Bargaining in Monetary Policy Committees

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1 Introduction

The vast majority of central banks use a committee structure to make monetary policy (**?**). Until recently, the Reserve Bank of New Zealand was one of the few exceptions, but now it too will use a committee rather than an individual decisionmaker. Monetary policy committees (MPCs) differ in size and formal and informal operating protocols, and the implications of this heterogeneity are still being examined.[1] **??** describes the pronounced global trend toward committee decisionmaking as a hallmark of recent central banking, but notes that "practice ran well ahead of theory" (**?**, p. X).

In an important contribution to the theory of monetary policy committee decisionmaking, Riboni and Ruge-Murcia (2010) study four voting protocols that MPCs may use: a consensus-based (supermajority) protocol, an agenda-setting protocol in which the chairman makes proposals that are subject to simple majority vote[2], a dictator protocol, and a median voter protocol. The dictator and median voter protocols are referred to as "frictionless" because the interest rate decisions reflect the preferences of the Chairman or the median member, respectively, and thus are observationally equivalent to decisionmaking by an individual. But the consensus and agenda-setting protocols involve bargaining frictions

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[1] Studies on the size of MPCs include **??**, ...

[2] See **?**.

with important implications for the size and frequency of interest rate adjustments. In particular, both imply a *status quo* bias, as there will be an inaction region in which the MPC keeps rates unchanged.

Riboni and Ruge-Murcia estimate the parameters of these models using data on interest rates, inflation, and unemployment for the Federal Reserve (Fed), the Bank of Canada (BoC), the Bank of England (BoE), the European Central Bank (ECB), and the Swedish Riksbank (Riksbank). Each of these MPCs appears to operate at least partially by consensus, though all but the BoC formally operate by simple majority rule. Informal aspects of committee decisionmaking can help explain key features of the data, including high interest rate autocorrelation.

As Riboni and Ruge-Murcia (2010, p. 366) note, a limitation of their modeling framework is that "Because the statistical models of the interest rate implied by these protocols are non-nested, standard tests do not have conventional distributions and cannot be used to discriminate across models." In this paper, we model an extremely general decisionmaking framework that nests, as special cases, the decisionmaking protocols considered in Riboni and Ruge-Murcia (2010). Since our framework does nest the protocols, we can estimate by maximum likelihood the key parameters describing these and a much broader possible set of protocols.

For example, one parameter corresponds to the agenda-setting power of the chairman, or probability that the chairman makes a proposal. This is 0 in the consensus and median voter models, 1 in the agenda-setting and dictator model, but can in theory take any value from 0 to 1.

We estimate these key decisionmaking parameters for the Fed, BoC, BoE, ECB, and Riksbank. We find...[KEY RESULTS HERE]

We also study the case of New Zealand. Though the RBNZ had a single monetary policymaker from [YEAR]-2018, the monetary policymaker was advised by a committee [DETAILS]. We test whether the RBNZ data is consistent with the frictionless models, or whether there is evidence of consensus. Even though the advisory committee had no formal role, maybe informally the individual central banker did try to work by some version of consensus...

Our results are related to a larger empirical literature on monetary policy committees...

Experimental studies have shown that committees can outperform individuals in monetary policy simulations (**??**).

We also contribute to a broader literature on decisionmaking and bargaining in committees. This literature often focuses on legislative committees. [ADD]

2 Theory

We model monetary policy decisionmaking by committee members that differ in their preferences over inflation, and hence in their preferred policy decisions. Our framework is based on the approach in Riboni and Ruge-Murcia (2010), but, importantly, is more general in that it nests the four decisionmaking protocols as special cases.

2.1 Preferences

There is a monetary policy committee that consists of a chairman C and a continuum $i \in [0,1]$ of other members.[3] Committee members are distinguished by their ideal inflation target. Let $\hat{\pi}(i)$ be committee member i's ideal inflation rate, and let $\hat{\pi}_C$ be the chairman's ideal. Each committee member's ideal policy is a random draw from a distribution F, assumed continuously differentiable and admits a density f.[4] Since there are a continuum of committee members, F also represents the empirical distribution of ideal inflation targets.

Committee members have preferences over the rate of inflation that prevails, given by the standard quadratic loss function:

$$u(\pi_t, \hat{\pi}) = -(\pi_t - \hat{\pi})^2$$

where π_t is the inflation rate at time t.[5]

[3]The assumption of a continuum is simply to avoid 'integer problems' that arise in the bargaining game, when the size of the Committee is finite. See Parameswaran and Murray (2018).

[4]In the empirical section, for tractability, we assume that F is completely characterized by its mean μ and standard deviation s, and that $\mathbb{P}(\hat{\pi} < \pi) = F(\frac{\pi-\mu}{s})$. Hence, F is the normalized distribution function, and f is the normalized density.

[5]Riboni and Ruge-Murcia (2010) assume that all agents share the same inflation target, but have preferences given by the asymmetric linex function (see Varian (1974)), which we can think of as a generalization of quadratic preferences that admits non-zero prudence. We verify that the ideal policy (Taylor) rule for each agent is the same under both formulations —and thus our departure from Riboni and Ruge-Murcia (2010) is not severe. Indeed, to say that agents share the same inflation target in their model is a little misleading; this is true if there is no uncertainty about the mapping from policy variables (interest rate) to inflation. However, introducing uncertainty, the heterogeneity in prudence causes different agents to target different levels of expected inflation.

2.2 Economic Environment

The behavior of the economy is described in terms of a Phillips curve (1) and an Aggregate Demand curve (2):

$$\pi_{t+1} = \pi_t + \alpha_1 y_t + \varepsilon_{t+1} \tag{1}$$

$$y_{t+1} = \beta_1 y_t - \beta_2(i_t - \pi_t - \iota) + \eta_{t+1} \tag{2}$$

where y_t is the output gap, ι is the long-run equilibrium real interest rate, $\alpha_1, \beta_2 > 0$ and $\beta_1 \in (0,1)$ are constant parameters, and ε_t and η_t are random innovations. These disturbances persist according to MA1 processes:

$$\varepsilon_t = \gamma u_{t-1} + u_t$$

$$\eta_t = \varsigma v_{t-1} + v_t$$

where $\gamma, \varsigma \in (-1,1)$, which ensures that the processes are invertible. u_t and v_t are assumed independent, and are normally distributed white noise processes with mean 0 and variances σ_u^2 and σ_v^2.

Combining Equations (1) and (2) gives:

$$\begin{aligned}\pi_{t+2} &= \underbrace{\alpha_1\beta_1\iota}_{A} + \underbrace{(1+\alpha_1\beta_2)}_{B}\pi_t + \underbrace{\alpha_1(1+\beta_1)}_{C} y_t - \underbrace{\alpha_1\beta_1}_{D} i_t \\ &\quad + \underbrace{\gamma u_t + \alpha_1\varsigma v_t}_{Z_t} + \underbrace{u_{t+2} + (1+\gamma)u_{t+1}) + \alpha_1 v_{t+1}}_{\xi_t} \\ &= A + B\pi_t + Cy_t - Di_t + Z_t + \xi_t \end{aligned} \tag{3}$$

Notice that, due to the lagged effect on the output gap, the current interest rate affects inflation two periods ahead.

