Algebraic expressions

AS Pure Mathematics (Year 1) – Unit P1

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Key Information

Key Formulae:

$$x^a \times x^b = x^{a+b}$$
$$x^a \div x^b = x^{a-b}$$
$$(x^a)^b = x^{ab}$$
$$x^{-n} = \frac{1}{x^n}$$
$$x^{\frac{1}{n}} = \sqrt[n]{x}$$
$$a^2 - b^2 = (a-b)(a+b)$$

Key Terms:

- **Expanding Brackets:** Multiplying brackets out
- **Factorising Brackets:** Putting expressions back into brackets
- **Surd:** A root of a number which can't be written as a whole number or fraction
- **Rationalising Denominator:** Removing the surd from the bottom of the fraction

Solution Bank:

Links to the Big Picture

P1. Algebraic expressions

- P1.1 Index Laws
- P1.2 Expanding Brackets
- P1.3 Factorising
- P1.4 Negative and fractional indices
- P1.5 Surds
- P1.6 Rationalising Denominators

Develops:

- GCSE algebraic manipulation

Leads to:

- P2 – Quadratics
- P6 – Algebraic Methods
- P9 – Integration
- P12 – Differentiation
- P13 – Integration

Exam Question

a Write $\sqrt{45}$ in the form $a\sqrt{5}$, where a is an integer. **(1 mark)**

b Express $\dfrac{2(3+\sqrt{5})}{(3-\sqrt{5})}$ in the form $b + c\sqrt{5}$, where b and c are integers. **(5 marks)**

Algebraic expressions 1

Overview

After completing this chapter you should be able to:

- Multiply and divide integer powers
- Expand a single term over brackets and collect like terms
- Expand the product of two or three expressions
- Factorise linear, quadratic and simple cubic expressions
- Know and use the laws of Indices
- Simplify and use the rules of surds
- Rationalise denominators

1.1 Index Laws

• You can use the laws of indices to simplify powers of the same base.

- $a^m \times a^n = a^{m+n}$
- $a^m \div a^n = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $(ab)^n = a^n b^n$

Example 1

Simplify these expressions:

a $x^2 \times x^5$ **b** $2r^2 \times 3r^3$ **c** $\frac{b^7}{b^4}$ **d** $6x^5 \div 3x^3$ **e** $(a^3)^3 \times 2a^2$ **f** $(3x^2)^3 \div x^4$

Example 2

Expand these expressions and simplify if possible:

a $-3x(7x-4)$ **b** $y^2(3-2y^3)$

c $4x(3x-2x^2+5x^3)$ **d** $2x(5x+3)-5(2x+3)$

Watch Out A minus sign outside brackets changes the sign of every term inside the brackets.

Example 3

Simplify these expressions:

a $\dfrac{x^7+x^4}{x^3}$ **b** $\dfrac{3x^2-6x^5}{2x}$ **c** $\dfrac{20x^7+15x^3}{5x^2}$

Independent Practice: Exercise 1A

1.2 Expanding Brackets

Finding the product

To find the **product** of two expressions you **multiply** each term in one expression by each term in the other expression.

Multiplying each of the 2 terms in the first expression by each of the 3 terms in the second expression gives $2 \times 3 = 6$ terms.

$x \times$

$$\begin{aligned}(x+5)(4x-2y+3) &= x(4x-2y+3)+5(4x-2y+3)\\ &= 4x^2-2xy+3x+20x-10y+15\\ &= 4x^2-2xy+23x-10y+15\end{aligned}$$

$5\times$

Simplify your answer by collecting like terms.

Example 4

Expand these expressions and simplify if possible:

a $(x+5)(x+2)$ **b** $(x-2y)(x^2+1)$ **c** $(x-y)^2$ **d** $(x+y)(3x-2y-4)$

Example 5

Expand these expressions and simplify if possible:

a $x(2x+3)(x-7)$ **b** $x(5x-3y)(2x-y+4)$ **c** $(x-4)(x+3)(x+3)$

Independent Practice: Exercise 1B

1.3 Factorising

You can write expressions as a **product of their factors.**

• **Factorising is the opposite of expanding brackets.**

Expanding Brackets

$$4x(2x+y) = 8x^2 + 4xy$$
$$(x+5)^3 = x^3 + 15x^2 + 75x + 125$$
$$(x+2y)(x-5y) = x^2 - 3xy - 10y^2$$

Factorising

Example 6

Factorise these expressions completely:

a $3x + 9$ **b** $x^2 - 5x$ **c** $8x^2 + 20x$ **d** $9x^2y + 15^xy^2$ **e** $3x^2 - 9xy$

• **A quadratic expression had the form $ax^2 + bx + c$ where a, b and c are real numbers and $a \neq 0$.**

Notation: Real numbers are all the positive and negative numbers, or zero, including fractions and surds.

