

Chapter 5

P08:

Solution:

- (a) $\frac{d\vec{p}}{dt}$ is zero.
- (b) Earth, cable, air
- (c) force due to cable pointing to the upper right, force due to air pointing left, force due to Earth pointing down
- (d) The vertical component of the tension must null out the package's weight. Therefore, $|\vec{F}_{\text{cable}}| \approx 9530 \text{ N}$.
- (e) $\vec{F}_{\text{cable}} = \langle 3609, 8820, 0 \rangle \text{ N}$
- (f) The force due to the air must null out the horizontal component of the tension. Therefore, $|\vec{F}_{\text{air}}| \approx -3609 \text{ N}$.
- (g) $\vec{F}_{\text{air}} = \langle -3609, 0, 0 \rangle \text{ N}$
- (h) Yes, the cable will likely break.

P11:

Solution:

Let the left wire be 1, the right wire be 2, and the vertical wire be 3.

- (a) $|\vec{F}_1| \approx 6098 \text{ N}$, $|\vec{F}_2| \approx 3980 \text{ N}$, $|\vec{F}_3| \approx 7840 \text{ N}$
- (b) For each wire, the strain will just be the stress divided by Young's modulus. Wire 1: $\frac{\Delta L}{L_o} \approx 0.049$, Wire 2: $\frac{\Delta L}{L_o} \approx 0.032$, Wire 3: $\frac{\Delta L}{L_o} \approx 0.063$

P14:

Solution:

This problem requires careful analysis of the forces acting on each block. Treat each block as the system.

- (a) Consider only the top block, block 2. The forces on block 2 are Earth's downward gravitational pull, the upward normal force from block 1, and the friction force from block 1, which is directed to the right. Note that the applied force F does NOT act on block 2. There is no vertical acceleration, so the normal force and the gravitational force have the same magnitude.

$$\begin{aligned}
 F_{\text{net},x} &= f_{21,x} = m_2 a_{2,x} \\
 f_{21,x} &= \mu_{k21} N_{2,y} = \mu_{k21} m_2 g \\
 \therefore a_{2,x} &= \mu_{k21} g \\
 &= (0.2)(9.8 \text{ m/s}^2) = 1.96 \text{ m/s}^2
 \end{aligned}$$

- (b) Consider only the bottom block, block 1. The forces on block 1 are Earth's downward gravitational pull, the downward force from block 2, the upward normal force from the floor, the friction force from block 2, and the friction force from the floor. The two friction forces point to the left. The downward forces are just the blocks' weights.

$$\begin{aligned}
 F_{\text{net},x} &= F - f_{1\text{floor},x} - f_{12,x} = m_1 a_{1,x} \\
 &= F - \mu_{k1\text{floor}} N_{1,y} - \mu_{k21} N_{2,y} \\
 &= F - \mu_{k1\text{floor}} (m_1 + m_2)g - \mu_{k21} m_2 g \\
 \therefore a_{1,x} &= \frac{F - \mu_{k1} m_1 g - (\mu_{k1} + \mu_{k2})m_2 g}{m_1} \\
 &= \frac{(350 \text{ N}) - (0.7)(35 \text{ kg})(9.8 \text{ m/s}^2) - (0.9)(10 \text{ kg})(9.8 \text{ m/s}^2)}{35 \text{ kg}} \\
 &= 0.62 \text{ m/s}^2
 \end{aligned}$$

(c) Now consider the system consisting of both blocks.

$$\begin{aligned}
 F_{\text{net},x} &= F - f_{\text{floor},x} - f_{12,x} = m_1 a_{1,x} \\
 &= F - \mu_{k1\text{floor}} N_{1,y} - \mu_{k21} N_{2,y} \\
 &= F - \mu_{k1\text{floor}} (m_1 + m_2)g - \mu_{k21} m_2 g \\
 \therefore a_{1,x} &= \frac{F - \mu_{k1} (m_1 + m_2)g}{(m_1 + m_2)} \\
 &= \frac{(350 \text{ N}) - (0.7)(45 \text{ kg})(9.8 \text{ m/s}^2)}{45 \text{ kg}} \\
 &= 0.92 \text{ m/s}^2
 \end{aligned}$$

For the upper block to not slip, the maximum static friction force on it must not be exceeded. At the boundary condition, just before the block slips, its acceleration is the same as the lower block's acceleration.

