Q8:

Solution:

B, C, D, E, and F show evidence of an interaction. In the case of B, speed changes (and therefore velocity changes). In the case of C through F, direction of motion changes (and therefore so does velocity). In the cases of A, velocity is constant and therefore no net interaction is indicated.

P31:

Solution:

(a)

$$\begin{aligned}
\vec{v}_{\text{avg}} &= \frac{\Delta \vec{r}}{\Delta t} \\
&= \frac{\vec{r}_{\text{f}} - \vec{r}_{\text{i}}}{\Delta t} \\
&= \frac{\langle -0.202, 0.054, 0.098 \rangle \text{ m} - \langle 0.2, -0.05, 0.1 \rangle \text{ m}}{2 \times 10^{-6} \text{s}} \\
&= \frac{\langle -0.402, 0.104, -0.002 \rangle \text{ m}}{2 \times 10^{-6} \text{s}} \\
&= \left\langle -2.01 \times 10^{5}, 5.2 \times 10^{4}, -1 \times 10^{3} \right\rangle \text{ m/s}
\end{aligned}$$

(b) Average speed is not always equal to the magnitude of average velocity unless the motion is linear. We can proceed with this assumption.

$$\begin{aligned}
\left| \vec{v}_{\text{avg}} \right| &= \sqrt{\left(-2.01 \times 10^{5}\right)^{2} + \left(5.2 \times 10^{4}\right)^{2} + \left(-1 \times 10^{3}\right)^{2}} \text{ m/s} \\
&= 2.08 \times 10^{5} \text{m/s}
\end{aligned}$$

P33:

Solution:

$$\begin{aligned}
\text{At } t = 0: &\quad \vec{r}_{\rm i} = \langle 0,0,0\rangle \\
\Delta t_1 = 200\,\text{s}: &\quad \hat{v} = \langle 1,0,0\rangle \\
\Delta t_2 = 300\,\text{s}: &\quad \hat{v} = \langle \cos(45\,^\circ), 0, \cos(45\,^\circ)\rangle \\
\Delta t_3 = 150\,\text{s}: &\quad \hat{v} = \langle \cos(60\,^\circ), 0, \cos(30\,^\circ)\rangle \\
&\quad \vec{v} = |\vec{v}|\,\hat{v} = (2\,\text{m/s})\,\hat{v}
\end{aligned}$$

Use the position update equation for each time interval.

(a)

$$\begin{aligned}
\Delta t_1 = 200\,\text{s}: \vec{r}_{\rm f} &= \vec{r}_{\rm i} + \vec{v}_{\rm avg}\Delta t \\
&= \langle 0,0,0\rangle + (2\,\text{m/s})\langle 1,0,0\rangle (200\,\text{s}) \\
&= \langle 400,0,0\rangle\,\text{m}
\end{aligned}$$

$$\begin{aligned}
\Delta t_2 = 300\,\text{s}: \vec{r}_{\rm f} &= \vec{r}_{\rm i} + \vec{v}_{\rm avg}\Delta t \\
&= \langle 400,0,0\rangle\,\text{m} + (2\,\text{m/s})\langle \cos(45\,^\circ), 0, \cos(45\,^\circ)\rangle (300\,\text{s}) \\
&= \langle 824,0,424\rangle\,\text{m}
\end{aligned}$$

$$\begin{aligned}
\Delta t_3 = 150\,\text{s}: \vec{r}_{\rm f} &= \vec{r}_{\rm i} + \vec{v}_{\rm avg}\Delta t \\
&= \langle 824,0,424\rangle\,\text{m} + (2\,\text{m/s})\langle \cos(60\,^\circ), 0, \cos(30\,^\circ)\rangle (150\,\text{s}) \\
&= \langle 974,0,684\rangle\,\text{m}
\end{aligned}$$

