Chapter 15

Q16:

Solution:

Option 4 is the best option.

P22:

Solution:

(a) each segment will have length $\Delta y \approx \frac{1.7\ \mathrm{m}}{8} \approx 0.2125\ \mathrm{m}$

(b) $\vec{\mathrm{r}}_{_3} = \langle 0, \frac{3}{2}\Delta y, 0\rangle \approx \langle 0, 0.31875, 0\rangle\ \mathrm{m}$

(c) $\Delta Q = \frac{Q}{N} \approx \frac{-2\times 10^{-8}\ \mathrm{C}}{8} \approx -2.5 \times 10^{-9}\ \mathrm{C}$

(d) Treating the segment as a particle and applying the expression for a particle's field gives $\Delta\vec{\mathrm{E}}_{_3} \approx \langle -34.61, 15.76, 0\rangle\ \mathrm{N/C}$. Note there are two components!

(e) By symmetry the arrow must perpendicular to the rod and must point toward the rod (negatively charged!), therefore choice (g) is the best.

P27:

Solution:

(a) Each chunk has position $\langle R\cos\theta, R\sin\theta, 0\rangle$ and contributes an electric field that will point from the origin to the rod (negatively charged rod).

(b) Treat each chunk as a particle and calculate $\Delta\vec{\mathrm{E}}$. The result is

$$\Delta\vec{\mathrm{E}} = \frac{1}{4\pi\varepsilon_{_o}}\frac{|Q|}{\alpha R^2}\Delta\theta\,\langle\cos\theta, \sin\theta, 0\rangle$$

(c) Evaluate each component's integral separately.

$$\mathrm{E}_{_x} = \frac{1}{4\pi\varepsilon_{_o}}\frac{|Q|}{\alpha R^2}\int_{\theta=0}^{\theta=\alpha}\cos\theta d\theta = \frac{1}{4\pi\varepsilon_{_o}}\frac{|Q|}{\alpha R^2}\sin\alpha$$

$$\mathrm{E}_{_y} = \frac{1}{4\pi\varepsilon_{_o}}\frac{|Q|}{\alpha R^2}\int_{\theta=0}^{\theta=\alpha}\sin\theta d\theta = \frac{1}{4\pi\varepsilon_{_o}}\frac{|Q|}{\alpha R^2}(1-\cos\alpha)$$

(d) Units are correct. $\vec{\mathrm{E}}$ points toward the rod's midpoint as dictated by symmetry. For small values of α, $\mathrm{E}_{_y} \approx 0$. For small α, the field approaches that of a particle as expected.

P30:

Solution:

This problem reduces to an integral over an angle. You can measure angles relative to the $+x-$axis but this solution measures angles relative to the $+y-$axis. Locate a piece of the curved rod by the angle θ is makes with the $+y-$axis.

(a)

$$\begin{aligned}
\vec{r} &= \vec{r}_{obs} - \vec{r}_{source} \\
\vec{r} &= \langle 0,0,0\rangle - \langle -R\sin\theta, R\cos\theta, 0\rangle \\
\vec{r} &= \langle R\sin\theta, -R\cos\theta, 0\rangle \\
|\vec{r}| &= R \\
\hat{r} &= \langle \sin\theta, -\cos\theta, 0\rangle \\
\Delta\vec{E} &= \frac{1}{4\pi\varepsilon_o}\frac{\Delta Q}{R^2}\langle \sin\theta, -\cos\theta, 0\rangle \\
\Delta Q &= Q\frac{\Delta\theta}{\pi} \\
\Delta\vec{E} &= \frac{1}{4\pi\varepsilon_o}\frac{Q}{\pi R^2}\Delta\theta\,\langle \sin\theta, -\cos\theta, 0\rangle \\
E_x &= \frac{1}{4\pi\varepsilon_o}\frac{Q}{\pi R^2}\int_{\theta=0}^{\theta=\pi}\sin\theta d\theta = \frac{1}{4\pi\varepsilon_o}\frac{2Q}{\pi R^2} \\
E_y &= \frac{1}{4\pi\varepsilon_o}\frac{Q}{\pi R^2}\int_{\theta=0}^{\theta=\pi} -\cos\theta d\theta = 0
\end{aligned}$$

The $y-$component must be zero by symmetry and performing the integral for E_y confirms this. Also, since Q is a negative number, E_x points to the left.

