$$J_1(i) = 2\tan^{-1}\left\{\frac{(R_i \mp \eta_i)(1+\sin\delta) \pm q_i\cos\delta}{\cos\delta}\right\}$$

$$\left\{\begin{array}{lcl}
\dfrac{\partial J_1}{\partial x} & = & \dfrac{q_i}{R_i(R_i+\eta_i)} \pm \dfrac{\tilde{y}_i}{R_i(R_i+\tilde{c}_i)} \\
\dfrac{\partial J_1}{\partial y} & = & -\dfrac{\xi\sin\delta}{R_i(R_i+\eta_i)} \mp \dfrac{\xi}{R_i(R_i+\tilde{c}_i)} \\
\dfrac{\partial J_1}{\partial z} & = & \pm\dfrac{\xi\cos\delta}{R_i(R_i+\eta_i)}
\end{array}\right.$$

$$J_2(i) = \log(R_i+\tilde{c}_i) \pm \sin\delta\log(R_i+\eta_i)$$

$$\left\{\begin{array}{lcl}
\dfrac{\partial J_2}{\partial x} & = & \dfrac{\xi}{R_i(R_i+\tilde{c}_i)} \pm \dfrac{\xi\sin\delta}{R_i(R_i+\eta_i)} \\
\dfrac{\partial J_2}{\partial y} & = & \dfrac{\tilde{y}_i}{R_i(R_i+\tilde{c}_i)} \pm \dfrac{\sin\delta\cos\delta}{R_i+\eta_i} \pm \dfrac{\tilde{y}_i\sin\delta}{R_i(R_i+\eta_i)} \\
\dfrac{\partial J_2}{\partial z} & = & -\dfrac{1}{R_i} + \dfrac{\sin^2\delta}{R_i+\eta_i} \mp \dfrac{\tilde{c}_i\sin\delta}{R_i(R_i+\eta_i)}
\end{array}\right.$$

$$K_1(i) = \tan\delta\left[\frac{\xi}{R_i+\tilde{c}_i} + J_1(i)\tan\delta\right]$$

$$\left\{\begin{array}{lcl}
\dfrac{\partial K_1}{\partial x} & = & \tan\delta\left[\dfrac{1}{R_i+\tilde{c}_i} - \dfrac{\xi^2}{R_i(R_i+\tilde{c}_i)^2} + \dfrac{\partial J_1}{\partial x}\tan\delta\right] \\
\dfrac{\partial K_1}{\partial y} & = & \tan\delta\left[-\dfrac{\xi\tilde{y}_i}{R_i(R_i+\tilde{c}_i)^2} + \dfrac{\partial J_1}{\partial y}\tan\delta\right] \\
\dfrac{\partial K_1}{\partial z} & = & \tan\delta\left[\dfrac{\xi}{(R_i+\tilde{c}_i)^2} + \dfrac{\xi\tilde{c}_i}{R_i(R_i+\tilde{c}_i)^2} + \dfrac{\partial J_1}{\partial z}\tan\delta\right]
\end{array}\right.$$

$$K_2(i) = \tan\delta\left[\frac{\tilde{y}_i}{R_i+\tilde{c}_i} \mp J_2(i)\tan\delta\right]$$

$$\left\{\begin{array}{lcl}
\dfrac{\partial K_2}{\partial x} & = & \tan\delta\left[-\dfrac{\xi\tilde{y}_i}{R_i(R_i+\tilde{c}_i)^2} \mp \dfrac{\partial J_2}{\partial x}\tan\delta\right] \\
\dfrac{\partial K_2}{\partial y} & = & \tan\delta\left[\dfrac{1}{R_i+\tilde{c}_i} - \dfrac{\tilde{y}_i^2}{R_i(R_i+\tilde{c}_i)^2} \mp \dfrac{\partial J_2}{\partial y}\tan\delta\right] \\
\dfrac{\partial K_2}{\partial z} & = & \tan\delta\left[\dfrac{\tilde{y}_i}{(R_i+\tilde{c}_i)^2} + \dfrac{\tilde{y}_i\tilde{c}_i}{R_i(R_i+\tilde{c}_i)^2} \mp \dfrac{\partial J_2}{\partial z}\tan\delta\right]
\end{array}\right.$$

$$K_3(i) = J_2(i)\tan\delta$$

$$\left\{\begin{array}{lcl}
\dfrac{\partial K_3}{\partial x} & = & \dfrac{\partial J_2}{\partial x}\tan\delta \\
\dfrac{\partial K_3}{\partial y} & = & \dfrac{\partial J_2}{\partial y}\tan\delta \\
\dfrac{\partial K_3}{\partial z} & = & \dfrac{\partial J_2}{\partial z}\tan\delta
\end{array}\right.$$

