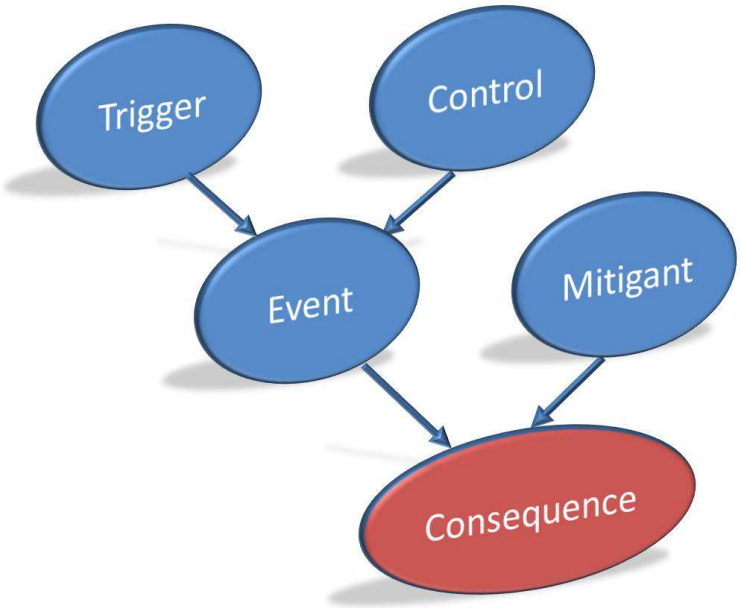
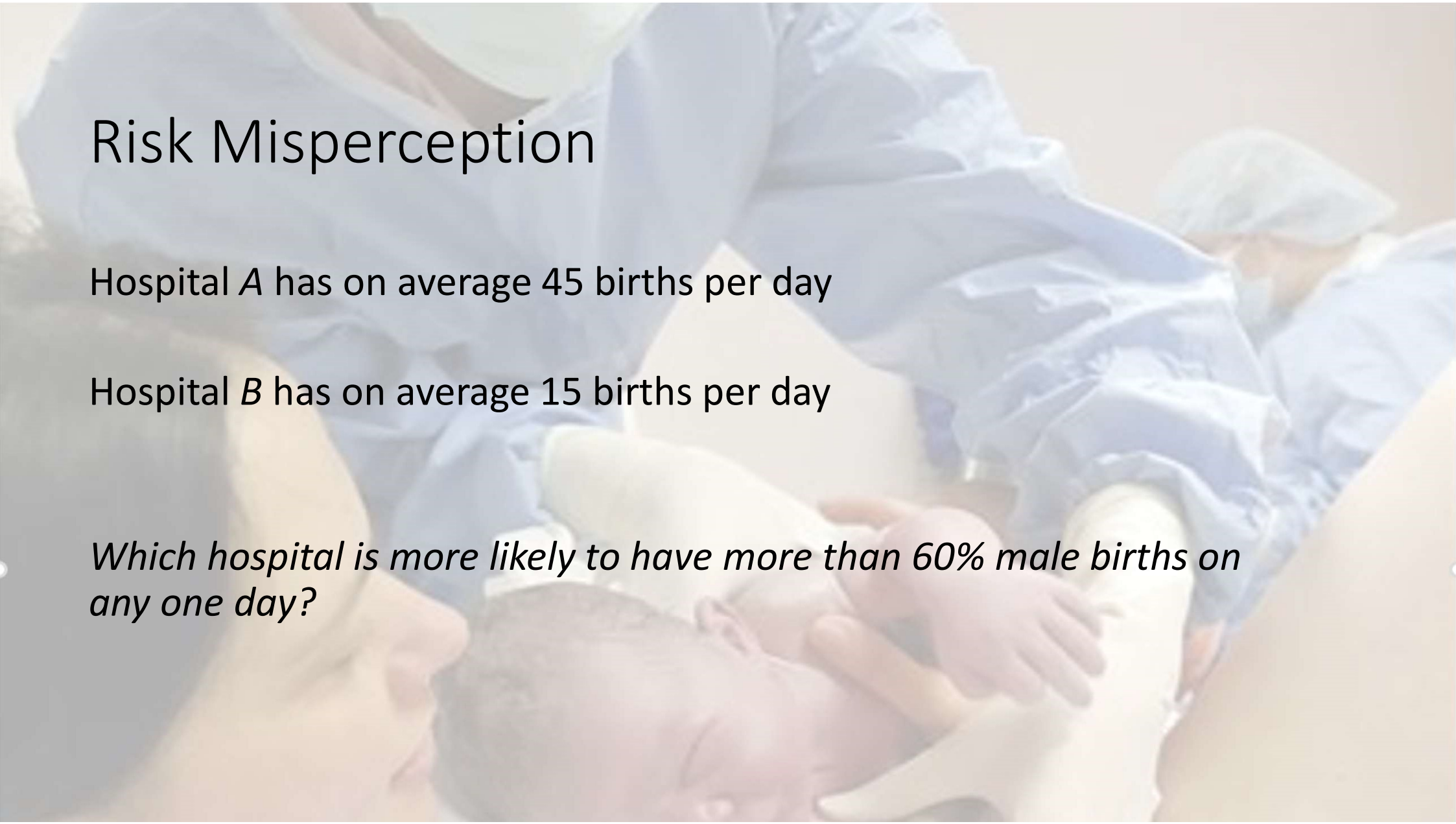


Risk and Decision Making for Data Science and AI

Lesson 5 Risk perception, framing and definitions

Norman Fenton

@ProfNFenton



Risk Misperception

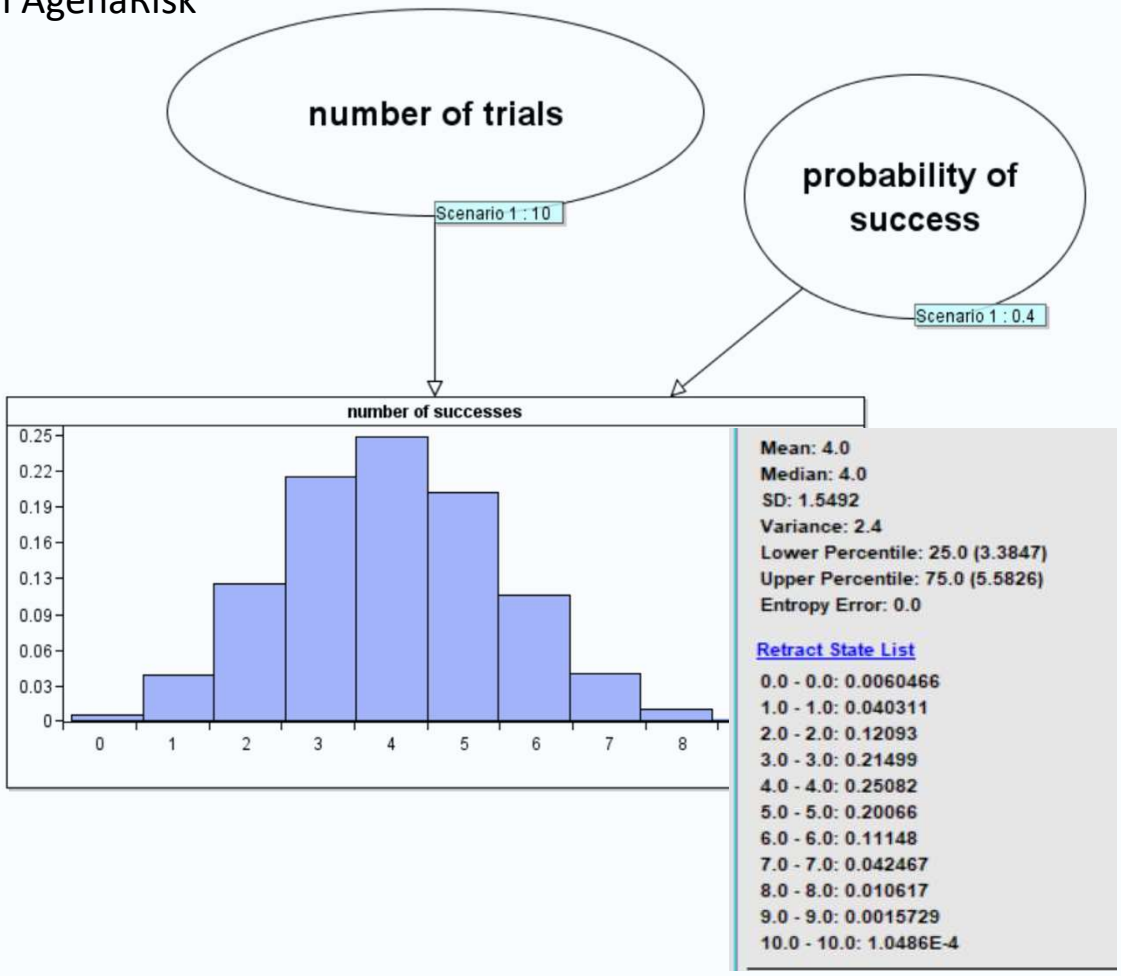
Hospital *A* has on average 45 births per day

Hospital *B* has on average 15 births per day

Which hospital is more likely to have more than 60% male births on any one day?

Binomial distribution

In AgenaRisk



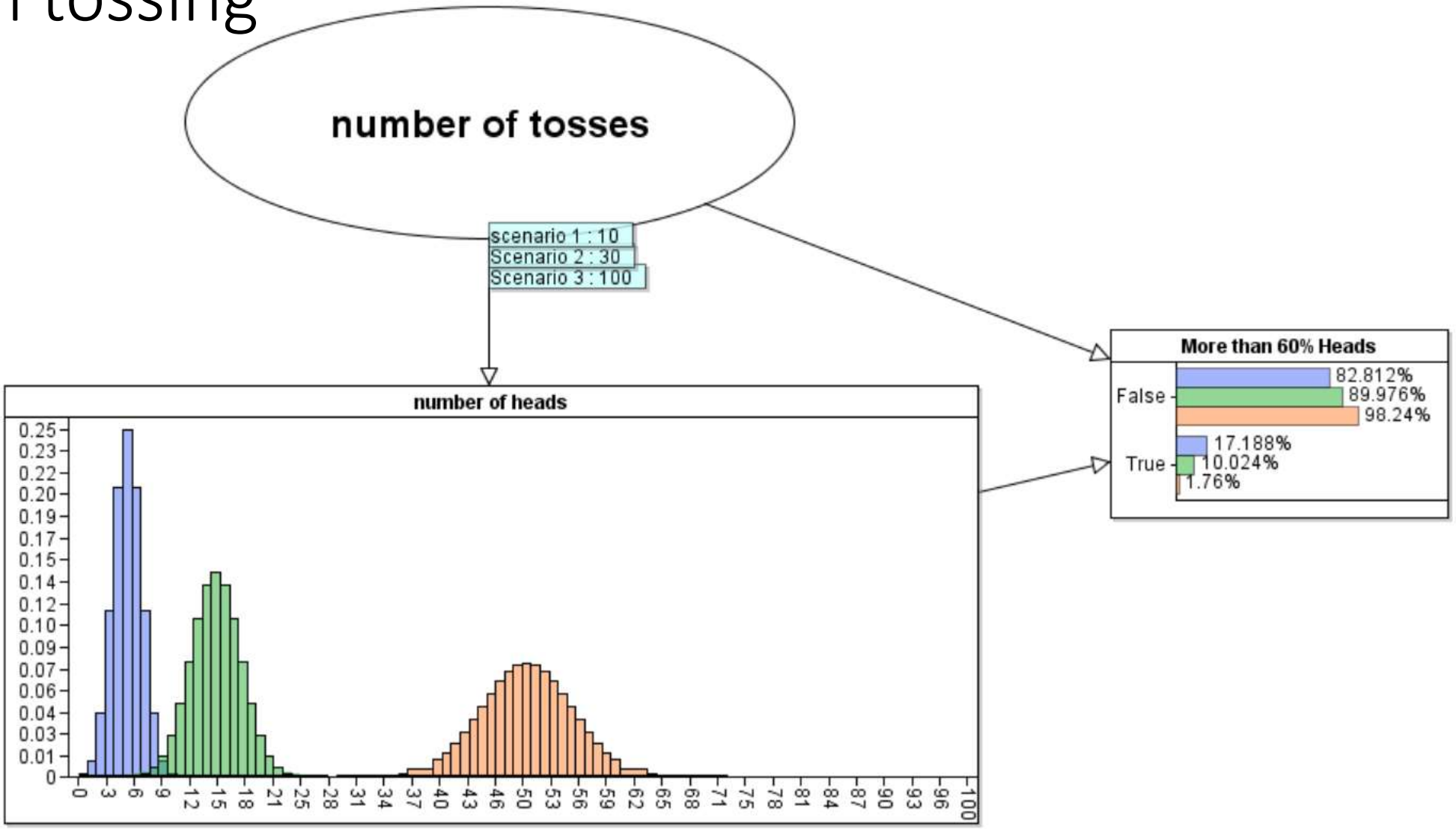
- Enter a value in each of the first three text boxes (the unshaded boxes).
- Click the **Calculate** button.
- The Calculator will compute Binomial and Cumulative Probabilities.

Probability of success on a single trial	0.4
Number of trials	10
Number of successes (x)	7
Binomial probability: P(X = x)	0.042467328
Cumulative probability: P(X < x)	0.9452381184
Cumulative probability: P(X ≤ x)	0.9877054464
Cumulative probability: P(X > x)	0.0122945536
Cumulative probability: P(X ≥ x)	0.0547618816

Calculate

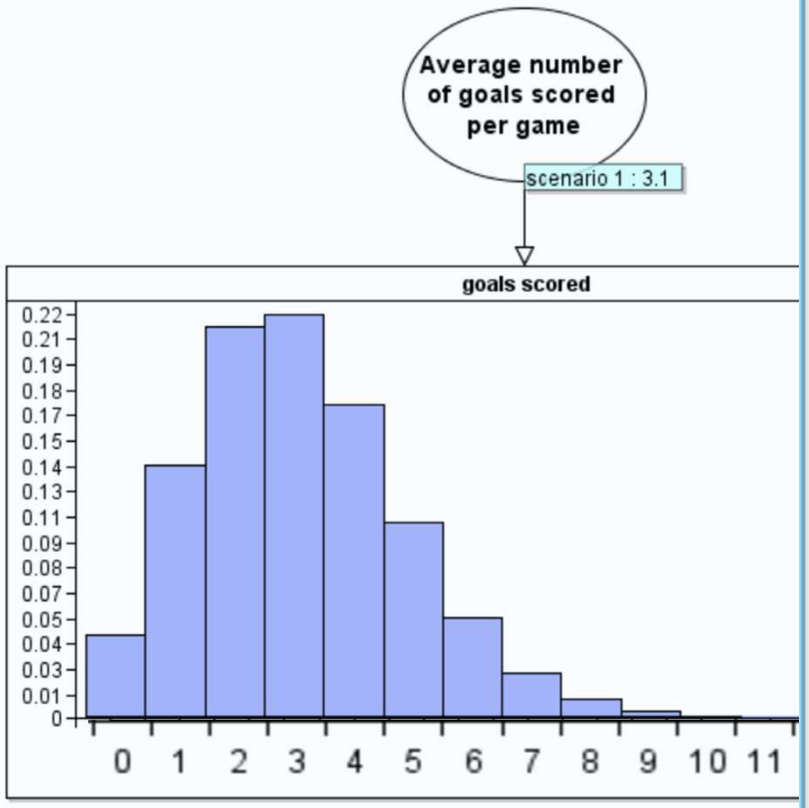
https://www.stattrek.com/online-calculator/binomial.aspx

Coin tossing



Poisson distribution

In AgenaRisk



scenario 1

Mean: 3.1
Median: 3.0
SD: 1.7607
Variance: 3.1
Lower Percentile: 25.0 (2.3017)
Upper Percentile: 75.0 (4.722)
Entropy Error: 0.0

Retract State List

0: 0.045049
1: 0.13965
2: 0.21646
3: 0.22368
4: 0.17335
5: 0.10748
6: 0.05553
7: 0.024592
8: 0.0095293
9: 0.0032823
10: 0.0010175
11: 2.8675E-4
12: 7.4078E-5
13: 1.7665E-5
14: 3.9115E-6
15: 8.0838E-7
16: 1.5662E-7
17: 2.8561E-8
18: 4.9188E-9
19: 8.0254E-10
20: 1.2439E-10
21: 1.8363E-11
22: 2.5875E-12
23: 3.4872E-13
24: 4.5075E-14
25: 5.5511E-15
26: 6.6613E-16
27: 1.1102E-16

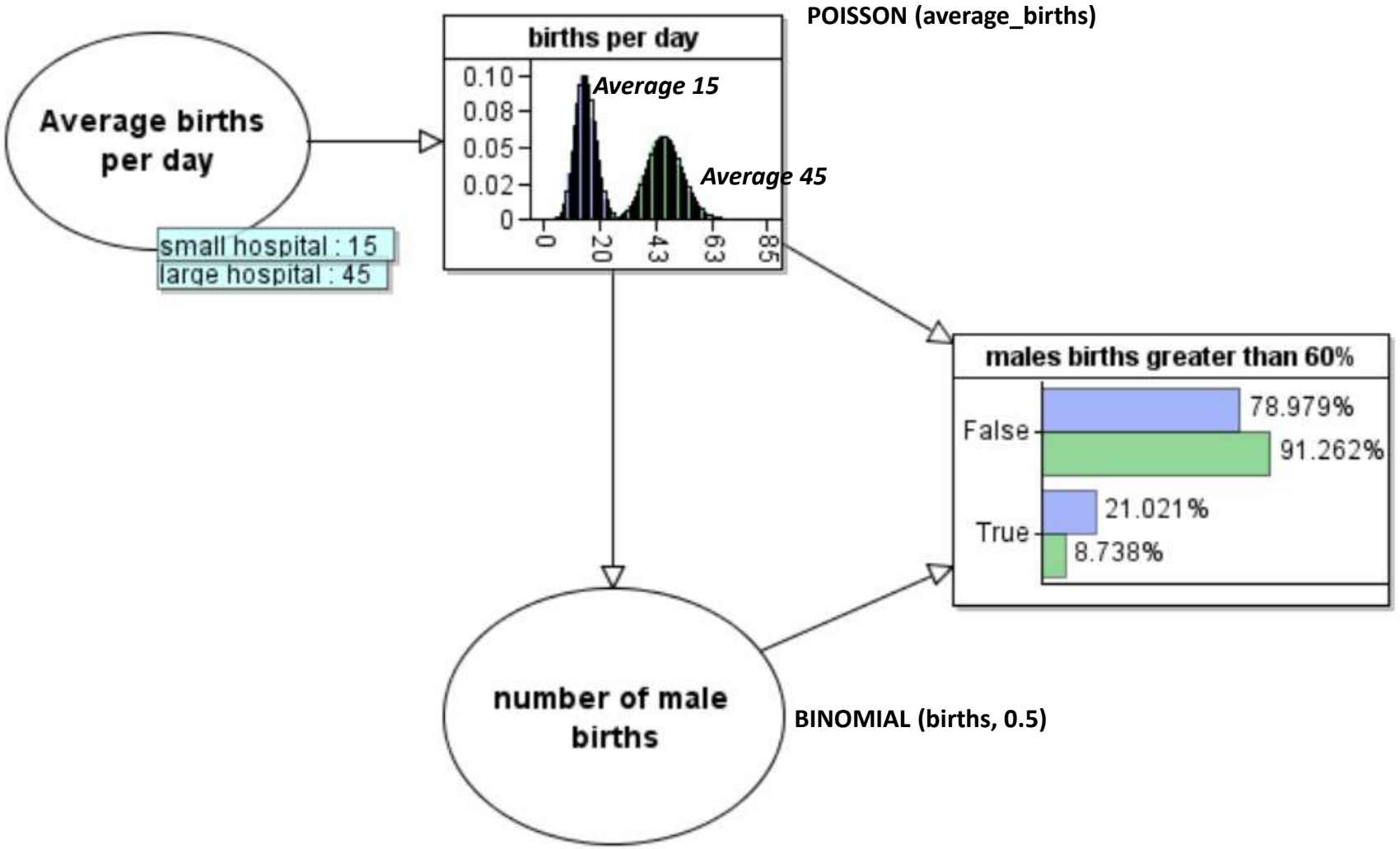
https://stattrek.com/online-calculator/poisson.aspx

- Enter a value in BOTH of the first two text boxes.
- Click the **Calculate** button.
- The Calculator will compute the Poisson and Cumulative Probabilities.

