

Fiscal Policy and Inequality

13. Tax Inefficiencies & Optimal taxation

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Outline

Tax Distortions and Deadweight Loss

Optimal Taxation

Ramsey Model

Taxes and Labor Supply

Mirrlees (1971)

Laffer Curve

Saez (2001)

Distortionary Effects of Taxation

- ▶ The introduction of a tax affects the behavior of economic agents, moving them *away* from the existing equilibrium
 - ▶ Hence, we say that taxes are **distortionary**

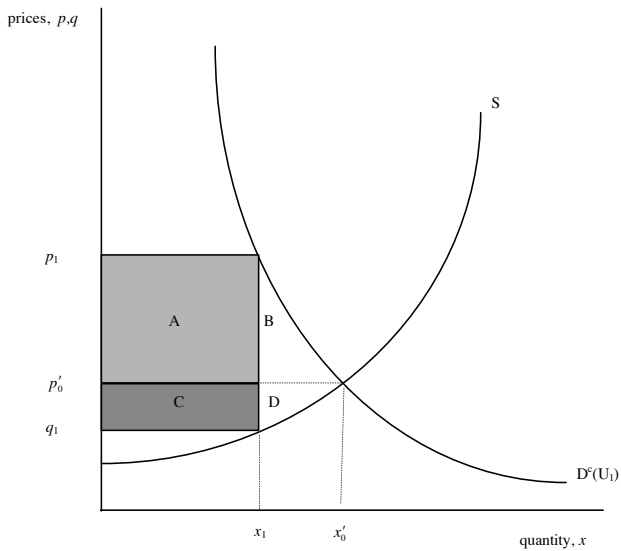
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- ▶ **Deadweight loss from (or excess burden) of taxation:**
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- ▶ While incidence analysis focuses on **prices**, efficiency analysis focuses on **quantities**

Deadweight Loss (DWL)



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- ▶ Let the consumer have income $Y = 1$

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$$z = \left(\frac{b}{a}\right) \left(\frac{p_x}{p_z}\right) x$$
- ▶ Budget constraint: $Y = p_x x + p_z z$
- ▶ Substitute optimal z into budget constraint:

$$x^* = \left(\frac{a}{a+b}\right) \frac{Y}{p_x}, \quad z^* = \left(\frac{b}{a+b}\right) \frac{Y}{p_z}$$

Simple example: solution

$$u(x_1, x_2) = x_1 x_2, P_1 = 1, P_2 = c, Y = 1$$

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- Consumption levels: $x_1 = \frac{1}{2}; x_2 = \frac{1}{2c}$
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- ▶ Consumption levels (demand): $x'_1 = \frac{1+T}{2}; x'_2 = \frac{1+T}{2(c+t)}$

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- ▶ Note that

$$\frac{\partial DWL}{\partial t} = \frac{t}{(2c+t)^3} > 0, \forall t > 0$$

- ▶ increasing taxes has a small but accelerating positive effect on on deadweight loss.

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Tax Distortions and Deadweight Loss

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Optimal Taxation

- ▶ “Optimal” tax theory must combine lessons from deadweight loss and tax incidence:
 - ▶ Optimal size of the pie? - **Efficiency**
 - ▶ How is the pie distributed? - **Equity**
- ▶ What is the best way to design taxes given equity and efficiency concerns?

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 - ▶ Does not induce any behavioural responses, so this is most efficient tax available
- ▶ **Equity perspective:** individualized lump-sum taxes
 - ▶ Tax higher-ability (higher earning power) individuals with larger lump sum
 - ▶ **Problem:** we cannot observe ability directly
 - ▶ Hence we tax outcomes, such as income or consumption
 - ▶ Creates distortions & inefficiency

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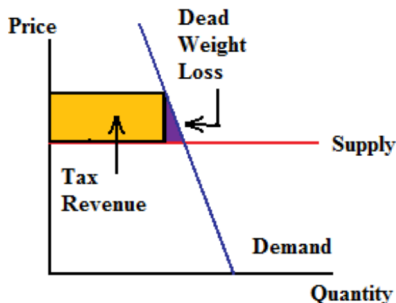
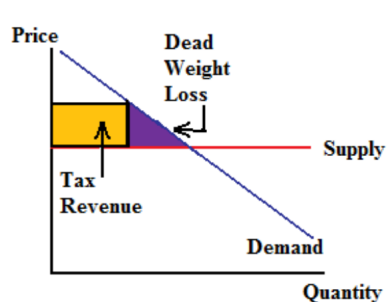
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 - ▶ no redistributive concerns.
4. Cannot tax all commodities: x_0 (leisure) untaxed

Ramsey (1927): Elastic vs Inelastic Demand

A. More Elastic Demand



- **Intuition:** tax inelastic goods to minimize efficiency costs

Ramsey (1927): Inverse Elasticity Rule

- ▶ The (simplified) Ramsey tax formula for the linear tax on good i is

$$\tau_i = \frac{\lambda}{\epsilon_i}$$

where λ is a parameter summarizing the value of government spending, and ϵ_i is the elasticity of demand for good i .

