Piezomagnetic potentials due to an inclined rectangular fault in a semi-infinite medium

Mitsuru Utsugi [1], Yasunori Nishida [1] and Yoichi Sasai [2]

[1] *Division of Earth and Planetary Sciences, Graduate School of Science, Hokkaido University, Sapporo 060-0810, Japan*, *Email: utsugi@ares.sci.hokudai.ac.jp*

[2] *Earthquake Research Institute, University of Tokyo*, *Bunkyo-ku, Tokyo 113-0032, Japan.*

SUMMARY

We present analytical solutions of the piezomagnetic potentials for strike-slip, dip-slip and tensile-opening fault motions with arbitrary dip and strike angles, which are applicable in various types of earthquakes. These solutions are expressed as the composition of elementary functions which are identical with the magnetic potentials produced by magnetic dipoles, quadrupoles and octupoles distributed on the fault plane and other planes. Therefore, the geomagnetic fields changes due to the piezomagnetic effect are expressed by the superposition of the fields produced by these equivalent sources.

Examples of calculated results show the following characteristic features of various types of fault motions: (1) The pattern of the geomagnetic field changes vary significantly different depending on the strike directions, although the maximum amplitude is almost the same for all directions. (2) The geomagnetic field change is a maximum at the dip angle of 90° for strike-slip and tensile-opening fault motions and at 45° for dip-slip fault motion.

Abbreviated title: Piezomagnetic potentials due to an inclined fault.

Key Words: seismomagnetic effect, piezomagnetic potential, inclined fault.

1 INTRODUCTION

Many scientists have been interested in the study of the geomagnetic field changes associated with seismic activity (i.e. seismomagnetism). Seismomagnetism is based on the piezomagnetic effects on rock magnetizations due to mechanical stress originating from earthquakes (Stacey, 1964; Nagata, 1970; Stacey and Johnston, 1972). Since Stacey(1964) presented a scheme for model calculation of the piezomagnetic fields due to dislocation, considerable progress has been made in this field as reviewed by Sasai(1994).

The geomagnetic field changes due to the piezomagnetic effects can be calculated by combining the magnetic dipole law and the Cauchy-Navier equation for elastic equilibrium through the constitution equation of piezomagnetism and Hook's law. Sasai(1991) implemented this process analytically and introduced a representation theorem for the piezomagnetic potentials. This theorem is given in terms of the surface integral of a function including the displacement and strain over the boundary surface of the strained body.

Sasai derived an analytical solution for the elementary piezomagnetic potential: the piezomagnetic potential due to a point dislocation source using the Green's function for the displacement obtained by Steketee(1958) and Maruyama(1964). He also formulated the piezomagnetic potentials due to a finite fault (Volterra's formula for the piezomagnetic potential). This formula has the form of integrated elementary piezomagnetic potentials over the fault plane. He solved this formula in the case of a vertical finite rectangular fault with uniform slip within a semi-infinite elastic medium. This result was successfully applied, for example, to interpret the geomagnetic field changes associated with the east-off Izu Peninsula earthquake(Sasai and Ishikawa, 1991) and the Landers earthquake of 1992 (Johnston *et al.*, 1994), both of which have almost vertical dip angles.

In this paper, we solve Volterra's formula in the case of an inclined finite rectangular fault and present analytical solutions for the piezomagnetic potentials due to such faulting observed outside the elastic medium, so as to be applicable to various types of earthquake mechanism.

Firstly, we make a coordinate transformation for the convenience of representation of a rectangular fault geometry which has arbitrary dip angle and strike direction. Secondly, we obtain an analytic form of the piezomagnetic potentials due to the inclined faulting by integrating the elementary piezomagnetic potentials over the fault plane. Thirdly, we derive the piezomagnetic fields by taking the gradient of the potentials and finally, we show some examples of calculated magnetic field variations due to fault motions.

2 FORMULATION OF PIEZOMAGNETIC POTENTIAL DUE TO AN INCLINED FAULT

2.1 Elementary piezomagnetic potential

From experimental studies, the relationship between uniaxial mechanical stress σ and the associated changes of the rock magnetization $\Delta\mathbf{J}$ is expressed as follows (Stacey, 1964; Nagata, 1970):

$$\Delta\mathbf{J}^{\parallel} = \beta\sigma\mathbf{J}_0^{\parallel}, \tag{1}$$

$$\Delta\mathbf{J}^{\perp} = -\frac{1}{2}\beta\sigma\mathbf{J}_0^{\perp}, \tag{2}$$

where the superscripts $\parallel$ and $\perp$ indicate the components parallel and perpendicular to the direction of applied mechanical stress, respectively. $\mathbf{J}_0$ is the initial magnetization of the rock and β is the stress sensitivity. Stacey et al.(1965) extended this formula to a three-dimensional stress state, obtaining

$$\Delta J_i\mathbf{e}_i = \beta J_i\left(\sigma_i - \frac{\sigma_j+\sigma_k}{2}\right)\mathbf{e}_i \quad (i,j,k = 1,2,3, i \neq j \neq k), \tag{3}$$

where $\mathbf{e}_i$ is unit vector directed in the i-th principal direction, σ_i is the principal stress, and J_i and ΔJ_i are the i-th component of magnetization and its increment, respectively. Using this formula, Sasai(1980) expressed $\Delta\mathbf{J}$ in terms of the stress tensor $\boldsymbol{\sigma}$ as follows:

$$\Delta\mathbf{J} = \frac{3}{2}\beta\boldsymbol{\sigma}'\cdot\mathbf{J}, \tag{4}$$

$$\sigma'_{kl} = \sigma_{kl} - \frac{1}{3}\sigma_0\delta_{kl},$$

where $\sigma_0 = \sigma_{11} + \sigma_{22} + \sigma_{33}$ and δ_{kl} is the Kronecker delta. Using eq. (4), we obtain the stress-induced magnetization due to a point dislocation source as follows:

$$\Delta\mathbf{J}_{kl} = C_x\mathbf{S}_{kl}^{(x)} + C_y\mathbf{S}_{kl}^{(y)} + C_z\mathbf{S}_{kl}^{(z)}$$

$$\begin{cases} (\mathbf{S}_{kl}^{(m)})_n &= \dfrac{3}{2}\left(\dfrac{\partial T_{kl}^m}{\partial x_n} + \dfrac{\partial T_{kl}^n}{\partial x_m}\right) - \delta_{mn}\mathrm{div}\mathbf{T}_{kl}, \\ C_m &= \dfrac{1}{2}\beta J_m \mu \dfrac{3\lambda + 2\mu}{\lambda + \mu}, \end{cases} \quad (m = x, y \,\mathrm{or}\, z), \tag{5}$$

where the suffixes k and l specify the type and the orientation of the point dislocation source, T_{kl}^m is the m-th component of the displacement vector due to the point dislocation source and $C_m\mathbf{S}_{kl}^{(m)}$ is the stress-induced magnetization produced by the m-th component of initial rock magnetization(J_m). λ and μ are the Lamé constants. Then, the elementary piezomagnetic potential, i.e., the piezomagnetic potential due to a point dislocation source of the type (kl) can be obtained from the magnetic dipole law of force:

$$\begin{aligned} \omega_{kl}^m(\boldsymbol{\xi}, \mathbf{r}) &= C_m \iiint_V \mathbf{S}_{kl}^{(m)}(\boldsymbol{\xi}, \mathbf{r}') \cdot \nabla\left(\frac{1}{R}\right) dV_{\mathbf{r}'}, \\ R &= |\mathbf{r}' - \mathbf{r}|, \end{aligned} \tag{6}$$

where ω_{kl}^m is the elementary piezomagnetic potential produced by the m-th component of initial magnetization of the crustal rock. $\mathbf{r}$ and $\boldsymbol{\xi}$ are the locations of the observation point and the point dislocation source, respectively. Applying Gauss's law to equation (6), Sasai(1991) presented the following result:

$$\begin{aligned} \omega_{kl}^m(\boldsymbol{\xi}, \mathbf{r}) &= C_m \iint_S \Big\{\Big(-\frac{\partial T_{kl}^m(\boldsymbol{\xi}, \mathbf{r}')}{\partial n'} + \frac{2(\lambda + \mu)}{3\lambda + 2\mu}\mathbf{S}_{kl}^{(m)}(\boldsymbol{\xi}, \mathbf{r}') \cdot \mathbf{n}'(\mathbf{r}')\Big)\frac{1}{R} \\ &\quad + T_{kl}^m(\boldsymbol{\xi}, \mathbf{r}')\frac{\partial}{\partial n'}\left(\frac{1}{R}\right)\Big\} dS_{\mathbf{r}'}, \end{aligned} \tag{7}$$

where S is the boundary layer of the volume V.

