CIS 3223 TMQ 4

Dr Anthony Hughes

Name: Solns

Temple ID (last 4 digits:

1 (16 pts) Give a big-θ bound for the solutions of the following recurrence relations.

(a) $T(n) = 2T(n/2) + n^{2/3}$

$\Theta(n)$

$a = 2 \quad b = 2 \quad d = \frac{2}{3}$

$\log_b a = \log_2 2 = 1$ } $d < \log_b a$, $T(n) = O(n^{\log_b a}) = \Theta(n)$

$d = \frac{2}{3}$

(b) $T(n) = 9T(n/3) + 500n^2$

$\Theta(n^2 \log n)$

$a = 9, \quad b = 3, \quad d = 2$

$\log_b a = \log_3 9 = 2$ } $d = \log_b a$, $T(n) = \Theta(n^2 \log n)$

$d = 2$

(c) $T(n) = 7T(n/4) + 8n$

$\Theta(n^{\log_4 7})$

$a = 7, \quad b = 4, \quad d = 1$

$\log_b a = \log_4 7$ } $d < \log_b a$, $T(n) = \Theta(n^{\log_4 7})$

$d = 1$

(d) $T(n) = T(n-1) + n^2$

$\Theta(n^3)$

$a = 1 \quad b = 1 \quad d = 2$

$T(n) = \Theta(n^{d+1}) = \Theta(n^3)$

2 (4 pts) Algorithm Z solves problems by dividing them into four subproblems of half the size, recursively solving each subproblem, and combining the solution in linear time. What is the runtime of algorithm Z (in big-O notation)?

$a = 4 \quad b = 2 \quad d = 1$

$\log_b a = \log_2 4 = 2 \quad \Big\} \quad d < \log_b a \qquad T(n) = \Theta(n^{\log_2 4}) = \Theta(n^2)$

$d = 1$

$\Theta(n^2)$

3 (**extra credit,** 2 pts) Let $N = 2221$. Find a power of 3 in the interval $[N, 3N]$.

8

Is there more than one power of 3 in the interval? yes (no)

$2221 = 12010001_3 < 100000000_3 = 3^8 < 3 \times 2221 = 6663$

8 digits

Note:

$$\begin{array}{ccccccccccc} & 1 & & & & & & & & & \\ 2221 & \longrightarrow & 740 & \longrightarrow & 246 & \longrightarrow & 82 & \longrightarrow & 27 & \longrightarrow & 9 \;\text{—}\; 3 \;\text{—}\; 1 \\ 1 & & 2 & & 0 & & 1 & & 0 & & 0 \quad 0 \quad 1 \end{array}$$