Elli Kiiski

An exercise from the course History of Mathematics
University of Helsinki

Menelaus's theorem

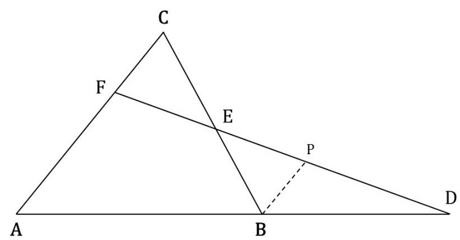

Figure 1: The setting for Menelaus's theorem.

Menelaus's theorem states that for a line intersecting triangle ABC in points D, E and F (see figure 1) holds

$$\frac{AD}{BD} \cdot \frac{BE}{CE} \cdot \frac{CF}{AF} = 1\,.$$

Let's prove this by drawing a line BP parallel to line AC and hence forming two sets of similar triangles: $ADF \sim BDP$ and $BPE \sim FEC$.

By the similarity we have $\frac{AF}{BP} = \frac{AD}{BD}$ and $\frac{BP}{CF} = \frac{BE}{CE}$. Multiplying these two equations with each other we get

$$\frac{AF}{BP} \cdot \frac{BP}{CF} = \frac{AD}{BD} \cdot \frac{BE}{CE} \quad \Leftrightarrow \quad \frac{AF}{CF} = \frac{AD}{BD} \cdot \frac{BE}{CE} \quad \Leftrightarrow \quad 1 = \frac{AD}{BD} \cdot \frac{BE}{CE} \cdot \frac{CF}{AF}\,,$$

proving the Menelaus's theorem.