Written Homework 3
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3.1.10. Determine if each of the following propositions is true or false. Justify each conclusion.

b. True. For each integer a, if $a \equiv 2 \mod 8$, then $a^2 \equiv 4 \mod 8$.
let $a \,\epsilon\, \mathbb{Z}$ and $a \equiv 2 \mod 8$, then $8 \,|\, a - 2$. Since $n|m \to n|ml$ for some $n, m, l \,\epsilon\, \mathbb{Z}$, then $\exists\, n$ S.T. $n = (a-2) * (a+2) = (a^2 - 2^2) = (a^2 - 4) \equiv 4 \mod 8$.

c. For each integer a, if $a^2 \equiv 4 \mod 8$, then $a \equiv 2 \mod 8$.
False. There are a total of 8 cases for a: $a = \{0, 1, 2, 3, 4, 5, 6, 7\}$. The one case that fufills the premis but not the conclusion is when $a = 6$. $\therefore a^2 = 36 \to 36 \mod 8 \equiv 4$ but $6 \mod 8 \equiv 6 \not\equiv 2$

3.2.10. Is the following proposition true or false? Justify.

For each integer n, n is even if and only if 4 divides n^2.

True.
First, since integer exponentiation is the equivalent of repetitive multiplication, $n^2 = n * n$. Second, odd numbers square to odd numbers: let x be an odd integer, then by defination, $x = 2k + 1, k\epsilon\mathbb{Z}$. Then:

$$\begin{aligned} x^2 &= (2k+1)^2 \\ &= 4k^2 + 4k + 1 \\ &= 2(2k^2 + 2k) + 1 \\ &= 2j + 1 \end{aligned}$$

Where $j \,\epsilon\, \mathbb{Z}$ since integers are closed under addition and multiplication and therefore x^2 is odd.
If $4 \,|\, n^2$, then $2 \,|\, n^2$ since $2|4$ and $n|m \to n|ml$ for some $n, m, l \,\epsilon\, \mathbb{Z}$

3.3.6. Are the following statements true or false? Justify.

c. For every pair of real numbers x and y, if $x + y$ is irrational, then x is irrational and y is irrational.

d. For every pair of real numbers x and y, if $x + y$ is irrational, then x is irrational or y is irrational.

3.4.7. Is the following proposition true or false? Justify your conclusion with a counterexample or a proof.

For each integer n, if n is odd, then $8|(n^2 - 1)$.