Linear Algebra cheat sheet

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1 Chapter 1

$\boxed{\text{Thm}}$ Given $\mathbf{A}\vec{x} = \vec{b}$, $\mathbf{A}$ is $m \times n$, the following statements are logically equivalent:

a) For each $\vec{b}$ in $\mathbb{R}^m$, $\mathbf{A}\vec{x} = \vec{b}$ has a solution.

b) Each $\vec{b}$ in $\mathbb{R}^m$ is a linear combination of columns of $\mathbf{A}$.

c) The columns of $\mathbf{A}$ span $\mathbb{R}^m$.

d) $\mathbf{A}$ has a pivot position in every row.

1.1 Transformations

$\boxed{\text{Def}}$ A transformation T is linear if:

a) $T(\vec{u} + \vec{v}) = T(\vec{u}) + T(\vec{v}), \forall \vec{u}, \vec{v} \in T$ check. does this mean each vector in the domain of T?

b) $T(c\vec{u}) = c\,T(\vec{u}), \forall c \in \mathbb{R}, \vec{u} \in T$

$\boxed{\text{Def}}$ A mapping $T : \mathbb{R}^n \to \mathbb{R}^m$ is **onto** $\mathbb{R}^m$ if each $\vec{b}$ in $\mathbb{R}^m$ is the image of at least one $\vec{x}$ in $\mathbb{R}^n$. This is assuming the mapping is Ax=b.

- pivots in every row iff columns span $\mathbb{R}^m$.

$\boxed{\text{Def}}$ A mapping $T : \mathbb{R}^n \to \mathbb{R}^m$ is **one-to-one** if each $\vec{b}$ in $\mathbb{R}^m$ is the image of at most one $\vec{x}$ in $\mathbb{R}^n$. This is assuming the mapping is Ax=b.

- pivots in every column iff columns are linearly independent.

2 Chapter 2

$\boxed{\text{Thm}}$ Matrix transpose, inverse identities:

a) $(\mathbf{A}^\intercal)^\intercal = \mathbf{A}$

b) $(\mathbf{A} + \mathbf{B})^\intercal = \mathbf{A}^\intercal + \mathbf{B}^\intercal$

c) $(r\,\mathbf{A})^\intercal = r\mathbf{A}^\intercal, \forall r \in \mathbb{R}$

d) $(\mathbf{AB})^\intercal = \mathbf{B}^\intercal\mathbf{A}^\intercal$

e) $(\mathbf{A}^{-1})^{-1} = \mathbf{A}$

f) $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$

g) $(\mathbf{A}^\intercal)^{-1} = (\mathbf{A}^{-1})^\intercal$

$\boxed{\text{Thm}}$ Inverse of a 2×2 matrix:

$$\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad \mathbf{A}^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$\boxed{\text{Thm}}$ Invertible Matrix Thereom: Given $\mathbf{A}$ is $n \times n$, the following statements are logically equivalent:

a) $\mathbf{A}$ is invertible.

b) $\mathbf{A}$ is row equivalent to $\mathbf{I}_{n\times n}$.

c) $\mathbf{A}$ has n pivot positions.

d) $\mathbf{A}\vec{x} = \vec{0}$ has only trivial solutions.

e) Columns of $\mathbf{A}$ form a linearly independent set.

f) $\mathbf{A}\vec{x} = \vec{b}$ has at least one solutions for each $\vec{b}$ in $\mathbb{R}^n$.

g) $\vec{x} \mapsto \mathbf{A}\vec{x}$ is one-to-one.

h) Columns of $\mathbf{A}$ span $\mathbb{R}^n$.

i) $\vec{x} \mapsto \mathbf{A}\vec{x}$ maps $\mathbb{R}^n$ onto $\mathbb{R}^n$.

j) $\exists\, \mathbf{C}_{n\times n}$ such that $\mathbf{CA} = \mathbf{I}$.

k) $\exists\, \mathbf{D}_{n\times n}$ such that $\mathbf{AD} = \mathbf{I}$.

l) $\mathbf{A}^\intercal$ is an invertible matrix.

m) Columns of $\mathbf{A}$ form a basis of $\mathbb{R}^n$.

n) Col $\mathbf{A} = \mathbb{R}^n$.

o) dim Col $\mathbf{A} = n$.

p) rank $\mathbf{A} = n$.

q) Nul $\mathbf{A} = \vec{0}$ check this...

r) dim Nul $\mathbf{A} = 0$.

s) 0 is not an eigenvalue of $\mathbf{A}$.

t) The determinant of $\mathbf{A}$ is not 0.