Intro to Databases (COMP_SCI 339)

06 B+Tree Index



ADMINISTRIVIA

Project #2 is due Sunday 2/4 @ 11:59pm

Exam #1 will be on 1/29 from 3:30-4:50pm

EXAM #1

Who: You

What: Exam #1

Where: Here

When: Monday 1/29 from 3:30-4:50pm

What to bring:

- → Pencil or pen with dark-colored ink
- → **One** double-sided 8.5x11" page of handwritten notes

DATA STRUCTURES

Internal Meta-data
Core Data Storage
Temporary Data Structures

Table Indexes

TABLE INDEXES

A <u>table index</u> is a replica of a subset of a table's attributes that are organized and/or sorted for efficient access using those attributes.

The DBMS ensures that the contents of the table and the index are logically synchronized.

TABLE INDEXES

It is the DBMS's job to figure out the best index(es) to use to execute each query.

There is a trade-off regarding the number of indexes to create per database.

- → Storage Overhead
- → Maintenance Overhead

TODAY'S AGENDA

B+Tree Overview
Use in a DBMS
Design Choices
Optimizations

B-TREE FAMILY

There is a specific data structure called a **B-Tree**.

People also use the term to generally refer to a class of balanced tree data structures:

- → **B-Tree** (1971)
- → **B+Tree** (1973)
- → **B*Tree** (1977?)
- → **B**link**-Tree** (1981)

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The Ubiquitese D-Tree

DOUBLAS COMES

Congrete: Science Department, Partice University, West Leftweite, Indiana 1790;

B-bress have become de facto, a standard for Me-organization. File inferes of users, dedicated database aratems, and general-summers around methods have all term proposed and amplemented using Ritress This paper writers Ritress and those why they bear here so recounted Indicessors the major-variations of the B-two, separably the B'-tree. contracting the relative mostle and more of each implementation. It illustrates a general propose notion method which uses a Ditrie.

Legenseds and Phonese T. tone, 24, year, 3" tree, 6th experiention, index

CR Catagorius: 3.23 1.71 430 434

INTRODUCTION

The secondary storage facilities available on large computer systems allow users to store update, and recall data from large collections of information called files. A computer must rottieve an item and place it in main memory before it can be procrossed. In order to make good use of the computer resources, one must organize files intelligently, making the retrieval process. officient.

The choice of a good file organisation depends on the kinds of recrieval to be performed. There are two broad classes of natrieval communes which can be illustweed by the following ramples:

Sequential: "From our employee file, prepare a list of all empleyees' names and addresses," and

"Proces was compleyed file, extract the information about employee J. Smith"

We car imagine a filing cabinet with three folders. drawers of folders, one folder for each errplayers. The drawers might be labeled "A- by considering last pursus as index entres.

might be labeled with the employees' last names. A requestial recessed receives the searcher to examine the settle file, one folder at a time. On the other hand, or moden request implies that the searcher. guided by the labels on the degwers and folders read only extract one folder

Associated with a large, randomly as consed file in a computer system is an index which, like the labels on the trewers and folders of the life cabinet, speech retrieval by directing the searcher to the small part. of the file containing the desired terr. Piguse I depicts a file and its index. An index may be physically integrated with the Mr. like the labels on employee to ders, or playsically separate. Ike the labels on the drawere. Usually the index itself is a file. If the inces file is large, mother index may be built on top of it to speed retrieved further, and so se. The resulting hierarchy is similar to the employee life, where the topmost incer consists of landson drowers, and the next level of index consists of labels on

Natural hierarchies, like the one formed "H.R." and "S.L." while the felders do not always produce the last perform-

formassion to rapp without the all or part of this material is greated provided that the regres are not made or distributed for denot communical advantages the NFM copposable notice and the title of the publication and as they argue, and notice as reconstant recovery in by parameters of the Association for Computing Machinery. To convertherwise, or to republish, removes a fee and/or specific personales. © 1875 ACM 4010-4802-09-9600-012 \$40.75

B+TREE

A **B+Tree** is a self-balancing tree data structure that keeps data sorted and allows searches, sequential access, insertions, and deletions always in **O(log n)**.

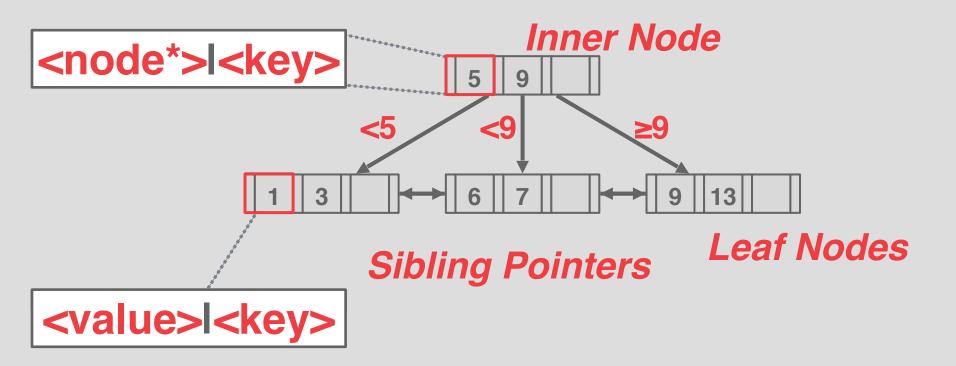
- → Generalization of a binary search tree, since a node can have more than two children.
- → Optimized for systems that read and write large blocks of data.

