

Question 5 - Homework 8 - Number Systems

Question: For the following two functions f and g , determine $g \circ f$ and $f \circ g$. For each of the two compositions state clearly what the domain, codomain and the rule are.

$$\begin{aligned} f : \mathbb{R} &\rightarrow \mathbb{R} \\ x &\mapsto \sin(3x) \\ g : \mathbb{R} &\rightarrow \mathbb{R} \\ x &\mapsto \begin{cases} e^{\frac{1}{x+2}} & , \text{ if } x \neq -2 \\ \pi & , \text{ if } x = -2 \end{cases} \end{aligned}$$

Solution:

When calculating $f \circ g$, we know $f \circ g = f \circ g(x) = f(g(x))$, where the function f is applied first, then g . Doing this for our corresponding functions, f and g , we get:

$$\begin{aligned} (f \circ g) : \mathbb{R} &\rightarrow [-1, 1] \\ x &\mapsto \begin{cases} \sin(3e^{\frac{1}{x+2}}) & x \neq -2 \\ \sin(3\pi) & x = -2 \end{cases} \end{aligned}$$

where the domain for $f \circ g$ is \mathbb{R} , the codomain is $[-1,1]$, and the rule is $f \circ g$, as defined above.

Similarly, for $g \circ f$, we know $g \circ f = g \circ f(x) = g(f(x))$, where the function g is applied first, then f . Doing this for our corresponding functions, g and f , we get:

$$\begin{aligned} f : \mathbb{R} &\rightarrow [e^{\frac{1}{3}}, e] \\ x &\mapsto e^{\frac{1}{\sin(3x)+2}} \end{aligned}$$

where the domain for $f \circ g$ is \mathbb{R} . For the codomain, we notice that $\sin(3x)$ oscillates between -1 and 1 which therefore, means $e^{\frac{1}{x+2}}$ also oscillates, but between $\frac{1}{3}$ and 1. Hence, $e^{\frac{1}{x+2}}$ will also oscillate between $e^{\frac{1}{3}}$ and e . Therefore, the codomain of $f \circ g$ is $[e^{\frac{1}{3}}, e]$. The rule is $f \circ g$, as defined above.