Homework set # 9 Daniel B. Szyld

1. (a)

$$v_1 = \begin{vmatrix} -1 \\ 1 \\ 0 \end{vmatrix} \quad v_2 = \begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix}$$

$$\|v_1\| = \sqrt{2} \quad \|v_2\| = 1 \quad v_1^T v_2 = -1$$

$$\cos\theta = \frac{-1}{\sqrt{2}} = \frac{-\sqrt{2}}{2} \quad \theta = \frac{3\pi}{4} = 135^\circ$$

(b) We observe that v_1 and v_2 lie in the horizontal plane (since the third component is zero)

Thus $v_3 = \begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix}$ is orthogonal to both v_1 and v_2

2. $$v_1 = \begin{vmatrix} 1 \\ 2 \\ -1 \\ 1 \end{vmatrix} \quad v_2 = \begin{vmatrix} 1 \\ -1 \\ -1 \\ 0 \end{vmatrix}$$

$$v_1^T v_2 = 1 - 2 + 1 + 0 = 0, \quad \cos\theta = 0, \quad \theta = \frac{\pi}{2}$$

3. Show that every vector $v \in R(A)$ is orthogonal to every vector $w \in N(A)$, when $A^TA = AA^T$.
Hint. use the fact that $R(A) = R(AA^T)$.

Let $v \in R(A) = R(AA^T)$, then $v = AA^Ty$ for some y
let us compute $v^Tw = y^TAA^Tw \underset{\text{A normal}}{=} y^TA^T\underbrace{Aw}_{=0} = y^TA^T 0 = 0$

q.e.d.