# Learning to Infer Graphics Programs from Hand-Drawn Images

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## Abstract

We introduce a model that learns to convert simple hand drawings into graphics programs written in a subset of LaTeX. The model combines techniques from deep learning and program synthesis. We learn a convolutional neural network that proposes plausible drawing primitives that explain an image. This set of drawing primitives is like an execution trace for a graphics program. From this trace we use program synthesis techniques to recover a graphics program with constructs like variable bindings, iterative loops, or simple kinds of conditionals. With a graphics program in hand, we can correct errors made by the deep network, cluster drawings by use of similar high-level geometric structures, and extrapolate drawings. Taken together these results are a step towards agents that induce useful, human-readable programs from perceptual input.

## 1 Introduction

How can an agent convert noisy, high-dimensional perceptual input to a symbolic, abstract object, such as a computer program? Here we consider this problem within a graphics program synthesis domain. We develop an approach for converting natural images, such as hand drawings, into executable source code for drawing the original image. The graphics programs in our domain draw simple figures like those found in machine learning papers (see Figure 1a).

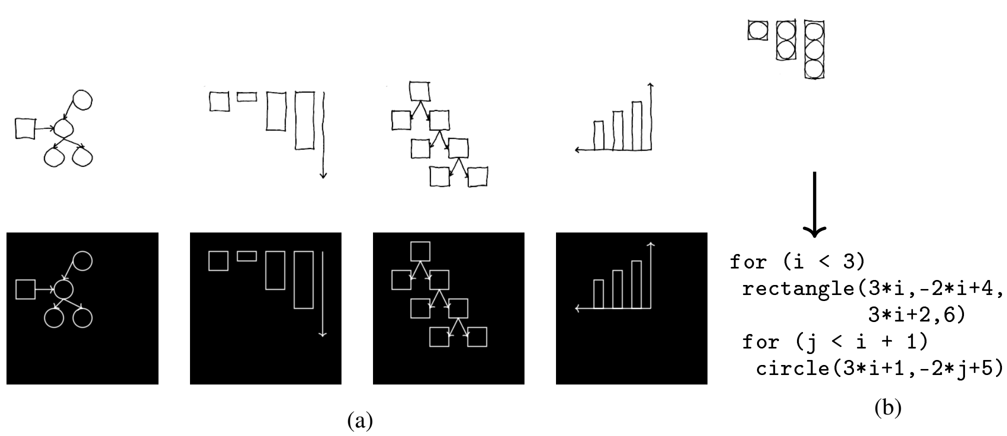


Figure 1: (a): Model learns to convert hand drawings (top) into LaTeX (bottom). (b) Synthesizes high-level *graphics program* from hand drawing.

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The key observation behind our work is that generating a programmatic representation from an image of a diagram actually involves two distinct steps that require different technical approaches. The first step involves identifying the components such as boxes, lines and arrows that make up the image. The second step involves identifying the high-level structure in how the components were drawn; for example, in Figure 1(b), it means identifying that there is a pattern in how the circles and rectangles are being drawn that is best described with two nested loops, and which can easily be extrapolated to a bigger diagram.

We present a hybrid architecture for inferring graphics programs that is structured around these two steps. For the first step, our approach uses a deep neural network to infer a sequence of primitive shape-drawing commands that can generate an image similar to the observed image. We refer to this sequence as a *program trace*, since it corresponds to the sequence of primitive commands that the desired program would have issued, but it lacks the high-level structure that determines how the program decided to issue those commands. For added robustness, we train the network to produce a proposal distribution of the most likely traces, and use stochastic search to find the trace that best matches the input. The network is trained from an automatically generated corpus of synthetic image generating programs.

