The University of Saskatchewan

Saskatoon, Canada

Department of Computer Science

CMPT 280– Intermediate Data Structures and Algorithms

Assignment 8

Date Due: April 8, 2022, 6:00pm

Total Marks: 36

General Instructions

- Assignments must be submitted using Canvas.
- Programs must be written in Java.
- VERY IMPORTANT: Canvas is very fragile when it comes to submitting multiple files. We insist that you package all of the files for all questions for the entire assignment into a ZIP archive file. This can be done with a feature built into the Windows explorer (Windows), or with the zip terminal command (LINUX and Mac). We cannot accept any other archive formats. This means no tar, no gzip, no 7zip, no RAR. Non-zip archives will not be graded. We will not grade assignments if these submission instructions are not followed.

1 Background

In this section we present material required for Question 1.

1.1 Union-find ADT

A *union-find* ADT (also called a *disjoint-set* ADT) keeps track of a set of elements which are partitioned into disjoint subsets. It is useful for establishing equivalencies of groups of items in a set about which nothing is known initially. For example, suppose we have an initial set of cities:

Vancouver, Edmonton, Regina, Saskatoon, Winnipeg, Toronto, Montreal, Calgary

Let's then suppose that we decide that Vancouver and Edmonton are "equivalent" (this can be defined in any number of ways), that Regina, Saskatoon, and Winnipeg are equivalent, and that Montreal and Calgary are equivalent. Now we would have four subsets of equivalent elements of our overall set:

$$\{\text{Vancouver}, \text{Edmonton}\}, \{\text{Regina}, \text{Saskatoon}, \text{Winnipeg}\}, \{\text{Toronto}\}, \{\text{Montreal}, \text{Calgary}\}$$

Note that since Toronto was not deemed equivalent to anything, it is in its own subset by itself. Now, let's suppose we want to find out which set a particular city is in. This is done by choosing from each subset a *representative* (also called an *equivalence-class label*) which acts as the identifier for that set. Suppose for the sake of simplicity, that we choose the first item in each set as its representative (shown in bold):

$$\{\textbf{Vancouver}, \text{Edmonton}\}, \{\textbf{Regina}, \text{Saskatoon}, \text{Winnipeg}\}, \{\textbf{Toronto}\}, \{\textbf{Montreal}, \text{Calgary}\}$$

If we were to now ask which subset Winnipeg belongs to, the answer would be Regina. Asking which subset an element belongs to is called the *find* operation. The find operation applied to an element returns the representative of the set to which it belongs, for example, find(Winnipeg) = Regina, or find(Calgary) = Montreal, or find(Vancouver) = Vancouver. The find operation is one of the two main operations supported by the Union-Find ADT.

The Union-Find ADT unsurprisingly supports a second operation called *union*. The union operation takes two elements as arguments, and establishes them as being "equivalent", meaning, they should be in the same set. So union(Edmonton, Calgary) would place Calgary and Edmonton in the same subset. But if Edmonton and Calgary are equivalent, then by transitivity, everything in the subsets to which Edmonton and Calgary belong must also be equivalent, so the union operation actually merges two subsets into one — so this is just familiar set union operation!. Thus, union(Edmonton, Calgary) would alter our group of subsets so they look like this:

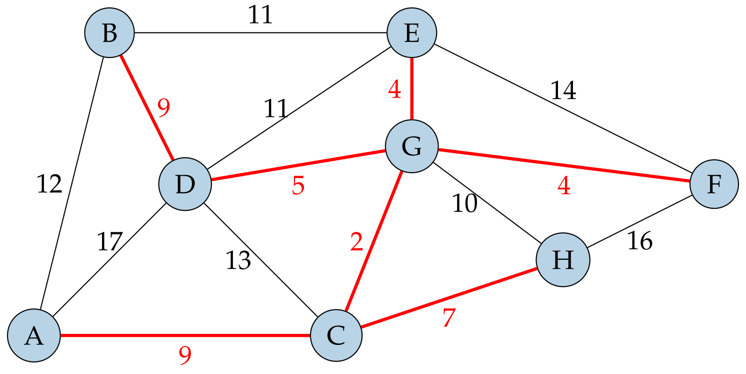
$$\{\textbf{Vancouver}, \text{Edmonton}, \text{Montreal}, \text{Calgary}\}, \{\textbf{Regina}, \text{Saskatoon}, \text{Winnipeg}\}, \{\textbf{Toronto}\}$$

So now Find(Calgary) would result in an answer of Vancouver. You may be wondering why we chose Vancouver as the representative element of the merged subset instead of Montreal. This is an implementation-level decision. In principle, either one could be chosen.

