$$\int u\,dv = uv - \int v\,du.$$

- 1214. $\int \ln x\,dx.$
- 1215. $\int xe^{2x}\,dx.$
- 1216. $\int \operatorname{arctg} x\,dx.$
- 1217. $\int x\cos x\,dx.$
- 1218. $\int x\operatorname{arctg} x\,dx.$
- 1219. $\int \frac{x}{\sin^2 x}\,dx.$
- 1220. $\int \frac{\ln x}{x^2}\,dx.$
- 1221. $\int \sqrt{x}\ln x\,dx.$
- 1222. $\int x^n \ln x\,dx.$
- 1223. $\int \frac{x\cos x}{\sin^3 x}\,dx.$
- 1224. $\int \arcsin x\,dx.$

$\checkmark\ \ln f(x)$ $\quad \checkmark\ P_n(x)\cdot a^x$ $\quad \checkmark\ a^x \cdot \sin x$

$\arccos f(x)$ $\quad \checkmark\ P_n(x)\cdot \sin x$

$$\frac{A}{(x^2+a^2)^n}$$

$$\int \underbrace{\ln x}_{u}\, \underbrace{dx}_{dv} = \left| \begin{matrix} \ln x = u & du = d\ln x = \frac{1}{x}\cdot dx \\ dx = d & V = x \end{matrix} \right| = \ln x\cdot x - \int x\cdot\frac{1}{x}\,dx =$$

$$= x\cdot\ln x - \int dx = x\cdot\ln x - x +$$

$$\int \underset{u}{x}\cdot \underbrace{e^{2x}\cdot dx}_{dv} = \left| \begin{matrix} u = x & du = dx \\ e^{2x}\cdot dx = dV & V = \frac{1}{2}e^{2x} \end{matrix} \right. = x\cdot\frac{1}{2}e^{2x} - \frac{1}{2}\int e^{2x}\cdot dx = x\cdot\frac{1}{2}e^{2x} - \frac{1}{2}\cdot\frac{1}{2}e^{2x} + C$$

$$\int e^{2x}dx = \frac{1}{2}\int e^{2x}\,d\,2x = \frac{1}{2}e^{2x}$$

$$\int \underset{u}{x}\cdot \cos x\,dx = \left| \begin{matrix} u = x & du = dx \\ \cos x\,dx = dV & v = \sin x \end{matrix} \right| = x\cdot\sin x - \int \sin x\,dx = x\cdot\sin x + \cos x + C$$

1229. $\int e^x \cos x\, dx. = \left| \begin{matrix} u = \cos x & du = -\sin x\, dx \\ e^x \cdot dx = dv & v = e^x \end{matrix} \right| =$

$= \cos x \cdot e^x + \int e^x \cdot \sin x\, dx = \left| \begin{matrix} u = \sin x & du = d\sin x = \cos x\, dx \\ e^x \cdot dx = dv & v = e^x \end{matrix} \right|$

1229. $\int e^x \cos x\, dx. = \cos x \cdot e^x + \sin x \cdot e^x - \int e^x \cdot \cos x\, dx$

$$2 \int e^x \cdot \cos x\, dx = \cos \cdot e^x + \sin x \cdot e^x$$

$$\int e^x \cdot \cos x\, dx = \frac{\cos x \cdot e^x + \sin x \cdot e^x}{2} + C$$

$$\int \underbrace{\arccos x}_{u}\, dx = \begin{matrix} \arccos x = u & du = d\arccos x = -\frac{1}{\sqrt{1-x^2}}\, dx \\ dx = d & V = x \end{matrix} =$$

$$= x \cdot \arccos x + \int x \cdot \frac{1}{\sqrt{1-x^2}}\, dx = x \cdot \arccos x + \frac{1}{2} \int \frac{dx^2}{\sqrt{1-x^2}} =$$

$$= x \cdot \arccos x - \frac{1}{2} \int (1-x^2)^{-\frac{1}{2}} d(1-x^2) =$$

$$= x \cdot \arccos x - \frac{1}{2} \cdot \frac{(1-x^2)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C$$

$$\int \sin \left(\sqrt{\ln x + 5}\right) dx$$

$$\int \frac{1\,dx}{(x^2+16)^2} = \frac{1}{16}\int \frac{16\cdot dx}{(x^2+16)^2} = \frac{1}{16}\int \frac{\left((x^2+16)-x^2\right)dx}{(x^2+16)^2} = \frac{1}{16}\int\left(\frac{1}{x^2+16} - \frac{x^2}{(x^2+16)^2}\right)dx$$