(b) From $t = 6.3\,\text{s}$ to $7.3\,\text{s}$:

$$\begin{aligned}
\Delta t &= 7.3\,\text{s} - 6.3\,\text{s} \\
&= 1.0\,\text{s} \\
\vec{r}_{\rm i} &= \langle -3.5, 9.4, 0\rangle\,\text{m} \\
\vec{r}_{\rm f} &= \langle 0.5, 1.7, 0\rangle\,\text{m}
\end{aligned}$$

$$\begin{aligned}
\vec{v}_{\rm avg} &= \frac{\Delta\vec{r}}{\Delta t} \\
&= \frac{\langle 0.5, 1.7, 0\rangle\,\text{m} - \langle -3.5, 9.4, 0\rangle\,\text{m}}{1.0\,\text{s}} \\
&= \langle 4, -7.7, 0\rangle\,\text{m/s}
\end{aligned}$$

(c) The best estimate for $\vec{v}$ at $t = 6.3\,\text{s}$ is the average velocity during the smallest possible time interval that includes $t = 6.3\,\text{s}$. Thus, the time interval from $t = 6.3\,\text{s}$ to $6.8\,\text{s}$ gives the best possible estimate in this case for the instantaneous velocity at $t = 6.3\,\text{s}$.

(d) Assume that the bee's average velocity between $t = 6.3\,\text{s}$ and $6.33\,\text{s}$ is approximately constant. From $t = 6.3\,\text{s}$ to $6.33\,\text{s}$:

$$\begin{aligned}
\Delta t &= 6.33\,\text{s} - 6.3\,\text{s} \\
&= 0.03\,\text{s} \\
\vec{r}_{\rm i} &= \langle -3.5, 9.4, 0\rangle\,\text{m} \\
\vec{v}_{\rm avg} &\approx \langle 4.4, -6.4, 0\rangle\,\text{m/s}
\end{aligned}$$

$$\begin{aligned}
\vec{v}_{\rm avg} &= \frac{\Delta\vec{r}}{\Delta t} \\
\Delta\vec{r} &= \vec{v}_{\rm avg}\Delta t \\
&= (\langle 4.4, -6.4, 0\rangle\,\text{m/s})(0.03\,\text{s}) \\
&= \langle 0.132, -0.192, 0\rangle\,\text{m}
\end{aligned}$$

P37:

Solution:

(a) From $t = 6.3\,\mathrm{s}$ to $6.8\,\mathrm{s}$:

$$\begin{aligned}\Delta t &= 6.8\,\mathrm{s} - 6.3\,\mathrm{s}\\ &= 0.5\,\mathrm{s}\\ \vec{r}_{\mathrm{i}} &= \langle -3.5, 9.4, 0\rangle\,\mathrm{m}\\ \vec{r}_{\mathrm{f}} &= \langle -1.3, 6.2, 0\rangle\,\mathrm{m}\end{aligned}$$

$$\begin{aligned}\vec{v}_{\mathrm{avg}} &= \frac{\Delta\vec{r}}{\Delta t}\\ &= \frac{\langle -1.3, 6.2, 0\rangle\,\mathrm{m} - \langle -3.5, 9.4, 0\rangle\,\mathrm{m}}{0.5\,\mathrm{s}}\\ &= \frac{\langle 2.2, -3.2, 0\rangle\,\mathrm{m}}{0.5\,\mathrm{s}}\\ &= \langle 4.4, -6.4, 0\rangle\,\mathrm{m/s}\end{aligned}$$

(b) The total duration of time is $200\,\mathrm{s} + 300\,\mathrm{s} + 150\,\mathrm{s} = 650\,\mathrm{s}$.

$$\begin{aligned}\vec{v}_{\mathrm{avg}} &= \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_{\mathrm{f}} - \vec{r}_{\mathrm{i}}}{\Delta t}\\ &= \frac{\langle 974, 0, 684\rangle\,\mathrm{m} - \langle 0,0,0\rangle}{650\,\mathrm{s}}\\ &= \langle 1.50, 0, 1.05\rangle\,\mathrm{m/s}\end{aligned}$$

P41:

