# Sequencing Legal DNA NLP for Law and Political Economy

5. Neural Nets and Word Embeddings

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  - $\triangleright$   $x_i$  is itself a compressed representation of the unprocessed document  $\mathcal{D}_i$ .
- Correspondingly: the parameters  $\hat{\theta}$  can also be understood as a compressed (or "learned") representation:
  - it contains information about the training corpus, the text features, and the outcomes.

# Information in $\hat{\theta}$

- ➤ Say we train a multinomial logistic regression on a bag-of-words representation of the documents.
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  - It contains  $n_x$  rows =  $n_y$ -vectors representing each word in the vocabulary.
- How to use this?
  - could cluster column vectors to understand which outcomes are similar/related.
  - could cluster row vectors to understand which features are similar/related.

### Preview of Word Embeddings

Let's say  $x_i$  is a bag-of-words representation for document i with length  $n_i$ . We can write

$$\mathbf{x}_i = \frac{1}{n_i} \sum_{l=1}^{n_i} \mathbf{x}_i^{[l]}$$

- / indexes words in the the document
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- Now let  $\theta^{[I]}$  be the row of  $\theta$  corresponding to the word  $w_I$ . We can write

$$\hat{\mathbf{y}}_i = \frac{1}{n_i} \sum_{l=1}^{n_i} \theta^{[l]}$$

the sum of the  $n_y$ -dimensional word representations (the row vectors from above).

- ▶ this is called the "continuous bag of words (CBOW)" representation.
- $\triangleright$   $\theta$  is a word embedding matrix.

### Outline

Intro to Neural Nets

Practicalities

Autoencoders

**Embedding Layers** 

Word Embeddings

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  - nothing like brains
- "Networks":
  - ▶ nothing to do with "networks" as normally understood in particular, nothing to do with network theory in math or social science.

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- ▶ Increase in computing power makes them computationally tractable, graphical processing units (GPUs, designed for video games) give you over 100x performance gain over CPUs.
- Training algorithms have improved small tweaks have made a huge impact.
- ➤ Some theoretical limitations of NNs have turned out to be benign in practice for example, they work well on non-convex functions.

#### Will it last?

- ► Three key principles of deep learning will persist:
  - Simplicity
    - feature engineering is obsolete
    - complex, brittle, engineering-heavy pipelines replaced with simple, end-to-end trainable models, composed of 5-6 tensor operations.

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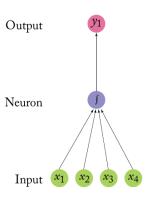
#### Scalability

- ▶ amenable to parallelization on GPUs or TPUs (tensor processing units)
- trained on batches of data, so can be scaled to datasets of arbitrary size.

#### Versatility and reusability

- can be trained on additional data without restarting from scratch, therefore amenable for continuous online learning.
- deep-learning models are repurposable and thus reusable

#### A "Neuron"



- ▶ A neuron multiplies each input by its weight, sums them, applies a non-linear function to the result, and passes the output.
  - ightharpoonup e.g., the  $\int$  shape indicates a sigmoid transformation.

#### In Notation

▶ The simplest neural network is called a perceptron:

$$MLP0(x) = x \cdot \omega$$

$$\mathbf{x} \in \mathbb{R}^{n_x}, \boldsymbol{\omega} \in \mathbb{R}^{n_x \times n_y}$$

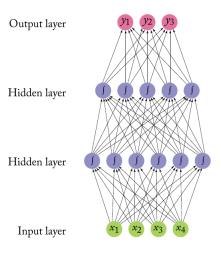
here,  $\omega$  is the matrix of weights in the layer.

▶ In more standard notation, there would be an additional constant (or "bias") term:

$$\mathsf{MLPO}(\mathbf{x}) = \alpha + \mathbf{x} \cdot \boldsymbol{\omega}$$

 $\triangleright$  We leave it out by assuming that x is de-meaned or has an extra column of ones.

### A Feed-Forward Neural Network



➤ A feed-forward network is simply a stack of linear models, separated by non-linear functions.

### Multi-Layer Perceptron

▶ An multi-layer perceptron (MLP) with one hidden layer is

$$\mathsf{MLP1}(\mathbf{x}) = \mathbf{g}(\mathbf{x} \cdot \boldsymbol{\omega}_1) \cdot \boldsymbol{\omega}_2$$
$$\mathbf{x} \in \mathbb{R}^{n_x}, \boldsymbol{\omega}_1 \in \mathbb{R}^{n_x \times n_1}, \boldsymbol{\omega}_2 \in \mathbb{R}^{n_1 \times n_y},$$

- $ightharpoonup n_1 = \text{dimensionality in first (and only) hidden layer}$
- $m{\omega}_1=$  set of learnable weights for the first linear transformation of the inputs.
- $\mathbf{g}(\cdot)$  = an element-wise non-linear function (an "activation function")
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### Multi-Layer Perceptron

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- $ightharpoonup \omega_2$  = weights on the second linear transformation leading to the output.
- MLP1 can approximate any continuous function on a closed and bounded subset of  $\mathbb{R}^n$ , and any mapping from one finite discrete space to another finite discrete space (Hornik et al 1989, Cybenko 1989).
  - ▶ But MLP1 would have to be exponentially large in some cases (Telgarsky 2016) .

