Q7:

Solution:

$\Delta\vec{p}$ is in the direction of the net force on the puck. Thus, $\Delta\vec{p}$ is in the direction of arrow (e).

P12:

Solution:

This is a straightforward application of the momentum principle. Choose the system to consist of the proton; the only significant interaction with the system is due to the HCl molecule. Further, assume non-relativistic speed so we can use Newton's expression for momentum. Since the net force only has an x component, we should not expect the proton's y or z components of momentum, and thus velocity, to change.

$$\begin{aligned}
\Delta\vec{p}_{\text{sys}} &= \vec{F}_{\text{net,sys}}\Delta t \\
\vec{p}_{\text{sys,f}} - \vec{p}_{\text{sys,i}} &= \vec{F}_{\text{net,sys}}\Delta t \\
\vec{p}_{\text{sys,f}} &= \vec{p}_{\text{sys,i}} + \vec{F}_{\text{net,sys}}\Delta t \\
\vec{v}_{\text{sys,f}} &\approx \frac{m_{\text{sys}}\,\vec{v}_{\text{sys,i}} + \vec{F}_{\text{net,sys}}\Delta t}{m_{\text{sys}}} \\
\vec{v}_{\text{sys,f}} &\approx \frac{\left(1.6726\times10^{-27}\,\text{kg}\right)\left(\langle 3600, 600, 0\rangle\,\text{m/s}\right) + \left\langle -1.12\times10^{-11}, 0, 0\right\rangle\text{N}\cdot\left(3.4\times10^{-14}\,\text{s}\right)}{\left(1.6726\times10^{-27}\,\text{kg}\right)} \\
\vec{v}_{\text{sys,f}} &\approx \frac{\left\langle 5.64\times10^{-24}, 1.00\times10^{-24}, 0\right\rangle\text{kg}\cdot\text{m/s}}{1.6726\times10^{-27}\,\text{kg}} \\
\vec{v}_{\text{sys,f}} &\approx \langle 3370, 600, 0\rangle\,\text{m/s}
\end{aligned}$$

Note that the proton's position didn't enter into this calculation.

P14:

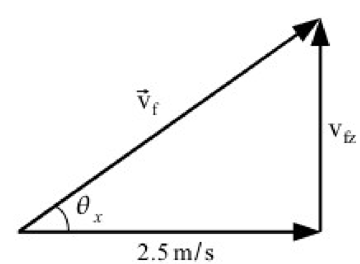
Solution:

(a) Image (B) is correct because the kick is brief and after the kick, $\vec{v}$ is constant so the block will move with a constant velocity.

(b) Only the y-component of $\vec{p}$ changes due to the kick because the kick is in the +y direction.

(c) $F_{net,x}$ is constant so $v_{fx} = v_{ix} = 2.5\,\text{m/s}$

Use a picture of $\vec{v}_{\text{f}}$ after the kick (shown in the figure below) to find v_{fz}.



$$\tan 22^\circ = \frac{v_{fz}}{v_{fx}}$$

$$v_{fz} = v_{fx} \tan 22^\circ$$

$$= (2.5\,\mathrm{m/s}) \tan 22^\circ$$

$$= 1.0\,\mathrm{m/s}$$

$$\vec{v}_f = \langle 2.5, 0, 1.0 \rangle\,\mathrm{m/s}$$

$$|\vec{v}| = \sqrt{(2.5)^2 + (0)^2 + (1.0)^2}\,\mathrm{m/s}$$

$$= 2.69\,\mathrm{m/s}$$

$$\widehat{v} = \frac{\vec{v}}{|\vec{v}|}$$

$$= \frac{\langle 2.5, 0, 1.0 \rangle\,\mathrm{m/s}}{2.69\,\mathrm{m/s}}$$

$$= \langle 0.929, 0, 0.372 \rangle$$

$\widehat{p}$ is the same as $\widehat{v}$, so

$$\widehat{p} = \langle 0.929, 0, 0.372 \rangle$$

(d)

$$p_{fx} = m v_{fx}$$

$$= (0.7\,\mathrm{kg})\,(2.5\,\mathrm{m/s})$$

$$= 1.75\,\mathrm{kg}\cdot\mathrm{m/s}$$

(e)

$$|\vec{p}| = m\,|\vec{v}|$$

$$= (0.7\,\mathrm{kg})\,(2.69\,\mathrm{m/s})$$

$$= 1.88\,\mathrm{kg}\cdot\mathrm{m/s}$$

(f)

$$p_{fz} = m v_{fz}$$

$$= (0.7\,\mathrm{kg})\,(1.0\,\mathrm{m/s})$$

$$= 0.7\,\mathrm{kg}\cdot\mathrm{m/s}$$

(g)

$$\vec{F}_{\text{net}} = \frac{\Delta \vec{p}}{\Delta t}$$

Choose the system to be the block of ice. The only component of the net force that is non-zero is the z-component.

