Learning to Infer Graphics Programs from Hand-Drawn Images

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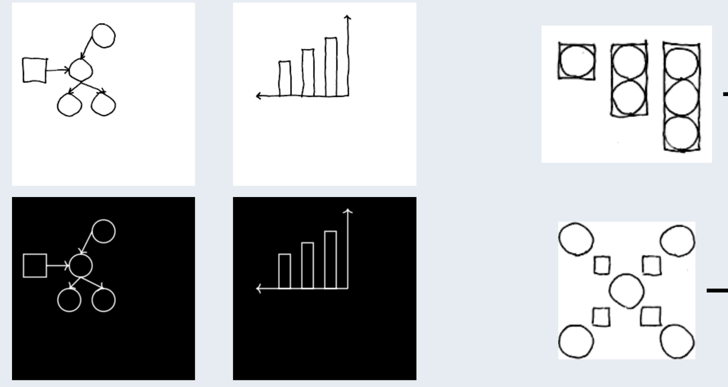
Hand drawings to high-level graphics procedures

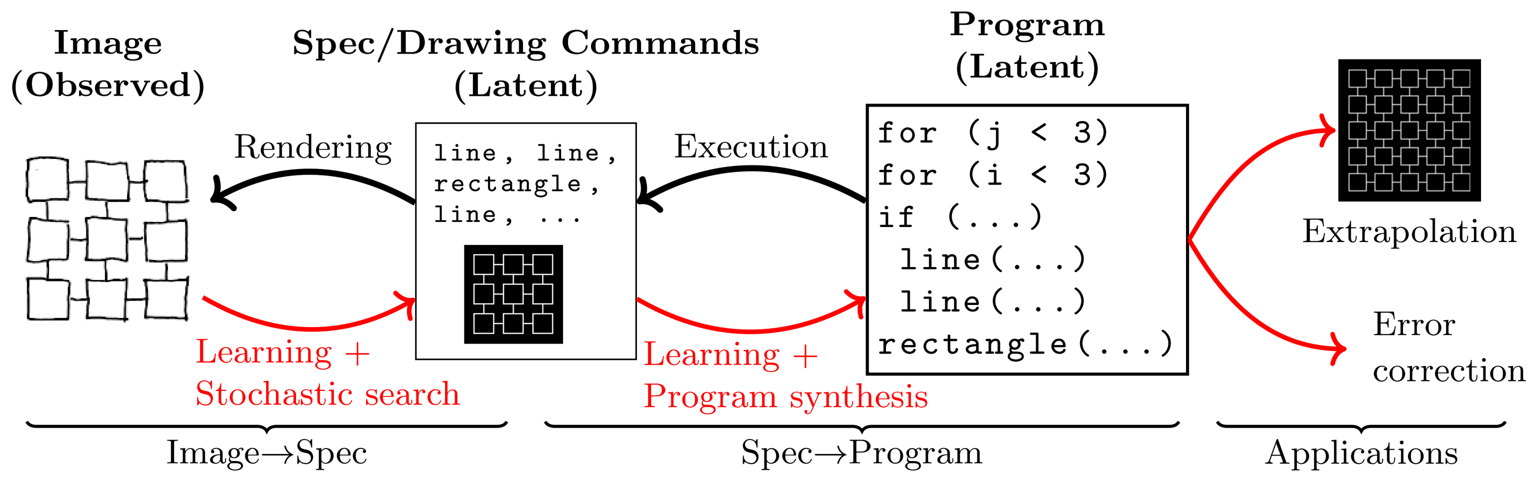
Convert image of hand drawing into LaTeX (left pair: top white is drawing, bottom black is LaTeX render). Infer high-level graphics program from drawing: captures higher-order structure like repetition, motifs, symmetries.



```
for (i < 3)
 rectangle(3*i,-2*i+4,
           3*i+2,6)
 for (j < i + 1)
  circle(3*i+1,-2*j+5)
```

