
for i in range(100):
  optimizer.zero grad()
                                                                          for i in range(100):
                                                                            optimizer.zero grad()
  y pred = net(X)
                                                                            y pred = net(X)
  loss = torch.mean((y- y pred.squeeze(1))**2)
  loss.backward()
                                                                            loss = loss fn(y pred.squeeze(1),y)
                                                                            loss.backward()
  optimizer.step()
                                                                            optimizer.step()
  losses.append(loss.item())
                                                                            losses.append(loss.item())
```

```
print('obtained w: ', w, 'true w:', w_true)
print('obtained b: ', b, 'true b:', b_true)

from matplotlib import pyplot as plt
plt.plot(losses)
print(list(net.parameters()))
```



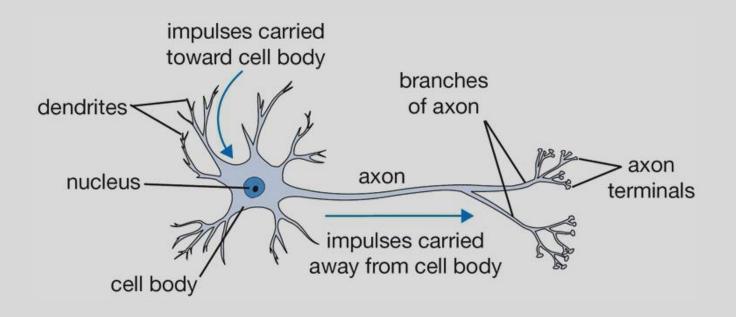


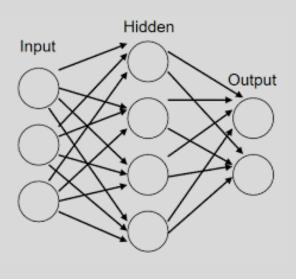
```
import torch
w true = torch. Tensor([1, 2, 3])
b true = 5
X = torch.randn(100, 3)
y = torch.mv(X, w true) + b true
gamma = 0.1
losses = []
net = torch.nn.Linear(in features = 3, out features = 1, bias = True)
optimizer = torch.optim.SGD(net.parameters(), lr=gamma)
loss fn = torch.nn.MSELoss()
for i in range(100):
 optimizer.zero grad()
 y pred = net(X)
 loss = loss fn(y pred.squeeze(1),y)
 loss.backward()
 optimizer.step()
 losses.append(loss.item())
print(list(net.parameters()))
from matplotlib import pyplot as plt
plt.plot(losses)
```

```
# Data Preparation.

