Written Homework 6

Name(s): Elliot Marshall

6.1.4. Let $f : \mathbb{Z} \to \mathbb{Z}$ be defined by $f(m) = 2m + 1$. (Be careful: the domain and codomain are $\mathbb{Z}$, not $\mathbb{R}$.)

(a) Evaluate $f(-7), f(-3), f(3)$, and $f(7)$.

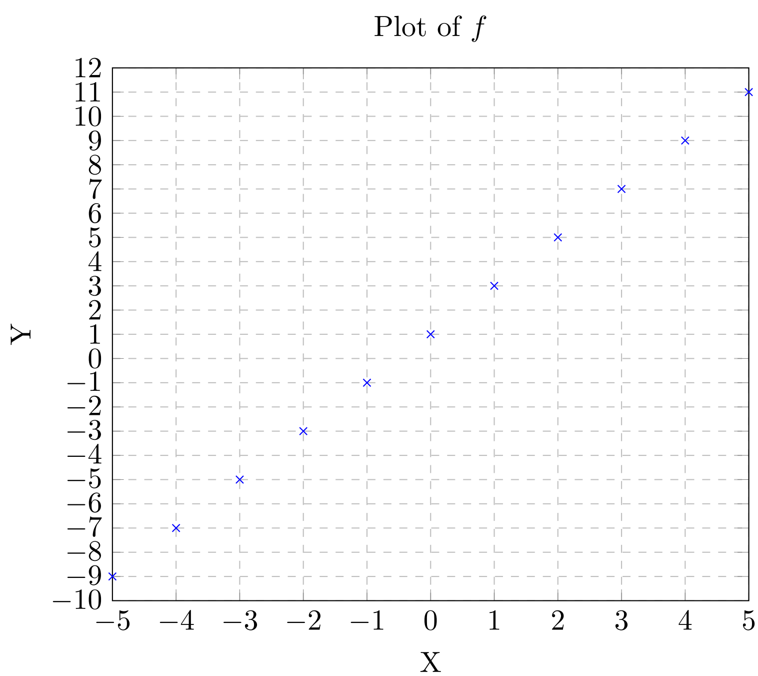
$$\text{f(-7)} = 2(-7) + 1 = -13$$
$$\text{f(-3)} = 2(-3) + 1 = -5$$
$$\text{f(3)} = 2(3) + 1 = 7$$
$$\text{f(7)} = 2(7) + 1 = 15$$

(b) Determine the set of all of the preimages of 5 and the set of all of the preimages of 4.

$$\text{preimages}(5) = \{2\}$$
$$\text{preimages}(4) = \emptyset$$

(c) Determine the range of the function f.
The range of f the set of all odd integers.

(d) Sketch a graph of the function f. Note: the graph will be an infinite set of points that lie on a line but not the whole line.



6.2.8. Let $g : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ be defined by $g(m, n) = (2m, m - n)$.

(a) Calculate $g(3, 5)$ and $g(-1, 4)$.

$$g(3,5) = (2(3), 3-5) = (6, -2)$$
$$g(-1,4) = (2(-1), -1-4) = (-2, -5)$$

(b) Determine all the preimages of $(0, 0)$. That is, find all $(m, n) \in \mathbb{Z} \times \mathbb{Z}$ such that $g(m, n) = (0, 0)$.
The set of preimages for m all satisfy $2m = 0$ which is equal to $\{0\}$. From this, we can now find the set of preimages of n which satisfy

$$m - n = 0$$
$$m = n$$
$$\text{and } m = 0$$
$$\therefore n = 0$$

Thus the set of preimages of $(0, 0)$ are $\{(0, 0)\}$.

(c) Determine the set of all preimages of $(8, -3)$.
The set of preimages for m in this case is $2m = 8 \to m = 4$. From this, we can now find the set of all preimages of n:

$$g(n) = m - n$$
$$g(n) - m = -n$$
$$m - g(n) = n$$
$$\text{from earlier, } m = 4 \text{ and } g(n) = -3$$
$$\therefore n = 4 - (-3) = 7$$

Thus the set of preimages of $(8, -3)$ are $\{(4, 7)\}$.

(d) Determine the set of all preimages of $(1, 1)$.
There are no perimages in for $(1, 1)$ because $g(m, n)$ is not a surjection of $\mathbb{Z}$
In order to get a preimage of $(1, 1)$, we need to expand the domain to $\mathbb{R}$ as $2m = 1 \to m = \frac{1}{2}$ and $\frac{1}{2} \notin \mathbb{Z}$

(e) Is the following proposition true or false? Justify your conclusion.

> For each $(s, t) \in \mathbb{Z} \times \mathbb{Z}$, there exists $(m, n) \in \mathbb{Z} \times \mathbb{Z}$ such that $g(m, n) = (s, t)$.
> False. Consider part d of this question.

6.3.8. (a) Let $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ be defined by $f(m, n) = 2m + n$. Is the function f an injection? Is the function f a surjection? Justify your conclusions.
The function f is not an injection as $f(1, 1) = f(2, -1) = 3$
However, the function f is a surjection:

$$\text{Case 1: y is even} \to m = \tfrac{y}{2} \to f(\tfrac{y}{2}, 0) = 2(\tfrac{y}{2}) + 0 = y$$
$$\text{Case 2: y is odd} \to m = \tfrac{y-1}{2} \to f(\tfrac{y-1}{2}, 1) = 2(\tfrac{y-1}{2}) + 1 = y$$

Thus the function f is a surjection.

(b) Let $g : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ be defined by $g(m, n) = 6m + 3n$. Is the function g an injection? Is the function g a surjection? Justify your conclusions.
The function g is not an injection as $g(1, 1) = 6(1) + 3(1) = g(2, -1) = 6(2) + 3(-1) = 9$
The function g is not a surjection as $g(m, n) = 3(2m + n)$ which means that g(m,n) must be divisible by 3. Consider 2 which is an integer and thus in the codomain of g (i.e. $\mathbb{Z}$), yet not divisible by 3 into an integer ($\frac{2}{3} \approx 0.6667$)