Matlab tutorial #1: Part 1:
Study of transfer functions using Matlab

Report only for the case study: due on Monday 7th of October 2013

In this tutorial we see how the Matlab control toolbox helps us to study the properties of transfer functions

You can define a transfer function by its numerator and denominator: **g=tf(num,den)**

1. **Calculation of poles and zeros**

The following functions can be used to find the poles and zeros of a transfer function or to find the transfer function given the poles and zeros:

Example 1: Given the following transfer function: $\frac{3s+21}{s^2+2s+10}$,

define the numerator n and the denominator (remember that polynomials can be defined using vectors in Matab), find the poles and zeros of the transfer function using the following command: **[z,p,k]=tf2zp(num,den)**. (Note: ***z*** for zero, ***p*** for poles, ***k*** for the gain). What is the factorized form of the function transfer?

Check how to use **zp2tf**.

2. **Decomposition in simple fractions**

Example 2: Given the transfer function: $\frac{s+6}{s^3+6s^2+8s}$

define the numerator n and the denominator, calculate the decomposition in simple fractions, using **[r,p]=residue(n,d)** (Note: ***p*** represents the poles and ***r*** the corresponding constants)

3. **Response to a step entry Matlab**

Function **step** is one of the most used functions for control design. Given a system described by a transfer function, the temporal response to a step entry signal can be represented, using the **step** function.

Example 3: Draw the temporal responses to a step entry for the transfer functions of examples 1 and 2.

Case study: Business Jet and Jet Fighter dynamics

Given the longitudinal transfer functions of a Business Jet and a Jet Fighter, both in cruise flight

1- **Calculate** the poles of each aircraft, **identify** and **describe** the **different modes** of the aircraft (separate the non-periodic and the oscillatory modes), **calculate** the **time constant τ** and the **damping factor ζ** (if needed).

2- **Conclude** on the stability of each aircraft.

3- **Plot** the responses of the velocity and the angle of attack (for the two aircrafts) for a displacement of 2º of the elevator.

Note that:

- Here u(s) and α(s) are directly the variations/perturbations of linear velocity and angle of attack (with their corresponding SI units), i.e. $U(t)=U_0(t)+u(t)$
- Be careful with the units (in the calculations and in the plots) and remember that 1 knot = 0.514 m/s

1. The Business Jet flies in cruise at 40000ft, with a 400kt velocity (U_0).
Its transfer functions for the velocity and the angle of attack are:

$$\frac{u(s)}{\delta_e(s)} = \frac{-379\,s^2 + 271888\,s + 24033}{\nabla}$$

$$\frac{\alpha(s)}{\delta_e(s)} = \frac{-42\,s^3 - 11939\,s^2 - 89\,s - 79}{\nabla}$$

with $\nabla = 676\,s^4 + 1359\,s^3 + 5540\,s^2 + 57\,s + 46$

- Calculate the final values of the variation of angle of attack and of the total velocity (U) (values for the steady state) using two different methods.

2. The Jet Fighter flies in cruise at 45000ft, with a 516kt velocity (U_0).
Its transfer functions for the velocity and the angle of attack are:

$$\frac{u(s)}{\delta_e(s)} = \frac{-247\,s^3 + 51\,s^2 - 218196\,s - 68073}{\nabla}$$

$$\frac{\alpha(s)}{\delta_e(s)} = \frac{-35\,s^3 + 5786\,s^2 + 11.5\,s + 22.5}{\nabla}$$

with $\nabla = 871\,s^4 + 608\,s^3 - 9065\,s^2 - 43\,s - 43$

- Is there any advantage to fly an unstable aircraft?
What solution would have the pilot in order to be able to fly this kind of aircraft?