Solution:

$$\begin{aligned}\vec{r}_{\mathrm{i}} &= \langle 7, 21, -17\rangle\,\mathrm{m}\\ \Delta t &= 3\,\mathrm{s}\\ \vec{v}_{\mathrm{avg}} &= \langle -11, 42, 11\rangle\,\mathrm{m/s}\\ y_{\mathrm{f}} &= ?\end{aligned}$$

$$\begin{aligned}\vec{r}_{\mathrm{f}} &= \vec{r}_{\mathrm{i}} + \vec{v}_{\mathrm{avg}}\Delta t\\ &= \langle 7, 21, -17\rangle\,\mathrm{m} + (\langle -11, 42, 11\rangle\,\mathrm{m/s})\,(3\,\mathrm{s})\\ &= \langle 7, 21, -17\rangle\,\mathrm{m} + \langle -33, 126, 33\rangle\,\mathrm{m}\\ &= \langle -26, 147, 16\rangle\,\mathrm{m}\end{aligned}$$

So $y_{\mathrm{f}} = 147\,\mathrm{m}$.

P43:

Solution:

(a)

$$\begin{aligned}\vec{v}_{\mathrm{avg\ AB}} &= \frac{\Delta\vec{r}}{\Delta t}\\ &= \frac{\vec{r}_{\mathrm{B}} - \vec{r}_{\mathrm{A}}}{\Delta t}\\ &= \frac{\langle 22.3, 26.1, 0\rangle\,\mathrm{m} - \langle 0,0,0\rangle}{1.0\,\mathrm{s} - 0.0\,\mathrm{s}}\\ &= \langle 22.3, 26.1, 0\rangle\,\mathrm{m/s}\end{aligned}$$

(b) From $t = 1.0\,\mathrm{s}$ to $t = 2.0\,\mathrm{s}$, assuming it travels with a constant velocity of $\langle 22.3, 26.1, 0\rangle\,\mathrm{m/s}$,

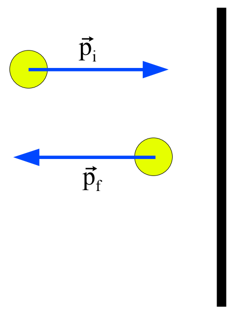
$$\begin{aligned}\vec{r}_{\mathrm{f}} &= \vec{r}_{\mathrm{i}} + \vec{v}_{\mathrm{avg}}\Delta t\\ &= \langle 22.3, 26.1, 0\rangle\,\mathrm{m} + (\langle 22.3, 26.1, 0\rangle\,\mathrm{m/s})\,(2.0\,\mathrm{s} - 1.0\,\mathrm{s})\\ &= \langle 22.3, 26.1, 0\rangle\,\mathrm{m} + \langle 22.3, 26.1, 0\rangle\,\mathrm{m}\\ &= \langle 44.6, 52.2, 0\rangle\,\mathrm{m}\end{aligned}$$

(c) $\vec{r}$ at point C is $\langle 40.1, 38.1, 0\rangle$ m which is not the same as what we predicted. We assumed constant velocity when making our prediction; however, in reality the velocity was not constant, but was decreasing in both the x and y directions. An approximation of constant velocity is only valid for small time intervals. For this projectile, $\Delta t = 1.0\,\mathrm{s}$ was not a small enough time interval to reasonably assume constant velocity.

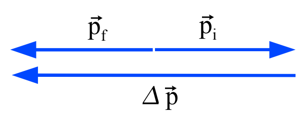
P51:

Solution:

(a) Draw a sketch of the situation, like the one shown in the figure below.



Sketch the change in momentum vector by drawing the initial and final momentum vectors tail to tail and drawing the change in momentum from the head of the initial momentum to the head of the final momentum, as shown in the figure below.



