

# Dynamic multilevel analyses

## Contents

<b>General comments</b>	<b>2</b>
<b>Multilevel Model 0</b>	<b>3</b>
Input MLM0 . . . . .	4
Output MLM0 . . . . .	4
Bayesian estimation . . . . .	5
<b>Multilevel Model 1</b>	<b>9</b>
Input MLM1 . . . . .	9
Output MLM1 . . . . .	10
<b>Multilevel Model 2</b>	<b>12</b>
Input MLM2 . . . . .	14
Output MLM2 . . . . .	15
<b>Multilevel Model 3</b>	<b>20</b>
Input MLM3 . . . . .	21
Output MLM3 . . . . .	21
<b>Multilevel Model 4</b>	<b>27</b>
Input MLM4 . . . . .	28
Output MLM4 . . . . .	29
<b>Multilevel Model 5</b>	<b>31</b>
Input MLM5 . . . . .	32
Output MLM5 . . . . .	33
<b>Multilevel Model 6</b>	<b>35</b>
Input MLM6 . . . . .	35
Output MLM6 . . . . .	36

Instead of analyzing the data for each person separately using replicated  $N = 1$  analyses, we can analyze the data of the 129 individuals in a single multilevel model.

In the book chapter we consider 6 dynamic multilevel models:

- **Multilevel models 1-3** are based on combining *time series modeling* with *multilevel modeling*
- **Multilevel models 4-6** are based on combining *times series modeling* and *multilevel modeling* with *structural equation modeling*

As an initial step, we determine the intraclass correlation of each variable in **Multilevel Model 0**. When discussing this model, we will also provide some additional information regarding Bayesian estimation which apply more generally to the analyses performed for the book chapter.

Below, we start by discussing some general aspects of the multilevel analyses; then we present each model, discuss how it is specified, and discuss the output.

## General comments

Dynamic multilevel modeling is based on decomposing the observed variables into a within-person part and a between-person part, that is

$$x_{it} = x_{it}^{(w)} + x_i^{(b)}$$

$$y_{it} = y_{it}^{(w)} + y_i^{(b)}$$

For both components a model is specified: At the within level a time series model is used to account for the dynamics in the repeated measures within a person, and at the between level the random effects can be related to each other, either through correlating them, or by including them in a regression model, a path model, and/or a factor model.

In Mplus, this approach requires the repeated measures within a person to be represented in long format (as was the case for the  $N = 1$  approach as well), with the data of all the individuals stacked. There has to be a specific variable that indicates to which individual the data belong (i.e., the clustering of the data).

The DATA command now becomes:

```
DATA: FILE = ChapterData1.dat;
```

The VARIABLE command is given as:

```
NAMES      =  ID prepost TimeHours U2P PA NA PEx NEx
              pa_pre na_pre U2P_pre PEx_pre NEx_pre
              pa_post na_post U2P_post PEx_post NEx_post
              ham_pre ham_post group;
CLUSTER     =  ID;
USEVAR      =  na_pre ue_pre;
MISSING     =  ALL(-999);
```

where the statement CLUSTER = ID is used to specify the variable that contains information about the clustering of the data.

Again, we are using the variable ue\_pre in our analyses, which is not part of the observed data file, but created using:

```
DEFINE:      ue_pre = -1*U2P_pre;
```

Note that we are no longer using the DEFINE command to standardized the variables; if we would use that approach, it would use the total observed variance (i.e., across all time points *and* participants). As we will see below—and as described in the book chapter—when we standardize parameters, we want this to be based on the variance at the level where the parameter is situated; moreover, we want the individual slopes to be standardized with the individual specific variances, rather than some average across individuals, or a variance that contains both within and between person variance.

As all the models we run here are multilevel models and we want to use Bayesian estimation, our ANALYSIS command will contain:

The command ANALYSIS is specified as:

```
ANALYSIS:    TYPE = TWOLEVEL;
              ESTIMATOR = BAYES;
              PROC = 2;
              BITER = (3000);
              BSEED = 6974;
              THIN = 5;
```

where the statement `TYPE = TWOLEVEL` ensures that the data are decomposed into a within-person component that varies over time, and a between-person component that does not vary over time. The other statements are all related to Bayesian estimation, and were discussed under the  $N = 1$  analyses.

To obtain the plots that are used to evaluate the Bayesian estimation, we include the `PLOT` command:

```
PLOT: TYPE = PLOT3;
```

This will allow us to look at trace plots for each parameter (i.e., the MCMC samples of the two chains that are run), posterior distributions of each parameter, the autocorrelation function for each parameter over the MCMC iterations.

If we include

```
FACTORS = ALL (500);
```

in the `PLOT` command, Mplus will run the MCMC algorithm for another 500\*100 iterations, and save every 100th draw of each factor score for *each person separately*. Note that factor scores include latent variables, but also random effects (i.e., means, slopes, and (residual) variances). Hence, this gives us information about the posterior distribution of each person separately.

Through using this option, we can plot the means or medians of these per person (e.g., in histograms univariately, or in scatter plots bivariately). Furthermore, it will also allow us to ask for a comparison of the factor scores (i.e., the individual's parameter estimates), as we will show below.

## Multilevel Model 0

The intraclass correlation indicates the proportion of variance that is due to stable between-person differences. It is computed as:

$$icc_y = \frac{var(y^{(b)})}{var(y^{(b)}) + var(y^{(w)})}$$

To estimate these quantities for each variable, we specify a bivariate, empty multilevel model, in which each variable is decomposed into a time-varying within-person component, and a time-invariant between-person component. This can be visualized as

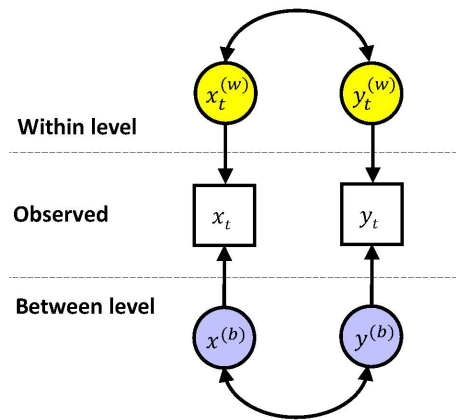


Figure 1:

This model allows us to estimate the variances at the within level and at the between level, and from these we can compute the intraclass correlations.

## Input MLM0

We make use of the `ANALYSIS` command such as presented above, indicating `TYPE = TWOLEVEL` and `ANALISYS = TWOLEVEL`. Subsequently, we need to define a model at the within level and at the between level. We use the `MODEL` command:

```
MODEL:
%WITHIN%
na_pre WITH ue_pre;
na_pre (wvar_na);
ue_pre (wvar_ue);

%BETWEEN%
na_pre WITH ue_pre;
na_pre (bvar_na);
ue_pre (bvar_ue);
```

As we have given the variances of each variable at each level a name, we can refer to them in the `MODEL CONSTRAINT` command where we create two new parameters, `icc_na` and `icc_ue`:

```
MODEL CONSTRAINT:
NEW (icc_na); icc_na = bvar_na/(bvar_na + wvar_na);
NEW (icc_ue); icc_ue = bvar_ue/(bvar_ue + wvar_ue);
```

As each of these new parameters is computed within each iteration in the MCMC algorithm, we will also obtain a posterior distribution for them.

## Output MLM0

The output for this analysis is can be found here.

This model provides the following parameter estimates:

### MODEL RESULTS

		Posterior	One-Tailed	95% C.I.			
		Estimate	S.D.	P-Value	Lower 2.5%	Upper 2.5%	Significance
Within Level							
NA_PRE	WITH						
UE_PRE		0.415	0.019	0.000	0.378	0.452	*
Variances							
NA_PRE		0.682	0.012	0.000	0.659	0.707	*
UE_PRE		2.842	0.052	0.000	2.742	2.945	*
Between Level							
NA_PRE	WITH						
UE PRE		0.171	0.045	0.000	0.090	0.266	*

Means						
NA_PRE	2.068	0.067	0.000	1.936	2.200	*
UE_PRE	-1.246	0.052	0.000	-1.346	-1.144	*
Variances						
NA_PRE	0.569	0.077	0.000	0.443	0.742	*
UE_PRE	0.287	0.047	0.000	0.213	0.392	*
New/Additional Parameters						
ICC_NA	0.455	0.033	0.000	0.392	0.523	*
ICC_UE	0.092	0.014	0.000	0.070	0.121	*

The part of the output that is of interest to us, are the last two lines where the intraclass correlations are computed for each variable. Note that by definition, the 95% CIs that are computed for variance related parameters (e.g., variances, residual variances, proportions of explained variance) will not cover zero; hence, when zero is not included in the CI of an intraclass correlation, this should not be interpreted as evidence that the intraclass correlation is different from zero. Instead, we may want to consider the lower bound and determine whether this is relatively close to zero or not, based also on the mean of the posterior distribution (i.e., the point estimate that is reported).

Here we see that the difference in the proportion between-person variance is quite large for these two variables: About 0.46 for negative affect, and only 0.09 for unpleasantness of events. The latter implies that the reported unpleasantness of events contains mostly within-person variation over time, whereas negative affect contains much more substantial differences between persons.

## Bayesian estimation

In generally, when using Bayesian estimation, we should be aware that a model may not have converged. While it is not possible to determine whether a model in fact did converged, there are several diagnostics that can be consulted to see whether there is evidence that the model did *not* converge.

As already mentioned, the `BITER = (3000)` statement can be used to ensure that Mplus draws at least 3000 samples in the MCMC algorithm; when the Potential Scale Reduction (PRS) statistic indicates that the algorithm did not converge yet, Mplus will continue beyond 3000 samples until the criterion is satisfied. If the user does not want additional samples, they should use `FBITER = 3000` instead.

The PRS criterion is shown on the screen while running the analysis; this information is saved in the output by asking for `TECH8` in the `OUTPUT` command. For the current analysis, this is

### TECHNICAL 8 OUTPUT

#### TECHNICAL 8 OUTPUT FOR BAYES ESTIMATION

CHAIN	BSEED	
1	6974	
2	534401	
ITERATION	POTENTIAL SCALE REDUCTION	PARAMETER WITH HIGHEST PSR
100	1.000	1
200	1.015	2
300	1.015	2
400	1.020	8
500	1.012	8
600	1.006	8

700	1.003	6
800	1.002	6
900	1.003	6
1000	1.004	6
1100	1.002	6
1200	1.001	6
1300	1.003	6
1400	1.002	6
1500	1.003	6
1600	1.004	6
1700	1.002	6
1800	1.001	6
1900	1.002	6
2000	1.001	6
2100	1.001	6
2200	1.000	6
2300	1.001	5
2400	1.001	5
2500	1.001	5
2600	1.000	5
2700	1.000	5
2800	1.000	2
2900	1.000	5
3000	1.000	5

The PSR is based on computing *for each model parameter* at each iteration: a) the total variance across both chains over all the iterations so far (i.e., both within and between chains); b) the variance within the two chains and averaging these; and c) dividing the former by the latter and taking the square root of this. Hence, when two chains have settled in the same area of the parameter space, the total variance will be (almost) identical to the within chain variance (or: between chain variance will be virtually zero), such that this criterion is very close to 1. The PSR that is reported here, is the largest one among all the model parameters; which parameter it is, is shown in the last column (which matches the parameter numbering in the **TECH1** output).

To compute the PSR, at least two chains are needed; this is the default number of chains used when doing Bayesian estimation in Mplus. The number can be changed in the **ANALYSIS** command, for instance by adding **CHAINS = 4;**.

In addition to the PSR criterion, the user should also check the trace plots of the model parameters. These can be viewed in Mplus, using the plotting option (from the plot menu, the option: Bayesian posterior parameter trace plot). An example of such a trace plot is given below.

It shows a red and a blue chain which are based on the two different seeds (see **TECH8** output). In this case, these overlap well, which means that the total variance will be (almost) identical to the within chain variance. Furthermore, the plot shows no evidence for a trend (a trajectory going up or down); if such a trend would be present, this would be considered a sign that the MCMC algorithm did not converge. Also, there are no extreme values, which would be some reason for concern about convergence (see the discussion of the results for Model 5 below).

Other plots that contain relevant information on the performance of the MCMC algorithm are the autocorrelation plots that can be obtained for each model parameter based on each chain, as shown below.

Preferably, these autocorrelations (i.e., the autocorrelation across samples of the MCMC draws) go to zero very quickly, as is the case here. If they remain higher for longer lags, this is a reason to: a) using the **THINNING** option; and/or b) increase the number of iterations.