$$F_{\text{net},x} \leq \mu_s m_2 g$$

P20:

Solution:

$$\begin{aligned}
 \vec{p} &= \langle -2.6 \times 10^{29}, -1.0 \times 10^{29}, 0 \rangle \text{ kg} \cdot \text{m/s} \\
 \vec{F} &= \langle -2.5 \times 10^{22}, -1.4 \times 10^{23}, 0 \rangle \text{ N} \\
 \vec{F}_{\parallel} &= (\vec{F} \cdot \hat{p}) \hat{p} \\
 &= \langle -6.87 \times 10^{22}, -2.64 \times 10^{22}, 0 \rangle \text{ N} \\
 \vec{F}_{\perp} &= \vec{F} - \vec{F}_{\parallel} \\
 &= \langle 4.37 \times 10^{22}, -1.14 \times 10^{23}, 0 \rangle \text{ N}
 \end{aligned}$$

The following VPython script was used for this problem.

```

from __future__ import division, print_function
from visual import *

p = vector(-2.6e29, -1.0e29, 0)
F = vector(-2.5e22, -1.4e23, 0)
phat = norm(p)
print(phat)

Fparallel = dot(F, phat)*phat
print(Fparallel)

Fperp = F - Fparallel
print(Fperp)

```

P24:

Solution:

(a) Because the proton's speed is constant, $\frac{d|\vec{p}|}{dt} \hat{p} = 0$.

(b)

$$\begin{aligned}
 \frac{d\vec{p}}{dt} &= |\vec{p}| \frac{d\hat{p}}{dt} \\
 \left| \frac{d\vec{p}}{dt} \right| &= |\vec{p}| \frac{|\vec{v}|}{R} \\
 &= \frac{m |\vec{v}|^2}{R} \text{ since the proton's speed is much less than } c. \\
 &= 1.04 \times 10^{-14} \text{ N}
 \end{aligned}$$

The direction of $|\vec{p}| \frac{d\hat{p}}{dt}$ is toward the center of the circle, which is arrow (h).

P35:

Solution:

The net force on the roller coaster at any instant is

$$\vec{\mathbf{F}}_{\text{net}} = \vec{\mathbf{F}}_{\text{track}} + \vec{\mathbf{F}}_{\text{grav}}$$

Since the roller coaster travels in a circle with constant speed,

$$\begin{aligned} \left| \vec{\mathbf{F}}_{\text{net}} \right| &= |\vec{p}| \frac{|\vec{v}|}{R} \\ &= \frac{m |\vec{v}|^2}{R} \text{ since the speed is much less than } c. \end{aligned}$$

At the top of the roller coaster, the net force on the roller coaster is toward the center of the circle which is in the downward ($-y$) direction. The gravitational force on the roller coaster is also in the downward ($-y$) direction. At the minimum speed needed to make it around the loop, $\vec{\mathbf{F}}_{\text{track}} = \vec{0}$ at the top of the loop. Thus,

$$\begin{aligned} \vec{\mathbf{F}}_{\text{net}} &= \vec{\mathbf{F}}_{\text{grav}} \\ \left| \vec{\mathbf{F}}_{\text{net}} \right| &= \left| \vec{\mathbf{F}}_{\text{grav}} \right| \\ \frac{m |\vec{v}|^2}{R} &= mg \\ |\vec{v}| &= \sqrt{Rg} \end{aligned}$$

A reasonable R is on the order of 10 m which gives a minimum speed of about 10 m/s.

P43:

Solution:

(a)

$$T = \frac{6.88 \text{ s}}{10 \text{ revolutions}} = 0.688 \text{ s}$$

$$\begin{aligned} |\vec{v}| &= \frac{2\pi R}{T} \\ &= 13.7 \text{ m/s} \end{aligned}$$

(b) Yes, the momentum vector changes because its direction changes. It is not moving in a straight line.

(c) The force by the spring on the ball changes its direction because this force is perpendicular to the ball's path.

$$(d) \left| \vec{\mathbf{F}}_{\text{spring}} \right| = k_s = 300 \text{ N}$$

(e)

$$\begin{aligned} \left| \vec{\mathbf{F}}_{\text{net}} \right| &= \frac{m |\vec{v}|^2}{R} \\ &= 300 \text{ N} \end{aligned}$$

Solving for mass gives $m = 32.8 \text{ kg}$.

