BASIC CONTROL SYSTEMS

02 LAPLACE TRANSFORM

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Presentation outline

- Integral transform
- Laplace transform
- ROC of Laplace
- Properties of Laplace transform
- Inverse Laplace transform

INTEGRAL TRANSFORM

Why do we use integral transform?

Simplification

What does an integral transform do?

Mapping from one domain to another

Originates from:

Solving differential equations

INTEGRAL TRANSFORM

Given a general integral transform $\boldsymbol{T}$
A function $f(m)$ in m-domain
A target domain: n-domain
A transformation kernel: $\boldsymbol{K}(m,n)$

The general integral transform:

$$\boldsymbol{L}[f(n)] = \int_a^b f(m)\boldsymbol{K}(m,n)\,\mathrm{d}m$$

We have mapped our function f from m domain to n domain using transformation $\boldsymbol{T}$

LAPLACE TRANSFORM

Laplace transform $\boldsymbol{L}$

A function $f(t)$ in t-domain

A target domain: s-domain

A transformation kernel: $\boldsymbol{K}(t,s) = e^{-st}$

Where $s = \sigma + j\omega$

The Laplace transform:

$$\boldsymbol{L}[f(t)] = \int_{-\infty}^{\infty} f(t)e^{-st}\mathrm{d}t$$

We have mapped our function f from t-domain to s-domain using $\boldsymbol{L}$.

This is call bilateral Laplace transform.

UNILATERAL LAPLACE TRANSFORM

Recall that we are only working with causal system!
So whatever before $t = 0$ we do not care, we start from 0!

The unilateral Laplace transform:

$$\boldsymbol{L}[f(t)] = \int_{0}^{\infty} f(t)e^{-st}\mathrm{d}t$$

A transformation kernel: $\boldsymbol{K}(t, s) = e^{-st}$
Where $s = \sigma + j\omega$

UNILATERAL LAPLACE TRANSFORM

The unilateral Laplace transform:

$$\boldsymbol{L}[f(t)] = \int_{0}^{\infty} f(t)e^{-st}\mathrm{d}t$$

A transformation kernel: $\boldsymbol{K}(t,s) = e^{-st}$
Where $s = \sigma + j\omega$

BUT WHY???

CONVERGE ! CONVERGE ! CONVERGE !

We are integrating from 0 to ∞.
So $f(t)\boldsymbol{K}(s,t)$ must converge for out integral to exist.

The unilateral Laplace transform:

$$\boldsymbol{L}[f(t)] = \int_0^{\infty} f(t)e^{-st}\mathrm{d}t$$

A transformation kernel: $\boldsymbol{K}(t,s) = e^{-st}$
Where $s = \sigma + j\omega$

REGION OF CONVERGENCE

The unilateral Laplace transform:

$$\boldsymbol{L}[f(t)] = \int_0^{\infty} f(t)e^{-st}\mathrm{d}t$$

We are integrating from 0 to ∞.
So $f(t)\boldsymbol{K}(s,t)$ must converge for out integral to exist.
We look at $f(t)$ and $\boldsymbol{K}(s,t)$ separately, $\boldsymbol{K}(s,t)$ first.

A transformation kernel:
$\boldsymbol{K}(t,s) = e^{-st}$
Where $s = \sigma + j\omega$

Assume: f(t) = 1

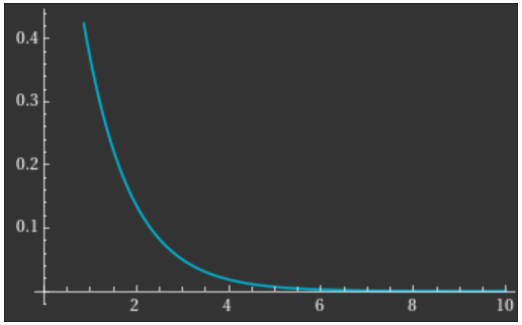
$$\int_0^{\infty} e^{-st}\mathrm{d}t = \int_0^{\infty} e^{-(\sigma+j\omega)t}\mathrm{d}t = \int_0^{\infty} e^{-\sigma t}e^{-j\omega t}\mathrm{d}t$$

Remember from complex analysis:

$$|e^{-j\omega t}| = 1$$

Then we only have:

$$e^{-\sigma t}$$



To converge: $\sigma > 0$

REGION OF CONVERGENCE

The unilateral Laplace transform:

$$\boldsymbol{L}[f(t)] = \int_0^{\infty} f(t)e^{-st}\mathrm{d}t$$

A transformation kernel:
$\boldsymbol{K}(t,s) = e^{-st}$
Where $s = \sigma + j\omega$

Assume: s = 0 + j0

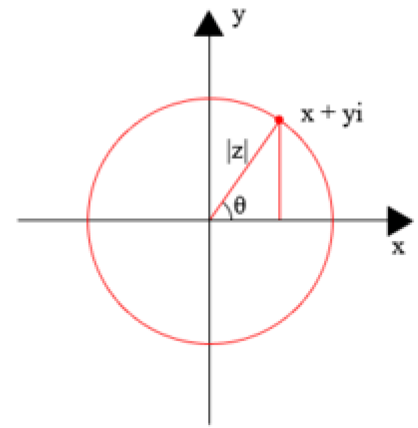
$$\int_0^{\infty} f(t)\,\mathrm{d}t$$

Without our kernel, f(t) should converge.

