Bayesian Regression

Priors and Pooling - Revisited with Last Weeks Data

Introduction

The objective is to examine the impact of pooling on a Bayesian regression models. We will develop 2 models: first with no pooling and next with partial pooling, followed by a discussion of results.

The data comes from the *Automobile Price Prediction.csv* dataset *(which you should be familiar with)*. The data are nested by model, and because our purpose is to examine the effect of pooling, we will only use 2 variables:

- model *(the grouping variable)*
- horsepower *(the indepedent variable)*

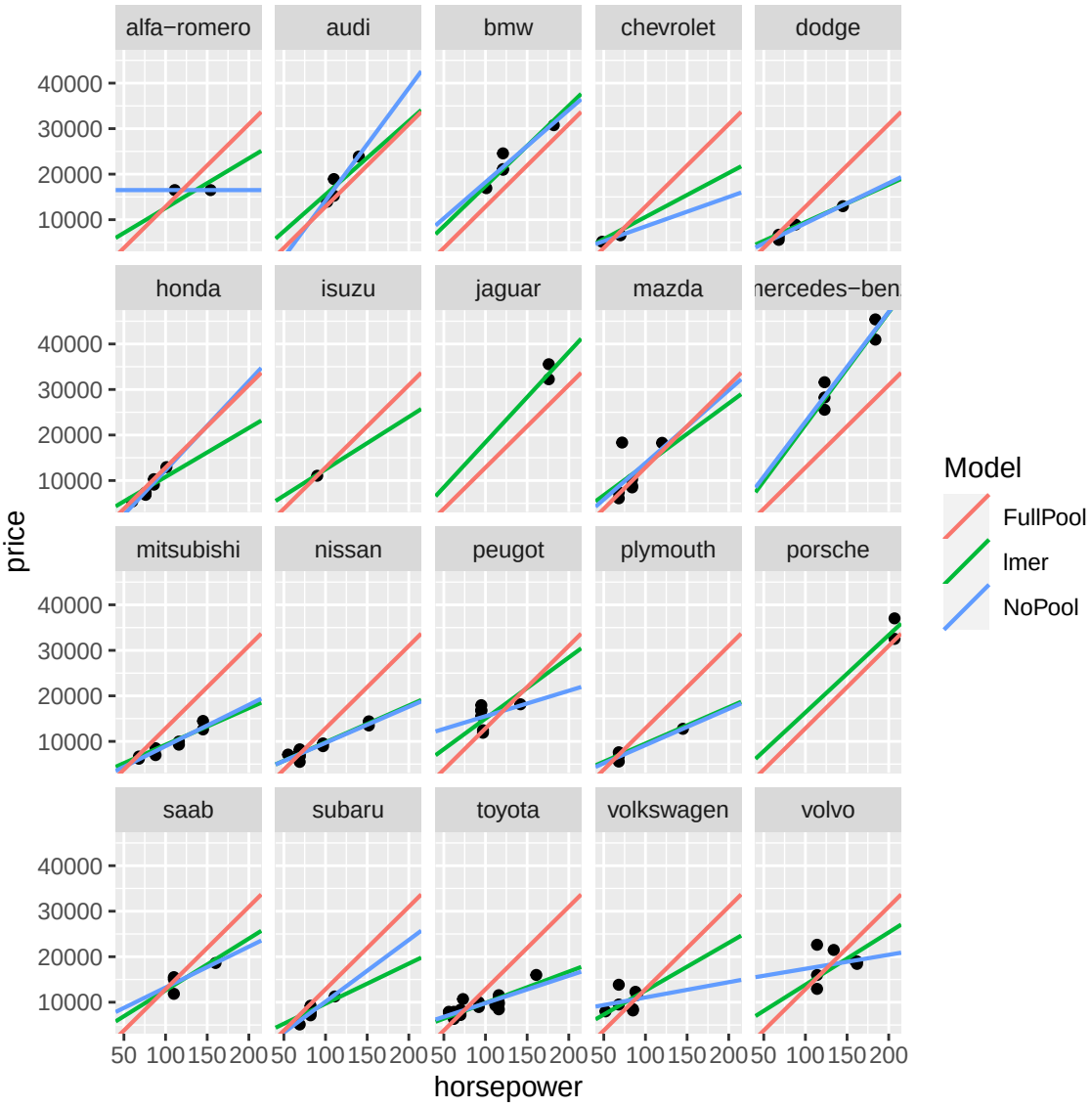
Full Pooling, No Pooling and Partial Pooling (using lmer)

