# Assignment3_EH

October 23, 2018

## 1 Assignment 3

#### 1.0.1 MACSS 3000, Dr. Evans

#### 1.0.2 Ellen Hsieh

```
In [1]: import numpy as np
        import matplotlib.pyplot as plt
        from matplotlib.ticker import MultipleLocator
```

#### 1.0.3 2. Simulating your income

**(a) Simulate 10,000 different realizations of my lifetime income**

$$\ln(inc_{2020}) = \ln(inc_0) + \ln(\varepsilon_{2020})$$
$$\ln(inc_t) = (1-\rho)[\ln(inc_0) + g(t-2020)] + \rho\ln(inc_{t-1}) + \ln(\varepsilon_t) \;\; \forall 2021 \leq t \leq 2059$$

```
In [2]: # create a simulation model for MACSS students lifetime income
        def macss_income_sim(p):

            """
            Requires a simulation profile, p, structured as a dictionary

            p = {
                'inc'          : 80000,       #starting income
                'rho'          : 0.4,         #dependence of todays income on last periods incom
                'mu'           : 0,           #mean
                'sd'           : 0.13,        #standard deviation
                'gr'           : 0.025,       #growth rate
                'st_year'      : int(2020),   #start year
                'wr_years'     : 40,          #years to work
                'num_draws'    : 10000        #simulations
            }
            """

            #set random seed
            np.random.seed(524)
```

```
        inc_errors = np.random.normal(p['mu'], p['sd'] , (p['wr_years'], p['num_draws']))

        #create a matrix of dim (wr_years, num_draws)
        ln_macss_inc = np.zeros((p['wr_years'], p['num_draws']))

        #fill the matrix
        ln_macss_inc[0, :] = np.log(p['inc']) + inc_errors[0, :]

        #loop and apply model
        for yr in range(1, p['wr_years']):
            ln_macss_inc[yr, :] = ((1- p['rho']) * (np.log(p['inc']) + p['gr']* yr) +
                                            p['rho'] * ln_macss_inc[yr - 1, :] + inc

        macss_inc = np.exp(ln_macss_inc)
        return macss_inc
```

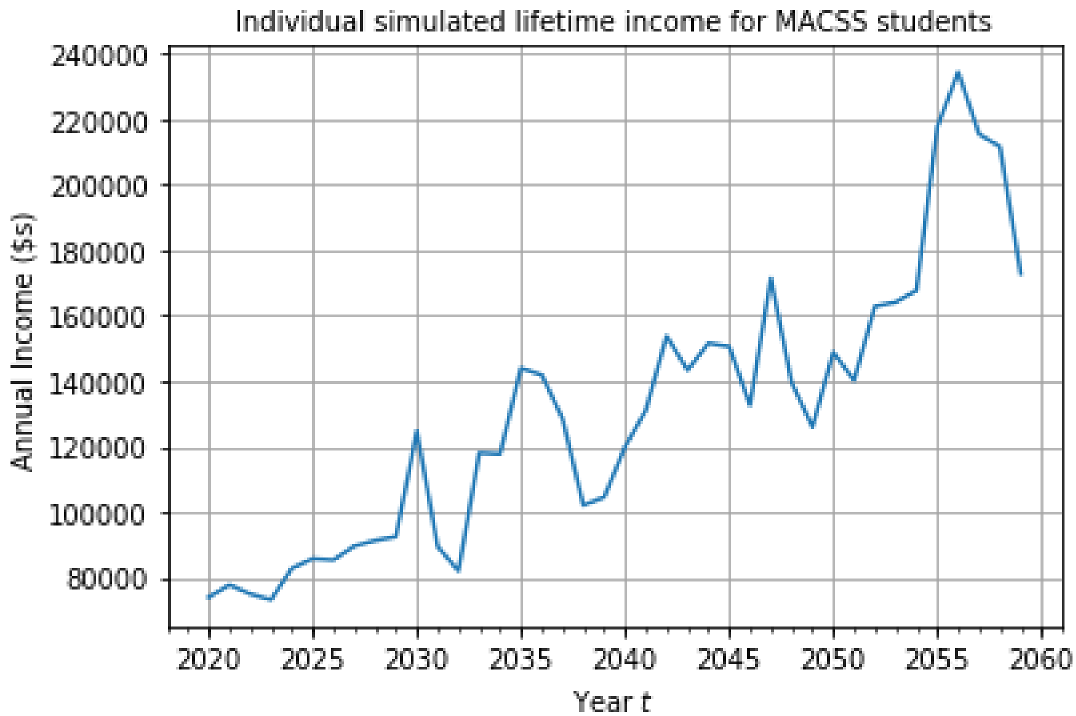
In [3]:
```
# simulate the lifetime income with initial income = 80000, mu = 0, sd = 0.13, g = 0.0
simulation_profile = {
    'inc'         : 80000,
    'rho'         : 0.4,
    'mu'          : 0,
    'sd'          : 0.13,
    'gr'          : 0.025,
    'st_year'     : int(2020),
    'wr_years'    : 40,
    'num_draws'   : 10000
    }

macss_inc = macss_income_sim(simulation_profile)
print(macss_inc)
```

```
[[ 66409.15585396  98274.13534194 101939.81109509 ...  98720.39690442
   72404.51636886  68710.32820307]
 [ 80020.53020329  67383.19350738  84557.85626308 ...  68247.7770509
   74518.33613244  80555.96068584]
 [ 75805.26636606  66134.42494243  91458.20304692 ...  67268.53350159
   90012.42673528  80645.62355527]
 ...
 [272690.56519108 217821.73027242 184724.24512469 ... 159922.45424852
  253961.68337673 209741.55004062]
 [231539.17420799 202509.15149494 197955.96626493 ... 199502.43481758
  210951.71828579 205420.27946389]
 [197895.95201384 165115.10025278 172644.86927513 ... 248654.44847819
  234237.14656466 221566.29879732]]
```

