

# 9.2.9

EE24BTECH11020 - Ellanti Rohith

**Question:** Using the method of integration find the area of region bounded by lines:  $2x + y = 4$ ,  $3x - 2y = 6$  and  $x - 3y + 5 = 0$ .

**Theoretical Solution:**

**By Integral:**

Let the three lines be defined as:

- 1) Line  $L_1 : 2x + y = 4$ ,
- 2) Line  $L_2 : 3x - 2y = 6$ ,
- 3) Line  $L_3 : x - 3y + 5 = 0$ .

$L_1 : 2x + y = 4$  and  $L_2 : 3x - 2y = 6$

$$\begin{pmatrix} 2 & 1 & 4 \\ 3 & -2 & 6 \end{pmatrix} \xrightarrow{R_2 \rightarrow R_2 - \frac{3}{2}R_1} \begin{pmatrix} 2 & 1 & 4 \\ 0 & -\frac{7}{2} & 0 \end{pmatrix} \xrightarrow{R_1 \div 2, R_2 \div -\frac{7}{2}} \begin{pmatrix} 1 & \frac{1}{2} & 2 \\ 0 & 1 & 0 \end{pmatrix} \xrightarrow{R_1 \rightarrow R_1 - \frac{1}{2}R_2} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \end{pmatrix}$$

Solution:  $x = 2$ ,  $y = 0$ .

$L_2 : 3x - 2y = 6$  and  $L_3 : x - 3y = -5$

$$\begin{pmatrix} 3 & -2 & 6 \\ 1 & -3 & -5 \end{pmatrix} \xrightarrow{R_2 \rightarrow R_2 - \frac{1}{3}R_1} \begin{pmatrix} 3 & -2 & 6 \\ 0 & -\frac{7}{3} & -7 \end{pmatrix} \xrightarrow{R_1 \div 3, R_2 \div -\frac{7}{3}} \begin{pmatrix} 1 & -\frac{2}{3} & 2 \\ 0 & 1 & 3 \end{pmatrix} \xrightarrow{R_1 \rightarrow R_1 + \frac{2}{3}R_2} \begin{pmatrix} 1 & 0 & 4 \\ 0 & 1 & 3 \end{pmatrix}$$

Solution:  $x = 4$ ,  $y = 3$ .

$L_1 : 2x + y = 4$  and  $L_3 : x - 3y = -5$

$$\begin{pmatrix} 2 & 1 & 4 \\ 1 & -3 & -5 \end{pmatrix} \xrightarrow{R_2 \rightarrow R_2 - \frac{1}{2}R_1} \begin{pmatrix} 2 & 1 & 4 \\ 0 & -\frac{7}{2} & -7 \end{pmatrix} \xrightarrow{R_1 \div 2, R_2 \div -\frac{7}{2}} \begin{pmatrix} 1 & \frac{1}{2} & 2 \\ 0 & 1 & 2 \end{pmatrix} \xrightarrow{R_1 \rightarrow R_1 - \frac{1}{2}R_2} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{pmatrix}$$

Solution:  $x = 1$ ,  $y = 2$ . The required area is the sum of:

- 1) The area bounded by  $L_1$  and  $L_3$  over the interval  $x = 1$  to  $x = 2$ , and
- 2) The area bounded by  $L_2$  and  $L_3$  over the interval  $x = 2$  to  $x = 4$ .

The required area can be calculated as the integral of the difference of the functions:

$$f(x) = \begin{cases} \frac{7}{3}x - \frac{7}{3} & \text{for } 1 \leq x \leq 2, \\ -\frac{3x}{2} + \frac{14}{3}, & \text{for } 2 < x \leq 4. \end{cases} \quad (2.1)$$

The area is given by:

$$\text{Area} = \int_1^2 \left( \frac{7}{3}x - \frac{7}{3} \right) dx + \int_2^4 \left( -\frac{3x}{2} + \frac{14}{3} \right) dx \quad (2.2)$$

Thus, the total area is:

$$\text{Area} = \frac{7}{6} + \frac{1}{3} = \frac{9}{6} = \frac{3}{2}. \quad (2.3)$$

Area =  $\frac{3}{2}$  square units.

**Simulation:**

Splitting the intervals with step size  $h = 0.01$

**Trapezoidal Rule:**

Summing area of all Trapezoids to give area under a given curve  $f(x)$

$$A \approx \frac{h}{2} ((f(x_0) + f(x_1)) + (f(x_1) + f(x_2)) + \cdots + (f(x_{n-1}) + f(x_n))) \quad (2.4)$$

$$A \approx \frac{h}{2} \left( f(x_0) + 2 \sum_{i=1}^{n-1} f(x_i) + f(x_n) \right) \quad (2.5)$$

$$A_{n+1} = A_n + \frac{h}{2} (y_n + y_{n+1}) \quad (2.6)$$

This is the required difference equation: (2.7)

$$x_{n+1} = x_n + h \quad (2.8)$$

$$y_{n+1} = y_n + hy'_n \quad (2.9)$$

$$A_{n+1} = A_n + \frac{h}{2} (2y_n + hy'_n) \quad (2.10)$$

$$y'_n = \begin{cases} \frac{7}{3} & \text{for } 1 \leq x_n \leq 2, \\ -\frac{3}{2}, & \text{for } 2 < x_n \leq 4. \end{cases} \quad (2.11)$$

By simulation, the answer turns out to be  $1.49933450000053 \approx \frac{3}{2}$ .

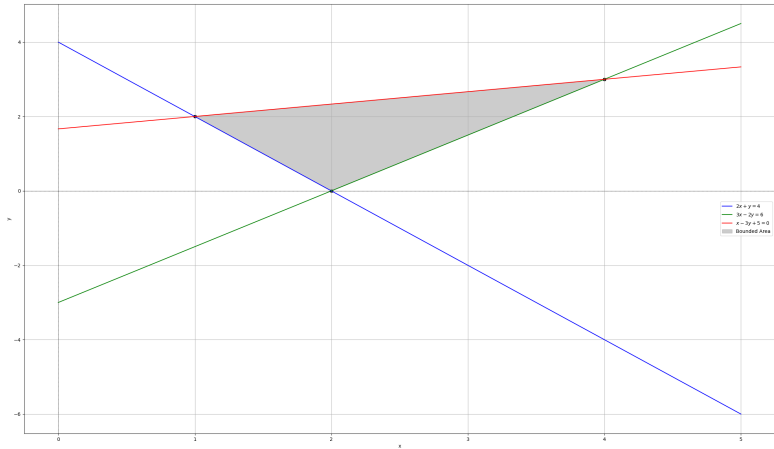


Fig. 2: Plot of three lines and the bounded area.