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Preface

The book root file bookex.tex gives a basic example of how to use LaTeX for preparation of a book. Note that all LaTeX commands begin with a backslash.

Each Chapter, Appendix and the Index is made as a *.tex file and is called in by the include command—thus ch1.tex is the name here of the file containing Chapter 1. The inclusion of any particular file can be suppressed by prefixing the line by a percent sign.

Do not put an enddocument command at the end of chapter files; just one such command is needed at the end of the book.

Note the tag used to make an index entry. You may need to consult Lamport's book [1] for details of the procedure to make the index input file; LATEX will create a pre-index by listing all the tagged items in the file bookex.idx then you edit this into a theindex environment, as index.tex.

2 LIST OF TABLES

Chapter 1

Introducción

El Software Libre y la Microelectrónica en la Argentina

1.1 Microelectrnica en la Universidades Argentinas

El objetivo de esta seccin ser brindar informacin sobre el estado actual de la microelectrnica en nuestro pas, obtener informacin sobre cuantos laboratorios realizan trabajos en microelectronica, cantas materias de grado y postgrado existen. Tambin se brindar un panorama sobre los posibles escenarios de desarrollo de la disciplina, en funcin de las licencias de software que las universidades adopten.

The Extension Problem Given an inclusion $A \stackrel{i}{\hookrightarrow} X$, and a map $A \stackrel{f}{\rightarrow} Y$, does there exist a map $f^{\dagger}: X \to Y$ such that f^{\dagger} agrees with f on A?

Here the appropriate source category for maps should be clear from the context and, moreover, commutativity through a candidate f^{\dagger} is precisely the restriction requirement; that is,

$$f^{\dagger}: f^{\dagger} \circ i = f^{\dagger}|_{A} = f$$
.

If such an f^{\dagger} exists¹, then it is called an **extension** of f and is said to **extend** f. In any diagrams, the presence of a dotted arrow or an arrow carrying a ? indicates a pious hope, in no way begging the question of its existence. Note that we shall usually omit \circ from composite maps.

The Lifting Problem Given a pair of maps $E \xrightarrow{p} B$ and $X \xrightarrow{f} B$, does there exist a map $f^{\circ}: X \to E$, with $pf^{\circ} = f$?

^{1†} suggests striving for perfection, crusading

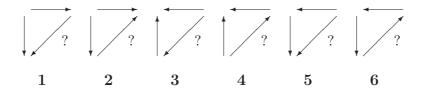
 β

β

N

Figure 1.1: The log-gamma family of densities with central mean $\langle N \rangle = \frac{1}{2}$ as a surface and as a contour plot.

That *all* existence problems about maps are essentially of one type or the other from these two is seen as follows. Evidently, all existence problems are representable by triangular diagrams and it is easily seen that there are only these six possibilities:



Chapter 2

Up to Homotopy is Good Enough

A log with nine holes—old Turkish riddle for a man

2.1 Introducing homotopy

In a topological category, a pair of maps $f, g: X \to Y$ which agree on $A \subseteq X$ is said to admit a **homotopy** H from f to g **relative to** A if there is a map

$$X \times \mathbb{I} \xrightarrow{H} Y : (x,t) \longmapsto H_t(x)$$

with $H_t(a) = H(a,t) = f(a) = g(a)$ for all $a \in A$, $H_0 = H(0,0) = f$, and $H_1 = H(0,1) = g$. Then we write $f \stackrel{H}{\sim} g$ (relA).

If $A = \emptyset$ or A is clear from the context (such as A = * for pointed spaces, cf. below), then we write $f \overset{H}{\sim} g$, or sometimes just $f \sim g$ and say that f and g are **homotopic**.

We can also think of H as either of:

• a 1-parameter family of maps

$$\{H_t: X \longrightarrow Y \mid t \in [0,1] \}$$
 with $H_0 = f$ and $H_1 = g$;

• a curve c_H from f to g in the function space Y^X of maps from X to Y

$$c_H:[0,1]\longrightarrow Y^X:t\longmapsto H_t.$$

We call f nullhomotopic or inessential if it is homotopic to a constant map. Intuitively, we picture H as a continuous deformation of the graph of f into that of g. The following is an easy exercise.

n	S^n	R^n
1	1	1
2	1	1
3	1	1
4	1	∞
5	1	1
6	1	1
7	28	1
8	2	1
9	8	1
10	6	1
11	992	1
12	1	1
13	3	1
14	2	1
15	16256	1
1		

Table 2.1: Numbers of distinct differentiable structures on real n-space and n-spheres

Proposition 2.1.1 For all $A \subseteq X$, $\sim (relA)$ is an equivalence relation on the set of maps from X to Y which agree on A.

Maps in the same equivalence class of $\sim (relA)$ are said to be **homotopic** (relA).

Bibliography

[1] L. Lamport. LATEX A Document Preparation System Addison-Wesley, California 1986.

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