Quantum Mechanics Assignment One

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Problem 2.17

This problem asks us to take the following spin-1 state and figure out the probabilities of measuring z- and x-component spins, and also to find their expectation value.

$$|\Psi\rangle \doteq \frac{1}{\sqrt{30}} \begin{pmatrix} 1 \\ 2 \\ 5i \end{pmatrix}$$

We use the normal $P_x = |\langle \Psi_x | \Psi \rangle|^2$ to find probabilities. It is simple to calculate the braket since both states are in the S_z representation.

$$P_1 = |\langle \Psi_1 | \Psi \rangle|^2 = \left| \frac{1}{\sqrt{30}} \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 5i \end{pmatrix} \right|^2 = \frac{1}{30}$$

$$P_0 = |\langle \Psi_0 | \Psi \rangle|^2 = \left| \frac{1}{\sqrt{30}} \begin{pmatrix} 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 5i \end{pmatrix} \right|^2 = \frac{4}{30}$$

$$P_{-1} = |\langle \Psi_{-1} | \Psi \rangle|^2 = \left| \frac{1}{\sqrt{30}} \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 5i \end{pmatrix} \right|^2 = \frac{25}{30}$$

$$\langle \Psi | S_z | \Psi \rangle = \frac{1}{30} \begin{pmatrix} 1 & 2 & -5i \end{pmatrix} \begin{pmatrix} \hbar & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\hbar \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 5i \end{pmatrix} = \frac{1}{30} \begin{pmatrix} 1 & 2 & -5i \end{pmatrix} \begin{pmatrix} \hbar \\ 0 \\ 5\hbar i \end{pmatrix} = \frac{1}{30} (1 + -25) = \frac{-4}{5}$$

We then want to compute the expectation value of the spin in the x-direction. We use the S_z representation of S_x:

$$\langle \Psi | S_x | \Psi \rangle = \frac{1}{30} \begin{pmatrix} 1 & 2 & -5i \end{pmatrix} \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 5i \end{pmatrix} = \frac{1}{30\sqrt{2}} \begin{pmatrix} 1 & 2 & -5i \end{pmatrix} \begin{pmatrix} 2 \\ 1 + 5i \\ 2 \end{pmatrix}$$

$$= \frac{4}{30\sqrt{2}}$$

Problem 5.11

This problem deals with an infinite square well wave which suddenly has its right potential wall move from L to $3L$. We are asked to find the probability of finding the wave in the ground or first excited state. This problem is basically asking us to find the c_1 and c_2 coefficients of the old wave function in the new well, and from them find probabilities.

$$\Psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right)$$
$$|E_1\rangle = \sqrt{\frac{2}{3L}} \sin\left(\frac{\pi x}{3L}\right)$$
$$|E_2\rangle = \sqrt{\frac{2}{3L}} \sin\left(\frac{2\pi x}{3L}\right)$$

We wish to find the c_1 and c_2 coefficients for this system.

$$\begin{aligned}
c_1 = \langle E_1|\Psi\rangle &= \int_0^{3L} \sqrt{\frac{2}{3L}} \sin\left(\frac{\pi x}{3L}\right) * \sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right) dx \\
&= \sqrt{\frac{4}{3L^3}} \int_0^{3L} \sin\left(\frac{\pi x}{3L}\right) \sin\left(\frac{\pi x}{L}\right) \\
&= \sqrt{\frac{2}{3L^3}} \left(\int_0^{3L} \cos\left(\frac{2\pi x}{3L}\right) dx - \int_0^{3L} \cos\left(\frac{4\pi x}{3L}\right) dx\right) \\
&= 0
\end{aligned}$$

$$\begin{aligned}
c_2 = \langle E_2|\Psi\rangle &= \int_0^{3L} \sqrt{\frac{2}{3L}} \sin\left(\frac{2\pi x}{3L}\right) * \sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right) dx \\
&= \sqrt{\frac{4}{3L^3}} \int_0^{3L} \sin\left(\frac{2\pi x}{3L}\right) \sin\left(\frac{\pi x}{L}\right) \\
&= \sqrt{\frac{2}{3L^3}} \left(\int_0^{3L} \cos\left(\frac{\pi x}{3L}\right) dx - \int_0^{3L} \cos\left(\frac{5\pi x}{3L}\right) dx\right) \\
&= 0
\end{aligned}$$

From this we can see that the probability of measuring the wave in the new tripled well to be in $|E_1\rangle$ or $|E_2\rangle$ is zero, since $P_n = |c_n|^2$. This result is to be expected, since the wave should be completely in the second excited state, $|E_3\rangle$. This is because extending the $\Psi(x)$ wave function from 0 to $3L$ results in a two-node wave function identical to the $|E_3\rangle$ state.

8.7

We begin with the potential and find the r-values at which $E = V$. We then calculate the probability using $P = \int_r^\infty \psi^* \psi dr$.

$$V(r) = \frac{-e^2}{4\pi\epsilon_0 r}$$

We first solve for the forbidden distance:

$$\begin{aligned}
V(r) &= E_2 \\
-\frac{Z\alpha\hbar c}{r} &= -\frac{1}{2n^2}\alpha^2 m_e c^2 \\
\frac{-1.44\text{eV nm}}{r} &= -3.4\text{eV} \\
r &= 2.36\text{nm}
\end{aligned}$$

Next we find the probability of our particles being further than this:

$$P_{200} = \int_{2.36}^{\infty} \int_{0}^{2\pi} \int_{0}^{\pi} \frac{1}{\sqrt{8\sqrt{2}\pi a_0^{3/2}}} \left(2 - \frac{r}{a_0}\right) e^{\frac{-r}{2a_0}} r^2 \sin(\theta) dr d\phi d\theta =$$

$$P_{21-1} = \int_{2.36}^{\infty} \int_{0}^{2\pi} \int_{0}^{\pi} \left(\frac{3r}{16\pi\sqrt{6}a_0^{5/2}}\right)^{1/2} \sin(\theta) e^{-i\phi} e^{-\frac{r}{2a_0}} r^2 \sin(\theta) dr d\phi d\theta =$$

$$P_{210} = \int_{2.36}^{\infty} \int_{0}^{2\pi} \int_{0}^{\pi} \left(\frac{3r}{8\pi\sqrt{6}a_0^{5/2}}\right)^{1/2} \cos(\theta) e^{-\frac{r}{2a_0}} r^2 \sin(\theta) dr d\phi d\theta =$$

$$P_{211} = \int_{2.36}^{\infty} \int_{0}^{2\pi} \int_{0}^{\pi} -\left(\frac{3r}{16\pi\sqrt{6}a_0^{5/2}}\right)^{1/2} \sin(\theta) e^{i\phi} e^{-\frac{r}{2a_0}} r^2 \sin(\theta) dr d\phi d\theta =$$