

1.2

b05902124

<http://stackoverflow.com/questions/4999448>

1.3 b05902124

2.1

b05902100

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2.2(b) b05902004

2.3 b05902108

3.2 b05902108

3.3 b05902108

3.4 b05902108

3.5 b05902108 / b05902120

Problem4

系籃學姊

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TA

<https://openhome.cc/Gossip/AlgorithmGossip/InFixPostfix.htm>

Problem5

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1.1

$$g(1) = n!$$

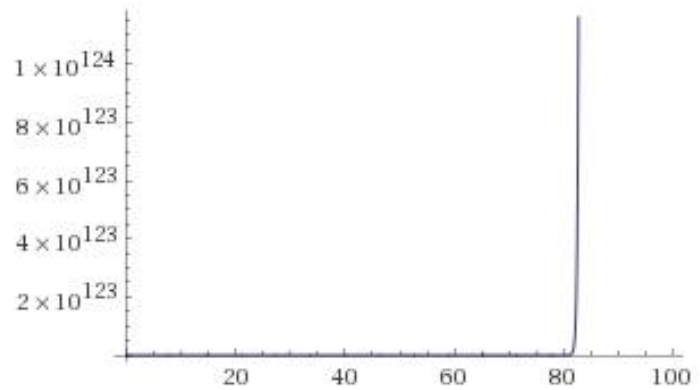
$$g(2) = 2n$$

$$g(3) = n^3 - n$$

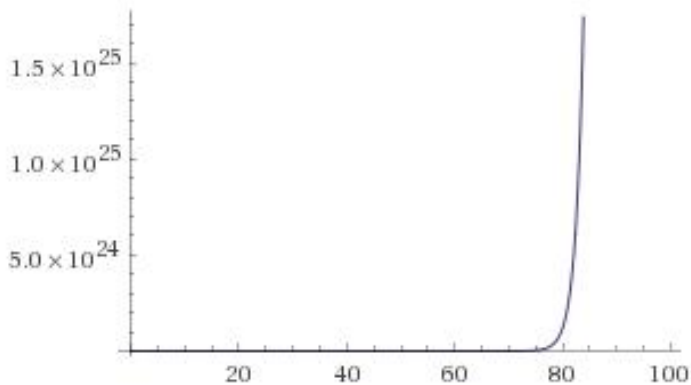
$$g(4) = e^{\log n} = n$$

$$g(5) = n^{1/\log n} = e$$

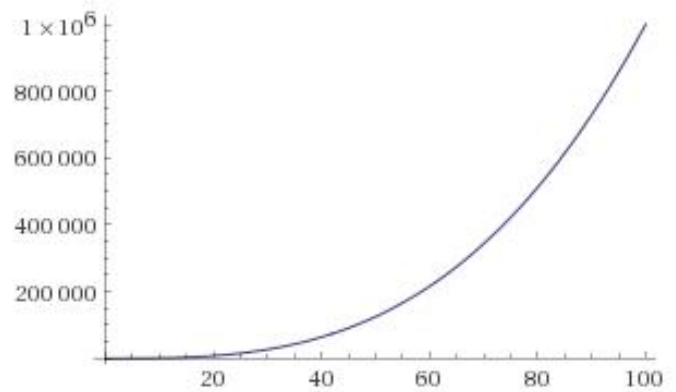
Plot: g(1)



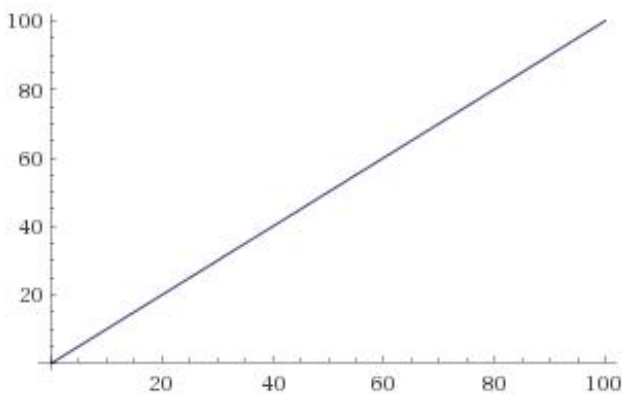
Plot: g(2)



