Intro to chapter 4

Use dot product for matrix multipl.

result

$$\underbrace{n \times \underbrace{m \quad m}_{} \times p}_{\text{result}}$$

eg
$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 6 & 3 \\ 5 & 2 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 1(6)+2(5)+3(4) & 10 \\ 4(6)+5(5)+6(4) & 28 \end{bmatrix}$$

$$2 \times 3 \quad 3 \times 2$$

$$= \begin{pmatrix} 28 & 10 \\ 73 & 28 \end{pmatrix}$$

1,1

1,2

$1(3)+2(2)+3(1)$
$=3+4+3 = 10$

2,1

$24+25+24$
$=48+25 = 73$

Cramer's Rule (deferred)

2,2

$4(3)+5(2)+6(1)$
$=12+10+6$
$=22+6 = 28$

Determinant of 2 × 2 matrix

$$2\times 2 \qquad \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1$$

eg
$$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 1(4)-2(3) = 4-6 = -2$$

Determinant of 3×3

$$\begin{array}{ccc} + & - & + \\ \end{array}$$

$$\begin{vmatrix} e^x & e^{2x} & e^{3x} \\ e^x & 2e^{2x} & 3e^{3x} \\ e^x & 4e^{2x} & 9e^{3x} \end{vmatrix}$$

$$= e^x \begin{vmatrix} 2e^{2x} & 3e^{3x} \\ 4e^{2x} & 9e^{3x} \end{vmatrix} - e^{2x} \begin{vmatrix} e^x & 3e^{3x} \\ e^x & 9e^{3x} \end{vmatrix}$$

$$+ e^{3x} \begin{vmatrix} e^x & 2e^{2x} \\ e^x & 4e^{2x} \end{vmatrix}$$

$$= e^x \left(18e^{5x} - 12e^{5x}\right) - e^{2x}\left(9e^{4x} - 3e^{4x}\right)$$

$$+ e^{3x}\left(4e^{3x} - 2e^{3x}\right)$$

$$= e^x(6e^{5x}) - e^{2x}(6e^{4x}) + e^{3x}(2e^{3x})$$

$$= 6e^{6x} - 6e^{6x} + 2e^{6x}$$

$$= 2e^{3x} \neq 0, \ \forall x.$$

Cramer's Rule

$$a_1 x + b_1 y = c_1$$
$$a_2 x + b_2 y = c_2$$

$$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1$$

$$D_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix} = c_1 b_2 - c_2 b_1$$

$$D_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} = a_1 c_2 - a_2 c_1$$

then Cramer's Rule:

$$x = \frac{D_x}{D} \quad \text{and } y = \frac{D_y}{D}$$

As long as $D \neq 0$.

eg $x + y = 0$

$-3x - 2y = 2$

$$\begin{bmatrix} 1 & 1 \\ -3 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$2x + 2y = 0$

$-3x - 2y = 2$

$-x = 2$

$x = -2$

$y = 2$

$$D = \begin{vmatrix} 1 & 1 \\ -3 & -2 \end{vmatrix} = -2 - (-3) = 1$$

$$D_x = \begin{vmatrix} 0 & 1 \\ 2 & -2 \end{vmatrix} = 0 - 2 = -2$$

$$D_y = \begin{vmatrix} 1 & 0 \\ -3 & 2 \end{vmatrix} = 2 - 0 = 2$$

$$x = \frac{D_x}{D} = \frac{-2}{1} = -2$$

$$y = \frac{D_y}{D} = \frac{2}{1} = 2$$