Wind Farm + Battery Portfolio: Comparative studies of Stochastic MILP and QAOA formulation

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1 Problem statement (data and assumptions): Version A

We consider a one-day horizon with $T = 24$ hourly time steps. A wind farm produces energy subject to uncertainty, represented by $S = 13$ equally-probable forecast scenarios. A battery energy storage system (BESS) can buy from and sell to the grid at the hourly market price.

Key simplification. The battery is assumed to buy and sell all its energy *from/to the grid at the market price*, *without physical netting with the wind farm.* Therefore, wind production does *not* constrain battery charging in the model.

2 Sets, indices, and parameters

Sets and indices

- Hours: $t \in \mathcal{T} := \{1, \dots, 24\}$.
- Scenarios: $s \in \mathcal{S} := \{1, \dots, 13\}$.

Scenario data

- p_t : electricity market price at hour t MWh, common to all scenarios.
- $w_{t,s}$: wind production at hour t under scenario s [MWh].
- Scenario probabilities: $\pi_s = \frac{1}{13}$ for all $s \in \mathcal{S}$.

Battery parameters

Parameter	Value
Energy capacity	$E^{\max} = 16$ MWh
Max charge power	$P^{\mathrm{ch,max}} = 5$ MW
Max discharge power	$P^{\mathrm{dis,max}} = 4$ MW
Charge efficiency	$\eta^{\mathrm{ch}} = 0.8$
Discharge efficiency	$\eta^{\mathrm{dis}} = 1.0$
Max full cycles/day	$N^{\max} = 2$
Initial SOC	$e_0 = 0$ MWh
Final SOC	$e_{24} = 0$ MWh

3 Day-ahead stochastic MILP formulation (expected revenue)

3.1 Decision variables (scenario-independent day-ahead schedule)

For each hour $t \in \mathcal{T}$:

- $P_t^{\mathrm{ch}} \geq 0$: charging power (MW).
- $P_t^{\mathrm{dis}} \geq 0$: discharging power (MW).
- $e_t \geq 0$: state of charge (SOC) at the end of hour t (MWh).
- $y_t \in \{0, 1\}$: operating mode, where $y_t = 1$ enables charging and $y_t = 0$ enables discharging.

3.2 Objective: maximize expected portfolio revenue

The wind farm sells all wind energy at the market price; the battery buys and sells from/to the grid at the same price:

$$\max \quad \sum_{s \in \mathcal{S}} \pi_s \sum_{t \in \mathcal{T}} p_t \, w_{t,s} \; + \sum_{t \in \mathcal{T}} p_t \left(P_t^{\mathrm{dis}} - P_t^{\mathrm{ch}} \right). \tag{1}$$

Remark. Under the "no netting" simplification, the first term is constant with respect to the battery decisions; the battery schedule is therefore driven by prices and battery constraints only. The full expected-revenue objective is shown for completeness.

3.3 Constraints

SOC dynamics (hourly energy balance). With one-hour time steps (MW $\equiv$ MWh per hour):

$$e_t = e_{t-1} + \eta^{\mathrm{ch}} P_t^{\mathrm{ch}} - \frac{1}{\eta^{\mathrm{dis}}} P_t^{\mathrm{dis}}, \quad \forall t \in \mathcal{T}. \tag{2}$$

With $\eta^{\mathrm{dis}} = 1$, the last term simplifies to $-P_t^{\mathrm{dis}}$.

SOC bounds (capacity).

$$0 \leq e_t \leq E^{\max}, \quad \forall t \in \mathcal{T}. \tag{3}$$

Charge/discharge power bounds.

$$0 \leq P_t^{\mathrm{ch}} \leq P^{\mathrm{ch,max}}, \quad \forall t \in \mathcal{T}, \tag{4}$$

$$0 \leq P_t^{\mathrm{dis}} \leq P^{\mathrm{dis,max}}, \quad \forall t \in \mathcal{T}. \tag{5}$$

Mutual exclusivity: cannot charge and discharge simultaneously. Using the binary y_t:

$$P_t^{\mathrm{ch}} \leq P^{\mathrm{ch,max}} \, y_t, \qquad \forall t \in \mathcal{T}, \tag{6}$$

$$P_t^{\mathrm{dis}} \leq P^{\mathrm{dis,max}} \, (1 - y_t), \qquad \forall t \in \mathcal{T}, \tag{7}$$

$$y_t \in \{0, 1\}, \qquad \forall t \in \mathcal{T}. \tag{8}$$

Initial and terminal SOC conditions.

$$e_0 = 0, \qquad e_{24} = 0. \tag{9}$$

Daily cycling limit (maximum 2 full cycles). A common linear proxy is to limit total discharged energy (equivalent full cycles):

$$\sum_{t \in \mathcal{T}} P_t^{\text{dis}} \le N^{\max} E^{\max} = 32 \text{ MWh}. \tag{10}$$

(Alternative conservative variants may also limit total charged energy; (10) is standard and remains linear.)

