Chapter 17

Q12:

Solution:

The ring acts as a magnetic dipole. Since current flows clockwise (looking from point A), \vec{B} along th axis of the loop points in the -x direction. Thus, $\hat{B}_{_{\rm C}}$ and $\hat{B}_{_{\rm A}}$ are <-1,0,0>. At points B, D, E, and F, \vec{B} is in the opposite direction of the dipole moment $\vec{\mu}$. Thus, $\hat{B}_{_{\rm B}}=\hat{B}_{_{\rm D}}=\hat{B}_{_{\rm E}}=(1,0,0)$.

P33:

Solution:

- (a) The conventional current is in the +x direction, <1,0,0>.
- (b) $\Delta l = \frac{1.3 \text{ m}}{8} = 0.1625 \text{ m}$
- (c) $\left| \Delta \vec{l} \right| = 0.1625 \,\mathrm{m}$
- (d) $\Delta \vec{l} = (0.1625 \text{ m}) < 1, 0, 0 > = (0.1625, 0, 0) \text{ m}$
- (e) $\vec{\mathbf{r}}_{4} = \langle \frac{-\Delta l}{2}, 0, 0 \rangle = \langle -0.08125, 0, 0 \rangle$ m

(f)

$$\begin{array}{lll} \vec{\rm r} & = & \vec{\rm r}_{_{\rm obs}} - \vec{\rm r}_{_4} \\ & = & \langle 0.081, 0.178, 0 \rangle \ {\rm m} - \langle -0.08125, 0, 0 \rangle \ {\rm m} \\ & = & \langle 0.16225, 0.178, 0 \rangle \ {\rm m} \end{array}$$

(g)

$$\begin{array}{lcl} |\vec{r}| & = & 0.2409 \; m \\ \\ \hat{r} & = & \frac{\vec{r}}{|\vec{r}|} = < 0.674, 0.739, 0 > \end{array}$$

- (h) $\Delta \vec{l} \times \hat{r} = \langle 0, 0, 0.120 \rangle$ m
- (i)

$$\begin{split} \Delta \vec{\mathrm{B}} &= \frac{\mu_o}{4\pi} I \frac{\Delta \vec{\mathrm{l}} \times \hat{\mathrm{r}}}{\left| \vec{\mathrm{r}} \right|^2} \\ &= \frac{\left(1 \times 10^{-7} \frac{\mathrm{T} \cdot \mathrm{m}}{\mathrm{A}}\right) (6.5 \; \mathrm{A}) (\langle 0, 0, 0.120 \rangle \; \mathrm{m})}{(0.1625 \; \mathrm{m})^2} \\ &= \langle 0, 0, 2.95 \times 10^{-6} \rangle \; \mathrm{T} \end{split}$$

Use the right-hand rule to verify the direction. With your thumb pointing to the right and fingers curling around your thumb, your fingers point in the +z direction at point A.

P36:

Solution:

- (a) Use the right-hand rule applied to each straight line segment (each side) of the triangle. In each case, the magnetic field due to a side is in the -z direction (east). Thus, the compass will be deflected eastward and will point northeast.
- (b) (3) is the only reasonable way to model the triangular loop.
- (c) Find \vec{B}_{side} , the magnetic field due to one side of the triangular loop. Multply by 3 to get \vec{B}_{loop} and multiply by N=3 to get \vec{B}_{not} due to the coil.

