CSCI 2824
Midterm Exam 1
Fall 2018

**Name:** Rubric

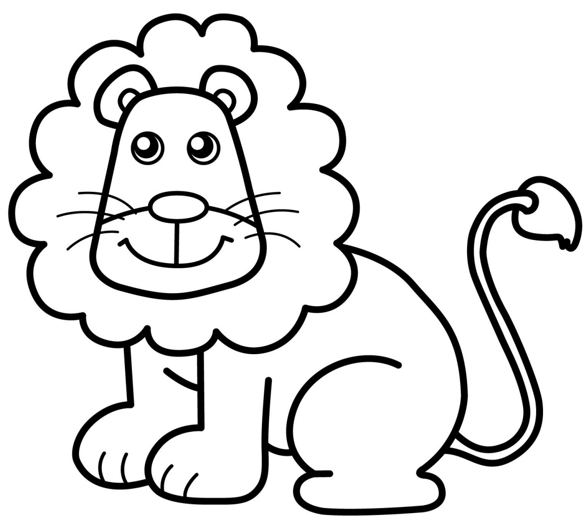
By writing my name I promise to abide by the Honor Code

**Section number:** ____________________

**Read the following:**

- **RIGHT NOW**! Write your name and section number on the top of your exam.
- You are allowed one 8 1/2 $\times$ 11in sheet of **handwritten** notes (both sides). No magnifying glasses!
- You may use a calculator provided that it cannot access the internet or store large amounts of data.
- You may **NOT** use a smartphone as a calculator.
- Clearly mark answers to multiple choice questions on the provided answer line.
- Mark only one answer for multiple choice questions. If you think two answers are correct, mark the answer that **best** answers the question. No justification is required for multiple choice questions.
- If you do not know the answer to a question, skip it and come back to it later.
- For free response questions you must clearly justify all conclusions to receive full credit. A correct answer with no supporting work will receive no credit.
- If you need more space for free-response questions, there are blank pages at the end of the exam. If you choose to use the extra pages, make sure to **clearly** indicate which problem you are continuing.
- You have **90 minutes** for this exam.

| Page | Points | Score |
|---|---|---|
| 2 | 12 | |
| 3 | 12 | |
| 4 | 16 | |
| 5 | 20 | |
| 6 | 20 | |
| 7 | 20 | |
| Total | 100 | |



1. (3 points) `Robot Rachel` is going to prison. Oh dear! There are 24 robot inmates in her cell block. What length bit-string is needed in order to assign each of them a unique binary inmate number?

   A. 6

   **B. 5**

   C. 4

   D. 3

   E. 2

1. ________ ________

2. (3 points) What rule of inference or logical fallacy is demonstrated by the following argument? *If $n$ is a real number such that $n > 1$, then $n^2 > 1$. Suppose that $n^2 > 1$. Then $n > 1$.*

   A. Denying the hypothesis

   B. Modus Ponens

   **C. Affirming the conclusion**

   D. Modus Tollens

   E. Contrapositive

2. ________ ________

3. (3 points) What can you say about the following argument? *If an integer $n$ is divisible by 2, then it is prime. 6 is not prime. Thus, 6 is not divisible by 2.*

   A. The argument is invalid. It is an instance of the fallacy of Denying the Hypothesis.

   B. The argument is invalid. It is an instance of the fallacy of Affirming the Conclusion.

   **C. The argument is valid. It is an example of Modus Tollens.**

   D. The argument is valid. It is an example of Modus Ponens.

3. ________ ________

4. (3 points) For the following set of premises, what relevant conclusion can be drawn? *"I am dreaming or hallucinating." "I am not dreaming." "If I am hallucinating, then I see elephants running down the road."*

   **A. I see elephants running down the road.**

   B. I am not hallucinating.

   C. I am dreaming.

   D. If I see elephants running down the road, then I am hallucinating.

4. ________ ________

5. (3 points) Let $A$ and $B$ by finite sets such that $A \subseteq B$. Consider the following statements. Which of the following is/are necessarily true?

   (1) $\emptyset \subseteq A$

   (2) $P(A) \subseteq P(B)$

   A. Statement (1) is true, but statement (2) is not necessarily true

   B. Statement (1) is not necessarily true, but statement (2) is true

   **C. Both statements are true**

   D. Both statements are not necessarily true

5. ______ ______

6. (3 points) Let $A = \{lions, tigers, bears, ohmy\}$ and $B = \{1, 2\}$. What is the correct choice for $A$ x $B$?

   A. $\{(1, lions), (1, tigers), (1, bears), (1, ohmy), (2, lions), (2, tigers), (2, bears), (2, ohmy)\}$

   B. $\{\emptyset, \{1, lions\}, \{1, tigers\}, \{1, bears\}, \{1, ohmy\}, \{2, lions\}, \{2, tigers\}, \{2, bears\}, \{2, ohmy\}\}$

   C. $\{\emptyset, \{lions, 1\}, \{tigers, 1\}, \{bears, 1\}, \{ohmy, 1\}, \{lions, 1\}, \{tigers, 2\}, \{bears, 2\}, \{ohmy, 2\}\}$

   **D.** $\{(lions, 1), (tigers, 1), (bears, 1), (ohmy, 1), (lions, 2), (tigers, 2), (bears, 2), (ohmy, 2)\}$

6. ______ ______

7. (3 points) Tony and Rachel got hungry while grading these exams and ordered a light snack of 20 pizzas. 8 pizzas have pepperoni as a topping, 6 pizzas have mushrooms, and 3 pizzas have both pepperoni and mushrooms. How many pizzas have **neither** pepperoni nor mushrooms as a topping?