### Two hidden layers

► Adding a second hidden layer gives

$$\begin{split} \mathsf{MLP2}(\pmb{x}) &= \pmb{g}_2(\pmb{g}_1(\pmb{x} \boldsymbol{\cdot} \pmb{\omega}_1) \boldsymbol{\cdot} \pmb{\omega}_2) \boldsymbol{\cdot} \pmb{\omega}_3 \\ \pmb{x} &\in \mathbb{R}^{n_{\!\scriptscriptstyle X}}, \pmb{\omega}_1 \in \mathbb{R}^{n_{\!\scriptscriptstyle X} \times n_1}, \pmb{\omega}_2 \in \mathbb{R}^{n_1 \times n_2}, \pmb{\omega}_3 \in \mathbb{R}^{n_2 \times n_y} \end{split}$$

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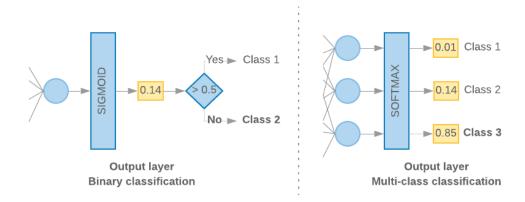
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- $ightharpoonup n_2=$  number of neurons in second hidden layer.
- MLP2 can be written in the following decomposed notation:

$$\mathsf{MLP2}(\mathbf{x}) = \\ \mathbf{h}_1 = \mathbf{g}_1(\mathbf{x} \cdot \boldsymbol{\omega}_1) \\ \mathbf{h}_2 = \mathbf{g}_2(\mathbf{h}_1 \cdot \boldsymbol{\omega}_2) \\ \mathbf{y} = \mathbf{h}_2 \cdot \boldsymbol{\omega}_3$$

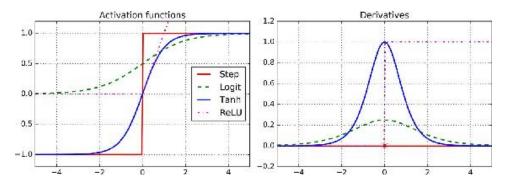
where  $h_l$  give hidden layers.

## Constructing the Last Layer



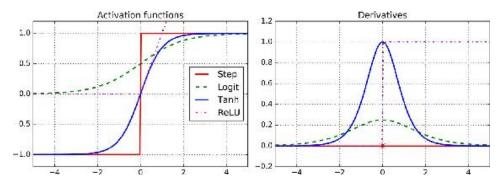
- ► MLPs will output a probability distribution across output classes.
  - can also output a real number, which would make a regression model.

## What to pick for g()



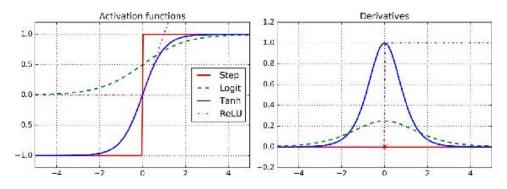
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- ▶ logistic function:  $logit(z) = \frac{1}{1 + exp(-z)}$
- ▶ hyperbolic tangent function:  $tanh(z) = 2\sigma(2z) 1$ 
  - ▶ ranges between -1 and 1 (rather than between 0 and 1, as the case with the logistic)
  - centered on zero, can speed up convergence

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  - ranges between -1 and 1 (rather than between 0 and 1, as the case with the logistic)
  - centered on zero, can speed up convergence
- ▶ ReLU (rectified linear unit) function:  $\max\{0, z\}$ ,
  - deceptively simple, fast to compute, and very effective in practice
  - gradient does not saturate to zero for large values (but is flat below zero)

### Backpropagation

- ▶ A crucial technology in deep learning is *backpropagation*, also known as reverse-mode automatic differentiation (autodiff).
  - intuitively, computes the network's output error and how much each neuron contributes to the error, so they can be updated to reduce error.

### Backpropagation

- ▶ A crucial technology in deep learning is *backpropagation*, also known as reverse-mode automatic differentiation (autodiff).
  - intuitively, computes the network's output error and how much each neuron contributes to the error, so they can be updated to reduce error.
  - Keras/TensorFlow will do this under the hood, just like how scikit-learn does gradient descent.
  - ▶ Appendix D of the Geron book has a decent explanation for how it works.

