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Edited version of an exercise from the course History of Mathematics
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Beer pong numbers

Beer pong numbers (also known as triangular numbers) are figurative numbers that can be represented by arranging red cups (or evenly spaced dots) into a shape of an equilateral triangle.

The series of beer pong numbers starts with $1, 3, 6, 10, 15, 21...$ and the nth pong number can be constructed followingly

$$T_n = 1 + 2 + 3 + ... + n = \sum_{k=1}^{n} k = \frac{n(n+1)}{2}.$$

Let's show that the sum of any two consecutive beer pong numbers is always a square number. We have

$$\begin{aligned} T_n + T_{n+1} &= \frac{n(n+1)}{2} + \frac{(n+1)(n+2)}{2} \\ &= \frac{1}{2}(n^2 + n + n^2 + 3n + 2) \\ &= n^2 + 2n + 1 \\ &= (n+1)^2, \end{aligned}$$

and as we see, the sum indeed makes a square of $(n + 1)$. Also, we notice that

$$T_{n+1} - T_n = \sum_{k=1}^{n+1} k - \sum_{k=1}^{n} k = n + 1,$$

which means that the sum of two consecutive beer pong numbers equals the square of their difference. Formally

$$T_n + T_{n+1} = (T_{n+1} - T_n)^2.$$

Figure 1 shows an example with $n = 2$, commonly known as overtime layout ($T_2 = 3$) plus 1vs1 layout ($T_3 = 6$).

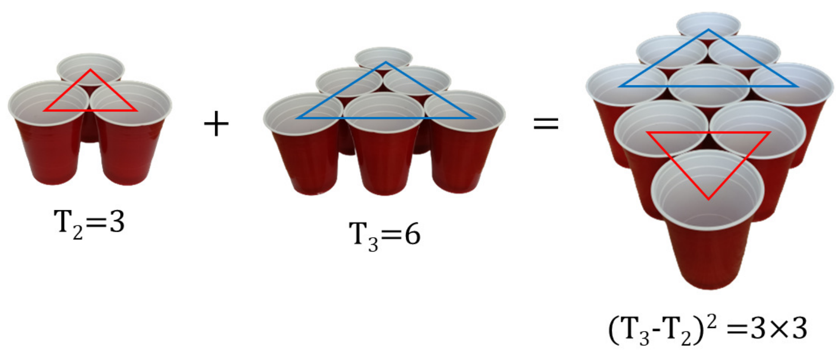

Figure 1: Sum of two consecutive beer pong numbers illustrated.