REGION OF CONVERGENCE

The unilateral Laplace transform:

$$\boldsymbol{L}[f(t)] = \int_0^\infty f(t)e^{-st}\mathrm{d}t$$



A transformation kernel:

$\boldsymbol{K}(t,s) = e^{-st}$

Where $s = \sigma + j\omega$

Assume: $s = 0 + j\omega$

$$\int_0^\infty f(t)e^{-j\omega t}\mathrm{d}t \text{ , and } |e^{-j\omega t}| = 1$$

We are traversing through the complex plane now, but our ROC still relies on the properties of f(t).

Note! This is not Fourier transform!!!

REGION OF CONVERGENCE

The unilateral Laplace transform:

$$\boldsymbol{L}[f(t)] = \int_0^{\infty} f(t)e^{-st}\mathrm{d}t$$

A transformation kernel:

$\boldsymbol{K}(t,s) = e^{-st}$

Where $s = \sigma + j\omega$

Assume: s = σ + jω

$$\int_0^{\infty} e^{-\sigma t} f(t) e^{-j\omega t}\mathrm{d}t \text{ , and } |e^{-j\omega t}| = 1$$

Finally, when $\sigma > 0$, we have an “envelop” $e^{-\sigma t}$ that make our transformation exist.

This gives us the desired properties of $f(t)$

1. $f(t)$ should be integrable and defined for $[0,\infty)$
2. $f(t)$ grows slower than e^{-st}

REGION OF CONVERGENCE THE "POLAR VIEW"

For any real number $\boldsymbol{r} = a\vec{x} + b\vec{y}$, the polar form:

$$\boldsymbol{r} = |\boldsymbol{r}|\angle\mathbf{r}$$

For any complex number $\boldsymbol{z} = a + \mathrm{j}b$, the polar form:

$$\boldsymbol{z} = |z|\hat{z},\ \hat{z}=e^{\mathrm{j}\theta}, \theta = \mathrm{Arg}(z)$$

For our $f(t)\boldsymbol{K}(s,t) = f(t)e^{-(\sigma+\mathrm{j}\omega)t}$:

We can obtain a polar representation:

$$|z|e^{\mathrm{i}\theta}$$

$$|z| = e^{-\sigma t}f(t)$$

$$\theta = -\omega t$$

Putting them together:

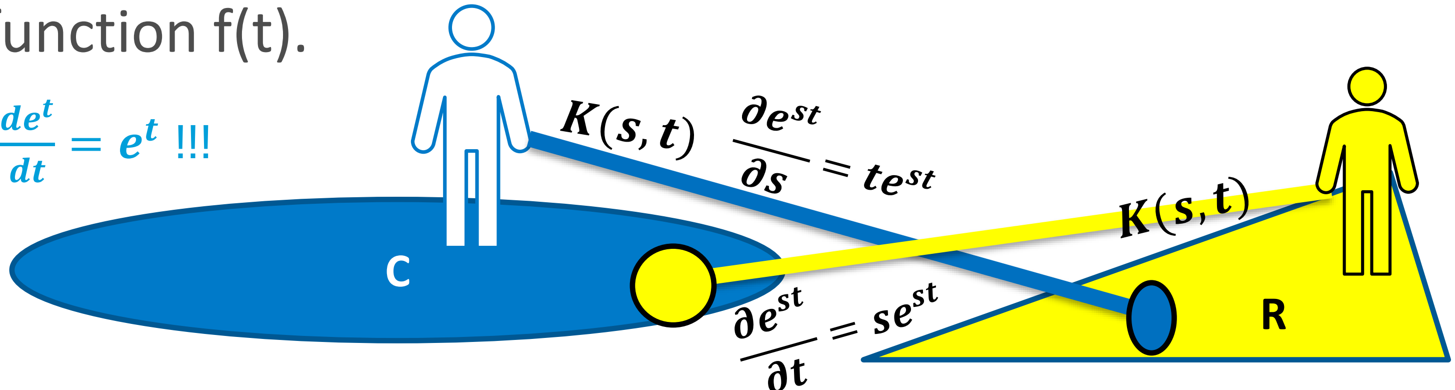
$$f(t)\boldsymbol{K}(s,t) = [e^{-\sigma t}f(t)]\, e^{\mathrm{j}(-\omega t)}$$

KERNEL OF LAPLACE

$$K(s,t) = f(t)e^{-(\sigma+j\omega)t}$$

This is a very beautiful **probing function** for measuring our function f(t).

$\frac{de^t}{dt} = e^t$!!!



This probe gives us very nice analytical properties such that we can map many time domain signals into complex space.

In the meantime, this probe still carries enough information such that we can recover the real signal from complex plane. (*Lerch's theorem*)

Laplace transform is **ONE-TO-ONE *(injective)***.

CONVERGE !
CONVERGE !
CONVERGE !