The following uses lm to generate a fully pooled model, lmlist to generate no pooled models, and lmer to generate partially pooled models:

```
library(tidyverse)
library(rstan)
library(shinystan)

rmse <- function(error)
{
  sqrt(mean(error^2))
}

# ------------------------ build mixed model - No Pooling -------------------- #

set.seed(103)

Autos <- read.csv("C:/Users/ellen/Documents/UH/Fall 2020/Class Materials/Section II/Pooling/Automobile
Autos <- rowid_to_column(Autos, var="SampleID") # this creates a primary key (you have to be careful wi
by_MakeStyle <- Autos %>% group_by(make) %>% dplyr::mutate(cnt = n()) %>% filter(cnt > 1)
train <- sample_frac(by_MakeStyle, .6) %>% ungroup()
train$make <- factor(train$make)
train$makeID <- as.integer(train$make)
test <- anti_join(by_MakeStyle, train, by = "SampleID")


map <- unique(data.frame(val = train$make, makeID = as.integer(train$make)))

# convert make to Factor to work with Stan

library(lme4)
lmerMod <- lmer(price ~ 1 + horsepower + (1 + horsepower | makeID), train)
lmerCoef <- coef(lmerMod)$`make`
p_a <- lmerCoef[,1]
```

```r
p_b <- lmerCoef[,2]


NoPoolCoef = lmList(price ~ horsepower | make, data = train, RMEL = FALSE) %>%
  coef() %>%
  rownames_to_column("Description")


NPCoef <- data.frame(Model = "NoPool",
                     make = NoPoolCoef$Description,
                     Intercept = NoPoolCoef$`(Intercept)`,
                     Slope = NoPoolCoef$horsepower)

lmerCoef1 = data.frame(coef(lmerMod)$make) %>%
  rownames_to_column("Make")

BayesMapLM <- data.frame(Model = "lmer",
                         make = unique(train$make),
                         Intercept = lmerCoef$`(Intercept)`,
                         Slope = lmerCoef$horsepower)

FPCoef = coef(lm(price ~ horsepower, data = train))

FPMap = data.frame(Model = "FullPool",
                         make = unique(train$make),
                         Intercept = FPCoef[1],
                         Slope = FPCoef[2])


BayesMap <- rbind(BayesMapLM, NPCoef, FPMap)
```

```r
p <- ggplot(data = train) +
  aes(x = horsepower, y = price) +
  geom_point() +
  geom_abline(data = BayesMap, aes(intercept = Intercept, slope = Slope, color = Model),
              size = .75) +
  facet_wrap("make")
p
```



Notice how many groups have no model for NoPool. Why?

