Training Models

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Different Methods for Different Goals

- **Supervised: Pursuing a known goal – prediction or classification.**
- Unsupervised: Unknown goal, let the computer summarize the data.

Approximating $Y = f(X)$

- We want to predict a real-valued outcome Y given X, that is, constructing an approximation of the function $f(X)$.
  - With high-dimensionality and multi-collinearity, normal regression methods do not work.
- Supervised learning:
  - regularized regression
  - random forests
  - cross-validation

OLS Regression

- Consider the linear model

$$Y_i = X_i'\beta + \epsilon_i$$

  where Y_i and all elements of X_i have been de-meaned and standardized to $s.d. = 1$.

- OLS assumptions:
  - X_i uncorrelated with ϵ_i
    - Let's just assume this for now; will come back later.
  - Columns of X_i are not highly collinear.
    - In the case of word/n-gram frequency data, this is not a good assumption.

Univariate OLS Regressions to Rank Predictive Features

- Consider the univariate regression

$$Y_i = \beta_w x_i^w + \epsilon_i$$

  for each text feature w (e.g., relative word or n-gram frequency).
- Can be estimated with OLS.
  - Can add fixed effects, or even better: residualize Y and X on fixed effects before running any regressions.
  - Robust or clustered standard errors is optional, if the goal is just to rank predictors or filter out noise features.

OLS Regression in Python statsmodels

- One could write a DO file to run these regressions in Stata.
  - But the loops and data saving would be tricky with so many feature variables.

```
import statsmodels.formula.api as smf

tstats, betas = [], []
for xvar in vocab: # loop through the words in vocab
    print(xvar)
    model = smf.ols('Y ~ %s' % xvar, data=df4)
    result = model.fit()
    tstats.append(result.tvalues[1])
    betas.append(result.params[1])

stats = list(zip(vocab, tstats))
stats.sort(key = lambda x: x[1], reverse=True)
stats[:10]
stats[-10:]
```

Ash-Morelli-Osnabrugge: Interpreting Topics

- In the analysis of New Zealand parliamentary speeches, we looked at the n-grams that were predictive of the topic classification.
  - Formally, for each topic k and each phrase m, regress

    $$\Pr(y_i = k) = \alpha + \delta_m x_i^m + \epsilon_i$$

    where $\Pr(y_i = k)$ is the predicted probability that speech i is topic k, and x_i^m is the tf-idf frequency of phrase m in speech i.
    - select phrases with highest positive t-statistic for δ_m

Agriculture and Education Topics

Estimating OLS

- OLS minimizes the mean squared error:

$$\min_{\hat{\theta}} \frac{1}{m} \sum_{i=1}^{m} (\boldsymbol{x}_i' \hat{\theta} - y_i)^2$$

  where $\boldsymbol{x}_i = (x_{i1}, x_{i2}, ..., x_{ip})$
- This has a closed form solution

$$\hat{\theta} = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{Y}$$

- Most machine learning models do not have a closed form solution.
  - objective must be minimized using some other optimization algorithm.

Gradient Descent

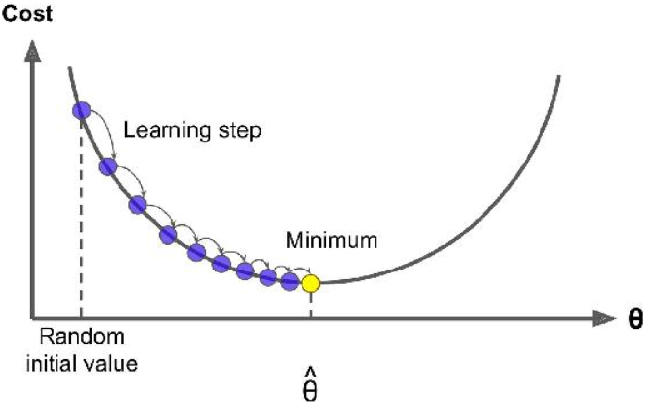
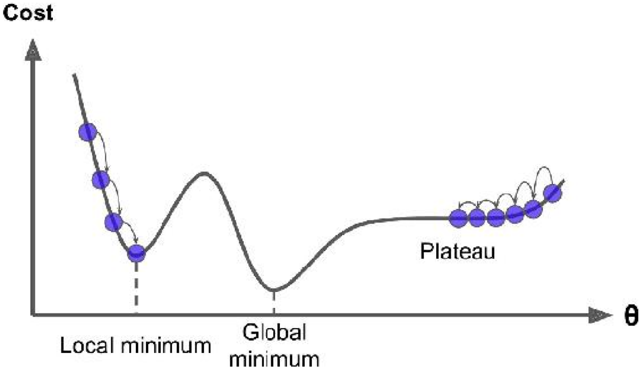