Let us consider a simple earth model: comprising a homogeneous and isotropic elastic half-space having a uniformly magnetized upper layer with a constant piezomagnetic stress sensitivity. We use the Cartesian coordinates as shown in Fig. 1: Here a semi-infinite elastic medium occupies z ≤ 0 and the magnetized region is limited to the layer $0 \leq z \leq H$, where H is the depth of the Curie point isotherm. In this earth model, ω_{kl}^m can be written as follows:

$$\omega_{kl}^m = \omega_{kl}^{m(0)} + \omega_{kl}^{m(K)} + \omega_{kl}^{m(H)}, \tag{8}$$

where the superscripts 0, K and H denote the contributions from the free-surface, the dislocation source and the Curie point isotherm, respectively.

Sasai(1991) solved eq. (7) and calculated ω_{kl}^m for each k, l and m when the observation point is located outside the medium($z \leq 0$). From his results, $\omega_{kl}^{m(0)}$ and $\omega_{kl}^{m(K)}$ are constructed from the term $R_1 = \sqrt{(x - \xi_1)^2 + (y - \xi_2)^2 + (\xi_3 - z)^2}$ (Sasai(1991) used the notation "ρ" instead of "R"), where (x, y, z) and (ξ_1, ξ_2, ξ_3) indicate the coordinates of the observation point and the dislocation source, respectively. On the contrary, $\omega_{kl}^{m(H)}$ changes its form, depending on the depth of the dislocation source ξ_3: when $\xi_3 < H$, $\omega_{kl}^{m(H)}$ is constructed from the terms $R_3 = \sqrt{(x - \xi_1)^2 + (y - \xi_2)^2 + (2H + \xi_3 - z)^2}$ and $R_2 = \sqrt{(x - \xi_1)^2 + (y - \xi_2)^2 + (2H - \xi_3 - z)^2}$, and when $H < \xi_3$, $\omega_{kl}^{m(H)}$ is made of the term R_3 and R_1 (see Sasai, 1991).

2.2 Finite rectangular source

The piezomagnetic potential due to a finite fault, $W(\mathbf{r})$, can be obtained by integrating ω_{kl}^{m} over the fault plane Σ:

$$\begin{aligned} W(\mathbf{r}) &= W^x(\mathbf{r}) + W^y(\mathbf{r}) + W^z(\mathbf{r}), \qquad (9)\\ W^m(\mathbf{r}) &= \iint_\Sigma \Delta u_k(\boldsymbol{\xi}) \cdot \omega_{kl}^m(\boldsymbol{\xi}, \mathbf{r}) \cdot \nu_l(\boldsymbol{\xi}) d\Sigma_{\boldsymbol{\xi}} \quad (m = x, y \,\mathrm{or}\, z), \qquad (10) \end{aligned}$$

where W^m is the piezomagnetic potential produced by the m-th component of initial rock magnetization. $\Delta\mathbf{u}$ and $\boldsymbol{\nu}_l$ denote a dislocation vector and a unit vector normal to the surface element $d\Sigma$, respectively. Because ω_{kl}^m obtained by Sasai(1991) are the solutions for the case that the observation point is located outside the medium, we also consider the same situation, i.e., $z \leq 0$.

We consider an inclined rectangular fault with length L, width W, dip angle δ, depth of upper burial d and a uniform dislocation $\Delta\mathbf{u} = (U_1, U_2, U_3)$ in a semi-infinite elastic medium. We introduce the new coordinates (ξ', η') as shown in Fig. 1. Therefore, ξ_1, ξ_2 and ξ_3 are replaced by $\xi', \eta' \cos\delta$ and $d + \eta' \sin\delta$, respectively. Then the terms R_1, R_2 and R_3 are written as follows:

$$R_i = \sqrt{(x - \xi')^2 + (y - \eta' \cos\delta)^2 + (D_i + \eta' \sin\delta)^2} \quad (i = 1, 2, 3),$$

where

$$D_1 = d - z, \quad D_2 = d - 2H + z \quad D_3 = d + 2H - z.$$

We introduce new variables defined by:

$$\begin{cases} \xi &= x - \xi', \\ \eta_i &= p_i - \eta', \\ p_i &= y \cos\delta - D_i \sin\delta, \\ q_i &= y \sin\delta + D_i \cos\delta. \end{cases} \quad (i = 1, 2, 3).$$

Using these variables, R_i can be more simply rewritten as follows:

$$R_i = \sqrt{\xi^2 + \eta_i^2 + q_i^2} \quad (i = 1, 2, 3),$$

The geometrical significance of the variables ξ, η_i and q_i are illustrated in Fig. 2. In this case, eq. (10) is rewritten as follows:

$$W^m = \int_x^{x-L} d\xi \int_{p_i}^{p_i - W} d\eta_i \Delta u_k \cdot \omega_{kl}^m(\xi, \eta_i, q_i) \cdot \nu_l, \qquad (11)$$

where the dislocation vector $\Delta\mathbf{u}$ and the normal vector $\boldsymbol{\nu}$ are expressed as

$$\begin{aligned} \Delta\mathbf{u} &= (U_1, U_3 \sin\delta - U_2 \cos\delta, -U_2 \sin\delta - U_3 \cos\delta) \quad , \qquad (12)\\ \boldsymbol{\nu} &= (0, \sin\delta, -\cos\delta) \quad . \qquad (13) \end{aligned}$$

Substituting eqs. (12) and (13) into eq. (11), W^m can be written in the form

$$W^m = \Big[U_1 S^m + U_2 D^m + U_3 T^m \Big], \qquad (14)$$

where S^m, D^m and T^m are the contributions from strike-slip, dip-slip and tensile opening, respectively. They are expressed by the following definite integrations:

$$
\begin{aligned}
S^m &= \int_x^{x-L} d\xi \int_{p_i}^{p_i-W} d\eta_i \left\{ \omega_{12}^m \sin\delta - \omega_{13}^m \cos\delta \right\} \quad , & (15) \\
D^m &= \int_x^{x-L} d\xi \int_{p_i}^{p_i-W} d\eta_i \left\{ \frac{1}{2}(\omega_{33}^m - \omega_{22}^m)\sin 2\delta + \omega_{23}^m \cos 2\delta \right\} \quad , & (16) \\
T^m &= \int_x^{x-L} d\xi \int_{p_i}^{p_i-W} d\eta_i \left\{ \omega_{22}^m \sin^2\delta - \omega_{23}^m \sin 2\delta + \omega_{33}^m \cos^2\delta \right\} \quad . & (17)
\end{aligned}
$$

2.3 Potential and geomagnetic changes

The integrations of eqs. (15), (16) and (17) can be evaluated analytically without any difficulty. The results are summarized in Appendix A. In this appendix, we divide S^m, D^m and T^m into two parts: contributions from the free-surface and the fault plane (which have subscript "0" in appendix A) and the Curie point isotherm. Further, we divide the contribution from the Curie point isotherm into three parts which comprise the terms R_3("$H0$"), R_2("HI") and R_1("$HIII$").

