Q13:

Solution:

(a) The neutrons have almost no interaction with the electrons in a block of uranium, but they do interact through the strong interaction with the uranium nuclei. A collision of a neutron with a uranium nucleus is an example of a low-mass projectile striking a massive target that is at rest (the uranium nucleus has about 235 times the mass of the neutron). If the interaction is elastic (kinetic energy constant), the neutron will bounce off with almost its original momentum and kinetic energy. The uranium nucleus may acquire as much as twice the neutron's momentum (this happens in a head-on collision), but due to the huge mass of the uranium nucleus compared to the neutron, the uranium nucleus acquires little kinetic energy $\frac{|\vec{p}|^2}{2M}$. So fast neutrons travel through uranium with little change in speed.

(b) Again, the neutrons have almost no interaction with the electrons in a block of carbon, but they do interact with the carbon nuclei, through the strong interaction. The mass of a carbon nucleus is only about 12 times the mass of the neutron, so much more kinetic energy can be transferred from the neutron to the carbon nucleus than is the case with uranium nuclei. Therefore carbon should be much more effective than uranium in slowing down fast neutrons.

(c) A water molecule (H_2O) contains two very low-mass nuclei–the two hydrogen nuclei, each consisting of a single proton. The mass of a proton is nearly equal to the mass of a neutron, so our studies of elastic collisions between two equal masses, one initially at rest, apply here. As an example, a head-on collision will leave the neutron nearly at rest, and the proton acquires the neutron's momentum and kinetic energy. For this reason water should be a better moderator than carbon.

A comment: In part (c) we showed that water is a good moderator (that is, it does a good job of slowing down fast neutrons). There is however a problem with ordinary water. A neutron can fuse with a proton to form a deuteron (a proton plus a neutron), the nucleus of "heavy hydrogen" or deuterium. If the neutron is captured in this way, the neutron cannot contribute to triggering fission in a uranium nucleus. Because of this neutron capture effect, some reactors use as a moderator "heavy water," water consisting of D_2O molecules (an oxygen atom plus two deuterium atoms). A neutron can fuse with a deuteron to form a triton (proton plus two neutrons), but it happens that the probability for this neutron capture reaction is quite low compared to the probability of neutron capture in ordinary hydrogen. There is a trade-off between ordinary hydrogen doing a better job of slowing down the fast neutrons and heavy hydrogen removing fewer neutrons from circulation.

P21:

Solution:

(a) The system is the car + truck. Use the Momentum Principle.

(b)

$$\begin{aligned}
\vec{p}_{\text{sys,i}} &= \vec{p}_{\text{sys,f}} \\
m\vec{v} + M\vec{V} &= (m+M)\vec{v}_f \\
\vec{v}_f &= \frac{m\vec{v} + M\vec{V}}{m+M} \\
\vec{v}_f &\approx \frac{(2300\text{ kg})\langle 38, 0, 0\rangle\text{ m/s} + (4300\text{ kg})\langle -16, 0, 27\rangle\text{ m/s}}{6600\text{ kg}} \\
\vec{v}_f &\approx \langle 2.82, 0, 17.59\rangle\text{ m/s}
\end{aligned}$$

(c) During the small time interval of the collision, the work done by the force of the road is negligible because the displacement through which friction acts (during this small time interval) is also small.

(d) Use the energy principle. There is no work done by external forces, but there is a change in internal energy.

$$\begin{aligned}
E_{\text{sys,f}} &= E_{\text{sys,i}} \\
K_{\text{f}} + \Delta E_{\text{int}} &= K_{\text{i}} \\
\Delta E_{\text{int}} &= K_{\text{i}} - K_{\text{f}} \\
\Delta E_{\text{int}} &= \frac{1}{2}(2300\text{ kg})(38\text{ m/s})^2 + \frac{1}{2}(4300\text{ kg})(31.38\text{ m/s})^2 - \frac{1}{2}(6600\text{ kg})(17.81\text{ m/s})^2 \\
&\approx 2.73 \times 10^{6}\text{ J}
\end{aligned}$$

(e) The collision is inelastic since there is a change in internal energy.

