Sequencing Legal DNA

NLP for Law and Political Economy

5. Neural Nets and Word Embeddings

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  - $\boldsymbol{x}_i$ is itself a compressed representation of the unprocessed document $\mathcal{D}_i$.
- Correspondingly: the parameters $\hat{\theta}$ can also be understood as a compressed (or "learned") representation:
  - it contains information about the training corpus, the text features, and the outcomes.

Information in $\hat{\theta}$

- Say we train a multinomial logistic regression on a bag-of-words representation of the documents.
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  - It contains n_y columns $= n_x$-vectors representing the outcome classes.

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  - It contains n_y columns $= n_x$-vectors representing the outcome classes.
  - It contains n_x rows $= n_y$-vectors representing each word in the vocabulary.
- How to use this?
  - could cluster column vectors to understand which outcomes are similar/related.
  - could cluster row vectors to understand which features are similar/related.

Preview of Word Embeddings

- Let's say $\boldsymbol{x}_i$ is a bag-of-words representation for document i with length n_i. We can write

$$\boldsymbol{x}_i = \frac{1}{n_i} \sum_{l=1}^{n_i} \boldsymbol{x}_i^{[l]}$$

  - l indexes words in the the document
  - each vector $\boldsymbol{x}_i^{[l]}$ is an n_x-dimensional one-hot vector – all entries are zero except the single entry corresponding to the word at l, which is 1.

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- Now let $\theta^{[l]}$ be the row of θ corresponding to the word w_l. We can write

$$\hat{\boldsymbol{y}}_i = \frac{1}{n_i} \sum_{l=1}^{n_i} \theta^{[l]}$$

  the sum of the n_y-dimensional word representations (the row vectors from above).

  - this is called the "continuous bag of words (CBOW)" representation.
  - θ is a word embedding matrix.

Outline

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  - nothing like brains
- “Networks”:
  - nothing to do with “networks” as normally understood – in particular, nothing to do with network theory in math or social science.

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- NNs frequently outperform other ML techniques on large and complex problems.

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- Increase in computing power makes them computationally tractable, graphical processing units (GPUs, designed for video games) give you over 100x performance gain over CPUs.
- Training algorithms have improved – small tweaks have made a huge impact.
- Some theoretical limitations of NNs have turned out to be benign in practice – for example, they work well on non-convex functions.

Will it last?

- Three key principles of deep learning will persist:
  - **Simplicity**
    - feature engineering is obsolete
    - complex, brittle, engineering-heavy pipelines replaced with simple, end-to-end trainable models, composed of 5-6 tensor operations.

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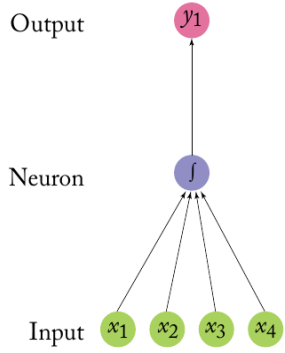
  - **Scalability**
    - amenable to parallelization on GPUs or TPUs (tensor processing units)
    - trained on batches of data, so can be scaled to datasets of arbitrary size.
  - **Versatility and reusability**
    - can be trained on additional data without restarting from scratch, therefore amenable for continuous online learning.
    - deep-learning models are repurposable and thus reusable

A "Neuron"



- A neuron multiplies each input by its weight, sums them, applies a non-linear function to the result, and passes the output.
  - e.g., the $\int$ shape indicates a sigmoid transformation.

In Notation

- The simplest neural network is called a perceptron:

$$\text{MLP0}(\boldsymbol{x}) = \boldsymbol{x} \cdot \boldsymbol{\omega}$$

$$\boldsymbol{x} \in \mathbb{R}^{n_x}, \boldsymbol{\omega} \in \mathbb{R}^{n_x \times n_y}$$

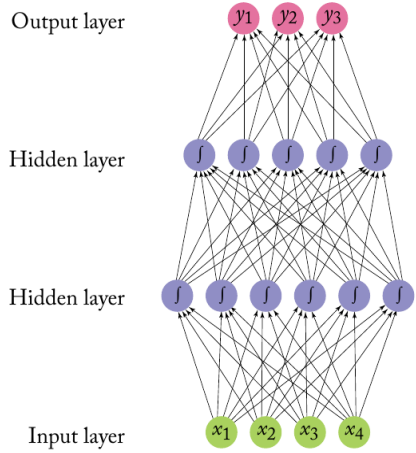
  here, $\boldsymbol{\omega}$ is the matrix of weights in the layer.

- In more standard notation, there would be an additional constant (or "bias") term:

$$\text{MLP0}(\boldsymbol{x}) = \alpha + \boldsymbol{x} \cdot \boldsymbol{\omega}$$

  - We leave it out by assuming that $\boldsymbol{x}$ is de-meaned or has an extra column of ones.

A Feed-Forward Neural Network



- A feed-forward network is simply a stack of linear models, separated by non-linear functions.

Multi-Layer Perceptron

- An multi-layer perceptron (MLP) with one hidden layer is

$$\text{MLP1}(\boldsymbol{x}) = \boldsymbol{g}(\boldsymbol{x} \cdot \boldsymbol{\omega}_1) \cdot \boldsymbol{\omega}_2$$

$$\boldsymbol{x} \in \mathbb{R}^{n_x}, \boldsymbol{\omega}_1 \in \mathbb{R}^{n_x \times n_1}, \boldsymbol{\omega}_2 \in \mathbb{R}^{n_1 \times n_y},$$

  - n_1 = dimensionality in first (and only) hidden layer
  - $\boldsymbol{\omega}_1$ = set of learnable weights for the first linear transformation of the inputs.
  - $\boldsymbol{g}(\cdot)$ = an element-wise non-linear function (an "activation function")
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  - $\boldsymbol{g}(\cdot)$ = an element-wise non-linear function (an "activation function")
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- MLP1 can approximate any continuous function on a closed and bounded subset of $\mathbb{R}^n$, and any mapping from one finite discrete space to another finite discrete space (Hornik et al 1989, Cybenko 1989).
  - But MLP1 would have to be exponentially large in some cases (Telgarsky 2016) .

