BASIC CONTROL SYSTEMS

01 FUNDAMENTALS CONCEPTS

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WHERE STUDENTS MATTER

A SIMPLE CASE

I would like to grab a bottle of water.

A SIMPLE CASE

I would like to grab a bottle of water.

I list what I have to do.

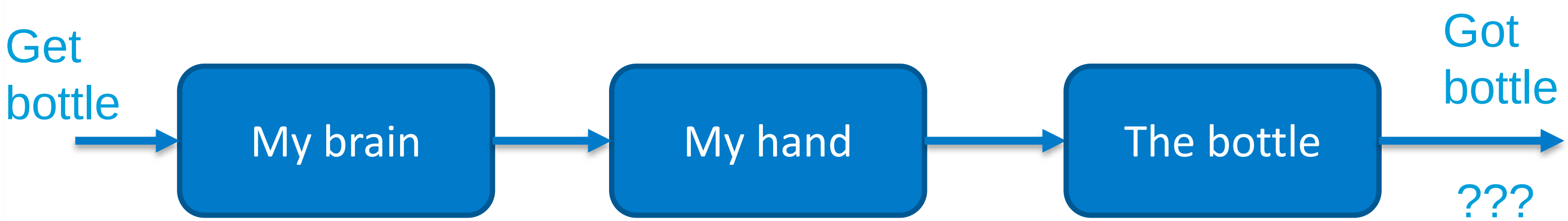
A SIMPLE CASE

I would like to grab a bottle of water.

I list what I have to do.

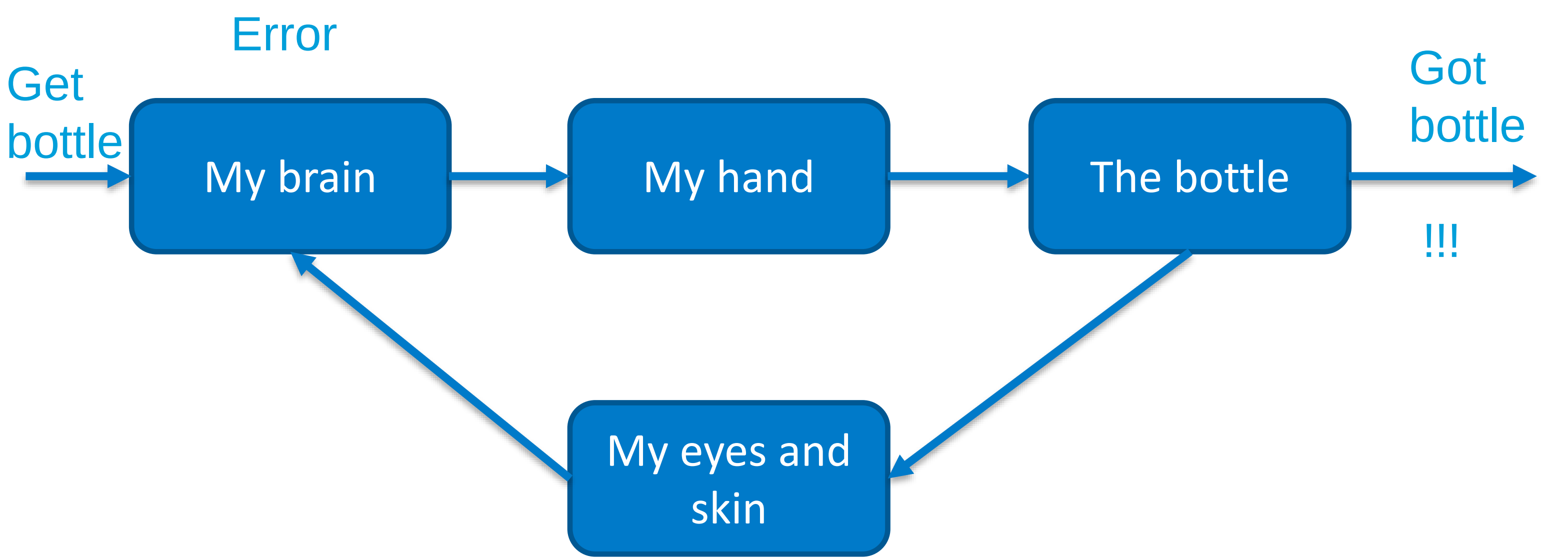
I wrap my hand around the bottle.
I apply force.
I grab bottle.

