Question 2 - Homework 7 - Number Systems

Show that for all a, $b \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $0.0ab0ab0ab..... = \dfrac{10a + b}{999}$

We can rewrite:

$$0.0ab0ab0ab..... = \frac{10a + b}{1000}(1 + \frac{1}{10^3} + \frac{1}{10^6} + ... + \frac{1}{10^{3n}})$$

$$= \frac{10a + b}{1000} \times \sum_{n=1}^{\infty} (\frac{1}{10})^{3n}$$

$$= \frac{10a + b}{1000} \times \frac{1}{1 - \dfrac{1}{10^3}}$$

$$= \frac{10a + b}{1000} \times \frac{1000}{999}$$

$$= \frac{10a + b}{999} \quad as\ required$$