§ 4.1.2 Homogeneous Equations

Def: p. 121 Homogeneous vs. nonhomogeneous

From this point forward,

Assume • $a_i(x)$ and $g(x)$ are continuous (on I), $i = 0, 1, 2, \ldots, n$.

• $a_n(x) \neq 0$, $\forall x$ in the interval.

Operator : Takes a function and converts it to another function

es $(f(x))' = f'(x)$

A linear operator is s.t.

$$L\{(f+g)(x)\} = L(f(x)) + L(g(x))$$

$$L\{c f(x)\} = c\, L\{f(x)\}$$

Differential operator:

$$\frac{dy}{dx} = Dy \qquad \frac{d^2y}{dx^2} = D^2y \quad \text{etc...}$$

eg $D(\cos 4x) = -4\sin(4x)$

In general, n^{th} order Differential operator is defined to be

$$L = a_n(x)D^n + a_{n-1}(x)D^{n-1} + \cdots + a_1(x)D + a_0(x).$$

eg $y'' + 5y' + 6y = 5x - 3$

can be written

$$D^2y + 5Dy + 6y = 5x - 3$$

Putting it all together:

Homogeneous n^{th} order DE can be written compactly as $L(y) = 0$.

Non hom: $L(y) = g(x)$.

er

Theorem 4.1.2 Suppposition Principle.

- Homog. DE. p.121

Summary

$y_1, \cdots, y_k$ Solutions $\Rightarrow$

$$y = c_1 y_1 + \cdots + c_k y_k$$

is also a solution to the DE.

Pf: $k = 2$

Let y_1, y_2 be solution of the homog.

L. DE $L(y) = 0$. $\Rightarrow$ $L(y_1) = 0$

$L(y_2) = 0$

Let $y = c_1 y_1 + c y_2$

So $L(c_1 y_1 + c y_2) = c_1 L(y_1) + c_2 L(y_2)$

$= 0 + 0$

$= 0$

Corollary to T4.1.2

(A) A constant multiple $y = c_1 y_1(x)$ of a solution $y_1(x)$ of a homo. Linear DE is also a solution.

(B) see book

The set of two functions $f_1(x)$ and $f_2(x)$ are linearly independent

$\Longleftrightarrow$ they are NOT constant multiples of each other.

Pf. in Book

Does a linear combination of n solutions give you all the solutions to a n^{th} order HLDE?

Only if $W \neq 0$ where p.121

Def: 4.1.2 Wronkian

$$W(y_1, y_2, \cdots, y_n) = \begin{vmatrix} y_1 & y_2 & \cdots & y_n \\ y_1' & y_2' & \cdots & y_n' \\ \vdots & & & \\ y_1^{(n-1)} & y_2^{(n-1)} & \cdots & y_n^{(n-1)} \end{vmatrix}$$

$\neq 0$

If $W(y_1, \dots, y_n) \neq 0$, then

$\{y_1, \dots, y_n\}$ is the fundamental set of solutions to HLDE and any solution is a linear combination of these n functions. i.e.

General solution of the HLDE is

$$y = c_1 y_1 + c_2 y_2 + \cdots + c_n y_n.$$

T. 4.1.3 Criterion for Linearly Independent Solutions --- $W \neq 0$

p. 124

Def 4.1.3 Fund'l Set of Solutions. $\{y_1, \dots, y_n\}$ LI p1.25

T. 4.1.4 Existence of A p. 125

T. 4.1.5 General Solution p. 126

HW : 23, 25, 26, 27, 28, 29

eg $y_1 = e^{3x}$, $y_2 = e^{-3x}$ are

Solutions to $y'' - 9y = 0$ on $(-\infty, \infty)$.

$$W(y_1, y_2) = \begin{vmatrix} e^{3x} & e^{-3x} \\ 3e^{3x} & -3e^{-3x} \end{vmatrix} = -3e^0 - 3e^0$$

$$= -6 \neq 0,$$

$\forall x \in (-\infty, \infty)$

So y_1, y_2 form a fund'l set of solutions to the HLDE and

the general solution is

$$y = C_1 e^{3x} + C_2 e^{-3x}$$

of HLDE on $(-\infty, \infty)$