Eleanor Barry

CSCE 350 Section 002

16 February 2021

Homework 3

2.2.2

a) True because the running time is $0(n^2)$ and so the condition $n^2 <= C * n^3$ is true for C.
b) True because the running time is $0(n^2)$ and so the condition $n^2 <= C * n^2$ is true for C.
c) False because there is no C or D for which $C * n^3 <= n^2 <= D * n^3$ for the equation given.
d) True because $a(n) = n^2$ and $b(n) = n$ so $n^2 >= C * n$ is true for C.

2.2.4

a) These values do not prove this fact with mathematical certainty because the order of growth is determined by Omicron, Omega, and Theta when n goes to infinity.
b)

$$\lim_{n\to\infty} \frac{\log(n)}{n} = \lim_{n\to\infty} \frac{(\log_2(n))'}{(n)'} = \log_2(e) \lim_{n\to\infty} \frac{1}{n} = \log_2(e)\left(\frac{1}{\infty}\right) = 0$$

therefore $\log(n) < n$ for order of growth

$$\lim_{n\to\infty} \frac{n}{(n)\log(n)} = \lim_{n\to\infty} \frac{1}{\log_2(n)} = \frac{1}{\log_2(\infty)} = 0$$

therefore $n < n\log(n)$ for order of growth

$$\lim_{n\to\infty} \frac{(n)\log(n)}{n^2} = \lim_{n\to\infty} \frac{\log(n)}{n} = \log_2(e) \lim_{n\to\infty} \frac{1}{n} = \log_2(e)\left(\frac{1}{\infty}\right) = 0$$

therefore $n\log(n) < n^2$ for order of growth

$$\lim_{n\to\infty} \frac{n^2}{n^3} = \lim_{n\to\infty} \frac{1}{n} = \frac{1}{\infty} = 0$$

therefore $n^2 < n^3$ for order of growth

$$\lim_{n\to\infty} \frac{n^3}{2^n} = \lim_{n\to\infty} \frac{(n^3)'}{(2^n)'} = \frac{3}{\ln(2)} \lim_{n\to\infty} \frac{(n^2)'}{(2^n)'} = \frac{6}{\ln^2(2)} \lim_{n\to\infty} \frac{(n)'}{(2^n)'} = \frac{6}{\ln^3(2)} \lim_{n\to\infty} = \frac{1}{2^n} = 0$$

therefore $n^3 < 2^n$ for order of growth

$$\lim_{n\to\infty} \frac{2^n}{n!} = \lim_{n\to\infty} \frac{2^n}{\sqrt{2\pi n}\left(\frac{n}{e}\right)^n} = \lim_{n\to\infty} \frac{1}{\sqrt{2\pi n}}\left(\frac{2e}{n}\right)^n = 0$$

therefore $2^n < n!$ for order of growth

Therefore, the functions are listed in increasing order of their order of growth

2.3.1

a.) $1+3+7+\ldots+999 = \sum_{i=1}^{500} 2i-1 = (2)\left(\frac{(500)(501)}{2}\right) - 500 = 250{,}500 - 500 = \boxed{250{,}000}$

b.) $2+4+8+16+\ldots+1024 = \sum_{i=1}^{10} 2^i - 1 = 2^{11} - 1 - 1 = 2{,}048 - 2 = \boxed{2{,}046}$

c.) $\sum_{i=3}^{n+1} 1 = (n+1) - 3 + 1 = n + 1 - 2 = \boxed{n-1}$

d.) $\sum_{i=3}^{n+1} i = \frac{(n+1)(n+2)}{2} - 3 = \boxed{\frac{n^2+3n-4}{2}}$

e.) $\sum_{i=0}^{n-1} (i+1) = \frac{n(n-1)}{2} + n - 1 + 1 = n + \boxed{\frac{n(n-1)}{2}}$

f.) $\sum_{j=1}^{n} 3^{j+1} = 3\left(\frac{3^{n+1}-1}{3-1} - 1\right) = 3\left(\frac{3^{n+1}-1}{2} - \frac{2}{2}\right) = \boxed{3\left(\frac{3^{n+1}-3}{2}\right)}$

