

# Additional Materials: Sensitivity and Specificity

Consider an Oracle that can predict HIV with the following:

**Sensitivity:**  $P(\text{Predict } + | \text{HIV}) = 99.9\%$

**Specificity:**  $P(\text{Predict } - | \text{healthy}) = 99.9\%$

Prevalence:  $P(\text{HIV}) = 0.01228\%$  (Singapore, 2012)

Is this an accurate test?

... suppose the “prediction” comes back positive: are you HIV infected? What is  $P(\text{HIV} | +)$ ?

# Bayes Theorem

$$\begin{aligned}
 P(HIV|+) &= \frac{P(+ \text{ and } HIV)}{P(+)} \\
 &= \frac{P(+|HIV)P(HIV)}{P(+|HIV)P(HIV) + P(+|healthy)P(healthy)} \\
 &= \frac{0.999 \times 0.0001228}{0.999 \times 0.0001228 + 0.001 \times 0.9998772} \\
 &= 10.93\%
 \end{aligned}$$

	Actual +ve	Actual -ve
Predicted +ve	TP	FP
Predicted -ve	FN	TN

**Sensitivity**      **Specificity**



**Sensitivity:**  $P(\text{Predict } +|HIV) = 99.9\%$   
**Specificity:**  $P(\text{Predict } -|healthy) = 99.9\%$

Now, think again, is this an accurate test?

# We need other measures!

- Consider a population of 1,000,000 people
- Prevalence of 0.01228% implies:
  - Number of HIV Positive  $\approx 123$
- Number of HIV Negative = 999,877

	Actual +ve	Actual -ve		Measure
<b>Predicted +ve</b>	$0.999 \times 123 \approx 123$	$999,877 - 998,877 = 1000$	<b><math>123/1123 \approx 10.95</math></b>	PPV (Precision)
<b>Predicted -ve</b>	$0.001 \times 123$	$0.999 \times 999,877 \approx 998,877$		NPV
	<b>0.999</b>	<b>0.999</b>		
<b>Measure</b>	Sensitivity (Recall)	Specificity		

# Other Quality Measures

- Positive Predictive Value, PPV (Precision):
  - Probability that subjects with a positive screening test truly have the disease
- Negative Predictive Value, NPV:
  - Probability that subjects with a negative screening test truly do not have the disease.
- Both are threshold dependent

# Other Quality Measures

NPV

PPV

Also known  
as Precision

Predicted No CHD (0)

Predicted CHD (1)

Actual No CHD (0)

853

77

Specificity

Actual CHD (1)

126

41

Sensitivity

Also known  
as Recall

- Consider the threshold value of 0.3:
  - PPV =  $TP / (TP + FP) = 41 / (41 + 77)$
  - NPV =  $TN / (TN + FN) = 853 / (853 + 126)$