Poisson random variable (x)	9
Average rate of success	3.1
Poisson Probability: P(X = 9)	0.00328
Cumulative Probability: P(X < 9)	0.99532
Cumulative Probability: P(X ≤ 9)	0.99860
Cumulative Probability: P(X > 9)	0.00140
Cumulative Probability: P(X ≥ 9)	0.00468

Calculate

Hospital births



The Case of (nurse) Ben Geen: Statistics of coincidence?

Convicted of murdering two patients and causing grievous bodily harm to 15 others while working at Horton General Hospital in Banbury, Oxfordshire in 2003 and 2004

Statistically driven investigation

During a two-month period at Horton General Hospital 18 cases of respiratory arrest (RA) were observed with Ben Geen present each time

There are on average only 2 to 3 RA events per month at a typical hospital like Horton. So seeing 18 in a 2-month period can certainly be considered ‘abnormally high’. And finding the same nurse on duty on each of these occasions is surely too much of a ‘coincidence’….certainly such ‘statistics’ drove the initial investigation.

But using the Poisson and Binomial distributions and taking account of all possible sequences like this throughout the country in a 4-year period (much like winning the lottery twice), it turns out not to be such an unusual sequence of events after all.

See the video, model and supporting material for details

Risk: Deciding ‘best’ outcome

Information about survival rates for two treatment choices for lung cancer, *radiation* or *surgery*:

Scenario 1: The five-year survival rate is higher with surgery than with radiation. For surgery 90% of patients survive beyond one month (one-month survival rate of 90%)

Scenario 2: The five-year survival rate is higher with surgery than with radiation. For surgery 10% of patients die within one month (one-month mortality rate of 10%)

84% of physicians presented with scenario 1 chose surgery. Only 50% of physicians presented with scenario 2 chose surgery.

…..***Problem framing*** for risk is critical

Relative or Absolute Risk?

The Daily News

Drinking cocoa increases risk of dying from rare disease by 100%

Would this story make you avoid drinking cocoa?

The Daily News

Drinking cocoa increases risk of dying from rare disease by 0.00000025%

Would this story make you avoid drinking cocoa?

…but both are 'true'

Relative or Absolute Risk?

Longitudinal study of life-style habits of 100 people known to have rare disease D

	Cocoa drinkers	Non cocoa drinkers
Did not die from D	10	60
Died from D	10	20

So 50% (10/20) cocoa drinkers died compared to 25% (20/80) of non-cocoa drinkers

As 50 is twice 25, we can say that the "risk" of dying is 100% greater for cocoa drinkers than for non-cocoa drinkers. But this is the **relative risk**

The **absolute risk** of dying is 25% greater for cocoa drinkers than for non-cocoa drinkers

So

(Relative) Risk increase	**100%**
(Absolute) Risk increase	**25%**

But this only applies to people with disease D.

Relative or Absolute Risk?

But only 1 in a million people have disease D

So the people in the study are ALL the people in a population of 100 million with D

	Cocoa drinkers*	Non cocoa drinkers
Did not die from D	19,999,990	79,999,980
Died from D	10	20

So 0.00005% (10/20,000,000) cocoa drinkers died from D compared to 0.000025% (20/80,000,000) of non-cocoa drinkers

So *for the population generally* the **relative risk** of dying is still 100% greater for cocoa drinkers (as 0.00005 is twice 0.000025)

But *for the population generally* the **absolute risk** of dying is only 0.000025% greater for cocoa drinkers than for non-cocoa drinkers.

So unless you ***know*** you have D

(Relative) Risk increase	**100%**
(Absolute) Risk increase	**0.000025%**

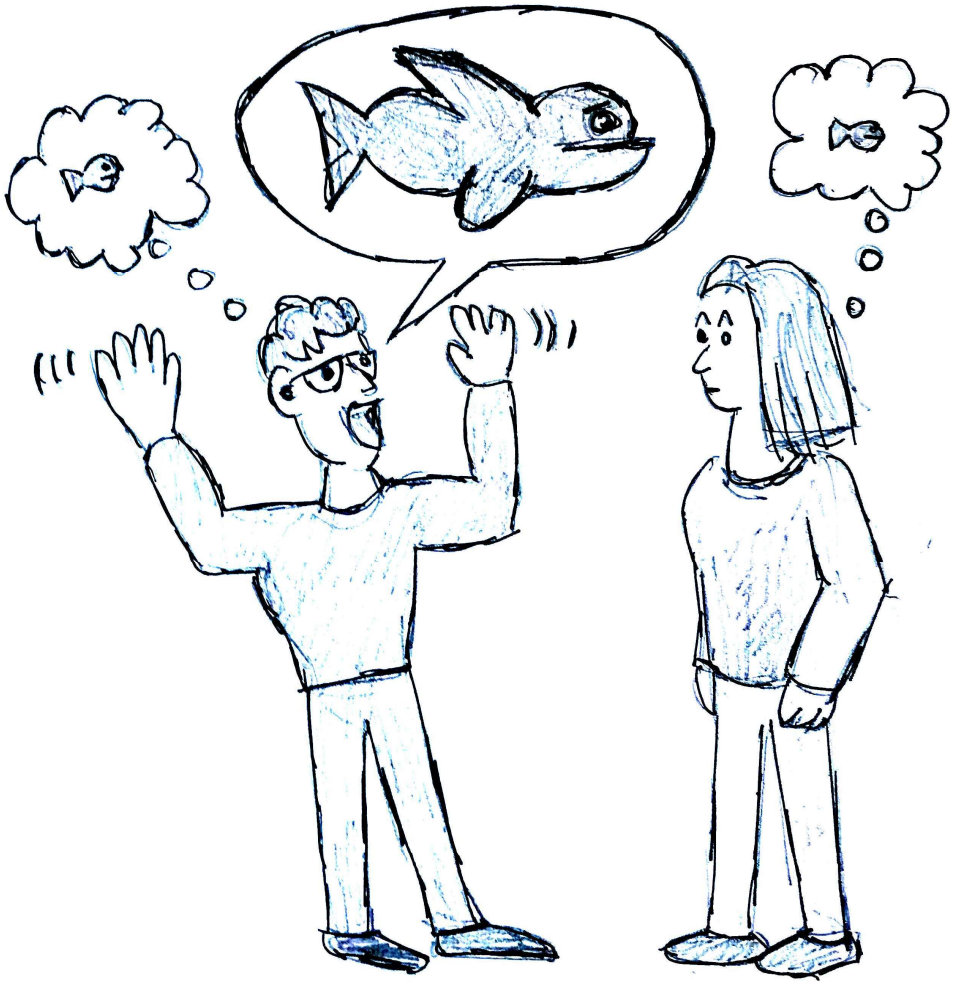
* We assume that 20% of the population drink cocoa

Relative or Absolute Risk?

Relative risk tends to provide a grossly exaggerated perception of risk to the general public

But it is the standard media ‘choice’ because it tends to provide sensational headlines

In what follows we explain, using further examples, why absolute risk is not only a more sensible measure of risk, but that it is also more meaningful in a natural causal way



Relative or Absolute Risk?

4 out of every 100 students on an MSc course who are members of a gym fail the course
5 out of every 100 students on an MSc course who are NOT gym members fail the course

	Members	Non-members
Failures	4%	5%

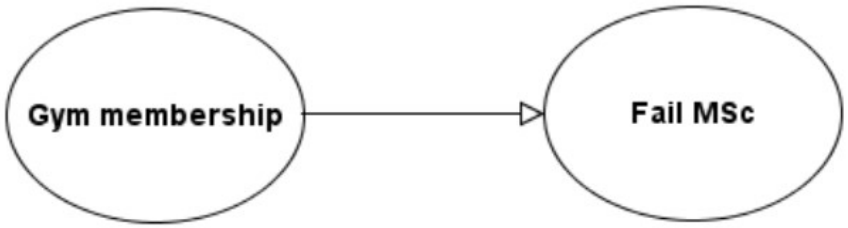
The **relative risk** of failing is 25% greater for non gym members than for gym members (since 5% is 25% greater than 4%). *Would you pay money to join?*

The **absolute risk** of failing is 1% greater for non gym members than for gym members

Relative and Absolute Risk as probabilities

Assume:

- Gym membership has a causal influence on success/failure
- There are no 'confounding' factors



The 1% absolute risk measure corresponds to a 1% increase in probability of failing MSc if student does not join a gym.

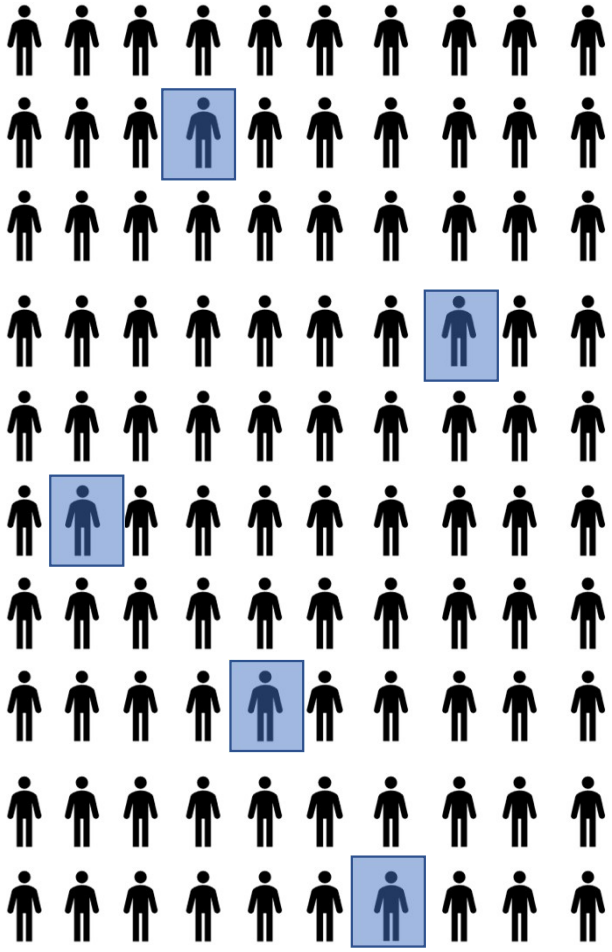
The 25% relative risk measure is far less 'meaningful'. It tells us about how much the posterior probability of a person being a non-gym member increases if you know they failed the MSc.

Specifically, if p_1, p_2 are the prior probabilities of non-gym and gym members respectively then

$$p_1' \text{ is } 25\% \text{ greater than } \frac{p_1}{p_2} \times p_2', \qquad i.e. \qquad p_1' = 1.25 \times \frac{p_1}{p_2} \times p_2'$$

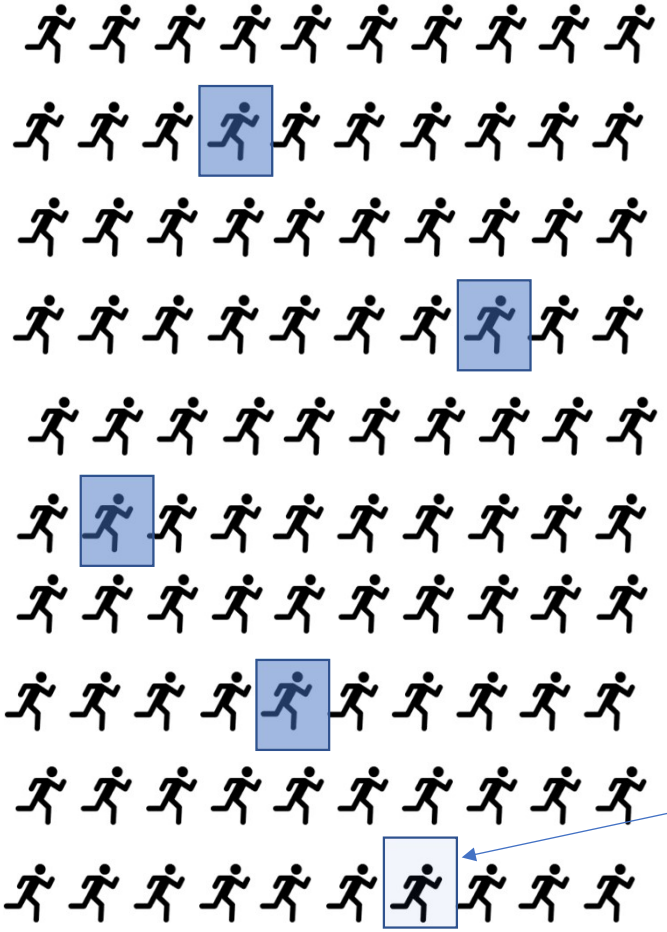
where p_1', p_2' are the posterior probabilities of non-gym and gym members respectively

Absolute risk measures are more obviously meaningful.



100 Non gym members

5 fail MSc



100 Gym members

4 fail MSc

So for every 100 gym members only one is ‘saved’ from failure by their gym membership

Relative and Absolute Risk

The Telegraph

Two glasses of wine a night triples risk of mouth cancer, Government warns

Drinking two large glasses of wine a day triples the risk of developing mouth cancer, a government campaign will warn.

Two glasses of wine a night triples the chance of mouth cancer, Government will warn Photo: ALAMY

By Laura Donnelly, Health Correspondent
9:45AM GMT 05 Feb 2012

500,000 deaths per year in UK
84 from mouth cancer

20% of those who died (i.e. 100,000) drank 2 glasses of wine

	Mouth cancer deaths	% deaths from mouth cancer
Wine drinkers (100,000)	36	0.036%
Non-wine drinkers (400,000)	48	0.012%

So the ***Relative risk*** is tripled

But the ***Absolute risk*** only increases 0.024%

Only 24 in every 100,000 regular wine drinkers would be saved from mouth cancer if they had not been wine drinkers

Relative and Absolute Risk: Look at the leaflet accompanying any drug

OESTROGEL® PUMP-PACK
0.06% Gel
17β-ESTRADIOL

Read all of this leaflet carefully before you start using this medicine because it contains important information for you.

- Keep this leaflet. You may need to read it again.
- If you have any further questions, ask your doctor or pharmacist.
- This medicine has been prescribed for you only. Do not pass it on to others. It may harm them, even if their signs of illness are the same as yours.
- If you get any side effects, talk to your doctor or pharmacist. This includes any possible side effects not listed in this leaflet. See section 4.

Effect of HRT on heart and circulation

Blood clots in a vein (thrombosis)

The risk of **blood clots in the veins** is about 1.3 to 3- times higher in HRT users than in non-users, especially during the first year of using it.

Other conditions

HRT will not prevent memory loss. There is some evidence of a higher risk of memory loss in women who start using HRT after the age of 65. Speak to your doctor for advice.

Stroke

The risk of getting stroke is about 1.5 times higher in HRT users than in non-users. The number of extra cases of stroke due to use of HRT will increase with age.

..no attempt anywhere to quantity the benefits

Standard definitions of risk and how to measure it

"an event that can have negative consequences" (Conversely an event that can have a positive impact is an ***opportunity****)*

Do these definitions help with risk assessment and decision-making?