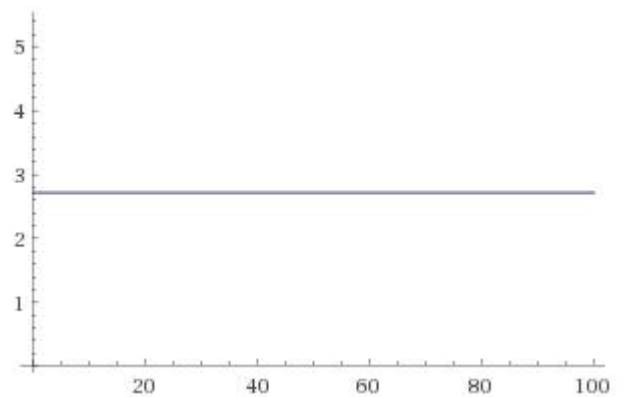
Plot: g(3)



Plot: g(4)



Plot: g(5)



1.2

$$f(n) = \frac{n!}{2^n} = \frac{n(n-1)(n-2)(n-3)\dots\dots 3 \times 2 \times 1}{2 \times 2 \times 2 \times 2 \dots\dots\dots 2 \times 2 \times 2}$$

→ Both of denominator and nominator have n numbers. When n goes to infinity, $f(n)$ goes to infinity.

$$\rightarrow n! > c(2^n)$$

$$\rightarrow n! = \omega(2^n)$$

$$g(n) = \frac{n!}{n^n} = \frac{n(n-1)(n-2)(n-3)\dots 3 \times 2 \times 1}{n \times n \times n \times n \dots n \times n \times n}$$

\rightarrow Both of denominator and nominator have n numbers. When n goes to infinity, $f(n) = 0$.

$$\rightarrow n! < c(n^n)$$

$$\rightarrow n! = o(2^n)$$

1.3

(a)

$$f(n) = O(g(n))$$

$$g(n) = \Omega(f(n))$$

$$\rightarrow f(n) \leq c \cdot g(n)$$

$$\rightarrow g(n) \geq c \cdot f(n)$$

$$\rightarrow (1/c) \cdot f(n) \leq g(n)$$

$$\rightarrow c' = 1/c \cdot c' \cdot g(n) \geq f(n)$$

$$\rightarrow c' = 1/c \cdot g(n) \geq c' \cdot f(n)$$

$$\rightarrow f(n) = O(g(n))$$

$$\rightarrow g(n) = \Omega(f(n))$$

(b)

$$f(n) = \theta(g(n))$$

$$f(n) = \Omega(g(n)) \cdot f(n) = O(g(n))$$

$$\rightarrow c_1 \cdot g(n) \leq f(n) \leq c_2 \cdot g(n)$$

$$\rightarrow f(n) \geq c_1 \cdot g(n) \cdot f(n) \leq c_2 \cdot g(n)$$

$$\rightarrow f(n) \geq c_1 \cdot g(n) \cdot f(n) \leq c_2 \cdot g(n)$$

$$\rightarrow c_1 \cdot g(n) \leq f(n) \leq c_2 \cdot g(n)$$

$$\rightarrow f(n) = \Omega(g(n)) \cdot f(n) = O(g(n))$$

$$\rightarrow f(n) = \theta(g(n))$$

(c)

$$f(n) = O(g(n))$$

$$\rightarrow f(n) \leq c \cdot g(n)$$

$$\rightarrow f(n) \cdot g(n) \leq c \cdot g(n)^2$$

$$\rightarrow f(n) \cdot g(n) = O(g(n)^2)$$

$$f(n) \cdot g(n) = O(g(n)^2)$$

$$\rightarrow f(n) \cdot g(n) \leq c \cdot g(n)^2$$

$$\rightarrow f(n) \leq c \cdot g(n)$$

$$\rightarrow f(n) = O(g(n))$$

(d)

$$f(n) = O(g(n))$$

$$\rightarrow f(n) \leq c \cdot g(n)$$

$$\rightarrow f(n)^2 \leq c^2 \cdot g(n)^2$$

$$\rightarrow c' = c^2 \cdot f(n)^2 \leq c' \cdot g(n)^2$$

$$\rightarrow f(n)^2 = O(g(n)^2)$$

$$f(n)^2 = O(g(n)^2)$$

$$\rightarrow f(n)^2 \leq c \cdot g(n)^2$$

$$\rightarrow f(n) \leq c^{1/2} \cdot g(n)$$

$$\rightarrow c' = c^{1/2} \cdot f(n) \leq c' \cdot g(n)$$

$$\rightarrow f(n) = O(g(n))$$

2.1

Binary_Search(A, N, k)

sort(A) 的 time complexity 是 $O(N \log N)$

因為：

k 為 while 執行的次數

$$N/2^k \leq 1 \rightarrow N \leq 2^k \rightarrow \log_2 N \leq k$$

$$N/2^{k-1} > 1 \rightarrow N > 2^k/2 \rightarrow \log_2 N > k-1 \rightarrow \log_2 N + 1 > k$$

$$\log_2 N \leq k < \log_2 N + 1$$

$$O(\log_2 N)$$

所以：

for loop 的 time complexity 是 $O(N\log N)$

c_1 : sort(A)

c_2 : for loop

c_3 : return

$$T(N) = c_1 * N\log N + c_2 * N\log N + c_3$$

Time complexity $\rightarrow O(N\log N)$

Count_Search(A, N, K, k)