$$\begin{split} \left| \vec{\mathbf{B}}_{\text{\tiny side}} \right| &= \frac{\mu_o}{4\pi} \frac{LI}{r(r^2 + \left(\frac{L}{2}\right)^2)^{\frac{1}{2}}} \\ &= \left(1 \times 10^{-7} \frac{\mathbf{T} \cdot \mathbf{m}}{\mathbf{A}}\right) \frac{(0.083 \text{ m})(0.62 \text{ A})}{(0.024 \text{ m})((0.024 \text{ m})^2 + \left(\frac{0.083 \text{ m}}{2}\right)^2)^{\frac{1}{2}}} \\ &= 2.05 \times 10^{-6} \text{ T} \end{split}$$

$$\left|\vec{\mathbf{B}}_{\mbox{\tiny loop}}\right| \ = \ 3B_{\mbox{\tiny side}} = 6.15 \times 10^{-6} \; \mathrm{T}$$

$$\begin{array}{lcl} B_{\mbox{\tiny coil}} & = & NB_{\mbox{\tiny loop}} \\ & = & 3(6.15 \times 10^{-6} \ \mbox{T}) \\ & = & 1.84 \times 10^{-5} \ \mbox{T} \end{array}$$

The compass deflection from north is

$$\theta = \tan^{-1} \left(\frac{B_{\text{coil}}}{B_{\text{Earth}}} \right)$$
 $\theta \approx 43^{\circ}$

P46:

Solution:

 \vec{B} is the superposition of $\vec{B}_{wire} + \vec{B}_{loop}$.

 \vec{B}_{wire} is in the -z direction and has a magnitude

$$\begin{array}{lcl} B_{\mbox{\tiny wire}} & \approx & \frac{\mu_{\mbox{\tiny o}}}{4\pi} \frac{2I}{r} \\ \\ & \approx & (1 \times 10^{-7} \, \frac{\mbox{T} \cdot \mbox{m}}{\mbox{A}}) \frac{2(3.5 \, \mbox{A})}{0.058 \, \mbox{m}} \\ \\ & \approx & 1.21 \times 10^{-5} \, \mbox{T} \\ \\ \mbox{so } \vec{B}_{\mbox{\tiny wire}} & \approx & \left< 0, 0, -1.21 \times 10^{-5} \right> \, \mbox{T} \end{array}$$

 \vec{B}_{loop} is in the -z direction and has a magnitude

$$\begin{array}{lll} B_{\mbox{\tiny loop}} & = & \frac{\mu_o}{4\pi} \frac{2IA}{R^3} \\ & = & (1\times 10^{-7}\,\frac{\mathrm{T}\cdot\mathrm{m}}{\mathrm{A}}) \frac{2(3.5\;\mathrm{A})\pi (0.058\;\mathrm{m})^2}{(0.058\;\mathrm{m})^3} \\ & = & 3.79\times 10^{-5}\;\mathrm{T} \\ \mathrm{so}\,\vec{\mathrm{B}}_{\mbox{\tiny loop}} & = & \left<0,0,-3.79\times 10^{-5}\right>\,\mathrm{T} \end{array}$$

$$\begin{array}{lcl} \vec{B}_{_{\rm net}} & = & \vec{B}_{_{\rm loop}} + \vec{B}_{_{\rm wire}} \\ & = & \left<0, 0, -3.79 \times 10^{-5}\right> \, T + \left<0, 0, -1.21 \times 10^{-5}\right> \, T \\ & = & \left<0, 0, -5.00 \times 10^{-5}\right> \, T \end{array}$$

P49:

Solution:

- (a) Neglect the upper straight line segments because they contribute zero field. Neglect the side segments because they are relatively short and far away and contribute negligible field. Thus, the net magnetic field is a superposition of the magnetic field due to the hemisphere and the lower straight line segment. Application of the right-hand rule shows that the field due to each of these parts of the wire is into the page (-z direction).
- (b)

$$\begin{array}{lcl} \vec{\mathrm{B}}_{\mathrm{net}} & = & \vec{\mathrm{B}}_{\mathrm{hemisphere}} + \vec{\mathrm{B}}_{\mathrm{straight/wire}} \\ \mathbf{B}_{net,z} & = & \mathbf{B}_{z,hemisphere} + \mathbf{B}_{z,straight/wire} \\ & = & \frac{\mu_o}{4\pi} \frac{I\pi}{R} + \frac{\mu_o}{4\pi} \frac{2I}{h} \end{array}$$

where we assume that $h \ll L$. Thus,

$$\left| \vec{\mathbf{B}} \right| = \frac{\mu_o}{4\pi} I \left(\frac{\pi}{R} + \frac{2}{h} \right)$$

