

1 (15 pts) Complete the following table by writing T or F in each box, where T represents “true ” and F represents “false ”. No justification required.

$f$		$g$	$f = O(g)$	$f = \Omega(g)$
$3^{n+2}$	$<$	$5^n$	T	F
$2n + \log n$	$=$	$n + (\log n)^2$	T	T
$2^n$	$<$	$5^{n/2}$	T	F
$n$	$>$	$(\log n)^{100}$	F	T
$n!$	$>$	$4^n$	F	T
$n^{0.1}$	$>$	$(\log n)^{10}$	F	T
$n$	$<$	$\sum_{k=1}^n \log k$	T	F

What is the most dominant function in the table?

$n!$

2 (5 pts) Give as good big- $\theta$  estimate for each of the following functions.

(a)  $f(n) = \frac{n^4 + 2^n}{2^n} (n^3 + \log(n^4 + 1))$

$\Theta(n^3 2^n)$

(b)  $f(n) = \frac{4n^2 \log(3^n + 1)}{n^4} + \frac{n^4}{n!} (n! + n3^n)$

$\Theta(n^4 n!)$

3 (Bonus 1 pt) Does  $f_n$  divide  $f_{2n}$ ,  $n \geq 1$

YES NO

$$\begin{pmatrix} f_{2n+1} & f_{2n} \\ f_{2n} & f_{2n-1} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^{2n} = \left( \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^n \right)^2 = \begin{pmatrix} f_{n+1} & f_n \\ f_n & f_{n-1} \end{pmatrix}^2 = \begin{pmatrix} * & f_{n+1}f_n + f_n f_{n-1} \\ * & * \end{pmatrix}$$

$$f_{2n} = f_{n+1}f_n + f_n f_{n-1} = f_n (f_{n+1} + f_{n-1})$$