The second step involves generating a high-level program capable of producing the program trace identified by the first phase. In principle, we could also train a neural network to learn this mapping. In practice, though, this would be challenging because it would no longer be sufficient to train the network with a synthetic corpus of randomly generated programs, since our goal is to generate programs that satisfy certain semantic constraints that ensure that we get the program that the user most likely intended. This paper argues that this stage is best achieved by *constraint-based program synthesis* []. Without the need for training, the program synthesizer can search the space of possible programs for one capable of producing the desired trace under a quantitative objective that maximizes the likelihood according to a prior distribution that aims to capture the user's preference.

## 2 Related work

Our work bears resemblance to the Attend-Infer-Repeat (AIR) system, which learns to decompose an image into its constituent objects [3]. AIR learns an iterative inference scheme which infers objects one by one and also decides when to stop inference; this is similar to our approach's first stage, which parses images into program execution traces. Our approach further produces interpretable, symbolic programs which generate those execution traces. The two approaches also differ in their architectures and training regimes: AIR learns a recurrent auto-encoding model via variational inference, whereas our parsing stage learns an autoregressive-style model from randomly-generated (execution trace, image) pairs. Finally, while AIR was evaluated on multi-MNIST images and synthetic 3D scenes, we focus on parsing and interpreting hand-drawn sketches.

Our image-to-execution-trace parsing architecture builds on prior work on controlling procedural graphics programs [4]. Given a program which generates random 2D recursive structures such as vines, that system learns a structurally-identical "guide program" whose output can be directed, via neural networks, to resemble a given target image. We adapt this method to a different visual domain (figures composed of multiple objects), using a broad prior over possible scenes as the initial program and viewing the execution trace through the guide program as a symbolic parse of the target image. We then show how to synthesize higher-level programs from these execution traces.

In the computer graphics literature, there have been other systems which convert sketches into procedural representations. One uses a convolutional network to match a sketch to the output of a parametric 3D modeling system [5]. Another uses convolutional networks to support sketch-based instantiation of procedural primitives within an interactive architectural modeling system [6]. Both systems focus on inferring fixed-dimensional parameter vectors. In contrast, we seek to automatically infer a structured, programmatic representation of a sketch which captures higher-level visual patterns.

Prior work has also applied sketch-based program synthesis to authoring graphics programs. In particular, Sketch-n-Sketch presents a bi-directional editing system in which direct manipulations to a program's output automatically propagate to the program source code [7]. We see this work as complementary to our own: programs produced by our method could be provided to a Sketch-n-Sketch-like system as a starting point for further editing.

The CogSketch [8] system also aims to have a high-level understanding of hand-drawn figures. Their primary goal is cognitive modeling (eg, they apply their system to solving IQ-test style visual reasoning problems), whereas we are interested in building an automated AI application (eg, in our system the user need not annotate which strokes correspond to which shapes; our neural network produces something equivalent to the annotations). A key similarity however is that both CogSketch and our system have as a goal to make it easier to produce nice-looking figures. Unsupervised Program Synthesis [9] is a related framework which was also applied to geometric reasoning problems. The goals of [9] were cognitive modeling, and they applied their technique to synthetic scenes used in human behavioral studies.

## 3 Neural architecture for inferring drawing execution traces

We developed a deep network architecture for efficiently inferring a execution trace, $T$, from an image, $I$. Our model constructs the trace one drawing command at a time. When predicting the next drawing command, the network takes as input the target image $I$ as well as the rendered output of previous drawing commands. Intuitively, the network looks at the image it wants to explain, as well as what it has already drawn. It then decides either to stop drawing or proposes another drawing command to add to the execution trace; if it decides to continue drawing, the predicted primitive is rendered to its "canvas" and the process repeats.

Figure 2 illustrates this architecture. We first pass a $256 \times 256$ target image and a rendering of the trace so far (encoded as a two-channel image) to a convolutional network. Given the features extracted by the convnet, a multilayer perceptron then predicts a distribution over the next drawing command to add to the trace; see Table 1. We predict the drawing command token-by-token, conditioning each token both on the image features and on the previously generated tokens. For example, the network first decides to emit the `circle` token conditioned on the image features, then it emits the $x$ coordinate of the circle conditioned on the image features and the `circle` token, and finally it predicts the $y$ coordinate of the circle conditioned on the image features, the `circle` token, and the $x$ coordinate. See supplement for the full details of the architecture, which we implemented in Tensorflow [10].

