



## Exam Information

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Exam: Basic Control Systems  
Regular (**TEST EXAM**)

Date: \_\_\_\_, \_\_\_\_, 2025

Time: \_\_:\_\_ till \_\_:\_\_

Duration:

3 hours

Teacher(s):

Hanshu Yu

Number of pages in this exam paper: **7**

(1 cover page + 4 main body + 1 formula sheet + 1 sketch sheet)

Number of problems in this test: **2**

Allowed material:

- Writing gear;
- Calculator;

Total raw score: 120 points.

Calculation of final scores:  $\text{MIN}(\text{total score}, 100) \times 0.5$

Note for grading: correct solutions without reasoning **do not** grant points.

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## Student Information

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Student name: \_\_\_\_\_

Student number: \_\_\_\_\_

Table number: \_\_\_\_\_

Final score: \_\_\_\_\_

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**DO NOT FORGET TO SUBMIT THE SKETCH SHEETS.**

**Good luck!**



Student name: \_\_\_\_\_

Student number: \_\_\_\_\_



### Problem 1 (57 points)

Open loop process  $H$  is described by an ordinary differential equation,

$$\frac{d y(t)}{dt} + 10y(t) = 9x(t)$$

The initial conditions are as follows:  $y(0) = 0, x(0) = 0$ .

- a) [5] Perform Laplace transform and find the transfer function  $H(s)$ .
- b) [5] Find step response  $y_{step}(t)$  in time domain.
- c) [5] Find  $y(t)$  if  $x(t) = \sin(\omega t)$ .
- d) [5] If the process is put in a negative unit feedback control loop together with a P-controller, find the poles and zeros of the closed-loop system if the P-controller gain is set to  $K = 10$ .
- e) [6] Find the time constant  $\tau_p$  of the system, and the percentage of decrease of the impulse response  $y_{impulse}(t)$  when  $t = \tau_p$ .
- f) [3] Is the closed-loop system stable when the controller gain  $K = 100$ ?
- g) [6] Draw root locus for the given system.
- h) [10] If there is a PD controller replacing the original P controller, how will the root locus change? Draw the new root locus and explain the effect when the PD controller's gain  $K$  becomes very large.
- i) [12] If there is a PID controller replacing the original P controller, prove mathematically that the steady-state error of the closed-loop system is zero.



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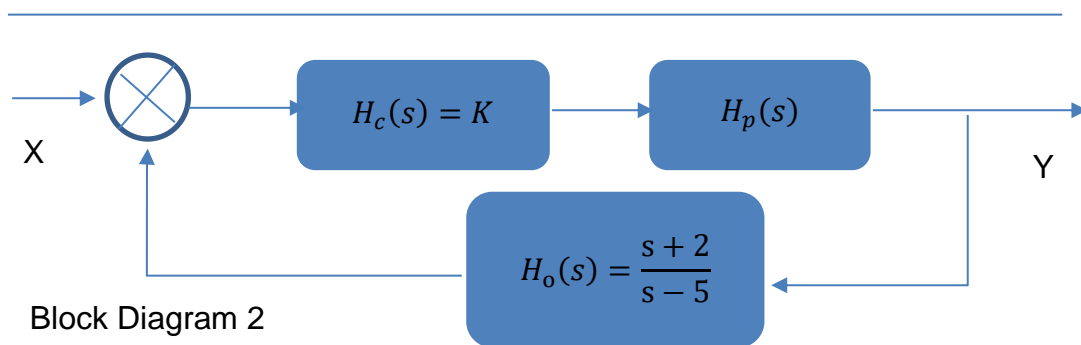
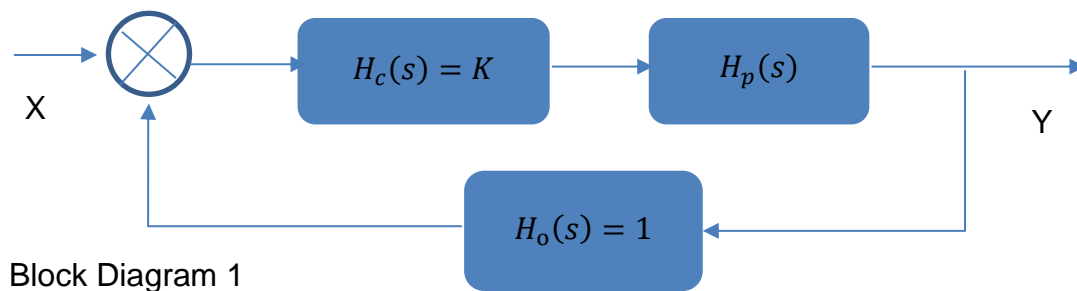


## Problem 2 (63 points)

Given a transfer function corresponds to an open-loop process:

$$H_c(s)H_p(s) = \frac{K(s+1)}{4s^2 + 64s + 32}, \text{ with } H_c(s) = K$$

- a) [4] Find poles and zeros of  $H_c(s)H_p(s)$ .
- b) [12] Draw bode plots for  $H_c(s)H_p(s)$  when  $H_c(s) = 10$ .
- c) [7] Given the 2 following block diagrams:



Find the closed-loop transfer function for these two block diagrams.

