

### Question 1

1. Basic set operations and relations, pairs, tuples and Cartesian product  $\times$ . Theorem about the cardinality of a power set  $\mathcal{P}(A)$  of a finite set  $A$ .
2. Theorem: A countable union of countable sets is countable.
3. Canonical normal forms for propositional formulas: CCNF, CDNF. Theorem about reduction to CCNF.

### Question 2

1. Mappings, functions: everywhere defined, injective, surjective, bijective, inverse mapping. Theorem about cardinality of a finite Cartesian product.
2. Carrying of a function.
3. Canonical normal forms for propositional formulas: CCNF, CDNF. Theorem about conversion to CCNF.

### Question 3

1. Mappings, functions: everywhere defined, injective, surjective, bijective, inverse mapping. Properties of composition of mappings, uniqueness of inverse mapping.
2. Notion of  $\lambda$ -term: operators of application and abstraction.
3. Correctness theorem for the propositional logic.

### Question 4

1. Binary relations, composition of binary relations, associativity of composition of binary relations.
2. Reductions in  $\lambda$ -calculus:  $\alpha$ -reduction.
3. Completeness theorem for the propositional logic.

### Question 5

1. Inverse binary relation, connection with the inverse mapping.
2. Reductions in  $\lambda$ -calculus:  $\beta$ -reduction.
3. Structures of signature  $\sigma$ .

### Question 6

1. Properties of binary relations: reflexivity, symmetry, transitivity, antisymmetry. Definition and characterization of these properties.
2. Reductions in  $\lambda$ -calculus:  $\eta$ -reduction.
3. Congruence on the structure of signature  $\sigma$ .

### Question 7

1. Equivalence relations, examples of equivalences. Equivalence classes, properties of equivalence classes, set partitions, lemma about equivalence classes and partitions.
2. Term rewriting in lambda calculus: call-by-value and call-by-name strategies.
3. Quotient structure of structure  $\mathcal{M}$  by congruence  $\theta$ .

### Question 8

1. Partial orders, examples of partial orders. Least/greatest, minimal/maximal elements in partial orders, upper/lower boundary, supremum/infimum of a set. Posets and losets.
2. Redexes, normal form of a  $\lambda$ -term. Church-Rosser theorem (without proof).
3. Homomorphisms, epimorphisms and isomorphisms of structures.

### Question 9

1. Posets and losets, lattices, proposition about losets and lattices. Boolean lattices, examples of Boolean lattice.
2. Combinatory terms, combinatory calculus, theorem about completeness of  $SKI$  basis.
3. Kernel of a homomorphism, theorem about homomorphisms.

### Question 10

1. Closure of a binary relation relative to some property, uniqueness of a closure.
2.  $Y$ -combinator, it's properties.
3. Substructures and superstructures.

### Question 11

1. Reflexive and symmetric closures of a binary relation, their existence.
2. Church numbers: definition, successor function.
3. Predicate logic: terms and formulas of a given signature. Relation  $\models$  between structures, formulas and interpretations of variables (semantics of predicate calculus).

### Question 12

1. Transitive closure of a binary relation, it's existence.
2. Church numbers: addition and multiplication.
3. Predicate calculus of a given signature. Notions of linear proof and deduction tree. Provability characterization theorem.

**Question 13**

1. Adjacency matrix of a binary relation, Floyd-Marshall algorithm.
2. Modelling of recursion with  $Y$ -combinator.
3. Syntactical equivalence in predicate calculus, replacement theorem.

**Question 14**

1. Transitive closure of a binary relation, it's existence.
2. Propositional logic: propositional formulas, tautological, satisfiable and unsatisfiable formulas.
3. Prenex normal forms, theorem about conversion to PNF.

**Question 15**

1. Transitive closure of a binary relation, it's existence.
2. Semantically equivalent propositional formulas: distributivity, and De-Morgan laws.
3. Correctness theorem for the predicate calculus.

**Question 16**

1. Toposort of a binary relation. Kahn's algorithm.
2. Replacement theorem for propositional formulas (semantic).
3. Theorem about the existence of a model (without proof), the completeness theorem for the predicate calculus.

**Question 17**

1. Relation of equinumerosity of sets, properties of this relation.
2. Normal forms of propositional formulas: DNF, CNF. Theorem about conversion to the normal form.
3. Skolem normal form, theorem about Skolemization.

**Question 18**

1. Cantor-Bernstein's theorem (without proof), Cantor's theorem.
2. Zhegalkin polynomial (ANF), theorem about ANF existence and uniqueness.
3. Herbrandt normal form, theorem about Herbrandization.

**Question 19**

1. Sets: finite, countable, uncountable. Characterization of finite sets.
2. The sequential propositional calculus: linear proofs, deduction trees, characterization theorem about proofs.
3. Unifiers, most general unifiers.

**Question 20**

1. Theorem about cardinality of a Cartesian square of a countable set.
2. Syntactical equivalence  $\equiv$ , syntactic form of replacement theorem.
3. Horn clauses, resolution rule. Soundness theorem for the resolution rule.

**Question 21**

1. Inverse binary relation, connection with the inverse mapping.
2. Term rewriting in lambda calculus: call-by-value and call-by-name strategies.
3. Conditions of program correctness: partial and total. Floyd method.

**Question 22**

1. Closure of a binary relation relative to some property, uniqueness of a closure.
2. Y-combinator, it's properties.
3. Semantics for a programming language: semantics of data types and operational semantics.

**Question 23**

1. Transitive closure of a binary relation, it's existence.
2. Church numbers: addition and multiplication.
3. Hoare triples, axiomatic semantics for a programming language. Correctness and completeness of axiomatic semantic.