The Missing Reallocation Effect: Decomposing The Declining Labour Share*

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Abstract

Since the mid-1980s, the US labour share fell considerably and the economy reallocated towards sectors with high labour shares. Previous studies claim this reallocation does not play a role in the decline of the labour share, so aggregate movements should be understood primarily as a within-sector phenomenon. By exploiting a decomposition method that explicitly accounts for simultaneous changes in the relative size of sectors and sectoral labour shares, I show reallocation offsets around half of the aggregate labour share decline - an order of magnitude larger than previous studies. My results imply that sectoral labour shares cannot be studied in isolation to gain insights into the fall of the aggregate labour share, the cause of which is not yet agreed upon in the literature.

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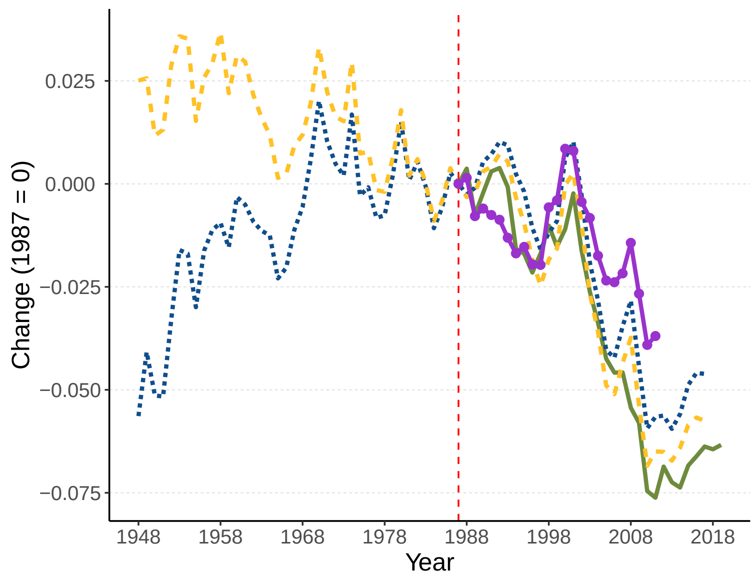
1 Introduction

Since the mid-1980s, the US labour share fell considerably. Figure 1 shows a clear downward trend for four measures of the labour share after 1987 [1]. A large macroeconomic literature has attempted to uncover the mechanisms driving the shift in the distribution of income away from labour, yet no clear consensus has been reached.

Figure 1. Aggregate labour share decline relative to 1987, percentage point change



Notes: The aggregate labour share equals labour income (self-employed labour income + payroll labour income) divided by GDP. The labour income of self-employed individuals is imputed four different ways. Section 2 contains the data description.
Source: Bureau of Economic Analysis (BEA), Bureau of Labour Statistics (BLS), and author's calculations.

The decline in the labour share coincided with a substantial reallocation of value-added towards sectors with high labour share levels, mainly in services (Boppart, 2014, Bridgman and Herrendorf, 2023, Gaggl et al., 2023, Herrendorf et al., 2013). All else equal, such reallocation should counterfactually raise the labour share, as the aggregate reflects the higher labour shares in the growing sectors to a greater extent. How much of the declining labour share is due to reallocation between sectors with different labour share levels and how much arises due to labour shares changing within sectors?

Despite the reallocation towards industries with high labour shares, numerous studies, including Dao et al. (2019), Elsby et al. (2013) and Karabarbounis and Neiman (2014), argue that between-sector compositional changes do not play a role in the decline of the US labour share, so aggregate movements should be understood primarily as a within-sector phenomenon. In this

[1] Section 2 discusses the four methods I use to measure the labour share.

paper, I show that the apparent lack of a reallocation effect is due to the specific decomposition method used in previous studies. First, I theoretically compare the commonly used shift-share (between-within) decomposition to the Haltiwanger (1997) (between-within-cross) decomposition (henceforth, I refer to it as the 'Haltiwanger decomposition'). The two methods are closely related because they both decompose the same change in the aggregate labour share. However, the additional cross term in the Haltiwanger decomposition explicitly captures the impact of covarying sectoral weights (relative size) and sectoral labour shares on the aggregate labour share. I theoretically show the shift-share between- and within-sector contributions include half of the Haltiwanger cross term. Therefore, the shift-share decomposition unintentionally counts half of the covariance impact as between-sector reallocation. The same holds for the within-sector contribution.

Second, I show the cross contribution is large and negative using US industry data and four different measures of the labour share. Following my theoretical result, the between- and within-sector contributions measured using the shift-share decomposition are downward biased because half of the negative covariance is added to each. Decomposing the declining payroll labour share using the shift-share method attributes only 4% to reallocation and the remaining 96% to within-sector mechanisms. A shift-share decomposition implies that to explain the path of the aggregate labour share one needs only to examine the determinants of labour shares within sectors. However, using the Haltiwanger decomposition - which removes the impact of co-movement between sectoral weights and sectoral labour shares - reallocation accounts for -47% of the total decline. The Haltiwanger decomposition recovers the positive reallocation effect missing from previous studies. In addition, the impact of within-sector mechanisms falls from 96% to 45% when using the Haltiwanger decomposition. I find similar results for three other measures of the aggregate labour share. The Haltiwanger decomposition highlights the quantitative importance of three channels operating in tandem to depress the labour share: reallocation towards industries with high labour share levels, declining labour shares within sectors, and the negative co-movement between sectoral weights and sectoral labour shares.

Related Literature. The negligible reallocation effect found in previous studies leads to authors proposing within-sector mechanisms to explain the falling labour share. Falling sectoral labour shares are related to the increased adoption of automation (Acemoglu and Restrepo, 2019), rising concentration (Barkai, 2020), intermediate inputs prices (Castro-Vincenzi and Kleinman, 2022), and sectoral heterogeneity in trade exposure (Elsby et al., 2013). Why these mechanisms would not also cause reallocation between sectors is not clear. For instance, increased trade exposure with China displaced manufacturing employment and automation is likely to promote growth heterogeneously across sectors that benefit differently from its adoption (Acemoglu and Restrepo, 2019, Autor et al., 2013). Since I find that reallocation and co-movements between sectors' size and labour shares are also important channels affecting the aggregate labour share, a methodology flexible enough to accommodate both is necessary to parse out which of the above mechanisms is driving the decline in the labour share.

