Things you must know:

Definition of average velocity	
Definition of momentum	The Momentum Principle
Definitions of particle energy, kinetic energy, and work	The Energy Principle

Multiparticle systems:

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots}{m_1 + m_2 + \ldots} \qquad \vec{P}_{tot} \approx M_{tot}\vec{v}_{cm}\ (v << c) \qquad K_{tot} = K_{trans} + K_{rel}$$

$$K_{rel} = K_{rot} + K_{vib} \qquad K_{trans} \approx \frac{1}{2}M_{tot}v_{cm}^2\ (v << c)$$

$$I = m_1 r_{1\perp}^2 + m_2 r_{2\perp}^2 + \ldots$$

$$K_{rot} = \frac{L_{rot}^2}{2I} = \frac{1}{2}I\omega^2$$

Other physical quantities:

$$\gamma \equiv \frac{1}{\sqrt{1-\left(\frac{|\vec{v}|}{c}\right)^2}} \qquad E^2 - (pc)^2 = \left(mc^2\right)^2$$

$$\vec{F}_{grav} = -G\frac{m_1 m_2}{|\vec{r}|^2}\hat{r} \qquad U_{grav} = -G\frac{m_1 m_2}{|\vec{r}|}$$

$\left|\vec{F}_{spring}\right| = k_s s$ opposite to the stretch $\qquad U_{spring} = \frac{1}{2}k_s s^2$ for ideal spring

$U_i \approx \frac{1}{2}k_{si}s^2 - E_M$ approx. interatomic pot. energy $\qquad \Delta E_{thermal} = mC\Delta T$

$E_N = -\frac{13.6\text{eV}}{N^2}$ where $N = 1, 2, 3\ldots$ (Hydrogen atom energy levels)

$E_N = N\hbar\omega_0 + E_0$ where $N = 0, 1, 2\ldots$ and $\omega_0 = \sqrt{\frac{k_{si}}{m_a}}$ (Quantized oscillator energy levels)

Constant	Symbol	Approximate Value
Speed of light	c	3×10^8 m/s
Gravitational constant	G	6.7×10^{-11} N $\cdot$ m^2/kg^2
Approx. grav field near Earth's surface	g	9.8 N/kg
Electron mass	m_e	9×10^{-31} kg
Proton mass	m_p	1.7×10^{-27} kg
Neutron mass	m_n	1.7×10^{-27} kg
Electric constant	$\frac{1}{4\pi\epsilon_0}$	9×10^9 N $\cdot$ m^2/C^2
Proton charge	e	1.6×10^{-19} C
Avogadro's number	N_A	6.02×10^{23} atoms/mol

milli	m	1×10^{-3}	kilo	K	1×10^{3}
micro	μ	1×10^{-6}	mega	M	1×10^{6}
nano	n	1×10^{-9}	giga	G	1×10^{9}
pico	p	1×10^{-12}	tera	T	1×10^{12}

Problem 1

A ball of unknown mass m is attached to a spring. In outer space, far from other objects, you hold the other end of the spring and swing the ball around in a circle of radius 1.5 m at constant speed.

1. You time the motion and observe that going around 10 times takes 6.88 seconds. What is the angular speed ω?
2. What is the speed of the ball?
3. Is the momentum of the ball changing or not? How can you tell?
4. If the momentum is changing, what interaction is causing it to change? If the momentum is not changing, why isnt it?
5. The relaxed length of the spring is 1.2 m, and its stiffness is 1000 N/m. While you are swinging the ball, since the radius of the circle is 1.5 m, the length of the spring is also 1.5 m. What is the magnitude of the force that the spring exerts on the ball?
6. What is the mass m of the ball?

Problem 2

An alpha particle with mass 6.8×10^{-27} kg is moving with a speed of 2.9×10^{8} m/s.

(a) What is the rest energy of this alpha particle?

(b) What is the total energy of this alpha particle?

(c) What is the kinetic energy of this alpha particle?

Problem 3

Jack and Jill are maneuvering a 3000 kg boat near a dock. Initially the boat's position is $\langle 2, 0, 3\rangle$ m and its speed is 1.3 m/s. As the boat moves to position $\langle 4, 0, 2\rangle$ m, Jack exerts a force $\langle -400, 0, 200\rangle$ N and Jill exerts a force $\langle 150, 0, 300\rangle$ N.

(a) How much work does Jack do?

(b) How much work does Jill do?

(c) Assuming that we can neglect the work done by the water on the boat, what is the final speed of the boat?

(d) Without doing any geometrical calculations, say what is the angle between the (vector) force that Jill exerts and the (vector) velocity of the boat. Explain briefly how you know this.

(e) What effect does Jill have on the boat's motion?

Problem 4

The short-lived neutral kaon K_S^0 is an unstable particle that can decay into a positive pion and a negative pion: $K_S^0 \to \pi^+ + \pi^-$. The rest energy of the kaon is 498 MeV (498×10^6 electron volts). The rest energy of either kind of charged pion is 140 MeV. Consider the case where the kaon is at rest when it decays, so the π^+ and π^- move in opposite directions with equal and opposite momenta, with speeds that are comparable to the speed of light.

(a) When the pions are far apart, what is the speed v of the π^+?