(b)

$$\begin{aligned}
\vec{p}_f &= \vec{p}_i + \vec{F}_{net}\Delta t = \langle 0,0,0\rangle + q\left\langle \frac{1}{4\pi\varepsilon_o}\frac{2Q}{\pi R^2}, 0, 0\right\rangle \\
\vec{p}_f &= \left\langle -\frac{1}{4\pi\varepsilon_o}\frac{4eQ}{\pi R^2}\Delta t, 0, 0\right\rangle
\end{aligned}$$

Remember that we've taken Q to be a negative number so the final momentum is actually points to the right.

P39:

Solution:

(a) The electron sea will shift to the right making the right side slightly negative and the left side slightly positive.

(b) The net electric field inside the foil is zero. $\vec{E}_{disk}$ points to the left and $\vec{E}_{foil}$ points to the right.

(c)

$$\begin{aligned}
\vec{E}_{net} &= \vec{E}_{disk} + \vec{E}_{foil} \\
\left|\vec{E}_{disk}\right| &\approx \frac{|Q|/\pi R^2}{2\varepsilon_o}\left(1 - \frac{d}{R}\right) \\
\left|\vec{E}_{foil}\right| &\approx \frac{|q|/\pi r^2}{\varepsilon_o} \\
\frac{|q|}{\pi\varepsilon_o r^2} &\approx \frac{|Q|}{2\pi\varepsilon_o R^2}\left(1 - \frac{d}{R}\right) \\
|q| &\approx |Q|\frac{r^2}{2R^2}\left(1 - \frac{d}{R}\right) \\
&\approx \left|3\times 10^{-5}\text{ C}\right|\frac{(0.02\text{ m})^2}{2(1.5\text{ m})^2}\left(1 - \frac{0.003\text{ m}}{1.5\text{ m}}\right) \\
&\approx 2.66\times 10^{-9}\text{ C}
\end{aligned}$$

P45:

Solution:

(a) As the particle enters the region between the plates, it will deflect downward due to its interaction with the electric field. Upon leaving the plates, its trajectory will not change at all and will remain linear.

(b) At low speeds, $\left|\vec{F}_{net}\right| \approx m\left|\vec{a}\right|$ or $\left|\vec{a}\right| \approx \frac{\left|\vec{F}_{net}\right|}{m} \approx \frac{|q|\left|\vec{E}\right|}{m} \approx 1.8 \times 10^{16}$ m/s^2

(c) Treat the plates as a capacitor, and get the magnitude of the charge from $\left|\vec{E}\right| \approx \frac{|Q|/A}{\varepsilon_o}$. The result is $|Q| \approx 3.18 \times 10^{-9}$ C. The upper plate must be negatively charged given the direction of $\vec{E}$.

P55:

Solution:

(a) Dipoles in the plastic will polarize and orient themselves radially, with their negative ends pointing toward the center.

(b) There will be no polarization inside the glass sphere since the net electric field there is zero.

(c) $\left|\vec{E}_P\right|$ will be unchanged.

(d) The metal will polarize so as to have negative charge on its inner surface and positive charge on its outer surface.

(e) The net electric field inside the metal is zero, so the charge on its inner surface must be the opposite of that on the glass sphere.

(f) $\vec{E}_P$ will be unchanged since the outer surface of the metal will be the same as that on the glass sphere.

Chapter 16

Q10:

Solution:

(a) $\Delta\vec{l} = \vec{r}_B - \vec{r}_C$ so $\Delta\vec{l}$ points in the +x direction.

(b) $\vec{E}$ is in the +x direction. Since $\vec{E}$ and $\Delta\vec{l}$ are in the same direction, $\vec{E}\cdot\Delta\vec{l}$ is positive and $\Delta V = V_D - V_C$ is negative.