Let $x_t = (\pi_t, y_t, i_{t-1}, \zeta_t)$ be a vector of observables at time t (where $Z_t = D\zeta_t$). For clarity, the period t shocks (u_t and v_t, and thus, ζ_t) are assumed observable to the agents at time t, although clearly the $t+1$ and $t+2$ period shocks will not be. (However, ζ_t is not observable to the econometrician, who takes it as a random variable.) Consider an agent with inflation target $\hat{\pi}$. The optimal period t interest rate from their perspective is the one that minimizes:

$$E_t[(\pi_{t+2} - \hat{\pi})^2 \,|\, x_t]$$

where the expectation is over ξ_t. Taking first order conditions, this implies that the optimal i_t satisfies: $E_t[\pi_{t+2} \,|\, x_t] = \hat{\pi}$. Finally, using (3) to substitute for π_{t+2}, we have:

$$\begin{aligned} i(x_t; \hat{\pi}) &= \underbrace{\frac{A - \hat{\pi}}{D}}_{a(\hat{\pi})} + \underbrace{\frac{B}{D}}_{b} \pi_t + \underbrace{\frac{C}{D}}_{c} y_t + \underbrace{\frac{1}{D} Z_t}_{\zeta_t} \\ &= a(\hat{\pi}) + b\pi_t + cy_t + \zeta_t \end{aligned} \tag{4}$$

Note —in the Appendix, we show that under the alternative preference formulation (outlined in Riboni and Ruge-Murcia (2010)),

$$i_t(x_t; \gamma) = a(\gamma) + b\pi_t + cy_t + \zeta_t$$

where b,c and ζ are exactly as above, and $a(\gamma) = \frac{1}{D}(A - \pi^* + \frac{1}{2}\gamma\sigma_\xi^2)$, where γ is the prudence parameter and π^* is the common inflation target. Clearly, setting $\hat{\pi} = \pi^* - \frac{1}{2}\gamma\sigma_\xi^2$ causes $a(\gamma) = a(\hat{\pi})$.

The optimal interest rate is the sum of four terms; it is responsive to the current inflation rate (π_t), the current output gap (y_t), and other innovations to the AD and Phillips Curves (ζ_t). Amongst these, notice that only the constant term, $a(\hat{\pi})$, is a (linear) function of $\hat{\pi}$. Hence, the heterogeneous inflation target of agents simply has the effect of linearly shifting their ideal interest rate, independent of macro-economic observables. Since u_t and v_t where normally distributed white noise processes, ζ_t also has a mean-zero normal distribution with conditional variance $\sigma_\zeta^2 = (\frac{\gamma}{\alpha_1\beta_2})^2\sigma_u^2 + (\frac{\varsigma}{\beta_2})^2\sigma_v^2$.

2.3 Bargaining

Let $S \subset [0,1] \cup \{C\}$ denote a coalition. A coalition S is *decisive* if the support of all members in S is sufficient to implement some policy. Let $\mathcal{S}$ denote the set of decisive coalitions.

This framework admits a broad range of decision rules, subject to the requirement that $\mathcal{S}$ be (i) *monotone* ($S \in \mathcal{S}$ and $S \subset S'$ implies $S' \in \mathcal{S}$), and (2) *proper* ($S \in \mathcal{S}$ implies $N \setminus S \notin \mathcal{S}$). Let $\lambda(S)$ be the fraction of committee members in coalition S. The framework admits, as special cases, each of the five regimes considered in Riboni and Ruge-Murcia (2010):

- Super-majority in Committee: $\mathcal{S} = \{S \subset [0,1] \cup \{C\} \,|\, \lambda(S) \geq q\}$ where $q \geq \frac{1}{2}$ is the required super-majority.

- Agenda Setting Chairman with Simple Majority in Committee: $\mathcal{S} = \left\{S \subset [0,1] \cup \{C\} \mid \lambda(S) \geq \frac{1}{2} \text{ and } C \in S\right\}$. Riboni and Ruge-Murcia (2010) divide this category into two sub-cases:
  - Hawkish chairmen (i.e. $\hat{\pi}_C < \hat{\pi}_{med}$).
  - Dovish chairmen (i.e. $\hat{\pi}_C > \hat{\pi}_{med}$).
- Simple Majority in Committee: $\mathcal{S} = \left\{S \subset [0,1] \cup \{C\} \mid \lambda(S) \geq \frac{1}{2}\right\}$.
- Dictatorial Chairman: $\mathcal{S} = \{S \subset [0,1] \cup \{C\} \mid C \in S\}$.

Of course, the framework also admits other decision procedures, including hybrid procedures, such as an Agenda Setting Chairman with Super-Majority in Committee, or a Chairman with (qualified)-veto power.

The bargaining protocol is the standard procedure in Baron and Ferejohn (1989) and Banks and Duggan (2006). There are potentially infinitely many intra-period bargaining rounds. Let p_C be the probability that the chairman makes the proposal in a given bargaining round. Similarly, let $(1-p_C)F(\hat{\pi})$ be the probability of a proposal from a committee member whose preference parameter is at most $\hat{\pi}$.[6] After observing the proposal, all players simultaneously vote to either accept or reject the proposal. Acceptance requires that the proposal receive the assent of some decisive coalition $S \in \mathcal{D}$. If so, the policy is implemented, and the bargaining game ends. In the event of disagreement, with probability $\delta \in [0,1)$, another proposer is randomly selected to make a proposal, and the process continues as described above. With probability $1-\delta$, bargaining terminates exogenously, and the status quo interest rate i_{t-1} is implemented.

A strategy for a player $j \in [0,1] \cup C$ is a pair $s_j = (y_j, A_j)$, where y_j is the policy proposed whenever j is as the proposer, and A_j is the set of policies that player j will accept ('the acceptance set'). We solve for stationary sub-game perfect equilibria. We limit attention to strategies that satisfy the weak dominance property; each player supports a proposal only if the utility from having the policy implemented is at least as large as the utility from disagreement.

[6]This formulation implicitly assumes that non-chair committee members are equally recognized to propose policies. In principle, we could allow different committee members to have different recognition probabilities, although this would make identification in the empirical exercise more challenging.

2.4 Equilibrium

In this section, I state equilibrium results without proof or derivation. For a more detailed account, see Parameswaran and Murray (2018) or Parameswaran and Rendleman (2018).

Let $L = \sup_{S\in\mathcal{S}} \inf_{i\in S}\{\hat{\pi}(i)\}$ and let $R = \inf_{S\in\mathcal{S}} \sup_{i\in S}\{\hat{\pi}\}$. We refer to L and R as the *left* and *right decisive agents*, respectively. The requirement that the decision rule is proper ensures that $\hat{\pi}_R \leq \hat{\pi}_R$. Although decisive coalitions need not be connected, since the Spence-Mirrlees condition holds, the coalitions that arise in equilibrium will be connected. It follows that the support of L and R is both necessary and sufficient for a proposal to be equilibrium consistent.

