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BIOSTAT 653
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Homework

$$\begin{aligned}
\gamma_h &= \text{Cov}(X_n, X_{n+h}) \\
&= \text{Cov}(X_n, \phi X_{n+h-1} + \epsilon_{n+h}) \\
&= \text{Cov}(X_n, \phi X_{n+h-1}) + \text{Cov}(X_n, \epsilon_{n+h}) \\
&= \phi\gamma_{h-1} + \underbrace{\text{Cov}(X_n, \epsilon_{n+h})}_{=0} \\
&= \phi\gamma_{h-1}
\end{aligned}$$

$$\begin{aligned}
\gamma_0 &= \text{Cov}(X_n, X_n) \\
&= \text{Var}(X_n) \\
&= \text{Var}(\phi X_{n-1} + \epsilon_n) \\
&= \phi^2 \text{Var}(X_n) + \text{Var}(\epsilon_n) \\
&= \phi^2\gamma_0 + \sigma^2
\end{aligned}$$

$$\begin{aligned}
\gamma_h &= \phi\gamma_{h-1} \\
&= \phi(\phi\gamma_{h-2}) \\
&= \phi^3\gamma_{h-3} \\
&\vdots \\
&= \phi^h\gamma_0 \\
&= \phi^h \frac{\sigma^2}{1-\phi^2}
\end{aligned}$$

$$\gamma_h = \phi\gamma_{h-1} = \phi(\phi\gamma_{h-2}) = \phi^3\gamma_{h-3} \cdots = \phi^h\gamma_0 = \phi^h \frac{\sigma^2}{1-\phi^2}$$