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Word Embeddings

### MLP baseline for Text Classification

Google Developers Advice

- 1. Calculate the number of samples/number of words per sample ratio.
- 2. If this ratio is less than 1500, tokenize the text as n-grams and use a simple multi-layer perceptron (MLP) model to classify them.
  - ▶ In the case of N-grams models, Google testers found that MLPs tended to out-perform logistic regression and gradient boosting machines.
  - ▶ A simple MLP is one of the models tried by Peterson and Spirling (2018).

### Keras Basics

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- "Dense" layer is the DNN baseline means that all neurons are connected.
- Output layer:
  - for regression, do not use an activation function
  - for binary classification, use activation='sigmoid'
  - for multi-class classification, use activation='softmax'

#### Loss function and metrics

- Loss function:
  - for regression, use mean\_squared\_error
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- Metrics:
  - ightharpoonup for classification, can use accuracy and  $F_1$
  - ightharpoonup for regression, use  $R^2$

Table 10-1. Typical regression MLP architecture

Hyperparameter	Typical value
# input neurons	One per input feature (e.g., 28 x 28 = 784 for MNIST)
# hidden layers	Depends on the problem, but typically 1 to 5
# neurons per hidden layer	Depends on the problem, but typically 10 to 100
# output neurons	1 per prediction dimension
Hidden activation	ReLU (or SELU, see Chapter 11)
Output activation	None, or ReLU/softplus (if positive outputs) or logistic/tanh (if bounded outputs)
Loss function	MSE or MAE/Huber (if outliers)

Table 10-2. Typical classification MLP architecture

Hyperparameter	Binary classification	Multilabel binary classification	Multiclass classification
Input and hidden layers	Same as regression	Same as regression	Same as regression
# output neurons	1	1 per label	1 per class
Output layer activation	Logistic	Logistic	Softmax
Loss function	Cross entropy	Cross entropy	Cross entropy

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- ▶ Standard practice is to use "Glorot" or "He" initialization:

Activation function	Uniform distribution [-r, r]	Normal distribution
Logistic	$r = \sqrt{\frac{6}{n_{\rm inputs} + n_{\rm outputs}}}$	$\sigma = \sqrt{\frac{2}{n_{\rm inputs} + n_{\rm outputs}}}$
Hyperbolic tangent	$r = 4\sqrt{\frac{6}{n_{\rm inputs} + n_{\rm outputs}}}$	$\sigma = 4\sqrt{\frac{2}{n_{\rm inputs} + n_{\rm outputs}}}$
ReLU (and its variants)	$r = \sqrt{2} \sqrt{\frac{6}{n_{\rm inputs} + n_{\rm outputs}}}$	$\sigma = \sqrt{2} \sqrt{\frac{2}{n_{\rm inputs} + n_{\rm outputs}}}$

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  - ▶ ReLU is a good baseline in the hidden layers.
- ► See Geron, pp. 322-323 for tools to help with tuning, e.g. Keras Tuner.
  - ► See also Smith (2018).

## Early stopping

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- evaluate model in test set.

#### Batch normalization

- Another trick to increase performance, speed up training, and regularize the model:
  - in between layers, zero-center and normalize the inputs to variance one.
  - normally done before a non-linear activation function

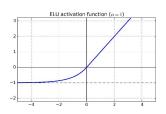
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- ► Another trick to increase performance, speed up training, and regularize the model:
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  - normally done before a non-linear activation function
- ▶ Does not work well in recurrent neural nets can use gradient clipping (hard constraint on gradient) instead.

#### **ELU** and **SELU**

#### Exponential linear unit

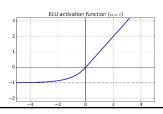
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SELU (Scaled ELU):

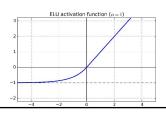
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- Requires standardized inputs.
- Gaussian initialization of neuron weights:
  - mean zero and standard deviation =  $n_X^{-1/2}$
- Set  $\lambda \approx 1.0507$ ,  $\alpha \approx 1.6732$ .
- →Then layers will be self-normalizing; weights will converge to mean zero and variance once (Klambauer et al 2017)

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▶ In general, SELU > ELU > ReLU.

## Optimizer and Learning Rate

- ► Choice of optimization algorithm is the topic of active research, which has shown that it can have a big impact on model performance.
  - ► See Geron, pp. 351-358. He recommends SGD with momentum, RMSProp, or Nadam.

# Optimizer and Learning Rate

- ► Choice of optimization algorithm is the topic of active research, which has shown that it can have a big impact on model performance.
  - See Geron, pp. 351-358. He recommends SGD with momentum, RMSProp, or Nadam.
- ▶ On the learning rate, see Geron pp. 359-364, recommending 1cycle scheduling:
  - start at  $\eta_0$ , increase linearly up to  $\eta_1$  halfway through training, then decrease linearly to  $\eta_0$  at the end.

