
Learning to Infer Graphics Programs from Hand-Drawn Images

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Abstract

1 We introduce a model that learns to convert simple hand drawings into graphics
2 programs written in a subset of \LaTeX . The model combines techniques from deep
3 learning and program synthesis. We learn a convolutional neural network that
4 proposes plausible drawing primitives that explain an image. This set of drawing
5 primitives is like an execution trace for a graphics program. From this trace we use
6 program synthesis techniques to recover a graphics program with constructs like
7 variable bindings, iterative loops, or simple kinds of conditionals. With a graphics
8 program in hand, we can correct errors made by the deep network, cluster drawings
9 by use of similar high-level geometric structures, and extrapolate drawings. Taken
10 together these results are a step towards agents that induce useful, human-readable
11 programs from perceptual input.

12 1 Introduction

13 How can an agent convert noisy, high-dimensional perceptual input to a symbolic, abstract object, such
14 as a computer program? Here we consider this problem within a graphics program synthesis domain.
15 We develop an approach for converting natural images, such as hand drawings, into executable source
16 code for drawing the original image. The graphics programs in our domain draw simple figures like
17 those found in machine learning papers (see Figure 1a).

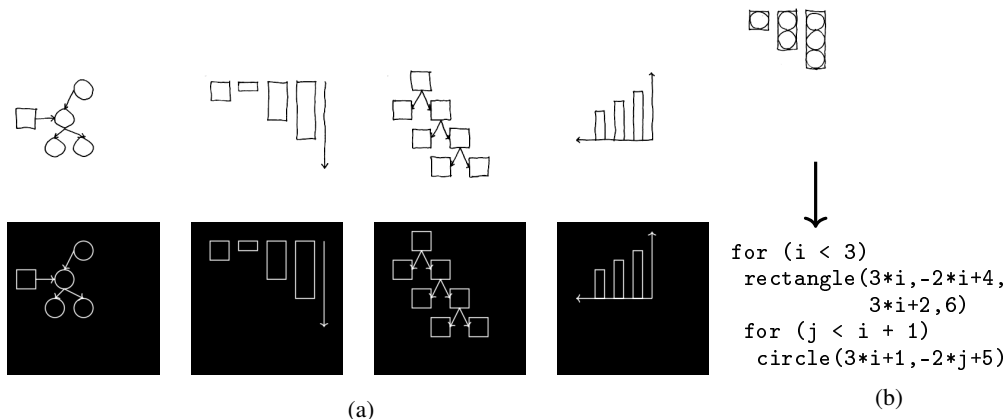


Figure 1: (a): Model learns to convert hand drawings (top) into \LaTeX (bottom). (b) Synthesizes high-level *graphics program* from hand drawing.

18 The key observation behind our work is that generating a programmatic representation from an image
 19 of a diagram actually involves two distinct steps that require different technical approaches. The first
 20 step involves identifying the components such as rectangles, lines and arrows that make up the image.
 21 The second step involves identifying the high-level structure in how the components were drawn. In
 22 Figure 1(b), it means identifying a pattern in how the circles and rectangles are being drawn that is
 23 best described with two nested loops, and which can easily be extrapolated to a bigger diagram.

24 We present a hybrid architecture for inferring graphics programs that is structured around these
 25 two steps. For the first step, our approach uses a deep network to infer a sequence of primitive
 26 shape-drawing commands that can generate an image similar to the observed image. We refer to this
 27 sequence as a *execution trace*, since it corresponds to the sequence of primitive commands that a
 28 program would have issued, but it lacks the high-level structure that determines how the program
 29 decided to issue those commands. For added robustness, we train the network to produce a proposal
 30 distribution of the most likely traces, and use stochastic search to find the trace that best matches the
 31 input. The network is trained from an automatically generated corpus of synthetic image generating
 32 programs.

33 The second step involves generating a high-level program capable of producing the program trace
 34 identified by the first phase. In principle, we could also train a neural network to learn this mapping.
 35 In practice, though, this would be challenging because it would no longer be sufficient to train the
 36 network with a synthetic corpus of randomly generated programs, since our goal is to generate
 37 programs that satisfy certain semantic constraints that ensure that we get the program that the user
 38 most likely intended. This paper argues that this stage is best achieved by *constraint-based program*
 39 *synthesis* []. Without the need for training, the program synthesizer can search the space of possible
 40 programs for one capable of producing the desired trace under a quantitative objective that maximizes
 41 the likelihood according to a prior distribution that aims to capture the user’s preference.

