

Here we have posterior distribution of velocity and image intensity:

$$P(v|I) \propto P(v)P(I|v) \quad I(x, y, t) = I(x + v_x \Delta t, y + v_y \Delta t, t + \Delta t) + \eta \quad (1)$$

Note that the image intensity is being subjected to the brightness constraint with noise. This allows us to take the Taylor series expansion:

$$\tilde{I}(x, y, t) \approx \frac{dI}{dx} \frac{dx}{dt} + \frac{dI}{dy} \frac{dy}{dt} + \frac{dI}{dt} \quad (2)$$

$$= I_x v_x + I_y v_y + I_t \quad (3)$$

We propose a prior distribution determined by the previous time step's posterior distribution:

$$P(v|I)_t \propto P(v|I)_{t-1} P(I|v) \quad (4)$$

This distribution is largely determined by the likelihood function, $P(I|v)$. The likelihood function is comprised of component functions calculated at each point in in the stimulus image. At point (x_i, y_i) and time t :

$$P(I(x_i, y_i, t)|v_i) \propto \exp \left(-\frac{1}{2\sigma^2} \int_{x,y} w_i(x, y) (\tilde{I}(x, y, t))^2 dx dy \right) \quad (5)$$

With $w_i(x, y)$ being a small Gaussian window centered at (x_i, y_i) . This is derived from the Taylor series expansion of (1). At each point in time:

$$P(I(x, y, t)|v) = \int_i P(I(x_i, y_i, t)|v_i) \quad (6)$$

Thus we have our final posterior distribution:

$$P(v|I)_t \propto P(v|I)_{t-1} \int_i P(I(x_i, y_i, t)|v_i) \quad (7)$$

In expanded and discrete form, the equation is given by

$$P(v|I)_t \propto \exp \left(\log(P(v|I)_{t-1}) - \frac{1}{2\sigma^2} \sum_i \sum_{x,y} w_i(x, y) (\tilde{I}(x, y, t))^2 \right) \quad (8)$$