In summary, the Union-Find data structure keeps track of a set of disjoint subsets of a set of elements. It supports the operations find(X) (look up the name of the subset to which element X belongs) and union(X,Y) (merge the subsets containing X and Y). In this assignment we will implement the union-find ADT using a directed, unweighted graph.

1.2 Minimum Spanning Tree

Given a connected, weighted, undirected graph, its minimum spanning tree consists of the subset of the graph's edges of smallest total weight such that the graph remains connected. Such a set of edges always forms a tree because if it weren't a tree there would be a cycle, which implies that it wouldn't be the minimum cost set of edges that keeps the graph connected because you could remove one edge from the cycle and the graph would still be connected. Here is a weighed, undirected graph, and its minimum spanning tree (denoted by thicker, red edges):



No other set of edges that keeps the above graph connected has a smaller sun of weights.

The minimum spanning tree has many applications since many optimization problems can be reduced to a minimum spanning tree algorithm. Suppose you have identified several sites at which to build network routers and you know what it would cost to connect each pair of network routers by a physical wire. You would like to know what is the cheapest possible way to connect all your routers. This is an instance of the minimum spanning tree problem.

Finding the minimum spanning tree isn't as straightforward as it might seem. There are various algorithms for finding the minimum spanning tree. We will be using Kruskal's algorithm which, conveniently, can be implemented efficiently with a union-find ADT.

1.3 Graph Classes in `lib280`

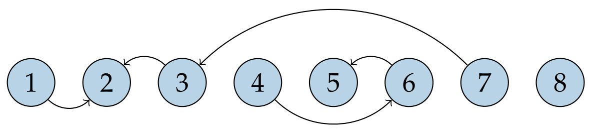
A breakdown of the graph class hierarchy in lib280 and the methods therein can be found in the tutorial 7 materials.

Question 1 (16 points):

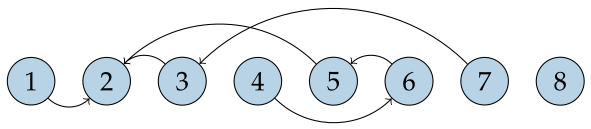
For this problem you will implement Kruskal's algorithm for finding the minimum spanning tree of an undirected weighted graph. Kruskal's algorithm uses a union-find data structure to keep track of subsets of vertices of the input graph G. Initially, every vertex of G is in a subset by itself. The intuition for Kruskal's algorithm is that the edges of the input graph G are sorted in ascending order of weight (smallest weights first), then each such edge (a, b) is examined in order, and if a and b are currently in different subsets we merge the two sets containing a and b and add (a, b) to the graph of the minimum spanning tree. This works because vertices in the same subset in the union-find structure are all connected. Once all of the vertices are in the same subset, we know that they are all connected. Since we always add the next smallest edge possible to the minimum spanning tree, the result is the smallest-cost set of edges that cause the graph to be completely connected, i.e. the minimum spanning tree! Here's Kruskal's algorithm, in pseudocode:

```
Algoirthm minimumSpanningTreeKruskal(G)
G - A weighted, undirected graph.

minST = an undirected, weighted graph with the same node set as G,
        but no edges.

UF = a union-find data structure containing the node set of G in which
      each node is initially in its own subset.

Sort the edges of G in order from smallest to largest weight.

for each edge e=(a,b) in sorted order
    if UF.find(a) != UF.find(b)
        minST.addEdge(a,b)
        set the weight of (a,b) in minST to the weight of (a,b) in G
        UF.union(a,b)

return minST
```

In order to implement Kruskal's algorithm you will first need to implement a union-find ADT. We can implement union-find with a directed (unweighted) graph F. Initially the graph has a node for each item in the set, and no edges. This makes the union operation very easy. The operation union(a,b) can be completed simply by adding the edge $(\text{find}(a), \text{find}(b))$ to F, that is, we add an edge that connects the representative elements of the subsets containing a and b. The find(a) operation then works by checking node a to see if it has an outgoing edge, if it does, we follow it and check the node we get to to see if it has an outgoing edge. We continue going in this fashion until we find a node that does not have an outgoing edge. That node is the representative element of the subset that contains a, and we would return that node. Here's an example of a directed graph that represents a set of subsets of the elements 1 through 8:



If we were to call find(7) on this graph, we would see that 7 has an edge to 3, which has an edge to 2, but 2 has no outgoing edge, so find(7) = 2. Similarly if we called find(4), we would follow the edge to node 6, then its outgoing edge to node 5, and find that 5 has no outgoing edge, so find(4) = 5. Overall, this graph represents that 1, 2, 3, and 7 are in the same subset, which has 2 as its representative element; that 4, 5, and 6 are in the same subset with representative element 5, and 8 is in a subset by itself. Now, suppose we do union(6, 1). This causes an edge to be added from find(6)=5 to find(1)=2, that is, an edge from 5 to 2:



This causes the subsets containing 6 and 1 to be merged, and the new merged subset has representative element 2. Convince yourself that if you call find() on any element except 8, you will get a result of 2 – follow the arrows from the starting node and you'll always end up at 2.

Here are the algorithms for the union and find operations using a graph as the underlying data structure:

```
Algorithm union(a, b)
a, b - elements whose subsets are to be merged

// If a and b are already in the same set, do nothing.
if find(a) == find(b)
   return

// Otherwise, merge the sets
add the edge (find(a), find(b)) to the union-find graph.



Algorithm find(a)
a - element for which we want to determine set membership

// Follow the chain of directed edges starting from a
x = a
while x has an outgoing edge (x,y) in the union-find graph
   x = y

// Since at this point x has no outgoing edge, it must be the
// representative element of the set to which a belongs, so...
return x
```

These are the simplest possible algorithms for union() and find(), and they don't result in the most efficient implementations. There are improvements that we could make, but to keep things simple, we won't bother with them. Eventually, I'll provide solutions that use the above algorithms, as well as an improved, more efficient solution for those who are interested.

Your Tasks

Well, that was a lot of stuff. Now we can finally get to what you actually have to do:

1. Import the project `Kruskal-Template` (provided) module into IntelliJ workspace. You may need to add the `lib280-asn8` project (also provided) as a module depdnency of the `Kruskal-Template` module (this process is covered in the self-guided tutorials on Moodle).

2. In the `UnionFind280` class in the `Kruskal-Template` project, complete the implementation of the methods `union()` and `find()`. Do not modify anything else except that you may add a `main` method to the `UnionFind` class for testing purposes.
3. In `Kruskal.java` complete the implementation of the `minSpanningTree` method. Do not modify anything else.
4. Run the main program in `Kruskal.java`. The pre-programmed input graph is the same as the one shown in Section 1.2. The input graph and the minimum spanning tree as computed by the `minSpanningTree()` method are displayed as output. Check the output to see if the minimum spanning tree that is output matches the one in Section 1.2.

Implementation Hints

- When implementing Kruskal's algorithm, you should be able to avoid having to write your own sorting algorithm, or putting the edges into an array to sort the edges by their weights. You can take advantage of ADTs already in `lib280-asn8a`. All you need is to put the edges in a dispenser which, when you remove an item, will always give you the edge with the smallest weight (hint: look in the lib280.tree package for `ArrayedMinHeap280`). Conveniently, `WeightedEdge280` objects are `Comparable` based on their weight.
- If necessary, you are allowed to modify the path to the input file in the provided main program in `Kruskal.java`. It can be a path to the input file relative to the working directory of the run configuration, or it can be an absolute path.

Question 2 (20 points):

For this question you will implement Dijkstra's algorithm. The implementation will be done within the `NonNegativeWeightedGraphAdjListRep280` class which you can find in the `lib280-asn8.graph` package. This class is an extension of `WeightedGraphAdjListRep280` which restricts the graph edges to have nonnegative weights. This works well for us since Dijkstra's algorithm can only be used on graphs with nonnegative weights.

1. Implement the `shortestPathDijkstra` method in `NonNegativeWeightedGraphAdjListRep280`. The method's javadoc comment explains the inputs and outputs of the method.
2. Implement the `extractPath` method in `NonNegativeWeightedGraphAdjListRep280`. The method's javadoc comment explains the inputs and outputs of the method.