$$= \frac{1}{16}\left[\int \frac{dx}{x^2+16} - \int \frac{x^2\,dx}{(x^2+16)^2}\right]$$

$$\int \frac{x^2\,dx}{(x^2+16)^2} = \int \frac{x\cdot x\,dx}{(x^2+16)^2} = \left| \begin{array}{ll} u = x & du = dx \\ dv = \frac{x\cdot dx}{(x^2+16)^2} = \frac{1}{2}\frac{d(x^2+16)}{(x^2+16)^2} = -\frac{1}{2}\,d\,\frac{1}{x^2+16} & \\ v = -\frac{1}{2}\frac{1}{x^2+16} & \end{array}\right.$$

$$/ \int \frac{x\,dx}{(x^2+16)^2} - \frac{1}{2}\int \frac{d(x^2+16)}{(x^2+16)^2} = \frac{1}{2}\int (x^2+16)^{-2}\,d(x^2+16) = -\frac{1}{2}\frac{1}{x^2+16} \,/$$

$$= x\cdot\left(-\frac{1}{2}\cdot\frac{1}{x^2+16}\right) + \frac{1}{2}\int \frac{1}{x^2+16}\cdot dx$$

1216. $\int \operatorname{arctg} x\, dx$. 1222. $\int x^n \ln x\, dx$.

1232. $\int \sqrt{1-x^2}\, dx$. 1235. $\int e^{\arcsin x}\, dx$.

1233. $\int \cos(\ln x)\, dx$.

$$\int \operatorname{arctg} x\, dx = \left| \begin{array}{ll} u = \operatorname{arctg} x & du = (\operatorname{arctg} x)'\cdot dx = \frac{1}{1+x^2}\,dx \end{array}\right| -$$

$\int \operatorname{arctg} x \quad \Big| \; dv = dx \quad v = x$

$$= x \cdot \operatorname{arctg} x - \int \frac{x\,dx}{1+x^2} = x \cdot \operatorname{arctg} x - \frac{1}{2}\int \frac{d(x^2+1)}{1+x^2} =$$

$$= x \cdot \operatorname{arctg} x - \frac{1}{2}\ln(x^2+1) + C$$

$$\int x^3 \cdot \ln x\,dx = \left| \begin{array}{lll} u = \ln x & du = d\ln x = \frac{1}{x}dx & \\ dv = x^3 dx & dv = d\frac{x^4}{4} & v = \frac{1}{4}x^4 \end{array} \right| = \ln x \cdot \frac{1}{4}x^4 - \frac{1}{4}\int x^4 \cdot \frac{1}{x}dx =$$

$$= \frac{1}{4}x^4 \ln x - \frac{1}{4}\int x^3 dx = \frac{1}{4}x \ln x - \frac{1}{4}\frac{x^4}{4} + C$$

1232. $\int \sqrt{1-x^2}\,dx$. *1235. $\int e^{\arcsin x}\,dx$.
1233. $\int \cos(\ln x)\,dx$.

$$\int \cos(\ln x)dx = \left| \begin{array}{ll} u = \cos(\ln x & du = d\cos(\ln x = -\sin(\ln x \cdot \frac{1}{x}dx \\ dx = d & v = x \end{array} \right|$$

$$= \cos(\ln x)\,x - \int \cancel{x}\sin(\ln x)\cdot\frac{1}{\cancel{x}}dx = \left| \begin{array}{ll} u = \sin(\ln x) & du = \cos(\ln x)\cdot\frac{1}{x}dx \\ dv = dx & v = x \end{array} \right|$$

$$= \cos(\ln x)\cdot x + \sin(\ln x)\cdot x - \int \cancel{x}\cos(\ln x)\cdot\frac{1}{\cancel{x}}dx$$

$$2\int \cos(\ln x)dx = \cos(\ln x)\cdot x + \sin(\ln x)\cdot x$$

$$\int \cos(\ln x)\,dx = \frac{(\cos(\ln x) + \sin(\ln x))\cdot x}{2} + C$$

$$\int \frac{(Ax+B)\,dx}{ax^2+bx+}$$

$\dfrac{P_n(x)}{Q_m(x)}$ $\quad n < m \qquad n \geq m$ выделить целую часть

1254. $\displaystyle\int \frac{5x^3+9x^2-22x-8}{x^3-4x}\,dx.$

$$\begin{array}{r|l} 5x^3+9x^2-22x-8 & x^3-4x \\ -\;5x^3-20x & \overline{5} \\ \hline 9x^2-2x-8 & \end{array}$$

