

Problem 1: Runtime Analysis

a. $\Theta(\log(\log(n)))$

- $i = 2$: takes $\Theta(1)$ time
- Since i is updated exponentially ($i = i * i$), after j iterations, i will equal to 2^{2^j}
- Loop is done when $i == n$, therefore, finding $k : 2^{2^k} = n$
- $k = \log(\log_2(n))$
- Inside the loop, it takes $\Theta(1)$ time
- $\Theta(\log(\log_2(n))) * \Theta(1) = \Theta(\log(\log_2(n)))$

b. $\Theta(n^3)$

- Outer loop from 1 to n takes $\Theta(n)$ time
- Runs $\text{pow}(i, 3)$ when the condition is true (i is a multiple of \sqrt{n})
- Each multiple of \sqrt{n} , loop runs i^3 times.
- Leading term is n^3

c. $\Theta(n^2 \log(n))$

- i runs from 1 to n takes $\Theta(n)$ time
- k runs from 1 to n takes $\Theta(n)$ time
- $A[k] == i$ comparison takes $\Theta(1)$ time
- $m = 1 \leq n$, with m being doubled each iteration takes $\Theta(\log(n))$ time
- Each iteration does something in $\Theta(1)$ time
- $\Theta(n) * \Theta(n) * \Theta(\log(n)) = \Theta(n^2 \log(n))$

d. $\Theta(n)$

- Initializing array takes $\Theta(1)$ time
- When $i == \text{size}(10)$, the array must be resized
- Resizing all iterations takes $\Theta(n)$ because the total elements copied is a geometric series
- $a[i] = i * i$ takes $\Theta(1)$ time
- $\Theta(n) * \Theta(1) = \Theta(n)$

Problem 2: Linked List Recursion Tracing

a. $\text{in1} = 1,2,3,4$ and $\text{in2} = 5,6$

- Starts with $\text{in1} \rightarrow 1$
- Because in1 is not a null pointer, $\text{in1} \rightarrow \text{next}$, resulting in a recursive call llrec with $\text{in1} \rightarrow \text{next}$
- $\text{in} \rightarrow 2$
- Same conditional checks as before, $\text{in1} \rightarrow \text{next}$ results in a recursive call llrec with $\text{in1} \rightarrow 3$
- Same conditional checks as before, $\text{in1} \rightarrow \text{next}$ results in a recursive call llrec with $\text{in1} \rightarrow 4$

- As $in1 \rightarrow 4$, the recursive call `llrec` results in $in1 \rightarrow \text{next}$, where $in1$ is now pointing to a null pointer
- Because $in1 \rightarrow \text{next}$ is null, function returns $in2$
- The function returns to goes back to previous recursion call and has $4 \rightarrow \text{next}$ to $5 \rightarrow 6$
- The function returns the second previous recursion call and sets $3 \rightarrow \text{next}$ to $4 \rightarrow 5 \rightarrow 6$
- The function returns the third previous recursion call and sets $2 \rightarrow \text{next}$ to $3 \rightarrow 4 \rightarrow 5 \rightarrow 6$
- The function returns the initial call and sets $1 \rightarrow \text{next}$ to $2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6$
- Final linked list is: $1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6$

b. $in1 = \text{nullptr}$ and $in2 = 2$?

- Function checks if $in1$ is a null pointer
- Function returns $in2$
- Final linked list: 2