CIS 3223 Homework 1

Dr Anthony Hughes

Name: Solutions
Temple ID (last 4 digits:

1 (32 pts) Complete the following table by writing "T"for true or "F "for false in each box. No justification required.

f	g	$f = O(g)$	$f = \Omega(g)$	$f = \Theta(g)$
$\log 2n$ =	$\log 10n$	T	T	T
$n \log n$ =	$2n \log 2n$	T	T	T
$n^{1/2}$ <	$n^{2/3}$	T	F	F
$n^{1.01}$ >	$n \log^2 n$	F	T	F
$n2^n$ <	3^n	T	F	F
$2^{\log n^2}$ <	$(\log n)^{\log n}$	T	F	F
$\sqrt{n}$ >	$(\log n)^5$	F	T	F
$\Sigma_{i=1}^n i^k$ >	n^k	F	T	F

$n^{0.1} > (\log n)^2$

$2^{\log n^2} = 2^{2\log n} = 4^{\log n}$

$\sqrt{n} = n^{\frac{1}{2}}$

$\int_1^n x^k \, dx = \frac{x^{k+1}}{k+1}\Big|_1^n = \Theta(n^{k+1})$

2 (6 pts) **Give** as good big$-\Theta$ estimate for each of the following functions.

(a) $f(n) = (n^2 + (\log(n^5 + \cancel{1}))(n! + n3^n))$ $\quad$ $\cancel{\Theta(n \log n)}$ $\quad$ $\boxed{\Theta(n! \log n)}$

$n^2 + (5 \log n)(n!)$

(b) $f(n) = (n + (\log \cancel{n})^5)(n^4 + \cancel{4n^2})(n\sqrt{n} + \cancel{1000})$ $\quad$ $\boxed{\Theta(n^{13/2})}$

$n \qquad n^4 \qquad n^{3/2}$

3 (4 pts) Evaluate

$$\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}^{12} = \begin{bmatrix} F_{11} & F_{12} \\ F_{12} & F_{13} \end{bmatrix} = \boxed{\begin{matrix} 89 & 144 \\ 144 & 233 \end{matrix}}$$

$$\begin{bmatrix} F_{n-1} & F_n \\ F_n & F_{n+1} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}^n$$

Note $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} c & d \\ a & b \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} d & c \\ b & a \end{bmatrix}$

4 (8 pts) Use strong induction to prove the following:

$$F_n \leq 1.7F_{n-1}, \quad n \geq 4$$

Base cases: Show true for $n = 4$ and 5:

$n = 4:$

lhs $= F_4 = 3$

rhs $= 1.7F_3 = 1.7 * 2 = 3.4$

So lhs $\leq$ rhs. True for $n = 4$

$n = 5:$

lhs $= F_5 = 5$

rhs $= 1.7F_4 = 1.7 * 3 = 5.1$

So lhs $\leq$ rhs. True for $n = 5$

Inductive case: Assume true for $n = s,\ 4 \leq s \leq k,\ k \geq 5$

Show true for $n = k + 1$:

lhs $= F_{k+1} = F_k + F_{k-1}$

$\leq 1.7 F_{k-1} + 1.7 F_{k-2}$ (Induction)

$= 1.7 (F_{k-1} + F_{k-2})$

$= 1.7 F_k$

rhs $= 1.7 \; F_{(k+1-1)} = 1.7 F_k$ [Note lhs $\leq 1.7 F_k$ = rhs]

So lhs $\leq$ rhs. True for $n = k+1$

(Bonus 2 pts) Why do we need to start with $n = 4$ (table useful)?

$n = 3$ lhs $= F_3 = 3$

rhs $= 1.7F_2 = 1.7 * 2 = 3.4$

$\Big\}$ lhs > rhs

not true for $n = 3$.