The distribution over the next drawing command factorizes as:

$$\mathbb{P}_\theta[t_1 t_2 \cdots t_K | I, T] = \prod_{k=1}^{K} \mathbb{P}_\theta[t_k | f_\theta(I, \text{render}(T)), \{t_j\}_{j=1}^{k-1}] \tag{1}$$

where $t_1 t_2 \cdots t_K$ are the tokens in the drawing command, $I$ is the target image, $T$ is an execution trace, $\theta$ are the parameters of the neural network, and $f_\theta(\cdot, \cdot)$ is the image feature extractor (convolutional network). The distribution over execution traces factorizes as:

$$\mathbb{P}_\theta[T|I] = \prod_{n=1}^{|T|} \mathbb{P}_\theta[T_n | I, T_{1:(n-1)}] \times \mathbb{P}_\theta[\texttt{STOP} | I, T] \tag{2}$$

where $|T|$ is the length of execution trace $T$, the subscripts on $T$ index drawing commands within the trace, and the `STOP` token is emitted by the network to signal that the execution trace explains the image.

We train the network by sampling execution traces $T$ and target images $I$ for randomly generated scenes and maximizing (2) with respect to $\theta$ by gradient ascent. Training does not require back-propagation across the entire sequence of drawing commands: drawing to the canvas 'blocks' the gradients, effectively offloading memory to an external visual store. In a sense, this model is like an autoregressive variant of AIR [3] without attention.

We trained the network on $10^5$ scenes, which takes a little less than a day on an Nvidia TitanX GPU.

This network suffices to "derender" synthetic images like those shown in Figure 3. We can perform a beam search decoding to recover what the network thinks is the most likely execution trace for images like these, recovering traces maximizing $\mathbb{P}_\theta[T|I]$. But, if the network makes a mistake (predicts an incorrect line of code), it has no way of recovering from the error. In order to derender an image with $n$ objects, it must correctly predict $n$ drawing commands – so its probability of success will decrease exponentially in $n$, assuming it has any nonzero chance of making a mistake. For added

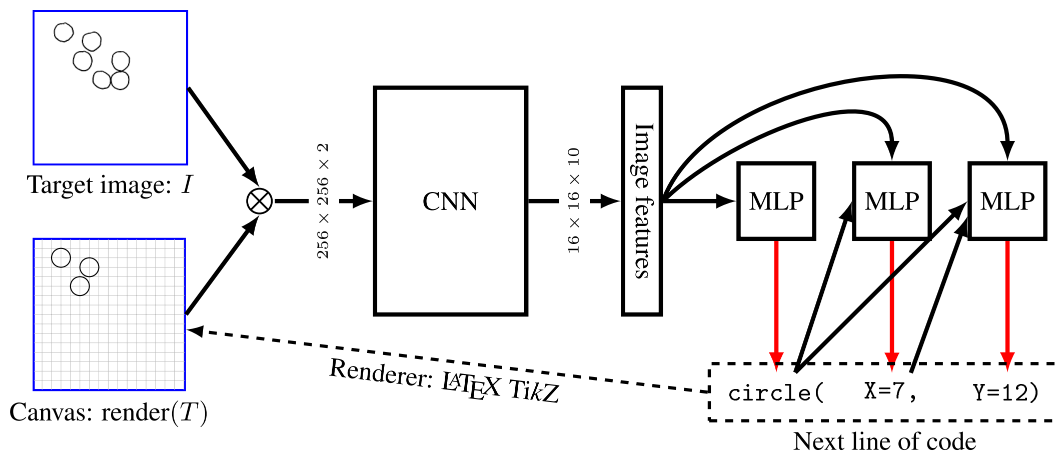



Figure 2: Our neural architecture for inferring the execution trace of a graphics program from its output. Blue: network inputs. Black: network operations. Red: samples from a multinomial. `Typewriter font`: network outputs. Renders snapped to a $16 \times 16$ grid, illustrated in gray.