Impatient guy from engineering department rushed to me:

"THESE ARE JUST MATHEMATICAL TRICKS! Mathematical rules and all! Does not make sense in practice!"

"How does these relate to application & real systems?"
"WHY should we care ????"

I say:

"Because of the physical meanings"

RECALL:
SYSTEMS: THE KEY TAKEOUT

No matter how much we simplify,
we are working with physical systems.

The mathematical tools you see later,
are describing the characteristics of the physical system.

(最重要的是物理系统自身的特性！）

ACTUAL PHYSICAL SYSTEMS OF INFINITE ENERGY DOES NOT EXIST!

Energy = Power over TIME.

$$\boldsymbol{E} = \int_0^\infty \boldsymbol{P}\,\mathrm{d}\boldsymbol{t}$$

As t-> ∞ , diverging power, infinite energy, impossible.
If this is the governing equation, the system will break!

Unstable!

If you sit in the lecture room,
you MUST agree that perpetual motion machine (永动机) does not exist.

EXAMPLE

$$f(t) = e^{at}$$

Laplace transform:

$$\mathbf{F}(s) = \boldsymbol{L}[f(t)] = \int_0^\infty f(t)e^{-st}\,\mathrm{d}t$$

$$= \int_0^\infty e^{at}e^{-st}\,\mathrm{d}t$$

$$= \int_0^\infty e^{(a-s)t}\,\mathrm{d}t$$

Without getting the exact F(s), we can already infer where is the ROC:

$$a - \sigma < 0$$

Thus:

$$\sigma > a$$

EXAMPLE

$$f(t) = e^{at}$$

Laplace transform:

$$\mathbf{F}(s) = \boldsymbol{L}[f(t)] = \int_0^\infty f(t)e^{-st}\,\mathrm{d}t$$

$$= \int_0^\infty e^{at}e^{-st}\,\mathrm{d}t$$

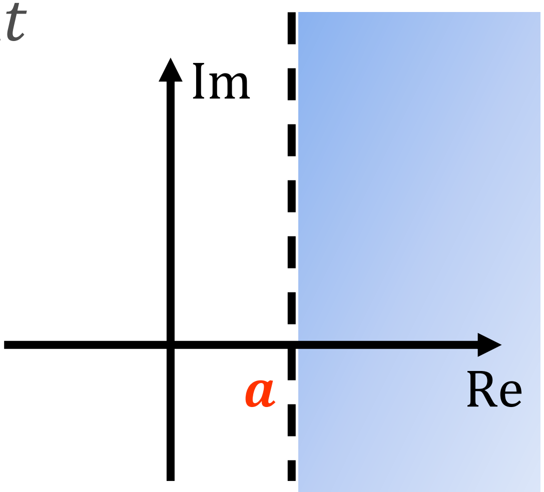
$$= \int_0^\infty e^{(a-s)t}\,\mathrm{d}t$$

The ROC:

$$a - \sigma < 0$$

Thus:

$$\sigma > a$$



EXAMPLE

$$f(t) = e^{at}$$

Laplace transform:

$$\mathbf{F}(s) = \boldsymbol{L}[f(t)] = \int_0^\infty f(t)e^{-st}\,\mathrm{d}t$$

$$= \int_0^\infty e^{(a-s)t}\,\mathrm{d}t$$

$$= \frac{1}{a-s}e^{(a-s)t}\Big|_{t=0}^{\infty}$$

$$= 0 - \frac{1}{a-s}$$

$$= \frac{1}{s-a}$$

LAPLACE TRANSFORM TABLE

1. $A \cdot 1(t)$	$\dfrac{A}{s}$
2. $\delta(t) \cdot 1(t)$	1
3. $t^n \cdot 1(t)$	$\dfrac{n!}{s^{n+1}}$
4. $e^{at} \cdot 1(t)$	$\dfrac{1}{s-a}$
5. $\sin(\omega t) \cdot 1(t)$	$\dfrac{\omega}{s^2+\omega^2}$
6. $\cos(\omega t) \cdot 1(t)$	$\dfrac{s}{s^2+\omega^2}$

LAPLACE TRANSFORM TABLE

$f(t)$	$F(s) = \mathcal{L}[f(t)]$	
$f(t) = 1$	$F(s) = \frac{1}{s}$	$s > 0$
$f(t) = e^{at}$	$F(s) = \frac{1}{(s-a)}$	$s > a$
$f(t) = t^n$	$F(s) = \frac{n!}{s^{(n+1)}}$	$s > 0$
$f(t) = \sin(at)$	$F(s) = \frac{a}{s^2 + a^2}$	$s > 0$
$f(t) = \cos(at)$	$F(s) = \frac{s}{s^2 + a^2}$	$s > 0$
$f(t) = \sinh(at)$	$F(s) = \frac{a}{s^2 - a^2}$	$s > \lvert a \rvert$
$f(t) = \cosh(at)$	$F(s) = \frac{s}{s^2 - a^2}$	$s > \lvert a \rvert$
$f(t) = t^n e^{at}$	$F(s) = \frac{n!}{(s-a)^{(n+1)}}$	$s > a$
$f(t) = e^{at}\sin(bt)$	$F(s) = \frac{b}{(s-a)^2 + b^2}$	$s > a$
$f(t) = e^{at}\cos(bt)$	$F(s) = \frac{(s-a)}{(s-a)^2 + b^2}$	$s > a$
$f(t) = e^{at}\sinh(bt)$	$F(s) = \frac{b}{(s-a)^2 - b^2}$	$s - a > \lvert b \rvert$
$f(t) = e^{at}\cosh(bt)$	$F(s) = \frac{(s-a)}{(s-a)^2 - b^2}$	$s - a > \lvert b \rvert$

