



Exam Information

Exam: Basic Control Systems

Regular (TEST EXAM)

Date: ___, ___, 2025

Time: ___:___ till ___:___

Duration:

3 hours

Teacher(s):

Hanshu Yu

Number of pages in this exam paper: 7

(1 cover page + 4 main body + 1 formula sheet + 1 sketch sheet)

Number of problems in this test: 2

Allowed material:

- Writing gear;
- Calculator;

Total raw score: 120 points.

Calculation of final scores: MIN(total score,100)*0.5

Note for grading: correct solutions without reasoning do not grant points.

Student Information

Student name: _____

Student number: _____

Table number: _____

Final score: _____

DO NOT FORGET TO SUBMIT THE SKETCH SHEETS.

Good luck!



Student name: _____



Student number: _____

Problem 1 (57 points)

Open loop process H is described by an ordinary differential equation,

$$\frac{d y(t)}{dt} + 10y(t) = 9x(t)$$

The initial conditions are as follows: $y(0) = 0, x(0) = 0$.

- a) [5] Perform Laplace transform and find the transfer function $H(s)$.
- b) [5] Find step response $y_{step}(t)$ in time domain.
- c) [5] Find $y(t)$ if $x(t) = \sin(\omega t)$.
- d) [5] If the process is put in a negative unit feedback control loop together with a P-controller, find the poles and zeros of the closed-loop system if the P-controller gain is set to $K = 10$.
- e) [6] Find the time constant τ_p of the system, and the percentage of decrease of the impulse response $y_{impulse}(t)$ when $t = \tau_p$.
- f) [3] Is the closed-loop system stable when the controller gain $K = 100$?
- g) [6] Draw root locus for the given system.
- h) [10] If there is a PD controller replacing the original P controller, how will the root locus change? Draw the new root locus and explain the effect when the PD controller's gain K becomes very large.
- i) [12] If there is a PID controller replacing the original P controller, prove mathematically that the steady-state error of the closed-loop system is zero.



Student name: _____



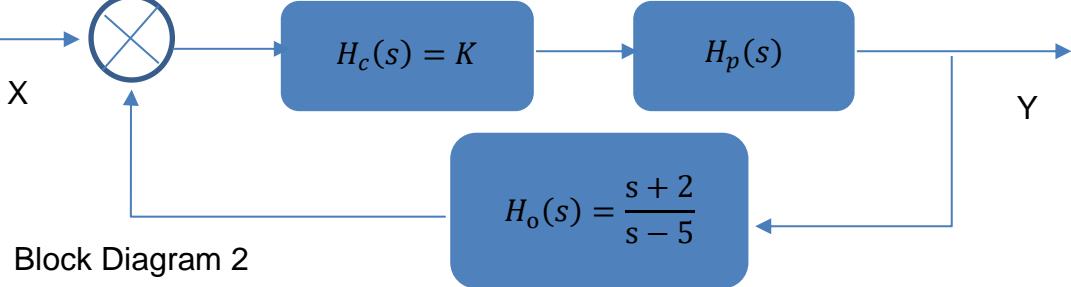
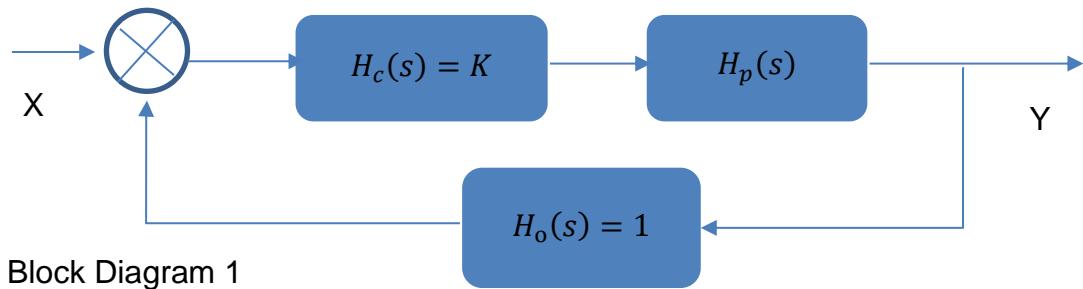
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Problem 2 (63 points)

Given a transfer function corresponds to an open-loop process:

$$H_c(s)H_p(s) = \frac{K(s+1)}{4s^2 + 64s + 32}, \text{ with } H_c(s) = K$$

- a) [4] Find poles and zeros of $H_c(s)H_p(s)$.
- b) [12] Draw bode plots for $H_c(s)H_p(s)$ when $H_c(s) = 10$.
- c) [7] Given the 2 following block diagrams:



Find the closed-loop transfer function for these two block diagrams.