P50:**Solution:**

- (a) The force by the seat on the rider can be upward (+y direction) or downward (−y direction) depending on how fast the roller coaster goes over the hill. The gravitational force by Earth on the rider is downward (−y direction). At the top of the hill, the net force on the rider is in the downward (−y direction), toward the center of the circle. To feel weightless, the force by the seat on the rider must be zero at the top of the hill.

$$\begin{aligned}
 \vec{F}_{\text{net}} &= \vec{F}_{\text{seat}} + \vec{F}_{\text{grav}} \\
 \left\langle 0, \frac{-m|\vec{v}|^2}{R}, 0 \right\rangle &= \langle 0, -mg, 0 \rangle \\
 \frac{m|\vec{v}|^2}{R} &= mg \\
 |\vec{v}| &= \sqrt{gR}
 \end{aligned}$$

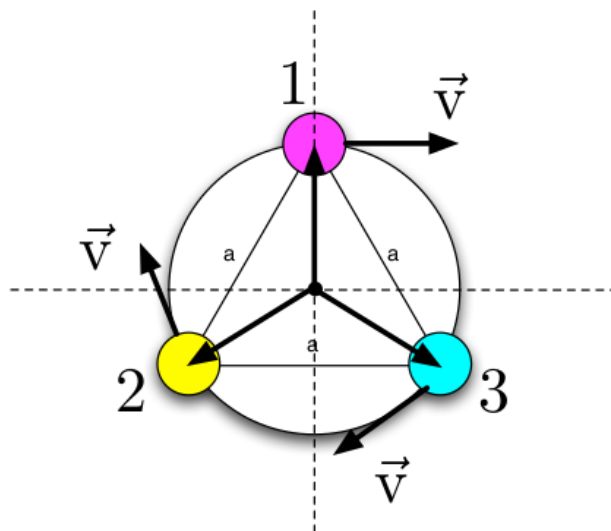
- (b) At the bottom of a hill (or a “dip”), the net force on the rider is upward. The force by the seat on the rider is also upward and in this case has a magnitude of $3mg$. The net force on the rider is

$$\begin{aligned}
 \vec{F}_{\text{net}} &= \vec{F}_{\text{seat}} + \vec{F}_{\text{grav}} \\
 \left\langle 0, \frac{m|\vec{v}|^2}{R}, 0 \right\rangle &= \langle 0, 3mg, 0 \rangle + \langle 0, -mg, 0 \rangle \\
 \frac{m|\vec{v}|^2}{R} &= 2mg \\
 |\vec{v}| &= \sqrt{2gR}
 \end{aligned}$$

P54:

Solution:

A picture of the situation is shown in the figure below. The vectors from the center of the circle to the stars are their position vectors.



The stars are equidistant from another and form an equilateral triangle of side length a . The stars move in uniform circular motion with speed v (that is much less than the speed of light) and radius r from the center of the circle. Therefore, the net force on each star has a magnitude

$$|\vec{F}_{\text{net}}| = \frac{m |\vec{v}|^2}{r}$$

and is directed toward the center of the circle. Select star 2 to be the “system”. Apply the Momentum Principle to star 2. We will need to calculate the gravitational force of each of the other stars on star 2; therefore, we need to know the positions of the stars at this instant. Define the $+x$ direction to the right and the $+y$ direction upward. Use direction cosines to get the unit vector for each star’s position (i.e. $\hat{r} = \langle \cos \theta_x, \cos \theta_y, 0 \rangle$). Note that because it is an equilateral triangle, the angles between the positions of the stars are 120° .

$$\begin{aligned} \vec{r}_1 &= r \langle \cos(90^\circ), \cos(0^\circ), 0 \rangle = r \langle 0, 1, 0 \rangle \\ \vec{r}_2 &= r \langle \cos(210^\circ), \cos(120^\circ), 0 \rangle = r \langle -0.866, -0.5, 0 \rangle \\ \vec{r}_3 &= r \langle \cos(330^\circ), \cos(240^\circ), 0 \rangle = r \langle 0.866, -0.5, 0 \rangle \end{aligned}$$

The vectors from star 2 to each of the other stars are:

$$\begin{aligned}
\vec{r}_{32} &= \vec{r}_3 - \vec{r}_2 \\
&= r < 1.723, 0, 0 > \\
&= \sqrt{3}r < 1, 0, 0 >
\end{aligned}$$

$$\begin{aligned}
\vec{r}_{12} &= \vec{r}_1 - \vec{r}_2 \\
&= r < 0, 1, 0 > - r < -0.866, -0.5, 0 > \\
&= r < 0.866, 1.5, 0 > \\
&= \sqrt{3}r < 0.5, 0.866, 0 >
\end{aligned}$$

