Week 6

Live Discussion Session

Starts at 2.05pm

Bayes v classical stats

One double headed coin in bag with 4 fair coins

Coin is tossed twice. Both Heads. What is the probability the coin is fair?

Classical stats (two extreme positions, neither makes sense):

- 2 heads out of 2 tosses. Probability of heads = 1. Hence coin 'must be double headed"; or
- Not enough data to reach any 'statistically significant' conclusion

But of course we must revise our belief in P(fair coin) after observing even this small amount of evidence

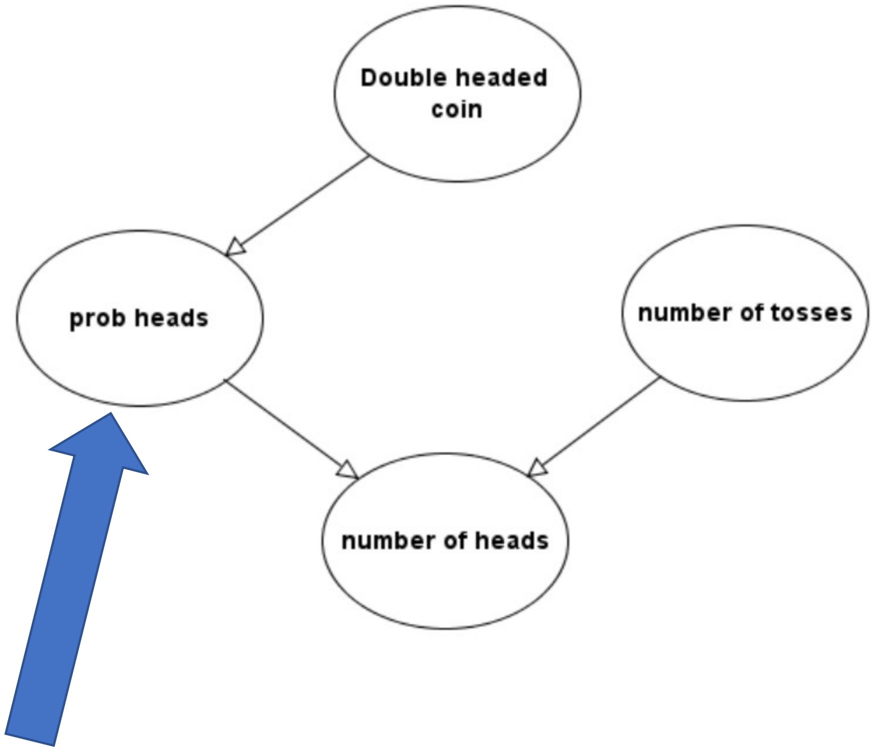
Let H be "fair coin", E = 2 heads out of 2
P(H) = 4/5   P(E|H) = ¼   P(E| not H) = 1

$$P(H|E) = \frac{P(E|H) \times P(H)}{P(E|H) \times P(H) + P(E|not\ H) \times P(not\ H)} = \frac{\frac{1}{4} \times \frac{4}{5}}{\frac{1}{4} \times \frac{4}{5} + \frac{1}{5}} = \frac{1}{2}$$

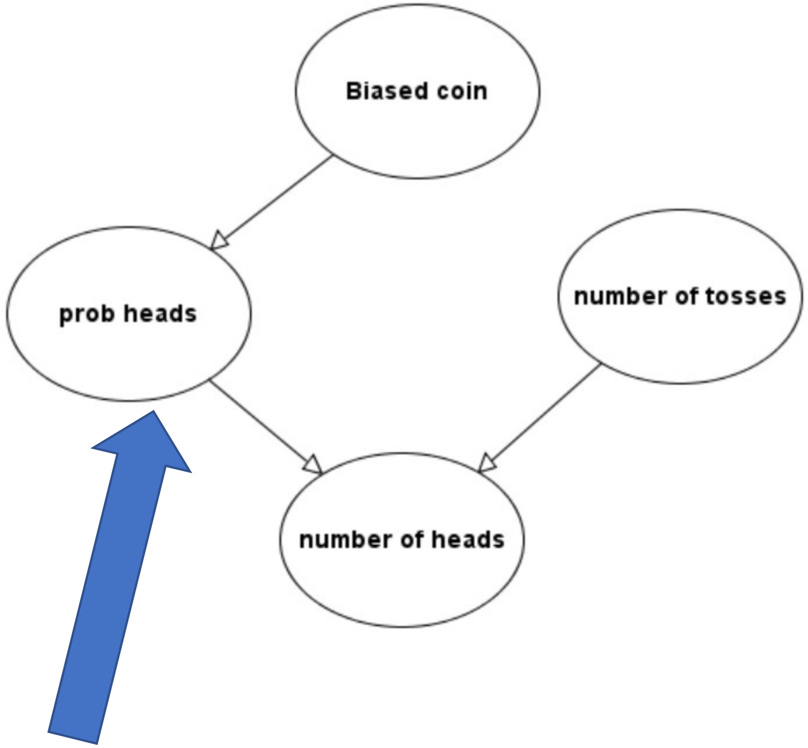
To achieve statistical significance at p-value 0.01 (1%) we must have P(E|H)<0.01

If we observe 6 out of 6 heads P(E|H) = 1/64 = 0.015625   NOT SIGNIFICANT!!!!!