The contribution from the Curie point isotherm is composed of $H0$ and HI, or $H0$ and $HIII$ depending on the depth of the fault according to the following rules:
Case 1: the fault is located above the Curie point isotherm ($d + W\sin\delta < H$):

$$
f = f_0|_x^{x-L}|_{p_1}^{p_1-W} + f_{H0}|_x^{x-L}|_{p_3}^{p_3-W} + f_{HI}|_x^{x-L}|_{p_2}^{p_2-W}. \tag{18}
$$

Case 2: the fault is located below the Curie point isotherm ($d > H$):

$$
f = f_0|_x^{x-L}|_{p_1}^{p_1-W} + f_{H0}|_x^{x-L}|_{p_3}^{p_3-W} + f_{HIII}|_x^{x-L}|_{p_1}^{p_1-W}. \tag{19}
$$

Case 3: the fault intersects the Curie point isotherm ($d < H < d + W\sin\delta$):

$$
\begin{aligned}
f &= f_0|_x^{x-L}|_{p_1}^{p_1-W} + f_{H0}|_x^{x-L}|_{p_3}^{p_3-W} \\
&+ f_{HI}|_x^{x-L}|_{p_2}^{p_2-w} + f_{HIII}|_x^{x-L}|_{p_1-w}^{p_1-W}.
\end{aligned} \tag{20}
$$

Here f means the general term of S^m, D^m or T^m, $w = (H-d)/\sin\delta$, and the notation $|_a^b|_c^d$ denotes the following calculation:

$$
f(\xi, \eta_i)|_a^b|_c^d \equiv f(a,c) - f(a,d) - f(b,c) + f(b,d), \tag{21}
$$

indicating the definite integration of eqs. (15), (16) and (17).

Some terms of these solutions become singular under the special conditions, as shown in Fig. 3. However, these singularities can be avoided by substituting these conditions into eq. (15),(16) and (17) directly to calculate the integrals. Then, we can obtain the following rules:

1. Replace J_1 with 0 when $\xi = 0$,

2. Replace $\log(R_i + \eta_i)$ with $-\log(R_i - \eta_i)$ and set all terms which contain $R_i + \eta_i$ in the denominators to be 0 when $R_i + \eta_i = 0$,

3. Replace $\tan^{-1}\left(\dfrac{\xi\eta_i}{q_i R_i}\right)$ with 0 when $q_i = 0$.

When the dip angle δ is $90°$, some terms which include $\sec\delta$ and $\tan\delta$ also become singular. The solution under this condition is listed in Sasai(1991).

Constructing S^m, D^m and T^m by eqs. (18), (19) and (20), and substituting S^m, D^m and T^m into (14) and (9), we obtain the piezomagnetic potentials due to a finite fault. Differentiation of the piezomagnetic potential with respect to x, y and z gives the x, y and z components of the geomagnetic field changes, respectively. Since we use the variables ξ, η_1, η_2 and η_3, the differential operations are rewritten as

$$\frac{\partial}{\partial x} \quad \rightarrow \quad \frac{\partial}{\partial \xi}, \tag{22}$$

$$\frac{\partial}{\partial y} \quad \rightarrow \quad \frac{\partial}{\partial y} + \frac{\partial p_i}{\partial y}\frac{\partial}{\partial \eta_i} + \frac{\partial q_i}{\partial y}\frac{\partial}{\partial q_i} = \frac{\partial}{\partial y} + \cos\delta\frac{\partial}{\partial \eta_i} + \sin\delta\frac{\partial}{\partial q_i}, \tag{23}$$

$$\frac{\partial}{\partial z} \quad \rightarrow \quad \frac{\partial}{\partial z} + \frac{\partial p_i}{\partial z}\frac{\partial}{\partial \eta_i} + \frac{\partial q_i}{\partial z}\frac{\partial}{\partial q_i} = \frac{\partial}{\partial z} \pm \sin\delta\frac{\partial}{\partial \eta_i} \mp \cos\delta\frac{\partial}{\partial q_i}, \tag{24}$$

where

$$\pm \equiv \begin{cases} + & (i = 1, 3), \\ - & (i = 2). \end{cases}$$

2.4 Magnetic source equivalents

The piezomagnetic potentials are represented by functions which are identical with the potentials produced by equivalent magnetic sources (magnetic dipoles, quadrupoles and octupoles). For example, the function

$$\log(R_i + \xi) = -\iint_{\Sigma} \frac{\eta_i}{R_i^3} d\xi d\eta_i \tag{25}$$

has the same form as the potential produced by dipoles distributed on the fault plane ($i = 1$) and two image planes ($i = 1$ or 2) as shown in Fig 4. The functions $M_1(i)$ and $M_{1,x}(i)$ have the same form as the derivatives of $\log(R_i + \xi)$ and $M_1(i)$ with respect to ξ, respectively. According to eqs. (22), we can obtain following equations:

$$M_1(i) \quad = \quad \frac{\partial \log(R_i + \xi)}{\partial x} = \frac{\partial \log(R_i + \xi)}{\partial \xi}, \tag{26}$$

$$M_{1,x}(i) \quad = \quad \frac{\partial M_1(i)}{\partial x} = \frac{\partial M_1(i)}{\partial \xi}. \tag{27}$$

Thus, $M_1(1)$ and $M_{1,x}(i)$ are the potentials produced by quadrupoles and octupoles distributed on the fault plane or the image planes, respectively. In the same manner, other functions appearing in eqs. (A-10) to (A-45) are also represented as the potentials produced by dipoles, quadrupoles and octupoles. Consequently the geomagnetic field changes due to the seismomagnetic effect are identical with the superposed magnetic fields produced by magnetic source equivalents distributed on the planes shown in Fig. 4.

3 NUMERICAL EXAMPLES

In this section, we show some numerical examples of geomagnetic field changes on the earth's surface. The numerical examples are focused on strike direction, fault depth and dip-angle dependencies of the geomagnetic field changes. The calculations are made for a fault having a 5×10 km plane. The dislocation of the fault is assumed to be 1m. Other model parameters are given in Table 1. In all cases, relatively intense magnetic changes are seen around the tips of the fault. This arises from the fact that the magnetic lines of force are disturbed near the tips of the fault because the magnetic source equivalents are located on the fault plane.

Fig. 5 shows the total intensity of the geomagnetic field changes due to N-S, SE-NW and E-W oriented strike-slip fault motion with dip-angle(δ) 75°. The pattern of the contours varies depending on the strike direction of the fault, although the maximum amplitude is almost the same.

Fig. 6 shows the total intensity of geomagnetic field changes due to a left-lateral strike slip fault motion with fault depths of (a) 0.5 km and (b) 5 km. We can see that the intensity of the geomagnetic changes is quite different; the intensity of case (a) is almost one order of magnitude larger than that of case (b). This figure shows that seismomagnetic changes can be more effectively observed in the case of shallower earthquakes.

Figs. 7 ,8 and 9 show the dip-angle dependence of the geomagnetic field changes due to E-W oriented left-lateral-strike, thrust and tensile-opening fault motions. We can see that, in the cases of strike-slip and tensile-opening fault motions, the geomagnetic changes increase as the dip angle increases. However, in the case of dip-slip fault motion, the intensity takes its maximum at $\delta = 45°$. Sasai(1991) showed that there are two types of elementary piezomagnetic potentials: for the types $(kl) = (33), (23)$ and (13), the contributions from the free-surface ($\omega_{kl}^{m(0)}$) and the dislocation source ($\omega_{kl}^{m(K)}$) exactly cancel each other, but for the types $(kl) = (11), (22)$ and (12) they do not. Especially, for the shallower earthquakes, the equivalent sources on the fault plane make the most important contributions to the geomagnetic field changes on the Earth's surface. Then, only ω_{11}^m, ω_{22}^m and ω_{12}^m can produce intense geomagnetic changes. Equations (15),(16) and (17) contain the effective terms $\omega_{11}^m \sin\delta$, $\omega_{22}^m \cos 2\delta$ and $\omega_{12}^m \sin^2\delta$. These terms attain their maxima at $\delta = 90°$ for strike-slip and tensile-opening, and at $\delta = 45°$ for dip-slip. Therefore, we can reasonably understand that the most intense magnetic field changes are generated near $\delta = 90°$ for strike-slip or tensile-opening fault motion and at $\delta = 45°$ for dip-slip fault motion.