For example, if the decision rule is characterized by:

- q-majority rule, then $\hat{\pi}_L = F^{-1}(1-q)$, $\hat{\pi}_R = F^{-1}(q)$, and $p_C = 0$.
- Hawkish agenda setting chairman with simple majority rule, then $\hat{\pi}_L = F^{-1}(\frac{1}{2})$, $\hat{\pi}_R = \hat{\pi}_C$, and $p_C = 1$.
- Dovish agenda setting chairman with simple majority rule, then $\hat{\pi}_L = \hat{\pi}_C$ and $\hat{\pi}_R = F^{-1}(\frac{1}{2})$, with $p_C = 1$.
- Simple majority rule, then $\hat{\pi}_L = F^{-1}(\frac{1}{2}) = \hat{\pi}_R$, and $p_C \in [0, 1)$.
- Dictatorial chairman, then $\hat{\pi}_L = \hat{\pi}_C = \hat{\pi}_R$, and $p_C \in (0, 1]$.

The bargaining equilibrium will depend crucially on the parameter δ. Our procedure characterizes the bargaining equilibrium for every $\delta \in [0, 1)$, and that characterizes the limit equilibrium as $\delta \to 1$. See Parameswaran and Murray (2018) for a justification of the focus on this limit.

Recall, $i(x_t, \hat{\pi}) = a(\hat{\pi}) + b\pi_t + cy_t + \zeta_t$ gives the ideal interest rate for an agent with inflation target $\hat{\pi}$, given period t fundamentals, x_t. Since preferences are quadratic:

$$\begin{aligned} U(x_t, i_t, \hat{\pi}) = E_t[u(\pi_{t+2}, \hat{\pi}; x_t, i_t)] &= -\left(A + B\pi_t + Cy_t - Di_t + Z_t - \hat{\pi}\right)^2 - \sigma_\xi^2 \\ &= -D^2\left(a(\hat{\pi}) + b\pi_t + cy_t + \zeta_t - i_t\right)^2 + \sigma_\xi^2 \end{aligned}$$

Let $b(x_t; \rho, \hat{\pi}_L, \hat{\pi}_R)$ denote the solution to the bilateral asymmetric Nash Bargaining problem between L and R, where the bargaining weights on L and R are ρ and $1-\rho$, respectively.

We have:

$$b(x_t; \rho, \hat{\pi}_L, \hat{\pi}_R) = \arg\max_{i_t} \left[U(x_t, i_t; \hat{\pi}_L) - U(x_t, i_{t-1}; \hat{\pi}_L)\right]^{\rho} \left[U(x_t, i_t; \hat{\pi}_R) - U(x_t, i_{t-1}, \hat{\pi}_R)\right]^{1-\rho} \tag{5}$$

Given our specific functional form, we can show that b is the solution to:

$$(i(x_t; \hat{\pi}_L) - b)(i(x_t; \hat{\pi}_R) - b) + \frac{1}{2}(b - i_{t-1}) \left[\rho i(x_t; \hat{\pi}_L) + (1-\rho) i(x_t; \hat{\pi}_R) - b\right] = 0 \tag{6}$$

Let $\underline{\rho} = (1 - p_C) F\left(\frac{\hat{\pi}_C - \mu}{s}\right)$ and $\overline{\rho} = (1 - p_C) F\left(\frac{\hat{\pi} - \mu}{s}\right) + p_C$. We are ready to state the main bargaining theorem.

Proposition 1. *The limit equilibrium is characterized as follows:*

1. *Suppose* $b(x_t; \underline{\rho}, \hat{\pi}_L, \hat{\pi}_R) < i(x_t, \hat{\pi}_C) < b(x_t; \overline{\rho}, \hat{\pi}_L, \hat{\pi}_R)$. *Then* $i^*(x_t) = i(x_t, \hat{\pi}_C)$.
2. *Otherwise, the equilibrium is characterized by a pair* (i^*, ρ^*) *satisfying:*

$$\begin{aligned} i^*(x_t) &= b(x_t, \rho^*(x_t); \hat{\pi}_L, \hat{\pi}_R) \\ \rho^*(x_t) &= (1 - p_C) F\left(\frac{i(x_t, \hat{\pi}_\mu) - i^*(x_t)}{s}\right) + p_C \mathbf{1}[\hat{\pi}_C < \Pi(x_t, i_t^*)]. \end{aligned}$$

Proposition (1) states that the equilibrium policy is characterized as the solution to a system of two equations. The first equation says that the equilibrium policy $i^*(x_t)$ is what results from Nash Bargaining between the left and right decisive players, given x_t and (endogenous) bargaining weight $\rho^*(x_t)$. The second equation pins down the bargaining weight in terms of the equilibrium policy. In particular, the bargaining weight of the left decisive agent is simply the collective recognition probability of all agents whose ideal policy is to the left of the equilibrium policy.[7]

Corollary 1. *If* $\hat{\pi}_L = \hat{\pi}_R$, *then* $i^*(x_t) = i(x_t, \hat{\pi}_L) = i(x_t, \hat{\pi}_R)$

Obviously, if the left and right decisive voters have the same preferences, then they will bargain to their ideal outcome.

[7] A caveat. The previous explanation was correct as long as the recognition probability of agents whose ideal policy coincided with the ideal policy is zero. This will be always be true, unless the equilibrium policy coincides with the chairman's ideal policy. In the latter case, we can think of the chairman splitting his support between the two factions. Then $\rho_C(x_t)$ would be the proportion of his support given to the left faction.

The *core* ($\mathcal{K}$) is the set of unbeatable policies given the decision rule (i.e. the set of outcomes for which there is not another policy strictly preferred by a decisive coalition). The core is straightforwardly $\mathcal{K}(x_t) = [i(x_t, \hat{\pi}_L), i(x_t, \hat{\pi}_R)]$. The next Corollary makes the obvious point that, if the status quo policy lies in the core, then the equilibrium outcome will simply be the status quo.

Corollary 2. *If $i_{t-1} \in \mathcal{K}(x_t) = [i(x_t, \hat{\pi}_L), i(x_t, \hat{\pi}_R)]$, then $i^*(x_t) = i_{t-1}$.*

If the status quo policy lies outside the core, then clearly there is an opportunity for a mutually beneficial bargain. Exhausting the gains from bargain requires that the chosen policy be in the core.

3 Empirical Analysis

3.1 The Data

Use the same data as Riboni and Ruge-Murcia (2010).

3.2 Likelihood Functions

We are interested in estimating the reduced form parameters that define the policy function.

The vector of parameters to be estimated is:

$$\theta = (a_R, a_L, a_C, b, c, \mu, s, p_C, \sigma)$$

Let $\omega_t = (y_t, \pi_t, i_{t-1})$, so that $x_t = (\omega_t, \zeta_t)$. Recall, x_t is observable to each committee member, and using this information, the Committee optimally chooses i_t. By contrast, the econometrician observes ω_t and i_t, but not ζ_t. Given the theory model described above, the econometrician treats ζ_t as a random variable with $\zeta_t \sim N(0, \sigma^2)$.