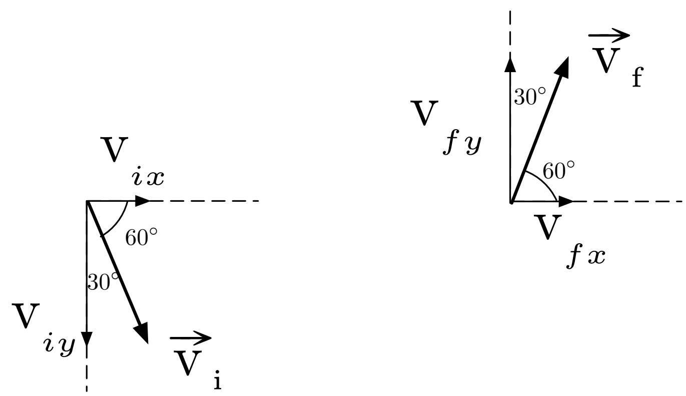
$$
\begin{aligned}
\vec{v}_{\mathrm{i}} &= \langle v_x, 0, 0\rangle \\
\vec{v}_{\mathrm{f}} &= \langle -v_x, 0, 0\rangle \\
\Delta\vec{p} &= \vec{p}_{\mathrm{f}} - \vec{p}_{\mathrm{i}} \\
&= m\left(\vec{v}_{\mathrm{f}} - \vec{v}_{\mathrm{i}}\right) \\
&= m\left(\langle -v_x, 0, 0\rangle - \langle v_x, 0, 0\rangle\right) \\
&= m\langle -2v_x, 0, 0\rangle \\
&= \langle -2mv_x, 0, 0\rangle
\end{aligned}
$$

The change in momentum is in the $-x$ direction which is consistent with the picture.

P53:

Solution:

It helps to sketch the velocity vector for the basketball before and after it hits the floor.



The angle of the vector with the $+y$ axis is $30\,°$, and the angle with the $+x$ axis is $60\,°$. The vector's components can be easily calculated using the cosine of each of these angles. Thus

P56:

Solution:

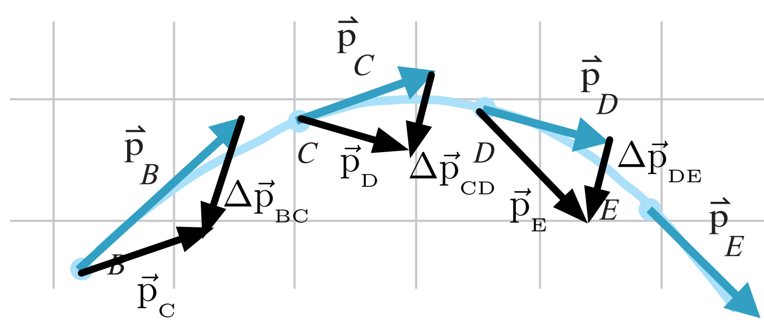
(a)

$$\begin{aligned}
\Delta\vec{p}_{\text{BC}} &= \vec{p}_{\text{C}} - \vec{p}_{\text{B}} \\
&= \langle 2.55, 0.97, 0\rangle\ \text{kg}\cdot\text{m/s} - \langle 3.03, 2.83, 0\rangle\ \text{kg}\cdot\text{m/s} \\
&= \langle -0.48, -1.86, 0\rangle\ \text{kg}\cdot\text{m/s} \\
\Delta\vec{p}_{\text{CD}} &= \vec{p}_{\text{D}} - \vec{p}_{\text{C}} \\
&= \langle 2.24, -0.57, 0\rangle\ \text{kg}\cdot\text{m/s} - \langle 2.55, 0.97, 0\rangle\ \text{kg}\cdot\text{m/s} \\
&= \langle -0.31, -1.54, 0\rangle\ \text{kg}\cdot\text{m/s}
\end{aligned}$$

$$\begin{aligned}
\Delta\vec{p}_{\text{DE}} &= \vec{p}_{\text{E}} - \vec{p}_{\text{D}} \\
&= \langle 1.97, -1.93, 0\rangle\ \text{kg}\cdot\text{m/s} - \langle 2.24, -0.57, 0\rangle\ \text{kg}\cdot\text{m/s} \\
&= \langle -0.27, -1.36, 0\rangle\ \text{kg}\cdot\text{m/s}
\end{aligned}$$

$$\begin{aligned}
\Delta\vec{p}_{\text{EF}} &= \vec{p}_{\text{F}} - \vec{p}_{\text{E}} \\
&= \langle 1.68, -3.04, 0\rangle\ \text{kg}\cdot\text{m/s} - \langle 1.97, -1.93, 0\rangle\ \text{kg}\cdot\text{m/s} \\
&= \langle -0.29, -1.11, 0\rangle\ \text{kg}\cdot\text{m/s}
\end{aligned}$$

(b) To sketch $\Delta\vec{p}_{\text{BC}}$, sketch $\vec{p}_{\text{C}}$ tail-to-tail at the location of $\vec{p}_{\text{B}}$ and sketch $\Delta\vec{p}_{\text{BC}}$ from the head of $\vec{p}_{\text{C}}$ to the head of $\vec{p}_{\text{B}}$. Do this for each of the other vectors as well. The results are shown in the figure below.