OPEN LOOP CONTROL SYSTEM



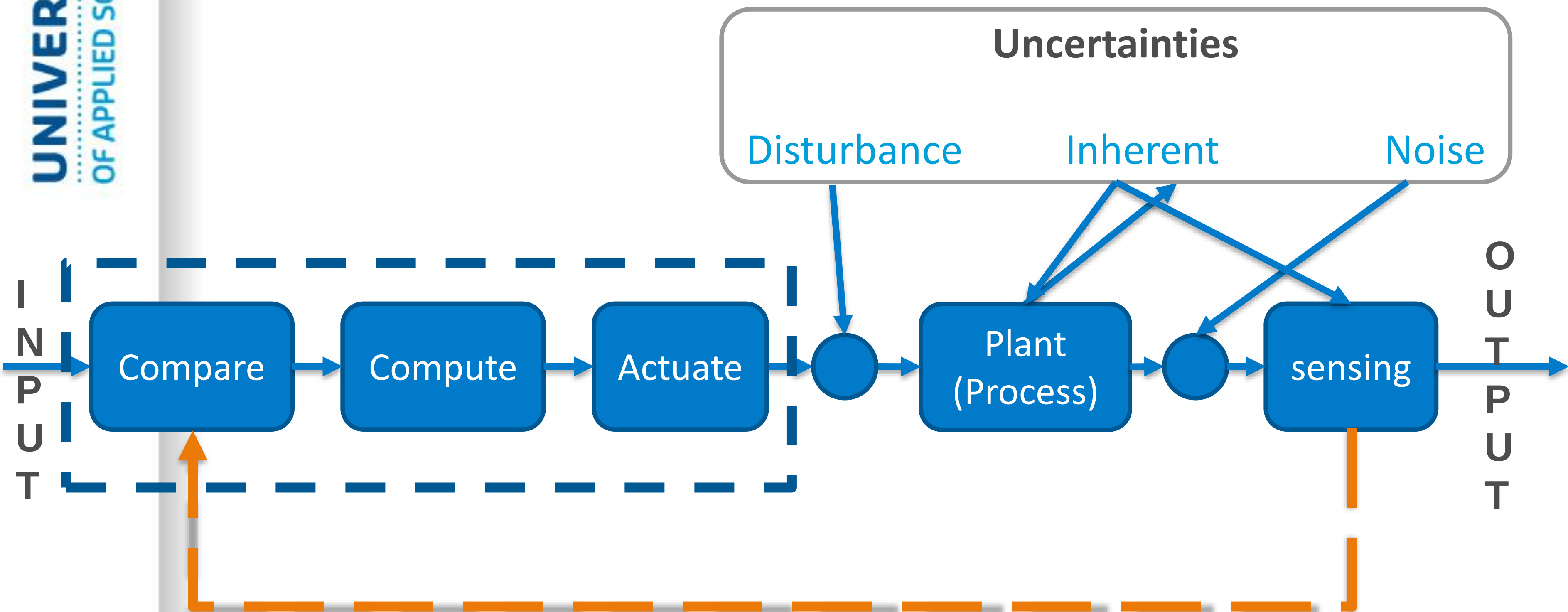
I don't know the force I need to apply,
I just grab,
I do not have my water.

CLOSED LOOP CONTROL SYSTEM



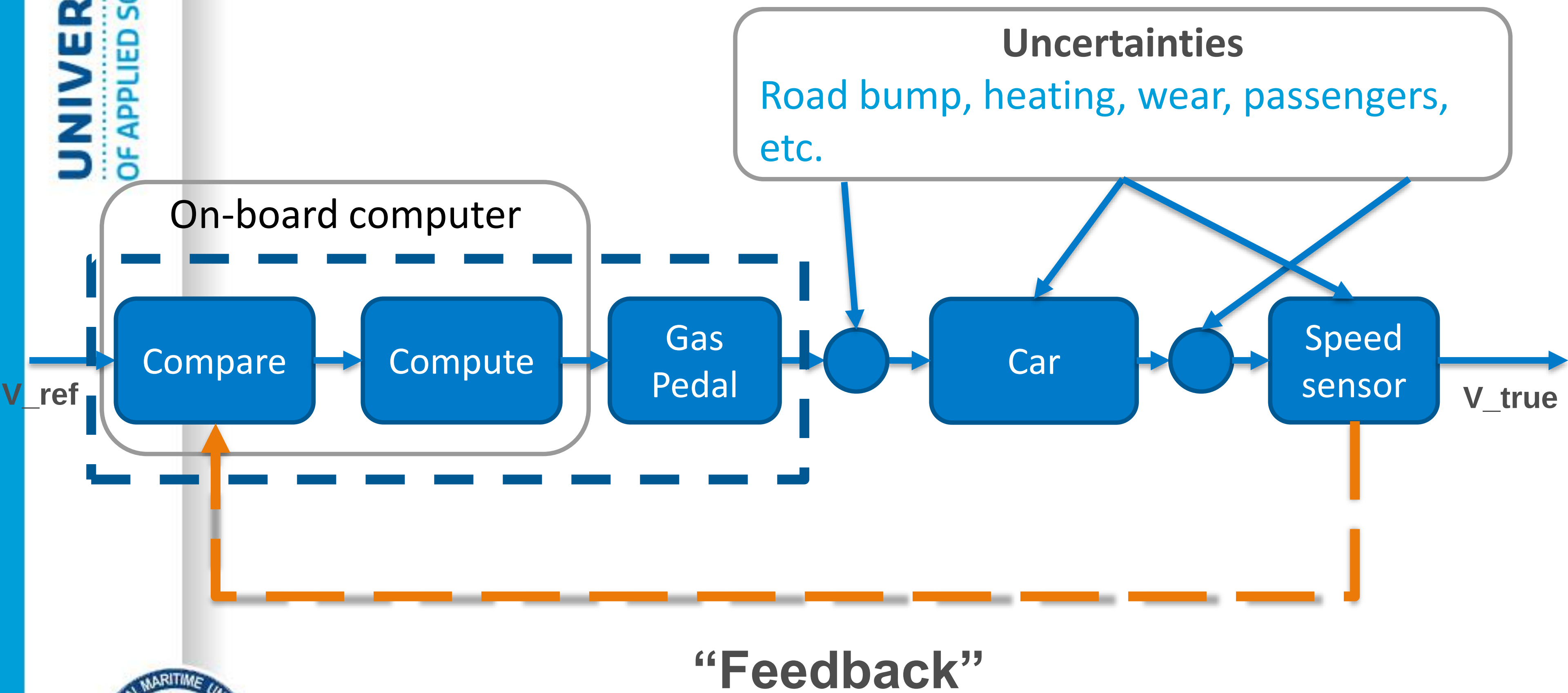
I don’t know the force I need to apply,
but I try, I feel, and I look,
I stop pressure when my hand apply just enough force,
I get my water.

