

$$J_1(i) = 2 \tan^{-1} \left\{ \frac{(R_i \mp \eta_i)(1 + \sin \delta) \pm q_i \cos \delta}{\cos \delta} \right\}$$

$$\begin{cases} \frac{\partial J_1}{\partial x} &= \frac{q_i}{R_i(R_i + \eta_i)} \pm \frac{\tilde{y}_i}{R_i(R_i + \tilde{c}_i)} \\ \frac{\partial J_1}{\partial y} &= -\frac{\tilde{y}_i}{R_i(R_i + \eta_i)} \mp \frac{\xi}{R_i(R_i + \tilde{c}_i)} \\ \frac{\partial J_1}{\partial z} &= \pm \frac{\xi}{R_i(R_i + \eta_i)} \end{cases}$$

$$J_2(i) = \log(R_i + \tilde{c}_i) \pm \sin \delta \log(R_i + \eta_i)$$

$$\begin{cases} \frac{\partial J_2}{\partial x} &= \frac{\xi}{R_i(R_i + \tilde{c}_i)} \pm \frac{\xi \sin \delta}{R_i(R_i + \eta_i)} \\ \frac{\partial J_2}{\partial y} &= \frac{\tilde{y}_i}{R_i(R_i + \tilde{c}_i)} \pm \frac{\sin \delta \cos \delta}{R_i + \eta_i} \pm \frac{\tilde{y}_i \sin \delta}{R_i(R_i + \eta_i)} \\ \frac{\partial J_2}{\partial z} &= -\frac{1}{R_i} + \frac{\sin^2 \delta}{R_i + \eta_i} \mp \frac{\tilde{c}_i \sin \delta}{R_i(R_i + \eta_i)} \end{cases}$$

$$K_1(i) = \tan \delta \left[\frac{\xi}{R_i + \tilde{c}_i} + J_1(i) \tan \delta \right]$$

$$\begin{cases} \frac{\partial K_1}{\partial x} &= \tan \delta \left[\frac{1}{R_i + \tilde{c}_i} - \frac{\xi^2}{R_i(R_i + \tilde{c}_i)^2} + \frac{\partial J_1}{\partial x} \tan \delta \right] \\ \frac{\partial K_1}{\partial y} &= \tan \delta \left[-\frac{\xi \tilde{y}_i}{R_i(R_i + \tilde{c}_i)^2} + \frac{\partial J_1}{\partial y} \tan \delta \right] \\ \frac{\partial K_1}{\partial z} &= \tan \delta \left[\frac{\xi}{(R_i + \tilde{c}_i)^2} + \frac{\xi \tilde{c}_i}{R_i(R_i + \tilde{c}_i)^2} + \frac{\partial J_1}{\partial z} \tan \delta \right] \end{cases}$$

$$K_2(i) = \tan \delta \left[\frac{\tilde{y}_i}{R_i + \tilde{c}_i} \mp J_2(i) \tan \delta \right]$$

$$\begin{cases} \frac{\partial K_2}{\partial x} &= \tan \delta \left[-\frac{\xi \tilde{y}_i}{R_i(R_i + \tilde{c}_i)^2} \mp \frac{\partial J_2}{\partial x} \tan \delta \right] \\ \frac{\partial K_2}{\partial y} &= \tan \delta \left[\frac{1}{R_i + \tilde{c}_i} - \frac{\tilde{y}_i^2}{R_i(R_i + \tilde{c}_i)^2} \mp \frac{\partial J_2}{\partial y} \tan \delta \right] \\ \frac{\partial K_2}{\partial z} &= \tan \delta \left[\frac{\tilde{y}_i}{(R_i + \tilde{c}_i)^2} + \frac{\tilde{y}_i \tilde{c}_i}{R_i(R_i + \tilde{c}_i)^2} \mp \frac{\partial J_2}{\partial z} \tan \delta \right] \end{cases}$$

$$K_3(i) = J_2(i) \tan \delta$$

$$\begin{cases} \frac{\partial K_3}{\partial x} &= \frac{\partial J_2}{\partial x} \tan \delta \\ \frac{\partial K_3}{\partial y} &= \frac{\partial J_2}{\partial y} \tan \delta \\ \frac{\partial K_3}{\partial z} &= \frac{\partial J_2}{\partial z} \tan \delta \end{cases}$$