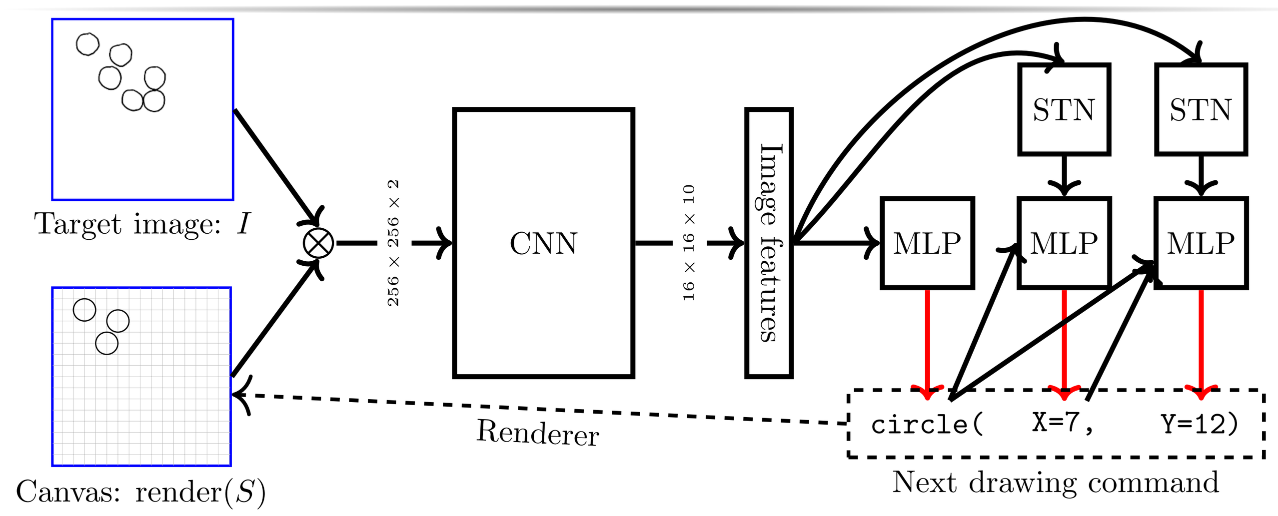
```
reflect(y=8)
 for(i<3)
  if(i>0)
   rectangle(3*i-1,2,3*i,3)
  circle(3*i+1,3*i+1)
```

Two-Stage Pipeline

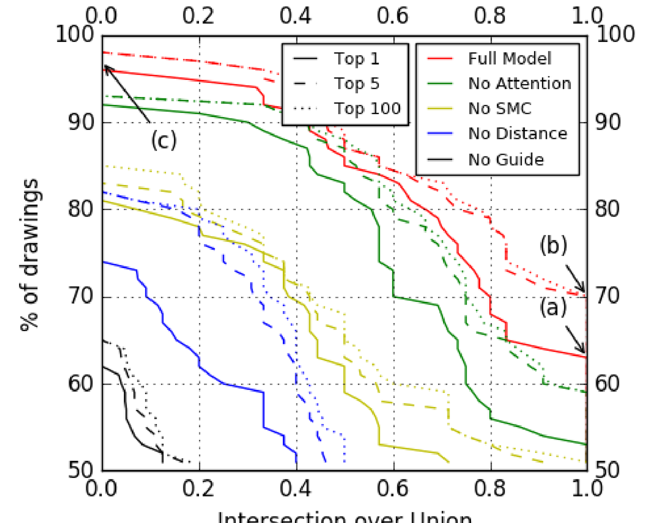


Black arrows: Top–down generative model; Program→Spec→Image. Red arrows: Bottom–up inference procedure. **Bold:** Random variables (image/spec/program)

Parsing Images into Drawing Commands (specs)



Blue: network inputs. Black: network operations. Red: draws from a multinomial. `Typewriter font`: network outputs. Renders on a 16 × 16 grid, shown in gray. STN: differentiable attention mechanism. **Combined with stochastic search (Sequential Monte Carlo)**



(Left) NN outputs vs ground truth on hand drawings (measured by IoU), as we consider larger sets of samples (1, 5, 100). (a) for 63% of drawings the model's top prediction is exactly correct; (b) for 70% of drawings the ground truth is in the top 5 model predictions; (c) for 4% of drawings all of the model outputs have no overlap with the ground truth. Red: the full model. Other colors: ablations. Model is at ceiling for synthetic LaTeX output.

Domain-Specific Language for Graphics Programs

We allow loops (`for`) with conditionals (`if`), vertical/horizontal reflections (`reflect`), variables (Var) and affine transformations ($\mathbb{Z}\times$Var$+\mathbb{Z}$).

