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Linear Algebra
Homework # 7

For (1)(a) and (2) all we need to do is to show that these sets are closed under addition and scalar multiplication (or not)

(1)(a) let $v, w \in S \cap T$ i.e.
$v \in S$, $v \in T$, $w \in S$, $w \in T$

What about ~~[illegible]~~ $v+w$? and αv, $\alpha \in \mathbb{R}$

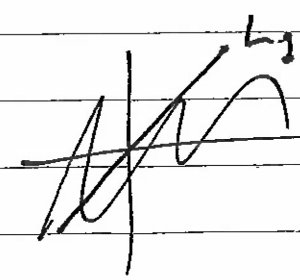
Well, $v+w \in S$ since S is a subspace and so is αv.
$v+w \in T$ and $\alpha v \in T$ since T is a subspace
$\Rightarrow$ $v+w \in S \cap T$, $\alpha v \in S \cap T$ $\forall \alpha \in \mathbb{R}$

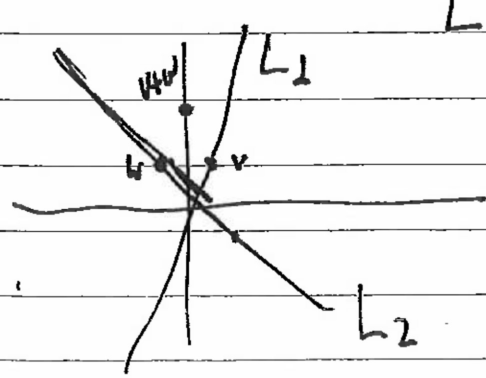
q.e.d.

b) Example: 2 lines in $\mathbb{R}^2$ (different lines)

For example $L_1 = \left\{ \alpha \begin{vmatrix} 1 \\ 2 \end{vmatrix}, \alpha \in \mathbb{R} \right\}$

$L_2 = \left\{ \alpha \begin{vmatrix} -1 \\ 1 \end{vmatrix}, \alpha \in \mathbb{R} \right\}$





Then $v = \begin{vmatrix} 1 \\ 2 \end{vmatrix} \in L_1$

$w = \begin{vmatrix} -1 \\ 1 \end{vmatrix} \in L_2$

$v+w = \begin{vmatrix} 0 \\ 3 \end{vmatrix} \notin L_1 \cup L_2$

(3) Let us consider a generic element ("vector") in Π_2, namely

$$p(x) = a_0 + a_1 x + a_2 x^2$$

We need to show that for <u>any</u> such $p(x)$ (or equivalent any $[a_0\ a_1\ a_2]^T$) there exist α_1 α_2 α_2 such that

$$p(x) = \alpha_1 p_1(x) + \alpha_2 p_2(x) + \alpha_3 p_3(x)$$

Let us see if this is the case

$$p(x) = a_0 x + a_1 x + a_2 x^2$$

$$= \alpha_1 (1+x) + \alpha_2 (1-x) + \alpha_3 (1-x^2)$$

$$= \alpha_1 + \alpha_1 x + \alpha_2 - \alpha_2 x + \alpha_3 - \alpha_3 x^2$$

$$= (\alpha_1 + \alpha_2 + \alpha_3) + (\alpha_1 - \alpha_2) x + (-\alpha_3) x^2$$

collecting terms

$$\alpha_1 + \alpha_2 + \alpha_3 = a_0$$

$$\alpha_1 - \alpha_2 = a_1$$

$$-\alpha_3 = a_2$$

or $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}$

The question again is

For any (right hand side) vector $\begin{vmatrix} a_0 \\ a_1 \\ a_2 \end{vmatrix}$

can I find a solution $\begin{vmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{vmatrix}$.

We know the answer is yes

if and only if the matrix $A = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 0 & 0 & -1 \end{vmatrix}$

is non singular

$$A = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 0 & 0 & -1 \end{vmatrix} \longrightarrow \begin{vmatrix} 1 & 1 & 1 \\ 0 & -2 & -1 \\ 0 & 0 & -1 \end{vmatrix}$$

$$m_{21} = -1$$

3 pivots. rank $A = 3$, A non singular

Thus $\{p_1\ p_2\ p_3\}$ span Π_2