THE STANDARD MODEL OF A FEEDBACK CONTROL SYSTEM

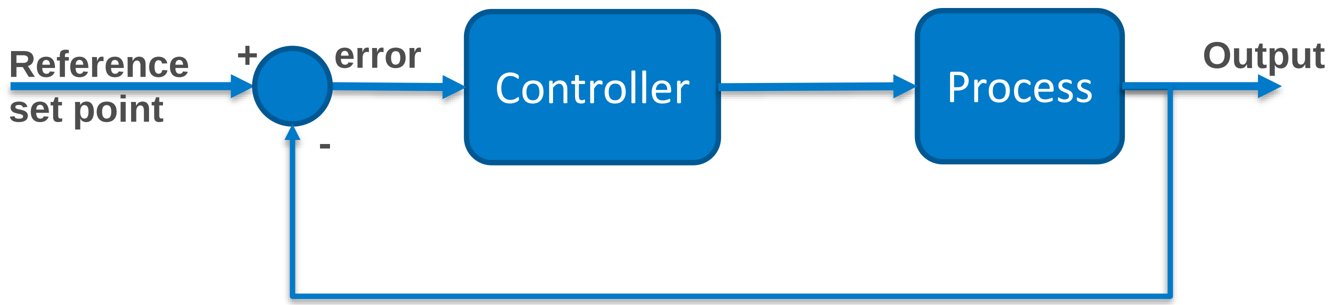


“Feedback”

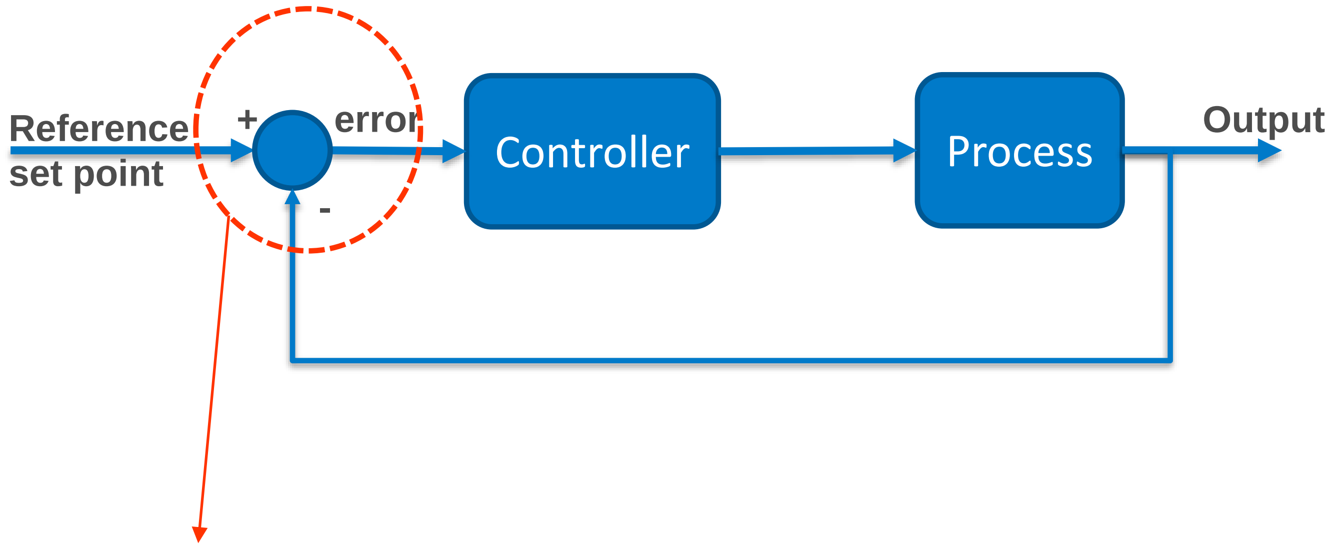
REAL WORLD EXAMPLE: CRUISE CONTROL OF A BUS

