STEM Games Day 2

1 Introduction

Sometimes it is difficult to write down the entire matrix, this is especially the case in $2D$ problems. For this reason it is important to develop methods that can be used when there is a way to quickly calculate Ax for any x. A good example of this is $2D$ convolution.

2 Iterative methods

Assume you have a linear system $Ax = b$. Define each step of the iteration as projecting the current guess onto the hyperplane defined by the $i - th$ equation (or by the i-th row of the matrix).

Task 2.1 *Sketch first* 5 *iterates for the system*

$$\begin{bmatrix} 7 & 8 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 11 \\ 3 \end{bmatrix}$$

starting with iteration $(0, 0)$.

Task 2.2 *Show that the sequence of errors* $||x_i - x_{gt}||_2^2$ *is strictly decreasing.*

Task 2.3 *Apply deconvolution using this method to konv1, konv2 and konv3 problems.*

3 Non-fixed projections

Cosine functions are a good subspace candidate for many problems, but it may be an even better idea to generate a subspace using the problem itself.

Look at the k dimensional subspace spanned by

$$\{b, Ab, A^2b, ..., A^{k-1}b\}$$

Task 3.1 *Apply deconvolution to problems konv1, konv2 and konv3 using this subspace.*

Task 3.2 *Show that an iterative method defined as*

$$x_{k+1} = \min ||Ax - b|| \quad x \in K_k$$

is updated in the direction of the steepest descent