

11.16.2.2.1

EE24BTECH11020
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Question

A die is thrown. Find the probability of that the outcome is less than 7.

Solution Outline

- 1 Define a random variable.
- 2 Devise the PMF and CDF of the random variable.
- 3 Deduce the required probability from the CDF expression.

Variables Used :

Variable name	Description
S	Sample space
X	Random variable corresponding to the number on die
$F_X(x)$	Cumulative distribution function (CDF)
$p_X(x)$	Probability Mass function (PMF)

Each outcome is equally likely.

Let X be the number obtained when the die is rolled.

$$X \in S$$

Event	Sample space
$p_X(1)$	$\{1\}$
$p_X(2)$	$\{2\}$
$p_X(3)$	$\{3\}$
$p_X(4)$	$\{4\}$
$p_X(5)$	$\{5\}$
$p_X(6)$	$\{6\}$

Since the die is fair, each outcome has an equal probability:

$$p_X(k) = \begin{cases} \frac{1}{6}, & k \in \{1, 2, 3, 4, 5, 6\} \\ 0, & \text{otherwise} \end{cases}$$

By the definition of the cumulative distribution function (CDF):

$$F_X(k) = P(X \leq k) = \sum_{i=-\infty}^k p_X(i)$$

Thus, the CDF is given by:

$$F_X(k) = \begin{cases} 0, & k < 1 \\ \frac{1}{6}, & 1 \leq k < 2 \\ \frac{2}{6}, & 2 \leq k < 3 \\ \frac{3}{6}, & 3 \leq k < 4 \\ \frac{4}{6}, & 4 \leq k < 5 \\ \frac{5}{6}, & 5 \leq k < 6 \\ 1 & k \geq 6 \end{cases}$$

We need to find:

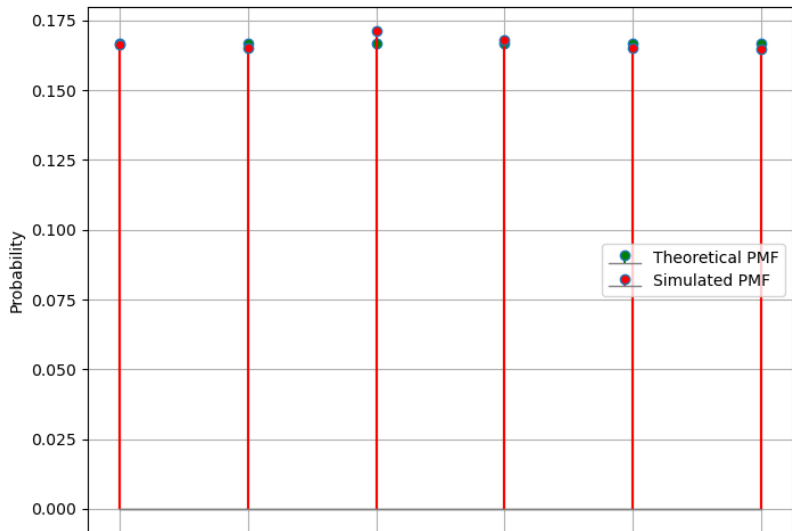
$$\begin{aligned} P(X < 7) &= P(X \leq 6) = \sum_{i=1}^6 P(X = i) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} \\ &= \frac{6}{6} = 1 \end{aligned}$$

Thus,

$$P(X < 7) = 1$$

- 1 Generate a random number using the **rand()** function.
- 2 Restrict random numbers from 1 to 6 by using `rand() % 6` operator.
- 3 Count the number of favourable outcomes by iterating for a large number of trails.
- 4 Divide it by the total number of trails to get the desired PMF.
- 5 CDF can then be simulated by summing the required PMFs.

PMF - Plot



CDF - Plot