$$K_4(i) = \cos \delta [K_2(i) - \sin \delta \log(R_i + \eta_i)]$$

$$\left\{ \begin{array}{l} \frac{\partial K_4}{\partial x} = \cos \delta \left[\frac{\partial K_2}{\partial x} - \frac{\xi \sin \delta}{R_i(R_i + \eta_i)} \right] \\ \frac{\partial K_4}{\partial y} = \cos \delta \left[\frac{\partial K_2}{\partial y} - \sin \delta \left\{ \frac{\cos \delta}{R_i + \eta_i} + \frac{\tilde{y}_i}{R_i(R_i + \eta_i)} \right\} \right] \\ \frac{\partial K_4}{\partial z} = \cos \delta \left[\frac{\partial K_2}{\partial z} - \sin \delta \left\{ \pm \frac{\sin \delta}{R_i + \eta_i} - \frac{\tilde{c}_i}{R_i(R_i + \eta_i)} \right\} \right] \end{array} \right]$$

$$K_5(i) = K_1(i) \cos \delta$$

$$\left\{ \begin{array}{l} \frac{\partial K_5}{\partial x} = \frac{\partial K_1}{\partial x} \cos \delta \\ \frac{\partial K_5}{\partial y} = \frac{\partial K_1}{\partial y} \cos \delta \\ \frac{\partial K_5}{\partial z} = \frac{\partial K_1}{\partial z} \cos \delta \end{array} \right.$$

$$K_6(i) = -J_1(i) \sin \delta$$

$$\left\{ \begin{array}{l} \frac{\partial K_6}{\partial x} = -\frac{\partial J_1}{\partial x} \sin \delta \\ \frac{\partial K_6}{\partial y} = -\frac{\partial J_1}{\partial y} \sin \delta \\ \frac{\partial K_6}{\partial z} = -\frac{\partial J_1}{\partial z} \sin \delta \end{array} \right.$$

$$K_7(i) = -K_4(i) \tan \delta$$

$$\left\{ \begin{array}{l} \frac{\partial K_7}{\partial x} = -\frac{\partial K_4}{\partial x} \tan \delta \\ \frac{\partial K_7}{\partial y} = -\frac{\partial K_4}{\partial y} \tan \delta \\ \frac{\partial K_7}{\partial z} = -\frac{\partial K_4}{\partial z} \tan \delta \end{array} \right.$$

$$K_8(i) = -K_5(i) \tan \delta$$

$$\left\{ \begin{array}{l} \frac{\partial K_8}{\partial x} = -\frac{\partial K_5}{\partial x} \tan \delta \\ \frac{\partial K_8}{\partial y} = -\frac{\partial K_5}{\partial y} \tan \delta \\ \frac{\partial K_8}{\partial z} = -\frac{\partial K_5}{\partial z} \tan \delta \end{array} \right.$$

$$K_9(i) = -K_6(i) \tan \delta$$

$$\left\{ \begin{array}{l} \frac{\partial K_9}{\partial x} = -\frac{\partial K_6}{\partial x} \tan \delta \\ \frac{\partial K_9}{\partial y} = -\frac{\partial K_6}{\partial y} \tan \delta \\ \frac{\partial K_9}{\partial z} = -\frac{\partial K_6}{\partial z} \tan \delta \end{array} \right.$$

$$\begin{cases}
\frac{\partial L_0}{\partial x} = -\frac{\xi(2R_i + \tilde{c}_i)}{R_i^3(R_i + \tilde{c}_i)^2} \mp \frac{\xi(2R_i + \eta_i)}{R_i^3(R_i + \eta_i)^2} \sin \delta \\
\frac{\partial L_0}{\partial y} = -\frac{\tilde{y}_i(2R_i + \tilde{c}_i)}{R_i^3(R_i + \tilde{c}_i)^2} \mp \frac{\tilde{y}_i(2R_i + \eta_i)}{R_i^3(R_i + \eta_i)^2} \sin \delta \mp \frac{\sin \delta \cos \delta}{R_i(R_i + \eta_i)^2} \\
\frac{\partial L_0}{\partial z} = \frac{1}{R_i^3} \pm \frac{\tilde{c}_i(2R_i + \eta_i)}{R_i^3(R_i + \eta_i)^2} \sin \delta - \frac{\sin^2 \delta}{R_i(R_i + \eta_i)^2}
\end{cases}$$

$$\begin{cases}
\frac{\partial L_1}{\partial x} = L_0 + \xi \frac{\partial L_0}{\partial x} \\
\frac{\partial L_1}{\partial y} = \xi \frac{\partial L_0}{\partial y} \\
\frac{\partial L_1}{\partial z} = \xi \frac{\partial L_0}{\partial z}
\end{cases}$$

$$\begin{cases}
\frac{\partial L_2}{\partial x} = \mp \frac{\xi \sin \delta}{R_i(R_i + \eta_i)^2} + \tilde{y}_i L_{0,x} \sec \delta \\
\frac{\partial L_2}{\partial y} = \mp \sin \delta \left\{ \frac{\cos \delta}{(R_i + \eta_i)^2} + \frac{\tilde{y}_i}{R_i(R_i + \eta_i)^2} \right\} + L_0 \sec \delta + \tilde{y}_i L_{0,y} \sec \delta \\
\frac{\partial L_2}{\partial z} = -\sin \delta \left\{ \frac{\sin \delta}{(R_i + \eta_i)^2} \mp \frac{\tilde{c}_i}{R_i(R_i + \eta_i)^2} \right\} + \tilde{y}_i L_{0,z} \sec \delta
\end{cases}$$