因為：

一個 for loop 的 time complexity 是 $O(N)$

c_1 : malloc

c_2 : for loop

c_3 : return

$$T(N) = c_1 + c_2 * N + c_3$$

Time complexity $\rightarrow O(N)$

因為：

最大的 k 可能是 $K + (K - 1)$

所以：

最糟的情況是開一個大小為 $2K$ 的動態陣列

Space complexity $\rightarrow O(K)$

I think that Count_Search is the better.

By time complexity, we know $O(N \log N) > O(N)$ when N goes to a large number.

Therefore, it will take less time than Binary_Search when N goes to a large number.

2.2

(a)

```
1  for i = 0 to (A.length - 1)
2      for j = 0 to (A.length - 1)
3          if A[i] + A[j] == k
4              M++;
```

(b)

```
1  merge(arr, reg, 0, N)
2  M = 0;
3  for i = 0 to N - 1
4      search = k - A[i];
5      left = 0;
6      right = N - 1;
7      while left <= right
8          mid = (left + right)/2;
9          if A[mid] == search
10             M++;
11         else if A[mid] > search
12             right = mid - 1;
```

```
13         else if A[mid] < search
14             left = mid + 1;
15 void merge(int arr[], int reg[], int start, int end)
16     if start >= end
17         return;
18     len = end - start + 1;
19     mid = (start + end)/2;
20     start1 = start;
21     end1 = mid;
22     start2 = mid + 1;
23     end2 = end;
24     merge(arr, reg, start1, end1);
25     merge(arr, reg, start2, end2);
26     k = start;
27     while start1 <= end1 and start2 <= end2
28         if arr[start1] < arr[start2]
29             reg[k] = arr[start1];
30             start1++;
31         else
32             reg[k] = arr[start2];
33             start2++;
```

```

34     k++;
35     while start1 <= end1
36         reg[k] = arr[start1];
37         k++;
38         start1++;
39     while start2 <= end2
40         reg[k] = arr[start2];
41         k++;
42         start2++;
43     for k = start to end
44         arr[k] = reg[k];
45     return;

```

2.3

```

1     for i = 0 to N - 3
2         now_k = k - A[i];
3         Count_Search (A, N, K, now_k);
4     Count_Search (A, N, K, k) {
5         B = malloc (K);
6         for i = 0 to N - 1
7             B[A[i]] = true
8         for i = 0 to N - 1

```



```
9     if B[k - A[i]]
10         return true;
11     return false;
12 }
```

3.1

push 1

push 2

push 3

pop

pop

push 4

pop

pop

push 5

pop

3.2

```
1  for i = 1 to n
2      if A[i] != i
3          return false;
4      return true;
```

Time complexity $\rightarrow O(n)$

3.3

依題意照 1, 2, 3, 4.....順序 push，若要 pop 某一個數字必定已經 push 所有比她小的數字，且遵守 stack last in first out 的規則，已 push 的數字由小到大，則 pop 出來的數字必是由大到小，所以判斷是否為 stack-valid，只要以一數為基準，被 pop 出來所有比她小的數字必須由大到小。

```
1  for i = 0 to n - 2
2      key = A[i];
3      for j = i + 1 to n - 1
4          if A[j] < A[i]
5              key--;
6          if A[i] != key
7              return false;
8  return true;
```

Time complexity $\rightarrow O(n^2)$

3.4

因為：

queue-valid 的唯一可能性是[1, 2, 3, 4, 5]，pseudo code 跟 3.2 一樣

所以：

只要設一個變數 i 從 1 開始跑，一次判斷一個數字再 i++，判斷順序是不是[1, 2, 3, 4, 5]

Space complexity $\rightarrow O(1)$

Time complexity $\rightarrow O(n)$

3.5

因為：

設三個變數， i 以哪個數字為基準、 key --數字依序遞減、 j 紀錄現在的位置，開始
往後跑，pseudo code 跟 3.3 一樣

所以：

再三個 space $\rightarrow O(1)$ additional space