Q9:

Solution:

(b) $Y_{_A} = Y_{_B}$ because both wires are made of pure copper.

Q14:

Solution:

Items (a) and (e) are true. At the lowest point, the spring is stretched more than it is at equilibrium. Thus $\left|\vec{F}_{\text{spring}}\right| > mg$ at the lowest point in the oscillation.

Q15:

Solution:

(a)

$$T = 2\pi\sqrt{\frac{m}{k}}$$
$$T \propto \sqrt{m}$$

If m is doubled, T increases by a factor $\sqrt{2}$. So if $T = 1\,\text{s}$, doubling the mass gives a period of $\sqrt{2}\,(1\,\text{s}) = 1.4\,\text{s}$

(b)

$$T = 2\pi\sqrt{\frac{m}{k}}$$
$$T \propto \frac{1}{\sqrt{k}}$$

Doubling the stiffness causes T to change by a factor of $\frac{1}{\sqrt{2}}$. Thus, if $T = 1\,\text{s}$, then doubling the stiffness results in a period of $0.71\,\text{s}$.

(c)

$$\begin{aligned}\frac{1}{k_{_\text{eff}}} &= N\frac{1}{k}\\ k &= Nk_{_\text{eff}}\\ &= 2k_{_\text{eff}}\end{aligned}$$

Thus, cutting the spring in half doubles the stiffness. Since

$$T \propto \frac{1}{\sqrt{k}}$$

doubling the stiffness changes T by a factor of $\frac{1}{\sqrt{2}} = 0.71$. Thus, a period of $1\,\text{s}$ becomes a period of $(0.71)\,(1\,\text{s}) = 0.71\,\text{s}$.

(d) Period is independent of amplitude. Therefore, the period will remain $1\,\text{s}$.

(e) Period is independent of g. Therefore, the period will remain $1\,\text{s}$.

P32:

Solution:

(a)

$$k_s \approx \frac{(415\,\mathrm{kg})\left(9.80\,\mathrm{m/s^2}\right)}{1.26 \times 10^{-2}\,\mathrm{m}} \approx 3.23 \times 10^{5}\,\mathrm{N/m}$$

(b)

$$N_{\text{chains}} \approx \frac{A_{\text{wire}}}{A_{\text{atom}}} \approx \frac{\left(0.15 \times 10^{-2}\,\mathrm{m}\right)^2}{\left(2.51 \times 10^{-10}\,\mathrm{m}\right)^2} \approx 3.57 \times 10^{13}$$

(c)

$$N_{\text{bonds in 1 chain}} \approx \frac{L}{d} \approx \frac{2.5\,\mathrm{m}}{2.51 \times 10^{-10}\,\mathrm{m}} \approx 9.96 \times 10^{9}$$

(d)

$$k_s \approx \frac{\left(3.23 \times 10^{5}\,\mathrm{N/m}\right)\left(9.96 \times 10^{9}\right)}{\left(3.57 \times 10^{13}\right)} \approx 90\,\mathrm{N/m}$$

P37:

Solution:

This is a straightforward application of the basic definition of Young's modulus.

$$Y = \frac{\left|\vec{F}\right|/A}{L/\Delta L}$$

$$\Delta L = \frac{\left|\vec{F}\right|/A}{Y/L}$$

$$\Delta L = \frac{\left|\vec{F}\right|/\left(\pi r^2\right)}{Y/L}$$

$$\Delta L \approx \frac{(10\,\mathrm{kg})\left(9.80\,\mathrm{m/s^2}\right)/\left(\pi\left(1.5 \times 10^{-3}\,\mathrm{m}\right)^2\right)}{\left(2 \times 10^{11}\,\mathrm{N/m^2}\right)/(3\,\mathrm{m})}$$

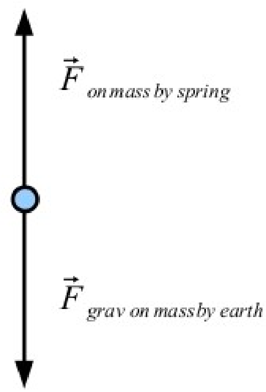
$$\Delta L \approx 2.1 \times 10^{-4}\,\mathrm{m} \approx 0.21\,\mathrm{mm}$$

P40:

Solution:

Spring force is F $= bs^3$

(a) Define the system as the hanging mass. Draw a free-body diagram.