| | |
|---|---|
| `circle`$(x, y)$ | Circle at $(x, y)$ |
| `rectangle`$(x_1, y_1, x_2, y_2)$ | Rectangle with corners at $(x_1, y_1)$ & $(x_2, y_2)$ |
| `line`$(x_1, y_1, x_2, y_2,$ arrow $\in \{0, 1\}$, dashed $\in \{0, 1\})$ | Line from $(x_1, y_1)$ to $(x_2, y_2)$, optionally with an arrow and/or dashed |
| `STOP` | Finishes execution trace inference |

Table 1: The deep network in (2) predicts drawing commands, shown above.

robustness as $n$ becomes large, we treat the neural network outputs as proposals for a Sequential Monte Carlo (SMC) sampling scheme [11]. For the SMC sampler, we use pixel-wise distance as a surrogate for a likelihood function. The SMC sampler is designed to produce samples from the distribution $\propto L(I|\text{render}(T))\mathbb{P}_\theta[T|I]$, where $L(\cdot|\cdot) : \text{image}^2 \to \mathcal{R}$ uses the distance between two images as a proxy for a likelihood. Figure 4 compares the neural network with SMC against the neural network by itself or SMC by itself. Only the combination of the two passes a critical test of generalization: when trained on images with $\leq 8$ objects, it successfully parses scenes with many more objects than the training data.

### 3.1 Generalizing to hand drawings

A practical application of our neural network is the automatic conversion of hand drawings into a subset of LaTeX. We train the model to generalize to hand drawings by introducing noise into the

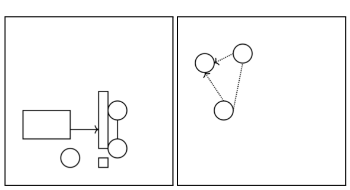


Figure 3: Network is trained to infer execution traces for randomly generated scenes like the two shown above. See supplement for details of the training data generation.

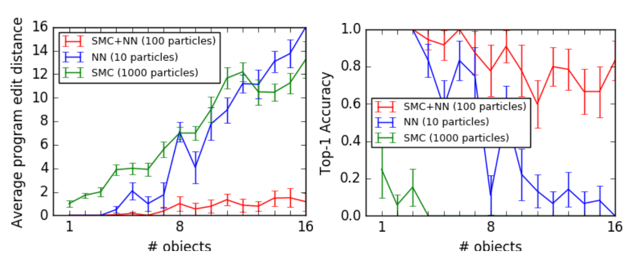
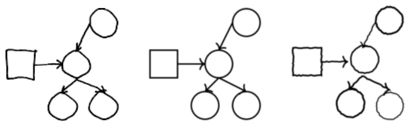



Figure 4: Using the model to parse latex output. The model is trained on diagrams with up to 8 objects. As shown above it generalizes to scenes with many more objects. Neither the stochastic search nor the neural network are sufficient on their own. # particles varies by model: we compare the models *with equal runtime* ($\approx 1$ sec/object)



(a) (b) (c)

Figure 5: (a): a hand drawing. (b): Rendering of the trace our model infers for (a). We can generalize to hand drawings like these because we train the model on images corrupted by a noise process designed to resemble the kind of noise introduced by hand drawings - see (c) for a noisy rendering of (b).