3.4 Complete MILP

The stochastic day-ahead MILP is given by objective (1) subject to constraints (2)–(10).

4 What Version A assumes (plain language)

Version A is the **"no netting"** (no physical coupling) model:

- The **wind farm** produces energy each hour (uncertain). We assume we **sell all wind energy to the market** at the market price.
- The **battery** is treated as an **independent trader**: it can **buy energy from the grid to charge** and **sell energy to the grid when discharging**, both at the hourly market price.
- There is **no physical link** between wind and the battery. In particular, the battery is **not limited by wind output** when charging.

As a result, total profit is simply:

$$\text{Total profit} = \text{Wind revenue} + \text{Battery arbitrage profit}.$$

5 Sets (time and scenarios)

- Hours: $t \in \{1, 2, \dots, 24\}$.
- Wind scenarios: $s \in \{1, 2, \dots, 13\}$.
- Scenario probabilities: $\pi_s = \frac{1}{13}$ for all s (equally likely scenarios).

6 Data (parameters)

From the input file:

- p_t: market price at hour t MWh.
- $w_{t,s}$: wind energy produced at hour t under scenario s MWh.

Battery constants:

Quantity	Value
Energy capacity	$E^{\max} = 16$ MWh
Max charge per hour	$P^{ch,\max} = 5$ MWh (MW for 1-hour steps)
Max discharge per hour	$P^{dis,\max} = 4$ MWh
Charge efficiency	$\eta^{ch} = 0.8$
Discharge efficiency	$\eta^{dis} = 1.0$
Initial SOC	$e_0 = 0$ MWh
Final SOC	$e_{24} = 0$ MWh

7 Decision variables (what we choose)

Battery decisions are **day-ahead** and therefore the same regardless of which wind scenario occurs.
For each hour t:

- $P_t^{ch} \geq 0$: energy bought from the grid to charge the battery [MWh].
- $P_t^{dis} \geq 0$: energy sold to the grid when discharging [MWh].
- $e_t \geq 0$: state of charge (SOC) at the *end* of hour t [MWh].
- $y_t \in \{0, 1\}$: operating mode (binary), where:
  - $y_t = 1$ means "charging mode" is allowed,
  - $y_t = 0$ means "discharging mode" is allowed.

8 Objective function (maximize expected revenue)

8.1 Wind revenue

If scenario s happens, the wind farm sells $w_{t,s}$ at price p_t, so the scenario wind revenue is:

$$\sum_{t=1}^{24} p_t \, w_{t,s}.$$

The **expected** wind revenue (average across the 13 equally likely scenarios) is:

$$\sum_{s=1}^{13} \pi_s \sum_{t=1}^{24} p_t \, w_{t,s}.$$

8.2 Battery arbitrage profit

In each hour t:

- discharging P_t^{dis} earns revenue $p_t P_t^{dis}$,
- charging P_t^{ch} costs $p_t P_t^{ch}$.

So the battery profit over the day is:

$$\sum_{t=1}^{24} p_t \left(P_t^{dis} - P_t^{ch} \right).$$

8.3 Total objective

$$\max \left[\sum_{s=1}^{13} \pi_s \sum_{t=1}^{24} p_t \, w_{t,s} \; + \; \sum_{t=1}^{24} p_t \left(P_t^{dis} - P_t^{ch} \right) \right].$$

Important observation (why wind scenarios do not affect the battery here). The expected wind revenue term uses only data $(p_t, w_{t,s})$ and contains *no battery decision variables.* Therefore, it is a **constant** with respect to the battery schedule. Adding a constant does not change which battery plan is optimal, so in Version A the optimal battery schedule is driven by prices and battery constraints only.