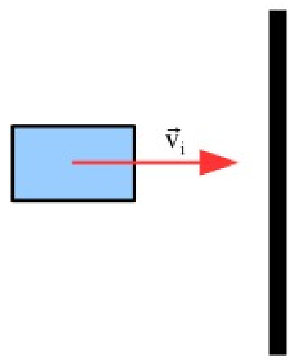
$$\begin{aligned}
F_{net,\,z} &= \frac{\Delta p_z}{\Delta t} \\
&= \frac{p_{fz} - p_{iz}}{\Delta t} \\
&= \frac{0.7\,\mathrm{kg\cdot m/s} - 0}{0.003\,\mathrm{s}} \\
&= 233\,\mathrm{N} \\
\left|\vec{F}_{\text{net}}\right| &= 233\,\mathrm{N}
\end{aligned}$$

This is called the average net force because we assumed that it is constant.

P17:

Solution:

Draw a sketch of the situation.



$$\begin{aligned}
m &= 2500\,\mathrm{kg} \\
v_{ix} &= 24\,\mathrm{m/s} \\
v_{fx} &= 0 \\
\Delta x &= 0.72\,\mathrm{m}
\end{aligned}$$

(a) Assuming constant $\vec{F}_{\text{net}}$ during the collision, then,

$$\begin{aligned}
v_{avg,\,x} &= \frac{v_{ix} + v_{fx}}{2} \\
&= \frac{24\,\mathrm{m/s} + 0}{2} \\
&= 12\,\mathrm{m/s}
\end{aligned}$$

(b)

$$\begin{aligned}
\Delta x &= v_{avg,\,x}\,\Delta t \\
\Delta t &= \frac{\Delta x}{v_{avg,\,x}} \\
&= \frac{0.72\,\mathrm{m}}{12\,\mathrm{m/s}} \\
&= 0.06\,\mathrm{s}
\end{aligned}$$

(c) Define the system to be the truck. The only significant force on the truck during the collision is the force by the wall. Apply the momentum principle.

$$\begin{aligned}
\vec{F}_{\text{net}} &= \frac{\Delta\vec{p}}{\Delta t} \\
\vec{F}_{\text{by wall}} &= \frac{\Delta\vec{p}}{\Delta t} \\
&= \frac{\vec{p}_{\text{f}} - \vec{p}_{\text{i}}}{\Delta t} \\
&= \frac{\langle 0,0,0\rangle - m\vec{v}_{\text{i}}}{\Delta t} \\
&= \frac{-\left(2500\,\text{kg}\right)\left(\langle 24,0,0\rangle\,\text{m/s}\right)}{0.06\,\text{s}} \\
&= -\left\langle 1.0\times 10^{6}, 0, 0\right\rangle \text{N} \\
\left|\vec{F}_{\text{by wall on truck}}\right| &= 1.0\times 10^{6}\,\text{N}
\end{aligned}$$

(d)

$$\begin{aligned}
\text{weight} &= mg \\
&= \left(2500\,\text{kg}\right)\left(9.8\,\text{N/kg}\right) \\
&= 2.45\times 10^{4}\,\text{N}
\end{aligned}$$

$$\begin{aligned}
\frac{\left|\vec{F}_{\text{by wall on truck}}\right|}{\left|\vec{F}_{\text{grav on truck}}\right|} &= \frac{1.0\times 10^{6}\,\text{N}}{2.45\times 10^{6}\,\text{N}} \\
&= 41
\end{aligned}$$

The wall exerts a force on the truck that is more than 40 times the weight of the truck.