## Regularization for Sparse Models

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- Another regularizer: "max-norm regularization":
  - ▶ constrain each neuron's weights  $\omega$  by  $||\omega||_2 \le r$ , where r is a hyperparameter.
- ▶ But usually its better/simpler to use dropout.

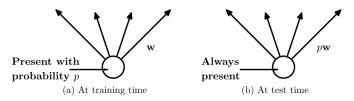


Figure 2: **Left**: A unit at training time that is present with probability p and is connected to units in the next layer with weights **w**. **Right**: At test time, the unit is always present and the weights are multiplied by p. The output at test time is same as the expected output at training time.

Source: Srivastava et al, JMLR 2014

#### An elegant regularization technique:

- ▶ at every training step, every neuron has some probability (typically p = 0.5) of being temporarily dropped out, so that it will be ignored at this step.
- ▶ at test time, neurons dont get dropped anymore but coefficients are down-weighted by p.

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- ▶ Approximately equivalent to averaging the output of *N* models (where *N* is the number of neurons).
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  - cannot co-adapt with neighboring neurons and must be independently useful.
  - cannot rely excessively on just a few input neurons; they have to pay attention to all input neurons.
  - makes the model less sensitive to slight changes in the inputs.
- ▶ If using SELU activiation functions, use alpha dropout (Klambauer et al 2017).

# Monte Carlo Dropout

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- ► Normal dropout:
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- ► Monte Carlo dropout:
  - ▶ at test time, continue to allow dropout but produce 100 predictions, and average them.

# Summary and Practical Guidelines

#### Table 11-3. Default DNN configuration

Hyperparameter	Default value
Kernel initializer	He initialization
Activation function	ELU
Normalization	None if shallow; Batch Norm if deep
Regularization	Early stopping (+ $\ell_2$ reg. if needed)
Optimizer	Momentum optimization (or RMSProp or Nadam)
Learning rate schedule	1cycle

#### Table 11-4. DNN configuration for a self-normalizing net

yperparameter	Default value
ernel initializer	LeCun initialization
activation function	SELU
formalization	None (self-normalization)
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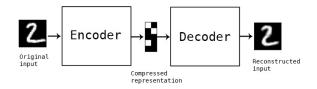
Word Embeddings

## Efficient Data Representations

Which of the following number sequences do you find the easiest to memorize?

- 40, 27, 25, 36, 81, 57, 10, 73, 19, 68
- 50, 48, 46, 44, 42, 40, 38, 36, 34, 32, 30, 28, 26, 24, 22, 20, 18, 16, 14

# Autoencoders ↔ Optimal Compression Algorithms



- "Autoencoder" refers to a class of deep neural network that performs domain-specific compression and dimension reduction.
  - ► They learn efficient encodings of the data, which can then be decoded back to a reconstruction a (minimally) lossy representation of the original data.
  - ► Can also randomly generate new data that look like the training data.

#### PCA is a linear autoencoder

```
from tensorflow import keras
encoder = keras.models.Sequential([keras.layers.Dense(2, input_shape=[3])])
decoder = keras.models.Sequential([keras.layers.Dense(3, input_shape=[2])])
autoencoder = keras.models.Sequential([encoder, decoder])
autoencoder.compile(loss="mse", optimizer=keras.optimizers.SGD(lr=0.1))
```

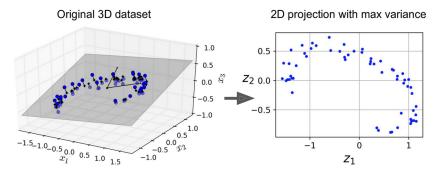


Figure 17-2. PCA performed by an undercomplete linear autoencoder

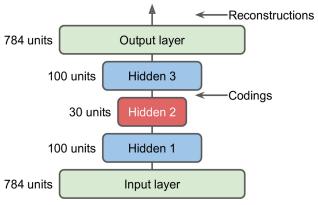


Figure 17-3. Stacked autoencoder

- ▶ Autoencoders work by stacking layers that gradually decrease in dimensionality to create the compressed representation (Z), and then gradually increase in dimensionality to try to reconstruct the input.
  - the autoencoder is implicitly solving the problem of maximizing entropy in the bottleneck layer.

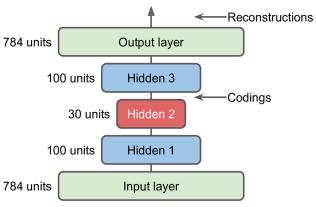


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  - the autoencoder is implicitly solving the problem of maximizing entropy in the bottleneck layer.
  - for symmetric autoencoders, **tying weights** of the encoding and decoding segments will speed up training and tends to improve performance.

