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11 April 2023
Due Thursday 13 April 2023

Linear Algebra, Math 2101-002
Homework set #12

1. Consider the following two vectors in $\mathbb{R}^4$ (the same as in homewrok 11)
$v_1 = \begin{vmatrix} 1 \\ 2 \\ -1 \\ 1 \end{vmatrix}$, $v_2 = \begin{vmatrix} 1 \\ -1 \\ -1 \\ 0 \end{vmatrix}$. Find a vector (in $\mathbb{R}^4$) which is orthogonal to *both* v_1 and v_2.

2. Let $v_1 = \begin{vmatrix} 1 \\ 2 \\ 3 \\ 1 \end{vmatrix}$, $v_2 = \begin{vmatrix} 0 \\ 0 \\ 1 \\ 2 \end{vmatrix}$.
(a) Let $S = span\{v_1, v_2\}$, i.e., the subspace generated by v_1 and v_2. Construnct an orthonormal basis for S.
(b) Let W be the set of all vectors which are orthogonal to both v_1 and v_2., i.e., $W = S^{\perp}$. Find a basis for W.
(c) Show that $w = \begin{vmatrix} 1 \\ 1 \\ 1 \\ 1 \end{vmatrix}$ does not lie in S. Show that w does not lie in W either. Explain why this is possible.
(d) Find the orthogonal projection P onto S, and Q, the orthogonal projection onto W. Check that $PQ = QP = 0$.
(e) Compute Pw and Qw and check that: 1. $Pw \in S$, 2. $Qw \in W$, 3. $(Pw)^T(Qw) = 0$,
4. $w = Pw + Qw$.