Fiscal Policy and Inequality

13. Tax Inefficiencies & Optimal taxation

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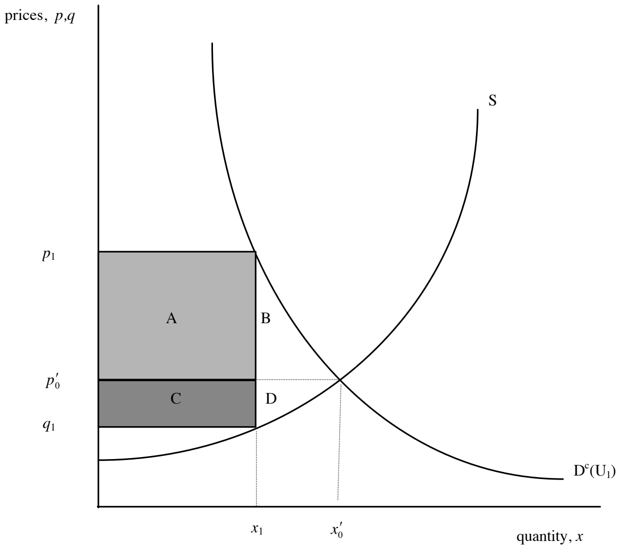
Distortionary Effects of Taxation

- The introduction of a tax affects the behavior of economic agents, moving them *away* from the existing equilibrium
  - Hence, we say that taxes are **distortionary**
- **Deadweight loss from (or excess burden) of taxation:**
  - If we introduce a tax, and then redistribute all the revenue, how much economic output is lost?
- While incidence analysis focuses on **prices**, efficiency analysis focuses on **quantities**

Deadweight Loss (DWL)



Simple example: setup

- Consider a two-good, one-consumer, one-firm economy
- Consumer's utility function is $U = u(x_1, x_2) = x_1 x_2$
- We only care about price ratios, so set $P_1 = 1$ and $P_2 = c > 0$
- Let the consumer have income $Y = 1$

Background: Cobb-Douglas utility

- Cobb-Douglas utility, general form: $u(x, z) = x^a z^b$
- Marginal rate of substitution: $MRS_{xz} = \frac{\partial u/\partial x}{\partial u/\partial z} = \frac{az}{bx}$
- Set MRS equal to price ratio to get optimal consumption: $z = \left(\frac{b}{a}\right)\left(\frac{p_x}{p_z}\right)x$
- Budget constraint: $Y = p_x x + p_z z$
- Substitute optimal z into budget constraint:

$$x^* = \left(\frac{a}{a+b}\right)\frac{Y}{p_x}\ ,\ z^* = \left(\frac{b}{a+b}\right)\frac{Y}{p_z}$$

Simple example: solution

$$u\left(x_1, x_2\right) = x_1 x_2, P_1 = 1, P_2 = c, Y = 1$$

- No taxation:
  - Consumption levels: $x_1 = \frac{1}{2}$; $x_2 = \frac{1}{2c}$
  - Consumer's utility: $U \equiv u\left(\frac{1}{2}, \frac{1}{2c}\right) = \frac{1}{4c}$
- With a tax t on good 2, $P_2' = c + t$. and new budget constraint after revenue returned as lump-sum transfer T:

$$x_1 + (c + t)x_2 \leq T + 1$$

  - Consumption levels (demand): $x_1' = \frac{1+T}{2}$; $x_2' = \frac{1+T}{2(c+t)}$

Simple example: Welfare loss

- In equilibrium: $T = tx_2'$. Therefore: $T = \frac{t}{2c+t}$
- Substitute into demands: $x_1' = \frac{t+c}{2c+t}$; $x_2' = \frac{1}{2c+t}$
- Overall consumer's utility: $U^t = \left(\frac{c+t}{2c+t}\right)\left(\frac{1}{2c+t}\right)$
- Difference in utility = **deadweight loss** (DWL):

$$\text{DWL} = U - U^t = \frac{t^2}{4c\,(2c+t)^2} > 0, \forall t > 0$$

- Note that

$$\frac{\partial \text{DWL}}{\partial t} = \frac{t}{(2c+t)^3} > 0, \forall t > 0$$

  - increasing taxes has a small but accelerating positive effect on on deadweight loss.

