

12.6.5.5.3

EE24BTECH11020 - Ellanti Rohith

Question: Find the absolute maximum and minimum value of the function

$$f(x) = 4x - \frac{x^2}{2} \quad (0.1)$$

in the interval $\left(-2, \frac{9}{2}\right)$.

Theoretical Solution:

Given function,

$$f(x) = 4x - \frac{x^2}{2} \quad (0.2)$$

1. First derivative:

$$f'(x) = 4 - x \quad (0.3)$$

2. Second derivative:

$$f''(x) = -1 \quad (0.4)$$

Critical Points:: To find the critical points, set $f'(x) = 0$:

$$4 - x = 0 \implies x = 4 \quad (0.5)$$

Nature of Critical Points: Since $f''(x) = -1 < 0$, $x = 4$ is a **local maximum**.

Evaluate Function at Critical Points and Boundaries: 1. At $x = -2$:

$$f(-2) = 4(-2) - \frac{(-2)^2}{2} = -8 - 2 = -10 \quad (0.6)$$

2. At $x = 4$:

$$f(4) = 4(4) - \frac{(4)^2}{2} = 16 - 8 = 8 \quad (0.7)$$

3. At $x = \frac{9}{2}$:

$$f\left(\frac{9}{2}\right) = 4\left(\frac{9}{2}\right) - \frac{\left(\frac{9}{2}\right)^2}{2} = 18 - \frac{81}{8} = \frac{144 - 81}{8} = \frac{63}{8} \quad (0.8)$$

Absolute Maximum and Minimum:

- Absolute Maximum: $f(4) = 8$
- Absolute Minimum: $f(-2) = -10$

Computational Solution:

Finding the maximum value of the function can also be done using the Gradient Ascent method:

$$x_{n+1} = x_n + \alpha f'(x_n) \quad (0.9)$$

Substituting $f'(x) = 4 - x$:

$$x_{n+1} = x_n + \alpha(4 - x_n) \quad (0.10)$$

Similarly, the minimum value can be found using the Gradient Descent method:

$$x_{n+1} = x_n - \alpha f'(x_n) \quad (0.11)$$

Substituting $f'(x) = 4 - x$:

$$x_{n+1} = x_n - \alpha(4 - x_n) \quad (0.12)$$

Let $\alpha = 0.01$ (learning rate). Then, using numerical iterations:

$$x_{min} = -2, \quad f(x_{min}) = -10 \quad (0.13)$$

$$x_{max} = 4, \quad f(x_{max}) = 8 \quad (0.14)$$

The absolute maximum value of the function is 8 at $x = 4$, and the absolute minimum value is -10 at $x = -2$.

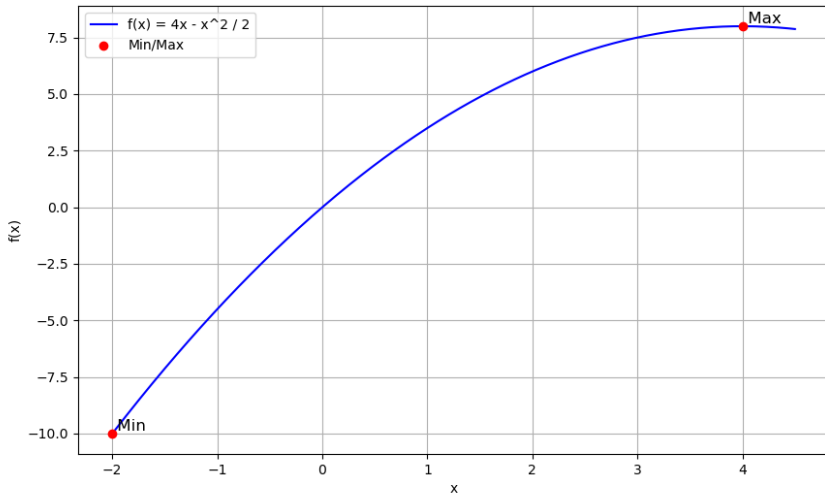


Fig. 0: Plot of the given question.