Figure 4-3. Gradient Descent

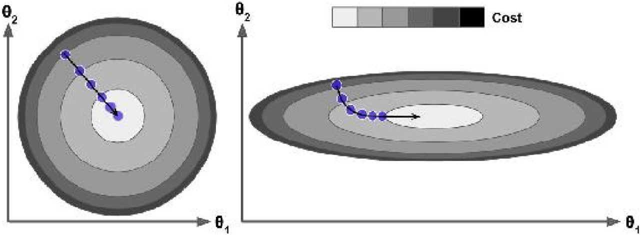
- Gradient descent measures the local gradient of the error function, and then steps in that direction.
  - Once the gradient equals zero, you have reached a minimum.

Gradient Descent Pitfalls



- Gradient descent will find a local minimum, not necessarily a global minimum. And it can get stuck on plateaus.

Gradient Descent and Scaling



- When using gradient descent, all features should be standardized to the same scale to speed up convergence.

Gradient Descent Implementation

- The partial derivative of MSE for feature j is

$$\frac{\partial \text{MSE}}{\partial \theta_j} = \frac{2}{m} \sum_{i=1}^{n} (\boldsymbol{x}_i' \hat{\theta} - y_i) x_{ij}$$

- The gradient is the vector of these partial derivatives is

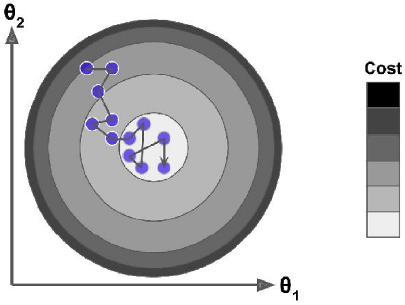
$$\nabla_\theta \text{MSE} = \begin{bmatrix} \frac{\partial \text{MSE}}{\partial \theta_0} \\ \frac{\partial \text{MSE}}{\partial \theta_0} \\ \vdots \\ \frac{\partial \text{MSE}}{\partial \theta_j} \end{bmatrix} = \frac{2}{m} \boldsymbol{X}'(\boldsymbol{X}'\theta - \boldsymbol{y})$$

- Gradient descent step:

$$\theta_{t+1} = \theta_t - \eta \nabla_\theta \text{MSE}$$

Stochastic Gradient Descent

- SGD picks a random instance in the training set and computes the gradient only for that single instance.

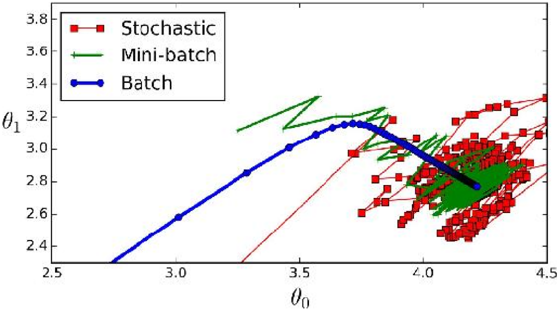



```
from sklearn.linear_model import SGDClassifier
sgd_clf = SGDClassifier(n_iter=50, penalty=None, eta0=.1)
sgd_clf.fit(X, y)
```

- Much faster to train, but can still bounce around even after it is close to the minimum.

Mini-Batch Gradient Descent

- A compromise between gradient descent and stochastic gradient descent is mini-batch gradient descent, which selects a small number of rows (a “mini-batch”) for gradient compute, rather than a single row.



The Problem of Overfitting

```
m = 100
X = 6 * np.random.rand(m,1) - 3
y = 0.5 * X ** 2 + X + 2 + np.random.randn(m,1)

from sklearn.preprocessing import PolynomialFeatures
poly_2 = PolynomialFeatures(degree=2) # also adds interactions
X_poly_2 = poly_2.fit_transform(X)

poly_30 = PolynomialFeatures(degree=300)
X_poly_30 = poly_300.fit_transform(X)

lin_reg = LinearRegression()
scores_1 = cross_val_score(lin_reg, X, y, cv=3, n_jobs=3)
scores_2 = cross_val_score(lin_reg, X_poly_2, y, cv=3, n_jobs=3)
scores_3 = cross_val_score(lin_reg, X_poly_300, y, cv=3, n_jobs=3)

print(scores_1.mean())
print(scores_2.mean())
print(scores_3.mean())
```