Optimal Taxation

- "Optimal" tax theory must combine lessons from deadweight loss and tax incidence:
  - Optimal size of the pie? - **Efficiency**
  - How is the pie distributed? - **Equity**
- What is the best way to design taxes given equity and efficiency concerns?

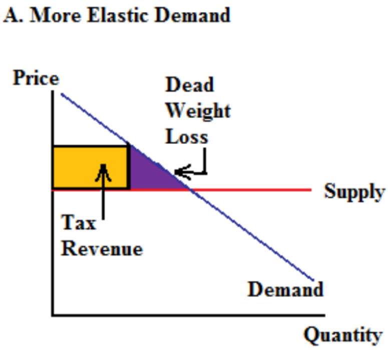
Optimal Taxation

- **Efficiency perspective**: finance the govt through lump-sum taxation
  - Fixed amount per person regardless of characteristics or actions
  - Does not induce any behavioural responses, so this is most efficient tax available
- **Equity perspective**: individualized lump-sum taxes
  - Tax higher-ability (higher earning power) individuals with larger lump sum
  - **Problem**: we cannot observe ability directly
    - Hence we tax outcomes, such as income or consumption
    - Creates distortions & inefficiency

Ramsey (1927): Tax Problem

1. No lump-sum or nonlinear taxes - only proportional tax τ_i on each good i
2. Raise total revenue $R = \sum_{i=1}^{n} \tau_i x_i$ to match fixed expenditures E
3. Maximize utility for agents – all agents identical with utility $u(x_0, x_1, ..., x_n)$
   - no redistributive concerns.
4. Cannot tax all commodities: x_0(leisure) untaxed

Ramsey (1927): Elastic vs Inelastic Demand



- **Intuition:** tax inelastic goods to minimize efficiency costs

Ramsey (1927): Inverse Elasticity Rule

- The (simplified) Ramsey tax formula for the linear tax on good i is
$$\tau_i = \frac{\lambda}{\epsilon_i}$$
where λ is a parameter summarizing the value of government spending, and ϵ_i is the elasticity of demand for good i.
    - Low $\epsilon_i \rightarrow$ high τ_i
    - Taxes on all goods, unless perfectly elastic demand
    - The ratio of the taxes on two goods is the inverse ratio of their demand elasticities:
$$\frac{\tau_i}{\tau_j} = \frac{\epsilon_j}{\epsilon_i}$$

Ramsey (1927): Limitations

- Restricted to linear taxes
- Does not take into account redistributive motives
    - Necessities usually more inelastic than luxuries
    - Thus, optimal Ramsey tax system is regressive

Taxes and Labor Supply

- In April 2010, the British government raised the top income tax rate from 40% to 50%
- Assuming that taxable income had remained constant after the reform, tax revenues would have increased by **6.8 billion** pounds (0.46% of GDP)
- Before the reform, the Treasury projected this reform to increase tax revenues by
  - £8.5 billion?
  - £6.8 billion?
  - £2.7 billion?
  - £0.68 billion?

Labor supply model

- The individual solves

$$\max_{\{c,h\}} u\left(c, h\right)$$

  subject to $c = wh + R$

  - c = consumption
  - h = hours worked (supply of labor)
  - $u\left(c, h\right)$ = utility function, with $u_c > 0$ and $u_h < 0$
    - people like consumption but don't like working
  - w = **after-tax** hourly wage
  - R = non-labor income

Labor supply elasticity

- Substituting for consumption:

$$\max_h u\left(wh + R, h\right)$$

- Solve with FOC's to get labor supply function, $h\left(w, R\right)$.
- Differentiate h with respect to wages/taxes to get labor supply elasticity

$$\varepsilon = \frac{\partial h}{\partial w}\frac{w}{h}$$

  - "a 1% increase in wages (or decrease in taxes) will change hours by ϵ%."

Substitution effect and income effect

- Substitution effect:
  - increasing wages makes each hour of work more valuable in terms of consumption.
  - increases hours worked.
- Income effect:
  - increasing wages gives me a higher overall income; I am now richer and therefore don't need to work as much.
  - decreases hours worked.
- Overall effect of wage increases (tax decrease) on hours worked:
  - could be positive or negative.

Individual Problem

- Individual solves

$$\max_{c,h} u\left(c, h\right)$$

subject to $c = wh - T\left(wh\right)$

  - $T(wh)$ is the tax imposed on person with wage w.