$$\begin{array}{r|l} 5 & 3 \\ 3 & \overline{1} \\ \hline 2 & \end{array} \qquad \frac{5}{3} = 1 + \frac{2}{3}$$

$$\frac{5x^3+9x^2-22x-8}{x^3-4x} = 5 + \frac{9x^2-2x-8}{x^3-4x}$$

$$\frac{P_n(x)}{Q_m(x)} = \frac{P_n(x)}{(x-a)^\alpha (x-b)^\beta \dots (x^2+cx+d)^\gamma \dots} = \frac{A}{x-a} + \frac{I}{(x-a)^2} + \dots + \frac{C}{(x-a)^\alpha} + \frac{D}{x-b} + \frac{E}{(x-b)^2} + $$

$$\alpha+\beta+\dots+\gamma = \dots$$

$$+ \frac{F}{(x-b)^\beta} + \dots\dots - \frac{Mx+K}{x^2+cx+d} + \frac{Lx+G}{(x^2+cx+d)^2} + \dots + \frac{}{(x^2+cx+d)^\gamma} + \dots$$

$9x^2-2x-8 \quad 9x^2 \; 2x \; 8$

$$\frac{5x^3+9x^2-22x-8}{x^3-4x} = 5 + \frac{9x^2-2x-8}{x^3-4x}$$

$$\boxed{\frac{9x^2-2x-8}{x^3-4x}} = \frac{9x^2-2x-8}{x\cdot(x^2-4)} = \frac{9x^2-2x-8}{x\cdot(x-2)(x+2)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+2} =$$

$(x-2)(x+2)$ $\quad x(x+2)$ $\quad x(x-2)$

$$= \boxed{\frac{A\cdot(x-2)(x+2) + Bx\cdot(x+2) + C\cdot x\cdot(x-2)}{x\cdot(x-2)(x+2)}}$$

$$9x^2-2x-8 = A\cdot(x-2)(x+2) + Bx\cdot(x+2) + C\cdot x\cdot(x-2)$$

$$9x^2-2x-8 = Ax^2-4A+Bx^2+B2x+Cx^2-C\cdot 2x$$

$$9x^2-2x-8 = x^2(A+B+C) + x\cdot(2B-2C) + (-4A$$

$$\begin{array}{l} x^2 \\ x \\ x^0 \end{array} \begin{cases} 9 = A+B+C \\ -2 = 2B-2C \\ -8 = -4A \end{cases}$$

$$9x^2-2x-8 = A\cdot(x-2)(x+2) + Bx\cdot(x+2) + C\cdot x\cdot(x-2)$$

$x=0$ $\quad -8 = A\cdot(-4) \Rightarrow A=2$

$x=2$ $\quad 9\cdot 4 - 2\cdot 2 - 8 = B\cdot 2\cdot 4 \Rightarrow B=3$

$x=-2$ $\quad 9\cdot 4 + 2\cdot 2 - 8 = C\cdot(-2)\cdot(-4) \Rightarrow C=4$

$$\boxed{\frac{9x^2-2x-8}{x^3-4x}} - \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+2} = \boxed{\frac{2}{x} + \frac{3}{x-2} + \frac{4}{x+2}}$$

$$I = \int\left(5 + \frac{2}{x} + \frac{3}{x-2} + \frac{4}{x+2}\right)dx = 5x + 2\ln|x| + 3\ln|x-2| + 4\ln|x+2| + C$$

1255. $\int \frac{x^2+x-1}{x^3+x^2-6x}\,dx. = \frac{1}{6}\int\frac{dx}{x} + \frac{1}{2}\int\frac{dx}{x-2} + \frac{1}{3}\int\frac{dx}{x+3} = \frac{1}{6}\ln|x| + \frac{1}{2}\ln|x-2| + \frac{1}{3}\ln|x+3| + C$

$$\frac{x^2+x-1}{x^3+x^2-6x} = \frac{x^2+x-1}{x(x^2+x-6)} = \frac{x^2+x-1}{x(x-2)(x+3)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+3} -$$

$$= \frac{A(x-2)(x+3) + B\cdot x\cdot(x+3) + C\cdot x(x-2)}{x\cdot(x-2)(x+3)}$$

$$\boxed{x^2+x-1 - A(x-2)(x+3) + B\cdot x\cdot(x+3) + C\cdot x\cdot(x-2)}$$

$x=0 \quad -1=-6A \quad A=\frac{1}{6}$

$x=2 \quad 5=10B \quad B=\frac{1}{2}$

$x=-3 \quad 5=15C \quad C=\frac{1}{3}$