We estimate the model via maximum likelihood, where the model parameters are chosen to maximize the likelihood of the implied sequence $\{\zeta_t\}$, assuming these are independent draws from the above normal distribution. To do so, we need to identify ζ_t, for each observation, and for every conjecture of the parameters θ. More Notation. Recall $i(x_t, \hat{\pi}; \theta) = \alpha(\hat{\pi}) + b\pi_t + cy_t + \zeta_t$ is the ideal interest for an agent with inflation target $\hat{\pi}$ given θ and all observable information at time t. Let $I(\omega_t, \hat{\pi}; \theta) = E_\zeta[i(\omega_t, \zeta_t, \hat{\pi}) \,|\, \omega_t, \theta] = a(\hat{\pi}) + b\pi_t + cy_t$ be the econometrician's best guess of the ideal interest rate of a type $\hat{\pi}$ agent. Thus $i(x_t, \hat{\pi}; \theta) = I(\omega_t, \hat{\pi}; \theta) + \zeta_t$. For notational simplicity, we write: $i^j(x_t) = i(x_t, \hat{\pi}_j)$ and $I^j(\omega_t) = I(\omega_t, \hat{\pi}_j)$ for $j \in \{L, R\}$. Note that $i^L \geq i^R$ and $I^L \geq I^R$, since $a(\hat{\pi})$ is decreasing in $\hat{\pi}$.

Similarly, let $A(i_t; x_t, \theta) = i_t - b\pi_t - cy_t - \zeta_t$ denote the value of a for which the chosen policy i_t is ideal.

There are two cases to consider: First, suppose $i_t = i_{t-1}$. Let $\Theta_0 = \{t \,|\, i_t = i_{t-1}\}$ and let $T_0 = |\Theta_0|$. By Corollary 2, we know that this outcome will arise whenever $i_{t-1} \in [i^R(x_t), i^L(x_t)]$. This implies that $\zeta_t \in [\zeta_L(\omega_t; \theta), \zeta_R(\omega_t; \theta)]$, where $\zeta_L(\omega_t, \theta) = i_{t-1} - I^L(\omega_t)$ and $\zeta_R(\omega_t, \theta) = i_{t-1} - I^R(\omega_t)$. Hence, if $i_t = i_{t-1}$, the realized of ζ_t must be contained in a particular interval.

[Insert lemma showing monotonicity of i_t in ζ_t. i.e. Show that $i^*(\omega_t, \zeta_t; \theta)$ is invertible.]

Second, suppose $i_t \neq i_{t-1}$. Let $\Theta_1 = \{t \,|\, i_t \neq i_{t-1}\}$ and let $T_1 = |\Theta_1|$. By Proposition 1, we know that, in such cases, i_t is the result of asymmetric Nash Bargaining between the decisive voters.

Some more setup: Let $\zeta(\omega_t, i_t, \rho; \theta)$ be the solution to:

$$\zeta^2 + 2\left[\frac{I^L(\omega_t) + I^R(\omega_t)}{2} - i_t + \frac{1}{4}\Delta i_t\right]\zeta + k(\rho) = 0 \tag{7}$$

where $k = (I^L(\omega_t) - i_t)(I^R(\omega_t) - i_t) + \frac{1}{2}\Delta i_t \left[\rho I^L(\omega_t) + (1-\rho)I^R(\omega_t) - i_t\right]$. Define $\rho_l(\theta) = (1 - p_C)F(\frac{\mu - a_C}{s})$ and $\rho_h(\theta) = p_C + (1 - p_C)F(\frac{\mu - a_C}{s})$.

By Proposition 1, there are two sub-cases to consider:

1. If $a_C \in [\zeta(\omega_t, i_t, \rho_l; \theta), \zeta(\omega_t, i_t, \rho_l; \theta)]$, then the chairman is pivotal. $\zeta_t = i_t - a_C - b\pi_t - cy_t$. (There is no need to find ρ_t.)

2. If $C \notin [\zeta(\omega_t, i_t, \rho_l; \theta), \zeta(\omega_t, i_t, \rho_l; \theta)]$, then ζ_t and ρ_t are jointly determined, with:

$$\zeta_t = \zeta(\omega_t, i_t, \rho_t; \theta) \tag{8}$$

$$\rho(\omega_t, i_t; \theta) = (1 - p_C)F\left(\frac{\mu - (i_t - b\pi_t - cy_- \zeta_t)}{s}\right) + p_C \mathbf{1}[i < a_C + b\pi_t + cy_t + \zeta_t] \tag{9}$$

Taken together, these imply that the log-likelihood function is:

$$l(\theta,\sigma) = -T_1 \ln\sigma - \frac{1}{2\sigma^2}\sum_{t\in\Theta_1}\zeta(\omega_t,\theta)^2 + \sum_{t\in\Theta_0}\ln\left[\Phi\left(\frac{\zeta_L(\omega_t,\theta)}{\sigma}\right) - \Phi\left(\frac{\zeta_R(\omega_t,\theta)}{\sigma}\right)\right] \tag{10}$$

3.3 Identification

[This section is wonky.]

Some concerns about identification:

- If $L = a_R$, then the equilibrium mapping depends only on (a_R, b, c, σ). Hence (a_C, p_C, μ, s) are not identifiable.
- If $a_C \notin (a_R, a_L)$, then we can at most identify whether $a_C < a_R$ or $a_C > a_L$.
- If $a_C = 1$, then the equilibrium mapping does not depend on (μ, s) and either a_L or a_R (whichever is closer to a_C) is not identifiable.

Otherwise, the equilibrium mapping is sensitive to all the parameters $(a_R, a_L, a_C, b, c, \mu, s,_C\ ,)$ and variation in i_t should reveal σ_ζ.

3.4 Testing the Model

Since we are able to estimate a general model that nests each of the Riboni and Ruge-Murcia (2010) decision procedures as special cases, we can test for evidence that those procedures are in operation. The relevant hypotheses would be:

- Consensus Model: $H_0:\ F(\frac{a_R-\mu}{s}) + F(a_R - \mu s) = 1$.
- Hawkish Agenda Setter: $H_0: p_C = 1$ and $F(\frac{\hat{\pi}_L-\mu}{s}) = 0.5$ and $\hat{\pi}_R = \hat{\pi}_C$.
- Dovish Agenda Setter: $H_0: p_C = 1$ and $F(\frac{\hat{\pi}_R-\mu}{s}) = 0.5$ and $\hat{\pi}_L = \hat{\pi}_C$.

4 Appendix

4.1 Alternative Preferences

Let $\gamma(i)$ represent each committee member's degree of absolute prudence, and let γ_C denote the chairman's prudence parameter. Let $F(\gamma)$ denote the cumulative distribution function over the preference parameter.

