## Section 3.6 Properties of Matrix Product.

**Distributive** (left and right)

$$A(B+C) = AB + AC$$

$$(D+E)F = DE + E \cdot F$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \left( \begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ -1 & 0 \end{vmatrix} \right) = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} \cdot \begin{vmatrix} 2 & 1 \\ -1 & 2 \end{vmatrix} = \begin{vmatrix} 0 & 5 \\ 2 & 11 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 5 \\ 3 & 11 \end{vmatrix} + \begin{vmatrix} -1 & 0 \\ -1 & 0 \end{vmatrix} = \begin{vmatrix} 0 & 5 \\ 2 & 11 \end{vmatrix}$$

**Associative** $A(BC) = (AB) \cdot C$

$$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} \left( \begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} \cdot \begin{vmatrix} 1 & 0 \\ -1 & 0 \end{vmatrix} \right) = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} \begin{vmatrix} 0 & 0 \\ -2 & 0 \end{vmatrix} = \left( \begin{matrix} -4 & 0 \\ -8 & 0 \end{matrix} \right|$$

$$\left( \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} \cdot \begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} \right) \cdot \begin{vmatrix} 1 & 0 \\ -1 & 0 \end{vmatrix} = \begin{bmatrix} 1 & 5 \\ 3 & 11 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix} = \begin{pmatrix} -4 & 0 \\ -8 & 0 \end{pmatrix}$$

Also $(\alpha B) . C = \alpha (BC) = B(\alpha C)$

$$2 \cdot \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} \cdot \begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} = \begin{vmatrix} 2 & 4 \\ 6 & 8 \end{vmatrix} \begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} = \begin{vmatrix} 2 & 10 \\ 6 & 22 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} \cdot 2 \begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} \begin{vmatrix} 2 & 2 \\ 0 & 4 \end{vmatrix} = \begin{vmatrix} 2 & 10 \\ 6 & 22 \end{vmatrix}$$

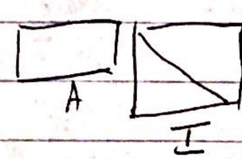
$$\boxed{\Rightarrow \alpha A \cdot \alpha B = \alpha^2 AB}$$

Identity matrix $\quad I = \begin{vmatrix} 1 & 0 & 0 & 0 & \cdots \\ 0 & 1 & & 0 & 0 \\ 0 & 0 & \ddots & & \\ \vdots & & & \ddots & \\ 0 & 0 & 0 & & 1 \end{vmatrix}$

$n \times n$

diagonal matrix $\quad$ ii entry $= 1$
ij entry $= 0 \quad i \neq j$

is such that $I . A = A$
$A \cdot I = A$

Recall A $n \times n$ square

$$A^2 = A \cdot A$$

$$A^3 = A^2 A = A A^2$$

$$A^n = \underbrace{A \cdots A}_{n \text{ times}}$$

define $A^0 = I$ (same as $a^0 = 1$
$a \cdot 1 = a$
$1 \cdot a = a$ )

$$A^r A^s = A^{r+s} \qquad (A^r)^s = A^{rs}$$

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Observe since $AB \neq BA$

$$(A+B)^2 = (A+B)(A+B) =$$

$$= A^2 + AB + BA + B^2$$

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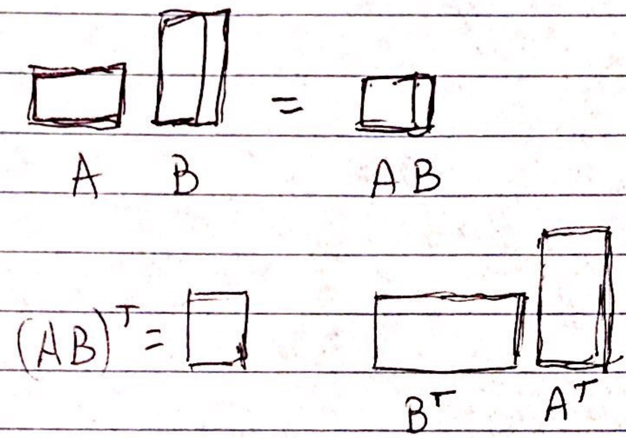
$$(AB)^T = B^T A^T$$




$\Rightarrow A^TA$ always symmetric

$(A^TA)^T = A^T(A^T)^T$
$= A^TA$

also $AA^T$

$(AA^T)^T = A^{T\,T}A^T = AA^T$

Recall $AB \neq BA$

But. Proposition: $\text{trace}(AB) = \text{trace}(BA)$

Proof. $\text{trace}(AB) = \sum_{i=1}^{n} (AB)_{ii} = \sum_{i=1}^{n} A_{i*}B_{*i} =$

$= \sum_{i}^{n} \sum_{k}^{m} a_{ik} b_{ki} = \sum_{i}^{n} \sum_{k}^{m} b_{ki} a_{ik} = \sum_{k}^{m} \sum_{i}^{n} b_{ki} a_{ik}$

$ab = ba$ in $\mathbb{R}$

$= \sum_{k=1}^{m} B_{k*} A_{*k} = \sum_{k=1}^{m} [BA]_{k} = \text{trace}(BA)$

Note $A_{m\times n}$ $B_{n\times m}$ $AB$ $n\times n$
$BA$ $m \times m$

$\text{trace}(ABC) = \text{trace}(CAB)$
$= \text{trace}(BCA)$

But $\text{trace}(ABC) \neq \text{trace}(BAC)$ !