CIS 3223 Miniquiz 7R | **Name:** Solutions

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1 (10 pts) Complete the following table by writing T or F in each box, where T represents "true " and F represents "false ". No justification required.

$\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0$

f	g	$f = O(g)$	$f = \Omega(g)$	$f = o(g)$
$n \log^2 n$ $<$	$n^{1.01}$	T	F	T
3^n $<$	$11^{n/2}$	T	F	T
n $>$	$(\log n)^{100}$	F	T	F
5^{n+2} $<$	7^n	T	F	T
$n^{1.01}$ $>$	$n \log^2 n$	F	T	F
n $<$	$\sum_{k=1}^{n} \log k$	T	F	T

$\sum_{k=1}^{n} \log k = \Theta(n \log n)$

What is the most dominant function in the table?

$11^{n/2}$

2 (3 pts) **Give** as good big$-\theta$ estimate for each of the following functions.

(a) $f(n) = (n^3 + 3^n)(n^2 + \log(n^7 + 1))$

$3^n \quad n^2$

$\Theta(n^2 3^n)$

(b) $f(n) = (2^n + 5^{n/2})(n + (\log n)^5)(n\sqrt{n} + 1000)$

$5^{n/2} \quad n \quad n^{3/2}$

$n^2\sqrt{n} \quad 5^{n/2}$

$\Theta(n^{5/2} 5^{n/2})$

3 (2 pts) Evaluate $\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^{16}$

$$= \begin{bmatrix} F_{17} & F_{16} \\ F_{16} & F_{15} \end{bmatrix}$$

$$\begin{bmatrix} 1597 & 987 \\ 987 & 610 \end{bmatrix}$$

4 (6 pts) Answer the following.

(a) Compute $-93 \bmod 19$

2

$\text{mod}(93, 19) = 17$

(b) Compute $13^6 \bmod 11$

9

$\text{mod}(13, 11) = 2$

$13^6 \equiv_{11} 2^6 \equiv_{11} 64 \equiv_{11} 9$

(c) Compute $2^{2024} \bmod 23$

1

$2024/22 = 92$

$\varphi(23) = 22$

$\text{mod}(2024, 22) = 0$

$2^{2024} \equiv_{23} 2^0 \equiv_{23} 1$

(d) Find the smallest non-zero integer b such that $10b \equiv 0 \pmod{35}$

7

$\gcd(10, 35) = 5$

$b = \frac{35}{5} = 7$

(e) Compute $\phi(143)$

120

$143 = 11 \times 13$

$\varphi(143) = \varphi(11)\varphi(13)$

$= 10 \times 12 = 120$

(f) Convert the base-3 number 121212_3 into a decimal integer.

451

$121212_3 = 12_3 \times 10101_3$

$= 5 \times 91$

$= 451$