FX derivatives (Market Maker) Technical report

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Abstract

This report documents an experiment simulating the role of an FX derivatives market maker, prioritizing client quoting, transaction execution, and hedging strategies to manage foreign currency risk exposure. We examine market rules, client quoting procedures, and hedging techniques based on interest rate parity and the Black-Scholes model. Practical application of theoretical concepts will demonstrated, including risk evaluation using 'The Greeks' and challenges in execution, such as missed trading opportunities and pricing errors, within a competitive market environment.

1 Introduction

The experiment undertaken within this report involves assuming the role of an FX derivates market maker, with the primary responsibilities including client quoting, transaction execution, and application of effective hedging strategies to reduce foreign currency risk exposure. Results and performance metrics will be analyzed and explanations of 'the Greeks' and their importance will be highlighted. Requirements of successfully traded transactions will be evaluated and the justification of models used to provide clients with fair and profitable quotes. This shall be done whilst ensuring delta risk is kept between $+-$ 100 Million. Real-time FX data will be used to provide clients with real-time quotes and any executed trades will be hedged to reduce foreign currency risk.

2 Market rules

Before we can start quoting clients, we need to understand what rules are in place within the market, to offer a valid executable offer to the client. Requirements that restrict our market maker include,

- The board lot of the FX market is 5 million USD - This is important as it will mean that we will not be able to completely hedge our position for every trade. For example, consider that effective hedging might be achieved at a size of 32 million USD. Due to this board lot size, we would be restricted to choosing between 30 and 35 Million USD for our hedging trade. Consequently, not all Foreign Exchange risk will be fully hedged and there will be some remaining Delta risk left from this trade.
- Tick sizes of FX and FX forward markets being both 0.001 - this means that any prices quoted to clients need to be given at 3 decimal places. When we calculate fair, ask and bid offers quite often we will be provided with a long series of numbers, we will have to round this to the nearest 3 decimal points and choose between the upper and lower price depending on how this would affect the Mtm (Mark-to-market) and Spot-Delta (M MXN).
- The tick size of the FX options market is 0.01% - this means that any FX options offered need to be priced at 2 decimal points.
- There is no board lot constraint in the FX forward market MXN and we must note that the market changes daily so prices used to offer any quotes will be provided at that day's rates.

3 Client Quoting

One of the main objectives of this investment experiment was to provide quotes to clients for forwards, puts, and calls. There are certain requirements that some of our clients require to trade, such as quoting at the money, providing a fair quote, or quoting at any price. It should be noted that due to the Interest Rate Parity principle, all of our quotes use arbitrage-free pricing. To calculate a forward contract we will either lend or borrow from the money market to obtain the forward rate. At time 0 we exchange the MXN with a spot rate of S and a notional value of N USD. For example, if we were borrowing MXN from the money market we would borrow $N * S$ MXN. We then exchange this amount into N USD and hold this USD until one year later. Then we exchange this using the forward contract with forward rate f. We will be able to use N USD to gain interest one year later from the USD money market. This amount of USD with the interest rate multiplied by the forward rate f will provide us with the amount of MXN. Finally, as we have borrowed MXN from the bank we must return this amount of MXN, this will be the amount that we initially borrowed multiplied by the interest rate for MXN. These two equations for MXN should equal (as shown below) and from this, we will be able to solve for the forward rate f of the contract using the continuously compounded rate, this methodology comes from the Interest rate parity principle.

