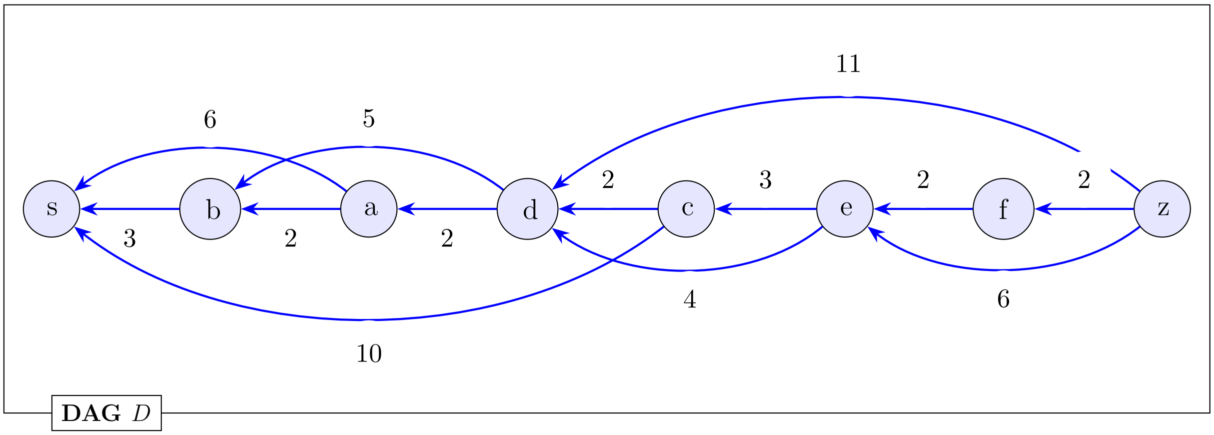
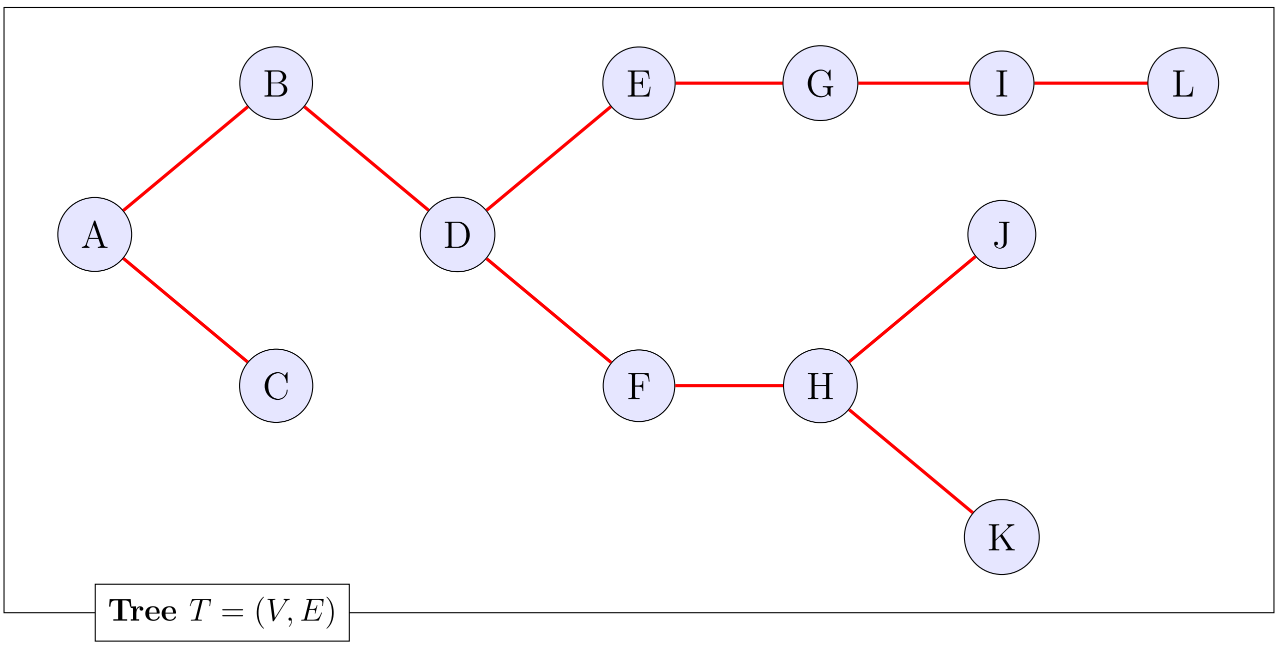
DATA SHEET





Assume T is directed away from the root A.

