

CIS 3223 TMQ 3

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Name: Solutions

Temple ID (last 4 digits):

1 (10 pts) Use the **modular exponentiation** algorithm to calculate $3^{25} \pmod{31}$.

6

$z = 1$ $25 = 11001_2$

| digit | power | z |
|-------|----------------------|------------------------------------|
| 1 | 3 | 3 |
| 0 | 9 | 3 |
| 0 | $81 \equiv_{31} 19$ | 3 |
| 1 | $361 \equiv_{31} 20$ | $20 \times 3 = 60 \equiv_{31} 29$ |
| 1 | $400 \equiv_{31} 28$ | $28 \times 29 = 812 \equiv_{31} 6$ |

$$-3 \times -2 \equiv_{31} 6$$

2 (10 pts) Consider an RSA key set with $N = 77$ and $e = 7$.

What value of d should be used for the secret key? 53

(-1 for -7)

$$N = 7 \times 11 \quad \varphi(N) = \varphi(7)\varphi(11) = 6 \times 10 = 60$$

$$\text{Want solns to } 60x + 7y = 1$$

$$\begin{array}{cccc} a & b & q & r \\ 60 & 7 & 8 & 4 \end{array} \quad \begin{array}{c} Q \\ \left[\begin{array}{cc} 0 & 1 \\ 1 & -8 \end{array} \right] \end{array}$$

$$\begin{array}{cccc} 7 & 4 & 1 & 3 \\ \left[\begin{array}{cc} 0 & 1 \\ 1 & -1 \end{array} \right] \left[\begin{array}{cc} 0 & 1 \\ 1 & -8 \end{array} \right] = \left[\begin{array}{cc} 1 & -8 \\ -1 & 9 \end{array} \right] \end{array}$$

$$\begin{array}{cccc} 4 & 3 & 1 & 1 \\ \left[\begin{array}{cc} 0 & 1 \\ 1 & -1 \end{array} \right] \left[\begin{array}{cc} 1 & -8 \\ -1 & 9 \end{array} \right] = \left[\begin{array}{cc} -1 & 9 \\ 2 & -17 \end{array} \right] \end{array}$$

$$\begin{array}{cccc} 3 & 1 & 3 & 0 \\ \left[\begin{array}{cc} 0 & 1 \\ 1 & -3 \end{array} \right] \left[\begin{array}{cc} -1 & 9 \\ 2 & -17 \end{array} \right] = \left[\begin{array}{cc} 2 & -17 \\ -7 & 60 \end{array} \right] \end{array} \quad \text{check}$$

Could $e = 5$ be chosen?

$$y = -7 \quad d = -7 + 60 = 53$$

yes no

Justify your answer.

$$\gcd(5, 60) = 5 \neq 1$$