# Network structure
# Optimizer
# Loss

# Iterate for updating network parameters.
# Initialize. gradients.
# Forward pass.
# Calculate loss.
# Backward pass (Calc. gradients.).
# Update network parameter.
```





$$y = \sigma(w_3(\sigma(w_2(\sigma(w_1x+b_1))+b_2) + b_3))$$

Why we need σ ?

$$y = \sigma(w_3(\sigma(w_2(\sigma(w_1x+b_1))+b_2)) + b_3))$$

$$y = w_3(w_2(w_1x+b_1)+b_2) + b_3$$

= $(w_3w_2w_1)x + (w_3w_2b_1+w_3b_2+b_3)$
= $wx+b$



Huge network converges to a simple linear regression task.

$$y = \sigma(w_3(\sigma(w_2(\sigma(w_1x+b_1))+b_2)) + b_3))$$

 σ : Sigmoid, Tanh, ReLu and so on...

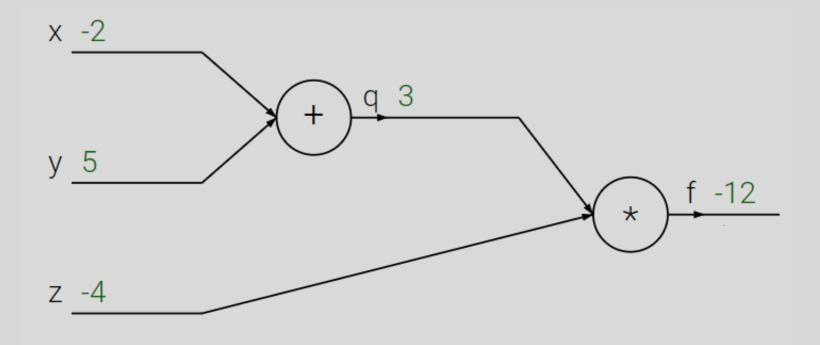
These functions are also differentiable.

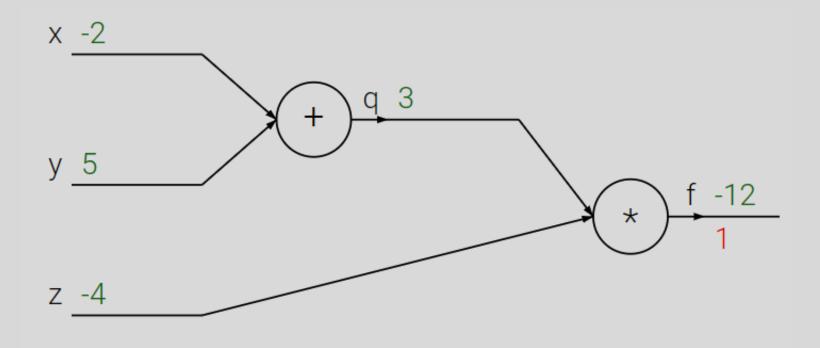
Error(**w**, **b**) =