INVERSE LAPLACE TRANSFORM

The time domain signal can be obtained from the frequency domain signal using inverse Laplace transform.

$$f(\mathrm{t}) = \boldsymbol{L}^{-1}\left[F(s)\right] = \frac{1}{2\pi \mathrm{j}} \int_{\sigma - \mathrm{j}\infty}^{\sigma + \mathrm{j}\infty} F(s) e^{st} \, \mathrm{d}s$$

Without transform table, you can use ***Cauchy's residue theorem*** for a contour integral with the closed contour of the integration as the region of convergence.

LAPLACE PROPERTIES

$$L[f(t)] = \int_{0}^{\infty} f(t)e^{-st}\mathrm{d}t$$

LINEARITY

$$\boldsymbol{a \cdot f(t) \Leftrightarrow \mathrm{a \cdot F(s)}}$$

$$\boldsymbol{f(t) + g(t) \Leftrightarrow \mathrm{F(s) + G(s)}}$$

$$\boldsymbol{a \cdot f(t) + b \cdot g(t) \Leftrightarrow \mathrm{a \cdot F(s) + b \cdot G(s)}}$$

LAPLACE PROPERTIES

$$L[f(t)] = \int_0^\infty f(t)e^{-st}\mathrm{d}t$$

TIME SCALING

$$\boldsymbol{f(at) \Leftrightarrow \frac{1}{|a|} \cdot \mathrm{F}\left(\frac{s}{a}\right)}$$

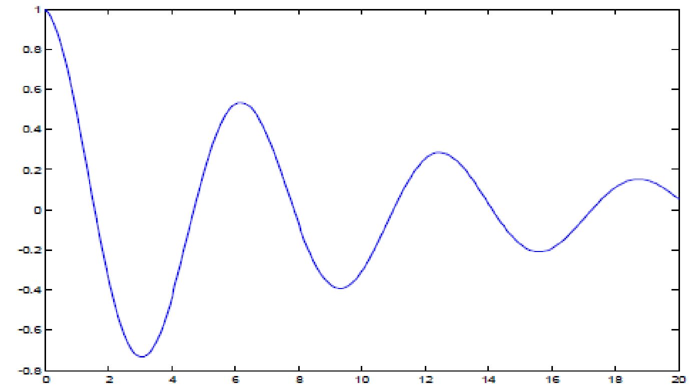
TIME SHIFTING

$$\boldsymbol{f(t-a) \Leftrightarrow e^{-as} \cdot \mathrm{F}(s)}$$

EXPONENTIAL SCALING (DAMPING)

$$\boldsymbol{e^{-at}f(t) \Leftrightarrow \mathrm{F}(s+a)}$$

Damping (example)



$$f(t) = e^{-at}cos(\omega t)$$

What do we need to find the laplace transform of f(t)?

Laplace transform theorems:

2. Damping $\quad e^{-at} \cdot f(t) \quad\quad F(s+a)$

Laplace transform table:

4. $e^{at} \cdot 1(t) \quad\quad \dfrac{1}{s-a}$

6. $cos(\omega t) \cdot 1(t) \quad\quad \dfrac{s}{s^2+\omega^2}$

$$\mathcal{L}\{e^{-at}\cos(\omega t)\} = \frac{(s+a)}{(s+a)^2+\omega^2}$$

LAPLACE PROPERTIES

$$\boldsymbol{L}[f(t)] = \int_0^{\infty} f(t)e^{-st}\mathrm{d}t$$

DIFFERENTATION

$$\boldsymbol{L}[f'(t)] = sF(s) - f(0)$$

$$\boldsymbol{L}[f''(t)] = s^2F(s) - sf(0) - f'(0)$$

General formula:

$$\boldsymbol{L}\left[f^{(k)}(t)\right] =$$

$$s^kF(s) - s^{k-1}f(0) - s^{k-2}f'^{(0)} \ldots - sf^{(k-2)}(0) - f^{(k-1)}(0)$$

Differentiation in t-domain in becomes an operator in s-domain.

LAPLACE PROPERTIES

$$\boldsymbol{L}[f(t)] = \int_{0}^{\infty} f(t)e^{-st}\mathrm{d}t$$

INTEGRAL

$$g(t) = \int_{0}^{t} f(\tau)\mathrm{d}\tau$$

With Laplace transform:

$$\boldsymbol{G}(s) = \frac{\mathbf{1}}{\boldsymbol{s}}\mathbf{F}(s)$$

Integration in t-domain in becomes an operator in s-domain.