$$N * (1 + r_{USD}) * f = N * S * (1 + r_{MXN}) \tag{1}$$

Before we can price a forward contract, we need to first calculate the day count fraction. This is the time to maturity divided by the number of days in a normal year (365), it is imperative to have this equation as it is used to work out the total currency interest over the period of our contract. To work out the forward rate we used the mid-spot rate multiplied by the exponential of the market rate of the quoted currency r_q times by the day count fraction δ_q (in our case this is MXN) this is divided by the exponential of the market rate of the base currency r_b multiplied by the day count fraction δ_b (in our case this is USD) as we can see below. This gave us the forward rate for the desired contract, however, for our clients we are required to provide a forward offer and a forward bid. Replacing the mid spot rate in the forward equation with the spot rate offer or bid we are then able to gain the forward offer or bid respectively. From this we are left with the fair quote, if we were to provide the clients a contract at this price, we would profit very little from it. As market makers, our source of profit comes from the bid-ask spread, so it is imperative that when there is no market requirement to provide a fair quote or at mid-price, we extend the bid-ask spread to provide more potential profit.

$$Fwd = Spot * \frac{1 + r_q\delta_q}{1 + r_b\delta_b} = Spot * \frac{exp(r_q\delta_q)}{exp(r_b\delta_b)} \tag{2}$$

From the Black-Scholes model provided within the Excel spreadsheet we can interpret this forward rate pricing as follows,

$$C(S, K, r_f, q, T) = Se^{-qT} * N(d_1) - N(d_2) * K * e^{-r_f T} \tag{3}$$

$$P(S, K, r_f, q, T) = N(-d_2) * K * e^{-r_f T} * -S_0 e^{-qT} * N(-d_1) \tag{4}$$

$$d_1 = \frac{ln(\frac{S_0}{K}) + (r_f + \frac{\sigma^2}{2}) * T}{\sigma\sqrt{T}}, d_2 = d_1 - \sigma\sqrt{T} \tag{5}$$

Where K is the strike price, S is the spot price, q is the interest rate (the interest rate for MXN), r_f is the risk-free rate (the interest rate for USD), T is the maturity and $N(x)$ is the standard normal CDF. This is risk free when $\sigma = 0$ and so we obtain $C = Se^{-qT} - Ke^{-r_f T}$ and $P = 0$, as the forward rate has no cash flow at time 0, the forward rate K is: $K(S, r_f, q, T) = S * exp((r_f - q)T)$

To provide quotes for FX options we can use the Black-Scholes formula as detailed above. This model assumes that the underlying asset follows a log-normal distribution. The Black-Scholes formula was appropriate to use as in our experiment we only dealt with European options. To provide quotes for put and call options longer than 1 year, linear interpolation was needed as we were only provided with 1 and 5-year at the money forward volatility. Firstly, we needed to calculate the rate of our contract for the bid and ask. This was done by using the 5-year day count minus the maturity of the contract, divided by the 5-year day count minus the 1-year day count. This provided us with the rate of our contract for a 5-year period, 1 minus this value gives us the rate of our contract for the 1-year period. Using these values, we can now calculate the volatility to use for the bid and offer of an options contract. To do this we multiply the 1-year volatility bid by the 5-year rate and add this to the 5-year volatility bid multiplied by the 1-year rate. This method is the same for ask volatility where we use volatility ask instead of bid. These are once again the fair quotes, this means that if no restrictions on setting a bid-ask are specified, it will be beneficial to extend the bid/ask spread to maximize profit. These values are inputted into the call and put option formula that is displayed above and as such, provides us with the fair contract price to offer to our client.

4 Hedging

The purpose of hedging contracts in the market maker setting is to insure against foreign exchange market rates as they change. This risk is categorized and measured by the Spot-Delta, this is the sensitivity of a contract price to the spot price of the market. We always hedge the Spot-Delta for each contract. It is imperative that all contracts are hedged as a singular entity and not hedged as a group of contracts. The importance of this is that when a contract is hedged separately, should the Delta move sharply, singular hedging would counteract this as the hedging is designed specifically for this contract. However, if one hedging position was used for multiple contracts, as each contract is different due to different maturities, it would not be possible to completely hedge this contract. As such this would expose the position to unnecessary risk that could be avoided.

