Exam Information

Exam: Basic Control Systems
Regular (TEST EXAM)
Date: ___, ____, 2025
Time: ___:___ till ___:___
Duration:
3 hours
Teacher(s):
Hanshu Yu
Number of pages in this exam paper: 7
(1 cover page + 4 main body + 1 formula sheet + 1 sketch sheet)
Number of problems in this test: 2

Allowed material:

- **Writing gear;**
- **Calculator;**

Total raw score: 120 points.

Calculation of final scores: MIN(total score,100)*0.5

Note for grading: correct solutions without reasoning do not grant points.

Student Information

Student name: ______________________

Student number: ______________________

Table number: ______________________

Final score:________

DO NOT FORGET TO SUBMIT THE SKETCH SHEETS.

Good luck!

Student name: ______________________

Student number: ______________________

Problem 1 (57 points)

Open loop process H is described by an ordinary differential equation,

$$\frac{\mathrm{d}\, y(t)}{dt} + 10y(t) = 9x(t)$$

The initial conditions are as follows: $y(0) = 0$, $x(0) = 0$.

a) [5] Perform Laplace transform and find the transfer function $H(s)$.

b) [5] Find step response $y_{step}(t)$ in time domain.

c) [5] Find $y(t)$ if $x(t) = sin(\omega t)$.

d) [5] If the process is put in a negative unit feedback control loop together with a P-controller, find the poles and zeros of the closed-loop system if the P-controller gain is set to K = 10.

e) [6] Find the time constant τ_p of the system, and the percentage of decrease of the impulse response $y_{impulse}(t)$ when $t = \tau_p$.

f) [3] Is the closed-loop system stable when the controller gain K = 100?

g) [6] Draw root locus for the given system.

h) [10] If there is a PD controller replacing the original P controller, how will the root locus change? Draw the new root locus and explain the effect when the PD controller's gain *K* becomes very large.

i) [12] If there is a PID controller replacing the original P controller, prove mathematically that the steady-state error of the closed-loop system is zero.

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Problem 2 (63 points)

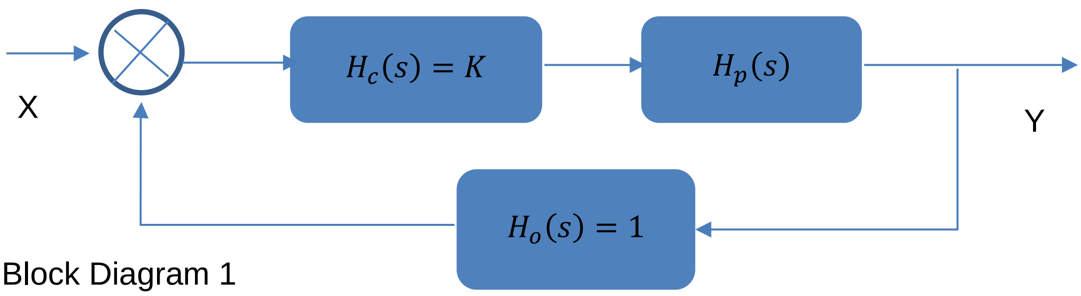
Given a transfer function corresponds to an open-loop process:

$$H_c(s)H_p(s) = \frac{K(s+1)}{4s^2 + 66s + 32}, \text{with } H_c(s) = K$$

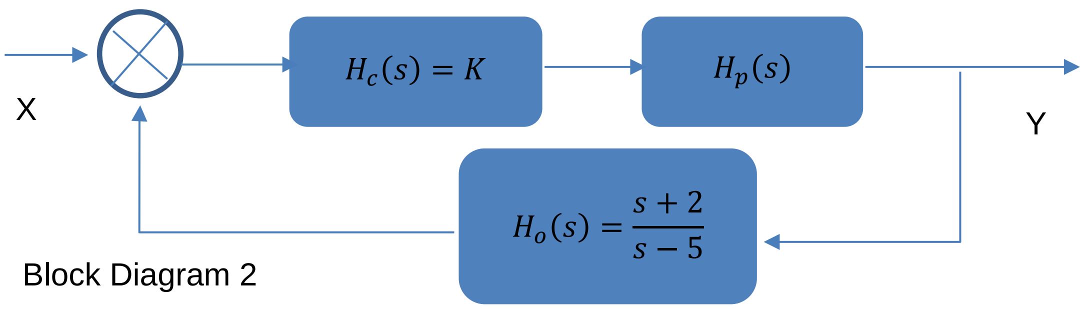
a) [4] Find poles and zeros of $H_c(s)H_p(s)$.

