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## Abstract (100-150)

# 1 Introduction (1000)

Gender gap in labour force participation is a world-wide phenomenon which is particularly pronounced in developing countries. Globally, the rate of labour force participation is about 75% among men while only 50% among women **(International Labour Organization, 2021)**. In regions such as Middle East, North Africa, and South Asia, the gender gap is even greater, with around 75% of men and 20% of women participating in the labour force **(International Labour Organization, 2021)**. In the search of potential measures to narrow the gender gap, its causes have been extensively studied with a recent attention on the prominent role of social norms. Researchers speculate that interventions aiming at changing social norms may be the key to achieve greater gender equality in labour force participation **(Bursztyn et al., 2020; Codazzi et al., 2018; Jayachandran, 2021)**.

A recent successful attempt in developing such social norm interventions was made by Bursztyn and colleagues **(2020)** in Saudi Arabia. In Saudi Arabia, gender norms exist that expect women to be absent from labour or segregated from men at workplace, and that women need approval from their male "guardian" (usually the husband or father) if they want to work outside the home **(Bursztyn et al., 2018)**. These allow us to make a reasonable speculation that the low female labour participation rate in Saudi Arabia **(28% in 2022, International Labour Organization)** may be because men don't allow their wives to work outside the home (WWOH) due to the internalisation of the gender norms (i.e., personal beliefs aligning with the norm that women should not work outside the home). Interestingly, however, Bursztyn and colleagues **(2020)** found that the critical factor was rather Saudi men's misperception of the injunctive norms regarding WWOH. Their findings revealed that 80-85% of surveyed Saudi men who were married and aged 18-35 reported to agree with the statement that women should be allowed to work outside the home, while more than three quarters of them underestimated this percentage.

Based on this result, Bursztyn and colleagues **(2020)** designed an intervention to correct men's misperception regarding WWOH found evidence supporting evidence for its effectiveness in changing their labour supply decisions. They found that men who received correct information of the true percentage of supporting men (vs. those who did not receive the information) were significantly more likely to sign up for job matching service for their wives immediately after the intervention. Their wives were also more likely to have applied and interviewed for a job three to five months after the intervention. The researchers also tested the effect of a similar intervention on Saudi women in a field setting, finding that women who received the information on the percentage of men supporting WWOH (vs. those who did not received the information) were more likely to take up a part-time job outside the home instead of a position to work at home.

The present research takes correcting norm misperception as a promising type of social norm intervention and seeks to explore its effectiveness when scaled up. Since large-scale policy intervention can usually be costly to implement and test, the present research aims to study, prior to implementation, the intervention strategy that is theoretically the most effective under appropriate assumptions via agent-based modelling (ABM). The reason for choosing ABM as the method of modelling the system of WWOH action is that the dynamic interaction among people's private belief and norm perception regarding WWOH, as well as their labour supply decision and action can be viewed as complex **(Johnson, 2009)**. One's labour supply decision and action can presumably influence their acquaintances' private belief and norm perception, and these influences in turn impact their acquaintances' labour supply decision and action as a function of private belief and norm perception, eventually creating a feedback loop among individuals. This nature of having agents interacting dynamically and irregularly over time makes the system of WWOH action irreducible to lower-level descriptions (e.g., mathematical equations) and qualifies it as a complex system **(Gustafsson & Sternad, 2010)**. ABMs are suitable for studying the dynamics and emergent properties of such systems, as well as simulate and explore the consequences of policy interventions to them **(Bailey et al., 2019; Madsen et al., 2019)**.

**give space for specific research questions**

# 2 Literature Review (2000)

## 2.1 Models of Belief and Action Dynamics

Belief and action changes among a group of interconnected and mutually influencing agents have been studied widely through mathematical and computational models. These models assume that agents hold beliefs regarding certain issues. At each time step, these beliefs are modified based on their neighbours' beliefs according to some rule. The dynamics of belief distribution in a population over time is studied. This section will first review various methods to formally model agents' beliefs and actions and the ways in which agents influence one another. Then, the topics that these models are usually applied to studied are introduced.