$K_4(i) = \cos\delta\,[K_2(i) - \sin\delta\log(R_i+\eta_i)]$

$$\left\{\begin{array}{lcl} \dfrac{\partial K_4}{\partial x} & = & \cos\delta\left[\dfrac{\partial K_2}{\partial x} - \dfrac{\xi\sin\delta}{R_i(R_i+\eta_i)}\right] \\ \dfrac{\partial K_4}{\partial y} & = & \cos\delta\left[\dfrac{\partial K_2}{\partial y} - \sin\delta\left\{\dfrac{\cos\delta}{R_i+\eta_i} + \dfrac{\tilde{y}_i}{R_i(R_i+\eta_i)}\right\}\right] \\ \dfrac{\partial K_4}{\partial z} & = & \cos\delta\left[\dfrac{\partial K_2}{\partial z} - \sin\delta\left\{\pm\dfrac{\sin\delta}{R_i+\eta_i} - \dfrac{\tilde{c}_i}{R_i(R_i+\eta_i)}\right\}\right] \end{array}\right.$$

$K_5(i) = K_1(i)\cos\delta$

$$\left\{\begin{array}{lcl} \dfrac{\partial K_5}{\partial x} & = & \dfrac{\partial K_1}{\partial x}\cos\delta \\ \dfrac{\partial K_5}{\partial y} & = & \dfrac{\partial K_1}{\partial y}\cos\delta \\ \dfrac{\partial K_5}{\partial z} & = & \dfrac{\partial K_1}{\partial z}\cos\delta \end{array}\right.$$

$K_6(i) = -J_1(i)\sin\delta$

$$\left\{\begin{array}{lcl} \dfrac{\partial K_6}{\partial x} & = & -\dfrac{\partial J_1}{\partial x}\sin\delta \\ \dfrac{\partial K_6}{\partial y} & = & -\dfrac{\partial J_1}{\partial y}\sin\delta \\ \dfrac{\partial K_6}{\partial z} & = & -\dfrac{\partial J_1}{\partial z}\sin\delta \end{array}\right.$$

$K_7(i) = -K_4(i)\tan\delta$

$$\left\{\begin{array}{lcl} \dfrac{\partial K_7}{\partial x} & = & -\dfrac{\partial K_4}{\partial x}\tan\delta \\ \dfrac{\partial K_7}{\partial y} & = & -\dfrac{\partial K_4}{\partial y}\tan\delta \\ \dfrac{\partial K_7}{\partial z} & = & -\dfrac{\partial K_4}{\partial z}\tan\delta \end{array}\right.$$

$K_8(i) = -K_5(i)\tan\delta$

$$\left\{\begin{array}{lcl} \dfrac{\partial K_8}{\partial x} & = & -\dfrac{\partial K_5}{\partial x}\tan\delta \\ \dfrac{\partial K_8}{\partial y} & = & -\dfrac{\partial K_5}{\partial y}\tan\delta \\ \dfrac{\partial K_8}{\partial z} & = & -\dfrac{\partial K_5}{\partial z}\tan\delta \end{array}\right.$$

$K_9(i) = -K_6(i)\tan\delta$

$$\left\{\begin{array}{lcl} \dfrac{\partial K_9}{\partial x} & = & -\dfrac{\partial K_6}{\partial x}\tan\delta \\ \dfrac{\partial K_9}{\partial y} & = & -\dfrac{\partial K_6}{\partial y}\tan\delta \\ \dfrac{\partial K_9}{\partial z} & = & -\dfrac{\partial K_6}{\partial z}\tan\delta \end{array}\right.$$

$$\left\{\begin{array}{lcl}
\dfrac{\partial L_0}{\partial x} & = & -\dfrac{\xi(2R_i+\tilde{c}_i)}{R_i^3(R_i+\tilde{c}_i)^2} \mp \dfrac{\xi(2R_i+\eta_i)}{R_i^3(R_i+\eta_i)^2}\sin\delta \\
\dfrac{\partial L_0}{\partial y} & = & -\dfrac{\tilde{y}_i(2R_i+\tilde{c}_i)}{R_i^3(R_i+\tilde{c}_i)^2} \mp \dfrac{\tilde{y}_i(2R_i+\eta_i)}{R_i^3(R_i+\eta_i)^2}\sin\delta \mp \dfrac{\sin\delta\cos\delta}{R_i(R_i+\eta_i)^2} \\
\dfrac{\partial L_0}{\partial z} & = & \dfrac{1}{R_i^3} \pm \dfrac{\tilde{c}_i(2R_i+\eta_i)}{R_i^3(R_i+\eta_i)^2}\sin\delta - \dfrac{\sin^2\delta}{R_i(R_i+\eta_i)^2}
\end{array}\right.$$

$$\left\{\begin{array}{lcl}
\dfrac{\partial L_1}{\partial x} & = & L_0 + \xi\dfrac{\partial L_0}{\partial x} \\
\dfrac{\partial L_1}{\partial y} & = & \xi\dfrac{\partial L_0}{\partial y} \\
\dfrac{\partial L_1}{\partial z} & = & \xi\dfrac{\partial L_0}{\partial z}
\end{array}\right.$$

