Hw2 Solution

Q11:

Solution:

If the stripe remains vertical, then translational angular momentum relative to A is nonzero, directed into the page. The rotational angular momentum is zero. If the wheel is welded to the rod, the stripe does not remain vertical the translational angular momentum relative to A is nonzero, and directed into the page. The rotational angular momentum is nonzero, also directed into the page.

P22:

Solution:

$$I = 2I_1 + 2I_2$$

(a)

$$\begin{aligned} I &= 2m_1 r_1^2 + 2m_2 r_2^2 \\ &= 2(0.29\ \text{kg})(0.08\ \text{m})^2 + 2(0.82\ \text{kg})(0.16\ \text{m})^2 \\ &= 0.0457\ \text{kg}\cdot\text{m}^2 \end{aligned}$$

(b)

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.9\ \text{s}} = 6.98\ \text{rad/s}$$

(c)

$$\begin{aligned} \left|\vec{L}\right| &= I\omega \\ &= 0.319\ \text{kg}\cdot\frac{\text{m}^2}{\text{s}} \end{aligned}$$

P27:

Solution:

(a)

$$\begin{aligned}\vec{p}_{total} &= m_1\vec{v}_1 + m_2\vec{v}_2 = m\left(\vec{v}_1 + \vec{v}_2\right) \\ &\approx \langle 29.4, 0, 0\rangle \text{ kg}\cdot\text{m/s}\end{aligned}$$

(b)

$$\begin{aligned}\vec{v}_{cm} &= \frac{\vec{p}_{total}}{2m} \\ &\approx \langle 49, 0, 0\rangle \text{ m/s}\end{aligned}$$

(c)

$$\begin{aligned}\vec{L}_{A,total} &= \vec{r}_1 \times \vec{p}_1 + \vec{r}_2 \times \vec{p}_2 \\ &\approx \langle 0, 0, -25.9\rangle \text{ kg}\cdot\text{m}^2\text{/s}\end{aligned}$$

(d)

$$\begin{aligned}\vec{L}_{rot} &= \vec{r}_{1,cm} \times \vec{p}_1 + \vec{r}_{2,cm} \times \vec{p}_2 \\ &\approx \langle 0, 0, 4.95\rangle \text{ kg}\cdot\text{m}^2\text{/s}\end{aligned}$$

(e)

$$\begin{aligned}\vec{L}_{trans,A} &= \vec{r}_{cm,A} \times \vec{p}_{total} \\ &\approx \langle 0, 0, -30.9\rangle \text{ kg}\cdot\text{m}^2\text{/s}\end{aligned}$$

Check: $\vec{L}_{A,total} = \vec{L}_{trans,A} + \vec{L}_{rot}$

(f)

$$\begin{aligned}\vec{p}_f &= \vec{p}_i + \vec{F}_{net}\Delta t \\ &\approx \langle 32.9, 0, 0\rangle \text{ kg}\cdot\text{m/s}\end{aligned}$$

P29:

Solution:

(a)

$$\begin{aligned}\left|\vec{L}_{rot}\right| &= I_{cm}\left|\vec{\omega}_2\right| \\ &= \frac{1}{2}md^2\left|\vec{\omega}_2\right| \\ &\approx 0.5\,(0.4\text{ kg})\,(0.6\text{ m})^2\,(20\text{ rad/s}) \\ &\approx 1.44\text{ kg}\cdot\text{m}^2\text{/s}\end{aligned}$$

directed into page

(b) $\left|\vec{L}_{trans,B}\right| \approx 9.72 \text{ kg}\cdot\text{m}^2\text{/s}$ directed into page

(c) Just add the rotational and translational angular momenta. $\left|\vec{L}_{total,B}\right| \approx 11.16 \text{ kg}\cdot\text{m}^2\text{/s}$ directed into page

P49:

Solution:

(a) Apply the Momentum Principle to the system of satellite and space junk. Since $\vec{F}_{net} = 0$, $\vec{p}_{sys}$ is constant. Define 1 to be the junk and 2 to be the satellite.

$$\vec{p}_{sys,i} = \vec{p}_{sys,f}$$

x:

$$\begin{aligned}
p_{1,i,x} + p_{2,i,x} &= p_{1,f,x} + p_{2,f,x} \\
m_1 v_{1,i,x} + m_2 v_{2,i,x} &= m_1 v_{1,f,x} + m_2 v_{2,f,x} \\
(4.1 \text{ kg})(-2200 \text{ m/s}) + (205 \text{ kg})(2600 \text{ m/s}) &= (4.1 \text{ kg})(-1300 \text{ m/s}) + (205 \text{ kg})v_{2,f,x} \\
v_{2,f,x} &= 2582 \text{ m/s}
\end{aligned}$$

y:

$$\begin{aligned}
p_{1,i,y} + p_{2,i,y} &= p_{1,f,y} + p_{2,f,y} \\
0 &= m_1 v_{1,f,y} + m_2 v_{2,f,y} \\
v_{2,f,y} &= \frac{-m_1 v_{1,f,y}}{m_2} \\
&= \frac{-(4.1 \text{ kg}(480 \text{ m/s}))}{205 \text{ kg}} \\
&= -9.6 \text{ m/s}
\end{aligned}$$