To factorise a quadratic expression:

• Find two factors of ac that add up to b ← For the expression $2x^2 + 5x - 3$, $ac = -6 = -1 \times 6$ and $-1 + 6 = 5 = b$

• Rewrite the b term as a sum of these two factors ← $2x^2 - x + 6x - 3$

• Factorise each pair of terms ← $= x(2x-1) + 3(2x-1)$

• Take out the common factor ← $= (x+3)(2x-1)$

• $x^2 - y^2 = (x+y)(x-y)$

Notation: An expression in the form $x^2 - y^2$ is called the **difference** of two sqaures.

Example 7

Factorise:

a $x^2 - 5x - 6$ **b** $x^2 + 6x + 8$ **c** $6x^2 - 11x - 10$ **d** $x^2 - 25$ **e** $4x^2 - 9y^2$

Example 8

Factorise completely:

a $x^3 - 2x^2$ **b** $x^3 - 25x$ **c** $x^3 + 3x^2 - 10x$

Independent Practice: Exercise 1C

1.4 Negative and fractional indices

Indices can be negative or fractions.

$x^{\frac{1}{2}} \times x^{\frac{1}{2}} = x^{\frac{1}{2}+\frac{1}{2}} = x^1 = x$

similarly $\underbrace{x^{\frac{1}{n}} \times x^{\frac{1}{n}} \times \ldots \times x^{\frac{1}{n}}}_{n \text{ terms}} = x^{\frac{1}{n}+\frac{1}{n}+\ldots+\frac{1}{n}} + x^1 = x$

• You can use the laws of indices with any rational power.

- $a^{\frac{1}{m}} = \sqrt[m]{a}$
- $a^{\frac{n}{m}} = \sqrt[m]{a^n}$
- $a^{-m} = \dfrac{1}{a^m}$
- $a^0 = 1$

Notation Rational numbers are those that can be written $\dfrac{a}{b}$ where a and b are integers.

Notation $a^{\frac{1}{2}} = \sqrt{a}$ is the positive square root of a. For example $9^{\frac{1}{2}} = \sqrt{9} = 3$ but $9^{\frac{1}{2}} \neq -3$.

Example 9

Simplify:

a $\dfrac{x^3}{x^{-3}}$ **b** $x^{\frac{1}{2}} \times x^{\frac{3}{2}}$ **c** $(x^3)^{\frac{2}{3}}$ **d** $2x^{1.5} \div 4x^{-0.25}$ **e** $\sqrt[3]{125x^6}$

Example 10

Evaluate:

a $9^{\frac{1}{2}}$ **b** $64^{\frac{1}{3}}$ **c** $49^{\frac{3}{2}}$ **d** $25^{-\frac{3}{2}}$

Example 11

Factorise completely:

a $y^{\frac{1}{2}}$ **b** $4y^{-1}$

Independent Practice: Exercise 1D

1.5 Surds

If n is an integer that is **not** a sqaure number, then any multiple of $\sqrt{n}$ is called a surd.
Examples of surds are: $\sqrt{2}$, $\sqrt{19}$ and $5\sqrt{2}$

Surds are examples of **irrational numbers**.
The decimal expansion of a surd is never-ending and never repeats, for example $\sqrt{2}$ = 1.414213562…

You can use surds to write exact answers to calculations.

• You can manipulate surds using these rules:

- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Notation Irrational numbers cannot be written in the form $\frac{a}{b}$ where a and b are integers. Surds are examples of **irrational numbers**.

Example 12

Simplify:

a $\sqrt{12}$ **b** $\frac{\sqrt{20}}{2}$ **c** $5\sqrt{6} - 2\sqrt{24} + \sqrt{294}$

Example 13

Simplify:

a $\sqrt{2}(5 - \sqrt{3})$ **b** $(2 - \sqrt{3})(5 + \sqrt{3})$

Independent Practice: Exercise 1E

1.6 Rationalising denominators

If a fraction has a surd in the denominator, it is sometimes useful to **rearrange** it so that the denominator is a rational number. This is called rationalising the denominator.

