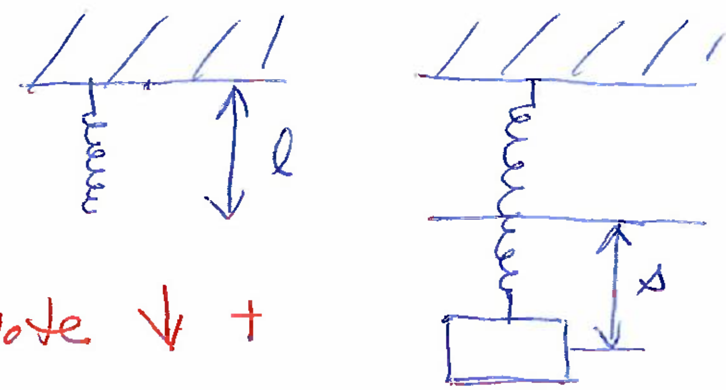
§5.1 Linear Models : IVP

Spring / Mass System (Intro)

Mount a flexible spring from a ceiling and attach a mass m on its end, which stretches the spring by length s.



Note $\downarrow +$

equilibrium:
$mg - ks = 0$

By Hooke's Law:

the spring itself exerts a restoring force F opposite to the direction of elongation and proportional to the amount of elongation s.

i.e. $\underline{F = -k s}$,

where $k > 0$ is the spring constant (depends on the spring.)

Units: $\dfrac{\text{units of force}}{\text{length}}$ eg lbs/ft, N/m

($N = kg \cdot m/s^2$)

With equilibrium, $\sum F = 0$.

$\therefore$ At equilibrium,

$$mg - ks = 0.$$

So $mg = ks$ and $k = \dfrac{mg}{s}$

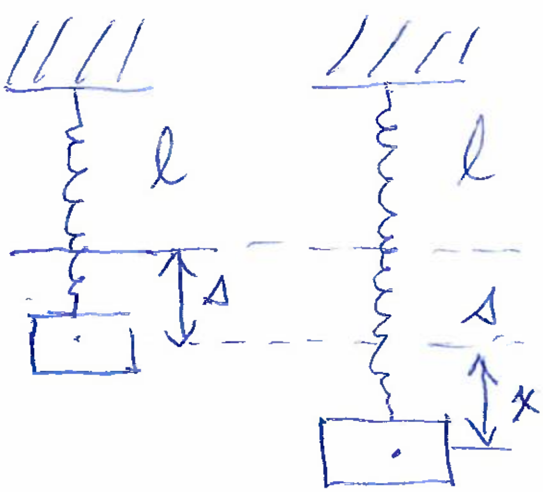
Now let the mass be

- initially displaced and released with some velocity ($v = 0$ if simply let go after displacement.)

AND/OR

- acted upon by an external force.

Get Newton's 2nd Law: $F = ma$



Let x be the displacement from equilibrium. Then

$$m \frac{d^2x}{dt^2} = \text{Net forces on the object.}$$

Consist of ?

$\downarrow +$

Net Forces

1) Weight of mass $\quad W = mg \quad \downarrow +$

2) Spring Force $F_{(s)} = -k(x+s)$ force that wants spring to go back to equilibrium.

my notation

3) Damping (resistive force)

$F_d = -\beta\, x'(t)$

always against motion.

proportional to the velocity

damping constant of the medium

4 cases. See next Page.

4) Outside force (if present - $F(t)$)

$$m x'' = mg - k(x+s) - \beta x' + F(t)$$

but $mg = ks$

So $\quad m x'' = -kx - \beta x' + F(t)$

Hence, $\quad m x'' + \beta x' + kx = F(t)$.

Note $x = x(t)$.

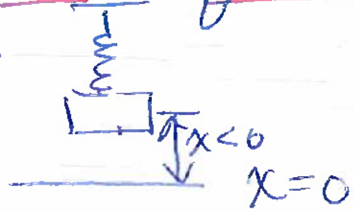
We look at different Cases. 5.1.1, 5.1.2, 5.1.3

Aside : 4 cases F_s , F_d

A) Motion is downward starting from

i) Above equilibrium

Contract first.



Motion $\downarrow$, $F_s \downarrow$, $F_d \uparrow$

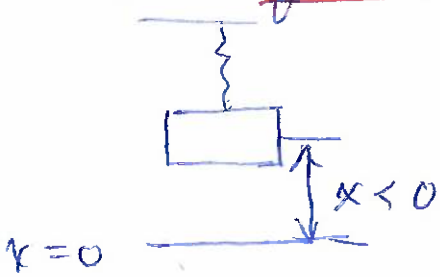
ii) Below equilibrium

Stretch first

Motion $\downarrow$, $F_s \uparrow$, $F_d \uparrow$

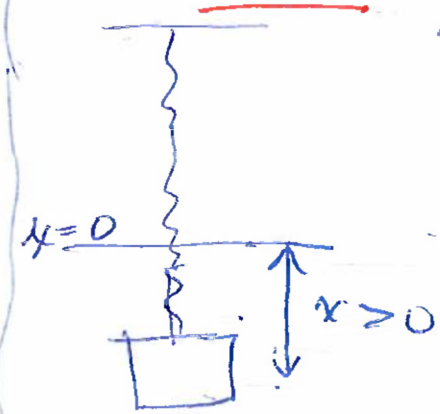
B) Motion is upward starting from

i) Above equilibrium



Motion $\uparrow$, $F_s \downarrow$, $F_d \downarrow$

ii) Below equilibrium



Motion $\uparrow$, $F_s \uparrow$, $F_d \downarrow$