- d) [5] Draw root-locus of the system in block diagram 1.

(Problem 2 continues on the next page)

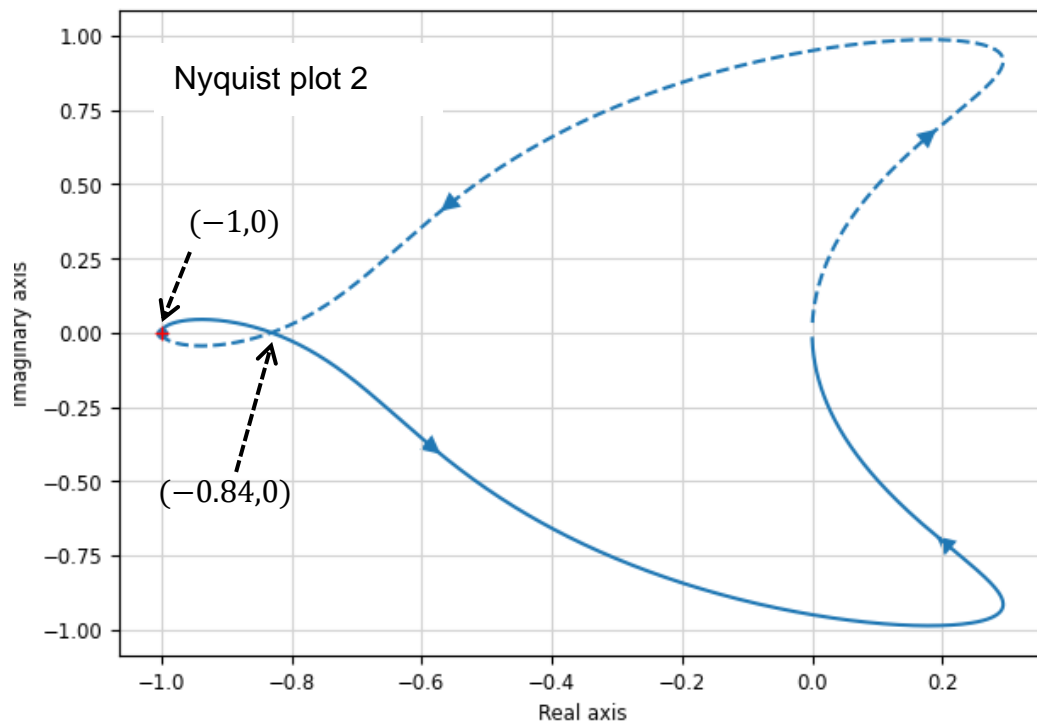
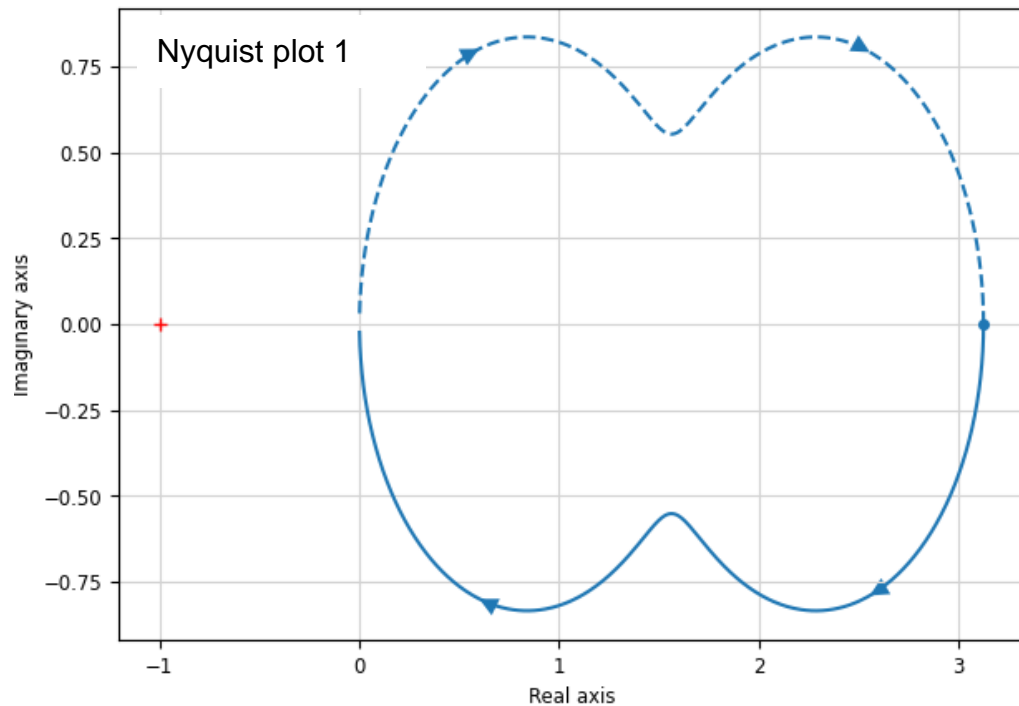


