2021 September 1 Shift 2

EE24BTECH11020 - Ellanti Rohith

1) Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function. Then,

$$\lim_{x \to \frac{\pi}{4}} \frac{\frac{\pi}{4} \int_2^{\sec^2 x} f(x)\, dx}{x^2 - \frac{\pi^2}{16}} \text{ is equal to:}$$

a) $f(2)$
b) $2f(2)$
c) $2f\left(\sqrt{2}\right)$
d) $4f(2)$

2) $\cos^{-1}(\cos(-5)) + \sin^{-1}(\sin(6)) - \tan^{-1}(\tan(12))$ is equal to:

a) $3\pi - 11$
b) $4\pi - 9$
c) $4\pi - 11$
d) $3\pi + 1$

3) Consider the system of linear equations:

$$-x + y + 2z = 0$$
$$3x - ay + 5z = 1$$
$$2x - 2y - az = 7$$

Let S_1 be the set of all $a \in \mathbb{R}$ for which the system is inconsistent, and S_2 the set for infinitely many solutions. Then:

a) $n(S_1) = 2,\ n(S_2) = 2$
b) $n(S_1) = 1,\ n(S_2) = 0$
c) $n(S_1) = 2,\ n(S_2) = 0$
d) $n(S_1) = 0,\ n(S_2) = 2$

4) Let the acute angle bisector of the two planes

$$x - 2y - 2z + 1 = 0$$
$$2x - 3y - 6z + 1 = 0$$

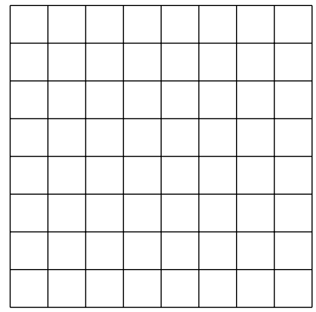
be the plane P. Then which of the following points lies on P ?

a) $\left(3, 1, -\frac{1}{2}\right)$
b) $(0, 2, -4)$
c) $\left(-2, 0, -\frac{1}{2}\right)$
d) $(4, 0, -2)$

5) Which of the following is equivalent to the Boolean expression $p \wedge \sim q$?

a) $\sim(q \to p)$
b) $\sim p \to \sim q$
c) $\sim(p \to \sim q)$
d) $\sim(p \to q)$

6) Two squares are chosen at random on a chessboard (see figure). The probability that they have a side in common is:



(1)

a) $\frac{2}{7}$

c) $\frac{1}{7}$

b) $\frac{1}{18}$

d) $\frac{1}{9}$

7) If $y = y(x)$ is the solution curve of the differential equation

$$x^2 \frac{dy}{dx} + y - \frac{1}{x} = 0$$

with $x > 0$ and $y(1) = 1$, then $y\left(\frac{1}{2}\right)$ is equal to:

a) $\frac{3}{2} - \frac{1}{\sqrt{e}}$

c) $3 + \frac{1}{\sqrt{e}}$

b) $3 + e$

d) $3 - e$

8) If n is the number of solutions of the equation.

$$2\cos x\left(4\sin\left(\frac{\pi}{4} + x\right)\sin\left(\frac{\pi}{4} - x\right) - 1\right) = 1,$$

$x \in [0, \pi]$ and S sum of all these solutions, Then ordered pair (n, S) is:

a) $(3, \frac{13\pi}{9})$ b) $(2, \frac{2\pi}{3})$ c) $(2, \frac{8\pi}{9})$ d) $(3, \frac{5\pi}{3})$

9) The function $f(x) = x^3 - 6x^2 + ax + b$ satisfies $f(2) = f(4) = 0$. Consider two statements:

(S_1) There exist $x_1, x_2 \in (2, 4)$ with $x_1 < x_2$, such that $f'(x_1) = -1$ and $f'(x_2) = 0$.

(S_2) There exist $x_3, x_4 \in (2, 4)$ with $x_3 < x_4$, such that f is decreasing in $(2, x_4)$, increasing in $(x_4, 4)$, and $2f'(x_3) = \sqrt{3}f'(x_4)$. Which of the following is correct?

a) Both (S_1) and (S_2) are true

c) Both (S_1) and (S_2) are false

b) (S_1) is false and (S_2) is true

d) (S_1) is true and (S_2) is false

10)

$$\text{Let } J_{n,m} = \int_0^{\frac{1}{2}} \frac{x^n}{x^m - 1}\, dx, \quad \forall n > m \text{ and } n, m \in \mathbb{N}.$$

Consider a matrix $A = [a_{ij}]_{3\times 3}$ where $a_{ij} = \begin{cases} J_{6+i,3} - J_{i+3,3} & i \leq j, \\ 0 & i > j. \end{cases}$ Then $\left|\text{adj}A^{-1}\right|$ is:

a) $(15)^2 \times 242$

b) $(15)^2 \times 234$

c) $(105)^2 \times 238$

d) $(105)^2 \times 236$

11) The area enclosed by the curves $y = \sin x - \cos x$ and $y = |\cos x - \sin x|$ between $x = 0$ and $x = \frac{\pi}{2}$ is:

a) $2\sqrt{2}(\sqrt{2} - 1)$

b) $2\sqrt{2}(1 + \sqrt{2})$

c) $4\sqrt{2}(\sqrt{2} - 1)$

d) $2\sqrt{2}(2 + \sqrt{2})$

12) The distance from the line $3y - 2z - 1 = 0 = 3x - z + 4$ to the point $(2, -1, 6)$ is:

a) $2\sqrt{6}$

b) $2\sqrt{5}$

c) $4\sqrt{2}$

d) $\sqrt{26}$

13) Consider the parabola with vertex $\left(\frac{1}{2}, \frac{3}{4}\right)$ and directrix $y = \frac{1}{2}$. Let P be the point where the parabola meets the line $x = -\frac{1}{2}$. If the normal to the parabola at P intersects it again at the point Q, then $(PQ)^2$ is equal to :

a) $\frac{75}{8}$

b) $\frac{125}{16}$

c) $\frac{25}{2}$

d) $\frac{15}{2}$

14) The number of pairs (a, b) of real numbers such that whenever α is a root of the equation $x^2+ax+b = 0$, $\alpha^2 - 2$ is also a root is:

a) 6

b) 2

c) 4

d) 8

15) Let $S_n = 1 \cdot (n-1) + 2 \cdot (n-2) + 3 \cdot (n-3) + \cdots + (n-1) \cdot 1$, for $n \geq 4$.

$$\text{The sum } \sum_{n=4}^{\infty} \left(\frac{2S_n}{n!} - \frac{1}{(n-2)!} \right) \text{is equal to:}$$

a) $\frac{e-1}{3}$

b) $\frac{e-2}{6}$

c) $\frac{e}{3}$

d) $\frac{e}{6}$