$$\begin{cases}
\frac{\partial M_1}{\partial x} = \frac{1}{R_i(R_i + \eta_i)} - \frac{\xi^2(2R_i + \eta_i)}{R_i^3(R_i + \eta_i)^2} \\
\frac{\partial M_1}{\partial y} = -\frac{\xi \cos \delta}{R_i(R_i + \eta_i)^2} - \frac{\xi \tilde{y}_i(2R_i + \eta_i)}{R_i^3(R_i + \eta_i)^2} \\
\frac{\partial M_1}{\partial z} = \mp \frac{\xi \sin \delta}{R_i(R_i + \eta_i)^2} + \frac{\xi \tilde{c}_i(2R_i + \eta_i)}{R_i^3(R_i + \eta_i)^2}
\end{cases}$$

$$\begin{cases}
\frac{\partial M_2}{\partial x} = \frac{\partial M_1}{\partial y} \\
\frac{\partial M_2}{\partial y} = \frac{\sin^2 \delta}{(R_i + \eta_i)^2} - \frac{\tilde{y}_i \cos \delta \pm \tilde{c}_i \sin \delta}{R_i(R_i + \eta_i)^2} - \frac{\tilde{y}_i^2(2R_i + \eta_i)}{R_i^3(R_i + \eta_i)^2} \\
\frac{\partial M_2}{\partial z} = \mp \frac{\sin \delta \cos \delta}{(R_i + \eta_i)^2} + \frac{\tilde{c}_i \cos \delta \mp \tilde{y}_i \sin \delta}{R_i(R_i + \eta_i)^2} + \frac{\tilde{y}_i \tilde{c}_i(2R_i + \eta_i)}{R_i^3(R_i + \eta_i)^2}
\end{cases}$$

$$\begin{cases}
\frac{\partial M_3}{\partial x} = \frac{\partial M_1}{\partial z} \\
\frac{\partial M_3}{\partial y} = \frac{\partial M_2}{\partial z} \\
\frac{\partial M_3}{\partial z} = \frac{\cos^2 \delta}{(R_i + \eta_i)^2} + \frac{\tilde{y}_i \cos \delta \pm \tilde{c}_i \sin \delta}{R_i(R_i + \eta_i)^2} - \frac{\tilde{c}_i^2(2R_i + \eta_i)}{R_i^3(R_i + \eta_i)^2}
\end{cases}$$

$$\begin{cases}
\frac{\partial N_1}{\partial x} = \frac{1}{R_i(R_i + \tilde{c}_i)} - \frac{\xi^2(2R_i + \tilde{c}_i)}{R_i^3(R_i + \tilde{c}_i)^2} \\
\frac{\partial N_1}{\partial y} = -\frac{\xi \tilde{y}_i(2R_i + \tilde{c}_i)}{R_i^3(R_i + \tilde{c}_i)^2} \\
\frac{\partial N_1}{\partial z} = \frac{\xi}{R_i^3}
\end{cases}$$

$$\begin{cases}
\frac{\partial N_2}{\partial x} = \frac{\partial N_1}{\partial y} \\
\frac{\partial N_2}{\partial y} = \frac{1}{R_i(R_i + \tilde{c}_i)} - \frac{\tilde{y}_i^2(2R_i + \tilde{c}_i)}{R_i^3(R_i + \tilde{c}_i)^2} \\
\frac{\partial N_2}{\partial z} = \frac{\tilde{y}_i}{R_i^3}
\end{cases}$$