P50:

Solution:

 $\vec{B}_{_{net}} = \vec{B}_{_1} + \vec{B}_{_2}. \text{ Since } \vec{B}_{_1} \text{ and } \vec{B}_{_2} \text{ are to the right, } \vec{B}_{_{net}} \text{ is to the right.}$

(a)

$$\begin{split} \left| \vec{\mathbf{B}}_{\text{coil}} \right| &= \frac{\mu_o}{4\pi} \frac{2IAN}{(r^2 + R^2)^{\frac{3}{2}}} \\ \left| \vec{\mathbf{B}}_{\text{l}} \right| &= \left(1 \times 10^{-7} \, \frac{\mathbf{T} \cdot \mathbf{m}}{\mathbf{A}} \right) \frac{2(2A)\pi (0.03 \, \mathbf{m})^2 (10)}{((0.10 \, \mathbf{m})^2 + (0.03 \, \mathbf{m})^2)^{\frac{3}{2}}} \\ &= 9.94 \times 10^{-6} \, \mathbf{T} \end{split}$$

$$\begin{array}{lcl} \vec{B}_{_{\rm net}} & = & \left< 9.94 \times 10^{-6}, 0, 0 \right> \, T + \left< 9.94 \times 10^{-6}, 0, 0 \right> \, T \\ & = & \left< 1.99 \times 10^{-5}, 0, 0 \right> \, T \\ \\ \left| \vec{B}_{_{\rm net}} \right| & = & 1.99 \times 10^{-5} \, \, T \end{array}$$

Direction is $\hat{B} = <1, 0, 0>$.

(b) Approximate formula gives

$$\begin{split} B_{_{\rm coil}} &\;\; \approx \;\; \frac{\mu_{_o}}{4\pi} \frac{2I(\pi R^2)}{r^3} \\ &\;\; \approx \;\; (1\times 10^{-7}\,\frac{{\rm T\cdot m}}{{\rm A}}) \frac{2(2A)\pi (0.03\,{\rm m})^2 (10)}{(0.1\,{\rm m})^3} \\ &\;\; \approx \;\; 1.13\times 10^{-5}\,{\rm T} \end{split}$$

% error =
$$\left(\frac{1.13 - 0.994}{0.994}\right) \times 100$$

 $\approx 14\%$

In this case, $z \approx 3R$, thus z is not much greater than R. If z = 10R, then the approximation is better and the error is less.

(c) In this case, \vec{B}_2 is in the -x direction. Since $\left|\vec{B}_1\right| = \left|\vec{B}_2\right|$ and $\left|\vec{B}_1\right|$ is in the +x direction, then $\vec{B}_{net} = \vec{B}_1 + \vec{B}_2 = 0$.

Chapter 18

P41:

Solution:

(a) Apply the Loop Rule (Conservation of Energy) to loop 2.

$$(V_F - V_C) + (V_C - V_D) + (V_D - V_E) + (V_E - V_F) = 0$$

$$-8 \text{ V} + (V_C - V_D) + 4.5 \text{ V} + 0 = 0$$

$$(V_C - V_D) = 3.5 \text{ V}$$

(b) $V_C > V_D$ so V_C must be the positive terminal since it is at a higher potential.

P42:

Solution:

The graph shows that the potential difference across the connecting wires is not negligible, even though the question suggests neglecting this potential difference. Therefore, we will not neglect it. By estimation, $V_B - V_A \approx -0.5$ V and $V_D - V_C \approx -0.5$ V. According to the Loop Rule,

$$(V_B - V_A) + (V_C - V_B) + (V_D - V_C) + (V_A - V_D) = 0$$

 $-0.5 \text{ V} + (V_C - V_B) + -0.5 \text{ V} + 3.5 \text{ V} = 0$
 $(V_C - V_B) \approx -2.5 \text{ V}$

The potential difference across the resistor is about 5 times the potential difference across the connecting wires.