42 2 Related work

43 Our work bears resemblance to the Attend-Infer-Repeat (AIR) system, which learns to decompose an
 44 image into its constituent objects [3]. AIR learns an iterative inference scheme which infers objects
 45 one by one and also decides when to stop inference; this is similar to our approach’s first stage, which
 46 parses images into program execution traces. Our approach further produces interpretable, symbolic
 47 programs which generate those execution traces. The two approaches also differ in their architectures
 48 and training regimes: AIR learns a recurrent auto-encoding model via variational inference, whereas
 49 our parsing stage learns an autoregressive-style model from randomly-generated (execution trace,
 50 image) pairs. Finally, while AIR was evaluated on multi-MNIST images and synthetic 3D scenes, we
 51 focus on hand-drawn sketches.

52 Our image-to-execution-trace parsing architecture builds on prior work on controlling procedural
 53 graphics programs [4]. Given a program which generates random 2D recursive structures such as
 54 vines, that system learns a structurally-identical “guide program” whose output can be directed, via
 55 neural networks, to resemble a given target image. We adapt this method to a different visual domain
 56 (figures composed of multiple objects), using a broad prior over possible scenes as the initial program
 57 and viewing the execution trace through the guide program as a symbolic parse of the target image.
 58 We then show how to synthesize higher-level programs from these execution traces.

59 In the computer graphics literature, there have been other systems which convert sketches into
 60 procedural representations. One uses a convolutional network to match a sketch to the output of a
 61 parametric 3D modeling system [5]. Another uses convolutional networks to support sketch-based
 62 instantiation of procedural primitives within an interactive architectural modeling system [6]. Both
 63 systems focus on inferring fixed-dimensional parameter vectors. In contrast, we seek to automatically
 64 infer a structured, programmatic representation of a sketch which captures higher-level visual patterns.

65 Prior work has also applied sketch-based program synthesis to authoring graphics programs. Sketch-
 66 n-Sketch is a bi-directional editing system in which direct manipulations to a program’s output
 67 automatically propagate to the program source code [7]. We see this work as complementary to our
 68 own: programs produced by our method could be provided to a Sketch-n-Sketch-like system as a
 69 starting point for further editing.

The CogSketch [8] system also aims to have a high-level understanding of hand-drawn figures. Their primary goal is cognitive modeling (eg, they apply their system to solving IQ-test style visual reasoning problems), whereas we are interested in building an automated AI application (eg, in our system the user need not annotate which strokes correspond to which shapes; our neural network produces something equivalent to the annotations). A key similarity however is that both CogSketch and our system have as a goal to make it easier to produce nice-looking figures. Unsupervised Program Synthesis [9] is a related framework which was also applied to geometric reasoning problems. The goals of [9] were cognitive modeling, and they applied their technique to synthetic scenes used in human behavioral studies.

3 Neural architecture for inferring drawing execution traces

We developed a deep network architecture for efficiently inferring a execution trace, T , from an image, I . Our model constructs the trace one drawing command at a time. When predicting the next drawing command, the network takes as input the target image I as well as the rendered output of previous drawing commands. Intuitively, the network looks at the image it wants to explain, as well as what it has already drawn. It then decides either to stop drawing or proposes another drawing command to add to the execution trace; if it decides to continue drawing, the predicted primitive is rendered to its “canvas” and the process repeats.

Figure 2 illustrates this architecture. We first pass a 256×256 target image and a rendering of the trace so far (encoded as a two-channel image) to a convolutional network. Given the features extracted by the convnet, a multilayer perceptron then predicts a distribution over the next drawing command to add to the trace; see Table 1. We predict the drawing command token-by-token, conditioning each token both on the image features and on the previously generated tokens. For example, the network first decides to emit the `circle` token conditioned on the image features, then it emits the x coordinate of the circle conditioned on the image features and the `circle` token, and finally it predicts the y coordinate of the circle conditioned on the image features, the `circle` token, and the x coordinate. See supplement for the full details of the architecture, which we implemented in Tensorflow [10]. The distribution over the next drawing command factorizes as:

$$\mathbb{P}_\theta[t_1 t_2 \cdots t_K | I, T] = \prod_{k=1}^K \mathbb{P}_\theta[t_k | f_\theta(I, \text{render}(T)), \{t_j\}_{j=1}^{k-1}] \quad (1)$$

where $t_1 t_2 \cdots t_K$ are the tokens in the drawing command, I is the target image, T is an execution trace, θ are the parameters of the neural network, and $f_\theta(\cdot, \cdot)$ is the image feature extractor (convolutional network). The distribution over execution traces factorizes as:

$$\mathbb{P}_\theta[T | I] = \prod_{n=1}^{|T|} \mathbb{P}_\theta[T_n | I, T_{1:(n-1)}] \times \mathbb{P}_\theta[\text{STOP} | I, T] \quad (2)$$

where $|T|$ is the length of execution trace T , the subscripts on T index drawing commands within the trace, and the `STOP` token is emitted by the network to signal that the trace explains the image.