B+TREE PROPERTIES

A B+Tree is an *M*-way search tree with the following properties:

- → It is perfectly balanced (i.e., every leaf node is at the same depth in the tree)
- → Every node other than the root is at least half-full M/2-1 ≤ #keys ≤ M-1
- → Every inner node with k keys has k+1 non-null children

B+TREE EXAMPLE

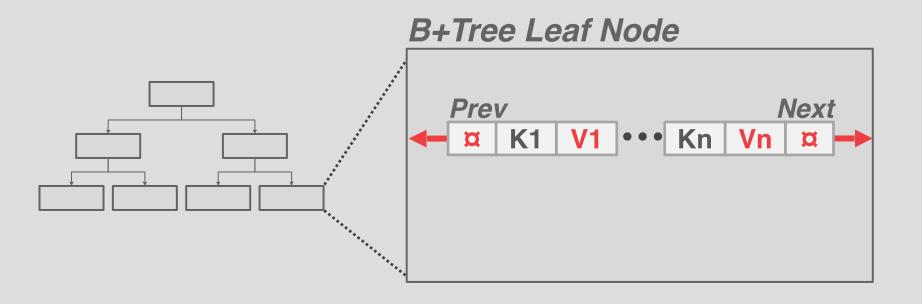


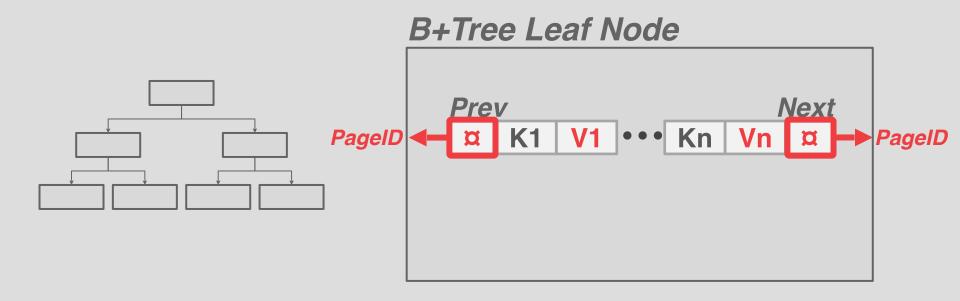
NODES

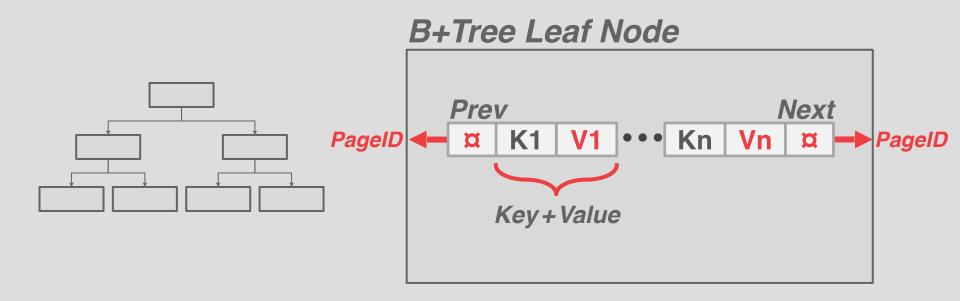
Every B+Tree node is comprised of an array of key/value pairs.

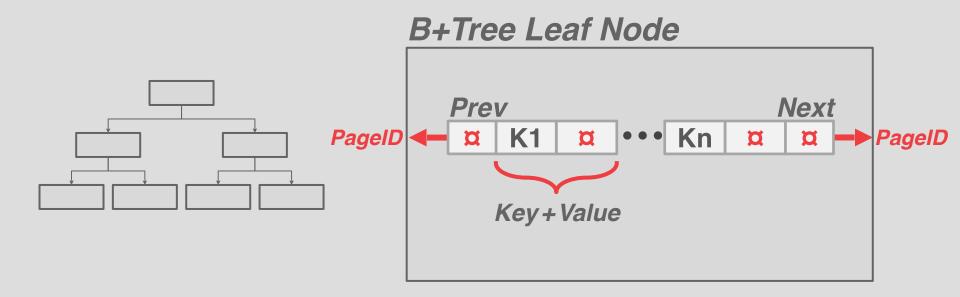
- → The keys are derived from the attribute(s) that the index is based on.
- → The values will differ based on whether the node is classified as an <u>inner node</u> or a <u>leaf node</u>.

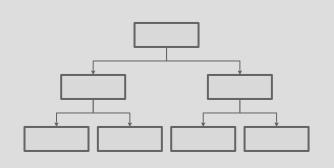
The arrays are (usually) kept in sorted key order.



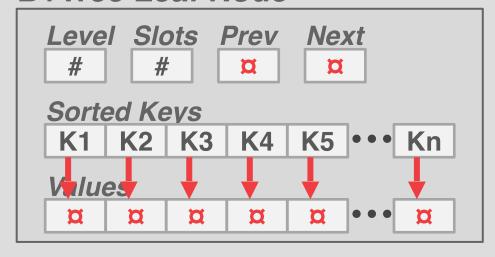








B+Tree Leaf Node



LEAF NODE VALUES

Approach #1: Record IDs

→ A pointer to the location of the tuple to which the index entry corresponds.









Approach #2: Tuple Data

- → The leaf nodes store the actual contents of the tuple.
- → Secondary indexes must store the Record ID as their values.









B-TREE VS. B+TREE

The original **B-Tree** from 1972 stored keys and values in all nodes in the tree.

→ More space-efficient, since each key only appears once in the tree.

A **B+Tree** only stores values in leaf nodes. Inner nodes only guide the search process.