As a final comment regarding Bayesian estimation, we point out that the prior distributions that were used

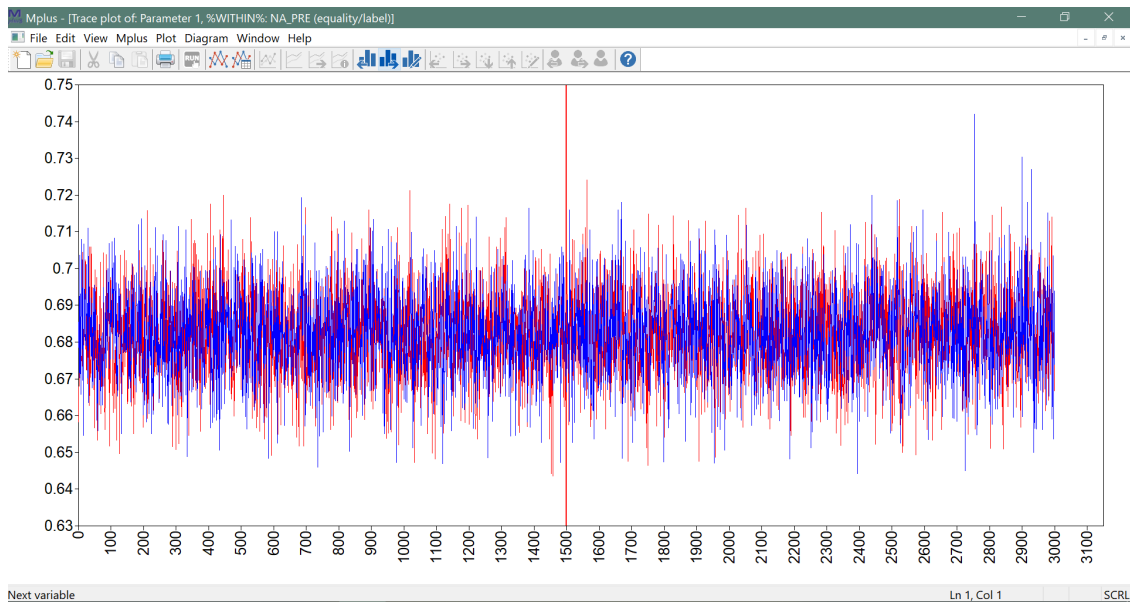


Figure 2:

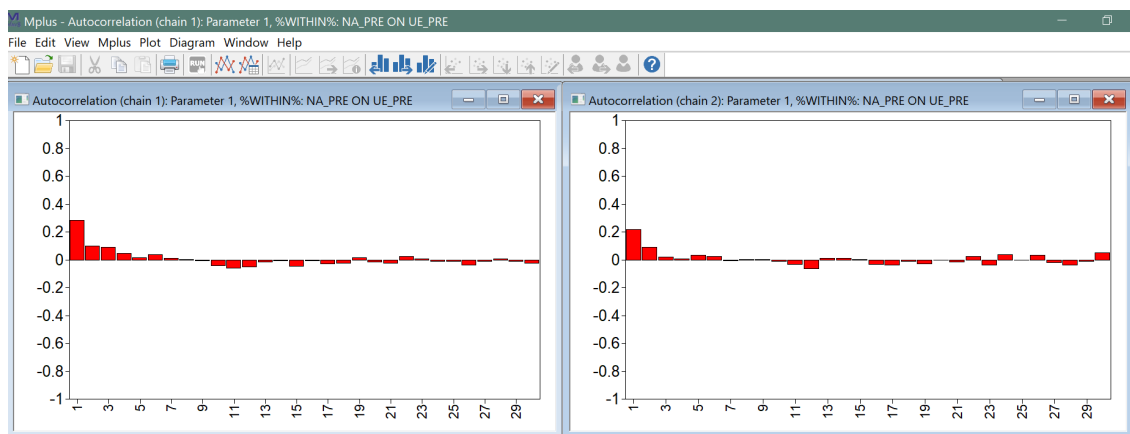


Figure 3:

for the model parameters, are also reported in the output at the end. For the current model these are

PRIORS FOR ALL PARAMETERS	PRIOR MEAN	PRIOR VARIANCE	PRIOR STD. DEV.
Parameter 1~IW(0.000,-3)	infinity	infinity	infinity
Parameter 2~IW(0.000,-3)	infinity	infinity	infinity
Parameter 3~IW(0.000,-3)	infinity	infinity	infinity
Parameter 4~N(0.000,infinity)	0.0000	infinity	infinity
Parameter 5~N(0.000,infinity)	0.0000	infinity	infinity
Parameter 6~IW(0.000,-3)	infinity	infinity	infinity
Parameter 7~IW(0.000,-3)	infinity	infinity	infinity
Parameter 8~IW(0.000,-3)	infinity	infinity	infinity

where the parameter numbers match the ones provided in the TECH1 output (based on the model matrices of the LISREL notation):

#### TECHNICAL 1 OUTPUT

##### PARAMETER SPECIFICATION FOR WITHIN

NU		
	NA_PRE	UE_PRE
	-----	-----
	0	0
THETA		
	NA_PRE	UE_PRE
	-----	-----
NA_PRE	1	
UE_PRE	2	3

##### PARAMETER SPECIFICATION FOR BETWEEN

NU		
	NA_PRE	UE_PRE
	-----	-----
	4	5
THETA		
	NA_PRE	UE_PRE
	-----	-----
NA_PRE	6	
UE_PRE	7	8

##### PARAMETER SPECIFICATION FOR THE ADDITIONAL PARAMETERS

NEW/ADDITIONAL PARAMETERS	
ICC_NA	ICC_UE



## Multilevel Model 1

Our first model is based on decomposing the data into a within-person and a between-person component, and specifying a time series model at the within level with fixed (i.e., non varying) slopes and residual variances. At the between level, we thus have only the two random effects from the decomposition, that is, the persons' means on negative affect and unpleasantness of events. This model can be represented as

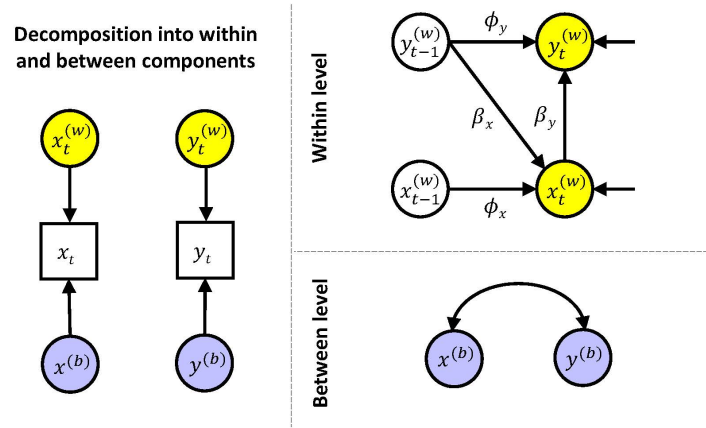


Figure 4:

At the within level we can write:

$$x_{it}^{(w)} = \phi_x x_{it-1}^{(w)} + \beta_x y_{it-1}^{(w)} + \zeta_{xit}$$

$$y_{it}^{(w)} = \phi_y y_{it-1}^{(w)} + \beta_y x_{it-1}^{(w)} + \zeta_{yit}$$

and at the between level we can write

$$x_i^{(b)} = \gamma_{x0} + u_{xi}$$

$$y_i^{(b)} = \gamma_{y0} + u_{yi}$$

### Input MLM1

To specify this model, we need to specify the model at the within level and at the between level. Since both variables that we use (na\_pre and ue\_pre) are time-varying, they will exist at both levels. At the within level we specify the same time series model that we ended our  $N = 1$  analyses with. The four regressions and the residual variances will be non-random, which means every person gets the same values for these. At the between level we therefor only have the two individual means of the two variables; we allow these to be correlated.

To allow for the lagged regressions, and ensure that the varying length of the time intervals is taken into account, we now add to the **VARIABLE** command the **LAGGED** and **TINTERVAL** option again, that is

```
LAGGED      = na_pre(1) ue_pre(1);
TINTERVAL   = TimeHours(1);
```

The model is now specified as:

MODEL:

```
%WITHIN%
na_pre ON na_pre&1; ! autoregression NA (lag 1)
na_pre ON ue_pre;   ! cross-regression from UE to NA (lag 0)
ue_pre ON ue_pre&1; ! autoregression UE (lag 1)
ue_pre on na_pre&1; ! cross-lagged regression from NA to UE (lag 1)

%BETWEEN%
! allow all random effects to be correlated
na_pre WITH ue_pre;
```

## Output MLM1

The output for this analysis is can be found here.

The parameter estimates for this model are organized by level, with the within level parameters presented first, and the between level parameters afterwards.

### MODEL RESULTS

		Posterior	One-Tailed	95% C.I.		
	Estimate	S.D.	P-Value	Lower 2.5%	Upper 2.5%	Significance
Within Level						
NA_PRE	ON					
NA_PRE&1	0.534	0.011	0.000	0.512	0.557	*
UE_PRE	0.110	0.006	0.000	0.099	0.121	*
UE_PRE	ON					
UE_PRE&1	0.154	0.020	0.000	0.116	0.194	*
NA_PRE&1	0.218	0.033	0.000	0.154	0.283	*
Residual Variances						
NA_PRE	0.445	0.009	0.000	0.429	0.464	*
UE_PRE	2.735	0.051	0.000	2.637	2.839	*
Between Level						
NA_PRE	WITH					
UE_PRE	0.157	0.045	0.000	0.079	0.259	*
Means						
NA_PRE	2.061	0.068	0.000	1.921	2.196	*
UE_PRE	-1.255	0.052	0.000	-1.357	-1.153	*
Variances						
NA_PRE	0.560	0.078	0.000	0.431	0.734	*
UE PRE	0.270	0.045	0.000	0.198	0.374	*

Hence, we see that for all four average slopes (i.e., ON statements) there is evidence that they deviate from zero. However, these are all unstandardized parameters.

Filling in the parameter estimates, the within level becomes:

$$UE_{it}^{(w)} = 0.15 * UE_{it-1}^{(w)} + 0.22 * NA_{it-1}^{(w)} + \zeta_{UEit} \quad var(\zeta_{UEit}) = 2.74$$

$$NA_{it}^{(w)} = 0.53 * NA_{it-1}^{(w)} + 0.11 * UE_{it}^{(w)} + \zeta_{NAit} \quad var(\zeta_{NAit}) = 0.46$$

and at the between level we can write

$$UE_i^{(b)} = 2.06 + u_{UE0i} \quad var(u_{UE0i}) = 0.27 \quad (1)$$

$$NA_i^{(b)} = -1.26 + u_{NA0i} \quad var(u_{NA0i}) = 0.56 \quad cov(u_{NA0i}, u_{UE0i}) = 0.16 \quad (2)$$

While autoregressive parameters are hardly affected by standardization (as the predictor and the outcome have the same variance), the cross-regressions are likely to change as a result of standardization. We may prefer the standardized results as they allow for an interpretation in terms of effect sizes. Alternatively, we may prefer the original, unstandardized results, as they allow for an interpretation close to the original scales that were used in measuring the variables.

The standardized results are obtained as part of the additional OUTPUT:

#### STANDARDIZED MODEL RESULTS

##### STDYX Standardization

		Posterior	One-Tailed	95% C.I.			
		Estimate	S.D.	P-Value	Lower 2.5%	Upper 2.5%	Significance
Within Level							
NA_PRE	ON						
	NA_PRE&1	0.534	0.011	0.000	0.512	0.557	*
	UE_PRE	0.221	0.011	0.000	0.199	0.243	*
UE_PRE	ON						
	UE_PRE&1	0.154	0.020	0.000	0.116	0.194	*
	NA_PRE&1	0.108	0.017	0.000	0.077	0.141	*
Residual Variances							
	NA_PRE	0.629	0.013	0.000	0.603	0.654	*
	UE_PRE	0.954	0.008	0.000	0.937	0.968	*
Between Level							
NA_PRE	WITH						
	UE_PRE	0.405	0.089	0.000	0.222	0.567	*
Means							
	NA_PRE	2.754	0.208	0.000	2.360	3.175	*
	UE_PRE	-2.414	0.216	0.000	-2.859	-2.006	*

Variances					
NA_PRE	1.000	0.000	0.000	1.000	1.000
UE_PRE	1.000	0.000	0.000	1.000	1.000

R-SQUARE

Within Level

Variable	Estimate	Posterior	One-Tailed	95% C.I.	
		S.D.	P-Value	Lower 2.5%	Upper 2.5%
NA_PRE	0.371	0.013	0.000	0.346	0.397
UE_PRE	0.046	0.008	0.000	0.032	0.063

The regression parameters (i.e., the ON statements) are standardized with the model implies variances at the within level. The residual variances at the within level represent the average proportion of unexplained variance; the corresponding proportions of explained variance can be found at the bottom as the R-SQUARE.