P23:

Solution:

(a)

$$\begin{aligned}
m_{\text{alpha}} &\approx 4\text{ m}_{\text{proton}} \approx 6.68 \times 10^{-27}\text{ kg} \\
K &= \frac{\text{p}^2}{2m} \\
\text{p}_{i,alpha} &= \sqrt{2m_{\text{alpha}} K_{\text{alpha}}} \\
&= \sqrt{2(6.68 \times 10^{-27}\text{ kg})(10 \times 10^{6}\text{ eV})\left(\frac{1.6 \times 10^{-19}\text{ J}}{1\text{ eV}}\right)} \\
&= 1.46 \times 10^{-19}\text{ kg} \cdot \text{m/s} \\
\text{p}_{i,alpha,x} &= +1.46 \times 10^{-19}\text{ kg} \cdot \text{m/s}
\end{aligned}$$

Use the results from 10.P.15. Both the Momentum Principle and Energy Principle are needed. Note that $m_{\text{gold}} \approx 197m_{\text{proton}} = 3.29 \times 10^{-25}$ kg. Label the alpha particle 1 and the gold nucleus 2.

$$\begin{aligned}
m_1 \text{v}_{1,f,x} &= \left(\frac{\frac{m_1}{m_2} - 1}{\frac{m_1}{m_2} + 1}\right) m_1 \text{v}_{1,i,x} \\
\text{p}_{1,f,x} &= \left(\frac{\frac{4}{197} - 1}{\frac{4}{197} + 1}\right) \text{p}_{1,i,x} \\
&= -0.96\text{p}_{1,i,x} \\
&= -0.96(1.46 \times 10^{-19}\text{ kg} \cdot \text{m/s}) \\
&= -1.40 \times 10^{-19}\text{ kg} \cdot \text{m/s}
\end{aligned}$$

(b) Use the Momentum Principle. Define the system to be the alpha particle and gold nucleus. $\vec{F}_{net} = 0$ so, the momentum of the system is constant.

$$
\begin{aligned}
\vec{p}_{sys,i} &= \vec{p}_{sys,f} \\
\vec{p}_{1,i} + \cancelto{0}{\vec{p}_{2,i}} &= \vec{p}_{1,f} + \vec{p}_{2,f} \\
\vec{p}_{2,f} &= \vec{p}_{1,i} - \vec{p}_{1,f} \\
p_{2,f,x} &= p_{1,i,x} - p_{1,f,x} \\
&= 1.46 \times 10^{-19}\ \mathrm{kg \cdot m/s}
\end{aligned}
$$

This is nearly 2 times the initial momentum of the alpha particle, as expected from the ping-pong-ball and bowling ball example.

(c)

$$
\begin{aligned}
K_{\mathrm{alpha,f}} &= \frac{p^2_{alpha,f}}{2m_{\mathrm{alpha}}} \\
&= \frac{(1.4 \times 10^{-19}\ \mathrm{kg \cdot m/s})^2}{2(6.68 \times 10^{-27}\ \mathrm{kg})} \\
&= 1.47 \times 10^{-12}\ \mathrm{J} \left(\frac{1\ \mathrm{eV}}{1.6 \times 10^{-19}\ \mathrm{J}}\right) \\
&= 9.17\ \mathrm{MeV}
\end{aligned}
$$

So the alpha particle lost 0.83 MeV of energy.

(d)

$$
\begin{aligned}
K_{\mathrm{gold,f}} &= \frac{p^2_{gold,f}}{2m_{\mathrm{gold}}} \\
&= \frac{(2.86 \times 10^{-19}\ \mathrm{kg \cdot m/s})^2}{2(3.29 \times 10^{-25}\ \mathrm{kg})} \\
&= 1.24 \times 10^{-13}\ \mathrm{J} \left(\frac{1\ \mathrm{eV}}{1.6 \times 10^{-19}\ \mathrm{J}}\right) \\
&= 0.78\ \mathrm{MeV}
\end{aligned}
$$

(e) Apply the Energy Principle. $K_{\text{alpha,f}} = 0$ at point of closest approach. Because $m_{\text{gold}} >> m_{\text{alpha}}$, $K_{\text{gold}} \approx 0$ at point of closest approach. It is a closed system.

