P11:

Solution:

According to Newton's law of gravitation, the gravitational force on M by m is directly proportional to the product of their masses and inversely proportional to the distance between them squared.

Tripling the mass of m will increase the force by a factor of 3.

Increasing the distance by a factor of 4 will decrease the force by 4^2, making the force smaller by $\left(\frac{1}{4}\right)^2 = \frac{1}{16}$.

Thus the objects will attract with a force

$$F' = \left(\frac{3}{16}\right) F$$

P19:

Solution:

(a) The relative position vector of the star, relative to the planet, is

$$\begin{aligned}\vec{r}_{\text{from planet to the star}} &= \vec{r}_{\text{star}} - \vec{r}_{\text{planet}} \\ &= \left\langle -2\times10^{11}, 3\times10^{11}, 0\right\rangle \text{m} - \left\langle 5\times10^{11}, -2\times10^{11}, 0\right\rangle \text{m} \\ &= \left\langle -7\times10^{11}, 5\times10^{11}, 0\right\rangle \text{m}\end{aligned}$$

(b) The distance is

$$\begin{aligned}\left|\vec{r}_{\text{from planet to the star}}\right| &= \sqrt{\left(7\times10^{11}\,\text{m}\right)^2 + \left(5\times10^{11}\,\text{m}\right)^2 + (0\,\text{m})^2} \\ &= 8.6\times10^{11}\,\text{m}\end{aligned}$$

(c) The direction of the relative position vector of the star, relative to the planet, is

$$\begin{aligned}\hat{r}_{\text{from planet to the star}} &= \frac{\left\langle -7\times10^{11}, 5\times10^{11}, 0\right\rangle \text{m}}{8.6\times10^{11}\text{m}} \\ &= \langle -0.81, 0.58, 0\rangle\end{aligned}$$

(d) Newton's law of gravitation is

$$\begin{aligned}\left|\vec{F}_{\text{grav on planet by star}}\right| &= G\frac{m_{\text{planet}}\, m_{\text{star}}}{|\vec{r}|^2} \\ &= \left(6.6738\times10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2\right)\frac{\left(4\times10^{24}\text{kg}\right)\left(5\times10^{30}\text{kg}\right)}{\left(8.6\times10^{11}\,\text{m}\right)^2} \\ &= 1.8\times10^{21}\,\text{N}\end{aligned}$$

(e) Because of the Principle of Reciprocity, the planet and star exert forces of equal magnitude on one another. Thus,

$$\begin{aligned}\left|\vec{F}_{\text{grav on planet by star}}\right| &= \left|\vec{F}_{\text{grav on star by planet}}\right| \\ &= 1.8\times10^{21}\,\text{N}\end{aligned}$$

(f) The (vector) force exerted on the planet by the star is

$$\begin{aligned}\vec{F}_{\text{on the planet by the star}} &= \left|\vec{F}_{\text{on the planet by the star}}\right| \hat{r}_{\text{from planet to star}} \\ &= \left(1.8\times10^{21}\,\text{N}\right)\langle -0.81, 0.58, 0\rangle \\ &= \left\langle -1.5\times10^{21}, 1.0\times10^{21}, 0\right\rangle \text{N}\end{aligned}$$

(g) The (vector) force exerted on the star by the planet is

$$\begin{aligned}\vec{F}_{\text{on the star by the planet}} &= \left|\vec{F}_{\text{on the star by the planet}}\right|\left(-\hat{r}_{\text{from planet to star}}\right)\\ &= \left(1.8\times10^{21}\,\mathrm{N}\right)\langle 0.81,-0.58,0\rangle\\ &= \left\langle 1.5\times10^{21},-1.0\times10^{21},0\right\rangle\,\mathrm{N}\end{aligned}$$

P25:

Solution:

