QEA3 Fidget Spinner Project

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1 Introduction

In pursuit of understanding ordinary differential equations (ODEs), we have modeled an active fidget spinner, sitting on a table, coming naturally to rest.

Figure 1: Image of our fidget spinner

2 Experimental Procedure

For our experiment, placed our fidget spinner flat down on a table [fig. 1]. Then we placed a phone, face up, at an elevated position above the fidget spinner in order to record it [fig. 2]. We then spin the spinner and press the record button in rapid succession, letting the spinner come to a complete rest. At this point, we stopped the recording and transfered the .mp4 image file onto a computer for analysis.

3 Video Analysis

A video is basically a series of pictures moving really fast. For our experiment, we need to convert those frames into timestamped angular velocities (ω, t). We first chose a small area near the edge of the spinner to focus on. This selected area would flash between the color of the fidget spinner and the color of the table, telling us the rotations per frame. Using the speed at which those flashes occurred

Figure 2: Image of experimental setup using a phone to record video

relative to the recording speed (24 frames per second (fps)), and a little bit of data processing, we could determine the angular velocity. This is shown in the following equation:

$$[\frac{rotation}{frame}] * [\frac{frame}{second}] = [\frac{rotations}{second}] = \omega(t)$$

Since the fidget spinner has three wings, we know that every three flashes after the first flash is one full rotation of the fidget spinner. This means we need to divide the calculated angular velocity by three to find the real angular velocity.

There is a possibility that the video and the rotation rate is the same. In this case, there would be no flashing because each time a picture is taken, the fidget spinner would be showing the same color. This is known as the stroboscopic effect. Luckily we did not encounter this issue.

4 Deriving Angular Acceleration

Angular velocity is how fast the fidget spinner spins. The units for angular velocity ($\omega(t)$) is rad/s while the units for angular acceleration ($\dot{\omega}(t)$) is rad/s^2. To approximate angular acceleration, we simply found the slope between the angular velocity at different timestamps (attained through the video analysis). Plotting the angular acceleration against angular velocity gives us the plot in figure 3.

The quadratic fit to the collected data is a good fit. We know this because the coefficient of determination (R^2) for this plot is relatively high. This tells us that the fit between our data and our fitted estimate is high.
Additionally, the plot can be visually inspected to ensure that the relationship is close in figure 3.

$$R^2 = 1 - RSS/TSS$$
$$\text{RSS} = \text{Residual sum of the squares}$$
$$\text{TSS} = \text{Total sum of the squares}$$
$$R^2 = 0.999$$

The fitted curve of figure 3 can be represented by the equation $\dot{\omega}(t) = a\omega(t)^2 + b\omega(t) + c$, with the constant values given in table 4. This is because we are assuming our ODE is based on a simplified dynamics model for a rotating body: $I\alpha = \sum \tau$ where $\sum \tau$ is made up of the quadratic drag ($-d_1\omega^2$), viscous drag ($-d_2\omega$), and contact friction ($-d_3$). This gives us:

$$\dot{\omega}(t) = a\omega(t)^2 + b\omega(t) + c$$

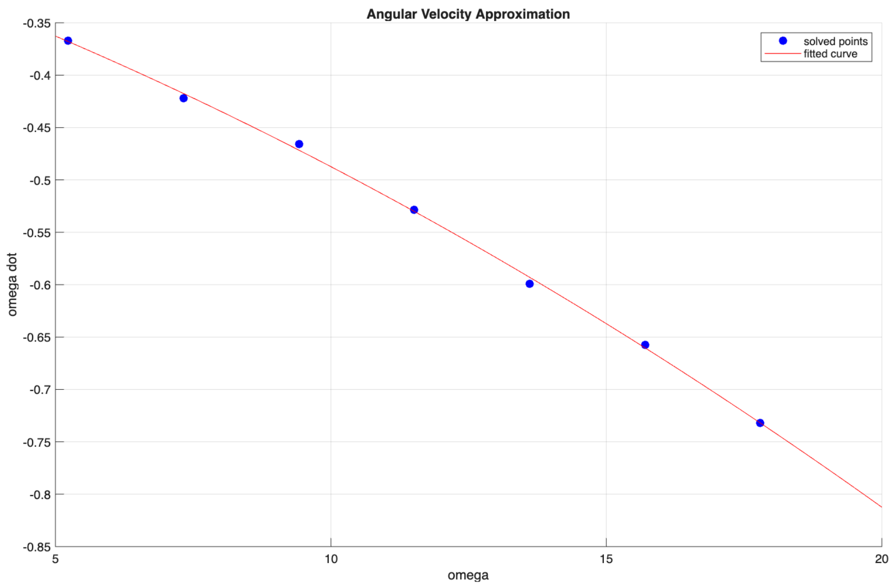

Figure 3: Plot of angular acceleration against angular velocity

where:

$$a = \frac{-d_1\omega^2}{I}, b = \frac{-d_2\omega}{I}, c = \frac{-d_3}{I}$$

Our equation, $\dot{\omega}(t) = a\omega(t)^2 + b\omega(t) + c$, is a first order equation because the highest order of derivative is $d\omega/dt$, which is a first order derivative. This equation is also nonlinear and forced because there is a exponent acting on $\dot{\omega}$ and there is a constant (c).

Quantity Description	Symbol	Value	Units
Estimated quadratic drag constant	a	0.0002	unitless
Estimated viscous damping constant	b	-0.0368	$N \cdot s/m$
Estimated Coulomb friction constant	c	-0.1423	unitless
Video frame rate	F_s	24	$frames/s$
Estimated initial angular velocity	$\omega(t=0)$	56.5255	rad/s

Table 1: Table describing various variables, their values, and units for the equation: $\dot{\omega}(t) = a\omega(t)^2 + b\omega(t) + c$

Now that we have an equation for our model and know all our constants, we can plot our model of angular velocity as a function of time and compare it to the data we gathered from the video analysis. This is shown in figure 4.

Looking at figure 4, we can see that the simulated curve from ode45 generally follows the flow of the data correctly. However, it contains a fairly consistent offset between the collected data and the simulated data. This could be explained by an initial condition that is slightly inaccurate; shifting the simulated plot up.

To further test the accuracy of our model, we created a quiver plot from our equation and depicted three simulation solutions for $\omega(t)$ with different initial conditions. This gives us the plot in figure 5. Looking at that plot, we can see that the simulations mostly follow the arrows on the quiver plot, further validating our model.

5 Comparison to Previous Models

While we collected data, we noticed that the fidget spinner does eventually stop rotating. This is different from other first order systems we have previously analyzed in that there is a known asymptote: $y = 0$. For example, the coffee cup example we looked at has an exponential curve of it cooling down but we can't really know what it cools down to because we don't have the final temperature readings.