$$\left\{\begin{array}{lcl}
\dfrac{\partial L_2}{\partial x} & = & \dfrac{\partial L_1}{\partial y}\sec\delta \\
\dfrac{\partial L_2}{\partial y} & = & \pm\left\{\dfrac{\sin\delta}{(R_i+\eta_i)^2} \mp \dfrac{\tilde{c}_i}{R_i(R_i+\eta_i)^2}\right\}\sin\delta\tan\delta + \left\{\dfrac{1}{R_i(R_i+\tilde{c}_i)} + \tilde{y}_i\dfrac{\partial L_0}{\partial y}\right\}\sec\delta \\
\dfrac{\partial L_2}{\partial z} & = & -\left\{\dfrac{\sin\delta}{(R_i+\eta_i)^2} \mp \dfrac{\tilde{c}_i}{R_i(R_i+\eta_i)^2}\right\}\sin\delta + \tilde{y}_i\dfrac{\partial L_0}{\partial z}\sec\delta
\end{array}\right.$$

$$\left\{\begin{array}{lcl}
\dfrac{\partial M_1}{\partial x} & = & \dfrac{1}{R_i(R_i+\eta_i)} - \dfrac{\xi^2(2R_i+\eta_i)}{R_i^3(R_i+\eta_i)^2} \\
\dfrac{\partial M_1}{\partial y} & = & -\dfrac{\xi\cos\delta}{R_i(R_i+\eta_i)^2} - \dfrac{\xi\tilde{y}_i(2R_i+\eta_i)}{R_i^3(R_i+\eta_i)^2} \\
\dfrac{\partial M_1}{\partial z} & = & \mp\dfrac{\xi\sin\delta}{R_i(R_i+\eta_i)^2} + \dfrac{\xi\tilde{c}_i(2R_i+\eta_i)}{R_i^3(R_i+\eta_i)^2}
\end{array}\right.$$

$$\left\{\begin{array}{lcl}
\dfrac{\partial M_2}{\partial x} & = & \dfrac{\partial M_1}{\partial y} \\
\dfrac{\partial M_2}{\partial y} & = & \dfrac{\sin^2\delta}{(R_i+\eta_i)^2} - \dfrac{\tilde{y}_i\cos\delta \pm \tilde{c}_i\sin\delta}{R_i(R_i+\eta_i)^2} - \dfrac{\tilde{y}_i^2(2R_i+\eta_i)}{R_i^3(R_i+\eta_i)^2} \\
\dfrac{\partial M_2}{\partial z} & = & \mp\dfrac{\sin\delta\cos\delta}{(R_i+\eta_i)^2} + \dfrac{\tilde{c}_i\cos\delta \mp \tilde{y}_i\sin\delta}{R_i(R_i+\eta_i)^2} + \dfrac{\tilde{y}_i\tilde{c}_i(2R_i+\eta_i)}{R_i^3(R_i+\eta_i)^2}
\end{array}\right.$$

$$\left\{\begin{array}{lcl}
\dfrac{\partial M_3}{\partial x} & = & \dfrac{\partial M_1}{\partial z} \\
\dfrac{\partial M_3}{\partial y} & = & \dfrac{\partial M_2}{\partial z} \\
\dfrac{\partial M_3}{\partial z} & = & \dfrac{\cos^2\delta}{(R_i+\eta_i)^2} + \dfrac{\tilde{y}_i\cos\delta \pm \tilde{c}_i\sin\delta}{R_i(R_i+\eta_i)^2} - \dfrac{\tilde{c}_i^2(2R_i+\eta_i)}{R_i^3(R_i+\eta_i)^2}
\end{array}\right.$$

$$\left\{\begin{array}{lcl}
\dfrac{\partial N_1}{\partial x} & = & \dfrac{1}{R_i(R_i+\tilde{c}_i)} - \dfrac{\xi^2(2R_i+\tilde{c}_i)}{R_i^3(R_i+\tilde{c}_i)^2} \\
\dfrac{\partial N_1}{\partial y} & = & -\dfrac{\xi\tilde{y}_i(2R_i+\tilde{c}_i)}{R_i^3(R_i+\tilde{c}_i)^2} \\
\dfrac{\partial N_1}{\partial z} & = & \dfrac{\xi}{R_i^3}
\end{array}\right.$$

$$\left\{\begin{array}{lcl}
\dfrac{\partial N_2}{\partial x} & = & \dfrac{\partial N_1}{\partial y} \\
\dfrac{\partial N_2}{\partial y} & = & \dfrac{1}{R_i(R_i+\tilde{c}_i)} - \dfrac{\tilde{y}_i^2(2R_i+\tilde{c}_i)}{R_i^3(R_i+\tilde{c}_i)^2} \\
\dfrac{\partial N_2}{\partial z} & = & \dfrac{\tilde{y}_i}{R_i^3}
\end{array}\right.$$

$$\left\{\begin{aligned}
\frac{\partial O_1}{\partial x} &= -\frac{\xi}{R_i^3}\\
\frac{\partial O_1}{\partial y} &= -\frac{\tilde{y}_i}{R_i^3}\\
\frac{\partial O_1}{\partial z} &= \frac{\tilde{c}_i}{R_i^3}
\end{aligned}\right.$$