P31:

Solution:

(a)

$$\begin{aligned}
\Delta\vec{l} &= \vec{r}_B - \vec{r}_A \\
&= \langle 0.4, 0, 0\rangle \text{ m} - \langle -0.3, 0, 0\rangle \text{ m} \\
&= \langle 0.7, 0, 0\rangle \text{ m}
\end{aligned}$$

(b)

$$\begin{aligned}
\Delta V &= -\vec{E}\cdot\Delta\vec{l} \\
&= -(E_x\Delta x + E_y\Delta y + E_z\Delta z) \\
&= -\left(-850\ \frac{\text{V}}{\text{m}}\right)(0.7\text{ m}) \\
&= 595\text{ V}
\end{aligned}$$

(c)

$$\begin{aligned}
\Delta U &= q\Delta V \\
&= (1.6\times10^{-19}\text{ C})(595\text{ V}) \\
&= 9.52\times10^{-17}\text{ J} \\
&= 595\text{ eV}
\end{aligned}$$

(d)

$$\begin{aligned}
\Delta U &= -9.52\times10^{-17}\text{ J} \\
&= -595\text{ eV}
\end{aligned}$$

Since q is negative.

P42:

Solution:

We can do an exact treatment with few approximations. Break the problem into three parts: (1) find v_f upon exiting the accelerating plates; (2) find $\vec{v}_f$ and Δy during the deflection plates; (3) find Δy during the region after the reflection plates before hitting the screen.

1.

$$\begin{aligned}
\Delta E &= 0 \\
\Delta U + \Delta K &= 0 \\
q\Delta V + \Delta K &= 0 \\
-e\Delta V + \Delta K &= 0 \\
\Delta K &= e\Delta V
\end{aligned}$$

Assume $K_{i} \approx 0$, so

$$\begin{aligned}
K_{f} &= (1.6 \times 10^{-19}\ \mathrm{C})(18000\ \mathrm{V}) \\
&= 2.88 \times 10^{-15}\ \mathrm{J} \\
K_{f} &= \frac{1}{2} m \mathrm{v}_{f}^{2} \\
\mathrm{v}_{f} &= \sqrt{\frac{2K}{m}} \\
&= \sqrt{\frac{2(2.88 \times 10^{-15}\ \mathrm{J})}{9.11 \times 10^{-31}\ \mathrm{kg}}} \\
&= 7.95 \times 10^{7}\ \mathrm{m/s}
\end{aligned}$$

This is relativistic. Thus, $K \approx \frac{1}{2}mv^2$ cannot be used. Instead, use $K = E - mc^2 = (\gamma - 1)mc^2$. Thus,

$$\begin{aligned}
\gamma &= \frac{K}{mc^2} + 1 \\
&= \frac{2.88 \times 10^{-15}\ \mathrm{J}}{(9.11 \times 10^{-31}\ \mathrm{kg})(3 \times 10^{8}\ \mathrm{m/s})} + 1 \\
&= 1.035 \\
\gamma &= \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}
\end{aligned}$$

So $\mathrm{v}_{f} = 7.74 \times 10^{7}$ m/s.

2. In between the deflecting plates,

$$\begin{aligned}
|\Delta V| &= \left|\vec{\mathrm{E}}\right| s \\
\left|\vec{\mathrm{E}}\right| &= \frac{40\ \mathrm{V}}{0.003\ \mathrm{m}} \\
&= 1.33 \times 10^{4}\ \frac{\mathrm{V}}{\mathrm{m}} \\
\mathrm{E}_{y} &= -1.33 \times 10^{4}\ \frac{\mathrm{V}}{\mathrm{m}}
\end{aligned}$$

$\vec{\mathrm{F}} = q\vec{\mathrm{E}}$, so

$$\begin{aligned}
\mathrm{F}_{y} &= (-1.6 \times 10^{-19}\ \mathrm{C})(-1.33 \times 10^{4}\ \frac{\mathrm{V}}{\mathrm{m}}) \\
&= 2.13 \times 10^{-15}\ \mathrm{N}
\end{aligned}$$