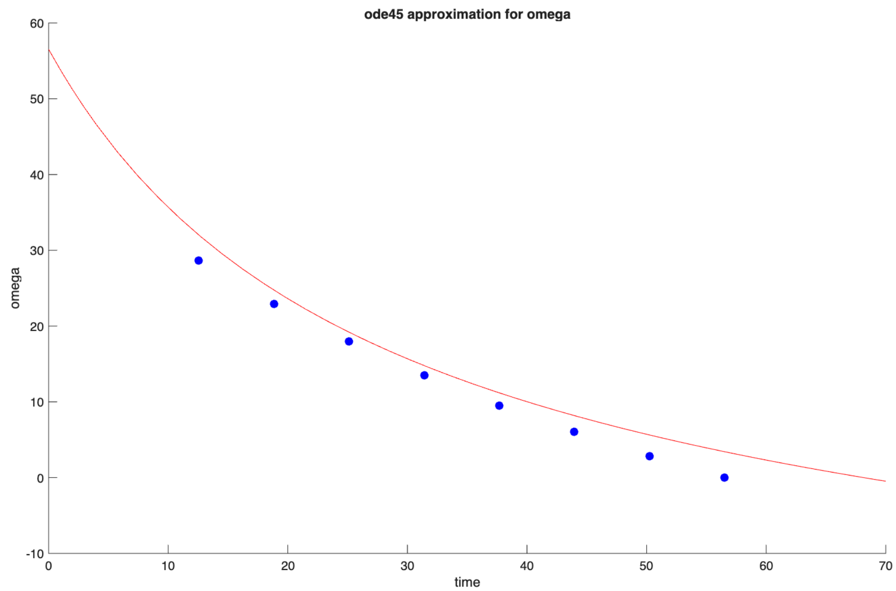

Figure 4: Plot of fitted curve from ODE45 of angular velocity by time

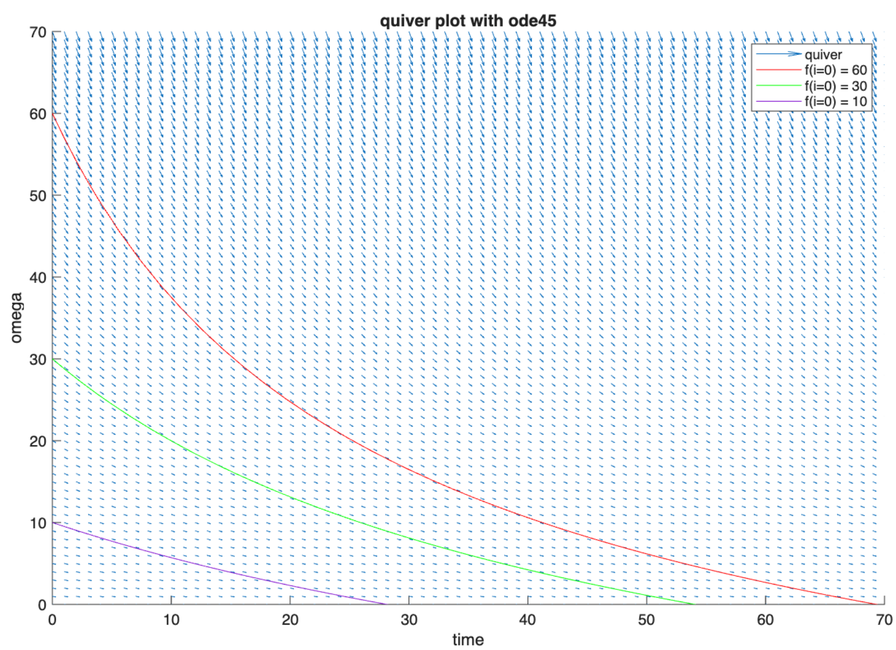

Figure 5: Plot of ode45 curve against a quiver grid

6 Changing Density

One way to make the fidget spinner spin for longer is by increasing its moment of inertia. In order to do that, we would have to make the fidget spinner out of a denser material, changing the ODE for the system to: $\frac{\rho_{new}}{\rho_{old}}\dot{\omega}(t) = a\omega(t)^2 + b\omega(t) + c$. In this equation, ρ_{new}/ρ_{old} is the density ratio of old and new components.

Given this equation, we can plot different variations of ρ_{new}/ρ_{old} (fig. 6) to see how much impact changing the density would have on the fidget spinner's spinning time.

Looking at figure 6, we can see that the fidget spinner would come to rest at time 41.4, 82.8, 165.5, and 330.9 for density ratios of .5, 1, 2, and 4, respectively.

Using these simulated results, we calculated a plot of $t_{stop}/t{*}_{stop}$ (time ratio) as a function of ρ_{new}/ρ_{old} (density ratio), where t_{stop} is the simulated stop time of a fidget spinner aka the time that

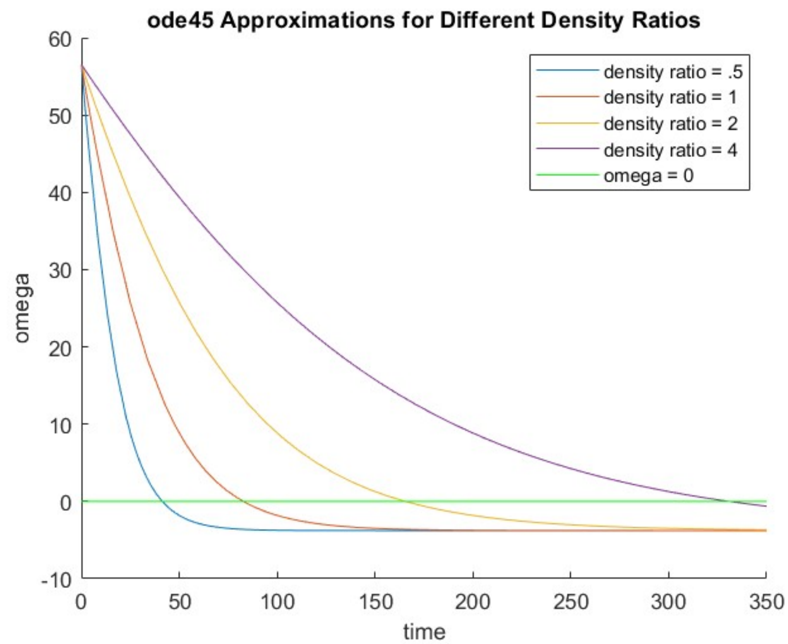

Figure 6: Plot of various approximations given different density ratios.