B+TREE - INSERT

Find correct leaf node L.
Insert data entry into L in sorted order.
If L has enough space, done!
Otherwise, split L keys into L and a new node L2

- → Redistribute entries evenly, copy up middle key.
- → Insert index entry pointing to L2 into parent of L.

To split inner node, redistribute entries evenly, but push up middle key.

B+TREE - DELETE

Start at root, find leaf L where entry belongs. Remove the entry.

If L is at least half-full, done!

If L has only M/2-1 entries,

- → Try to re-distribute, borrowing from sibling (adjacent node with same parent as L).
- → If re-distribution fails, merge L and sibling.

If merge occurred, must delete entry (pointing to L or sibling) from parent of L.

B+TREE VISUALIZATION

https://cmudb.io/btree

Source: David Gales (Univ. of San Francisco)

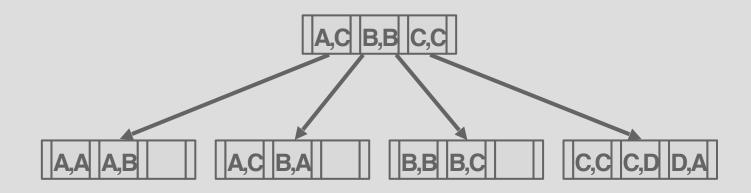
The DBMS can use a B+Tree index if the query provides any of the attributes of the search key.

Example: Index on <a,b,c>

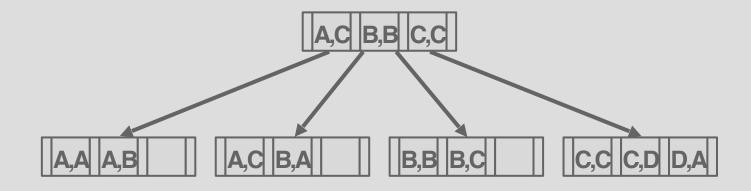
- → Supported (a=1 AND b=2 AND c=3)
- → Supported: (a=1 AND b=2)
- → Supported: (b=2), (c=3)

Not all DBMSs support this.

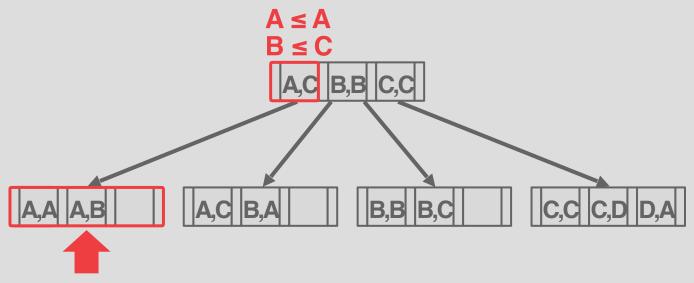
For a hash index, we must have all attributes in search key.



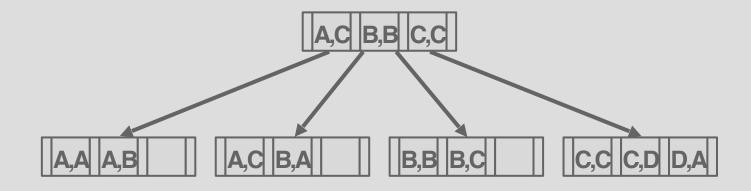
Find Key=(A,B)



Find Key=(A,B)



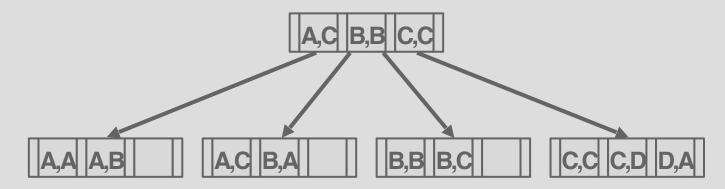
Find Key=(A,B)

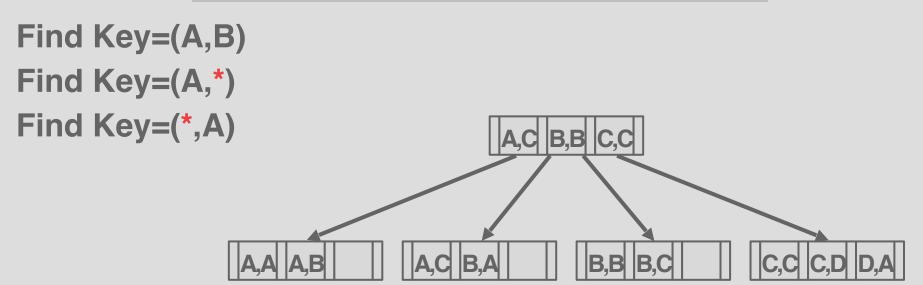


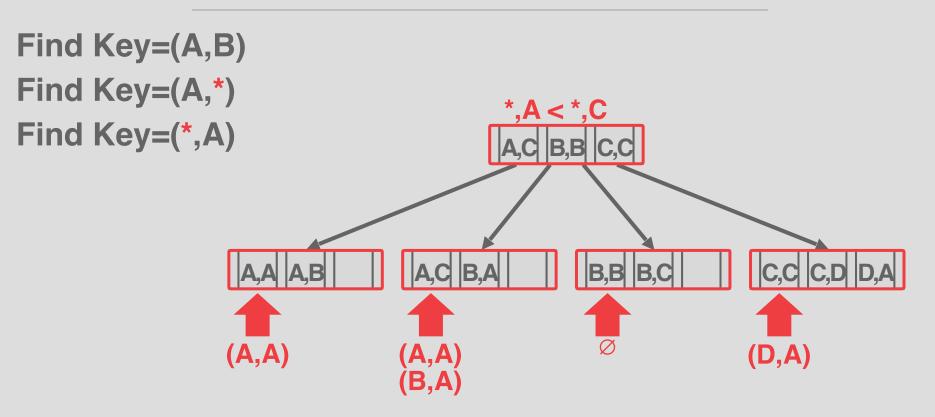
Find Key=(A,B) Find Key=(A,*) $A \leq A$ A,C B,B C,C A.C B,A B,B B,C