Model 2 - Partial Pooling

Now we'll use a Bayesian Model to tweak partial pooling

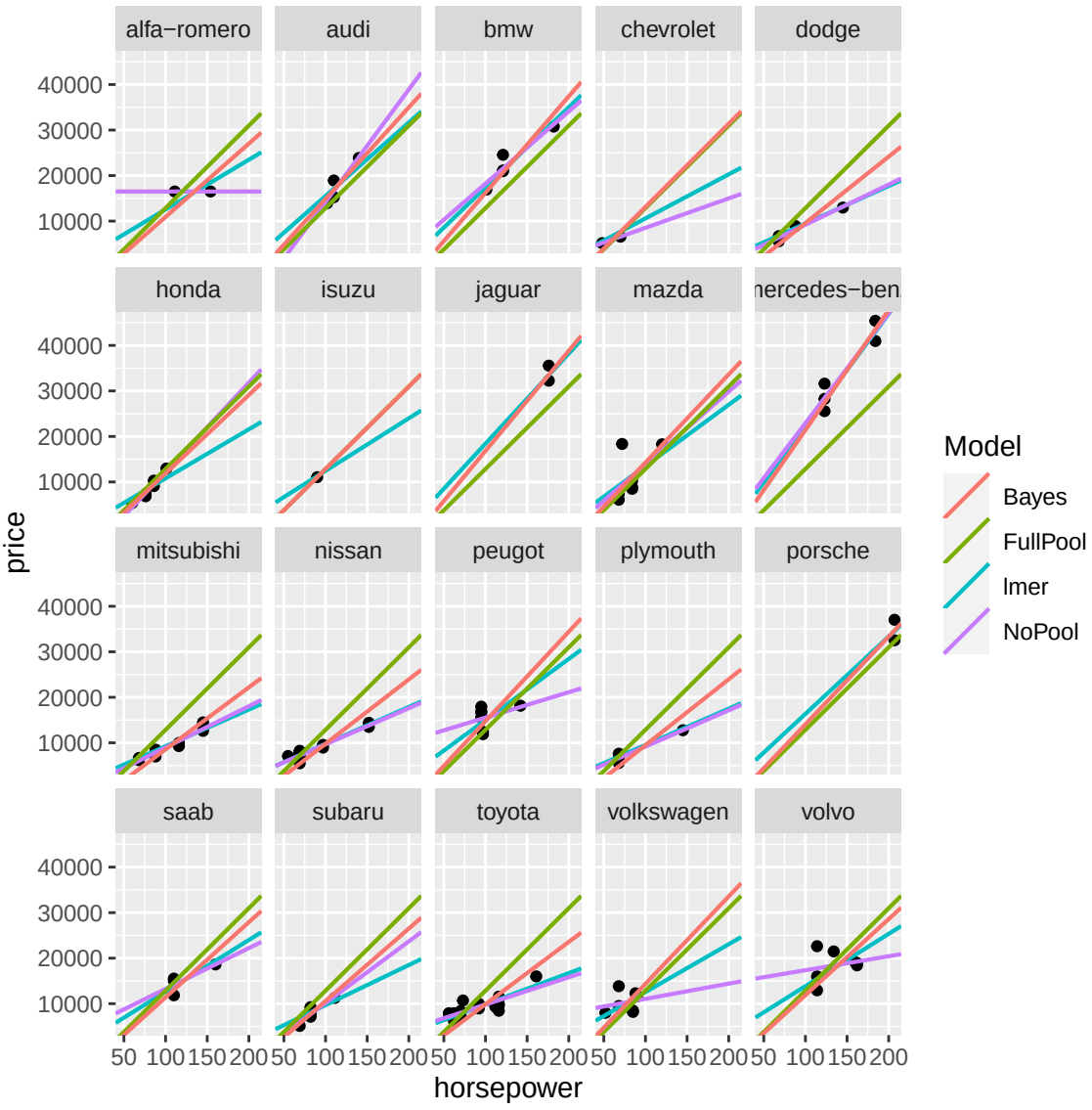
First lets use

```
p_aBU <- p_a
p_bBU <- p_b
#p_a <- rep(mean(p_a),length(p_a))
#p_b <- rep(mean(p_b),length(p_b))

p_a <- rep(FPCoef[1],length(p_a))
p_b <- rep(FPCoef[2],length(p_b))


stanModel2 <- '
```

```
data {
int<lower=0> N;
int<lower=0> J;
vector[N] y;
real x[N];
int make[N];
real p_a[J];
real p_b[J];
real p_aSigma;
real p_bSigma;

}
parameters {
real<lower = 0> sigma;
vector[J] a;
vector[J] b;
}
transformed parameters {
vector[N] y_hat;
for (i in 1:N)
y_hat[i] = a[make[i]] + b[make[i]] * x[i];
}
model {
target += normal_lpdf(y | y_hat,  sigma);
target += normal_lpdf(a | p_a, p_aSigma);
target += normal_lpdf(b | p_b, p_bSigma);
}
'

stanData <- list(
  N=nrow(train),
  J=length(unique(train$make)),
  y=train$price,
  x=train$horsepower,
  make=train$makeID,
  p_a = p_a,
  p_b = p_b,
  p_aSigma = 1000,
  p_bSigma = 50
)



fit2 <- stan(model_code = stanModel2, data = stanData, refresh = 0)

sumFit2 <- data.frame(summary(fit2))

# build on this
Intercept2 <- summary(fit2, pars = c("a"), probs = c(0.1, 0.9))$summary
Slope2 <- summary(fit2, pars = c("b"), probs = c(0.1, 0.9))$summary

BayesMap2 <- data.frame(Model = "Bayes",
                        make = unique(train$make),
```

```
                          Intercept = Intercept2[,1],
                          Slope = Slope2[,1])

BayesMap <- rbind(BayesMap, BayesMap2)

#BayesMap = filter(BayesMap, Model != "Bayes")
```