b) [12]Draw bode plots for $H_c(s)H_p(s)$ when $H_c(s) = 10$.

c) [7]Given the 2 following block diagrams:



Block Diagram 1



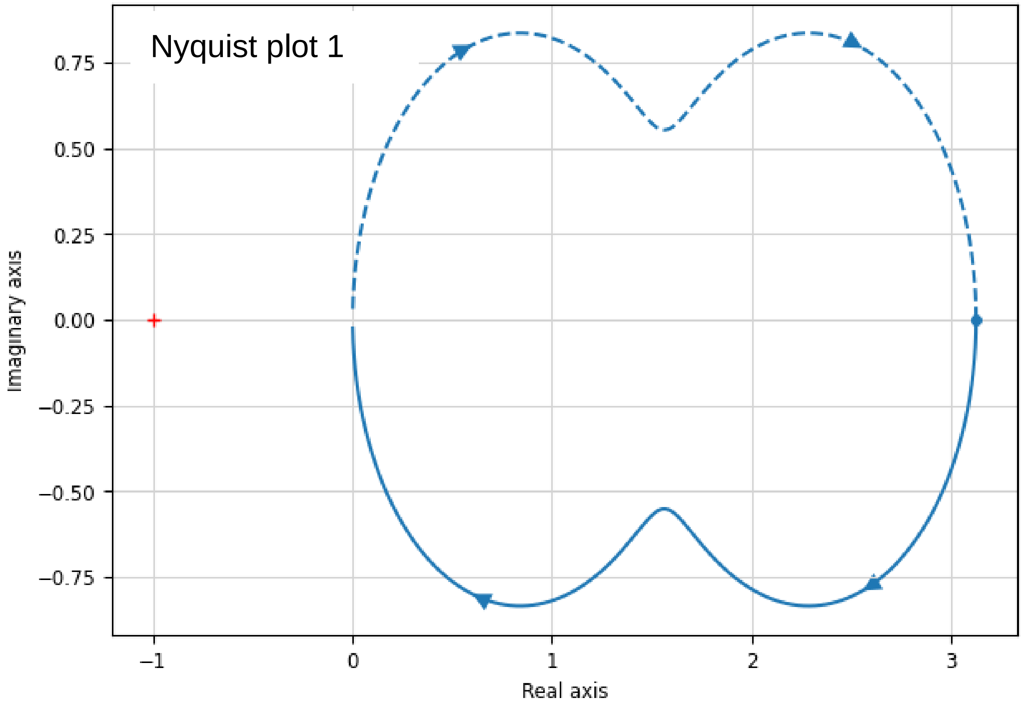
Block Diagram 2

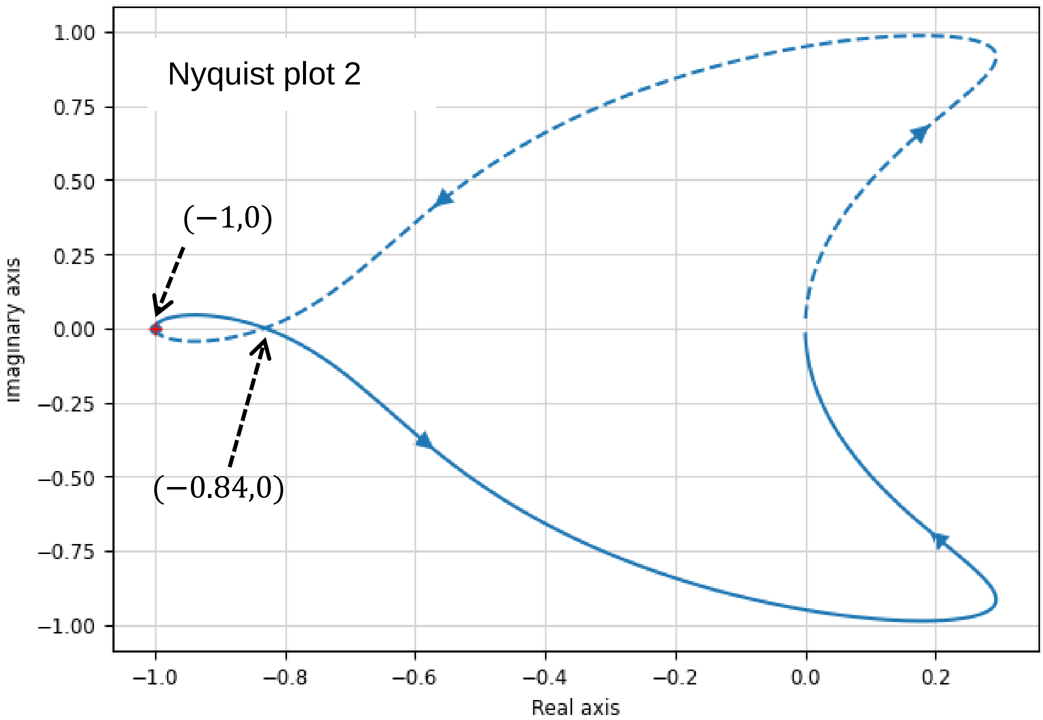
Find the closed-loop transfer function for these two block diagrams.