Bridgman and Herrendorf (2023) and Feijo Moreira (2022) find a positive reallocation effect on the aggregate labour share in two-sector structural models. However, neither paper explains why their results differ from previous seminal studies claiming reallocation has no effect. My paper bridges this gap.

Lastly, Autor et al. (2020) and Kehrig and Vincent (2021) decompose changes in sectoral labour shares at the establishment level using US Census data. The authors explicitly measure the impact of co-movement between establishments' size (reallocation) and their labour shares on different industries' labour shares. Both papers find that co-movements drive sectoral trends. The establishment-level data allows for a more granular analysis than the industry-level data I use. However, the establishment-level data is not consistently measured across sectors, whereas the industry-level data covers the entire economy.

2 Data

The aggregate labour share in year t, λ_t, can be written as the weighted sum of N sectoral labour shares

$$\lambda_t = \sum_{i=1}^{N} \omega_{it} \lambda_{it}. \tag{1}$$

The sectoral weights and labour shares are defined as

$$\omega_{it} = \frac{VA_{it}}{\sum_{i=1}^{N} VA_{it}} \tag{2}$$

and

$$\lambda_{it} = \frac{WL_{it}}{VA_{it}} \tag{3}$$

in which VA_{it} and WL_{it} denote sector i's gross value-added and labour income, respectively, in year t. I use value-added data from the Bureau of Economic Analysis (BEA) industry accounts. Labour income in sector i in year t equals the sum of payroll workers' labour income WL_{it}^{p} and self-employed persons' labour incomes WL_{it}^{s}. The first is measured using payroll tax records. However, the labour-capital split is not recorded for self-employed persons' total income. How should labour income be divided from returns on capital investments for self-employed individuals? Following the literature, I exploit four different assumptions for imputing self-employed labour income, resulting in the four labour share definitions I use throughout the paper.

The first is the 'same-wage-distribution' approach used in the BEA-BLS integrated industry-level production account (Eldridge et al., 2020). First, assume the average wages for self-employed individuals and payroll workers are equal within skill×demographic cells j, for each sector i and year t. Second, the labour income WL^s_{jit} of self-employed individuals equals the labour income of payroll workers WL^p_{jit} corrected by the ratio of hours worked (which are recorded for both)

$$WL^s_{jit} = WL^p_{jit} \times \frac{L^s_{jit}}{L^p_{jit}} \tag{4}$$

in which L^p_{jit} and L^p_{jit} correspond to the total hours worked for payroll workers and self-employed individuals in cell j, sector i and year t, respectively. Second, the 'same-labour-share' approach by Mendieta-Muñoz et al. (2021) corrects total self-employed individuals' income $Y^s_{it} \equiv WL^s_{it} + RK^s_{it}$ (the labour-capital split is not known) by the payroll labour share $\frac{WL^p_{it}}{VA_{it}}$

$$WL^s_{it} = \frac{WL^p_{it}}{VA_{it}} \times Y^s_{it}. \tag{5}$$

Third, the 'economy-wide labour share' assumption subtracts the total self-employed persons' income from the denominator in the payroll labour share in equation (5)

$$WL^s_{it} = \frac{WL^p_{it}}{VA_{it} - Y^s_{it}} \times Y^s_{it}. \tag{6}$$

Lastly, since the first three definitions impute self-employed persons' labour income, they are subject to potential measurement error. To avoid mismeasurement, I examine the payroll labour share $\frac{WL^p_{it}}{VA_{it}}$. The payroll labour share represents the part of the aggregate labour share which is unambiguously labour income.

The final choice required to specify ω_{it} and λ_{it} is the level at which sectors i are defined. Aggregating the economy into fewer sectors reduces the extent to which reallocation can affect the aggregate labour share. For example, reallocation across NAICS 3-digit industries is always measured as a within-sector contribution if the industries are within the same NAICS 2-digit classification, even if the labour shares are constant within the 3-digit industries[2]. Therefore, aggregating the economy into coarser sectors loads onto the within-sector contribution. Ideally, I would use establishment-level information, but this kind of data is generally not available for the universe of firms in the US economy, and only within certain major sectors. To compromise, and for comparison to previous studies, I use sectors classified at the NAICS 3-digit classification for the 'same-wage-distribution' and 'payroll labour share' measures. For the 'same-labour-share' and 'economy-wide' assumptions, I can only use data aggregated into 14 sectors because total

[2] Appendix A1 demonstrates that aggregating NAICS 3-digit sectors into NAICS 2-digit classifications eliminates the entire between-sector effect, even though the effect is positive at the 2-digit level.

self-employed income Y^s_{it} is not measured at a more disaggregated level. Table 1 shows how the average sectoral labour share falls from the first 5 years to the last 5 years for each of the four labour share definitions.

Table 1. Average sectoral labour share for each definition.

Measure	Mean (first five)	Mean (last five)	Years	Number of sectors
Same-wage-distribution	0.69	0.61	1987-2019	60
Same-labour-share	0.69	0.65	1987-2017	14
Economy-wide	0.78	0.68	1987-2017	14
Payroll	0.65	0.62	1987-2011	60

Source: see Appendix A.1 for the data sources of each measure.