(e) We assumed a constant force by the wall on the truck during the collision. This assumption allows us to calculate the average velocity of the truck using:

$$\vec{v}_{\text{avg}} = \frac{\vec{v}_{\text{i}} + \vec{v}_{\text{f}}}{2}$$

P23:

Solution:

The ball is rolling in the xz-plane . Before you kick it, its velocity and momentum are in the $+z$ direction. Your kick imparts a momentum component in the $-x$ direction, so that immediately after your kick the ball's velocity and momentum have two nonzero components. The net force on the ball is opposite the direction of the momentum. Assume time starts passing immediately after your kick and at that instant, the ball is at the origin. Carry out a step by step calculation as before.

At $t = 0\,\mathrm{s}$ we have $\vec{r}=\langle 0,0,0\rangle\,\mathrm{m}$, $\vec{v} = \langle -3.023, 0, 2.2\rangle\,\mathrm{m/s}$, $m = 0.43\,\mathrm{kg}$, $\vec{p} = \langle -1.3, 0, 0.946\rangle\,\mathrm{kg \cdot m/s}$, $\hat{p} = \langle -0.809, 0, 0.588\rangle$, and $\vec{F}_{\mathrm{net}} = \langle 0.202, 0, -0.147\rangle\,\mathrm{N}$.

With these initial conditions, the following VPython program does the calculations and thus at $t = 1.5\,\mathrm{s}$ we have $\vec{r} \approx \langle -3.83, 0, 2.79\rangle\,\mathrm{m}$.

```
from __future__ import division , print_function
from visual import *

m = 0.43
r = vector(0,0,0)
print ("r=",r)
v = vector(-3.023,0,2.2)
print ("v=",v)
p = m*v
print ("p=",p)
phat = norm(p)
print ("phat=",phat)
Fnet = -0.25 * phat
print ("Fnet=",Fnet)
dt = 0.5
t = 0
print ("t=",t,"r=",r,"v=",v)
while (t<01.5):
    p = p + Fnet * dt
    v = p/m
    r = r + (p/m) * dt
    t += dt
    print ("t=",t,"r=",r,"v=",v)
```

P30:

Solution:

(a) p_x starts at zero, briefly increases in the +x direction, and then steadily decreases to zero as the cart slows to a stop. The graph that depicts this motion is graph (2).

(b) p_x starts at zero and steadily increases until it quickly decreases when it is caught. This is shown in graph (4).

(c) While pushed, p_x briefly increases. After being released, p_x decreases to zero (as the cart slows to a stop) and then increases in the -x direction (i.e. becomes more negative) as it speeds up while traveling in the -x direction. This is shown in graph (1).

P31:

Solution:

(a) After releasing the cart, the force in the cart (friction) is constant and in the $-x$ direction. This is shown in graph (6). The frictional force on the cart becomes zero when the cart stops.

(b) The force by the air on the cart (resulting from the turning fan) is constant in the $+x$ direction. A large force by your hand on the cart is in the $-x$ direction when stopping the cart. This is shown in graph (7).

(c) After the initial push in the $+x$ direction, the force by air on the cart (due to the turning fan) is in the $-x$ direction. This is shown in graph (5).

P38:

Solution:

There are two regions to consider. (1) in between the plates where $\vec{F}_{net} = \langle 0, +F, 0\rangle$ and is constant. (2) after leaving the plates where $\vec{F}_{net} = 0$ and $\vec{v}$ is constant.

Calculate Δy in region (1) and Δy in region (2). Add them together for the total Δy. Define $y_i = 0$ to be the vertical position of the electron before going through the plates.

In region (1),

$$\begin{aligned}
p_{fy} &= p_{iy} + F_{net,\,y}\Delta t \\
v_{fy} &= v_{iy} + \frac{F_{net,\,y}}{m}\Delta t \\
&= v_{iy} + \frac{F}{m}\Delta t \\
&= 0 + \frac{F}{m}\Delta t
\end{aligned}$$

$$\begin{aligned}
v_{avg,\,y} &= \frac{v_{iy} + v_{fy}}{2} \\
y_f - y_i &= v_{avg,\,y}\Delta t \\
\Delta y &= \left(\frac{v_{iy} + v_{fy}}{2}\right)\Delta t \\
&= \frac{0 + \frac{F}{m}\Delta t}{2}\Delta t \\
&= \frac{1}{2}\frac{F}{m}\Delta t^2
\end{aligned}$$