Find Key=(A,B) Find Key=(A,*) $A \leq A$ A,C B,B C,C A,C B,A B,B B,C $(\mathsf{A},^*) \leq (\mathsf{B},^*)$

Find Key=(A,B) Find Key=(A,*)

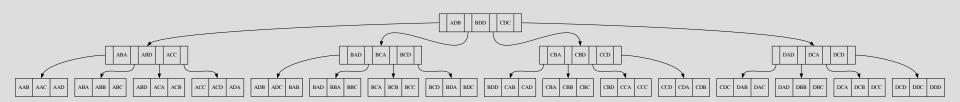






Example: Index on <col1,col2,col3>

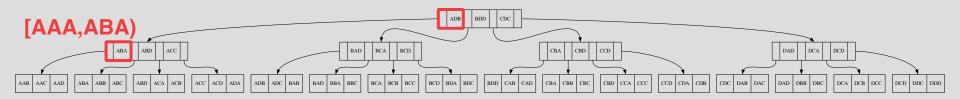
- → Column Values: {A,B,C,D}
- → Supported: col2 = B



Example: Index on <col1,col2,col3>

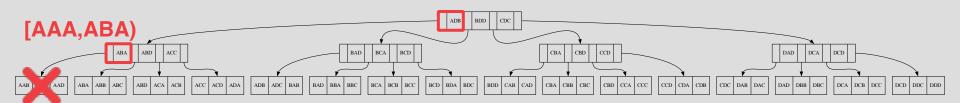
→ Column Values: {A,B,C,D}

→ Supported: col2 = B

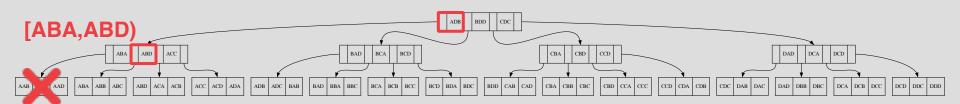


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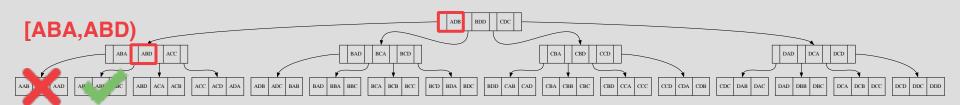
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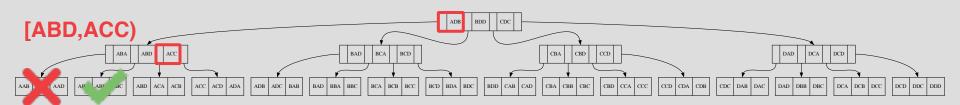
- → Column Values: {A,B,C,D}
- → Supported: col2 = B



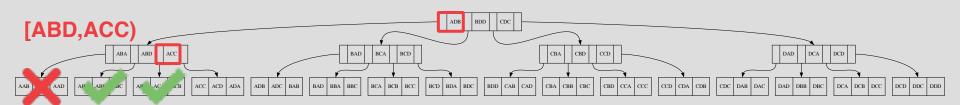
- → Column Values: {A,B,C,D}
- → Supported: col2 = B



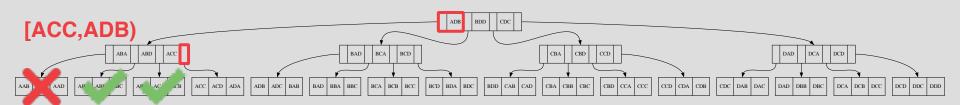
- → Column Values: {A,B,C,D}
- → Supported: col2 = B



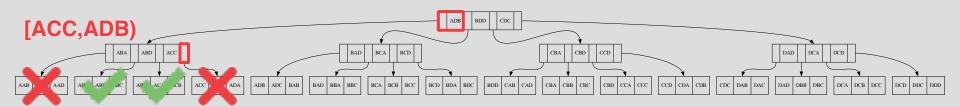
- → Column Values: {A,B,C,D}
- → Supported: col2 = B



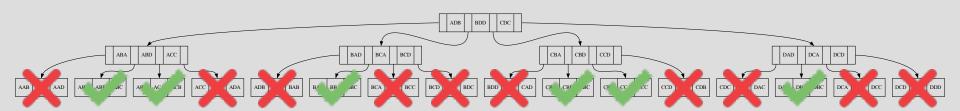
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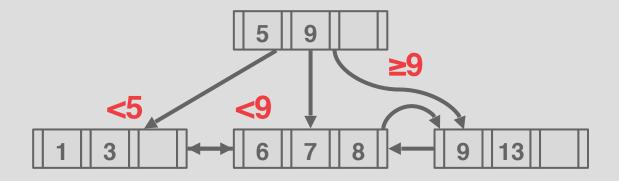
B+TREE - DUPLICATE KEYS