Let's assume this experiment takes place near Earth's surface so we can approximate gravitational interactions between Earth and object as $\left|\vec{F}_{\text{object,Earth}}\right| \approx m_{\text{object}}g$. Let Δt be the very short duration of contact, and assume that Δt is so short that the downward change in the ball's momentum due to its interaction with Earth during contact is much smaller than the upward change in the ball's momentum due to its interaction with the scale. The ball initially has a downward momentum $p_{y,\text{ball,i}} = p$ just before hitting the scale. Shortly after interacting with the scale, the ball has an upward momentum $p_{y,\text{ball,f}}$. So the ball undergoes a change in momentum of $\Delta\vec{p}_{\text{ball}} = \vec{p}_{\text{ball,f}} - \vec{p}_{\text{ball,i}} \approx p_{y,\text{ball,f}} - p_{y,\text{ball,i}}$ which, remembering that (a) we're assuming the ball rebounds to its initial height and (b) $p_{y,\text{ball,i}}$ is a negative number, is just $-2\vec{p}_{\text{ball,i}}$ or just $-2p$. (Note that p is a negative number!) This change in momentum is, remember, almost exclusively due to the ball's interaction with the scale and is directed upward. Therefore, the average force on the ball due to its interaction with the scale must also be upward.

Let T be the long time between the instants when the ball is at its maximum height (it's really the period of the ball's oscillatory motion), neglecting any energy dissipation. The momentum gives us the magnitude of the average force immediately as $\left|\vec{F}_{\text{avg}}\right| \approx \frac{\left|\vec{p}_{\text{ball}}\right|}{T} \approx \frac{-2p}{T}$ (remember p is a negative number!).

In each fall, the magnitude of the ball's momentum increases from 0 to $-p$ (remember p is a negative number!) due to its gravitational interaction with Earth. This happens within a duration of $T/2$. Applying the momentum principle to the ball's downward motion allows us to express T in terms of known quantities.

$$\begin{aligned}\Delta p_{y,\text{ball}} &\approx -p - 0 \approx \left|\vec{F}_{\text{ball,Earth}}\right|\Delta t\\ &\approx -p \approx m_{\text{ball}}\,g\frac{T}{2}\end{aligned}$$

Solving for T gives $T \approx \frac{-2p}{m_{\text{ball}}g}$, and now we can get a final expression for the magnitude of the average force:

$$\left|\vec{F}_{\text{avg}}\right| \approx \frac{-2p}{T} \approx \frac{-2p}{\left(\frac{-2p}{m_{\text{ball}}g}\right)} \approx m_{\text{ball}}g$$

This surprising result happens to be exactly the same magnitude of the ball-scale interaction if the ball were at rest on the scale! This unusual result will play a role later in our study of gases.

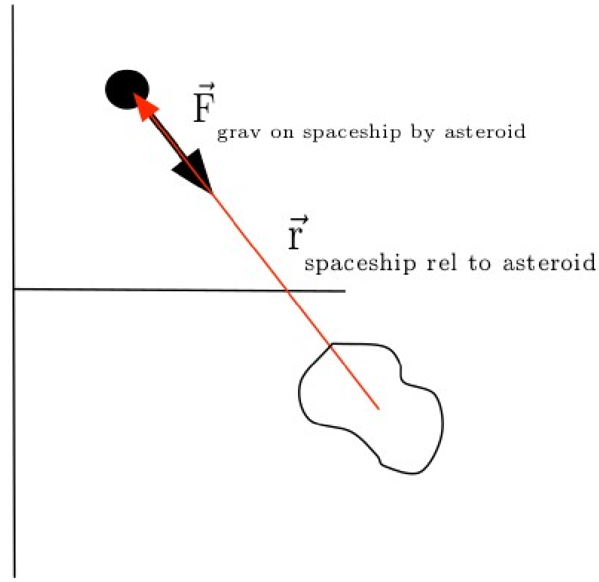
Alternatively, we can get this surprising result another way. Let T be the duration of the ball's fall from rest back to that same place (again at rest). The net change in the ball's momentum is zero. Let $\vec{F}_{\text{avg}}$ be the average force on the ball due to the scale. Apply the momentum principle.

$$\begin{aligned}\Delta\vec{p}_{\text{ball}} &= \vec{F}_{\text{avg}}T + \vec{F}_{\text{ball,Earth}}T\\ 0 &\approx \left|\vec{F}_{\text{avg}}\right|T - m_{\text{ball}}gT\\ \therefore \left|\vec{F}_{\text{avg}}\right| &\approx m_{\text{ball}}g\end{aligned}$$

P27:

Solution:

(a) Sketch a picture of the situation in 2-D, like the one shown in the figure below.



