Chapter 1 : Intro to DE

§1.1 Definitions & Terminology

Def 1.1.1 Differential Equation: An equation containing the derivatives of one or more unknown functions (or dependent variables), WRT one or more independent variables is said to be a differential equation.

solve for

I) Types of DE

a) Ordinary Differential Equation: ODE - single independent variable.

Partial Differential Equation : two or more independent variables.

(An ODE can have more than one unknown function).

eg ODE $\frac{dy}{dx} + 5y = 0$

eg Partial DE $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

b) Order of the DE : order of highest derivative.

eg $\dfrac{d^3y}{dt^3} + y^5 = \cos t$ Third order

nothing to do with order

eg $y' = 2 - y$, 1st order

eg $M(x,y)\,dx + N(x,y)\,dy = 0$ 1st order

but either x or y can be the independent variable.

differential form
(DO NOT worry about this for now)

1.1.9
HW #9 (both ways).

Normal form of a DE (do NOT worry about this)

$$\frac{d^n y}{dx^n} = f(x, y, y', y'', \ldots, y^{n-1})$$

eg $\dfrac{dy}{dx} = f(x,y)$

eg $\dfrac{d^2y}{dx^2} = f(x, y, y')$

c) Linear vs Non linear DE

Def: An n^{th} order DE is said to be linear if it is linear in $y, y', \ldots, y^{(n)}$.

linear n^{th}-order DE

$$a_n(x)\frac{d^n y}{dx^n} + a_{n-1}(x)\frac{d^{n-1}y}{dx^{n-1}} + \cdots + a_1(x)\frac{dy}{dx} + a_0(x)y = g(x) \qquad (6)$$

Note: 1) All powers of y and its derivatives are 1^{st} degree.

2) All the coefficients are fns of the independent variable only.

Def: A nonlinear DE is not linear.

Examples (next page)

eg $t^2 y'' + \ln t \, y' + e^t y = \arctan t$

is linear

eg $(1+y) y'' + \ln t \, y' + e^t y = \arctan t$

is nonlinear

$1+y$ is not a fn of the independent variable t.

eg $\dfrac{d^3 y}{dx^3} + y^5 = \cos x$ nonlinear

HW 1-7 classification of ODE
order, linear or nonlinear.

II) Solution of a DE

Def: Any fn ϕ, defined on an interval I and possessing at least n derivatives that are continuous on I, which when substituted into an n-th order DE reduces the equation to an identity, is said to be a solution of the equation on the interval I. (will come back to this).

HW 11-18 To check to see if a function is a solution of a DE, plug it into the equation.

eg $y'' - \tan t \, y' = 1$

Prove $y = \ln(\sec t)$ is a solution to the DE.

$$y = \ln(\sec t)$$

$$y' = \frac{\sec t \tan t}{\sec t} = \tan t$$

$$y'' = \sec^2 t$$

$$y'' - \tan t \, y' = 1$$

$y = \ln(\sec t) \implies \sec^2 t - \tan t(\tan t) -$

$= \sec^2 t - \tan^2 t$

$= \tan^2 t + 1 - \tan^2 t$

✓

$= 1$

$\therefore y$ is a solution.

HW 33-34

Solutions of the form $y = e^{rt}$

$y' = re^{rt}$

$y'' = r^2 e^{rt}$

etc...

Do the same as above and solve for r.

HW 35-36 $\quad y = x^m \quad y' = mx^{m-1}$

~~etc.~~

Solve for m.

Note: $x^m \neq 0$ for any $x \neq 0$.

need an identity true for all x

Note: A solution of a DE that is identically 0 (i.e. $y=0$) on an interval I is said ~~said~~ to be a trivial solution

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III) Solution Curve. The graph of a solution ϕ of an ODE is called a solution curve.

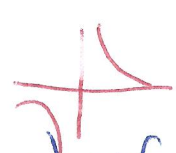
Since ϕ is differentiable on I, it is continuous on I.

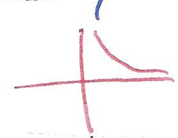
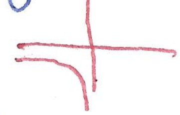
must Be

for us solution of DE.

eg $y = \frac{1}{x}$ is a solution of $xy' + x = 0$ (verify).

But when we talk about solution of DE, we are talking about an open (connected) interval. Unlike domains, can only have 1 interval.



 OR 

$D_{\frac{1}{x}} = (-\infty,0) \cup (0,\infty)$, but $y = \frac{1}{x}$ is a solution either on $(-\infty,0)$ OR $(0,\infty)$.