#### Reconstruction from encoded vector



 $Figure\ 17\text{--}4.\ Original\ images\ (top)\ and\ their\ reconstructions\ (bottom)$ 

#### Autoencoders for Data Visualization



Figure 17-5. Fashion MNIST visualization using an autoencoder followed by t-SNE

- Decent baseline for visualizing the encodings:
  - use an autoencoder to compress your data to relatively low dimension (e.g. 32 dimensions)
  - then use t-SNE for mapping the compressed data to a 2D plane.

# Denoising Autoencoders

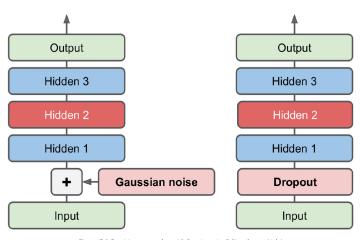


Figure 17-8. Denoising autoencoders, with Gaussian noise (left) or dropout (right)

#### Outline

Intro to Neural Nets

Practicalities

Autoencoders

**Embedding Layers** 

Word Embeddings

## What is an Embedding?

- A (relatively) low-dimensional vector representation of a categorical variable.
  - > spatial location of the vector encodes predictive information about the category.

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  - instead of including a fifty-dimensional categorical variable, include two-dimensional latitude and longitude

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- e.g., trying to predict how employment responds to economic growth with data from U.S. states:
  - instead of including a fifty-dimensional categorical variable, include two-dimensional latitude and longitude
  - or initialize each state to a random two-dimensional vector, and let the model decide where to move the states to improve prediction on your task (e.g. ).

# An embedding layer is just matrix multiplication

An embedding layer can be represented as

- w, a categorical variable (e.g., representing a word)
  - one-hot vector with a single item equaling one.
  - ► The input to the embedding layer.

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  - The output of the embedding layer.
- $\blacktriangleright$  An embedding function  $v(\cdot)$ , defined by matrix E, learnable by the DNN

## The Embedding Matrix *E*

- ► The model learns the weights of the embedding matrix in the same way that it would learn any model parameters.
- $\triangleright$  The rows of the matrix correspond to vectors for the  $n_w$  categories.
  - ► These are the "word vectors" that people talk about when they mention word embeddings or Word2Vec.

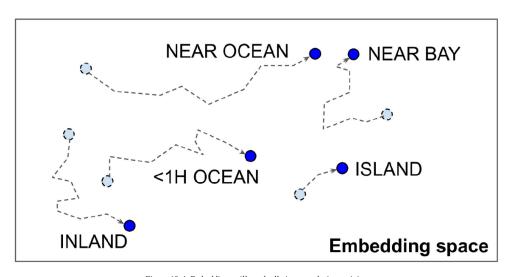
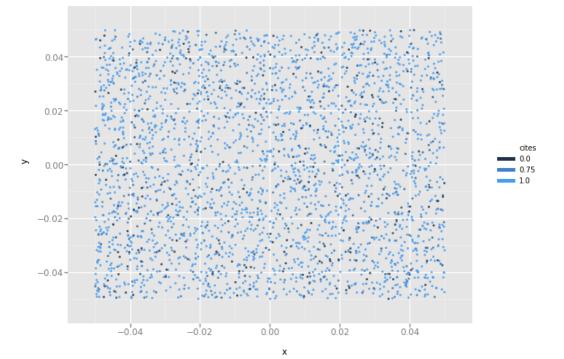
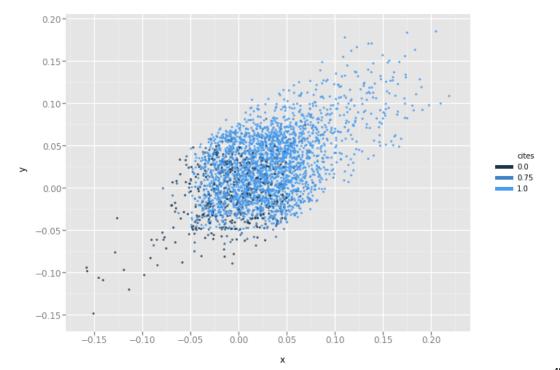
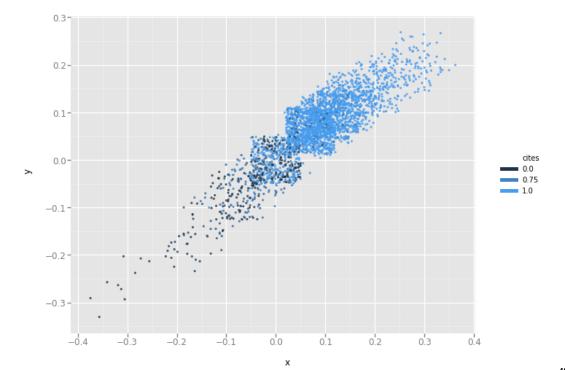


Figure 13-4. Embeddings will gradually improve during training







## Embedding Layers versus Dense Layers

► An embedding layer is statistically equivalent to a fully-connected dense layer with sparse data set as input and linear activation.