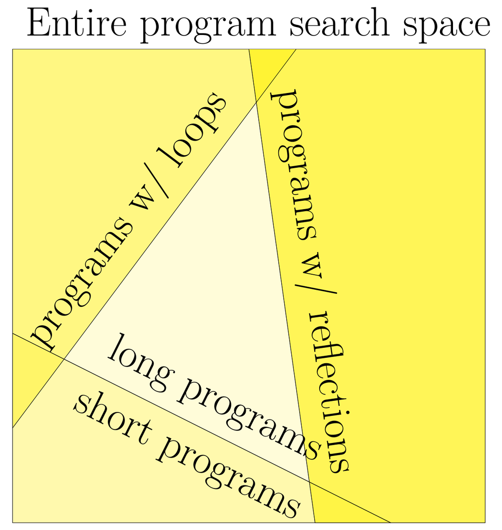
Program→ Statement; · · · ; Statement
Statement→ `circle`(Expression,Expression)
Statement→ `rectangle`(Expression,Expression,Expression,Expression)
Statement→ `line`(Expression,Expression,Expression,Expression,Boolean,Boolean)
Statement→ `for`($0 \leq$ Var $<$ Expression) { `if` (Var > 0) { Program }; Program }
Statement→ `reflect`(Axis) { Program }
Expression→ $\mathbb{Z}\times$Var$+\mathbb{Z}$
Axis→ `X` = $\mathbb{Z}$ | `Y` = $\mathbb{Z}$
$\mathbb{Z}$ → an integer

Learning & Constraint-Based Program Synthesis

Sketch: state-of-the-art program synthesizer. Solar-Lezama 2008.
Solves, for spec S & program p:

$$\text{program}(S) = \operatorname*{arg\,min}_{p\in\text{DSL, s.t. } p \text{ consistent w/ } S} \text{cost}(p)$$

Learn policy π_θ to accelerate program synthesizer's search.
$\pi_\theta(\cdot|S) \in \Delta^\Sigma$, where $\Sigma \ni \sigma$ a set of synthesis problem (i.e., $\sigma \in \Sigma$ is a sketch)
Inference strategy: Timeshare according to $\pi_\theta(\cdot|S)$, like in Levin Search



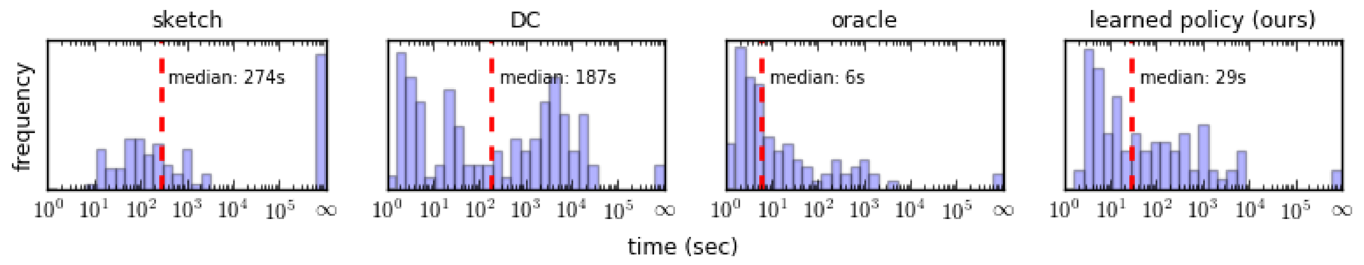
π_θ(short, no loop/reflect) =
π_θ(long, loops) =
π_θ(long, no loop/reflect) =
π_θ(long, reflects) =
etc.

Figure 1: The bias-optimal search algorithm divides the entire (intractable) program search space in to (tractable) program subspaces (written σ), each of which contains a restricted set of programs. For example, one subspace might be short programs which don't loop. The policy π predicts a distribution over program subspaces: weight assigned by π indicated by shading

Differentiable loss ($\mathcal{D}$ a corpus of synthesis problems):

$$\text{Loss}(\theta;\mathcal{D}) = \mathbb{E}_{S\sim\mathcal{D}}\left[\min_{\sigma\in\text{BEST}(S)} \frac{t(\sigma|S)}{\pi_\theta(\sigma|S)}\right] + \lambda\|\theta\|_2^2 \quad (1)$$