### 2.1.1 Formal Representation of Beliefs and Actions

Different models of belief and action dynamics model agents' beliefs and actions and the ways in which they are affected by those of other agents differently **(Hassani et al., 2022)**. The most simplistic models represent agents' beliefs using binary variables, the classic ones of which include the Ising model **(Li et al., 2019)**, the voter model **(Holley & Liggett, 1975)**, and the Sznajd model **(Sznajd-Weron & Sznajd, 2000)**. Although these models and their variations adopt different rules to update agents' beliefs, the updating usually results in the agents being memoryless about their previous beliefs. For example, in the voter model, an agent $i$ is randomly chosen at each time step, together with one of its neighbour $j$, and the agent $i$ then abandons its previous belief takes the opinion of its neighbour $j$.

Other simple models represent agents' beliefs as continuous variables that take the value of a real number. These are exemplified by the classic Degroot model **(Berger, 1981)** and the bounded confidence model **(Rainer & Krause, 2002)**. The models of this type usually updates agents' beliefs using a weighted average of each individual's belief and those of their neighbour(s). For example, in the bounded confidence model, an agent $i$ is randomly chosen at each time step $t$, together with one of its neighbour $j$, who hold the beliefs $\sigma_i(t)$ and $\sigma_j(t)$, respectively. When the condition $|\sigma_i(t) - \sigma_j(t)| < \epsilon$ is met, the agent $i$ updates its belief according to $\sigma_i(t+1) = (1-\alpha)\sigma_i(t) + \alpha\sigma_j(t)$, and the agent $j$ according to $\sigma_j(t+1) = (1-\alpha)\sigma_j(t) + \alpha\sigma_i(t)$. In other words, when the selected agents hold similar enough beliefs according to a threshold $\epsilon$, they update their beliefs as the weighted average of their neighbour's and their own previous beliefs based on a convergence parameter $\alpha$.

More sophisticated models recognise the distinction between agents' private beliefs and public actions, modelling the former as a continuous variable and the latter a discrete one. The Continuous Opinions and Discrete Actions (CODA) model **(Martins, 2013)** and the more recent social network opinions and actions evolutions (SNOAEs) model **(Zhan et al., 2022)** are examples of this category. To take the CODA model as an example, it models private beliefs as $P(A) \in [0, 1]$, the probability of the $A$ being the best alternative, which is also the probability of an agent publicly displaying the action $A$. Agents have no access to other agents' private beliefs but only their public actions, which is used to update one's own private beliefs via the Bayes' theorem. For example, upon observing a neighbour $j$ displaying the action $A$ at the time step $t$, an agent update its private beliefs according to the rule $P_{t+1}(A) = P(A|a_j = +1) \propto P_t(A)P(a_j = +1|A)$, where $P(A|a_j = +1)$ denotes the probability of $A$ being the best action conditioned on the neighbour $j$ displaying $A$, and $P(a_j = +1|A)$ the probability of the neighbour $j$ displaying $A$ conditioned on $A$ being the best action (which is modelled as a constant).

It is important to notice that there is a recent trend to build psychologically realistic models to represent belief and behaviour dynamics among a group of agents **(Duggins, 2017; Gavrilets, 2021; Tverskoi et al., 2023)**. Still assuming each agent's beliefs and actions are affected by their neighbours, these model further consider the influence of social norm, the psychological tendency to conform to peers, external authorities, as well as material cost-benefit considerations **(Gavrilets, 2021)**. To incorporate these psychological and social complexities, a recent unifying modelling framework **(Gavrilets, 2021)** not only considers the distinction between agents' private beliefs and public actions, but also model agents' perception of others' private beliefs and public actions as two separate variables. This allows the representation of various psychological factors in updating beliefs and actions, including cognitive dissonance in taking a private belief (i.e., the aversion towards misaligned belief and action), social projection in perceiving others' private beliefs (i.e., the tendency to project one's own private belief onto others), and compliance with authority.

### 2.1.2 Model Applications

Models of belief and action dynamics have been applied to studying multiple themes associated with opinion change in a population. These include but are not limited to consensus reaching and polarisation **(Acemoglu & Ozdaglar, 2010; Hassani et al., 2022; Jager & Amblard, 2004; Duggins, 2017)**, the spread of (mis)information **(Watts, 2002; Watts & Dodds, 2007; Pilditch et al.,**

**2022)**, and echo chamber formation **(Madsen et al., 2018; Fränken & Pilditch, 2021)**. Applying mathematical or computational models, researchers are mostly interested in the assumptions and conditions under which certain phenomena can arise.