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- e) [10] Given the following Nyquist plots where Nyquist plot 1 corresponds to block diagram 1 and Nyquist plot 2 corresponds to block diagram 2 in question d).



Analyze the stability of the systems displayed in block diagrams 1 and 2.

(Problem 2 continues on the next page)



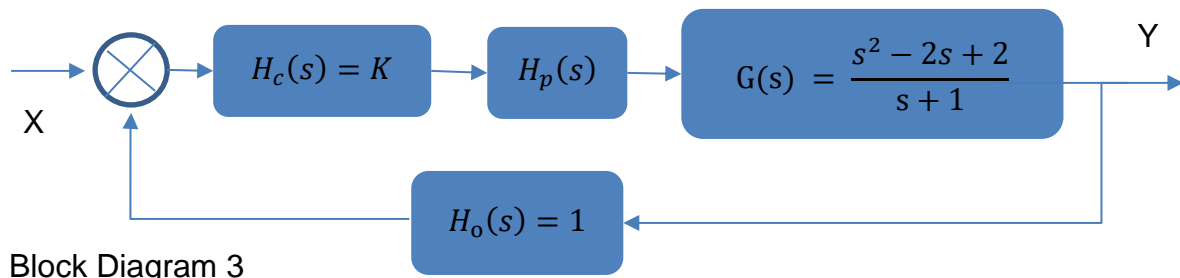
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- f) [15] In Nyquist plot 2, the proportional controller gain  $H_c(s) = 80$ . An engineer would like to increase the controller gain  $K$  from 0 to  $\infty$ . Specify at what range of  $K$  the system has: zero unstable pole; one unstable pole; two unstable poles. Motivate your answer.

- g) [10] A new block diagram:



Draw the root locus for  $H_c(s)H_p(s)G(s)$  and find the controller gain  $K$  such that the closed-loop system is marginally stable.