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  - batch updating with regularization and dropout do not work well on sparse data.
- ▶ Geron (pg. 433) advises using one-hot encoding for less than 10 items, embedings for more than 50, and in between it's unclear.

## MLP output layer as embedding matrix

- Consider the output layer for an MLP:
  - last hidden layer l, with dimensionality  $n_l$
  - ightharpoonup output classes with dimensionality  $n_y$
  - layer  $I = n_I \times n_y$  matrix of weights  $E_y$

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- ightharpoonup columns of  $E_y$  give  $n_y$  outcome class vectors.
  - vector similarities between columns indicate model's learned similarities between the output classes (Goldberg pg. 99)

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- The embedding layer replaces the list of sparse one-hot vectors with a list of  $n_E$ -dimensional ( $n_E << n_w$ ) dense vectors

$$\mathbf{X} = \begin{bmatrix} x_1 & \dots & x_L \end{bmatrix}$$

where

$$\underbrace{x_j}_{n_E \times 1} = \underbrace{E}_{n_E \times n_w} \cdot \underbrace{w_j}_{n_w \times 1}$$

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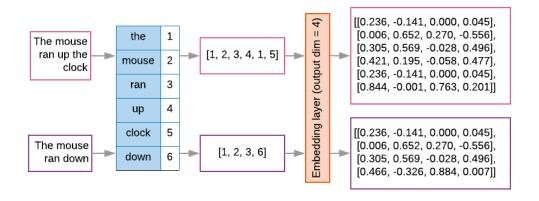
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where

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▶ This **X** matrix is then flattened into an  $L*n_E$  vector for input to the next layer.

#### Illustration



## Continuous Bag-of-Words Representation

- $\blacktriangleright$  Let  $w_i$  be the one-hot representation of feature j
- ▶ The dot product  $w_j \cdot E$  selects the row of E corresponding to j:

$$v(w_j) = \boldsymbol{w}_j \cdot \boldsymbol{E}$$

And a document can be represented as

$$\mathbf{x}_i = \mathsf{CBOW}(x_1, ..., x_{n_i}) = \sum_{j=1}^{n_i} w_j \cdot \mathbf{E} = (\sum_{j=1}^{n_i} w_j) \cdot \mathbf{E}$$

the sum over the word vectors in the document.

# Word Embeddings can be used for any NLP Task

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  - ▶ DNN will learn the optimal vector for each word in predicting the political party.

#### Word2Vec & GloVe

- "Word embeddings" often refers to Word2Vec or GloVe these are particular (popular) models for producing word embeddings.
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  - ▶ the goal: represent the meaning of words by the neighboring words their **contexts**.
  - rather than predicting some metadata (such as citations) they predict the co-occurence of neighboring words.
- "You shall know a word by the company it keeps":
  - ▶ "He filled the wampimuk, passed it around and we all drunk some."
  - "We found a little, hairy wampimuk sleeping behind the tree."

#### Words and Contexts

A long line of NLP research aims to capture the distributional properties of words using a **word-context matrix** M:

- ▶ each row w represents a **word** (e.g. "income"), each column c represents a linguistic **context** in which words can occur (e.g. "corporate \_\_\_\_ tax").
  - A matrix entry  $M_{[w,c]}$  quantifies the strength of association between a word and a context in a large corpus.

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  - normally, these vectors have a spatial interpretation → geometric distances between word vectors reflect semantic distances between words.
  - $\blacktriangleright$  Different definitions of contexts and different measures of association  $\rightarrow$  different types of word vectors.

- ► The simplest definition of context is neighboring words:
  - for "the tabby cat": we get (w = "the", c = "tabby"), (w = "tabby", c = "the"), (w = "tabby", c = "cat"), and (w = "cat", c = "tabby")

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  - **•** ...
- Etc.

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- ▶ Point-wise mutual information (PMI) normalized for high frequency:

$$(w,c) = \log \frac{\Pr(w,c)}{\Pr(w)\Pr(c)} = \log \frac{\frac{\#(w,c)}{n_D}}{\frac{\#(w)}{n_D} \frac{\#(c)}{n_D}} = \log \frac{n_D \#(w,c)}{\#(w) \#(c)}$$

where #(w) and #(c) are the corpus counts for w and c, respectively.

### Issues with PMI

$$f_M(w,c) = PMI(w,c) = \log \frac{\#(w,c)}{\#(w)\#(c)}$$

- ▶ Issue 1: with PMI(·), unseen (w,c) pairs will take value  $-\infty$ .
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- Issue 2: PMI assigns high value to rare word-context pairs.
  - so impose a minimum count threshold on (w,c) pairs; below the threshold, set to zero.

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  - if c is drawn from from the vocabulary of a reasonably large corpus, the associated word vectors  $\{v_1 = \mathbf{M}_{[w_1,:]}, v_2 = \mathbf{M}_{[w_2,:]}, ...\}$  are too high-dimensional to be useful.