We train the network by sampling execution traces T and target images I for randomly generated scenes and maximizing (2) with respect to θ by gradient ascent. Training does not require back-propagation across the entire sequence of drawing commands: drawing to the canvas ‘blocks’ the gradients, effectively offloading memory to an external visual store. In a sense, this model is like an autoregressive variant of AIR [3] without attention. We trained the network on 10^5 scenes, which takes a little less than a day on an Nvidia TitanX GPU.

<code>circle</code> (x, y)	Circle at (x, y)
<code>rectangle</code> (x_1, y_1, x_2, y_2)	Rectangle with corners at (x_1, y_1) & (x_2, y_2)
<code>line</code> (x_1, y_1, x_2, y_2 ,	Line from (x_1, y_1) to (x_2, y_2) ,
$\text{arrow} \in \{0, 1\}$, $\text{dashed} \in \{0, 1\}$)	optionally with an arrow and/or dashed
<code>STOP</code>	Finishes execution trace inference

Table 1: The deep network in (2) predicts drawing commands, shown above.

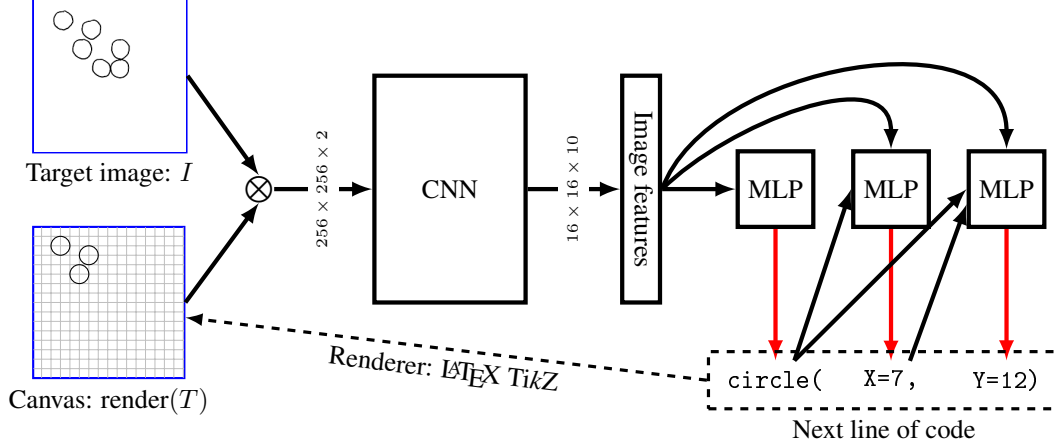


Figure 2: Our neural architecture for inferring the execution trace of a graphics program from its output. **Blue**: network inputs. **Black**: network operations. **Red**: samples from a multinomial. Typewriter font: network outputs. Renders snapped to a 16×16 grid, illustrated in gray.

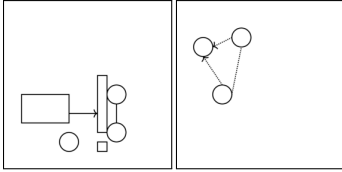


Figure 3: Network is trained to infer execution traces for randomly generated scenes like the two shown above. See supplement for details of the training data generation.

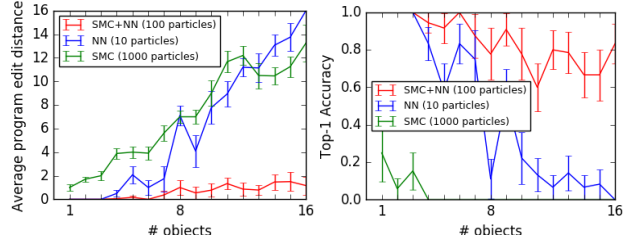


Figure 4: Using the model to parse latex output. The model is trained on diagrams with up to 8 objects. As shown above it generalizes to scenes with many more objects. Neither the stochastic search nor the neural network are sufficient on their own. # particles varies by model: we compare the models *with equal runtime* (≈ 1 sec/object)

