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31 January 2023
Due Tuesday 7 February 2023

Linear Algebra, Math 2101-002
Homework set #3

1. Given the matrices

$$A = \begin{vmatrix} 1 & 2 \\ -1 & 0 \end{vmatrix}, \quad B = \begin{vmatrix} 3 & 2 & 1 \\ -1 & 0 & 1 \end{vmatrix},$$

(a) Compute $C = A.B$. (b) Compute A^T and B^T. (c) Check that $C^T = B^T.A^T$.

2. Consider the vectors $v = \begin{vmatrix} 1 \\ 2 \\ -1 \end{vmatrix}, \quad w = \begin{vmatrix} 0 \\ 1 \\ 2 \end{vmatrix}$ as 3×1 matrices and compute:
(a) $w^T.v$, (b) $v^T.w$, (c) $v.w^T$ (d) $w.v^T$, (e) $v^T.v$, and (f) $v.v^T$.

3. Use the results from 2. Call $\alpha = v^T.v$. Let $I = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$ be the 3×3 identity matrix.
(a) Compute the matrix $P = I - \frac{1}{\alpha} v.v^T$.
(b) Compute $P.v$ and $P.w$.
(c) Compute $P^2 = P.P$.

4. Prove that for any $n \times 1$ vectors v and w. If $\alpha = v^T.v$ and $P = I - \frac{1}{\alpha} v.v^T$,
(a) $P^T = P$
(b) $P^2 = P$
(c) $P.v = 0$, and
(d) If $v^T.w = 0$, then $P.w = w$.