Apply the Momentum Principle. The system remains at rest (i.e. in equilibrium).

$$
\begin{aligned}
\vec{F}_{\text{net}} &= \frac{\mathrm{d}\vec{p}}{\mathrm{d}t} \\
\vec{F}_{\text{spring}} + \vec{F}_{\text{grav}} &= 0 \\
\vec{F}_{\text{spring}} &= -\vec{F}_{\text{grav}} \\
\left\langle 0, bs^3, 0 \right\rangle &= -\left\langle 0, -mg, 0 \right\rangle
\end{aligned}
$$

Examine the y-component only.

$$
\begin{aligned}
bs^3 &= mg \\
b &= \frac{mg}{s^3}
\end{aligned}
$$

Where $s = L - L_0 = 29\,\text{cm} - 25\,\text{cm} = 4\,\text{cm} = 0.04\,\text{m}$.

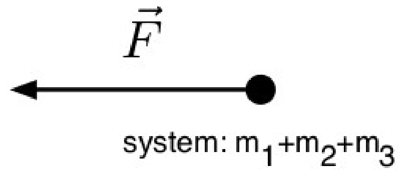
$$
\begin{aligned}
b &= \frac{(0.018\,\text{kg})\left(9.80\,\text{m/s}^2\right)}{\left(0.04\,\text{m}^3\right)^3} \\
&= 2760\,\text{N/m}^3
\end{aligned}
$$

(b) The following ideas were used in the analysis for part (a). The Momentum Principle The fact that the gravitational force acting on an object near Earth's surface is approximately mg. The rate of change of momentum of the system is zero.

P41:

Solution:

(a) Define the system to be m_1,m_2, and m_3 together. Sketch a free-body diagram (see the figure below.).



Apply the Momentum Principle to the system.

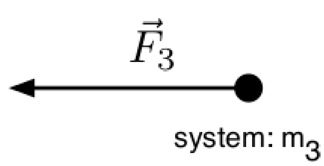
$$\vec{F}_{net} = \frac{d\vec{p}}{dt}$$

Write the x-component of the Momentum Principle.

$$-F = m_{system}\frac{dv_x}{dt}$$
$$\frac{dv_x}{dt} = \frac{-F}{m_{system}}$$
$$\frac{dv_x}{dt} = \frac{-F}{(m_1 + m_2 + m_3)}$$

Note that the acceleration will be the same for all parts in the system as well. So m_1,m_2, and m_3 all have the same acceleration.

(b) For the left end, define the system to be m_3. Sketch a free-body diagram (see the figure below.).



$\vec{F}_3$ is the compression force on the right side of m_3 by m_2.

Apply the Momentum Principle to the system.

$$\vec{F}_{net} = \frac{d\vec{p}}{dt}$$
$$-F_3 = m_3\frac{dv_x}{dt}$$

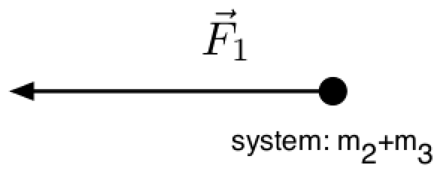
Substitute the acceleration of the system.

$$-F_3 = m_3 \left(\frac{-F}{m_1 + m_2 + m_3} \right)$$

$$F_3 = \left(\frac{m_3}{m_1 + m_2 + m_3} \right) F$$

Note that this is same magnitude as the force on m_2 by m_3.

To find the force on m_1 by m_2, note that it is the same as the force on m_2 by m_1. Define the system to be m_2 and m_3 together. Sketch a free-body diagram of the system (see the figure below.).



$\vec{F}_1$ is the compression force at the right end of m_2 due to contact with m_1.