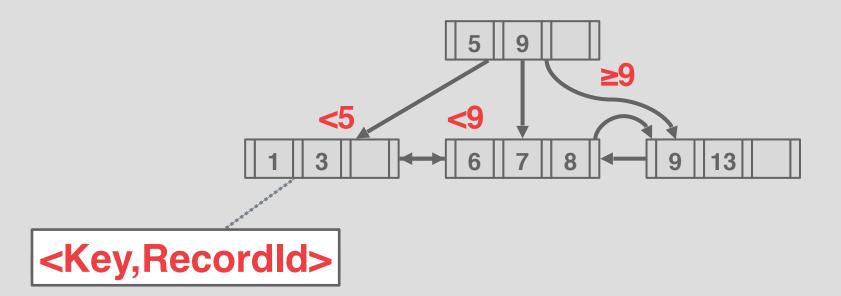
Approach #1: Append Record ID

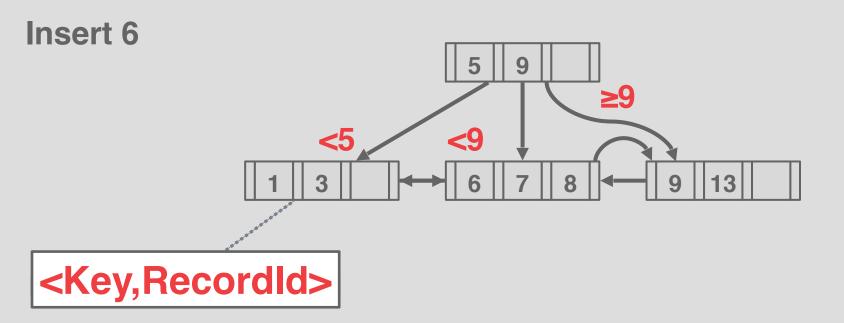
- → Add the tuple's unique Record ID as part of the key to ensure that all keys are unique.
- → The DBMS can still use partial keys to find tuples.

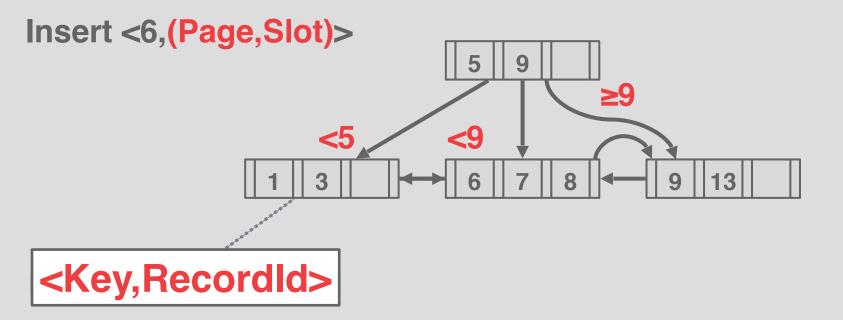
Approach #2: Overflow Leaf Nodes

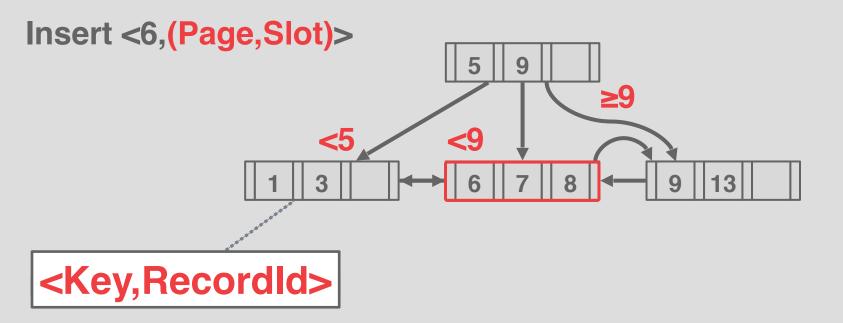
- → Allow leaf nodes to spill into overflow nodes that contain the duplicate keys.
- → This is more complex to maintain and modify.