For instance, a study reviews a series of mathematical models to answer the question about consensus reaching, information aggregation, and the spread of misinformation. ***More specifically, it answers the question about under what conditions 1) agents can hold beliefs in an agreement even when they start with different views, 2) information can be aggregated in the population so that agents holding incorrect beliefs can end up with correct beliefs, and 3) prominent agents can spread misinformation. The study considers both Bayesian (i.e., agents update their beliefs via the Bayes' theorem) and non-Bayesian models (i.e., agents update their beliefs via other rules) (consider remove)*** and concludes that in both types of models there is a tendency towards reaching consensus, but agents' beliefs are not effectively corrected through information aggregation. Misinformation spreads with limited extent in both types of models, which is due to the limited influence of misinformation on Bayesian agents in Bayesian models, and the lack of persistent disagreements in the population in non-Bayesian models.

In addition to belief updating mechanisms, other research has also studied how network structures can influence the emergence of phenomena related to opinion change. ***Focusing on information cascade, a phenomenon where a group of individuals make the same decision sequentially,*** a simulation study finds that the distribution of the size of cascades depends on the connectivity of the network, so does the type of agents who can easily trigger a cascade **(Watts, 2002)**. A subsequent simulation study further points out that influential individuals, i.e., those who have the highest number of connections to others, are not sufficient for triggering cascades, but the existence of a group of other individuals who are easily influenced by the influential ones is critical **(Watts & Dodds, 2007)**.

## 2.2 Agent-Based Models of Pluralistic Ignorance

There has been a growing interest in using ABMs to study the conditions under which the phenomenon called pluralistic ignorance (PI) can arise. This line of research usually defines PI as the situation where the majority of people in a population express opinions different from their private beliefs **(e.g., Seeme et al., 2016; Wang et al., 2014; Ye et al., 2019)**. The seminal work on this topic focuses on the effects of the properties of the network on holding inconsistent private beliefs and expressed opinions **(Centola et al., 2005)**. The research inserts, as an initial condition, a few agents who hold a private belief held by few individuals in the population (referred to as "true believers") and enforce other agents to adopt the same belief. It reveals that holding inconsistent private beliefs and expressed opinions (which is equivalent to enforcing the belief on other agents in the context of this research) cannot spread widely in the population if the population is fully connected, if the true believers are scattered in the population rather than clustered, or if ties in the network are randomly rewired, breaking those that originally exist between local neighbours.

Recent studies start to include in their models psychological processes that have been theorised by research in social psychology. Two processes that have been mostly attended to are social conformity and cognitive dissonance **(e.g., Wang et al., 2014; Seeme, 2019)**. For instance, a research models the change in agents' expressed opinions as a result of the influence from the opinions in the groups formed among the neighbours, representing people's psychological tendency to conform to the group **(Wang et al., 2014)**. Following the cognitive dissonance theory, the model asks the agents change their private attitudes when the group influence is moderate. When the group influence is strong, the model represents the situation where people are aware of the groups influence and doesn't ask agents to change their private beliefs. Under these assumptions, the research finds that there is a widespread inconsistency between agents' private beliefs and expressed opinions, that is, pluralistic ignorance exists by definition.

Another example is a study that considers social conformity and cognitive dissonance assuming the rationality of agents **(Seeme, 2019)**. It models the agents as having a utility function that is proportional to the rewards from high conformity with its opinion group (expressing similar opinions with the mean opinion of the group) and the rewards from low cognitive dissonance (holding similar private beliefs and expressed opinions). Agents update their private beliefs and expressed opinions by maximising the value of the utility function. The study finds that different degrees of pluralistic ignorance arise under different conditions. When all agents make the update simultaneously, the more opinion groups in the population, the more agents demonstrate inconsistency between private beliefs and expressed opinions; when the updating happens sequentially (i.e., agents update one by one), all agents end up demonstrating inconsistency between private beliefs and expressed opinions.