For each vertex $u \in V$, let

> $T(u)$ denote the subtree of T with root u,
>
> $L(u)$ = size of the maximal independent set contained in $T(u)$,
>
> $S(u)$ = true if u is included in $T(u)$ and false otherwise.

CIS 3223 Homework 8

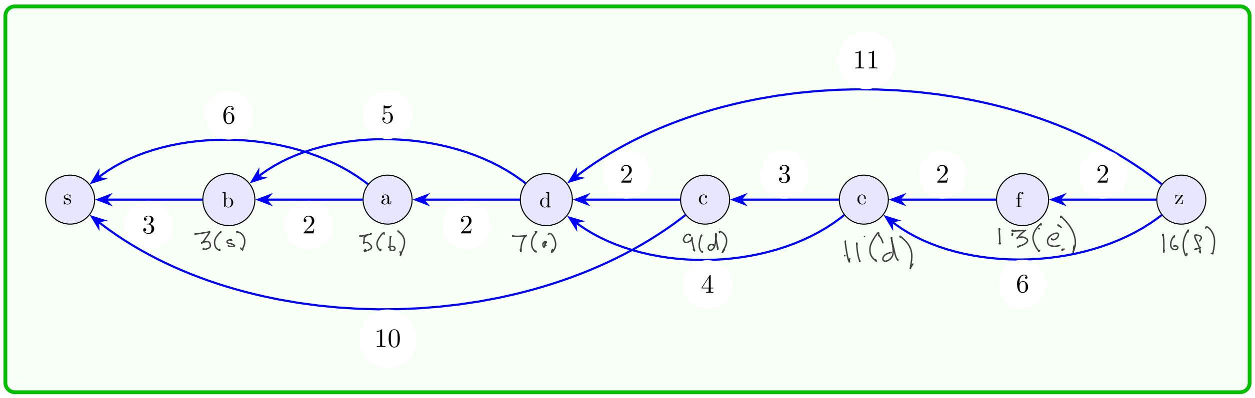
Name: Solutions Solutions
Temple ID (last 4 digits:

Dr Anthony Hughes

Ver 2

1 (12 pts) Consider the dag D.

Draw the reverse dag.



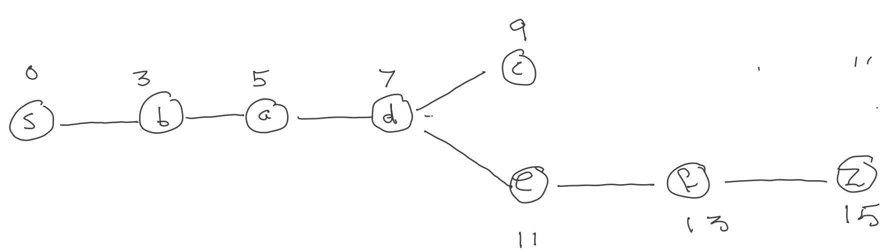
For each vertex $u \in V$, let dist(v) = shortest path from s to v. Complete the following table.

d(s) = 0
d(b) = 3 + d(s) = 3 + 0 = 3
d(a) = 6 + d(s) = 6
= 2 + d(b) = 2 + 3 = 5.
d(d) = 2 + d(a) = 2 + 5 = 7
= 5 + d(b) = 5 + 3 = 8
d(c) = 10 + d(s) = 10 + 0 = 10
= 2 + d(d) = 2 + 7 = 9

d(e) = 3 + d(c) = 3 + 9 = 12
= 4 + d(d) = 4 + 7 = 11
d(f) = 2 + d(e) = 2 + 11 = 13
d(z) = 11 + d(d) = 11 + 7 = 18
= 6 + d(e) = 6 + 11 = 17.
= 2 + d(f) = 2 + 13 = 15

parent	s	s	b	a	d	d	e	f
dist	0	3	5	7	9	11	13	15
vertex	s	b	a	d	c	e	f	z

Draw the tree (horizontally) indicating the shortest path from s.