9 Battery constraints (the physics)

9.1 (A) SOC (energy) balance each hour

SOC at the end of hour t equals SOC at the end of hour $t-1$, plus stored energy from charging, minus energy removed by discharging:

$$e_t = e_{t-1} + \eta^{ch} P_t^{ch} - \frac{1}{\eta^{dis}} P_t^{dis}, \qquad \forall t = 1, \ldots, 24.$$

With $\eta^{ch} = 0.8$ and $\eta^{dis} = 1.0$, this becomes:

$$e_t = e_{t-1} + 0.8 \, P_t^{ch} - P_t^{dis}.$$

9.2 (B) SOC bounds (cannot overfill or go negative)

$$0 \le e_t \le E^{\max} = 16, \qquad \forall t.$$

9.3 (C) Charge and discharge limits (power constraints)

$$0 \le P_t^{ch} \le P^{ch,\max} = 5, \qquad \forall t,$$
$$0 \le P_t^{dis} \le P^{dis,\max} = 4, \qquad \forall t.$$

9.4 (D) Cannot charge and discharge in the same hour

We use the binary mode variable y_t to enforce "either charge or discharge (or idle)."

$$P_t^{ch} \le P^{ch,\max} y_t = 5 y_t, \qquad \forall t,$$
$$P_t^{dis} \le P^{dis,\max} (1 - y_t) = 4(1 - y_t), \qquad \forall t,$$
$$y_t \in \{0, 1\}, \qquad \forall t.$$

How to read this.

- If $y_t = 1$ (charging mode), then $P_t^{ch} \le 5$ and $P_t^{dis} \le 0$ (so discharging is forced to zero).
- If $y_t = 0$ (discharging mode), then $P_t^{ch} \le 0$ (charging forced to zero) and $P_t^{dis} \le 4$.

9.5 (E) Start empty and end empty

$$e_0 = 0, \qquad e_{24} = 0.$$

This makes the schedule "daily" and prevents ending the day with leftover energy in the battery.

9.6 (F) Optional: maximum 2 full cycles per day (simple proxy)

A common linear proxy limits total discharged energy:

$$\sum_{t=1}^{24} P_t^{dis} \leq 2E^{\max} = 32.$$

Interpretation: the battery cannot discharge more than two times its full capacity across the day.

10 Why this is a MILP

The model is a **Mixed-Integer Linear Program (MILP)** because:

- all equations and inequalities are **linear** in the decision variables,
- there is a **binary** variable y_t (integer), while the power and SOC variables are continuous.

11 Key takeaway

In Version A, the battery operates like a **price-arbitrage machine**:

- charge when prices are low,
- discharge when prices are high,

subject to battery constraints. Wind uncertainty affects the expected wind revenue, but **does not change the optimal battery schedule** because the battery is not physically coupled to wind in this simplified model.

12 Model idea (Version B in words)

Version B explicitly **physically couples** the wind farm and the battery. In each hour and scenario, wind energy can be:

- **sold** to the grid,
- **used to charge** the battery,
- **curtailed**.

The battery schedule (charge/discharge amounts) is decided **day-ahead** (before knowing which wind scenario occurs). After the scenario is realized, the operator can decide how much of the planned charging comes from **wind** versus **grid** (recourse), subject to any limits.

13 Parameters (given data)

- p_t : electricity market price in hour t MWh (common to all scenarios).
- $w_{t,s}$: wind energy produced in hour t under scenario s MWh.
- Scenario probabilities: $\pi_s = \frac{1}{13}$ for all $s \in \mathcal{S}$ (equally likely).

Battery parameters

Quantity	Value
Energy capacity	$E^{\max} = 16$ MWh
Max charge input per hour	$U^{\max} = 5$ MWh
Max discharge per hour	$D^{\max} = 4$ MWh
Charge efficiency	$\eta^{ch} = 0.8$
Discharge efficiency	$\eta^{dis} = 1.0$
Max full cycles/day	$N^{\max} = 2$
Initial SOC	$e_0 = 0$
Final SOC	$e_{24} = 0$

Optional parameter (grid charging cap)

- $G_t^{\max} \geq 0$: maximum grid energy allowed to feed the charger in hour t [MWh].

Special cases:

- **Wind-only charging:** set $G_t^{\max} = 0$ for all t.
- **(Nearly) unlimited grid charging:** choose $G_t^{\max}$ very large.

14 Decision variables

14.1 First-stage (day-ahead, scenario-independent)

These decisions are fixed **before** the wind scenario is known.

For each $t \in \mathcal{T}$:

- $U_t \geq 0$: **total** energy sent into the battery charger (charging input) [MWh].
- $D_t \geq 0$: battery discharge sold to the grid [MWh].
- e_t: state of charge (SOC) at the end of hour t [MWh].
- $y_t \in \{0, 1\}$: mode (binary): $y_t = 1$ charging mode, $y_t = 0$ discharging mode.

14.2 Second-stage (recourse, scenario-dependent)

After scenario s is realized, for each (t, s):

- $x_{t,s} \geq 0$: wind sold to the grid [MWh].
- $z_{t,s} \geq 0$: wind sent to the battery charger [MWh].

- $g_{t,s} \geq 0$: grid energy sent to the battery charger [MWh].
- $c_{t,s} \geq 0$: curtailed wind [MWh].