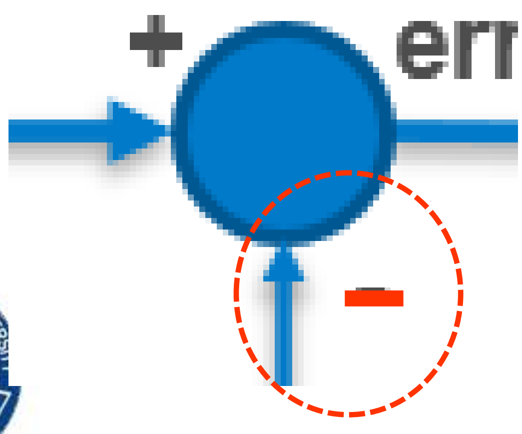


SIMPLE CLOSED LOOP CONTROL WITH UNIT FEEDBACK



SIMPLE CLOSED LOOP CONTROL WITH UNIT FEEDBACK





Minus sign: negative feedback.
Plus sign: positive feedback

We only work with negative feedback in this course.

WITH OR WITHOUT FEEDBACK

With feedback we can:

- deal with system dynamics
- be robust to uncertainty
- modular operation
- gain even more knowledge of environment

But the trouble feedback brings:

- increase complexity
- potential to bring unstable
- noise amplification

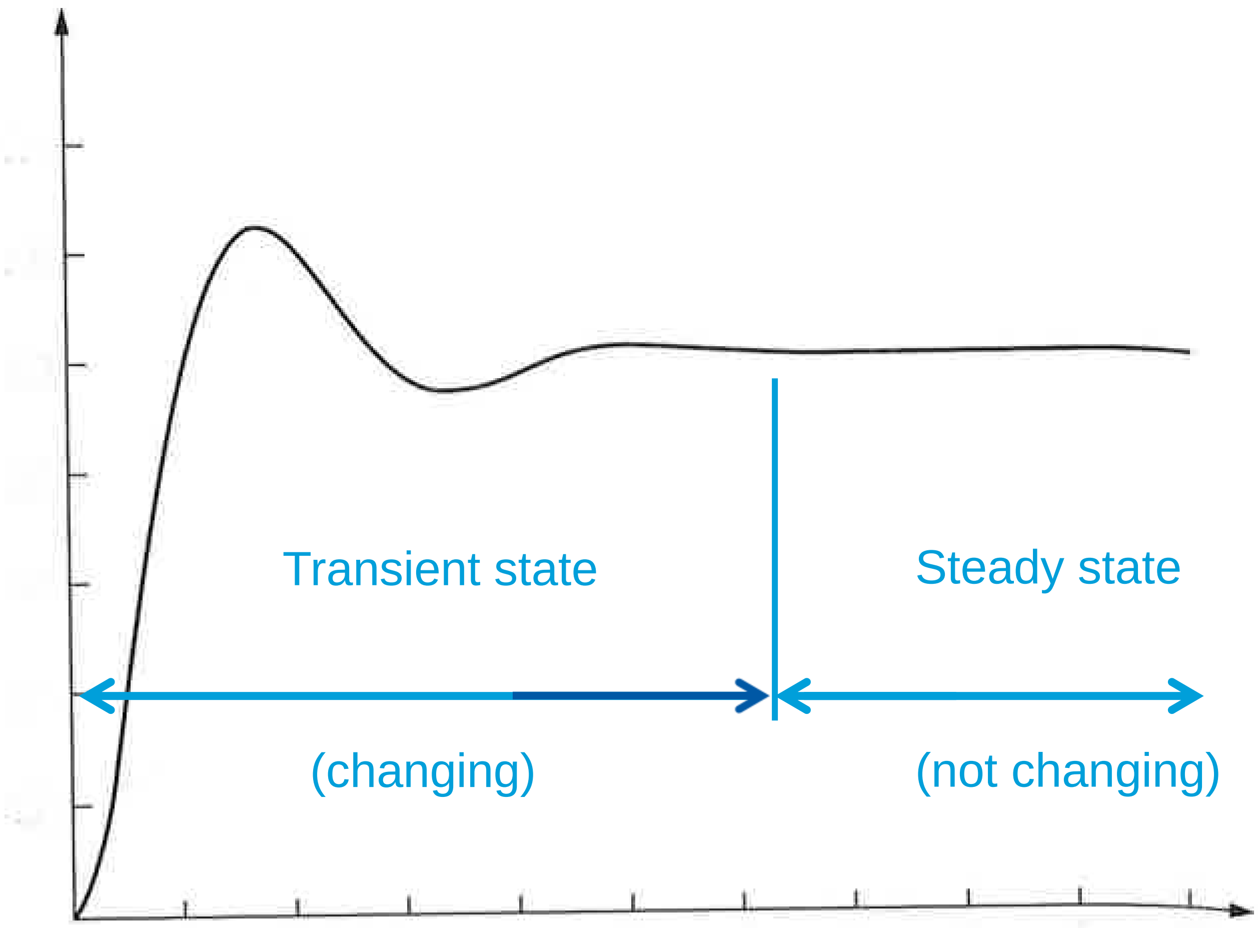
TRANSFER FUNCTION

$$OUTPUT = INPUT \ * G(s)$$

$$G(s) \ = \frac{OUTPUT}{INPUT}$$

Blackbox

TRANSIENT & STEADY



CONTROL

(roughly explained)

The process of design & implementing algorithms in engineered target system to achieve a desired output or system state.