Calculate the gravitational force by star 1 on star 2 and the gravitational force by star 3 on star 2. The net force on star 2 is the sum of these forces.

$$\begin{aligned}
\vec{F}_{32} &= \frac{Gmm}{|\vec{r}_{32}|} \hat{r}_{32} \\
&= \frac{Gmm}{3r^2} < 1, 0, 0 >
\end{aligned}$$

$$\begin{aligned}
\vec{F}_{12} &= \frac{Gmm}{|\vec{r}_{12}|} \hat{r}_{12} \\
&= \frac{Gmm}{3r^2} < 0.5, 0.866, 0 >
\end{aligned}$$

The net force on star 2 is

$$\begin{aligned}
\vec{F}_{\text{net}} &= \vec{F}_{32} + \vec{F}_{12} \\
&= \frac{Gmm}{3r^2} < 1, 0, 0 > + \frac{Gmm}{3r^2} < 0.5, 0.866, 0 > \\
&= \frac{Gmm}{3r^2} < 1.5, 0.866, 0 >
\end{aligned}$$

Its magnitude is:

$$\begin{aligned}
|\vec{F}_{\text{net}}| &= \frac{Gmm}{3r^2} \sqrt{(1.5)^2 + (0.866)^2 + (0)^2} \\
&= \frac{Gmm}{3r^2} \sqrt{(1.5)^2 + (0.866)^2 + (0)^2} \\
&= \frac{Gmm}{3r^2} \sqrt{3} \\
&= \frac{Gmm}{\sqrt{3}r^2}
\end{aligned}$$

According to the Momentum Principle, the net force is equal to $d\vec{p}/dt$ which has a magnitude of v^2/r . Thus,

$$\begin{aligned}\frac{m |\vec{v}|^2}{r} &= \frac{Gmm}{\sqrt{3}r^2} \\ \cancel{m} \frac{|\vec{v}|^2}{r} &= \frac{G\cancel{m}m}{\sqrt{3}r^2} \\ |\vec{v}|^2 &= \frac{Gm}{\sqrt{3}r}\end{aligned}$$

The speed of a star in uniform circular motion is $v = 2\pi r/T$. Substitute this for the speed and solve for the period of the star.

$$\begin{aligned}\frac{4\pi^2 r^2}{T^2} &= \frac{Gm}{\sqrt{3}r} \\ T^2 &= \sqrt{3} \frac{4\pi^2}{GM} r^3\end{aligned}$$

Note that this looks like Kepler's law for a star orbiting a central object of mass m except for the constant $\sqrt{3}$. The additional gravitational force due to the second star in the three-body problem increases the period compared to a star orbiting a central body in the two-body problem.

The period of the star will be

$$T = \left(\sqrt{3} \frac{4\pi^2 r^3}{GM} \right)^{1/2}$$

Chapter 6

Q07:

Solution:

To be associated with a potential energy function, the work done by a force around a path from A to B must be path independent. (There are other criteria too, but if a force fails this test, you know that it has no potential energy function.) Chose point A to be at the top of the circle. Choose point B at the bottom of the circle. If you select a path clockwise from A to B, then

$$\begin{aligned} \text{clockwise; } W &= \int_i^f \vec{F} \cdot d\vec{r} \\ &= +\pi R F \end{aligned}$$

where R is the radius and $\vec{F} \cdot d\vec{r} = +Fdr$ since the force is in the same direction as a small displacement $d\vec{r}$. If you select a path counterclockwise from A to B, then

$$\begin{aligned} \text{counterclockwise; } W &= \int_i^f \vec{F} \cdot d\vec{r} \\ &= -\pi R F \end{aligned}$$

where R is the radius and $\vec{F} \cdot d\vec{r} = -Fdr$ since the force is in the opposite direction as a small displacement $d\vec{r}$.

Because the integral depends on the path, then the force does not have an associated potential energy function.