Apply the Momentum Principle to the system.

$$\vec{F}_{\text{net}} = \frac{\mathrm{d}\vec{p}}{\mathrm{d}t}$$

$$-F_1 = m_{\text{system}} \frac{\mathrm{d}v_x}{\mathrm{d}t}$$

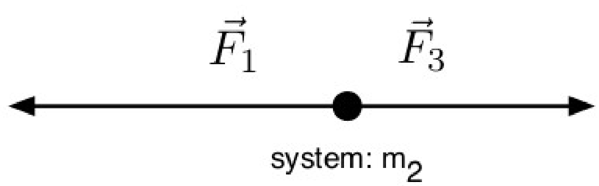
$$-F_1 = (m_2 + m_3) \frac{\mathrm{d}v_x}{\mathrm{d}t}$$

Substitute the x-acceleration from part (a).

$$-F_1 = (m_1 + m_2) \left(\frac{-F}{m_1 + m_2 + m_3} \right)$$

$$F_1 = \left(\frac{m_2 + m_3}{m_1 + m_2 + m_3} \right) F$$

Note that $F_1 > F_3$. Sketch a free-body diagram for m_2.



Because $F_1 > F_3$, then m_2 accelerates to the left, as expected.

(c) When sketching the free-body diagram in part (a), the direction of the force on the system is the same whether you pull on block 3 or push on block 1. Thus, $\vec{F_{net,3}} = \langle -F, 0, 0 \rangle$. The only difference is that if you pull on block 3, interatomic bonds will stretch. If you push on block 1, interatomic bonds will compress. But the magnitudes and directions of the net force on block 3 will be the same in the two cases.

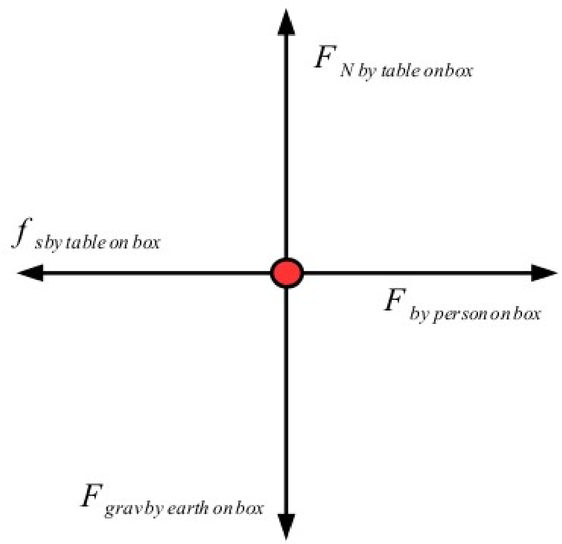
P44:

Solution:

(a) To start the box moving, you must apply a force parallel to the surfaces in contact that is greater than the maximum static force. Thus,

$$f_{s,\max} = \mu_s F_N$$

Apply the Momentum Principle. Define the system to be the box. Draw a free-body diagram for the box (see the figure below.).



$$\vec{F}_{\text{net}} = \frac{\mathrm{d}\vec{p}}{\mathrm{d}t}$$

At the instant just before it starts to move,

$$\vec{F}_{\text{net}} = 0$$

In the y-direction,

$$\begin{aligned}
F_{net,\,y} &= 0 \\
F_N + -mg &= 0 \\
F_N &= mg \\
&= (3\,\text{kg})\left(9.80\,\text{m/s}^2\right) \\
&= 29.4\,\text{N}
\end{aligned}$$

In the x-direction,

$$\begin{aligned}
F_{by\ person\ on\ box} + -f_{s,\text{max}} &= 0 \\
F_{by\ person\ on\ box} &= f_{s,\text{max}} \\
&= \mu_s F_N \\
&= (0.3)\,(29.4\,\text{N}) \\
&= 8.82\,\text{N}
\end{aligned}$$

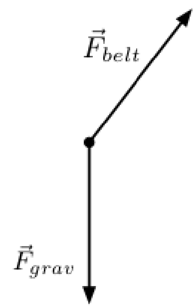
(b) To move at constant speed, the box is in equilibrium with $\vec{F}_{\text{net}} = 0$, but the frictional force is kinetic friction. Define the system to be the box, and apply the Momentum Principle.

$$\begin{aligned}
\vec{F}_{\text{net}} &= 0 \\
F_N &= 29.4\,\text{N} \\
F_{by\ person\ on\ box} &= f_{\text{k}} \\
&= \mu_{\text{k}} F_N \\
&= (0.2)\,(29.4\,\text{N}) \\
&= 5.9\,\text{N}
\end{aligned}$$