3 The Missing Reallocation Effect

I now use the industry-level data to demonstrate why the absence of a reallocation effect in previous studies is puzzling. The aggregate labour share in year t, λ_t, can be written as the weighted sum of sectoral labour shares

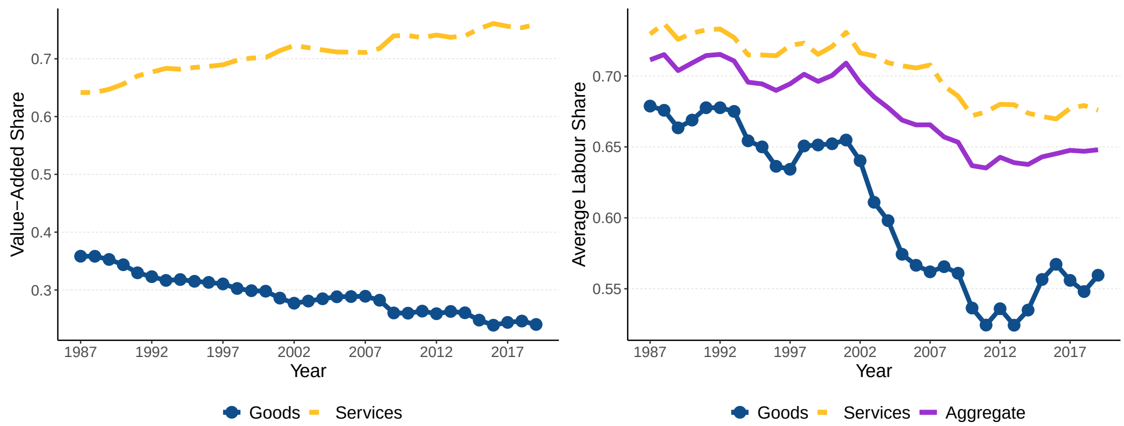
$$\lambda_t = \sum_{i=1}^{N} \omega_{it} \lambda_{it} \tag{7}$$

in which ω_{it} denote a sector's value-added-to-GDP weight in year t, and λ_{it} is sector i's labour share of value-added.

First, towards which sectors does the economy reallocate since 1987? The left panel in figure 2 shows the share of value-added shifted towards service-producing industries since 1987, which is already well documented in the literature (e.g. Bridgman and Herrendorf (2023), Herrendorf et al. (2013)). Second, how do labour share levels and trends differ between growing and shrinking sectors? The right panel shows the average labour share in services sectors is higher and declines slower than in goods sectors. Goods-producing industries see a much sharper drop in their average labour share from 1987 onwards. All else equal, the reallocation of value-added towards higher labour share sectors in services should partially offset the decline in the labour share. Indeed, in the right panel of figure 2, the fall in the aggregate labour share is muted relative to the larger decline in the average labour share in goods-producing sectors.

Third, I conduct a simple counterfactual exercise in which I fix sectoral labour shares λ_{it} at their 1987 values, allowing only the weights ω_{it} to change. The counterfactual labour share is

Figure 2. Value-added and average labour shares of goods and services.



Notes: The left panel shows the value-added-to-GDP share of goods and service sectors. The right panel shows the average labour share within goods and service sectors.
Source: BEA-BLS integrated industry-level production account and author's calculations.

given by

$$\lambda_t^{\text{counterfactual}} = \sum_{i=1}^{N} \omega_{it} \lambda_{i1987}. \tag{8}$$

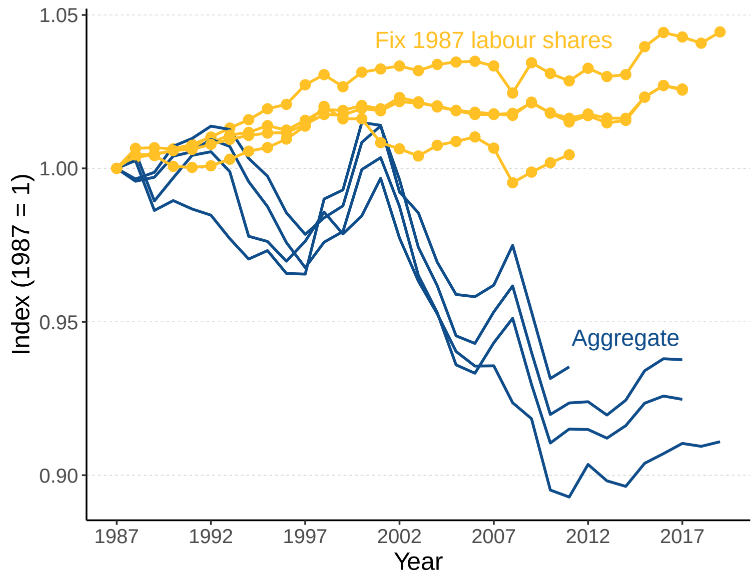
Figure 3 shows the counterfactual path of each labour share definition, all indexed to the 1987 aggregate labour share. For all four definitions, albeit to different extents, the counterfactual labour share is higher than the actual path. These counterfactuals suggest the labour share would have risen since 1987 if only sectoral weights (reallocation) had changed since then. Why is the positive reallocation effect missing from decompositions used in previous studies? To answer the question, in the next section I describe the the decomposition framework I apply to the industry data.

4 Decomposition Framework

How much of the declining labour share arises due to reallocation between sectors with different labour share levels and how much comes from labour shares changing within sectors? I use both the shift-share and the Haltiwanger decomposition to decompose changes in the aggregate labour share. Most studies of the declining labour share use the shift-share method, whereas the Haltiwanger decomposition has not been used to the best of my knowledge.

Using the weighted-average definition of the labour share in equation (7), the shift-share method decomposes changes in the labour share, $\Delta\lambda_t = \lambda_t - \lambda_{t-1}$, into a term capturing reallocation, arising from $\Delta\omega_{it}$, and a term capturing the effect of changing sectoral labour

Figure 3. Counterfactual labour shares holding sectoral labour shares fixed



Notes: The four lines sloping downward represent the path of the aggregate labour share for the four methods I use to impute self-employed labour income. The four lines sloping upward show the counterfactual paths of the aggregate labour share had sectoral labour shares stayed fixed at their 1987 values: $\lambda_t^{\text{counterfactual}} = \sum_{i=1}^{N} \omega_{it} \lambda_{i,1987}$. All lines are indexed to the respective 1987 labour share.
Source: Bureau of Economic Analysis (BEA), Bureau of Labour Statistics (BLS), and author's calculations.

shares, derived from $\Delta\lambda_{it}$.