The WITH statement at the between level can be interpreted as the correlation between the (model implied) means of individuals on negative affect and unpleasantness of events. This correlation of 0.405 indicates that individuals with a higher trait level for negative affect also tend to report more unpleasantness of events on average over time.

The MEANS in the standardized solution indicate the number of standard deviations that the grand mean differs from zero. In general this is less informative and can be ignored. Furthermore, since there are not predictors of the random means at the between level, the standardized variances of the two variables are simply 1.

Compared to the highly idiographic approach that we used before (the replicated  $N = 1$  analyses), the current model is extremely restrictive: While we allow for individual differences in means, everything else is constrained across individuals, meaning that we assume each individual is characterized by the exact same dynamics and sensitivity to external influences. This is not very realistic; therefore, in all the following model we relax these constraints and allow for random slopes and random residual variances.

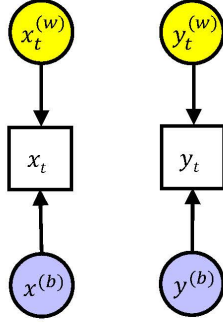
## Multilevel Model 2

In the second model (and all subsequent models) we allow for random slopes and random residual variances. This adds six more random effects to the model. The eight random effects are allowed to covary with each other at the between level. This model can be represented as

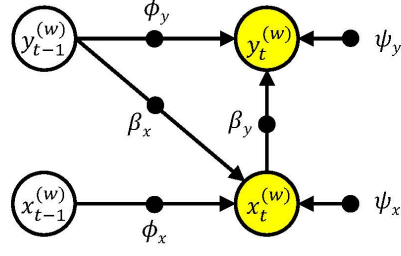
Note that filled circles at the within level indicate these parameters are random, meaning every individual has their own values for them. These individual parameters are modeled as latent variables at the between level (where there are open circles that correspond to the filled circles from the within level). The arrow system connecting the eight random effects implies they are all allowed to be correlated. These random effects are assumed to come from a multivariate normal distribution. Note that the residual variances from the within level are log transformed at the between level; this implies that when there is a negative value for this quantity at the between level, this is associated with a within-person residual variance between 0 and 1.

The model above can be expressed at the within level as:

Decomposition into within and between components



Within level



Between level

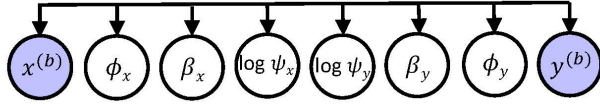


Figure 5:

$$\begin{aligned} y_{it}^{(w)} &= \phi_{yi} y_{it-1}^{(w)} + \beta_{yi} x_{it}^{(w)} + \zeta_{yit} \\ x_{it}^{(w)} &= \phi_{xi} x_{it-1}^{(w)} + \beta_{xi} y_{it-1}^{(w)} + \zeta_{xit} \end{aligned}$$

and at the between level as:

$$\begin{aligned} y_i^{(b)} &= \gamma_{y0} + u_{y0i} \\ x_i^{(b)} &= \gamma_{x0} + u_{x0i} \\ \phi_{yi} &= \gamma_{y1} + u_{y1i} \\ \phi_{xi} &= \gamma_{x1} + u_{x1i} \\ \beta_{yi} &= \gamma_{y2} + u_{y2i} \\ \beta_{xi} &= \gamma_{x2} + u_{x2i} \\ \log(\psi_{yi}) &= \gamma_{y3} + u_{y3i} \\ \log(\psi_{xi}) &= \gamma_{x3} + u_{x3i} \end{aligned}$$

where the  $\gamma$ 's are the fixed (or averaged) effects, and the  $u$ 's are the individual deviations from these means. These random components are allowed to be correlated. Specifically, we have

$$\begin{bmatrix} u_{y0i} \\ u_{x0i} \\ u_{y1i} \\ u_{x1i} \\ u_{y2i} \\ u_{x2i} \\ u_{y3i} \\ u_{x3i} \end{bmatrix} \sim MN \left( \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_{y0}^2 & \tau_{x0y0} & \tau_{y1x0} & \tau_{x1y0} & \tau_{y2x0} & \tau_{x2y0} & \tau_{y3x0} & \tau_{x3y0} \\ \tau_{x0y0} & \tau_{x0}^2 & \tau_{y1x0} & \tau_{x1x0} & \tau_{y2x0} & \tau_{x2x0} & \tau_{y3x0} & \tau_{x3x0} \\ \tau_{y1x0} & \tau_{y1x0} & \tau_{y1}^2 & \tau_{x1y1} & \tau_{y2y1} & \tau_{x2y1} & \tau_{y3y1} & \tau_{x3y1} \\ \tau_{x1y0} & \tau_{x1x0} & \tau_{x1y1} & \tau_{x1}^2 & \tau_{y2x1} & \tau_{x2x1} & \tau_{y3x1} & \tau_{x3x1} \\ \tau_{y2x0} & \tau_{y2x0} & \tau_{y2y1} & \tau_{y2x1} & \tau_{y2}^2 & \tau_{x2y2} & \tau_{y3y2} & \tau_{x3y2} \\ \tau_{x2y0} & \tau_{x2x0} & \tau_{x2y1} & \tau_{x2x1} & \tau_{x2y2} & \tau_{x2}^2 & \tau_{y3x2} & \tau_{x3x2} \\ \tau_{y3x0} & \tau_{y3x0} & \tau_{y3y1} & \tau_{y3x1} & \tau_{y3y2} & \tau_{y3x2} & \tau_{y3}^2 & \tau_{x3y3} \\ \tau_{x3y0} & \tau_{x3x0} & \tau_{x3y1} & \tau_{x3x1} & \tau_{x3y2} & \tau_{x3x2} & \tau_{x3y3} & \tau_{x3}^2 \end{bmatrix} \right)$$

that is, the random effects are assumed to come from a multivariate normal distribution, where the diagonal elements of the covariance matrix ( $\tau_{y0}^2$  to  $\tau_{x3}^2$ ) represent their variances, and the off-diagonal elements represent the covariances.

## Input MLM2

To allow for random slopes and residual variances, we have to change the **TYPE** option in the **ANALYSIS** command to

```
ANALYSIS: TYPE = TWOLEVEL RANDOM;
```

where **TWOLEVEL** ensures we will decompose the time-varying variables in a within-person component that varies over time, and a between-person component that is invariant over time, and **RANDOM** allows for the additional random effects.

These random slopes and residual variances are specified in the **%WITHIN%** model, using a vertical bar **|**; left of this, the name of the random effect is given, and right of it, an expression is given of the parameter that needs to be included as random. The names that are given at the within level, can then be used to model relations between the random effects at the between level. The current model is specified as:

```
MODEL:
%WITHIN%
phi_na | na_pre ON na_pre&1; ! autoregression NA (lag 1)
b_ae | na_pre ON ue_pre;      ! cross-regression from UE to NA (lag 0)
phi_ue | ue_pre ON ue_pre&1; ! autoregression UE (lag 1)
b_ea | ue_pre ON na_pre&1;    ! cross-lagged regression from NA to UE (lag 1)
var_na | na_pre;              ! random residual variance NA
var_ue | ue_pre;              ! random residual variance UE

%BETWEEN%
! allow all random effects to be correlated
na_pre ue_pre phi_na-var_ue WITH ue_pre phi_na-var_ue;
```

The first four lines of the within level model are used to specify the random slopes; the last two lines of the within level model are used to specify the random residual variances (note that at the within level these are variances, while at the between level these are log transformed).

At the between level all eight random effects are allowed to be correlated.

We will also ask for additional output, through

```
OUTPUT: TECH1 TECH8 STDYX FSCOMPARISON STAND(CLUSTER);
```

```
PLOT: TYPE = PLOT3;
      FACTOR = ALL (500);
```

where

**TECH1** provides information about where the free model parameters are (when representing the model using a LISRL format)

**TECH8** contains the information about the potential scale reduction parameter over every 100 iterations of the MCMC algorithm

**STDYX** provides the standardized results

**FSCOMPARISONS** provides comparisons between all 129 participants on their estimated scores for the random effects; this option requires the use of **FACTOR = ALL (500)** as part of the **PLOT** command, as this ensures that the MCMC runs an addition—in this case—500 iterations of the MCMC algorithm and saves the individually sampled parameters (i.e., the eight random effects in this model).

STAND(CLUSTER) gives the standardized results per person

## Output MLM2

The output for this analysis is can be found here.

In contrast to the previous model, in which the autoregressions, cross-regressions, and residual variances were constrained to be invariant across individuals, the current model allows for individual differences. This implies that for each of the 129 persons, in every iteration of the MCMC algorithm, six individual parameters are sampled. This will often result in a warning given by Mplus that we obtained here:

```
WARNING: PROBLEMS OCCURRED IN SEVERAL ITERATIONS IN THE COMPUTATION OF THE STANDARDIZED
ESTIMATES FOR SEVERAL CLUSTERS. THIS IS MOST LIKELY DUE TO AR COEFFICIENTS GREATER
THAN 1 OR PARAMETERS GIVING NON-STATIONARY MODELS. SUCH POSTERIOR DRAWS ARE REMOVED.
THE FOLLOWING CLUSTERS HAD SUCH PROBLEMS:
120 66 39 134 116 144 28 117 3 141
```

This warning is related to the fact that if the autoregressive parameter in a first-order autoregressive model is 1 or larger than 1 (or -1 or smaller than -1), the process is not stationary, meaning: It does not vary around a constant with the same amount of variance over time. To check whether there is a problem with this analysis, we can: a) look at the sequences of the individuals; and b) check their individual posterior distributions (which we can plot since we used the `FACTOR = ALL(500)` statement). We may also decide to run the analyses without these individual, to see whether this has an effect on the overall results.

Since all the slopes and residual variances are random, there are no model parameters at the within level. Instead, all the parameter of this model are: 8 fixed (averaged) effects, 8 variances of these random effects, and  $7*8/2=28$  covariances between these 8 random effects.

### MODEL RESULTS

	Estimate	Posterior S.D.	One-Tailed P-Value	95% C.I.		Significance
				Lower 2.5%	Upper 2.5%	
Within Level						
Between Level						
NA_PRE WITH						
PHI_NA	0.029	0.020	0.072	-0.012	0.068	
B_AE	0.030	0.008	0.000	0.017	0.047	*
PHI_UE	0.007	0.024	0.374	-0.039	0.055	
B_EA	-0.062	0.038	0.036	-0.145	0.005	
VAR_NA	0.505	0.091	0.000	0.352	0.705	*
VAR_UE	-0.001	0.045	0.491	-0.087	0.087	
UE_PRE WITH						
PHI_NA	0.011	0.016	0.246	-0.020	0.044	
B_AE	0.007	0.005	0.069	-0.002	0.018	
PHI_UE	0.013	0.018	0.238	-0.024	0.047	
B_EA	-0.005	0.030	0.436	-0.069	0.051	
VAR_NA	0.179	0.065	0.002	0.063	0.318	*
VAR_UE	0.104	0.037	0.001	0.036	0.182	*
PHI_NA WITH						
B_AE	-0.002	0.002	0.146	-0.007	0.002	
PHI_UE	-0.003	0.007	0.335	-0.018	0.011	