$$\begin{cases}
\frac{\partial O_1}{\partial x} = -\frac{\xi}{R_i^3} \\
\frac{\partial O_1}{\partial y} = -\frac{\tilde{y}_i}{R_i^3} \\
\frac{\partial O_1}{\partial z} = \frac{\tilde{c}_i}{R_i^3} \\
\frac{\partial O_2}{\partial x} = \frac{\partial O_1}{\partial y} \\
\frac{\partial O_2}{\partial y} = \frac{1}{R_i(R_i + \xi)} - \frac{\tilde{y}_i^2(2R_i + \xi)}{R_i^3(R_i + \xi)^2} \\
\frac{\partial O_2}{\partial z} = \frac{\tilde{y}_i \tilde{c}_i(2R_i + \xi)}{R_i^3(R_i + \xi)^2} \\
\frac{\partial O_3}{\partial x} = \frac{\partial O_1}{\partial z} \\
\frac{\partial O_3}{\partial y} = \frac{\partial z}{\partial O_2} \\
\frac{\partial O_3}{\partial z} = \frac{1}{R_i(R_i + \xi)} - \frac{\tilde{c}_i^2(2R_i + \xi)}{R_i^3(R_i + \xi)^2} \\
\frac{\partial P_1}{\partial x} = \frac{\xi q_i(2R_i + \eta_i)}{R_i^3(R_i + \eta_i)^2} \\
\frac{\partial P_1}{\partial y} = \pm \frac{\tilde{c}_i}{R_i^3} + \frac{\sin \delta}{R_i(R_i + \eta_i)} - \frac{\xi^2(2R_i + \eta_i)}{R_i^3(R_i + \eta_i)^2} \sin \delta \\
\frac{\partial P_1}{\partial z} = \pm \left\{ \frac{\tilde{y}_i}{R_i^3} - \frac{\cos \delta}{R_i(R_i + \eta_i)} + \frac{\xi^2(2R_i + \eta_i)}{R_i^3(R_i + \eta_i)^2} \cos \delta \right\} \\
\frac{\partial P_2}{\partial x} = \frac{\partial P_1}{\partial y} \\
\frac{\partial P_2}{\partial y} = \pm \frac{\tilde{y}_i \tilde{c}_i(2R_i + \xi)}{R_i^3(R_i + \xi)^2} - \frac{\xi \sin \delta \cos \delta}{R_i(R_i + \eta_i)^2} - \frac{\xi \tilde{y}_i(2R_i + \eta_i)}{R_i^3(R_i + \eta_i)^2} \sin \delta \\
\frac{\partial P_2}{\partial z} = \pm \left\{ \frac{1}{R_i(R_i + \xi)} - \frac{\tilde{c}_i^2(2R_i + \xi)}{R_i^3(R_i + \xi)^2} \right\} \mp \frac{\xi \sin^2 \delta}{R_i(R_i + \eta_i)^2} + \frac{\xi \tilde{c}_i(2R_i + \eta_i)}{R_i^3(R_i + \eta_i)^2} \sin \delta \\
\frac{\partial P_3}{\partial x} = \frac{\partial P_1}{\partial z} \\
\frac{\partial P_3}{\partial y} = \frac{\partial z}{\partial P_2} \\
\frac{\partial P_3}{\partial z} = \mp \frac{\tilde{y}_i \tilde{c}_i(2R_i + \xi)}{R_i^3(R_i + \xi)^2} + \frac{\xi \sin \delta \cos \delta}{R_i(R_i + \eta_i)^2} \mp \frac{\xi \tilde{c}_i(2R_i + \eta_i)}{R_i^3(R_i + \eta_i)^2} \cos \delta \\
\frac{\partial M_{1,y}}{\partial x} = -\frac{\cos \delta}{R_i(R_i + \eta_i)^2} + \frac{\xi^2(3R_i + \eta_i)}{R_i^3(R_i + \eta_i)^3} \cos \delta \\
\quad - \frac{\tilde{y}_i(2R_i + \eta_i)}{R_i^3(R_i + \eta_i)^2} + \frac{\xi^2 \tilde{y}_i(8R_i^2 + 9\eta_i R_i + 3\eta_i^2)}{R_i^5(R_i + \eta_i)^3} \\
\frac{\partial M_{1,y}}{\partial y} = -\frac{2\xi \sin^2 \delta}{R_i(R_i + \eta_i)^3} + \frac{\xi(\tilde{y}_i \cos \delta \pm \tilde{c}_i \sin \delta)(3R_i + \eta_i)}{R_i^3(R_i + \eta_i)^3} \\
\quad + \frac{\xi \tilde{y}_i^2(8R_i^2 + 9\eta_i R_i + 3\eta_i^2)}{R_i^5(R_i + \eta_i)^3} \\
\frac{\partial M_{1,y}}{\partial z} = \pm \frac{2\xi \sin \delta \cos \delta}{R_i(R_i + \eta_i)^3} - \frac{\xi(\tilde{c}_i \cos \delta \mp \tilde{y}_i \sin \delta)(3R_i + \eta_i)}{R_i^3(R_i + \eta_i)^3} \\
\quad - \frac{\xi \tilde{y}_i \tilde{c}_i(8R_i^2 + 9\eta_i R_i + 3\eta_i^2)}{R_i^5(R_i + \eta_i)^3}
\end{cases}$$