LAPLACE PROPERTIES

$$\boldsymbol{L}[f(t)] = \int_{0}^{\infty} f(t)e^{-st}\mathrm{d}t$$

From CONVOLUTION to MULTIPLICATION

$$f(t) * g(t) = \int_{0}^{t} f(\tau)g(t-\tau)\,\mathrm{d}\tau$$

With Laplace transform:

$$\boldsymbol{f}(t) * \boldsymbol{g}(t) \Leftrightarrow \mathbf{F}(s)\mathbf{G}(s)$$

Convolution in t-domain in becomes multiplication in s-domain.

LAPLACE PROPERTIES

$$\boldsymbol{L}[f(t)] = \int_0^\infty f(t)e^{-st}\mathrm{d}t$$

INITIAL VALUE THEOREM

$$\lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s)$$

Proof1:

f is causal & bounded such that $\lim_{t\to 0^+} f(t) \to \alpha$, we play with definition

$$F(s) = \int_0^\infty f(t)e^{-st}\mathrm{d}t = \int_0^\infty f\left(\frac{t}{s}\right)e^{-s\frac{t}{s}}\mathrm{d}\frac{t}{s} = \int_0^\infty \frac{1}{s}f\left(\frac{t}{s}\right)e^{-t}\mathrm{d}t$$

$$sF(s) = \int_0^\infty f\left(\frac{t}{s}\right)e^{-t}\mathrm{d}t$$

Based on Lebesgue's dominated convergence theorem, we can have

$$\lim_{s\to\infty} sF(s) = \int_0^\infty \alpha e^{-t}\mathrm{d}t = \alpha = \lim_{t\to 0^+} f(t)$$

LAPLACE PROPERTIES

$$\boldsymbol{L}[f(t)] = \int_0^\infty f(t)e^{-st}\mathrm{d}t$$

INITIAL VALUE THEOREM

$$\lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s)$$

Proof2:

f is causal & bounded such that $\lim_{t\to 0^+} f(t) \to \alpha$, we play with definition

$$F(s) = \int_0^\infty f(t)e^{-st}\mathrm{d}t = \int_0^\infty f\left(\frac{t}{s}\right)e^{-s\frac{t}{s}}\mathrm{d}\frac{t}{s} = \int_0^\infty \frac{1}{s} f\left(\frac{t}{s}\right)e^{-t}\mathrm{d}t$$

$$sF(s) = \int_0^\infty f\left(\frac{t}{s}\right)e^{-t}\mathrm{d}t$$

We select a δ ∈ R sufficiently close to 0 such that $\int_\delta^\infty e^{-t}\mathrm{d}t < \epsilon$ that is arbitrarily small and $\lim_{s\to\infty} f\left(\frac{t}{s}\right) = \alpha$ for $t \in (0,\delta]$, we may also conclude that:

$$\lim_{s\to\infty} sF(s) = \int_0^\infty \alpha e^{-t}\mathrm{d}t = \alpha = \lim_{t\to 0^+} f(t)$$

LAPLACE PROPERTIES

$$\boldsymbol{L}[f(t)] = \int_0^\infty f(t)e^{-st}\mathrm{d}t$$

FINAL VALUE THEOREM

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$$

Proof:

f is continuously differentiable and bounded, and f' is absolutely integrable.

$\lim_{t\to\infty} f(t)$ exists and finite.

$$\lim_{s\to 0} sF(s) = \lim_{s\to 0} L[\frac{\mathrm{d}f(t)}{\mathrm{d}t}] = \lim_{s\to 0}\int_0^\infty \frac{\mathrm{d}f(t)}{\mathrm{d}t}e^{-st}\mathrm{d}t$$

As obviously $\lim_{s\to 0} e^{-st} = 1$, we have:

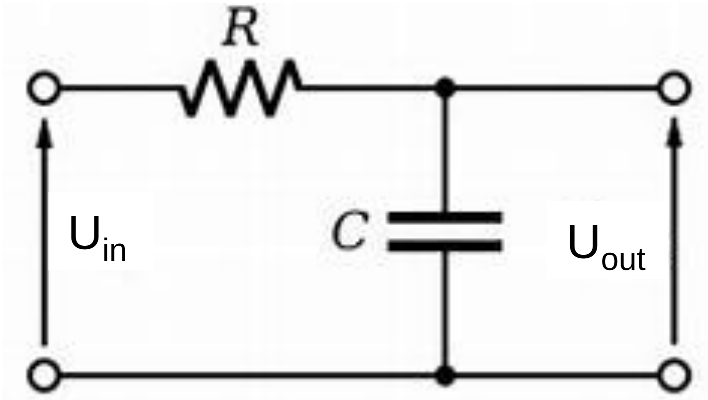
$$\lim_{s\to 0}\int_0^\infty \frac{\mathrm{d}f(t)}{\mathrm{d}t}e^{-st}\mathrm{d}t = f(t)\Big|_0^\infty = f(\infty) - \cancel{f(0)}$$

LAPLACE PROPERTIES

We have seen the incredible symmetry between time domain and the frequency domain.