Risk Registers and their deficiencies

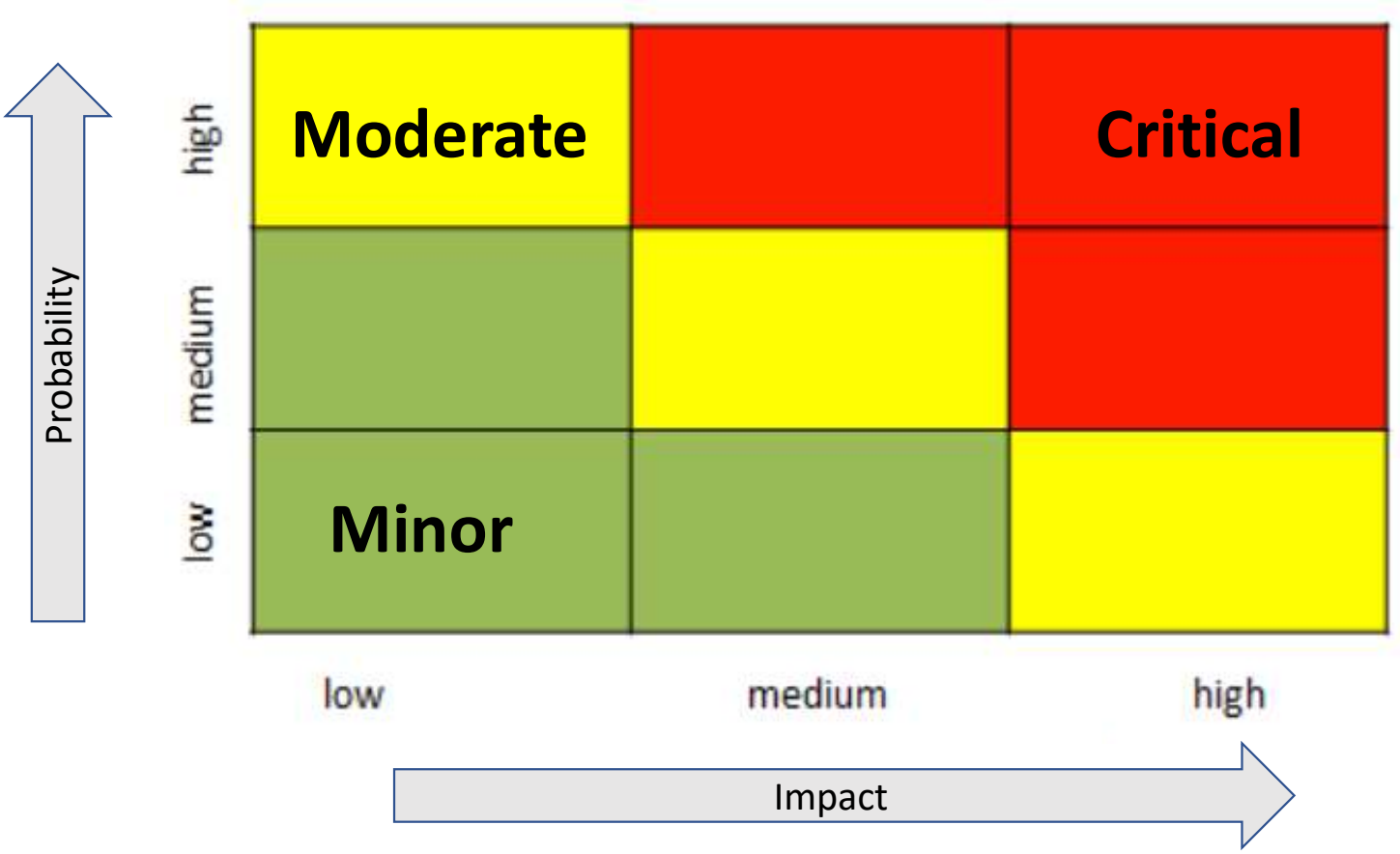
Risk	Probability	Impact	Risk score
1. Key staff leave	4	5	20
2. Cannot deliver core requirements against customer deadline	3	5	15
3. Technical requirements too complex	3	4	12
4. Customer fails to make up-front payment	2	5	10
……..			

Paradox: The more you think about risk for a new project, the higher the cumulative risk score. So best way to get project go-ahead is to ignore or under-report risks

Inconsistent: Different projects or business divisions assess risk differently

Risks are not independent

Heat maps


Probability
high
medium
low
Moderate
Critical
Minor
low
medium
high
Impact

Probability of damage during foreseeable lifetime of the product		Severity of injury			
		1	2	3	4
High	>50 %	H	S	S	S
	> 1/10	M	S	S	S
	> 1/100	M	S	S	S
	> 1/1 000	L	H	S	S
	> 1/10 000	L	M	H	S
	> 1/100 000	L	L	M	H
	> 1/1 000 000	L	L	L	M
Low	< 1/1 000 000	L	L	L	L

1 - First aid required
2 - Visit to A&E required
3 - Hospital stay required
4 - Life changing damage

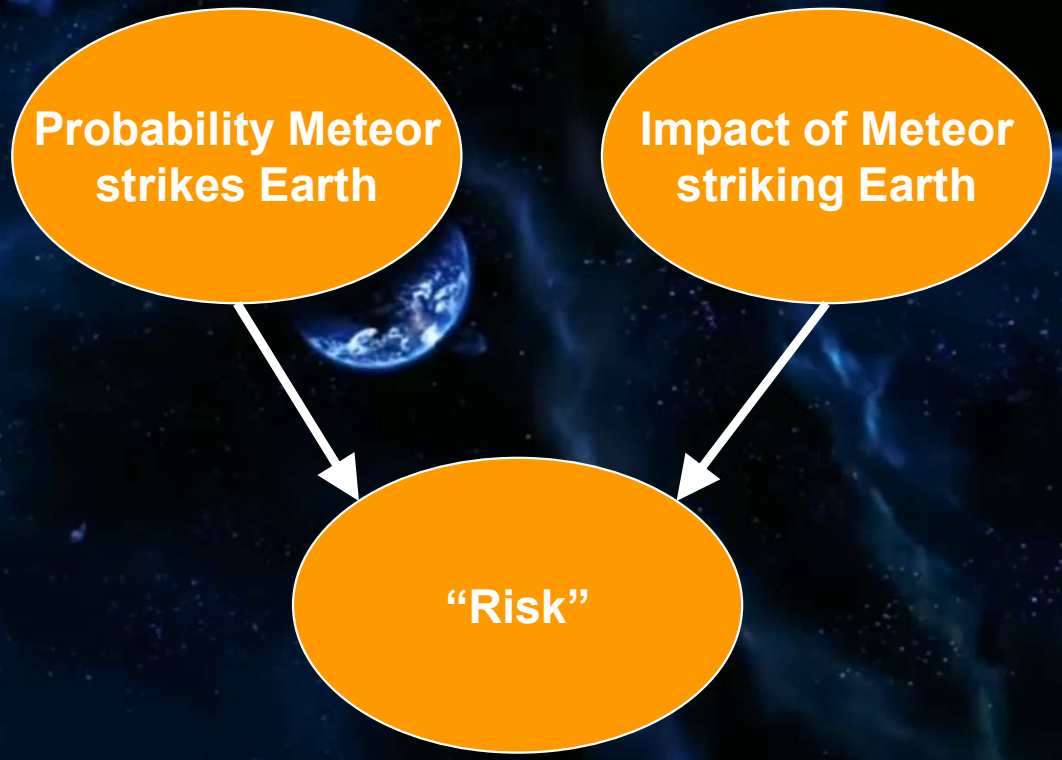
S — Serious Risk
H — High risk
M — Medium risk
L — Low risk

The EU *RAPEX* risk assessment guidelines for product safety (2019)

https://eur-lex.europa.eu/eli/dec/2019/417/oj

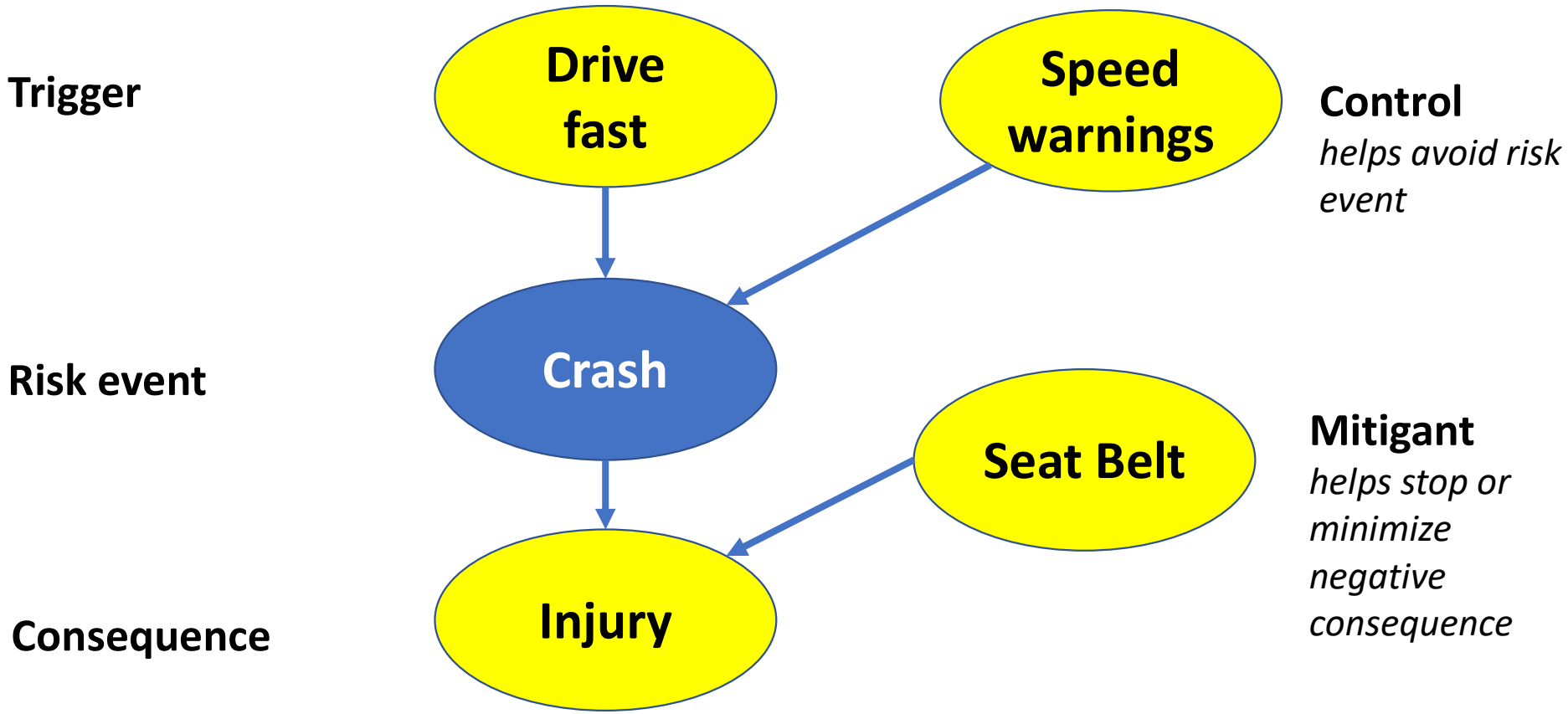
Risk = Probability x Impact?

The ‘standard approach’ makes no sense at all



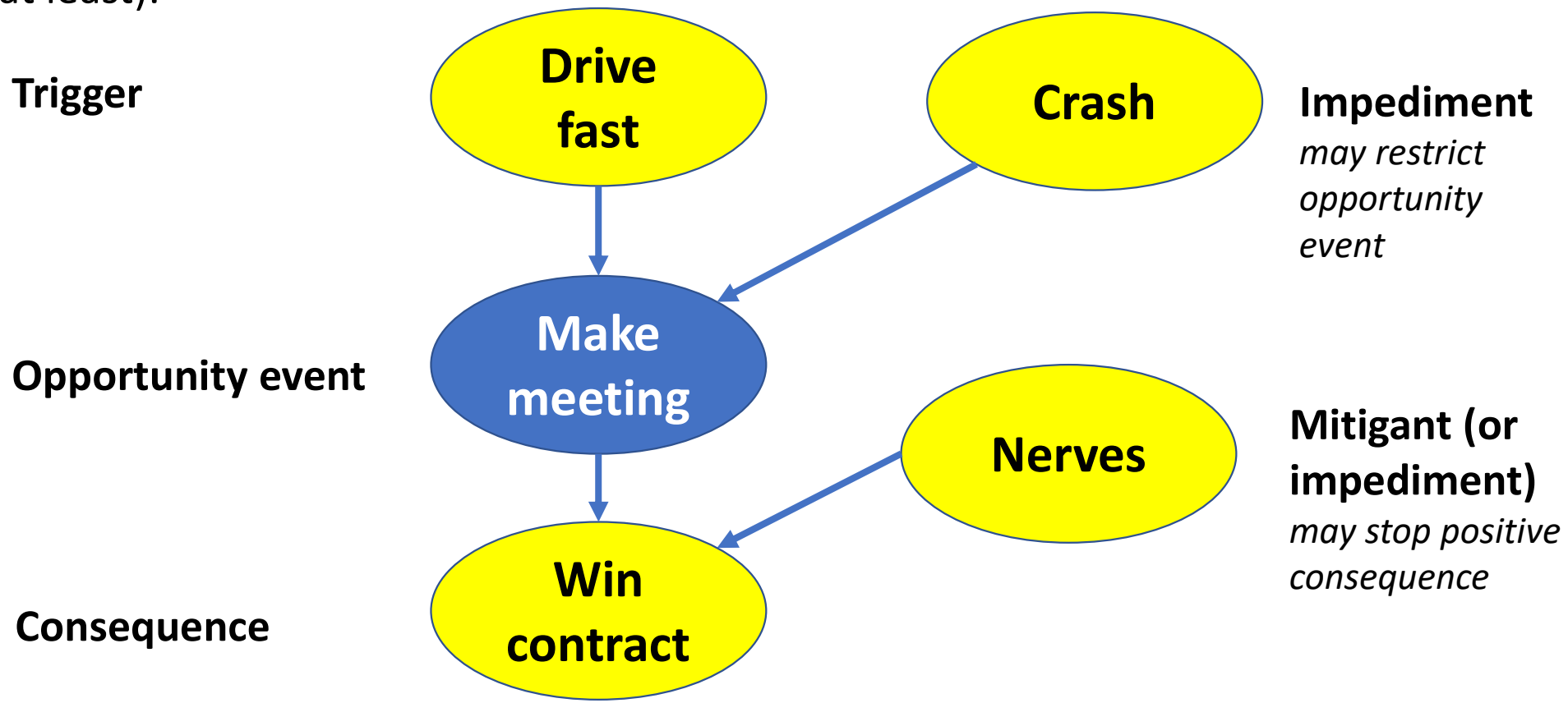
Risk using causal analysis

A **risk** is an *event* that can be characterised by a causal chain of events involving (at least):

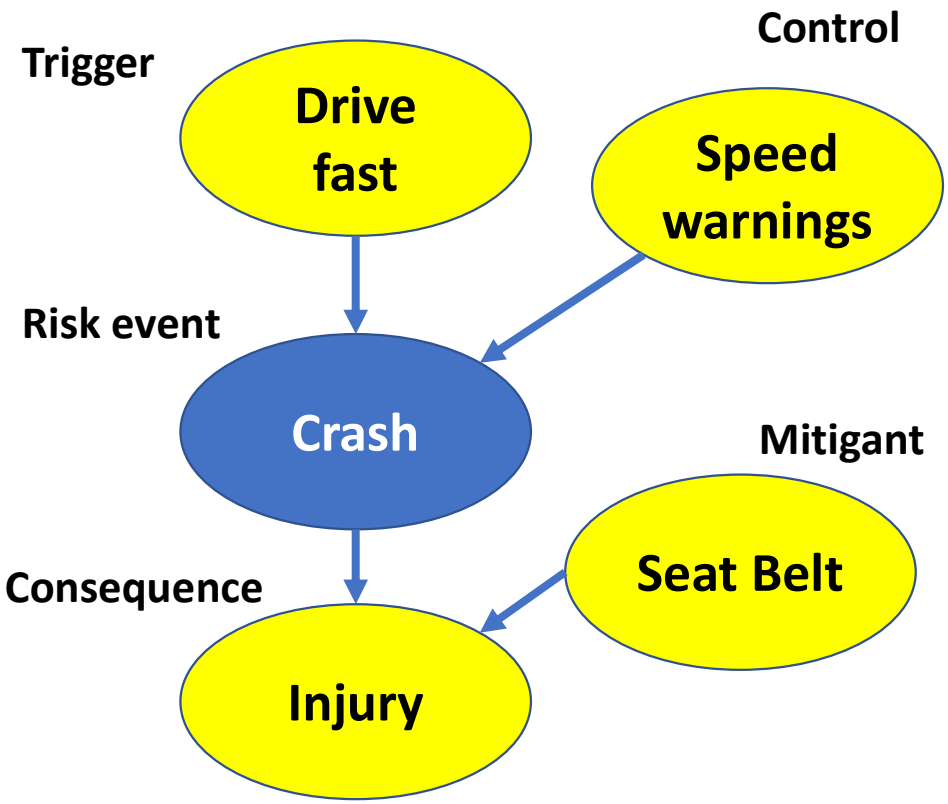


Risk using causal analysis

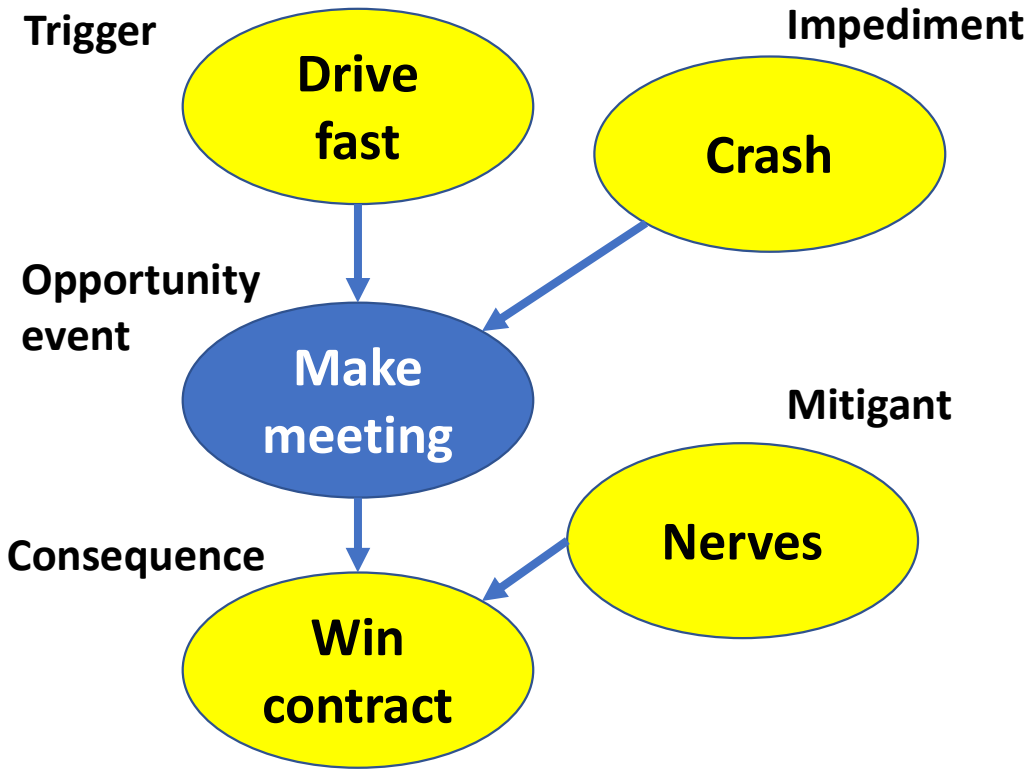
An **opportunity** is an *event* that can be characterised by a causal chain of events involving (at least):