The moment of inertia of the satellite is

$$\begin{aligned}
I_{sat} &= \frac{2}{5}MR^2 \\
&= \frac{2}{5}(205 \text{ kg})(4.8 \text{ m})^2 \\
&= 1890 \text{ kg} \cdot \text{m}^2
\end{aligned}$$

The rotational angular momentum of the satellite before the collision is

$$\begin{aligned} L_{rot,z} &= -I\omega_1 \\ &= -(1890\ \mathrm{kg \cdot m^2})(2\ \mathrm{rad/s}) \\ &= -3780\ \mathrm{kg} \cdot \frac{\mathrm{m}^2}{\mathrm{s}} \end{aligned}$$

Let's label the center of the satellite as point D. Then the angular momentum of the space junk about point D before the collision is

$$\begin{aligned} \vec{L}_{D,trans} &= \vec{r}_D \times \vec{p}_{1,i} \\ L_{D,trans,z} &= +r_\perp |\vec{p}| \\ &= +(4.8\ \mathrm{m})(4.1\ \mathrm{kg})(2200\ \mathrm{m/s}) \\ &= +4.33 \times 10^4\ \mathrm{kg} \cdot \frac{\mathrm{m}^2}{\mathrm{s}} \end{aligned}$$

$$\vec{L}_{D,trans,i} = \langle 0, 0, 4.33 \times 10^4 \rangle\ \mathrm{kg} \cdot \frac{\mathrm{m}^2}{\mathrm{s}}$$

The angular momentum of the space junk about point D after the collision is

$$\begin{aligned} \vec{L}_{D,trans} &= \vec{r}_D \times \vec{p}_{1,f} \\ &= < 0, R, 0 > \times (4.1\ \mathrm{kg})\langle -1300, 480, 0 \rangle\ \mathrm{m/s} \\ &= \langle 0, 0, 2.56 \times 10^4 \rangle\ \mathrm{kg} \cdot \frac{\mathrm{m}^2}{\mathrm{s}} \end{aligned}$$

Since the net torque on the satellite/junk system about D is 0, then $\vec{L}$ for the system about point D is constant. Use this to calculate $\vec{L}_{z,rot,f}$ for the satellite. Label the junk as 1 and the satellite as 2. Then,

$$\begin{aligned} \vec{L}_{sys,f} &= \vec{L}_{sys,i} \\ L_{1,trans,i,z} + L_{2,rot,i,z} &= L_{1,trans,f,z} + L_{2,rot,f,z} \\ 4.33 \times 10^4\ \mathrm{kg} \cdot \frac{\mathrm{m}^2}{\mathrm{s}} - 3780\ \mathrm{kg} \cdot \frac{\mathrm{m}^2}{\mathrm{s}} &= 2.56 \times 10^4\ \mathrm{kg} \cdot \frac{\mathrm{m}^2}{\mathrm{s}} + L_{2,rot,f,z} \\ L_{z,rot,f} &= 1.39 \times 10^4\ \mathrm{kg} \cdot \frac{\mathrm{m}^2}{\mathrm{s}} \end{aligned}$$

Note that the rotational angular momentum of the satellite is in the positive z direction, thus the satellite is now spinning counterclockwise as a result of the collision. Its angular speed is

$$
\begin{aligned}
L_{z,rot,f} &= I\omega_f \\
\omega_f &= \frac{1.39 \times 10^4\ \mathrm{kg} \cdot \frac{\mathrm{m}^2}{\mathrm{s}}}{1890\ \mathrm{kg} \cdot \mathrm{m}^2} \\
&= 7.35\ \mathrm{rad/s}
\end{aligned}
$$

(b) The change in the internal energy of the system as a result of the collision is:

$$
\begin{aligned}
\Delta E_{\mathrm{sys}} &= 0 \\
\Delta K_{\mathrm{rot}} + \Delta K_{\mathrm{trans}} + \Delta E_{\mathrm{internal}} &= 0 \\
\Delta E_{\mathrm{internal}} &= -\Delta K_{\mathrm{rot}} - \Delta K_{\mathrm{trans}} \\
&= -(\Delta K_{\mathrm{rot}} + \Delta K_{\mathrm{trans}})
\end{aligned}
$$