$\mathrm{F}_{x} = 0$ so v_{x} is constant, 7.74×10^{7} m/s. The time interval to cross the plates is given by

$$\begin{aligned}
\mathrm{v}_x &= \frac{\Delta x}{\Delta t} \\
\Delta t &= \frac{\Delta x}{\mathrm{v}_x} \\
&= \frac{L}{\mathrm{v}_x} \\
&= \frac{0.08 \text{ m}}{7.74 \times 10^{7} \text{ m/s}} \\
&= 1.03 \times 10^{-9} \text{ s}
\end{aligned}$$

v_y after exiting the plates is found from the Momentum Principle.

$$\begin{aligned}
\mathrm{F}_{net,y} &= \frac{m\Delta \mathrm{v}_y}{\Delta t} \\
&= \frac{m(\mathrm{v}_{f,y} - \mathrm{v}_{i,y})}{\Delta t} \\
&= \frac{m\mathrm{v}_{f,y}}{\Delta t} \\
\mathrm{v}_{f,y} &= \frac{\mathrm{F}_{net,y}\Delta t}{m} \\
&= \frac{(2.13 \times 10^{-15} \text{ N})(1.03 \times 10^{-9} \text{ s})}{9.11 \times 10^{-31} \text{ kg}} \\
&= 2.42 \times 10^{6} \text{ m/s}
\end{aligned}$$

To get Δy, use the mean velocity.

$$\begin{aligned}
\mathrm{v}_{avg,y} &= \frac{(\mathrm{v}_{f,y} + \mathrm{v}_{i,y})}{2} \\
&= \frac{\mathrm{v}_{f,y}}{2} \\
&= \frac{2.42 \times 10^{6} \text{ m/s}}{2} \\
&= 1.21 \times 10^{6} \text{ m/s}
\end{aligned}$$

$\mathrm{v}_{avg,y} = \frac{\Delta y}{\Delta t}$, so

$$\begin{aligned}
\Delta y &= \mathrm{v}_{avg,y}\Delta t \\
&= (1.21 \times 10^{6} \text{ m/s})(1.03 \times 10^{-9} \text{ s}) \\
&= 0.00125 \text{ m}
\end{aligned}$$

3. After leaving the deflection plates, the velocity of the electron is constant. Find the time interval to hit the screen.

$$\begin{aligned} \mathrm{v}_x &= \frac{\Delta x}{\Delta t} \\ \Delta t &= \frac{\Delta x}{\mathrm{v}_x} \\ &= \frac{0.3\ \mathrm{m}}{7.74 \times 10^{7}\ \mathrm{m/s}} \\ &= 3.88 \times 10^{-9}\ \mathrm{s} \end{aligned}$$

$$\begin{aligned} \mathrm{v}_y &= \frac{\Delta y}{\Delta t} \\ \Delta y &= \mathrm{v}_y \Delta t \\ &= (2.42 \times 10^{6}\ \mathrm{m/s})(3.88 \times 10^{-9}\ \mathrm{s}) \\ &= 0.00939\ \mathrm{m} \end{aligned}$$

The total deflection is the deflection while traveling through the plates plus the y-displacement after leaving the deflection plates.

$$\begin{aligned} \Delta y &= \Delta y_1 + \Delta y_2 \\ &= 0.00125\ \mathrm{m} + 0.00939\ \mathrm{m} \\ &= 0.0106\ \mathrm{m} \end{aligned}$$

This is about 1 cm. Note that neglecting the y-displacement between the deflecting plates would have resulted in roughly 10% error.