• The rules to rationalise denominators are:

- For fractions in the form $\frac{1}{\sqrt{a}}$, multiply the numerator and denominator by $\sqrt{a}$.
- For fractions in the form $\frac{1}{a+\sqrt{a}}$, multiply the numerator and denominator by $a-\sqrt{a}$.
- For fractions in the form $\frac{1}{a-\sqrt{a}}$, multiply the numerator and denominator by $a+\sqrt{a}$.

Example 14

Rationalise the denominator of:

a $\frac{1}{\sqrt{3}}$ **b** $\frac{1}{3+\sqrt{2}}$ **c** $\frac{\sqrt{5}+\sqrt{2}}{3+\sqrt{2}}$ **d** $\frac{1}{(1-\sqrt{3})^2}$

Independent Practice: Exercise 1F

Summary of key points

1. You can use the laws of indices to simplify powers of the **same base**.
   - $a^m \times a^n = a^{m+n}$
   - $a^m \div a^n = a^{m-n}$
   - $(a^m)^n = a^{mn}$
   - $(ab)^n = a^n b^n$
2. Factorising is the opposite of expanding brackets.
3. A quadratic expression has the form $ax^2 + bx + c$ where a, b, and c are real numbers and $a \neq 0$.
4. $x^2 - y^2 = (x+y)(x-y)$
5. You can use the laws of indices with any rational power.
   - $a^{\frac{1}{m}} = \sqrt[m]{a}$
   - $a^{\frac{n}{m}} = \sqrt[m]{a^n}$
   - $a^{-m} = \dfrac{1}{a^m}$
   - $a^0 = 1$
6. You can manipulate surds using the rules:
   - $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
   - $\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}$
7. The rules to rationalise denominators are:
   - For fractions in the form $\dfrac{1}{\sqrt{a}}$, multiply the numerator and denominator by $\sqrt{a}$.
   - For fractions in the form $\dfrac{1}{a+\sqrt{a}}$, multiply the numerator and denominator by $a - \sqrt{a}$.
   - For fractions in the form $\dfrac{1}{a-\sqrt{a}}$, multiply the numerator and denominator by $a + \sqrt{a}$.