Need to observe at least 7 out of 7 heads   P(E|H) = 1/128 = 0.0078125

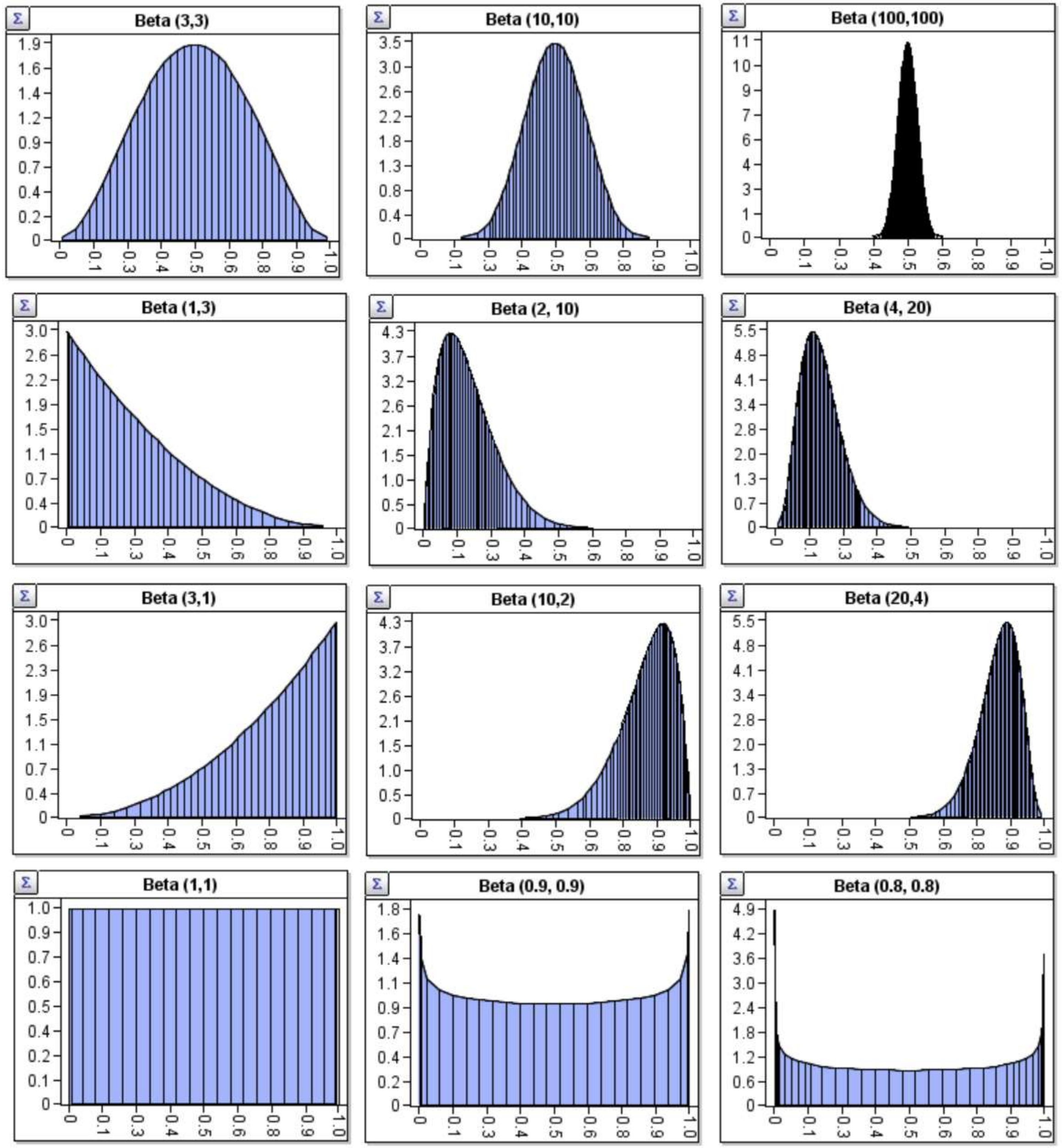


Double headed coin	False	True
Expressions	Arithmetic(0.5)	Arithmetic(1.0)