Typically you control a system using controller & control loops.

CONTROL

When there is uncertainty,
apply control!

SYSTEM

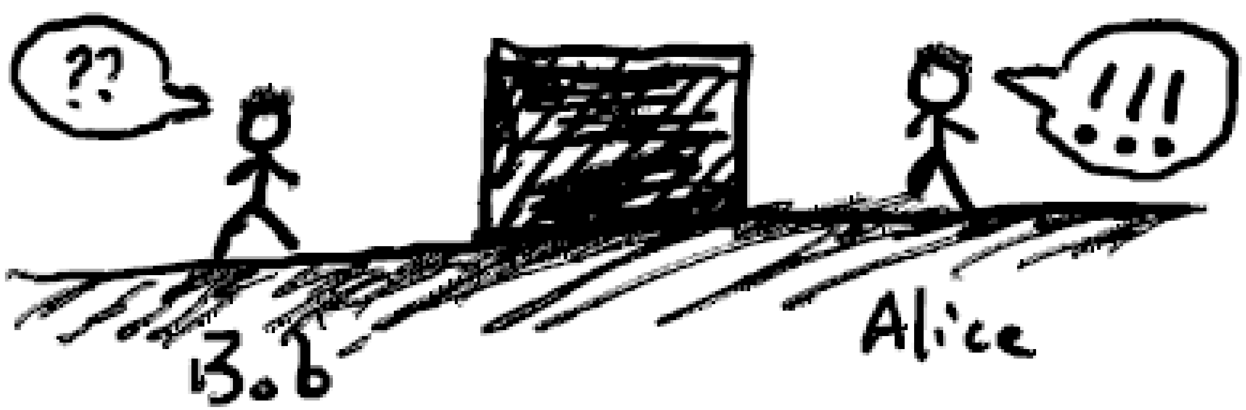
Generally,
An object or a series of interacting objects of your interest.

In which you usually can discover:

- Input(s)
- Output(s)
- Object(s) as “building block(s)”
  - The physical characteristics are **crucial**

SYSTEM - BRAINSTORM

Is a large piece of metal on the ground a system?



SYSTEMS - SCOPE OF THIS COURSE

We only deal with:

Causal LTI SISO systems

- **Causal:** output only depends on the past and present (input), not the future
- **L**: Linear systems or systems that can be linearized
- **TI**: Time-invariant
- **SISO**: Single Input Single Output

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CAUSAL LTI SISO SYSTEMS

Properties:

Given a system that yields $x_{(t)} \mapsto y_{(t)}$

1. Homogeneity

$$\alpha \cdot x_{(t)} \mapsto \alpha \cdot y_{(t)}\ , \alpha \in \mathbf{R}$$

2. Additive

Given : $x_{1(t)} \mapsto y_{1(t)},\ x_{2(t)} \mapsto y_{2(t)}$, we have:

$$(x_1 + x_2)\_(t) \mapsto (y_1 + y_2)\_(t)$$

3. Time invariance

$$x_{(t+a)} \mapsto y_{(t+a)},\ a \in \mathbf{R}$$

4. Causality

the system remains stationary before t_0

(we almost always take $t_0 = 0$)

$\forall\ t_1 \neq t_2$ and $t_1, t_2 < t_0$, we have:

$x_{(t_1)} = x_{(t_2)}$ and $y_{(t_1)} = y_{(t_2)}$

SYSTEMS: THE KEY TAKEOUT

No matter how much we simplify,
we are working with physical systems.

The mathematical tools you see later,
are describing the characteristics of the physical system.

（最重要的是物理系统自身的特性！）

STABILITY

finite input

system

finite output

STABLE !

STABILITY (MATHEMATICAL DESCRIPTION)

Given a system $h(t)$,

for $h(t)$ to be stable, the impulse response of $h(t)$ should be absolutely integrable.

$$\int_{-\infty}^{\infty} |h(t)|\,\mathrm{d}t < \infty$$

In our case, for causal systems:

$$\int_{0}^{\infty} |h(t)|\,\mathrm{d}t < \infty$$

STABILITY – BIBO

More formally we have:

- **B**ounded-**I**nput **B**ounded-**O**utput (**BIBO**) stable
- Marginally stable
- Conditionally stable
- Uniformly stable
- Asymptotically stable
- Unstable

finite input

system

finite output

BEYOND STABILITY - OTHER CHARACTERISTICS

- Stability

- Robustness
- Sensitivity

- Observability
- Controllability
- Reachability
- Stabilizability
- Reconstructability
- Detectability
- ……

MODELLING

You have a physical system
You have learned some physics

Determine the input and output

Write the relevant equations

Derive the ODE

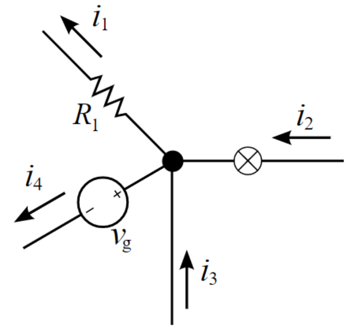
(Next step)
Solve it using Laplace transform

Modelling: Electric circuits

Kirchhoff's circuit laws deal with the conservation of charge and energy in electrical circuits.