d) [5] Draw root-locus of the system in block diagram 1.

(Problem 2 continues on the next page)

e) [10] Given the following Nyquist plots where Nyquist plot 1 corresponds to block diagram 1 and Nyquist plot 2 corresponds to block diagram 2 in question d).



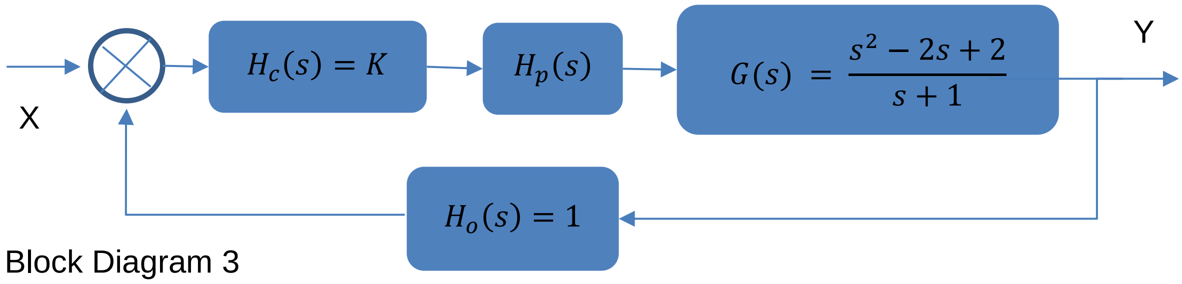


Analyze the stability of the systems displayed in block diagrams 1 and 2.

(Problem 2 continues on the next page)

f) [15] In Nyquist plot 2, the proportional controller gain $H_c(s) = 80$. An engineer would like to increase the controller gain K from 0 to ∞. Specify at what range of K the system has: zero unstable pole; one unstable pole; two unstable poles. Motivate your answer.

g) [10] A new block diagram:



Block Diagram 3

Draw the root locus for $H_c(s)H_p(s)G(s)$ and find the controller gain K such that the closed-loop system is marginally stable.