$$\frac{1}{N} \sum_{i=1}^{N} (\sigma(\mathbf{w}_3)(\sigma(\mathbf{w}_2)(\sigma(\mathbf{w}_1)(\mathbf{x} + \mathbf{b}_1)) + \mathbf{b}_2)) + \mathbf{b}_3)) - \mathbf{y}_i)^2$$
,

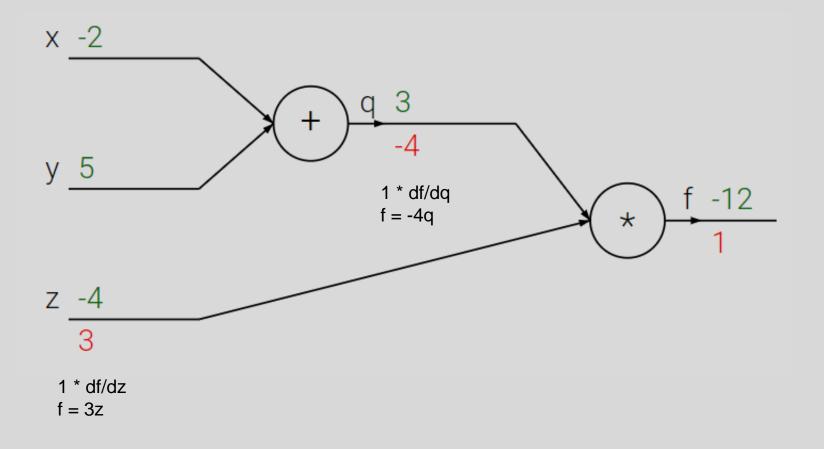
$$w_i^{(t+1)} = w_i^{(t)} - \epsilon \frac{\partial}{\partial w_i} \text{Error}(\mathbf{w}, \mathbf{b})$$

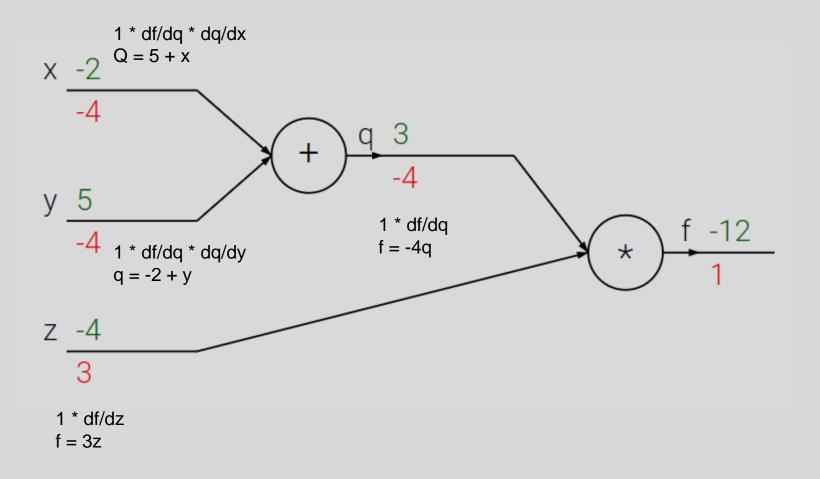
$$\mathbf{b_i}^{(t+1)} = \mathbf{b_i}^{(t)} - \epsilon \frac{\partial}{\partial \mathbf{b_i}} \mathsf{Error}(\mathbf{w}, \mathbf{b})$$

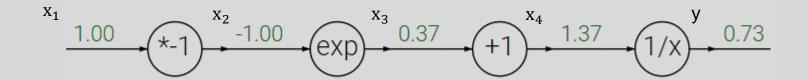
 ϵ : Learning rate (small value e.g. 0.001)

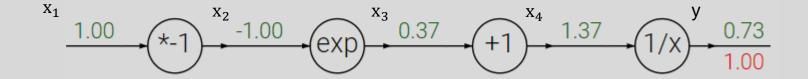


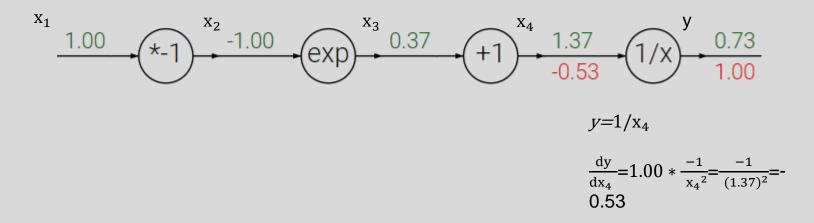


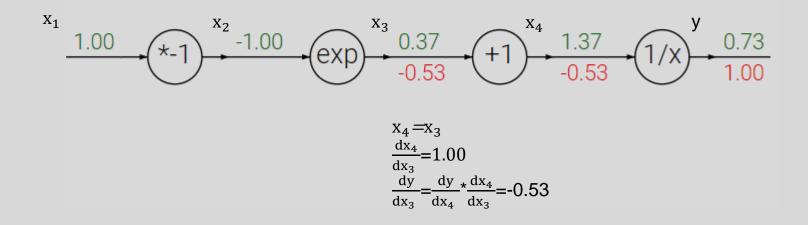


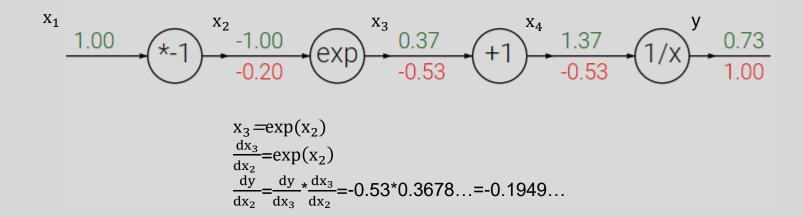


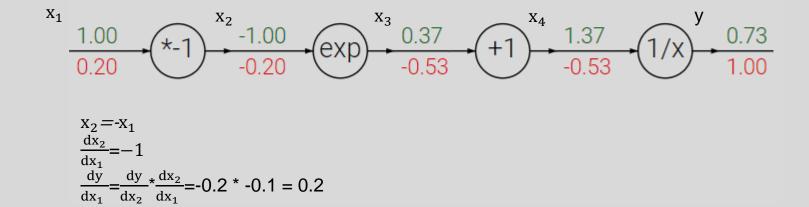