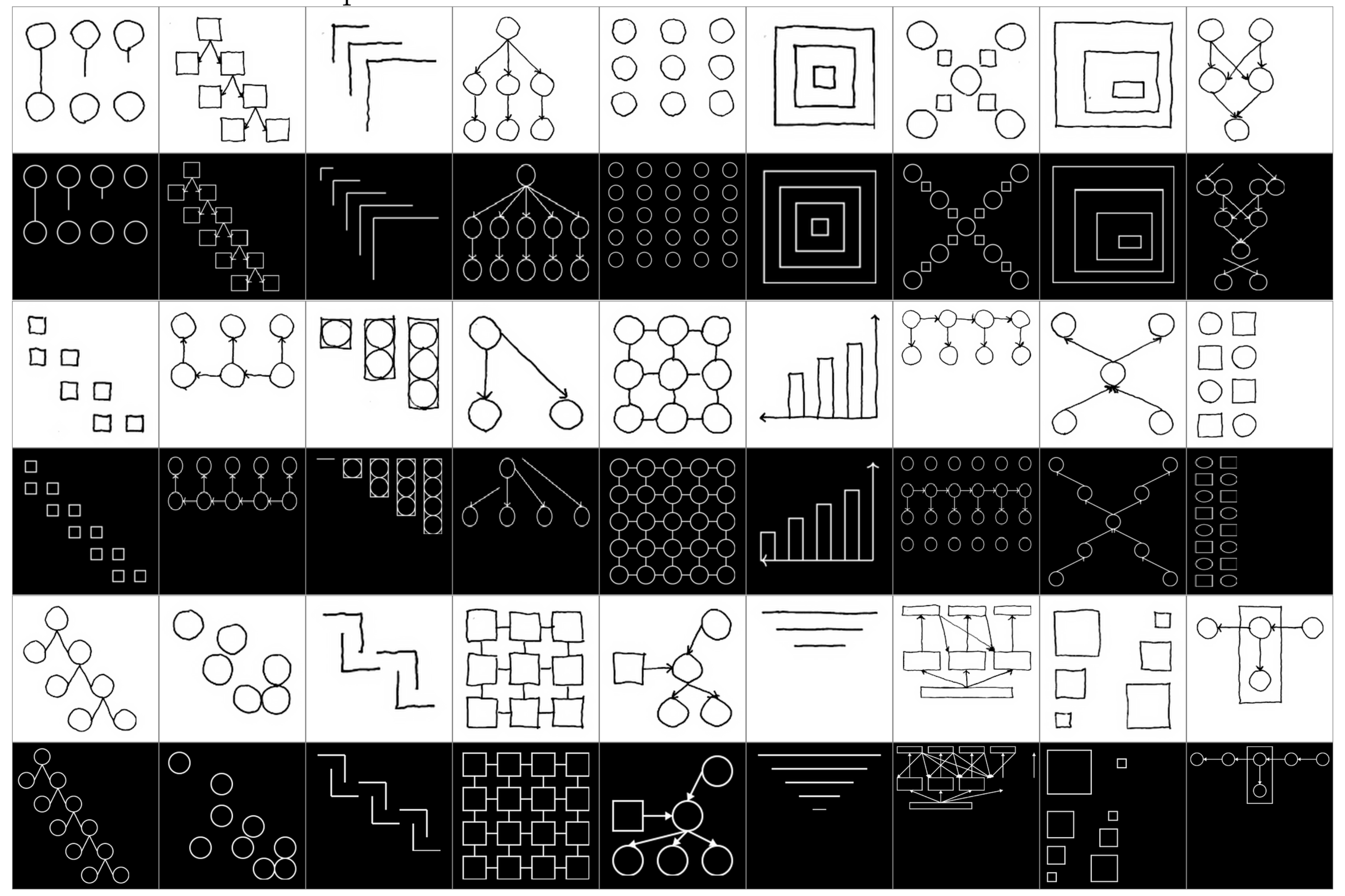
where $\sigma \in$ BEST(S) if a minimum cost program for S is in σ.