108 This network suffices to “derender” synthetic images like those shown in Figure 3. We can perform a
 109 beam search decoding to recover what the network thinks is the most likely execution trace for images
 110 like these, recovering traces maximizing $\mathbb{P}_\theta[T|I]$. But, if the network makes a mistake (predicts an
 111 incorrect line of code), it has no way of recovering from the error. In order to derender an image
 112 with n objects, it must correctly predict n drawing commands – so its probability of success will
 113 decrease exponentially in n , assuming it has any nonzero chance of making a mistake. For added
 114 robustness as n becomes large, we treat the neural network outputs as proposals for a Sequential
 115 Monte Carlo (SMC) sampling scheme [11]. For the SMC sampler, we use pixel-wise distance as
 116 a surrogate for a likelihood function. The SMC sampler is designed to produce samples from the
 117 distribution $\propto L(I|\text{render}(T))\mathbb{P}_\theta[T|I]$, where $L(\cdot|\cdot) : \text{image}^2 \rightarrow \mathcal{R}$ uses the distance between two
 118 images as a proxy for a likelihood. Figure 4 compares the neural network with SMC against the
 119 neural network by itself or SMC by itself. Only the combination of the two passes a critical test of
 120 generalization: when trained on images with ≤ 8 objects, it successfully parses scenes with many
 121 more objects than the training data.

122 3.1 Generalizing to hand drawings

123 A practical application of our neural network is the automatic conversion of hand drawings into a
 124 subset of \LaTeX . We train the model to generalize to hand drawings by introducing noise into the
 125 renderings of the training target images. We designed this noise process to introduce the kinds of
 126 variations found in hand drawings (Figure 5; see supplement for details). Our neurally-guided SMC
 127 procedure used pixel-wise distance as a surrogate for a likelihood function ($L(\cdot|\cdot)$ in section 3). But

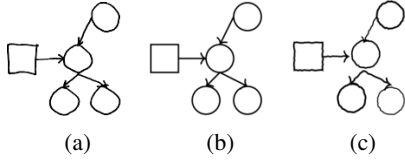


Figure 5: (a): a hand drawing. (b): Rendering of the trace our model infers for (a). We can generalize to hand drawings like these because we train the model on images corrupted by a noise process designed to resemble the kind of noise introduced by hand drawings - see (c) for a noisy rendering of (b).

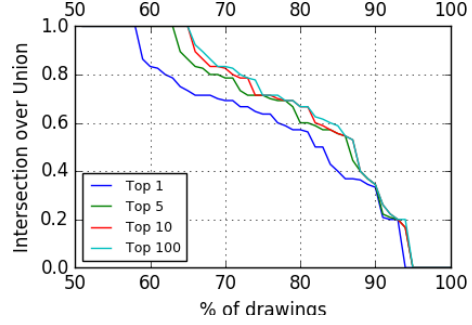


Figure 6: How close are the model’s outputs to the ground truth on hand drawings, as we consider larger sets of samples (1,5,10,100)? Distance to ground truth trace measured by the intersection over union of predicted vs. ground truth traces (sets of drawing commands).

128 pixel-wise distance fares poorly on hand drawings, which never exactly match the model’s renders.
 129 So, for hand drawings, we *learn* a surrogate likelihood function, $L_{\text{learned}}(\cdot|\cdot)$. The density $L_{\text{learned}}(\cdot|\cdot)$
 130 is predicted by a convolutional network that we train to predict the distance between two traces
 131 conditioned upon their renderings. We train our likelihood surrogate to approximate the symmetric
 132 difference, which is the number of drawing commands by which two traces differ:

$$-\log L_{\text{learned}}(\text{render}(T_1)|\text{render}(T_2)) \approx |T_1 - T_2| + |T_2 - T_1| \quad (3)$$

133 Intuitively, Eq. 3 says that $L_{\text{learned}}(\cdot|\cdot)$ approximates the distance between the trace we want and the
 134 trace we have so far. Pixel-wise distance metrics are sensitive to the details of how arrows, dashes,
 135 and corners are drawn – but we wish to be invariant to these details. So, we learn a distance metric
 136 over images that approximates the distance metric in the search space over traces.

137 We drew 100 figures by hand; see figure 7. These were drawn carefully but not perfectly. Our model
 138 assumes that objects are snapped to a 16×16 grid, so we made the drawings on graph paper.