4 CONCLUSIONS

In this study, we expanded Sasai's solutions (Sasai,1991) and developed the analytical solutions of the piezomagnetic potentials due to an inclined rectangular fault. Using our solutions, we can calculate the geomagnetic changes due to various types of earthquake with arbitrary slip-direction, seismic moment, strike-direction and dip-angle. The solutions consist of the contributions from free-surface, Curie point isotherm and fault plane, and have the same form as the superposition of magnetic potentials produce by some magnetic source equivalents. The model calculations revealed the following features of the seismomagnetic effect:

1. The magnetic changes are most intense in the case of shallower earthquakes.

2. Relatively intense magnetic changes are seen around the tips of the fault.
3. The pattern of the geomagnetic field changes varies significantly depending on the strike direction, although the maximum amplitude is almost the same.
4. The geomagnetic field changes depend strongly on the dip-angle of the fault plane: the amplitude of field change attains maximum at a dip angle of 90° for strike-slip and tensile-opening fault motions and at 45° for dip-slip fault motion.

ACKNOWLEDGMENTS

We thank Dr. F. Chivas and anonymous referees for their helpful comments on the manuscript.

REFERENCES

Johnston, M.J.S., Mueller R.J. & Sasai Y., 1994. Magnetic field observations in the near-field the 28 June 1992 M_w 7.3 Landers, California, earthquake, *Bull. Seism. Soc. Am.,* **84**, 792-798.

Maruyama, T., 1964. Statical elastic dislocations in an infinite and semi-infinite medium, *Bull. Earthq. Res. Inst., Univ. Tokyo,* **42**, 289-368.

Nagata, T., 1970. Basic magnetic properties of rocks under mechanical stresses, *Tectonophysics*, **9**, 167-195.

Sasai, Y., 1980. Application of the elasticity theory of dislocations to tectonomagnetic modeling, *Bull. Earthq. Res. Inst., Univ. Tokyo,* **58**, 387-447.

Sasai, Y., 1991. Tectonomagnetic modeling on the basis of linear piezomagnetic effect, *Bull. Earthq. Res. Inst., Univ, Tokyo,* **66**, 585-722.

Sasai, Y., 1994. Resolution of contradiction between seismomagnetic models, *J. Geomag. Geoelectr.*, **46**, 329-340.

Sasai, Y. & Ishikawa, Y., 1991. Tectonomagnetic signals related to the seismomagnetic activity in the Izu Peninsula, *J. Phys. Earth*, **39**, 299-319.

Stacey, F.D., 1964. The seismomagnetic effect, *Pageoph.*, **58**, 5-22.

Stacey, F.D., Barr, K.G. & Robson, G.R., 1965. The volcanomagnetic effect, *Pageoph.*, **62**, 96-104.

Stacey, F.D. & Johnston, M.J.S., 1972. Theory of the piezomagnetic effect in titanomagnetite-bearing rock, *Pageoph.*, **97**, 146-155.

Steketee, J. A., 1958. On Volterra's dislocations in a semi-infinite medium, *Can. J. Phys.,* **36**, 192-205.

Caption of figures

Figure 1. Cartesian coordinate system and geometry of the fault model used in this study. L: fault length; W: fault width; d: depth of the upper edge of the fault; δ: dip-angle of fault.

Figure 2. Geometrical meaning of variables (ξ, η_i, q_i).

Figure 3. Locations where the mathematical singularities appear in the solution of the piezomagnetic potentials. This figure shows the case of $i = 1$.

Figure 4. Magnetic source equivalents distributed on the thick and dashed lines.
(a) Case 1 : The fault located above the Curie depth $(d + W \sin \delta < H)$.
(b) Case 2 : The fault located below the Curie depth $(d + W \sin \delta < H)$.

Figure 5. Changes of geomagnetic total force due to strike-slip faulting with dip-angle 75°. The orientations of the faults are (a) E-W, (b) NE-SW and (c) N-S. The fault depth is 0.5 km. Rectangles represent the fault plane projected onto the horizontal surface. Upper edges are shown by bold lines. Bold and thin arrows show the fault motion of hanging wall and footwall, respectively. Solid and dotted contours represent the positive and negative changes with contour intervals of 0.5 nT.

Figure 6. Changes of geomagnetic total force due to left-lateral strike-slip fault with a depth of (a) 0.5 km and (b) 5 km. The dip-angle δ is 75°. Units are nT.

Figure 7. Changes of geomagnetic total force due to left-lateral strike-slip fault with dip angles of (a) 75°, (b) 45° and (c) 15°. Units are nT.

Figure 8. Changes of geomagnetic total force due to dip-slip fault with dip angles of (a) 75°, (b) 45° and (c) 15°. Units are nT.

Figure 9. Total geomagnetic field changes due to a tensile-opening fault motion with dip angles of (a) 75°, (b) 45° and (c) 15°. Units are nT.

Table 1. Parameters used for the calculation of magnetic field changes.

Fault length	$2L$	10	km
Fault width	W	5	km
Dislocation	Δu	1	m
Rigidity	μ	3.5×10^{11}	cgs
Poisson's ratio	ν	0.25	
Average magnetization	J	1	A/m
Stress sensitivity	β	1.0×10^{-4}	bar^{-1}
Curie depth	H	15	km
Average magnetic inclination		45	degree

Table 1

Appendix A : Piezomagnetic potential

The solutions of the piezomagnetic potentials are shown in the later sections A.1, A.2 and A.3. The notations which will appear in eqs (A-10) to (A-45) indicate the magnetic potentials produced by magnetic source equivalents, such as dipole, quadrupole and octupole terms, as follows:

Dipole terms:

$$\left\{\begin{array}{l} K_1(i) = \tan\delta\left[\dfrac{\xi}{R_i+\tilde{c}_i} + J_1(i)\tan\delta\right], \\ K_2(i) = \tan\delta\left[\dfrac{\tilde{y}_i}{R_i+\tilde{c}_i} \mp J_2(i)\tan\delta\right], \\ K_3(i) = J_2(i)\tan\delta, \\ K_4(i) = \cos\delta\left[K_2(i) - \sin\delta\log(R_i+\eta_i)\right], \\ K_5(i) = K_1(i)\cos\delta, \\ K_6(i) = -J_1(i)\sin\delta, \\ K_7(i) = -K_4(i)\tan\delta, \\ K_8(i) = -K_5(i)\tan\delta, \\ K_9(i) = -K_6(i)\tan\delta. \end{array}\right. \tag{A-1}$$

$$\left\{\begin{array}{lcl} J_1(i) & = & 2\tan^{-1}\left\{\dfrac{(R_i\mp\eta_i)(1+\sin\delta)\pm q_i\cos\delta}{\xi\cos\delta}\right\}, \\ J_2(i) & = & \log(R_i+\tilde{c}_i)\pm\sin\delta\log(R_i+\eta_i), \end{array}\right. \tag{A-2}$$