P29:

Solution:

$$\begin{aligned} W_1 &\approx \langle 30, 0, 0 \rangle \text{ N} \bullet \langle 2.3, 0, 0 \rangle \text{ m} \approx 69 \text{ J} \\ W_2 &\approx \langle 15, 0, 0 \rangle \text{ N} \bullet \langle 8, 0, 0 \rangle \text{ m} \approx 120 \text{ J} \\ W_{\text{net}} &= W_1 + W_2 \approx 189 \text{ J} \end{aligned}$$

P33:

Solution:

$$\begin{aligned} W_1 &\approx \langle 250, 400, -170 \rangle \text{ N} \bullet \langle 6, 7, -4 \rangle \text{ m} \approx 4980 \text{ J} \\ W_2 &\approx \langle 140, 250, 150 \rangle \text{ N} \bullet \langle 4, -7, 5 \rangle \text{ m} \approx -440 \text{ J} \\ W &= W_1 + W_2 \approx 4540 \text{ J} \\ K_f &= K_i + W \\ |\vec{v}_f| &= \sqrt{|\vec{v}_i|^2 + \frac{2W}{m}} \\ &\approx \sqrt{(3.5 \text{ m/s})^2 + \frac{2(4540 \text{ J})}{100 \text{ kg}}} \\ &\approx 10.2 \text{ m/s} \end{aligned}$$

P41:

Solution:

Steps 1, 2, 5, and 6 must be included, but not necessarily in that order.

$$\begin{aligned}W &= \vec{F} \bullet \Delta \vec{r} \\&\approx \langle 1.6 \times 10^{-13}, 0, 0 \rangle \text{ N} \bullet \langle 2, 0, 0 \rangle \text{ m} \\&\approx 3.2 \times 10^{-13} \text{ J} \\E_f &= E_i + W \\&\approx \frac{1}{\sqrt{1 - \frac{0.91c^2}{c^2}}} (9 \times 10^{-31} \text{ kg}) (3 \times 10^8 \frac{\text{m}}{\text{s}})^2 + 3.2 \times 10^{-13} \text{ J} \\&\approx 5.15 \times 10^{-13} \text{ J}\end{aligned}$$

Solve for speed as a function of energy (final energy, that is).

$$\begin{aligned}E &= \gamma mc^2 \\ \gamma &= \frac{E}{mc^2} = \frac{1}{\sqrt{1 - \frac{|\vec{v}|^2}{c^2}}} \\ \frac{|\vec{v}|}{c} &= \sqrt{1 - \frac{mc^2}{E}} \\ \frac{|\vec{v}|}{c} &\approx \sqrt{1 - \frac{(9 \times 10^{-31} \text{ kg})(3 \times 10^8 \frac{\text{m}}{\text{s}})^2}{5.15 \times 10^{-13} \text{ J}}} \\ \frac{|\vec{v}|}{c} &\approx 0.99\end{aligned}$$

Significant figures are important in this problem.

P45:

Solution:

(a)

$$\begin{aligned}E_{i, \text{rest}} &= mc^2 \\&= (3.894028 \times 10^{-25} \text{ kg})(2.99792 \times 10^8 \text{ m/s})^2 \\&= 3.499767 \times 10^{-8} \text{ J}\end{aligned}$$

(b)

$$\begin{aligned}E_{\text{rest}, \alpha} + E_{\text{rest}, \text{new nucleus}} &= m_{\alpha} c^2 + m_{\text{nucleus}} c^2 \\&= (6.640678 \times 10^{-27} \text{ kg})(2.99792 \times 10^8 \text{ m/s})^2 + (3.827555 \times 10^{-25} \text{ kg})(2.99792 \times 10^8 \text{ m/s})^2 \\&= 5.968326 \times 10^{-10} \text{ J} + 3.440024 \times 10^{-8} \text{ J} \\&= 3.499708 \times 10^{-8} \text{ J}\end{aligned}$$

(c) The rest energy decreased.

(d)

$$\begin{aligned}K &= E_{\text{rest}, i} - E_{\text{rest}, f} \\&= 5.95 \times 10^{-12} \text{ J} \\&= 3.72 \text{ MeV}\end{aligned}$$

P58:

Solution:

(a)

$$\begin{aligned}U_i + K_i &= \cancel{U_f}^0 + K_f \\K_i &= K_f - U_i \\\frac{1}{2}mv_i^2 &= \frac{1}{2}mv_f^2 - \frac{-GMm}{r_i} \\v_i &= \sqrt{v_f^2 + \frac{2GM}{r_i}} \\&= \sqrt{1000^2 + \frac{2(6.7 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2})(0.6 \times 10^{24} \text{ kg})}{3.4 \times 10^6 \text{ m}}} \\v_i &= 4960 \text{ m/s}\end{aligned}$$

