The cascading failure network building

Each project is composed of different tasks, and the completion of upstream tasks is the basis for the beginning of downstream tasks (Crucitti P, 2004;69:045104). Therefore, in order to capture such a structure, the whole project is abstracted into a directed network, G. The nodes p of the network are the tasks that make up the project, and the edges e between the two nodes are the links of tasks. The direction of the nodes is from the upstream task to the downstream task. The project could be then described as,

$$G = (p, e)$$

- Value Assignment

  i. The node

  a. Initial load of nodes

First, each node in the network is assigned an initial load that can represent its characteristics. In this study, the initial load of a node is set based on its degree, that is,

$$initial\_load = (d_p)^{\alpha}$$

α is the adjustment parameter in the simulation process, and d_p is the total weighted degree of the node p, including the sum of the weighted in and out degrees of p, which stand for its connection with its upstream and downstream task respectively.

$$d_p = \sum_{upstream\ j\ to\ p} w_j + \sum_{p\ to\ downstream\ i} w_i$$

  b. Capacity of nodes

The capacity of a node represents the maximum load that a node can afford. For the task in a project, if its load exceeds its capacity, the project has no ability to do it anymore. Here, it is assumed that the capacity of the task is linearly related to the initial load of the node. Since the capacity must be greater than the initial load, it can be expressed as:

$$C_p = Intial\_load(p) \times (1 + \beta)$$
$$= (d_p)^{\alpha} \times (1 + \beta)$$

β is the adjustment parameter in the simulation process.

  c. Weight of nodes

In construction projects, the main factors that affect the completion of each task are the duration, cost and manpower. A long-time duration, high cost and great labour cost are the criteria for important tasks (Adachi, 2007). Therefore, in the project network, the weights of nodes will be expressed by the sum of time, cost and manpower after standardization, that is,

$$w_p = normalised(duration) + normalised(cost) + normalised(manpower)$$

ii. The edge

The weight of the edge is used to represent the importance of the relationship between two tasks, which is calculated by getting the average value of the weight of the two nodes the edges linked with.

$$w_{p1,p2} = (w_{p1} + w_{p2})/2$$

- The Principle of Failure spread

In cascading networks, the failure of the upstream task transmits its load to downstream tasks. The load received by each downstream task mainly depends on the importance of the relationship between them, that is, the proportion that their edge weight accounts in the sum of the weight coming out from the upstream task. For example, if p_1 is an initial failed task and p_2 is one of its downstream tasks, then, after p_1 fails, the load of if p_2 will be,

$$\boldsymbol{load(p_2) = initial_load(p_2) + trans_load_{p1to\ p2}}$$

$$= initial_load(p_2) + \frac{w_{p1p2}}{\sum w_{p1p_{out}}} \times initial\_load(p_1)$$

Here, the nodes with 'out' are the downstream tasks of p_1. If the load of p_2 does not exceed its capacity after p_1 fails, it means that p_2 could recover from the failure of p_1 without causing its own failure. On the contrary, if the load of p_2 exceeds its capacity, p_2 will be affected by the failure of p_1 and spread its load downward to form cascading failure. In order to evaluate the effect of cascading failure result, the index CF is mainly used (Guo, 127 (2019)), which is related to the ratio of the number of tasks directly affected by the failure of p_1 to the total number of failed tasks, that is,

$$CF = \frac{N_{f_i}}{N_f(N-1)}$$

N_{f_i} is the number of nodes failed directly caused by the failure of the seed node, and N_f is the number of all failed nodes, N is the number of all nodes. The larger the CF value, the smaller the scope of failure diffusion and the lower the cascading effect. When CF is equal to $\frac{1}{N-1}$, means that, the failure will not be spread to any downstream nodes of the seed nodes. In this case, the failure will be fixed, and all the projects involved could overcome and recover from this little disturbance.

