Answers to ~~Homework~~ #1 Linear Algebra 2101.

(1.) (a) A, B symetric, i.e. $A^T = A$, $B^T = B$
and they commute, i.e. $AB = BA$
prove that $AB = (AB)^T = B^T A^T$

$$(AB)^T = B^T A^T = B \cdot A = AB \qquad \underline{q.e.d}$$

property of transpose of the product

since they are symetric

since they commute.

(1) (b). For example let $A = \begin{vmatrix} 2 & 2 \\ 2 & 1 \end{vmatrix}$, $B = \begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix}$

they are both symetric but

$$A \cdot B = \begin{vmatrix} 2 & 2 \\ 2 & 1 \end{vmatrix} \begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix} = \begin{vmatrix} 0 & 0 \\ 1 & -1 \end{vmatrix}$$

and $B \cdot A = \begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix} \begin{vmatrix} 2 & 2 \\ 2 & 1 \end{vmatrix} = \begin{vmatrix} 0 & 1 \\ 0 & -1 \end{vmatrix} \neq AB$

(2) We proved that both AA^T and A^TA are symetric but they may not be equal to each other
For example

$$A = \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} \qquad A^T = \begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix}$$

$$A A^T = \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} \cdot \begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} = \begin{vmatrix} 5 & 2 \\ 2 & 1 \end{vmatrix}$$

$$A A^T = \begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 2 \\ 2 & 5 \end{vmatrix}$$