Two hidden layers

- Adding a second hidden layer gives

$$\text{MLP2}(\boldsymbol{x}) = \boldsymbol{g}_2(\boldsymbol{g}_1(\boldsymbol{x} \cdot \boldsymbol{\omega}_1) \cdot \boldsymbol{\omega}_2) \cdot \boldsymbol{\omega}_3$$

$$\boldsymbol{x} \in \mathbb{R}^{n_x}, \boldsymbol{\omega}_1 \in \mathbb{R}^{n_x \times n_1}, \boldsymbol{\omega}_2 \in \mathbb{R}^{n_1 \times n_2}, \boldsymbol{\omega}_3 \in \mathbb{R}^{n_2 \times n_y}$$

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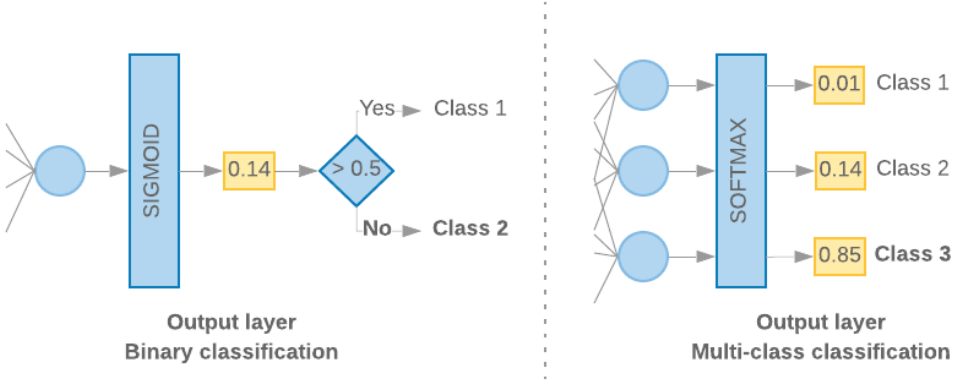
  - n_2= number of neurons in second hidden layer.
- MLP2 can be written in the following decomposed notation:

$$\begin{aligned} \text{MLP2}(\boldsymbol{x}) = & \\ \boldsymbol{h}_1 &= \boldsymbol{g}_1(\boldsymbol{x} \cdot \boldsymbol{\omega}_1) \\ \boldsymbol{h}_2 &= \boldsymbol{g}_2(\boldsymbol{h}_1 \cdot \boldsymbol{\omega}_2) \\ \boldsymbol{y} &= \boldsymbol{h}_2 \cdot \boldsymbol{\omega}_3 \end{aligned}$$

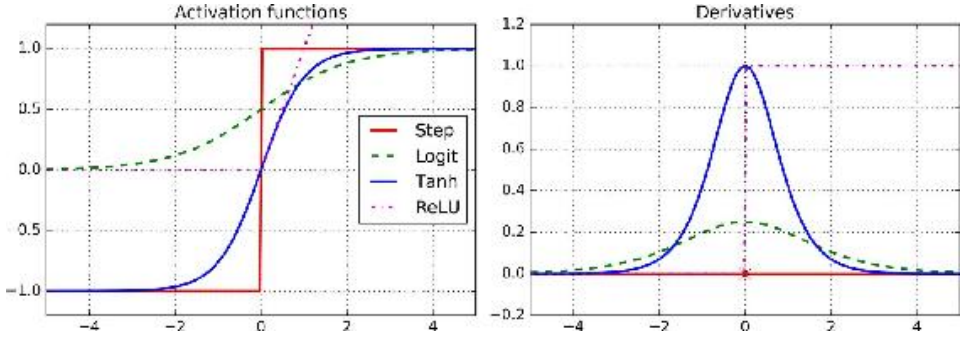
where $\boldsymbol{h}_l$ give hidden layers.

Constructing the Last Layer



- MLPs will output a probability distribution across output classes.
  - can also output a real number, which would make a regression model.

What to pick for $g(\,)$



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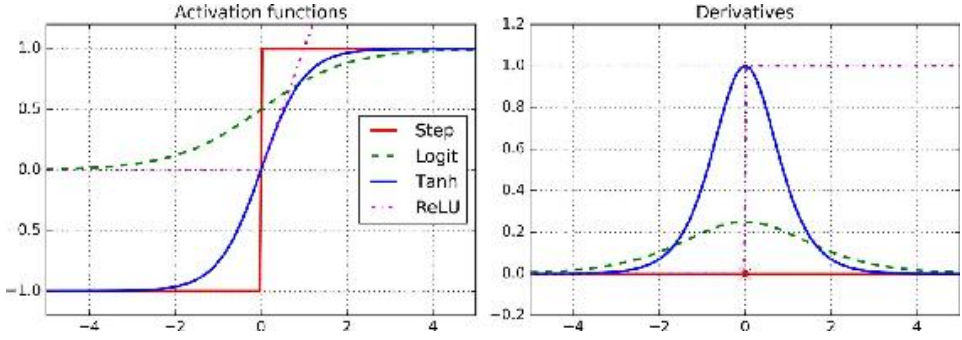

- logistic function: $\text{logit}(z) = \frac{1}{1+\exp(-z)}$
- hyperbolic tangent function: $\tanh(z) = 2\sigma(2z) - 1$
  - ranges between -1 and 1 (rather than between 0 and 1, as the case with the logistic)
  - centered on zero, can speed up convergence

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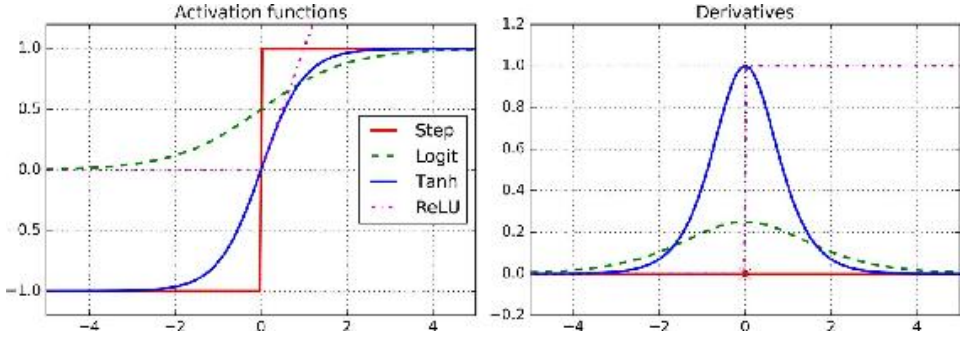

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  - ranges between -1 and 1 (rather than between 0 and 1, as the case with the logistic)
  - centered on zero, can speed up convergence
- ReLU (rectified linear unit) function: $\max\{0, z\}$,
  - deceptively simple, fast to compute, and very effective in practice
  - gradient does not saturate to zero for large values (but is flat below zero)

Backpropagation

- A crucial technology in deep learning is *backpropagation*, also known as *reverse-mode automatic differentiation (autodiff)*.
  - intuitively, computes the network's output error and how much each neuron contributes to the error, so they can be updated to reduce error.

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- A crucial technology in deep learning is *backpropagation*, also known as *reverse-mode automatic differentiation (autodiff)*.
  - intuitively, computes the network's output error and how much each neuron contributes to the error, so they can be updated to reduce error.
  - Keras/TensorFlow will do this under the hood, just like how scikit-learn does gradient descent.
  - Appendix D of the Geron book has a decent explanation for how it works.

Outline

MLP baseline for Text Classification

Google Developers Advice

1. Calculate the number of samples/number of words per sample ratio.
2. If this ratio is less than 1500, tokenize the text as n-grams and use a simple multi-layer perceptron (MLP) model to classify them.

- In the case of N-grams models, Google testers found that MLPs tended to out-perform logistic regression and gradient boosting machines.
- A simple MLP is one of the models tried by Peterson and Spirling (2018).