The Problem of Overfitting

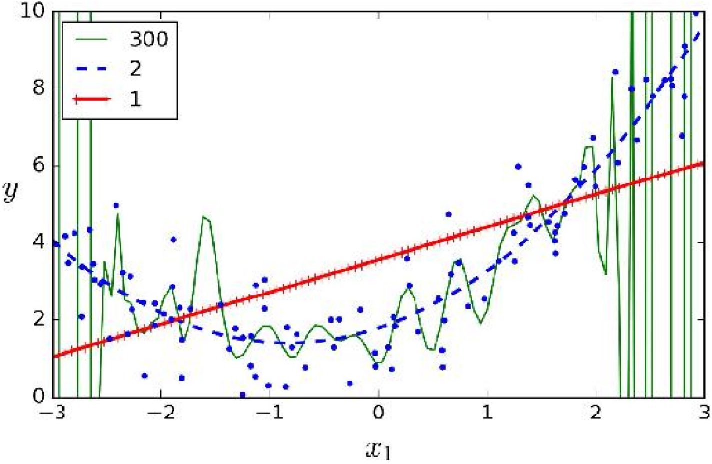


Figure 4-14. High-degree Polynomial Regression

Bias-Variance Tradeoff

- *Bias*
  - Error due to wrong assumptions, such as summing data is linear when it is quadratic.
  - Will underfit the training data
- *Variance*
  - Error due to excess sensitivity to small variations in the training data.
  - A model with high variance is likely to overfit the training data.
- *Irreducible error*
  - Error due to noise in the data.

Training Notes

- In general, increasing a model's complexity will increase variance and reduce bias.
- If the model is underfitting:
  - adding more training data will not help.
  - use a more complex model
- If the model is overfitting:
  - adding more training data may help
  - or use regularization

Lasso, Ridge, and Elastic Net

- Lasso and ridge regression are tools for dealing with large feature sets where:
  - models have multicollinearity that causes bias
  - models tend to overfit
  - models are computationally costly to fit
- These algorithms work by constraining estimated parameter sizes.

Lasso Regresison

- The Lasso cost function is

$$J(\theta) = \mathrm{MSE}(\theta) + \alpha_1 \sum_{i=1}^{n} |\theta_i|$$

  - i indexes over n features
  - α_1 is a hyperparameter setting the strength of the L1 penalty

- Lasso automatically performs feature selection and outputs a sparse model.

```python
from sklearn.linear_model import Lasso
lasso_reg = Lasso(alpha=0.1)
lasso_reg.fit(X,y)
```

Ridge Regression

- The Ridge cost function is

$$J(\theta) = \text{MSE}(\theta) + \alpha_2 \frac{1}{2} \sum_{i=1}^{n} \theta_i^2$$

  - i indexes over n features
  - α_2 is a hyperparameter setting the strength of the L2 penalty

- It turns out that the Ridge estimator, like OLS, has a closed-form solution:

$$\hat{\theta}_{\text{Ridge}} = (X'X + \alpha_2 \boldsymbol{I}_n)^{-1} X'\boldsymbol{y}$$

  where $\boldsymbol{I}_n$ is the identity matrix.
  - But it can also be solved by (stochastic) gradient descent.

```python
from sklearn.linear_model import Ridge
ridge_reg = Ridge(alpha=1)
ridge_reg.fit(X,y)
```

Elastic Net

- Elastic Net uses both L1 and L2 penalties:

$$J(\theta) = \mathrm{MSE}(\theta) + \lambda\alpha_1 \sum_{i=1}^{n} |\theta_i| + (1-\lambda)\alpha_2 \frac{1}{2} \sum_{i=1}^{n} \theta_i^2$$

  - in general, elastic net is preferred to lasso, which can behave erratically when the number of features is greater than the number of rows, or when some features are highly collinear.