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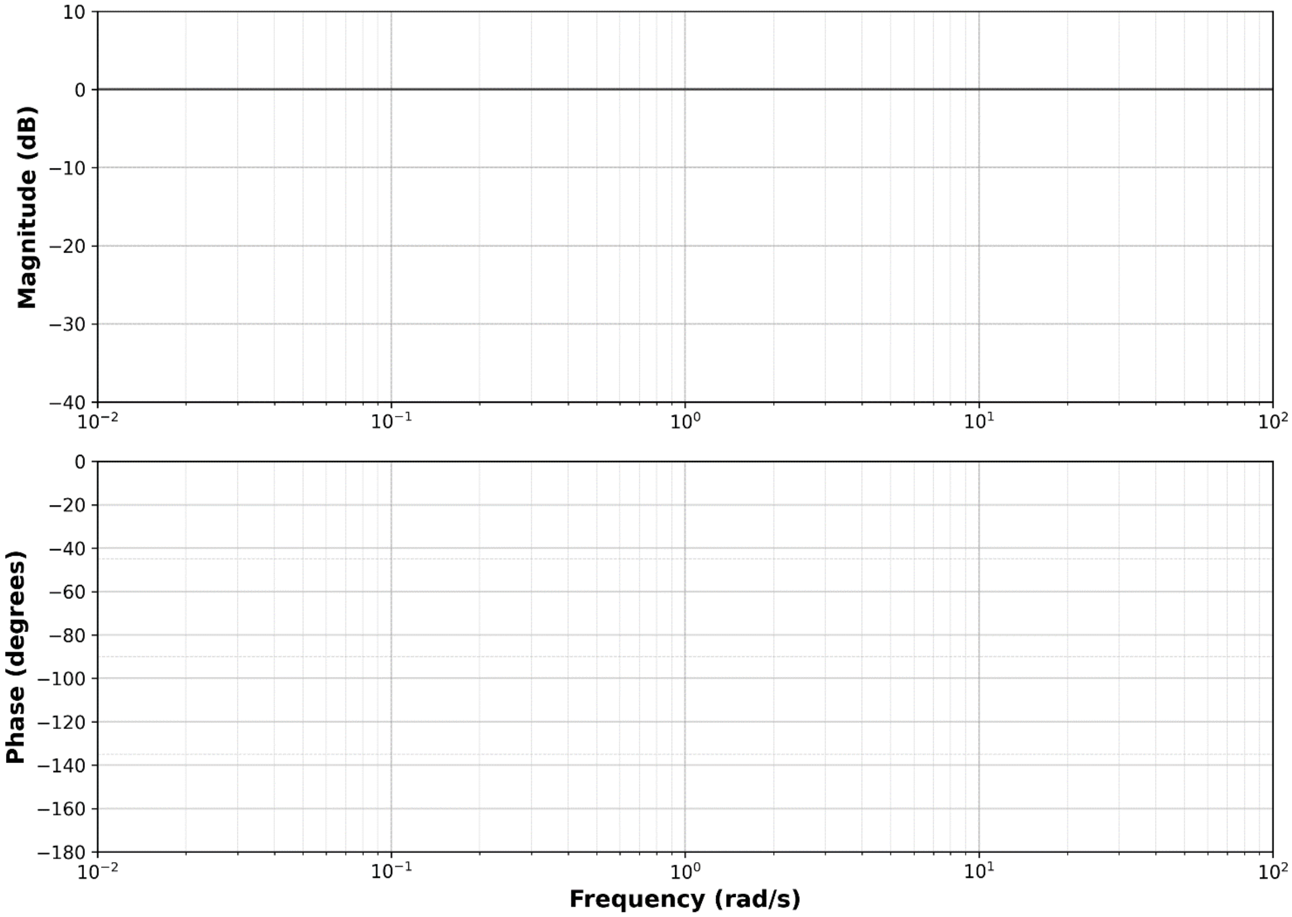
Student number: ______________________________

Bode Plot Sketch Sheet



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Formula Sheet

Laplace transform table and properties

1. $A \cdot 1(t)$	$\frac{A}{s}$
2. $\delta(t) \cdot 1(t)$	1
3. $t^n \cdot 1(t)$	$\frac{n!}{s^{n+1}}$
4. $e^{at} \cdot 1(t)$	$\frac{1}{s-a}$
5. $\sin(\omega t) \cdot 1(t)$	$\frac{\omega}{s^2+\omega^2}$
6. $\cos(\omega t) \cdot 1(t)$	$\frac{s}{s^2+\omega^2}$

1. **Linearity**	$a \cdot f(t) + b \cdot g(t)$	$a \cdot F(s) + b \cdot G(s)$
2. **Frequency shift**	$e^{-at} f(t)$	$F(s+a)$
3. **Time shift**	$f(t-a) \cdot 1(t-a)$	$e^{-as} \cdot F(s)$
4. **Scaling**	$f(at)$	$1/a\, F(s/a)$
5. **Differentiation**	$f^{(n)}(t)$	$s^n \cdot F(s) - s^{n-1} \cdot f(0) - s^{n-2} \cdot f'(0) - \ldots - s^0 \cdot f^{(n-1)}(0)$
6. **Initial**	$f(0) = \lim_{t \to 0} f(t)$	$f(0) = \lim_{s \to \infty} sF(s)$
7. **Final**	$f(\infty) = \lim_{t \to \infty} f(t)$	$f(\infty) = \lim_{s \to 0} sF(s)$

Logarithmic scale: Decibel

$$\mathrm{dB} = 20 \log_{10} linear$$

$$linear = 10^{\frac{dB}{20}}$$

Root Locus Rules

1. Where does the root locus start and end?

   Start at poles of OLTF, ends at finite zeros or infinity of OLTF. Number of branches = number of poles

2. Where is the locus on the real axis?

   To the left of an odd number of real axis poles & zeros

3. What is the shape of root locus?

   $$\beta = \frac{\sum p - \sum z}{\#p - \#n} \text{ (Centroid of asymptotes)}$$

   $$\phi_l = \frac{(2l+1)180^o}{\#p-\#n}, l = 0,1, \ldots (\#p - \#n - 1)$$

   (Angles of asymptotes)

   The root locus is symmetric about the real axis!

4. Where does the root locus break in or out?

   Solve: $\frac{\mathrm{d}\,(1+KL(s))}{\mathrm{d}\,s} = 0$