

# MATH 547: STOCHASTIC PROCESSES

Winter 2023

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COURSE SCHEDULE: Tuesday and Thursday, 1:05 pm - 2:25 pm.

TEXTBOOKS: There is no official textbook for the course. The following are good references that could be used as a supplement:

*Probability: theory and examples*. **Durrett**. Available online via the [Author's website](#). Good general, measure-theoretic probability text with an emphasis on discrete time stochastic processes.

*Probability with Martingales*. **Williams**. Slightly idiosyncratic book with a very thorough development of discrete martingale theory, far beyond what is available in Durrett.

*Stochastic processes*. **Sheldon Ross**. Non-measure-theoretic introduction to stochastic processes. This covers most material from the class, with emphasis on probabilistic (as opposed to measure-theoretic) details.

This course develops the main topics in (discrete time) stochastic process theory: Markov chains, random walks and martingales.

This is a preliminary outline of the course content, which may be abridged as the course develops.

- Introduction to Markov chains
  - Classification of states
  - Stationarity & the long-time behaviour of finite state Markov chains
  - Markov chain Monte Carlo
  - Regeneration & the strong Markov property
  - Card shuffling & the rate of convergence to stationarity
- Random walks and martingales
  - Doob decomposition and discrete stochastic calculus
  - Martingale convergence
  - The bracket process and L2 martingales
- Infinite state Markov chains
  - Transience and recurrence
  - The Polya theorem for simple random walks.

- The stochastic approximation theorem
- Convergence analysis of SGD
- Optional stopping theorem
  - Hitting time problems for random walks
  - The reflection principle for symmetric random walks
  - Maximal inequalities
  - Applications of maximal inequalities
- Azuma’s inequality and exponential concentration
  - IID concentration
  - The stochastic travelling salesman problem
  - The chromatic number
- Branching processes
  - Mean properties,
  - The probability generating function
  - Extinction and survival
  - Growth rate (the Kesten-Stigum theorem)
  - Cascades and chaos

PREREQUISITE: Math 356 Probability or equivalent.

#### GRADING SCHEME

**Caveat.** If the resources provided by the university (in the form of teaching assistants and graders) is inadequate, the grading scheme may be modified.

Your grade will be calculated as the maximum of the following two formulas:

Formula 1:

- Marked homework assignments: 50%
- Midterm: 20%
- Final: 30%

Formula 2:

- Marked homework assignments: 50%
- Midterm: 0%
- Final: 50%

Solutions will be judged on mathematical correctness, completeness, and also on clarity of exposition.

The book *Mathematical writing for undergraduate students* is a useful resource for learning the basics of clear mathematical writing. It is available at via the McGill library catalogue at [McGill Library Link](#).

Students may work in groups, but must each write up their assignment solutions on their own.

McGill’s “student rights and responsibilities” web page has this to say:

Peer learning should be encouraged, since it helps students learn to teach. Instructors should explain effective peer teaching strategies such as working in pairs, sharing comments on work, and brainstorming solutions to problems in groups. Sharing completed work is not an acceptable peer learning technique. If a student has copied the answers of another student, the incident must be documented and the material sent to the course instructor, who will contact the appropriate disciplinary officer.

–From [Official McGill Policy](#)

## LANGUAGE

In accord with McGill University's Charter of Students Rights, students in this course have the right to submit in English or in French any written work that is to be graded.

Conformément à la Charte des droits de l'étudiant de l'Université McGill, chaque étudiant a le droit de soumettre en français ou en anglais tout travail écrit devant être noté (sauf dans le cas des cours dont l'un des objets est la maîtrise d'une langue).

## ACADEMIC INTEGRITY

McGill University values academic integrity. Therefore all students must understand the meaning and consequences of cheating, plagiarism and other academic offences under the Code of Student Conduct and Disciplinary Procedures (see [Official McGill page](#) for more information). This, in particular, excludes usage of websites such as Chegg, Course Hero, Bartleby, or Scribd for the purpose of answering assignment and exam questions, which will be strictly enforced.

L'université McGill attache une haute importance à l'honnêteté académique. Il incombe par conséquent à tous les étudiants de comprendre ce que l'on entend par tricherie, plagiat et autres infractions académiques, ainsi que les conséquences que peuvent avoir de telles actions, selon le Code de conduite de l'étudiant et des procédures disciplinaires (pour de plus amples renseignements, veuillez consulter le [site](#)).