- For elastic net, as with Lasso and Ridge, the hyperparameters can be selected by cross-validation.

```
from sklearn.linear_model import ElasticNetCV
enet_reg = ElasticNetCV(alphas=[.01,.1,1],
                        l1_ratio=[.01,.1,.5,.9, .99, 1])
enet_reg.fit(X,y)
```

Scaling while maintaining sparsity

- Regularization penalties are designed to work with scaled data.
  - An important feature of text data is sparsity, which is lost when taking out the mean:

$$\tilde{x}_i = \frac{x_i - \bar{\boldsymbol{x}}}{\mathrm{SD}[\boldsymbol{x}]}$$

- Solution:

$$\tilde{x}_i = \frac{x_i}{\mathrm{SD}[\boldsymbol{x}]}$$

```
from sklearn.preprocessing import StandardScaler
sparse_scaler = StandardScaler(with_mean=False)
X_sparse = sparse_scaler.fit_transform(X)
```

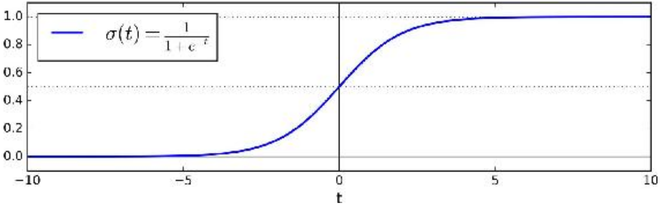
Logistic Regression

- Like OLS, logistic regression computes a weighted sum of the input features to predict the output.
  - But rather than output the sum directly, it transforms the sum using the logist function.

$$\hat{p} = \Pr(Y_i = 1) = \sigma(\theta' \boldsymbol{x})$$

where $\sigma(\cdot)$ is the signmoid function

$$\sigma(a) = \frac{1}{1 + \exp(-a)}$$



- For the binary version of the classifier, the prediction $\hat{Y} \in \{0, 1\}$ is determined by whether $\hat{p} \gtrless .5$.

Logistic Regression Cost Function

- The cost function to minimize is the

$$J(\theta) = \underbrace{-\frac{1}{m}}_{\text{negative}} \sum_{i=1}^{m} [\underbrace{y_i}_{y_i=1} \underbrace{\log(\hat{p}_i)}_{\text{log prob} y_i=1} + \underbrace{(1-y_i)}_{y_i=0} \underbrace{\log(1-\hat{p}_i)}_{\text{log prob} y_i=0}]$$

  - this does not have a closed form solution
  - but it is convex, so gradient descent will find the global minimum.

- Just like linear models, logistic can be regulared with L1 or L2 penalties, e.g.:

$$J(\theta) = \text{log loss} + \alpha_2 \frac{1}{2} \sum_{i=1}^{n} \theta_i^2$$

Multinomial Logistic Regression

- Logistic can be generalized to multiple classes.
  - When given an instance $\boldsymbol{x}_i$, multinomial logistic computes a score $s_k(\boldsymbol{x}_i)$ for each class k,

$$s_k(\boldsymbol{x}_i) = \theta'_k \boldsymbol{x}_i$$

  - If there are n features and K output classes, there is a $K \times n$ parameter matrix Θ, where the parameters for each class are stored as rows.
- Using the scores, probabilities for each class are computed using the softmax function

$$\hat{p}_k(\boldsymbol{x}_i) = \Pr(Y_i = k) = \frac{\exp(s_k(\boldsymbol{x}_i))}{\sum_{j=1}^{K} \exp(s_j(\boldsymbol{x}_i))} = \frac{e^{\theta_k \boldsymbol{x}_i}}{\sum_{j=1}^{K} e^{\theta_j \boldsymbol{x}_i}}$$

  - And the prediction $Y_i \in \{1, ..., K\}$ is determined by the highest-probability category.

Multinomial Logistic Cost Function

- The binary cost function generalizes to the cross entropy

$$J(\theta) = \underbrace{-\frac{1}{m}}_{\text{negative}} \sum_{i=1}^{m} \sum_{k=1}^{K} \underbrace{\mathbf{1}[y_i = k]}_{y_i = k} \underbrace{\log(\hat{p}_k(\boldsymbol{x}_i))}_{\text{log prob} y_i = k}$$

  - again, this is convex, so gradient descent will find the global minimum.

```
from sklearn.linear_model import LogisticRegression
logistic = LogisticRegression(C=1, # default L2 penalty
                              class_weight='balanced')

scores = cross_val_score(logistic,
                         X,
                         y,
                         cv=3,
                         n_jobs=3)

scores.mean(), scores.std()
```

Comparative Manifesto Project Corpus

- 44,020 annotated English-language political statements
  - hundreds of political party platforms from English-speaking countries.
- Each statement gets a CMP code, e.g. “decentralization”, “education”
  - 45 topics
    - some topics are somewhat esoteric, such as “marxist analysis”
  - We normalized to 19 broader, more interpretable topics