To hedge a forward contract two different strategies are used depending on the position of the contract. When the client longs a forward contract we will short-sell the spot and lend the base currency in the money market, this is equivalent to a longing a zero-coupon bond. The opposite can be said when a client shorts the forward contract, we long the spot and borrow the base currency, this is equivalent to shorting a zero-coupon bond. To find the amount of spot required for our hedging we must take the opposite position of the quoted currency at maturity. We then use this to find the discounted quoted currency at time 0. We will then be able to apply the spot rate to find the amount of base currency required to hedge. We must once again note that we will not be able to fully hedge our position given the restriction of trading a board lot of 5 Million for the base currency.

Hedging options are slightly different than hedging forwards as we do not need to take the opposite position of the quoted currency as it may not be exercised. When the client shorts an FX put option we will short-sell the spot and lend the base currency in the money market, this is equivalent to longing a zero-coupon bond. The opposite positions are held when we are shorting an FX call option. That is, we long the spot and borrow the base currency in the money market, this is equivalent to shorting a zero-coupon bond. To find the amount we require to be hedged we find the amount of quoted currency's Delta, such that it offsets the contract value changes due to the spot market changes. This is called Delta hedging. This is needed for hedging options as when the spot rate changes our base currency value and option value change, so we must hedge in this way. For every hedge that is completed, we cost the firm money, costing the bid-ask spread and paying the brokers.

5 'The Greeks'

'The Greeks' are commonly used to evaluate the different risks within the derivatives market and used to explain profit and loss. For this market experiment, we only evaluate the major Greeks, including,

- Delta explained (Δ) – This is the sensitivity of a contract price relative to the spot price and is calculated by the previous Delta multiplied by the change of the spot rate. As this is the change of \$1 in the underlying assets price, we will need to multiply this value by 1 million as our contracts are provided in millions. This Delta is then used as a hedge ratio for creating a Delta-neutral position and eliminating the risk of changing FX rates.

- Vega explained (ν) – Is the sensitivity of a contract price compared to its implied volatility and measures the PnL from a 1% change in volatility. As there is no change in implied volatility for forward contracts this will only contribute towards our PnL explained for the option contracts. To calculate the Vega explained, we take the previous Vega multiplied by the change of the at-the-money implied volatility (ATMF).

- Gamma explained (Γ) – Is the sensitivity of Delta to the spot price, unlike the other Greeks this is a second-order price sensitivity. This is used to indicate how stable a specific contract Delta is. The higher the Gamma the more rapidly the Delta could change in response to movement in the underlying price. Gamma increases in magnitude the closer the contract approaches expiration and options that are further away from maturity will have much smaller Gamma, as these are less sensitive to Delta changes. It is sometimes known to hedge for Gamma as well as Delta to be Delta-Gamma neutral, meaning that as the underlying price changes the Delta remains close to 0. However, in this experiment, we only hedge for Delta. Gamma explained is calculated by computing 0.5 multiplied by the second derivative of the change in the spot rate multiplied by the previous Gamma. Like Delta, this is also multiplied by 1 million for the same reason.

- Rho explained (ρ) – This is the sensitivity of the contract price to the risk-free rate. For FX derivatives we have two types of Rho's the local interest rate Rho and the foreign interest rate Rho. In the local currency rate, this is the change of the disc rate multiplied by the previous local currency Rho. However, for the foreign currency, Rho is the change of the forward rate multiplied by the previous Rho for the foreign currency. As Rho is measured by a 1% change we divide this equation by 0.01% to suit the measurements within our Excel spreadsheet. Rho will be highest when there is an at-the-money option with a long time until expiration.

- Theta explained (θ) – This is the sensitivity of a contract price when compared to the maturity also known as the time to decay. Theta indicates the amount a contract will decrease as the time to maturity decreases. When contracts are priced at-the-money, Theta increases. The opposite is true when contracts are in or out of the money. As a contract advances closer to maturity it will have an accelerating time decay. Normally, contracts with a short position will normally have a positive Theta whereas contracts with a long position will tend to have a negative Theta. This indicates how much of an options value is being lost. Within our Spreadsheet Theta can be explained by the carry (This is calculated by using the notional MXN multiplied by the daily Theta function built into Excel), multiplied by the change in the change of time to maturity.