To know Δt, use the x-motion. Since $F_{net,\,x} = 0$, v_x is constant and is v_0.

$$\begin{aligned}
\Delta x &= v_x \Delta t \\
d &= v_0 \delta t \\
\Delta t &= \frac{d}{v_0}
\end{aligned}$$

Substitute to get Δy for region (1).

$$\begin{aligned}
\Delta y &= \frac{1}{2}\frac{F}{m}\Delta t^2 \\
&= \frac{1}{2}\frac{F}{m}\left(\frac{d}{v_0}\right)^2
\end{aligned}$$

Now for region (2),

The velocity is constant and is

$$\vec{v} = \left\langle v, 0 \, . \, \frac{F}{m}\Delta t \right\rangle 0$$

where $\Delta t = \frac{d}{v_0}$ from region (1). Thus,

$$\begin{aligned}
\vec{v} &= \left\langle v, 0 \, . \, \frac{F}{m}\frac{d}{v_0} \right\rangle 0 \\
\Delta y &= v_{avg,\, y}\, \Delta t \\
&= \frac{F}{m}\frac{d}{v_0}\Delta t
\end{aligned}$$

Where Δt is the time interval in region (2).

To get Δt in region (2), use the x-motion. Note that $v_x = v_0$ throughout the motion since $F_{net,\, x} = 0$.

$$\begin{aligned}
\Delta x &= v_{avg,\, x}\, \Delta t \\
L &= v_0 \Delta t \\
\Delta t &= \frac{L}{v_0}
\end{aligned}$$

Substitute into the expression for Δy.

$$\begin{aligned}
\Delta y &= \left(\frac{F}{m}\frac{d}{v_0}\right)\left(\frac{L}{v_0}\right) \\
\Delta y &= \frac{F}{m}\frac{dL}{v_0^2}
\end{aligned}$$

The total y-displacement across regions (1) and (2) is

$$\begin{aligned}
\Delta y_{\text{total}} &= \frac{1}{2}\frac{F}{m}\left(\frac{d}{v_0}\right)^2 + \frac{F}{m}\frac{dL}{v_0^2} \\
&= \frac{F}{mv_0^2}\left(\frac{1}{2}d^2 + dL\right)
\end{aligned}$$

Notice that the dimensions on each term are the same.

$$\frac{\text{force}}{\text{mass}}\frac{\text{distance}^2}{\text{speed}^2}$$

Notice that the dimensions on each term are the same.

$$\frac{\text{force}}{\text{mass}}\frac{\text{distance}^2}{\text{speed}^2}$$

So that gives us some confidence. Now, check that the units result in meters.

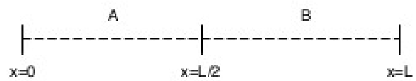
$$\frac{\text{N}}{\text{kg}}\frac{\text{m}^2}{\frac{\text{m}^2}{\text{s}^2}}=\frac{\cancel{\text{kg}}\frac{\text{m}}{\cancel{\text{s}^2}}\cancel{\text{m}^2}}{\cancel{\text{kg}}\frac{\cancel{\text{m}^2}}{\cancel{\text{s}^2}}}=\text{m}$$

So the units do give meters which is correct.

P39:

Solution:

First, sketch a picture of the test track. Let's call region A the first half of the track ($0 \le x < L/2$). Let's call region B the last half of the track ($L/2 < x \le L$). Let's call the length of the track L.



The cars have different accelerations, so let's call their x-accelerations a_{1x} and a_{2x} respectively. For constant net force, use these equations

$$
\begin{aligned}
v_{x,\mathrm{f}} &= v_{x,\mathrm{i}} + a_x \Delta t \\
\Delta x &= v_{x,\mathrm{i}} \Delta t + \frac{1}{2} a_x (\Delta t)^2
\end{aligned}
$$

Begin with Car 1. You have to analyze its motion during each region (A and B) separately since its acceleration is different in the two regions. In region B, $a_1 = 0$.