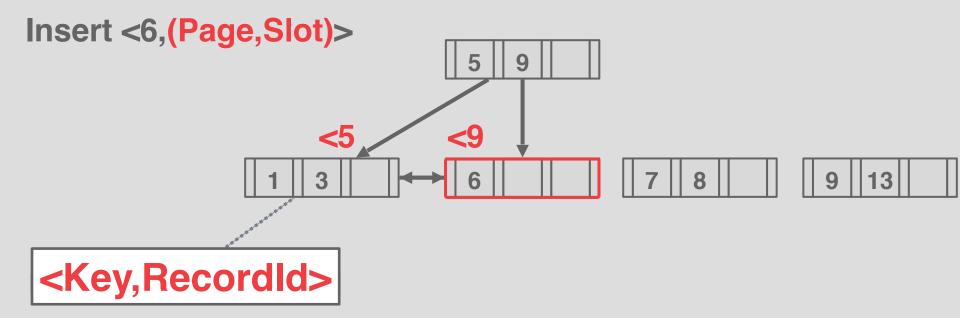


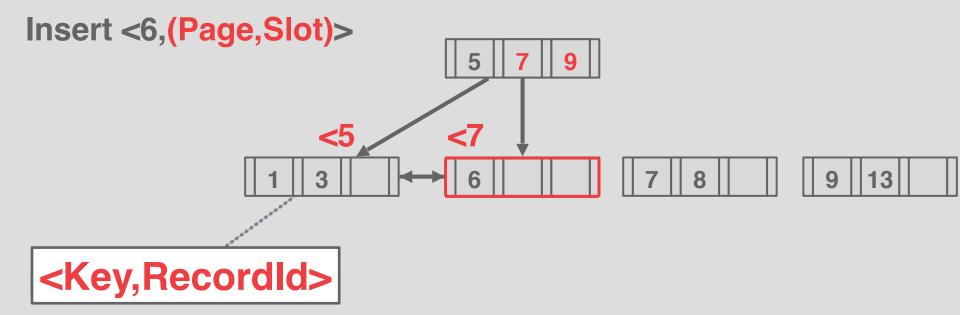


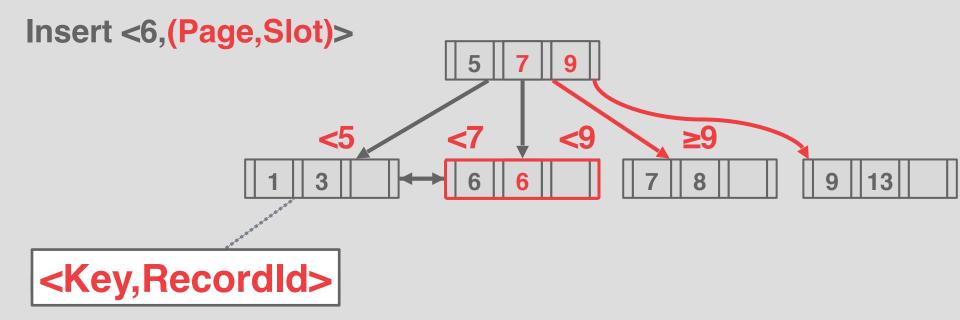


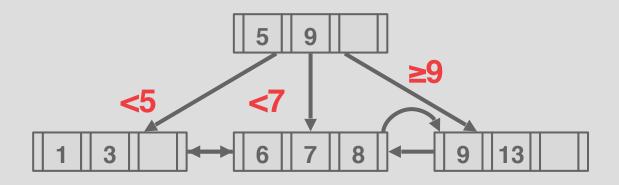


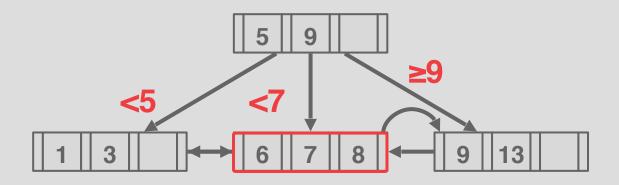


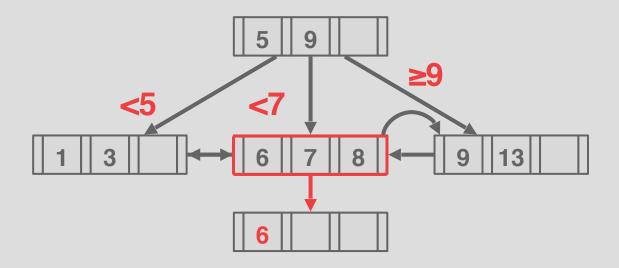




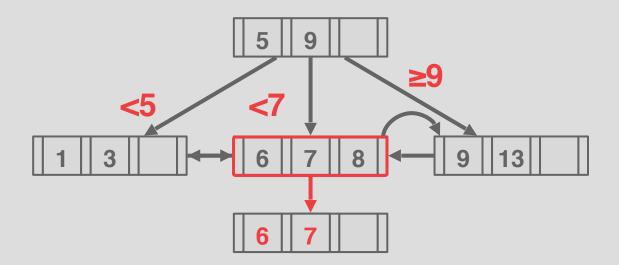






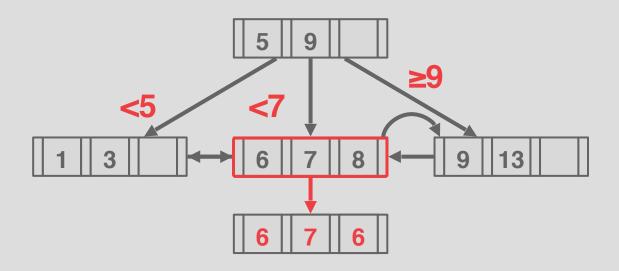


Insert 6



Insert 6

Insert 7



CLUSTERED INDEXES

The table is stored in the sort order specified by the primary key.

→ Can be either heap- or index-organized storage.

Some DBMSs always use a clustered index.

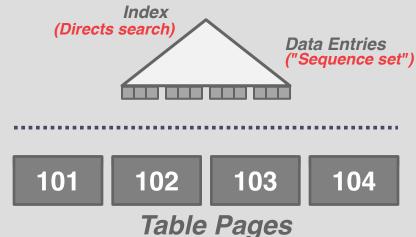
→ If a table does not contain a primary key, the DBMS will automatically make a hidden primary key.

Other DBMSs cannot use them at all.

CLUSTERED B+TREE

Traverse to the left-most leaf page and then retrieve tuples from all leaf pages.

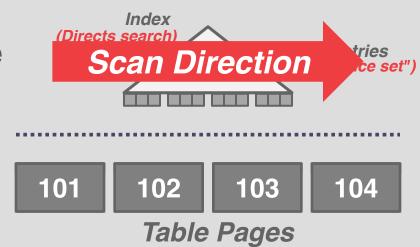
This will always be better than sorting data for each query.