The conventional current through the resistor is $I = neAuE = neAu\frac{\Delta V}{L}$. We need to know the properties of the resistor in order to calculate the current through the resistor. No, there is not enough information to determine the current I in the circuit.

Chapter 19

P44:

Solution:

(a) The electric potential outside a uniformly charged sphere is that of a point particle at the center of the sphere, $V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$. Inside the sphere, the electric field is zero. Thus, at r = r, $V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$.

At $r\geq r_{_2},\,V=rac{1}{4\piarepsilon_o}rac{Q}{r}+rac{1}{4\piarepsilon_o}rac{(-Q)}{r}=0.$ Just inside $r_{_2},\,V=rac{1}{4\piarepsilon_o}rac{Q}{r_{_2}}.$ Thus,

$$\begin{split} \Delta V &=& V_{_2} - V_{_1} \\ &=& \frac{1}{4\pi\varepsilon_o} \frac{Q}{r_{_2}} - \frac{1}{4\pi\varepsilon_o} \frac{Q}{r_{_1}} \\ &=& \frac{1}{4\pi\varepsilon_o} Q(\frac{1}{r_{_2}} - \frac{1}{r_{_1}}) \end{split}$$

Solve for Q to get

$$Q = \frac{4\pi\epsilon_0}{(\frac{1}{r_2} - \frac{1}{r_2})}\Delta V$$

Comparing to $Q = C\Delta V$ gives

$$C = \frac{4\pi\epsilon_{_{0}}}{(\frac{1}{r_{_{2}}} - \frac{1}{r_{_{1}}})}$$

(b)

Since $r_{_1}\approx r_{_2}=R,$ then $r_{_1}r_{_2}\approx R^2$ and

$$C \approx \frac{\epsilon_0 4\pi R^2}{s}$$

$$C \approx \frac{\epsilon_0 A}{s}$$

P56:

Solution:

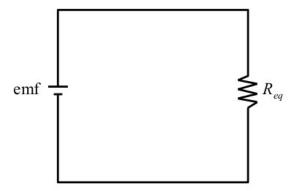
(a)

$$\begin{array}{ccc} \frac{1}{R_{_{1,2}}} & = & \frac{1}{R_{_{1}}} + \frac{1}{R_{_{2}}} \\ & = & \frac{1}{31} \frac{1}{\Omega} + \frac{1}{47 \; \Omega} \\ R_{_{1,2}} & = & 18.7 \; \Omega \end{array}$$

(b)

$$\begin{array}{rcl} R_{_{\rm eq}} & = & R_{_3} + R_{_{1,2}} \\ & = & 52\;\Omega + 18.7\;\Omega \\ & = & 70.7\;\Omega \end{array}$$

(c) Replace the circuit with the equivalent circuit in the figure below.



Apply the loop rule:

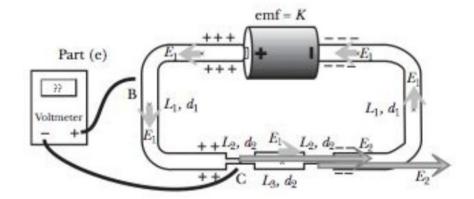
$$\begin{array}{rcl} {\rm emf} - \Delta V_{_{\rm R}} & = & 0 \\ & {\rm emf} & = & IR_{_{\rm eq}} \\ I & = & \dfrac{7.4\ {\rm V}}{70.7\ \Omega} \\ & = & 0.105\ {\rm A} \end{array}$$

This is the current through the battery. Since the battery and $R_{_3}$ are in series, then the current through $R_{_3}$ is also 0.105 A.