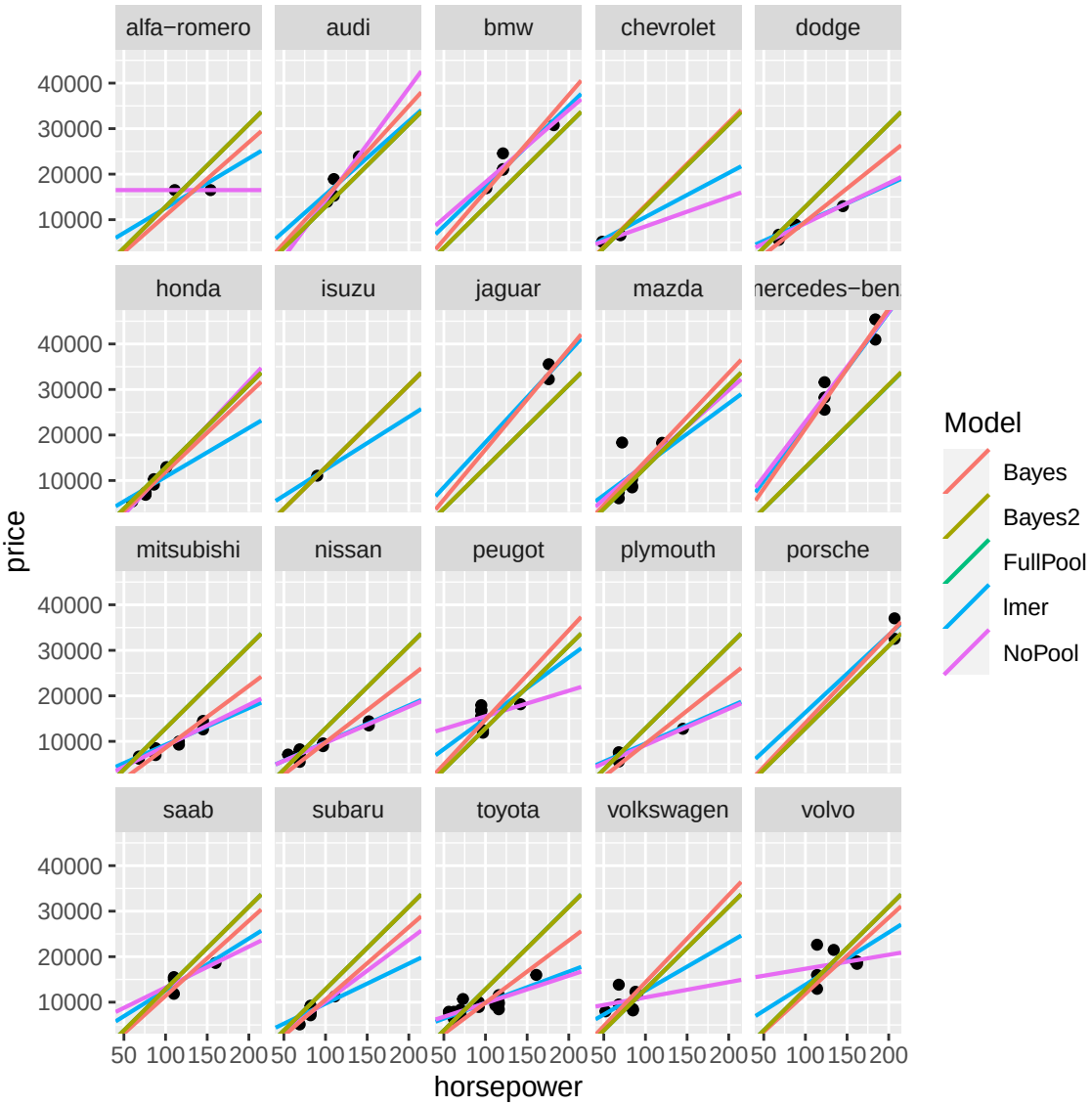
```
p <- ggplot(data = train) +
  aes(x = horsepower, y = price) +
  geom_point() +
  geom_abline(data = BayesMap, aes(intercept = Intercept, slope = Slope, color = Model),
              size = .75) +
  facet_wrap("make")
p
```