Apply Newton's law of gravitation. The relative position vector of the spaceship, relative to the asteroid, is

$$
\begin{aligned}
\vec{r}_{\text{from asteroid to spaceship}} &= \vec{r}_{\text{spaceship}} - \vec{r}_{\text{asteroid}} \\
&= \left\langle 3 \times 10^5, 7 \times 10^5, -4 \times 10^5 \right\rangle \text{m} - \left\langle 9 \times 10^5, -3 \times 10^5, -12 \times 10^5 \right\rangle \text{m} \\
&= \left\langle -6 \times 10^5, 10 \times 10^5, 8 \times 10^5 \right\rangle \text{m}
\end{aligned}
$$

The distance between the asteroid and spaceship is

$$
\begin{aligned}
\left|\vec{r}_{\text{from asteroid to spaceship}}\right| &= \sqrt{\left(-6 \times 10^5 \text{ m}\right)^2 + \left(10 \times 10^{11} \text{ m}\right)^2 + \left(-16 \times 10^5 \text{ m}\right)^2} \\
&= 1.41 \times 10^6 \text{ m}
\end{aligned}
$$

The unit vector is

$$
\begin{aligned}
\hat{r}_{\text{from asteroid to spaceship}} &= \frac{\left\langle -6 \times 10^5, 10 \times 10^5, 8 \times 10^5 \right\rangle}{1.41 \times 10^6 \text{m}} \\
&= \langle -0.424, 0.707, 0.566 \rangle
\end{aligned}
$$

Newton's law of gravitation is

$$
\begin{aligned}
\left|\vec{F}_{\text{grav on spaceship by asteroid}}\right| &= G\frac{m_{\text{spaceship}}\,m_{\text{asteroid}}}{|\vec{r}|^2} \\
&= \left(6.6738\times10^{-11}\,\mathrm{N\cdot m^2/kg^2}\right)\frac{\left(1.4\times10^{3}\,\mathrm{kg}\right)\left(7\times10^{15}\,\mathrm{kg}\right)}{\left(1.41\times10^{6}\,\mathrm{m}\right)^2} \\
&= 3.29\times10^{-4}\,\mathrm{N}
\end{aligned}
$$

The (vector) force exerted on the spaceship by the asteroid is

$$
\begin{aligned}
\vec{F}_{\text{grav on spaceship by asteroid}} &= \left|\vec{F}_{\text{on spaceship by asteroid}}\right|\left(-\hat{r}_{\text{from asteroid to spaceship}}\right) \\
&= \left(3.29\times10^{-4}\,\mathrm{N}\right)\left(-\langle -0.424, 0.707, 0.566\rangle\right) \\
&= \left\langle 1.39\times10^{-4}, -2.32\times10^{-4}, -1.86\times10^{-4}\right\rangle \mathrm{N}
\end{aligned}
$$

In the xy-plane , the force points to the right and downward which is in agreement with the force vector drawn in the figure below.

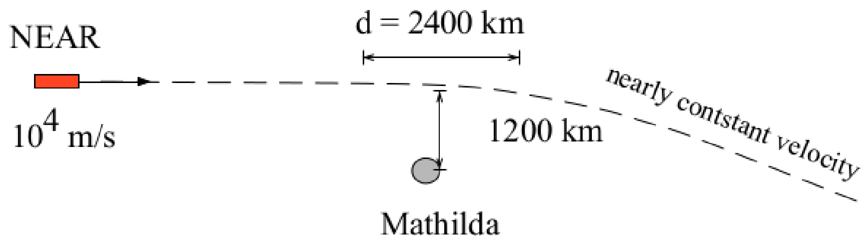
(b) Apply the momentum principle

$$
\begin{aligned}
\vec{p}_{\text{final}} - \vec{p}_{\text{initial}} &= \vec{F}_{\text{net}}\Delta t \\
&= \left(\left\langle 1.39\times10^{-4}, -2.32\times10^{-4}, 1.86\times10^{-4}\right\rangle \mathrm{N}\right)(538\,\mathrm{s} - 532\,\mathrm{s}) \\
&= \left\langle 8.4\times10^{-4}, 1.40\times10^{-3}, 1.12\times10^{-3}\right\rangle \mathrm{kg\cdot m/s}
\end{aligned}
$$