$$\left\{ \begin{array}{l}
\frac{\partial M_{1,z}}{\partial x} = \pm \frac{\xi^2(3R_i + \eta_i)}{R_i^3(R_i + \eta_i)^3} \sin \delta \\
\quad + \frac{\tilde{c}_i(2R_i + \eta_i)}{R_i^3(R_i + \eta_i)^2} - \frac{\xi^2 \tilde{c}_i(8R_i^2 + 9\eta_i R_i + 3\eta_i^2)}{R_i^5(R_i + \eta_i)^3} \\
\quad \mp \frac{R_i(R_i + \eta_i)^2}{\sin \delta} \\
\frac{\partial M_{1,z}}{\partial y} = \frac{\partial M_{1,y}}{\partial z} \\
\frac{\partial M_{1,z}}{\partial z} = -\frac{2\xi \cos^2 \delta}{R_i(R_i + \eta_i)^3} - \frac{\xi(\tilde{y}_i \cos \delta \pm \tilde{c}_i \sin \delta)(3R_i + \eta_i)}{R_i^3(R_i + \eta_i)^3} \\
\quad + \frac{\xi \tilde{c}_i^2(8R_i^2 + 9\eta_i R_i + 3\eta_i^2)}{R_i^5(R_i + \eta_i)^3} \\
\frac{\partial M_{2,y}}{\partial x} = \frac{\partial M_{1,y}}{\partial y} \\
\frac{\partial M_{2,y}}{\partial y} = -\frac{2\tilde{y}_i \sin^2 \delta}{R_i(R_i + \eta_i)^3} - \frac{\cos \delta}{R_i(R_i + \eta_i)^2} + \frac{\tilde{y}_i(\tilde{y}_i \cos \delta \pm \tilde{c}_i \sin \delta)(3R_i + \eta_i)}{R_i^3(R_i + \eta_i)^3} \\
\quad - \frac{2\tilde{y}_i(2R_i + \eta_i)}{R_i^3(R_i + \eta_i)^2} + \frac{\tilde{y}_i^3(8R_i^2 + 9\eta_i R_i + 3\eta_i^2)}{R_i^5(R_i + \eta_i)^3} \\
\quad - \frac{2 \sin^2 \delta \cos \delta}{(R_i + \eta_i)^3} + \frac{2(\tilde{y}_i \cos \delta \pm \tilde{c}_i \sin \delta)}{R_i(R_i + \eta_i)^3} \cos \delta + \frac{\tilde{y}_i^2(3R_i + \eta_i)}{R_i^3(R_i + \eta_i)^3} \cos \delta \\
\frac{\partial M_{2,y}}{\partial z} = \frac{2\tilde{c}_i \sin^2 \delta}{R_i(R_i + \eta_i)^3} \pm \frac{\sin \delta}{R_i(R_i + \eta_i)^2} - \frac{\tilde{c}_i(\tilde{y}_i \cos \delta \pm \tilde{c}_i \sin \delta)(3R_i + \eta_i)}{R_i^3(R_i + \eta_i)^3} \\
\quad - \frac{\tilde{y}_i^2 \tilde{c}_i(8R_i^2 + 9\eta_i R_i + 3\eta_i^2)}{R_i^5(R_i + \eta_i)^3} \\
\quad \mp \frac{2 \sin^3 \delta}{(R_i + \eta_i)^3} \pm \frac{2(\tilde{y}_i \cos \delta \pm \tilde{c}_i \sin \delta)}{R_i(R_i + \eta_i)^3} \sin \delta \pm \frac{\tilde{y}_i^2(3R_i + \eta_i)}{R_i^3(R_i + \eta_i)^3} \sin \delta \\
\frac{\partial M_{2,z}}{\partial x} = \frac{\partial M_{1,y}}{\partial z} \\
\frac{\partial M_{2,z}}{\partial y} = \frac{\partial M_{2,y}}{\partial z} \\
\frac{\partial M_{2,z}}{\partial z} = -\frac{2\tilde{y}_i \cos^2 \delta}{R_i(R_i + \eta_i)^3} + \frac{\cos \delta}{R_i(R_i + \eta_i)^2} - \frac{\tilde{y}_i(\tilde{y}_i \cos \delta \pm \tilde{c}_i \sin \delta)(3R_i + \eta_i)}{R_i^3(R_i + \eta_i)^3} \\
\quad + \frac{\tilde{y}_i \tilde{c}_i^2(8R_i^2 + 9\eta_i R_i + 3\eta_i^2)}{R_i^5(R_i + \eta_i)^3} \\
\quad - \frac{2 \cos^3 \delta}{(R_i + \eta_i)^3} - \frac{2(\tilde{y}_i \cos \delta \pm \tilde{c}_i \sin \delta)}{R_i(R_i + \eta_i)^3} \cos \delta + \frac{\tilde{c}_i^2(3R_i + \eta_i)}{R_i^3(R_i + \eta_i)^3} \cos \delta \\
\frac{\partial M_{3,z}}{\partial x} = \frac{\partial M_{1,z}}{\partial z} \\
\frac{\partial M_{3,z}}{\partial y} = \frac{\partial M_{2,z}}{\partial z} \\
\frac{\partial M_{3,z}}{\partial z} = \frac{2\tilde{c}_i \cos^2 \delta}{R_i(R_i + \eta_i)^3} \mp \frac{\sin \delta}{R_i(R_i + \eta_i)^2} + \frac{\tilde{c}_i(\tilde{y}_i \cos \delta \pm \tilde{c}_i \sin \delta)(3R_i + \eta_i)}{R_i^3(R_i + \eta_i)^3} \\
\quad + \frac{2\tilde{c}_i(2R_i + \eta_i)}{R_i^3(R_i + \eta_i)^2} - \frac{\tilde{c}_i^3(8R_i^2 + 9\eta_i R_i + 3\eta_i^2)}{R_i^5(R_i + \eta_i)^3} \\
\quad \mp \frac{2 \sin \delta \cos^2 \delta}{(R_i + \eta_i)^3} \mp \frac{2(\tilde{y}_i \cos \delta \pm \tilde{c}_i \sin \delta)}{R_i(R_i + \eta_i)^3} \sin \delta \pm \frac{\tilde{c}_i^2(3R_i + \eta_i)}{R_i^3(R_i + \eta_i)^3} \sin \delta
\end{array} \right.$$

