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Mar 18, 2025

Hashmaps Assignment

1) **Recurrence:**

We are given the recurrence relation:

T(n)=3T(n/4)+4n

We can then expand it by repeated substitution.

Substitute T(n/4):

T(n/4) = 3T(n/16)+4(n/4)

T(n)=3(3T(n/16)+4(n/4))+4n

Now we will expand it:

=9T(n/16)+3⋅4(n/4)+4n

=9T(n/16)+12(n/4)+4n

=9T(n/16)+3n+4n

=9T(n/16)+7n

We will then substitute T(n/16):

T(n/16)=3T(n/64)+4(n/16)

T(n)=9(3T(n/64)+4(n/16))+7n

=27T(n/64)+9⋅4(n/16)+7n

=27T(n/64)+9n/4+7n

From the expansions, we can observe the pattern:

$$T(n) = 3^k T(n/4^k) + \sum_{i=0}^{k-1} 4n(3^i/4^i)$$

The summation term simplifies to a geometric series:

$$S_k = \sum_{i=0}^{k-1} 4n(3/4)^i$$

The sum of a geometric series with ratio r = 3/4:

$$S_k = 4n\,(1 - (3/4)^k / (1 - (3/4)) = 16n(1 - (3/4)^k)$$

Now, when setting the base case T(1)=Θ(1), we will solve for k when $n/4^k$= 1:

$$n = 4^k$$

$$k = log_4 n$$

Thus, substituting k:

$$T(n) = 3^{log_4 n} T(1) + 16n(1 - (3/4)^{log_4 n})$$

Approximating:

$$3^{log_4 n} = n^{log_4 3}$$

Since $(3/4)^{log_4 n}$ approaches 0 as n gets larger, we get:

$$T(n) = \theta(n^{log_4 3}) + 16n$$

Since $n^{log_4 3}$ grows faster than 16n, the final result is:

$$T(n) = \theta(n^{log_4 3})$$

For solving with the master theorem,

The recurrence has the form:

T(n)=aT(n/b)+f(n)

where:

- a=3,

- b=4,
- f(n)=4n

We will then calculate for:

$log_b a = log_4 3$

Comparing f(n) and $n^{log_4 3}$

- $f(n) = 4n = \theta(n^1)$
- $log_4 3 \approx 0.792$

Since f(n)=$\theta(n^1)$ and 1 > $log_4 3$, this falls under **Case 2** of the Master Theorem, where f(n) dominates.

We can then check the regularity condition:
f(n)=4n is polynomially larger than $n^{log_4 3}$, so the final bound is determined by f(n), giving:

T(n)=Θ(n)

In conclusion, our answers:

- **By repeated substitution:** T(n)=Θ($n^{log_4 3}$).
- **By the Master Theorem:** T(n)=Θ(n).
  Since the Master Theorem gives the tighter bound, we can conclude:

T(n)=Θ(n)

2) <u>Master Theorem:</u>

a) $T(n) = 3T(n/5) + n^2$
   i) A = 3, b = 5, f(n) = n^2
   ii) We can then compute $log_5 3 \approx 0.68$
   iii) Compare with $f(n) = n^2$: since 2 > 0.68, f(n) dominates.
   iv) After checking the regularity condition: f(n) is polynomially larger, so Case 3 applies.
   v) Solution: T(n) = $\theta(n^2)$

b) $T(n) = 4T(n/3) + 7n$
   i) a = 4, b = 3, f(n) = 7n

ii) Compute $log_3 4 \approx 1.26$

iii) Compare with f(n) = O(n): since 1.26 > 1, O($n^{log_b a}$) dominates.

iv) Apply Case 1.

v) Solution: T(n) = $\theta(n^{log_3 4}) = \theta(n^{1.26})$

c) T(n) = 5T(n/4)+10

i) a = 5, b = 4, f(n) = O(1)

ii) Compute $log_4 5 \approx 1.16$

iii) Compare with f(n): Since O(1) grows slower than O($n^{1.16}$), apply Case 1.

iv) Solution T(n) = $\theta(n^{log_4 5})$=$\theta(n^{1.16})$

d) T(n) = 9T(n/3) + n^4

i) a = 9, b = 3, f(n) = n^4

ii) Compute $log_3 9$ = 2

iii) Compare with f(n) = n^4: since 4 > 2, f(n) dominates.

iv) After checking the regularity condition: f(n) is polynomially larger, so apply Case 3.

v) Solution: T(n) = $\theta(n^4)$

e) T(n) = 6T(n/8)+ n^3

i) a = 6, b = 8, f(n) = n^3

ii) Compute $log_8 6 \approx 0.87$

iii) Compare with f(n) = n^3: since 3 > 0.87, f(n) dominates.

iv) After checking the regularity condition: f(n) is polynomially larger, so apply Case 3.

v) Solution: T(n) = $\theta(n^3)$

3) <u>Radix Sort:</u>

Given List:
CAP, COL, USD, SUN, JPY, VEE, ROW, JOB, COX, LOL, RAT, WOW, DOD, CAR, FIG, PIG, VIS, LOW, LOX, VEA, CAD, DOG, TSL

- Step 1: Sort by the last letter.

Sort based on the third character:
A → CAD, VEA
B → JOB
D → DOD, USD, DOG
G → FIG, PIG

L → COL, LOL, TSL
N → SUN
O → COX, LOX
P → CAP
R → CAR
S → VIS
T → RAT
W → ROW, LOW, WOW
Y → JPY

Sorted by last letter:
CAD, VEA, JOB, DOD, USD, DOG, FIG, PIG, COL, LOL, TSL, SUN, COX, LOX, CAP, CAR, VIS, RAT, ROW, LOW, WOW, JPY

- Step 2: Sort by the second letter (middle)

Sort based on the second character:
A → CAD, CAP, CAR, RAT
O → COL, COX, JOB, LOL, LOW, LOX, ROW, WOW
I → FIG, PIG, VIS
U → SUN, USD
E → VEA, VEE
S → TSL
P → JPY
D → DOD, DOG

Sorted by second letter:
CAD, CAP, CAR, RAT, COL, COX, JOB, LOL, LOW, LOX, ROW, WOW, FIG, PIG, VIS, SUN, USD, VEA, VEE, TSL, JPY, DOD, DOG

- Step 3: Sort by the first letter (leftmost)

Sort based on the first character:
C → CAD, CAP, CAR, COL, COX
D → DOD, DOG
F → FIG
J → JOB, JPY
L → LOL, LOW, LOX
P → PIG
R → RAT, ROW
S → SUN
T → TSL
U → USD
V → VEA, VEE, VIS
W → WOW

Final Sorted Order: CAD, CAP, CAR, COL, COX, DOD, DOG, FIG, JOB, JPY, LOL, LOW, LOX, PIG, RAT, ROW, SUN, TSL, USD, VEA, VEE, VIS, WOW