Written Homework 2

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2.1.7. Construct truth tables for $P \wedge (Q \vee R)$ and $(P \wedge Q) \vee (P \wedge R)$. What do you observe?

P	Q	R	Q∨R	P∧(Q∨R)
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	F
F	T	F	T	F
F	F	T	T	F
F	F	F	F	F

P	Q	R	(P∧Q)	(P∧R)	(P∧Q) ∨ (P∧R)
T	T	T	T	T	T
T	T	F	T	F	T
T	F	T	F	T	T
T	F	F	F	F	F
F	T	T	F	F	F
F	T	F	F	F	F
F	F	T	F	F	F
F	F	F	F	F	F

I observer that these two expressions, $P \wedge (Q \vee R)$ and $(P \wedge Q) \vee (P \wedge R)$ are equivalent. This is due to the distributive principle.

2.2.6. Use truth tables to prove the following logical equivalency from Theorem 2.8:

$$[(P \vee Q) \to R] \equiv (P \to R) \wedge (Q \to R).$$

P	Q	R	P∨Q
T	T	T	T
T	T	F	F
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	F
F	F	T	T
F	F	F	T

P	Q	R	(P→R)	(Q→R)	(P→R) ∧ (Q→R)
T	T	T	T	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	T	T

As one can see, the pattern TFTFTFTT holds for both logical expressions which means they are equivalent.

2.2.9. Use previously proven logical equivalencies to prove each of the following logical equivalencies.

(a) $[\neg P \to (Q \wedge \neg Q)] \equiv P$
$(Q \wedge \neg Q)$ will always be false because it is a comparison of the two possible states. Therefor, if P is True, then ¬P is false and the expression becomes False → False which is a true statement and therefore logically equivalent to P. On the other hand, if P is False, then the statement $\neg P$ is True and the logical statement is True → True which is itself, True.

(c) $\neg(P \leftrightarrow Q) \equiv (P \wedge \neg Q) \vee (Q \wedge \neg P)$
The logical expression "if and only if" has truth values of True if the proposition and conclusion are of equal logical signs. Therefore, the negation of the statement $(P \leftrightarrow Q)$ is true if and only if P and Q have *opposit* signs. On the RHS of the equivalence expression, we have P and $\neg Q$ or Q and $\neg P$. Each one of these sub-expressions is true if and only if P and Q have opposit logical signs. Then if at least one of them is true, the whole RHS expression is true. So Both sides are true if and only if P and Q have opposit logical signs and is false if they are the same which makes them equivalent.

(e) $(P \to Q) \to R \equiv (\neg P \to R) \wedge (Q \to R)$ For a given logical implication, it is True unless the Premise is True and the Conclusion is False. Therefore, the first term on the RHS is true *unless* P is False *and* R is False. Oppositly, the second term on the RHS is True *unless* Q is True and R is False. Combining these two, the total logical value is True unless P is False, Q is True, and R is False. On the LHS, if a Premise P is False and a Conclusion Q is True, the expression is True. However, if the Premise term $(P \to Q)$ is True but the Conclusion (R) is False, this is the only situation where the whole LHS is False. Checking the value of the variables, we get $P = False$, $Q = True$, and $R = False$ which so happens to be the only situation wherein the RHS is False. Therefore these two expressions are equivalent.

2.4.9. An integer m is said to have the *divides property* provided that for all integers a and b, if m divides ab, then m divides a or m divides b.

(a) Using the symbols for quantifiers, write what it means to say that the integer m has the divides property.
$(\exists m \in \mathbb{Z} : m \mid ab \to m \mid a \vee m \mid b)$

(b) Using the symbols for quantifiers, write what it means to say that the integer m does not have the divides property.
$(\exists m \in \mathbb{Z} : m \not| ab \to m \not| a \wedge m \not| b)$

(c) Write an English sentence stating what it means to say that the integer m does not have the divides property.
For an integer m, if m does not divide an integer a and m does not divide an integer b, then m does not divide the integer ab.