Definition 1 (Shift-Share Decomposition). *The change in the labour share, $\Delta\lambda_t = \lambda_t - \lambda_{t-1}$, can be decomposed into two terms*

$$\Delta\lambda_t = \underbrace{\sum_{i=1}^{N} \Delta\omega_{it}\tilde{\lambda}_{it}}_{\textit{Between}} + \underbrace{\sum_{i=1}^{N} \tilde{\omega}_{it}\Delta\lambda_{it}}_{\textit{Within}} \tag{9}$$

in which $\Delta x_{it} = x_{it} - x_{it-1}$ and $\tilde{x}_{it} = \dfrac{x_{it-1} + x_{it}}{2}$.

Both terms in the shift-share decomposition keep the bases defined at their arithmetic means $\tilde{\lambda}_{it}$ and $\tilde{\omega}_{it}$, respectively. Alternatively, the Haltiwanger decomposition uses bases defined at their $t-1$ values and, therefore, splits changes in the aggregate labour share into a between, within, and cross term.

Definition 2 (Haltiwanger Decomposition). *The change in the labour share, $\Delta\lambda_t = \lambda_t - \lambda_{t-1}$,* *can be decomposed into three terms*

$$\Delta\lambda_t = \underbrace{\sum_{i=1}^{N} \Delta\omega_{it}\lambda_{it-1}}_{\textit{Between}} + \underbrace{\sum_{i=1}^{N} \omega_{it-1}\Delta\lambda_{it}}_{\textit{Within}} + \underbrace{\sum_{i=1}^{N} \Delta\omega_{it}\Delta\lambda_{it}}_{\textit{Cross}} \tag{10}$$

in which $\Delta x_{it} = x_{it} - x_{it-1}$.

Using either decomposition, the 'Between' term is used to measure the effect of reallocation between sectors with different labour shares. The 'Within' terms capture the effect of changing labour shares within industries. The 'Cross' term in the Haltiwanger decomposition captures the impact of co-movement of sectoral weights and labour shares. Theorem 1 demonstrates how the two decompositions are related since they are both exact decompositions of changes in the labour share $\Delta\lambda_t$.

Theorem 1. *By adding half of the Haltiwanger cross term to the Haltiwanger between-sector term, you get the shift-share between term.*

$$\underbrace{\sum_{i=1}^{N} \Delta\omega_{it}\tilde{\lambda}_{it}}_{\textit{Shift-Share Between}} = \underbrace{\sum_{i=1}^{N} \Delta\omega_{it}\lambda_{it-1}}_{\textit{Haltiwanger Between}} + \frac{1}{2}\underbrace{\sum_{i=1}^{N} \Delta\omega_{it}\Delta\lambda_{it}}_{\textit{Haltiwanger Cross}} .$$

The same relationship holds for the within-sector terms.

$$\underbrace{\sum_{i=1}^{N} \tilde{\omega}_{it}\Delta\lambda_{it}}_{\textit{Shift-Share Within}} = \underbrace{\sum_{i=1}^{N} \omega_{it-1}\Delta\lambda_{it}}_{\textit{Haltiwanger Within}} + \frac{1}{2}\underbrace{\sum_{i=1}^{N} \Delta\omega_{it}\Delta\lambda_{it}}_{\textit{Haltiwanger Cross}} .$$

Proof. See Appendix **??**. □

Theorem 1 demonstrates the shift-share between-sector term, which aims to capture the effect of reallocation, also captures half of the impact of co-movements between ω_{it} and λ_{it}. The same logic applies to the within-sector terms. Moreover, a non-zero Haltiwanger cross term leads to different conclusions about the impact of between-sector reallocation and within-sector mechanisms depending on the decomposition method used. For example, suppose the Haltiwanger between-sector term is positive, which means reallocation leads to a counterfactually higher aggregate labour share. Using the shift-share decomposition instead can result in a zero reallocation effect being measured if the Haltiwanger cross term is large and negative, because it subtracts from the positive Haltiwanger between-sector term, via Theorem 1. Additionally, the negative cross term leads to negative within-sector mechanisms being overcounted. Therefore, the Haltiwanger decomposition provides a more suitable measure of reallocation and within-sector causes when sectoral weights and labour shares covary, as I will show in the next section using the industry-level data.

Furthermore, the Haltiwanger method is preferable to the shift-share method when there are trends in either λ_{it} or ω_{it}. For instance, a downward trend in λ_{it} pushes the arithmetic mean, $\tilde{\lambda}_{it}$, down each period, meaning the shift-share between term captures reallocation and sector-level trends in labour shares (a within-sector phenomenon). Díez-Catalán (2018), Elsby et al. (2013) and Gutierrez (2017) show the labour share is trending down in virtually every sector and rising in a few. The presence of sectoral trends does not pose an issue when using the Haltiwanger decomposition because the bases are fixed at the previous period values.

By examining only the shift-share and Haltiwanger decompositions, I omit several other commonly used decomposition techniques. For example, changes in the aggregate labour share can also be decomposed as in Olley and Pakes (1996). I use the shift-share method since it is typically used in studies that decompose the declining labour share. Moreover, I use the Haltiwanger method because, as Theorem 1 shows, it is closely related to the shift-share method, but additionally accounts for co-movements between sectors' weights and labour shares.

5 Decomposition Results

Here I decompose the four series of the aggregate labour share using both the shift-share and Haltiwanger methods for the industry-level data.