$$\left\{\begin{aligned}
\frac{\partial O_2}{\partial x} &= \frac{\partial O_1}{\partial y}\\
\frac{\partial O_2}{\partial y} &= \frac{1}{R_i(R_i+\xi)} - \frac{\tilde{y}_i^2(2R_i+\xi)}{R_i^3(R_i+\xi)^2}\\
\frac{\partial O_2}{\partial z} &= \frac{\tilde{y}_i\tilde{c}_i(2R_i+\xi)}{R_i^3(R_i+\xi)^2}
\end{aligned}\right.$$

$$\left\{\begin{aligned}
\frac{\partial O_3}{\partial x} &= \frac{\partial O_1}{\partial z}\\
\frac{\partial O_3}{\partial y} &= \frac{\partial O_2}{\partial z}\\
\frac{\partial O_3}{\partial z} &= \frac{1}{R_i(R_i+\xi)} - \frac{\tilde{c}_i^2(2R_i+\xi)}{R_i^3(R_i+\xi)^2}
\end{aligned}\right.$$

$$\left\{\begin{aligned}
\frac{\partial P_1}{\partial x} &= \frac{\xi q_i(2R_i+\eta_i)}{R_i^3(R_i+\eta_i)^2}\\
\frac{\partial P_1}{\partial y} &= \pm\frac{\tilde{c}_i}{R_i^3} + \frac{\sin\delta}{R_i(R_i+\eta_i)} - \frac{\xi^2(2R_i+\eta_i)}{R_i^3(R_i+\eta_i)^2}\sin\delta\\
\frac{\partial P_1}{\partial z} &= \pm\left\{\frac{\tilde{y}_i}{R_i^3} - \frac{\cos\delta}{R_i(R_i+\eta_i)} + \frac{\xi^2(2R_i+\eta_i)}{R_i^3(R_i+\eta_i)^2}\cos\delta\right\}
\end{aligned}\right.$$

$$\left\{\begin{aligned}
\frac{\partial P_2}{\partial x} &= \frac{\partial P_1}{\partial y}\\
\frac{\partial P_2}{\partial y} &= \pm\frac{\tilde{y}_i\tilde{c}_i(2R_i+\xi)}{R_i^3(R_i+\xi)^2} - \frac{\xi\sin\delta\cos\delta}{R_i(R_i+\eta_i)^2} - \frac{\xi\tilde{y}_i(2R_i+\eta_i)}{R_i^3(R_i+\eta_i)^2}\sin\delta\\
\frac{\partial P_2}{\partial z} &= \pm\left\{\frac{1}{R_i(R_i+\xi)} - \frac{\tilde{c}_i^2(2R_i+\xi)}{R_i^3(R_i+\xi)^2}\right\} \mp \frac{\xi\sin^2\delta}{R_i(R_i+\eta_i)^2} + \frac{\xi\tilde{c}_i(2R_i+\eta_i)}{R_i^3(R_i+\eta_i)^2}\sin\delta
\end{aligned}\right.$$

$$\left\{\begin{aligned}
\frac{\partial P_3}{\partial x} &= \frac{\partial P_1}{\partial z}\\
\frac{\partial P_3}{\partial y} &= \frac{\partial P_2}{\partial z}\\
\frac{\partial P_3}{\partial z} &= \mp\frac{\tilde{y}_i\tilde{c}_i(2R_i+\xi)}{R_i^3(R_i+\xi)^2} + \frac{\xi\sin\delta\cos\delta}{R_i(R_i+\eta_i)^2} \mp \frac{\xi\tilde{c}_i(2R_i+\eta_i)}{R_i^3(R_i+\eta_i)^2}\cos\delta
\end{aligned}\right.$$

$$\left\{\begin{aligned}
\frac{\partial M_{1,y}}{\partial x} &= -\frac{\cos\delta}{R_i(R_i+\eta_i)^2} + \frac{\xi^2(3R_i+\eta_i)}{R_i^3(R_i+\eta_i)^3}\cos\delta\\
&\quad -\frac{\tilde{y}_i(2R_i+\eta_i)}{R_i^3(R_i+\eta_i)^2} + \frac{\xi^2\tilde{y}_i(8R_i^2+9\eta_iR_i+3\eta_i^2)}{R_i^5(R_i+\eta_i)^3}\\
\frac{\partial M_{1,y}}{\partial y} &= -\frac{2\xi\sin^2\delta}{R_i(R_i+\eta_i)^3} + \frac{\xi(\tilde{y}_i\cos\delta\pm\tilde{c}_i\sin\delta)(3R_i+\eta_i)}{R_i^3(R_i+\eta_i)^3}\\
&\quad +\frac{\xi\tilde{y}_i^2(8R_i^2+9\eta_iR_i+3\eta_i^2)}{R_i^5(R_i+\eta_i)^3}\\
\frac{\partial M_{1,y}}{\partial z} &= \pm\frac{2\xi\sin\delta\cos\delta}{R_i(R_i+\eta_i)^3} - \frac{\xi(\tilde{c}_i\cos\delta\mp\tilde{y}_i\sin\delta)(3R_i+\eta_i)}{R_i^3(R_i+\eta_i)^3}\\
&\quad -\frac{\xi\tilde{y}_i\tilde{c}_i(8R_i^2+9\eta_iR_i+3\eta_i^2)}{R_i^5(R_i+\eta_i)^3}
\end{aligned}\right.$$