$$\begin{cases}
\frac{\partial N_{1,z}}{\partial x} = \frac{1}{R_i^3} - 3 \frac{\xi^2}{R_i^5} \\
\frac{\partial N_{1,z}}{\partial y} = -3 \frac{\xi \tilde{y}_i}{R_i^5} \\
\frac{\partial N_{1,z}}{\partial z} = 3 \frac{\xi \tilde{c}_i}{R_i^5} \\
\frac{\partial N_{2,z}}{\partial x} = \frac{\partial N_{1,z}}{\partial y} \\
\frac{\partial N_{2,z}}{\partial y} = \frac{1}{R_i^3} - 3 \frac{\tilde{y}_i^2}{R_i^5} \\
\frac{\partial N_{2,z}}{\partial z} = 3 \frac{\tilde{y}_i \tilde{c}_i}{R_i^5} \\
\frac{\partial O_{2,y}}{\partial x} = -\frac{1}{R_i^3} + 3 \frac{\tilde{y}_i^2}{R_i^5} \\
\frac{\partial O_{2,y}}{\partial y} = -\frac{3 \tilde{y}_i (2R_i + \xi)}{R_i^3 (R_i + \xi)^2} + \frac{\tilde{y}_i^3 (8R_i^2 + 9\xi R_i + 3\xi^2)}{R_i^5 (R_i + \xi)^3} \\
\frac{\partial O_{2,y}}{\partial z} = \frac{\tilde{c}_i (2R_i + \xi)}{R_i^3 (R_i + \xi)^2} - \frac{\tilde{y}_i^2 \tilde{c}_i (8R_i^2 + 9\xi R_i + 3\xi^2)}{R_i^5 (R_i + \xi)^3} \\
\frac{\partial O_{2,z}}{\partial x} = -3 \frac{\tilde{y}_i \tilde{c}_i}{R_i^5} \\
\frac{\partial O_{2,z}}{\partial y} = \frac{\partial O_{2,y}}{\partial z} \\
\frac{\partial O_{2,z}}{\partial z} = -\frac{\tilde{y}_i (2R_i + \xi)}{R_i^3 (R_i + \xi)^2} + \frac{\tilde{y}_i \tilde{c}_i^2 (8R_i^2 + 9\xi R_i + 3\xi^2)}{R_i^5 (R_i + \xi)^3} \\
\frac{\partial O_{3,z}}{\partial x} = -\frac{1}{R_i^3} + 3 \frac{\tilde{c}_i^2}{R_i^5} \\
\frac{\partial O_{3,z}}{\partial y} = \frac{\partial O_{2,z}}{\partial z} \\
\frac{\partial O_{3,z}}{\partial z} = \frac{3 \tilde{c}_i (2R_i + \xi)}{R_i^3 (R_i + \xi)^2} - \frac{\tilde{c}_i^3 (8R_i^2 + 9\xi R_i + 3\xi^2)}{R_i^5 (R_i + \xi)^3} \\
\frac{\partial P_{1,y}}{\partial x} = \mp 3 \frac{\xi \tilde{c}_i}{R_i^5} - \frac{3\xi (2R_i + \eta_i)}{R_i^3 (R_i + \eta_i)^2} \sin \delta + \frac{\xi^3 (8R_i^2 + 9\eta_i R_i + 3\eta_i^2)}{R_i^5 (R_i + \eta_i)^3} \sin \delta \\
\frac{\partial P_{1,y}}{\partial y} = \mp 3 \frac{\tilde{y}_i \tilde{c}_i}{R_i^5} - \frac{\tilde{y}_i (2R_i + \eta_i)}{R_i^3 (R_i + \eta_i)^2} \sin \delta + \frac{\xi^2 \tilde{y}_i (8R_i^2 + 9\eta_i R_i + 3\eta_i^2)}{R_i^5 (R_i + \eta_i)^3} \sin \delta \\
\frac{\partial P_{1,y}}{\partial z} = \mp \frac{\sin \delta \cos \delta}{R_i (R_i + \eta_i)^2} + \frac{\xi^2 (3R_i + \eta_i)}{R_i^3 (R_i + \eta_i)^3} \sin \delta \cos \delta \\
\mp \frac{1}{R_i^3} \pm 3 \frac{\tilde{c}_i^2}{R_i^5} + \frac{\tilde{c}_i (2R_i + \eta_i)}{R_i^3 (R_i + \eta_i)^2} \sin \delta - \frac{\xi^2 \tilde{c}_i (8R_i^2 + 9\eta_i R_i + 3\eta_i^2)}{R_i^5 (R_i + \eta_i)^3} \sin \delta \\
\mp \frac{\sin^2 \delta}{R_i (R_i + \eta_i)^2} \pm \frac{\xi^2 (3R_i + \eta_i)}{R_i^3 (R_i + \eta_i)^3} \sin^2 \delta
\end{cases}$$

