Consequence-theoretic semantics and entailment

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Consequence-theoretic semantics

Model-theoretic semantics is a broad tradition in formal semantics.

MTS takes notions like truth, denotation, etc as basic.

MTS's key formal tools come from model theory:
domains, relational structures, satisfaction.

These are taken to relate directly to semantically significant statuses.

Proofs have relevance only indirectly,
via these models.

Example

MTS and 'and':

- Formal bits: $[\![A \wedge B]\!]^{g,w} = T$ iff $[\![A]\!]^{g,w} = [\![B]\!]^{g,w} = T$
- Interpretation: $[\![A]\!]^{g,w}$ is the truth value of A as uttered in context g, in possible world w

Proof-theoretic semantics is a different tradition in formal semantics

PTS takes notions like inference, warrant, etc as basic.

PTS's key formal tools come from proof theory:
proof systems, rules, reduction procedures.

These are taken to relate directly to semantically significant statuses.

Models have relevance only indirectly,
via these proofs.

Example

PTS and ‘and’:

- Formal bits: $\wedge$I: $\dfrac{A \qquad B}{A \wedge B}$ $\wedge E_1$: $\dfrac{A \wedge B}{A}$ $\wedge E_2$: $\dfrac{A \wedge B}{B}$
- Interpretation: A canonical warrant for $A \wedge B$ is a pair: a warrant for A and a warrant for B

Both are applicable to semantics of formal languages
as well as semantics of natural languages.

Dowty, Wall, Peters 1981:

"In constructing the semantic component of a grammar, we are attempting to account...[for speakers'] judgements of synonymy, entailment, contradiction, and so on" (2, emphasis added).

Consequence-theoretic semantics gives a third option.

CTS takes consequence as basic.

Key formal tools come from direct explorations
of consequence relations themselves.

Models and proofs both have relevance only indirectly,
via consequence.

For this even to get off the ground,
we need some understanding of consequence
that doesn't depend on either models or proofs.

(or on truth, warrant, denotation, inference,...)

Example

CTS and ‘and’:

- Formal bits:
$$A \wedge B, \Gamma \vdash \Delta \quad \text{iff} \quad A, B, \Gamma \vdash \Delta$$
$$\Gamma \vdash \Delta, A \wedge B \quad \text{iff} \quad (\Gamma \vdash \Delta, A \text{ and also } \Gamma \vdash \Delta, B)$$
- Interpretation: ??

Drawing on Restall,
I prefer to do this work in terms of positions and bounds.

A position is a set of assertions and denials.

In conversations, we take up positions
and attribute positions to each other.

Some positions are in bounds, and some are out of bounds.

An out of bounds position is no good, self-undermining, doesn’t fit with itself, incoherent, etc, as a matter of meaning.

The bounds themselves are a social kind: constituted by our practices.

We can use this to understand one kind of consequence, which I call bounds consequence:

Conclusions Δ are a bounds consequence of premises Γ iff: the position that asserts Γ and denies Δ is out of bounds.

Example

CTS and 'and':

- Formal bits: $A \wedge B, \Gamma \vdash \Delta$ iff $A, B, \Gamma \vdash \Delta$
  $\Gamma \vdash \Delta, A \wedge B$ iff $(\Gamma \vdash \Delta, A$ and also $\Gamma \vdash \Delta, B)$
- Interpretation: It's oob to assert $A \wedge B$ iff
  it's oob to assert both A and B, and

  It's oob to deny $A \wedge B$ iff
  it's oob to deny A and oob to deny B

Models and proofs both matter only indirectly,
as ways to study the bounds.

Proofs are certificates of oobness,
and models are certificates of ibness.

A problem: entailment

Dowty, Wall, Peters 1981:

"In constructing the semantic component of a grammar, we are attempting to account…[for speakers'] judgements of synonymy, entailment, contradiction, and so on" (2, emphasis added).

There might seem to be an easy path to entailment:
say that Γ entails A iff
the position that asserts Γ and denies A is oob.

That is, iff A is a bounds consequence of Γ.

If some controversial assumptions hold,
this view is extensionally right.
But even if this is so, it's more or less a coincidence.