Keras Basics

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- "Dense" layer is the DNN baseline – means that all neurons are connected.
- Output layer:
  - for regression, do not use an activation function
  - for binary classification, use `activation='sigmoid'`
  - for multi-class classification, use `activation='softmax'`

Loss function and metrics

- Loss function:
  - for regression, use `mean_squared_error`
  - for binary classification, use `binary_crossentropy`
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  - for regression, use `mean_squared_error`
  - for binary classification, use `binary_crossentropy`
  - for multi-class classification, use `sparse_categorical_crossentropy`
- Metrics:
  - for classification, can use accuracy and F_1
  - for regression, use R^2

Table 10-1. Typical regression MLP architecture

Hyperparameter	Typical value
# input neurons	One per input feature (e.g., 28 x 28 = 784 for MNIST)
# hidden layers	Depends on the problem, but typically 1 to 5
# neurons per hidden layer	Depends on the problem, but typically 10 to 100
# output neurons	1 per prediction dimension
Hidden activation	ReLU (or SELU, see Chapter 11)
Output activation	None, or ReLU/softplus (if positive outputs) or logistic/tanh (if bounded outputs)
Loss function	MSE or MAE/Huber (if outliers)

Table 10-2. Typical classification MLP architecture

Hyperparameter	Binary classification	Multilabel binary classification	Multiclass classification
Input and hidden layers	Same as regression	Same as regression	Same as regression
# output neurons	1	1 per label	1 per class
Output layer activation	Logistic	Logistic	Softmax
Loss function	Cross entropy	Cross entropy	Cross entropy

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- Standard practice is to use "Glorot" or "He" initialization:

Activation function	Uniform distribution [–r, r]	Normal distribution
Logistic	$r = \sqrt{\frac{6}{n_{\text{inputs}} + n_{\text{outputs}}}}$	$\sigma = \sqrt{\frac{2}{n_{\text{inputs}} + n_{\text{outputs}}}}$
Hyperbolic tangent	$r = 4\sqrt{\frac{6}{n_{\text{inputs}} + n_{\text{outputs}}}}$	$\sigma = 4\sqrt{\frac{2}{n_{\text{inputs}} + n_{\text{outputs}}}}$
ReLU (and its variants)	$r = \sqrt{2}\sqrt{\frac{6}{n_{\text{inputs}} + n_{\text{outputs}}}}$	$\sigma = \sqrt{2}\sqrt{\frac{2}{n_{\text{inputs}} + n_{\text{outputs}}}}$

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  - ReLU is a good baseline in the hidden layers.
- See Geron, pp. 322-323 for tools to help with tuning, e.g. Keras Tuner.
  - See also Smith (2018).

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Batch normalization

- Another trick to increase performance, speed up training, and regularize the model:
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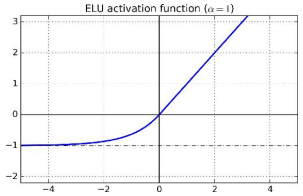
- Another trick to increase performance, speed up training, and regularize the model:
  - in between layers, zero-center and normalize the inputs to variance one.
  - normally done before a non-linear activation function
- Does not work well in recurrent neural nets – can use gradient clipping (hard constraint on gradient) instead.

ELU and SELU

Exponential linear unit

$$\text{ELU}(z) = \begin{cases} \alpha(\exp(z) - 1) & z < 0 \\ z & z \geq 0 \end{cases}$$



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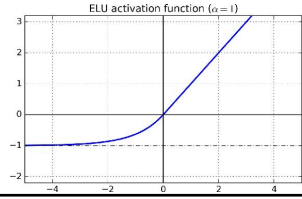

SELU (Scaled ELU):

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- Requires standardized inputs.
- Gaussian initialization of neuron weights:
  - mean zero and standard deviation = $n_x^{-1/2}$
- Set $\lambda \approx 1.0507, \alpha \approx 1.6732$.

- $\rightarrow$Then layers will be self-normalizing; weights will converge to mean zero and variance once (Klambauer et al 2017)

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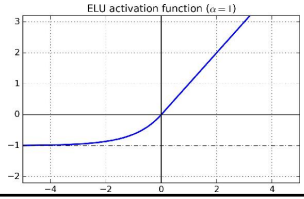

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- In general, SELU $>$ ELU $>$ ReLU.

Optimizer and Learning Rate

- Choice of optimization algorithm is the topic of active research, which has shown that it can have a big impact on model performance.
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  - See Geron, pp. 351-358. He recommends SGD with momentum, RMSProp, or Nadam.
- On the learning rate, see Geron pp. 359-364, recommending 1cycle scheduling:
  - start at η_0, increase linearly up to η_1 halfway through training, then decrease linearly to η_0 at the end.

Regularization for Sparse Models

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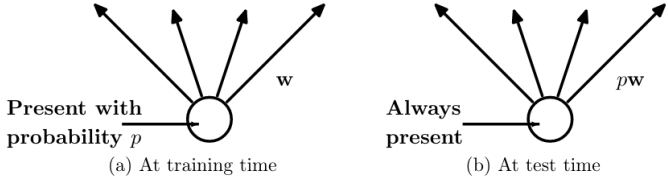
- But usually its better/simpler to use dropout.

Dropout



(a) At training time (b) At test time

Figure 2: **Left**: A unit at training time that is present with probability p and is connected to units in the next layer with weights **w**. **Right**: At test time, the unit is always present and the weights are multiplied by p. The output at test time is same as the expected output at training time.

Source: Srivastava et al, JMLR 2014

An elegant regularization technique:

- at every training step, every neuron has some probability (typically $p = 0.5$) of being temporarily dropped out, so that it will be ignored at this step.
- at test time, neurons dont get dropped anymore but coefficients are down-weighted by p.

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  - makes the model less sensitive to slight changes in the inputs.
- If using SELU activiation functions, use alpha dropout (Klambauer et al 2017).

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- Normal dropout:
  - at test time, neurons dont get dropped but coefficients are down-weighted by p.
- Monte Carlo dropout:
  - at test time, continue to allow dropout but produce 100 predictions, and average them.

Summary and Practical Guidelines

Table 11-3. Default DNN configuration

Hyperparameter	Default value
Kernel initializer	He initialization
Activation function	ELU
Normalization	None if shallow; Batch Norm if deep
Regularization	Early stopping (+ℓ_2 reg. if needed)
Optimizer	Momentum optimization (or RMSProp or Nadam)
Learning rate schedule	1cycle

Table 11-4. DNN configuration for a self-normalizing net

Hyperparameter	Default value
Kernel initializer	LeCun initialization
Activation function	SELU
Normalization	None (self-normalization)
Regularization	Alpha dropout if needed
Optimizer	Momentum optimization (or RMSProp or Nadam)
Learning rate schedule	1cycle

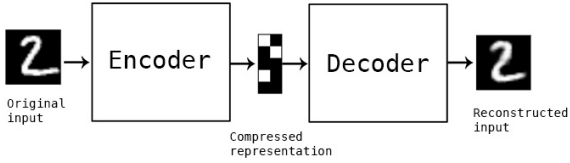
Outline

Efficient Data Representations

Which of the following number sequences do you find the easiest to memorize?