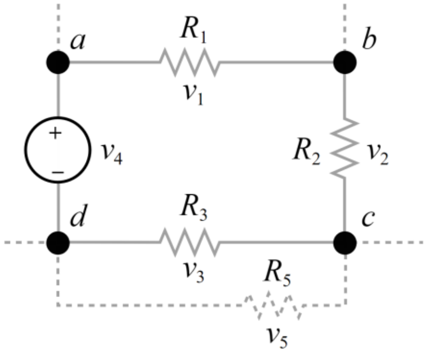
Kirchhoff's current law:
The current entering any junction is equal to the current leaving that junction. $i_2 + i_3 = i_1 + i_4$

$$\sum_{k=1}^{n} I_k = 0$$

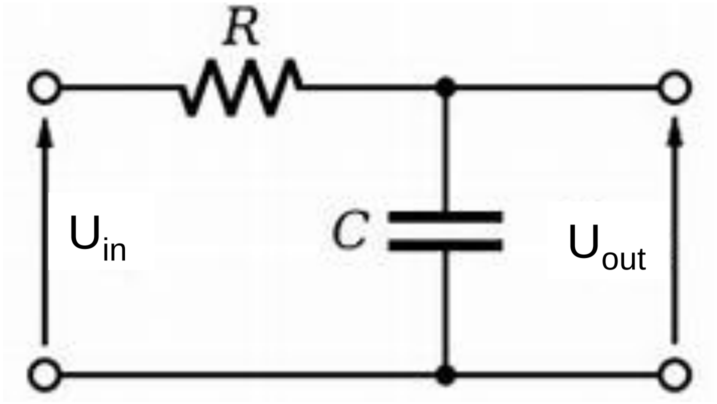


Kirchhoff's voltage law:
The sum of all the voltages around a loop is equal to zero.
$v_1 + v_2 + v_3 + v_4 = 0$

$$\sum_{k=1}^{n} V_k = 0$$



Modelling example: RC low-pass filter



This is a RC low-pass filter(LPF).

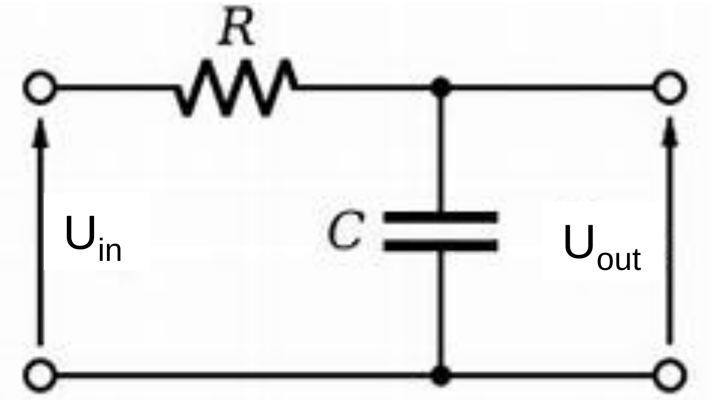
You have a voltage source U_{in} .

You want to know U_{out} ,the voltage after the RC-LPF.

You know R and C values. The initial condition is 0.

What should you do?

Modelling example: RC low-pass filter



Circuit analysis!

$$I_{total} = I_R = I_C$$

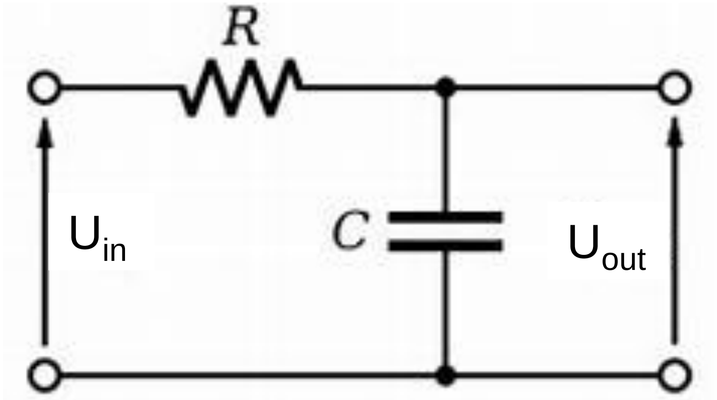
$$U_{in} = U_R + U_C = U_R + U_{out}$$

$$U_C = \frac{1}{C}\int_0^1 I(\tau)\,d\tau \Rightarrow \mathrm{I_C} = \mathrm{C}\frac{\mathrm{d}U_C}{\mathrm{d}t}$$

$$U_R = I_R R \Rightarrow \mathrm{I_R} = \frac{U_R}{R}$$

$$\frac{U_R}{R} = \mathrm{C}\frac{\mathrm{d}U_C}{\mathrm{d}t}$$

Modelling example: RC low-pass filter



Circuit analysis! -> Differential Equation!