Symmetry between causal view of risk and opportunity

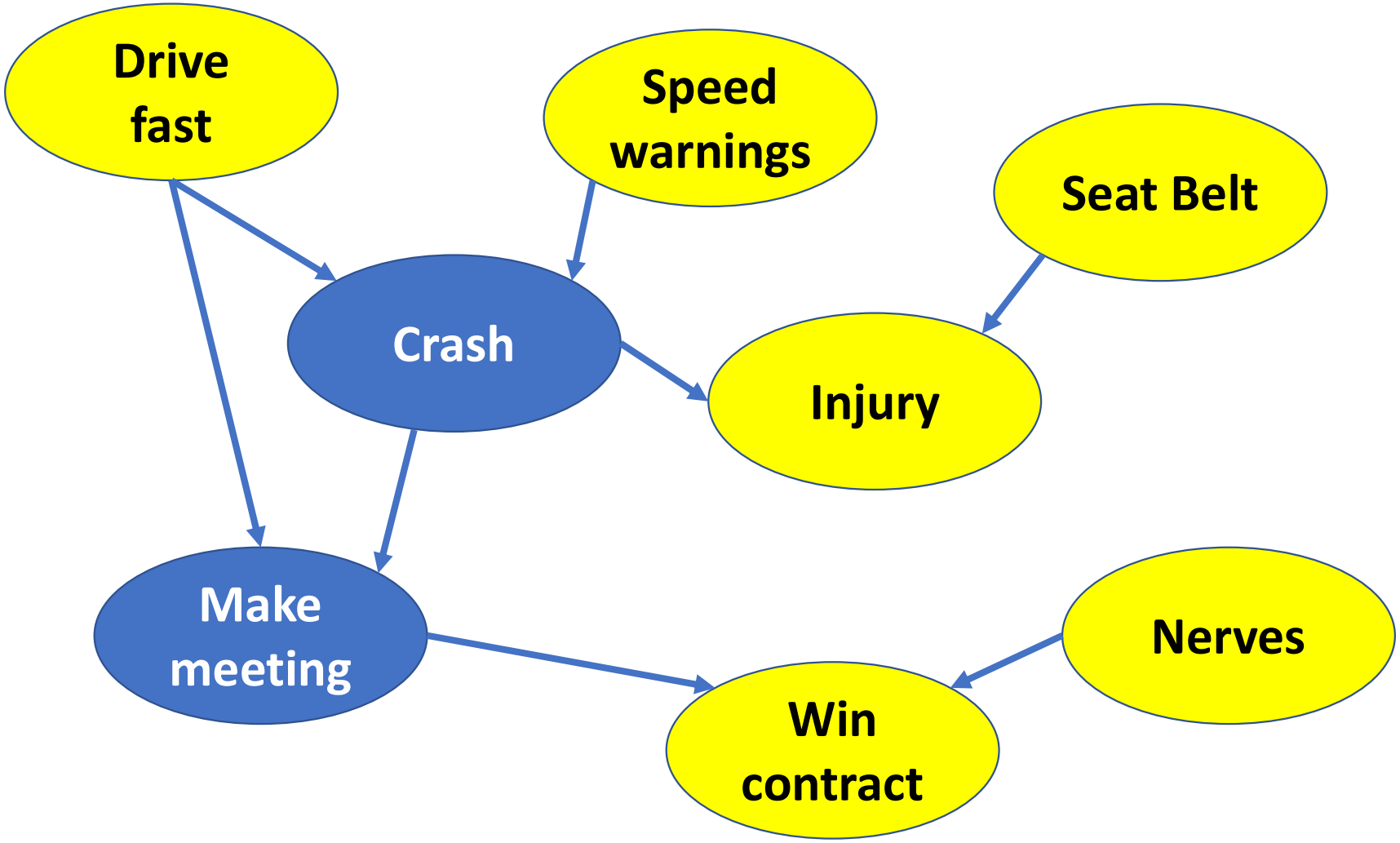


RISK

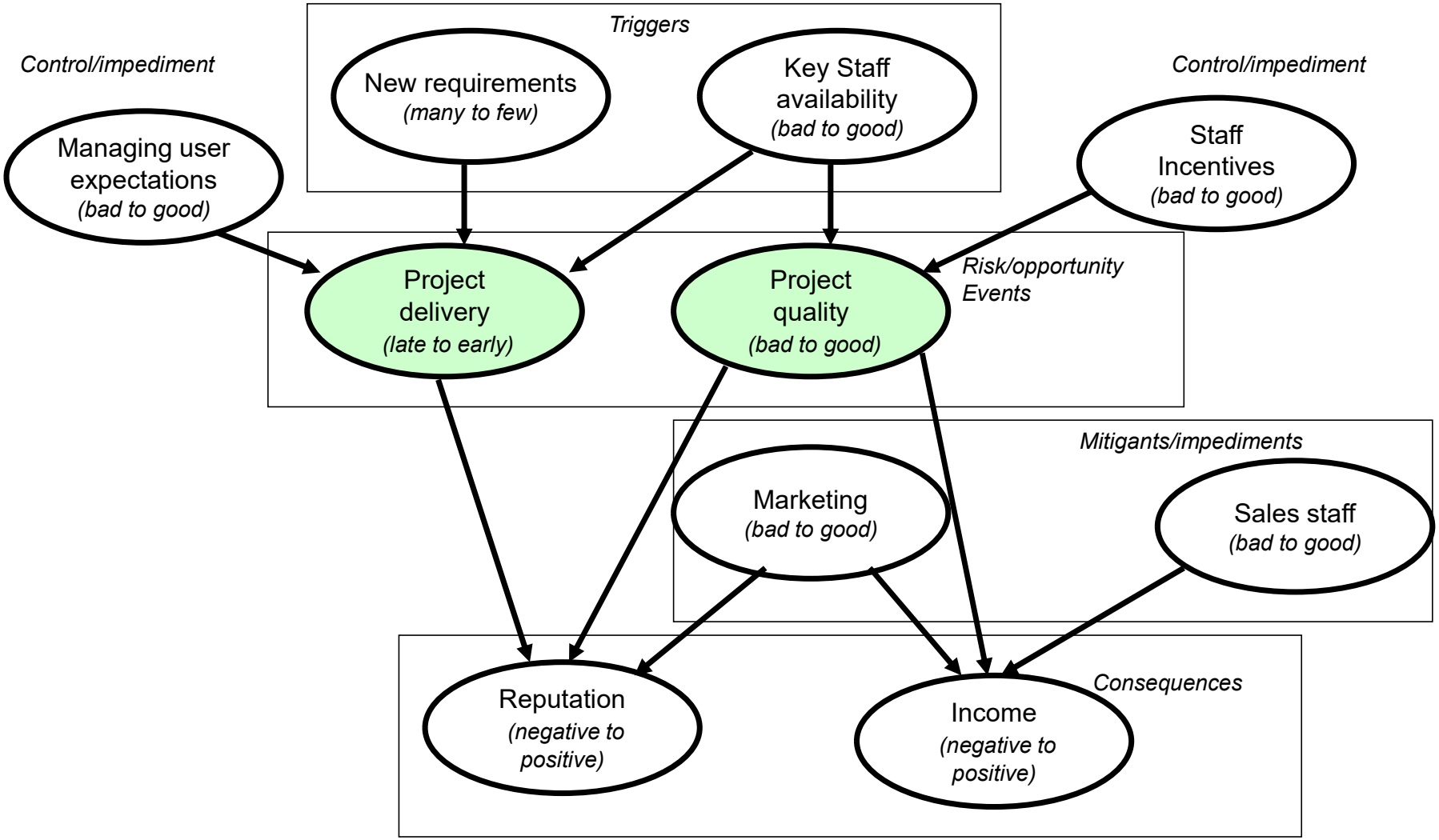


OPPORTUNITY

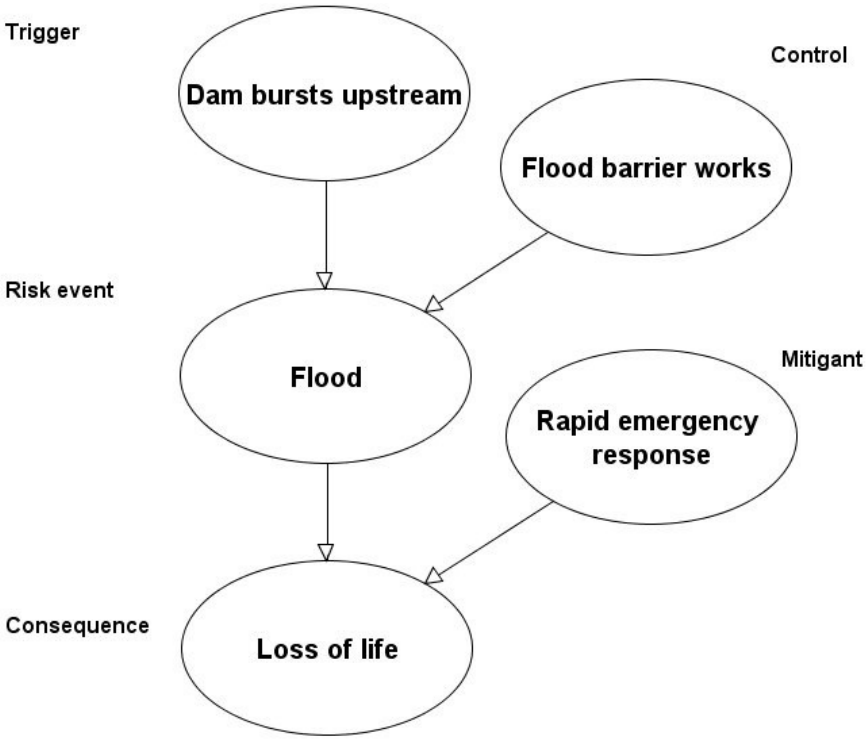
Risks and opportunities: unified causal model



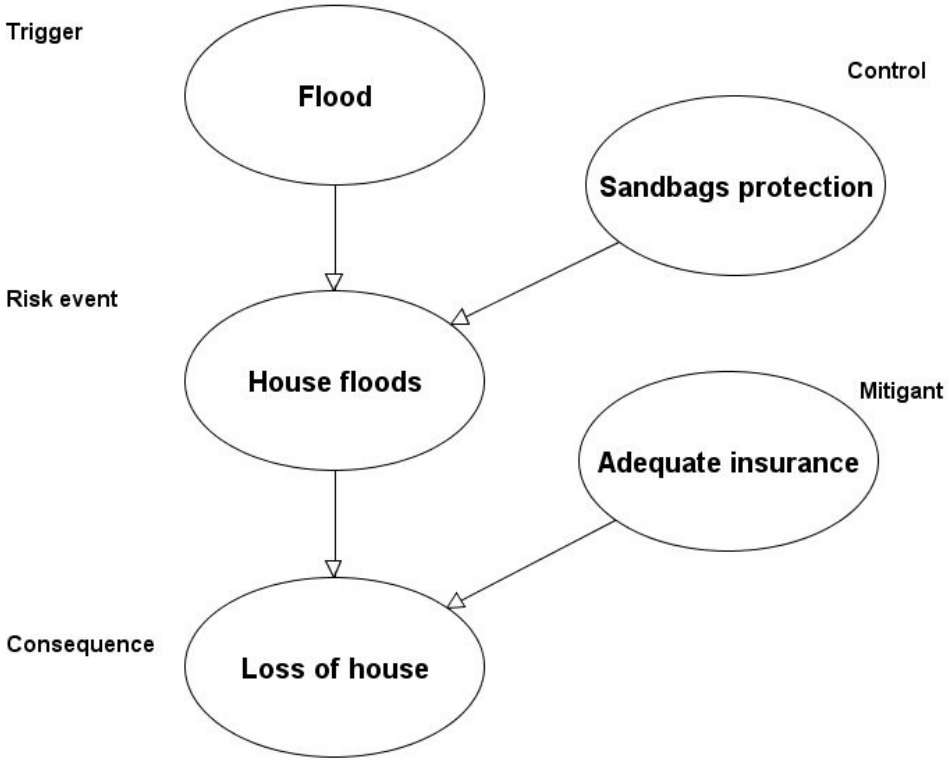
Neutral causal taxonomy for risks and opportunities



Risk from different perspectives: Flooding

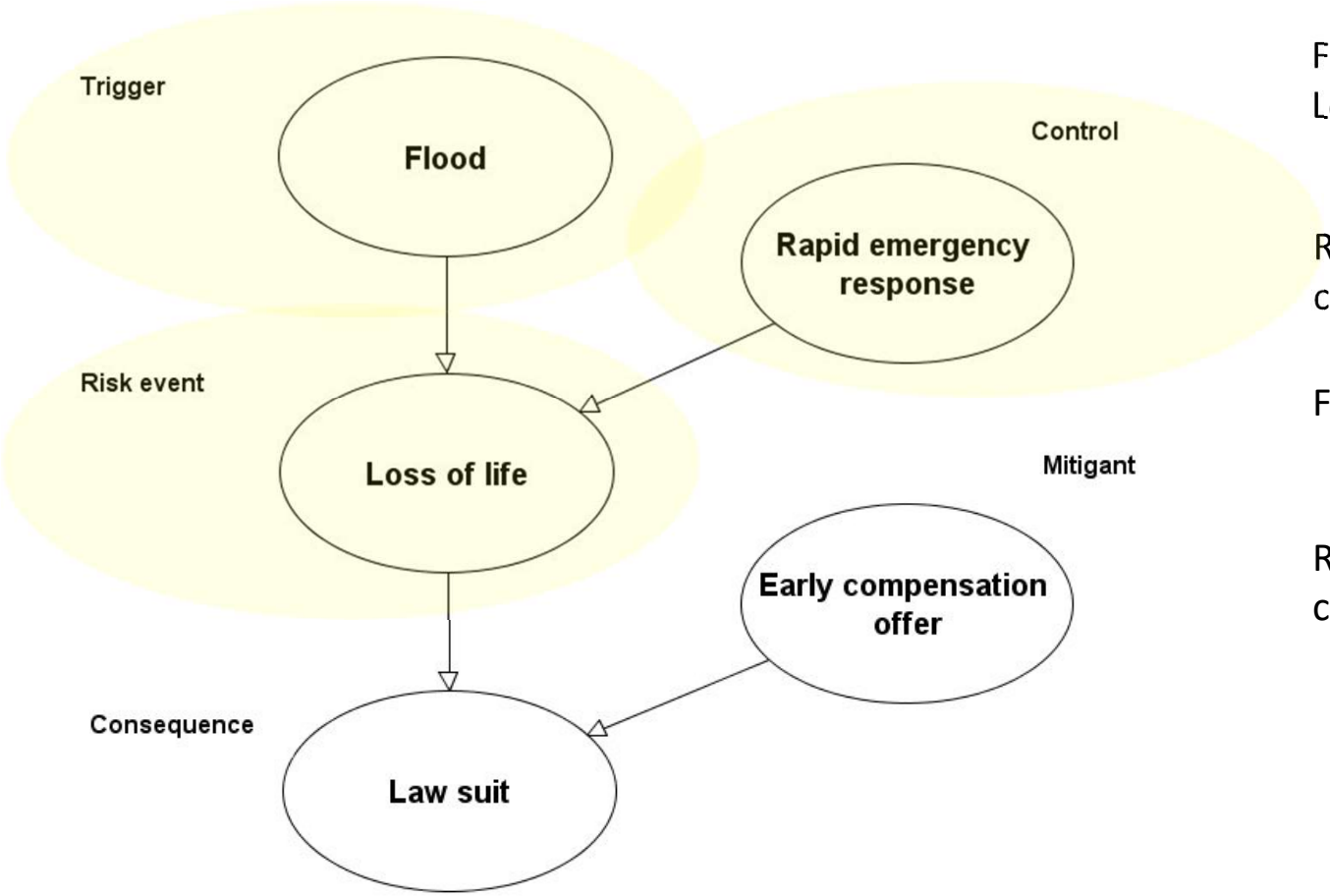


Local Authority Perspective



Householder Perspective

Interchangeability of concepts based on perspectives



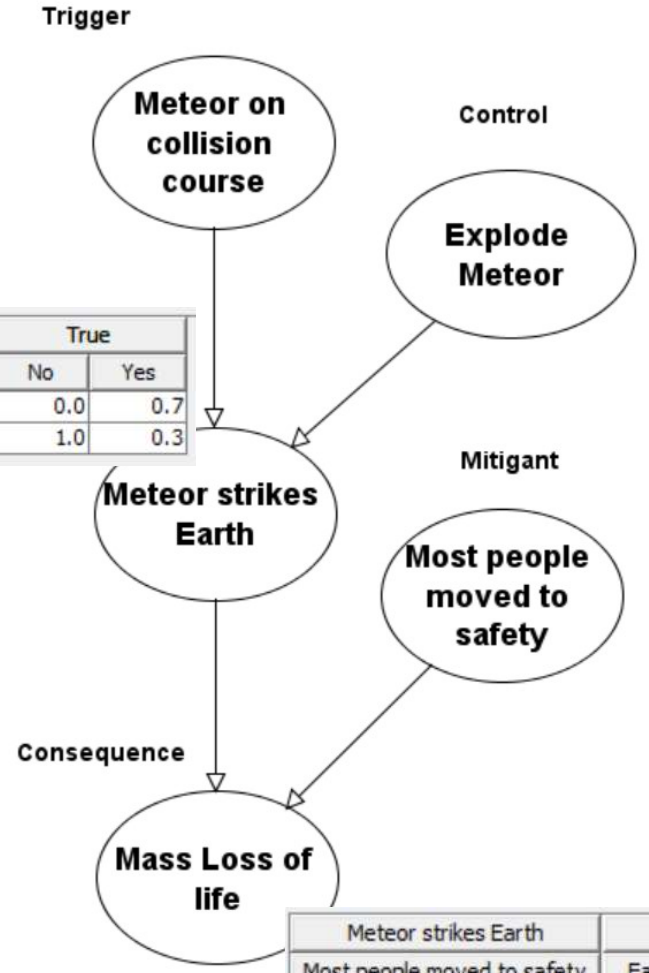
From the perspective of the Local Authority's Lawyers

Risk event is 'loss of life' (previously consequence)

Flood is now the trigger

Rapid emergency response becomes a control rather than a mitigant

Armageddon: meteor strike risk

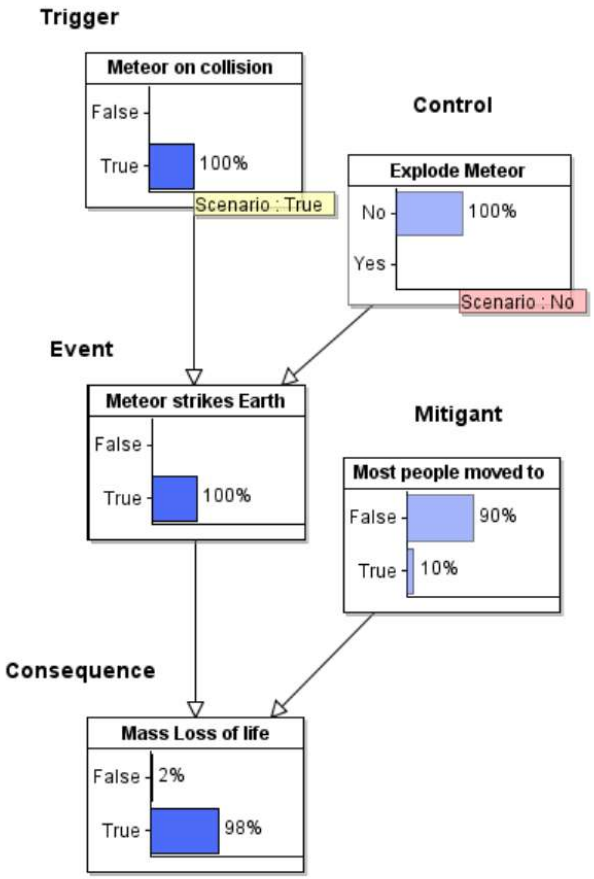


Meteor on collision course	False		True	
Explode Meteor	No	Yes	No	Yes
False	1.0	1.0	0.0	0.7
True	0.0	0.0	1.0	0.3