P47:

Solution:

(a) Assume that the thickness of the central plate in negligible. If Δx_1 is the distance from A to the central plate, then

$$\begin{aligned}\Delta V_1 &= -\mathrm{E}_{1,x}\Delta x \\ &= -(725\ \frac{\mathrm{V}}{\mathrm{m}})(0.4\ \mathrm{m}) \\ &= -290\ \mathrm{V}\end{aligned}$$

If Δx_2 is from the plate to point B, then

$$\begin{aligned}\Delta V_2 &= -\mathrm{E}_{2,x}\Delta x_2 \\ &= -(-425\ \frac{\mathrm{V}}{\mathrm{m}})(0.2\ \mathrm{m}) \\ &= 85\ \mathrm{V}\end{aligned}$$

$$\begin{aligned}\Delta V &= \Delta V_1 + \Delta V_2 \\ &= -290\ \mathrm{V} + 85\ \mathrm{V} \\ &= -205\ \mathrm{V}\end{aligned}$$

This is along a path from A to B, so $\vec{\mathrm{V}}_{\mathrm{B}} - \vec{\mathrm{V}}_{\mathrm{A}} = -205$ V.

(b)

$$\begin{aligned}\Delta V &= V_{\mathrm{A}} - V_{\mathrm{B}} \\ &= -(V_{\mathrm{B}} - V_{\mathrm{A}}) \\ &= -(-205\ \mathrm{V}) \\ &= 205\ \mathrm{V}\end{aligned}$$

(c)

$$
\begin{aligned}
\Delta E &= 0 \\
\Delta U + \Delta K &= 0 \\
q\Delta V + \Delta K &= 0 \\
(-e)\Delta V + \Delta K &= 0 \\
\Delta K &= e\Delta V \\
&= (1.6 \times 10^{-19}\ \mathrm{C})(-205\ \mathrm{V}) \\
&= -3.28 \times 10^{-17}\ \mathrm{J}
\end{aligned}
$$

(d) The electron must at least reach the center of the plate. After this, the force by the electric field will accelerate it to the right. The minimum kinetic energy at point A in order to just reach the center of the plate (with K=0 at the center of the plate) is

$$
\begin{aligned}
\Delta E &= 0 \\
\Delta U + \Delta K &= 0 \\
q\Delta V + \Delta K &= 0 \\
(-e)\Delta V + \Delta K &= 0 \\
\Delta K &= e\Delta V \\
&= (1.6 \times 10^{-19}\ \mathrm{C})(-290\ \mathrm{V}) \\
K_f - K_i &= -4.64 \times 10^{-17}\ \mathrm{J} \\
0 - K_i &= -4.64 \times 10^{-17}\ \mathrm{J} \\
K_i &= 4.64 \times 10^{-17}\ \mathrm{J} \\
&= 290\ \mathrm{eV}
\end{aligned}
$$

P54:

Solution:

(a) $\Delta K + \Delta U = 0$ since it is a closed system.

(b) Note that $\Delta V_{\text{AB}} = 0$, $\Delta V_{\text{CD}} = 0$, and $\Delta V_{\text{FG}} = 0$ since the conductors are in equilibrium. Thus,

$$\begin{aligned} \Delta V_{\text{AB}} &= \Delta V_{\text{BC}} + \Delta V_{\text{DF}} \\ V_{\text{B}} - V_{\text{A}} &= (V_{\text{C}} - V_{\text{B}}) + (V_{\text{F}} - V_{\text{D}}) \end{aligned}$$

If $\Delta K = -5.2 \times 10^{-18}$ J, then $\Delta U = 5.2 \times 10^{-18}$ J since $\Delta U + \Delta K = 0$ for a closed system.

(c)

$$\begin{aligned} \Delta U &= q\Delta V \\ \Delta V &= \frac{\Delta U}{q} \\ &= \frac{\Delta U}{(-e)} \\ &= \frac{5.2 \times 10^{-18}\ \text{J}}{-1.6 \times 10^{-19}\ \text{C}} \\ &= -32.5\ \text{V} \\ V_{\text{G}} - V_{\text{A}} &= -32.5\ \text{V} \end{aligned}$$

(d)

$$\begin{aligned} V_{\text{G}} - V_{\text{A}} &= (V_{\text{C}} - V_{\text{B}}) + (V_{\text{F}} - V_{\text{D}}) \\ -32.5\ \text{V} &= -16\ \text{V} + (V_{\text{F}} - V_{\text{D}}) \\ (V_{\text{F}} - V_{\text{D}}) &= -32.5\ \text{V} + 16\ \text{V} \\ &= -16.5\ \text{V} \end{aligned}$$

