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An exercise from the course History of Mathematics
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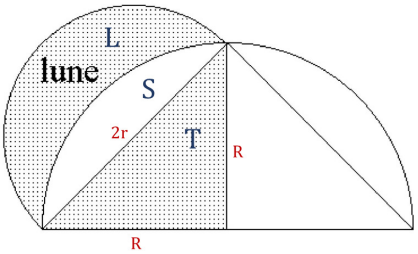
Area of lune

Lune of Hippocratus is a shape bounded by arcs of two circles, as shown in the picture here.



Let R be the radius of the bigger circle and r the radius of the smaller. Also, let T, S and L equal the areas of the triangle, segment and lune respectively (see the picture).

By simple geometry, we have

$$2r = \sqrt{2}R \Leftrightarrow r = \frac{\sqrt{2}}{2}R$$

and

$$S = \frac{1}{4}\pi R^2 - T.$$

Finally, we get

$$L = \frac{1}{2}\pi r^2 - S = \frac{1}{2}\pi(\frac{\sqrt{2}}{2}R)^2 - S = \frac{1}{4}\pi R^2 - (\frac{1}{4}\pi R^2 - T) = T.$$

Hence that area of the lune (L) equals the area of the triangle (T).