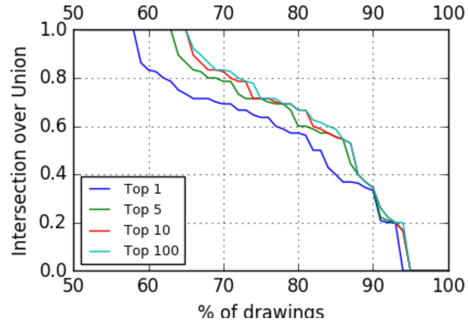


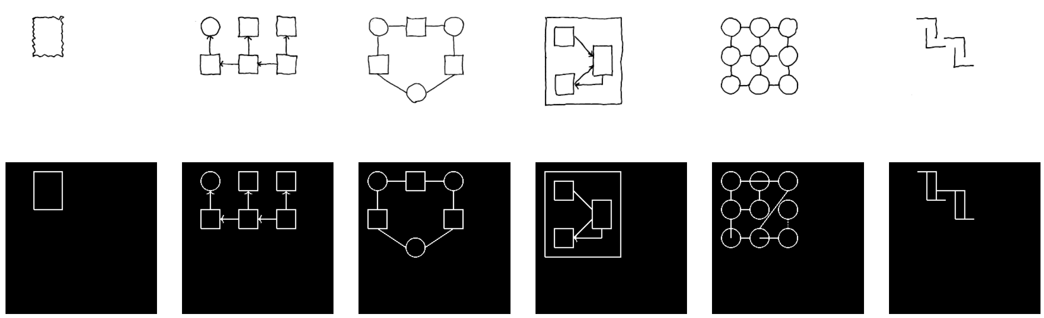

Figure 6: How close are the model's outputs to the ground truth on hand drawings, as we consider larger sets of samples (1,5,10,100)? Distance to ground truth trace measured by the intersection over union of predicted vs. ground truth traces (sets of drawing commands).

renderings of the training target images. We designed this noise process to introduce the kinds of variations found in hand drawings (Figure 5; see supplement for details). Our neurally-guided SMC procedure used pixel-wise distance as a surrogate for a likelihood function ($L(\cdot|\cdot)$ in section 3). But pixel-wise distance fares poorly on hand drawings, which never exactly match the model's renders. So, for hand drawings, we *learn* a surrogate likelihood function, $L_{\text{learned}}(\cdot|\cdot)$. The density $L_{\text{learned}}(\cdot|\cdot)$ is predicted by a convolutional network that we train to predict the distance between two traces conditioned upon their renderings. We train our likelihood surrogate to approximate the symmetric difference, which is the number of drawing commands by which two traces differ:

$$-\log L_{\text{learned}}(\text{render}(T_1)|\text{render}(T_2)) \approx |T_1 - T_2| + |T_2 - T_1| \quad (3)$$

Intuitively, Eq. 3 says that $L_{\text{learned}}(\cdot|\cdot)$ approximates the distance between the trace we want and the trace we have so far. Pixel-wise distance metrics are sensitive to the fine details of how and exactly where arrows, dashes, and corners are drawn – but we wish to be invariant to these details. So, we learn a distance metric over images that approximates the distance metric in the search space over traces.

We drew 100 figures by hand; see figure 7. These were drawn reasonably carefully but not perfectly. Because our model assumes that objects are snapped to a $16 \times 16$ grid, we made the drawings on graph paper.



(a) Noisy input (b) A graphical model (c) A figure from a deep learning textbook (d) Near miss (e) Failing on a Ising model (f) Illusory contours

Figure 7: Example drawings above model outputs. See also Fig. 1. Stochastic search (SMC) can help correct for these errors, as can the program synthesizer (Section 4.1)

For each drawing we annotated a ground truth trace, and evaluated the model by asking it to sample many candidate traces for each drawing. For 58% of the drawings the Top-1 most likely sample exactly matches the ground truth; as we consider more samples the model encounters traces that are closer to the ground truth annotation (Fig. 6). Because our current model sometimes makes mistakes on hand drawings, we envision the current system working as follows: a user sketches a diagram, and the system responds by proposing a few candidate interpretations. The user could then select the one closest to their intention and edit it if necessary.

## 4 Synthesizing graphics programs from execution traces

Although the execution trace of a graphics program describes the parts of a scene, it fails to encode higher-level features of the image, such as repeated motifs or symmetries. A *graphics program* better describe structures like these, and we now take as our goal to synthesize simple graphics programs from their execution traces.

