

Section 3.1 Dilution Examples

Example 2: A tank initially holds 100 gal of a brine solution containing 1 lb of salt. At $t = 0$, another brine solution containing 1 lb of salt per gallon is poured into the tank at the rate of 3 gal/min, while the well-stirred mixture leaves the tank at the same rate. Find

- (a) the amount of salt in the tank at any given time t .
- (b) the time at which the mixture in the tank contains 2 pounds of salt.

a) Work solving the equation in class using an integrating factor.

$$\frac{dA}{dt} = 1 \frac{\text{lb}}{\text{gal}} \cdot 3 \frac{\text{gal}}{\text{min}} - \frac{A(t)}{100 + 3t - 3t} \frac{\text{lb}}{\text{gal}} \cdot 3 \frac{\text{gal}}{\text{min}}, A(0) = 1$$

$$\frac{dA}{dt} + 0.03A = 3$$

$$A(t) = Ce^{-0.03t} + 100$$

$$1 = Ce^0 + 100$$

$$C = -99$$

$$A(t) = -99e^{-0.03t} + 100$$

b)

$$2 = -99e^{-0.03t} + 100$$

$$e^{-0.03t} = \frac{98}{99}$$

$$t = -\frac{1}{0.03} \ln\left(\frac{98}{99}\right) \approx 0.338 \text{ min}$$

See next page for next example.

Example 3: A 50-gal tank initially contains 10 gal of fresh water. At $t = 0$, a brine solution containing 1 lb of salt per gallon is poured into the tank at the rate of 4 gal/min, while the well-stirred mixture leaves the tank at the rate of 2 gal/min. Find

- (a) the amount of salt in the tank at any given time t .
- (b) the amount of time required for overflow to occur, and
- (c) the amount of salt in the tank at the time of overflow.

a) Work solving the equation in class using an integrating factor.

$$\frac{dA}{dt} = 1 \frac{\text{lb}}{\text{gal}} \cdot 4 \frac{\text{gal}}{\text{min}} - \frac{A(t)}{10 + 4t - 2t} \frac{\text{lb}}{\text{gal}} \cdot 2 \frac{\text{gal}}{\text{min}}, A(0) = 0$$

$$\frac{dA}{dt} + \frac{2}{10 + 2t} A = 4$$

$$A(t) = \frac{40t + 4t^2 + C}{10 + 2t}$$

$$0 = \frac{C}{10}$$

$$C = 0$$

$$A(t) = \frac{40t + 4t^2}{10 + 2t}$$

b)

$$V(t) = 10 + 2t$$

$$50 = 10 + 2t$$

$$40 = 2t$$

$$t = 20 \text{ min}$$

c)

$$A(20) = \frac{40(20) + 4(20)^2}{10 + 2(20)} = 48 \text{ lb}$$

Note: $10 + 2(20) = 50$ gallons