In [4]:
```
# plot the lifetime income path of individual 214
%matplotlib inline
```

```python
p = simulation_profile
year_vec = np.arange(p['st_year'], p['st_year'] + p['wr_years'])
individual = 214
fig, ax = plt.subplots()
plt.plot(year_vec, macss_inc[:, individual])
minorLocator = MultipleLocator(1)
ax.xaxis.set_minor_locator(minorLocator)
plt.grid(b=True, which='major', color='0.65', linestyle='-')
plt.title('Individual simulated lifetime income for MACSS students', fontsize=10)
plt.xlabel(r'Year $t$')
plt.ylabel(r'Annual Income (\$s)')
```

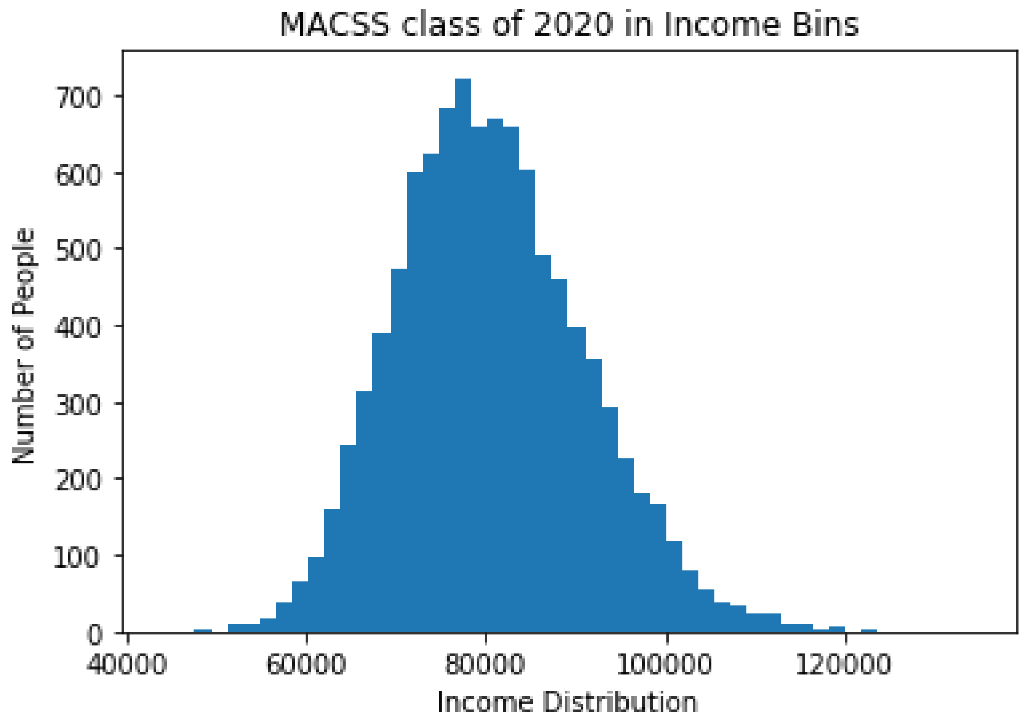
Out[4]: Text(0,0.5,'Annual Income (\\$s)')