We constrain the space of allowed programs by writing down a context free grammar over a space of programs. Although it might be desirable to synthesize programs in a Turing-complete language such as Lisp or Python, a more tractable approach is to specify what in the program languages community is called a Domain Specific Language (DSL) [12]. Our DSL (Table 2) encodes prior knowledge of what graphics programs tend to look like.

| | |
|---:|---|
| Program$\rightarrow$ | Command; $\cdots$; Command |
| Command$\rightarrow$ | `circle`(Expression,Expression) |
| Command$\rightarrow$ | `rectangle`(Expression,Expression,Expression,Expression) |
| Command$\rightarrow$ | `line`(Expression,Expression,Expression,Expression,Boolean,Boolean) |
| Command$\rightarrow$ | `for`($0 \leq$ Var $<$ Expression) `{ if` (Var $> 0$) `{` Program `};` Program `}` |
| Command$\rightarrow$ | `reflect`(Axis) `{` Program `}` |
| Expression$\rightarrow$ | $\mathcal{Z}$ `*` Var `+` $\mathcal{Z}$ |
| Var$\rightarrow$ | A free (unused) variable |
| $\mathcal{Z}\rightarrow$ | an integer |
| Axis$\rightarrow$ | `X =` $\mathcal{Z}$ |
| Axis$\rightarrow$ | `Y =` $\mathcal{Z}$ |

Table 2: Grammar over graphics programs. We allow loops (`for`) with conditionals (`if`), vertical/horizontal reflections (`reflect`), variables (Var) and affine transformations ($\mathcal{Z}$ `*` Var `+` $\mathcal{Z}$).

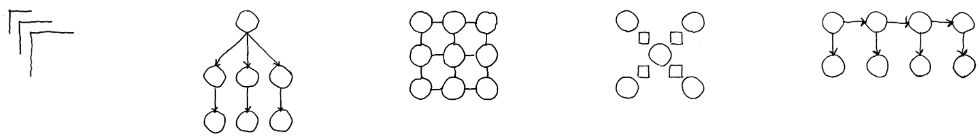
Given the DSL and a trace $T$, we want to recover a program that both evaluates to $T$ and, at the same time, is the "best" explanation of $T$. For example, we might prefer more general programs or, in the spirit of Occam's razor, prefer shorter programs. We wrap these intuitions up into a cost function over programs, and seek the minimum cost program consistent with $T$:

$$\text{program}(T) = \underset{\substack{p \in \text{DSL} \\ p \text{ evaluates to } T}}{\arg\min} \quad \text{cost}(p) \tag{4}$$

We define the cost of a program to be the number of statements it contains, where a statement is a "Command" in Table 2. We also penalize using many different numerical constants; see supplement.

The constrained optimization problem in equation 4 is intractable in general, but there exist efficient-in-practice tools for finding exact solutions to program synthesis problems like these. We use the state-of-the-art Sketch tool [1]. Describing Sketch's program synthesis algorithm is beyond the scope of this paper; see [1]. At a high level, Sketch takes as input a space of programs, along with a specification of the program's behavior and optionally a cost function. It translates the synthesis problem into a constraint satisfaction problem, and then uses a SAT solver to find a minimum cost program satisfying the specification. In exchange for not having any guarantees on how long it will take to find a minimum cost solution, it comes with the guarantee that it will always find a globally optimal program.