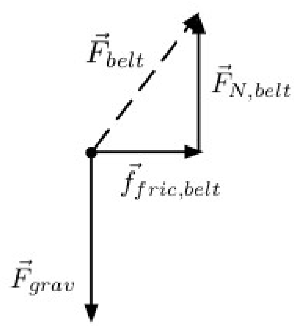
P48:

Solution:

(a) Define the system to be the box. When the box is released, the frictional force by the conveyor belt on the box has a magnitude $f_{\text{fric}} = \mu_k F_N$. It is in the direction that the conveyor belt is traveling. If this is the $+x$ direction, then the external forces on the box are:



The force by the conveyor belt on the box has two components: (1) a frictional component that is parallel to the conveyor belt and (2) a normal component that is perpendicular to the conveyor belt. Sketch these components on the force diagram.



Note that these are simply components of the force by the belt. There are only two external forces acting on the box.

Apply the Momentum Principle.

$$\vec{F}_{\text{net}} = \frac{\Delta \vec{p}}{\Delta t}$$

Write it separately for the x and y components.

$$
\begin{aligned}
\text{x:} \qquad \vec{F}_{\text{net,x}} &= \frac{\Delta p_x}{\Delta t} \\
f_{\text{fric}} &= \frac{\Delta p_x}{\Delta t} \\
\mu_{\text{k}} F_{\text{N}} &= \frac{\Delta p_x}{\Delta t}
\end{aligned}
$$

$$
\begin{aligned}
\text{y:} \qquad \vec{F}_{\text{net,y}} &= \frac{\Delta p_y}{\Delta t} \\
F_{\text{N}} + F_{\text{grav,y}} &= 0 \\
F_{\text{N}} - mg &= 0 \\
F_{\text{N}} &= mg = (8\,\text{kg})\,(9.8\,\text{N/kg}) = 78.4\,\text{N}
\end{aligned}
$$

The box has an initial x-velocity of zero and a final x-velocity of 5 m/s. Apply the Momentum Principle in the x-direction, and solve for Δt.

$$
\begin{aligned}
\mu_{\text{k}} F_{\text{N}} &= \frac{\Delta p_x}{\Delta t} \\
\mu_{\text{k}} F_{\text{N}} &= \frac{m\left(v_{x,\,f} - 0\right)}{\Delta t} \\
\Delta t &= \frac{m v_{x,\,f}}{\mu_{\text{k}} F_{\text{N}}} \\
&= \frac{(8\,\text{kg})\,(5\,\text{m/s})}{(0.6)\,(78.4\,\text{N})} \\
\Delta t &= 0.85\,\text{s}
\end{aligned}
$$

(b) The box has an initial x-velocity of zero and a final x-velocity of 5 m/s.

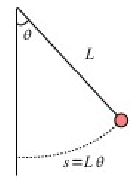
$$
\begin{aligned}
v_{x,\text{avg}} &= \frac{\Delta x}{\Delta t} \\
\frac{v_{x,\text{f}} + v_{x,\text{i}}}{2} &= \frac{\Delta x}{\Delta t} \\
\Delta x &= \left(\frac{v_{x,\text{f}}}{2}\right)\Delta t \\
&= \left(\frac{5\,\text{m/s}}{2}\right)(0.85\,\text{s}) \\
\Delta x &= 2.1\,\text{m}
\end{aligned}
$$

After traveling this distance of 2.1 m (in 0.85 s), the box has the same velocity as the belt and no longer slips on the belt. Then the acceleration of the box is zero, and it's velocity is constant. Thus according to the Momentum Principle, the net force on the box is zero. This means that there is no frictional force on the box after it is in equilibrium.

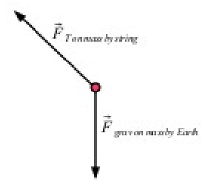
P61:

Solution:

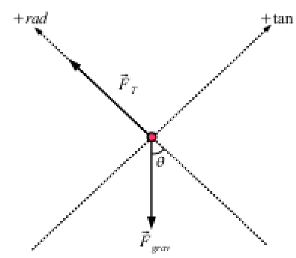
(a) Begin with a sketch of the system, as shown below.



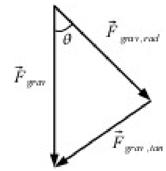
Define the system to be the mass m. Apply the Momentum Principle to the system. Sketch the forces on the system, as shown below.