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- ▶ Low $\epsilon_i \rightarrow$ high τ_i
- ▶ Taxes on all goods, unless perfectly elastic demand
- ▶ The ratio of the taxes on two goods is the inverse ratio of their demand elasticities:

$$\frac{\tau_i}{\tau_j} = \frac{\epsilon_j}{\epsilon_i}$$

Ramsey (1927): Limitations

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- ▶ Restricted to linear taxes
- ▶ Does not take into account redistributive motives
 - ▶ Necessities usually more inelastic than luxuries
 - ▶ Thus, optimal Ramsey tax system is regressive

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Taxes and Labor Supply

- ▶ In April 2010, the British government raised the top income tax rate from 40% to 50%
- ▶ Assuming that taxable income had remained constant after the reform, tax revenues would have increased by **6.8 billion** pounds (0.46% of GDP)
- ▶ Before the reform, the Treasury projected this reform to increase tax revenues by
 - ▶ £8.5 billion?
 - ▶ £6.8 billion?
 - ▶ £2.7 billion?
 - ▶ £0.68 billion?

Labor supply model

- ▶ The individual solves

$$\max_{\{c,h\}} u(c, h)$$

subject to $c = wh + R$

- ▶ c = consumption
- ▶ h = hours worked (supply of labor)
- ▶ $u(c, h)$ = utility function, with $u_c > 0$ and $u_h < 0$
 - ▶ people like consumption but don't like working
- ▶ w = **after-tax** hourly wage
- ▶ R = non-labor income

Labor supply elasticity

- ▶ Substituting for consumption:

$$\max_h u(wh + R, h)$$

- ▶ Solve with FOC's to get labor supply function, $h(w, R)$.
- ▶ Differentiate h with respect to wages/taxes to get labor supply elasticity

$$\epsilon = \frac{\partial h}{\partial w} \frac{w}{h}$$

- ▶ "a 1% increase in wages (or decrease in taxes) will change hours by $\epsilon\%$."

Substitution effect and income effect

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 - ▶ decreases hours worked.
- ▶ Overall effect of wage increases (tax decrease) on hours worked:
 - ▶ could be positive or negative.

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- ▶ Individual optimization:

$$w(1 - T'(\cdot)) \frac{\partial u}{\partial c} = \frac{\partial u}{\partial h}$$

- ▶ marginal benefit of working equals marginal cost of working.

Government Problem

- ▶ Government chooses $T(wh)$ to maximize:

$$\sum_w G(u(wh - T(wh), h))$$

- ▶ $G(\cdot)$ is increasing and concave
- ▶ summation is over all individuals, indexed by w

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- ▶ $G(\cdot)$ is increasing and concave
- ▶ summation is over all individuals, indexed by w
- ▶ Government budget constraint:

$$\sum_w T(wh) = E$$

- ▶ Incentive compatibility constraint (taxpayer will optimize):

$$w(1 - T'(\cdot)) \frac{\partial u}{\partial c} = \frac{\partial u}{\partial h}$$

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- ▶ With equal weights and diminishing marginal utility, we would equalize everyone's income
 - ▶ Utilitarianism leads to communism!
- ▶ Is maximizing total ex-post utility the right objective function?
 - ▶ Deep debate dating back to Rawls, Nozick, Sen...

Main Results

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 2. $T'(\cdot) \geq 0$: Non-trivial. Rules out wage subsidies.
 3. $T'(\cdot) = 0$ at the bottom of the skill distribution (assuming everyone works)
 4. $T'(\cdot) = 0$ at the top of the skill distribution (if skill distribution is bounded)

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 - ▶ Models with asymmetric information
- ▶ But little impact on **practical tax policy**
- ▶ Recently, connected to empirical tax literature:
 - ▶ Diamond (AER, 1998), Saez (REStud, 2001)
 - ▶ Sufficient statistic formulas in terms of elasticities

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Laffer Curve: Revenue Maximizing Rate

- ▶ Useful benchmark for optimal rate
- ▶ Let tax revenue be $R(\tau) = \tau \cdot z((1 - \tau))$
 - ▶ Notice: reported income $z(\cdot)$ is a *function* of net-of-tax rate $(1 - \tau)$

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- ▶ Useful benchmark for optimal rate
- ▶ Let tax revenue be $R(\tau) = \tau \cdot z((1 - \tau))$
 - ▶ Notice: reported income $z(\cdot)$ is a *function* of net-of-tax rate $(1 - \tau)$
- ▶ $R(\tau)$ has an inverse-U shape:
 - ▶ No taxes: $R(\tau = 0) = 0$
 - ▶ Confiscatory taxes: $R(\tau = 1) = 0$