5.1 Payroll Labour Share

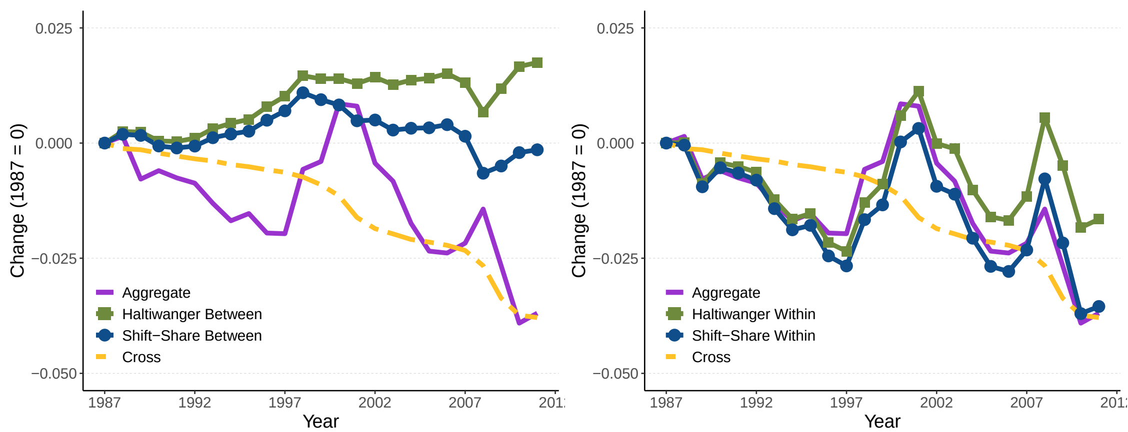
I begin by decomposing the payroll labour share. The advantage of examining the payroll labour share is that payroll labour income represents the part of the labour share accruing unambiguously to labour. Figure 4 shows the cumulative paths of the between, within, and cross terms estimated for each year using both the shift-share and Haltiwanger decomposition.

The left panel in figure 4 shows the cumulative between-sector terms for each decomposition. The shift-share decomposition replicates the results from Elsby et al. (2013) and estimates that between-sector reallocation adds a cumulative -0.1pp to the labour share from 1987 to 2011, around 4% of the total decline. Using the shift-share decomposition, one would conclude there is a zero reallocation effect. However, the cross effect estimated in the Haltiwanger decomposition is negative and growing over time. Via Theorem 1, a negative cross term reduces the shift-share between-sector term relative to the Haltiwanger between-sector term. The Haltiwanger decomposition estimates that reallocation cumulatively adds 1.7pp to the payroll labour share, around -47% of the total decline. The Haltiwanger decomposition recovers a positive reallocation effect, in line with the counterfactual evidence presented in Section 3 and in contrast to previous studies.

Moreover, the negative within-sector contribution in the shift-share decomposition is overestimated due to the negative cross term. In the right panel, The shift-share decomposition apportions 96% of the total decline in the labour share to within-sector mechanisms, whereas the Haltiwanger decomposition apportions only 45%. Since the between, within, and cross contributions add up to 100%, the Haltiwanger decomposition implies co-movements between sectoral weights and labour shares account for 102% of the decline in the labour share. The negative co-movement between ω_{it} and λ_{it} is the dominant factor in the declining payroll labour share.

Figure 4. Cumulative decomposition of the payroll labour share

(a) Between-sector contributions **(b)** Within-sector contributions



Notes: Cumulative between, within and cross terms for both decomposition methods using the 'payroll labour share' approach, 1987 to 2011. '0.05' on the y-axis corresponds to 5 percentage points (pp). The Aggregate and Cross lines are equivalent across both panels.
Source: BEA, Elsby et al. (2013) replication package, and author's calculations.

The Haltiwanger cross term is negative since changes in value-added affect sectoral weights and sectoral labour shares in opposite directions: VA_{it} appears in the numerator and denominator, respectively. Appendix A.3 shows the cross term is negative for virtually every sector analysed. Given the quantitative importance of the cross term, my results imply value-added dynamics within sectors are important for the path of the aggregate payroll labour share. Kehrig and Vincent (2021) report a similar finding using establishment-level data in the US manufacturing sector.

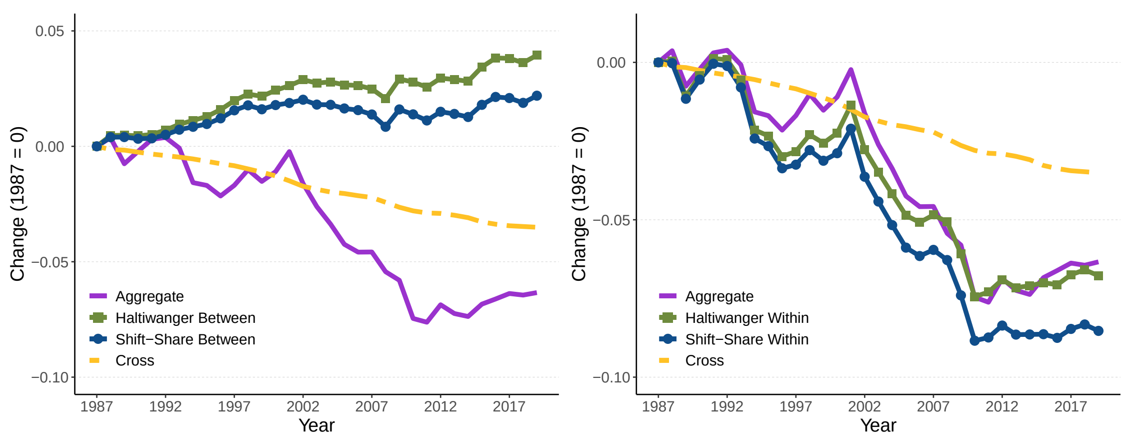
5.2 Accounting For Self-Employed Labour Income

While the payroll labour share captures the part of the aggregate labour share accruing unambiguously to labour income, how do the results differ when self-employed labour income is accounted for? Figures 5, 6, and 7 plot the cumulative shift-share and Haltiwanger decompositions for the three self-employed labour income imputation methods I use. In contrast to the 'payroll labour share' results, all three shift-share decompositions estimate a positive and non-negligible reallocation effect. Looking at the three left panels in figures 5, 6, and 7, between-sector changes add to the aggregate labour share measures. Nevertheless, each panel shows the cross effect is negative, so, as a result, the shift-share decomposition undercounts the positive reallocation effect and overcounts the negative within-sector effect. Netting out the impact of the co-movement, the Haltiwanger decomposition lines show that reallocation adds between 14.5pp and 27.7pp more to the aggregate labour share than the shift-share method estimates. Similarly, the Haltiwanger decomposition estimates that declining labour shares within sectors account for between 14.5pp and 27.7pp less than the shift-share method estimates.