1A Exercise 1A

1. Simplify these expressions:

a $x^3 \times x^4$

b $2x^3 \times 3x^2$

c $\frac{k^3}{k^2}$

d $\frac{4p^3}{2p}$

e $\frac{3x^3}{3x^2}$

f $(y^2)^5$

g $10x^5 \div 2x^3$

h $(p^3)^2 \div p^4$

i $(2a^3)^2 \div 2a^3$

j $8p^4 \div 4p^3$

k $2a^4 \times 3a^5$

l $\frac{21a^3b^7}{7ab^4}$

m $9x^2 \times 3(x^2)^3$

n $3x^3 \times 2x^2 \times 4x^6$

o $7a^4 \times (3a^4)^2$

p $(4y^3)^3 \div 2y^3$

q $2a^3 \div 3a^2 \times 6a^5$

r $3a^4 \times 2a^5 \times a^3$

2. Expand and simplify if possible:

a $9(x-2)$

b $x(x+9)$

c $-3y(4-3y)$

d $x(y+5)$

e $-x(3x+5)$

f $-5x(4x+1)$

g $(4x+5)x$

h $-3y(5-2y^2)$

i $-2x(5x-4)$

j $(3x-5)x^2$

k $3(x+2)+(x-7)$

l $5x-6-(3x-2)$

m $4(c+3d^2)-3(2c+d^2)$

n $(r^2+3t^2+9)-(2t^2+3t^2-4)$

o $x(3x^2-2x+5)$

p $7y^2(2-5y+3y^2)$

q $-2y^2(5-7y+3y^2)$

r $7(x-2)+3(x+4)-6(x-2)$

s $5x-3(4-2x)+6$

t $3x^2-x(3-4x)+7$

u $4x(x+3)-2x(3x-7)$

v $3x^2(2x+1)-5x^2(3x-4)$

3. Simplify these fractions:

a $\frac{6x^4+10x^6}{2x}$

b $\frac{3x^5-x^7}{x}$

c $\frac{2x^4-4x^2}{4x}$

d $\frac{8x^3+5x}{2x}$

e $\frac{7x^7+5x^2}{5x}$

f $\frac{9x^5-5x^3}{3x}$

1B Exercise 1B

1. Expand and simplify if possible:

a $(x+4)(x+7)$

b $(x-3)(x+2)$

c $(x-2)^2$

d $(x-y)(2x+3)$

e $(x+3y)(4x-y)$

f $(2x-4y)(3x+y)$

g $(2x-3)(x-4)$

h $(3x+2y)^2$

i $(2x+8y)(2x+3)$

j $(x+5)(2x+3y-5)$

k $(x-1)(3x-4y-5)$

l $(x-4y)(2x+y+5)$

m $(x+2y-1)(x+3)$

n $(2x+2y+3)(x+6)$

o $(4-y)(4y-x+3)$

p $(4y+5)(3x-y+2)$

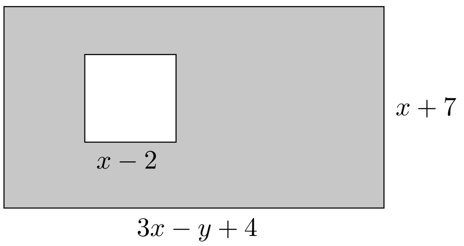
q $(5y-2x+3)(x-4)$

r $(4y-x-2)(5-y)$

2. Expand and simplify if possible:

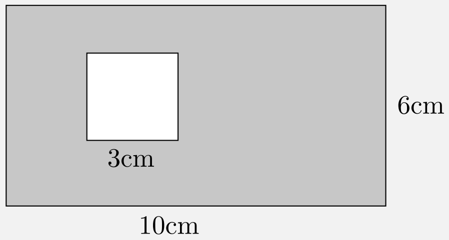
a $5(x+1)(x-4)$ **b** $7(x-2)(2x+5)$ **c** $3(x-3)(x-3)$
d $x(x-y)(x+y)$ **e** $x(2x+y)(3x+4)$ **f** $y(x-5)(x+1)$
g $y(3x-2y)(4x+2)$ **h** $y(7-x)(2x-5)$ **i** $x(2x+y-3)(2x+1)$
j $x(x+2)(x+3y-4)$ **k** $y(2x+y-1)(x+5)$ **l** $y(3x+2y-3)(2x+1)$
m $x(2x+3)(x+y-5)$ **n** $2x(3x-1)(4x-y-3)$ **o** $3x(x-2y)(2x+3y+5)$
p $(x+3)(x+2)(x+1)$ **q** $(x+2)(x-4)(x+3)$ **r** $(x+3)(x-1)(x-5)$
s $(x-5)(x-4)(x-3)$ **t** $(2x+1)(x-2)(x+1)$ **u** $(2x+3)(3x-1)(x+2)$
v $(3x-2)(2x+1)(3x-2)$ **w** $(x+y)(x-y)(x-1)$ **x** $(2x-3y)^3$

3. The digram shows a rectangle with a square cut out.
The rectangle has length $3x-y+4$ and width $x+7$.
The square has length $x-2$.
Find an expanded and simplified expression for the shaded area.



Problem-solving

Use the strategy as you use if the lengths were given as number.



4. A cuboid has dimensions $x+2$, $2x-1$cm and $2x+3$cm.
Show that the volume of the cuboid is $4x^3+12x^2+5x-6\text{cm}^3$.

5. Given that $(2x+5y)(3x-y)(2x+y)=ax^3+bx^2y+cxy^2+dy^3$, where a, b, c and d are constants, find the values of a, b, c and d.

1C Exercise 1C

1. Expand and simplify if possible:

a $4x+8$ **b** $6x-24$ **c** $20x+15$
d $2x^2+4$ **e** $4x^2+20$ **f** $6x^2-18x$
g x^2-7x **h** $2x^2+4x$ **i** $3x^2-x$
j $6x^2-2x$ **k** $10y^2-5y$ **l** $35x^2-28x$
m x^2+2x **n** $3y^2+2y$ **o** $4x^2+12x$
p $5y^2-20y$ **q** $9xy^2+12x^2y$ **r** $6ab-2ab^2$
s $5x^2-25xy$ **t** $12x^2y+8xy^2$ **u** $15y-20yz^2$
v $12x^2-30$ **w** xy^2-x^2y **x** $12y^2-4yx$

