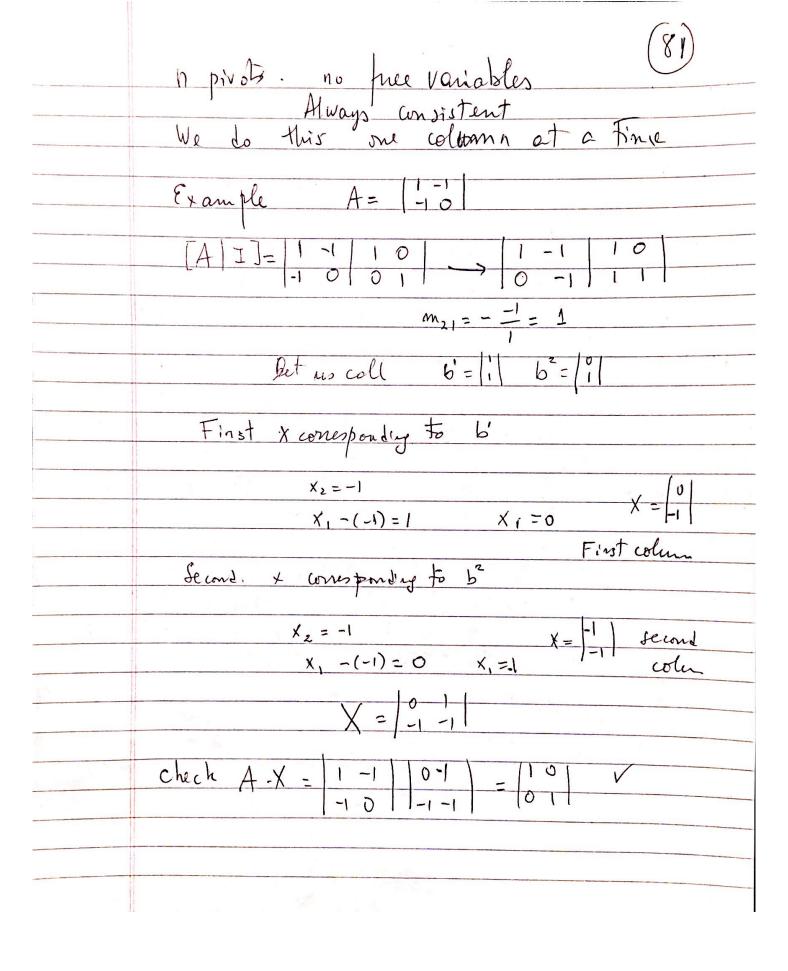
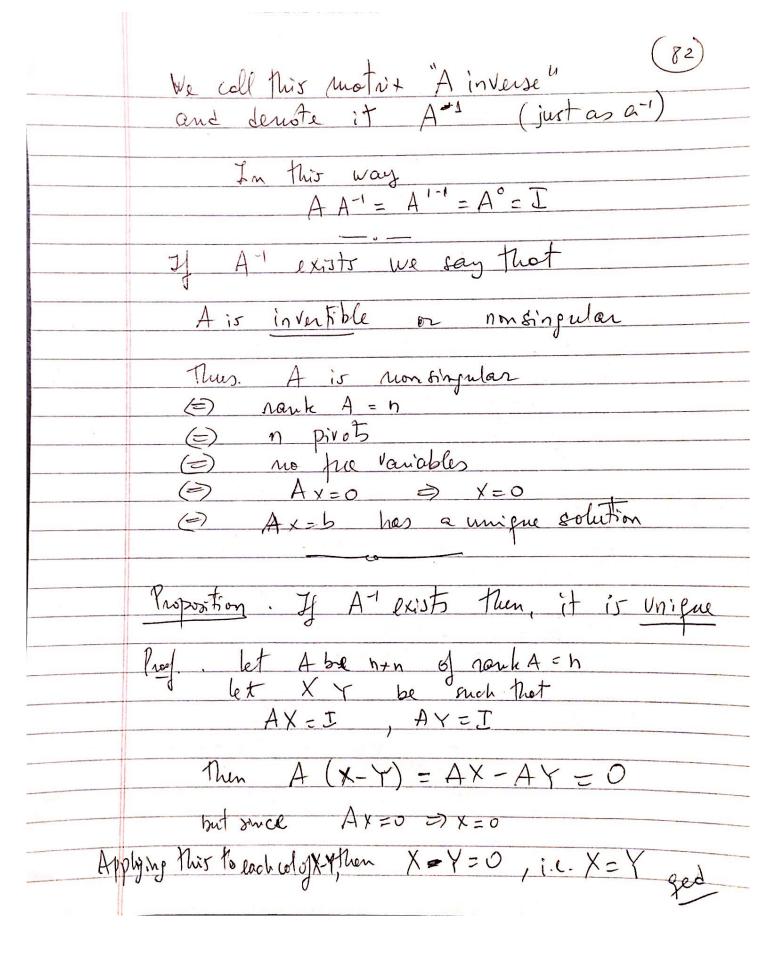
Section 3.7 Matrix Inversion In real numbers a E IR a = 0 = 1 b, b = 0 $\Rightarrow a.b=1$ b-a=1Indeed b=== a" Same with square matrices of full rank If Ais nxn and rank A=n IX rum of rank n so that XA = AX = I I nen identity First we look into finding X so that AX=I Consider Dupmented Matrix [A I] do Gassian elimination > [U/Z] Since A it full rank U = I upper twongular n pivots Obtain X by back substitution UX = Z AX = I





(83) Proposition. A nxn nonsingular

if AX = I then XA = IProof. First note that X must be nonsingular otherwise 7 v=0 with Xv=0 but V=IV= AXV= A0=0 V=0 contradiction. X nonsingular, then $\exists X^{-1}$ so that X = I $A \times -T \Rightarrow A \times X^{-1} = T \times^{-1} = X^{-1}$ $A = X^{-1}$ multiply by X => XA = XX = I XA=I q.ed Note, we just showed $(A^{-1})^{-1} = A$ Also Note $(A^2)^{-1} = (A^{-1})^2$ and (A") = (AT)

Proposition let A, B be nxn monsingular,
Then A.B um singular.
Proof. It suffices to exhibit (AB) 7 So that AB (AB) 1=I
Well (AB) = B-'A-1
$AB B^{7}A^{-1} = A (BB^{-1}) A^{-1}$ = $A I A^{-1} = AA^{-1} = I$ fed.
ADBA' = A(BB')A' $-ATA' - AA' - T$
fed.
$(A B)^{-1} = B^{-1}A^{-1}$
$(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$
(A, A2 An) = An An A2 A,

85 Anxn y Ais not invertible (mondingular) we say that A is singular (=) Jx+0 > Ax=0 (=) Nank A < n (=) Free Yayables Example A= |22 7 00 A= 001 A = 100 200 200 200 In general triangular motrix with some Licyonal entry =0, singular In particular diagrand matrix [00] Not I'=I sma I.I =I