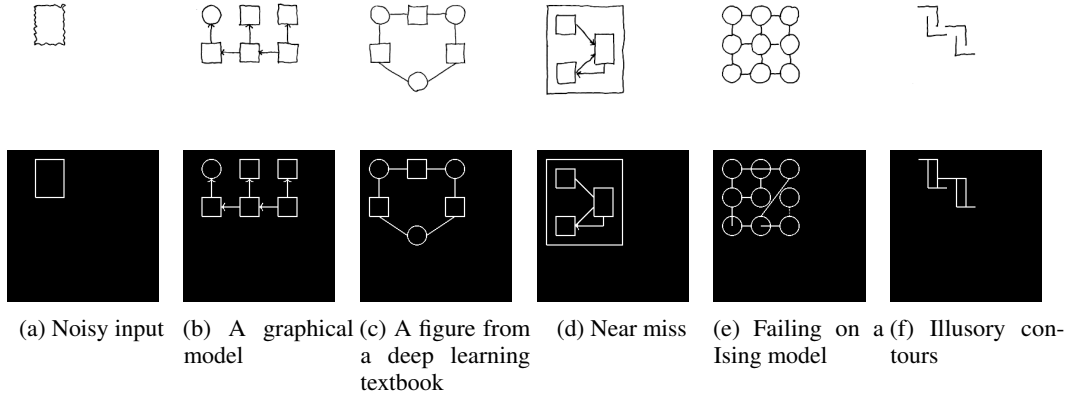


Figure 7: Example drawings above model outputs. See also Fig. 1. Stochastic search (SMC) can help correct for these errors, as can the program synthesizer (Section 4.1)

139 For each drawing we annotated a ground truth trace, and evaluated the model by asking it to sample
 140 many candidate traces for each drawing. For 58% of the drawings the Top-1 most likely sample
 141 exactly matches the ground truth; as we consider more samples the model encounters traces that are
 142 closer to the ground truth annotation (Fig. 6). Because our current model sometimes makes mistakes,
 143 we envision it working as follows: a user sketches a diagram, and the system responds by proposing
 144 a few candidate interpretations. The user could then select one and edit it if necessary.

145 4 Synthesizing graphics programs from execution traces

146 Although the execution trace of a graphics program describes the parts of a scene, it fails to encode
 147 higher-level features of the image, such as repeated motifs or symmetries. A *graphics program* better
 148 describe structures like these, and we now take as our goal to synthesize simple graphics programs
 149 from their execution traces.

150 We constrain the space of allowed programs by writing down a context free grammar over a space of
 151 programs. Although it might be desirable to synthesize programs in a Turing-complete language such
 152 as Lisp or Python, a more tractable approach is to specify what in the program languages community
 153 is called a Domain Specific Language (DSL) [12]. Our DSL (Table 2) encodes prior knowledge of
 154 what graphics programs tend to look like.

Program	→	Command; ...; Command
Command	→	circle(Expression, Expression)
Command	→	rectangle(Expression, Expression, Expression, Expression)
Command	→	line(Expression, Expression, Expression, Expression, Boolean, Boolean)
Command	→	for($0 \leq \text{Var} < \text{Expression}$) { if ($\text{Var} > 0$) { Program }; Program }
Command	→	reflect(Axis) { Program }
Expression	→	$\mathcal{Z} * \text{Var} + \mathcal{Z}$
Var	→	A free (unused) variable
\mathcal{Z}	→	an integer
Axis	→	$X = \mathcal{Z} \mid Y = \mathcal{Z}$

Table 2: Grammar over graphics programs. We allow loops (for) with conditionals (if), vertical/horizontal reflections (reflect), variables (Var) and affine transformations ($\mathcal{Z} * \text{Var} + \mathcal{Z}$).

155 Given the DSL and a trace T , we want to recover a program that both evaluates to T and, at the same
 156 time, is the “best” explanation of T . For example, we might prefer more general programs or, in the
 157 spirit of Occam’s razor, prefer shorter programs. We wrap these intuitions up into a cost function
 158 over programs, and seek the minimum cost program consistent with T :

$$\text{program}(T) = \arg \min_{p \in \text{DSL, s.t. } p \text{ evaluates to } T} \text{cost}(p) \quad (4)$$

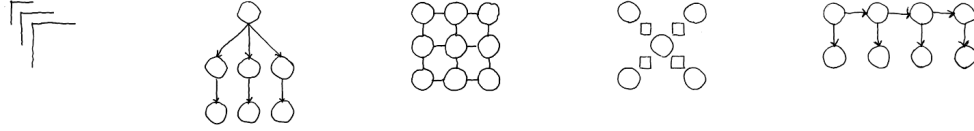
159 We define the cost of a program to be the number of statements it contains, where a statement is a
 160 “Command” in Table 2. We also penalize using many different numerical constants; see supplement.

161 The constrained optimization problem in equation 4 is intractable in general, but there exist efficient-
 162 in-practice tools for finding exact solutions to program synthesis problems like these. We use the
 163 state-of-the-art Sketch tool [1]. Describing Sketch’s program synthesis algorithm is beyond the
 164 scope of this paper; see [1]. At a high level, Sketch takes as input a space of programs, along with
 165 a specification of the program’s behavior and optionally a cost function. It translates the synthesis
 166 problem into a constraint satisfaction problem, and then uses a SAT solver to find a minimum cost
 167 program satisfying the specification. In exchange for not having any guarantees on how long it will
 168 take to find a minimum cost solution, it comes with the guarantee that it will always find a globally
 169 optimal program.