B_EA	0.006	0.013	0.303	-0.021	0.030	
VAR_NA	0.023	0.028	0.195	-0.030	0.080	
VAR_UE	0.001	0.015	0.472	-0.028	0.032	
B_AE WITH						
PHI_UE	0.003	0.002	0.091	-0.002	0.008	
B_EA	-0.007	0.004	0.017	-0.016	-0.001	*
VAR_NA	0.040	0.009	0.000	0.024	0.061	*
VAR_UE	-0.006	0.005	0.136	-0.016	0.005	
PHI_UE WITH						
B_EA	-0.008	0.013	0.244	-0.036	0.016	
VAR_NA	0.026	0.029	0.189	-0.031	0.082	
VAR_UE	0.012	0.015	0.202	-0.017	0.042	
B_EA WITH						
VAR_NA	-0.147	0.062	0.003	-0.294	-0.043	*
VAR_UE	0.025	0.022	0.106	-0.016	0.072	
VAR_NA WITH						
VAR_UE	0.068	0.058	0.112	-0.043	0.187	
NA_PRE WITH						
UE_PRE	0.178	0.050	0.000	0.090	0.287	*
Means						
NA_PRE	2.035	0.068	0.000	1.903	2.172	*
UE_PRE	-1.261	0.056	0.000	-1.369	-1.154	*
PHI_NA	0.470	0.024	0.000	0.421	0.516	*
B_AE	0.111	0.008	0.000	0.095	0.127	*
PHI_UE	0.128	0.026	0.000	0.076	0.177	*
B_EA	0.255	0.051	0.000	0.164	0.362	*
VAR_NA	-1.160	0.088	0.000	-1.329	-0.982	*
VAR_UE	0.873	0.052	0.000	0.767	0.977	*
Variances						
NA_PRE	0.556	0.084	0.000	0.421	0.744	*
UE_PRE	0.321	0.054	0.000	0.232	0.440	*
PHI_NA	0.051	0.010	0.000	0.035	0.074	*
B_AE	0.004	0.001	0.000	0.002	0.006	*
PHI_UE	0.032	0.010	0.000	0.015	0.055	*
B_EA	0.070	0.032	0.000	0.029	0.149	*
VAR_NA	0.976	0.146	0.000	0.733	1.311	*
VAR_UE	0.292	0.047	0.000	0.215	0.396	*

Note that the **Means** are the fixed (or averaged) effects. Specifically for the slopes it is of interest to see that these show evidence for being different from zero. For the log residual variances we see that the fixed effect for negative affect is negative; this implies that on average the residual variance for this variable is smaller than 1 (as we would have to take the exponent to transform it back to the variance, that is,  $\exp(-1.160)=0.313$ ).

The **Variances** indicate the variability across the participants for each of these random effects. While the variance for the slopes are relatively small, it is important to realize that—especially for autoregressive parameters—the scale on which individual differences for these parameters can occur is very small.

The **WITH** statements contain all the covariances; these are in general difficult to interpret, as they are scale



dependent. However, checking whether zero falls in the 95% CI gives an indication of whether there is evidence that the covariance differs from zero.

In addition to the above parameter estimates, we also get the standardized estimates. We discuss the standardized results for each level separately below.

## STANDARDIZED MODEL RESULTS

### STDYX Standardization

	Estimate	Posterior S.D.	One-Tailed P-Value	95% C.I.		
				Lower 2.5%	Upper 2.5%	Significance
Within-Level Standardized Estimates Averaged Over Clusters						
PHI_NA   NA_PRE ON NA_PRE&1	0.470	0.013	0.000	0.443	0.495	*
B_AE   NA_PRE ON UE_PRE	0.232	0.011	0.000	0.210	0.252	*
PHI_UE   UE_PRE ON UE_PRE&1	0.129	0.020	0.000	0.089	0.166	*
B_EA   UE_PRE ON NA_PRE&1	0.102	0.017	0.000	0.071	0.137	*
VAR_NA   NA_PRE	0.637	0.012	0.000	0.613	0.659	*
VAR_UE   UE_PRE	0.922	0.010	0.000	0.902	0.941	*

As indicate here, the standardized results at the within level are based on standardizing the parameters per person first (using their model implied variances, rather than their observed variances!), and then taking the average across all 129 participants. Since this is done within each iteration of the MCMC algorithm, this results in a posterior distribution for each of these averaged standardized parameters. Hence, the Posterior S.D. indicates the amount of variability of this averaged standardized parameter, and the lower and upper bound define the 95% credibility interval of it.

It is important to realize that—in contrast to what we saw in the replicated  $N = 1$  analyses—this has nothing to do with variability across individuals!

The standardized results at the between level contain the following:

### Between Level

NA_PRE WITH						
PHI_NA	0.172	0.114	0.072	-0.071	0.390	
B_AE	0.647	0.104	0.000	0.415	0.814	*
PHI_UE	0.055	0.172	0.374	-0.280	0.386	
B_EA	-0.329	0.166	0.036	-0.616	0.028	
VAR_NA	0.692	0.056	0.000	0.568	0.786	*
VAR_UE	-0.002	0.108	0.491	-0.210	0.208	
UE_PRE WITH						

PHI_NA	0.085	0.122	0.246	-0.154	0.321	
B_AE	0.210	0.135	0.069	-0.066	0.456	
PHI_UE	0.125	0.176	0.238	-0.243	0.457	
B_EA	-0.032	0.191	0.436	-0.408	0.347	
VAR_NA	0.324	0.099	0.002	0.115	0.504	*
VAR_UE	0.346	0.103	0.001	0.130	0.525	*
PHI_NA WITH						
B_AE	-0.158	0.142	0.146	-0.410	0.146	
PHI_UE	-0.080	0.176	0.335	-0.410	0.274	
B_EA	0.111	0.212	0.303	-0.304	0.524	
VAR_NA	0.106	0.120	0.195	-0.136	0.331	
VAR_UE	0.009	0.120	0.472	-0.225	0.247	
B_AE WITH						
PHI_UE	0.278	0.196	0.091	-0.138	0.631	
B_EA	-0.440	0.188	0.017	-0.765	-0.042	*
VAR_NA	0.645	0.086	0.000	0.448	0.789	*
VAR_UE	-0.172	0.147	0.136	-0.434	0.135	
PHI_UE WITH						
B_EA	-0.175	0.248	0.244	-0.574	0.408	
VAR_NA	0.146	0.160	0.189	-0.178	0.443	
VAR_UE	0.130	0.155	0.202	-0.178	0.433	
B_EA WITH						
VAR_NA	-0.582	0.163	0.003	-0.825	-0.192	*
VAR_UE	0.185	0.147	0.106	-0.107	0.472	
VAR_NA WITH						
VAR_UE	0.130	0.104	0.112	-0.082	0.328	
NA_PRE WITH						
UE_PRE	0.428	0.090	0.000	0.230	0.586	*
Means						
NA_PRE	2.732	0.212	0.000	2.327	3.160	*
UE_PRE	-2.233	0.212	0.000	-2.686	-1.841	*
PHI_NA	2.092	0.247	0.000	1.649	2.625	*
B_AE	1.750	0.229	0.000	1.357	2.266	*
PHI_UE	0.722	0.191	0.000	0.386	1.143	*
B_EA	0.973	0.234	0.000	0.589	1.487	*
VAR_NA	-1.171	0.124	0.000	-1.415	-0.935	*
VAR_UE	1.616	0.160	0.000	1.325	1.944	*
Variances						
NA_PRE	1.000	0.000	0.000	1.000	1.000	
UE_PRE	1.000	0.000	0.000	1.000	1.000	
PHI_NA	1.000	0.000	0.000	1.000	1.000	
B_AE	1.000	0.000	0.000	1.000	1.000	
PHI_UE	1.000	0.000	0.000	1.000	1.000	
B_EA	1.000	0.000	0.000	1.000	1.000	
VAR_NA	1.000	0.000	0.000	1.000	1.000	
VAR_UE	1.000	0.000	0.000	1.000	1.000	

The WITH statements are the correlations between the random effects. Note these are the correlations between the unstandardized random effects; these may differ considerably from the correlations between the individually standardized random effects. The reason for this is that slopes are standardized with the participant's (model implied) variances, and since these may be different for individuals, the standardized random slopes may show a very different ordering of individuals than the ordering of individuals based on the unstandardized random slopes. A similar issue may occur for the residual variances. Furthermore, the correlation reported here between the residual variances and other random effects are based on the log transformed residual variances.

The individually standardized parameters are obtained by including STAND(CLUSTER) in the OUTPUT command. This leads to additional output for each separate person, that looks like this:

#### WITHIN-LEVEL STANDARDIZED MODEL RESULTS FOR CLUSTER 120

##### STDYX Standardization

	Estimate	Posterior S.D.	One-Tailed P-Value	95% C.I.		Significance
				Lower 2.5%	Upper 2.5%	
PHI_NA   NA_PRE ON NA_PRE&1	0.498	0.170	0.005	0.135	0.804	*
BETA_NA   NA_PRE ON UE_PRE	0.231	0.071	0.000	0.102	0.386	*
PHI_UE   UE_PRE ON UE_PRE&1	0.163	0.168	0.165	-0.195	0.482	
BETA_UE   UE_PRE ON NA_PRE&1	0.027	0.171	0.441	-0.328	0.358	
PSI_NA   NA_PRE	0.673	0.162	0.000	0.286	0.908	*
PSI_UE   UE_PRE	0.937	0.082	0.000	0.704	0.997	*

#### WITHIN-LEVEL R-SQUARE FOR CLUSTER 120

Variable	Estimate	Posterior S.D.	One-Tailed P-Value	95% C.I.	
				Lower 2.5%	Upper 2.5%
NA_PRE	0.327	0.162	0.000	0.092	0.713
UE_PRE	0.063	0.082	0.000	0.003	0.294

We used this information to plot the individually standardized cross-regressions against each other in Figure 7 in the book chapter.

The unstandardized individual parameter estimates can also be obtained, using FSCOMPARISON in the OUTPUT command (combined with the PLOT: FACTOR = ALL (500) command). It generates the following output

#### BETWEEN-LEVEL FACTOR SCORE COMPARISONS

##### Results for Factor PHI\_NA

Ranking	Cluster	Factor Score	Ranking	Cluster	Factor Score	Ranking	Cluster	Factor Score
1	39	0.856	2	60	0.818	3	3	0.816
4	141	0.805	5	116	0.801	6	144	0.800
...								
124	88	0.167	125	12	0.156	126	17	0.151
127	99	0.150	128	118	0.129	129	35	0.105

These are the individual estimates for  $\phi_{yi}$  (i.e., autoregression for negative affect). They are included from highest to lowest, with **Cluster** referring to the clustering variable (here the variable ID).

The individual estimates are also included for all the other random effects in the model ( $\beta_{yi}$ ,  $\phi_{xi}$ ,  $\beta_{xi}$ ,  $\log(\psi_{yi})$ ,  $\log(\psi_{xi})$ ,  $y_i^{(b)}$ , and  $x_i^{(b)}$ ). Combined with the **PLOT: PLOT = TYPE3** command, this also provides us with the opportunity to plot the estimates of the individual random effects against each other (see for instance the middle panel of Figure 7 of the book chapter). Such scatter plots can be created in Mplus for review; the data in the plot can then be easily saved by right-clicking the plot and choosing save, which allows for further editing outside of Mplus.

### Multilevel Model 3

In the third multilevel model, we include an observed predictor for the random effects at the between level.

**Decomposition into within and between components**

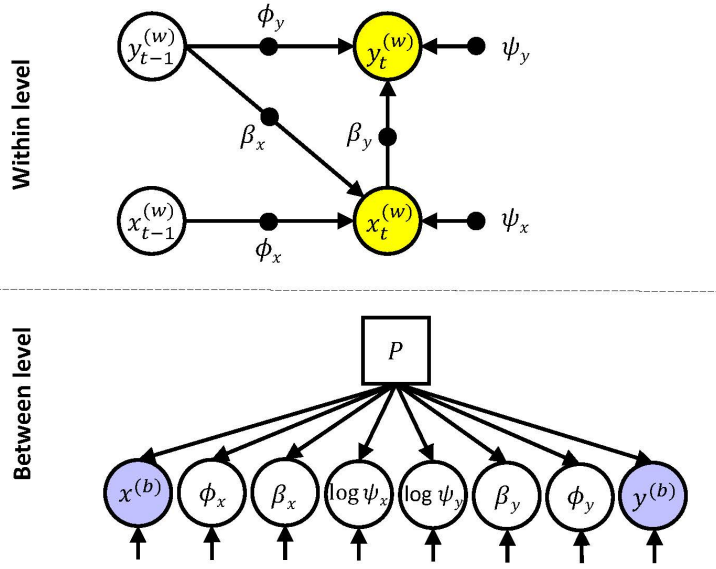
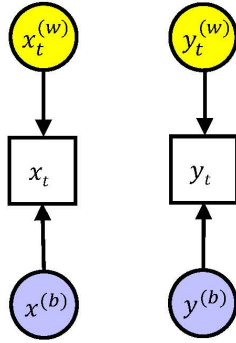


Figure 6:

The within level of this model is as before (Model 2); the between level can be expressed as:

$$\begin{aligned}
x_i^{(b)} &= \gamma_{x00} + \gamma_{x01}P_i + u_{x0i} \\
y_i^{(b)} &= \gamma_{y00} + \gamma_{y01}P_i + u_{y0i} \\
\phi_{xi} &= \gamma_{x10} + \gamma_{x11}P_i + u_{x1i} \\
\phi_{yi} &= \gamma_{y10} + \gamma_{y11}P_i + u_{y1i} \\
\beta_{xi} &= \gamma_{x20} + \gamma_{x21}P_i + u_{x2i} \\
\beta_{yi} &= \gamma_{y20} + \gamma_{y21}P_i + u_{y2i} \\
\log(\psi_{xi}) &= \gamma_{x30} + \gamma_{x31}P_i + u_{x3i} \\
\log(\psi_{yi}) &= \gamma_{y30} + \gamma_{y31}P_i + u_{y3i}
\end{aligned}$$

where the intercepts ( $\gamma_{x00}$  to  $\gamma_{y30}$ ) are the expected score for the random effects when the covariate  $P_i$  is zero; and the regression coefficients ( $\gamma_{x01}$  and  $\gamma_{y31}$ ) indicate the degree to which the random effects can be predicted from the covariate  $P_i$ .