- Individual optimization:

$$w\left(1 - T'(\cdot)\right)\frac{\partial u}{\partial c} = \frac{\partial u}{\partial h}$$

  - marginal benefit of working equals marginal cost of working.

Government Problem

- Government chooses $T(wh)$ to maximize:

$$\sum_w G(u(wh - T(wh), h)$$

  - $G(\cdot)$ is increasing and concave
  - summation is over all individuals, indexed by w

- Government budget constraint:

$$\sum_w T(wh) = E$$

- Incentive compatibility constraint (taxpayer will optimize):

$$w\left(1 - T'(\cdot)\right)\frac{\partial u}{\partial c} = \frac{\partial u}{\partial h}$$

Overview

- Govt maximizes ***weighted*** sum of utilities of ex-post consumption
- With equal weights and diminishing marginal utility, we would equalize everyone's income
  - Utilitarianism leads to communism!
- Is maximizing total ex-post utility the right objective function?
  - Deep debate dating back to Rawls, Nozick, Sen...

Main Results

- Mirrlees formulas are complicated, only a few general results:
  1. $T'(\cdot) \leq 1$: Obvious, because otherwise no one works
  2. $T'(\cdot) \geq 0$: Non-trivial. Rules out wage subsidies.
  3. $T'(\cdot) = 0$ at the bottom of the skill distribution (assuming everyone works)
  4. $T'(\cdot) = 0$ at the top of the skill distribution (if skill distribution is bounded)

Mirrlees (1971): Results

- Mirrlees model had big impact in fields like contract theory
  - Models with asymmetric information
- But little impact on **practical tax policy**
- Recently, connected to empirical tax literature:
  - Diamond (AER, 1998), Saez (REStud, 2001)
  - Sufficient statistic formulas in terms of elasticities

Laffer Curve: Revenue Maximizing Rate

- Useful benchmark for optimal rate
- Let tax revenue be $R(\tau) = \tau \cdot z((1-\tau))$
  - Notice: reported income $z(\cdot)$ is a *function* of net-of-tax rate $(1-\tau)$
- $R(\tau)$ has an inverse-U shape:
  - No taxes: $R(\tau = 0) = 0$
  - Confiscatory taxes: $R(\tau = 1) = 0$

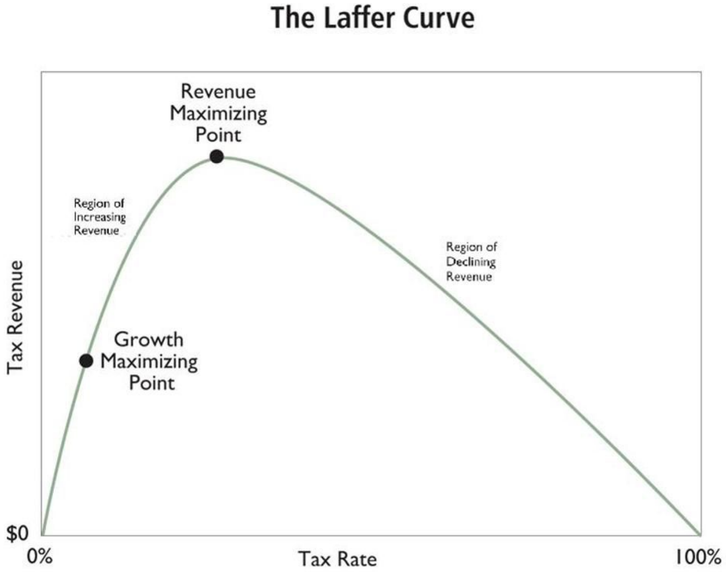
Laffer Curve: Revenue Maximizing Rate

- Revenue maximizing rate, τ^*:

$$\begin{aligned}
R'(\tau^*) &= 0 \\
z - \tau \frac{\partial z}{\partial (1-\tau)} &= 0 \\
z \left[\frac{(1-\tau)}{z} \right] - \tau \underbrace{\frac{\partial z}{\partial (1-\tau)} \left[\frac{(1-\tau)}{z} \right]}_{\varepsilon} &= 0 \\
1 - \tau - \tau\varepsilon &= 0 \\
\Rightarrow \tau^* &= \frac{1}{1+\varepsilon}
\end{aligned}$$

- **Strictly** inefficient to have $\tau > \tau^*$ (Why?)