- d) [5] Draw root-locus of the system in block diagram 1.

(Problem 2 continues on the next page)

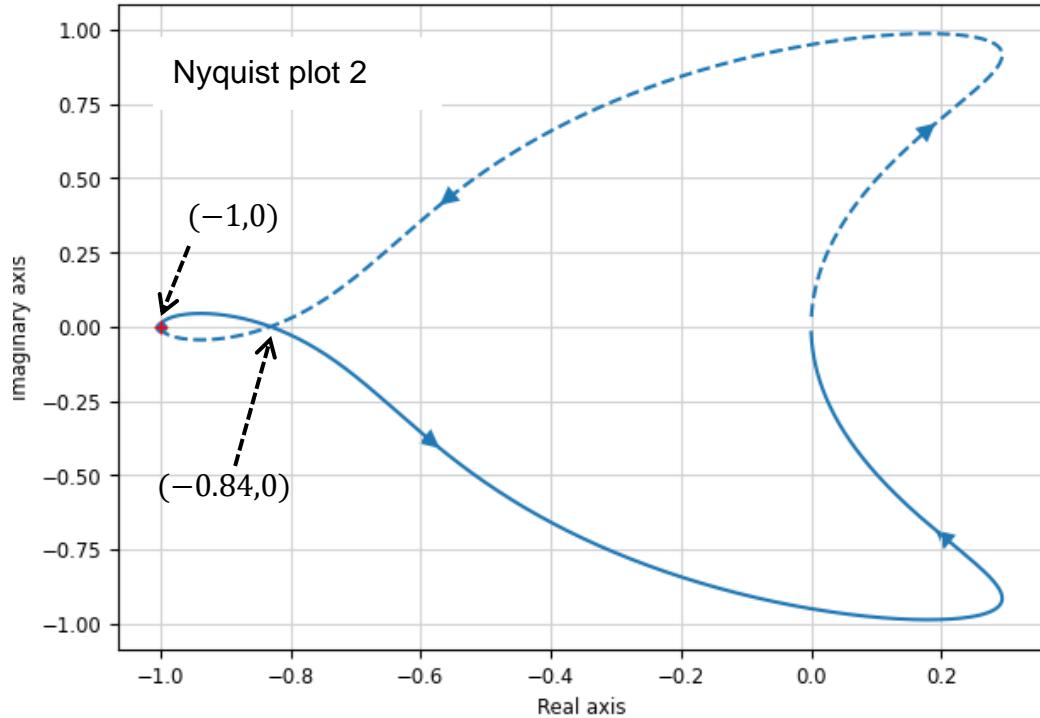
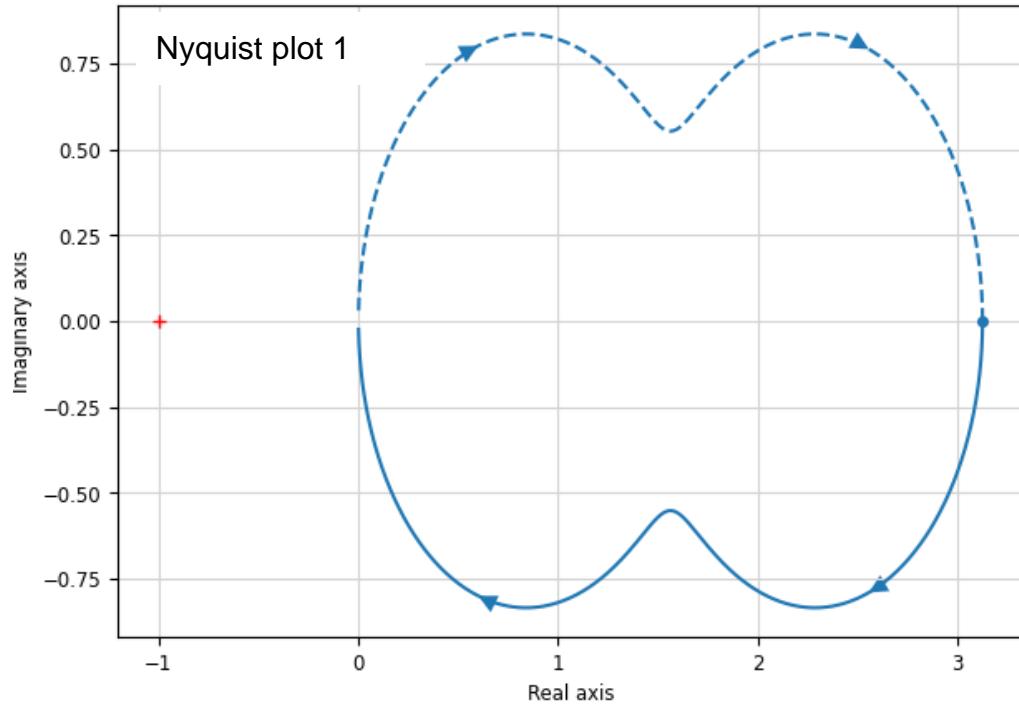