$$
\left\{
\begin{aligned}
\frac{\partial M_{1,z}}{\partial x} &= \pm\frac{\xi^2(3R_i+\eta_i)}{R_i^3(R_i+\eta_i)^3}\sin\delta \\
&\quad +\frac{\tilde{c}_i(2R_i+\eta_i)}{R_i^3(R_i+\eta_i)^2}-\frac{\xi^2\tilde{c}_i(8R_i^2+9\eta_iR_i+3\eta_i^2)}{R_i^5(R_i+\eta_i)^3} \\
&\quad \mp\frac{\sin\delta}{R_i(R_i+\eta_i)^2} \\
\frac{\partial M_{1,z}}{\partial y} &= \frac{\partial M_{1,y}}{\partial z} \\
\frac{\partial M_{1,z}}{\partial z} &= -\frac{2\xi\cos^2\delta}{R_i(R_i+\eta_i)^3}-\frac{\xi(\tilde{y}_i\cos\delta\pm\tilde{c}_i\sin\delta)(3R_i+\eta_i)}{R_i^3(R_i+\eta_i)^3} \\
&\quad +\frac{\xi\tilde{c}_i^2(8R_i^2+9\eta_iR_i+3\eta_i^2)}{R_i^5(R_i+\eta_i)^3}
\end{aligned}
\right.
$$

$$
\left\{
\begin{aligned}
\frac{\partial M_{2,y}}{\partial x} &= \frac{\partial M_{1,y}}{\partial y} \\
\frac{\partial M_{2,y}}{\partial y} &= -\frac{2\tilde{y}_i\sin^2\delta}{R_i(R_i+\eta_i)^3}-\frac{\cos\delta}{R_i(R_i+\eta_i)^2}+\frac{\tilde{y}_i(\tilde{y}_i\cos\delta\pm\tilde{c}_i\sin\delta)(3R_i+\eta_i)}{R_i^3(R_i+\eta_i)^3} \\
&\quad -\frac{2\tilde{y}_i(2R_i+\eta_i)}{R_i^3(R_i+\eta_i)^2}+\frac{\tilde{y}_i^3(8R_i^2+9\eta_iR_i+3\eta_i^2)}{R_i^5(R_i+\eta_i)^3} \\
&\quad -\frac{2\sin^2\delta\cos\delta}{(R_i+\eta_i)^3}+\frac{2(\tilde{y}_i\cos\delta\pm\tilde{c}_i\sin\delta)}{R_i(R_i+\eta_i)^3}\cos\delta+\frac{\tilde{y}_i^2(3R_i+\eta_i)}{R_i^3(R_i+\eta_i)^3}\cos\delta \\
\frac{\partial M_{2,y}}{\partial z} &= \frac{2\tilde{c}_i\sin^2\delta}{R_i(R_i+\eta_i)^3}\pm\frac{\sin\delta}{R_i(R_i+\eta_i)^2}-\frac{\tilde{c}_i(\tilde{y}_i\cos\delta\pm\tilde{c}_i\sin\delta)(3R_i+\eta_i)}{R_i^3(R_i+\eta_i)^3} \\
&\quad -\frac{\tilde{y}_i^2\tilde{c}_i(8R_i^2+9\eta_iR_i+3\eta_i^2)}{R_i^5(R_i+\eta_i)^3} \\
&\quad \mp\frac{2\sin^3\delta}{(R_i+\eta_i)^3}\pm\frac{2(\tilde{y}_i\cos\delta\pm\tilde{c}_i\sin\delta)}{R_i(R_i+\eta_i)^3}\sin\delta\pm\frac{\tilde{y}_i^2(3R_i+\eta_i)}{R_i^3(R_i+\eta_i)^3}\sin\delta
\end{aligned}
\right.
$$

$$
\left\{
\begin{aligned}
\frac{\partial M_{2,z}}{\partial x} &= \frac{\partial M_{1,y}}{\partial z} \\
\frac{\partial M_{2,z}}{\partial y} &= \frac{\partial M_{2,y}}{\partial z} \\
\frac{\partial M_{2,z}}{\partial z} &= -\frac{2\tilde{y}_i\cos^2\delta}{R_i(R_i+\eta_i)^3}+\frac{\cos\delta}{R_i(R_i+\eta_i)^2}-\frac{\tilde{y}_i(\tilde{y}_i\cos\delta\pm\tilde{c}_i\sin\delta)(3R_i+\eta_i)}{R_i^3(R_i+\eta_i)^3} \\
&\quad +\frac{\tilde{y}_i\tilde{c}_i^2(8R_i^2+9\eta_iR_i+3\eta_i^2)}{R_i^5(R_i+\eta_i)^3} \\
&\quad -\frac{2\cos^3\delta}{(R_i+\eta_i)^3}-\frac{2(\tilde{y}_i\cos\delta\pm\tilde{c}_i\sin\delta)}{R_i(R_i+\eta_i)^3}\cos\delta+\frac{\tilde{c}_i^2(3R_i+\eta_i)}{R_i^3(R_i+\eta_i)^3}\cos\delta
\end{aligned}
\right.
$$