Define a coordinate system with the radial axis perpendicular to the object's path and directed toward the pivot and the tangential axis tangent to the path, as shown below.



With this coordinate system, write $\vec{F}_{\text{grav}}$ using the right triangle shown below.



$$\begin{aligned} F_{grav,\,tan} &= -\left|\vec{F}_{\text{grav}}\right| \sin\theta \\ F_{grav,\,rad} &= -\left|\vec{F}_{\text{grav}}\right| \cos\theta \end{aligned}$$

The net force on the mass m is

$$\begin{aligned} \vec{F}_{\text{net}} &= \vec{F}_{\text{grav}} + \vec{F}_{\text{T}} \\ &= \left\langle -\left|\vec{F}_{\text{grav}}\right| \sin\theta, F_{T} - \left|\vec{F}_{\text{grav}}\right| \cos\theta, 0 \right\rangle \end{aligned}$$

where the first component is the tangential component and the second component is the radial component. Thus, the only component of the net force in the tangential direction is $-\left|\vec{F}_{\text{grav}}\right| \sin\theta$.

Write the Momentum Principle

$$\vec{F}_{\text{net}} = \frac{\mathrm{d}\vec{p}}{\mathrm{d}t}$$

Express this in the tangential direction and substitute for $F_{net,\,tan}$. Solve for $\frac{\mathrm{d}p_{tan}}{\mathrm{d}t}$.

$$\begin{aligned} F_{net,\,tan} &= \frac{\mathrm{d}p_{tan}}{\mathrm{d}t} \\ -\left|\vec{F}_{\text{grav}}\right| \sin\theta &= \frac{\mathrm{d}p_{tan}}{\mathrm{d}t} \\ \frac{\mathrm{d}p_{tan}}{\mathrm{d}t} &= -mg\sin\theta \end{aligned}$$

Substitute $\theta = \frac{s}{L}$ since arc length is $s = L\theta$

$$\frac{\mathrm{d}p_{tan}}{\mathrm{d}t} = -mg\sin\frac{s}{L}$$

Because the pendulum's momentum is never in the radial direction, $p = p_{\text{tan}}$ and

$$\frac{\mathrm{d}p}{\mathrm{d}t} = -mg\sin\frac{s}{L}$$

(b) For small angles, $\sin\theta \approx \theta$, and

$$\frac{dp_{tan}}{dt} \approx -mg\frac{s}{L}$$

The tangential component of momentum is $p_{tan} = mv_{tan}$, where $v_{tan} = \frac{ds}{dt}$. Thus the Momentum Principle gives

$$\begin{aligned}
\frac{dp_{tan}}{dt} &= m\frac{dv_{tan}}{dt} = -mg\frac{s}{L} \\
m\frac{d}{dt}\frac{ds}{dt} &= -mg\frac{s}{L} \\
\cancel{m}\frac{d^2s}{dt^2} &= -\cancel{m}g\frac{s}{L} \\
\frac{d^2s}{dt^2} + \frac{g}{L}s &= 0
\end{aligned}$$

(c) Compare this to the Momentum Principle applied to a mass-spring system where

$$\frac{d^2x}{dt^2} + \frac{k_s}{m}x = 0$$

and

$$\omega = \sqrt{\frac{k_s}{m}}$$

The equations for the pendulum and the mass-spring system have the same form. Thus, for the pendulum

$$\omega = \sqrt{\frac{g}{L}}$$

Using $\omega = 2\pi f$ and $f = 1/T$, solve for the period.

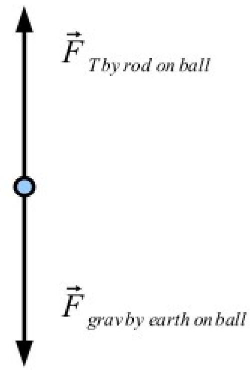
$$\begin{aligned}
\omega &= \frac{2\pi}{T} \\
T &= \frac{2\pi}{\omega} \\
&= 2\pi\sqrt{\frac{L}{g}}
\end{aligned}$$

(d) A simple experiment can be constructed with a mass and string. Use a stopwatch to measure the time for 10 oscillations (or whatever number you choose). Measure Δt for N oscillations. Then $T = \Delta t/N$. Calculate the period from the theory ($T = 2\pi\sqrt{\frac{L}{g}}$) and compare your experimental and theoretical results.