Student name: _____



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- e) [10] Given the following Nyquist plots where Nyquist plot 1 corresponds to block diagram 1 and Nyquist plot 2 corresponds to block diagram 2 in question d).



Analyze the stability of the systems displayed in block diagrams 1 and 2.

(Problem 2 continues on the next page)



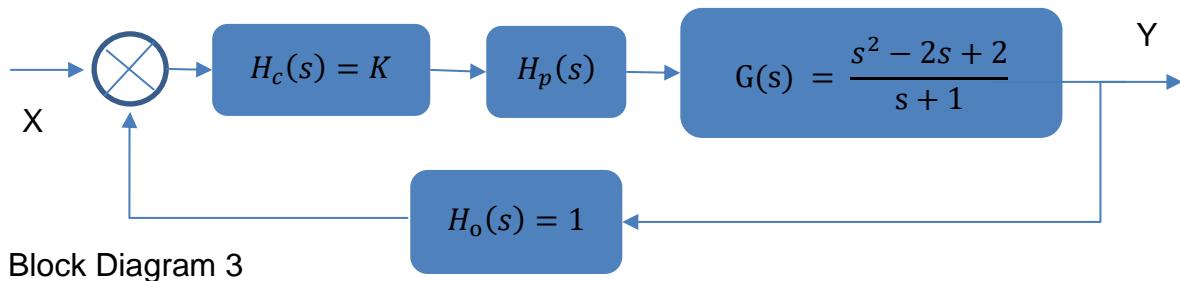
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- f) [15] In Nyquist plot 2, the proportional controller gain $H_c(s) = 80$. An engineer would like to increase the controller gain K from 0 to ∞ . Specify at what range of K the system has: zero unstable pole; one unstable pole; two unstable poles. Motivate your answer.

- g) [10] A new block diagram:



Draw the root locus for $H_c(s)H_p(s)G(s)$ and find the controller gain K such that the closed-loop system is marginally stable.

