Eleanor Barry

CSCE 350 Section 002

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Homework 4

2.4.1

a.) $x(n) = x(n-1) + 5$ for $n > 1$, $x(1) = 0$

$x(n) = x(n-2) + 5 + 5$

$x(n) = x(n-3) + 5 + 5 + 5$

$\downarrow$

$x(n) = x(n-i) + 5i$

$n - i = 1$, $i = n - 1$

$x(n) = x(1) + 5(n-1)$

$x(n) = 0 + 5n - 5 \rightarrow \boxed{x(n) = 5(n-1)}$

b.) $x(n) = 3x(n-1)$ for $n > 1$, $x(1) = 4$

$x(n) = 3^2 x(n-2)$

$x(n) = 3^3 x(n-3)$

$\downarrow$

$x(n) = 3^i x(n-i)$

$n - i = 1$, $i = n - 1$

$x(n) = 3^{n-1} x(1) \longrightarrow \boxed{x(n) = (4)(3^{n-1})}$

c.) $x(n) = x(n-1) + n$ for $n > 0$, $x(0) = 0$

$x(n) = x(n-2) + (n-1) + n$

$x(n) = x(n-3) + (n-2) + (n-1) + n$

$\downarrow$

$x(n) = x(n-i) + (n-(i-1)) + (n-(i-2)) \ldots + n$

$x(n) = x(0) + 1 + 2 + 3 + \ldots + n \rightarrow \boxed{x(n) = \frac{n(n+1)}{2}}$

d.) $x(n) = x(n/2) + n$ for $n > 1$, $x(1) = 1$ (solve for $n = 2^k$)

$x(n) = x(2^{k-1}) + 2^k$

$x(n) = x(2^{k-2}) + 2^{k-1} + 2^k$

$x(n) = x(2^{k-3}) + 2^{k-2} + 2^{k-1} + 2^k$

$\downarrow$

$x(n) = x(2^{k-i}) + 2^{k-i+1} + 2^{k-1+2} \ldots + 2^k$

$x(n) = x(2^{k-k}) + 2^{k-k+1} + 2^{k-k+2} + \ldots + 2^k$

$x(n) = 1 + 2^1 + 2^2 + \ldots + 2^k = 2^{k+1} - 1$

$x(n) = 2(2^k) - 1 \rightarrow \boxed{x(n) = 2n - 1}$

e.) $x(n) = x(n/3)+1$ for $n>1$, $x(1)=1$ (Solve for $n=3^k$)

$x(n) = x(3^{k-1})+1$

$x(n) = x(3^{k-2})+2$

$x(n) = x(3^{k-3})+3$

$\downarrow$

$x(n) = x(3^{k-i})+i$

$x(n) = x(3^{k-k})+k$

$x(n) = x(1)+k \rightarrow \boxed{x(n) = 1+\log_3(n)}$

2.4.3

a.) Solve a recurrance relation for algorithm

$M(n) = M(n-1)+2$

$M(n) = M(n-2)+2+2$

$M(n) = M(n-3)+2+2+2$

$M(n) = M(n-4)+2+2+2+2$

$\downarrow$

$M(n) = M(n-i)+2i$

$\downarrow$

$M(n) = M(1)+2(n-1) \rightarrow \boxed{M(n) = 2(n-1)}$

b.) Nonrecursive algorithm for computing sum

```
S=1
for i=2 to n
    S=S+(i)(i)(i)
return S
```

$$\sum_{i=2}^{n} 2 = 2\sum_{i=2}^{n} 1 = \boxed{2(n-1)}$$

The nonrecursive algorithm returns the same result as the recurrance relation.

2.4.4

a.) Recurrance relation for values and solve

$\boxed{Q(n)=Q(n-1)+2n-1 \text{ for } n>1, Q(1)=1}$

↓

$Q(2)=1+2(2)-1=4$

$Q(3)=4+2(3)-1=9$

$Q(4)=9+2(4)-1=16$

↓

$Q(n)=n^2$ check: $Q(n)=Q(n-1)^2+2n-1=n^2$

$Q(1)=1^2=1$ ✓ so $\boxed{Q(n)=n^2}$

b.) Recurrance relation for # of multiplications and solve

$\boxed{M(n)=M(n-1)+1 \text{ for } n>1, M(1)=0}$

↓

$M(2)=0+1=1$

$M(3)=1+1=2$

$M(4)=2+1=3$

↓

$M(n)=n-1$

check: $M(1)=1-1=0$ ✓ so $\boxed{M(n)=n-1}$

c.) Recurrance relation for additions/subtractions and solve

$\boxed{C(n)=C(n-1)+3 \text{ for } n>1, C(1)=0}$

↓

$C(2)=0+3=3$

$C(3)=3+3=6$

$C(4)=6+3=9$

↓

$C(n)=3(n-1)$

check: $C(2)=3(1)=3$ ✓ so $\boxed{C(n)=3(n-1)}$