## Input MLM3

The observed predictor `ham_pre` is a baseline covariate, with between-person variance only. We need to specify it as a between variable in the **VARIABLE** command. For this, we also need to indicate it is used in the analysis. We do this by specifying

```
USEVAR      =  na_pre ham_pre ue_pre;
BETWEEN     =  ham_pre;
```

Note that in the **USEVAR** option, we first need to indicate all the observed variables (from the data file) that are included in our analysis, and then the variables that are created using the **DEFINE** command (here: `ue_pre`). Furthermore, with **BETWEEN = ham\_pre** we indicate that the variable `ham_pre` only exists at the between level.

When we regress the random effects at the between level on this observed predictor, the intercepts in these regression equations represent the expected score when the predictor is zero. To make this an easy to interpret parameter, we grand mean center the between level predictor: That way, the intercept is the average random effect (i.e., the expected individual score for someone who is average with respect to the time-invariant covariate). We do this within the **DEFINE** command, by adding:

```
STANDARDIZE(ham_pre);
```

The **ANALYSIS** command remains unchanged (except for choosing a different seed), and the within part of **MODEL** command also remains unchanged. However, the between part of the **MODEL** command is now changed to

```
%BETWEEN%
! regress all random effects on observed predictor
na_pre ue_pre phi_na-psi_ue ON ham_pre;
```

Note that with this specification, Mplus will include a covariance for the residuals of the between components of the observed variables (akin to stating: **na\_pre WITH ue\_pre** at the between level); this is a default setting. We may also decide to allow all residuals to be correlated, as will be shown below.

## Output MLM3

The output for this analysis is can be found [here](#).

The parameter estimates obtained for this model are:

# MODEL RESULTS

		Posterior	One-Tailed	95% C.I.		
	Estimate	S.D.	P-Value	Lower 2.5%	Upper 2.5%	Significance
Within Level						
Between Level						
PHI_NA ON HAM_PRE	-0.013	0.022	0.278	-0.057	0.032	
BETA_NA ON HAM_PRE	0.020	0.007	0.006	0.005	0.033	*
PHI_UE ON HAM_PRE	0.035	0.024	0.075	-0.015	0.082	
BETA_UE ON HAM_PRE	-0.086	0.038	0.014	-0.160	-0.011	*
PSI_NA ON HAM_PRE	0.415	0.077	0.000	0.264	0.572	*
PSI_UE ON HAM_PRE	-0.039	0.048	0.207	-0.134	0.058	
NA_PRE ON HAM_PRE	0.303	0.058	0.000	0.191	0.420	*
UE_PRE ON HAM_PRE	0.114	0.053	0.014	0.013	0.217	*
UE_PRE WITH NA_PRE	0.133	0.040	0.000	0.061	0.222	*
Intercepts						
NA_PRE	2.012	0.061	0.000	1.897	2.133	*
UE_PRE	-1.285	0.054	0.000	-1.390	-1.182	*
PHI_NA	0.476	0.022	0.000	0.430	0.518	*
BETA_NA	0.100	0.007	0.000	0.086	0.115	*
PHI_UE	0.132	0.025	0.000	0.084	0.181	*
BETA_UE	0.233	0.040	0.000	0.158	0.315	*
PSI_NA	-1.165	0.075	0.000	-1.314	-1.021	*
PSI_UE	0.872	0.050	0.000	0.775	0.971	*
Residual Variances						
NA_PRE	0.413	0.062	0.000	0.308	0.555	*
UE_PRE	0.275	0.046	0.000	0.201	0.379	*
PHI_NA	0.042	0.008	0.000	0.029	0.059	*
BETA_NA	0.003	0.001	0.000	0.002	0.005	*
PHI_UE	0.024	0.008	0.000	0.010	0.043	*
BETA_UE	0.030	0.017	0.000	0.005	0.071	*
PSI_NA	0.717	0.101	0.000	0.558	0.946	*

PSI_UE	0.256	0.041	0.000	0.190	0.350	*
--------	-------	-------	-------	-------	-------	---

The ON statements are the estimates of the regression coefficients (i.e.,  $\gamma_{x01}$  and  $\gamma_{y31}$ ), and the **Intercepts** are the estimates of the intercepts at the between level (i.e.,  $\gamma_{x00}$  and  $\gamma_{y30}$ ). The latter can be interpreted as the means across individuals, as we grand mean centered the predictor  $P_i$  (here: Ham\_pre).

The **Residual variances** are the estimates of the between level residuals (i.e., variances of  $u_{x0i}$  and  $u_{yi}$ ). Furthermore, the WITH statement is the default covariance between the residuals of the between components  $y_i^{(b)}$  and  $x_i^{(b)}$ .

In addition to the unstandardized estimates, we also obtained the standardized estimates. Note that while the current model has no parameters at the within level (as all the parameters are random), the standardized solution contains within level standardized parameters for all the random slopes and residual variances, as these are standardized per person and then average across individuals, as can be seen below:

#### STANDARDIZED MODEL RESULTS

##### STDYX Standardization

	Estimate	Posterior S.D.	One-Tailed P-Value	95% C.I.		Significance
				Lower 2.5%	Upper 2.5%	
Within-Level Standardized Estimates Averaged Over Clusters						
PHI_NA   NA_PRE ON NA_PRE&1	0.476	0.013	0.000	0.450	0.502	*
BETA_NA   NA_PRE ON UE_PRE	0.218	0.011	0.000	0.197	0.239	*
PHI_UE   UE_PRE ON UE_PRE&1	0.131	0.021	0.000	0.090	0.172	*
BETA_UE   UE_PRE ON NA_PRE&1	0.106	0.017	0.000	0.074	0.139	*
PSI_NA   NA_PRE	0.639	0.012	0.000	0.615	0.663	*
PSI_UE   UE_PRE	0.924	0.010	0.000	0.906	0.943	*

At the between level we get between level parameter that are standardized; this means these average regression parameters are standardized with between level variances:

##### Between Level

PHI_NA ON HAM_PRE	-0.047	0.076	0.278	-0.189	0.109	
BETA_NA ON HAM_PRE	0.252	0.094	0.006	0.067	0.432	*
PHI_UE ON HAM_PRE	0.157	0.116	0.075	-0.065	0.404	

BETA_UE	ON						
HAM_PRE		-0.336	0.162	0.014	-0.676	-0.047	*
PSI_NA	ON						
HAM_PRE		0.329	0.057	0.000	0.208	0.435	*
PSI_UE	ON						
HAM_PRE		-0.055	0.067	0.207	-0.182	0.078	
NA_PRE	ON						
HAM_PRE		0.317	0.058	0.000	0.199	0.429	*
UE_PRE	ON						
HAM_PRE		0.152	0.069	0.014	0.017	0.285	*
UE_PRE	WITH						
NA_PRE		0.399	0.093	0.000	0.200	0.568	*
Intercepts							
NA_PRE		2.969	0.217	0.000	2.547	3.410	*
UE_PRE		-2.416	0.222	0.000	-2.889	-2.011	*
PHI_NA		2.324	0.252	0.000	1.867	2.841	*
BETA_NA		1.808	0.249	0.000	1.372	2.354	*
PHI_UE		0.839	0.248	0.000	0.472	1.400	*
BETA_UE		1.264	0.433	0.000	0.750	2.456	*
PSI_NA		-1.297	0.119	0.000	-1.531	-1.075	*
PSI_UE		1.713	0.163	0.000	1.416	2.046	*
Residual Variances							
NA_PRE		0.900	0.037	0.000	0.816	0.960	*
UE_PRE		0.977	0.022	0.000	0.918	1.000	*
PHI_NA		0.996	0.010	0.000	0.963	1.000	*
BETA_NA		0.936	0.048	0.000	0.813	0.995	*
PHI_UE		0.975	0.045	0.000	0.836	1.000	*
BETA_UE		0.887	0.122	0.000	0.544	0.997	*
PSI_NA		0.892	0.037	0.000	0.811	0.957	*
PSI_UE		0.996	0.009	0.000	0.967	1.000	*

The standardized intercepts indicate the amount of between level standard deviations that the average parameter deviates from zero. The standardized residual variances indicate the proportion of unexplained variance at the between level.

We also get the  $R^2$  at each level:

R-SQUARE

Within-Level R-Square Averaged Across Clusters

Variable	Estimate	Posterior	One-Tailed	95% C.I.	
		S.D.	P-Value	Lower 2.5%	Upper 2.5%
NA_PRE	0.361	0.012	0.000	0.337	0.384
UE_PRE	0.076	0.010	0.000	0.057	0.094

Between Level



Variable	Estimate	Posterior S.D.	One-Tailed P-Value	95% C.I.	
				Lower 2.5%	Upper 2.5%
NA_PRE	0.100	0.037	0.000	0.039	0.184
UE_PRE	0.023	0.022	0.000	0.000	0.081

Variable	Estimate	Posterior S.D.	One-Tailed P-Value	95% C.I.	
				Lower 2.5%	Upper 2.5%
PHI_NA	0.004	0.010	0.000	0.000	0.037
BETA_NA	0.064	0.048	0.000	0.005	0.187
PHI_UE	0.025	0.045	0.000	0.000	0.163
BETA_UE	0.113	0.122	0.000	0.003	0.456
PSI_NA	0.108	0.037	0.000	0.043	0.189
PSI_UE	0.004	0.009	0.000	0.000	0.033

Note that the lower bound of the CIs for these  $R^2$ 's will never be smaller than zero; while some are printed as 0.000, they will also not be actually zero, as we are using Bayesian estimation.