$$
\left\{
\begin{aligned}
\frac{\partial M_{3,z}}{\partial x} &= \frac{\partial M_{1,z}}{\partial z} \\
\frac{\partial M_{3,z}}{\partial y} &= \frac{\partial M_{2,z}}{\partial z} \\
\frac{\partial M_{3,z}}{\partial q_i} &= \frac{2\tilde{c}_i\cos^2\delta}{R_i(R_i+\eta_i)^3}\mp\frac{\sin\delta}{R_i(R_i+\eta_i)^2}+\frac{\tilde{c}_i(\tilde{y}_i\cos\delta\pm\tilde{c}_i\sin\delta)(3R_i+\eta_i)}{R_i^3(R_i+\eta_i)^3} \\
&\quad +\frac{2\tilde{c}_i(2R_i+\eta_i)}{R_i^3(R_i+\eta_i)^2}-\frac{\tilde{c}_i^3(8R_i^2+9\eta_iR_i+3\eta_i^2)}{R_i^5(R_i+\eta_i)^3} \\
&\quad \mp\frac{2\sin\delta\cos^2\delta}{(R_i+\eta_i)^3}\mp\frac{2(\tilde{y}_i\cos\delta\pm\tilde{c}_i\sin\delta)}{R_i(R_i+\eta_i)^3}\sin\delta\pm\frac{\tilde{c}_i^2(3R_i+\eta_i)}{R_i^3(R_i+\eta_i)^3}\sin\delta
\end{aligned}
\right.
$$

$$
\left\{\begin{array}{lcl}
\dfrac{\partial N_{1,z}}{\partial x} & = & \dfrac{1}{R_i^3} - 3\dfrac{\xi^2}{R_i^5} \\
\dfrac{\partial N_{1,z}}{\partial y} & = & -3\dfrac{\xi\tilde{y}_i}{R_i^5} \\
\dfrac{\partial N_{1,z}}{\partial z} & = & 3\dfrac{\xi\tilde{c}_i}{R_i^5}
\end{array}\right.
$$

$$
\left\{\begin{array}{lcl}
\dfrac{\partial N_{2,z}}{\partial x} & = & \dfrac{\partial N_{1,z}}{\partial y} \\
\dfrac{\partial N_{2,z}}{\partial y} & = & \dfrac{1}{R_i^3} - 3\dfrac{\tilde{y}_i^2}{R_i^5} \\
\dfrac{\partial N_{2,z}}{\partial z} & = & 3\dfrac{\tilde{y}_i\tilde{c}_i}{R_i^5}
\end{array}\right.
$$

$$
\left\{\begin{array}{lcl}
\dfrac{\partial O_{2,y}}{\partial x} & = & -\dfrac{1}{R_i^3} + 3\dfrac{\tilde{y}_i^2}{R_i^5} \\
\dfrac{\partial O_{2,y}}{\partial y} & = & -\dfrac{3\tilde{y}_i(2R_i+\xi)}{R_i^3(R_i+\xi)^2} + \dfrac{\tilde{y}_i^3(8R_i^2+9\xi R_i+3\xi^2)}{R_i^5(R_i+\xi)^3} \\
\dfrac{\partial O_{2,y}}{\partial z} & = & \dfrac{\tilde{c}_i(2R_i+\xi)}{R_i^3(R_i+\xi)^2} - \dfrac{\tilde{y}_i^2\tilde{c}_i(8R_i^2+9\xi R_i+3\xi^2)}{R_i^5(R_i+\xi)^3}
\end{array}\right.
$$

$$
\left\{\begin{array}{lcl}
\dfrac{\partial O_{2,z}}{\partial x} & = & -3\dfrac{\tilde{y}_i\tilde{c}_i}{R_i^5} \\
\dfrac{\partial O_{2,z}}{\partial y} & = & \dfrac{\partial O_{2,y}}{\partial z} \\
\dfrac{\partial O_{2,z}}{\partial z} & = & -\dfrac{\tilde{y}_i(2R_i+\xi)}{R_i^3(R_i+\xi)^2} + \dfrac{\tilde{y}_i\tilde{c}_i^2(8R_i^2+9\xi R_i+3\xi^2)}{R_i^5(R_i+\xi)^3}
\end{array}\right.
$$

$$
\left\{\begin{array}{lcl}
\dfrac{\partial O_{3,z}}{\partial x} & = & -\dfrac{1}{R_i^3} + 3\dfrac{\tilde{c}_i^2}{R_i^5} \\
\dfrac{\partial O_{3,z}}{\partial y} & = & \dfrac{\partial O_{2,z}}{\partial z} \\
\dfrac{\partial O_{3,z}}{\partial z} & = & \dfrac{3\tilde{c}_i(2R_i+\xi)}{R_i^3(R_i+\xi)^2} - \dfrac{\tilde{c}_i^3(8R_i^2+9\xi R_i+3\xi^2)}{R_i^5(R_i+\xi)^3}
\end{array}\right.
$$