Committee members have preferences over the rate of inflation that prevails, given by the asymmetric linex function (see Varian (1974)):

$$U(\pi_t, \gamma) = \frac{-\exp\left(\gamma(\pi_t - \pi^*)\right) + \gamma(\pi_t - \pi^*) + 1}{\gamma^2}$$

where π_t is the inflation rate at time t, π^* is the inflation target, and γ is a preference parameter that captures the agent's degree of prudence. If $\gamma > 0$, the agent weights positive deviations of π from its target by more than negative deviations – the agent is averse to upside risk. Similarly, if $\mu < 0$, the agent is averse to down-side risk – she overweights negative deviations from the target to positive ones. In the limit as $\mu \to 0$, the agent's preferences become symmetric (and, in fact, as $\gamma \to 0$, $U \to -(\pi_t - \pi^*)^2$). Notice that U satisfies the Spence-Mirrlees condition, in the sense that $\gamma' > \gamma$ implies $U_\pi(\pi, \gamma') < U_\pi(\pi, \gamma)$ (or, equivalently, $U_{\pi,\gamma} < 0$).

Let $x_t = (\pi_t, y_t, i_{t-1}, \zeta_t)$ be a vector of observables at time t. Consider an agent with preference parameter γ. The optimal period t interest rate from their perspective is the one that maximizes:

$$E_t[U(\pi_{t+2}, \gamma)|\ x_t]$$

Taking first order conditions, this implies that the optimal i_t satisfies:

$$E_t[\pi_{t+2}|\ x_t] = \pi^* - \frac{1}{2}\gamma\sigma_\pi^2$$

where $\sigma_\pi^2 = (1 + (1+\gamma)^2)\sigma_u^2 + \alpha_1^2\sigma_v^2$ is the conditional variance in π_{t+2}. (For details of the derivation, see Riboni and Ruge-Murcia (2010).) Finally, using (3) to substitute for π_{t+2}, we have:

$$i(x_t; \gamma) = a(\gamma) + b\pi_t + cy_t + \zeta_t \tag{11}$$

where

$$
\begin{aligned}
a(\gamma) &= \iota - \frac{1}{\alpha_1\beta_2}\pi^* + \frac{\gamma}{2\alpha_1\beta_2}\sigma_\pi^2 \\
b &= 1 + \frac{1}{\alpha_1\beta_2} \\
c &= \frac{1+\beta_1}{\beta_2} \\
\zeta_t &= \frac{\gamma}{\alpha_1\beta_2}u_t + \frac{\varsigma}{\beta_2}v_t
\end{aligned}
$$

The optimal interest rate is the sum of four terms; it is responsive to the current interest rate (π_t), the current output gap (y_t), and other innovations in the AD and Phillips Curves (ζ_t). Amongst these, notice that only the constant term, $a(\gamma)$, is a (linear) function of γ. Hence, the risk (prudence) preferences of agents simply has the effect of linearly shifting their ideal interest rate, independent of macro-economic observables. Since u_t and v_t where normally distributed white noise processes, ζ_t also has a mean-zero normal distribution with conditional variance $\sigma_\zeta^2 = (\frac{\gamma}{\alpha_1\beta_2})^2\sigma_u^2 + (\frac{\varsigma}{\beta_2})^2\sigma_v^2$.

Finally, note that, since $i(x_t;\gamma)$ is monotone in γ for every x_t, that $i(x_t,\cdot)$ is invertible in γ. Let $r(i_t,x_t)$ be the unique function defined by:

$$
i\left(x_t, r(i(x_t,\gamma),x_t)\right) = \gamma
$$

$r(i_t,x_t)$ gives the prudence parameter for an agent whose ideal policy at time t, given observables x_t, is i_t.

4.2 Proofs

5 Technical Appendix

[Not for published version]

5.1 Log Likelihood and Gradient Ascent

Hence $\theta = (a_L, a_R, a_C, b, c, \mu, s, p_C)$. We must additionally estimate σ, the standard deviation of ζ.

The likelihood function is:

$$l(\theta,\sigma) = -T_1 \ln(\sigma) - \frac{1}{2\sigma^2}\sum_{t\in\Theta_1}\zeta(\omega_t,i_t,\theta)^2 + \sum_{t\in\Theta_0}\ln\left[\Phi\left(\frac{\zeta_R(\omega_t,\theta)}{\sigma}\right) - \Phi\left(\frac{\zeta_L(\omega_t,\theta)}{\sigma}\right)\right]$$

We use gradient ascent to solve the MLE. The gradient vector is:

$$\frac{\partial l(\theta,\sigma)}{\partial\theta} = -\frac{1}{\sigma^2}\sum_{t\in\Theta_1}\zeta_t\frac{\partial\zeta(\omega_t,i_t,\theta)}{\partial\theta} + \frac{1}{\sigma}\sum_{t\in\Theta_0}\frac{\phi\left(\frac{\zeta_R(\omega_t,\theta)}{\sigma}\right)\frac{\partial\zeta_{Rt}}{\partial\theta} - \phi\left(\frac{\zeta_L(\omega_t,\theta)}{\sigma}\right)\frac{\partial\zeta_{Lt}}{\partial\theta}}{\Phi\left(\frac{\zeta_R(\omega_t,\theta)}{\sigma}\right) - \Phi\left(\frac{\zeta_L(\omega_t,\theta)}{\sigma}\right)}$$

and

$$\frac{\partial l(\theta,\sigma)}{\partial\sigma} = -\frac{T_1}{\sigma} + \frac{1}{\sigma^3}\sum_{t\in\Theta_1}\zeta(\omega_t,i_t,\theta)^2 - \frac{1}{\sigma^2}\sum_{t\in\Theta_0}\frac{\phi\left(\frac{\zeta_R(\omega_t,\theta)}{\sigma}\right)\cdot\zeta_{Rt} - \phi\left(\frac{\zeta_L(\omega_t,\theta)}{\sigma}\right)\cdot\zeta_{Lt}}{\Phi\left(\frac{\zeta_R(\omega_t,\theta)}{\sigma}\right) - \Phi\left(\frac{\zeta_L(\omega_t,\theta)}{\sigma}\right)}$$

where we use the abbreviations $\zeta(\omega_t,i_t,\theta) = \zeta_t$, $\zeta_L(\omega_t,\theta) = \zeta_{Lt}$ and $\zeta_R(\omega_t,\theta) = \zeta_{Rt}$ for convenience.

