# 10.3.6.1.6

EE24BTECH11020 - Ellanti Rohith

**Question:** A die is thrown. Find the probability of following event: The outcome is less than 7.

**Solution:**The sample space for a fair six-sided die is:

$$S = \{1, 2, 3, 4, 5, 6\}$$

| Variable name | Description |
|---|---|
| **S** | Sample space |
| **X** | Random variable corresponding to the number on die |
| $F_{\mathbf{X}}(x)$ | Cumulative distribution function ( CDF ) |
| $p_{\mathbf{X}}(x)$ | Probability Mass function ( PMF ) |

Each outcome is equally likely.

Let $X$ be the number obtained when the die is rolled.

$$X \in S$$

| Event | Sample space |
|---|---|
| $p_{\mathbf{X}}(1)$ | $\{1\}$ |
| $p_{\mathbf{X}}(2)$ | $\{2\}$ |
| $p_{\mathbf{X}}(3)$ | $\{3\}$ |
| $p_{\mathbf{X}}(4)$ | $\{4\}$ |
| $p_{\mathbf{X}}(5)$ | $\{5\}$ |
| $p_{\mathbf{X}}(6)$ | $\{6\}$ |

Since the die is fair, each outcome has an equal probability:

$$p_X(k) = \begin{cases} \frac{1}{6}, & k \in \{1, 2, 3, 4, 5, 6\} \\ 0, & \text{otherwise} \end{cases}$$

By the definition of the cumulative distribution function (CDF):

$$F_X(k) = P(X \leq k) = \sum_{i=-\infty}^{k} p_X(i)$$

Thus, the CDF is given by:

$$F_X(k) = \begin{cases} 0, & k < 1 \\ \frac{1}{6}, & 1 \le k < 2 \\ \frac{2}{6}, & 2 \le k < 3 \\ \frac{3}{6}, & 3 \le k < 4 \\ \frac{4}{6}, & 4 \le k < 5 \\ \frac{5}{6}, & 5 \le k < 6 \\ 1, & k \ge 6 \end{cases}$$

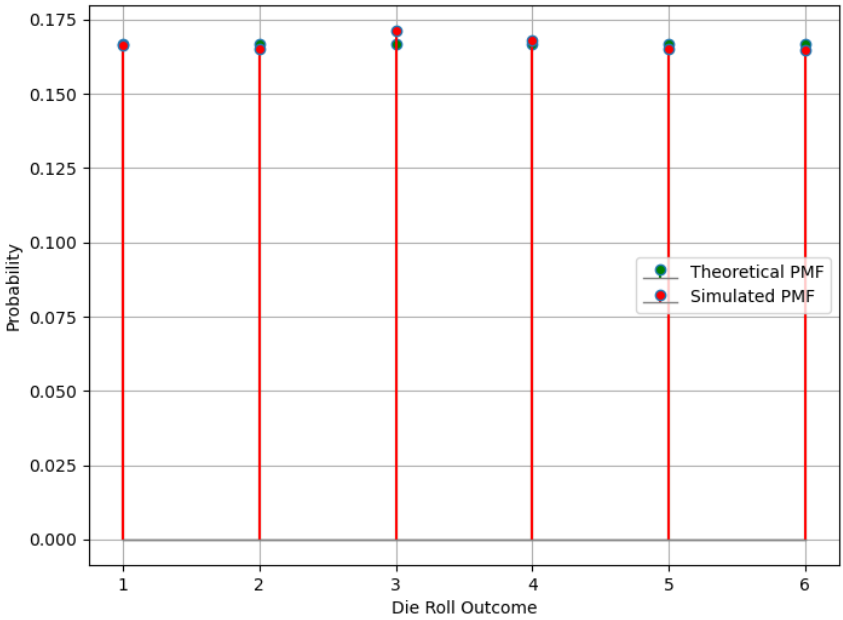
We need to find:

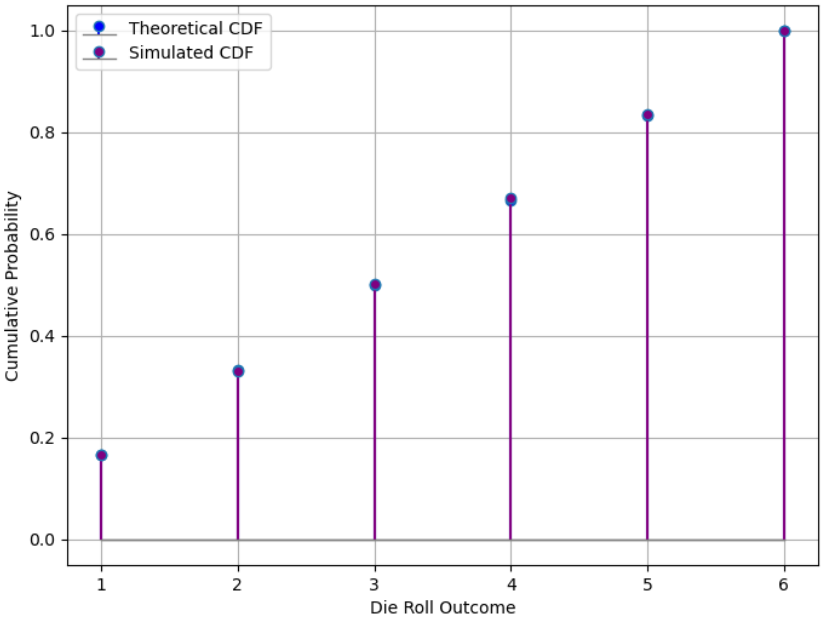
$$P(X < 7) = P(X \le 6) = \sum_{i=1}^{6} P(X = i) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$$

$$= \frac{6}{6} = 1$$

Thus,

$$P(X < 7) = 1$$




Theoretical CDF
Simulated CDF
Cumulative Probability
1.0
0.8
0.6
0.4
0.2
0.0
1
2
3
4
5
6
Die Roll Outcome