Digital Image Processing - Homework #3

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Problem 3.34.

We can simplify the problem by considering a single scan line through the bars in the image.Seeing that the separation of the vertical bars is 20 pixels, and these bars are 5 pixels wide. Besides, the image was blurred using square box kernels of size 25. The responses of the mask are the average of the pixels it compasses. It shows that when the mask moves one pixel to the right, it loses one value of the bar on the left and get an identical one on the right simultaneously. Therefore, the responses of the mask never change as long as it is contained within the vertical bars and not near the edges of the bars. However, this phenomenon doesn't happen when using square box kernels of size 23 or 45 since their responses change due to the width of the bars and their separation.

Problem 3.42.

$$f(x,y) - \nabla^2 f(x,y)$$

$$= f(x,y) - [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)]$$

$$= 6f(x,y) - [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) + f(x,y)]$$

$$= 5\{1.2f(x,y) - \frac{1}{5}[f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) + f(x,y)]\}$$

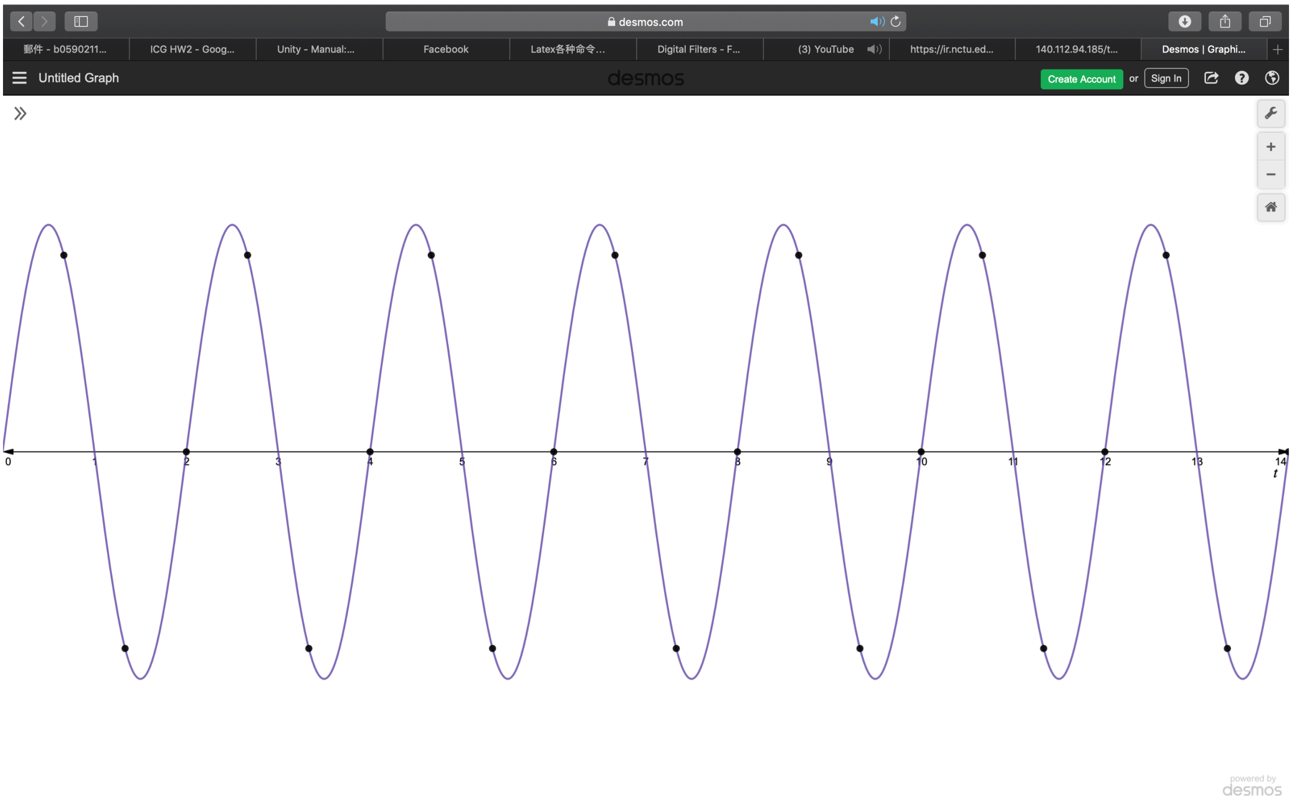
$$= 5[1.2f(x,y) - \bar{f}(x,y)]$$

$$\approx 5[f(x,y) - \bar{f}(x,y)] = 5 \times g_{mask}(x,y)$$

Therefore, we know that subtracting the Laplacian from an image is proportional to the unsharp mask.