$$\frac{U_R}{R} = \mathrm{C}\frac{\mathrm{d}U_C}{\mathrm{d}t} \Rightarrow U_R = \ \mathrm{RC}\frac{\mathrm{d}U_C}{\mathrm{d}t}$$

$$U_{in} = U_R + U_{out}\,, U_C = U_{out}$$

$$\Rightarrow U_{in} = \ \mathrm{RC}\frac{\mathrm{d}U_{out}}{\mathrm{d}t} + U_{out}$$

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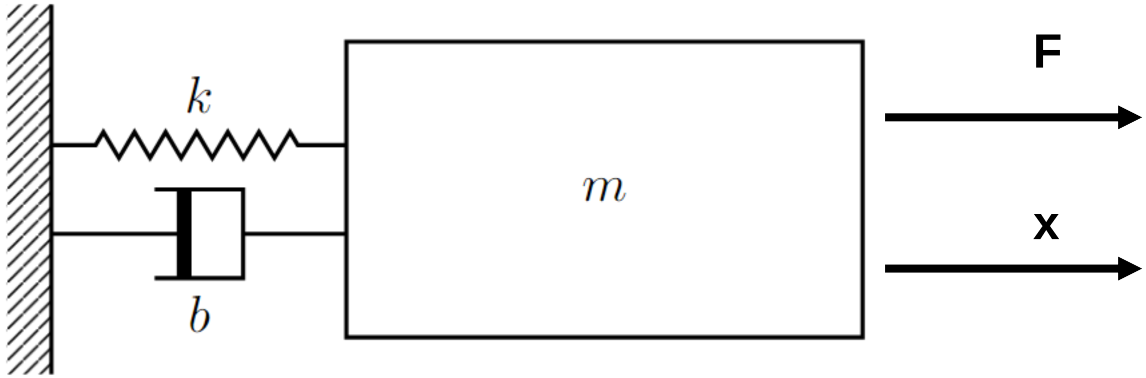
ELECTRICAL SYSTEMS – RLC CIRCUITS

	Voltage - Current
Resistor	$\mathrm{U}(t) = I(t)R$
Capacitor	$\mathrm{U}(t) = \frac{1}{C}\int_0^1 I(\tau)\, d\tau$
Inductor	$\mathrm{U}(t) = L\frac{d\, I(t)}{d\, t}$

Modelling example: Mechanical system



The force applied on m is F.

The velocity of m is v.

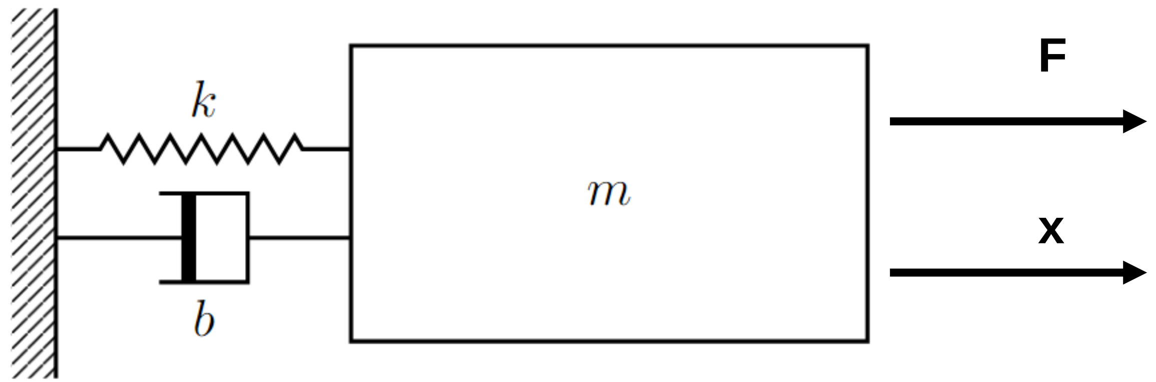
The displacement of m is x.

I pull the block m from sitting still, I want to know how it moves, in this case velocity.