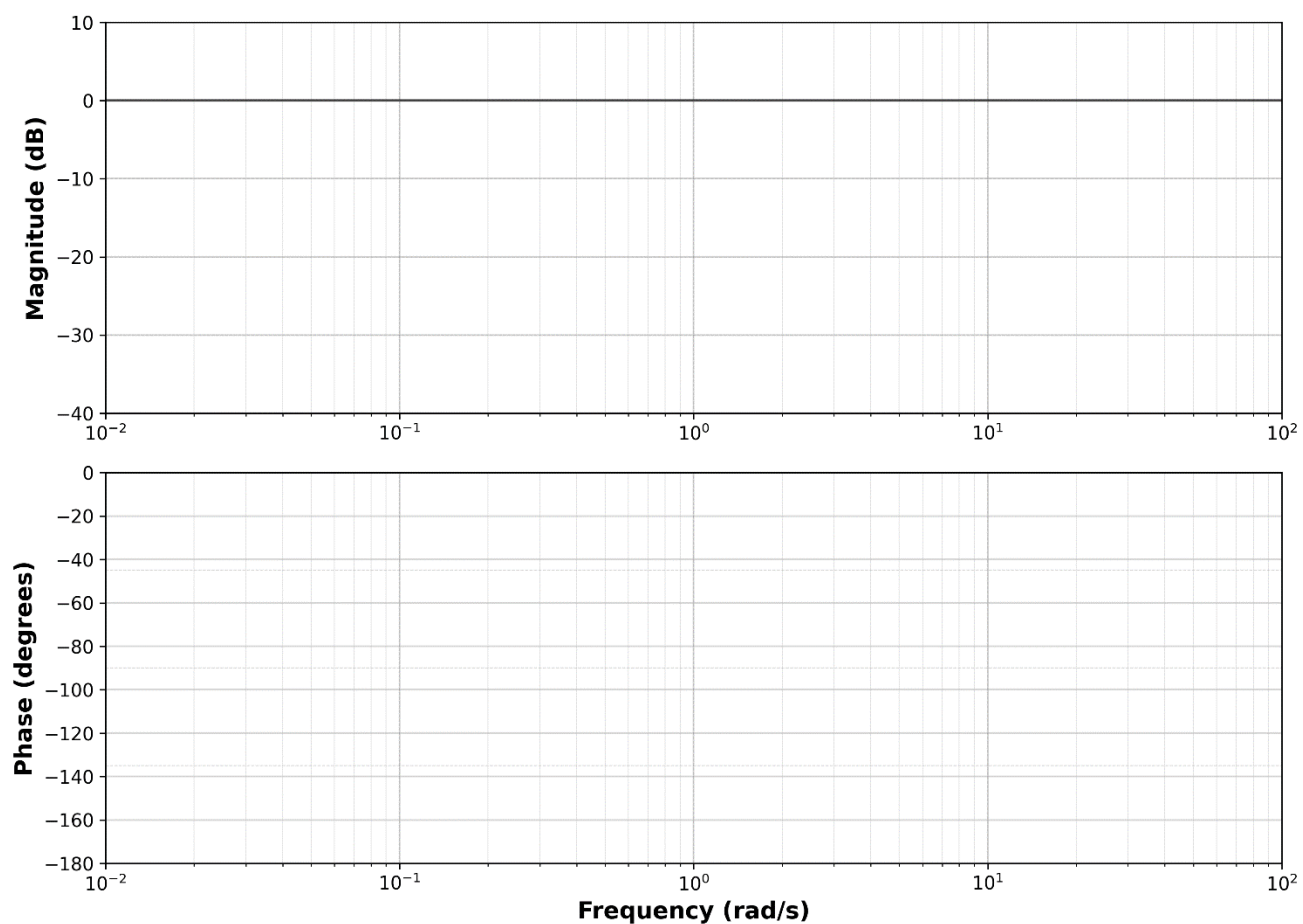


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## Bode Plot Sketch Sheet





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## Formula Sheet

### Laplace transform table and properties

1. $A \cdot 1(t)$	$\frac{A}{s}$	1. <b>Linearity</b>	$a \cdot f(t) + b \cdot g(t)$	$a \cdot F(s) + b \cdot G(s)$
2. $\delta(t) \cdot 1(t)$	1	2. <b>Frequency shift</b>	$e^{-at} \cdot f(t)$	$F(s+a)$
3. $t^n \cdot 1(t)$	$\frac{n!}{s^{n+1}}$	3. <b>Time shift</b>	$f(t-a) \cdot 1(t-a)$	$e^{-as} \cdot F(s)$
4. $e^{at} \cdot 1(t)$	$\frac{1}{s-a}$	4. <b>Scaling</b>	$f(at)$	$1/a \cdot F(s/a)$
5. $\sin(\omega t) \cdot 1(t)$	$\frac{\omega}{s^2 + \omega^2}$	5. <b>Differentiation</b>	$f^{(n)}(t)$	$s^n \cdot F(s) - s^{n-1} \cdot f(0) - s^{n-2} \cdot f'(0) - \dots - s^0 \cdot f^{(n-1)}(0)$
6. $\cos(\omega t) \cdot 1(t)$	$\frac{s}{s^2 + \omega^2}$	6. <b>Initial</b>	$f(0) = \lim_{t \rightarrow 0} f(t)$	$f(0) = \lim_{s \rightarrow \infty} sF(s)$
		7. <b>Final</b>	$f(\infty) = \lim_{t \rightarrow \infty} f(t)$	$f(\infty) = \lim_{s \rightarrow 0} sF(s)$

### Logarithmic scale: Decibel

$$\text{dB} = 20 \log_{10} \text{linear}$$

$$\text{linear} = 10^{\frac{\text{dB}}{20}}$$

### Root Locus Rules

- Where does the root locus start and end?  
Start at poles of OLTF, ends at finite zeros or infinity of OLTF. Number of branches = number of poles
- Where is the locus on the real axis?  
To the left of an odd number of real axis poles & zeros
- What is the shape of root locus?

$$\beta = \frac{\sum p - \sum z}{\#p - \#n} \text{ (Centroid of asymptotes)}$$

$$\phi_l = \frac{(2l+1)180^\circ}{\#p - \#n}, l = 0, 1, \dots (\#p - \#n - 1)$$

(Angles of asymptotes)

The root locus is symmetric about the real axis!

- Where does the root locus break in or out?

$$\text{Solve: } \frac{d(1+KL(s))}{ds} = 0$$