P31:

Solution:

(a) If Mathilde were not present, the NEAR spacecraft would travel through space with constant momentum. However, Mathilde will interact gravitationally with NEAR, deflecting the spacecraft's path slightly toward the asteroid as the spacecraft approaches and passes the asteroid. The interaction will probably have more effect on NEAR's direction than its speed.



(b) NEAR's change in momentum resulting from this encounter can be approximated by estimating the net force on NEAR due to Mathilde, $\vec{F}_{\text{NEAR,Mathilde}}$, along with an estimate of the duration, Δt, of the interaction. The magnitude and the direction of the force on NEAR both vary with time, so we have no choice but to use rather crude estimates. Let's take Δt represent the duration of NEAR's travel across the 2.4×10^6 m distance in the diagram. Let's take the force to have an approximate magnitude corresponding to a distance of 1.2×10^6 m. Since we're really only interested in NEAR's deviation from an otherwise straight trajectory, let's also treat the force as having only a y component, which is another very crude approximation. But wait! We also need an estimate of Mathilde's mass. We can get this from its assumed mass density and rough physical dimensions.

$$M_{\text{Mathilde}} \approx \left(7 \times 10^4\,\text{m}\right)\left(5 \times 10^4\,\text{m}\right)\left(5 \times 10^4\,\text{m}\right)\left(3 \times 10^3\,\text{kg/m}^3\right) \approx 5.25 \times 10^{17}\,\text{kg}$$

Now let's apply the momentum principle.

$$\begin{aligned}\Delta p_{y,\text{NEAR}} &\approx F_{y,\text{NEAR,Mathilde}}\,\Delta t \approx -\left(G\frac{M_{\text{NEAR}}\,M_{\text{Mathilde}}}{\left|\vec{r}_{\text{NEAR,Mathilde}}\right|^2}\right)\left(\frac{d}{\left|\vec{v}_{\text{NEAR}}\right|}\right)\\ &\approx -\left(6.6738 \times 10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(\frac{(805\,\text{kg})\left(5.25 \times 10^{17}\,\text{kg}\right)}{\left(1.2 \times 10^6\,\text{m}\right)^2}\right)\left(\frac{\left(2.4 \times 10^6\,\text{m}\right)}{\left(1 \times 10^4\,\text{m/s}\right)}\right) \approx -5\,\text{kg}\cdot\text{m/s}\end{aligned}$$

Be certain you understand where the negative sign came from!

(c) In one day, NEAR will travel approximately $\left(1 \times 10^4\,\text{m/s}\right)(24 \times 60 \times 60\,\text{s}) \approx 9 \times 10^8\,\text{m}$. As NEAR gets farther and farther away from Mathilde, the y component of its momentum (and its velocity) is nearly constant. Therefore, in one day NEAR's deviation from it's initial trajectory will be approximately

$$\Delta y_{\text{NEAR}} \approx \left(\frac{\Delta p_{y,\text{NEAR}}}{M_{\text{NEAR}}}\right)(1\,\text{day}) \approx \left(\frac{-5\,\text{kg}\cdot\text{m/s}}{805\,\text{kg}}\right)(24 \times 60 \times 60\,\text{s}) \approx -500\,\text{m}$$

(d) Astronomers observed a deviation significantly less than was predicted. Gravitational attraction is proportional to mass, and mass is proportional to density, so a smaller deviation indicated a smaller mass. NEAR's mass was constant, so Mathilde's mass must have been less than the predicted value.