170 Why synthesize a graphics program, if the execution trace already suffices to recover the objects in
 171 an image? Within our domain of hand-drawn figures, graphics program synthesis has several uses:

172 4.1 Correcting errors made by the neural network

173 The program synthesizer can help correct errors from the execution trace proposal network by favoring
 174 execution traces which lead to more concise or general programs. For example, one generally prefers
 175 figures with perfectly aligned objects over figures whose parts are slightly misaligned – and precise
 176 alignment lends itself to short programs. Concretely, we estimated a prior over programs. Then,
 177 given the few most likely traces output by the neurally guided sampler, we reranked them according
 178 to the prior probability of their programs. Our neurally guided sampler could only do better on 7
 179 drawings by looking at the Top-100 sampled traces (see Fig. 6), precluding a statistically significant
 180 analysis of how much learning a prior over programs could help correct errors. But, learning this prior



```

for(i<3){
  line(i,-1*i+6,
2*i+2,-1*i+6)
  line(i,-2*i+4,
i,-1*i+6)
}

circle(4,10)
for(i<3){
  circle(-3*i+7,5)
  circle(-3*i+7,1)
  line(-3*i+7,4,
-3*i+7,2,
  arrow=True)
  line(4,9,
-3*i+7,6,
  arrow=True)
}

for(i<3)
  for(j<3)
    if(j>0)
      line(-3*j+8,
-3*i+7,-3*j+9,
-3*i+7)
      line(-3*i+7,-3*j+8,
-3*i+7,
-3*j+9)
      circle(-3*j+7,-3*i+7)

reflect(y=8){
  for(i<3){
    for(j<3){
      if(i>0){
        rectangle(3*i-1,
2,3*i,3)}
        circle(3*i+1,
3*i+1)
      }
    }
  }
}

for(i<4){
  line(-4*i+13,4,
-4*i+13,2,
  arrow=True)
  for(j<3){
    if(j>0){
      circle(-4*i+13,
4*j+-3)}
      line(-4*j+10,5,
-4*j+12,5,
  arrow=True)
    }
  }
}

```

Figure 8: Example drawings (top row) and the programs we synthesize from their ground truth traces (bottom row). Notice the nested loops in the Ising model (middle), special case conditionals for the HMM (rightmost), combination of symmetry and iteration in middle left, and affine transformation in the leftmost figure.

181 does sometimes help correct mistakes made by the neural network, and also occasionally introduces
 182 mistakes of its own; see Fig. 9 for a representative example of the kinds of corrections that it makes.
 183 See supplement for details.

184 4.2 Modeling similarity between drawings

185 Modeling an image using a program opens up new ways of
 186 measuring similarity between drawings. For example, we
 187 might say that two drawings are similar if they both contain
 188 repetitions of length 4, or if they share the same reflectional
 189 symmetry, or if they are both organized according to a grid-
 190 like structure.

191 We measure the similarity between two drawings by ex-
 192 tracting features of the best programs that describe them.
 193 Our features are counts of the number of times that dif-
 194 ferent components in the DSL were used (Table 2). We
 195 project these features down to a 2-dimensional subspace
 196 using nonnegative matrix factorization (NMF: [13]); see
 197 Fig.10. One could use many alternative similarity metrics
 198 between drawings which would capture pixel-level while
 199 missing high-level geometric similarities. We used our
 200 learned distance metric between execution traces, $L_{\text{learned}}(\cdot, \cdot)$, and projected to a 2-dimensional
 201 subspace using multidimensional scaling (MDS: [14]). This reveals similarities between the objects
 202 in the drawings, while missing similarities at the level of the program.

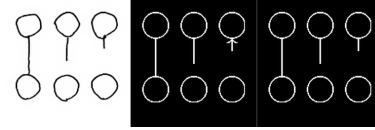


Figure 9: Left: hand drawing. Center: interpretation favored by the deep network. Right: interpretation favored after learning a prior over programs. Our learned prior favors shorter, simpler programs, thus continuing the pattern of not having an arrow is preferred.

203 4.3 Extrapolating figures

204 Having access to the source code of a graphics program facilitates coherent, high-level edits to the
 205 figure generated by that program. For example, we can change all of the circles to squares or make all
 206 of the lines be dashed. We can also **extrapolate** figures by increasing the number of times that loops
 207 are executed. Extrapolating repetitive visual patterns comes naturally to humans, and building this
 208 ability into an application is practical: imagine hand drawing a repetitive graphical model structure
 209 and having our system automatically induce and extend the pattern. Fig. 12 shows extrapolations of
 210 programs synthesized from ground truth traces; see supplement for our full set of extrapolations.