- Attack set

Because of the variety of tasks in the project, it is more effective to control cascade failures caused by understanding how the cascading effect caused be different tasks failure would be like. Since in complex networks, centrality measurement is the most commonly used method to figure the network structure out (Zhao L, 2004). The popular measures of centrality are degree and betweenness. Therefore, it makes sense to use two kinds of nodes to attack the network, which are,

(1) The node with the greatest betweenness: betweenness indicates the importance of nodes in the network.

(2) The node with the greatest degree: the nodes with the most extensive impact.

- Network building

This simulation process involves a network containing three projects and two companies.

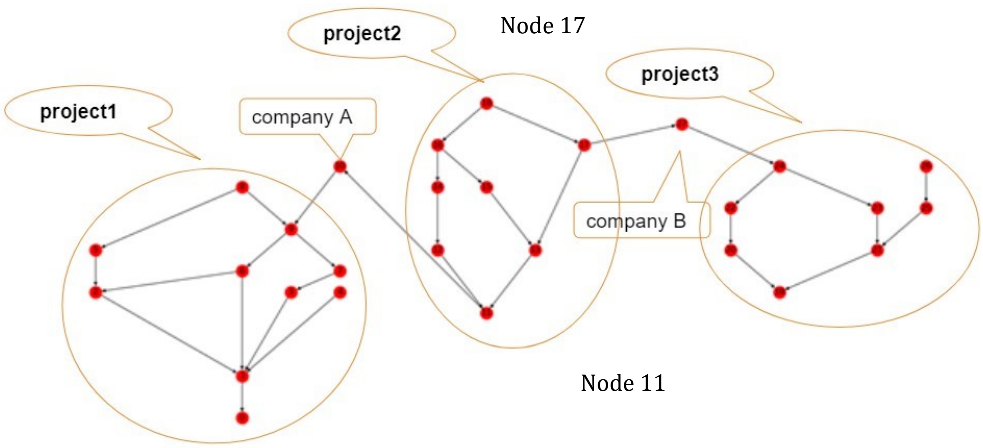


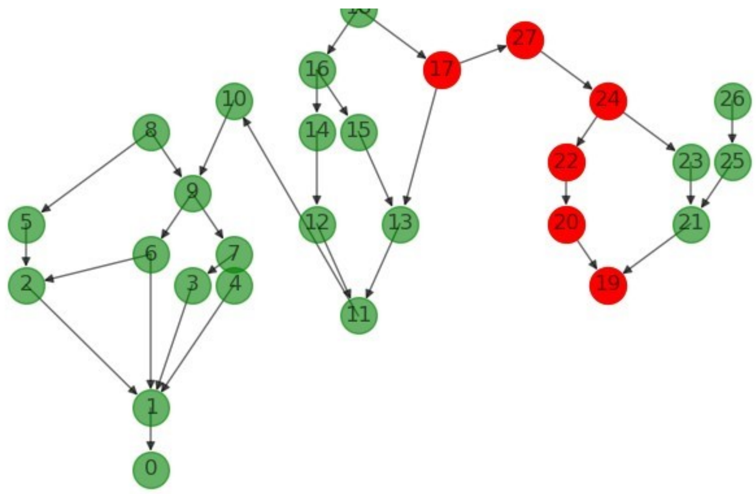
Figure 10 Projects Network

Company A is a subcontractor of both Project A and Project B, while Company B is a subcontractor of Project 2 and Project 3, respectively. In this simulation, the cascading effect is studied under the assumption that treating the failure of tasks in project 2 as the initial failure nodes, which are seed nodes to attack the network. After calculating and sorting the betweenness and degree, the node with the greatest betweenness in project two is node11, while the node with the largest degree (weighted) is node 17. Therefore, node 11 and 17 are used as the seed nodes to attack the network.