P63:

Solution:

Apply the Momentum Principle to the ball. Define the system to be the ball. Draw a free-body diagram, as shown in the figure below.



Since the body is in equilibrium,

$$
\begin{aligned}
\vec{F}_{\text{net}} &= \frac{\Delta \vec{p}}{\Delta t} \\
&= 0 \\
\vec{F}_{\text{T}} + \vec{F}_{\text{grav}} &= 0 \\
\vec{F}_{\text{T}} &= -\vec{F}_{\text{grav}} \\
&= -\langle 0, -mg, 0\rangle \\
&= \langle 0, mg, 0\rangle \\
&= \left\langle 0, (41\,\text{kg})\left(9.80\,\text{m/s}^2\right), 0\right\rangle \\
&= \langle 0, 402, 0\rangle\,\text{N} \\
\left|\vec{F}_{\text{T}}\right| &= 402\,\text{N}
\end{aligned}
$$

Calculate Young's Modulus,

$$
\begin{aligned}
\frac{F_T}{A} &= Y\frac{\Delta L}{L} \\
Y &= \frac{F_T}{A}\frac{L}{\Delta L}
\end{aligned}
$$

The cross-sectional area of the rod is

$$
\begin{aligned}
A &= (1.5\,\mathrm{mm})\,(3.1\,\mathrm{mm}) \\
&= \left(1.5 \times 10^{-3}\,\mathrm{m}\right)\left(3.1 \times 10^{-3}\,\mathrm{m}\right) \\
&= 4.65 \times 10^{-6}\,\mathrm{m}^2 \\
Y &= \left(\frac{402\,\mathrm{N}}{4.65 \times 10^{-6}\,\mathrm{m}^2}\right)\left(\frac{2.6\,\mathrm{m}}{0.002898\,\mathrm{m}}\right) \\
&= 7.8 \times 10^{10}\,\mathrm{N/m}^2
\end{aligned}
$$

Calculate the diameter of a silver atom. Assume a simple cubic lattice. Find the volume of a cube taken up by a spherical atom.

$$
\begin{aligned}
\rho &= \left(10.5\,\mathrm{g/cm}^3\right)\left(\frac{1\,\mathrm{kg}}{1000\,\mathrm{g}}\right)\left(\frac{(100\,\mathrm{cm})^3}{1\,\mathrm{m}^3}\right) = 1.05 \times 10^{4}\,\mathrm{kg/m}^3 \\
M &= 108\,\mathrm{g/mol} = 0.108\,\mathrm{kg/mol}
\end{aligned}
$$

$$
\begin{aligned}
V &= \left(\frac{1\,\mathrm{m}^3}{1.05 \times 10^4\,\mathrm{kg}}\right)\left(\frac{0.108\,\mathrm{kg}}{1\,\mathrm{mol}}\right)\left(\frac{1\,\mathrm{mol}}{6.02 \times 10^{23}\,\mathrm{atom}}\right) \\
&= 1.71 \times 10^{-29}\,\mathrm{m}^3 \\
d &= V^{\frac{1}{3}} \\
&= 2.56 \times 10^{-10}\,\mathrm{m}
\end{aligned}
$$

The interatomic bond stiffness is

$$
\begin{aligned}
k_s &= Yd \\
&= \left(7.8 \times 10^{10}\,\mathrm{N/m}^2\right)\left(2.56 \times 10^{-10}\,\mathrm{m}\right) \\
&= 20.0\,\mathrm{N/m}
\end{aligned}
$$

Calculate the speed of sound in silver.

$$
\mathrm{v} = \sqrt{\frac{k_s}{m_s}}\,d
$$

The mass of an atom is

$$m_a = (0.108\,\mathrm{kg/mol})\left(\frac{1\,\mathrm{mol}}{6.02\times 10^{23}\mathrm{atom}}\right)$$
$$= 1.79\times 10^{-25}\mathrm{kg/atom}$$

$$\mathrm{v} = \sqrt{\frac{20\,\mathrm{N/m}}{1.79\times 10^{-25}\mathrm{kg}}}\left(2.56\times 10^{10}\,\mathrm{m}\right)$$
$$= 2710\,\mathrm{m/s}$$