$$
\begin{aligned}
\text{Car 1, Region A:} \qquad \Delta x &= v_{x,\mathrm{i}} \Delta t + \frac{1}{2} a_x (\Delta t)^2 \\
&= \frac{1}{2} a_x (\Delta t)^2 \\
\frac{L}{2} &= \frac{1}{2} a_1 \Delta t_{\mathrm{A}} \\
\Delta t_{\mathrm{A}} &= \sqrt{\frac{L}{a_1}} \\
v_{x,\mathrm{f}} &= v_{x,\mathrm{i}} + a_x \Delta t \\
&= a_1 \sqrt{\frac{L}{a_1}} \\
v_{x,\mathrm{f}} &= \sqrt{a_1 L}
\end{aligned}
$$

$$
\begin{aligned}
\text{Car 1, Region B:} \qquad \Delta x &= v_x \Delta t \\
\frac{L}{2} &= \sqrt{a_1 L} \Delta t_{\mathrm{B}} \\
\Delta t_{\mathrm{B}} &= \frac{1}{2} \sqrt{\frac{L}{a_1}}
\end{aligned}
$$

Thus, the total time for Car 1 to travel down the track is $\Delta t = \sqrt{\frac{L}{a_1}} + \frac{1}{2}\sqrt{\frac{L}{a_1}} = \frac{3}{2}\sqrt{\frac{L}{a_1}}$.

Now analyze Car 2. It has constant acceleration for the entire length of the track. There's no need to analyze regions A and B separately.

$$
\begin{aligned}
\text{Car 2:} \qquad \Delta x &= v_{x,\text{i}} \Delta t + \frac{1}{2} a_x (\Delta t)^2 \\
&= \frac{1}{2} a_x (\Delta t)^2 \\
L &= \frac{1}{2} a_2 \Delta t^2 \\
\Delta t &= \sqrt{\frac{2L}{a_2}} \\
v_{x,\text{f}} &= v_{x,\text{i}} + a_x \Delta t \\
&= a_2 \sqrt{\frac{2L}{a_2}} \\
v_{x,\text{f}} &= \sqrt{2 a_2 L}
\end{aligned}
$$

(a) Using these results we can answer the questions. First find the average speed of each car. Use the conventional definition of average speed:

$$
\text{average speed} = \frac{\text{distance traveled}}{\text{time elapsed}}
$$

Note that this does not generally give the same result as $\left|\vec{v}_{\text{avg}}\right| = \left|\frac{\vec{v}_\text{i} + \vec{v}_\text{f}}{2}\right|$, except in the case of constant acceleration (such as Car 2).

Both cars travel the distance L in the time Δt. Thus, they have the same average speed, and the ratio of their average speeds is 1.

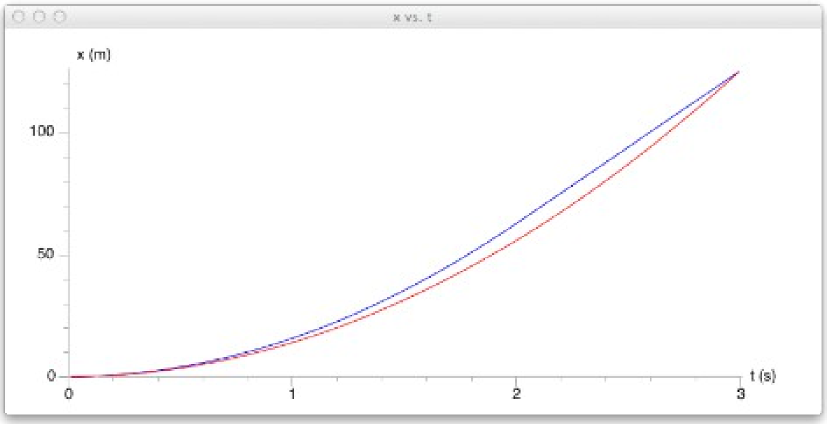
(b) We have already solved for the total time traveled by each car in terms of the distance L and their accelerations. They reach the end of the track in the same time. Set their times equal to each other and solve for the ratio of accelerations.