(e)

$$\begin{aligned} \Delta V &= -\text{E}_x \Delta x \\ \text{E}_x &= \frac{-\Delta V}{\Delta x} \\ &= -\frac{(-16.5\ \text{V})}{0.002\ \text{m}} \\ &= 8250\ \frac{\text{V}}{\text{m}} \end{aligned}$$

P63:

Solution:

Assume that parts A and B are sufficiently close to the rod and disk that "near field" approximations can be used. Assume no contribution to the E-field from polarized molecules in the glass and plastic.

$$\begin{aligned}\Delta V &= V_{B} - V_{A} \\ &= \Delta V_{rod} + \Delta V_{disk}\end{aligned}$$

$$\begin{aligned}\Delta V_{rod} &= -\int \vec{E}\cdot d\vec{l} \\ &= -\int_A^B E_r \, dr \\ &= -\int_A^B \frac{1}{4\pi\epsilon_0}\frac{2\frac{Q}{L}}{r}\, dr \\ &= -\frac{1}{4\pi\epsilon_0}\frac{2Q}{L}\ln r\Big|_A^B \\ &= -\frac{1}{4\pi\epsilon_0}\frac{2Q}{L}(\ln(d-h) - \ln d) \\ &= -\frac{1}{4\pi\epsilon_0}\frac{2Q}{L}(\ln\frac{d-h}{d}) \\ &= \frac{1}{4\pi\epsilon_0}\frac{2Q}{L}\ln\frac{d}{d-h}\end{aligned}$$

$$\begin{aligned}\Delta V_{disk} &= -E_y \Delta y \\ &\approx -\frac{\frac{-Q}{A}}{2\epsilon_0}h \\ &= \frac{\frac{Q}{A}}{2\epsilon_0}h\end{aligned}$$

The potential difference due to both the disk and the rod is

$$\begin{aligned}\Delta V &= V_{B} - V_{A} \\ &\approx \frac{1}{4\pi\epsilon_0}\frac{2Q}{L}\ln\left(\frac{d}{d-h}\right) + \frac{\frac{Q}{A}}{2\epsilon_0}h \\ &\approx \frac{1}{4\pi\epsilon_0}\frac{Q}{R}\ln\left(\frac{d}{d-h}\right) + \frac{\frac{Q}{A}}{2\epsilon_0}h\end{aligned}$$

Note that we assumed that $\vec{E}_{disk} \approx$ constant for points very close to the disk.

P83:

Solution:

Assume that the polarization of the plastic does not affect the E-field within regions 1-2 and 3-4 (the vacuum gaps). The E-field in these regions is solely due to the plates. Since the charge on the plates stays the same, then the voltage across the vacuum gaps remains the same. Since the E-field is constant in these regions, the potential is linear and you can use the ratio of the width of the region to the width of the plates. Thus,

$$\begin{aligned}
\Delta V_{12} &= \Delta V_0 \left(\frac{0.5\text{mm}}{2\text{mm}}\right) \\
&= \frac{1}{4}(1000\ \text{V}) \\
&= 250\ \text{V}
\end{aligned}$$

Since region 3-4 has the same width as 1-2, then $\Delta V_{34} = \Delta V_{12} = 250$ V.

Inside the plastic, $\Delta V = \Delta V_{vacuum}/K = \Delta V_{23,plates}/K$. As a result,

$$\begin{aligned}
\Delta V_{23} &= \frac{\Delta V_0 \left(\frac{1\ \text{mm}}{2\ \text{mm}}\right)}{K} \\
&= \frac{1/2(1000\ \text{V})}{5} \\
&= 100\ \text{V}
\end{aligned}$$

The total potential difference across the plates is

$$\begin{aligned}
\Delta V_{14} &= \Delta V_{12} + \Delta V_{23} + \Delta V_{34} \\
&= 250\ \text{V} + 100\ \text{V} + 250\ \text{V} \\
&= 600\ \text{V}
\end{aligned}$$

As expected, the potential difference decreased as a result of inserting the plastic.