   A. 3

   B. 8

   **C. 9**

   D. 11

7. ______ ______

8. (3 points) Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{0, 3, 6\}$. Find $A - B$.

   A. $\{0, 1, 2, 4, 5, 6\}$

   B. $\{0, 6\}$

   C. $\{3\}$

   **D.** $\{1, 2, 4, 5\}$

8. ______ ______

## Begin: Short-answer problems

9. (4 points) You and Rachel are at the zoo, and she tells you this about the monkeys: "For all of the monkeys in this zoo, it is the case that monkeys with a short tail always have big fangs."

   Let the domain of discourse be all monkeys in the zoo, and translate Rachel's rule into a quantifier statement. Be sure to define all propositional functions you might use.

10. (4 points) Consider the following quantifier statements, where the domain for all variables is the integers. Label each of the following as True (T) or False (F) by circling the appropriate letter **clearly**. No justification is needed.

    1 pt/each

    a. $\forall x\ \exists y\ (xy = 0)$ T F

    b. $\forall x\ \exists y\ \exists z\ (xy = z)$ T F

    c. $\exists x\ \exists y\ \forall z\ (xy = z)$ T F

    d. $\exists x\ \exists z\ \forall y\ (xy = z)$ T F

11. (4 points) Consider the two propositions below regarding the empty set, $\emptyset$. For each, (1) determine if the statement is true or false, and (2) justify your answer.

    +2 a. $\emptyset \in \{0, 1\}$

    +2 b. $\emptyset \subset \{0, 1\}$

12. (4 points) Determine if the following argument is valid or invalid. Then, select the appropriate rule of inference or logical fallacy that it demonstrates. Mark your answers **clearly** using the boxes provided.

    *If I work all night on my Discrete Structures homework, then I can answer all of the exam questions. If I answer all of the exam questions, then I will receive a good grade. Therefore, if I work all night on my Discrete Structures homework, then I will receive a good grade.*

**Solution:**

☐ **Valid**

☐ Invalid

+2

☐ denying the hypothesis
☐ affirming the conclusion
☐ **hypothetical syllogism**
☐ disjunctive syllogism
☐ resolution
☐ modus ponens
☐ modus tollens

+2

13. (20 points) Use truth tables to solve the following problems.

Be sure to show **all work** (i.e., all relevant columns of the truth table). You **must** use a truth table to receive points.

8 (a) Prove the following logical equivalence **using a truth table**.

$$p \to q \equiv (p \land \neg q) \to (q \land \neg q)$$

**Solution**: $p \to q \equiv (p \land \neg q) \to (q \land \neg q)$

$\}$ Correct values in each column +6

Circling (drawing arrows to matching TT columns is fine

Since they have the same column in the truth table, the two are logically equivalent.

12 (b) Suppose the following set of compound propositions represents a set of constraints. Use a **truth table** to determine if the set of constraints is **satisfiable**. Clearly state and fully justify your conclusion. Again, be sure to show **all relevant columns** of the truth table, including intermediate work.

$$p \to \neg q \qquad q \to r \qquad r \land p$$

14. (20 points) Prove the following. Show **all** work, and be sure to name which type(s) of proof you use.

10 (a) Prove that $n$ is even if and only if $n^2 - 6n + 5$ is odd.

10 (b) Suppose $n$ is a positive integer and define the integer $m = 1 + (-1)^n(2n - 1)$.
Prove that 4 divides $m$ (in other words, show that $m$ is a multiple of 4).

15. (20 points) Prove the following using the permitted logical equivalences and rules of inference. Be sure to cite which rules or equivalences you use on each line of your proofs, and **use only one rule/equivalence per line**. Recall that there is only a specific set of equivalences and rules of inference that you are permitted to use. You may **not** use truth tables.

Note: in the exam version, there was a typo. So, grade gently.

6/ (a) Suppose $p$, $q$ and $r$ are all propositions. Prove the following logical equivalence.

$$(p \to q) \land r \equiv \neg(r \to (p \land \neg q))$$

14/ (b) Suppose we have propositional functions $P(x)$, $Q(x)$ and $R(x)$, for variable $x$ defined on some domain. Prove that the following argument is valid. Be sure to cite which rules of inference or logical equivalences you use on each line of your proof, indicate which line numbers are involved in each step, and use only one rule/equivalence per line.

$$\begin{array}{rl} \text{Premise:} & \neg\exists x\ \neg(P(x) \to \neg Q(x)) \\ \text{Premise:} & \exists x\ (P(x) \land R(x)) \\ \hline \text{Conclusion:} & \exists x\ (\neg Q(x) \land R(x)) \end{array}$$