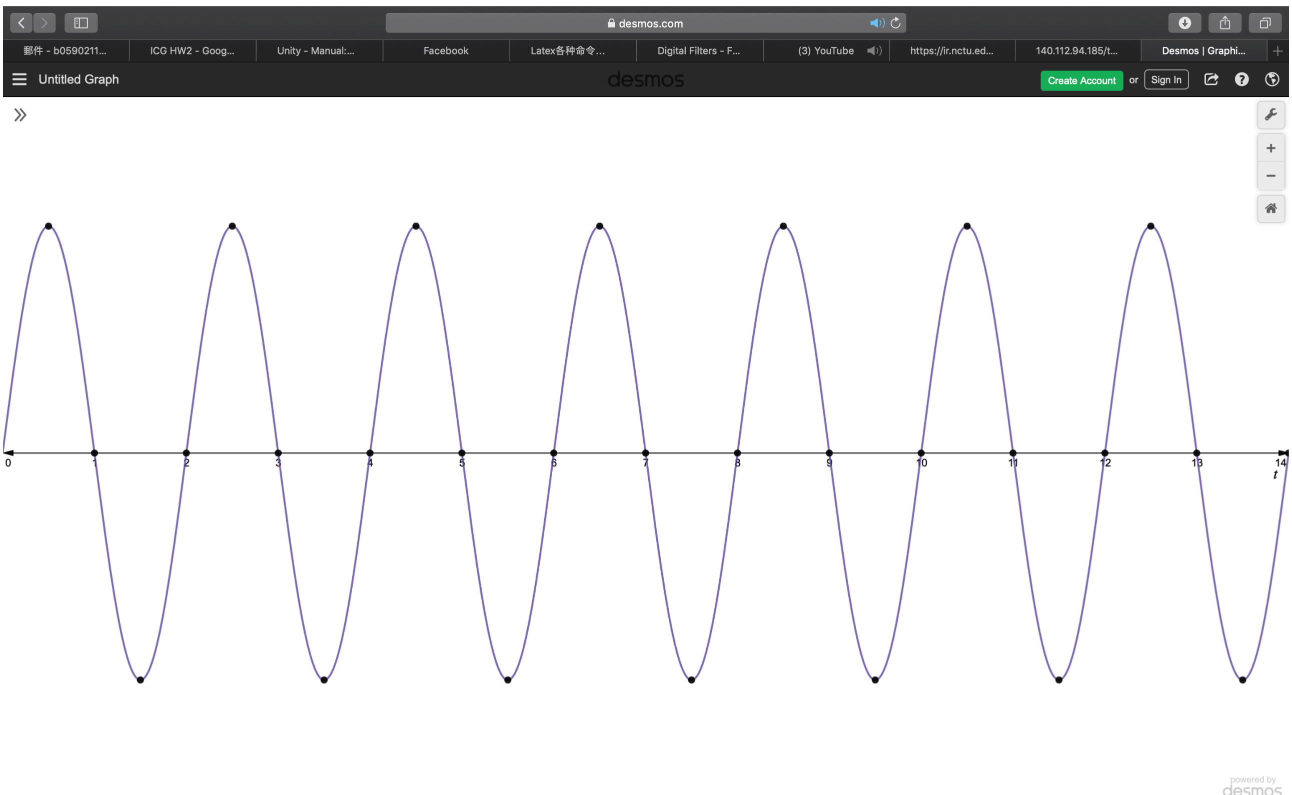
Problem 4.6.

(a) Set sampling rate $= \frac{3}{2}$ (Nyquist rate $= 1$)



(b) Because there are 11 dots from 0 to 22, the sampling rate is about $\frac{1}{2}$.

(c) Set sampling rate $= 2$



Problem 4.9.

(a) $\mathfrak{F}\{\cos(2\pi\mu_0 t)\} = \dfrac{1}{2}[\delta(\mu - \mu_0) + \delta(\mu + \mu_0)]$

proof:

$$\mathfrak{F}^{-1}\{\frac{1}{2}[\delta(\mu - \mu_0) + \delta(\mu + \mu_0)]\}$$

$$= \frac{1}{2} \times [\mathfrak{F}^{-1}\{\delta(\mu - \mu_0)\} + \mathfrak{F}^{-1}\{\delta(\mu + \mu_0)\}]$$

$$= \frac{1}{2} \times [\int_{-\infty}^{\infty} \delta(\mu - \mu_0)e^{2j\pi\mu t}\ d\mu + \int_{-\infty}^{\infty} \delta(\mu + \mu_0)e^{2j\pi\mu t}\ d\mu]$$

$$= \frac{1}{2} \times [e^{2j\pi\mu_0 t} + e^{2j\pi(-\mu_0)t}]$$

$$= \frac{1}{2} \times \{[\cos(2\pi\mu_0 t) + j\sin(2\pi\mu_0 t)] + [\cos(2\pi\mu_0 t) - j\sin(2\pi\mu_0 t)]\}$$

$$= \cos(2\pi\mu_0 t)$$

$$\therefore \mathfrak{F}\{\cos(2\pi\mu_0 t)\} = \frac{1}{2}[\delta(\mu - \mu_0) + \delta(\mu + \mu_0)]$$