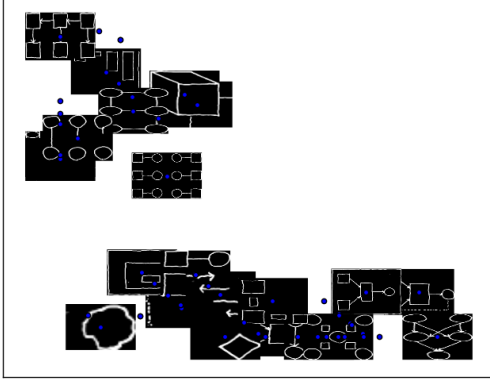


Figure 10: NMF on features of the programs that were synthesized for each image. Horizontal component roughly corresponds to “symmetry” while vertical component roughly corresponds to “loopyness”, with images on the diagonal having both of these.

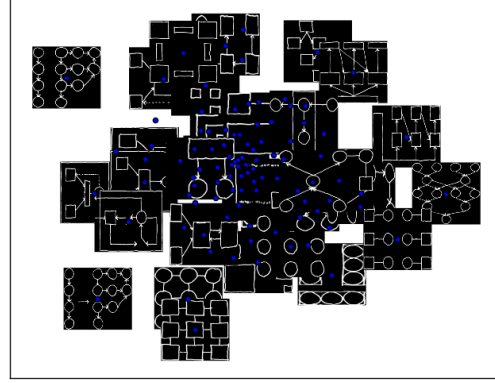


Figure 11: MDS on drawings using the learned distance metric, $L_{\text{learned}}(\cdot|\cdot)$. Drawings with similar looking parts in similar locations are clustered together.

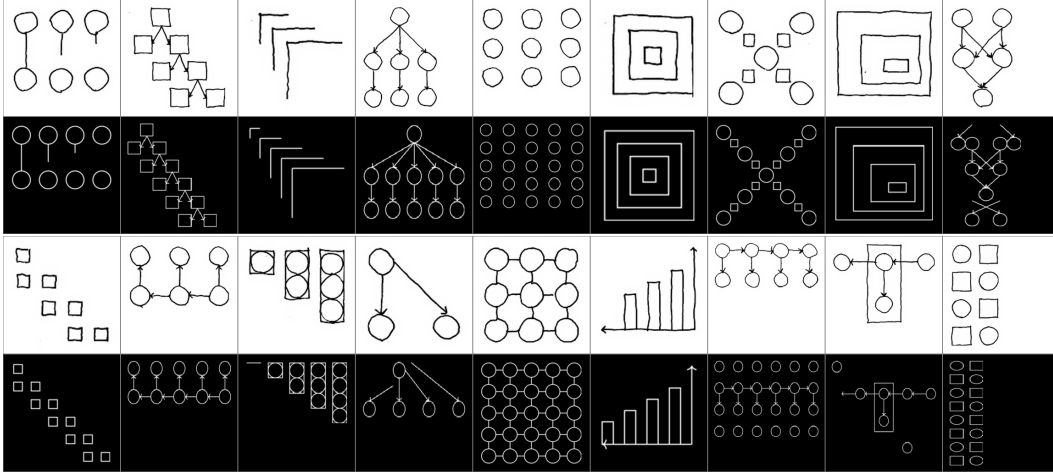


Figure 12: Top, white: hand drawings. Bottom, black: extrapolations produced by running loops for extra iterations.

5 Conclusion

We have presented a system for inferring graphics programs which generate \LaTeX -style figures from hand-drawn images. The system uses a combination of deep neural networks and stochastic search to parse drawings into symbolic execution traces; it then feeds these traces to a general-purpose program synthesis engine to infer a structured graphics program. We evaluated our model’s performance at parsing novel images, and we demonstrated its ability to extrapolate from provided drawings and to organize them according to high-level geometric features.

There are many directions for future work. In the parsing phase, the proposal network currently samples positional variables on a discrete grid. More general types of drawings could be supported by instead sampling from continuous distributions, e.g. using Mixture Density Networks [15]. The proposal network also currently handles only a very small subset of \LaTeX drawing commands, though there is no reason that it could not be extended to handle more with a higher-capacity network. Exploring more sophisticated network architectures, including ones that utilize attention [16], could also help correct some of the errors the network makes. In the synthesis phase, a more expressive DSL—including subroutines, recursion, and symmetry groups beyond reflections—would allow the

system to effectively model a wider variety of graphical phenomena. The synthesizer itself could also be the subject of future work: the system currently uses the general-purpose Sketch synthesizer, which can take minutes to hours to run, whereas program synthesizers which are custom-built for special problem domains can run much faster or even interactively [17].