$$\left\{ \begin{array}{l}
\frac{\partial P_{1,z}}{\partial x} = \pm \left[-3 \frac{\xi \tilde{y}_i}{R_i^5} + \frac{3\xi(2R_i + \eta_i)}{R_i^3(R_i + \eta_i)^2} \cos \delta - \frac{\xi^3(8R_i^2 + 9\eta_i R_i + 3\eta_i^2)}{R_i^5(R_i + \eta_i)^3} \cos \delta \right] \\
\frac{\partial P_{1,z}}{\partial P_{1,y}} = \frac{\partial z}{\partial y} \\
\frac{\partial P_{1,z}}{\partial z} = \pm 3 \frac{\tilde{y}_i \tilde{c}_i}{R_i^5} \mp \frac{\tilde{c}_i(2R_i + \eta_i)}{R_i^3(R_i + \eta_i)^2} \cos \delta \pm \frac{\xi^2 \tilde{c}_i(8R_i^2 + 9\eta_i R_i + 3\eta_i^2)}{R_i^5(R_i + \eta_i)^3} \cos \delta \\
\quad + \frac{\sin \delta \cos \delta}{R_i(R_i + \eta_i)^2} - \frac{\xi^2(3R_i + \eta_i)}{R_i^3(R_i + \eta_i)^3} \sin \delta \cos \delta \\
\frac{\partial P_{3,y}}{\partial x} = \frac{\partial P_{1,y}}{\partial z} \\
\frac{\partial P_{3,y}}{\partial y} = \mp \frac{\tilde{y}_i(2R_i + \xi)}{R_i^3(R_i + \xi)^2} \pm \frac{\tilde{y}_i \tilde{c}_i^2(8R_i^2 + 9\xi R_i + 3\xi^2)}{R_i^5(R_i + \xi)^3} \\
\quad \pm \frac{\xi \tilde{y}_i(3R_i + \eta_i)}{R_i^3(R_i + \eta_i)^3} \sin^2 \delta - \frac{\xi \tilde{y}_i \tilde{c}_i(8R_i^2 + 9\eta_i R_i + 3\eta_i^2)}{R_i^5(R_i + \eta_i)^3} \sin \delta \\
\quad \pm \frac{2\xi \sin^2 \delta \cos \delta}{R_i(R_i + \eta_i)^3} - \frac{\xi \tilde{c}_i(3R_i + \eta_i)}{R_i^3(R_i + \eta_i)^3} \sin \delta \cos \delta \\
\frac{\partial P_{3,y}}{\partial z} = \pm \frac{3\tilde{c}_i(2R_i + \xi)}{R_i^3(R_i + \xi)^2} \mp \frac{\tilde{c}_i^3(8R_i^2 + 9\xi R_i + 3\xi^2)}{R_i^5(R_i + \xi)^3} \\
\quad \mp \frac{\xi \tilde{c}_i(3R_i + \eta_i)}{R_i^3(R_i + \eta_i)^3} \sin^2 \delta - \frac{\xi(2R_i + \eta_i)}{R_i^3(R_i + \eta_i)^2} \sin \delta + \frac{\xi \tilde{c}_i^2(8R_i^2 + 9\eta_i R_i + 3\eta_i^2)}{R_i^5(R_i + \eta_i)^3} \sin \delta \\
\quad + \frac{2\xi \sin^3 \delta}{R_i(R_i + \eta_i)^3} \mp \frac{\xi \tilde{c}_i(3R_i + \eta_i)}{R_i^3(R_i + \eta_i)^3} \sin^2 \delta \\
\frac{\partial P_{3,z}}{\partial x} = \frac{\partial P_{1,z}}{\partial z} \\
\frac{\partial P_{3,z}}{\partial y} = \frac{\partial z}{\partial y} \\
\frac{\partial P_{3,z}}{\partial z} = \pm \frac{\tilde{y}_i(2R_i + \xi)}{R_i^3(R_i + \xi)^2} \mp \frac{\tilde{y}_i \tilde{c}_i^2(8R_i^2 + 9\xi R_i + 3\xi^2)}{R_i^5(R_i + \xi)^3} \\
\quad + \frac{\xi \tilde{c}_i(3R_i + \eta_i)}{R_i^3(R_i + \eta_i)^3} \sin \delta \cos \delta \pm \frac{\xi(2R_i + \eta_i)}{R_i^3(R_i + \eta_i)^2} \cos \delta \mp \frac{\xi \tilde{c}_i^2(8R_i^2 + 9\eta_i R_i + 3\eta_i^2)}{R_i^5(R_i + \eta_i)^3} \cos \delta \\
\quad \mp \frac{2\xi \sin^2 \delta \cos \delta}{R_i(R_i + \eta_i)^3} + \frac{\xi \tilde{c}_i(3R_i + \eta_i)}{R_i^3(R_i + \eta_i)^3} \sin \delta \cos \delta
\end{array} \right.$$