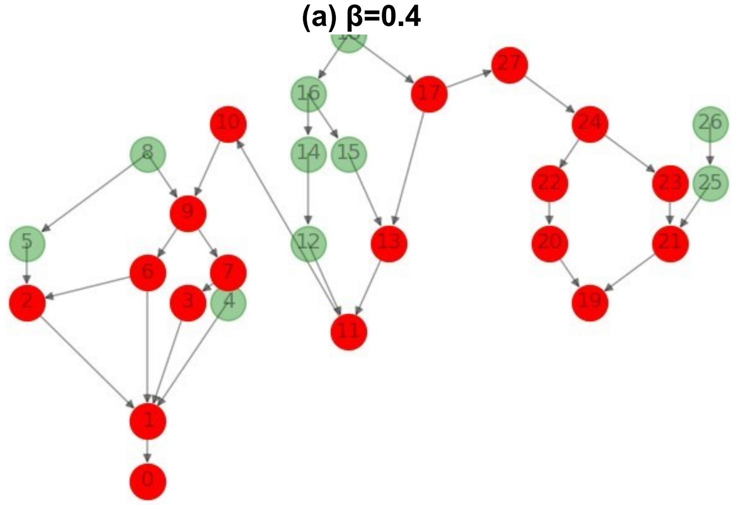
- Simulation Result

In order to show the cascading failure process more clearly, the simulation with taking node 17 as the seed node, setting α as 0.4 is described here as an example, as figure 10.

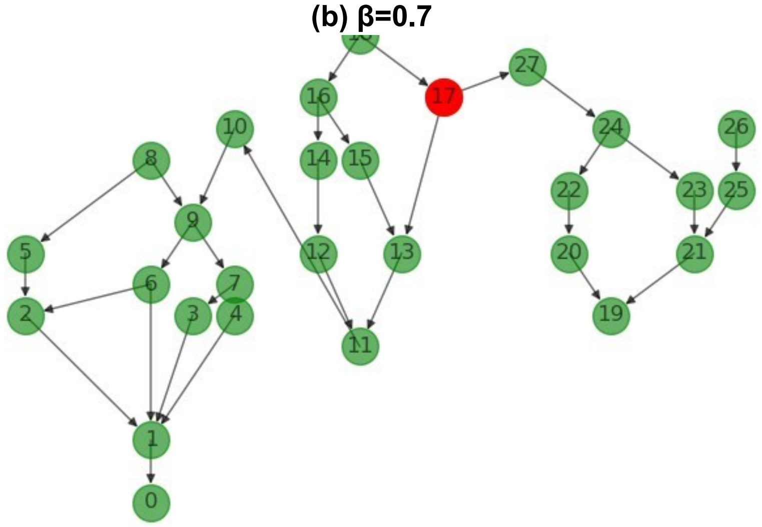
As can be seen from the figure, keeping the load of the task remains unchanged, the cascading effect would decrease with the capacity of nodes increasing. When the capacity of Task 17 is 1.4 times the initial load, the failure of Task 17 is spread by the companies involved in two projects, which results in the failure of three projects at the same time. When the capacity is 1.7 times the initial load, the failure of Task 17 only leads to the failure of Project 3 through Company B. When the capacity is twice the initial load, the failure of Task 17 will be repaired directly without any bad result.



(a) β=0.4



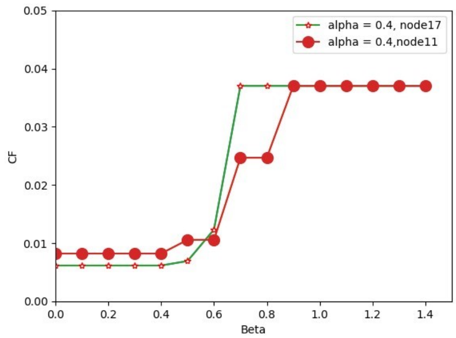
(b) β=0.7

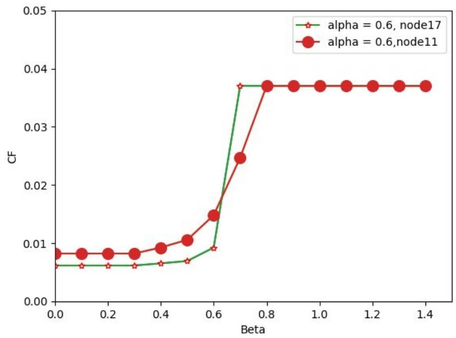


(c) β=1

Figure 11 Cascading failure spreading process

In order to study the trend of cascading effect, five different values are assigned to α, which are 0.4, 0.6, 0.8, 1.0, 1.5 and β is get in range (0,1.5,0.1) in the fitting process. The simulation results are as follows,