$$\begin{aligned}
E_{\text{i}} &= E_{\text{f}} \\
K_{\text{i}} + \cancelto{0}{U_{\text{elec,i}}} &= \cancelto{0}{K_{\text{f}}} + U_{\text{elec,f}} \\
K_{\text{i,alpha}} &= U_{\text{elec,f}} \\
K_{\text{i,alpha}} &= \frac{1}{4\pi\epsilon_0} \frac{q_{\text{alpha}} q_{\text{gold}}}{r_{\text{f}}} \\
(10 \text{ MeV}) \left(\frac{1.6 \times 10^{-19} \text{ J}}{1 \text{ eV}} \right) &= \left(9 \times 10^9 \, \frac{\text{Nm}^2}{\text{C}^2} \right) \left(\frac{2(79)(1.6 \times 10^{-19} \text{ C})^2}{r_{\text{f}}} \right) \\
r_{\text{f}} &= 2.28 \times 10^{-14} \text{ m}
\end{aligned}$$

P25:

Solution:

Apply the Momentum Principle. Assuming that the Σ^- particle is initially at rest, then $\vec{p}_{\text{i}} = \vec{p}_{\text{f}} = 0$. After decay,

$$\begin{aligned}
\vec{p}_{\text{sys,f}} &= 0 \\
\vec{p}_{\text{n}} + \vec{p}_{\pi^-} &= 0 \\
\vec{p}_{\text{n}} &= -\vec{p}_{\pi^-} \\
|\vec{p}_{\text{n}}| &= |\vec{p}_{\pi^-}|
\end{aligned}$$

Apply the Energy Principle. It is a closed system.

$$\begin{aligned}
\Delta E_{\text{sys}} &= 0 \\
E_{\text{i}} &= E_{\text{f}} \\
E_{rest,\Sigma^-} &= E_{\text{n}} + E_{\pi^-} \\
m_{\Sigma^-} c^2 &= E_{\text{n}} + E_{\pi^-} \\
E_{\text{n}} + E_{\pi^-} &= 1196 \text{ MeV}
\end{aligned}$$

Momentum and energy for the neutron are related by

$$\begin{aligned}
E_{\text{n}}^2 - (\text{p}_n c)^2 &= (m_{\text{n}} c^2)^2 \\
E_{\text{n}}^2 - (\text{p}_n c)^2 &= (939 \text{ MeV})^2
\end{aligned}$$

And for the π^-,

$$E_{\pi^-} - (\text{p}_{\pi^-} c)^2 = (140 \text{ MeV})^2$$

We now have four equations and four unknowns that can be solved for E_{n}, E_{π^-}, p_n, and p_{π^-}.

P28:

Solution:

Initially, the pion is at rest and the momentum of the system is zero. After decaying, the muon has a velocity in the $-x$ direction and the neutrino has a velocity in the $+x$ direction (for example) so that the total momentum of the system is also zero. (In other words, the two decay particles must travel in opposite directions in order for the momentum of the system to be conserved.)

$$\begin{aligned}
E_{rest,pion} &= 139.6 \text{ MeV} \\
E_{rest,muon} &= 105.7 \text{ MeV} \\
E_{rest,neutrino} &= 0 \qquad \text{thus it only has kinetic energy, and } E = pc \\
K_{muon,f} &= ? \\
K_{neutrino,f} &= ?
\end{aligned}$$

Define the system to be all particles and apply the Momentum Principle.

$$\begin{aligned}
p_{ix} &= p_{fx} \\
0 &= p_{muon,x} + p_{neutrino,x} \\
p_{muon,x} &= -p_{neutrino,x} \\
|p_{muon,x}| &= |p_{neutrino,x}| \\
p_{muon} &= \frac{E_{neutrino}}{c} \\
\gamma_f m v &= \frac{E_{neutrino}}{c}
\end{aligned}$$

Apply the Energy Principle to the system.

$$\begin{aligned}
E_i &= E_f \\
E_{rest,pion} &= E_{muon} + E_{neutrino} \\
E_{rest,pion} &= \gamma E_{rest,muon} + E_{neutrino}
\end{aligned}$$