The pseudocode for Dijkstra's algorithm is reproduced below.

```
Algoirthm dijkstra(G, s)
G is a weighted graph with non-negative weights.
s is the start vertex.
Postcondition: v.tentativeDistance is the length of the
               shortest path from s to v.
               v.predecessorNode is the node that appears before v
               on the shortest path from s to v.

Let V be the set of vertices in G.

For each v in V
    v.tentativeDistance = infinity
    v.visited = false
    v.predecessorNode = null

s.tentativeDistance = 0

while there is an unvisited vertex
    cur = the unvisited vertex with the smallest tentative distance.
    cur.visited = true

    // update tentative distances for adjacent vertices if needed
    // note that w(i,j) is the cost of the edge from i to j.
    For each z adjacent to cur
        if (z is unvisited and z.tentativeDistance >
                               cur.tentativeDistance + w(cur,z) )
            z.tentativeDistance = cur.tentativeDistance + w(cur,z)
            z.predecessorNode = cur
```

Implementation Hints

Even though the pseudocode implies that `tentativeDistance`, `visited` and `predecessorNode` are properties of vertices and perhaps should be stored in vertex objects, it is easiest to just use a set of parallel arrays in the implementation of Dijstra's algorithm, much like the way we represented these as arrays during the in-class examples. E.g. an array `boolean visited[]` such that if `visited[i]` is true, it means that vertex `i` has been visited. This is quite easy to use since vertices are always numbered 1 through *n*.

Sample Output

If you done things right, then you should get the following outputs for start vertices 1 and 9 respectively.

```
Enter the number of the start vertex:
1
The length of the shortest path from vertex 1 to vertex 1 is: 0.0
Not reachable.
The length of the shortest path from vertex 1 to vertex 2 is: 1.0
The path to 2 is: 1, 2
The length of the shortest path from vertex 1 to vertex 3 is: 3.0
The path to 3 is: 1, 3
The length of the shortest path from vertex 1 to vertex 4 is: 23.0
The path to 4 is: 1, 3, 5, 6, 4
The length of the shortest path from vertex 1 to vertex 5 is: 7.0
The path to 5 is: 1, 3, 5
The length of the shortest path from vertex 1 to vertex 6 is: 16.0
The path to 6 is: 1, 3, 5, 6
The length of the shortest path from vertex 1 to vertex 7 is: 42.0
The path to 7 is: 1, 3, 5, 6, 4, 8, 9, 7
The length of the shortest path from vertex 1 to vertex 8 is: 31.0
The path to 8 is: 1, 3, 5, 6, 4, 8
The length of the shortest path from vertex 1 to vertex 9 is: 36.0
The path to 9 is: 1, 3, 5, 6, 4, 8, 9


Enter the number of the start vertex:
9
The length of the shortest path from vertex 9 to vertex 1 is: 36.0
The path to 1 is: 9, 8, 4, 6, 5, 3, 1
The length of the shortest path from vertex 9 to vertex 2 is: 35.0
The path to 2 is: 9, 8, 4, 6, 5, 3, 2
The length of the shortest path from vertex 9 to vertex 3 is: 33.0
The path to 3 is: 9, 8, 4, 6, 5, 3
The length of the shortest path from vertex 9 to vertex 4 is: 13.0
The path to 4 is: 9, 8, 4
The length of the shortest path from vertex 9 to vertex 5 is: 29.0
The path to 5 is: 9, 8, 4, 6, 5
The length of the shortest path from vertex 9 to vertex 6 is: 20.0
The path to 6 is: 9, 8, 4, 6
The length of the shortest path from vertex 9 to vertex 7 is: 6.0
The path to 7 is: 9, 7
The length of the shortest path from vertex 9 to vertex 8 is: 5.0
The path to 8 is: 9, 8
The length of the shortest path from vertex 9 to vertex 9 is: 0.0
Not reachable.
```

2 Files Provided

lib280-asn8: A copy of lib280 which includes:

- solutions to assignment 7;
- graph classes necessary for questions 1 and 2.

GraphAdjListRep280 and WeightedGraphAdjListRep280 which you'll use in Question 1
Kruskal-template An IntelliJ module with templates template for question 1.
NonNegativeWeightedGraphAdjListRep280 class for Question 2.

3 What to Hand In

Hand in a ZIP archive containing **only** the following files:

UnionFind280.java Your completed union-find class from Question 1
Kruskal.java Your completed implementation of Kruskal's algorithm from Question 1.
NonNegativeWeightedGraphAdjListRep280.java Your completed implementation of Dijkstra's algorithm from Question 2.