15 Objective: maximize expected revenue

In scenario s and hour t:

- selling wind $x_{t,s}$ yields revenue $p_t x_{t,s}$,
- selling battery discharge D_t yields revenue $p_t D_t$,
- buying grid energy $g_{t,s}$ to charge costs $p_t g_{t,s}$.

Therefore, the expected (average) revenue is:

$$\max \quad \sum_{s \in \mathcal{S}} \pi_s \sum_{t \in \mathcal{T}} p_t \Big(x_{t,s} + D_t - g_{t,s} \Big). \tag{11}$$

16 Constraints

16.1 A) Wind energy balance (physical split)

All wind in hour t and scenario s must be allocated to selling, battery charging, or curtailment:

$$x_{t,s} + z_{t,s} + c_{t,s} = w_{t,s}, \qquad \forall t \in \mathcal{T},\ \forall s \in \mathcal{S}. \tag{12}$$

16.2 B) Charger supply balance (coupling wind/grid to planned charging)

The day-ahead charging input U_t must be supplied by wind-to-charger plus grid-to-charger in each scenario:

$$z_{t,s} + g_{t,s} = U_t, \qquad \forall t \in \mathcal{T},\ \forall s \in \mathcal{S}. \tag{13}$$

This equation is the core **coupling** between the wind scenario and the battery: when $w_{t,s}$ is low, less wind can be routed to the charger and more grid energy (if allowed) is needed to meet the planned charging U_t.

16.3 C) Optional grid charging cap (makes wind uncertainty "bite")

$$0 \leq g_{t,s} \leq G_t^{\max}, \qquad \forall t \in \mathcal{T},\ \forall s \in \mathcal{S}. \tag{14}$$

If $G_t^{\max} = 0$, then $g_{t,s} = 0$ and (13) forces $z_{t,s} = U_t$; combined with (12) this implies $U_t \leq w_{t,s}$ (feasible for every scenario).

16.4 D) Battery SOC dynamics (deterministic because U_t, D_t are day-ahead)

With 1-hour time steps:

$$e_t = e_{t-1} + \eta^{ch} U_t - \frac{1}{\eta^{dis}} D_t, \qquad \forall t \in \mathcal{T}. \tag{15}$$

With $\eta^{ch} = 0.8$ and $\eta^{dis} = 1.0$, this becomes $e_t = e_{t-1} + 0.8 U_t - D_t$.

16.5 E) Battery bounds (capacity and power limits)

$$0 \le e_t \le E^{\max}, \qquad \forall t \in \mathcal{T}, \tag{16}$$
$$0 \le U_t \le U^{\max}, \qquad \forall t \in \mathcal{T}, \tag{17}$$
$$0 \le D_t \le D^{\max}, \qquad \forall t \in \mathcal{T}. \tag{18}$$

16.6 F) Mutually exclusive operation (no charge and discharge simultaneously)

Using the binary variable y_t:

$$U_t \le U^{\max} y_t, \qquad \forall t \in \mathcal{T}, \tag{19}$$
$$D_t \le D^{\max}(1 - y_t), \qquad \forall t \in \mathcal{T}, \tag{20}$$
$$y_t \in \{0, 1\}, \qquad \forall t \in \mathcal{T}. \tag{21}$$

16.7 G) Initial and final SOC

$$e_0 = 0, \qquad e_{24} = 0. \tag{22}$$

16.8 H) Cycle limit (simple linear proxy)

A standard linear proxy for "at most $N^{\max}$ full cycles per day" is to bound total discharged energy:

$$\sum_{t \in \mathcal{T}} D_t \le N^{\max} E^{\max} = 32 \text{ MWh}. \tag{23}$$

17 Complete Version B model

The complete (two-stage) stochastic MILP for Version B is:

- Objective: (11)
- Constraints: (12)–(23)
- Variable domains: $U_t, D_t, e_t \in \mathbb{R}$ with bounds above; $y_t \in \{0, 1\}$; and $x_{t,s}, z_{t,s}, g_{t,s}, c_{t,s} \ge 0$.

18 Interpretation note

If grid charging is *unlimited* and priced the same as wind sales (same p_t), then using 1 MWh of wind to charge the battery instead of selling it has an opportunity cost p_t, but it also saves buying 1 MWh from the grid at the same price p_t. These cancel. In that case, the optimal day-ahead battery schedule tends to depend mainly on prices and battery constraints, similarly to Version A. To make wind uncertainty materially influence battery decisions, constraints like (14) with a small $G_t^{\max}$ (or $G_t^{\max} = 0$ for wind-only charging) are commonly used.