P41:

Solution:

(a) The total initial momentum of the system is the sum of the momenta of the two balls. Name them A and B.

$$\begin{aligned}
\vec{p}_{\text{sys,i}} &= \vec{p}_{\text{A,i}} + \vec{p}_{\text{B,i}} \\
&= m_{\text{A}} \vec{v}_{\text{A,i}} + m_{\text{B}} \vec{v}_{\text{B,i}} \\
&= 0.3\,\text{kg}\langle 4, -3, 2\rangle\,\text{m/s} + 0.5\,\text{kg}\langle 2, 1, 4\rangle\,\text{m/s} \\
&= \langle 1.2, -0.9, 0.6\rangle\,\text{kg}\cdot\text{m/s} + \langle 1, 0.5, 2\rangle\,\text{kg}\cdot\text{m/s} \\
&= \langle 2.2, -0.4, 2.6\rangle\,\text{kg}\cdot\text{m/s}
\end{aligned}$$

(b) The system is near Earth, so the gravitational force on the system is

$$\begin{aligned}
\vec{F}_{\text{grav by Earth on sys}} &= m\vec{g} \\
&= \langle 0, -mg, 0\rangle \\
&= \left\langle 0, -\left(0.3\,\text{kg} + 0.5\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right), 0\right\rangle \\
&= \langle 0, -7.84, 0\rangle\,\text{N}
\end{aligned}$$

(c) Use the momentum principle

$$\begin{aligned}
\Delta\vec{p}_{\text{sys}} &= \langle 2.2, -0.4, 2.6\rangle\,\text{kg}\cdot\text{m/s} + \left(\langle 0, -7.84, 0\rangle\,\text{N}\right)(0.1\,\text{s}) \\
&= \langle 2.2, -0.4, 2.6\rangle\,\text{kg}\cdot\text{m/s} + \langle 0, -0.784, 0\rangle\,\text{kg}\cdot\text{m/s} \\
&= \langle 2.2, -1.2, 2.6\rangle\,\text{kg}\cdot\text{m/s}
\end{aligned}$$

Note that the x and z momenta of the system are constant (i.e. they did not change) since the net force on the system is only in the y-direction.

P45:

Solution:

(a) Let's take system = both rocks + string. We're told the string's mass is negligible, which means the string's contribution to the system's momentum is also negligible. Therefore, we only need to account for the momenta of the two rocks. Let's also assume both rocks have velocities with magnitudes negligible compared to light speed, so we can use the Newtonian approximation for momentum.

$$\begin{aligned}
\vec{p}_{\text{sys}} &= \vec{p}_{\text{rock 1}} + \vec{p}_{\text{rock 2}} + \cancelto{0}{\vec{p}_{\text{string}}} \approx \vec{p}_{\text{rock 1}} + \vec{p}_{\text{rock 2}} \\
&\approx m_{\text{rock 1}} \vec{v}_{\text{rock 1}} + m_{\text{rock 2}} \vec{v}_{\text{rock 2}} \\
&\approx (0.1\,\text{kg})\,\langle 0, 5, 0\rangle\,\text{m/s} + (0.25\,\text{kg})\,\langle 7.5, 0, 0\rangle\,\text{m/s} \\
&\approx \langle 0, 0.5, 0\rangle\,\text{kg}\cdot\text{m/s} + \langle 1.875, 0, 0\rangle\,\text{kg}\cdot\text{m/s} \\
&\approx \langle 1.875, 0.5, 0\rangle\,\text{kg}\cdot\text{m/s}
\end{aligned}$$

(b) The total momentum can be expressed as $\vec{P} = M\vec{v}_{CM}$ where M is the total mass of the system and $\vec{P}$ is the total momentum of the system. The center of mass velocity is

$$\begin{aligned}
\vec{v}_{CM} &= \frac{\vec{P}}{M} \\
&= \frac{\langle 1.875, 0.5, 0\rangle\,\text{kg}\cdot\text{m/s}}{(0.1\,\text{kg} + 0.25\,\text{kg})} \\
&= \langle 5.36, 1.43, 0\rangle\,\text{m/s}
\end{aligned}$$

P49:

Solution:

(a) Let's choose system = all clay and apply the momentum principle. Let's also use the Newtonian approximation for momentum.