Table 2 indicates the number of percentage points that the shift-share decomposition undercounts the effect of reallocation and overcounts the effect of declining labour shares within sectors relative to the Haltiwanger decomposition. Regardless of the labour share definition, using the shift-share method severely undercounts the positive effect of reallocation and loads onto the within-sector contribution. From the cumulative Haltiwanger decompositions in figures 4, 5, 6, and 7, the quantitative importance of all three channels - between, within, and cross - for changes in the aggregate labour share emerges. Sectoral labour shares cannot be studied in isolation to examine the decline of the aggregate labour share.

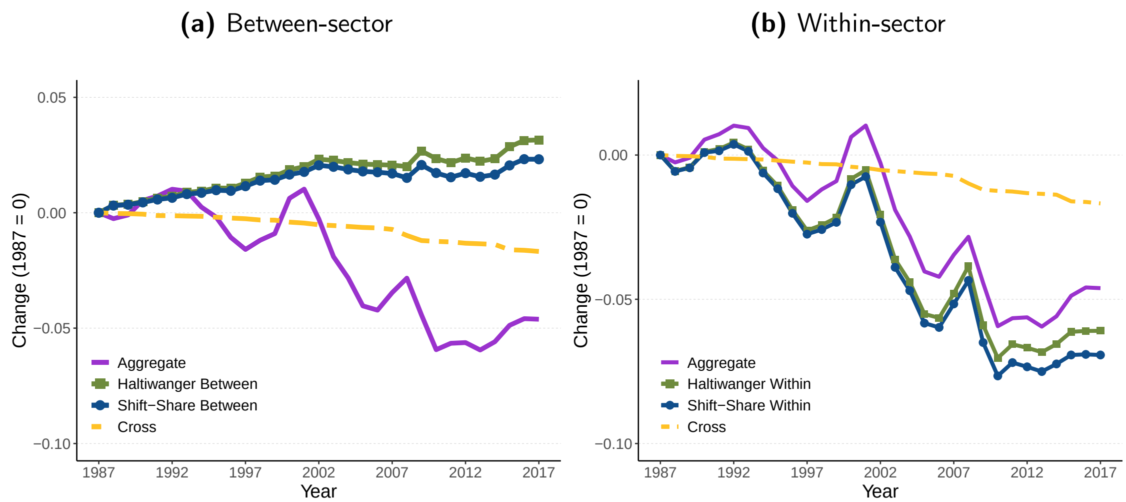
Figure 5. Cumulative decompositions of aggregate labour share

(a) Between-sector **(b)** Within-sector



Notes: Cumulative between, within and cross terms for both decomposition methods using the 'same-wage-distribution' assumption, 1987-2019. '0.05' on the y-axis corresponds to 5 percentage points (pp). The scale of the left and right panels are different, but the Aggregate and Cross lines are equivalent.
Source: BEA-BLS integrated industry-level production account and author's calculations.

Figure 6. Cumulative decompositions of aggregate labour share

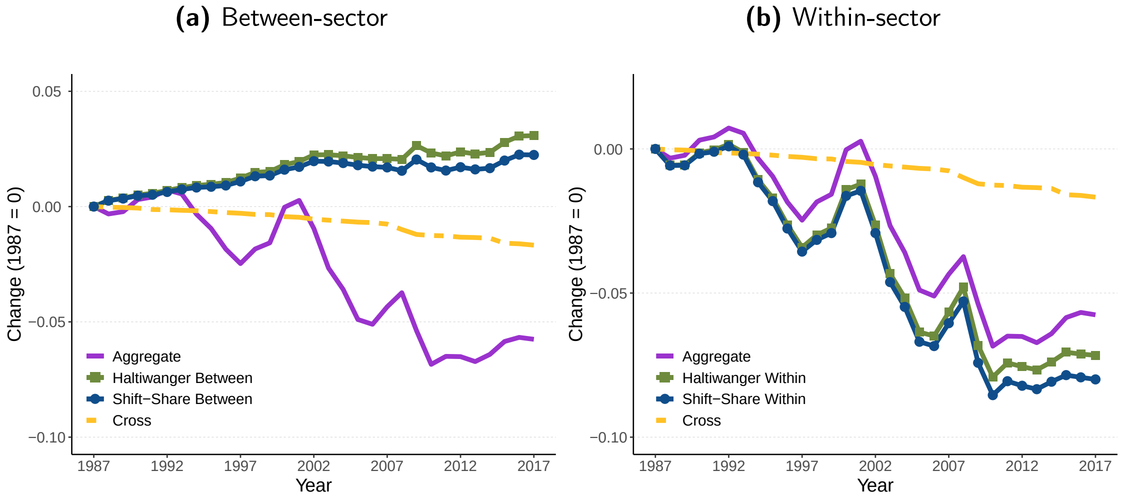


Notes: Cumulative between, within and cross terms for both decomposition methods using the 'same-labour-share' assumption, 1987-2017. '0.05' on the y-axis corresponds to 5 percentage points (pp). The scale of the left and right panels are different, but the Aggregate and Cross lines are equivalent.
Source: BEA, BLS, Mendieta-Muñoz et al. (2021) replication package, and author's calculations.

6 Conclusion

Reallocation towards high-labour-share sectors offsets around half of the decline in the aggregate labour share since the mid-1980s, an order of magnitude larger than previous studies. Declining labour shares within sectors are still an important force driving the decline in the aggregate labour share, but the contributions are not as large as previously thought. The reason is the commonly used shift-share decomposition does not explicitly account for simultaneous reallocation between sectors with different labour share levels and within-sector changes in labour shares. As a result, the shift-share decomposition terms intended to capture reallocation and within-sector contributions also capture the co-movement between sectoral weights and labour shares. By using the Haltiwanger decomposition, I explicitly account for the mismeasurement and recover the missing reallocation effect.

Figure 7. Cumulative decompositions of aggregate labour share



Notes: Cumulative between, within and cross terms for both decomposition methods using the 'economy-wide labour share' assumption, 1987-2017. '0.05' on the y-axis corresponds to 5 percentage points (pp). The scale of the left and right panels are different, but the Aggregate and Cross lines are equivalent.
Source: BEA, BLS, Mendieta-Muñoz et al. (2021) replication package, and author's calculations.