Recall $\zeta_j(\omega_t,\theta) = i_{t-1} - a_j - b_{\pi t} - cy_t$ for $j\in\{L,R\}$. We need expressions for $\frac{\partial\zeta_t}{\partial\theta}$, $\frac{\partial\zeta_{Lt}}{\partial\theta}$, and $\frac{\partial\zeta_{Rt}}{\partial\theta}$. The latter two are straight forward, we have:

$$\begin{bmatrix} \frac{\partial\zeta_{Lt}}{\partial a_R} & \frac{\partial\zeta_{Rt}}{\partial a_R} \\ \frac{\partial\zeta_{Lt}}{\partial a_L} & \frac{\partial\zeta_{Rt}}{\partial a_L} \\ \frac{\partial\zeta_{Lt}}{\partial a_C} & \frac{\partial\zeta_{Rt}}{\partial a_C} \\ \frac{\partial\zeta_{Lt}}{\partial b} & \frac{\partial\zeta_{Rt}}{\partial b} \\ \frac{\partial\zeta_{Lt}}{\partial c} & \frac{\partial\zeta_{Rt}}{\partial c} \\ \frac{\partial\zeta_{Lt}}{\partial \mu} & \frac{\partial\zeta_{Rt}}{\partial \mu} \\ \frac{\partial\zeta_{Lt}}{\partial s} & \frac{\partial\zeta_{Rt}}{\partial s} \\ \frac{\partial\zeta_{Lt}}{\partial p_C} & \frac{\partial\zeta_{Rt}}{\partial p_C} \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \\ 0 & 0 \\ -\pi_t & -\pi_t \\ -y_t & -y_t \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Finding $\frac{\partial\zeta_t}{\partial\theta}$ is more complicated, since $\zeta(\omega_t,i_t,\theta)$ is implicitly defined.For notational brevity, denote $I_t^j = I^j(\omega_t)$. Let $\zeta(\omega_t,i_t,\rho;\theta)$ be the solution to:

$$\zeta_t^2 + 2\left[\frac{I_t^L + I_t^R}{2} - i_t + \frac{1}{4}\Delta i_t\right]\zeta_t + (I_t^L - i_t)(I_t^R - i_t) + \frac{1}{2}\Delta i_t\left[(\rho I_t^L + (1-\rho)I_t^R) - i_t\right] = 0$$

As above, define $\rho_l = (1-p_C)F\left(\frac{\mu - a_C}{s}\right)$ and $\rho_h = p_C + (1-p_C)F\left(\frac{\mu - a_C}{s}\right)$. Recall $A(\zeta,\omega_t,i_t;\theta) = i_t - b\pi_t - cy_t - \zeta)$.

Then, by Proposition 1 and Corollary 2, when $t \in \Theta_1$, then:

- If $a_C \in [A(\zeta(\rho_l), \omega_t, i_t; \theta), A(\zeta(\rho_h), \omega_t, i_t; \theta)]$, then $\zeta_t = i_t - a_C - b\pi_t - cy_t$. Let Θ_1^C denote the set of t for which this regime prevails.

- If $a_C \notin [\zeta(\omega_t, i_t, \rho_l; \theta), \zeta(\omega_t, i_t, \rho_h; \theta)]$, then: (ζ_t, ρ_t) are the solutions to:

$$\zeta_t^2 + 2\left[\frac{I_t^L + I_t^R}{2} - i_t + \frac{1}{4}\Delta i_t\right]\zeta_t + (I_t^L - i_t)(I_t^R - i_t) + \frac{1}{2}\Delta i_t\left[(\rho_t I_t^L + (1-\rho_t)I_t^R) - i_t\right] = 0$$

and

$$p_C \mathbf{1}[a_C > A(i_t, x_t; \zeta_t; \theta)] + (1 - p_C)F\left(\frac{\mu - A(i_t, x_t; \zeta_t; \theta)}{s}\right) - \rho_t = 0$$

where $A_t = A(i_t, x_t; \zeta_t; \theta) = i_t - b\pi_t - cy_t - \zeta_t$. Let $\Theta_1^{\neg C}$ be the set of t for which this regime prevails.

If $t \in \Theta_1^C$, then the gradient is again straight-forward. We have:

$$\begin{bmatrix} \frac{\partial \zeta_t}{\partial a_R} \\ \frac{\partial \zeta_t}{\partial a_L} \\ \frac{\partial \zeta_t}{\partial a_C} \\ \frac{\partial \zeta_t}{\partial b} \\ \frac{\partial \zeta_t}{\partial c} \\ \frac{\partial \zeta_t}{\partial \mu} \\ \frac{\partial \zeta_t}{\partial s} \\ \frac{\partial \zeta_t}{\partial p_C} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -1 \\ -\pi_t \\ -y_t \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The complicated case is when $t \in \Theta_1^{\neg C}$. We cannot apply gradient ascent directly to this system, since it includes a (discontinuous) indicator function. As a work around, we take a continuous approximation of the indicator. Let G be the CDF of some standardized distribution (e.g. Standard Normal, Standard Logistic) that is symmetric about its mean (of 0) and with variance 1. Let $\epsilon > 0$ be small. Then:

$$\mathbf{1}[a_C > A_t] \approx G\left(\frac{a_C - A_t}{\epsilon}\right)$$

and this approximation can be made arbitrarily precise. Let $g(\cdot)$ be the pdf of G.

Then, by the implicit function theorem:

$$\begin{bmatrix} \frac{\partial \zeta_t}{\partial a_R} & \frac{\partial \rho_t}{\partial a_r} \\ \frac{\partial \zeta_t}{\partial a_L} & \frac{\partial \rho_t}{\partial a_L} \\ \frac{\partial \zeta_t}{\partial a_C} & \frac{\partial \rho_t}{\partial a_C} \\ \frac{\partial \zeta_t}{\partial b} & \frac{\partial \rho_t}{\partial b} \\ \frac{\partial \zeta_t}{\partial c} & \frac{\partial \rho_t}{\partial c} \\ \frac{\partial \zeta_t}{\partial \mu} & \frac{\partial \rho_t}{\partial \mu} \\ \frac{\partial \zeta_t}{\partial s} & \frac{\partial \rho_t}{\partial s} \\ \frac{\partial \zeta_t}{\partial p_C} & \frac{\partial \rho_t}{\partial p_C} \end{bmatrix} \begin{bmatrix} 2\left(\zeta_t + \frac{I_t^L+I_t^R}{2} - i_t\right) + \frac{1}{2}\Delta i_t & \frac{p_C}{\epsilon} g(\frac{a_C-A_t}{\epsilon}) + \frac{1-p_C}{s} f\left(\frac{\mu-A_t}{s}\right) \\ \frac{1}{2}\Delta i_t (I_t^L - I_t^R) & -1 \end{bmatrix} = - \begin{bmatrix} \zeta_t + (I_t^L - i) + \frac{1}{2}(1-\rho_t)\Delta i_t & 0 \\ \zeta_t + (I_t^R - i) + \frac{1}{2}\rho_t)\Delta i_t & 0 \\ 0 & \frac{p_C}{\epsilon} g\left(\frac{a_C-A_t}{\epsilon}\right. \\ \pi_t\left[2\left(\zeta_t + \frac{I_t^L+I_t^R}{2} - i_t\right) + \frac{1}{2}\Delta i_t\right] & \pi_t\left[\frac{p_C}{\epsilon} g\left(\frac{a_C-A_t}{\epsilon}\right) + \frac{1-p_C}{s}\right. \\ y_t\left[2\left(\zeta_t + \frac{I_t^L+I_t^R}{2} - i_t\right) + \frac{1}{2}\Delta i_t\right] & y_t\left[\frac{p_C}{\epsilon} g\left(\frac{a_C-A_t}{\epsilon}\right) + \frac{1-p_C}{s}\right. \\ 0 & \frac{1-p_C}{s} f\left(\frac{\mu-A_t}{s}\right. \\ 0 & -\frac{1-p_C}{s} \cdot \frac{\mu-A_t}{s} f\left(\right. \\ 0 & G\left(\frac{a_C-A_t}{\epsilon}\right) - F\left(\right. \end{bmatrix}$$