## 2.3 Social Norm Interventions in Empirical Studies

The strategy adopted by **Bursztyn and colleagues (2020)** to correct Saudi Arabian men's misperception regarding the injunctive norm about WWOH is by providing information about the true percentage of men supporting WWOH, which is a form of social norm intervention. There have been numerous attempts to design social norm interventions that provide people with information about peers in order to change behaviours **(Yamin et al., 2019; Miller & Prentice, 2016)**. These interventions can be generally categorised into two types **(Yamin et al., 2019)**. One type provides people with summary information regarding the beliefs or actions in a population, usually in a statistical format such as "82% of married Saudi men aged 18–35 agreed that women should be allowed to work outside the home" **(Bursztyn et al., 2020; Yamin et al., 2019)**. The other type directly exposes people to the expressed opinions or actions of peers via various means such as groups discussions and interactive educational sessions **(Yamin et al., 2019)**. This section will introduce examples of each type to provide a brief sketch of existing intervention approaches, and this will serve as a blueprint for model building in the current research.

The interventions that distribute summary information about peers usually take two forms: distributing either the same information to every target individual or personalised information combining the the summary information with comments about how individuals' beliefs or actions compare to their peers **(Yamin et al., 2019; Miller & Prentice, 2016)**. Examples for the former include the intervention to Saudi men's misperception about the WWOH norms **(Bursztyn and colleagues, 2020)**. Another example is a study that gave the information that most other students were concerned about climate change to those who were themselves concerned about this topic, which resulted in those student being more willing to participate in a discussion on climate change **(Geiger & Swim, 2016)**. Social norm interventions of the latter form follows the following structure: "You said you drink 10 drinks per week and that you think the typical student drinks 15. The actual average is 4.6 drinks. You drink more than 80% of other college students" **(Neighbors et al. 2011)**. This type of personalised summary information has been applied to and found generally effective in changing alcohol consumption, tax compliance, and sustainable behaviours **(Miller et al., 2013; Schultz, 1999; Hallsworth et al., 2017)**.

The interventions that directly exposes people to the expressed opinions or actions of peers to change their norm perception can take various forms. The most prominent one in the literature is integrated interventions combining features of group discussions, workshops, and interactive educational activities **(Gidycz et al., 2011; Prince et al., 2015)**. For example, a sexual assault prevention program included a norm correction component in a group discussion and workshop setting, which were found to increase male students' perception of peers' positive attitudes towards bystander intervention and decrease their sexual aggressive behaviours **(Gidycz et al., 2011)**. Another intervention technique that is increasingly being explored is to change the public actions of salient individuals so that changes of larger scales will happen in the population **(Paluck et al., 2012; Paluck et al., 2016)**. These individuals are identified through social network analyses and were administered training sessions before leading public demonstrations of beliefs and actions that are (perceived to be) unpopular. This method is shown to be effective in reducing harassment and conflict behaviours in schools by inducing the norm perception that these behaviours are unacceptable and the discussions about them are encouraged **(Paluck et al., 2012; Paluck et al., 2016)**.

## 2.4 The Current Research

*Some words summarising the current model*

To my best knowledge, few models of belief and action dynamics or models of pluralistic ignorance have conceptualised norm perception as an independent belief that can be revised based on observations of other people **(see exceptions Gavrilets, 2021; Tverskoi et al., 2023)**. The models of belief and action dynamics primarily focus on the change in beliefs about a certain issue itself rather than those about the norm **(e.g., Rainer & Krause, 2002; Martins, 2013)**, while models of pluralistic ignorance usually model the influence of norm as a result of direct observations of others **(e.g., Wang et al., 2014; Seeme, 2019)**. However, perception of social norm is a good candidate to be modelled by models of probabilistic inference. This is because it has the structure of a piece of abstract knowledge that is inferred from limited observational data, as we can state what we believe to be most others' beliefs while we are only able to observe (at most) the beliefs and actions of our acquaintances. Cognitive scientists have used various probabilistic models to understand this type of belief **(Tenenbaum et al., 2011; Chater et al., 2006)**. The current research adopts the Bayesian inference model to model norm perception, which is widely adopted in research of human perception and memory and has been shown adequate in modelling inference about everyday events **(Griffiths and Tenenbaum, 2006; Tavoni et al., 2022)**.