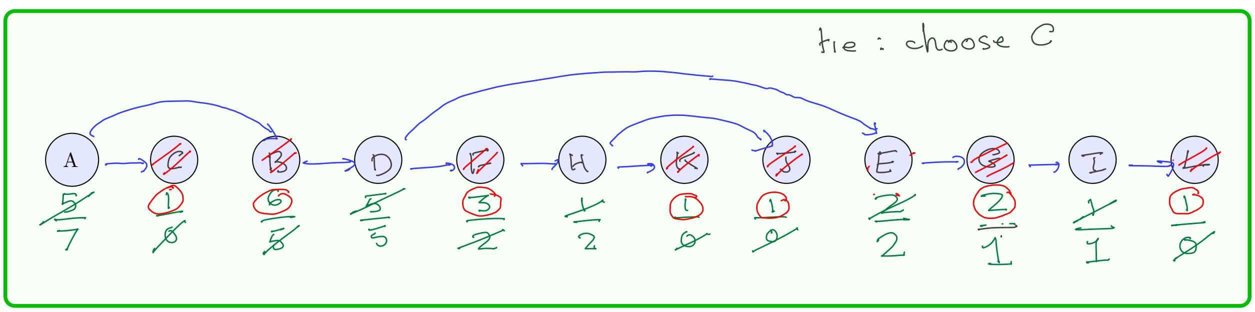
2 (16 pts) Consider the tree $T = (V, E)$ and assume that the neighbors of each vertex are listed alphabetically.

(a) Run the DFS topological ordering algorithm on the graph. Use alphabetical ordering.

parent	vertex	pre	post
H	J	14	15
H	K	16	17
F	H	13	18
I	L	7	8
G	I	6	9
E	G	5	10
D	E	4	11
D	F	12	19
B	D	3	20
A	B	2	21
A	C	22	23
A	A	1	24

A	C	B	D	F	H	K	J	E	G	I	L
24	23	22	20	19	18	17	15	11	10	9	8

Redraw the graph with the vertices listed in descending post order (topological sort). $\frac{\text{include}}{\text{exclude}}$ $\frac{I+G}{C}$



Traverse through the topological sort of V in reverse order and complete the following table:

L	7	6	1	5	2	3	2	2	1	1	1	1
S	0	1	1	0	0	1	1	0	0	1	1	1
V	A	B	C	D	E	F	G	H	I	J	K	L

If $S(u) = 1$ where u is the parent of v, reset $S(v) = 0$ if $S(v) = 1$. No resets.

Identify a maximal independent set: B, C, E, F, J I, L Size: 7

3 (16 pts) Consider the following 0-1 knapsack problem with $W = 11$

Item	Weight	Value ($)
1	4	17
2	6	28
3	2	9
4	5	21
5	3	14

Complete the table and determine the solution.

0	1	2	3	4	5	6	7	8	9	10	11	
0	0	0	0	17	17	17	17	17	17	17	17	~~item 1~~
0	0	0	0	0	0	0	0	0	0	17	17	item 2 : 6L
0	0	0	0	0	0	28	28	28	28	45	45	
0	0	0	0	17	17	28	28	28	28	45	45	item 3 : 2L
0	0	0	0	0	0	17	17	28	28	28	28	
0	0	9	9	9	9	26	26	37	37	35	35	
0	0	9	9	17	17	28	28	37	37	45	45	~~item 4~~
0	0	0	0	0	0	0	9	9	17	17	28	
0	0	0	0	0	21	21	30	30	38	38	49	
0	0	9	9	17	21	28	30	37	38	45	49	item 5 : 3L
0	0	0	0	0	9	9	17	21	28	30	37	
0	0	0	14	14	23	23	31	35	42	44	51	
0	0	9	14	17	23	28	31	37	42	45	51	

Max value $\boxed{51}$ Items: 2, 3, 5

What is the max value if $W = 9$ and only the first four items can be taken? $\boxed{38}$

4 (6 pts) Let $T = (V, E)$ be a tree. Use induction to show that $|E| = |V| - 1$

Induction on $n := |V|$

Base case $n = 1$ Tree consists of one node, no edges

$|V| = 1$, $|E| = 0$ True for $n = 1$

Inductive case: Assume true for $n = k$, $k \geq 1$

Show true for $n = k+1$

Let $T = (V, E)$ be a tree with $|V| = k$.

Let b be a leaf in T, and a the parent of b.

Let $T_1 = (V_1, E_1)$, $V_1 = V \setminus \{b\}$, $E_1 = E \setminus \{ab\}$

Then $|V_1| = k$, and so $|E_1| = |V_1| - 1$.

But $V = V_1 \cup \{b\}$, $E = E_1 \cup \{ab\}$ and

$|E| = |E_1| + 1 = |V_1| - 1 + 1 = (|V_1| + 1) - 1 = |V| - 1$

So true for $n = k$

5 ([Google] extra credit, 2 pts) Why are manhole covers round?

Width of any cross-section through the center = diameter. Lip prevents cover from falling in (not the only shape).