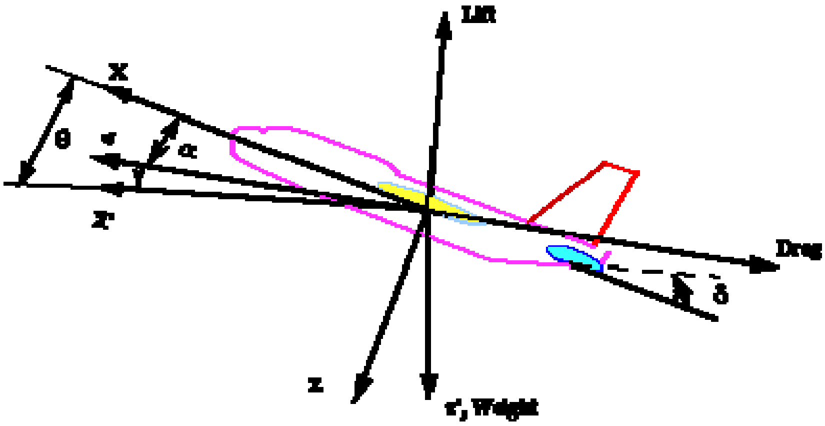
Matlab tutorial #1: Part 2:

Modelling a simple pitch controller

Report: due on Monday 14th of October 2013



The longitudinal equations of motion of an aircraft can be written as:

$$\begin{aligned}
\dot{\alpha}(t) &= -0.313\,\alpha(t) + 56.7\,q(t) + 0.232\,\delta_e(t) \\
\dot{q}(t) &= -0.0139\,\alpha(t) - 0.426\,q(t) + 0.0203\,\delta_e(t) \\
\dot{\theta}(t) &= 56.7\,q(t)
\end{aligned}$$

For this system, the input will be the elevator deflection angle, and the output will be the pitch angle. These values are taken from the data from one of the Boeing's commercial aircraft.

1. Transfer function

- Prove (here you do not need to use Matlab) that:

$$G(s) = \frac{\theta(s)}{\delta_e(s)} = \frac{1.151s + 0.1774}{s^3 + 0.739s^2 + 0.921s}$$

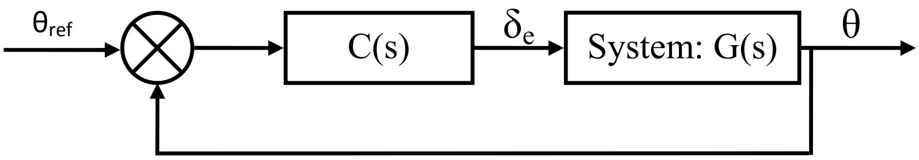
2. Parametrical study

Now, we are ready to observe the system characteristics using Matlab. First, let's obtain the open-loop response of the system to a step input, and determine which system characteristics need improvement. Let the input δ_e be 0.2 rad (11º).

Create a new m-file to enter the needed commands: plot the response of the pitch angle for the given input (use function `step`) in open-loop.

- What can you say about the stability?
- Calculate the poles (with Matlab!) of the transfer function in order to justify the stability study of the open-loop system.

To solve this problem, a feedback controller will be added to improve the system performance. The figure shown below is the block diagram of a typical unit feedback system.



Using the function `feedback` calculate the transfer function of the closed loop system (using a controller of gain 2: K=2).

Plot the response of the pitch angle for the given input (reference pitch angle θ_{ref} = 0.1 rad=5.5º) in closed-loop.

From the plot measure

- the overshoot M,
- the peak time t_p,
- the period T,
- the error. Justify the value of the error by studying the expression of the transfer function G(s).

Note that by clicking on the graph of the time response you obtain the exact value of time and amplitude.

- Deduce the value of the damping factor and the natural frequency.
- Try several values of K and conclude on the effect of the value of K (this would be a proportional controller) on the overshoot, the peak time and the damping factor.