- 40, 27, 25, 36, 81, 57, 10, 73, 19, 68
- 50, 48, 46, 44, 42, 40, 38, 36, 34, 32, 30, 28, 26, 24, 22, 20, 18, 16, 14

Autoencoders $\leftrightarrow$ Optimal Compression Algorithms



- "Autoencoder" refers to a class of deep neural network that performs domain-specific compression and dimension reduction.
  - They learn efficient encodings of the data, which can then be decoded back to a *reconstruction* – a (minimally) lossy representation of the original data.
  - Can also randomly generate new data that look like the training data.

PCA is a linear autoencoder

```
from tensorflow import keras

encoder = keras.models.Sequential([keras.layers.Dense(2, input_shape=[3])])
decoder = keras.models.Sequential([keras.layers.Dense(3, input_shape=[2])])
autoencoder = keras.models.Sequential([encoder, decoder])

autoencoder.compile(loss="mse", optimizer=keras.optimizers.SGD(lr=0.1))
```

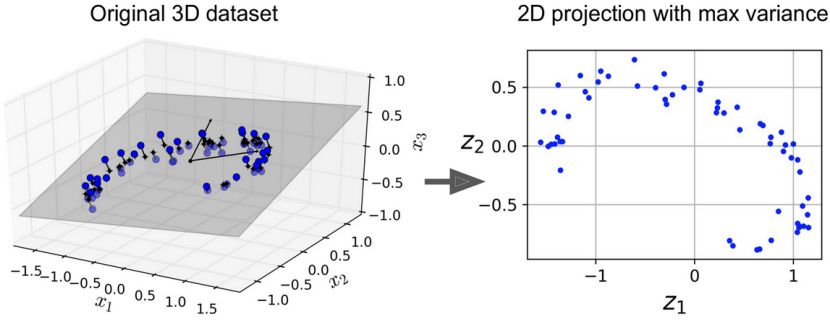


Figure 17-2. PCA performed by an undercomplete linear autoencoder

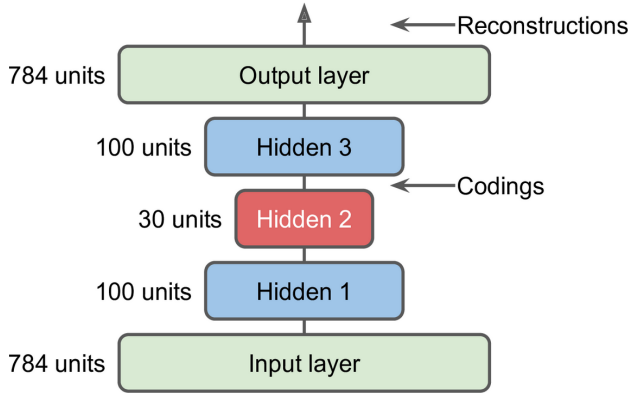
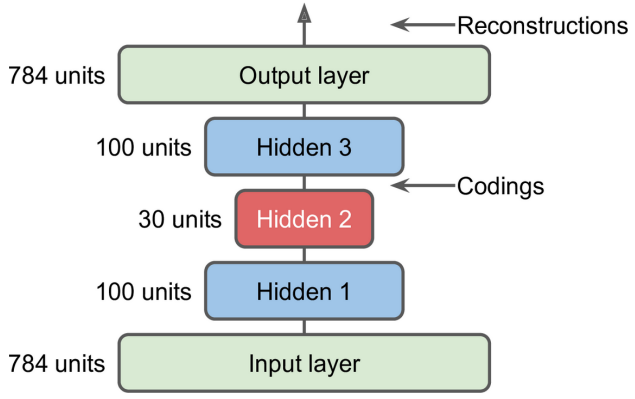


Figure 17-3. Stacked autoencoder

- Autoencoders work by stacking layers that gradually decrease in dimensionality to create the compressed representation (Z), and then gradually increase in dimensionality to try to reconstruct the input.
  - the autoencoder is implicitly solving the problem of maximizing entropy in the bottleneck layer.



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  - for symmetic autoencoders, **tying weights** of the encoding and decoding segments will speed up training and tends to improve performance.

Reconstruction from encoded vector

Figure 17-4. Original images (top) and their reconstructions (bottom)

Autoencoders for Data Visualization

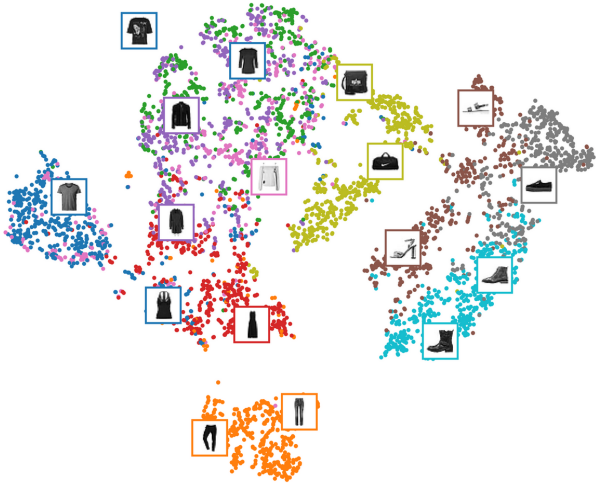

Figure 17-5. Fashion MNIST visualization using an autoencoder followed by t-SNE

- Decent baseline for visualizing the encodings:
  - use an autoencoder to compress your data to relatively low dimension (e.g. 32 dimensions)
  - then use t-SNE for mapping the compressed data to a 2D plane.

Denoising Autoencoders

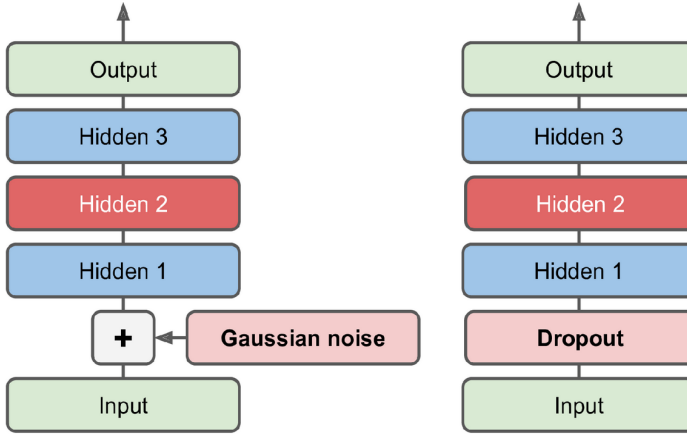

Figure 17-8. Denoising autoencoders, with Gaussian noise (left) or dropout (right)

Outline

What is an Embedding?

- A (relatively) low-dimensional vector representation of a categorical variable.
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- e.g., trying to predict how employment responds to economic growth with data from U.S. states:
  - instead of including a fifty-dimensional categorical variable, include two-dimensional latitude and longitude
  - or initialize each state to a random two-dimensional vector, and let the model decide where to move the states to improve prediction on your task (e.g.).

An embedding layer is just matrix multiplication

- An embedding layer can be represented as

$$x = v(w)$$

$$\underbrace{x}_{n_E \times 1} = \underbrace{E}_{n_E \times n_w} \cdot \underbrace{w}_{n_w \times 1}$$

  - w, a categorical variable (e.g., representing a word)
    - one-hot vector with a single item equaling one.
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    - The output of the embedding layer.
  - An embedding function $v(\cdot)$, defined by matrix E, learnable by the DNN

The Embedding Matrix E

- The model learns the weights of the embedding matrix in the same way that it would learn any model parameters.
- The rows of the matrix correspond to vectors for the n_w categories.
  - These are the "word vectors" that people talk about when they mention word embeddings or Word2Vec.