(c) $|\Delta\vec{p}_{\text{BC}}|$ is greatest because both Δp_x and Δp_y are greatest (in magnitude) for the interval from B to C.

P60:

Solution:

$$
\begin{aligned}
m &= 400\,\text{kg}\\
\vec{r}_{\text{i}} &= \left\langle 0, 3\times 10^{4}, -6\times 10^{4}\right\rangle \text{m}\\
\vec{p} &= \left\langle 6\times 10^{3}, 0, -3.6\times 10^{3}\right\rangle \text{kg}\cdot\text{m/s}\\
\Delta t &= 2\,\text{min} = 120\,\text{s}\\
\vec{r}_{\text{f}} &= ?
\end{aligned}
$$

Since $|\vec{v}| << c$, then $\vec{p} \approx m\vec{v}$.

$$
\begin{aligned}
\vec{v} &\approx \frac{\vec{p}}{m} = \frac{\left\langle 6\times 10^{3}, 0, -3.6\times 10^{3}\right\rangle \text{kg}\cdot\text{m/s}}{400\,\text{kg}}\\
&\approx \langle 15, 0, -9\rangle\,\text{m/s}
\end{aligned}
$$

To find the final position, use the position update equation.

$$
\begin{aligned}
\vec{r}_{\text{f}} &= \vec{r}_{\text{i}} + \vec{v}\Delta t\\
&= \left\langle 0, 3\times 10^{4}, -6\times 10^{4}\right\rangle \text{m} + \left(\langle 15, 0, -9\rangle\,\text{m/s}\right)(120\,\text{s})\\
&= \left\langle 1.08\times 10^{3}, 3\times 10^{4}, -6.11\times 10^{4}\right\rangle \text{m}
\end{aligned}
$$

P63:

Solution:

$$
\begin{aligned}
m_{\text{proton}} &= 1.67\times 10^{-27}\text{kg}\\
|\vec{p}| &= \gamma m|\vec{v}|\\
&= \frac{m|\vec{v}|}{\sqrt{1-\frac{|\vec{v}|^2}{c^2}}}\\
&= \frac{\left(1.67\times 10^{-27}\text{kg}\right)(0.9999)\left(3\times 10^{8}\text{m/s}\right)}{\sqrt{1-(0.9999)^2}}\\
&= 3.61\times 10^{-17}\ \text{Kg}\cdot\text{m/s}
\end{aligned}
$$

P66:

Solution:

$$\begin{aligned} m_{\text{electron}} &= 9 \times 10^{-31} \text{kg} \\ |\vec{v}| &= 0.996c \\ \hat{v} &= \langle -.655, -0.492, -0.573 \rangle \end{aligned}$$

(a)

$$\begin{aligned} \gamma &= \frac{1}{\sqrt{1 - \frac{|\vec{v}|^2}{c^2}}} \\ &= \frac{1}{1 - (0.996)^2} \\ &= 11.2 \end{aligned}$$

(b)

$$\begin{aligned} |\vec{v}| &= 0.996c \\ &= (0.996)\left(3 \times 10^{8} \text{m/s}\right) \\ &= 2.988 \times 10^{8} \text{m/s} \end{aligned}$$

(c)

$$\begin{aligned} |\vec{p}| &= \gamma m |\vec{v}| \\ &= (11.2)\left(9 \times 10^{-31} \text{kg}\right)\left(2.988 \times 10^{8} \text{m/s}\right) \\ &= 3.0 \times 10^{-21} \text{kg} \cdot \text{m/s} \end{aligned}$$