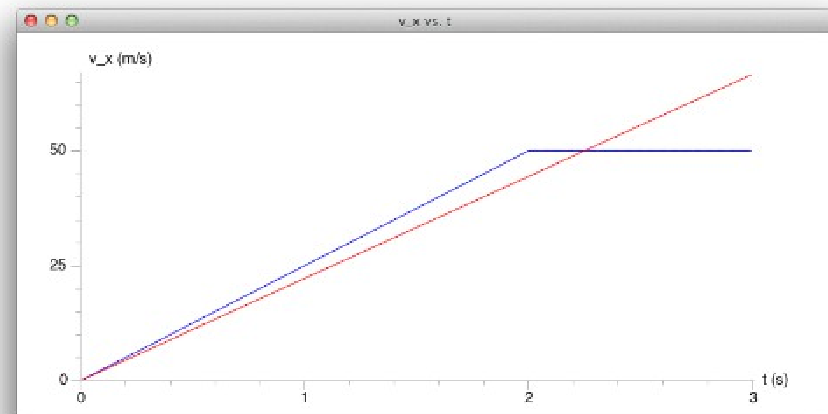
$$
\begin{aligned}
\Delta t_1 &= \Delta t_2 \\
\frac{3}{2}\sqrt{\frac{L}{a_1}} &= \sqrt{\frac{2L}{a_2}} \\
\frac{9}{4}\frac{L}{a_1} &= \frac{2L}{a_2} \\
\frac{a_1}{a_2} &= \frac{9}{8}
\end{aligned}
$$

(c) The ratio of their final x-velocities is

$$
\begin{aligned}
\frac{v_{x,\text{f1}}}{v_{x,\text{f2}}} &= \frac{\sqrt{a_1 L}}{\sqrt{2 a_2 L}} \\
&= \sqrt{\frac{a_1 L}{2 a_2 L}} = \sqrt{\frac{a_1}{2 a_2}} \\
&= \sqrt{\frac{9}{16}} = \frac{3}{4}
\end{aligned}
$$

It is nice to check our work with a VPython program. Using $L = 100\,\mathrm{m}$ and $a_1 = 25\,\mathrm{m/s^2}$, a VPython program produced the following graphs of $x(t)$ and $v_x(t)$ for the two cars. By knowing that $a_1 = \frac{9}{8}a_2$, you should be able to figure out which curves (on each graph) represent Car 1 and Car 2, respectively. From the graphs you should also be able to measure the total time to reach the end of the track and the final x-velocity of each cart. Then you can compare the values to what you calculate from the equations, using $L = 100\,\mathrm{m}$ and $a_1 = 25\,\mathrm{m/s^2}$.





P43:

Solution:

Begin with $\vec{r}$=$\langle 0.0.0798, 0\rangle$ m, $\vec{v}$= $\langle 0.0877, 0\rangle$ m/s, $\vec{p}$=$\langle 0, 0.0307, 0\rangle$ kg · m/s, $\Delta t = 0.1$ s. Actual calculations were done in Python.

Here's the first time step.

$$
\begin{aligned}
\vec{F}_{\text{net}} &= \left\langle 0, -mg - k_s\left(L - L_o\right), 0\right\rangle \\
&= \langle 0, -\left(0.350\,\text{kg}\right)\left(9.80\,\text{N/kg}\right) - \left(55\,\text{N/m}\right)\left(-0.1502\,\text{m}\right), 0\rangle \\
&= \langle 0, 4.831, 0\rangle\,\text{N} \\
\vec{p} &= \vec{p} + \vec{F}_{\text{net}}\Delta t \\
&= \langle 0, 0.5138, 0\rangle\,\text{kg}\cdot\text{m/s} \\
\vec{r} &= \vec{r} + \left(\frac{\vec{p}}{m}\right)\Delta t \\
&= \langle 0, 0.2266, 0\rangle\,\text{m}
\end{aligned}
$$

Here's the second time step, using final value from first time step as new initial values. Note that the force changed because the amount of stretch changed.

$$
\begin{aligned}
\vec{F}_{\text{net}} &= \left\langle 0, -mg - k_s\left(L - L_o\right), 0\right\rangle \\
&= \langle 0, -\left(0.350\,\text{kg}\right)\left(9.80\,\text{N/kg}\right) - \left(55\,\text{N/m}\right)\left(-0.0034\,\text{m}\right), 0\rangle \\
&= \langle 0, -3.243, 0\rangle\,\text{N} \\
\vec{p} &= \vec{p} + \vec{F}_{\text{net}}\Delta t \\
&= \langle 0, 0.1895, 0\rangle\,\text{kg}\cdot\text{m/s} \\
\vec{r} &= \vec{r} + \left(\frac{\vec{p}}{m}\right)\Delta t \\
&= \langle 0, 0.2807, 0\rangle\,\text{m}
\end{aligned}
$$