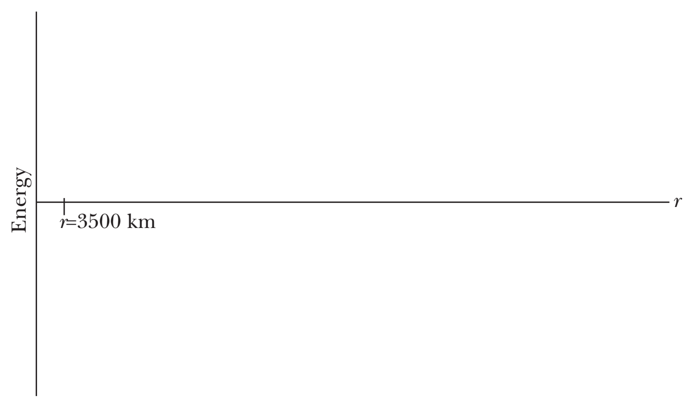
(b) When the pions are far apart, what is the kinetic energy of the π^+, in MeV?

Problem 5

You blast off from Mars, and you turn off the rockets when you are 3500 km (3.5×10^6 m) from the center of Mars, well above its thin atmosphere and headed away from the planet. You intend to leave Mars for good, and by the time you get very far away you want to be coasting at a speed of 1800 m/s. Mars has a mass of 6.4×10^{23} kg.

(a) Draw a graph of kinetic energy, of potential energy, and of kinetic plus potential energy for this process. Label each of the three curves. The center-to-center distance is marked for the place where you turn off the rockets; start your drawing at that place.

Be sure you label the curves to identify them!

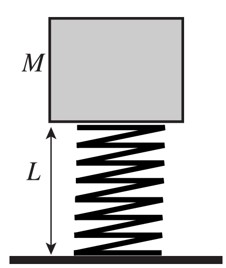


(b) Calculate the speed you must have when you are 3500 km (3.5×10^6 m) from the center of Mars in order that your speed when you are very far from Mars is 1800 m/s. Explain carefully and completely, starting from a fundamental principle.

(c) What approximations or idealizations did you need to make to calculate the speed?

Problem 6

A spring with stiffness k and relaxed length L stands vertically on a table. You hold a mass M just barely touching the top of the spring.

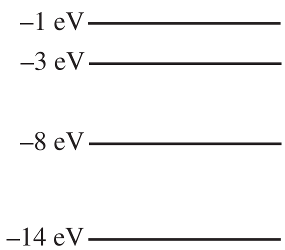


(a) You very slowly let the mass down onto the spring a certain distance, and when you let go, the mass doesn't move. How much did the spring compress? How much work did you do?

(b) Now you again hold the mass just barely touching the top of the spring, and then let go. What is the maximum compression of the spring? State what approximations and simplifying assumptions you made.

(c) Next you push the mass down on the spring so that the spring is compressed an amount s, then let go, and the mass starts moving upward and goes quite high. When the mass is a height of $2L$ above the table, what is its speed?

Problem 7



A particular microscopic object has energy levels, $K + U$, shown in the diagram. A bottle contains a large number of these objects. A high energy electron beam is run through the bottle, such that an object could be in any of the energy levels at any given time. What are the energies of the photons emitted by this collection of objects? **Choose the answer below that contains all the possible energies, and no others.**

(A) 1 eV, 4 eV, 5 eV

(B) 6 eV, 7 eV, 11 eV

(C) 8 eV, 12 eV, 13 eV

(D) 1 eV, 4 eV, 5 eV, 8 eV, 12 eV, 13 eV

(E) 1 eV, 4 eV, 5 eV, 6 eV, 7 eV, 8 eV, 11 eV, 12 eV, 13 eV

Problem 8

A horizontal spring-mass system has low friction, spring stiffness 200 N/m, and mass 0.4 kg. The system is released with an initial compression of the spring of 10 cm and an initial speed of the mass of 3 m/s.

(a) What is the maximum stretch during the motion?

(b) What is the maximum speed during the motion?

(c) Demonstrate quantitatively that it is reasonable to ignore quantum effects in this system.

(d) Now suppose that there is energy dissipation of 0.01 J per cycle of the spring-mass system. What is the average power input in watts required to maintain a steady oscillation?

Problem 9

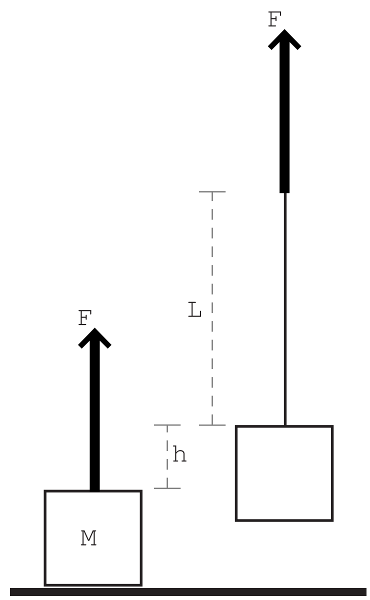
A bottle contains a gas consisting of carbon monoxide molecules (CO). Two CO molecules initially in their ground states collide head on, each with initial kinetic energy 0.8 eV. After the collision both molecules are in the 3^{rd} excited state above the vibrational ground state. The effective stiffness of the CO interatomic bond is quite high, about 2000 N/m.

(a) Calculate approximately the kinetic energy that each molecule has after the collision.

(b) What are the photon energies that these excited molecules may emit?

Problem 10

A box contains machinery that can rotate. The total mass of the box plus machinery is M. A string wound around the machinery comes out through a small hole in the top of the box. Initially the box sits on the ground, and the machinery inside the box is not rotating. Then you pull upwards on the string with a force whose magnitude F is constant. At an instant when you have pulled a length of string L out of the box, the box has risen a height h.



(a) Consider the point particle system and calculate the speed of the box at this instant. Start from a fundamental principle, and show all your work. Express your answer in terms of the given quantities.

(b) Consider the real system and calculate the rotational kinetic energy of the machinery inside the box at this instant. Start from a fundamental principle, and show all your work. Express your answer in terms of the given quantities.