9-9.2-37

EE24BTECH11020 - Ellanti Rohith

Question:
Draw the rough rough sketch of region $\begin{pmatrix} x & y \end{pmatrix}$: $y^2 < 6ax$ and $x^2 + y^2 < 16a^2$

Variable	Description
$\mathbf{e}$	Eccentricity of conic
$\mathbf{F}$	Focus of conic
$\mathbf{I}$	Identity matrix
$\mathbf{n}^\top \mathbf{x} = c$	Equation of directrix
$\mathbf{n}$	Slope of normal to directrix
f	$\|\mathbf{n}\|^2 \|\mathbf{F}\|^2 - c^2 e^2$
$\mathbf{V}$	A symmetric matrix given by eigenvalue decomposition
$\mathbf{u}$	Vertex of conic with same directrix

TABLE I: Variables Used

Solution: The general equation of a parabola with directrix $\mathbf{n}^\top \mathbf{x} = c$ is given by,

$$g(\mathbf{x}) = \mathbf{x}^\top \mathbf{V} \mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{1}$$

$$\mathbf{V} = \|\mathbf{n}\|^2 \mathbf{I} - e^2 \mathbf{n}\mathbf{n}^\top \tag{2}$$

$$\mathbf{u} = ce^2 \mathbf{n} - \|\mathbf{n}\|^2 \mathbf{F} \tag{3}$$

$$f = \|\mathbf{n}\|^2 \|\mathbf{F}\|^2 - c^2 e^2 \tag{4}$$

for the parabola $y^2 = 6ax$, equation of directrix is, $\begin{pmatrix} 2 & 0 \end{pmatrix} \mathbf{x} = -3a$

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{5}$$

$$\mathbf{u} = \begin{pmatrix} -3a \\ 0 \end{pmatrix} \tag{6}$$

$$f = 0 \tag{7}$$

The given circle can be expressed as conics with parameters

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{8}$$

$$\mathbf{u} = 0 \tag{9}$$

$$f = -16a^2 \tag{10}$$

The intersection of two conics with parameters $\mathbf{V}_i, \mathbf{u}_i, f_i, i = 1, 2$ is defined as,

$$\mathbf{x}^\top (\mathbf{V}_1 - \mathbf{V}_2) \mathbf{x} + 2 (\mathbf{u}_1 - \mathbf{u_2})^\top \mathbf{x} + (f_1 - f_2) = 0 \tag{11}$$

On solving we get the points of intersection to be $2a\begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix}, 2a\begin{pmatrix} 1 \\ -\sqrt{3} \end{pmatrix}$

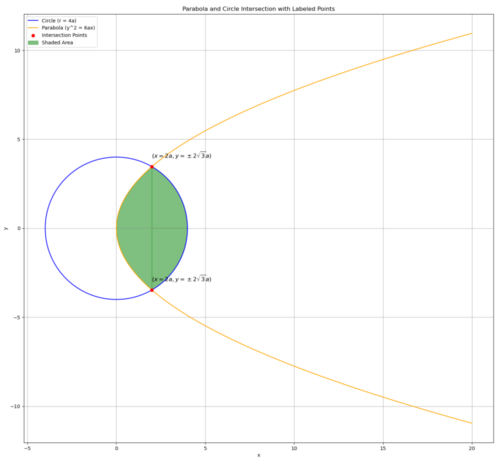

Fig. 1: Sketch of given region