Hence:

$$\begin{bmatrix} \frac{\partial \zeta_t}{\partial a_R} & \frac{\partial \rho_t}{\partial a_r} \\ \frac{\partial \zeta_t}{\partial a_L} & \frac{\partial \rho_t}{\partial a_L} \\ \frac{\partial \zeta_t}{\partial a_C} & \frac{\partial \rho_t}{\partial a_C} \\ \frac{\partial \zeta_t}{\partial b} & \frac{\partial \rho_t}{\partial b} \\ \frac{\partial \zeta_t}{\partial c} & \frac{\partial \rho_t}{\partial c} \\ \frac{\partial \zeta_t}{\partial \mu} & \frac{\partial \rho_t}{\partial \mu} \\ \frac{\partial \zeta_t}{\partial s} & \frac{\partial \rho_t}{\partial s} \\ \frac{\partial \zeta_t}{\partial p_C} & \frac{\partial \rho_t}{\partial p_C} \end{bmatrix} = -\frac{1}{det} \begin{bmatrix} \zeta_t + (I_t^L - i) + \frac{1}{2}(1-\rho_t)\Delta i_t & 0 \\ \zeta_t + (I_t^R - i) + \frac{1}{2}\rho_t)\Delta i_t & 0 \\ 0 & \frac{p_C}{\epsilon} g\left(\frac{a_C-A_t}{\epsilon}\right) \\ \pi_t\left[2\left(\zeta_t + \frac{I_t^L+I_t^R}{2} - i_t\right) + \frac{1}{2}\Delta i_t\right] & \pi_t\left[\frac{p_C}{\epsilon} g\left(\frac{a_C-A_t}{\epsilon}\right) + \frac{1-p_C}{s} f\left(\frac{\mu-A_t}{s}\right)\right] \\ y_t\left[2\left(\zeta_t + \frac{I_t^L+I_t^R}{2} - i_t\right) + \frac{1}{2}\Delta i_t\right] & y_t\left[\frac{p_C}{\epsilon} g\left(\frac{a_C-A_t}{\epsilon}\right) + \frac{1-p_C}{s} f\left(\frac{\mu-A_t}{s}\right)\right] \\ 0 & \frac{1-p_C}{s} f\left(\frac{\mu-A_t}{s}\right) \\ 0 & -\frac{1-p_C}{s} \cdot \frac{\mu-A_t}{s} f\left(\frac{\mu-A_t}{s}\right) \\ 0 & G\left(\frac{a_C-A_t}{\epsilon}\right) - F\left(\frac{\mu-A_t}{s}\right) \end{bmatrix} \begin{bmatrix} -1 & -\frac{p_C}{\epsilon} g(\frac{a_C-A_t}{\epsilon}) - \frac{1-p_C}{s} f\left(\right. \\ -\frac{1}{2}\Delta i_t (I_t^L - I_t^R) & \left(\zeta_t + \frac{I_t^L+I_t^R}{2} - i_t\right) + \frac{1}{2}\Delta \end{bmatrix}$$

where $det = -\left[2\left(\zeta_t + \frac{I_t^L+I_t^R}{2} - i_t\right) + \frac{1}{2}\Delta i_t + \frac{1}{2}\Delta i_t (I_t^L - I_t^R)\left(\frac{p_C}{\epsilon} g(\frac{a_C-A_t}{\epsilon}) - \frac{1-p_C}{s} f\left(\frac{\mu-A_t}{s}\right)\right)\right]$

Simplifying, we have:

$$\begin{bmatrix} \frac{\partial \zeta_t}{\partial a_R} \\ \frac{\partial \zeta_t}{\partial a_L} \\ \frac{\partial \zeta_t}{\partial a_C} \\ \frac{\partial \zeta_t}{\partial b} \\ \frac{\partial \zeta_t}{\partial c} \\ \frac{\partial \zeta_t}{\partial \mu} \\ \frac{\partial \zeta_t}{\partial s} \\ \frac{\partial \zeta_t}{\partial p_C} \end{bmatrix} = \begin{bmatrix} -\frac{\zeta_t + (I_t^L - i) + \frac{1}{2}(1-\rho_t)\Delta i_t}{2\left(\zeta_t + \frac{I_t^L+I_t^R}{2} - i_t\right) + \frac{1}{2}\Delta i_t + \frac{1}{2}\Delta i_t (I_t^L - I_t^R)\left[\frac{p_C}{\epsilon} g(\frac{a_C-A_t}{\epsilon}) - \frac{1-p_C}{s} f\left(\frac{\mu-A_t}{s}\right)\right]} \\ -\frac{\zeta_t + (I_t^R - i) + \frac{1}{2}\rho_t \Delta i_t}{2\left(\zeta_t + \frac{I_t^L+I_t^R}{2} - i_t\right) + \frac{1}{2}\Delta i_t + \frac{1}{2}\Delta i_t (I_t^L - I_t^R)\left[\frac{p_C}{\epsilon} g(\frac{a_C-A_t}{\epsilon}) - \frac{1-p_C}{s} f\left(\frac{\mu-A_t}{s}\right)\right]} \\ -\frac{\frac{p_C}{\epsilon} g\left(\frac{a_C-A_t}{\epsilon}\right)}{2\left(\zeta_t + \frac{I_t^L+I_t^R}{2} - i_t\right) + \frac{1}{2}\Delta i_t + \frac{1}{2}\Delta i_t (I_t^L - I_t^R)\left[\frac{p_C}{\epsilon} g(\frac{a_C-A_t}{\epsilon}) - \frac{1-p_C}{s} f\left(\frac{\mu-A_t}{s}\right)\right]} \\ -\pi_t \\ -y_t \\ \frac{-\frac{1-p_C}{2s}\Delta i_t (I_t^L - I_t^R) f\left(\frac{\mu-A_t}{s}\right)}{2\left(\zeta_t + \frac{I_t^L+I_t^R}{2} - i_t\right) + \frac{1}{2}\Delta i_t + \frac{1}{2}\Delta i_t (I_t^L - I_t^R)\left[\frac{p_C}{\epsilon} g(\frac{a_C-A_t}{\epsilon}) - \frac{1-p_C}{s} f\left(\frac{\mu-A_t}{s}\right)\right]} \\ \frac{\frac{1-p_C}{2s} \cdot \frac{\mu-A_t}{s}\Delta i_t (I_t^L - I_t^R) f\left(\frac{\mu-A_t}{s}\right)}{2\left(\zeta_t + \frac{I_t^L+I_t^R}{2} - i_t\right) + \frac{1}{2}\Delta i_t + \frac{1}{2}\Delta i_t (I_t^L - I_t^R)\left[\frac{p_C}{\epsilon} g(\frac{a_C-A_t}{\epsilon}) - \frac{1-p_C}{s} f\left(\frac{\mu-A_t}{s}\right)\right]} \\ -\frac{\frac{1}{2}\Delta i_t (I_t^L - I_t^R)\left[G\left(\frac{a_C-A_t}{\epsilon}\right) - F\left(\frac{\mu-A_t}{s}\right)\right]}{2\left(\zeta_t + \frac{I_t^L+I_t^R}{2} - i_t\right) + \frac{1}{2}\Delta i_t + \frac{1}{2}\Delta i_t (I_t^L - I_t^R)\left[\frac{p_C}{\epsilon} g(\frac{a_C-A_t}{\epsilon}) - \frac{1-p_C}{s} f\left(\frac{\mu-A_t}{s}\right)\right]} \end{bmatrix}$$