(b) U_f and $K_f = 0$, so

$$\begin{aligned}U_i + K_i &= 0 \\K_i &= -U_i \\\frac{1}{2}mv_i^2 &= -\left(\frac{-GMm}{r_i}\right) \\v_i &= \sqrt{\frac{2GM}{r_i}} \\&= 4860 \text{ m/s}\end{aligned}$$

P64:

Solution:

$$\begin{aligned}E_i &= E_f \\U_i + K_i &= U_f + K_f \\K_f &= U_i - U_f + K_i \\\frac{1}{2}mv_f^2 &= \frac{-GMm}{r_i} - \frac{GMm}{r_f} + \frac{1}{2}mv_i^2 \\\frac{1}{2}v_f^2 &= -GM_{\text{sun}} \left(\frac{1}{r_i} - \frac{1}{r_f} \right) + \frac{1}{2}v_i^2 \\&= -3.33 \times 10^9 \frac{\text{J}}{\text{kg}} + 3.34 \times 10^9 \frac{\text{J}}{\text{kg}} \\v_f &= 4420 \text{ m/s}\end{aligned}$$

P75:

Solution:

(a)

$$\begin{aligned}E_i &= E_f \\E_{\text{rest,U}} &= 2E_{\text{rest,Pd}} + 2K_{\text{Pd}} \\m_{\text{U}}c^2 &= 2m_{\text{Pd}}c^2 + 2K_{\text{Pd}} \\K_{\text{Pd}} &= \frac{1}{2}(m_{\text{U}}c^2 - 2m_{\text{Pd}}c^2) \\&= \frac{1}{2}((235.996)(1.6603 \times 10^{-27} \text{ kg})(2.99792 \times 10^8 \text{ m/s})^2 - 2(117.894)(1.6603 \times 10^{-27} \text{ kg})(2.99792 \times 10^8 \text{ m/s})^2) \\&= \frac{1}{2}(3.52153 \times 10^{-8} \text{ J} - 3.51843 \times 10^{-8} \text{ J}) \\&= \frac{1}{2}(3.103 \times 10^{-11} \text{ J}) \\&= 1.55 \times 10^{-11} \text{ J}\end{aligned}$$

Assuming that $|\vec{v}| \ll c$, then

$$\begin{aligned}K &= \frac{1}{2}m|\vec{v}|^2 \\v_{\text{Pd}} &= 1.26 \times 10^7 \text{ m/s}\end{aligned}$$

This is less than 10% of c , so it is reasonable to use the non-relativistic approximation.

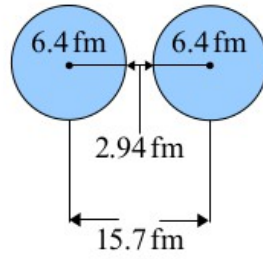
(b) Immediately after fission, the Pd nuclei are at rest. Each one has a charge of $46(1.60218 \times 10^{-19} \text{ C}) = 7.37003 \times 10^{-18} \text{ C}$.

$$\begin{aligned}E_i &= E_f \\U_i &= 2K_{\text{f,Pd}} \\\frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_i} &= 3.103 \times 10^{-11} \text{ J} \\r_i &= 1.57 \times 10^{-14} \text{ m}\end{aligned}$$

(c)

$$\begin{aligned}R &= (1.3 \times 10^{-15} \text{ m})(118)^{\frac{1}{3}} \\&= 6.38 \times 10^{-15} \text{ m}\end{aligned}$$

The distance between the centers of the nuclei is $15.7 \times 10^{-15} \text{ m}$. Thus the gap between the surfaces is $15.7 - 2(6.38) = 2.95 \times 10^{-15} \text{ m}$. A sketch is shown in the figure below. Note that $10^{-15} \text{ m} = 1 \text{ fm}$.



- (d) Each U atom produces 2 Pd atoms with $2(1.55 \times 10^{-11} \text{ J})$ of kinetic energy. The total kinetic energy produced by 1 mol of U atoms is

$$\left(\frac{6.023 \times 10^{23} \text{ U atoms}}{\text{mol}} \right) \left(\frac{2(1.55 \times 10^{-11} \text{ J})}{\text{U atom}} \right) = 1.9 \times 10^{13} \text{ J}$$