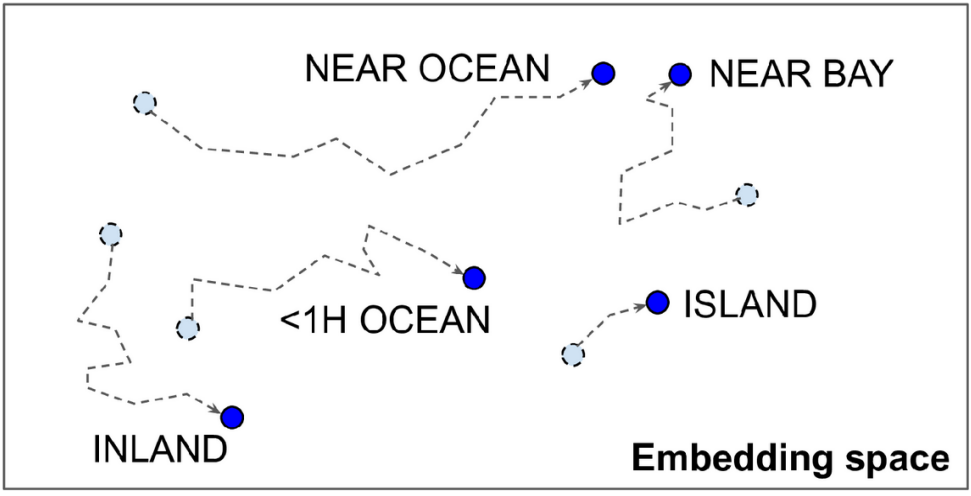
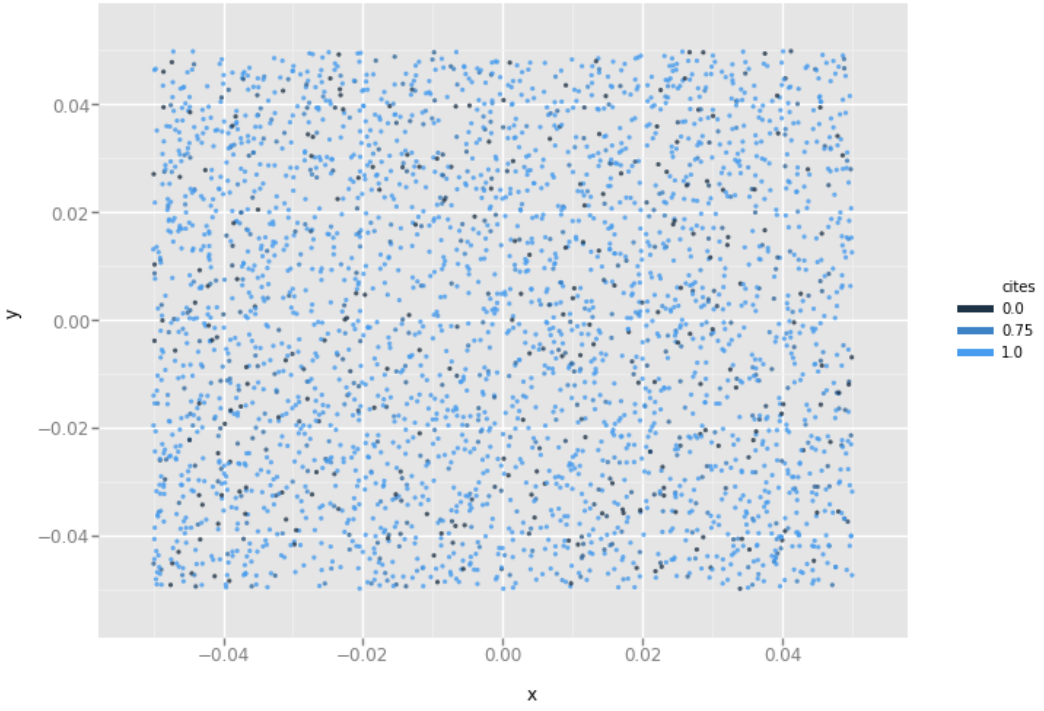


Figure 13-4. Embeddings will gradually improve during training


0.04
0.02
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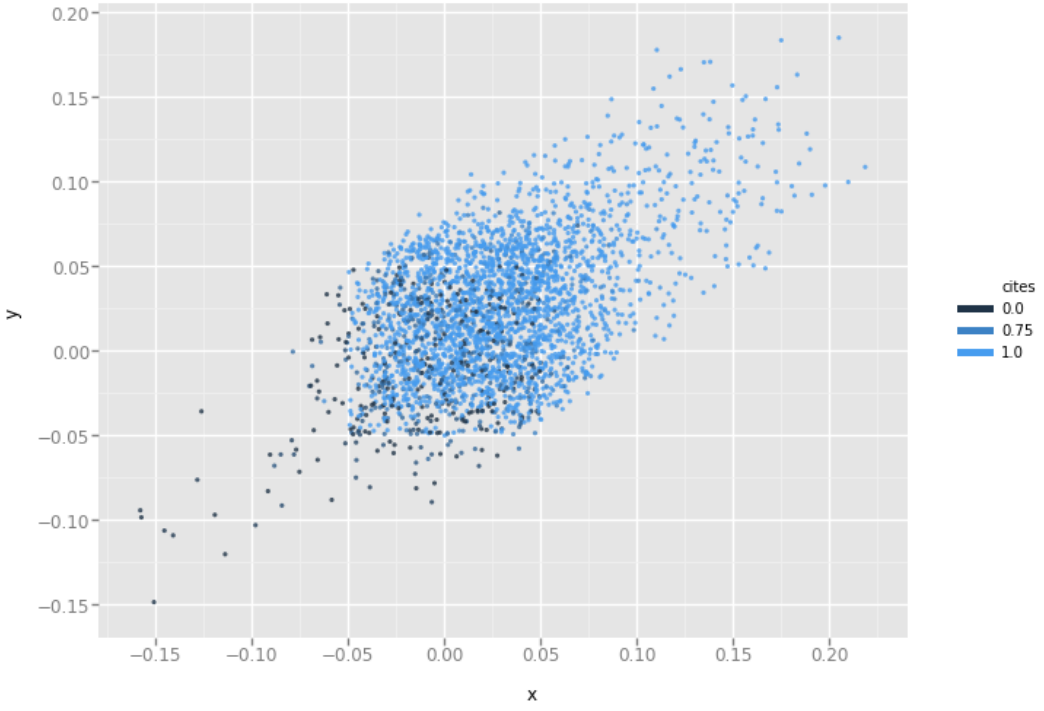
0.20
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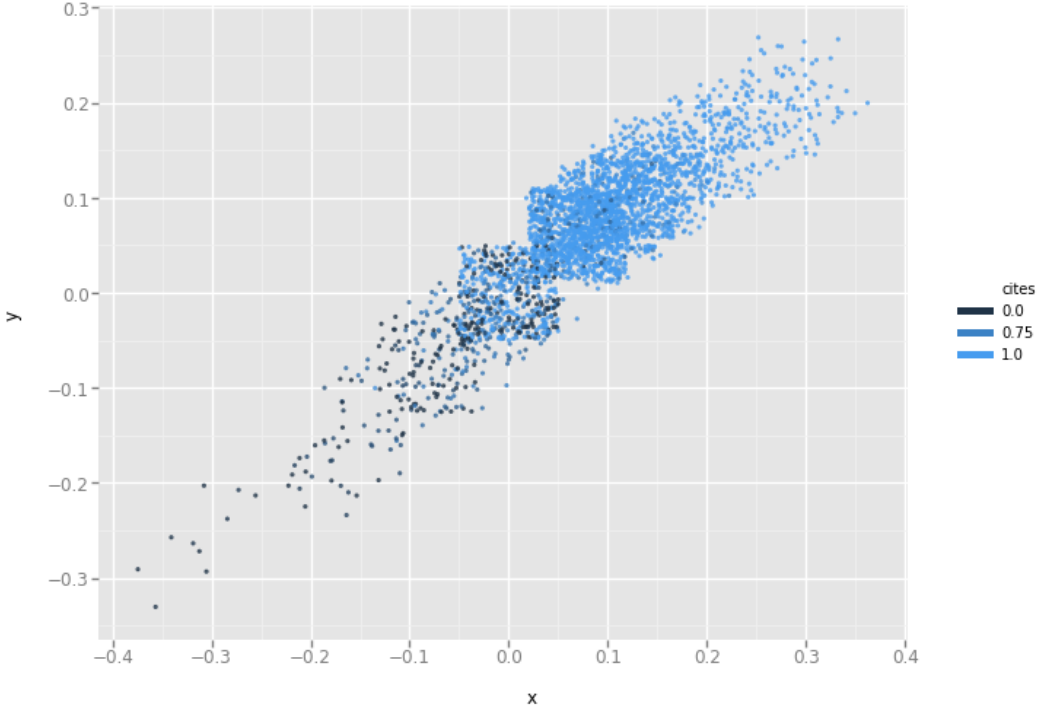
0.3
0.2
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0.0
−0.1
−0.2
−0.3
y
−0.4
−0.3
−0.2
−0.1
0.0
0.1
0.2
0.3
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x
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0.0
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Embedding Layers versus Dense Layers