The change in rotational kinetic energy is

$$
\begin{aligned}
\Delta K_{\mathrm{rot}} &= K_{\mathrm{rot,f}} - K_{\mathrm{rot,i}} \\
&= \frac{1}{2} I(\omega_f^2 - \omega_1^2) \\
&= \frac{1}{2}(1890\ \mathrm{kg} \cdot \mathrm{m}^2)((7.35\ \mathrm{rad/s})^2 - (2\ \mathrm{rad/s})^2) \\
&= 4.73 \times 10^5\ \mathrm{J}
\end{aligned}
$$

The change in translational kinetic energy is

$$
\begin{aligned}
\Delta K_{\mathrm{trans}} &= K_{\mathrm{trans,f}} - K_{\mathrm{trans,i}} \\
&= \frac{1}{2} m \mathrm{v}_{1,f}^2 + \frac{1}{2} M \mathrm{v}_{2,f}^2 - (\frac{1}{2} m \mathrm{v}_{1,i}^2 + \frac{1}{2} M \mathrm{v}_{2,i}^2) \\
&= \frac{1}{2} m(\mathrm{v}_{1,f}^2 - \mathrm{v}_{1,i}^2) + \frac{1}{2} M(\mathrm{v}_{2,f}^2 - \mathrm{v}_{2,i}^2) \\
&= \frac{1}{2}(4.1\ \mathrm{kg})((-1300\ \mathrm{m/s})^2 + (480\ \mathrm{m/s})^2 - (2200\ \mathrm{m/s})^2) + \\
&\qquad \frac{1}{2}(205\ \mathrm{kg})((2582\ \mathrm{m/s})^2 + (-9.6\ \mathrm{m/s})^2 - (2600\ \mathrm{m/s})^2) \\
&= -5.99 \times 10^6\ \mathrm{J} - 9.55 \times 10^6\ \mathrm{J} \\
&= -1.55 \times 10^7\ \mathrm{J}
\end{aligned}
$$

The change in the internal energy of the system as a result of the collision is

$$
\begin{aligned}
\Delta E_{\mathrm{internal}} &= -\Delta K_{\mathrm{rot}} - \Delta K_{\mathrm{trans}} \\
&= -(\Delta K_{\mathrm{rot}} + \Delta K_{\mathrm{trans}}) \\
&= -(4.73 \times 10^5\ \mathrm{J} + -1.55 \times 10^7\ \mathrm{J}) \\
&= 1.51 \times 10^7\ \mathrm{J}
\end{aligned}
$$

P62:

Solution:

There are two ways to approach this problem:

(1) Apply both the Momentum Principle and Angular Momentum Principle.

(2) Apply the Energy Principle to the point particle system and the real system of the yo-yo.

The following solution uses the first approach. Choose the system to be the yo-yo. Apply the Momentum Principle.

$$\begin{aligned}
\vec{F}_{net} &= \frac{\Delta \vec{p}}{\Delta t} \\
F_{net,y} &= \frac{\Delta p_y}{\Delta t} \\
(F_T - M_{total} g) &= M_{total} \frac{\Delta v_{CM}}{\Delta t}
\end{aligned}$$

As the wheel is displaced -y, it rotates a distance $2\pi r$ in the +z direction (i.e. counterclockwise). Thus, $v_{CM,y} = -\omega_z r$ and $\Delta v_{CM,y} = -r\omega_z$.

$$(F_T - M_{total} g) = -M_{total} r \frac{\Delta \omega_z}{\Delta t}$$

Now, apply the Momentum Principle. Calculate $\vec{\tau}$ and $\vec{L}$ about the CM of the disk.

$$\begin{aligned}
\vec{\tau}_{net} &= \frac{\Delta \vec{L}}{\Delta t} \\
\tau_{net,z} &= \frac{\Delta L_z}{\Delta t} \\
F_T r &= I \frac{\Delta \omega_z}{\Delta t} \\
F_T r &= (2(\frac{1}{2} M R^2) + \frac{1}{2} m r^2) \frac{\Delta \omega_z}{\Delta t}
\end{aligned}$$

Solve for $\frac{\Delta \omega_z}{\Delta t}$ from the Momentum Principle and substitute.

$$\frac{\Delta \omega_z}{\Delta t} = \frac{M_{total} g - F_T}{M_{total} r}$$

Substitute this quantity into the Angular Momentum Principle.

$$\begin{aligned}
F_T r &= (M R^2 + \frac{1}{2} m r^2) \frac{(M_{total} g - F_T)}{M_{total} r} \\
\frac{F_T M_{total} r}{M R^2 + \frac{1}{2} m r^2} &= M_{total} - F_T \\
F_T &= \frac{M_{total} g}{\frac{M_{total} r^2}{M R^2 + \frac{1}{2} m r^2} + 1}
\end{aligned}$$