P59:

Solution:

(a) The electric field E_1 is small and uniform throughout the thick wires. From the current node rule applied to the thick and thin wires, we know that the electric field E_2 is much larger and uniform throughout the thin wires, since $neA_1uE_1 = neA_2uE_2$. A sketch is shown in the figure below.



- (b) Roughly, we expect a small gradient along the thick wires (small E_1) and a large gradient along the thin wires (large E_2). By symmetry the central thick wire has very little charge. The surface charge is sketched in the figure below.
- (c) Applying the loop rule (conservation of energy) to the circuit gives:
- (d) If $d_2 \ll d_1$, then $\frac{d_2^2}{d_1^2} \approx zero$, so the first expression for i gives

$$i \ \approx \ \frac{n(1/4)\pi d_2^2 u K}{2L_2}$$

Essentially this is the same result as if we had ignored ΔV for each think wire.

(e) The potential difference read by the voltmeter is $\Delta V = E_2(\frac{L_2}{2})$. The electric field E_2 is approximately $K/(2L_2)$ since the thin wires are in series and we are neglecting the potential drop across the thick wires. Thus, the voltmeter will read $\Delta V = K/(2L_2)(\frac{L_2}{2}) = (1/4)K$. The sign is positive because the positive lead was connected to the high-potential side of the wire.

P64:

Solution:

(a) $P=I\Delta V.$ Since $\Delta V_{_{\rm R}}={\rm emf}$ (loop rule) and $\Delta V_{_{\rm R}}=IR,$ then

$$\begin{split} P &= & \left(\frac{\Delta V_{_{\rm R}}}{R}\right) \Delta V_{_{\rm R}} \\ &= & \frac{(\Delta V_{_{\rm R}})^2}{R} \\ R_{_1} &= & \frac{(\Delta V_{_{\rm R}})^2}{P} = \frac{(10\ {\rm V})^2}{5\ {\rm W}} = 20\ \Omega \\ R_{_2} &= & \frac{(10\ {\rm V})^2}{20\ {\rm W}} = 5\ \Omega \end{split}$$

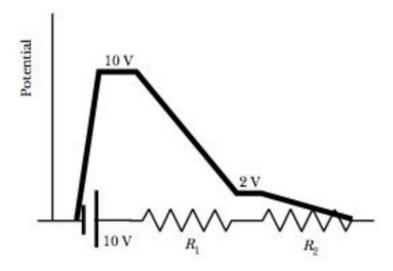
(b)

$$\begin{array}{rcl} \Delta V & = & EL \\ E & = & \frac{\Delta V}{L} = \frac{10 \text{ V}}{(3 \times 10^{-3} \text{ m})} = 3.33 \times 10^4 \frac{\text{V}}{\text{m}} \end{array}$$

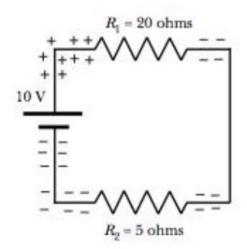
(c)

$$\begin{split} \text{emf} - IR_{_1} - IR_{_2} &= 0 \\ I &= \frac{\text{emf}}{(R_{_1} + R_{_2})} = \frac{10 \text{ V}}{20 \ \Omega} = 0.4 \text{ A} \\ \Delta V_{_1} &= IR_{_1} = (0.4 \text{ A})(20 \ \Omega) = 8 \text{ V} \\ \Delta V_{_2} &= IR_{_2} = (0.4 \text{ A})(5 \ \Omega) = 2 \text{ V} \end{split}$$

If the negative terminal of the battery is ground (V=0), then the electric potential V drops from 10 V to 2 V to zero. It is 2 V between $R_{_1}$ and $R_{_2}$, 10 V at x=0, and 0 after $R_{_2}$. A sketch of the graph is shown in the figure below.



(d) A sketch of the surface charge distribution is shown in the figure below.