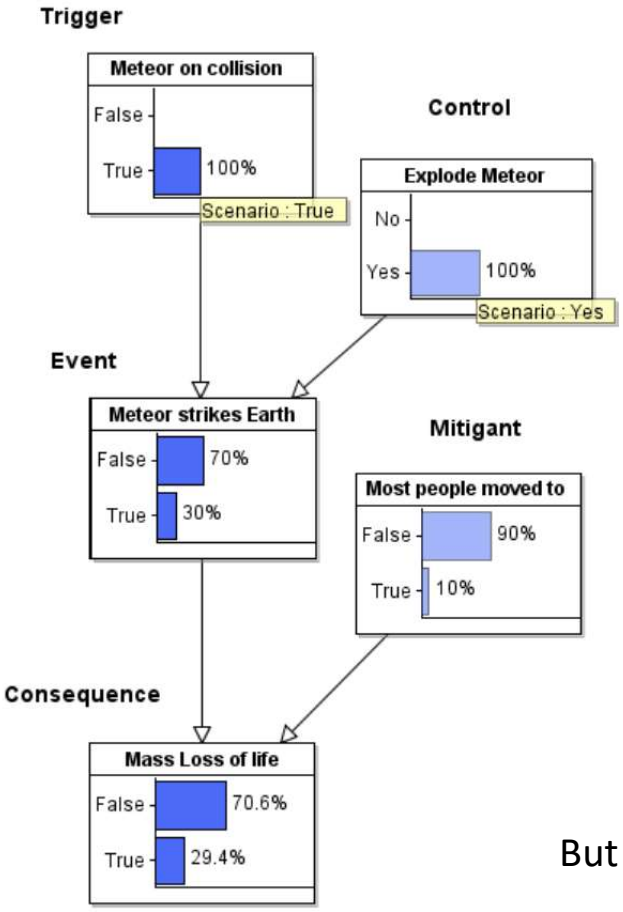
Meteor strikes Earth	False		True	
Most people moved to safety	False	True	False	True
False	1.0	1.0	0.0	0.2
True	0.0	0.0	1.0	0.8

Armageddon Bayesian Network

No NASA mission (best case outcome)



NASA mission (best case outcome)



But at what cost?

…. controls have costs and consequences have costs and/or benefits

Hence, risk-based decision-making must take account not just of probabilities of outcomes but also their UTILTY, i.e. costs, benefits and trade-offs between these

Utility, expected utility and optimal decisions

The notion of UTILITY unifies costs and benefits. Typically utility is expressed in financial units (e.g. £, $)

Example: suppose it costs £100 to play a game with a die, and you win £500 if the number 6 is rolled
Then the game has two outcomes:
'win' which has utility 400
'lose' which has utility -100

Deciding whether or not it is worth playing the game involves the notion of ***expected utility***.
This is ultimately what the 'risk = probability × impact' definition is based on. It an expected utility measure.
If an outcome A has probability p of happening and utility u when it does, then:

the ***expected utility*** of *A,* which we write as E(*A*), is defined as $u \times p$

So, assuming you decide to play the game above
E('win') = $400 \times 1/6 = 66.66$
E('lose') = $-100 \times 5/6 = -83.33$

In general. if there are n possible outcomes *A*1, *A*2, …, *A*n of a decision then:

the ***total expected utility*** of the decision is defined as the sum E(A1) + E(A2) + … + E(An)

So the decision to play the game has total expected utility 66.66 - 83.33 = - 16.66
The decision to **not play the game** has just one outcome "no play" expected utility 0.

The ***'optimal' decision*** is the one which ***maximizes total expected utility***

So the optimal decision is to **not play the game**
However, if you value the excitement of simply playing the game as having utility of 20, then
utility ('win')= 420 and utility ('lose')= -80
so the total expected utility of playing is
$(420 \times 1/6) + (-80 \times 5/6) = 3.33$
and the decision which maximizes total expected utility is **play the game**.

Example: To hire or not hire a marquee….

A charity is planning a major fundraising summer barbeque event.

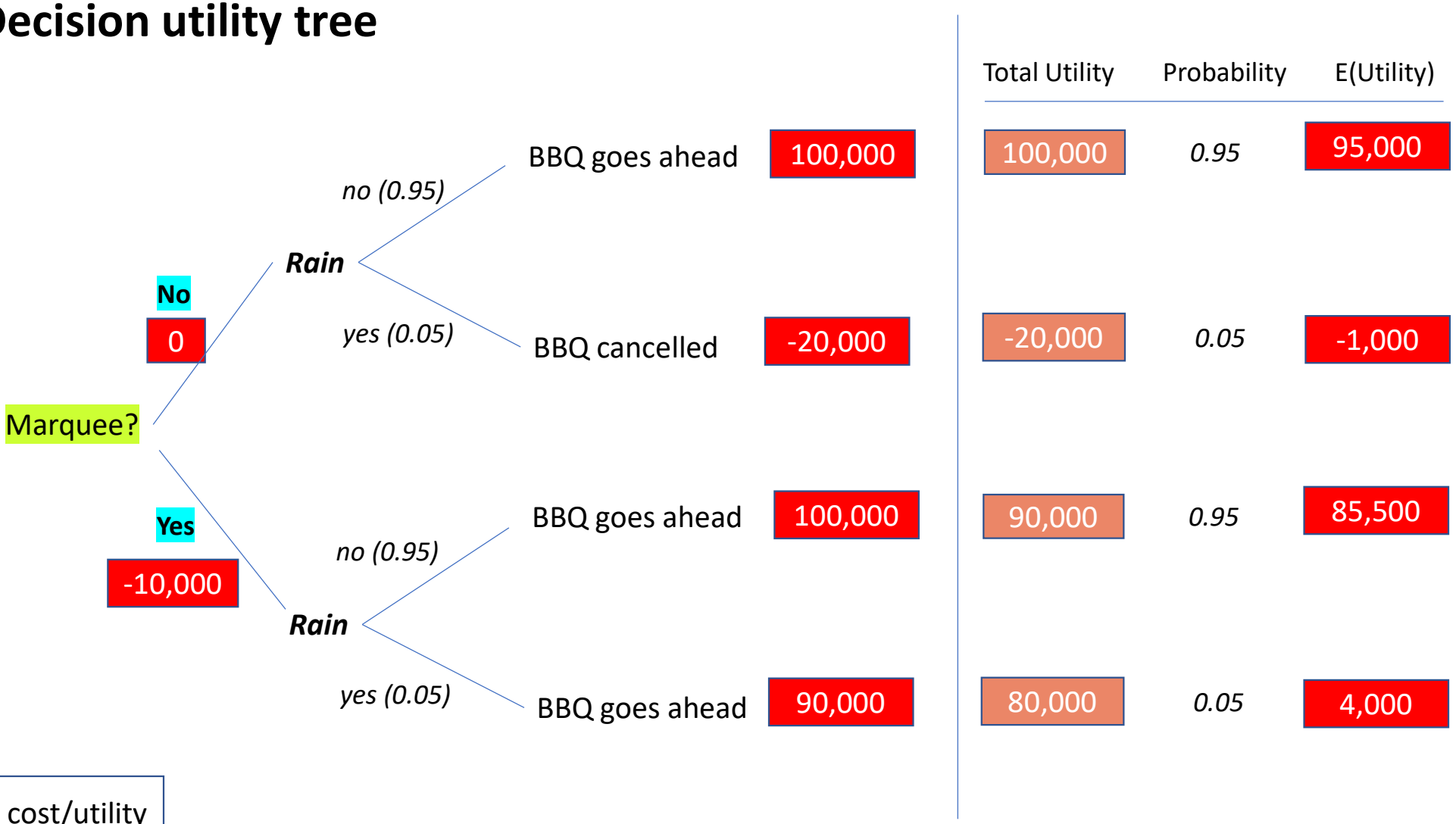
Although it rarely rains (only a 5% chance) the charity is considering hiring a large marquee which will cost $10,000. Hence its utility is -10,000.

The barbeque only gets cancelled if it rains on the day and there is no marquee. If that happens the charity loses $20,000 in cost of food and staff

If the barbeque goes ahead it will make a profit on the day of $100,000 (after cost of food and staff) – so its utility is 100,000, although if it rains this is reduced to a profit of £90,000 as fewer people attend when it rains

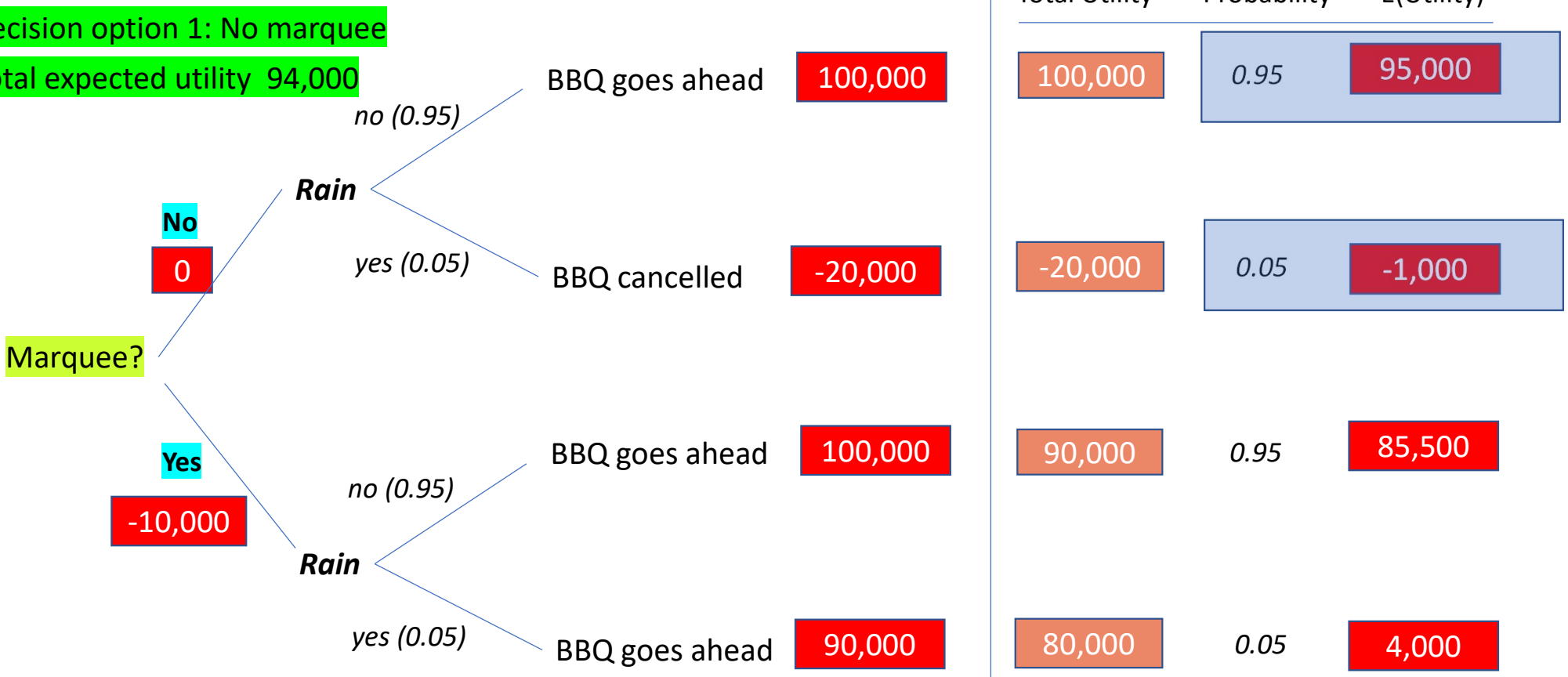
Should the charity hire the marquee or not?

Decision utility tree



-3	cost/utility
0.8	probability

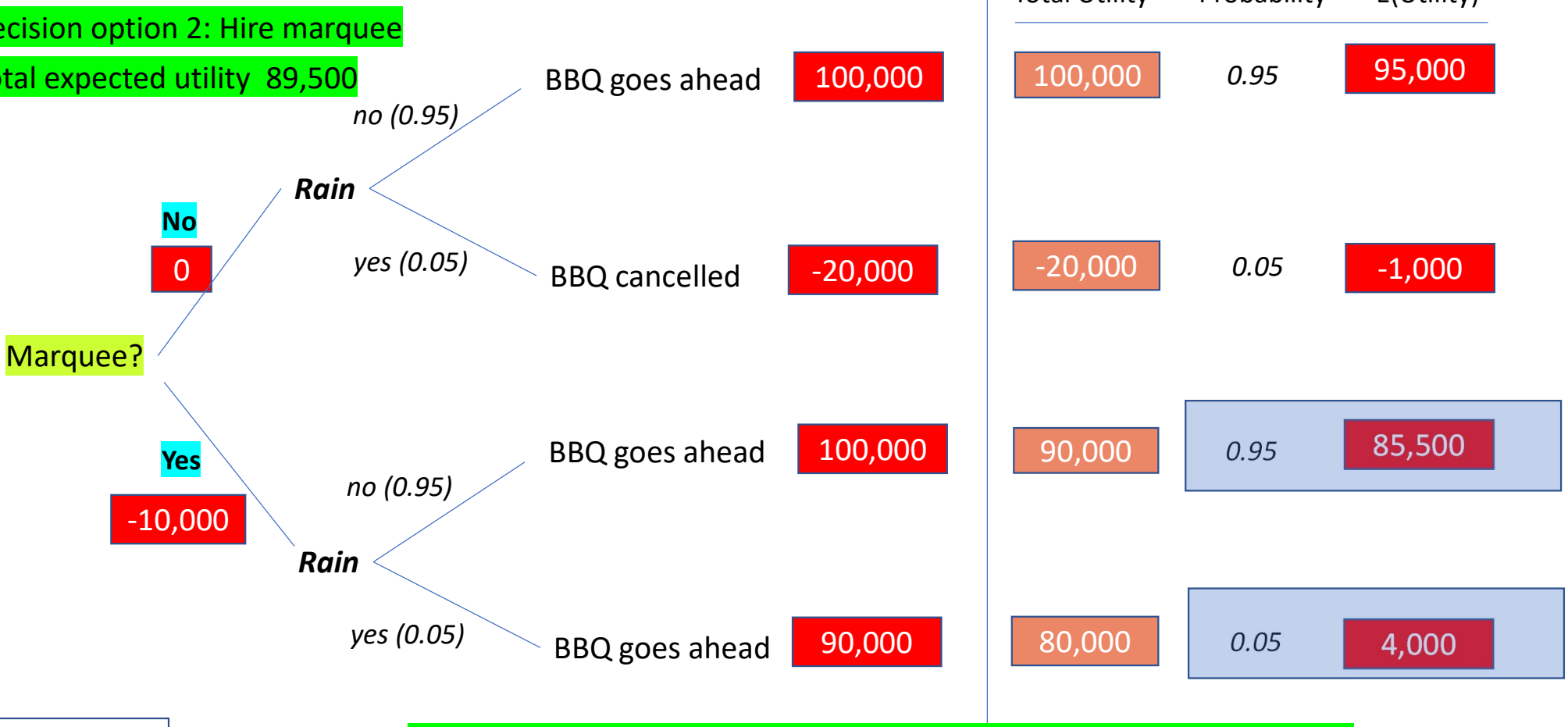
Decision utility tree



-3 cost/utility

0.8 probability

Decision utility tree



-3 cost/utility

0.8 probability

So optimal decision is option 1: Do not hire marquee

(but try calculating it when p(rain) is 0.1)

A more complex decision problem

It is not raining when time to leave for work, but on days like this there is still a 20% chance of rain

Carrying an umbrella is a pain, so has utility -3

However, there is a website that provides very accurate forecasts (assume for simplicity it is perfect, i.e. it only rains when website says it will)

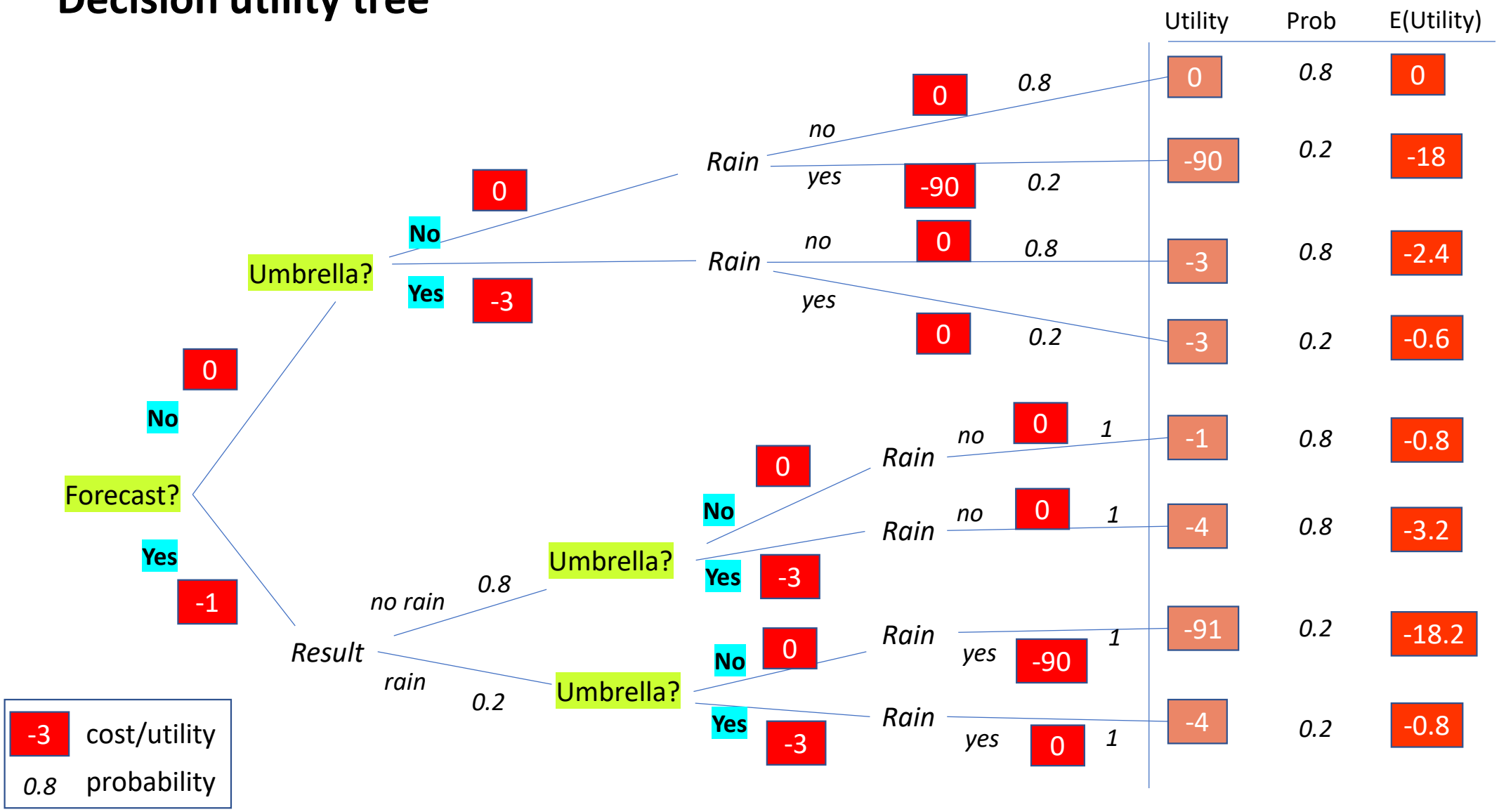
It's a bit of hassle finding the website, so getting the forecast has a utility of -1

The utility of getting caught in the rain with no umbrella is -90

There are multiple possible decisions here.