Quadrupole terms:

$$L_0(i) = \frac{1}{R_i(R_i+\tilde{c}_i)} \pm \frac{\sin\delta}{R_i(R_i+\eta_i)}, \tag{A-3}$$

$$\left\{\begin{array}{l} L_1(i) = \xi L_0, \\ L_2(i) = \pm\dfrac{\sin\delta}{R_i+\eta_i} + \tilde{y}_i L_0\sec\delta, \end{array}\right. \tag{A-4}$$

$$\left\{\begin{array}{l} M_1(i) = \dfrac{\xi}{R_i(R_i+\eta_i)}, \\ M_2(i) = \dfrac{\cos\delta}{R_i+\eta_i} + \dfrac{\tilde{y}_i}{R_i(R_i+\eta_i)}, \\ M_3(i) = \pm\dfrac{\sin\delta}{R_i+\eta_i} - \dfrac{\tilde{c}_i}{R_i(R_i+\eta_i)}, \end{array}\right. \tag{A-5}$$

$$\left\{\begin{array}{l} N_1(i) = \dfrac{\xi}{R_i(R_i+\tilde{c}_i)}, \\ N_2(i) = \dfrac{\tilde{y}_i}{R_i(R_i+\tilde{c}_i)}, \end{array}\right. \tag{A-6}$$

$$\left\{\begin{array}{l} O_1(i) = \dfrac{1}{R_i}, \\ O_2(i) = \dfrac{\tilde{y}}{R_i(R_i+\xi)}, \\ O_3(i) = -\dfrac{\tilde{c}_i}{R_i(R_i+\xi)}, \end{array}\right. \tag{A-7}$$

$$
\begin{cases}
P_1(i) = -\dfrac{q_i}{R_i(R_i+\eta_i)}, \\
P_2(i) = \dfrac{\xi\sin\delta}{R_i(R_i+\eta_i)} \mp \dfrac{\tilde{c}_i}{R_i(R_i+\xi)}, \\
P_3(i) = \mp\left\{\dfrac{\xi\cos\delta}{R_i(R_i+\eta_i)} + \dfrac{\tilde{y}_i}{R_i(R_i+\xi)}\right\}.
\end{cases}
\tag{A-8}
$$

Octupole terms:

The notation f_{i,x_j} indicates $\dfrac{\partial f_i}{\partial x_j}$ $\quad (x_j = x, y, z)$

$$
\begin{cases}
M_{1,y}(i) = -\dfrac{\xi\cos\delta}{R_i(R_i+\eta_i)^2} - \dfrac{\xi\tilde{y}_i(2R_i+\eta_i)}{R_i^3(R_i+\eta_i)^2}, \\
M_{1,z}(i) = \mp\dfrac{\xi\sin\delta}{R_i(R_i+\eta_i)^2} + \dfrac{\xi\tilde{c}_i(2R_i+\eta_i)}{R_i^3(R_i+\eta_i)^2}, \\
M_{2,y}(i) = \dfrac{\sin^2\delta}{(R_i+\eta_i)^2} - \dfrac{\tilde{y}_i\cos\delta \pm \tilde{c}_i\sin\delta}{R_i(R_i+\eta_i)^2} - \dfrac{\tilde{y}_i^2(2R_i+\eta_i)}{R_i^3(R_i+\eta_i)^2}, \\
M_{2,z}(i) = \mp\dfrac{\sin\delta\cos\delta}{(R_i+\eta_i)^2} + \dfrac{\tilde{c}_i\cos\delta \mp \tilde{y}_i\sin\delta}{R_i(R_i+\eta_i)^2} + \dfrac{\tilde{y}_i\tilde{c}_i(2R_i+\eta_i)}{R_i^3(R_i+\eta_i)^2}, \\
M_{3,y}(i) = M_{2,z}, \\
M_{3,z}(i) = \dfrac{\cos^2\delta}{(R_i+\eta_i)^2} + \dfrac{\tilde{y}_i\cos\delta \pm \tilde{c}_i\sin\delta}{R_i(R_i+\eta_i)^2} - \dfrac{\tilde{c}_i^2(2R_i+\eta_i)}{R_i^3(R_i+\eta_i)^2}, \\
N_{1,z}(i) = \dfrac{\xi}{R_i^3}, \\
N_{2,z}(i) = \dfrac{\tilde{y}_i}{R_i^3}, \\
O_{2,y}(i) = \dfrac{1}{R_i(R_i+\xi)} - \dfrac{\tilde{y}_i^2(2R_i+\xi)}{R_i^3(R_i+\xi)^2}, \\
O_{2,z}(i) = \dfrac{\tilde{y}_i\tilde{c}_i(2R_i+\xi)}{R_i^3(R_i+\xi)^2}, \\
O_{3,z}(i) = \dfrac{1}{R_i(R_i+\xi)} - \dfrac{\tilde{c}_i^2(2R_i+\xi)}{R_i^3(R_i+\xi)^2}, \\
P_{1,y}(i) = \pm\dfrac{\tilde{c}_i}{R_i^3} + \dfrac{\sin\delta}{R_i(R_i+\eta_i)} - \dfrac{\xi^2\sin\delta(2R_i+\eta_i)}{R_i^3(R_i+\eta_i)^2}, \\
P_{1,z}(i) = \pm\left[\dfrac{\tilde{y}_i}{R_i^3} - \dfrac{\cos\delta}{R_i(R_i+\eta_i)} + \dfrac{\xi^2\cos\delta(2R_i+\eta_i)}{R_i^3(R_i+\eta_i)^2}\right], \\
P_{3,y}(i) = \pm\left[\dfrac{1}{R_i(R_i+\xi)} - \dfrac{\tilde{c}_i^2(2R_i+\xi)}{R_i^3(R_i+\xi)^2}\right] \mp \dfrac{\xi\sin^2\delta}{R_i(R_i+\eta_i)^2} + \dfrac{\xi\tilde{c}_i\sin\delta(2R_i+\eta_i)}{R_i^3(R_i+\eta_i)^2}, \\
P_{3,z}(i) = \mp\dfrac{\tilde{y}_i\tilde{c}_i(2R_i+\xi)}{R_i^3(R_i+\xi)^2} + \dfrac{\xi\sin\delta\cos\delta}{R_i(R_i+\eta_i)^2} \mp \dfrac{\xi\tilde{c}_i\cos\delta(2R_i+\eta_i)}{R_i^3(R_i+\eta_i)^2}.
\end{cases}
\tag{A-9}
$$

where the notations using above are as follows;

$$
\alpha = \frac{\lambda+\mu}{\lambda+2\mu}
$$

$$
\begin{aligned}
\tilde{y}_i &= \eta_i\cos\delta + q_i\sin\delta, \\
\tilde{c}_i &= \pm(q_i\cos\delta - \eta_i\sin\delta), \\
p_i &= y\cos\delta - D_i\sin\delta, \\
q_i &= y\sin\delta + D_i\cos\delta, \\
D_1 &= d - z, \quad D_2 = d - 2H + z \quad D_3 = d + 2H - z.
\end{aligned}
$$

A.1 Strike-slip

$$
\begin{aligned}
S^m \quad = \quad & S_0^m|_x^{x-L}|_{p_1}^{p_1-W} + S_{H0}^m|_x^{x-L}|_{p_3}^{p_3-W} \\
& + \begin{cases} S_{HI}^m|_x^{x-L}|_{p_2}^{p_2-W} & (d > H) \\ S_{HI}^m|_x^{x-L}|_{p_2}^{p_2-w} + S_{HIII}^m|_x^{x-L}|_{p_1-w}^{p_1-W} & (d < H < d + W\sin\delta) \\ S_{HIII}^m|_x^{x-L}|_{p_1}^{p_1-W} & (d + W\sin\delta < H) \end{cases} \\
& (m = x, y, z,\ w = (H-d)/\sin\delta)
\end{aligned}
$$