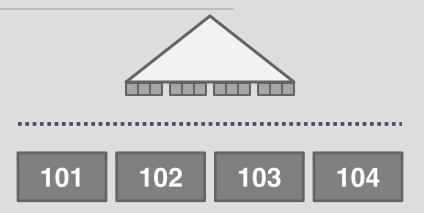
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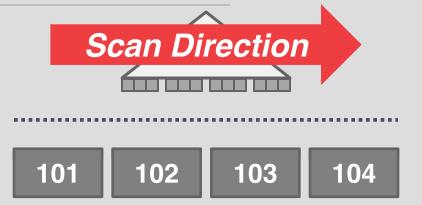
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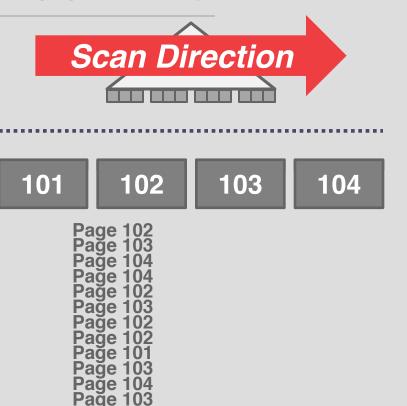
Retrieving tuples in the order they appear in a non-clustered index is inefficient due to redundant reads.



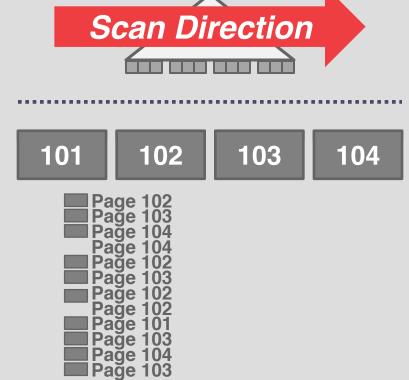
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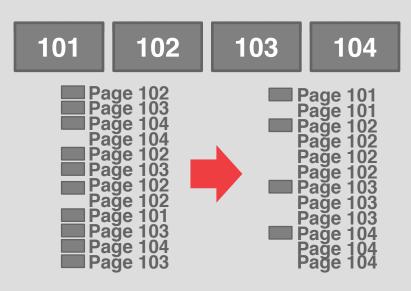


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B+TREE DESIGN CHOICES

Node Size
Merge Threshold
Variable-Length Keys
Intra-Node Search



NODE SIZE

The slower the storage device, the larger the optimal node size for a B+Tree.

- → HDD: ~1MB
- → SSD: ~10KB
- → In-Memory: ~512B

Optimal sizes can vary depending on the workload

→ Leaf Node Scans vs. Root-to-Leaf Traversals

MERGE THRESHOLD

Some DBMSs do not always merge nodes when they are half full.

Delaying a merge operation may reduce the amount of reorganization.

It may also be better to just let smaller nodes exist and then periodically rebuild entire tree.

VARIABLE-LENGTH KEYS

Approach #1: Pointers

→ Store the keys as pointers to the tuple's attribute.

Approach #2: Variable-Length Nodes

- → The size of each node in the index can vary.
- → Requires careful memory management.

Approach #3: Padding

→ Always pad the key to be max length of the key type.

Approach #4: Key Map / Indirection

→ Embed an array of pointers that map to the key + value list within the node.

INTRA-NODE SEARCH

Approach #1: Linear

- → Scan node keys from beginning to end.
- → Use SIMD to vectorize comparisons.

Approach #2: Binary

→ Jump to middle key, pivot left/right depending on comparison.

Approach #3: Interpolation

→ Approximate location of desired key based on known distribution of keys.

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Find Key=8



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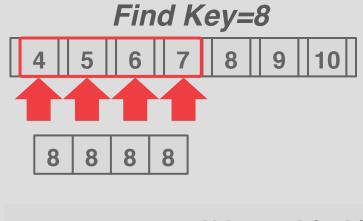
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OPTIMIZATIONS

Prefix Compression

Deduplication

Suffix Truncation

Pointer Swizzling

Bulk Insert

Buffer Updates

Many more...

PREFIX COMPRESSION

Sorted keys in the same leaf node are likely to have the same prefix.

robbed robbing robot

Instead of storing the entire key each time, extract common prefix and store only unique suffix for each key.

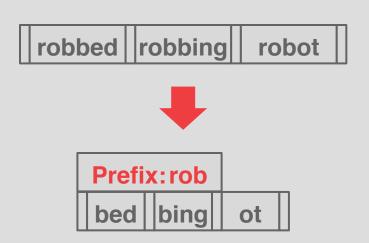
→ Many variations.

PREFIX COMPRESSION

Sorted keys in the same leaf node are likely to have the same prefix.

Instead of storing the entire key each time, extract common prefix and store only unique suffix for each key.

→ Many variations.



DEDUPLICATION

Non-unique indexes can end up storing multiple copies of the same key in leaf nodes.

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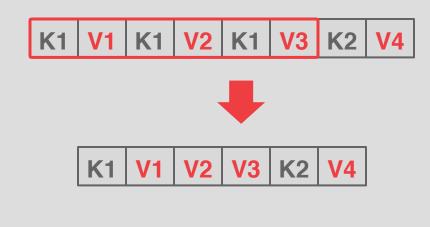
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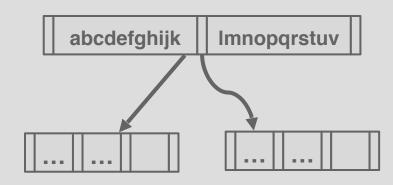


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The keys in the inner nodes are only used to "direct traffic".

→ We don't need the entire key.

