§ 2.2 Separable Equations (First-Order)

Definitions from § 1.1

A solution in which the dependent variable is expressed solely in terms of the independent variable and constants is said to be an explicit solution.

$$y = \phi(x)$$

A relation $G(x,y) = 0$ is said be an implicit solution of an ordinary DE on an interval I, ... (see book).

Most basic type of SE:

$$\frac{dy}{dx} = F(x)$$

Ex

$$\frac{dy}{dx} = x^2$$

$$dy = x^2\,dx$$

$$\int 1\,dy = \int x^2\,dx$$

$$y = \frac{1}{3}x^3 + C$$

In this course,

1^{st} order separable ΔE is of the form $\frac{dy}{dx} = g(x)\,h(y)$.

Rewrite as $\frac{dy}{h(y)} = g(x)\,dx$ and integrate both sides.

Note: These are sometimes written as

$$M(y)\frac{dy}{dx} + N(x) = 0$$

but after algebra, we get

$$M(y)\,dy = -N(x)\,dx.$$

OR

$$A(x)\,dx + B(y)\,dy = 0$$, etc...

Separable Equation is one where you can collect everything with x on one side of the equation and everything in y on the other.

eg $(1+x)\,dy - y\,dx = 0$

$$\int \frac{dy}{y} = \int \frac{dx}{1+x}$$

$$\ln|y| = \ln|1+x| + C_1$$

$$|y| = e^{\ln|1+x| + c_1}$$

$$|y| = e^{\ln|1+x|} \cdot e^{c_1} = |1+x|\,e^{c_1}$$

$$y = \pm e^{c_1}(1+x)$$

$$\therefore\ y = c(1+x)$$

eg $\frac{dy}{dx} = \frac{x^2+2}{2y}$

Similar to HW #12

$2y\,dy = (x^2+2)\,dx$

$y^2 = \frac{1}{3}x^3 + 2x + C$

explicit. $y = \pm\sqrt{\frac{1}{3}x^3 + 2x + C}$

(IF given an initial condition, this determin if + or −).

§ 2.2 (cont.)

eg $\underbrace{y'}_{\frac{dy}{dx}} = x + xy^2$

$\frac{dy}{dx} = x + xy^2$

$\frac{dy}{dx} = x(1+y^2)$

$\int \frac{dy}{1+y^2} = \int x\,dx$

$\underbrace{\tan^{-1}(y) = \frac{1}{2}x^2 + C}_{\text{implicit solution}}$

HW 13, 23 similar

explicit solution : $\underbrace{y = \tan\left(\frac{1}{2}x^2 + c\right)}_{\text{explicit solution}}$

eg IVP

Solve the IVP and determine the interval for which the solution is defined.

$$e^x - y\frac{dy}{dx} = 0, \quad y(0) = -1$$

$$y\frac{dy}{dx} = e^x \qquad\qquad \frac{e^x}{y} = \frac{dy}{dx}$$

$$y\,dy = e^x\,dx$$

$$\frac{1}{2}y^2 = e^x + C$$

$$\frac{1}{2}(-1)^2 = e^0 + C \quad \text{Since } y(0) = -1$$

$$\frac{1}{2} = 1 + C \implies C = -\frac{1}{2}$$

$$\therefore \frac{1}{2}y^2 = e^x - \frac{1}{2}$$

$$y^2 = 2e^x - 1 \implies y = \pm\sqrt{2e^x - 1}$$

But $y(0) = -1$. So the particular solution is

$$y = -\sqrt{2e^x - 1}.$$

I : D_y, $D_{y'}$

$$2e^x - 1 \cancel{\geq} 0$$
$$2e^x \cancel{\geq} 1$$
$$e^x \cancel{\geq} \frac{1}{2}$$
$$x \cancel{\geq} \ln\left(\frac{1}{2}\right) = \ln 2$$

$$I = (-\ln 2, \infty)$$

$$y' = -\frac{2e^x}{\sqrt{2e^x - 1}}$$

OR $\dfrac{e^x}{y} = \dfrac{dy}{dx}$

$y \neq 0$

Hw 31-34 Hints.

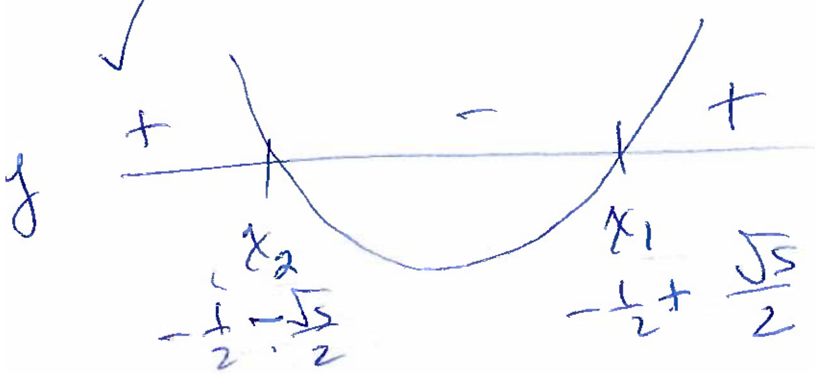
31) $y = -\sqrt{x^2+x-1}$

$y(-2) = -1$

$\boxed{}\, x^2+x-1 > 0$

$x_1 = -\frac{1}{2} + \frac{\sqrt{5}}{2}$

$x = -\frac{1}{2} - \frac{\sqrt{5}}{2}$



≈ -1.618

$I = \left(-\infty, -\frac{1}{2} - \frac{\sqrt{5}}{2}\right)$

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33) $y = -\ln(2-e^x)$

$2 - e^x > 0$

$-e^x > -2$

$e^x < 2$

$x < \ln 2$

$I = (-\infty, \ln 2)$

34) $y = \sqrt{2\cos x - 1}$

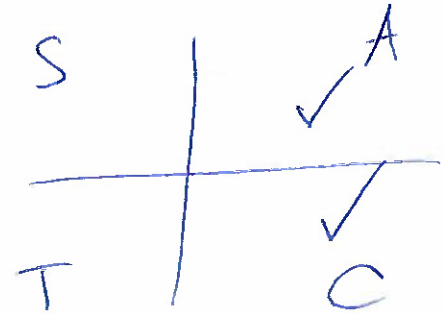
$2\cos x - 1 > 0$

$2\cos x > 1$

$\cos x > \frac{1}{2}$

$y(0) = 1$

$x = 0$



$I = \left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$