(e)
$$I=0.4$$
 A (from part (c)) and $i=\frac{I}{e}=\frac{0.4}{1.6\times10^{-19}}$ C = 2.5×10^{18} electrons per second.

(f)

$$\begin{array}{rcl} P & = & (\mathsf{emf})I \\ & = & (10\;\mathrm{V})(0.4\;\mathrm{A}) = 4\;\mathrm{W} \end{array}$$

P68:

Solution:

(a)

$$\begin{array}{rcl} \Delta V & = & \mathrm{emf} - I r_{_{\mathrm{int}}} \\ 0 & = & \mathrm{emf} - I r_{_{\mathrm{int}}} \\ \\ r_{_{\mathrm{int}}} & = & \frac{\mathrm{emf}}{I} = \frac{9 \, \mathrm{V}}{18 \, \mathrm{A}} = \frac{1}{2} \Omega \end{array}$$

$$\begin{array}{rcl} P & = & (\mathsf{emf})I \\ & = & (9\,\mathrm{V})(18\,\mathrm{A}) = 162\,\mathrm{W} \end{array}$$

(d)

$$\begin{split} \Delta V_{_{\mathrm{R}}} &=& \mathrm{emf} - Ir_{_{\mathrm{int}}} \\ IR &=& \mathrm{emf} - Ir_{_{\mathrm{int}}} \\ I(R+r_{_{\mathrm{int}}}) &=& \mathrm{emf} \\ I &=& \frac{\mathrm{emf}}{R+r_{_{\mathrm{int}}}} = \frac{9 \ \mathrm{V}}{(10 \ \Omega + 0.5 \ \Omega)} = 0.857 \ \mathrm{A} \end{split}$$

(e)

$$P = \Delta V I$$
= $\left(\frac{I}{R}\right) I$
= $\frac{I^2}{R} = \frac{(0.857 \text{ A})^2}{10 \Omega} = 0.0735 \text{ W}$

(f) It reads $\Delta V = \text{emf} - Ir_{\text{int}} = 9 \text{ V} - (0.857 \text{ A})(\frac{1}{2}\Omega) = 8.57 \text{ V}$

P75:

Solution:

- (a) In the final state of static equilibrium there is no E inside the resistor and wires, so the round-trip potential difference is $(+\text{emf}) + (-\Delta V_{cap}) = 0$. Therefore $Q = C\Delta V_{cap} = C\text{emf}$.
- (b) Field argument: Polarized molecules in plastic contribute a field just outside the capacitor that is in the opposite direction to the fringe field of the plates, so the field due to the capacitor (plates and plastic) is reduced. The net field had been zero and is now nonzero due to the reduction in the field contributed by the capacitor, so current flows. The current will run until the fringe field of the capacitor again is large enough to cancel the field of all other charges.

Potential argument: E inside gap is reduced by a factor of 1/K, so $\Delta V_{cap} = Es$ is reduced to $\Delta V_{cap} = \text{emf}/K$. That means that the loop rule requires a potential difference across the resistor, and hence current will run until the potential difference across the capacitor is again equal to emf.

(c) E inside gap is reduced by a factor of 1/K, so $\Delta V_{cap} = Es$ is reduced to $\Delta V_{cap} = \text{emf}/K$. The loop rule becomes (+emf) + (-emf/K) + (-RI) = 0, and the initial current is

$$I = \frac{\text{emf}}{R} \left(1 - \frac{1}{K} \right)$$

(d) The effective capacitance has changed:

$$C_{new} = \frac{Q}{\Delta V}$$

$$= \frac{Q}{Es}$$

$$= \frac{Q}{\frac{Q/A}{K\varepsilon_0}s}$$

$$= \frac{K\varepsilon_0 A}{s} = KC_0$$

$$Q_{new} = C_{new}(emf) = KC_0(emf)$$

There is now a factor of K more charge on the plates than before the dielectric was inserted.