- Differentiation in one becomes multiplication in another.
- Exponential scaling in one domain becomes shifting in the other.

Modelling example: RC low-pass filter



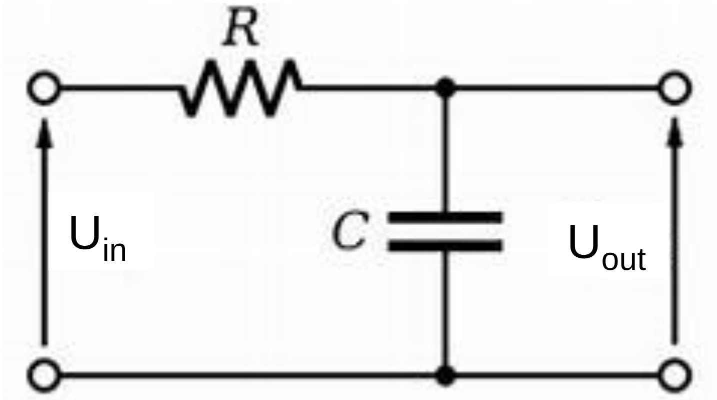
Differential Equation! -> Laplace transform!

$$U_{in}(t) = \mathrm{RC}\frac{\mathrm{d}U_{out}}{\mathrm{d}t} + U_{out}(t)$$

$$U_{in}(s) = \mathrm{RC}sU_{out} + U_{out}(s)$$

$$U_{in} = \mathrm{RC}sU_{out} + U_{out}$$

Modelling example: RC low-pass filter



Laplace transform! -> Transfer function!

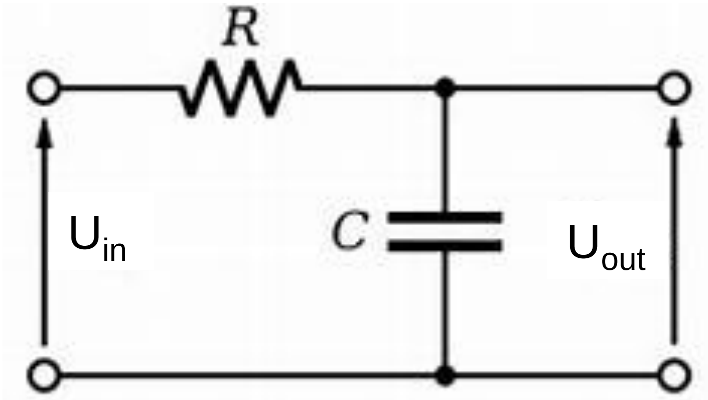
$$\mathrm{OUTPUT} = \mathrm{INPUT} \cdot \mathrm{TF}$$

$$\Rightarrow$$

$$\mathrm{TF} = \frac{OUTPUT}{INPUT}$$

$$\boxed{H(s) = \frac{1}{1 + RCs}}$$

Modelling example: RC low-pass filter



Time domain solution?

Reverse Laplace transform!

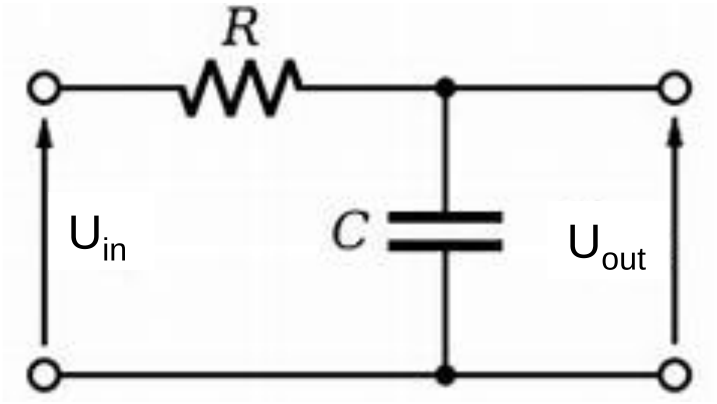
$$H(s) = \frac{1}{1 + RCs}$$

$$U_{out}(s) = H(s)U_{in}(s)$$

We assume $U_{in} = \frac{1}{s}$ (Constant)

$$U_{out}(s) = \frac{1}{s(1 + RCs)}$$

Modelling example: RC low-pass filter



Reverse Laplace transform!

$$U_{out}(s) = \frac{1}{s(1 + RCs)}$$

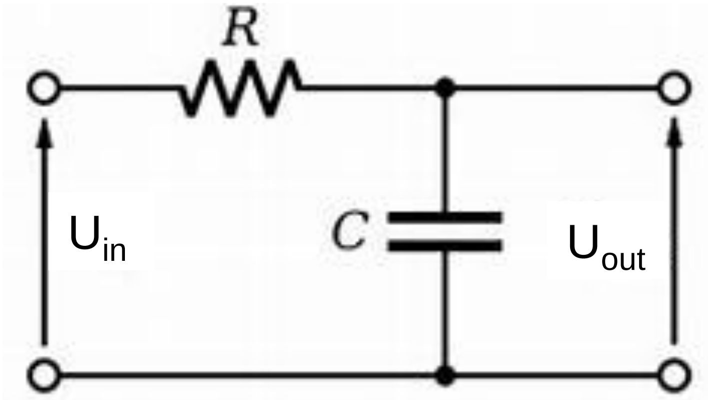
Partial fraction decomposition:

$$U_{out}(s) = \frac{A}{s} + \frac{B}{1 + RCs}$$

$$A + (ARC + B)s \;=\; 1$$

$$A = \; 1, \mathrm{B} = -RC$$

Modelling example: RC low-pass filter



$$U_{out}(s) = \frac{A}{s} + \frac{B}{1 + RCs}$$

$$A + (ARC + B)s \; = \; 1$$

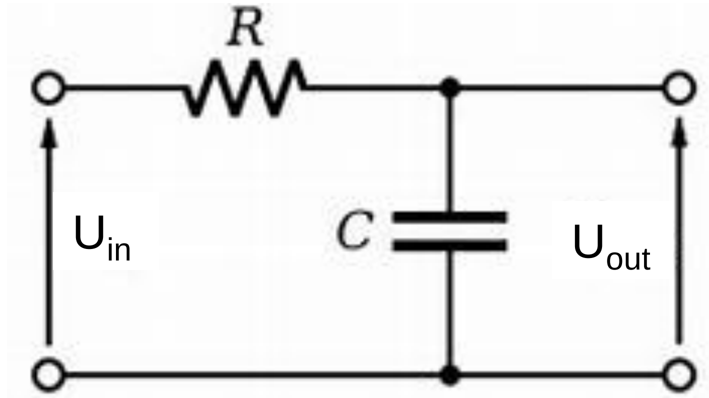
$$A = \; 1, \mathrm{B} = -RC$$

$$U_{out}(s) = \frac{1}{s} - \frac{RC}{1 + RCs} = \frac{1}{s} - \frac{1}{\frac{1}{RC} + s}$$

Inverse! Check transform table!

$$U_{out}(t) = 1 - e^{-\frac{t}{RC}}$$

Modelling example: RC low-pass filter



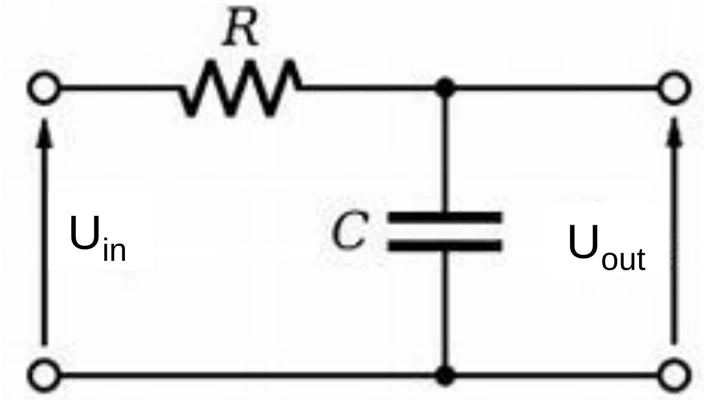
Circuit analysis! **"The smart way"**

Time - domain	S-domain
Integration	Multiplicative operator '$\frac{1}{s}$'
Differentiation	Multiplicative operator 's'

ELECTRICAL SYSTEMS – RLC CIRCUITS

	Voltage - Current	Impedance (Laplace transformed)
Resistor	$\mathrm{U}(t) = I(t)R$	R
Capacitor	$\mathrm{U}(t) = \frac{1}{C}\int_0^1 I(\tau)\, d\tau$	$\frac{1}{Cs}$
Inductor	$\mathrm{U}(t) = L\frac{d\, I(t)}{d\, t}$	Ls

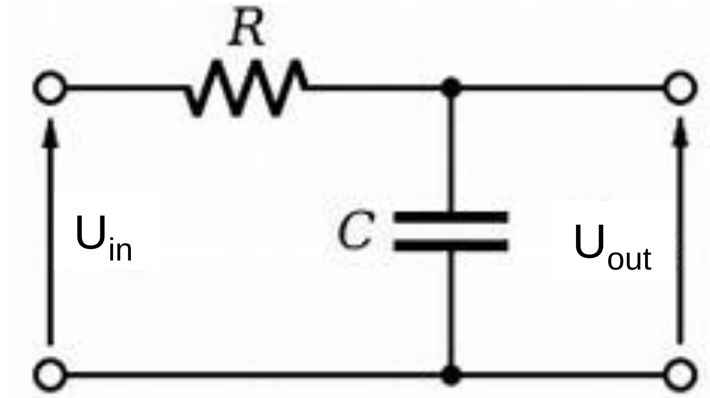
Modelling example: RC low-pass filter



Circuit analysis! **“The smart way”**

Resistive components	S-domain Impedance
Resistor	R
Inductor	Ls
Capacitor	$\frac{1}{Cs}$

Modelling example: RC low-pass filter



Circuit analysis!