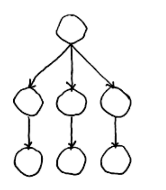
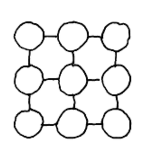
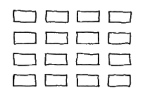
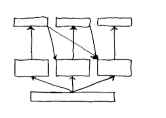
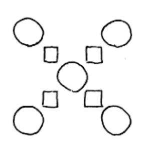
Time to synthesize a minimum cost program. Sketch: out-of-the-box performance of Sketch. DC: Deep–Coder style baseline that predicts program components, trained like Balog 2016. Oracle: upper bounds the performance of any bias–optimal search policy. ∞ = timeout. Red dashed line is median time

Extrapolating Drawings

Automatically extrapolate by increasing loop bounds. Top white: Hand drawing. Bottom black: extrapolation.

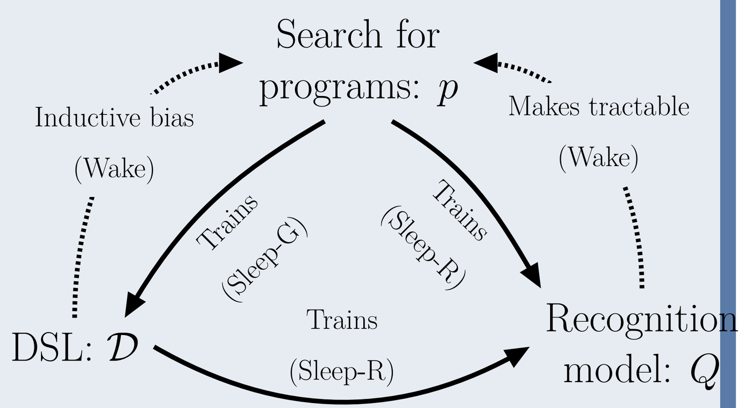


Example System Outputs

Drawing	Spec	Program	Compression factor
	`Line(2,15, 4,15)` `Line(4,9, 4,13)` `Line(3,11, 3,14)` `Line(2,13, 2,15)` `Line(3,14, 6,14)` `Line(4,13, 8,13)`	`for(i<3)` `line(i,-1*i+6, 2*i+2,-1*i+6)` `line(i,-2*i+4,i,-1*i+6)`	$\frac{6}{3} = 2$x
	`Line(5,13,2,10,arrow)` `Circle(5,9)` `Circle(8,5)` `Line(2,8, 2,6,arrow)` `Circle(2,5)` ... *etc.* ...; *13 lines*	`circle(4,10)` `for(i<3)` `circle(-3*i+7,5)` `circle(-3*i+7,1)` `line(-3*i+7,4,-3*i+7,2,arrow)` `line(4,9,-3*i+7,6,arrow)`	$\frac{13}{6} = 2.2$x
	`Circle(5,8)` `Circle(2,8)` `Circle(8,11)` `Line(2,9, 2,10)` `Circle(8,8)` `Line(3,8, 4,8)` `Line(3,11, 4,11)` ... *etc.* ...; *21 lines*	`for(i<3)` `for(j<3)` `if(j>0)` `line(-3*j+8,-3*i+7, -3*j+9,-3*i+7)` `line(-3*i+7,-3*j+8, -3*i+7,-3*j+9)` `circle(-3*j+7,-3*i+7)`	$\frac{21}{6} = 3.5$x
	`Rectangle(1,10,3,11)` `Rectangle(1,12,3,13)` `Rectangle(4,8,6,9)` `Rectangle(4,10,6,11)` ... *etc.* ...; *16 lines*	`for(i<4)` `for(j<4)` `rectangle(-3*i+9,-2*j+6, -3*i+11,-2*j+7)`	$\frac{16}{3} = 5.3$x
	`Line(3,10,3,14,arrow)` `Rectangle(11,8,15,10)` `Rectangle(11,14,15,15)` `Line(13,10,13,14,arrow)` ... *etc.* ...; *16 lines*	`for(i<3)` `line(7,1,5*i+2,3,arrow)` `for(j<i+1)` `if(j>0)` `line(5*j-1,9,5*i,5,arrow)` `line(5*j+2,5,5*j+2,9,arrow)` `rectangle(5*i,3,5*i+4,5)` `rectangle(5*i,9,5*i+4,10)` `rectangle(2,0,12,1)`	$\frac{16}{9} = 1.8$x
	`Circle(2,8)` `Rectangle(6,9, 7,10)` `Circle(8,8)` `Rectangle(6,12, 7,13)` `Rectangle(3,9, 4,10)` ... *etc.* ...; *9 lines*	`reflect(y=8)` `for(i<3)` `if(i>0)` `rectangle(3*i-1,2,3*i,3)` `circle(3*i+1,3*i+1)`	$\frac{9}{5} = 1.8$x