Why synthesize a graphics program, if the execution trace already suffices to recover the objects in an image? Within our domain of hand-drawn figures, graphics program synthesis has several uses:



```
for(i<3){
line(i,-1*i+6,
2*i+2,-1*i+6)
line(i,-2*i+4,
i,-1*i+6)
```

```
circle(4,10)
for(i<3){
circle(-3*i+7,5)
circle(-3*i+7,1)
line(-3*i+7,4,
-3*i+7,2,
arrow=True)
line(4,9,
-3*i+7,6,
arrow=True)
```

```
for(i<3)
for(j<3)
if(j>0)
line(-3*j+8,
-3*i+7,-3*j+9,
-3*i+7)
line(-3*i+7,-3*j+8,
-3*i+7,
-3*j+9)
circle(-3*j+7,-3*i+7)
```

```
reflect(y=8){
for(i<3){
if(i>0){
rectangle(3*i-1,
2,3*i,3)}
circle(3*i+1,
3*i+1)
```

```
for(i<4){
line(-4*i+13,4,
-4*i+13,2,
arrow=True)
for(j<3){
if(j>0){
circle(-4*i+13,
4*j+-3)}
line(-4*j+10,5,
-4*j+12,5,
arrow=True)
```

Figure 8: Example drawings (top row) and the programs we synthesize from their ground truth traces (bottom row). Notice the nested loops in the Ising model (middle), special case conditionals for the HMM (rightmost), combination of symmetry and iteration in middle left, and affine transformation in the leftmost figure.

### 4.1 Correcting errors made by the neural network

The program synthesizer can help correct errors from the execution trace proposal network by favoring execution traces which lead to more concise or general programs. For example, one generally prefers figures with perfectly aligned objects over figures whose parts are slightly misaligned – and precise alignment lends itself to short programs. Concretely, we estimated a prior over programs. Then, given the few most likely traces output by the neurally guided sampler, we reranked them according to the prior probability of their programs. Our neurally guided sampler could only do better on 7 drawings by looking at the Top-100 sampled traces (see Fig. 6), precluding a statistically significant analysis of how much learning a prior over programs could help correct errors. But, learning this prior does sometimes help correct mistakes made by the neural network, and also occasionally introduces mistakes of its own; see Fig. 9 for a representative example of the kinds of corrections that it makes. See supplement for details.

### 4.2 Modeling similarity between drawings

Modeling an image using a program opens up new ways of measuring similarity between drawings. For example, we might say that two drawings are similar if they both contain repetitions of length 4, or if they share the same reflectional symmetry, or if they are both organized according to a grid-like structure.

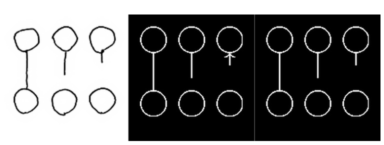


Figure 9: Left: hand drawing. Center: interpretation favored by the deep network. Right: interpretation favored after learning a prior over programs. Our learned prior favors shorter, simpler programs, thus continuing the pattern of not having an arrow is preferred.

We measure the similarity between two drawings by extracting features of the best programs that describe them. Our features are counts of the number of times that different components in the DSL were used (Table 2). We project these features down to a 2-dimensional subspace using nonnegative matrix factorization (NMF: [13]); see Fig.10. One could use many alternative similarity metrics between drawings which would capture pixel-level while missing high-level geometric similarities. We used our learned distance metric between execution traces, $L_{\text{learned}}(\cdot|\cdot)$, and projected to a 2-dimensional subspace using multidimensional scaling (MDS: [14]). This reveals similarities between the objects in the drawings, while missing similarities at the level of the program.

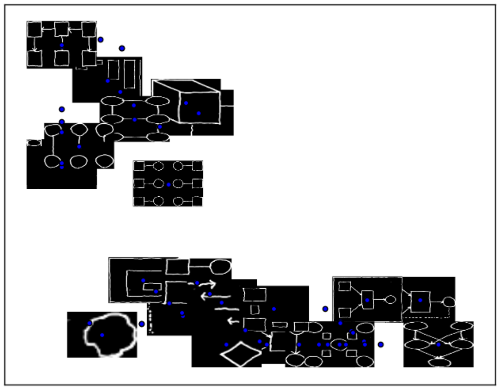


Figure 10: NMF on features of the programs that were synthesized for each image. Horizontal component roughly corresponds to "symmetry" while vertical component roughly corresponds to "loopyness", with images on the diagonal having both of these.