$$\begin{aligned}
&\vec{p}_{\text{sys,i}} = \vec{p}_{1,\text{i}} + \vec{p}_{2,\text{i}} \approx m_1 \vec{v}_{1,\text{i}} + m_2 \vec{v}_{2,\text{i}} \\
&\vec{p}_{\text{sys,i}} \approx (0.03\,\text{kg})\,(\langle 3, 3, -3\rangle\,\text{m/s}) + (0.03\,\text{kg})\,(\langle -3, 0, -3\rangle\,\text{m/s}) \\
&\vec{p}_{\text{sys,i}} \approx \langle 0.09, 0.09, -0.09\rangle\,\text{kg}\cdot\text{m/s} + \langle -0.09, 0, -0.09\rangle\,\text{kg}\cdot\text{m/s} \approx \langle 0, 0.09, -0.18\rangle\,\text{kg}\cdot\text{m/s}
\end{aligned}$$

(b) Just after the collision, the clay exists as one lump whose mass is the sum of the individual masses. Apply the momentum principle to the system just after the collision.

$$\vec{p}_{\text{sys,f}} \approx \vec{p}_{\text{sys,f}} \approx (m_1 + m_2)\,\vec{v}_{\text{f}}$$

But we can't evaluate this because we don't know $\vec{v}_{\text{f}}$. If, however, we assume there are no significant interactions on our system, then the momentum principle predicts that $\vec{p}_{\text{sys,f}} = \vec{p}_{\text{sys,i}}$. Therefore, $\vec{p}_{\text{sys,f}} = \langle 0, 0.09, -0.18\rangle\,\text{kg}\cdot\text{m/s}$.

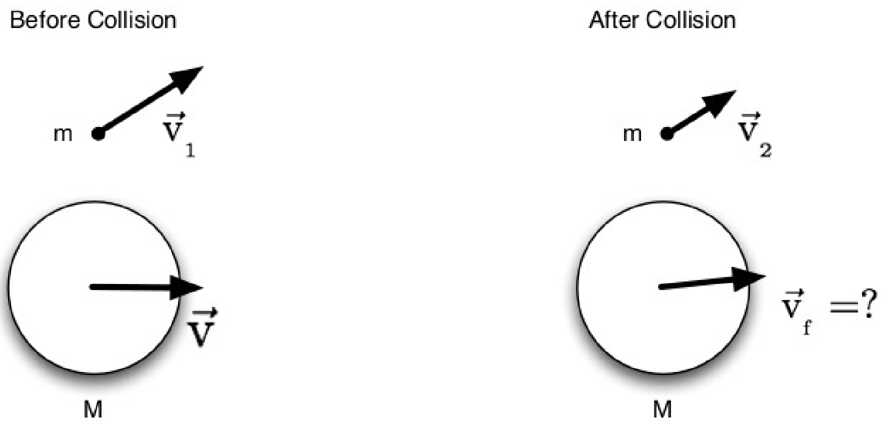
(c) Solve the above application of the momentum principle for $\vec{v}_{\text{f}}$.

$$\begin{aligned}
\vec{p}_{\text{sys,f}} &\approx (m_1 + m_2)\,\vec{v}_{\text{f}} \\
\vec{v}_{\text{f}} &\approx \frac{\vec{p}_{\text{sys,i}}}{(m_1 + m_2)} \approx \frac{\langle 0, 0.09, -0.18\rangle\,\text{kg}\cdot\text{m/s}}{0.06\,\text{kg}} \approx \langle 0, 1.5, -3\rangle\,\text{m/s}
\end{aligned}$$

P59:

Solution:

First, sketch a picture showing the objects before and after the collision (see the figure below.).



Though the satellite is rotating, treat it as a particle. Define the system to be the meteor and satellite. Apply the momentum principle with net external force equal to zero.

$$
\begin{aligned}
\vec{p}_{\text{sys,f}} &= \vec{p}_{\text{sys,i}} + \cancelto{0}{\vec{F}_{\text{net}}}\Delta t \\
\vec{p}_{\text{sys,f}} &= \vec{p}_{\text{sys,i}} \\
\vec{p}_{\text{meteor,f}} + \vec{p}_{\text{satellite,f}} &= \vec{p}_{\text{meteor,i}} + \vec{p}_{\text{satellite,i}} \\
m\vec{v}_{\text{meteor,f}} + M\vec{v}_{\text{satellite,f}} &= m\vec{v}_{\text{meteor,i}} + M\vec{v}_{\text{satellite,i}} \\
m\left\langle v_{2x}, v_{2y}, 0\right\rangle + M\left\langle v_{fx}, v_{fy}, 0\right\rangle &= m\left\langle v_{1x}, v_{1y}, 0\right\rangle + M\left\langle v, 0, 0\right\rangle
\end{aligned}
$$