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  - such that  $\tilde{\mathbf{M}} = \mathbf{WC}'$  is the best rank- $n_E$  approximation of  $\mathbf{M}$ .

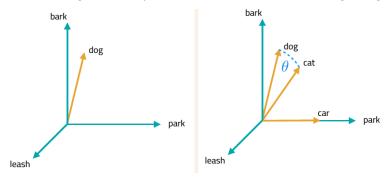
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## Word Similarity

- ▶ Once words are represented as vectors  $\{v_1 = \boldsymbol{M}_{[w_1,:]}, v_2 = \boldsymbol{M}_{[w_2,:]},...\}$ , we can use linear algebra to understand the relationships between words:
  - ▶ Words that are geometrically close to each other are similar: e.g. "dog" and "cat":



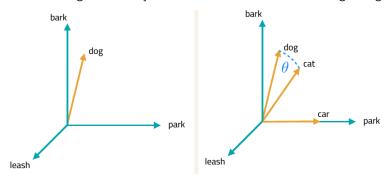
▶ The standard metric for comparing vectors is cosine similarity:

$$\cos\theta = \frac{v_1 \cdot v_2}{||v_1||||v_2||}$$

alternatives include e.g. Jaccard similarity (Goldberg 2017)

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  - ▶ Words that are geometrically close to each other are similar: e.g. "dog" and "cat":



▶ The standard metric for comparing vectors is cosine similarity:

$$\cos\theta = \frac{v_1 \cdot v_2}{||v_1||||v_2||}$$

- ▶ alternatives include e.g. Jaccard similarity (Goldberg 2017)
- ▶ Thanks to linearity, can compute similarities between groups of words by

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  - to fake (randomly sampled) ones ("The <u>prescription of</u> fox <u>is advised</u> for this diagnosis")
- Word2Vec learns embedding vectors for the target word ("fox") and context words (neighbors of "fox") to distinguish true from false samples.

# Word2Vec Objective

- ▶ The dataset is a collection of context pairs indexed by *i*:
  - $\triangleright$   $y_i = 1$  means correct (it appeared in the corpus)
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  - ► This random drawing of incorrect pairs is called "negative sampling". It is a form of unsupervised learning.
- Let  $\hat{y}_i(w,c;\theta)$  be the predicted probability of a correct pair. Word2Vec minimizes the binary cross-entropy

$$L(\theta) = -\sum_{i=1}^{n_D} [y_i \log \hat{y}_i(w, c; \theta) + [1 - y_i] \log(1 - \hat{y}_i(w, c; \theta))]$$

We have an embedding matrix for target words,  $\boldsymbol{E}_w$ , and for context words,  $\boldsymbol{E}_c$ .

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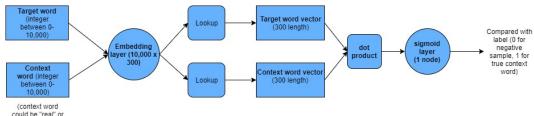
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- If we take  $\tilde{M} = E_w E_c'$ , Levy and Goldberg (2014) show that word2vec is equivalent to factorizing a matrix M with items

$$\mathbf{M}_{[w,c]} = \mathsf{PMI}(w,c) - \log a$$

where a is a constant calibrating the amount of negative sampling.

## GloVe Embeddings

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$$\min_{\boldsymbol{W},\boldsymbol{C}} \sum_{(\boldsymbol{w},\boldsymbol{c})} \phi(\#(\boldsymbol{w},\boldsymbol{c})) (\boldsymbol{w} \cdot \boldsymbol{c} - \log(\#(\boldsymbol{w},\boldsymbol{c})))^2$$

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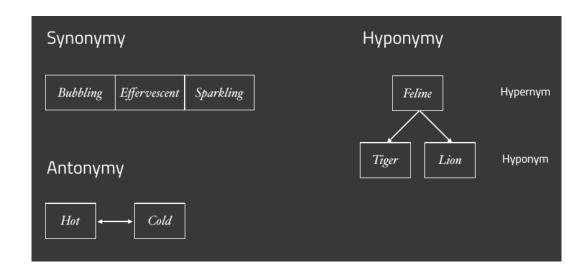
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- $\phi(\cdot)$  is weighting function to down-weight frequent words.
- Minimizes squared difference between:
  - dot product of word vectors, w · c
  - empirical co-occurrence,  $\log(\#(w,c))$
- Intuitively: words that co-occur have high correlation (dot product)

Word Embeddings Encode Linguistic Relations

#### Word Embeddings Encode Linguistic Relations



# Similarity vs. Relatedness (Budansky and Hirst, 2006)

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- Word embeddings will recover one or both of these relations, depending on how contexts and associated are constructed.

#### Most similar words to dog, depending on context window size



► Small windows pick up substitutable words; large windows pick up topics.

