Daniel B. Szyld

20 April 2023
Due Tuesday 25 April 2023

Linear Algebra, Math 2101-002
Homework set #13

(1) Let $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 43/25 & -24/25 \\ 0 & -24/25 & 57/25 \end{bmatrix}$.

Compute all eigenvalues and eigenvectors of A. Check that the eigenvalues λ and the eigenvectors v satisfy $Av = \lambda v$, and in this case, for any two different eigenvalues λ_1, λ_2 with corresponding eigenvectors v_1, v_2, we have $v_1^T v_2 = 0$.

(2) (Extra credit).
Prove that if $A = A^T$, and $\lambda_1, \lambda_2 \in \sigma(A)$, $\lambda_1 \neq \lambda_2$, i.e., if the symmetric matrix A has two *distinct* eigenvalues, then, the corresponding eigenvectors are orthogonal. In other words, if $Av_1 = \lambda_1 v_1$ and $Av_2 = \lambda v_2$, then $v_1^T v_2 = 0$.