- An embedding layer is statistically equivalent to a fully-connected dense layer with sparse data set as input and linear activation.

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- Why use an embedding layer rather than a dense layer?
  - embedding layers are much faster for this purpose
  - batch updating with regularization and dropout do not work well on sparse data.
- Geron (pg. 433) advises using one-hot encoding for less than 10 items, embedings for more than 50, and in between it's unclear.

MLP output layer as embedding matrix

- Consider the output layer for an MLP:
  - last hidden layer l, with dimensionality n_l
  - output classes with dimensionality n_y
  - layer $l = n_l \times n_y$ matrix of weights E_y

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- columns of E_y give n_y outcome class vectors.
  - vector similarities between columns indicate model's learned similarities between the output classes (Goldberg pg. 99)

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- The embedding layer replaces the list of sparse one-hot vectors with a list of n_E-dimensional ($n_E << n_w$) dense vectors

$$\boldsymbol{X} = \begin{bmatrix} x_1 & \dots & x_L \end{bmatrix}$$

where

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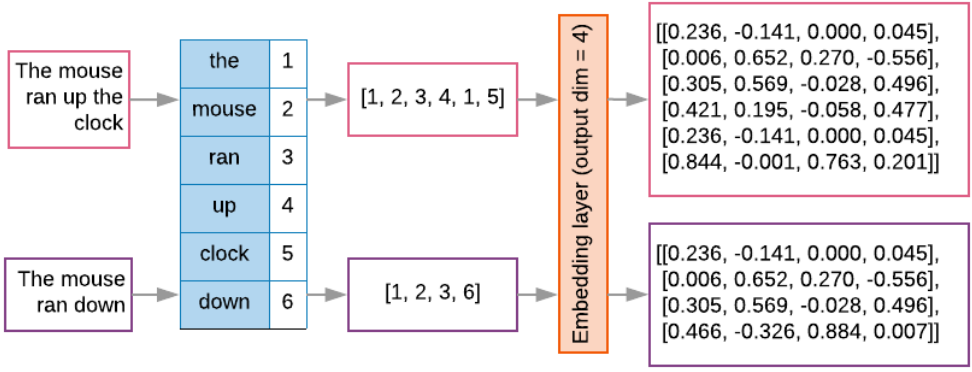
- This $\boldsymbol{X}$ matrix is then flattened into an $L * n_E$ vector for input to the next layer.

Illustration



Continuous Bag-of-Words Representation

- Let w_j be the one-hot representation of feature j
- The dot product $w_j \cdot \boldsymbol{E}$ selects the row of $\boldsymbol{E}$ corresponding to j:

$$v(w_j) = \boldsymbol{w}_j \cdot \boldsymbol{E}$$

- And a document can be represented as

$$\boldsymbol{x}_i = \text{CBOW}(x_1, ..., x_{n_i}) = \sum_{j=1}^{n_i} w_j \cdot \boldsymbol{E} = (\sum_{j=1}^{n_i} w_j) \cdot \boldsymbol{E}$$

  - the sum over the word vectors in the document.

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  - DNN will learn the optimal vector for each word in predicting the political party.

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- "You shall know a word by the company it keeps":
  - "He filled the **wampimuk**, passed it around and we all drunk some."
  - "We found a little, hairy **wampimuk** sleeping behind the tree."

Words and Contexts

A long line of NLP research aims to capture the distributional properties of words using a **word-context matrix** $\boldsymbol{M}$:

- each row w represents a **word** (e.g. "income"), each column c represents a linguistic **context** in which words can occur (e.g. "corporate ___ tax").
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  - Different definitions of contexts and different measures of association $\rightarrow$ different types of word vectors.

Defining the context

- The simplest definition of context is neighboring words:
  - for "the tabby cat": we get $(w = \text{"the"}, c = \text{"tabby"})$, $(w = \text{"tabby"}, c = \text{"the"})$, $(w = \text{"tabby"}, c = \text{"cat"})$, and $(w = \text{"cat"}, c = \text{"tabby"})$

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  - ...
- Etc.

Association Measures

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- **Point-wise mutual information** (PMI) normalized for high frequency:

$$(w, c) = \log \frac{\Pr(w, c)}{\Pr(w)\Pr(c)} = \log \frac{\frac{\#(w,c)}{n_D}}{\frac{\#(w)}{n_D}\frac{\#(c)}{n_D}} = \log \frac{n_D \#(w, c)}{\#(w)\#(c)}$$

where $\#(w)$ and $\#(c)$ are the corpus counts for w and c, respectively.