A.1.1 Horizontal magnetization in x-direction

$$
\frac{2}{C_x} S_0^x = 2K_1(1) \tag{A-10}
$$

$$
\begin{aligned}
\frac{2}{C_x} S_{H0}^x = & -2K_1(3) + \frac{\alpha(2\alpha+1)}{4\alpha-1} K_1(3) \\
& - \frac{3\alpha}{4\alpha-1} \tan^{-1}\left(\frac{\xi\eta_3}{q_3 R_3}\right) - \frac{2\alpha(1-\alpha)}{4\alpha-1} J_1(3) \\
& - \frac{2\alpha(1-\alpha)}{4\alpha-1} \left[q' M_1(3) + (z-H) L_1(3) \tan\delta \right] \\
& - \frac{2\alpha(2\alpha+1)}{4\alpha-1} H \left[M_1(3)\cos\delta - L_1(3)\tan\delta \right] - \frac{12\alpha}{4\alpha-1} H L_1(3) \tan\delta \\
& + \frac{12\alpha^2}{4\alpha-1} H \left[(q' + H\cos\delta) M_{1,z}(3) - (z-2H) M_{1,y}(3) \sin\delta \right]
\end{aligned} \tag{A-11}
$$

$$
\begin{aligned}
\frac{2}{C_x} S_{HI}^x = & \frac{\alpha(2\alpha+1)}{4\alpha-1} K_1(2) - \frac{3\alpha}{4\alpha-1} \tan^{-1}\left(\frac{\xi\eta_2}{q_2 R_2}\right) - \frac{2\alpha(1-\alpha)}{4\alpha-1} J_1(2) \\
& - \frac{6\alpha^2}{4\alpha-1} \left[q' M_1(2) + (z-H) L_1(2) \tan\delta \right]
\end{aligned} \tag{A-12}
$$

$$
\begin{aligned}
\frac{2}{C_x} S_{HIII}^x = & -\frac{\alpha(2\alpha+1)}{4\alpha-1} K_1(1) + \frac{3\alpha}{4\alpha-1} \tan^{-1}\left(\frac{\xi\eta_1}{q_1 R_1}\right) + \frac{2\alpha(1-\alpha)}{4\alpha-1} J_1(1) \\
& + \frac{2\alpha(1-\alpha)}{4\alpha-1} \left[q' M_1(1) + (z-H) L_1(1) \tan\delta \right]
\end{aligned} \tag{A-13}
$$

A.1.2 Horizontal magnetization in y-direction

$$
\frac{2}{C_y} S_0^y = 2K_2(1) \tag{A-14}
$$

$$
\begin{aligned}
\frac{2}{C_y} S_{H0}^y = & -2K_2(3) + \frac{\alpha(2\alpha+1)}{4\alpha-1} K_2(3) \\
& + \frac{\alpha(2\alpha+1)}{4\alpha-1} \sin\delta \log(R_3 + \eta_3) + \frac{2\alpha(1-\alpha)}{4\alpha-1} J_2(3)
\end{aligned}
$$

$$
\begin{aligned}
& - \frac{2\alpha(1-\alpha)}{4\alpha-1}\left[q' M_2(3) + (z-H)L_2(3)\sin\delta\right] \\
& - \frac{2\alpha(2\alpha+1)}{4\alpha-1} H \left[M_2(3)\cos\delta - L_2(3)\sin\delta\right] - \frac{12\alpha}{4\alpha-1} H L_2(3)\sin\delta \\
& + \frac{12\alpha^2}{4\alpha-1} H \left[(q' + H\cos\delta)M_{2,z}(3) - (z-2H)M_{2,y}(3)\sin\delta + M_3(3)\sin\delta\right]
\end{aligned}
\tag{A-15}
$$

$$
\begin{aligned}
\frac{2}{C_y} S^y_{HI} &= \frac{\alpha(2\alpha+1)}{4\alpha-1} K_2(2) + \frac{3\alpha(1-2\alpha)}{4\alpha-1}\sin\delta\log(R_2+\eta_2) - \frac{2\alpha(1-\alpha)}{4\alpha-1} J_2(2) \\
& - \frac{6\alpha^2}{4\alpha-1}\left[q' M_2(2) + (z-H)L_2(2)\sin\delta\right]
\end{aligned}
\tag{A-16}
$$

$$
\begin{aligned}
\frac{2}{C_y} S^y_{HIII} &= -\frac{\alpha(2\alpha+1)}{4\alpha-1} K_2(1) - \frac{\alpha(2\alpha+1)}{4\alpha-1}\sin\delta\log(R_1+\eta_1) - \frac{2\alpha(1-\alpha)}{4\alpha-1} J_2(1) \\
& + \frac{2\alpha(1-\alpha)}{4\alpha-1}\left[q' M_2(1) + (z-H)L_2(1)\sin\delta\right]
\end{aligned}
\tag{A-17}
$$

A.1.3 Vertical magnetization

$$
\frac{2}{C_z} S^z_0 = 2K_3(1) \tag{A-18}
$$

$$
\begin{aligned}
\frac{2}{C_z} S^z_{H0} &= -2K_3(3) - \frac{\alpha(2\alpha-5)}{4\alpha-1} K_3(3) \\
& - \alpha\cos\delta\log(R_3+\eta_3) - \frac{2\alpha(1-\alpha)}{4\alpha-1}\left[q' M_3(3) - (z-H)M_2(3)\sin\delta\right] \\
& + 2\alpha H\left[M_3(3)\cos\delta + M_2(3)\sin\delta\right] - \frac{12\alpha}{4\alpha-1} H M_2(3)\sin\delta \\
& - \frac{12\alpha^2}{4\alpha-1} H\left[(q' + H\cos\delta)M_{3,z}(3) - (z-2H)M_{3,y}(3)\sin\delta + M_2(3)\sin\delta\right]
\end{aligned}
\tag{A-19}
$$

$$
\begin{aligned}
\frac{2}{C_z} S^z_{HI} &= \frac{\alpha(2\alpha-5)}{4\alpha-1} K_3(2) + \alpha\cos\delta\log(R_2+\eta_2) \\
& + \frac{6\alpha^2}{4\alpha-1}\left[q' M_3(2) - (z-H)M_2(2)\sin\delta\right]
\end{aligned}
\tag{A-20}
$$

$$
\begin{aligned}
\frac{2}{C_z} S^z_{HIII} &= \frac{\alpha(2\alpha-5)}{4\alpha-1} K_3(1) + \alpha\cos\delta\log(R_1+\eta_1) \\
& + \frac{2\alpha(1-\alpha)}{4\alpha-1}\left[q' M_3(1) - (z-H)M_2(1)\sin\delta\right]
\end{aligned}
\tag{A-21}
$$

A.2 Dip-slip

$$
\begin{aligned}
D^m &= D_0^m|_x^{x-L}|_{p_1}^{p_1-W} + D_{H0}^m|_x^{x-L}|_{p_3}^{p_3-W} \\
&+ \begin{cases} D_{HI}^m|_x^{x-L}|_{p_2}^{p_2-W} & (d > H) \\ D_{HI}^m|_x^{x-L}|_{p_2}^{p_2-w} + D_{HIII}^m|_x^{x-L}|_{p_1-w}^{p_1-W} & (d < H < d + W\sin\delta) \\ D_{HIII}^m|_x^{x-L}|_{p_1}^{p_1-W} & (d + W\sin\delta < H) \end{cases} \\
&(m = x, y, z,\ w = (H-d)/\sin\delta)
\end{aligned}
$$