Steinberger (2011):

“Take the example of the classical theoremhood of the law of the excluded middle. [This] would have to be rendered as ‘It is incoherent to deny $A \vee \neg A$’. But surely this is not what is intended; even the intuitionist can happily agree that it is incoherent to deny (every instance of) $A \vee \neg A$. [We need] a way of expressing that $A \vee \neg A$ can always be correctly asserted” (353).

If asserting Γ and denying A is oob,
that's just a prohibition.

But entailment should enable us to go on.

In the limiting case of empty Γ,
it should ensure that A is assertible.

Bounds consequence doesn't do this.

A solution: implicit assertion

Positions are just collections of acts (assertions and denials).

One position leaves open another position iff
the union of the two positions is in bounds.

Someone who's done all the acts in one of these
can go on to do all the acts in the other, and will be in bounds.

Two positions are equivalent iff
they leave exactly the same positions open.

They might contain different acts,
but they constrain their adopters' future acts
in just the same way.

A position implicitly asserts *A* iff:
it's equivalent to the position you'd get by adding an assertion of *A*.

A is implicitly asserted when adding a genuine assertion of *A*
to that position would leave all the same things open.

Implicit assertion is characterized via positions and bounds, not truth or inference.

It is a broad notion, subsuming assertion proper.

(A worse but more accurate name would be ‘at-least-implicit assertion’.)

Now, say Γ entails A iff:
every position that (implicitly) asserts all of Γ implicitly asserts A.

When Γ entails A, if you've already asserted Γ,
you close off no further options for yourself by going on to assert A.

Suppose the earlier theory of conjunction's assertion conditions: asserting $A \wedge B$ is oob iff asserting A with B is oob.

It follows that any position asserting A and B implicitly asserts $A \wedge B$.

That is: A, B together entail $A \wedge B$.
Having asserted A, B, you may as well have asserted $A \wedge B$.

That assumes only the assertion conditions for conjunction.

Without any assumptions on the bounds,
we can't conclude that $A \wedge B$ is a bounds consequence of A, B.

Even so, we can reach the entailment claim.

Or suppose that the empty set entails A.

This means that any position whatever implicitly asserts A.

If this is so, assertions of A are free;
they close off no options at all.

This is just what we were after.

No matter the bounds, entailment is

reflexive
(A entails A),

diluting
(if Γ entails A, then $\Gamma \cup \Sigma$ entails A),

and cuttable
(if Γ entails A, and $\Sigma \cup \{A\}$ entails C, then $\Sigma \cup \Gamma$ entails C).

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Whether bounds consequence obeys these depends on the bounds.

Steinberger's point stands:
what we want from entailment is some permissive, positive status,
and bounds consequence is not directly fit for purpose.

Entailment via implicit assertion provides this,
using only the raw materials of positions and bounds.

Structural properties

Bounds consequence is set-set and entailment is set-formula, so they're not directly comparable.

But the two need not be counterparts.
That is, we need not have Γ entails A iff A is a bc of Γ.

Connections between entailment and bc depend on what the bounds are like.

Say that the bounds are nondefeasible iff
whenever a position is out of bounds,
every position containing it is also out of bounds.

Say that the bounds are clashing iff
any two-act position that asserts and denies the very same thing
is out of bounds.

Say that the bounds are extensible iff
whenever a position is in bounds, for any A,
either extending the position with an assertion of A is ib,
or extending the position with a denial of A is ib.

All these properties are controversial!

If the bounds are nondefeasible and clashing,
then entailment implies bounds consequence.

If you shouldn't assert and deny the same thing,
no matter what other acts you've done,
then you shouldn't deny what you've implicitly asserted.

If the bounds are nondefeasible and extensible,
then bounds consequence implies entailment.

If being in bounds means there's always some safe stand to take on *A*,
and if there's no way to rescue an oob position by adding more,
then if you can't deny *A* you've implicitly asserted it.

So if the bounds are
nondefeasible, clashing, and extensible,
then entailment is (the counterpart of) bounds consequence.

But: each of these assumptions is controversial,
and this is just an extensional match anyhow.

It's entailment that's entailment,
whether or not it lines up with bounds consequence.

Steinberger's point stands:
what we want from entailment is some permissive, positive status,
and bounds consequence is not directly fit for purpose.

Entailment via implicit assertion provides this,
using only the raw materials of positions and bounds.

Whether bounds consequence lines up with entailment
is a separate question,
a question about what the bounds are like.