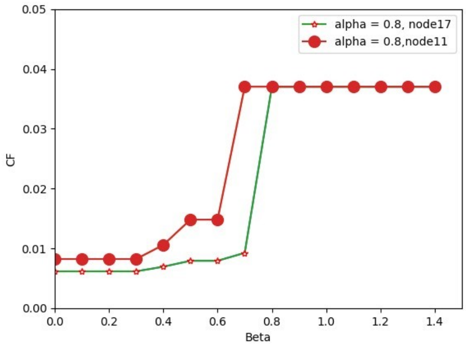
(a) **(b)**

As can be seen from Figure (a), cascading failure caused by node 11 has the most extensive impact when the capacity of node is less than 1.4 times of the initial load. When the node capacity is between 1.4 and 1.9 times the node initial load, the CF value increases with the increase of the capacity, that is to say, the impact range of cascading failure decreases. When the capacity is greater than or equal to 1.9 times the initial load, the failure of node 11 is within the scope of the project could afford. As long as it is repaired, it will not have an impact, and there will be no cascading effect.

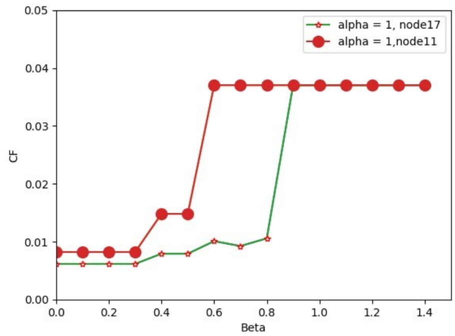
For node 17, as for node 11, the cascading effect spread the most widely when the capacity is less than 1.4 times the initial load. However, with the increase of node capacity, the CF value rapidly reaches its maximum after a short period of increase, which means capacity reaches the threshold that the project can repair the node failure after that.

The basic trend of the two lines in Figure b is similar to Figure a, except that the capacity threshold of node 11 decreases to 1.8 times the initial load. What is more, the

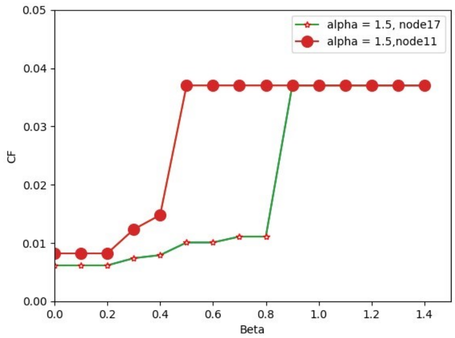
CF value of Node 11 increases more slowly in the growth stage before the capacity reaching the threshold.



(c)



(d)



(e)

Figure 12 Simulation Result

In Figure c, the capacity threshold of node 11 continues to decrease to 1.7 times the initial load, while the capacity threshold of node 17 increases to 1.8 times. There is another change happened in the growth stage of the CF value of node 17, which contains 5 times increases before reaching the low limit, that has an obvious difference from 3 times in Figure b.

In Figure d and Figure e, the difference of capacity thresholds between the two lines is increasing. Node 11 has been reduced to 1.5 times, while Node 17 has increased to 1.9 times. In addition, the growth stage of CF value for node 11 becomes shorter, from 4 times increase to only 3 times before the failure can be repaired, while that of node 17 has increase to 7 times growth.

Therefore, based on the comprehensive analysis of the above five figures, as α increases, the capacity threshold of node 11 decreases continuously, and the growth stage if the CF value, that is, the decreasing stage of cascading effect has a shortening trend. However, the capacity threshold of node 17 is increasing, and the growth stage of CF value become longer. Nevertheless, the rate of growth, which is represented by the slope of the growth stage, has not changed significantly.

The simulation of cascaded networks shows that the failure of a single task in a project can affect the performance of other tasks, even influence the result of several projects.