We now have two equations and two unknowns. First, solve for $E_{neutrino}$ in the Momentum Principle.

$$\begin{aligned}
\gamma_f m v &= \frac{E_{neutrino}}{c} \\
E_{neutrino} &= \gamma_f m v c \\
E_{neutrino} &= \gamma_f m c^2 \frac{v}{c} \\
E_{neutrino} &= \gamma_f E_{rest,muon} \frac{v}{c}
\end{aligned}$$

Now substitute into the Energy Principle and solve for v, the speed of the muon after the decay. Note that $\gamma = 1/\sqrt{1-(v/c)^2}$.

$$\begin{aligned}
E_{rest,pion} &= \gamma E_{rest,muon} + E_{neutrino} \\
&= \gamma_f E_{rest,muon} + \gamma_f E_{rest,muon} \frac{v}{c} \\
&= \gamma_f E_{rest,muon} \left(1 + \frac{v}{c}\right) \\
\frac{E_{rest,pion}}{E_{rest,muon}} &= \frac{1 + \frac{v}{c}}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}
\end{aligned}$$

For the purpose of doing the algebra, it is convenient to set $\beta = v/c$ and $\alpha = E_{rest,pion}/E_{rest,muon} = 1.32$. Of course, you can always use a calculator or computer algebra system to solve for v.

$$\begin{aligned}
\alpha &= \frac{1+\beta}{\sqrt{1-(\beta^2)}} \\
\alpha^2 &= \frac{(1+\beta)^2}{1-\beta^2} \\
&= \frac{(1+\beta)(1+\beta)}{(1-\beta)(1+\beta)} \\
&= \frac{(1+\beta)}{(1-\beta)} \\
\alpha^2 - \beta\alpha^2 &= 1+\beta \\
\beta(\alpha^2+1) &= \alpha^2 - 1 \\
\beta &= \frac{\alpha^2-1}{(\alpha^2+1)} \\
&= \frac{1.32^2-1}{(1.32^2+1)} \\
&= 0.271 \\
v &= 0.271c
\end{aligned}$$

Calculate γ_f for the muon after the decay.

$$\begin{aligned}
\gamma &= \frac{1}{\sqrt{1-(v/c)^2}} \\
&= \frac{1}{\sqrt{1-0.271^2}} \\
&= 1.039
\end{aligned}$$

Now can calculate the kinetic energy of the muon and the neutrino after the decay. Since $E = \gamma mc^2 = mc^2 + K$, then $K = (\gamma - 1)mc^2$ for the muon. For the neutrino, use the Momentum Principle or Energy Principle.

$$\begin{aligned}
K_{muon} &= (\gamma-1)mc^2 \\
&= (1.039-1)(105.7 \text{ MeV}) \\
&= 4.1 \text{ MeV}
\end{aligned}$$

For the neutrino, from the Momentum Principle:

$$\begin{aligned}
E_{neutrino} &= \gamma_f E_{rest,muon}\frac{v}{c} \\
&= (1.039)(105.7 \text{ MeV})(0.271) \\
&= 30 \text{ MeV}
\end{aligned}$$

P32:

Solution:

The goal is to find the velocity of the system (of car and truck) after the collision in the frame of reference of Earth, using the center-of-momentum frame of reference. Because the objects are stuck together, the velocity of the stuck-together system (in the reference frame of Earth) is the center of mass velocity. And since the momentum of the system is conserved, $\vec{v}_{CM}$ is constant (before and after the collision). Therefore, if we find the center-of-mass velocity before the collision, then we know the velocity of the system after the collision.

$$\begin{aligned}
\vec{v}_{CM} &= \frac{m_1\vec{v}_{1i} + m_2\vec{v}_{2i}}{m_1 + m_2} \\
&= \frac{(2300\ \text{kg})(<38,0,0>\ \text{m/s}) + (4300\ \text{kg})(<-16,0,27>\ \text{m/s})}{2300\ \text{kg} + 4300\ \text{kg}} \\
&= <2.82, 0, 17.6>\ \text{m/s}
\end{aligned}$$

Checking with the solution to P21, we see that the answers agree.