Table 2. Percentage point under- or over-counting of shift-share relative to Haltiwanger decomposition

	Between-sector undercounted	Within-sector overcounted
A. Payroll labour share. *Years*: 1987-2011	-51.3	51.3
B. Same-wage-distribution. *Years*: 1987-2019	-27.7	27.7
C. Same-labour-share. *Years*: 1987-2017	-18.2	18.2
D. Economy-wide. *Years*: 1987-2017	-14.5	14.5

Notes: The 'Between-sector undercounted' column subtracts the percentage of the labour share decline accounted for by the shift-share between-sector term from the percentage accounted for by the Haltiwanger between term. The 'Within-sector overcounted' column subtracts the shift-share within-sector term from the Haltiwanger within-sector term. I calculate the amount for each labour share definition and sample period.
Source: BEA, BLS, and author's calculations.

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A Appendix Tables And Figures

A.1 Data Sources And Construction

Table A1 shows the data source and any sample notes for each of the four labour share definitions.

Appendix Table A1. Information about samples.

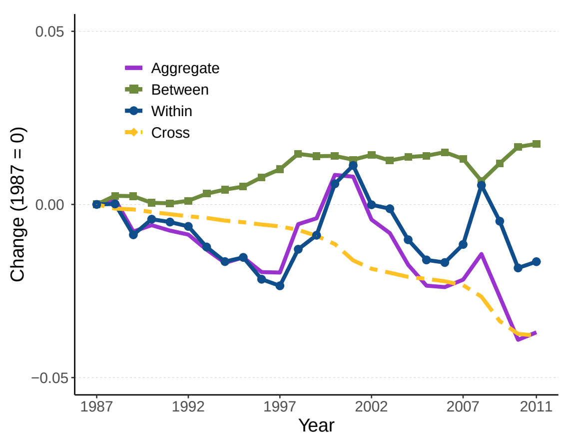
Measure	Source	Years	Notes
Payroll	BEA & Elsby et al. (2013)	1987-2011	Non-farm private sector
Same-wage-distribution	BEA-BLS integrated industry-level production account	1987-2019	Private sector excluding real estate
Same-labour-share	BEA & (Mendieta-Muñoz et al., 2021)	1987-2017	Private sector excluding real estate
Economy-wide	BEA & (Mendieta-Muñoz et al., 2021)	1987-2017	Private sector excluding real estate

A.2 Effect Of Aggregating Sectoral Data

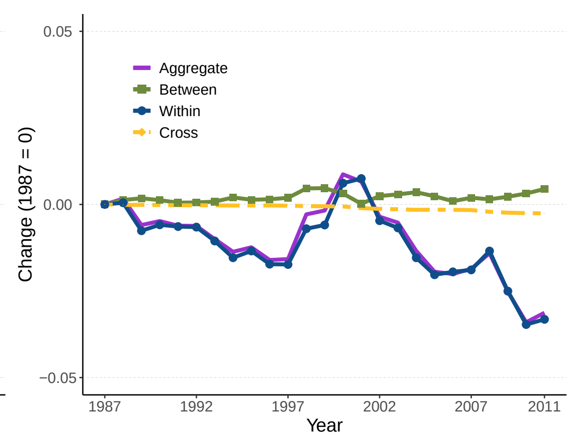
Aggregating the economy into coarser sector classifications washes out the reallocation effect. The left panel in figure A1 plots the cumulative Haltiwanger decomposition using the 'payroll labour share' approach and uses data from 60 sectors defined at the NAICS 3-digit level. There is a positive reallocation effect. The decomposition in the right panel in figure A1 uses weights and labour shares aggregated into ten NAICS 2-digit industries. Both the between-sector and cross terms disappear - the Between and Cross lines are flat - even though reallocation is quantitatively important at the 3-digit classification level. The reason is that between-sector reallocation between any two NAICS 3-digit industries will always be counted as a within-sector contribution at the NAICS 2-digit level when the 3-digit industries are in the same 2-digit classification. Aggregating the economy into coarser definitions loads onto the within-sector contribution. I attempt to nullify the issue by using the most disaggregated data possible for each labour share definition.

Appendix Figure A1. Between-sector effect disappears when aggregating NAICS 3-digit to NAICS 2-digit

(a) NAICS 3-digit sectors



(b) NAICS 2-digit sectors



Notes: Cumulative between, within, cross terms for the Haltiwanger method using the payroll labour share. The left panel uses data at the NAICS 3-digit level (60 sectors) and the right panel uses the same data but aggregated into NAICS 2-digit level (10 sectors). '0.05' on the y-axis corresponds to 5 percentage points (pp).
Source: BEA and the Elsby et al. (2013) replication package.

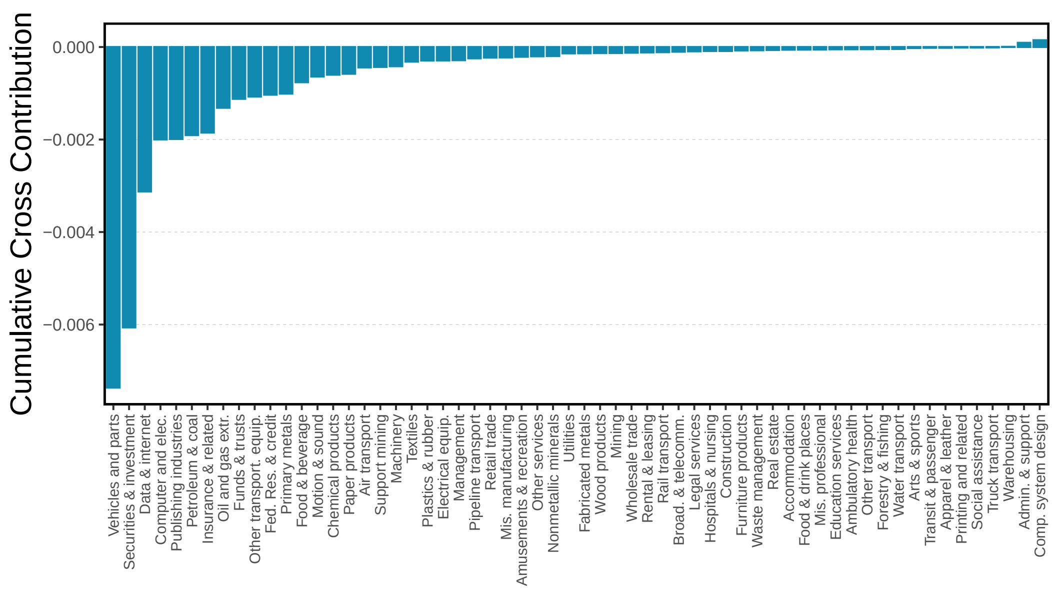
A.3 What Is The Cumulative Haltiwanger Cross Contribution For Each Sector?