If we allow the residuals of the random effects at the between level to be all correlated, we get the following standardized results:

#### STANDARDIZED MODEL RESULTS

##### STDYX Standardization

	Estimate	Posterior S.D.	One-Tailed P-Value	95% C.I.		Significance
				Lower 2.5%	Upper 2.5%	
Within-Level Standardized Estimates Averaged Over Clusters						
PHI_NA   NA_PRE ON NA_PRE&1	0.468	0.013	0.000	0.443	0.494	*
BETA_NA   NA_PRE ON UE_PRE	0.230	0.011	0.000	0.209	0.253	*
PHI_UE   UE_PRE ON UE_PRE&1	0.127	0.021	0.000	0.085	0.169	*
BETA_UE   UE_PRE ON NA_PRE&1	0.104	0.018	0.000	0.070	0.140	*
PSI_NA   NA_PRE	0.638	0.012	0.000	0.613	0.660	*
PSI_UE   UE_PRE	0.921	0.010	0.000	0.901	0.941	*
Between Level						
PHI_NA ON HAM_PRE	-0.036	0.074	0.306	-0.182	0.105	

BETA_NA	ON						
HAM_PRE		0.236	0.086	0.003	0.063	0.401	*
PHI_UE	ON						
HAM_PRE		0.139	0.103	0.087	-0.061	0.342	
BETA_UE	ON						
HAM_PRE		-0.250	0.113	0.016	-0.466	-0.026	*
PSI_NA	ON						
HAM_PRE		0.313	0.060	0.000	0.185	0.419	*
PSI_UE	ON						
HAM_PRE		-0.055	0.067	0.199	-0.184	0.081	
NA_PRE	ON						
HAM_PRE		0.300	0.060	0.000	0.175	0.409	*
UE_PRE	ON						
HAM_PRE		0.144	0.069	0.018	0.008	0.280	*
NA_PRE	WITH						
PHI_NA		0.220	0.111	0.031	-0.011	0.433	
BETA_NA		0.578	0.118	0.000	0.309	0.778	*
PHI_UE		-0.005	0.167	0.487	-0.320	0.343	
BETA_UE		-0.191	0.181	0.147	-0.515	0.191	
PSI_NA		0.618	0.067	0.000	0.472	0.734	*
PSI_UE		0.041	0.107	0.352	-0.169	0.254	
UE_PRE	WITH						
PHI_NA		0.106	0.123	0.208	-0.140	0.334	
BETA_NA		0.135	0.137	0.172	-0.135	0.388	
PHI_UE		0.091	0.171	0.317	-0.239	0.415	
BETA_UE		0.044	0.200	0.417	-0.331	0.451	
PSI_NA		0.255	0.108	0.013	0.031	0.454	*
PSI_UE		0.373	0.102	0.000	0.156	0.556	*
PHI_NA	WITH						
BETA_NA		-0.153	0.147	0.164	-0.425	0.155	
PHI_UE		-0.064	0.178	0.352	-0.417	0.286	
BETA_UE		0.090	0.233	0.354	-0.339	0.565	
PSI_NA		0.150	0.116	0.107	-0.086	0.371	
PSI_UE		0.002	0.118	0.493	-0.224	0.236	
BETA_NA	WITH						
PHI_UE		0.233	0.202	0.125	-0.175	0.626	
BETA_UE		-0.355	0.203	0.052	-0.721	0.083	
PSI_NA		0.579	0.093	0.000	0.381	0.743	*
PSI_UE		-0.134	0.156	0.201	-0.440	0.183	
PHI_UE	WITH						
BETA_UE		-0.128	0.256	0.320	-0.571	0.431	
PSI_NA		0.064	0.164	0.346	-0.252	0.389	

PSI_UE	0.183	0.166	0.135	-0.155	0.497	
BETA_UE WITH						
PSI_NA	-0.473	0.204	0.023	-0.792	-0.011	*
PSI_UE	0.148	0.162	0.185	-0.160	0.475	
PSI_NA WITH						
PSI_UE	0.188	0.106	0.041	-0.021	0.386	
NA_PRE WITH						
UE_PRE	0.375	0.098	0.000	0.164	0.551	*
Intercepts						
NA_PRE	2.880	0.218	0.000	2.449	3.308	*
UE_PRE	-2.233	0.212	0.000	-2.661	-1.838	*
PHI_NA	2.074	0.243	0.000	1.627	2.596	*
BETA_NA	1.798	0.245	0.000	1.364	2.330	*
PHI_UE	0.722	0.193	0.000	0.392	1.152	*
BETA_UE	0.986	0.233	0.000	0.596	1.528	*
PSI_NA	-1.236	0.118	0.000	-1.481	-1.018	*
PSI_UE	1.618	0.165	0.000	1.304	1.944	*
Residual Variances						
NA_PRE	0.910	0.035	0.000	0.833	0.969	*
UE_PRE	0.979	0.021	0.000	0.922	1.000	*
PHI_NA	0.997	0.009	0.000	0.967	1.000	*
BETA_NA	0.944	0.042	0.000	0.839	0.996	*
PHI_UE	0.980	0.033	0.000	0.883	1.000	*
BETA_UE	0.938	0.058	0.000	0.783	0.998	*
PSI_NA	0.902	0.036	0.000	0.825	0.965	*
PSI_UE	0.996	0.010	0.000	0.966	1.000	*

## Multilevel Model 4

The addition of SEM to the combination of time series modeling and multilevel modeling, offers additional opportunities for modeling the variables at the between level. Specifically, we can now include a latent variable there, for instance with the random effects as its indicators, but we can also specify mediation models or other path models, either with or without a factor model.

We begin with specifying a latent variable that uses the eight random effects as its indicators. This latent variable captures what they have in common with each other.

The within level of this model is as before (Model 2); the between level can be expressed as:

Decomposition into within and between components

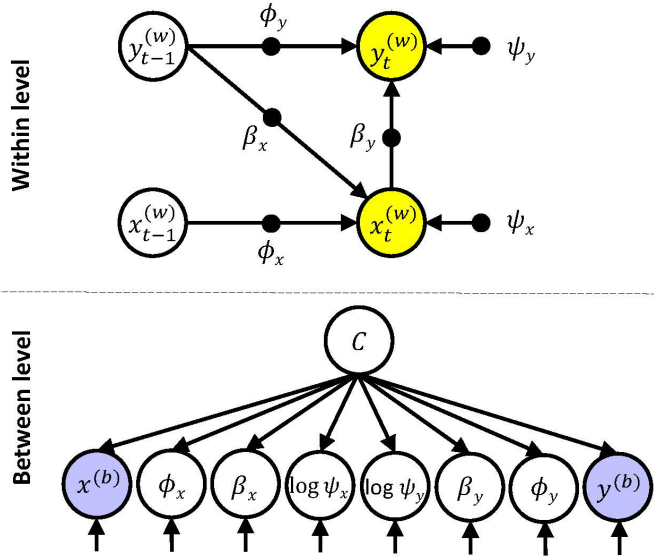
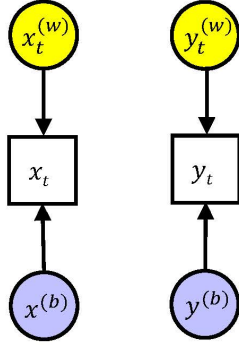


Figure 7:

$$\begin{aligned}
 x_i^{(b)} &= \gamma_{x00} + \gamma_{x01}C_i + u_{x0i} \\
 y_i^{(b)} &= \gamma_{y00} + \gamma_{y01}C_i + u_{y0i} \\
 \phi_{xi} &= \gamma_{x10} + \gamma_{x11}C_i + u_{x1i} \\
 \phi_{yi} &= \gamma_{y10} + \gamma_{y11}C_i + u_{y1i} \\
 \beta_{xi} &= \gamma_{x20} + \gamma_{x21}C_i + u_{x2i} \\
 \beta_{yi} &= \gamma_{y20} + \gamma_{y21}C_i + u_{y2i} \\
 \log(\psi_{xi}) &= \gamma_{x30} + \gamma_{x31}C_i + u_{x3i} \\
 \log(\psi_{yi}) &= \gamma_{y30} + \gamma_{y31}C_i + u_{y3i}
 \end{aligned}$$

where the intercepts ( $\gamma_{x00}$  to  $\gamma_{y30}$ ) are average values of the random effects; and the regression coefficients ( $\gamma_{x01}$  and  $\gamma_{y31}$ ) are the factor loadings.

## Input MLM4

The model is specified similar to Multilevel Model 2; it does not include any between level covariates. The within level of this model is as specified in Models 2 and 3. The between level of the `MODEL` command is specified as:

```

%BETWEEN%
! Specify a common factor for the random effects
c BY na_pre@1 ue_pre phi_na-psi_ue*.1;

```

where the latent variable is created with the `BY` statement. The usual Mplus default of fixing the first factor loading to 1 for identification, does not work here; hence we fix the first factor loading ourselves by stating `na_pre@1`. We use the `*.1` to specify starting values.

## Output MLM4

The output for this analysis is can be found here.

Again, this model does not contain any model parameter at the within level. The unstandardized parameters that we obtain are:

### MODEL RESULTS

		Posterior	One-Tailed	95% C.I.			
		Estimate	S.D.	P-Value	Lower 2.5%	Upper 2.5%	Significance
Within Level							
Between Level							
C	BY						
	NA_PRE	1.000	0.000	0.000	1.000	1.000	
	UE_PRE	0.379	0.099	0.000	0.197	0.574	*
C	BY						
	PHI_NA	0.045	0.044	0.142	-0.038	0.134	
	BETA_NA	0.072	0.014	0.000	0.046	0.101	*
	PHI_UE	0.047	0.050	0.165	-0.051	0.149	
	BETA_UE	-0.154	0.087	0.036	-0.339	0.015	
	PSI_NA	1.270	0.200	0.000	0.929	1.702	*
	PSI_UE	0.050	0.096	0.305	-0.138	0.237	
Intercepts							
	NA_PRE	2.030	0.063	0.000	1.908	2.157	*
	UE_PRE	-1.278	0.053	0.000	-1.382	-1.173	*
	PHI_NA	0.473	0.023	0.000	0.426	0.517	*
	BETA_NA	0.108	0.007	0.000	0.094	0.123	*
	PHI_UE	0.135	0.026	0.000	0.085	0.185	*
	BETA_UE	0.233	0.043	0.000	0.148	0.319	*
	PSI_NA	-1.154	0.084	0.000	-1.318	-0.989	*
	PSI_UE	0.874	0.050	0.000	0.778	0.968	*
Variances							
C		0.347	0.080	0.000	0.216	0.527	*
Residual Variances							
	NA_PRE	0.127	0.049	0.000	0.034	0.233	*
	UE_PRE	0.225	0.040	0.000	0.162	0.316	*
	PHI_NA	0.040	0.008	0.000	0.028	0.059	*
	BETA_NA	0.001	0.001	0.000	0.000	0.003	*
	PHI_UE	0.023	0.008	0.000	0.010	0.041	*
	BETA_UE	0.028	0.019	0.000	0.002	0.074	*
	PSI_NA	0.310	0.087	0.000	0.160	0.503	*
	PSI_UE	0.257	0.039	0.000	0.194	0.348	*

The BY statements contain the factor loading estimates; the CIs indicate whether there is evidence that the factor loading deviates from zero. Note that, in contrast to having an observed predictor as we had in Model 3, where by default the residuals of the individual means (i.e., the between components  $x_i^{(b)}$  and  $y_i^{(b)}$ ) covaried, here all residuals are by default uncorrelated.

The standardized results are:

# STANDARDIZED MODEL RESULTS

## STDYX Standardization

	Estimate	Posterior S.D.	One-Tailed P-Value	95% C.I.		Significance
Within-Level Standardized Estimates Averaged Over Clusters						
PHI_NA   NA_PRE ON NA_PRE&1	0.473	0.013	0.000	0.446	0.498	*
BETA_NA   NA_PRE ON UE_PRE	0.229	0.011	0.000	0.207	0.251	*
PHI_UE   UE_PRE ON UE_PRE&1	0.135	0.021	0.000	0.095	0.175	*
BETA_UE   UE_PRE ON NA_PRE&1	0.102	0.017	0.000	0.070	0.137	*
PSI_NA   NA_PRE	0.641	0.012	0.000	0.617	0.664	*
PSI_UE   UE_PRE	0.927	0.010	0.000	0.906	0.945	*
Between Level						
C BY						
NA_PRE	0.856	0.062	0.000	0.721	0.964	*
UE_PRE	0.427	0.096	0.000	0.224	0.595	*
C BY						
PHI_NA	0.130	0.121	0.142	-0.105	0.362	
BETA_NA	0.757	0.099	0.000	0.531	0.914	*
PHI_UE	0.179	0.182	0.165	-0.192	0.522	
BETA_UE	-0.488	0.243	0.036	-0.891	0.051	
PSI_NA	0.804	0.062	0.000	0.663	0.902	*
PSI_UE	0.057	0.108	0.305	-0.158	0.261	
Intercepts						
NA_PRE	2.941	0.221	0.000	2.530	3.396	*
UE_PRE	-2.420	0.222	0.000	-2.875	-2.006	*
PHI_NA	2.313	0.264	0.000	1.816	2.855	*
BETA_NA	1.928	0.253	0.000	1.501	2.462	*
PHI_UE	0.867	0.233	0.000	0.501	1.423	*
BETA_UE	1.200	0.542	0.000	0.690	2.595	*
PSI_NA	-1.234	0.123	0.000	-1.491	-1.000	*
PSI_UE	1.709	0.159	0.000	1.400	2.018	*

Variances						
C	1.000	0.000	0.000	1.000	1.000	
Residual Variances						
NA_PRE	0.267	0.104	0.000	0.070	0.479	*
UE_PRE	0.818	0.079	0.000	0.646	0.950	*
PHI_NA	0.982	0.037	0.000	0.868	1.000	*
BETA_NA	0.426	0.143	0.000	0.165	0.718	*
PHI_UE	0.963	0.075	0.000	0.727	1.000	*
BETA_UE	0.761	0.217	0.000	0.207	0.997	*
PSI_NA	0.353	0.097	0.000	0.186	0.559	*
PSI_UE	0.993	0.020	0.000	0.931	1.000	*

The standardized results at the within level are as before: They are based on standardizing the parameters per person first, and then taking the average across all individuals within each iteration in the MCMC algorithm. The standardized results at the between level are based on standardizing parameters with between-person variances within each iteration of the MCMC algorithm. The standardized factor loadings (the BY statements) can be considered also as correlations between the random effects (the indicators) and the latent variable; for four out of eight we see evidence that they differ from zero. The standardized residual variances represent the proportion of unexplained variance in each random effect.

