Written Homework 1
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1.1.5. Let P be the statement "Student X passed every assignment in Calculus I," and let Q be the statement "Student X received a grade of C or better in Calculus I."

(a) What does it mean for P to be true? What does it mean for Q to be true?

(1) For P to be true, the student must have achieved a passing grade on every assignment they attempted (presumably a 60% or higher) in Calculus I.

(2) For Q to be true, the student must have had an average grade of C or higher in Calculus I.

(b) Suppose that Student X passed every assignment in Calculus I and received a grade of B−, and that the instructor made the statement $P \to Q$. Would you say that the instructor lied or told the truth?

– The Instructor told the truth because the student apparently achieved an assignment average (including tests and quizzes as assignments) greater than a C and the student (presumably) passed the class.

(c) Suppose that Student X passed every assignment in Calculus I and received a grade of C−, and that the instructor made the statement $P \to Q$. Would you say that the instructor lied or told the truth?

– In this scenario, the instructor could have lied or not based on the exact grads the student got. For example, if the student received a D or C− on enough assignments, they would have still passed every assignment (P is true) yet still received a grade worse than a C ($\neg Q$). Alternatively, if the student received an F on any assignment, then P would be false and therefore Q could be either True or False while the statement $P \to Q$ is True because the premise is false

(d) Now suppose that Student X did not pass two assignments in Calculus I and received a grade of D, and that the instructor made the statement $P \to Q$. Would you say that the instructor lied or told the truth?

– In this scenario, the instructor told the truth as the premise (P) is false.

(e) How are Parts (5b), (5c), and (5d) related to the truth table for $P \to Q$?

– Part (5b) has both a True premise and conclusion and is therefore true. Part (5c) has two possibilities, the first of which has a True premise but a false conclusion resulting in a lie from the instructor while the second possibility has a false premise which results in a True statement from the instructor regardless of whether or not the conclusion is also True.

1.1.7. Following is a statement of a theorem which can be proven using the quadratic formula. For this theorem, a, b, and c are real numbers.

> **Theorem** If f is a quadratic function of the form $f(x) = ax^2 + bx + c$ and $ac < 0$, then the function f has two x-intercepts

Using **only** this theorem, what can be concluded about the functions given by the following formulas?

(a) $g(x) = -8x^2 + 5x - 2$: Because $-8 * -2 = 16 > 0$, there are not two x-intercepts for this equation.

(b) $h(x) = -\dfrac{1}{3}x^2 + 3x$: Because $-\frac{1}{3} * 0 = 0 \not< 0$, there are not two x-intercepts for this equation.

(c) $k(x) = 8x^2 - 5x - 7$: Because $8 * -7 = -56 < 0$, there are two x-intercepts for this equation.

(d) $j(x) = -\dfrac{71}{99}x^2 + 210$: Because $-\frac{71}{99} * 210 \approx -150.61 < 0$ there are two x-intercepts for this equation

(e) $f(x) = -4x^2 - 3x + 7$: Because $-4 * 7 = -28 < 0$ there are two x-intercepts for this equation

(f) $F(x) = -x^4 + x^3 + 9$: Because $-1 * 9 = -28 < 0$ there are two x-intercepts for this equation

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1.2.5a. Construct a know-show table and write a complete proof for the statement: If m is an even integer, then $3m^2 + 2m + 3$ is an odd integer.

Step	Know	Reason P
m is even	Hypothesis	
P_1	m = 2k where k is some integer	definition of integer
P_2	$3m^2 + 2m + 3 = 3(2k)^2 + 2(2k) + 3$	substitution
P_3	$3 * 4k^2 + 4k + 3$	algebra
P_4	$12k^2 + 4k + 3$	algebra
P_5	$12k^2 + 4k + 2 + 1$	algebra
P_6	$2(6k^2 + 2k + 1) + 1$	algebra
Q_2	$6k^2 + 2k + 1$ is an integer, q	closure properties of integers
Q_1	there exists an odd integer $n = 2(q) + 1$	definition of odd integer
Q	$3m^2 + 2m + 3$ is odd	conclusion
Step	Show	Reason

Theorem: If m is an even integer, then $3m^2 + 2m + 3$ is an odd integer which by definition is made up of $2k + 1$ where k is some integer.
Proof: First we assume that m is an even integer and will prove that $3m^2 + 2m + 3$ is an odd integer.

By the definition of an even integer from page 16 in the text, we can conclude that there exists some integer n such that $m = 2n$. From here, we can substitute $2n$ for m:

$$3m^2 + 2m + 3 = 3(2n)^2 + 2(2n) + 3$$

Then, Using algebra, we obtain

$$\begin{gathered} = 3 * 4n^2 + 4n + 3 \\ = 12n^2 + 4n + 3 \\ = 12n^2 + 4n + 2 + 1 \\ = 2(6n^2 + 2n + 1) + 1 \end{gathered}$$

Since the integers are closed under addition, multiplication, and exponentiation, we can conclude that $6n^2 + 2n + 1$ is also an integer. Hence $3m^2 + 2m + 3$ has been written in the form of $2k+1$, or the definition of an odd integer. Therefore if m is an even integer, $3m^2 + 2m + 3$ must be an odd integer.

1.2.7. Are the following statements true or false? Justify your conclusions.

(a) If a, b, and c are integers, then $ab + ac$ is an even integer.

- False. If a is odd and only one of the other integers is odd, then $ab+ac$ is an odd integer
  For example, let $a = 3$,$b = 4$, and $c = 5$, then:
  $$\begin{aligned} ab + ac &= 3 * 4 + 3 * 5 \\ &= 12 + 15 \\ &= 27 \end{aligned}$$
  which is an odd number.

(b) If b and c are odd integers and a is an integer, then $ab + ac$ is an even integer.

- True. Let $a = k$, $b = 2i + 1$ (where i is some integer), and $c = 2j + 1$ (where j is some integer)
  Then:
  $$\begin{aligned} ab + ac &= k(2i + 1) + k(2j + 1) \\ &= (2ki + k) + (2kj + k) \\ &= 2ki + 2kj + 2k \\ &= 2(ki + kj + 1) \end{aligned}$$

Since integers are closed under addition and multiplication, $ki + kj + 1$ must be an integer, and since the definition of an even integer is $2n$ where n is some integer, the Premis stated above must be true.