Target Word	BoW5	BoW2	Deps
batman	nightwing	superman	superman
	aquaman	superboy	superboy
	catwoman	aquaman	supergirl
	superman	catwoman	catwoman
	manhunter	batgirl	aquaman
hogwarts	dumbledore	evernight	sunnydale
	hallows	sunnydale	collinwood
	half-blood	garderobe	calarts
	malfoy	blandings	greendale
	snape	collinwood	millfield
turing	nondeterministic	non-deterministic	pauling
	non-deterministic	finite-state	hotelling
	computability	nondeterministic	heting
	deterministic	buchi	lessing
	finite-state	primality	hamming
florida	gainesville	fla	texas
	fla	alabama	louisiana
	jacksonville	gainesville	georgia
	tampa	tallahassee	california
	lauderdale	texas	carolina
object-oriented	aspect-oriented	aspect-oriented	event-driven
	smalltalk	event-driven	domain-specific
	event-driven	objective-c	rule-based
	prolog	dataflow	data-driven
	domain-specific	4gl	human-centered
dancing	singing	singing	singing
	dance	dance	rapping
	dances	dances	breakdancing
	dancers	breakdancing	miming
	tap-dancing	clowning	busking

## Parts of Speech and Phrases

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- ▶ The default model only works by word, but "new york  $\neq$  "new" + "york"
  - it makes sense to tokenize phrases together (see Week 2 lecture) before training.

▶ The trivial or obvious features of a word are not mentioned in standard corpora.

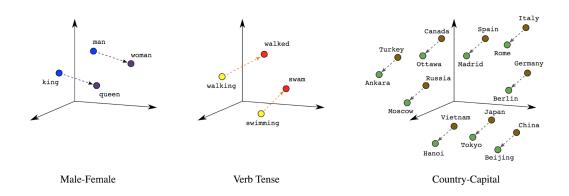
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- Relatedly, antonyms are often rated similarly, be careful with that.

# $Vector\ Directions \leftrightarrow Meaning$

► Intriguingly, word2vec algebra can depict conceptual, analogical relationships between words:



 $vec(king) - vec(man) + vec(woman) \approx vec(queen)$ 

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More generally: The analogy  $a_1:b_1::a_2:b_2$  can be solved (that is, find  $b_2$  given  $a_1,b_1,a_2$ ) by

$$\arg\max_{b_2\in V}\cos(b_2,a_2-a_1+b_1)$$

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- Often works better with normalized vectors (so that one long vector doesnt wash out the others)
- ▶ Levy and Goldberg (2014) recommend the following "CosMul" metric which tends to perform better:

$$\arg\max_{b_2 \in V} \frac{\cos(b_2, a_2)\cos(b_2, b_1)}{\cos(b_2, a_1) + \epsilon}$$

- requires normalized, non-negative vectors (can transform using (x+1)/2)
- $ightharpoonup \epsilon$  is a small smoothing parameter.

#### Word embeddings vs topic models

#### Ben Schmidt:

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- ▶ Word embedding models ignore information about individual documents to better understand the relationships between words.

## Tokenizing for Embeddings

- embeddings work better with more information about the placement of words in sentences.
  - don't drop stopwords/function-words
  - should include tokens for start of sentence and end of sentence
  - should include a special token for out-of-vocabulary words
    - or replace with the part of speech tag

#### Word Dropout

- ▶ When training models, words can be randomly replaced with the unknown symbol with some small probability (lyyer et al 2015).
- ▶ Prevents models from relying too much on particular words.

## K-means clustering with Word Embeddings

Income Tax (Pensions Topic and Health Care Topic)



Sales Tax (Retail Topic and Health Care Topic)



Clustered phrases affecting tax revenues (Ash 2018); Green words tend to increase revenues; red words tend to decrease revenues.

#### Word Mover Distance

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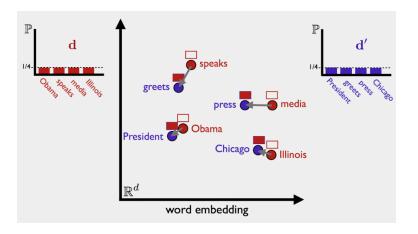
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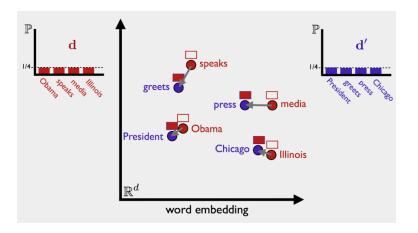
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- Requires measure of distance between words (word embeddings).
  - see wmd package in Python.

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- ightharpoonup d (obama speaks media illinois) is orthogonal to d' (president greets press chicago):
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- Can initialize prediction model using pre-trained embeddings.

# Tips for using pre-trained embeddings

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  - in second run, un-freeze the embedding layer for fine tuning.