For each sector, I sum the cross contribution over each year of the 'payroll labour share' sample

$$\underbrace{\sum_{t=1}^{T} \Delta\omega_{jt}\Delta\lambda_{jt}}_{\text{Haltiwanger: Cross}}$$

Figure A2 shows the cumulative Haltiwanger cross contribution is negative for virtually every sector. The reason is that, holding labour income WL^{p}_{it} constant, changes in value-added VA_{it} generate opposite movements in a sector's weight and labour share: $\Delta\omega_{it} > 0 \implies \Delta\lambda_{it} < 0$.

Appendix Figure A2. Size of cumulative cross contribution for each sector



Notes: Each bar indicates the cumulative sum of each sector's Haltiwanger cross contribution

$$\underbrace{\sum_{t=1}^{T} \Delta\omega_{jt}\Delta\lambda_{jt}}_{\text{Haltiwanger: Cross}}$$

Source: BEA, Elsby et al. (2013) replication package, and author's calculations.

B Theorem 1 Proof

The Haltiwanger decomposition for the change in the labour share, $\Delta\lambda_t = \lambda_t - \lambda_{t-1}$, is

$$\Delta\lambda_t = \underbrace{\sum_{i=1}^{N} \Delta\omega_{it}\lambda_{it-1}}_{\text{Between}} + \underbrace{\sum_{i=1}^{N} \omega_{it-1}\Delta\lambda_{it}}_{\text{Within}} + \underbrace{\sum_{i=1}^{N} \Delta\omega_{it}\Delta\lambda_{it}}_{\text{Cross}}$$

in which $\Delta x_{it} = x_{it} - x_{it-1}$. Divide the 'Cross' term into two

$$\Delta\lambda_t = \underbrace{\sum_{i=1}^{N} \Delta\omega_{it}\lambda_{it-1}}_{\text{Between}} + \underbrace{\sum_{i=1}^{N} \omega_{it-1}\Delta\lambda_{it}}_{\text{Within}} + \underbrace{\frac{1}{2}\sum_{i=1}^{N} \Delta\omega_{it}\Delta\lambda_{it}}_{\equiv \text{ Cross: A}} + \underbrace{\frac{1}{2}\sum_{i=1}^{N} \Delta\omega_{it}\Delta\lambda_{it}}_{\equiv \text{ Cross: B}}. \tag{B1}$$

Add 'Cross: A' to the 'Within' component in equation (B1)

$$\begin{aligned} &= \frac{1}{2}\sum_{i=1}^{N} \Delta\omega_{it}\Delta\lambda_{it} + \sum_{i=1}^{N} \omega_{it-1}\Delta\lambda_{it} \\ &= \sum_{i=1}^{N} \frac{\Delta\omega_{it}}{2}\Delta\lambda_{it} + \sum_{i=1}^{N} \omega_{it-1}\Delta\lambda_{it}. \end{aligned}$$

Collect like terms

$$\begin{aligned} &= \sum_{i=1}^{N} \left(\frac{\Delta\omega_{it}}{2} + \omega_{it-1}\right)\Delta\lambda_{it} \\ &= \sum_{i=1}^{N} \left(\frac{\omega_{it} - \omega_{it-1}}{2} + \omega_{it-1}\right)\Delta\lambda_{it} \\ &= \sum_{i=1}^{N} \left(\frac{\omega_{it} + \omega_{it-1}}{2}\right)\Delta\lambda_{it} \\ &= \sum_{i=1}^{N} \tilde{\omega}_{it}\Delta\lambda_{it} \end{aligned} \tag{B2}$$

in which I denote $\tilde{x}_{it}$ as the arithmetic mean of x_{it-1} and x_{it}. The last line in equation (B2) is the within-sector term in shift-share decomposition. Next, add 'Cross: B' to the 'Between' component in equation (B1)

$$= \frac{1}{2}\sum_{i=1}^{N} \Delta\omega_{it}\Delta\lambda_{it} + \sum_{i=1}^{N} \Delta\omega_{it}\lambda_{it-1}$$
$$= \sum_{i=1}^{N} \Delta\omega_{it}\frac{\Delta\lambda_{it}}{2} + \sum_{i=1}^{N} \Delta\omega_{it}\lambda_{it-1}.$$

Collect like terms

$$\begin{aligned}
&= \sum_{i=1}^{N} \left(\frac{\Delta\lambda_{it}}{2} + \lambda_{it-1}\right)\Delta\omega_{it} \\
&= \sum_{i=1}^{N} \left(\frac{\lambda_{it} - \lambda_{it-1}}{2} + \lambda_{it-1}\right)\Delta\omega_{it} \\
&= \sum_{i=1}^{N} \left(\frac{\lambda_{it} + \lambda_{it-1}}{2}\right)\Delta\omega_{it} \\
&= \sum_{i=1}^{N} \Delta\omega_{it}\tilde{\lambda}_{it}
\end{aligned} \tag{B3}$$

in which $\tilde{x}_{it}$ denotes the arithmetic mean of x_{it-1} and x_{it}. The last line in equation (B3) is the between-sector term in the shift-share decomposition. □