$\omega(t)$ first reaches zero, and $t*_{stop}$ is the simulated stop time when $\rho_{new}/\rho_{old} = 1$. This is shown in figure 7 and produces an interesting graph because it seems that this function is linear.

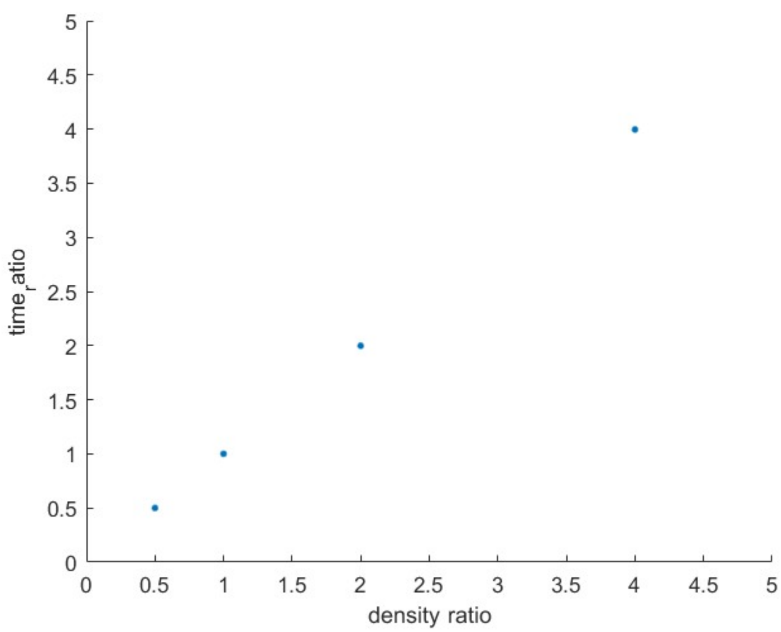

Figure 7: Plot of the density ratio vs. the time ratio

Instead of using always solving equation (1), we can represent the density relationship without time by substituting $\tilde{t}$ in. This allows us to clearly see the effects of the density ratio on the time it takes for the fidget spinner to spin.

First, begin with

$$\frac{\rho_{new}}{\rho_{old}}\dot{\omega}(t) = a\omega(t)^2 + b\omega(t) + c \tag{1}$$

and define $\tilde{t}$ as:

$$\tilde{t} = a\frac{t}{\rho_{new}/\rho_{old}} \tag{2}$$

Using the chain rule of derivatives, we can convert $\dot{\omega}(t)$ into $\dot{\omega}(\tilde{t})$ form.

$$\dot{\omega}(t) = \frac{d\omega(t)}{dt} = \frac{d\dot{\omega}(\tilde{t})}{d\tilde{t}}\frac{d\tilde{t}}{dt} = \frac{d\dot{\omega}(\tilde{t})}{d\tilde{t}}\frac{\rho_{old}}{\rho_{new}} \tag{3}$$

Plugging this into equation (1) gives us our new differential in terms of t tilde by canceling the densities.

$$\frac{\rho_{new}}{\rho_{old}}\frac{\rho_{old}}{\rho_{new}}\dot{\omega}(\tilde{t}) = a\omega(\tilde{t})^2 + b\omega(\tilde{t}) + c \tag{4}$$

$$\frac{d\dot{\omega}(\tilde{t})}{d\tilde{t}} = a\omega(\tilde{t})^2 + b\omega(\tilde{t}) + c \tag{5}$$

Now we can see that the t_{stop}/t^*_{stop} is the same as ρ_{new}/ρ_{old} equation just scaled differently along the x-axis. Looking at the plot in figure 7, we can predict the stop time for the fidget spinner if it were made out of diamond or gold. Knowing our current fidget spinner is made of ABS plastic where $\rho \approx 1.2g/cm^3$ and $\rho \approx 3.5g/cm^3$ for diamond and $\rho \approx 19.32g/cm^3$ for gold, we can find our two density ratios: 2.917 and 16.1 for diamond and gold, respectively. Plugging in these two values into the function given by figure 7, we get 241.5 seconds and 1333.08 seconds for diamond and gold respectively.

$$t_{new} = t_{old}\frac{\rho_{new}}{\rho_{old}} \tag{6}$$

7 Conclusion

Changing the density is an effective way of letting the fidget spinner spin longer as it is a linear relationship between the ratio of the change in density and the ratio of the time it takes to come to rest.