A.2.1 Horizontal magnetization in x-direction

$$
\frac{2}{C_x} D_0^x = -2K_4(1) \quad \text{(A-22)}
$$

$$
\begin{aligned}
\frac{2}{C_x} D_{H0}^x &= 2K_4(3) - \frac{\alpha(2\alpha+1)}{4\alpha-1}K_4(3) + \frac{2\alpha(1-\alpha)}{4\alpha-1}\left[\sin 2\delta \log(R_3+\eta_3) - J_2(3)\sec\delta\cos 2\delta\right] \\
&+ \frac{2\alpha(1-\alpha)}{4\alpha-1}\left[q'O_1(3) + (z-H)N_2(3)\sin\delta\right] \\
&+ \frac{2\alpha(2\alpha-5)}{4\alpha-1}H\left[O_1(3)\cos\delta - N_2(3)\sin\delta\right] + \frac{12\alpha}{4\alpha-1}HM_2(3) \\
&- \frac{12\alpha^2}{4\alpha-1}H\left[(q'+H\cos\delta)O_{2,z}(3) + (z-2H)N_{2,z}(3)\sin\delta\right] \quad \text{(A-23)}
\end{aligned}
$$

$$
\begin{aligned}
\frac{2}{C_x} D_{HI}^x &= -\frac{\alpha(2\alpha+1)}{4\alpha-1}K_4(2) + \frac{2\alpha(1-\alpha)}{4\alpha-1}\left[\sin 2\delta \log(R_2+\eta_2) + J_2(2)\sec\delta\cos 2\delta\right] \\
&+ \frac{6\alpha^2}{4\alpha-1}\left[q'O_1(2) + (z-H)N_2(2)\sin\delta\right] \quad \text{(A-24)}
\end{aligned}
$$

$$
\begin{aligned}
\frac{2}{C_x} D_{HIII}^x &= \frac{\alpha(2\alpha+1)}{4\alpha-1}K_4(2) - \frac{2\alpha(1-\alpha)}{4\alpha-1}\left[\sin 2\delta \log(R_1+\eta_1) - J_2(1)\sec\delta\cos 2\delta\right] \\
&- \frac{2\alpha(1-\alpha)}{4\alpha-1}\left[q'O_1(1) + (z-H)N_2(1)\sin\delta\right] \quad \text{(A-25)}
\end{aligned}
$$

A.2.2 Horizontal magnetization in y-direction

$$
\frac{2}{C_y} D_0^y = 2K_5(1) \quad \text{(A-26)}
$$

$$
\begin{aligned}
\frac{2}{C_y} D_{H0}^y &= -2K_5(3) + \frac{\alpha(2\alpha+1)}{4\alpha-1}K_5(3) - \alpha\sin\delta\log(R_3+\xi) \\
&+ \frac{\alpha(2\alpha+1)}{4\alpha-1}\cos\delta\tan^{-1}\left(\frac{\xi\eta_3}{q_3R_3}\right) - \frac{2\alpha(1-\alpha)}{4\alpha-1}J_1(3)\sec\delta\cos 2\delta
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad \frac{2\alpha(1-\alpha)}{4\alpha-1}\left[q' O_2(3) - (z-H)\{N_1(3) - O_3(3)\}\sin\delta\right] \\
&+ \quad \frac{2\alpha(2\alpha+1)}{4\alpha-1} H \left[O_2(3)\cos\delta + \{N_1(3) - O_3(3)\}\sin\delta\right] - \frac{12\alpha}{4\alpha-1} H L_1(3)\sin\delta \\
&- \quad \frac{12\alpha^2}{4\alpha-1} H \left[(q' + H\cos\delta) O_{2,z}(3) - (z-2H)\{N_{1,z}(3) - O_{3,z}(3)\}\sin\delta + O_3(3)\sin\delta\right]
\end{aligned} \tag{A-27}
$$

$$
\begin{aligned}
\frac{2}{C_y} D^y_{HI} \quad &= \quad \frac{\alpha(2\alpha+1)}{4\alpha-1} K_5(2) + \alpha \sin\delta \log(R_2 + \xi) \\
&+ \quad \frac{\alpha(2\alpha+1)}{4\alpha-1} \cos\delta \tan^{-1}\left(\frac{\xi\eta_2}{q_2 R_2}\right) - \frac{2\alpha(1-\alpha)}{4\alpha-1} J_1(2)\sec\delta\cos 2\delta \\
&+ \quad \frac{6\alpha^2}{4\alpha-1}\left[q' O_2(2) - (z-H)\{N_1(2) - O_3(2)\}\sin\delta\right]
\end{aligned} \tag{A-28}
$$

$$
\begin{aligned}
\frac{2}{C_y} D^y_{HIII} \quad &= \quad -\frac{\alpha(2\alpha+1)}{4\alpha-1} K_5(1) + \alpha \log(R_1 + \xi) \\
&- \quad \frac{\alpha(2\alpha+1)}{4\alpha-1} \cos\delta \tan^{-1}\left(\frac{\xi\eta_1}{q_1 R_1}\right) + \frac{2\alpha(1-\alpha)}{4\alpha-1} J_1(1)\sec\delta\cos 2\delta \\
&- \quad \frac{2\alpha(1-\alpha)}{4\alpha-1}\left[q' O_2(1) - (z-H)\{N_1(3) - O_3(1)\}\sin\delta\right]
\end{aligned} \tag{A-29}
$$

A.2.3 Vertical magnetization

$$
\frac{2}{C_z} D^z_0 \quad = \quad 2K_6(1) \tag{A-30}
$$

$$
\begin{aligned}
\frac{2}{C_z} D^z_{H0} \quad &= \quad -2K_6(3) - \frac{\alpha(2\alpha-5)}{4\alpha-1} K_6(3) \\
&+ \quad \alpha\cos\delta\log(R_3 + \xi) + \frac{\alpha(2\alpha+1)}{4\alpha-1}\sin\delta\tan^{-1}\left(\frac{\xi\eta_3}{q_3 R_3}\right) \\
&+ \quad \frac{2\alpha(1-\alpha)}{4\alpha-1}\left[q' O_3(3) - (z-H) O_2(3)\sin\delta\right] \\
&- \quad 2\alpha H\left[O_3(3)\cos\delta + O_2(3)\sin\delta\right] - \frac{12\alpha}{4\alpha-1} H M_1(3)\sin\delta\cos\delta \\
&+ \quad \frac{12\alpha^2}{4\alpha-1} H\left[(q' + H\cos\delta) O_{3,z}(3) - (z-2H) O_{2,z}(3)\sin\delta - O_2(3)\sin\delta\right]
\end{aligned} \tag{A-31}
$$

$$
\begin{aligned}
\frac{2}{C_z} D^z_{HI} \quad &= \quad -\frac{\alpha(2\alpha-5)}{4\alpha-1} K_6(2) - \alpha\cos\delta\log(R_2 + \xi) + \frac{\alpha(2\alpha+1)}{4\alpha-1}\sin\delta\tan^{-1}\left(\frac{\xi\eta_2}{q_2 R_2}\right) \\
&- \quad \frac{6\alpha^2}{4\alpha-1}\left[q' O_3(2) - (z-H) O_2(2)\sin\delta\right]
\end{aligned} \tag{A-32}
$$

$$
\frac{2}{C_z} D^z_{HIII} \quad = \quad \frac{\alpha(2\alpha-5)}{4\alpha-1} K_6(1) - \alpha\cos\delta\log(R_1 + \xi) - \frac{\alpha(2\alpha+1)}{4\alpha-1}\sin\delta\tan^{-1}\left(\frac{\xi\eta_1}{q_1 R_1}\right)
$$

$$- \frac{2\alpha(1-\alpha)}{4\alpha-1}\left[q'O_3(1)-(z-H)O_2(1)\sin\delta\right] \qquad \text{(A-33)}$$

A.3 Tensile-opening

$$\begin{aligned}
T^m &= T_0^m|_x^{x-L}|_{p_1}^{p_1-W} + T_{H0}^m|_x^{x-L}|_{p_3}^{p_3-W} \\
&\quad + \begin{cases} T_{HI}^m|_x^{x-L}|_{p_2}^{p_2-W} & (d>H) \\ T_{HI}^m|_x^{x-L}|_{p_2}^{p_2-w} + T_{HIII}^m|_x^{x-L}|_{p_1-w}^{p_1-W} & (d<H<d+W\sin\delta) \\ T_{HIII}^m|_x^{x-L}|_{p_1}^{p_1-W} & (d+W\sin\delta<H) \end{cases} \\
&\quad (m=x,y,z,\ w=(H-d)/\sin\delta)
\end{aligned}$$