$$
\left\{\begin{array}{lcl}
\dfrac{\partial P_{1,y}}{\partial x} & = & \mp 3\dfrac{\xi\tilde{c}_i}{R_i^5} - \dfrac{3\xi(2R_i+\eta_i)}{R_i^3(R_i+\eta_i)^2}\sin\delta + \dfrac{\xi^3(8R_i^2+9\xi R_i+3\xi^2)}{R_i^5(R_i+\xi)^3}\sin\delta \\
\dfrac{\partial P_{1,y}}{\partial y} & = & \mp 3\dfrac{\tilde{y}_i\tilde{c}_i}{R_i^5} - \dfrac{\tilde{y}_i(2R_i+\eta_i)}{R_i^3(R_i+\eta_i)^2}\sin\delta + \dfrac{\xi^2\tilde{y}_i(8R_i^2+9\eta_i R_i+3\eta_i^2)}{R_i^5(R_i+\eta_i)^3}\sin\delta \\
 & & -\dfrac{\sin\delta\cos\delta}{R_i(R_i+\eta_i)^2} + \dfrac{\xi^2(3R_i+\eta_i)}{R_i^3(R_i+\eta_i)^3}\sin\delta\cos\delta \\
\dfrac{\partial P_{1,y}}{\partial z} & = & \mp\dfrac{1}{R_i^3} \pm 3\dfrac{\tilde{c}_i^2}{R_i^5} + \dfrac{\tilde{c}_i(2R_i+\eta_i)}{R_i^3(R_i+\eta_i)^2}\sin\delta - \dfrac{\xi^2\tilde{c}_i(8R_i^2+9\eta_i R_i+3\eta_i^2)}{R_i^5(R_i+\eta_i)^3}\sin\delta \\
 & & \mp\dfrac{\sin^2\delta}{R_i(R_i+\eta_i)^2} \pm \dfrac{\xi^2(3R_i+\eta_i)}{R_i^3(R_i+\eta_i)^3}\sin^2\delta
\end{array}\right.
$$

$$
\left\{
\begin{array}{lcl}
\dfrac{\partial P_{1,z}}{\partial x} & = & \pm\left[-3\dfrac{\xi\tilde{y}_i}{R_i^5} + \dfrac{3\xi(2R_i+\eta_i)}{R_i^3(R_i+\eta_i)^2}\cos\delta - \dfrac{\xi^3(8R_i^2+9\eta_iR_i+3\eta_i^2)}{R_i^5(R_i+\eta_i)^3}\cos\delta\right] \\
\dfrac{\partial P_{1,z}}{\partial y} & = & \dfrac{\partial P_{1,y}}{\partial z} \\
\dfrac{\partial P_{1,z}}{\partial z} & = & \pm 3\dfrac{\tilde{y}_i\tilde{c}_i}{R_i^5} \mp \dfrac{\tilde{c}_i(2R_i+\eta_i)}{R_i^3(R_i+\eta_i)^2}\cos\delta \pm \dfrac{\xi^2\tilde{c}_i(8R_i^2+9\eta_iR_i+3\eta_i^2)}{R_i^5(R_i+\eta_i)^3}\cos\delta \\
 & & +\dfrac{\sin\delta\cos\delta}{R_i(R_i+\eta_i)^2} - \dfrac{\xi^2(3R_i+\eta_i)}{R_i^3(R_i+\eta_i)^3}\sin\delta\cos\delta
\end{array}
\right.
$$

$$
\left\{
\begin{array}{lcl}
\dfrac{\partial P_{3,y}}{\partial x} & = & \dfrac{\partial P_{1,y}}{\partial z} \\
\dfrac{\partial P_{3,y}}{\partial y} & = & \mp\dfrac{\tilde{y}_i(2R_i+\xi)}{R_i^3(R_i+\xi)^2} \pm \dfrac{\tilde{y}_i\tilde{c}_i^2(8R_i^2+9\xi R_i+3\xi^2)}{R_i^5(R_i+\xi)^3} \\
 & & \pm\dfrac{\xi\tilde{y}_i(3R_i+\eta_i)}{R_i^3(R_i+\eta_i)^3}\sin^2\delta - \dfrac{\xi\tilde{y}_i\tilde{c}_i(8R_i^2+9\eta_iR_i+3\eta_i^2)}{R_i^5(R_i+\eta_i)^3}\sin\delta \\
 & & \pm\dfrac{2\xi\sin^2\delta\cos\delta}{R_i(R_i+\eta_i)^3} - \dfrac{\xi\tilde{c}_i(3R_i+\eta_i)}{R_i^3(R_i+\eta_i)^3}\sin\delta\cos\delta \\
\dfrac{\partial P_{3,y}}{\partial z} & = & \pm\dfrac{3\tilde{c}_i(2R_i+\xi)}{R_i^3(R_i+\xi)^2} \mp \dfrac{\tilde{c}_i^3(8R_i^2+9\xi R_i+3\xi^2)}{R_i^5(R_i+\xi)^3} \\
 & & \mp\dfrac{\xi\tilde{c}_i(3R_i+\eta_i)}{R_i^3(R_i+\eta_i)^3}\sin^2\delta - \dfrac{\xi(2R_i+\eta_i)}{R_i^3(R_i+\eta_i)^2}\sin\delta + \dfrac{\xi\tilde{c}_i^2(8R_i^2+9\eta_iR_i+3\eta_i^2)}{R_i^5(R_i+\eta_i)^3}\sin\delta \\
 & & +\dfrac{2\xi\sin^3\delta}{R_i(R_i+\eta_i)^3} \mp \dfrac{\xi\tilde{c}_i(3R_i+\eta_i)}{R_i^3(R_i+\eta_i)^3}\sin^2\delta
\end{array}
\right.
$$

