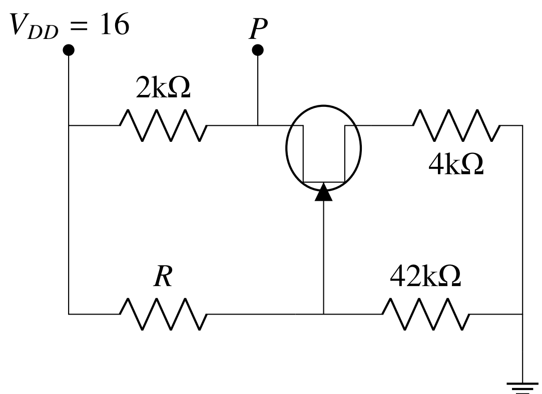
2007 PH 69-85

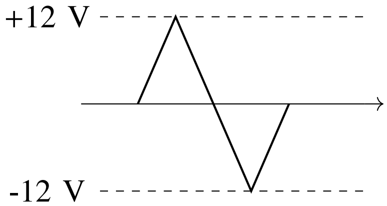
EE24BTECH11020 - Ellanti Rohith

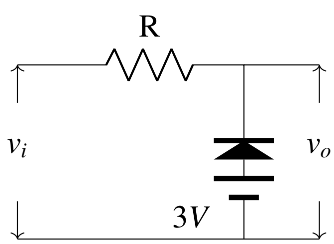
1) In the circuit shown, the voltage at test point P is 12V and the voltage between gate and source is -2V. The value of R (in kΩ) is [GATE 2007]

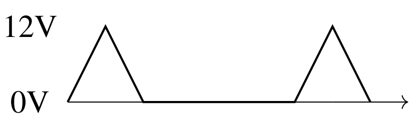


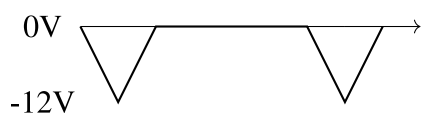
a) 42 b) 48 c) 56 d) 70

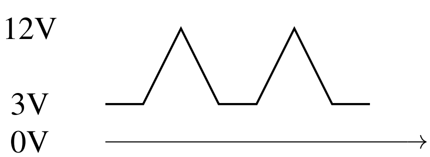
2) When an input voltage V_i, of the form shown, is applied to the circuit given below, the output voltage V_o is of the form [GATE 2007]

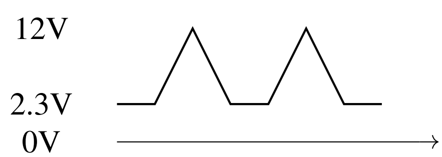




a) 

c) 

b) 

d) 

Common Data for Questions 71, 72, 73:
A particle of mass m is confined in the ground state of a one-dimensional box, extending from $x = -2L$ to $x = +2L$. The wavefunction of the particle in this state is

$$\psi(x) = \psi_0 \cos\left(\frac{\pi x}{4L}\right)$$

where ψ_0 is a constant. [GATE 2007]

3) The normalization factor ψ_0 of this wavefunction is

a) $\frac{\sqrt{2}}{L}$ b) $\frac{1}{\sqrt{4L}}$ c) $\frac{1}{\sqrt{2L}}$ d) $\frac{1}{\sqrt{L}}$

4) The energy eigenvalue corresponding to this state is

a) $\frac{\hbar^2\pi^2}{2mL^2}$ b) $\frac{\hbar^2\pi^2}{4mL^2}$ c) $\frac{\hbar^2\pi^2}{16mL^2}$ d) $\frac{\hbar^2\pi^2}{32mL^2}$

5) The expectation value of p^2 (p is the momentum operator) in this state is

a) 0
b) $\frac{\hbar^2\pi^2}{32L^2}$
c) $\frac{\hbar^2\pi^2}{16L^2}$
d) $\frac{\hbar^2\pi^2}{8L^2}$

Common Data for Questions 74, 75:
The Fresnel relations between the amplitudes of incident and reflected electromagnetic waves at an interface between air and a dielectric of refractive index μ_r are

$$\frac{E_{\|}^{\text{reflected}}}{E_{\|}^{\text{incident}}} = \frac{\cos r - \mu\cos i}{\cos r + \mu\cos i}, \quad \frac{E_{\perp}^{\text{reflected}}}{E_{\perp}^{\text{incident}}} = \frac{\mu\cos r - \cos i}{\mu\cos r + \cos i}$$

The subscripts $\|$ and $\perp$ refer to polarization, parallel and normal to the plane of incidence respectively. Here, i and r are the angles of incidence and refraction respectively. [GATE 2007]

6) The condition for the reflected ray to be completely polarized is

a) $\mu\cos i = \cos r$
b) $\cos i = \mu\cos r$
c) $\mu\cos i = -\cos r$
d) $\cos i = -\mu\cos r$