- Don't get the forecast and don't take umbrella
- Don't get the forecast and take umbrella
- Get the forecast but take the umbrella anyway
- Get the forecast but don't take the umbrella anyway
- Get the forecast and only take umbrella if it predicts no rain
- Get the forecast and only take umbrella if it predicts rain

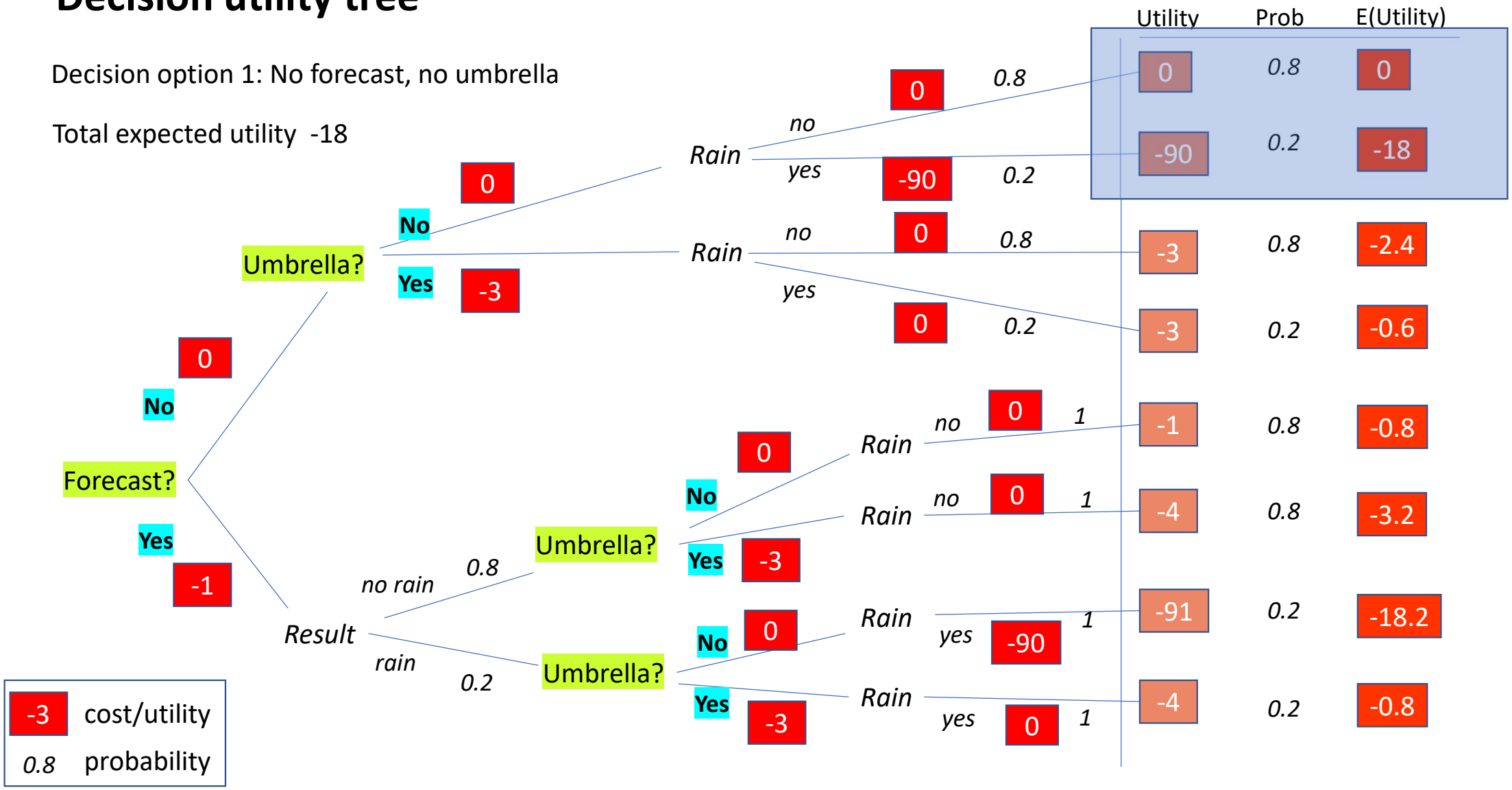
Decision utility tree



Decision utility tree

Decision option 1: No forecast, no umbrella

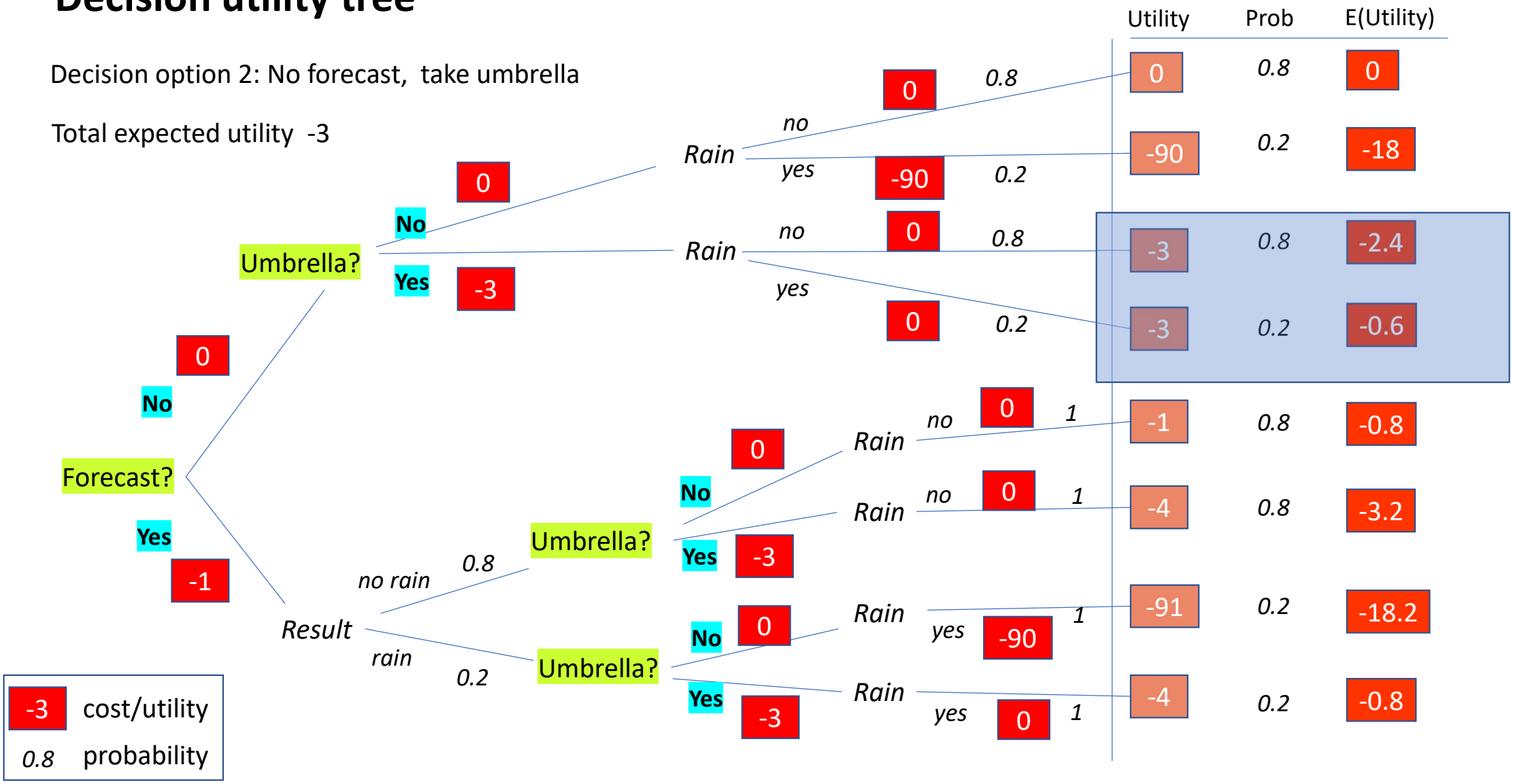
Total expected utility -18



Decision utility tree

Decision option 2: No forecast, take umbrella

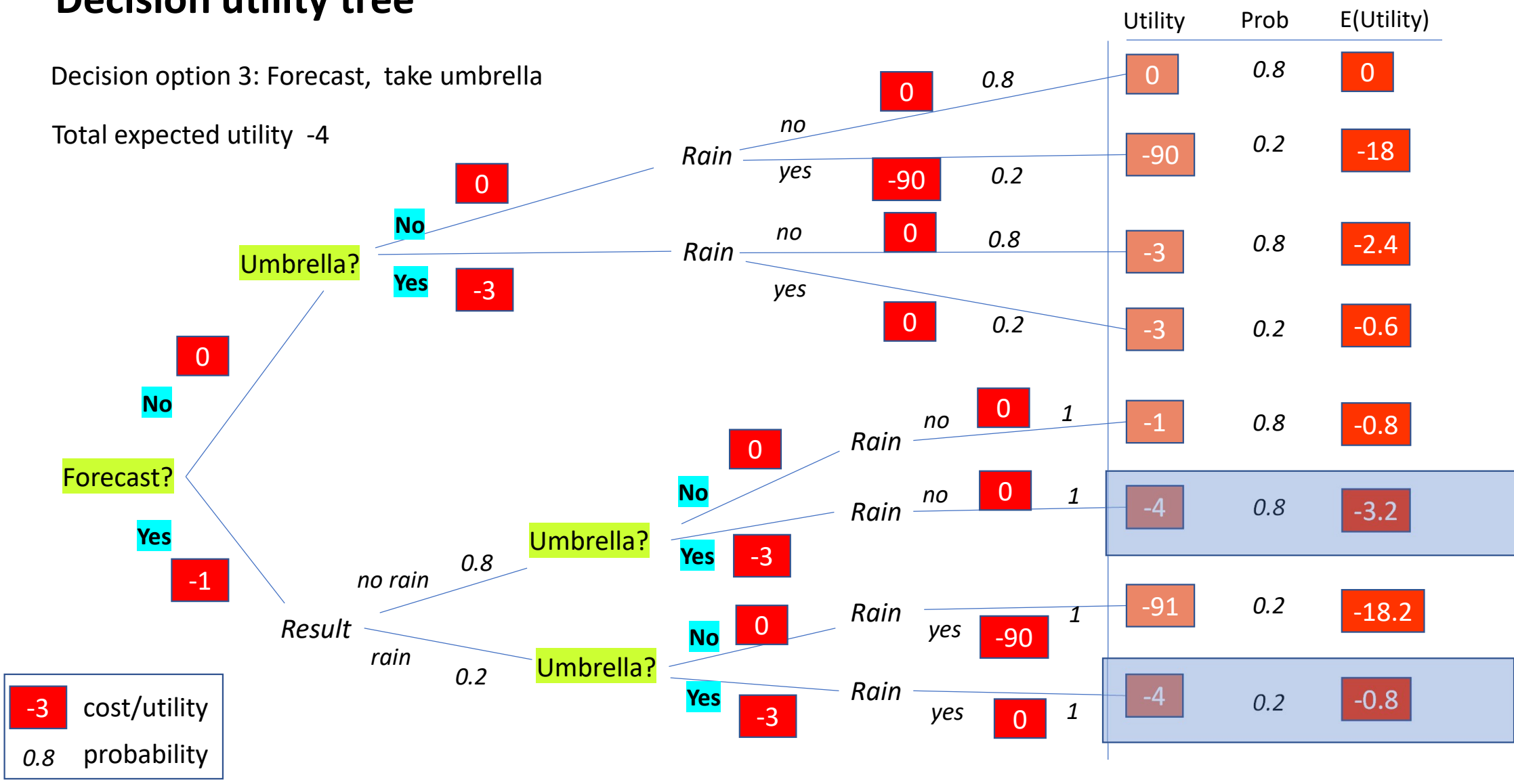
Total expected utility -3



Decision utility tree

Decision option 3: Forecast, take umbrella

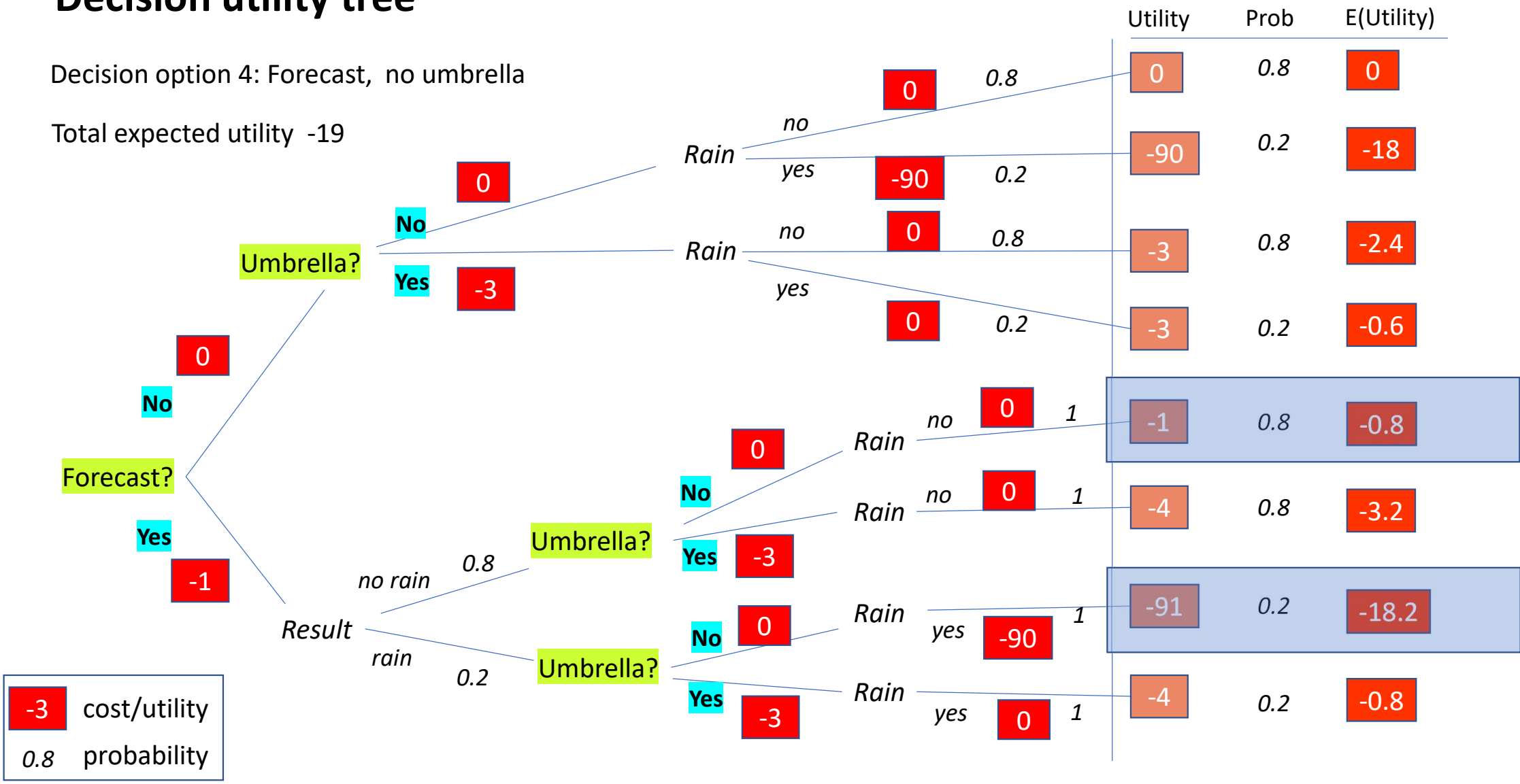
Total expected utility -4



Decision utility tree

Decision option 4: Forecast, no umbrella

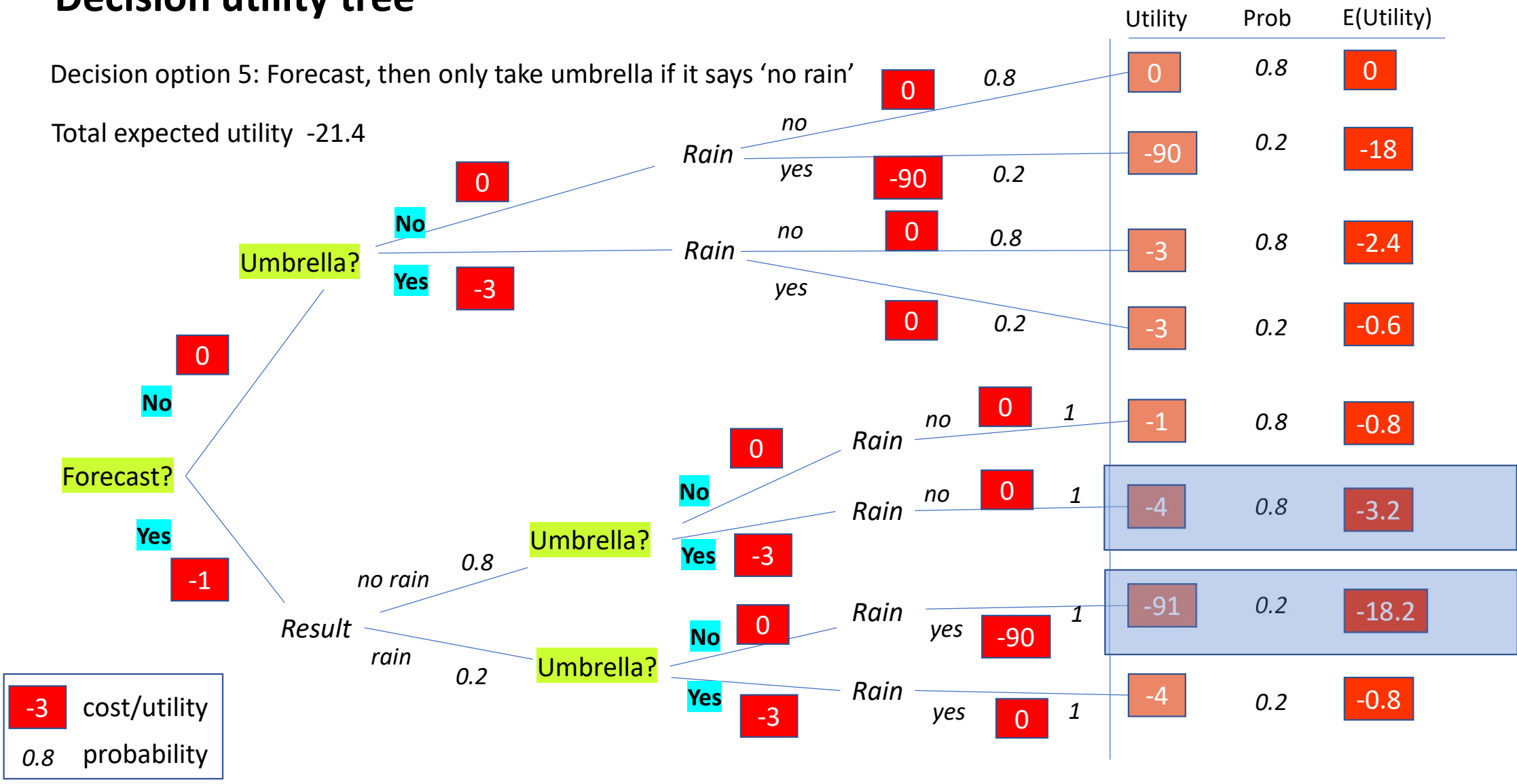
Total expected utility -19



Decision utility tree

Decision option 5: Forecast, then only take umbrella if it says 'no rain'