$$
\left\{
\begin{array}{lcl}
\dfrac{\partial P_{3,z}}{\partial x} & = & \dfrac{\partial P_{1,z}}{\partial z} \\
\dfrac{\partial P_{3,z}}{\partial y} & = & \dfrac{\partial P_{3,y}}{\partial z} \\
\dfrac{\partial P_{3,z}}{\partial z} & = & \pm\dfrac{\tilde{y}_i(2R_i+\xi)}{R_i^3(R_i+\xi)^2} \mp \dfrac{\tilde{y}_i\tilde{c}_i^2(8R_i^2+9\xi R_i+3\xi^2)}{R_i^5(R_i+\xi)^3} \\
 & & +\dfrac{\xi\tilde{c}_i(3R_i+\eta_i)}{R_i^3(R_i+\eta_i)^3}\sin\delta\cos\delta \pm \dfrac{\xi(2R_i+\eta_i)}{R_i^3(R_i+\eta_i)^2}\cos\delta \mp \dfrac{\xi\tilde{c}_i^2(8R_i^2+9\eta_iR_i+3\eta_i^2)}{R_i^5(R_i+\eta_i)^3}\cos\delta \\
 & & \mp\dfrac{2\xi\sin^2\delta\cos\delta}{R_i(R_i+\eta_i)^3} + \dfrac{\xi\tilde{c}_i(3R_i+\eta_i)}{R_i^3(R_i+\eta_i)^3}\sin\delta\cos\delta
\end{array}
\right.
$$

$\log(R_i+\xi)$

$$
\left\{
\begin{array}{lcl}
\dfrac{\partial}{\partial x}\log(R_i+\xi) & = & \dfrac{1}{R_i} \\
\dfrac{\partial}{\partial y}\log(R_i+\xi) & = & \dfrac{\tilde{y}_i}{R_i(R_i+\xi)} \\
\dfrac{\partial}{\partial z}\log(R_i+\xi) & = & -\dfrac{\tilde{c}_i}{R_i(R_i+\xi)}
\end{array}
\right.
$$

$\log(R_i+\eta_i)$

$$
\left\{
\begin{array}{lcl}
\dfrac{\partial}{\partial x}\log(R_i+\eta_i) & = & \dfrac{\xi}{R_i(R_i+\eta_i)} \\
\dfrac{\partial}{\partial y}\log(R_i+\eta_i) & = & \dfrac{\cos\delta}{R_i+\eta_i} + \dfrac{\tilde{y}_i}{R_i(R_i+\eta_i)} \\
\dfrac{\partial}{\partial z}\log(R_i+\eta_i) & = & \pm\dfrac{\sin\delta}{R_i+\eta_i} - \dfrac{\tilde{c}_i}{R_i(R_i+\eta_i)}
\end{array}
\right.
$$

$\log(R_i + \tilde{c}_i)$

$$\left\{\begin{array}{lcl}
\dfrac{\partial}{\partial x}\log(R_i+\tilde{c}_i) & = & \dfrac{\xi}{R_i(R_i+\tilde{c}_i)} \\
\dfrac{\partial}{\partial y}\log(R_i+\tilde{c}_i) & = & \dfrac{\tilde{y}_i}{R_i(R_i+\tilde{c}_i)} \\
\dfrac{\partial}{\partial z}\log(R_i+\tilde{c}_i) & = & -\dfrac{1}{R_i}
\end{array}\right.$$

$\tan^{-1}\left(\dfrac{\xi\eta_i}{q_i R_i}\right)$

$$\left\{\begin{array}{lcl}
\dfrac{\partial}{\partial x}\tan^{-1}\left(\dfrac{\xi\eta_i}{q_i R_i}\right) & = & -\dfrac{q_i}{R_i(R_i+\xi)} \\
\dfrac{\partial}{\partial y}\tan^{-1}\left(\dfrac{\xi\eta_i}{q_i R_i}\right) & = & \dfrac{\xi\sin\delta}{R_i(R_i+\eta_i)} \mp \dfrac{\tilde{c}_i}{R_i(R_i+\xi)} \\
\dfrac{\partial}{\partial z}\tan^{-1}\left(\dfrac{\xi\eta_i}{q_i R_i}\right) & = & \mp\dfrac{\xi\cos\delta}{R_i(R_i+\eta_i)} \mp \dfrac{\tilde{y}_i}{R_i(R_i+\xi)}
\end{array}\right.$$