7) For normal incidence at an air-glass interface with $\mu = 1.5$, the fraction of energy reflected is given by

a) 0.40 b) 0.20 c) 0.16 d) 0.04

Statement for Linked Answer Questions 76 & 77:
In the laboratory frame, a particle P of rest mass m_0 is moving in the positive x direction with a speed of $\frac{5c}{19}$. It approaches an identical particle Q, moving in the negative x direction with a speed of $\frac{2c}{5}$. [GATE 2007]

8) The speed of the particle P in the rest frame of the particle Q is

a) $\frac{7c}{95}$ b) $\frac{13c}{85}$ c) $\frac{3c}{5}$ d) $\frac{63c}{95}$

9) The energy of the particle P in the rest frame of the particle Q is

a) $\frac{1}{2}m_0c^2$ b) $\frac{5}{4}m_0c^2$ c) $\frac{19}{13}m_0c^2$ d) $\frac{11}{9}m_0c^2$

Statement for Linked Answer Questions 78 & 79:
The atomic density of a solid is $5.85\times10^{28}\text{m}^{-3}$. Its electrical resistivity is $1.6\times10^{-8}\,\Omega\,\text{m}$. Assume that electrical conduction is described by the Drude model (classical theory), and that each atom contributes one conduction electron. [GATE 2007]

10) The drift mobility (in $\text{m}^2\text{V}^{-1}\text{s}^{-1}$) of the conduction electrons is

a) 6.67×10^{-3} b) 6.67×10^{-6} c) 7.63×10^{-3} d) 7.63×10^{-6}

11) The relaxation time (mean free time), in seconds, of the conduction electrons is

a) 3.98×10^{-15}
b) 3.79×10^{-14}
c) 2.84×10^{-12}
d) 2.64×10^{-11}

Statement for Linked Answer Questions 80 & 81:
A sphere of radius R carries a polarization $\mathbf{P} = k\mathbf{r}$, where k is a constant and $\mathbf{r}$ is measured from the center of the sphere. [GATE 2007]

12) The bound surface and volume charge densities are given, respectively, by

a) $-k|r|$ and $3k$
b) $k|r|$ and $-3k$
c) $k|r|$ and $-4\pi kR$
d) $-k|r|$ and $4\pi kR$

13) The electric field $\mathbf{E}$ at a point $\mathbf{r}$ outside the sphere is given by

a) $\mathbf{E} = 0$
b) $\mathbf{E} = \frac{kR(R^2-r^2)}{6\varepsilon_0 r^3}\hat{r}$
c) $\mathbf{E} = \frac{kR(R^2-r^2)}{\varepsilon_0 r^3}\hat{r}$
d) $\mathbf{E} = \frac{3k(r-R)}{4\pi\varepsilon_0 r^4}\hat{r}$

Statement for Linked Answer Questions 82 & 83:
An ensemble of quantum harmonic oscillators is kept at a finite temperature $T = \frac{1}{k_B\beta}$. [GATE 2007]

14) The partition function of a single oscillator with energy levels $\left(n + \frac{1}{2}\right)\hbar\omega$ is given by

a) $Z = \frac{e^{-\beta\hbar\omega/2}}{1-e^{-\beta\hbar\omega}}$
b) $Z = \frac{e^{-\beta\hbar\omega/2}}{1+e^{-\beta\hbar\omega}}$
c) $Z = \frac{1}{1-e^{-\beta\hbar\omega}}$
d) $Z = \frac{1}{1+e^{-\beta\hbar\omega}}$

15) The average number of energy quanta of the oscillators is given by

a) $\langle n \rangle = \frac{1}{e^{\beta\hbar\omega}-1}$
b) $\langle n \rangle = \frac{e^{-\beta\hbar\omega}}{e^{\beta\hbar\omega}-1}$
c) $\langle n \rangle = \frac{1}{e^{\beta\hbar\omega}+1}$
d) $\langle n \rangle = \frac{e^{-\beta\hbar\omega}}{e^{\beta\hbar\omega}+1}$

Statement for Linked Answer Questions 84 & 85:
A $16\,\mu A$ beam of alpha particles, having cross-sectional area $10^{-4}\,\text{m}^2$, is incident on a rhodium target of thickness $1\,\mu$m. This produces neutrons through the reaction:

$$\alpha + {}^{100}Rh \to {}^{101}Pd + 3n$$

[GATE 2007]

16) The number of alpha particles hitting the target per second is

a) 0.5×10^{14} b) 1.0×10^{14} c) 2.0×10^{20} d) 4.0×10^{20}

17) The neutrons are observed at the rate of $1.806 \times 10^8\,\text{s}^{-1}$. If the density of rhodium is approximated as $10^4\,\text{kg}\,\text{m}^{-3}$, the cross-section for the reaction (in barns) is

a) 0.1 b) 0.2 c) 0.4 d) 0.8