```
import torch
num data = 1000
num epoch = 10000
x = torch.randn(num data, 1)
y = (x**2) + 3
net = torch.nn.Sequential(
    torch.nn.Linear(1,6),
    torch.nn.ReLU(),
    torch.nn.Linear(6, 10),
    torch.nn.ReLU(),
    torch.nn.Linear(10, 6),
    torch.nn.ReLU(),
    torch.nn.Linear(6, 1),
loss func = torch.nn.MSELoss()
optimizer = torch.optim.SGD(net.parameters(), lr=0.01)
losses = []
for i in range(num epoch):
  optimizer.zero grad()
  output = net(x)
  loss = loss func(output, y)
  loss.backward()
  optimizer.step()
  losses.append(loss.item())
```

Make data.

Multi-layer perceptron structure.

Define loss function and optimizer.

Optimize network through iterations.

```
import torch
num data = 1000
num epoch = 10000
x = torch.randn(num data, 1)
y = (x**2) + 3
net = torch.nn.Sequential(
    torch.nn.Linear(1,6),
    torch.nn.ReLU(),
    torch.nn.Linear(6, 10),
    torch.nn.ReLU(),
    torch.nn.Linear(10, 6),
    torch.nn.ReLU(),
    torch.nn.Linear(6, 1),
loss func = torch.nn.MSELoss()
optimizer = torch.optim.SGD(net.parameters(), lr=0.01)
losses = []
for i in range(num epoch):
  optimizer.zero grad()
  output = net(x)
  loss = loss func(output, y)
  loss.backward()
  optimizer.step()
  losses.append(loss.item())
```



Make data.



Multi-layer perceptron structure.

Define loss function and optimizer.

Optimize network through iterations.

```
import torch
num data = 1000
num epoch = 10000
x = torch.randn(num data, 1)
y = (x**2) + 3
net = torch.nn.Sequential(
    torch.nn.Linear(1,6),
    torch.nn.ReLU(),
    torch.nn.Linear(6, 10),
    torch.nn.ReLU(),
    torch.nn.Linear(10, 6),
    torch.nn.ReLU(),
    torch.nn.Linear(6, 1),
loss func = torch.nn.MSELoss()
optimizer = torch.optim.SGD(net.parameters(), lr=0.01)
losses = []
for i in range(num epoch):
  optimizer.zero grad()
  output = net(x)
  loss = loss func(output, y)
  loss.backward()
  optimizer.step()
  losses.append(loss.item())
```

Make data.

Multi-layer perceptron structure.

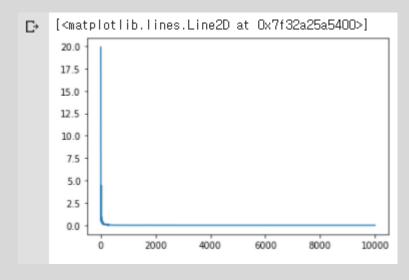
Define loss function and optimizer.

Optimize network through iterations.

```
import torch
num data = 1000
num epoch = 10000
x = torch.randn(num data, 1)
                                                                            Make data.
y = (x**2) + 3
net = torch.nn.Sequential(
    torch.nn.Linear(1,6),
    torch.nn.ReLU(),
    torch.nn.Linear(6, 10),
                                                                            Multi-layer perceptron structure.
    torch.nn.ReLU(),
    torch.nn.Linear(10, 6),
    torch.nn.ReLU(),
    torch.nn.Linear(6, 1),
loss func = torch.nn.MSELoss()
                                                                            Define loss function and optimizer.
optimizer = torch.optim.SGD(net.parameters(), lr=0.01)
losses = []
for i in range(num epoch):
  optimizer.zero grad()
                                                                            Optimize network through iterations.
  output = net(x)
  loss = loss func(output, y)
  loss.backward()
  optimizer.step()
```

losses.append(loss.item())

from matplotlib import pyplot as plt
plt.plot(losses)



```
x = torch.randn(5, 1)
y = (x**2) + 3
y_pred = net(x)

print(y)
print(y_pred)
```

