$$V_0(h_0, d_0) = \max_{c_t, h_{t+1}} \sum_{j=0}^{J-1} \beta^j u(c_t, h_{t+1}) + \beta^{J-1}\psi(h_J)$$

$$\text{st.} \quad c_t + px_t + \pi_t = y_t$$

$$x_t = h_{t+1} - (1-\delta)h_t$$

$$\pi_t = (1+r)d_t - d_{t+1}$$

1. Other assets missing
   ~ Stocks, etc.
2. Financial sector; i.e. if prices fall
   → financial contagion
3. No idiosyncratic risk to income

p is MC $\{p_1, p_2\}$ → $Q = \begin{pmatrix} 1-\varepsilon & \varepsilon \\ \varepsilon & 1-\varepsilon \end{pmatrix}$

4. House prices are not endogenous in the model
5. Not general eq^m
6. ph is cts.
   → large investment
   → less houses bought straight away
   → ownership is constant in model

1. Define parameter values

$$\beta, \delta, \xi, J, y, d_0$$

2. Solve model (tractable)

$$\hat{r} = \sum_{j=0}^{J-1} \frac{1}{(1+r)^j} y - d_0 \quad (\text{PV of income})$$

$$\kappa_3 = (1 - \beta(1-\delta)) \cdot \frac{1-\varepsilon}{\varepsilon}$$

Fabian's Derivation

$$\mathcal{L} = \sum_{j=0}^{J-1} \beta^j \left\{ u(c_t, h_{t+1}) + \beta^{J-1}\psi(h_J) + \lambda_t \left(y + (1+r)d_t - d_{t+1} - p(h_{t+1} - (1-\delta)h_t) - c_t \right) \right\}$$

FOC

c_t: $\quad u_c(c_t, h_{t+1}) = \lambda_t$

h_{t+1}: $\quad u_h(c_t, h_{t+1}) - \lambda_t p + \beta(1-\delta)\lambda_{t+1} p = 0$,

d_{t+1}: $\quad \lambda_t = \beta\lambda_{t+1}(1+r) \implies \lambda_t = \lambda_{t+1}$

Assume: $\beta(1+r) = 1$

$$\Rightarrow \quad u_h(c_t, h_{t+1}) = \lambda_t p(1 - \beta(1-\delta))$$

$$\frac{u_h(c_t, h_{t+1})}{u_c(c_t, h_{t+1})} = p(1-\beta(1-\delta))$$

Using utility fn. from class: $u(c,s) = c^{1-\xi} s^{\xi}$

$\cdot\ s = h - \varphi \bar{h}$

$$\Rightarrow u(c,h) = c^{1-\xi} h'^{\xi}$$

$$u_c(c,h) = (1-\xi) c^{-\xi} h'^{\xi}$$

$$u_h(c,h) = \xi c^{1-\xi} h'^{\xi-1}$$

$$\text{FOC} \rightarrow \frac{\xi c^{1-\xi} h'^{\xi-1}}{(1-\xi) c^{-\xi} h'^{\xi}} = p(1-\beta(1-\delta))$$

$$\frac{c}{h'} = p(1-\beta(1-\delta))\frac{1-\xi}{\xi} \equiv pK_3 \rightarrow c = ph'K_3 \quad (1)$$

Want solution for ph now:

Iterate BC:

$$c_t + p(h_{t+1} - (1-\delta)h_t) + d_{t+1} = (1+r)d_t + y$$

With $c_t = ph_{t+1}K_3$

$$ph_{t+1}K_3 + ph_{t+1} - p(1-\delta)h_t = \pi_t + y$$

$$\Theta(J,r) = \sum_{j=0}^{J-1} \frac{1}{(1+r)^j}$$

$$\Upsilon = \sum_{j=0}^{J-1} \frac{y}{(1+r)^j} - d_0$$

$$ph' = \frac{\sum_{j=0}^{J-1} \frac{y}{(1+r)^j} - d_0}{\left(1-\delta + \sum_{j=0}^{J-1} \frac{(\delta + K_3)}{(1+r)^j}\right)}$$

$$ph' = \frac{\pi + y + p(1-\delta)h}{K_3 + 1} = \frac{\pi + y}{K_3 + 1} + \frac{(1-\delta)}{K_3+1} ph$$

$$= \frac{\pi + y}{\kappa_3 + 1} + \frac{(1-\delta)}{\kappa_3+1}\left(\frac{\pi+y}{\kappa_3+1} + \frac{(1-\delta)}{\kappa_3+1} p h^{-}\right)$$

Shock to housing prices, at some $j \leq J$

- say $p_1 > p_0$

  $\Rightarrow h \downarrow$, since price of maintenance rises

- deaccumulate debt quicker
  - cannot end with positive assets, so payback, etc.
  - quicker because

Cons. Response over Lifecycle

- grows with age $\qquad (p_1 = q\text{-}by\text{-}p \cdot p_0)$

$$\text{num} = (1-\delta) \cdot q\text{-}by\text{-}p + \sum_{j=0}^{J-\tilde{j}} \frac{1}{(1+r)^j}(\delta + \kappa_3)$$

$$\text{den} = (1-\delta) + \sum_{j=0}^{J-\tilde{j}} \frac{1}{(1+r)^j}(\delta + \kappa_3)$$

$$c\_res = \frac{\text{num}}{\text{den}} - 1 \qquad \& \quad c = ph\kappa_3$$

$$\frac{c_j(a,h,p_1) - c_j(a,h,p_0)}{c_j(a,h,p_0)}$$

Q1 Choose $n \geq 1$ statistics from SCF. Choose n parameters from model; and set s.t. SCF matches model

Data

1.	Debt	1.07597e5
2.	Houses	2.23356e5