Now, let's tighten up the priors a bit and see what that does to pooling:

```r
stanData <- list(
  N=nrow(train),
  J=length(unique(train$make)),
  y=train$price,
  x=train$horsepower,
  make=train$makeID,
  p_a = p_a,
  p_b = p_b,
  p_aSigma = 10,
  p_bSigma = 1
)



fit3 <- stan(model_code = stanModel2, data = stanData, refresh = 0)

sumFit3 <- data.frame(summary(fit3))

# build on this
Intercept3 <- summary(fit3, pars = c("a"), probs = c(0.1, 0.9))$summary
Slope3 <- summary(fit3, pars = c("b"), probs = c(0.1, 0.9))$summary


BayesMap3 <- data.frame(Model = "Bayes2",
                        make = unique(train$make),
                        Intercept = Intercept3[,1],
                        Slope = Slope3[,1])

BayesMap <- rbind(BayesMap, BayesMap3)
```

```r
p <- ggplot(data = train) +
  aes(x = horsepower, y = price) +
  geom_point() +
  geom_abline(data = BayesMap, aes(intercept = Intercept, slope = Slope, color = Model),
              size = .75) +
  facet_wrap("make")
p
```



Notice how the p-pool regression lines varies less across models *(notice how the slopes are more consistent)*. Also notice how, in test data, the groups with less data has less impact on the partailly pooled model.

Analysis

There are many possible combinations of pooling and with most data, even a simple dataset like this one. The usage of Bayesian priors gives us great flexiblity in controlling the effect of pools *(note that we can set a prior mean AND variance for EACH grouping)*. To restate a few of the advantages:

- Crossed effects let us differentiate pricing between models *(a shopper expecting to buy a Mercedes based on an average of all models is going to be very disappointed)*. So we have the ability to target expected values.
- Partial pooling lets us tune effects for each group - data tends to normalize inter-group, as well in intra-group and inter-group. In many cases, neither no-pooling nor complete pooling will be a good approach.
- Partial pooling lets us create predictions for groups that have little data *(a no pooled model will fail if there are few data points)*

- Generalization. Using nested models with priors gives us the ability to generalize models in a very targeted way - by group, by paramter. This level of control is just not possible with any other approach to modeling.