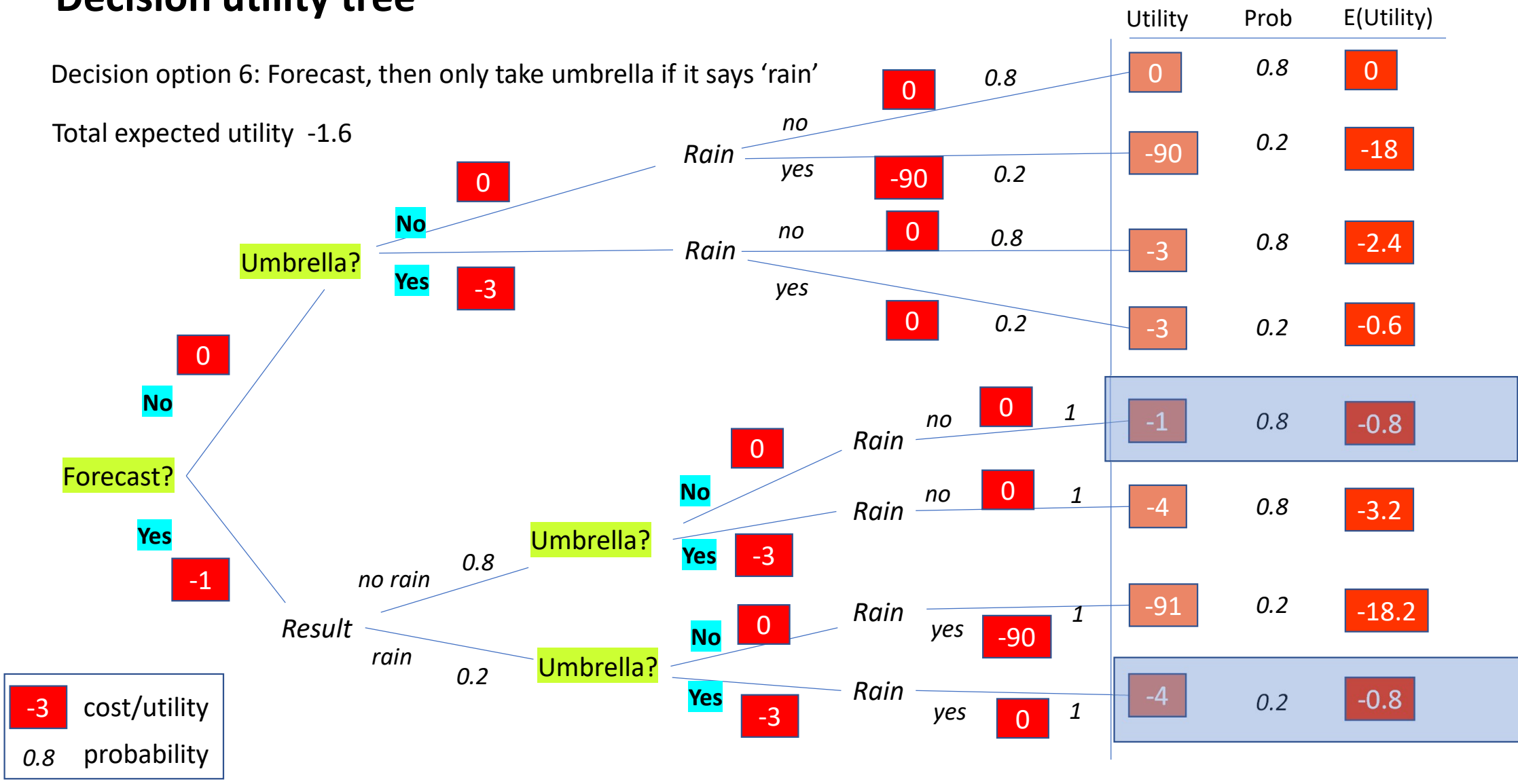
Total expected utility -21.4



Decision utility tree

Decision option 6: Forecast, then only take umbrella if it says 'rain'

Total expected utility -1.6



Decision options summary

Decision option	Expected utility
1. No forecast, no umbrella	-18
2: No forecast, umbrella	-3
3: Forecast, umbrella	-4
4: Forecast, no umbrella	-19
5: Forecast, then only take umbrella if test says ‘no rain’	-4
6: Forecast, then only take umbrella if test says ‘rain’	-1.6

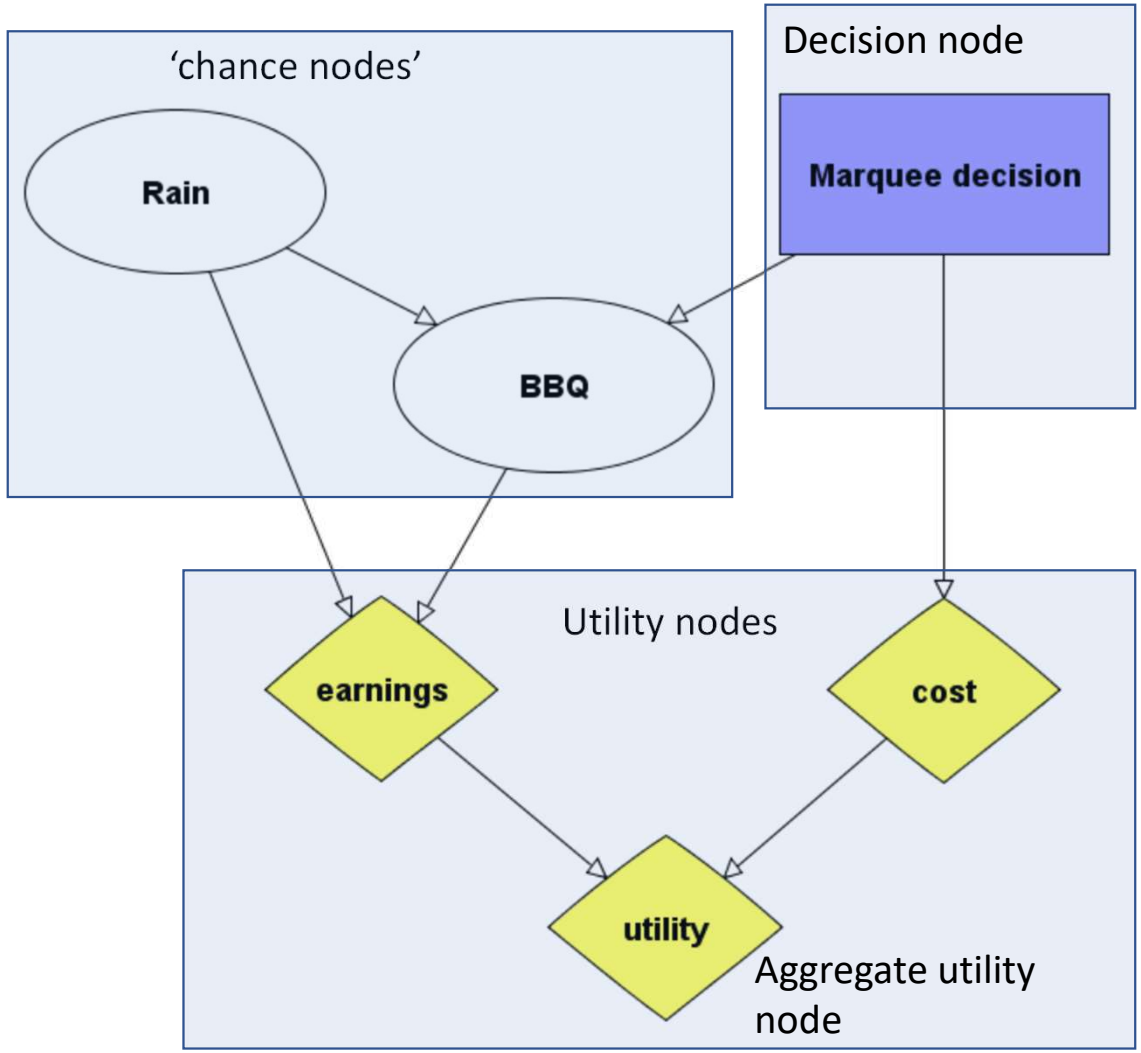
Choose the decision that maximizes expected utility

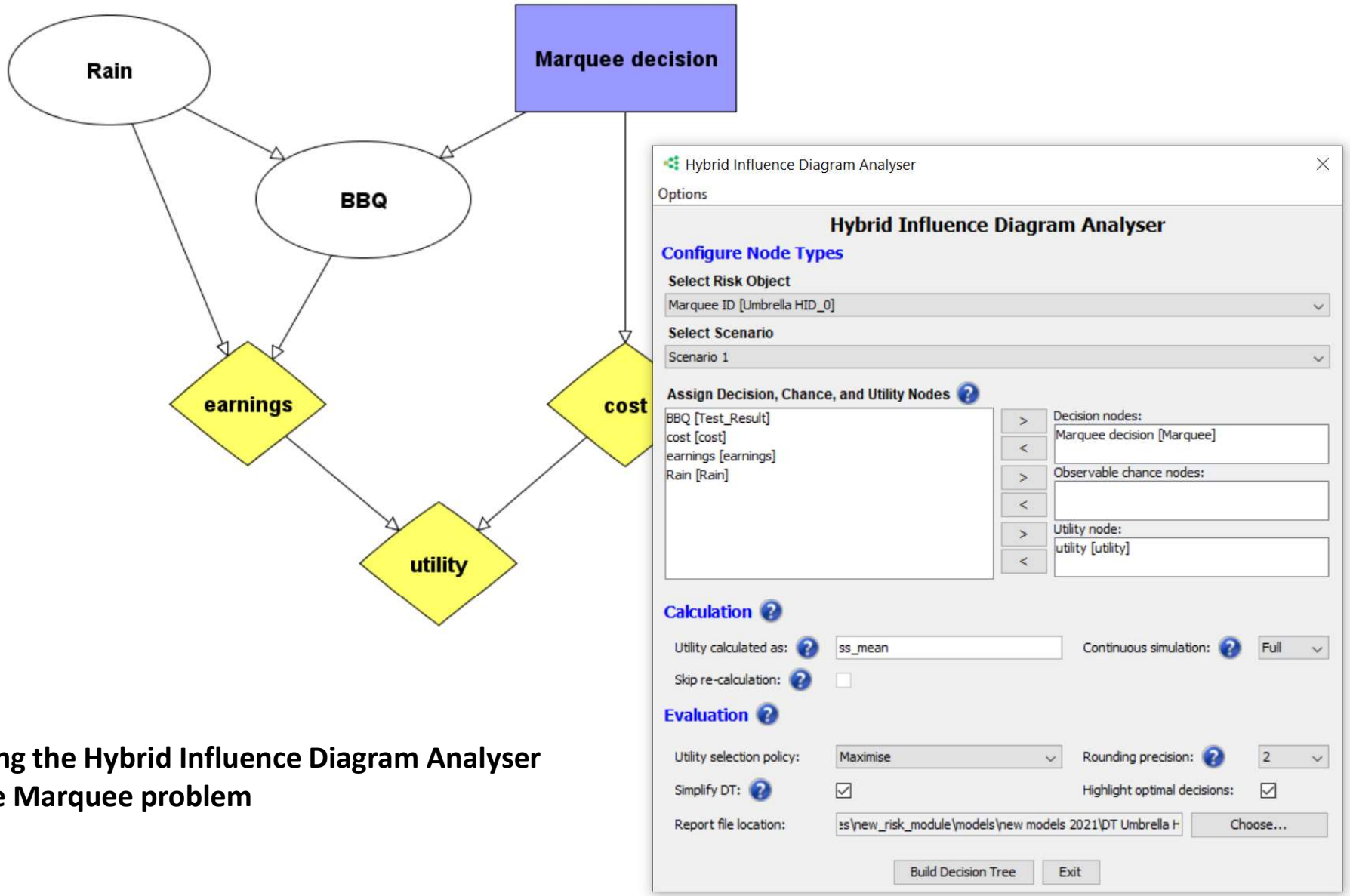
Influence diagrams

As more decisions and outcomes are added using decision trees become exponentially complex
Hence decision trees are not generally a feasible way of 'solving' decision problems
A more compact solution is provided by ***influence diagrams***
An influence diagram is simply a Bayesian network with some additional types of nodes:

- Regular BN nodes (called 'chance nodes' in an influence diagram)
- Decision nodes
- Utility nodes (one of which represents the 'aggregate utility')

There are algorithms that automatically compute the optimal decision (and associated utility) in an influence diagram
AgenaRisk does this computation and outputs a simplified decision tree showing optimal path





Running the Hybrid Influence Diagram Analyser for the Marquee problem

…produces this html output in your browser

Decision Tree for: Marquee ID [Umbrella HID_0]