(b) $\mathfrak{F}\{\sin(2\pi\mu_0 t)\} = \dfrac{1}{2j}[\delta(\mu - \mu_0) - \delta(\mu + \mu_0)]$

proof:

$$\mathfrak{F}^{-1}\{\frac{1}{2j}[\delta(\mu - \mu_0) - \delta(\mu + \mu_0)]\}$$

$$= \frac{1}{2j} \times [\mathfrak{F}^{-1}\{\delta(\mu - \mu_0)\} - \mathfrak{F}^{-1}\{\delta(\mu + \mu_0)\}]$$

$$= \frac{1}{2j} \times [\int_{-\infty}^{\infty} \delta(\mu - \mu_0)e^{2j\pi\mu t}\ d\mu - \int_{-\infty}^{\infty} \delta(\mu + \mu_0)e^{2j\pi\mu t}\ d\mu]$$

$$= \frac{1}{2j} \times [e^{2j\pi\mu_0 t} - e^{2j\pi(-\mu_0)t}]$$

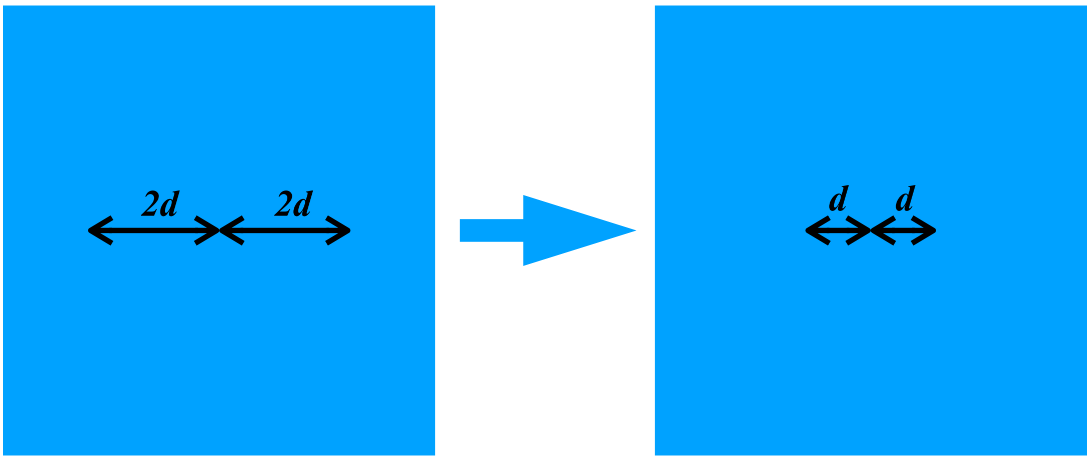
$$= \frac{1}{2j} \times \{[\cos(2\pi\mu_0 t) + j\sin(2\pi\mu_0 t)] - [\cos(2\pi\mu_0 t) - j\sin(2\pi\mu_0 t)]\}$$

$$= \sin(2\pi\mu_0 t)$$

$$\therefore \mathfrak{F}\{\sin(2\pi\mu_0 t)\} = \frac{1}{2j}[\delta(\mu - \mu_0) - \delta(\mu + \mu_0)]$$

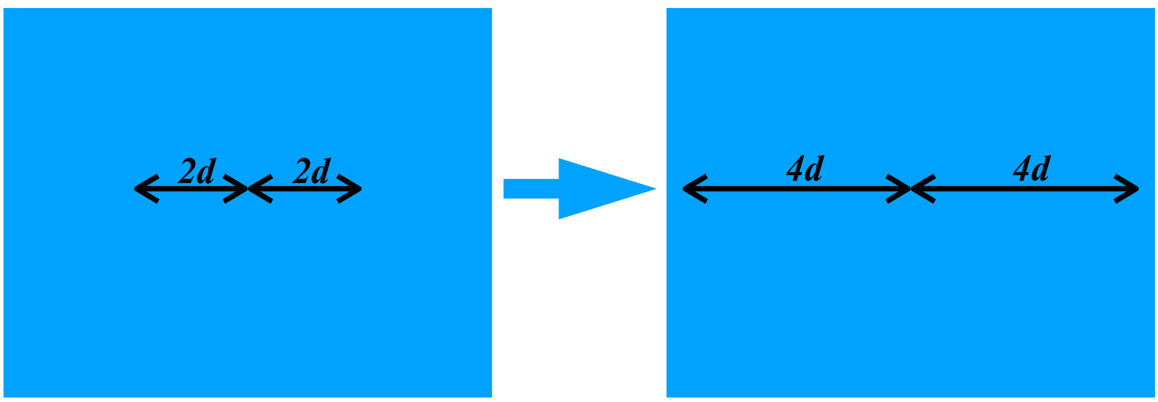
Problem 4.21.

(a) Because the stripes of an image become twice longer than before, the frequency becomes half.



(b) Because the image can be completely superimposed by the triangle waves in the horizontal direction, the component of frequency domain only appears on one axis.

(c) Similar to Question(a), because the stripes of an image become half, the frequency becomes twice longer than before.



(d) *dc term* = $F(0,0) = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)$. *No matter how wide are the stripes, black pixels and white pixels account for half of the image respectively, so* $F(0,0) = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)$ *never changes. And then dc terms in (a) and (c) are the same.*