NB - notice that $\frac{\partial \zeta_t}{\partial s} = -\frac{I(\omega_t,\mu)+\zeta_t-i_t}{s} \cdot \frac{\partial \zeta_t}{\partial \mu}$

5.2 Information Matrix - RRM Consensus

Let $\theta = (a_L, a_R, b, c)$ and σ be the parameters to be estimated. Defined $z_t^- = i_t - a_R - b\pi_t - cy_t$ and $z_t^+ = i_t - a_L - b\pi_t - cy_t$ as the implied shocks when $i_t < i_{t-1}$ and $i_t > i_{t-1}$, respectively. Let $g_t^- = -\frac{\partial g_t^-}{\partial \theta} = (0, 1, \pi_t, y_t)^T$ (which is a column vector) and similarly define $g_t^+ = -\frac{\partial g_t^+}{\partial \theta} = (1, 0, \pi_t, y_t)^T$. Let $z_t^R = i_{t-1} - a_R - b\pi_t - cy_t$ and $z_t^L = i_{t-1} - a_L - b\pi_t - cy_t$. Finally, let $\zeta_t^- = \frac{z_t^-}{\sigma}$ and define ζ_t^+, ζ_t^L and ζ_t^R, analogously.

The score vector is:

$$V(Z, x, \theta) = \begin{cases} V^- & \text{if } Z < z^R(x, \theta) \\ V^0 & \text{if } Z \in [z^R(x, \theta), z^L(x, \theta)] \\ V^+ & \text{if } Z > z^L(x, \theta) \end{cases}$$

where $V^- = \begin{bmatrix} \frac{1}{\sigma}\frac{Z}{\sigma}g^- \\ -\frac{1}{\sigma} + \frac{1}{\sigma}(\frac{Z}{\sigma})^2 \end{bmatrix}$, and $V^0 = \begin{bmatrix} -\frac{1}{\sigma}\frac{\phi(\zeta^L)g^+ - \phi(\zeta^R)g^-}{\Phi(\zeta^L) - \Phi(\zeta^R)} \\ -\frac{1}{\sigma}\frac{\zeta^L\phi(\zeta^L) - \zeta^R\phi(\zeta^R)}{\Phi(\zeta^L) - \Phi(\zeta^R)} \end{bmatrix}$, and $V^+ = \begin{bmatrix} \frac{1}{\sigma}\frac{Z}{\sigma}g^+ \\ -\frac{1}{\sigma} + \frac{1}{\sigma}(\frac{Z}{\sigma})^2 \end{bmatrix}$.

Confirm that $E[V] = 0$. [Hint —use fact that $\int_{-\infty}^{\zeta^R}(\zeta)\phi(\zeta)d\zeta = -\phi(\zeta^R)$ and $\int_{\zeta^L}^{\infty}(\zeta)\phi(\zeta)d\zeta = \phi(\zeta^L)$.]

Define the following:

$$H_t^-(\theta, \sigma) = \frac{1}{\sigma^2}\begin{bmatrix} \Phi(\zeta_t^R) \cdot g_t^-(g_t^-)^T & -2\phi(\zeta_t^R) \cdot g_t^- \\ -2\phi(\zeta_t^R) \cdot (g_t^-)^T & 1 - 3(1 - \zeta_t^R)\phi(\zeta_t^R) \end{bmatrix}$$

and:

$$H_t^+(\theta, \sigma) = \frac{1}{\sigma^2}\begin{bmatrix} [1 - \Phi(\zeta_t^L)] \cdot g_t^+(g_t^+)^T & 2\phi(\zeta_t^L) \cdot g_t^+ \\ 2\phi(\zeta_t^L) \cdot (g_t^+)^T & 1 - 3(1 + \zeta_t^L)\phi(\zeta_t^L) \end{bmatrix}$$

and:

$$H_t^0(\theta, \sigma) = \frac{1}{\sigma^2}\begin{bmatrix} H_t^0(\theta, \theta) & H_t^0(\theta, \sigma) \\ H_t^0(\theta, \sigma)^T & H_t^0(\sigma, \sigma) \end{bmatrix}$$

where:

- $H_t^0(\theta,\theta) = \frac{[\phi(\zeta_t^L)g^+\phi(\zeta_t^R)g^-][\phi(\zeta_t^L)g^+-\phi(\zeta_t^R)g^-]^T}{\Phi(\zeta_t^L)-\Phi(\zeta_t^R)} - [\zeta_t^L\phi(\zeta_t^L)g_t^+ - \zeta_t^R\phi(\zeta_t^R)g_t^-]$
- $H_t^0(\theta,\sigma) = \frac{[\zeta_t^L\phi(\zeta_t^L)-\zeta_t^R\phi(\zeta_t^R)][\phi(\zeta_t^L)g^+-\phi(\zeta_t^R)g^-]^T}{\Phi(\zeta_t^L)-\Phi(\zeta_t^R)} - \left[\phi(\zeta_t^L)g_t^+ - \phi(\zeta_t^R)g_t^-\right] - [(\zeta_t^L)^2\phi(\zeta_t^L)g_t^+ - (\zeta_t^R)^2\phi(\zeta_t^R)g_t^-]$
- $H_t^0(\sigma,\sigma) = \frac{\left(\zeta^L\phi(\zeta^L)-\zeta^R\phi(\zeta^R)\right)}{\Phi(\zeta^L)-\Phi(\zeta^R)} - [\zeta^L\phi(\zeta^L) - \zeta^R\phi(\zeta^R)g^-] - [\zeta_t^L(1-(\zeta_t^L)^2)\phi(\zeta_t^L) - \zeta_t^R(1 - (\zeta_t^R)^2)\phi(\zeta_t^R)]$

Then the Fisher Information is:

$$I(\theta,\sigma) = \sum_t \left[H_t^-(\theta,\sigma) + H_t^0(\theta,\sigma) + H_t^+(\theta,\sigma)\right]$$

The asymptotic variance of the estimators is $I(\theta,\sigma)^{-1}$.

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