Model: charity_marquee_ID.cmpx

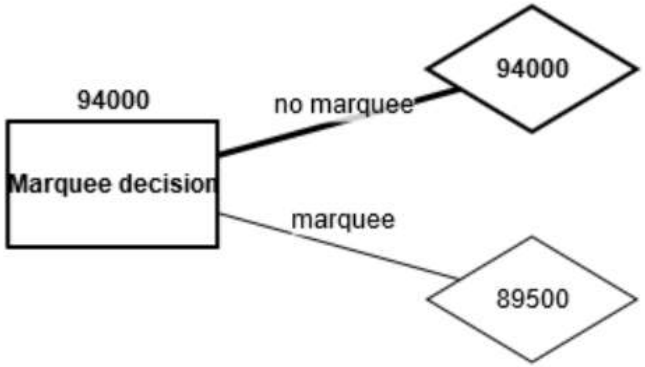
Generated: 07/02/21 22:38

Scenario: Scenario 1

Total build time: 17 ms

Image size: 307x178

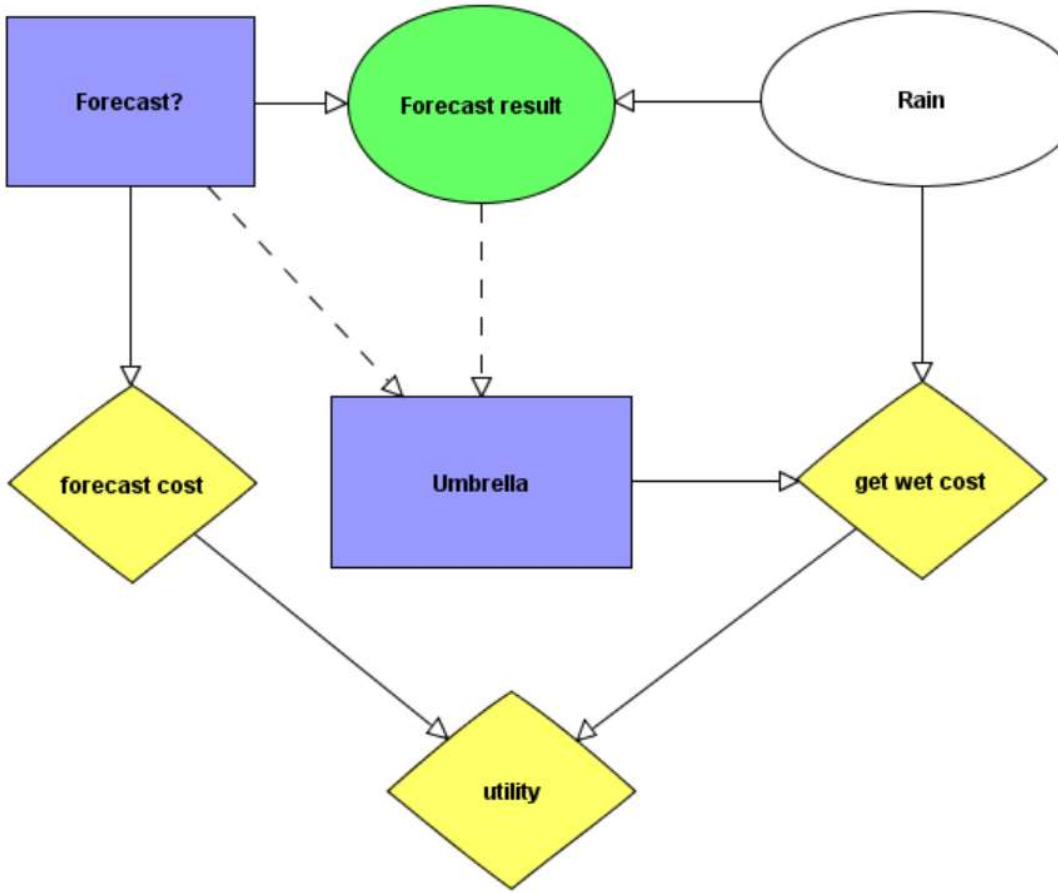
[+] Adjust graph settings

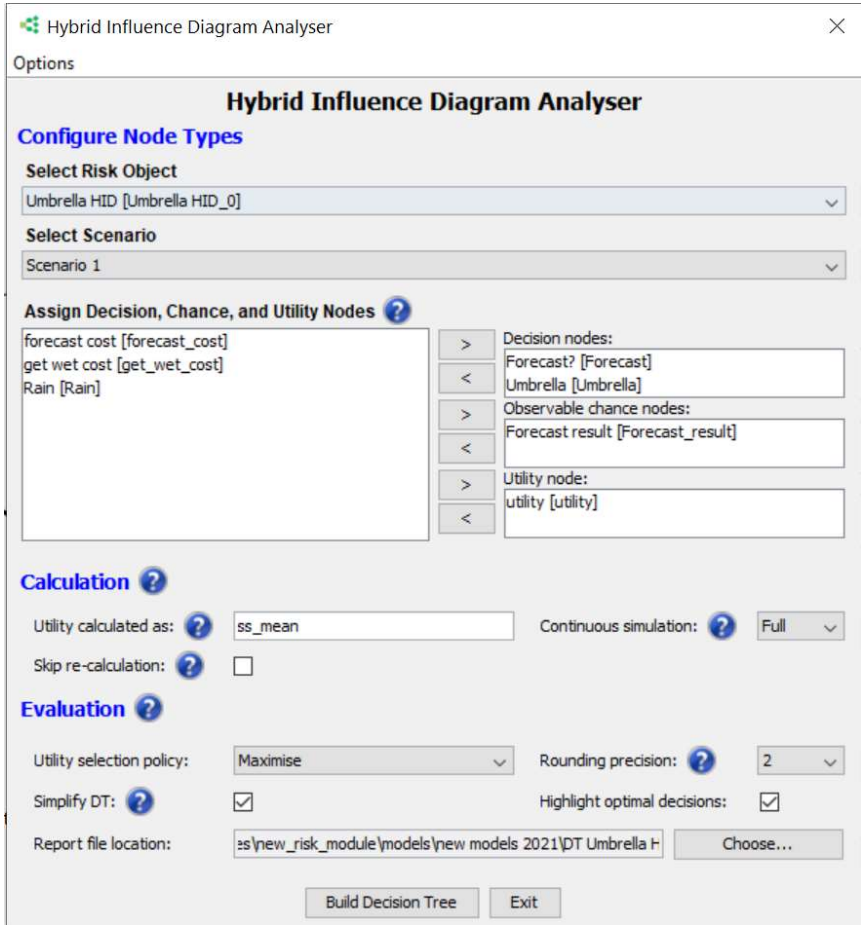


Please note that to open this report on a different device or operating system you may need to save the diagram as an image or save this report as an HTML page from your browser.

[+] Duration log

[+] Copyright and References





Running the Hybrid Influence Diagram Analyser for the umbrella problem

…produces this html output in your browser

Decision Tree for: Umbrella HID [Umbrella HID_0]

Model: Umbrella influence diagram.cmpx

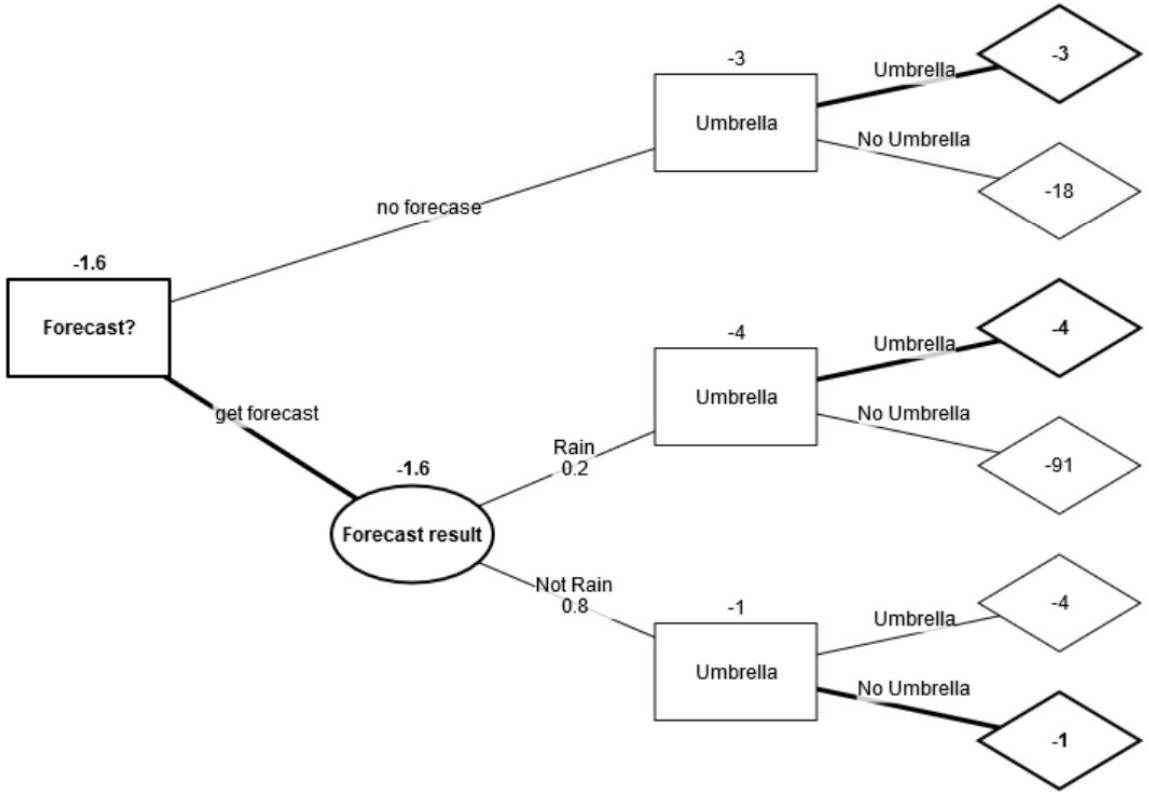
Generated: 07/02/21 22:51

Scenario: Scenario 1

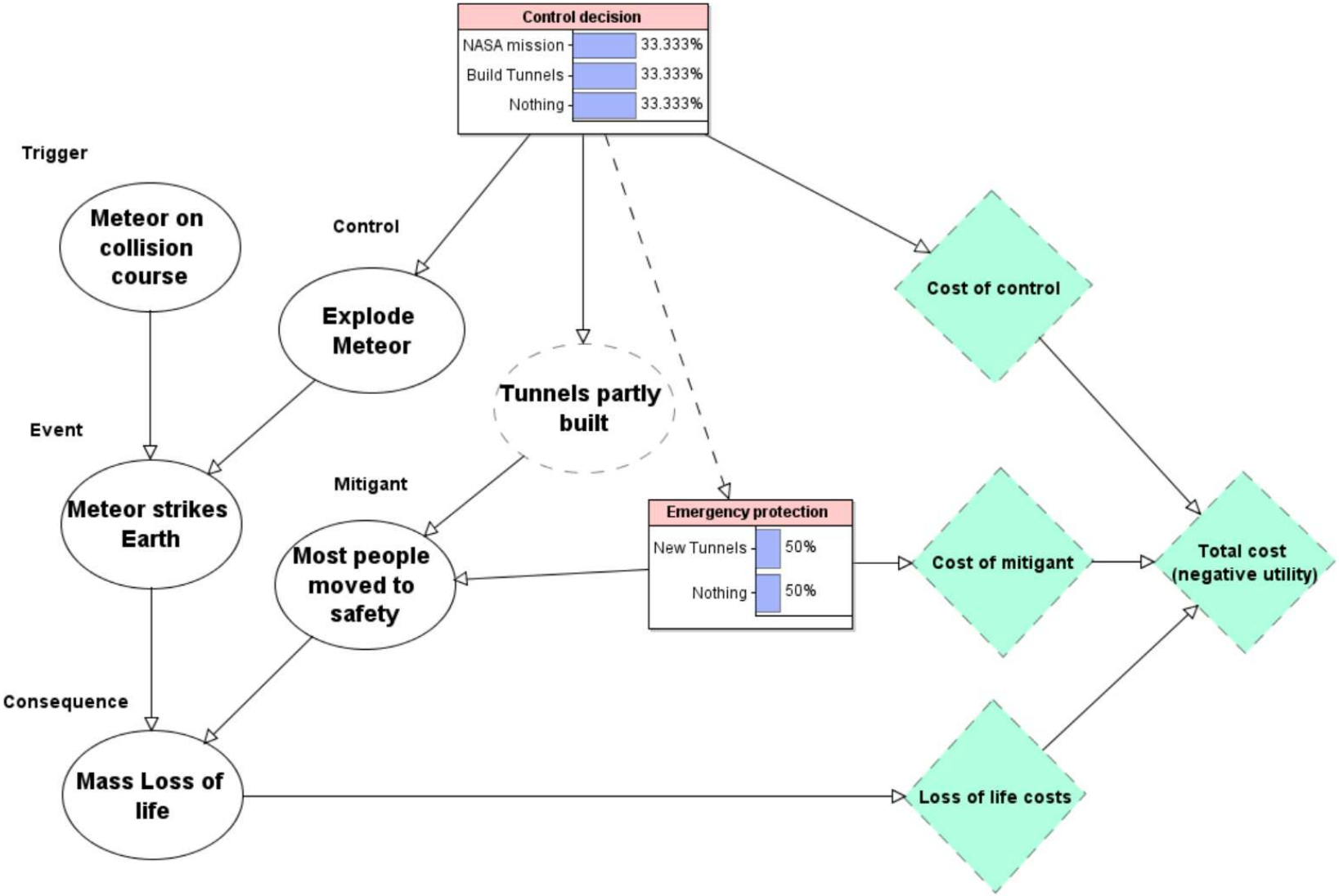
Total build time: 26 ms

Image size: 707x495

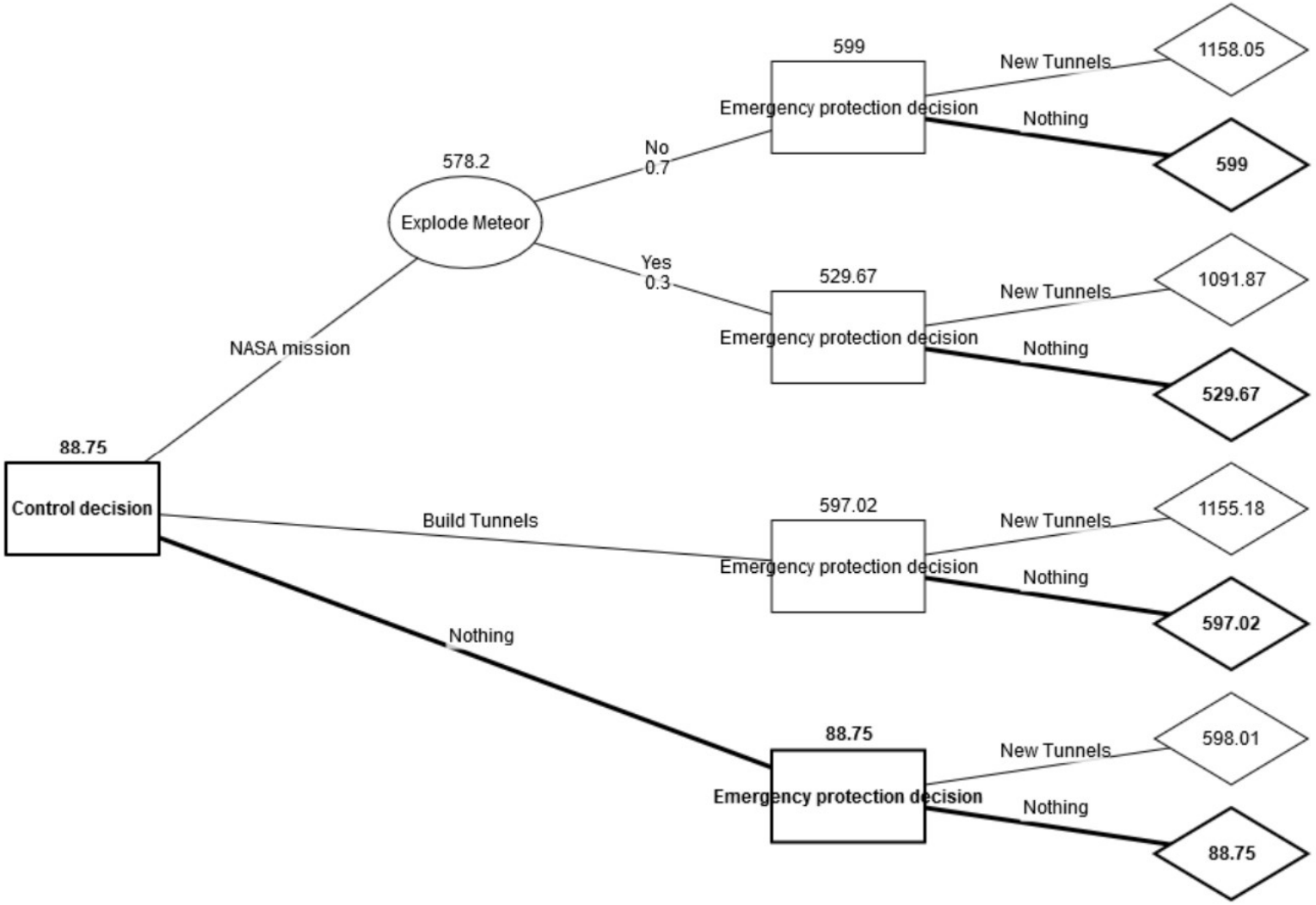
[+] Adjust graph settings



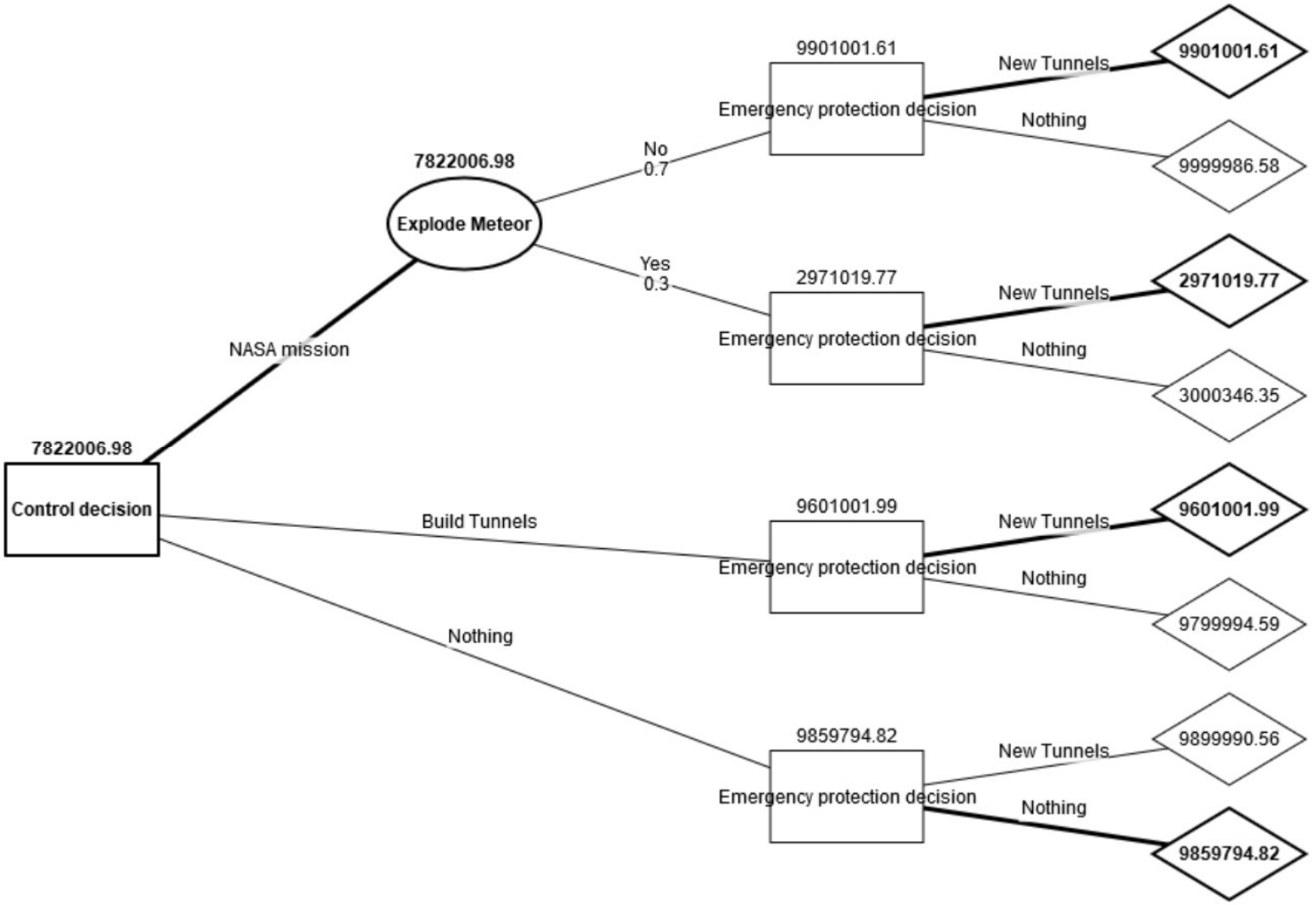
Armageddon Influence Diagram



Armageddon: no meteor alert



Armageddon: meteor on collision course



Key points from this lesson

- ‘Correct’ problem framing is critical to risk perception
- Most media reports that quantify risk based on empirical studies, use ‘relative’ risk rather than ‘absolute’ risk
- Relative risk generally exaggerates the true risk, so absolute risk should be used instead
- Standard definitions of risk that drive ‘risk registers’ are oversimplistic and often highly misleading
- To properly quantify risk, we must think of risk in terms of a causal model where the risk event is conditioned on one or more trigger and one or more controls and where the consequence is conditioned on mitigating actions
- For rational decision-making we must incorporate the notion of utility and cost-benefit. Influence diagrams enable us to do this efficiently.