§ 7.1   Intro   (NOT needed)

Transform takes one function into another.

eg $(3x^2)' = 6x$; $\int 3x^2 \, dx = x^3 + C$

Differentiation   ;   Integration

Suppose $F(x,y)$ is a function of two variables.

Taking the definite integral of F WRT one variable transforms $F(x,y)$ into a function of the other variable.

eg $\int_1^2 2xy^2 \, dx = y^2 \left[x^2 \right]_1^2 = y^2(4-1) = 3y^2$

Combine the two ideas: Integral Transform

$\int_a^b K(s,t) \, F(t) \, dt$ will transform a

Kernel of transform

function F of variable t to a function F of variable s.

One more piece is needed.

Recall

Improper Integrals : limits of definite integrals.
↪ extends to fns of more than one variable.

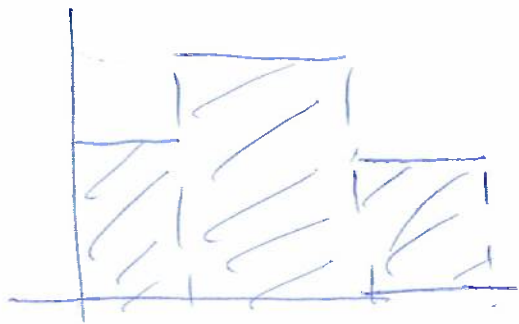$$\int_0^\infty K(s,t)\, \mathcal{F}(t)\, dt = \lim_{T \to \infty} \int_0^T K(s,t)\, \mathcal{F}(t)\, dt$$

If the limit exists, the integral is said to converge. Otherwise, it diverges.

⤷ the limit will exist for certain s.

Laplace Transform is of this type. ↖

Def : Piecewise Continuous function on $[a,b]$.
— function with finitely many jump discontinuities — PW cont fns are integrable.

eg

$\mathcal{F} > 0$

$\int_a^b \mathcal{F}(x)\, dx$ is area under the curve of $\mathcal{F}$.