2. Factorise:

a $x^2 + 4x$ **b** $2x^2 + 6x$ **c** $x^2 + 11x + 24$

d $x^2 + 8x + 12$ **e** $x^2 + 3x - 40$ **f** $x^2 - 8x + 12$

g $x^2 + 5x + 6$ **h** $x^2 - 2x - 24$ **i** $x^2 - 3x - 10$

j $x^2 + x - 20$ **k** $2x^2 + 5x + 2$ **l** $3x^2 + 10x - 8$

m $5x^2 - 16x + 3$ **n** $6x^2 - 8x - 8$

o $2x^2 + 7x - 15$ **p** $2x^4 + 14x + 24$

q $x^2 - 4$ **r** $x^2 - 49$

s $4x^2 - 25$ **t** $9x^2 - 25y^2$ **u** $36x^2 - 4$

v $2x^2 - 50$ **w** $6x^2 - 10x + 4$ **x** $15x^2 + 42x - 9$

> **Hint** For part n, take 2 out as a common factor first. For part p, let $y = x^2$.

3. Factorise completely:

a $x^3 + 2x$ **b** $x^3 - x^2 + x$ **c** $x^3 - 5x$

d $x^3 - 9x$ **e** $x^3 - x^2 - 12x$ **f** $x^3 + 11x^2 + 30x$

g $x^3 - 7x^2 + 6x$ **h** $x^3 - 64x$ **i** $2x^3 - 5x^2 - 3x$

j $2x^3 + 13x^2 + 15x$ **k** $x^3 - 4x$ **l** $3x^3 + 27x^2 + 60x$

4. Factorise completely $x^4 - y^4$.

> **Problem-solving**
>
> Watch out for terms that can be written as a function of a function: $x^4 = (x^2)^2$.

5. Factorise completely $6x^3 + 7x^2 - 5x$. **(2 marks)**

> **Challenge**
>
> Write $4x^4 - 13x^2 + 9$ as the product of four linear factors.

1D Exercise 1D

1. Simplify

a $x^3 \div x^{-2}$ **b** $x^5 \div x^7$ **c** $x^{\frac{3}{2}} \times x^{\frac{5}{2}}$

d $(x^2)^{\frac{3}{2}}$ **e** $(x^3)^{\frac{5}{3}}$ **f** $3x^{0.5} \times 4x^{-0.5}$

g $9x^{\frac{2}{3}} \div 3x^{\frac{1}{6}}$ **h** $5x^{\frac{7}{5}} \div x^{\frac{2}{5}}$ **i** $3x^4 \times 2x^{-5}$

j $\sqrt{x} \times \sqrt[3]{x}$ **k** $\left(\sqrt{x}\right)^3 \times \left(\sqrt[3]{x}\right)^4$ **l** $\dfrac{\sqrt[3]{x}}{\sqrt{x}}$

2. Evaluate:

a $25^{\frac{1}{2}}$ **b** $81^{\frac{3}{2}}$ **c** $27^{\frac{1}{3}}$

d 4^{-2} **e** $9^{-\frac{1}{2}}$ **f** $(-5)^{-3}$

g $\left(\dfrac{3}{4}\right)^0$ **h** $1296^{\frac{3}{4}}$ **i** $\left(\dfrac{25}{16}\right)^{\frac{3}{2}}$

j $\left(\dfrac{27}{8}\right)^{\frac{2}{3}}$ **k** $\left(\dfrac{6}{5}\right)^{-1}$ **l** $\left(\dfrac{343}{512}\right)^{-\frac{2}{3}}$

3. Simplify:

a $(64x^{10})^{\frac{1}{2}}$ **b** $\dfrac{5x^3 - 2x^2}{x^5}$ **c** $(125x^{12})^{\frac{1}{3}}$ **d** $\dfrac{x + 4x^3}{x^3}$

e $\dfrac{2x + x^2}{x^4}$ **f** $\left(\dfrac{4}{9}x^4\right)^{\frac{3}{2}}$ **g** $\dfrac{9x^2 - 15x^5}{3x^3}$ **h** $\dfrac{5x + 3x^2}{15x^3}$