## Multilevel Model 5

In this model, we include an observed predictor  $P_i$  and an observed outcome variable  $O_i$  at the between level, and use the eight random effects to model indirect effects.

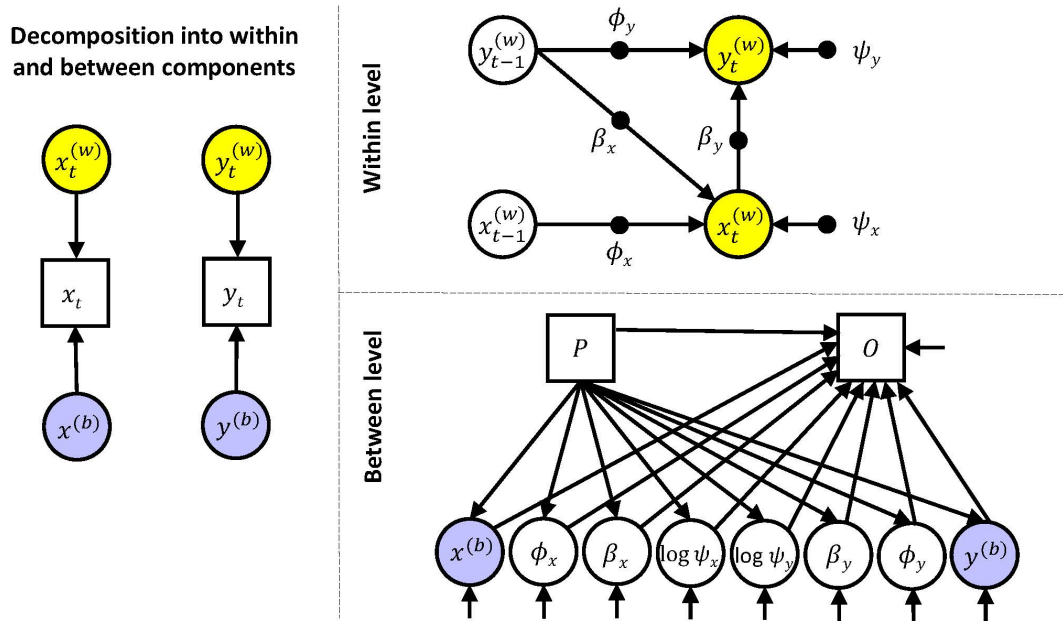


Figure 8:

In this model, the within level is as in Models 2, 3, and 4, while the between level is similar to that of Model 3 but with the additional expression for the distal outcome  $O_i$ , that is:

$$\begin{aligned}
x_i^{(b)} &= \gamma_{x00} + \gamma_{x01}P_i + u_{x0i} \\
y_i^{(b)} &= \gamma_{y00} + \gamma_{y01}P_i + u_{y0i} \\
\phi_{xi} &= \gamma_{x10} + \gamma_{x11}P_i + u_{x1i} \\
\phi_{yi} &= \gamma_{y10} + \gamma_{y11}P_i + u_{y1i} \\
\beta_{xi} &= \gamma_{x20} + \gamma_{x21}P_i + u_{x2i} \\
\beta_{yi} &= \gamma_{y20} + \gamma_{y21}P_i + u_{y2i} \\
\log(\psi_{xi}) &= \gamma_{x30} + \gamma_{x31}P_i + u_{x3i} \\
\log(\psi_{yi}) &= \gamma_{y30} + \gamma_{y31}P_i + u_{y3i} \\
O_i &= \tau_0 + \tau_1P_i + \tau_2x_i^{(b)} + \tau_3y_i^{(b)} + \tau_4\phi_{xi} + \tau_5\phi_{yi} + \tau_6\beta_{xi} + \tau_7\beta_{yi} + \tau_8\log(\psi_{xi}) + \tau_9\log(\psi_{yi}) + \epsilon_i
\end{aligned}$$

## Input MLM5

We now include two time-invariant variables at the between level, that is, depression measured prior to the first episode of ESM measures, and depression afterwards (measured prior to the second episode of ESM measures).

Hence, in the VARIABLE command, we now have

```
USEVAR      =   na_pre ham_pre ham_post ue_pre;
BETWEEN     =   ham_pre ham_post;
```

The MODEL command for the within part remains the same. At the between level, we include the observed outcome variable and regress this on the observed predictor and the eight random effects. We now give labels to the regression parameters that involve the random effects such that we can use these parameters in the subsequent MODEL CONSTRAINT command, that is:

```
%BETWEEN%
! regress all random effects on observed predictor
na_pre ue_pre phi_na-psi_ue ON ham_pre (a1-a8);

! regress observed outcome on all random effects and observed predictor      ham_post ON ham_pre;
ham_post ON na_pre ue_pre phi_na-psi_ue (b1-b8);
```

In the MODEL CONSTRAINT command, we can now compute the indirect effects, by multiplying the two coefficients associated with each random effect:

```
MODEL CONSTRAINT:
  new (i_bc_na); i_bc_na=a1*b1;      ! Indirect effect through between component NA
  new (i_bc_ue); i_bc_ue=a2*b2;      ! Indirect effect through between component UE
  new (i_phi_na); i_phi_na=a3*b3;     ! Indirect effect through autoregression phi_na
  new (i_bet_na); i_bet_na=a4*b4;     ! Indirect effect through cross-regression beta_na
  new (i_phi_ue); i_phi_ue=a5*b5;     ! Indirect effect through autoregression phi_ue
  new (i_bet_ue); i_bet_ue=a6*b6;     ! Indirect effect through cross-regreesion beta_ue
  new (i_psi_na); i_psi_na=a7*b7;     ! Indirect effect through log of the residual variance of NA
  new (i_psi_ue); i_psi_ue=a8*b8;     ! Indirect effect through log of the residaul variance of UE
```

Note that these lines do not actually constrain any parameters in the model; they are merely used to compute new quantities based on the model parameters. This will happen within each iteration of the MCMC algorithm, such that we will get a posterior distribution for each of these indirect effects, which we can use to determine whether there is evidence they differ from zero or not.



## Output MLM5

The output for this analysis is can be found here.

The model results in the following parameter estimates:

### MODEL RESULTS

		Estimate	Posterior S.D.	One-Tailed P-Value	95% C.I. Lower 2.5%    Upper 2.5%		Significance
Within Level							
Between Level							
PHI_NA	ON						
HAM_PRE		-0.014	0.022	0.264	-0.058	0.028	
BETA_NA	ON						
HAM_PRE		0.020	0.007	0.003	0.006	0.033	*
PHI_UE	ON						
HAM_PRE		0.035	0.024	0.070	-0.012	0.082	
BETA_UE	ON						
HAM_PRE		-0.085	0.036	0.011	-0.155	-0.013	*
PSI_NA	ON						
HAM_PRE		0.415	0.077	0.000	0.265	0.567	*
PSI_UE	ON						
HAM_PRE		-0.041	0.049	0.205	-0.137	0.058	
HAM_POST	ON						
PHI_NA		-0.171	0.548	0.372	-1.290	0.873	
BETA_NA		1.865	3.287	0.281	-4.235	8.957	
PHI_UE		-0.076	1.326	0.478	-3.018	2.277	
BETA_UE		2.137	7.085	0.095	-2.100	16.495	
PSI_NA		0.106	0.152	0.240	-0.193	0.405	
PSI_UE		0.007	0.218	0.485	-0.404	0.445	
NA_PRE	ON						
HAM_PRE		0.306	0.057	0.000	0.193	0.416	*
UE_PRE	ON						
HAM_PRE		0.114	0.050	0.014	0.016	0.212	*
HAM_POST	ON						
HAM_PRE		0.437	0.571	0.028	-0.035	1.541	
NA_PRE		0.108	0.195	0.284	-0.263	0.502	
UE_PRE		-0.010	0.226	0.483	-0.448	0.431	
Intercepts							
HAM_POST		-0.800	1.826	0.207	-3.892	1.248	
NA_PRE		2.009	0.059	0.000	1.896	2.128	*

UE_PRE	-1.290	0.052	0.000	-1.390	-1.188	*
PHI_NA	0.476	0.023	0.000	0.431	0.519	*
BETA_NA	0.100	0.007	0.000	0.086	0.114	*
PHI_UE	0.134	0.025	0.000	0.083	0.182	*
BETA_UE	0.236	0.037	0.000	0.164	0.312	*
PSI_NA	-1.169	0.077	0.000	-1.317	-1.020	*
PSI_UE	0.879	0.049	0.000	0.783	0.976	*

#### Residual Variances

HAM_POST	0.779	0.208	0.000	0.245	1.106	*
NA_PRE	0.397	0.060	0.000	0.299	0.530	*
UE_PRE	0.255	0.042	0.000	0.186	0.349	*
PHI_NA	0.041	0.008	0.000	0.028	0.060	*
BETA_NA	0.003	0.001	0.000	0.001	0.005	*
PHI_UE	0.022	0.008	0.000	0.008	0.040	*
BETA_UE	0.021	0.018	0.000	0.001	0.067	*
PSI_NA	0.708	0.099	0.000	0.545	0.936	*
PSI_UE	0.254	0.041	0.000	0.188	0.346	*

We only find evidence of an effect if ham\_pre on the the distal outcome ham\_post; none of the random effects seems to predict ham\_post. When considering the indirect effects we asked Mplus to compute, we find:

#### New/Additional Parameters

I_BC_NA	0.031	0.061	0.284	-0.080	0.160
I_BC_UE	-0.001	0.028	0.482	-0.060	0.059
I_PHI_NA	0.001	0.015	0.430	-0.025	0.039
I_BET_NA	0.033	0.070	0.283	-0.088	0.200
I_PHI_UE	-0.001	0.057	0.476	-0.130	0.107
I_BET_UE	-0.160	0.559	0.105	-1.260	0.187
I_PSI_NA	0.042	0.065	0.240	-0.081	0.177
I_PSI_UE	0.000	0.014	0.500	-0.033	0.029

which also implies we have no evidence that any of the indirect effects differs from zero. However, the trace plots for some of the parameters in this model give rise to concern, as they show rather extreme values are sampled for a number of iterations in the MCMC algorithm, as shown below

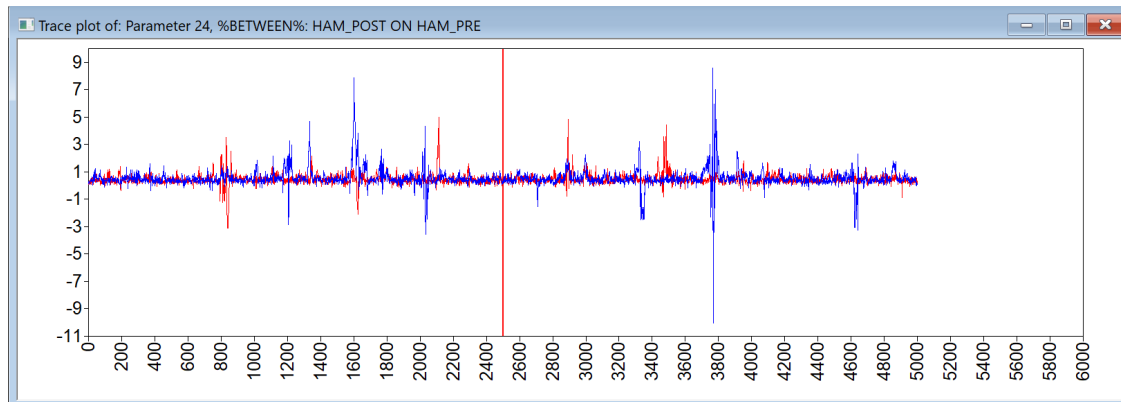


Figure 9:

As explained in the book chapter, this is an indication that the current model is too complex given the data, and we could proceed with either fixing certain parameters to zero, or specifying more informative priors. However, we decided to simplify the model through modeling the indirect effect through a single common

factor of the random effects, rather than through all eight random effects separately.

## Multilevel Model 6

This final dynamic multilevel model combines path analysis and factor analysis at the between level; as such it makes full use of the SEM component of DSEM.