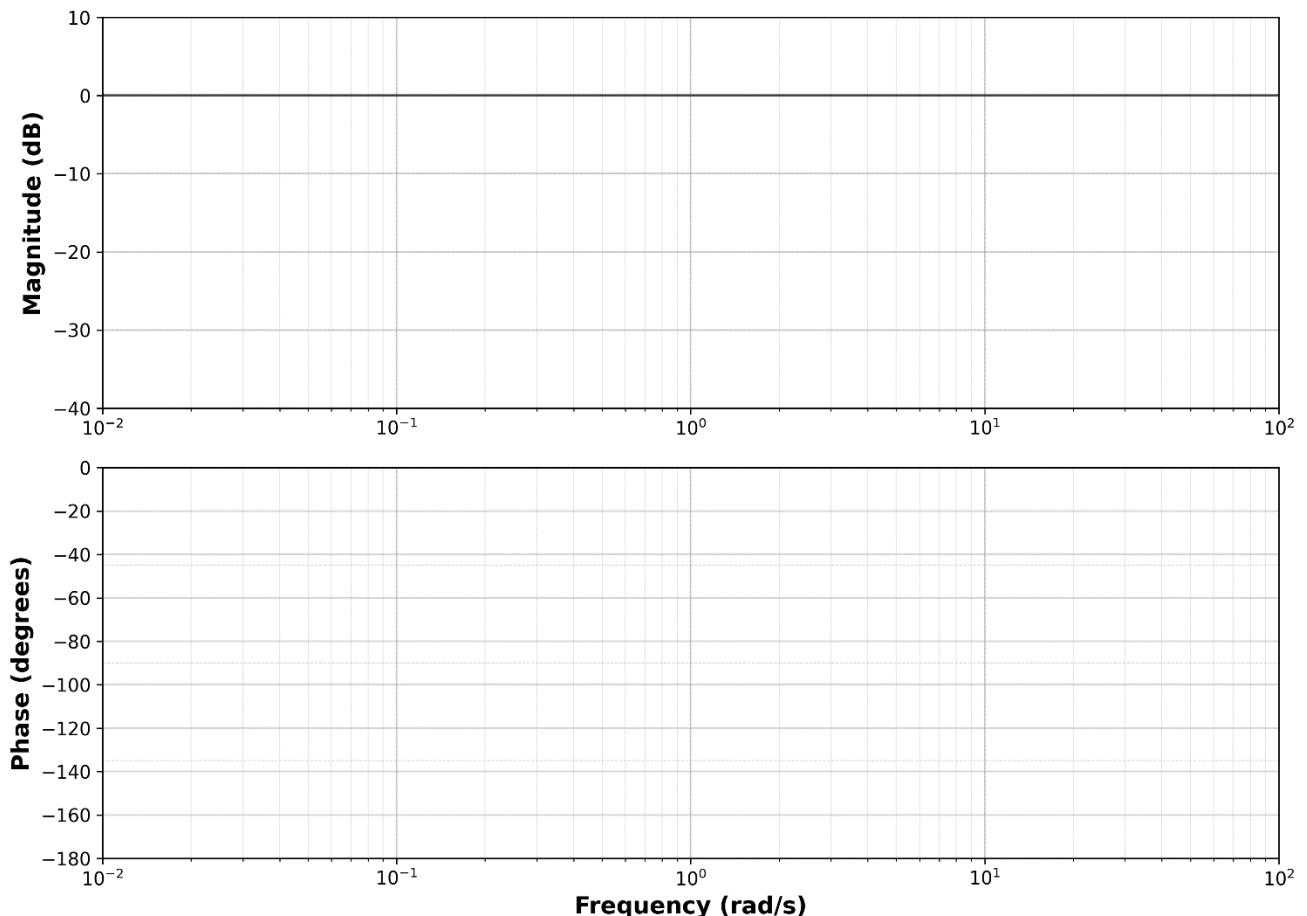


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Student number: _____

Bode Plot Sketch Sheet





Student name: _____



Student number: _____

Formula Sheet

Laplace transform table and properties

1. $A \cdot l(t)$	$\frac{A}{s}$	1. Linearity	$a \cdot f(t) + b \cdot g(t)$	$a \cdot F(s) + b \cdot G(s)$
2. $\delta(t) \cdot l(t)$	1	2. Frequency shift	$e^{-at} f(t)$	$F(s+a)$
3. $t^n \cdot l(t)$	$\frac{n!}{s^{n+1}}$	3. Time shift	$f(t-a) \cdot l(t-a)$	$e^{-ax} \cdot F(s)$
4. $e^{at} \cdot l(t)$	$\frac{1}{s-a}$	4. Scaling	$f(at)$	$1/a F(s/a)$
5. $\sin(\omega t) \cdot l(t)$	$\frac{\omega}{s^2 + \omega^2}$	5. Differentiation	$f^{(n)}(t)$	$s^n \cdot F(s) - s^{n-1} f'(0) - s^{n-2} f''(0) - \dots - s^0 f^{(n-1)}(0)$
6. $\cos(\omega t) \cdot l(t)$	$\frac{s}{s^2 + \omega^2}$	6. Initial	$f(0) = \lim_{t \rightarrow 0} f(t)$	$f(0) = \lim_{s \rightarrow \infty} sF(s)$
		7. Final	$f(\infty) = \lim_{t \rightarrow \infty} f(t)$	$f(\infty) = \lim_{s \rightarrow 0} sF(s)$

Logarithmic scale: Decibel

$$\text{dB} = 20 \log_{10} \text{linear}$$

$$\text{linear} = 10^{\frac{\text{dB}}{20}}$$

Root Locus Rules

1. Where does the root locus start and end?

Start at poles of OLTF, ends at finite zeros or infinity of OLTF. Number of branches = number of poles

2. Where is the locus on the real axis?

To the left of an odd number of real axis poles & zeros

3. What is the shape of root locus?

$$\beta = \frac{\sum p - \sum z}{\#p - \#n} \quad (\text{Centroid of asymptotes})$$

$$\phi_l = \frac{(2l+1)180^\circ}{\#p - \#n}, l = 0, 1, \dots (\#p - \#n - 1)$$

(Angles of asymptotes)

The root locus is symmetric about the real axis!

4. Where does the root locus break in or out?

$$\text{Solve: } \frac{d(1+KL(s))}{ds} = 0$$