

9-9.2-37

EE24BTECH11020 - Ellanti Rohith

Question:

Draw the rough rough sketch of region $(x \ y)$: $y^2 < 6ax$ and $x^2 + y^2 < 16a^2$

Variable	Description
e	Eccentricity of conic
\mathbf{F}	Focus of conic
\mathbf{I}	Identity matrix
$\mathbf{n}^\top \mathbf{x} = c$	Equation of directrix
\mathbf{n}	Slope of normal to directrix
f	$\ \mathbf{n}\ ^2 \ \mathbf{F}\ ^2 - c^2 e^2$
\mathbf{V}	A symmetric matrix given by eigenvalue decomposition
\mathbf{u}	Vertex of conic with same directrix

TABLE I: Variables Used

Solution: The general equation of a parabola with directrix $\mathbf{n}^\top \mathbf{x} = c$ is given by,

$$g(\mathbf{x}) = \mathbf{x}^\top \mathbf{V} \mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \quad (1)$$

$$\mathbf{V} = \|\mathbf{n}\|^2 \mathbf{I} - e^2 \mathbf{n} \mathbf{n}^\top \quad (2)$$

$$\mathbf{u} = ce^2 \mathbf{n} - \|\mathbf{n}\|^2 \mathbf{F} \quad (3)$$

$$f = \|\mathbf{n}\|^2 \|\mathbf{F}\|^2 - c^2 e^2 \quad (4)$$

for the parabola $y^2 = 6ax$, equation of directrix is, $\begin{pmatrix} 2 & 0 \end{pmatrix} \mathbf{x} = -3a$

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad (5)$$

$$\mathbf{u} = \begin{pmatrix} -3a \\ 0 \end{pmatrix} \quad (6)$$

$$f = 0 \quad (7)$$

The given circle can be expressed as conics with parameters

$$\mathbf{V} = \begin{pmatrix} 4a & 0 \\ 0 & 4a \end{pmatrix} \quad (8)$$

$$\mathbf{u} = 0 \quad (9)$$

$$f = -16a^2 \quad (10)$$

The intersection of two conics with parameters $\mathbf{V}_i, \mathbf{u}_i, f_i, i = 1, 2$ is defined as,

$$\mathbf{x}^\top (\mathbf{V}_1 - \mathbf{V}_2) \mathbf{x} + 2(\mathbf{u}_1 - \mathbf{u}_2)^\top \mathbf{x} + (f_1 - f_2) = 0 \quad (11)$$

On solving we get the points of intersection to be $2a\left(\frac{1}{\sqrt{3}}\right), 2a\left(-\frac{1}{\sqrt{3}}\right)$

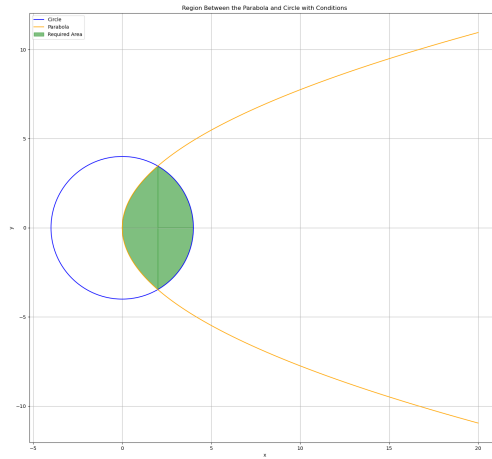


Fig. 1: Sketch of given region