Featurizing the Statements

- Each row includes a CMP code and a statement.
  - The statements are plain text.
- Standard featurization steps:
  - remove capitalization, punctuation, stopwords
  - construct n-grams up to length 3
  - remove n-grams appearing in less than 10 statements or more than 40 percent of statements
    - $M = 7,646$ features
  - compute tf-idf-weighted n-gram frequencies

Regularized Logit Model

- N rows, M text features, K policy topics
- Probability model:

$$P(Y_i = c) = \frac{e^{\beta_c X_i}}{\sum_{k=1}^{K} e^{\beta_k X_i}},$$

  where $c \in 1,...,K$ are the topic labels and β is an $M \times K$ matrix of parameters.
- Cost function:

$$J(\beta) = -\frac{1}{N}\left[\sum_{i=1}^{N}\sum_{k=1}^{K} \mathbf{1}\{y^i = k\} \log \frac{e^{\beta_k X_i}}{\sum_{l=1}^{K} e^{\beta_l X_i}}\right] + \gamma \sum_{j=1}^{M}\sum_{k=1}^{K} \beta_{jk}^2$$

- γ = strength of L2 penalty
  - $\gamma^* = 1/2$ selected by 3-fold cross-validation grid search.

Prediction Model

- Given a chunk of text, the logistic model computes a probability distribution over policy topics.
  - harnesses expert knowledge about political topics from Manifesto Project
- Validation of accuracy: predict the CMP code in a held-out sample of manifesto corpus statements
  - In-sample accuracy = 71%, Out-of-sample accuracy = 53%
    - quite good given there are 19 policy areas – choosing randomly would be correct 5% of the time; choosing top category (other topic) would be correct 15% of the time.

Confusion Matrix

	Admin.	Agri.	Cult.	De-cent.	Econ.	Educ.	Freed.	Intnl.	Labor	Milit.	Natl Way Life	Demos	Other	Party Pol.	Quality	Target	Tech	Morals	Wel-fare	Total True
Adminis-tration	270	5	2	7	77	12	71	7	5	3	4	1	78	11	36	3	11	0	101	**704**
Agricul-ture	5	114	0	3	24	3	7	4	2	0	2	2	23	1	56	0	7	0	22	**275**
Culture	8	3	155	5	22	21	22	4	0	1	19	4	56	3	18	7	17	1	26	**392**
Decentral-ization	27	2	6	73	16	11	24	1	0	1	3	1	32	7	20	0	13	2	22	**261**
Economics	59	12	5	4	715	13	29	16	19	2	11	4	138	30	96	1	49	2	126	**1331**
Education	7	1	9	1	17	461	8	0	0	0	1	4	75	10	10	2	37	3	69	**715**
Freedom	51	0	9	20	41	13	642	19	3	16	12	6	126	34	16	4	14	7	89	**1122**
Interna-tionalism	8	1	4	3	33	2	45	245	2	14	11	1	46	9	19	3	3	0	26	**475**
Labor Groups	11	2	2	0	36	7	12	2	92	0	2	3	20	3	13	0	6	1	40	**252**
Military	7	0	1	0	7	1	24	27	2	118	5	0	32	2	10	0	9	1	13	**259**
Nat'l Way of Life	4	1	5	3	38	3	32	17	2	6	88	6	61	21	19	2	2	4	44	**358**
Non-Econ Demo Grps	7	1	7	0	15	15	30	1	5	2	5	95	36	8	4	5	5	5	104	**350**
Other Topic	38	14	17	13	78	48	86	25	20	12	20	26	1263	39	71	20	48	7	159	**2004**
Party Poli-tics	20	5	5	2	44	2	48	9	4	5	10	0	80	183	25	4	10	2	53	**511**
Quality of Life	21	17	9	4	102	9	13	14	7	3	7	1	112	16	684	0	42	0	34	**1095**
Target Groups	11	2	7	2	7	17	24	2	4	0	8	1	28	4	1	67	8	1	63	**257**
Tech & In-fra	22	8	5	8	57	31	6	6	5	1	4	2	85	1	62	1	413	0	40	**757**
Trad'l Morality	2	0	2	0	6	9	21	2	0	0	8	3	25	4	1	1	0	61	40	**185**
Welfare	33	6	11	3	93	32	62	3	10	1	9	26	142	37	29	11	23	7	1173	**1711**
Total Predicted	**611**	**194**	**261**	**151**	**1428**	**710**	**1206**	**404**	**182**	**185**	**229**	**186**	**2458**	**423**	**1190**	**131**	**717**	**104**	**2244**	