Input F

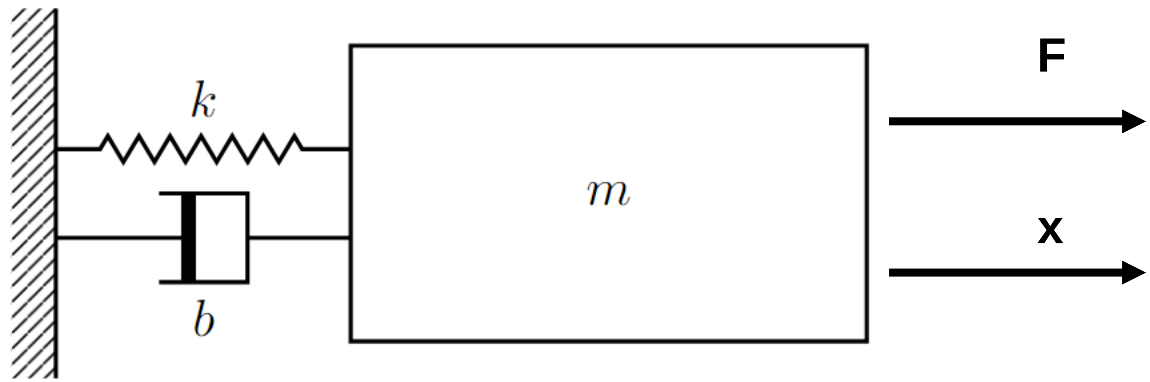
Output v

Modelling example: Mechanical system

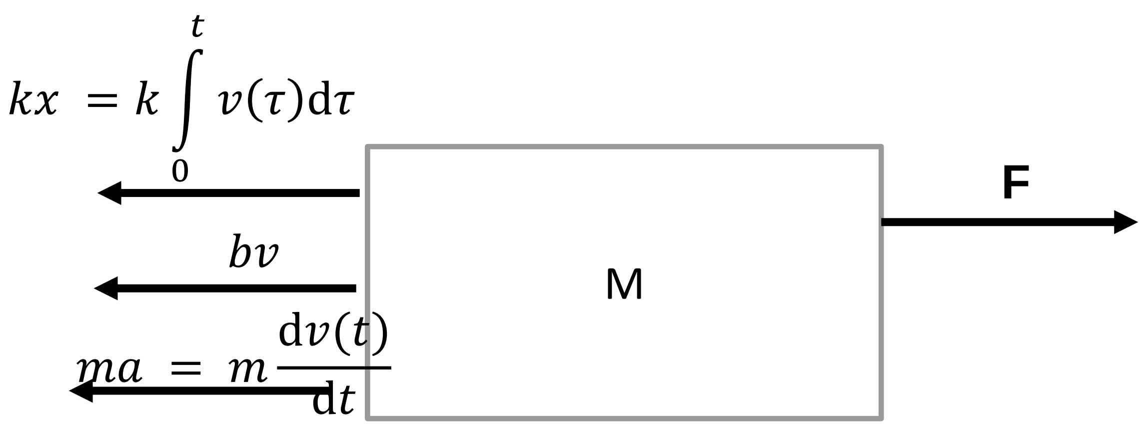


Analysis! Free body diagram.

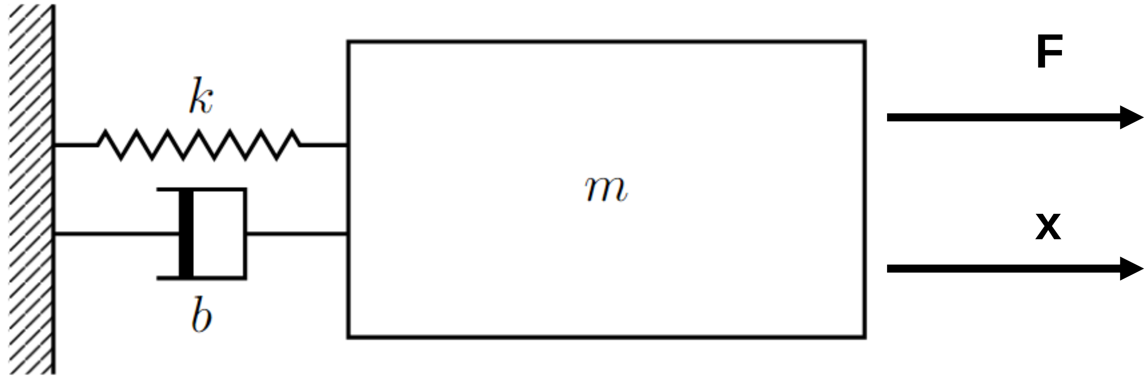
Modelling example: Mechanical system



Analysis! Free body diagram.



Modelling example: Mechanical system



Write differential equation

$$F(t) = k\int_0^t v(\tau)\mathrm{d}\tau + bv(t) + m\frac{\mathrm{d}v(t)}{\mathrm{d}t}$$

\reals

MECHANICAL SYSTEMS – TRANSLATING SYSTEM

	Force - Velocity
Damper (Viscous friction)	$F = b\mathrm{v}$
Spring	$F = k\int_0^t v(\tau)\mathrm{d}\tau$
Mass (Inertia)	$F = m\frac{\mathrm{d}v(t)}{\mathrm{d}t}$

\reals

MECHANICAL SYSTEMS – TRANSLATING SYSTEM

	Force – Displacement
Damper (Viscous friction)	$F = b\dfrac{\mathrm{d}x(t)}{\mathrm{d}t}$
Spring	$F = k\mathrm{x}$
Mass (Inertia)	$F = m\dfrac{\mathrm{d}^2x(t)}{\mathrm{d}t^2}$

SUMMARY

- Getting what we want from a system: **control**
- We like: **causal** & **linear** systems
- Open loop & closed loop control system
    - **"The standard model"**
- Still changing? Transient
- Stop changing? Steady
- (BIBO) Stability
- Modelling physical systems with ordinary differential equations.

HOMEWORK

Read the lecture note:

Part 1 Introduction to fundamental concepts

Try to solve exercise problems in section 1.6

Stage ONE exercise:

- Problem 5 (you can skip subquestion 3 for now)
- Problem 8