12.6.5.5.3

EE24BTECH11020 - Ellanti Rohith

Question: Find the absolute maximum and minimum value of the function

$$f(x) = 4x - \frac{x^2}{2} \tag{0.1}$$

in the interval $\left(-2, \frac{9}{2}\right)$.

Theoretical Solution:

Given function,

$$f(x) = 4x - \frac{x^2}{2} \tag{0.2}$$

1. First derivative:

$$f'(x) = 4 - x \tag{0.3}$$

2. Second derivative:

$$f''(x) = -1 \tag{0.4}$$

Critical Points:: To find the critical points, set $f'(x) = 0$:

$$4 - x = 0 \implies x = 4 \tag{0.5}$$

Nature of Critical Points: Since $f''(x) = -1 < 0$, $x = 4$ is a **local maximum**. Evaluate Function at Critical Points and Boundaries: 1. At $x = -2$:

$$f(-2) = 4(-2) - \frac{(-2)^2}{2} = -8 - 2 = -10 \tag{0.6}$$

2. At $x = 4$:

$$f(4) = 4(4) - \frac{(4)^2}{2} = 16 - 8 = 8 \tag{0.7}$$

3. At $x = \frac{9}{2}$:

$$f\left(\frac{9}{2}\right) = 4\left(\frac{9}{2}\right) - \frac{\left(\frac{9}{2}\right)^2}{2} = 18 - \frac{81}{8} = \frac{144 - 81}{8} = \frac{63}{8} \tag{0.8}$$

Absolute Maximum and Minimum:

- Absolute Maximum: $f(4) = 8$
- Absolute Minimum: $f(-2) = -10$

Computational Solution:

Finding the maximum value of the function can also be done using the Gradient Ascent method:

$$x_{n+1} = x_n + \alpha f'(x_n) \tag{0.9}$$

Substituting $f'(x) = 4 - x$:

$$x_{n+1} = x_n + \alpha(4 - x_n) \tag{0.10}$$

Similarly, the minimum value can be found using the Gradient Descent method:

$$x_{n+1} = x_n - \alpha f'(x_n) \tag{0.11}$$

Substituting $f'(x) = 4 - x$:

$$x_{n+1} = x_n - \alpha(4 - x_n) \tag{0.12}$$

Let $\alpha = 0.01$ (learning rate). Then, using numerical iterations:

$$x_{min} = -2, \quad f(x_{\min}) = -10 \tag{0.13}$$

$$x_{max} = 4, \quad f(x_{\max}) = 8 \tag{0.14}$$

The absolute maximum value of the function is 8 at $x = 4$, and the absolute minimum value is -10 at $x = -2$.

Computational Solution using cvxpy: We can find absolute maximum and minimum using *cvxpy* module in python. Gives $x_{max} = 4, \quad f(x_{\max}) = 8$. However, Python throws an error because the function being minimized is concave. The objective function must be convex for minimization. As a result, we can instead check the endpoints of the boundary within the given interval. At $x = -2$:

$$f(-2) = 4(-2) - \frac{(-2)^2}{2} = -8 - 2 = -10 \tag{0.15}$$

At $x = \frac{9}{2}$:

$$f\left(\frac{9}{2}\right) = 4\left(\frac{9}{2}\right) - \frac{\left(\frac{9}{2}\right)^2}{2} = 18 - \frac{81}{8} = \frac{144 - 81}{8} = \frac{63}{8} \tag{0.16}$$

$$f(-2) < f\left(\frac{9}{2}\right) \tag{0.17}$$

Thus, Minimum occurs at $x = -2$.

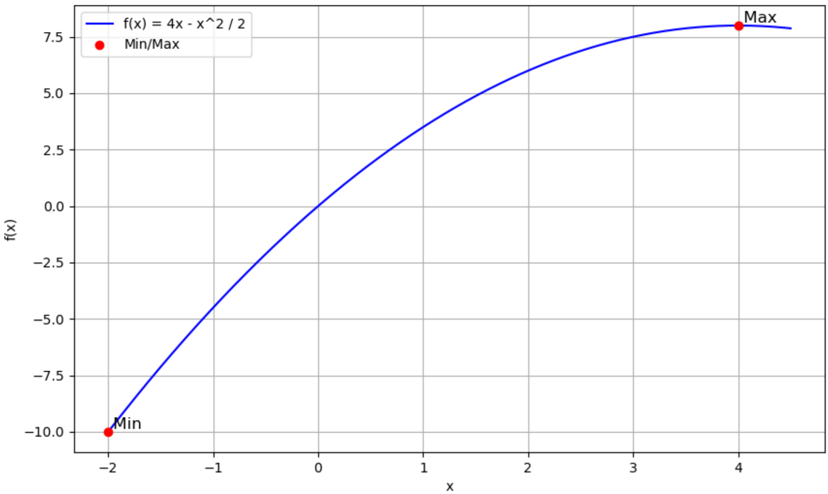

Fig. 0: Plot of the given question.