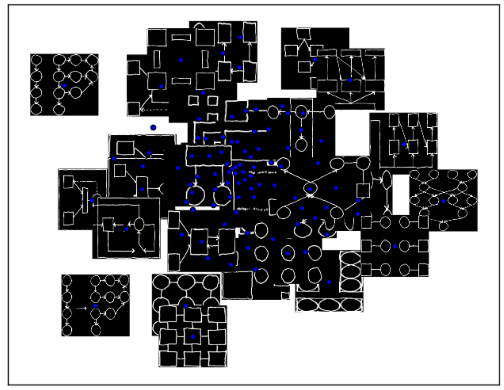


Figure 11: MDS on drawings using the learned distance metric, $L_{\text{learned}}(\cdot|\cdot)$. Drawings with similar looking parts in similar locations are clustered together.

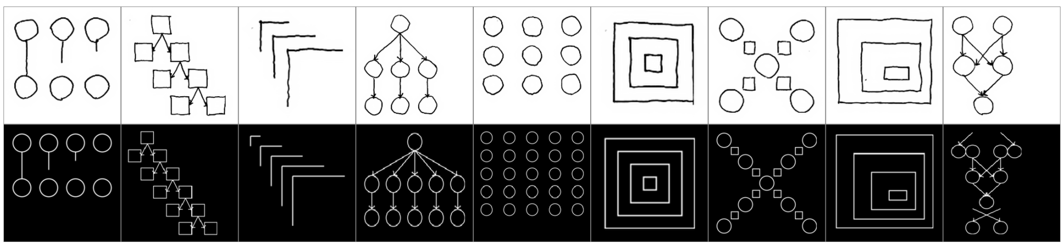


Figure 12: Top: hand drawings. Bottom: extrapolations produced by running loops for extra iterations. Rightmost pair is an illustrative failure case.

### 4.3 Extrapolating figures

Having access to the source code of a graphics program facilitates coherent, high-level edits to the figure generated by that program. For example, we can change all of the circles to squares or make all of the lines be dashed. We can also **extrapolate** figures by increasing the number of times that loops are executed. Extrapolating repetitive visuals patterns comes naturally to humans, and building this ability into an application is practical: imagine hand drawing a repetitive graphical model structure and having our system automatically induce and extend the pattern. Fig. 12 shows extrapolations of programs synthesized from ground truth traces; see supplement for our full set of extrapolations.

## 5 Conclusion

We have presented a system for inferring graphics programs which generate LaTeX-style figures from hand-drawn images. The system uses a combination of deep neural networks and stochastic search to parse drawings into symbolic execution traces; it then feeds these traces to a general-purpose program synthesis engine to infer a structured graphics program. We evaluated our model's performance at parsing novel images, and we demonstrated its ability to extrapolate from provided drawings and to organize them according to high-level geometric features.

There are many directions for future work. In the parsing phase, the proposal network currently samples positional variables on a discrete grid. More general types of drawings could be supported by instead sampling from continuous distributions, e.g. using Mixture Density Networks [15]. The proposal network also currently handles only a very small subset of LaTeXdrawing commands, though there is no reason that it could not be extended to handle more with a higher-capacity network. Exploring more sophisticated network architectures, including ones that utilize attention [16], could also help correct some of the errors the network makes. In the synthesis phase, a more expressive DSL—including subroutines, recursion, and symmetry groups beyond reflections—would allow the system to effectively model a wider variety of graphical phenomena. The synthesizer itself could also be the subject of future work: the system currently uses the general-purpose Sketch synthesizer, which can take minutes to hours to run, whereas program synthesizers which are custom-built for special problem domains can run much faster or even interactively [17].

In the not-too-distant future, we believe it should be possible to produce professional-looking figures just by drawing them and then letting an artificially-intelligent agent write the corresponding code. More generally, we believe that the two-phase system we have proposed—parsing into execution traces, then searching for a low-cost symbolic program which generates those traces—may be a useful paradigm for other domains in which agents must programmatically reason about noisy perceptual input.

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