Be sure to check the units to verify that you will get Newtons. In this case, F_T has the units of the numerator, $M_{total}g$, which is Newtons.

$$F_T = \frac{(2M+m)g}{\frac{(2M+m)r^2}{MR^2+\frac{1}{2}mr^2}+1}$$

P77:

Solution:

Consider the electron to orbit the nucleus in a circular orbit of radius r with a speed much less than c. Apply the momentum principle to the electron. The only force on the electron is the electric force F by the nucleus.

$$\begin{aligned} |F_{net}| &= p\frac{v}{r} \\ F &= \frac{mv^2}{r} \end{aligned}$$

In the Bohr model, the angular momentum of the electron is quantized and equal to $L = N\hbar$. Since $L = rp = rmv$ for a circular orbit, then

$$\begin{aligned} rmv &= N\hbar \\ v &= \frac{N\hbar}{mr} \end{aligned}$$

Substitute v into the momentum principle.

$$\begin{aligned} F &= \frac{mv^2}{r} \\ &= \frac{m}{r}\left(\frac{N\hbar}{mr}\right)^2 \\ &= \frac{N^2\hbar^2}{mr^3} \end{aligned}$$

The force F is the electric force on the electron by the nucleus. The electron's charge has a magnitude e, and in this case, the nucleus has a charge $2e$.

$$\begin{aligned} F &= \frac{1}{4\pi\varepsilon_o}\frac{q_1q_2}{r^2} \\ F &= \frac{1}{4\pi\varepsilon_o}\frac{2e^2}{r^2} \end{aligned}$$

Substitute the electric force into the momentum principle and solve for r.

$$\begin{aligned} F &= \frac{N^2\hbar^2}{mr^3} \\ \frac{1}{4\pi\varepsilon_o}\frac{2e^2}{r^2} &= \frac{N^2\hbar^2}{mr^3} \\ r &= \frac{N^2\hbar^2}{2me^2\left(\frac{1}{4\pi\varepsilon_o}\right)} \end{aligned}$$

We know from the hydrogen atom that

$$\frac{\hbar^2}{me^2\left(\frac{1}{4\pi\varepsilon_o}\right)} = 0.529 \times 10^{-10} \text{ m}$$

which is called the Bohr radius. Thus, for the singly ionized helium atom

$$\begin{aligned} r &= \frac{1}{2}(0.529 \times 10^{-10} \text{ m})N^2 \\ r &= (0.264 \times 10^{-10} \text{ m})N^2 \end{aligned}$$

That fact that the ground state orbital radius is less for singly ionized helium than for hydrogen makes sense. Helium has two protons which exert twice as large a force on the electron than hydrogen. This causes the electron to orbit closer to the nucleus than for hydrogen.

Now let's calculate the energy levels. The energy of the electron and helium nucleus is

$$\begin{aligned} E &= K + U_{elec} \\ &= \frac{1}{2}mv^2 + \frac{1}{4\pi\varepsilon_o}\frac{(+2e)(-e)}{r} \\ &= \frac{1}{2}Fr + \frac{1}{4\pi\varepsilon_o}\frac{(+2e)(-e)}{r} \\ &= \frac{1}{2}\frac{1}{4\pi\varepsilon_o}\frac{2e^2}{r} - \frac{1}{4\pi\varepsilon_o}\frac{2e^2}{r} \\ &= -\frac{1}{4\pi\varepsilon_o}\frac{e^2}{r} \end{aligned}$$

Substitute the radii of the orbits. Then

$$\begin{aligned} E &= -\frac{1}{4\pi\varepsilon_o}\frac{e^2}{r} \\ &= -\frac{1}{4\pi\varepsilon_o}\frac{e^2}{(0.2645 \times 10^{-10} \text{ m})N^2} \\ &= \frac{-8.71 \times 10^{-18} \text{ J}}{N^2}\left(\frac{1 \text{ eV}}{1.6 \times 10^{-19} \text{ J}}\right) \\ &= \frac{-54.4 \text{ eV}}{N^2} \end{aligned}$$

The ground state ($N = 1$) is -54.4 eV. The first excited state ($N = 2$) is -13.6 eV. The energy of a photon emitted in a transition from $N = 2$ to $N = 1$ is $54.4 - 13.6 = 40.8$ eV.

These results differ from hydrogen in that each energy state for the He^+ atom is 4 times lower than the equivalent state for hydrogen. The highest energy photon emitted (for a transition from $N = 2$ to $N = 1$) for the He^+ atom is 4 times greater than the equivalent transition for hydrogen.