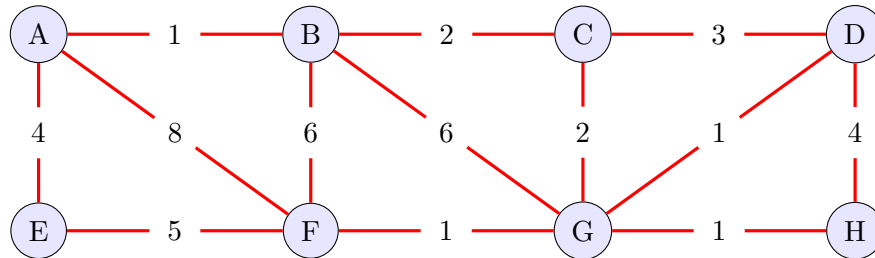


1 (9 pts) For the following graph use Kruskal's algorithm to find a **minimum**-cost spanning tree, and then determine the minimum cost. Use alphabetical ordering.



Construct a hash table using the lengths of the edges to store the edges.

Edge List (Hash Table)

1	AB	DG	FG	GH	
2	BC	CG			
3	CD				
4	AE	DH			
5	EF				
6	BG	BF			
8	AF				

Sorted Edge List

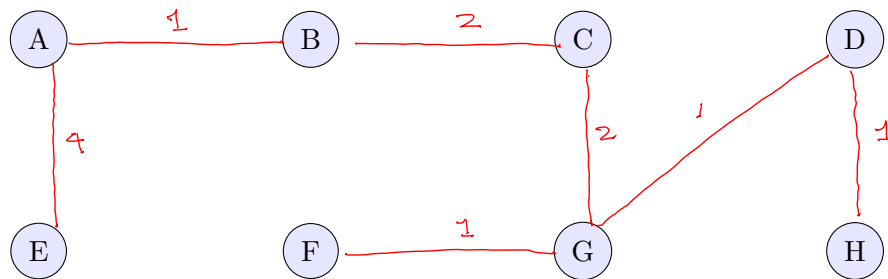
1	AB	DG	FG	GH	
2	BC	CG			
3	<del>CD</del>				
4	AE	DH			
5	EF				
6	BF	BG			
8	AF				

4  
4  
4  

---

12

Minimum cost spanning tree



Cost 12

2 (2 pts) If an undirected graph  $G$  has  $n$  nodes, what is the runtime for the function **find**?

$O(\log n)$

$O(n)$

$O(n \log n)$

$O(n^2)$

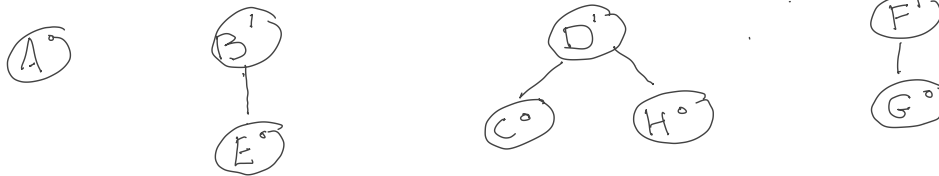
$O(2^n)$

3 (9 pts) The current state  $S$  of the **union by rank** representation of disjoint subsets of the set of vertices  $\{A, B, C, D, E, F, G, H\}$  is given by

pi	A	B	D	D	B	F	F	D	<del>B</del>
	A	B	C	D	E	F	G	H	<del>H</del>

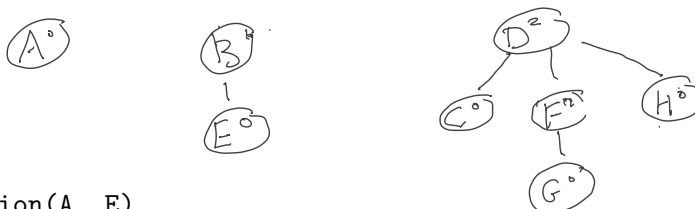
rank	0	1	0	1	0	<del>0</del>	0	0	<del>1</del>
	A	B	C	D	E	F	G	H	<del>H</del>

(a) Draw the corresponding trees representing the sets.

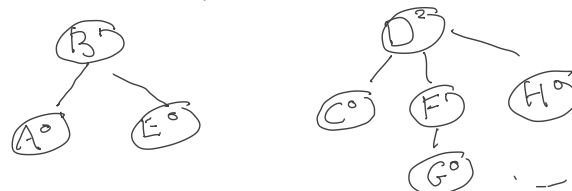


(b) Consider the following **SEQUENCE** of operations. Draw the corresponding trees representing the sets after each of the operations (use alphabetical order):

**union(C, F)**  $\text{find}(C) = D$   $\text{find}(F) = F$   $\text{rank}(D) = 1$   $\text{rank}(F) = 1$  join F to D  
 $\text{rank}(D) = 2$



**union(A, E)**  $\text{find}(A) = A$ ,  $\text{find}(E) = B$   $\text{rank}(A) = 0$   $\text{rank}(B) = 1$  join A to B



**union(C, D)**  $\text{find}(C) = D$   $\text{find}(D) = D$  ignore

**union(A, F)**  $\text{find}(A) = B$   $\text{find}(F) = D$   $\text{rank}(B) = 1$   $\text{rank}(D) = 2$  join B to D

