2023 January 29 Shift-1

EE24BTECH11020 - Ellanti Rohith

1) The domain of $f(x) = \frac{\log_{(x+1)}(x-2)}{e^{2\log_e x}-(2x+3)}$, $x \in \mathbb{R}$, is: (Jan 2023)

   a) $\mathbb{R} - \{1, -3\}$ b) $(2, \infty) - \{3\}$ c) $(1, \infty) - \{3\}$ d) $\mathbb{R} - \{-3\}$

2) Let $f : \mathbb{R} \to \mathbb{R}$ be a function such that:

$$f(x) = \frac{x^2 + 2x + 1}{x^2 + 1}$$

   Then: (Jan 2023)
   a) $f(x)$ is many-one in $(-\infty, -1)$
   b) $f(x)$ is many-one in $(1, \infty)$
   c) $f(x)$ is one-one in $[1, \infty)$ but not in $(-\infty, \infty)$
   d) $f(x)$ is one-one in $(-\infty, \infty)$

3) For two non-zero complex numbers z_1 and z_2, if:

$$\text{Re}(z_1 z_2) = 0 \quad \text{and} \quad \text{Re}(z_1 + z_2) = 0,$$

   then which of the following are possible?
   a) $\text{Im}(z_1) > 0$ and $\text{Im}(z_2) > 0$
   b) $\text{Im}(z_1) < 0$ and $\text{Im}(z_2) > 0$
   c) $\text{Im}(z_1) > 0$ and $\text{Im}(z_2) < 0$
   d) $\text{Im}(z_1) < 0$ and $\text{Im}(z_2) < 0$
   Choose the correct answer from the options given below: (Jan 2023)

   a) B and D b) B and C c) A and B d) A and C

4) Let $\lambda \neq 0$ be a real number. Let α, β be the roots of the equation $14x^2 - 31x + 3\lambda = 0$ and α, γ be the roots of the equation $35x^2 - 53x + 4\lambda = 0$. Then $\frac{3\alpha}{\beta}$ and $\frac{4\alpha}{\gamma}$ are the roots of the equation:(Jan 2023)

   a) $7x^2 + 245x - 250 = 0$
   b) $7x^2 - 245x + 250 = 0$
   c) $49x^2 - 245x + 250 = 0$
   d) $49x^2 + 245x + 250 = 0$

5) Consider the system of equations:

$$\alpha x + 2y + z = 1$$
$$2\alpha x + 3y + z = 1$$
$$3x + \alpha y + 2z = \beta$$

   For some $\alpha, \beta \in \mathbb{R}$, which of the following is NOT correct? (Jan 2023)
   a) No solution if $\alpha = -1$ and $\beta \neq 2$
   b) No solution for $\alpha = -1$ for all $\beta \in \mathbb{R}$
   c) No solution for $\alpha = 3$ and $\beta \neq 2$
   d) Solution for all $\alpha \neq -1$ and $\beta = 2$

6) Let α and β be real numbers. Consider a 3×3 matrix A such that: $A^2 = 3A + \alpha\text{I}$. If $A^4 = 21A + \beta\text{I}$ Then: (Jan 2023)

a) $\alpha = 1$ b) $\alpha = 4$ c) $\beta = 8$ d) $\beta = -8$

7) Let $x = 2$ be a root of the equation $x^2 + px + q = 0$ and

$$f(x) = \begin{cases} \frac{1-\cos(x^2-4px+q^2+8q+16)}{(x-2p)^4} & , \quad x \neq 2p \\ 0 & , \quad x = 2p \end{cases}$$

Then $\lim_{x\to 2p^+} [f(x)]$, where [.] denotes the greatest integer function, is (Jan 2023)

a) 2 b) 1 c) 0 d) -1

8) Let $f(x) = x + \frac{a}{\pi^2-4}\sin x + \frac{b}{\pi^2-4}\cos x, \quad x \in \mathbb{R}$ be a function which satisfies

$$f(x) = x + \int_0^{\pi/2} \sin(x+y) f(y)\, dy.$$ Then $(a+b)$ is equal to:

(Jan 2023)

a) $-\pi(\pi+2)$ b) $-2\pi(\pi+2)$ c) $-2\pi(\pi-2)$ d) $-\pi(\pi-2)$

9) Let

$$A = \left\{(x, y) \in \mathbb{R}^2 : y \geq 0, 2x \leq y \leq \sqrt{4-(x-1)^2}\right\}$$

$$B = \left\{(x, y) \in \mathbb{R}\times\mathbb{R} : 0 \leq y \leq \min\left\{2x, \sqrt{4-(x-1)^2}\right\}\right\}$$

Then the ratio of the area of A to the area of B is (Jan 2023)

a) $\frac{\pi-1}{\pi+1}$

b) $\frac{\pi}{\pi-1}$

c) $\frac{\pi}{\pi+1}$

d) $\frac{\pi+1}{\pi-1}$

10) Let Δ be the area of the region $\left\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 21, y^2 \leq 4x, x \geq 1\right\}$. Then $\frac{1}{2}\left(\Delta - 21\sin^{-1}\frac{2}{\sqrt{7}}\right)$ is equal to (Jan 2023)

a) $2\sqrt{3} - \frac{1}{3}$

b) $\sqrt{3} - \frac{2}{3}$

c) $2\sqrt{3} - \frac{2}{3}$

d) $\sqrt{3} - \frac{4}{3}$

11) A light ray emits from the origin at an angle of $30°$ with the positive x-axis. After geting reflected by the line $x + y = 1$, then intersects the x-axis at Q. Then the abscissa of Q is (Jan 2023)

a) $\frac{2}{(\sqrt{3}-1)}$

b) $\frac{2}{(3+\sqrt{3})}$

c) $\frac{2}{(3-\sqrt{3})}$

d) $\frac{2}{2(\sqrt{3}+1)}$

12) Let B and C be the two points on the line $y + x = 0$ such that B and C are symmetric with respect to the origin. Suppose A is a point on $y - 2x = 2$ such that $\triangle ABC$ is an equilateral triangle. Then, the area of the $\triangle ABC$ is (Jan 2023)

a) $3\sqrt{3}$ b) $2\sqrt{3}$ c) $\frac{8}{\sqrt{3}}$ d) $\frac{10}{\sqrt{3}}$

13) The tangents at points $A(4, -11)$ and $B(8, -5)$ on the circle: $x^2 + y^2 - 3x + 10y - 15 = 0$ intersect at C. Find the radius of the circle with center C and line joining A and B as its tangent, is equal to (Jan 2023)

a) $3\sqrt{3}$ b) $2\sqrt{3}$ c) $\frac{8}{\sqrt{3}}$ d) $\frac{10}{\sqrt{3}}$

14) Let $[x]$ denote the greatest integer $\leq x$. Consider the function $f(x) = \max\{x^2, 1 + [x]\}$. Then the value of the integral $\int_0^2 f(x)\,dx$ is: (Jan 2023)

a) $\frac{5+4\sqrt{2}}{3}$ c) $\frac{1+5\sqrt{2}}{3}$

b) $\frac{8+4\sqrt{2}}{3}$ d) $\frac{4+5\sqrt{2}}{3}$

15) If the vectors $\overrightarrow{\mathbf{a}} = \lambda\mathbf{i} + \mu\mathbf{j} + 4\mathbf{k}$, $\overrightarrow{\mathbf{b}} = -2\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$ and $\overrightarrow{\mathbf{c}} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ are coplanar and the projection of $\overrightarrow{\mathbf{a}}$ on $\overrightarrow{\mathbf{b}}$ is $\sqrt{54}$ units, then the sum of all possible values of $\lambda + \mu$ is equal to (Jan 2023)

a) 6 b) 18 c) 0 d) 24