$$\log(R_i + \xi)$$

$$\left\{ \begin{array}{l}
\frac{\partial}{\partial x} \log(R_i + \xi) = \frac{1}{R_i} \\
\frac{\partial}{\partial y} \log(R_i + \xi) = \frac{\tilde{y}_i}{R_i(R_i + \xi)} \\
\frac{\partial}{\partial z} \log(R_i + \xi) = -\frac{\tilde{c}_i}{R_i(R_i + \xi)}
\end{array} \right.$$

$$\log(R_i + \eta_i)$$

$$\left\{ \begin{array}{l}
\frac{\partial}{\partial x} \log(R_i + \eta_i) = \frac{\xi}{R_i(R_i + \eta_i)} \\
\frac{\partial}{\partial y} \log(R_i + \eta_i) = \frac{\tilde{y}_i}{R_i + \eta_i} + \frac{\tilde{y}_i}{R_i(R_i + \eta_i)} \\
\frac{\partial}{\partial z} \log(R_i + \eta_i) = \pm \frac{\sin \delta}{R_i + \eta_i} - \frac{\tilde{c}_i}{R_i(R_i + \eta_i)}
\end{array} \right.$$

$$\log(R_i + \tilde{c}_i)$$

$$\begin{cases} \frac{\partial}{\partial x} \log(R_i + \tilde{c}_i) &= \frac{\xi}{R_i(R_i + \tilde{c}_i)} \\ \frac{\partial}{\partial y} \log(R_i + \tilde{c}_i) &= \frac{\tilde{y}_i}{R_i(R_i + \tilde{c}_i)} \\ \frac{\partial}{\partial z} \log(R_i + \tilde{c}_i) &= -\frac{1}{R_i} \end{cases}$$

$$\tan^{-1} \left(\frac{\xi \eta_i}{q_i R_i} \right)$$

$$\begin{cases} \frac{\partial}{\partial x} \tan^{-1} \left(\frac{\xi \eta_i}{q_i R_i} \right) &= -\frac{q_i}{R_i(R_i + \xi)} \\ \frac{\partial}{\partial y} \tan^{-1} \left(\frac{\xi \eta_i}{q_i R_i} \right) &= \frac{\xi \sin \delta}{R_i(R_i + \eta_i)} \mp \frac{\tilde{c}_i}{R_i(R_i + \xi)} \\ \frac{\partial}{\partial z} \tan^{-1} \left(\frac{\xi \eta_i}{q_i R_i} \right) &= \mp \frac{\xi \cos \delta}{R_i(R_i + \eta_i)} \mp \frac{\tilde{y}_i}{R_i(R_i + \xi)} \end{cases}$$

$$O_{1,z} = \frac{\tilde{c}_i}{R_i^3}$$

$$\begin{cases} \frac{\partial O_{1,z}}{\partial x} &= -3 \frac{\xi \tilde{c}_i}{R_i^5} \\ \frac{\partial O_{1,z}}{\partial y} &= -3 \frac{\tilde{y}_i \tilde{c}_i}{R_i^5} \\ \frac{\partial O_{1,z}}{\partial z} &= -\frac{1}{R_i^3} + 3 \frac{\tilde{c}_i^2}{R_i^5} \end{cases}$$