**Written Homework 7**
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7.1.9. Let $R$ be the relation on $\mathbb{Z}$ where for all $a, b \in \mathbb{Z}$, $a\,R\,b$ if and only if $|a - b| \leq 2$.

(a) Use set builder notation to describe the relation $R$ as a set of ordered pairs.

$$a\,R\,b = \{(a, b) : a, b \subseteq \mathbb{Z}, -2 \leq a - b \leq 2\}$$

(b) Determine the domain and range of the relation $R$.

$$\text{Domain(R)} = \{u \in a | (u, y) \in R \text{ for at least one } y \in b\}$$
$$\text{Range(R)} = \{v \in b | (x, v) \in R \text{ for at least one } x \in a\}$$

(c) Use the roster method to specify the set of all integers $x$ such that $x\,R\,5$ and the set of all integers $x$ such that $5\,R\,x$.

$$x\,R\,5 = \{3, 4, 5, 6, 7\}$$
$$5\,R\,x = \{3, 4, 5, 6, 7\}$$

(d) If possible, find integers $x$ and $y$ such that $x\,R\,8$, $8\,R\,y$, but $x \not R\, y$.

x = 6, y = 10

(e) If $a \in \mathbb{Z}$, use the roster method to specify the set of all $x \in \mathbb{Z}$ such that $x\,R\,a$.

$$x\,R\,a = \{x - 2, x - 1, x, x + 1, x + 2\}$$

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7.2.15. Define the relation $\approx$ on $\mathbb{R} \times \mathbb{R}$ as follows: For $(a, b), (c, d) \in \mathbb{R} \times \mathbb{R}$, $(a, b) \approx (c, d)$ if and only if $a^2 + b^2 = c^2 + d^2$.

(a) Prove that $\approx$ is an equivalence relation on $\mathbb{R} \times \mathbb{R}$.
To prove that $\approx$ is an equivalence relation on $\mathbb{R} \times \mathbb{R}$,
it must be reflexive, symmetric, and transitive.

Reflexive: let $(a, a) \in \mathbb{R}$. Then $a^2 + a^2 = a^2 + a^2$ Then $(a, b) \approx (a, a)$

Symmetric: let $(a, b)(c, d) \in \mathbb{R} \times \mathbb{R}$. Then $a^2 + b^2 = c^2 + d^2$ and Thus $c^2 + d^2 = a^2 + b^2$.
Therefore $(a, b) \approx (c, d) = (c, d) \approx (a, b)$

Transitive: let $(a, b), (c, d), (e, f) \in \mathbb{R} \times \mathbb{R}$. Then $a^2 + b^2 = c^2 + d^2$ and $c^2 + d^2 = e^2 + f^2$.
Therefore $a^2 + b^2 = e^2 + f^2$ and $(a, b) \approx (c, d)$ and $(c, d) \approx (e, f)$
Quorum Est Demonstrandum.

(b) List four different elements of the set

$$C = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid (x, y) \approx (4, 3)\}.$$

5: (0,5) $= 0^2 + 5^2 = 5^2 = 25$
4: (4,3) $= 4^2 + 3^2 = 16 + 9 = 25$
3: (3,4) $= 3^2 + 4^2 = 9 + 16 = 25$

$\sqrt{21}$: $(2,\sqrt{21}) = 2^2 + \sqrt{21}^2 = 4 + 21 = 25$
$\sqrt{24}$: $(1,\sqrt{24}) = 1^2 + \sqrt{24}^2 = 1 + 24 = 25$

(c) Give a geometric description of the set $C$.
For each $x, y \in \mathbb{Z}$, $x^2 + y^2 = 25$. This apperes to be a circle with radius $\sqrt{25}$ or 5.

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7.3.7. Define the relation $\sim$ on $\mathbb{R}$ as follows:

For $x, y \in \mathbb{R}$, $x \sim y$ if and only if $x - y \in \mathbb{Q}$.

(a) Prove that $\sim$ is an equivalence relation on $\mathbb{R}$.
To prove that $\approx$ is an equivalence relation on $\mathbb{R} \times \mathbb{R}$,
it must be reflexive, symmetric, and transitive.

Reflexive: let $x \in \mathbb{R}$. Then $x - x \in \mathbb{Q}$ Then $x \sim x$ which is true.

Symmetric: let $x, y \in \mathbb{R}$. Then $x - y \in \mathbb{Q}$ and $y - x \in \mathbb{Q}$. Therefore $x \sim y = y \sim x$. This is also true.

Transitive: let $x, y, z \in \mathbb{R}$. Then $x - y \in \mathbb{Q}$ and $y - z \in \mathbb{Q}$. Therefore $x - z \in \mathbb{Q}$ and $x \sim y$ and $y \sim z$ therefore $x \sim z$.
Quorum Est Demonstrandum.

(b) List four different real numbers that are in the equivalence class of $\sqrt{2}$.

1: $\sqrt{2} + 1 \approx 2.4142$
2: $\sqrt{2} - 1 \approx 0.4142$
3: $\sqrt{2} - 2 \approx -0.5858$
4: $\sqrt{2} - 3 \approx -1.5858$

(c) If $a \in \mathbb{Q}$, what is the equivalence class of $a$?
The equivalence class of $a$ for positive numbers is any number whose decimal is the same as $a$, while for negetive numbers it is any negetive number whose decimal is the same as 1- $a$

(d) Prove that $\left[\sqrt{2}\right] = \left\{r + \sqrt{2} \mid r \in \mathbb{Q}\right\}$.

(e) If $a \in \mathbb{Q}$, prove that there is a bijection from $[a]$ to $\left[\sqrt{2}\right]$.