A.3.1 Horizontal magnetization in x-direction

$$\frac{2}{C_x}T_0^x = -2K_7(1) \qquad \text{(A-34)}$$

$$\begin{aligned}
\frac{2}{C_x}T_{H0}^x &= 2K_7(3) - \frac{\alpha(2\alpha+1)}{4\alpha-1}K_7(3) \\
&+ \frac{4\alpha(1-\alpha)}{4\alpha-1}\sin\delta\log(R_3+\tilde{c}_3) - \alpha\log(R_3+\eta_3) \\
&+ \frac{2\alpha(1-\alpha)}{4\alpha-1}\left[q'P_1(3)-(z-H)\{M_2(3)+N_2(3)\sin\delta\}\tan\delta\right] \\
&+ \frac{2\alpha(2\alpha-5)}{4\alpha-1}H\left[P_1(3)\cos\delta+\{M_2(3)+N_2(3)\sin\delta\}\tan\delta\right] + \frac{12\alpha(1-\alpha)}{4\alpha-1}HM_3(3) \\
&- \frac{12\alpha^2}{4\alpha-1}H\left[(q'+H\cos\delta)P_{1,z}(3)-(z-2H)P_{1,y}(3)\sin\delta\right] \qquad \text{(A-35)}
\end{aligned}$$

$$\begin{aligned}
\frac{2}{C_x}T_{HI}^x &= -\frac{\alpha(2\alpha+1)}{4\alpha-1}K_7(2) + \alpha\log(R_2+\eta_2) - \frac{4\alpha(1-\alpha)}{4\alpha-1}\sin\delta\log(R_2+\tilde{c}_2) \\
&+ \frac{6\alpha^2}{4\alpha-1}\left[q'P_1(2)+(z-H)\{M_2(2)-N_2(2)\sin\delta\}\tan\delta\right] \qquad \text{(A-36)}
\end{aligned}$$

$$\begin{aligned}
\frac{2}{C_x}T_{HIII}^x &= \frac{\alpha(2\alpha+1)}{4\alpha-1}K_7(1) + \alpha\log(R_1+\eta_1) - \frac{4\alpha(1-\alpha)}{4\alpha-1}\sin\delta\log(R_1+\tilde{c}_1) \\
&- \frac{2\alpha(1-\alpha)}{4\alpha-1}\left[q'P_1(1)-(z-H)\{M_2(1)+N_2(1)\sin\delta\}\tan\delta\right] \qquad \text{(A-37)}
\end{aligned}$$

A.3.2 Horizontal magnetization in y-direction

$$\frac{2}{C_y}T_0^y = 2K_8(1) \qquad \text{(A-38)}$$

$$
\begin{aligned}
\frac{2}{C_y}T^y_{H0} &= -2K_8(3)+\frac{\alpha(2\alpha+1)}{4\alpha-1}K_8(3) \\
&- \frac{\alpha(2\alpha+1)}{4\alpha-1}\sin\delta\tan^{-1}\left(\frac{\xi\eta_3}{q_3R_3}\right)-\alpha\cos\delta\log(R_3+\xi)+\frac{4\alpha(1-\alpha)}{4\alpha-1}J_1(3)\sin\delta \\
&+ \frac{2\alpha(1-\alpha)}{4\alpha-1}\left[q'\{O_3(3)+M_1(3)\sin\delta\}-(z-H)\{O_2(3)-L_1(3)\tan\delta\}\sin\delta\right] \\
&- 2\alpha H\left[\{O_3(3)+M_1(3)\sin\delta\}\cos\delta+\{O_2(3)-L_1(3)\tan\delta\}\sin\delta\right] \\
&+ \frac{12\alpha(1-\alpha)}{4\alpha-1}HL_1(3)\sin\delta\tan\delta \\
&+ \frac{12\alpha^2}{4\alpha-1}H\Big[(q'+H\cos\delta)\{O_{2,y}(3)+M_{1,y}(3)\cos\delta\} \\
&\quad +(z-2H)\{O_{2,z}(3)+M_{1,y}(3)\sin\delta\}+M_1(3)\sin^2\delta\cos\delta\Big] \qquad \text{(A-39)}
\end{aligned}
$$

$$
\begin{aligned}
\frac{2}{C_y}T^y_{HI} &= \frac{\alpha(2\alpha+1)}{4\alpha-1}K_8(2)+\alpha\cos\delta\log(R_2+\xi) \\
&- \frac{\alpha(2\alpha+1)}{4\alpha-1}\sin\delta\tan^{-1}\left(\frac{\xi\eta_2}{q_2R_2}\right)+\frac{4\alpha(1-\alpha)}{4\alpha-1}J_1(2)\sin\delta \\
&- \frac{6\alpha^2}{4\alpha-1}\left[q'\{O_3(2)-M_1(2)\sin\delta\}-(z-H)\{O_2(2)+L_1(2)\tan\delta\}\sin\delta\right]
\end{aligned}
\qquad \text{(A-40)}
$$

$$
\begin{aligned}
\frac{2}{C_y}T^y_{HIII} &= -\frac{\alpha(2\alpha+1)}{4\alpha-1}K_8(1)+\alpha\cos\delta\log(R_1+\xi) \\
&+ \frac{\alpha(2\alpha+1)}{4\alpha-1}\sin\delta\tan^{-1}\left(\frac{\xi\eta_1}{q_1R_1}\right)-\frac{4\alpha(1-\alpha)}{4\alpha-1}J_1(1)\sin\delta \\
&- \frac{2\alpha(1-\alpha)}{4\alpha-1}\left[q'\{O_3(1)+M_1(1)\sin\delta\}-(z-H)\{O_2(1)-L_1(1)\tan\delta\}\sin\delta\right]
\end{aligned}
\qquad \text{(A-41)}
$$

A.3.3 Vertical magnetization

$$
\frac{2}{C_z}T^z_0 = 2K_9(1) \qquad \text{(A-42)}
$$

$$
\begin{aligned}
\frac{2}{C_z}T^z_{H0} &= -2K_9(3)-\frac{\alpha(2\alpha-5)}{4\alpha-1}K_9(3) \\
&- \alpha\sin\delta\log(R_3+\xi)+\frac{\alpha(2\alpha+1)}{4\alpha-1}\cos\delta\tan^{-1}\left(\frac{\xi\eta_3}{q_3R_3}\right) \\
&+ \frac{2\alpha(1-\alpha)}{4\alpha-1}\left[q'P_3(3)-(z-H)P_2(3)\sin\delta\right] \\
&+ \frac{2\alpha(2\alpha+1)}{4\alpha-1}H\left[P_3(3)\cos\delta+P_2(3)\sin\delta\right]+\frac{12\alpha}{4\alpha-1}HM_1(3)\sin^2\delta
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad \frac{12\alpha^2}{4\alpha-1}\left[(q'+H\cos\delta)P_{3,z}(3)-(z-2H)P_{3,y}(3)+2P_3(3)\sin\delta\right] && \text{(A-43)}\\
\frac{2}{C_z}T^z_{HI} &= -\frac{\alpha(2\alpha-5)}{4\alpha-1}K_9(2)+\alpha\sin\delta\log(R_2+\xi)+\frac{\alpha(2\alpha+1)}{4\alpha-1}\cos\delta\tan^{-1}\left(\frac{\xi\eta_2}{q_2R_2}\right)\\
&- \quad \frac{6\alpha^2}{4\alpha-1}\left[q'P_3(2)-(z-H)P_2(2)\sin\delta\right] && \text{(A-44)}\\
\frac{2}{C_z}T^z_{HIII} &= \frac{\alpha(2\alpha-5)}{4\alpha-1}K_9(1)+\alpha\sin\delta\log(R_1+\xi)-\frac{\alpha(2\alpha+1)}{4\alpha-1}\cos\delta\tan^{-1}\left(\frac{\xi\eta_1}{q_1R_1}\right)\\
&- \quad \frac{2\alpha(1-\alpha)}{4\alpha-1}\left[q'P_3(1)-(z-H)P_2(1)\sin\delta\right] && \text{(A-45)}
\end{aligned}
$$

where

$$q' = y\sin\delta + (d-H)\cos\delta.$$