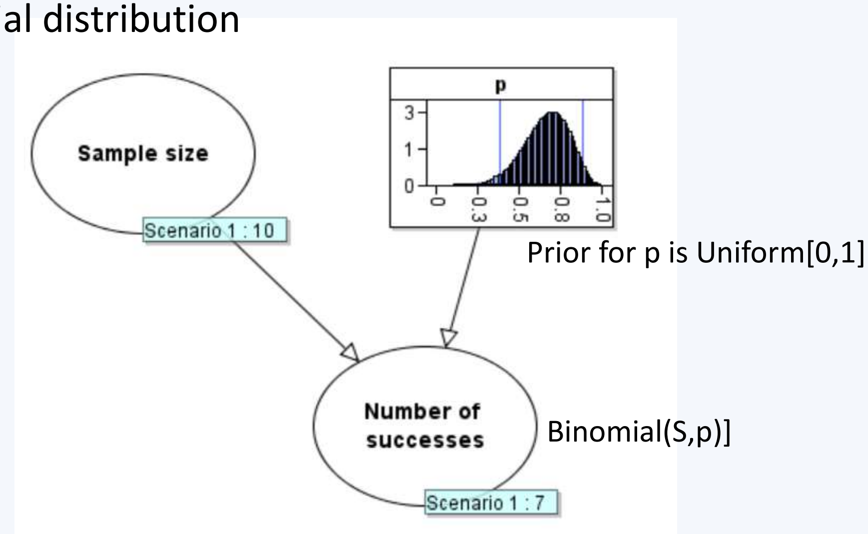


Biased coin	False	True
Expressions	Uniform(0.499,0.501)	Uniform(0.501,1.0)

Beta(alpha, beta, 0, 1) distribution with range of alpha and beta parameters

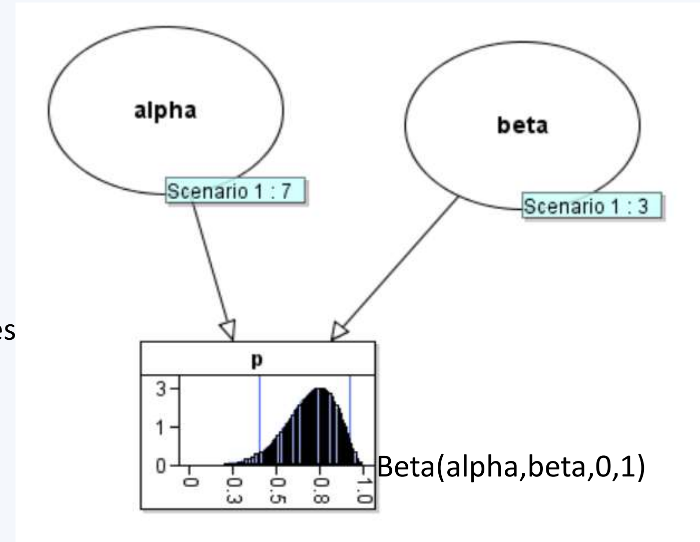


Binomial distribution



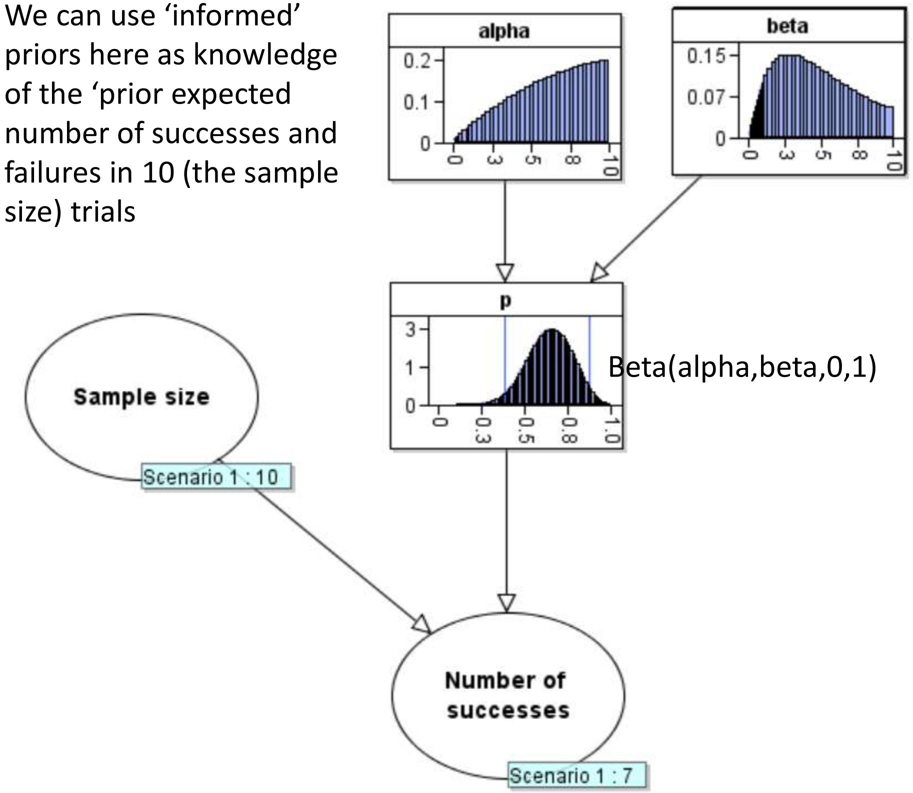
Beta distribution

Think of:
alpha: number of successes
beta: number of failures
in *alpha+beta* trials



“Beta-Binomial” distribution

We can use ‘informed’ priors here as knowledge of the ‘prior expected number of successes and failures in 10 (the sample size) trials



Meta analysis: combining results from different studies/samples where we cannot assume they come from 'same' population