*although studies that combine opinion dynamics and normative influence start to exist, few have applied those in understand real-world phenomenon. aim to study whether the current states of PI will persist under model assumptions. getting ideas form empirical research, will further see if pluralistic ignorance can be reduced by different interventions.*

ABMs are a type of computational model that simulates in a synthetic environment the actions and interactions of autonomous agents, whose behaviours determine the evolution of the entire system **(Bandini et al., 2009)**. ABMs have been deployed in research in a wide range of fields including climate science **(Simmonds et al., 2019)**, ecology **(McLane et al., 2011)**, epidemiology **(de Mooij et al., 2022)**, economics **(Grazzini & Richiardi, 2015)**, finance **(Bonabeau, 2022; Chen et al., 2017,)**, and social sciences **(Epstein & Axtell, 1996; Schelling, 2006)**. Recently, findings from psychological and cognitive science have also been integrated in building more psychologically realistic agent-based models to study political opinion dynamics **(Duaggins, 2017)**, the formation of echo chamber **(Fränken & Pilditch, 2021; Madsen et al., 2018)**, and interventions to the spread of misinformation **(Pilditch et al., 2022)**. These recent developments exemplify the ability of ABMs in modelling a system where multiple individual-level processes, such as interpersonal influence, social conformity, and commitment to previous beliefs, together influence the emergent properties of the system. Since the system of WWOH actions is presumably such a system, ABM is able to model it (???).

## 3 Methodology (2000-3000)

### 3.1 Model Setup

The current research builds an model populated by $n = 100$ agents who are connected by undirected ties and thus form a social network on a two-dimensional $61 \times 61$ lattice. The social network structure follows the small-world network model, which is characterised by being both highly clustered and having low average path lengths **(Watts & Strogatz, 1998)**. This network model is chosen because many real-world social networks, such as email messages, academic co-authorship, and Facebook following, resemble the small-world network, which makes this model a suitable baseline assumption for modelling an unknown network **(Newman, 2003; Ugander et al., 2011)**. The network is programmed by generating agents on a circle, wiring all agents with their closest two neighbours, and rewiring $p_r = 0.5$ of all ties **(Watts & Strogatz, 1998; Wilensky, 2015)**.

#### 3.1.1 Initial Setup

Agents are initialised with a belief regarding WWOH $B_i$ ($B_i \in [0, 1]$), a norm perception $N_i$ (a random variable with a normal distribution with the mean $\mu_{N_i}$ and standard deviation $\sigma_{N_i}$), and a WWOH action $A_i$ ($A_i \in \{0, 1\}$). The variable $B_i$ represents the private belief regarding to what extent women should be allowed to work outside the home. Agents with $B_i \in [0.5, 1]$ are regarded as WWOH supporters and the others are regarded as non-supporters. As the initial condition, $p_s = 0.8$ of all agents are generated as supporters, and the $B_i$ of these agents are drawn from the upper half of the normal distribution with $\mu_B = 0.5$, $\sigma_B = 0.2$. The other agents have a $B_i$ from the lower half of this distribution.

The initial norm perception of each agent is determined by five samples drawn from a normal distribution with $\mu_{N_{prior}} = 0.5$, $\sigma_{N_{prior}} = 0.2$. The mean of the five sampled values provides the prior mean of the norm perception (i.e., $\mu_{N_i}$ of the distribution representing each agent's norm perception) and the standard deviation provides the initial uncertainty about the perception (i.e., $\sigma_{N_i}$).

A certain proportion $p_A = 0.05$ of WWOH supporters demonstrate WWOH actions. The value of $p_s$ and $p_A$ are drawn from the empirical statistics about Saudi Arabian men's beliefs and actions regarding WWOH **(Bursztyn et al., 2020)**.

### 3.1.2 Updating Procedures

At each time step, agents reach out to all linked neighbours and can communicate with each of them with a probability $p_c$. If an agent communicate with a neighbour, the agent updates both its belief and norm perception regarding WWOH based on the neighbour's belief; if the communication doesn't happen, the agent updates its norm perception using the inferred belief from the neighbour's action. Figure 1 gives the flow chart of the procedure. This approach follows the CODA model and its extensions in terms of modelling beliefs as a continuous variable and actions a binary one and updating beliefs depending on whether other agents' beliefs are accessible **(Martins, 2008; Zhan et al., 2022)**. This is aligned with the aim of the current study to model the WWOH action, which is a binary decision, while also capturing the possibly continuous nature of beliefs regarding WWOH.