4. **a** Find the value of $81^{\frac{1}{4}}$. **(1 mark)**

b Simplify $x\left(2x^{-\frac{1}{3}}\right)^4$. **(2 marks)**

5. Given that $y = \dfrac{1}{8}x^3$ express each of the following in the form kx^n, where k and n are constants.

a $y^{\frac{1}{3}}$ **(2 marks)**

b $\dfrac{1}{2}y^{-2}$ **(2 marks)**

1E Excercise 1E

1. Do not use your calculator for this exercise. Simplify:

a $\sqrt{28}$ **b** $\sqrt{72}$ **c** $\sqrt{50}$

d $\sqrt{32}$ **e** $\sqrt{90}$ **f** $\dfrac{\sqrt{12}}{2}$

g $\dfrac{\sqrt{27}}{3}$ **h** $\sqrt{20} + \sqrt{80}$ **i** $\sqrt{200} + \sqrt{18} - \sqrt{72}$

j $\sqrt{175} + \sqrt{63} - 2\sqrt{28}$ **k** $\sqrt{28} - 2\sqrt{63} + \sqrt{7}$ **l** $\sqrt{80} - 2\sqrt{20} + 3\sqrt{45}$

m $3\sqrt{80} - 2\sqrt{20} + 5\sqrt{45}$ **n** $\dfrac{\sqrt{44}}{\sqrt{11}}$ **o** $\sqrt{12} + 3\sqrt{48} + \sqrt{75}$

2. Expand and simplify if possible:

a $\sqrt{3}(2 + \sqrt{3})$ **b** $\sqrt{5}(3 - \sqrt{3})$ **c** $\sqrt{2}(4 - \sqrt{5})$

d $(2 - \sqrt{2})(3 + \sqrt{5})$ **e** $(2 - \sqrt{3})(3 - \sqrt{7})$ **f** $(4 + \sqrt{5})(2 + \sqrt{5})$

g $(5 - \sqrt{3})(1 - \sqrt{3})$ **h** $(4 + \sqrt{3})(2 - \sqrt{3})$ **i** $(7 - \sqrt{11})(2 + \sqrt{11})$

3. Simplify $\sqrt{75} - \sqrt{12}$ giving your answer in the form $a\sqrt{3}$, where a is an integer. **(3 marks)**

1F Exercise 1F

1. Simplify:

a $\dfrac{1}{\sqrt{5}}$ **b** $\dfrac{1}{\sqrt{11}}$ **c** $\dfrac{1}{\sqrt{2}}$ **d** $\dfrac{\sqrt{3}}{\sqrt{15}}$

e $\dfrac{\sqrt{12}}{\sqrt{48}}$ **f** $\dfrac{\sqrt{5}}{\sqrt{80}}$ **g** $\dfrac{\sqrt{12}}{\sqrt{156}}$ **h** $\dfrac{\sqrt{7}}{\sqrt{63}}$

2. Rationalise the denominators and simplify:

a $\dfrac{1}{1 + \sqrt{3}}$ **b** $\dfrac{1}{2 + \sqrt{5}}$ **c** $\dfrac{1}{3 - \sqrt{7}}$ **d** $\dfrac{4}{3 - \sqrt{5}}$ **e** $\dfrac{1}{\sqrt{5} - \sqrt{3}}$

f $\dfrac{3 - \sqrt{2}}{4 - \sqrt{5}}$ **g** $\dfrac{5}{2 + \sqrt{5}}$ **h** $\dfrac{5\sqrt{2}}{\sqrt{8} - \sqrt{7}}$ **i** $\dfrac{11}{3 + \sqrt{11}}$ **j** $\dfrac{\sqrt{3} - \sqrt{7}}{\sqrt{3} + \sqrt{7}}$

k $\dfrac{\sqrt{17} - \sqrt{11}}{\sqrt{17} + \sqrt{11}}$ **l** $\dfrac{\sqrt{41} + \sqrt{29}}{\sqrt{41} - \sqrt{29}}$ **m** $\dfrac{\sqrt{2} - \sqrt{3}}{\sqrt{3} - \sqrt{2}}$

4. Simplify $\dfrac{3-2\sqrt{5}}{\sqrt{5}-1}$ giving you answer in the form $p+q\sqrt{5}$, where p and q are rational number. **(4 marks)**

Problem-solving

You can check that you answer is in the correct form but writing down the values of p and q and check that they are rational numbers.