**(b) Plot a histogram with 50 bins of year t = 2020 initial income for each of the 10,000 simulations**

In [5]:
```python
# plot the histogram
plt.hist(macss_inc[0,:], bins=50)
plt.xlabel("Income Distribution")
plt.ylabel("Number of People")
plt.title("MACSS class of 2020 in Income Bins")
```

Out[5]: Text(0.5,1,'MACSS class of 2020 in Income Bins')




This distriction is normally distributed but a little bit skewed to the right.

```
In [6]: income = macss_inc[0, :]
        len(income [income > 100000]) / len(income)
```

```
Out[6]: 0.0417
```

4.17% of the class will earn more than $100,000 in the first year.

```
In [7]: income = macss_inc[0, :]
        len(income [income < 70000]) / len(income)
```

```
Out[7]: 0.1512
```

15.12% of class will earn less than 70,000 in the first year.

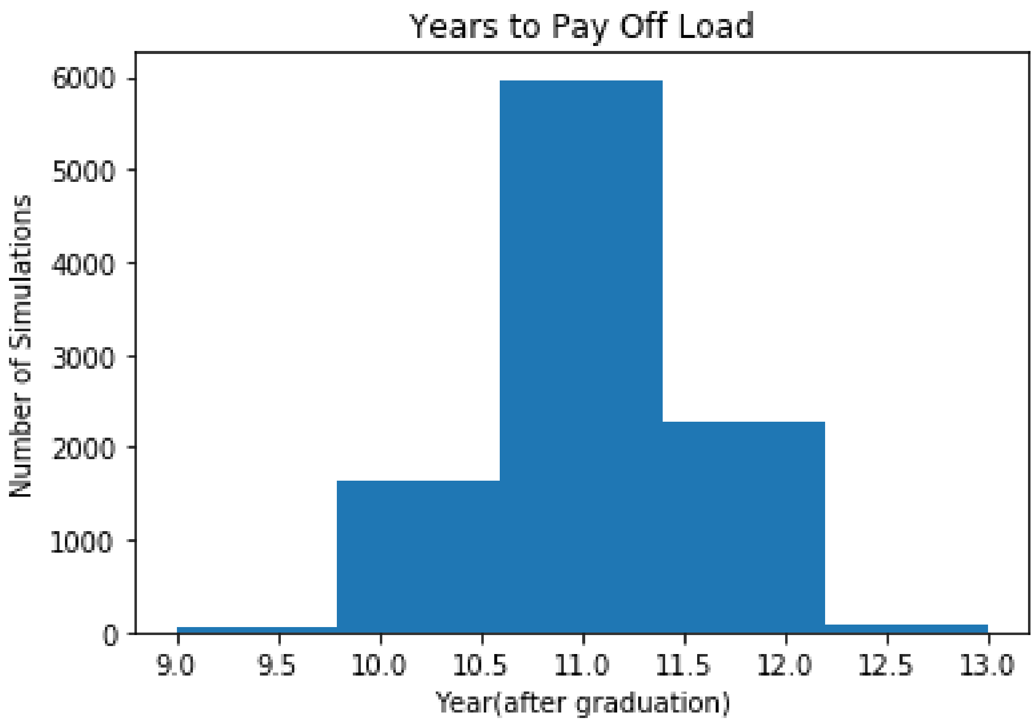
**(c) Plot the histogram of how many years it takes to pay the loan in each of your 10,000 simulations**

```
In [8]: # calculate the number of years to pay off the loan
        loan = 0.1 * macss_inc

        year_to_payoff = []
        for i in range(10000):
            paid=loan[:,i][0]
```

```
        for j in range(1,40):
            if paid < 95000:
                paid = paid + loan[:,i][j]
            else:
                year_to_payoff.append(j)
                break
```

```
In [9]: # plot the histogram
        plt.hist(year_to_payoff, bins=5)
        plt.xlabel("Year(after graduation)")
        plt.ylabel("Number of Simulations")
        plt.title("Years to Pay Off Load")
```

```
Out[9]: Text(0.5,1,'Years to Pay Off Load')
```




```
In [10]: # calculate the percentage of the simulations which can pay off the loan in 10 years
         perc_before_10y = 0

         for i in range(10000):
             if year_to_payoff[i] <= 10:
                 perc_before_10y += 1

         perc_before_10y/10000
```

```
Out[10]: 0.1678
```

16.78% of the simulations are able to pay off the loan in 10 years.

