Written Homework 5
Name(s): Elliot Marshall

5.1.9. Let P, Q, R, and S be subsets of a universal set U. Assume that $(P - Q) \subseteq (R \cap S)$.

(a) Complete the following sentence:

For each $x \in U$, if $x \in (P - Q)$, then ...

For each $x \in U$, if $x \in (P - Q)$, then $x \in R$ and $x \in S$.

(b) Write a useful negation of the statement in part (a).

For each $x \in U$, if $x \in (P - Q)$ then $x \notin R$ or $x \notin S$.

(c) Write the contrapositive of the statement in part (a).

For each $x \in U$, if $x \notin R$ or $x \notin S$ then $x \notin (P - Q)$.

5.2.11. Let A, B, C, and D be subsets of some universal set U. Are the following propositions true or false? Justify your conclusions.

(a) If $A \subseteq B$ and $C \subseteq D$ and A and C are disjoint, then B and D are disjoint.

False. It does not follow that two disjoint subsets necessitates that their supersets are disjoint.
For example, let $A = \{1, 3\}$, $B = \{1, 3, 5\}$, $C = \{2, 4\}$, and $D = \{2, 4, 5\}$. Then $A \subseteq B$, $C \subseteq D$, A and C are disjoint, while B and D are not disjoint as $5 \in B$ and $5 \in D$.

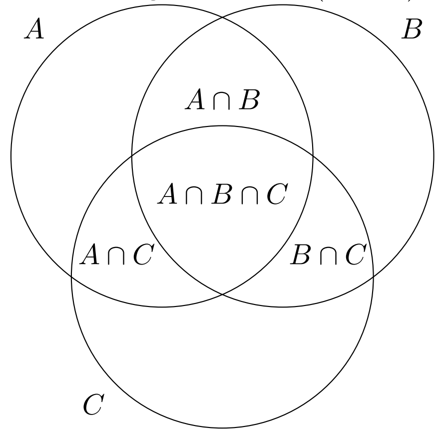
(b) If $A \subseteq B$ and $C \subseteq D$ and B and D are disjoint, then A and C are disjoint.

True. If two sets are truly disjoint, then their respective subsets must by defination share no common values, thus making those subsets disjoint as well. Furthermore, A and D along with B and C are each disjoint.

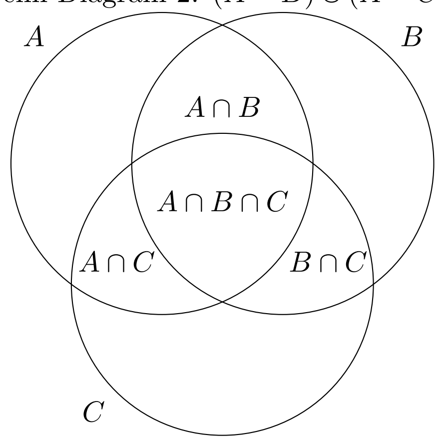
5.3.8 Let A, B, and C be subsets of some universal set U.

(a) Draw two general Venn diagrams for the sets A, B, and C. On one, shade the region that represents $A - (B - C)$, and on the other, shade the region that represents $(A - B) \cup (A - C^c)$. Based on the Venn diagrams, make a conjecture about the relationship between the sets $A - (B - C)$ and $(A - B) \cup (A - C^c)$. (Are the two sets equal? If not, is one of the sets a subset of the other set?)

Venn Diagram 1: $A - (B - C)$



Venn Diagram 2: $(A - B) \cup (A - C^c)$



Conjecture: $A - (B - C) = (A - B) \cup (A - C^c)$

(b) Prove the conjecture from part (a).

Proof: Let $x \in A - (B - C)$. Then $x \in A$ and $x \notin (B - C)$
$\to x \in A$ and $x \in C$ while $x \notin B$.
$\to (x \in A$ and $x \notin B)$ or $(x \in A$ and $x \in C)$
$\to x \in (A - B)$ or $(x \in A$ and $x \notin C^c)$
$\to x \in (A - B) \cup x \in (A - C^c)$
$\to x \in (A - B) \cup (A - C^c)$. QED