1G Mixed Exercise 1

1. Simplify:

   a $y^3 \times y^5$ **b** $3x^2 \times 2x^5$ **c** $(4x^2)^3$ **d** $4b^2 \times 3b^3 \times b^4$

2. Expand and simplify if possible:

   a $(x+3)(x-5)$ **b** $(2x-7)(3x+1)$ **c** $(2x+5)(3x-y+2)$

3. Expand and simplify if possible:

   a $x(x+4)(x-1)$ **b** $(x+2)(x-3)(x+7)$ **c** $(2x+3)(x-2)(3x-1)$

4. Expand the brackets:

   a $3(5y+4)$ **b** $5x^2(3-5x+2x^2)$ **c** $5x(2x+3)-2x(1-3x)$ **d** $3x^2(1+3x)-2x(3x-2)$

5. Factorise these expressions completely:

   a $3x^2+4x$ **b** $4y^2+10y$ **c** $x^2+xy+xy^2$ **d** $8xy^2+10x^2y$

6. Factorise:

   a x^2+3x+2 **b** $3x^2+6x$ **c** $x^2-2x-35$ **d** $2x^2-x-3$
   e $5x^2-13x-6$ **f** $6-5x-x^2$

7. Factorise:

   a $2x^3+6x$ **b** x^3-36x **c** $2x^3+7x^2-15x$

8. Simplify:

   a $9x^3 \div 3x^{-3}$ **b** $\left(4^{\frac{3}{2}}\right)^{\frac{1}{3}}$ **c** $3x^{-2} \times 2x^4$ **d** $3x^{\frac{1}{3}} \div 6x^{\frac{2}{3}}$

9. Evaluate:

   a $\dfrac{3}{\sqrt{63}}$ **b** $\left(\dfrac{225}{289}\right)^{\frac{3}{2}}$

10. Simplify:

    a $\dfrac{3}{\sqrt{63}}$ **b** $\sqrt{20}+2\sqrt{45}-\sqrt{80}$

11. **a** Find the value of $35x^2+2x-48$.

    b By factorising the expression, show that your answer to part **a** can be written as the product of two prime factors.

12. Expand and simplify if possible:

    a $\sqrt{2}(3+\sqrt{5})$ **b** $(2-\sqrt{5})(5+\sqrt{3})$ **c** $(6-\sqrt{2})(4-\sqrt{7})$

13. Rationalise the denominator and simplify:

    a $\dfrac{1}{\sqrt{3}}$ **b** $\dfrac{1}{\sqrt{2}-1}$ **c** $\dfrac{3}{\sqrt{3}-2}$ **d** $\dfrac{\sqrt{23}-\sqrt{37}}{\sqrt{23}+\sqrt{37}}$ **e** $\dfrac{1}{(2+\sqrt{3})^2}$ **f** $\dfrac{1}{(4-\sqrt{7})^2}$

14. **a** Given that $x^3-x^2-17x-15=(x+3)(x^2+bx+c)$, where b and c are constants, work out the values of b and c.

    b Hence, fully factorise $x^3-x^2-17x-15$.

15. Given that $y = \frac{1}{64}x^3$ express each of the following in the form kn^n, where k and n are constants.

a $y^{\frac{1}{3}}$

b $4y^{-1}$

16. Show that $\frac{5}{\sqrt{75} - \sqrt{50}}$ can be written in the form $\sqrt{a} + \sqrt{b}$, where a and b are integers. **(5 marks)**

17. Expand and simplify $(\sqrt{11} - 5)(5 - \sqrt{11})$. **(2 marks)**

18. Factorise completely $x - 64x^3$. **(3 marks)**

19. Express 27^{2x+1} in the form 3^y, stating y in terms of x. **(2 marks)**

20. Solve the equation $8 + x\sqrt{12} = \frac{8x}{\sqrt{3}}$
Give your answer in the form $a\sqrt{b}$, where a and b are integers. **(4 marks)**

21. A rectangle has a length of $(1 + \sqrt{3})$cm and area of $\sqrt{12}$cm^2.
Calculate the width of the rectangle in cm.
Express you answer in the form $a + b\sqrt{3}$, where a and b are integers to be found.

22. Show that $\frac{(2 - \sqrt{x})^2}{\sqrt{x}}$ can be written as $4x^{-\frac{1}{2}} - 4 + x^{\frac{1}{2}}$. **(2 marks)**

23. Given $243\sqrt{3} = 3^a$, find the value of a. **(3 marks)**

24. Given that $\frac{4x^3 + x^{\frac{5}{2}}}{\sqrt{x}}$ can be written in the form $4x^a + x^b$, write down the value of a and the value of b. **(2 marks)**

Challenge

a Simplify $\left(\sqrt{a} + \sqrt{b}\right)\left(\sqrt{a} - \sqrt{b}\right)$

b Hence show that $\frac{1}{\sqrt{1} + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{4}} + ... + \frac{1}{\sqrt{24} + \sqrt{25}} = 4$