Issues with PMI

$$f_M(w, c) = \text{PMI}(w, c) = \log \frac{\#(w, c)}{\#(w)\#(c)}$$

- Issue 1: with PMI$(\cdot)$, unseen (w, c) pairs will take value $-\infty$.
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- Issue 2: PMI assigns high value to rare word-context pairs.
  - so impose a minimum count threshold on (w, c) pairs; below the threshold, set to zero.

$\boldsymbol{M}$ is too high-dimensional

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  - if c is drawn from from the vocabulary of a reasonably large corpus, the associated word vectors $\{v_1 = \boldsymbol{M}_{[w_1,:]}, v_2 = \boldsymbol{M}_{[w_2,:]}, ...\}$ are too high-dimensional to be useful.

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Word Similarity

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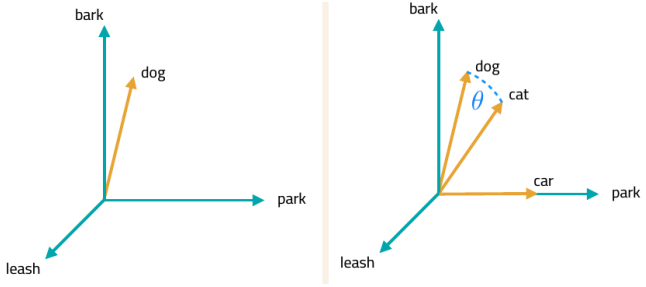

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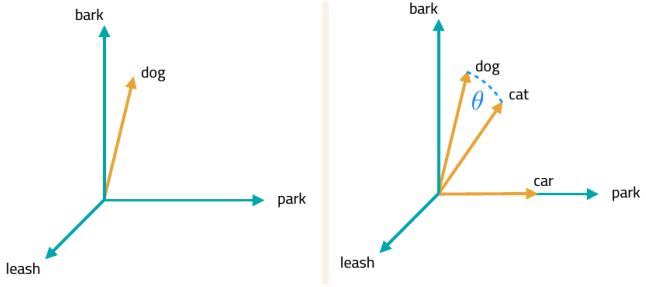

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- Thanks to linearity, can compute similarities between groups of words by averaging the groups.

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- Let $\hat{y}_i(w, c; \theta)$ be the predicted probability of a correct pair. Word2Vec minimizes the binary cross-entropy

$$L(\theta) = -\sum_{i=1}^{n_D} [y_i \log \hat{y}_i(w, c; \theta) + [1 - y_i] \log(1 - \hat{y}_i(w, c; \theta))]$$

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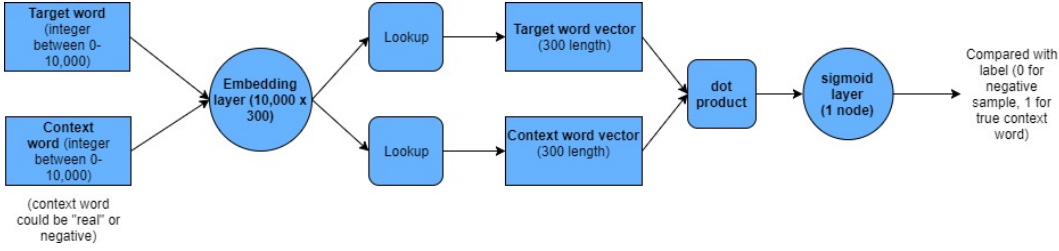

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- If we take $\tilde{M} = E_w E_c'$, Levy and Goldberg (2014) show that word2vec is equivalent to factorizing a matrix $\boldsymbol{M}$ with items

$$\boldsymbol{M}_{[w,c]} = \text{PMI}(w, c) - \log a$$

where a is a constant calibrating the amount of negative sampling.

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- Minimizes **squared difference** between**:**
  - **dot product of word vectors,** $\boldsymbol{w}\cdot\boldsymbol{c}$
  - **empirical co-occurrence,** $\log(\#(w,c))$
- Intuitively: words that co-occur have high correlation (dot product)

Word Embeddings Encode Linguistic Relations

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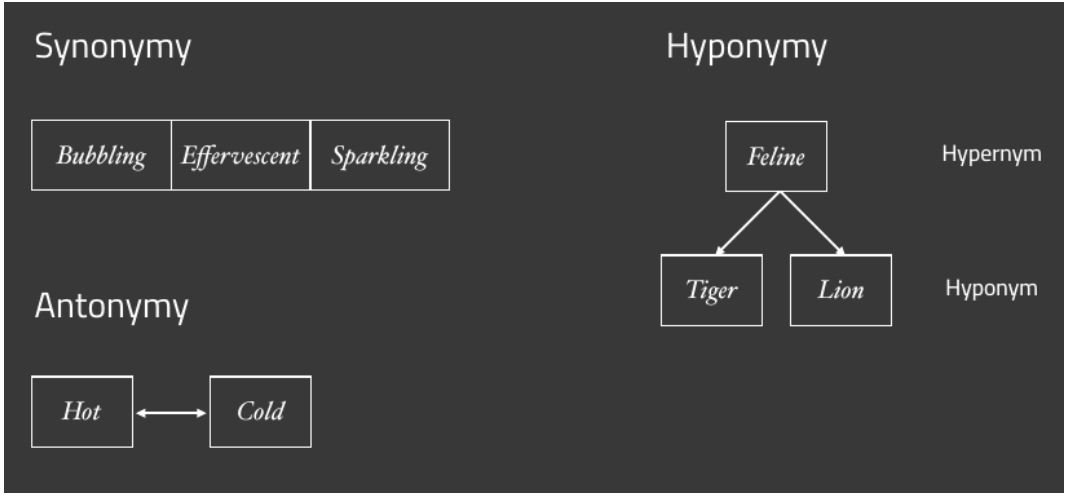

Similarity vs. Relatedness (Budansky and Hirst, 2006)

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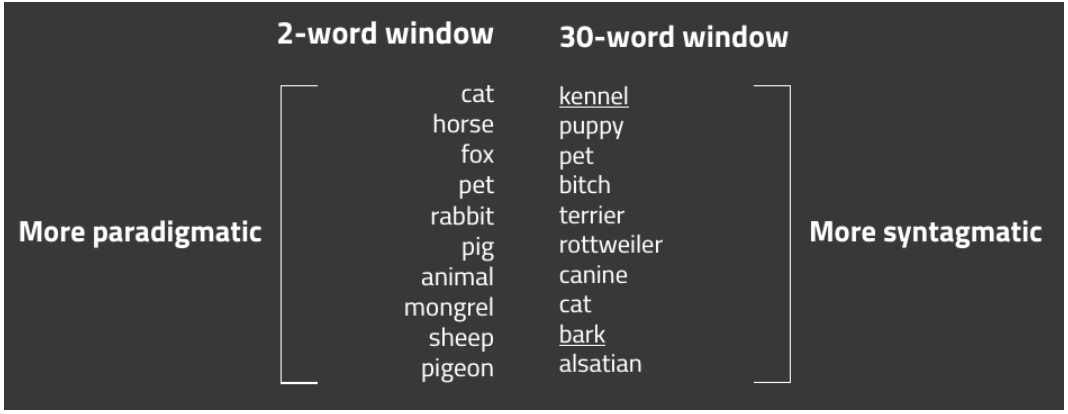
- Word embeddings will recover one or both of these relations, depending on how contexts and associated are constructed.