**(d) Plot the new histogram of how many years it takes to pay your loan of according to new income and standard deviation**

```
In [11]: # new prifile to simulate
         new_simulation_profile = {
             'inc'           : 90000,
             'rho'           : 0.4,
             'mu'            : 0,
             'sd'            : 0.17,
             'gr'            : 0.025,
             'st_year'       : int(2020),
             'wr_years'      : 40,
             'num_draws'     : 10000
             }

         new_macss_inc = macss_income_sim(new_simulation_profile)
         print(new_macss_inc)
```

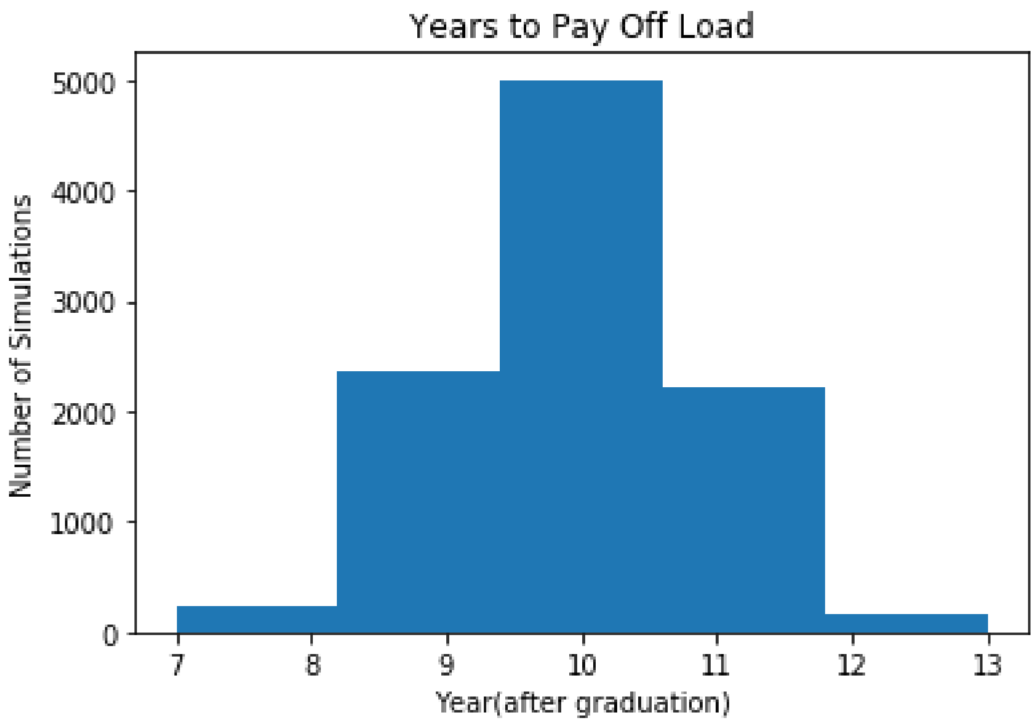
```
[[ 70550.46142451 117783.33011091 123561.20729139 ... 118483.24080508
   78992.81966812  73764.25171169]
 [ 89615.63768821  71575.56495871  96317.75493523 ...  72778.88084775
   81644.3347736   90400.57899801]
 [ 82955.30101689  69396.06916251 106035.55593099 ...  70956.3661129
  103848.93176006  89949.09077038]
 ...
 [338309.11761165 252187.52025149 203293.03644369 ... 168361.21927259
  308250.29858492 240024.49205936]
 [271061.07048342 227502.32436192 220836.5697397  ... 223095.32811759
  239983.96514044 231788.44418303]
 [219057.46748997 172865.33333479 183245.71710131 ... 295275.8618388
  273090.00167035 253934.86273481]]
```

```
In [12]: # calculate the new number of years to pay off the loan
         loan = 0.1 * new_macss_inc

         new_year_to_payoff = []
         for i in range(10000):
             paid=loan[:,i][0]
             for j in range(1,40):
                 if paid < 95000:
                     paid = paid + loan[:,i][j]
                 else:
                     new_year_to_payoff.append(j)
                     break
```

```
In [13]: # plot the new histogram
         plt.hist(new_year_to_payoff, bins=5)
         plt.xlabel("Year(after graduation)")
```

```
plt.ylabel("Number of Simulations")
plt.title("Years to Pay Off Load")
```

Out[13]: Text(0.5,1,'Years to Pay Off Load')




In [14]:
```
# new percentage of simulations which can pay off the loan in 10 years
new_perc_before_10y = 0

for i in range(10000):
    if new_year_to_payoff[i] <= 10:
        new_perc_before_10y += 1

new_perc_before_10y/10000
```

Out[14]: 0.7602

76.02% of the simulations are able to pay off the loan in 10 years.