6 Analysis of results

Now that we have established the methodology that the market maker uses, we can analyze and evaluate how this theory works in practice. Throughout this experiment, there were many opportunities to provide a variety of quotes to the client including forwards, calls, and puts at a variety of positions. On the first market day, we can notice that there was no explained PnL. This was due to the Greeks summing up to provide the explanation for profit and loss, and the absence of a previous market day sends all the Greeks to 0. Over the duration of our experiment, we can see that the Spot-Delta is effectively hedged for each singular contract. We can see the effect this has on the older trades as they progress throughout the market, with some market days having a total Spot-Delta as low as 1.08. This was well within the experiments' requirement and enabled us to see that we have achieved suitable risk-management and well-hedged positions. We must notice that there is never a scenario where our contracts are fully hedged as explained previously. However, we were still able to manage the Spot-Delta efficiently enough that it never moved more than 50 points around 0 on any of the market days. As previously mentioned, the only contracts that can be explained by Gamma and Vega were the call and put option contracts. Notably, we can see that Client F has the highest Vega, which means that this contract is very sensitive to changes in volatility, we can see that this sensitivity isn't as sharp in other contracts. Evaluating the options contracts, we can see that client D has the highest Gamma out of all the option contracts. This tells us that as the market days advance the overall delta for this option will begin to increase and we can observe this in the Excel sheet. In order to combat this we could consider re-hedging, but in our case, this was not necessary as our delta risk was already managed appropriately. It can be seen throughout all the option contracts that the lower the Gamma the smaller the increase of Delta there is as the market advances. There was not much trend with Rho, however we can notice that on market days were there was no MXN Rho there was USD Rho and vice versa.

On some of the market days, we can observe some unexplained profit and loss with the maximum on all market days being -$14,071. This is related to the fact that the options risk profile is more complicated than the forwards. With options, the pricing formula is non-linear and produces this unexplained profit and loss and we can see this change as we progress from forward contracts to option contracts. This is due to a Non-linear change of price and the dramatic change of Greeks, most notably Vega and Gamma. On the final day of the experiment, it can be seen that we have a final Money-to-market of $6,913,128, and on the last two market days where no trades occurred, we made a continuous loss. This was due to the costs associated with hedging the contracts, so these are unavoidable. The majority of this profit came from Client F with our most profitable market day being Feb-27 with a PnL of $6,453,263. All contracts that were exercised were correctly hedged and recorded within the trades blotter. Within the quotes blotter we can see that there exist three trades that were not executed, leaving us with a total trade execution of around 63%. Two of these trades were not executed due to the speed of providing the quotes, this was an important lesson to be learned, as within the market if a client is not provided with a quote quickly they will leave and find someone else to quote their contract. We are competing with the other market makers, and when providing quotes we should be punctual and accurate to maximize the chance of execution. We can see that for client G, we accidentally provided a quote that was not profitable. This was due to negligence, and by using the wrong maturity to calculate the price of the contract we provided an advantageous quote for the client, this can be seen within the Excel spreadsheet and we did lose money on this trade. For client F we were not able to trade due to the pricing difference we had when compared to other market makers. We had provided a bid-ask spread that was large, and as such we, were beaten by another market maker providing a better quote for the client.

7 Conclusion

Overall, using all the methodologies in a practical setting we were successful throughout this market experiment. Totaling with a closing profit of $6,913,128 and the Spot-Delta remaining at 43.79 with all objects successfully achieved. If this experiment were to be conducted again, more time and care when providing quotes would be taken and punctuality doing this would be increased due to the experience we have gained. Additionally, we would consider using new techniques to optimize our risk management, such as the previously mentioned Delta-Gamma hedging and re-hedging of positions.

7.1 references

Investopedia the Greeks - https://www.investopedia.com/terms/g/greeks.asp