3.2.24

EE24BTECH11020 - Ellanti Rohith

Question: A triangle ABC can be constructed in which $BC = 6cm, \angle B = 30°$ and $AC - AB = 4cm$.

Solution:

Symbol	Description
a	length of side BC
b	length of side CA
c	length of side AB
$\angle A$	angle at vertex A
$\angle B$	angle at vertex B
$\angle C$	angle at vertex C
K	$AC - AB = b - c$

TABLE 0: Variables Used

Using the cosine formula in $\triangle ABC$,

$$(K + c)^2 = a^2 + c^2 - 2ac\cos B \tag{1}$$

$$\implies c = \frac{a^2 - K^2}{2(K + a\cos B)} \tag{2}$$

Substituting the values of K = 4 , a = 6 , and $\cos B = \cos 30° = \frac{\sqrt{3}}{2}$:

$$c = \frac{6^2 - 4^2}{2(4 + 6\cos 30°)} \tag{3}$$

$$c = \frac{36 - 16}{2\left(4 + \left(6 \times \frac{\sqrt{3}}{2}\right)\right)} \tag{4}$$

$$c = \frac{10}{4 + 3\sqrt{3}} \tag{5}$$

The coordinates of $\triangle ABC$ can then be expressed as:

$$\mathbf{A} = c\begin{pmatrix}\cos B \\ \sin B\end{pmatrix}, \mathbf{B} = \begin{pmatrix}0 \\ 0\end{pmatrix}, \mathbf{C} = \begin{pmatrix}a \\ 0\end{pmatrix} \tag{6}$$

$$\mathbf{A} = \frac{10}{4 + 3\sqrt{3}}\begin{pmatrix}\frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}}\end{pmatrix}, \mathbf{B} = \begin{pmatrix}0 \\ 0\end{pmatrix}, \mathbf{C} = \begin{pmatrix}6 \\ 0\end{pmatrix} \tag{7}$$

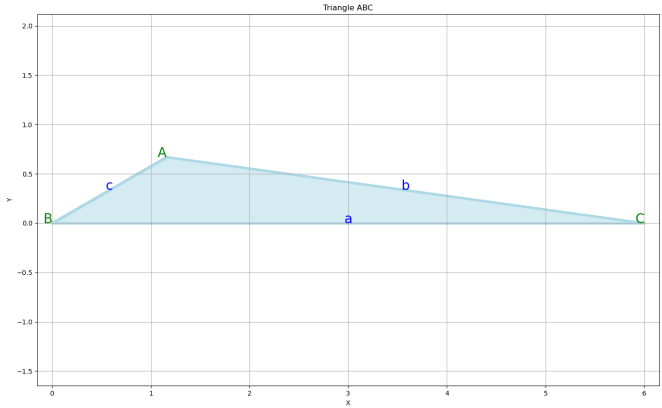

Fig. 0: Triangle with $BC = 6cm, \angle B = 30°$ and $AC - AB = 4cm$