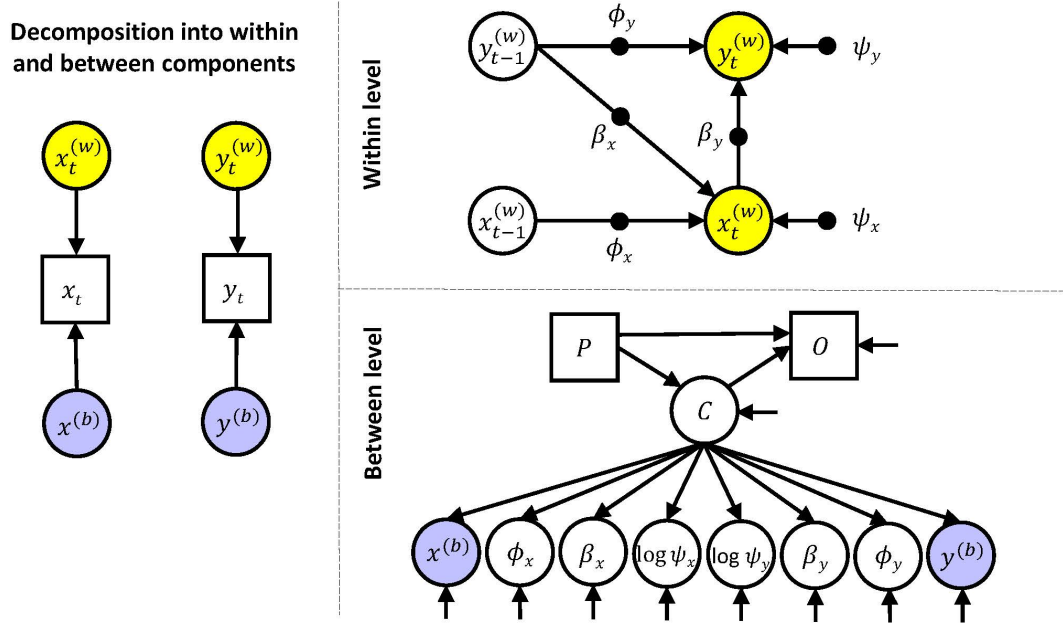


Figure 10:

Hence, this model can be expressed as Model 4, with the addition

$$\begin{aligned} O_i &= \tau_0 + \tau_1 P_i + \tau_2 C_i + \epsilon_{O_i} \\ C_i &= \tau_3 P_i + \epsilon_{C_i} \end{aligned}$$

The effect of  $P_i$  then is the direct effect  $\tau_1$  plus the indirect effect  $\tau_2 * \tau_3$ .

## Input MLM6

At the within level the model is specified as before. At the between level we specify the model as

```
%BETWEEN%
! Specify a common factor for the random effects
c BY na_pre@1 ue_pre phi_na-psi_ue*.1;

! Formulate a path model for the observed predictor
! the observed outcome, and the common factor
c ON ham_pre (s);
ham_post ON c ham_pre (b d);
```

Subsequently, we cause the `MODEL CONSTRAINT` command again to compute the indirect effect as an additional parameter, through

`MODEL CONSTRAINT:`

```
new (indirect); indirect=a*b; ! Indirect effect through common factor c
```

Again, this is not really a constraint on the actual model, but is simply a way to compute a new quantity based on model parameters.

## Output MLM6

The output for this analysis is can be found here.

We obtain the following parameter estimates for this model:

MODEL RESULTS

		Estimate	Posterior S.D.	One-Tailed P-Value	95% C.I. Lower 2.5% Upper 2.5%		Significance
Within Level							
Between Level							
C	BY						
	NA_PRE	1.000	0.000	0.000	1.000	1.000	
	UE_PRE	0.379	0.104	0.000	0.193	0.600	*
C	BY						
	PHI_NA	0.041	0.046	0.185	-0.050	0.132	
	BETA_NA	0.073	0.013	0.000	0.050	0.101	*
	PHI_UE	0.059	0.053	0.116	-0.041	0.170	
	BETA_UE	-0.185	0.092	0.014	-0.395	-0.024	*
	PSI_NA	1.367	0.197	0.000	1.041	1.826	*
	PSI_UE	0.042	0.099	0.334	-0.152	0.246	
C	ON						
	HAM_PRE	0.289	0.054	0.000	0.184	0.399	*
HAM_POST	ON						
C		0.390	0.210	0.030	-0.019	0.803	
HAM_POST	ON						
	HAM_PRE	0.263	0.104	0.008	0.052	0.467	*
Intercepts							
	HAM_POST	0.014	0.086	0.439	-0.155	0.184	
	NA_PRE	2.015	0.058	0.000	1.903	2.133	*
	UE_PRE	-1.281	0.052	0.000	-1.381	-1.175	*
	PHI_NA	0.475	0.023	0.000	0.428	0.517	*
	BETA_NA	0.108	0.007	0.000	0.094	0.122	*
	PHI_UE	0.133	0.024	0.000	0.085	0.179	*
	BETA_UE	0.245	0.044	0.000	0.160	0.334	*
	PSI_NA	-1.169	0.077	0.000	-1.322	-1.017	*
	PSI_UE	0.874	0.050	0.000	0.778	0.972	*

Residual Variances						
HAM_POST	0.907	0.123	0.000	0.707	1.180	*
NA_PRE	0.144	0.046	0.000	0.057	0.238	*
UE_PRE	0.226	0.038	0.000	0.162	0.311	*
C	0.240	0.057	0.000	0.142	0.363	*
PHI_NA	0.041	0.008	0.000	0.029	0.061	*
BETA_NA	0.001	0.001	0.000	0.000	0.003	*
PHI_UE	0.022	0.009	0.000	0.007	0.042	*
BETA_UE	0.027	0.019	0.000	0.002	0.075	*
PSI_NA	0.270	0.075	0.000	0.132	0.429	*
PSI_UE	0.257	0.039	0.000	0.192	0.348	*

New/Additional Parameters					
INDIRECT	0.109	0.063	0.030	-0.006	0.244

It shows that while the common factor can be predicted from the baseline covariate ham\_pre, it does not predict the distal outcome ham\_post. This is also in agreement with the result we find for the indirect path that we computed as an additional parameter, for which there is no evidence that it differs from zero. The distal outcome ham\_post can be predicted from the baseline covariate ham\_pre though (regression coefficient is 0.26).

We can consider the standardized results to see what the relative sizes of parameters are:

#### STANDARDIZED MODEL RESULTS

##### STDYX Standardization

	Estimate	Posterior S.D.	One-Tailed P-Value	95% C.I. Lower 2.5%    Upper 2.5%		Significance
Within-Level Standardized Estimates Averaged Over Clusters						
PHI_NA   NA_PRE ON NA_PRE&1	0.474	0.014	0.000	0.447	0.500	*
BETA_NA   NA_PRE ON UE_PRE	0.229	0.011	0.000	0.209	0.250	*
PHI_UE   UE_PRE ON UE_PRE&1	0.134	0.020	0.000	0.093	0.171	*
BETA_UE   UE_PRE ON NA_PRE&1	0.105	0.016	0.000	0.073	0.138	*
PSI_NA   NA_PRE	0.640	0.013	0.000	0.615	0.663	*
PSI_UE   UE PRE	0.927	0.010	0.000	0.907	0.946	*

##### Between Level

C BY

NA_PRE	0.813	0.064	0.000	0.673	0.928	*
UE_PRE	0.392	0.094	0.000	0.203	0.555	*
C	BY					
PHI_NA	0.104	0.115	0.185	-0.125	0.327	
BETA_NA	0.736	0.105	0.000	0.517	0.930	*
PHI_UE	0.211	0.180	0.116	-0.156	0.557	
BETA_UE	-0.508	0.224	0.014	-0.940	-0.068	*
PSI_NA	0.813	0.056	0.000	0.694	0.912	*
PSI_UE	0.043	0.101	0.334	-0.158	0.235	
C	ON					
HAM_PRE	0.386	0.063	0.000	0.255	0.500	*
HAM_POST	ON					
C	0.203	0.105	0.030	-0.010	0.396	
HAM_POST	ON					
HAM_PRE	0.184	0.072	0.008	0.036	0.323	*
Intercepts						
HAM_POST	0.014	0.084	0.439	-0.153	0.178	
NA_PRE	3.084	0.222	0.000	2.657	3.546	*
UE_PRE	-2.468	0.218	0.000	-2.917	-2.062	*
PHI_NA	2.302	0.263	0.000	1.819	2.857	*
BETA_NA	2.013	0.284	0.000	1.550	2.663	*
PHI_UE	0.866	0.303	0.000	0.481	1.592	*
BETA_UE	1.254	0.392	0.000	0.706	2.262	*
PSI_NA	-1.308	0.124	0.000	-1.559	-1.072	*
PSI_UE	1.712	0.163	0.000	1.403	2.049	*
Residual Variances						
HAM_POST	0.890	0.045	0.000	0.785	0.962	*
NA_PRE	0.338	0.103	0.000	0.139	0.547	*
UE_PRE	0.846	0.072	0.000	0.691	0.959	*
C	0.851	0.048	0.000	0.750	0.934	*
PHI_NA	0.987	0.030	0.000	0.893	1.000	*
BETA_NA	0.459	0.152	0.000	0.134	0.733	*
PHI_UE	0.954	0.086	0.000	0.681	1.000	*
BETA_UE	0.741	0.230	0.000	0.116	0.993	*
PSI_NA	0.339	0.089	0.000	0.168	0.518	*
PSI_UE	0.995	0.016	0.000	0.943	1.000	*

R-SQUARE

Within-Level R-Square Averaged Across Clusters

Variable	Estimate	Posterior	One-Tailed	95% C.I.	
		S.D.	P-Value	Lower 2.5%	Upper 2.5%
NA_PRE	0.360	0.013	0.000	0.337	0.385
UE_PRE	0.073	0.010	0.000	0.054	0.093

#### Between Level

Variable	Estimate	Posterior	One-Tailed	95% C.I.	
		S.D.	P-Value	Lower 2.5%	Upper 2.5%
HAM_POST	0.110	0.045	0.000	0.038	0.215
NA_PRE	0.662	0.103	0.000	0.452	0.860
UE_PRE	0.154	0.072	0.000	0.041	0.308

Variable	Estimate	Posterior	One-Tailed	95% C.I.	
		S.D.	P-Value	Lower 2.5%	Upper 2.5%
C	0.149	0.048	0.000	0.065	0.250
PHI_NA	0.013	0.030	0.000	0.000	0.107
BETA_NA	0.541	0.152	0.000	0.267	0.866
PHI_UE	0.046	0.086	0.000	0.000	0.314
BETA_UE	0.259	0.230	0.000	0.007	0.881
PSI_NA	0.661	0.089	0.000	0.482	0.832
PSI_UE	0.005	0.016	0.000	0.000	0.057

Here we specifically consider the  $R^2$ 's of the variables, as it indicates the proportion of explained variance. At the within level we see that averaged across participants, 36% of the variance in negative affect can be explained by negative affect one hour earlier, and by the unpleasantness of events since the previous beep. In contrast, only 7% of the variance in unpleasantness of events can be predicted from unpleasantness of events in the preceding interval, and from negative affect one hour earlier.

At the between level, each of the dependent variables has only one predictor, which implies that the proportions of explained variance directly indicate how much variance is explained by that predictor. It shows that ham\_pre explains almost 15% variance in the common factor or the random effects c, and 11% in the distal outcome ham\_post. Furthermore, we see that the common factor c explains more than half of the individual differences in

- the mean of negative affect (i.e., 66.2% in  $y_i^{(b)}$ )
- the sensitivity in negative affect to unpleasantness of events as captured by the random slope from events to affect (i.e., 54.1% in  $\beta_{yi}$ )
- the sensitivity in negative affect to additional, unmeasured factors, as captured by the log residual variance of negative affect (i.e., 66.1% in  $\log(\psi_{yi})$ )

The other random effect that are partly explained by the common factor are:

- the mean of unpleasantness of events (i.e., 15.4% in  $x_i^{(b)}$ )
- the spill-over of negative affect into unpleasantness of events (i.e., 25.9% in  $\beta_{xi}$ )

Note that as with variances, the  $R^2$ 's have CIs that never include zero. Hence, to determine whether there is evidence that a predictor really explains variance in the outcome, we should consider the CIs of the regression parameters (i.e., factor loadings and/or structural parameters).