CIS 3223 Homework 4

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1 (12 pts) (a) N = 289 289 is a perfect square **true** **false**

L	U	U-L	U-L > 1	M = ⌊ L + U)/2⌋	M^2	Action
1	289	288	True	145	21025	U = M
1	145	145	True	73	5329	U = 73
1	73	72	True	37	1369	U = 37
1	37	36	True	19	361	U = 19
1	19	18	True	10	100	L = 10
10	19	9	True	14	196	L = 14
14	19	5	True	16	256	L = 16
16	19	3	True	17	289	

(b) N = 360 360 is a perfect square **true** **false**

L	U	U-L	U-L > 1	M = ⌊ L + U)/2⌋	M^2	Action
1	360	359	True	180	32400	U = 180
1	180	179	True	90	8100	U = 90
1	90	89	True	45	2025	U = 45
1	45	44	True	23	529	U = 23
1	23	22	True	12	144	L = 12
12	23	11	True	17	289	L = 17
17	23	6	True	20	400	U = 20
17	20	3	True	18	324	L = 18
18	20	2	True	19	361	U = 19
18	19	1	False			

2 (12 pts) Give a big-θ bound for the solutions of the following relations (show work).

$\boxed{\Theta(n^2)}$

(a) $T(n) = 4T(n/2) + 1$

$a = 4 \quad b = 2 \quad d = 0$

$\left.\begin{array}{l} \log_b a = \log_2 4 = 2 \\ d = 0 \end{array}\right\}$

$d < \log_b a \qquad T(n) = O(n^{\log_b a}) = \Theta(n^2)$

$\boxed{\Theta(n^{3/2})}$

(b) $T(n) = 7T(n/4) + n^{3/2}$

$a = 7 \quad b = 4 \quad d = 3/2$

$\log_b a = \log_4 7 < \log_4 8 = \frac{3}{2}$

$d = 3/2$

$d > \log_b(a) \qquad T(n) = \Theta(n^d) = \Theta(n^{3/2})$

$\boxed{\Theta(n^2 \log n)}$

(c) $T(n) = 9T(n/3) + n^2$

$a = 9 \quad b = 3 \quad d = 2$

$\log_b a = \log_3 9 = 2$

$d = 2$

$d = \log_b a \qquad T(n) = \Theta(n^d \log n) = \Theta(n^2 \log n)$

The Master Theorem (subtract and conquer)

3 (6 pts) Give a big-θ bound for the solutions of the following relations (show work).

(a) $T(n) = 4T(n-2) + n$

$\boxed{\Theta(n2^n)}$

$a = 4 \quad b = 2 \quad d = 1$

$a > 1 \quad T(n) = \Theta(n\, a^{n/b})$

$= \Theta(n\, 4^{n/2})$

$= \Theta(n\, (4^{1/2})^n)$

$= \Theta(n\, 2^n)$

(b) $T(n) = T(n-1) + 2$

$\boxed{\Theta(n)}$

$a = 1 \quad b = 1 \quad d = 0$

$T(n) = \Theta(n^{d+1}) = \Theta(n^{0+1}) = \Theta(n)$

4 (10 pts) P71 Q4 (Show work. Circle choice)

Algorithm A $T(n) = 5(\frac{n}{2}) + O(n)$

$a = 5 \quad b = 2 \quad d = 1$

$\left.\begin{array}{l}\log_b a = \log_2 5 > 2 \\ d = 1\end{array}\right\}$ $d < \log_b a$

$T = O(n^{\log_b a}) = O(n^{\log_2 5})$

Algorithm B $T(n) = 2T(n-1) + O(1)$

$a = 2 \quad b = 1 \quad d = 0$

$a > 1 \quad T(n) = O(n^d a^{n/b}) = O(n^0 2^{n/1}) = O(2^n)$

Algorithm C $T(n) = 9T(n/3) + O(n^2)$

$a = 9 \quad b = 3 \quad d = 2$

$\left.\begin{array}{l}\log_b a = \log_3 9 = 2 \\ d = 2\end{array}\right\}$ $d = \log_b a$

$T(n) = O(n^d \log n) = O(n^2 \log n)$

$n^{\log_2 5}, \; 2^n, \; n^3 \log n$

$\log_2 5 > 2$,

$n^2 = o(n^{\log_2 5})$

$n^2 \log n = o(n^{\log_2 5})$

$n^2 = o(2^n)$

$n^2 \log n = o(2^n)$

Algorithm C

Algorithm A: $O(n^{\log_2 5})$	Algorithm B: $O(2^n)$	Algorithm C: $O(n^2 \log n)$

5 (5 pts). Exercise 1.31 P41

(a) # bits $= \lceil \log_2 N! \rceil \sim \log_2 N!$

$$\log_2 N! = \log_2(1 \cdot 2 \cdot 3 \cdots N) = \sum_{i=1}^{N} \log i$$

$$= \Theta\left(\int_1^N \log x \, dx\right) = \Theta\left(x \log x \Big|_1^N\right) = \boxed{\Theta(N \log N)}$$

(b) function y = fact(N)

if N==1 return 1;

Z=1;

for i = 2 to N

Z = Z * i;

return Z;

runtime ignoring multiplications = $O(N)$

Z is at most N! -- N log N bits

i is at most N -- log N bits

$Z * i = O(N \log N * \log N)$

$= O(N(\log N)^2)$

[m bit * n bit is $O(mn)$]

runtime counting multiplication $= O(N * N(\log N)^2) = O(N^2(\log N)^2)$

6 (5 pts). Exercise 1.2 P38

base-10 digits $d_{10}(N) = \lceil \log_{10} N \rceil < 1 + \log_{10} N$

base-2 digits $d_2(N) = \lceil \log_2 N \rceil \gtrsim \log_2 N$

$$\frac{d_2(N)}{d_{10}(N)} < \frac{1 + \log_2(N)}{\log_{10}(N)} = \frac{1 + \log_2 10 \log_{10}(N)}{\log_{10}(N)}$$

$$= \frac{1}{\log_{10}(N)} + \log_2 10 \leq 4$$

$$\leq \frac{1}{2} + \log_2 10 = 0.5 + 3.3219 \leq 4 \quad (N \geq 100)$$

$$\lim_{N \to \infty} \frac{1}{\log_{10} N} = 0 \qquad \lim_{N \to \infty} \frac{d_2(10)}{d_{10}(10)} = \log_2 10$$