9.2.9

EE24BTECH11020 - Ellanti Rohith

Question: Using the method of integration find the area of region bounded by lines: $2x + y = 4, 3x - 2y = 6$ and $x - 3y + 5 = 0$.

Theoretical Solution:
By Integral:
Let the three lines be defined as:

- Line $L_1 : 2x + y = 4$,
- Line $L_2 : 3x - 2y = 6$,
- Line $L_3 : x - 3y + 5 = 0$.

The intersection points of the lines are as follows:

- Intersection of L_1 and L_2 is $A(2, 0)$.
- Intersection of L_2 and L_3 is $B(4, 3)$.
- Intersection of L_1 and L_3 is $C(1, 2)$.

The required area is the sum of:

1) The area bounded by L_1 and L_3 over the interval $x = 1$ to $x = 2$, and
2) The area bounded by L_2 and L_3 over the interval $x = 2$ to $x = 4$.

The required area can be calculated as the integral of the difference of the functions:

$$f(x) = \begin{cases} \frac{7}{3}x - \frac{7}{3} & \text{for } 1 \leq x \leq 2, \\ -\frac{3x}{2} + \frac{14}{3}, & \text{for } 2 < x \leq 4. \end{cases} \tag{2.1}$$

The area is given by:

$$\text{Area} = \int_1^2 \left(\frac{7}{3}x - \frac{7}{3} \right) dx + \int_2^4 \left(-\frac{3x}{2} + \frac{14}{3} \right) dx \tag{2.2}$$

Thus, the total area is:

$$\text{Area} = \frac{7}{6} + \frac{1}{3} = \frac{9}{6} = \frac{3}{2}. \tag{2.3}$$

Area $= \frac{3}{2}$ square units.
Simulation:
Splitting the intervals with step size $h = 0.01$
Trapezoidal Rule:

Summing area of all Trapezoids to give area under a given curve $f(x)$

$$A \approx \frac{h}{2}\left(\left(f(x_0)+f(x_1)\right)+\left(f(x_1)+f(x_2)\right)+\cdots+\left(f(x_{n-1})+f(x_n)\right)\right) \tag{2.4}$$

$$A \approx \frac{h}{2}\left(f(x_0)+2\sum_{i=1}^{n-1} f(x_i)+f(x_n)\right) \tag{2.5}$$

$$A_{n+1} = A_n + \frac{h}{2}\left(y_n + y_{n+1}\right) \tag{2.6}$$

This is the required difference equation: (2.7)

$$A_{n+1} = A_n + \frac{h}{2}\left(2y_n + hy_n'\right) \tag{2.8}$$

$$y_n' = \begin{cases} \frac{7}{3} & \text{for } 1 \le x \le 2, \\ -\frac{3}{2}, & \text{for } 2 < x \le 4. \end{cases} \tag{2.9}$$

By simulation, the answer turns out to be $1.49933450000053 \approx \frac{3}{2}$.

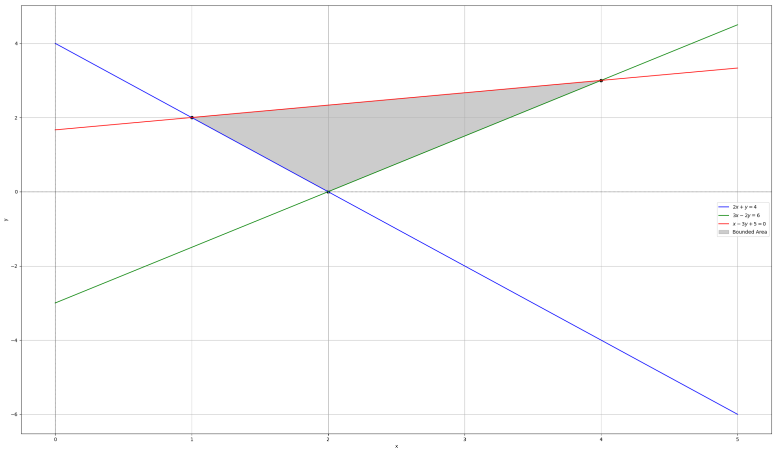

Fig. 2: Plot of three lines and the bounded area.