In the not-too-distant future, we believe it should be possible to produce professional-looking figures just by drawing them and then letting an artificially-intelligent agent write the corresponding code. More generally, we believe that the two-phase system we have proposed—parsing into execution traces, then searching for a low-cost symbolic program which generates those traces—may be a useful paradigm for other domains in which agents must programmatically reason about noisy perceptual input.

References

- [1] Armando Solar Lezama. *Program Synthesis By Sketching*. PhD thesis, EECS Department, University of California, Berkeley, Dec 2008.
- [2] Leonardo De Moura and Nikolaj Bjørner. Z3: An efficient smt solver. In *Tools and Algorithms for the Construction and Analysis of Systems*, pages 337–340. Springer, 2008.
- [3] SM Eslami, N Heess, and T Weber. Attend, infer, repeat: Fast scene understanding with generative models. arxiv preprint arxiv:..., 2016. URL <http://arxiv.org/abs/1603.08575>.
- [4] Daniel Ritchie, Anna Thomas, Pat Hanrahan, and Noah Goodman. Neurally-guided procedural models: Amortized inference for procedural graphics programs using neural networks. In *Advances In Neural Information Processing Systems*, pages 622–630, 2016.
- [5] Haibin Huang, Evangelos Kalogerakis, Ersin Yumer, and Radomir Mech. Shape synthesis from sketches via procedural models and convolutional networks. *IEEE transactions on visualization and computer graphics*, 2017.
- [6] Gen Nishida, Ignacio Garcia-Dorado, Daniel G. Aliaga, Bedrich Benes, and Adrien Bousseau. Interactive sketching of urban procedural models. *ACM Trans. Graph.*, 35(4), 2016.
- [7] Brian Hempel and Ravi Chugh. Semi-automated svg programming via direct manipulation. In *Proceedings of the 29th Annual Symposium on User Interface Software and Technology*, UIST ’16, pages 379–390, New York, NY, USA, 2016. ACM.
- [8] Kenneth Forbus, Jeffrey Usher, Andrew Lovett, Kate Lockwood, and Jon Wetzel. Cogsketch: Sketch understanding for cognitive science research and for education. *Topics in Cognitive Science*, 3(4):648–666, 2011.
- [9] Kevin Ellis, Armando Solar-Lezama, and Josh Tenenbaum. Unsupervised learning by program synthesis. In *Advances in Neural Information Processing Systems*, pages 973–981, 2015.
- [10] Martín Abadi, Ashish Agarwal, Paul Barham, Eugene Brevdo, Zhifeng Chen, Craig Citro, Greg S. Corrado, Andy Davis, Jeffrey Dean, Matthieu Devin, Sanjay Ghemawat, Ian Goodfellow, Andrew Harp, Geoffrey Irving, Michael Isard, Yangqing Jia, Rafal Jozefowicz, Lukasz Kaiser, Manjunath Kudlur, Josh Levenberg, Dan Mané, Rajat Monga, Sherry Moore, Derek Murray, Chris Olah, Mike Schuster, Jonathon Shlens, Benoit Steiner, Ilya Sutskever, Kunal Talwar, Paul Tucker, Vincent Vanhoucke, Vijay Vasudevan, Fernanda Viégas, Oriol Vinyals, Pete Warden, Martin Wattenberg, Martin Wicke, Yuan Yu, and Xiaoqiang Zheng. TensorFlow: Large-scale machine learning on heterogeneous systems, 2015. Software available from tensorflow.org.
- [11] Arnaud Doucet, Nando De Freitas, and Neil Gordon, editors. *Sequential Monte Carlo Methods in Practice*. Springer, 2001.
- [12] Oleksandr Polozov and Sumit Gulwani. Flashmeta: A framework for inductive program synthesis. *ACM SIGPLAN Notices*, 50(10):107–126, 2015.
- [13] Daniel D Lee and H Sebastian Seung. Learning the parts of objects by non-negative matrix factorization. *Nature*, 401(6755):788–791, 1999.
- [14] Michael AA Cox and Trevor F Cox. Multidimensional scaling. *Handbook of data visualization*, pages 315–347, 2008.
- [15] Christopher M. Bishop. Mixture Density Networks. Technical report, 1994.

- 276 [16] Volodymyr Mnih, Nicolas Heess, Alex Graves, et al. Recurrent models of visual attention. In *Advances in*
277 *neural information processing systems*, pages 2204–2212, 2014.
- 278 [17] Vu Le and Sumit Gulwani. Flashextract: a framework for data extraction by examples. In *ACM SIGPLAN*
279 *Notices*, volume 49, pages 542–553. ACM, 2014.