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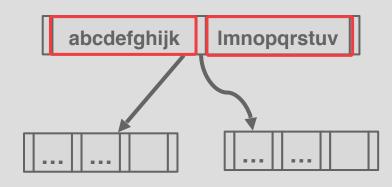


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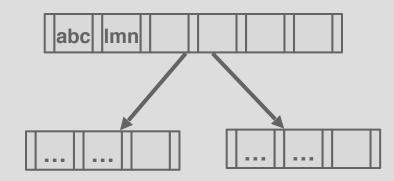


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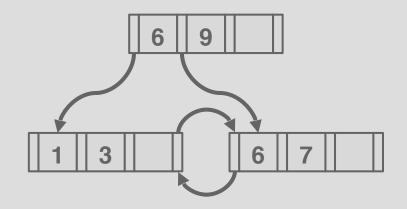
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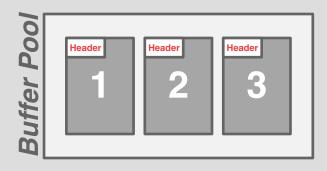
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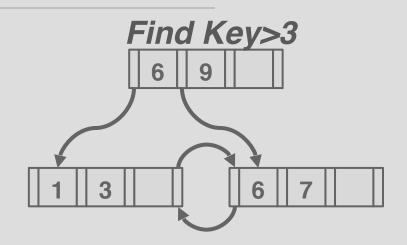
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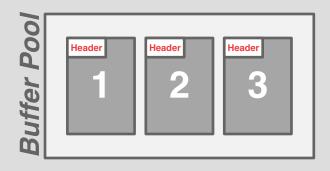
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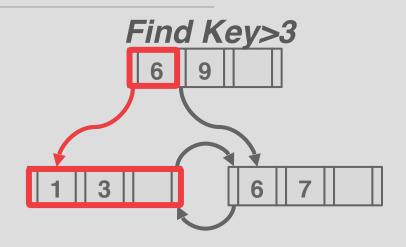


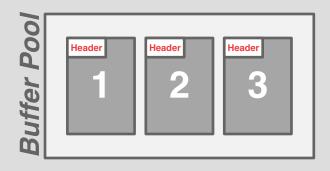
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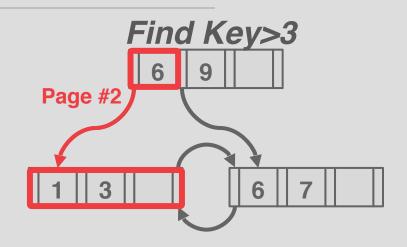


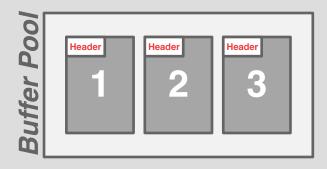
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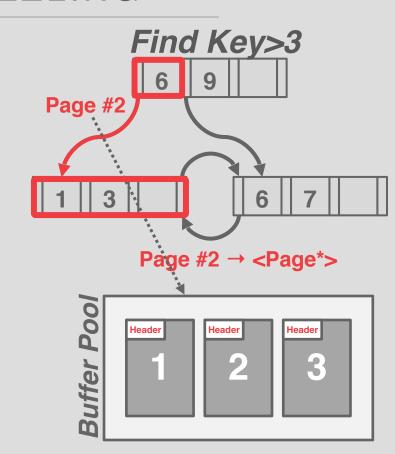


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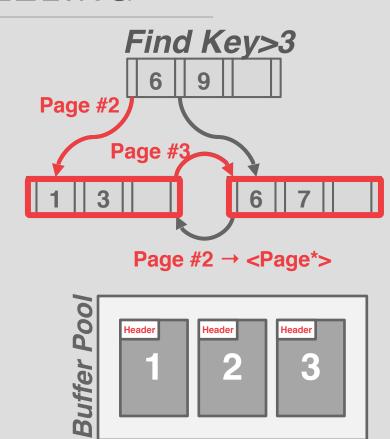




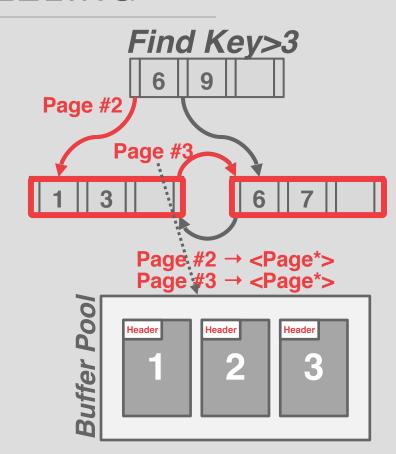
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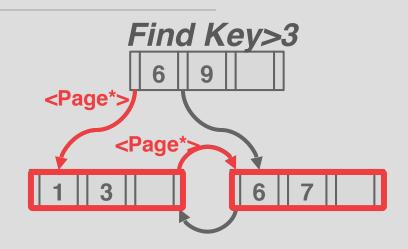
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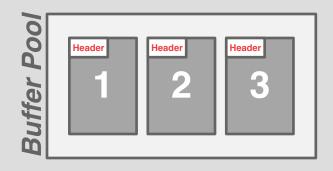


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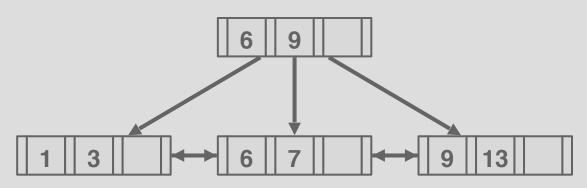
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CONCLUSION

The B+Tree is (almost) always a good choice for an index in your DBMS.

NEXT CLASS

Index Concurrency Control