CSCI 2824
Midterm Exam
Fall 2017

Name: ______________________

By writing my name I promise to abide by the Honor Code

Read the following:

- **RIGHT NOW**! Write your name on the top of your exam.
- You are allowed one 8 1/2 $\times$ 11in sheet of **handwritten** notes (both sides). No magnifying glasses!
- You may use a calculator provided that it cannot access the internet or store large amounts of data.
- You may **NOT** use a smartphone as a calculator.
- Clearly mark answers to multiple choice questions on the provided answer line.
- Mark only one answer for multiple choice questions. If you think two answers are correct, mark the answer that **best** answers the question. No justification is required for multiple choice questions.
- If you do not know the answer to a question, skip it and come back to it later.
- For free response questions you must clearly justify all conclusions to receive full credit. A correct answer with no supporting work will receive no credit.
- If you need more space for free-response questions, there are blank pages at the end of the exam. If you choose to use the extra pages, make sure to **clearly** indicate which problem you are continuing.
- You have **90 minutes** for this exam.

Page	Points	Score
2	12	
3	12	
4	9	
5	6	
6	20	
7	20	
8	20	
For Luck	1	1
Total	100	

1. (3 points) What rule of inference or logical fallacy is demonstrated by the following argument? *Either Chris got enough sleep last night, or he is grouchy today. Chris is going to work out or he did not get enough sleep last night. Thus, Chris is going to work out or he is grouchy today.*

   A. disjunctive syllogism

   B. affirming the conclusion

   C. modus tollens

   D. resolution

   E. hypothetical syllogism

1. ______ ______

2. (3 points) Let the propositional function $S(x, y)$ represent the statement "x is standing in line behind y", and let the domain for x and y be the set of all people standing in line at an ice cream shop. Represent the following statement using quantifiers and S: "Everyone is standing in line behind someone else."

3. (3 points) What can you say about the following argument? *If an integer n is not prime then it can be written as a product of two or more primes.* 17 *is not prime. Therefore* 17 *can be written as the product of two or more primes.*

   A. The argument is valid. It is an example of Modus Ponens

   B. The argument is valid. It is an example of Modus Tollens

   C. The argument is invalid because 17 is prime

   D. The argument is invalid. It is an instance of the fallacy of Denying the Hypothesis

3. ______ ______

4. (3 points) Let x be an element of nonempty set A, and suppose $x \neq \emptyset$. Consider the following statements.

   (1) $\emptyset \in A$

   (2) $\emptyset \subset A$

   Which of the following is/are necessarily true?

   A. Statement (1) is true, but statement (2) is not necessarily true

   B. Statement (1) is not necessarily true, but statement (2) is true

   C. Both statements are true

   D. Both statements are not necessarily true

4. ______ ______

5. (3 points) An exam given in two subject areas: Discrete Structures (D) and Data Science (S), was taken by 20 students. The following table shows how many students failed each subject and in their various combinations (e.g. 9 people failed Discrete and 5 people failed both Discrete and Data Science). How many people **passed** both sections of the exam?

D	S	D and S
9	8	5

A. 3
B. 8
C. 12
D. 13

5. ______ ______

6. (3 points) Suppose A and B are finite sets such that there are m distinct elements in A and n distinct elements in B, and $A \subseteq B$. What is the cardinality of the power set $\mathcal{P}(A \cap B)$?

A. $m \cdot n$
B. 2^m
C. 2^n
D. 2^{m+n}

6. ______ ______

7. (3 points) Let $A = (-\infty, -3]$, $B = [3, \infty)$, and $C = [-3, 3]$. Decide which **one** of the following is true:

A. $A \cap C$ **is finite**
B. $A \cap C$ is countably infinite
C. $A \cap C$ is uncountably infinite
D. $A \cap C$ is the empty set

7. ______ ______

8. (3 points) Consider the function $f : \mathbb{Z} \to \mathbb{Z}$ defined by $f(n) = n^2 + 1$. Decide which **one** of the following is true:

A. $f(n)$ is both one-to-one and onto
B. $f(n)$ is one-to-one but not onto
C. $f(n)$ is onto but not one-to-one
D. $f(n)$ **neither one-to-one nor onto**

8. ______ ______

9. (3 points) Suppose you have an algorithm `foo` that takes as input a sequence of numbers of length n and does something with it. You do some timing benchmarks for `foo` by testing it on sequences of increasing length and get the following table (with times shown in seconds).

n	times
2	2.0
4	8.0
8	32.0

Based on the timings, what can you conclude is the complexity of the procedure?

A. `foo` is order n

B. `foo` is order n^2

C. `foo` is order n^3

D. `foo` is order $\log 2$

9. _____ _____

10. (3 points) Consider **element-wise** matrix multiplication of $n \times n$ matrices A and B (`elemMult(A,B)`), where $C[i][j]$ (the element in row i and column j of matrix C) is given by $A[i][j] \cdot B[i][j]$. What is the complexity of `elemMult(A,B)`, where complexity is measured by the number of multiplications needed?

```
procedure elemMult(A, B, n):
   C = zeros(n,n)
   for i from 1 to n:
      for j from 1 to n:
         C[i][j] = A[i][j]*B[i][j]
   return C
```

A. `elemMult` is order n^4

B. `elemMult` is order n^3

C. `elemMult` is order n^2

D. `elemMult` is order n

10. _____ _____

11. (3 points) What is the **smallest** integer p such that $f(n) = 7n^3 + (\log(n))^5 + n\log(n^3)$ is $\mathcal{O}(n^p)$?

A. $p = 3$

B. $p = 4$

C. $p = 5$

D. $p = 6$

11. _____ _____

12. (3 points) Select the answer that is a closed form solution to this recurrence relation: $a_n = 2a_{n-1} + 1$, $a_0 = 0$

    A. $a_n = 2^n + 1$

    B. $a_n = 2^n - \frac{1}{2}$

    C. $a_n = 2n - 1$

    D. $a_n = 2^n - 1$

12. ______ ______

13. (3 points) Suppose a and t are integers, s and m are positive integers, and that $as + mt \equiv 2 \pmod{m}$. Which of the following necessarily must be true?

    A. $as \equiv 1 \pmod{m}$

    B. $as \equiv 2 \pmod{m}$

    C. $as = 1$

    D. $as + mt = 0$

13. ______ ______

14. (20 points) On the Island of Knights and Knaves live two types of people: Knights who always tell the truth and Knaves who always lie. As you are exploring the Island of Knights and Knaves you encounter two people named A and B. A tells you "If I am a Knight, then B and I are both Knights."

   (a) Clearly define any propositional variables that you intend to use.

   (b) Translate the statement that A makes into symbolic logic using the propositional variables you defined above.

   (c) Use a **truth table** to determine the natures of A and B. Clearly state and justify your conclusion.

15. (20 points) Prove the following. Show **all** work, and be sure to name which type(s) of proof you use.

(a) Let x be a three-digit number. Prove that if the sum of the digits of x is divisible by 9, then x is also divisible by 9.

For example, $x = 369$*. The sum of the digits of* x *is* $3 + 6 + 9 = 18$*, which is divisible by 9. Thus,* $x = 369$ *is divisible by 9.*

(b) Prove that if $3n^2 + 2n + 1$ is even, then n is odd.

16. (20 points) Consider the function $f(n) = 3n^3 + n\log(n^3) - n^2$.

(a) Find a tight big-$\mathcal{O}$ bound for $f(n)$. Be sure to specify your values of C and k in the definition of big-$\mathcal{O}$.

(b) Find a tight big-Ω bound for $f(n)$. Be sure to specify your values of C and k in the definition of big-Ω.

(c) Can you state that $f(n)$ is $\Theta(g(n))$ for some function $g(n)$? If so, state $g(n)$ and briefly justify your reasoning.