Agent $i$, upon communicating with and accessing the belief of neighbour $j$, updating its own belief with bounded confidence. The bounded confidence model is able to capture the ubiquitous psychological phenomenon of being more likely to be influenced by people like ourselves while keeps the computation simple **(Cialdini & Goldstein, 2004)**. Previous research has successfully nested the bounded confidence model of opinion dynamics in the framework of the CODA model in the context of a social network, which serves as a foundation for the current model **(Zhan et al., 2022)**. Specifically, agent $i$ updates its belief based on neighbour $j$'s belief using the following equation:

$$B_i' = \begin{cases} B_i, & \text{if } |B_i - B_j| > \epsilon_i \\ B_i + \alpha(B_j - B_i), & \text{if } |B_i - B_j| \leq \epsilon_i \end{cases} \tag{1}$$

where $\alpha \in (0, 0.5]$ is the convergence parameter and $\epsilon_i$ is the bounded confidence threshold of the agent $i$, which is a value drawn from an exponential distribution with the mean $\epsilon_{mean}$ and is a fixed value over time.

Agents update their norm perception $N_i$ following the Bayesian inference model **(Krauß et**

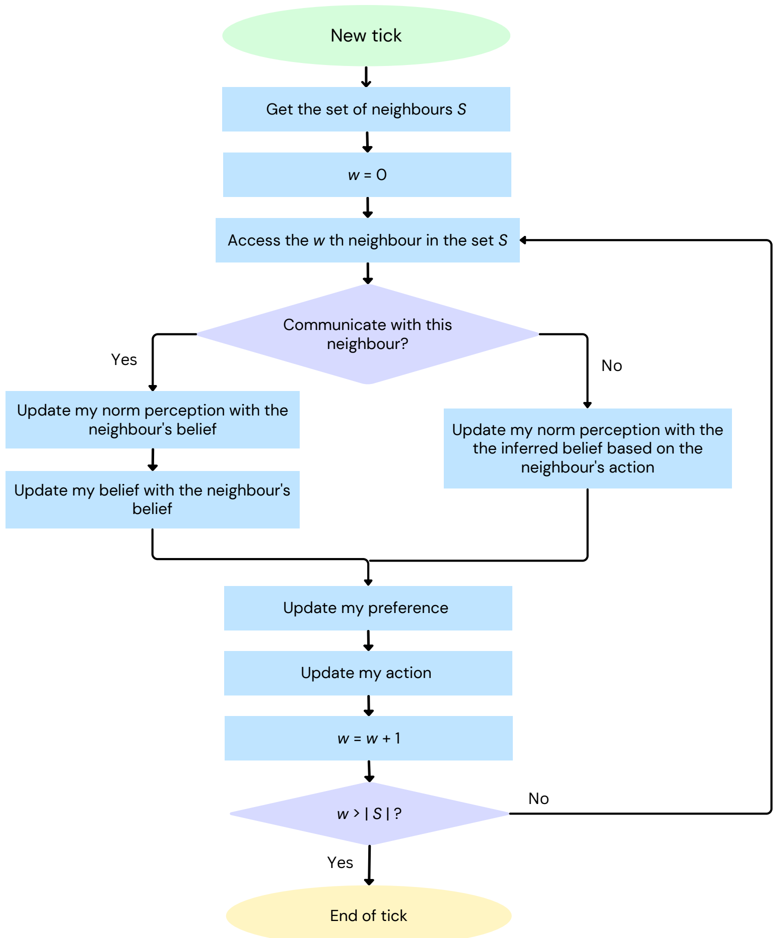


**Figure 1: Main updating procedure.** The flow chart shows agents' behaviours at each time step.

**al.,1999).** Using Bayes' theorem, we can update the conditional probability of the agent $i$'s norm perception given evidence using the following equation:

$$f(N_i \mid E_i) = \frac{f(N_i)f(E_i \mid N_i)}{\int f(N_i)f(E_i \mid N_i)dN_i}, \tag{2}$$

