

**Written Homework 7**

Name(s):

7.1.9. Let  $R$  be the relation on  $\mathbb{Z}$  where for all  $a, b \in \mathbb{Z}$ ,  $a R b$  if and only if  $|a - b| \leq 2$ .

- (a) Use set builder notation to describe the relation  $R$  as a set of ordered pairs.
  - (b) Determine the domain and range of the relation  $R$ .
  - (c) Use the roster method to specify the set of all integers  $x$  such that  $x R 5$  and the set of all integers  $x$  such that  $5 R x$ .
  - (d) If possible, find integers  $x$  and  $y$  such that  $x R 8$ ,  $8 R y$ , but  $x \not R y$ .
  - (e) If  $a \in \mathbb{Z}$ , use the roster method to specify the set of all  $x \in \mathbb{Z}$  such that  $x R a$ .
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7.2.15. Define the relation  $\approx$  on  $\mathbb{R} \times \mathbb{R}$  as follows: For  $(a, b), (c, d) \in \mathbb{R} \times \mathbb{R}$ ,  $(a, b) \approx (c, d)$  if and only if  $a^2 + b^2 = c^2 + d^2$ .

- (a) Prove that  $\approx$  is an equivalence relation on  $\mathbb{R} \times \mathbb{R}$ .
- (b) List four different elements of the set

$$C = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid (x, y) \approx (4, 3)\}.$$

- (c) Give a geometric description of the set  $C$ .
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7.3.7. Define the relation  $\sim$  on  $\mathbb{R}$  as follows:

For  $x, y \in \mathbb{R}$ ,  $x \sim y$  if and only if  $x - y \in \mathbb{Q}$ .

- (a) Prove that  $\sim$  is an equivalence relation on  $\mathbb{R}$ .
  - (b) List four different real numbers that are in the equivalence class of  $\sqrt{2}$ .
  - (c) If  $a \in \mathbb{Q}$ , what is the equivalence class of  $a$ ?
  - (d) Prove that  $[\sqrt{2}] = \{r + \sqrt{2} \mid r \in \mathbb{Q}\}$ .
  - (e) If  $a \in \mathbb{Q}$ , prove that there is a bijection from  $[a]$  to  $[\sqrt{2}]$ .
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