g.) $\sum_{i=1}^{n} \sum_{j=1}^{n} ij = \sum_{i=1}^{n} i \sum_{j=1}^{n} j = \left(\frac{n(n+1)}{2}\right)\left(\frac{n(n+1)}{2}\right) = \boxed{\frac{n^2(n+1)^2}{4}}$

h.) $\sum_{i=1}^{n} \frac{1}{i(i+1)} = \sum_{i=1}^{n} \left(\frac{1}{i} - \frac{1}{i+1}\right) = \frac{1}{1} - \frac{1}{n+1} = \boxed{\frac{n}{n+1}}$

2.3.2

a.) $\sum_{i=0}^{n-1} (i^2+1)^2 = \sum_{i=0}^{n-1} i^4 + \sum_{i=0}^{n} 1 + \sum_{i=0}^{n-1} i^2 = \sum_{i=0}^{n} i^4 + (n-1+1) + \frac{2n(n+1)(2n+1)}{6}$

$\sum_{i=0}^{n} i^4 = \frac{2n(n+1)(2n+1)}{6}$

$= \frac{1}{30} n^5 + n + \frac{2n(n+1)(2n+1)}{6} = \frac{1}{30} n^5 + n + \frac{1}{3} n^3 = \Theta(n^5) + \Theta(n) + \Theta(n^3) \approx \boxed{\Theta(n^5)}$

b.) $\sum_{i=2}^{n-1} \lg i^2 = 2\sum_{i=2}^{n-1} \lg i = 2\sum_{i=1}^{n-1} \lg i - 2\log_2(n)$

$= \Theta(n\log_2(n)) - \Theta(\log_2(n)) \approx \boxed{\Theta(n\log_2(n))}$

c.) $\sum_{i=1}^{n} (i+1)(2^{i-1}) = \frac{1}{2}\left[\sum_{i=1}^{n} (i)(2^i) + \sum_{i=0}^{n} (2^i)\right]$

$= \Theta(n2^n) + \Theta(2^n) \approx \boxed{\Theta(n2^n)}$

d.) $\sum_{i=0}^{n-1} \sum_{j=0}^{i-1} (i+j) = \sum_{i=0}^{n-1} \sum_{j=0}^{i-1} i + \sum_{j=0}^{i-1} j = \sum_{i=0}^{n-1} (i) \sum_{j=0}^{i-1} (i)(1) + \frac{(i-1)(i-1+1)}{2} = \sum_{i=0}^{n-1} (i)(i-1+0+1) + \frac{(i-1)(i)}{2}$

$S = \sum_{i=0}^{n-1} \frac{(i-1)(i)}{2} = \sum_{i=0}^{n-1} \left(\frac{3i^2-i}{2}\right) = \frac{3}{2}\sum_{i=0}^{n-1} (i^2) - \frac{1}{2}\sum_{i=0}^{n-1} (i)$

$= \Theta(n^3) - \Theta(n^2) \approx \boxed{\Theta(n^3)}$

2.3.6

a) This algorithm computes whether the matrix is asymmetric and symmetric and returns a Boolean. True means that the matrix is a symmetric matrix and false means that the matrix is an asymmetric matrix.
b) Its basic operation is a comparison of two matrix elements.
c) The basic operation is executed (n(n-1))/2 times worst case. I found this by solving $\sum_{i=0}^{n-2} \sum_{j=i+1}^{n-1} 1$
d) The efficiency class of this algorithm is $\Theta(n^2)$.
e) The given algorithm does not need any improvements because in worst case the runtime is still optimal for the algorithm.

2.3.12

$n=0$, squares $=1$ $n=1$, squares $=5$ $n=2$, squares $=13$ $n=3$, squares $=25$ etc.

$2\sum_{i=1}^{n}(2i-1)+(2n+1) = \boxed{2n^2+2n+1}$