where $f(N_i)$ denotes the prior probability of norm perception of agent $i$, $E_i$ the evidence (either agent $j$'s belief or inferred belief from its action), and $f(E_i \mid N_i)$ the conditional probability of the evidence given $N_i$. Assuming normal distributions for both $f(N_i)$ and $f(E_i \mid N_i)$, the following updating rules for the mean and standard deviation of agents' norm perception can be derived (see the appendices for the derivation):

$$\mu'_{N_i} = \lambda \mu_{N_i} + (1 - \lambda) E_i, \tag{3}$$

$$\sigma'^2_{N_i} = \lambda \sigma^2_{N_i}, \tag{4}$$

where $\lambda = \frac{c_i^2}{c_i^2 + \sigma^2_{N_i}}$. The variable $c_i$ is agent $i$'s confidence in its norm perception (i.e., the standard deviation of the conditional probability of the evidence, $f(E_i \mid N_i)$), which is drawn from an exponential distribution with the mean $c_{mean}$ and is a fixed value over time.

Since agents may or may not communicate with neighbour $j$, the value of $E$ is given by the following equation:

$$E_i = \begin{cases} B_j, & \text{if agent } i \text{ communicates with agent } j \\ P(B_j \mid A_j), & \text{if agent } i \text{ doesn't communicate with agent } j \end{cases} \tag{5}$$

$P(B_j \mid A_j = 1) = 0.9$ and $P(B_j \mid A_j = 0) = 0.1$ by stipulation in the simulations reported in the current research.

After updating their belief and norm perception, agents update preference and action regarding WWOH based on these two values. The preference is determined by the equation

$$P_i = r_i B_i + (1 - r_i) N_i, \tag{6}$$

where $r_i$ represents each agent's resistance to social norm, which is drawn from an exponential distribution with the mean $r_{mean}$ and is a fixed value over time. The WWOH action is determined by the equation:

$$A_i = \begin{cases} 0, & \text{if } P_i \in [0, 0.5) \\ 1, & \text{if } P_i \in [0.5, 1] \end{cases} \tag{7}$$

### 3.2 Simulations

## 4 Results (2000-2500)

## 5 Discussion and Conclusion (1000-1500)

## Reference

# Appendices

## The Derivation of Equation 3 and 4

Since agents' norm perception $N_i$ is a random variable with a normal distribution with the mean $\mu_{N_i}$ and standard deviation $\sigma_{N_i}$, we have the probability density function (PDF) of $N_i$ as:

$$f(N_i) \propto \text{exp}(-\frac{(N_i - \mu_{N_i})^2}{2\sigma_{N_i}^2}).$$

The conditional PDF of the evidence given agent $i$'s prior norm perception in Equation 2 is given by

$$f(E_i \mid N_i) \propto \text{exp}(-\frac{(E_i - N_i)^2}{2c_i^2}).$$

Following the Equation 2, we have the posterior PDF of agent $i$'s norm perception as

$$\begin{aligned} f(N_i \mid E_i) &= C \cdot \text{exp}(-\frac{(N_i - \mu_{N_i})^2}{2\sigma_{N_i}^2})\text{exp}(-\frac{(E_i - N_i)^2}{2c_i^2}) \\ &= C \cdot \text{exp}(-\frac{(N_i - \mu_{N_i})^2}{2\sigma_{N_i}^2} - \frac{(E_i - N_i)^2}{2c_i^2}) \\ &= C \cdot \text{exp}(-\frac{(N_i - \mu'_{N_i})^2}{2\sigma'^2_{N_i}} - \frac{(E_i - \mu_{N_i})^2}{2(\sigma_{N_i}^2 + c_i^2)}), \end{aligned}$$

where $C$ is a normalising constant. Since the second term in the exponent does not involve the random variable $N_i$, we can incorporate it into the constant and rewrite the expression as

$$f(N_i \mid E_i) \propto \text{exp}(-\frac{(N_i - \mu'_{N_i})^2}{2\sigma'^2_{N_i}}),$$

where

$$\mu'_{N_i} = \frac{E_i\sigma_{N_i}^2 + \mu_{N_i}c_i^2}{\sigma_{N_i}^2 + c_i^2} \text{ and } \sigma'^2_{N_i} = \frac{\sigma_{N_i}^2 c_i^2}{\sigma_{N_i}^2 + c_i^2}.$$