

Exercises for section 2.1

2.1.1. Reduce each of the following matrices to row echelon form, determine the rank, and identify the basic columns.

$$(a) \begin{pmatrix} 1 & 2 & 3 & 3 \\ 2 & 4 & 6 & 9 \\ 2 & 6 & 7 & 6 \end{pmatrix} \quad (b) \begin{pmatrix} 1 & 2 & 3 \\ 2 & 6 & 8 \\ 2 & 6 & 0 \\ 1 & 2 & 5 \\ 3 & 8 & 6 \end{pmatrix} \quad (c) \begin{pmatrix} 2 & 1 & 1 & 3 & 0 & 4 & 1 \\ 4 & 2 & 4 & 4 & 1 & 5 & 5 \\ 2 & 1 & 3 & 1 & 0 & 4 & 3 \\ 6 & 3 & 4 & 8 & 1 & 9 & 5 \\ 0 & 0 & 3 & -3 & 0 & 0 & 3 \\ 8 & 4 & 2 & 14 & 1 & 13 & 3 \end{pmatrix}$$

2.1.2. Determine which of the following matrices are in row echelon form:

$$(a) \begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 4 \\ 0 & 1 & 0 \end{pmatrix}, \quad (b) \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$$(c) \begin{pmatrix} 2 & 2 & 3 & -4 \\ 0 & 0 & 7 & -8 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad (d) \begin{pmatrix} 1 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

2.1.3. Suppose that A is an $m \times n$ matrix. Give a short explanation of why each of the following statements is true.

- (a) $\text{rank}(A) \leq \min\{m, n\}$.
- (b) $\text{rank}(A) < m$ if one row in A is entirely zero.
- (c) $\text{rank}(A) < m$ if one row in A is a multiple of another row.
- (d) $\text{rank}(A) < m$ if one row in A is a combination of other rows.
- (e) $\text{rank}(A) < n$ if one column in A is entirely zero.

2.1.4. Let $A = \begin{pmatrix} .1 & .2 & .3 \\ .4 & .5 & .6 \\ .7 & .8 & .901 \end{pmatrix}$.

- (a) Use exact arithmetic to determine $\text{rank}(A)$.
- (b) Now use 3-digit floating-point arithmetic (without partial pivoting or scaling) to determine $\text{rank}(A)$. This number might be called the "3-digit numerical rank."
- (c) What happens if partial pivoting is incorporated?

2.1.5. How many different "forms" are possible for a 3×4 matrix that is in row echelon form?

2.1.6. Suppose that $[A|b]$ is reduced to a matrix $[E|c]$.

- (a) Is $[E|c]$ in row echelon form if E is?
- (b) If $[E|c]$ is in row echelon form, must E be in row echelon form?