Write the above equation in component form in the x and y directions.

x:

$$
\begin{aligned}
mv_{2x} + Mv_{fx} &= mv_{1x} + Mv \\
Mv_{fx} &= mv_{1x} + Mv - mv_{2x} \\
v_{fx} &= \frac{mv_{1x} + Mv - mv_{2x}}{M} \\
v_{fx} &= \frac{m\left(v_{1x} - v_{2x}\right) + Mv}{M}
\end{aligned}
$$

Note that $v_{fx} > v$ as a result of the collision. Thus, the satellite will be moving faster in the +x direction due to the collision. Now for y,

y:

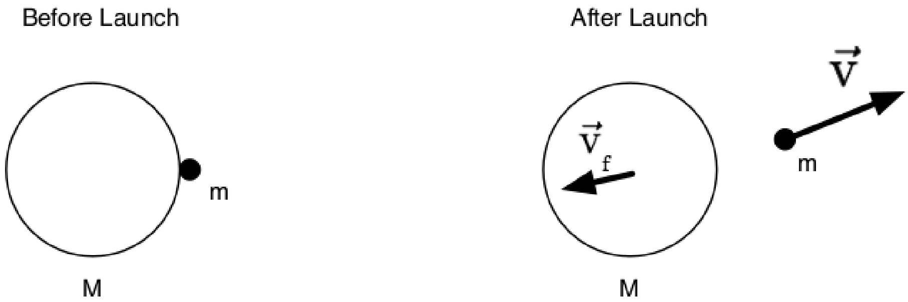
$$
\begin{aligned}
mv_{2y} + Mv_{fy} &= mv_{1y} + 0 \\
Mv_{fy} &= mv_{1y} - mv_{2y} \\
v_{fy} &= \frac{mv_{1y} - mv_{2y}}{M} \\
v_{fy} &= \frac{m\left(v_{1y} - v_{2y}\right)}{M}
\end{aligned}
$$

Note that v_{fy} is positive, thus after the collision, the satellite's velocity has a slight "upward" component in the +y direction.

P61:

Solution:

Sketch a picture of the system before and after the package is launched, as shown in the figure below.



Treat the objects as point particles. Define the system to be the space station and the package. Assume that $m << M$ so that the center of mass of the system before the package is launched is very close to the center of mass of the space station. With this assumption, the initial momentum of the system (before the launch) is zero. Also, assume that the net external force on the system is zero. Apply the momentum principle.

$$\begin{aligned}\vec{p}_{\text{sys,f}} &= \vec{p}_{\text{sys,i}} + \cancelto{0}{\vec{F}_{\text{net}}}\Delta t \\ \vec{p}_{\text{station,f}} + \vec{p}_{\text{package,f}} &= \vec{p}_{\text{station,i}} + \vec{p}_{\text{package,i}} \\ &= 0 + 0\end{aligned}$$

Thus,

$$\begin{aligned}\vec{p}_{\text{station,f}} &= -\vec{p}_{\text{package,f}} \\ M\vec{v}_{\text{f}} &= -m\vec{v} \\ \vec{v}_{\text{f}} &= -\left(\frac{m}{M}\right)\vec{v} \\ \vec{v}_{\text{f}} &= -\left(\frac{m}{M}\right)\langle v\cos\theta, v\sin\theta, 0\rangle\end{aligned}$$

Written in component form:

$$v_{\text{fx}} = -\left(\frac{m}{M}\right) v\,\cos\theta$$

and

$$v_{\text{fy}} = -\left(\frac{m}{M}\right) v\,\sin\theta$$

As a result of launching the package the space station recoils in the opposite direction with equal magnitude and opposite momentum as the package.