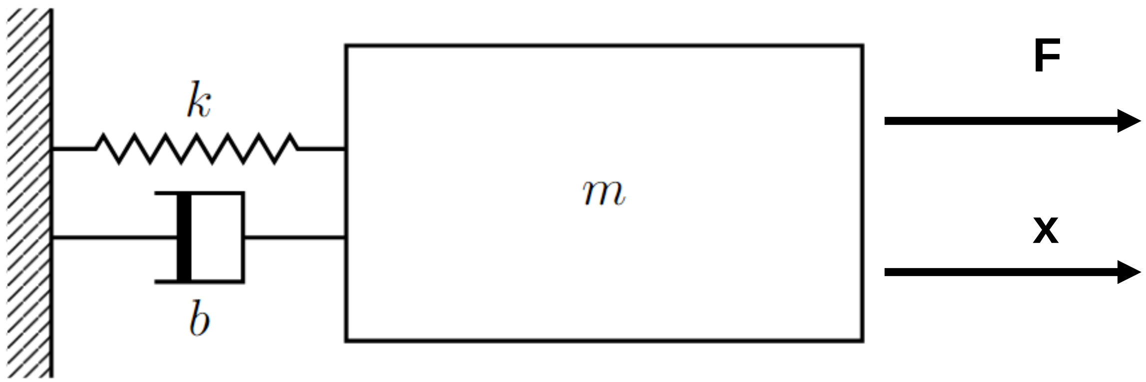
Total resistance: $R + \frac{1}{Cs}$ over U_{in}

U_{out} is the voltage over resistance $\frac{1}{Cs}$.

Like a simple pure resistor circuit:

$$\frac{U_{out}}{U_{in}} = \frac{\frac{1}{Cs}}{R + \frac{1}{Cs}} = \frac{1}{1 + RCs}$$

Modelling example: Mechanical system



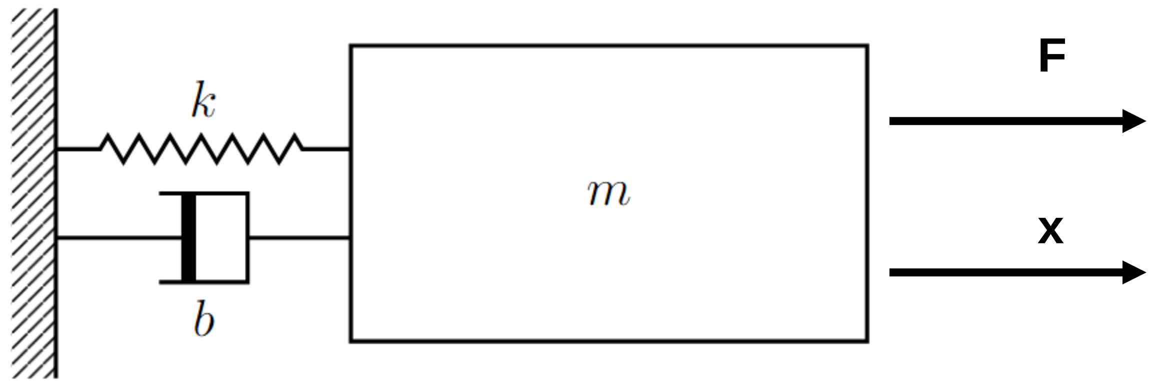
Write differential equation

$$F(t) = k\int_0^t v(\tau)\mathrm{d}\tau + bv(t) + m\frac{\mathrm{d}v(t)}{\mathrm{d}t}$$

Laplace transform

$$F(s) = \frac{k}{s}V(s) + bV(s) + msV(s)$$

Modelling example: Mechanical system



$$F(s) = \frac{k}{s}V(s) + bV(s) + msV(s)$$

Transfer function

$$\frac{V(s)}{F(s)} = \frac{1}{\frac{k}{s} + b + ms}$$

$$= \frac{s}{ms^2 + bs + k}$$

\reals

MECHANICAL SYSTEMS – TRANSLATING SYSTEM

	Force - Velocity	Impedance (Laplace transformed)
Damper (Viscous friction)	$F = b\text{v}$	b
Spring	$F = k\int_0^t v(\tau)\mathrm{d}\tau$	$\frac{k}{s}$
Mass (Inertia)	$F = m\frac{\mathrm{d}v(t)}{\mathrm{d}t}$	ms

MECHANICAL SYSTEMS – TRANSLATING SYSTEM

	Force – Displacement	Impedance (Laplace transformed)
Damper (Viscous friction)	$F = b\frac{\mathrm{d}x(t)}{\mathrm{d}t}$	bs
Spring	$F = kx$	k
Mass (Inertia)	$F = m\frac{\mathrm{d}^2x(t)}{\mathrm{d}t^2}$	ms^2

SUMMARY

$$L[f(t)] = \int_0^\infty f(t)e^{-st}\mathrm{d}t$$

Transform table

Region of convergence

Properties:

- Linearity
- Symmetry between t-domain and s-domain
- From differentiation and integral to operators in s-domain
- Initial & final value theorem

We may use Laplace transform to conveniently solve ODE

SUMMARY

The physics laws governs our systems’ performance

Analyze your system based on these physics laws

Eventually we can use ODE to describe our desired input and output

Laplace transform is a handy tool for analysis and finding solutions