§ 4.1.3. Non homogeneous Equations

If y_p: particular solution

of $a_n(x)\dfrac{d^n y}{dx^n} + \cdots + a_1(x)\dfrac{dy}{dx} + a_0(x)y = g(x)$ $\neq 0$

and $y_1, \cdots, y_n$ is a fund'l set of solutions to the associated homogeneous equation (6) on I.

Then the general solution of the in equation on I is

$$y = \underbrace{c_1 y_1 + c_2 y_2 + \cdots + c_n y_n}_{y_h} + y_p(x)$$

$$y = y_c + y_p$$

$$y = y_h + y_p$$

In Example 9 :

$$y_h = c_1 e^{x} + c_2 e^{2x} + c_3 e^{3x}$$ was shown to be the general solution of the by showing $W \neq 0$.

$$y''' - 6y'' + 11y' - 6y = 0.$$

Example 10 : $y''' - 6y'' + 11y' - 6y = 3x$

Above is the complementary function to this nonhomogeneous equation.

Claim : $y_p = -\frac{11}{2} - \frac{1}{2}x$ is a particular solution of the nonhomogeneous DE.

Pf : $y_p' = \frac{1}{2}$, $y_p'' = 0$

$$0 - 6(0) + 11\left(-\tfrac{1}{2}\right) - 6\left(-\tfrac{11}{12} - \tfrac{1}{2}x\right)$$
$$= -\frac{11}{2} + \frac{11}{2} + 3x \stackrel{\checkmark}{=} 3x .$$

So the general solution of the nonhomog. DE is

$$y = y_h + y_p$$

$$y = c_1 e^{x} + c_2 e^{2x} + c_3 e^{3x} - \frac{11}{12} - \frac{1}{2}x .$$

HW : 31, 33 Show $W \neq 0$ for y_c, Prove y_p is a particular solution of the NHDE.

Write $y = y_c + y_p$

T. 4.17 Superposition Principle - Nonhomog. Equations

$y_{p_1}, y_{p_2}, \cdots, y_{p_k}$ are particular solutions of the following, respectively -

$$L(y) = g_1(x)$$
$$L(y) = g_2(x)$$
$$\vdots$$
$$L(y) = g_k(x)$$

Then the particular solution of

$L(y) = g_1(x) + g_2(x) + \cdots + g_k(x)$ is

$y_p(x) = y_{p_1}(x) + \cdots + y_{p_k}(x)$.

See Example 11 for HW problems 35 & 36.