Most similar words to dog, depending on context window size



- Small windows pick up substitutable words; large windows pick up topics.

Target Word	BoW5	BoW2	Deps
batman	nightwing aquaman catwoman superman manhunter	superman superboy aquaman catwoman batgirl	superman superboy supergirl catwoman aquaman
hogwarts	dumbledore hallows half-blood malfoy snape	evernight sunnydale garderobe blandings collinwood	sunnydale collinwood calarts greendale millfield
turing	nondeterministic non-deterministic computability deterministic finite-state	non-deterministic finite-state nondeterministic buchi primality	pauling hotelling heting lessing hamming
florida	gainesville fla jacksonville tampa lauderdale	fla alabama gainesville tallahassee texas	texas louisiana georgia california carolina
object-oriented	aspect-oriented smalltalk event-driven prolog domain-specific	aspect-oriented event-driven objective-c dataflow 4gl	event-driven domain-specific rule-based data-driven human-centered
dancing	singing dance dances dancers tap-dancing	singing dance dances breakdancing clowning	singing rapping breakdancing miming busking

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- The default model only works by word, but “new york $\neq$ ”new” + “york”
  - it makes sense to tokenize phrases together (see Week 2 lecture) before training.

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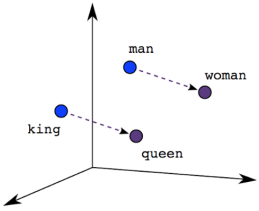
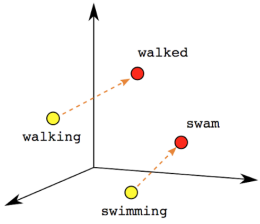
- Relatedly, antonyms are often rated similarly, be careful with that.

Vector Directions ↔ Meaning

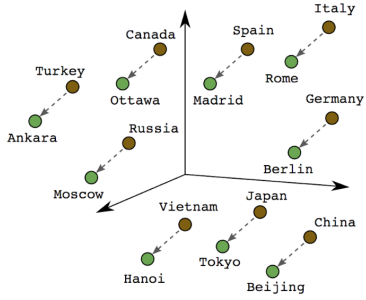
- Intriguingly, word2vec algebra can depict conceptual, analogical relationships between words:



Male-Female



Verb Tense



Country-Capital

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- Often works better with normalized vectors (so that one long vector doesnt wash out the others)
- Levy and Goldberg (2014) recommend the following "CosMul" metric which tends to perform better:

$$\arg\max_{b_2 \in V} \frac{\cos(b_2, a_2)\cos(b_2, b_1)}{\cos(b_2, a_1) + \epsilon}$$

  - requires normalized, non-negative vectors (can transform using $(x+1)/2$)
  - ϵ is a small smoothing parameter.

Word embeddings vs topic models

Ben Schmidt:

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- Word embedding models ignore information about individual documents to better understand the relationships between words.

Tokenizing for Embeddings

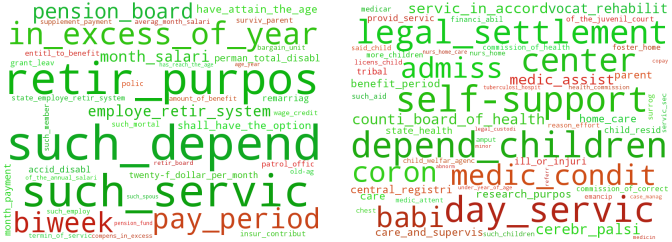
- embeddings work better with more information about the placement of words in sentences.
  - don't drop stopwords/function-words
  - should include tokens for start of sentence and end of sentence
  - should include a special token for out-of-vocabulary words
    - or replace with the part of speech tag

Word Dropout

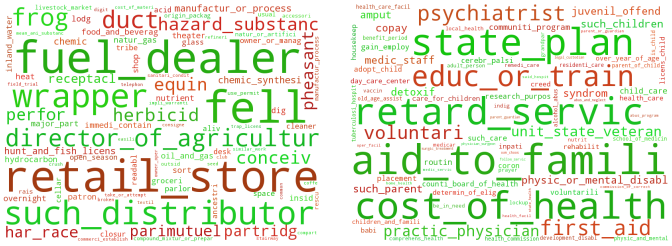
- When training models, words can be randomly replaced with the unknown symbol with some small probability (Iyyer et al 2015).
- Prevents models from relying too much on particular words.

K-means clustering with Word Embeddings

Income Tax (Pensions Topic and Health Care Topic)



Sales Tax (Retail Topic and Health Care Topic)



- Clustered phrases affecting tax revenues (Ash 2018); Green words tend to increase revenues; red words tend to decrease revenues.

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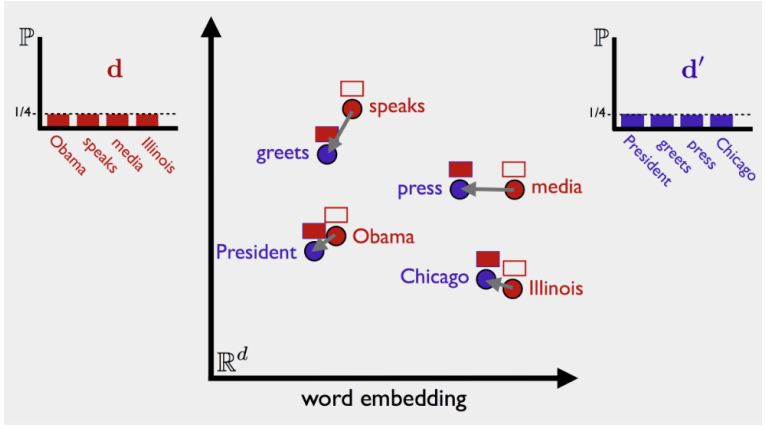
- Requires measure of distance between words (word embeddings).
  - see wmd package in Python.

Illustration



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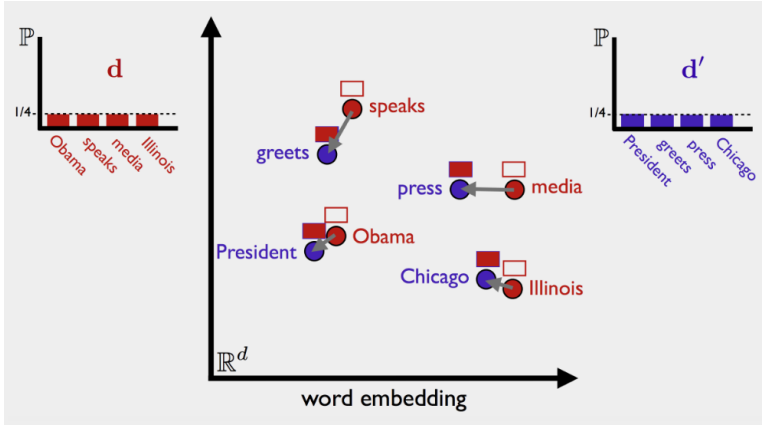

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  - Word mover distance sums the shortest distances between the words in the documents.

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- Can initialize prediction model using pre-trained embeddings.

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