Laffer Curve: Revenue Maximizing Rate

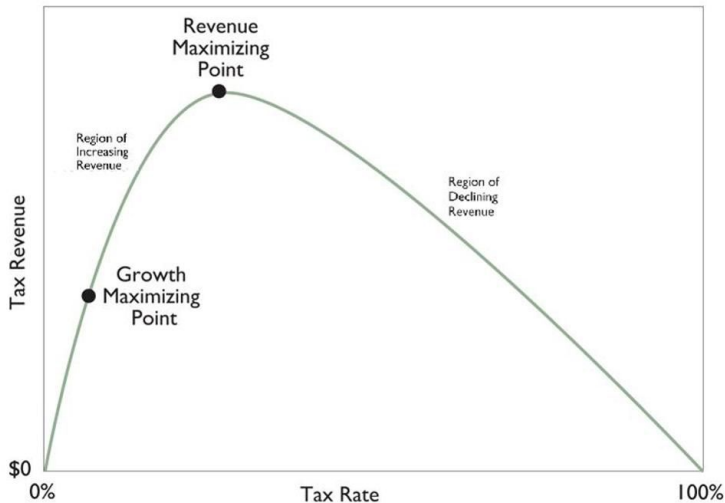
- Revenue maximizing rate, τ^* :

$$\begin{aligned}R'(\tau^*) &= 0 \\z - \tau \frac{\partial z}{\partial (1 - \tau)} &= 0 \\z \left[\frac{(1 - \tau)}{z} \right] - \tau \underbrace{\frac{\partial z}{\partial (1 - \tau)} \left[\frac{(1 - \tau)}{z} \right]}_{\varepsilon} &= 0 \\1 - \tau - \tau \varepsilon &= 0 \\\Rightarrow \tau^* &= \frac{1}{1 + \varepsilon}\end{aligned}$$

- **Strictly** inefficient to have $\tau > \tau^*$ (Why?)

Laffer Curve: Revenue Maximizing Rate

The Laffer Curve



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Using Elasticities to Derive Optimal Tax Rates

- ▶ Saez (2001) derives optimal tax rate τ using “perturbation” argument
- ▶ Assumptions:
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 - ▶ N individuals above earnings z^*
 - ▶ Let $z^m((1 - \tau))$ be average income function of these individuals

Effect of increasing top-brakcey tax rate

- ▶ Three effects of small $\Delta\tau > 0$ reform above z^* :

1. **Mechanical increase** in tax revenue:

$$\Delta M = N \cdot [z^m - z^*] \Delta\tau$$

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2. **Behavioural response**:

$$\begin{aligned} \Delta B = N_{\tau} \Delta z^m &= N_{\tau} \left(-\Delta\tau \frac{\Delta z^m}{\Delta(1-\tau)} \right) \\ &= -N \frac{\tau}{1-\tau} \bar{\varepsilon} z^m \Delta\tau \end{aligned}$$

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3. **Welfare effect**:

$$\Delta W = -\bar{g}\Delta M$$

where $\bar{g} \in [0, 1]$ is government value on rich consumption (relative to value of government expenditure, or value of lump sum transfers to everyone else)

Optimal Income Tax Rate

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$$\frac{\tau^{*top}}{1 - \tau^{*top}} = \frac{(1 - \bar{g}) \left[\frac{z^m}{z^*} - 1 \right]}{\bar{\varepsilon} \left(\frac{z^m}{z^*} \right)}$$

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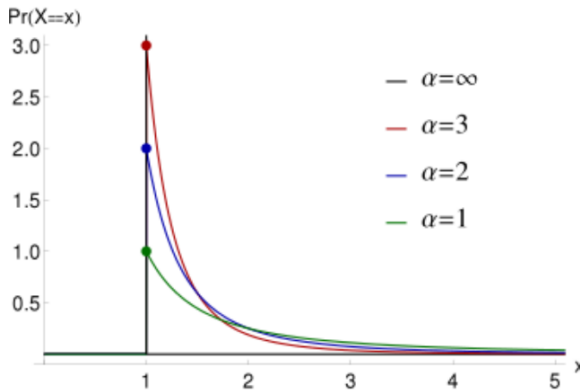
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 - ▶ $\downarrow \bar{g}$: less weight on welfare of the rich
 - ▶ $\downarrow \bar{\varepsilon}$: lower elasticity of taxable income
 - ▶ $\uparrow \frac{z^m}{z^*}$: higher income inequality

Pareto Distribution



- ▶ Assume income follows a Pareto distribution with parameter a
 - ▶ Then $\frac{z^m}{z^*}$ is approximated by $\left(\frac{a}{a-1}\right)$

Saez (2001): Optimal Income Tax Rate

- Simplified formula:

$$\tau^{*top} = \frac{1 - \bar{g}}{1 - \bar{g} + \varepsilon a}$$

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- ▶ Simplified formula:

$$\tau^{*top} = \frac{1 - \bar{g}}{1 - \bar{g} + \varepsilon a}$$

- ▶ In the United States, $\frac{z^m}{z^*} \approx 3$ (average income of rich is three times the top tax bracket threshold)
 - ▶ pareto parameter given by $3 = \frac{a}{a-1} \Rightarrow a = 1.5$

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Saez (2001): Optimal Income Tax Rate

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- ▶ Society decides value of \bar{g} (relative weight of rich on SWF)

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- ▶ Assuming $a = 1.5$:

	$\varepsilon = 0.2$	$\varepsilon = 0.5$	$\varepsilon = 1$
$\bar{g} = 0$	0.77	0.57	0.40
$\bar{g} = 0.5$	0.62	0.40	0.25