Laffer Curve: Revenue Maximizing Rate



Using Elasticities to Derive Optimal Tax Rates

- Saez (2001) derives optimal tax rate τ using "perturbation" argument
- Assumptions:
  - no income effects on labor elasticity
    - Diamond (1998) shows this is a key theoretical simplification
  - N individuals above earnings z^*
    - Let $z^m((1-\tau))$ be average income function of these individuals

Effect of increasing top-brakcey tax rate

- Three effects of small $\Delta\tau > 0$ reform above z^*:
  1. **Mechanical increase** in tax revenue:

$$\Delta M = N \cdot [z^m - z^*] \Delta\tau$$

  2. **Behavioural response**:

$$\begin{aligned} \Delta B = N\tau\Delta z^m &= N\tau \left(-\Delta\tau \frac{\Delta z^m}{\Delta(1-\tau)} \right) \\ &= -N \frac{\tau}{1-\tau} \bar{\varepsilon} z^m \Delta\tau \end{aligned}$$

  3. **Welfare effect**:

$$\Delta W = -\bar{g} \Delta M$$

where $\bar{g} \in [0, 1]$ is government value on rich consumption (relative to value of government expenditure, or value of lump sum transfers to everyone else)

Optimal Income Tax Rate

- Optimal tax rate equalizes marginal gains and losses:

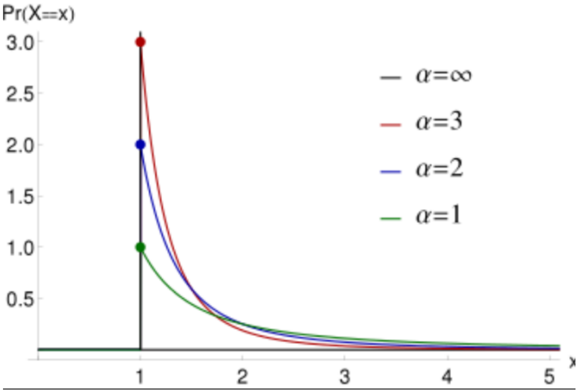
$$\Delta M + \Delta W + \Delta B = 0$$

- After some algebra:

$$\frac{\tau^{*top}}{1-\tau^{*top}} = \frac{(1-\bar{g})\left[\frac{z^m}{z^*}-1\right]}{\bar{\varepsilon}\left(\frac{z^m}{z^*}\right)}$$

  - Top tax rate τ^{*top} is higher when:
    - $\downarrow \bar{g}$: less weight on welfare of the rich
    - $\downarrow \bar{\varepsilon}$: lower elasticity of taxable income
    - $\uparrow \frac{z^m}{z^*}$: higher income inequality

Pareto Distribution



- Assume income follows a Pareto distribution with parameter a
  - Then $\frac{z^m}{z^*}$ is approximated by $\left(\frac{a}{a-1}\right)$

Saez (2001): Optimal Income Tax Rate

- Simplified formula:

$$\boxed{\tau^{*top} = \frac{1-\bar{g}}{1-\bar{g}+\varepsilon a}}$$

  - In the United States, $\frac{z^m}{z^*} \approx 3$ (average income of rich is three times the top tax bracket threshold)
    - pareto parameter given by $3 = \frac{a}{a-1} \Rightarrow a = 1.5$
  - We can estimate ε (next lecture)
  - Society decides value of $\bar{g}$ (relative weight of rich on SWF)

Connection to Revenue Maximizing Tax Rate

- How to set $\bar{g}$?
  - Revenue-maximizing top tax rate can be calculated by setting $\bar{g} = 0$
  - Rawlsian social welfare function: $\bar{g} = 0$ for any $z^* > \min(z)$
  - Utilitarian social welfare function: $\bar{g} = u_c(z^m) \to 0$ when $z^* \to \infty$
- If $\bar{g} = 0$, we obtain $\tau^{*top} = \tau^{max} = \frac{1}{1+\bar{\varepsilon}a}$
- Assuming $a = 1.5$:

	$\varepsilon = 0.2$	$\varepsilon = 0.5$	$\varepsilon = 1$
$\bar{g} = 0$	0.77	0.57	0.40
$\bar{g} = 0.5$	0.62	0.40	0.25