NEW: Wake-Sleep DSL learning with DreamCoder

- **Wake:** Synthesize programs from DSL + neural net ('recognition model') to guide search
- **Sleep-G:** Grow DSL, using grammar induction algorithms
- **Sleep-R:** Train neural net on programs found during waking ('experience replay') and samples from DSL ('dreaming')

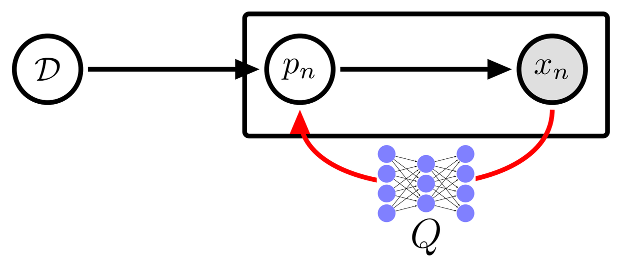


DreamCoder outputs for three different task domains

	List Functions		Text Editing	Symbolic Regression	
Programs & Tasks	`[7 2 3]`→`[7 3]` `[1 2 3 4]`→`[3 4]` `[4 3 2 1]`→`[4 3]` $f(\ell)$ =(`f1` ℓ (λ (x) (> x 2)))	`[7 3]`→`False` `[3]`→`False` `[9 0 0]`→`True` `[0]`→`True` `[0 7 3]`→`True` $f(\ell)$ =(f_3 ℓ 0)	+106 769-438→106.769.438 +83 973-831→83.973.831 f(s) =(f_0 "." "-" (f_0 "." " " (cdr s)))	 f(x) =(f_1 x)	 f(x) =(f_6 x)
	`[2 7 8 1]`→8 `[3 19 14]`→19 $f(\ell)$ =(f_2 ℓ)		Temple Anna H →TAH Lara Gregori→LG f(s) =(f_2 s)	 f(x) =(f_4 x)	 f(x) =(f_3 x)
DSL	$f_1(\ell$,p) =(foldr ℓ nil (λ (x a) (if (p x) (cons x a) a))) (f_1: *Higher-order filter function*)		f_0(s,a,b) =(map (λ (x) (if (= x a) b x)) s) (f_0: *substitutes characters*)	f_0(x) =(+ x real) f_1(x) =(f_0 (* real x)) f_2(x) =(f_1 (* x (f_0 x))) f_3(x) =(f_0 (* x (f_2 x))) f_4(x) =(f_0 (* x (f_3 x))) (f_4: *4th order polynomial*)	
	$f_2(\ell)$ =(foldr ℓ 0 (λ (x a) (if (> a x) a x))) (f_2: *Maximum element in list* ℓ)		f_1(s,c) =(foldr s s (λ (x a) (cdr (if (= c x) s a)))) (f_1: *Drop characters from* s *until* c *reached*)		
	$f_3(\ell$,k) =(foldr ℓ (is-nil ℓ) (λ (x a) (if a a (= k x)))) (f_3: *Whether* ℓ *contains* k)		f_2(s) =(unfold s is-nil car (λ (z) (f_1 z " "))) (f_2: *Abbreviates words*)	f_5(x) =(/ real x) f_6(x) =(f_5 (f_0 x)) (f_6: *rational function*)	

Tasks from three domains we apply our algorithm to, each followed by the programs DreamCoder discovers for them. Bottom: Several examples from learned DSL. Notice that learned DSL primitives can call each other, and that DreamCoder rediscovers higher-order functions like filter (f_1 under List Functions)

Bayesian framing



$$p_n^* = \underbrace{\arg\max_{p_n} \mathbb{P}[x_n|p_n]\mathbb{P}[p_n|\mathcal{D}^*]}_{\text{Wake}}$$

$$\mathcal{D}^* = \underbrace{\arg\max_{\mathcal{D}} \mathbb{P}[\mathcal{D}]\prod_n\sum_{p_n}\mathbb{P}[x_n|p_n]\mathbb{P}[p_n|\mathcal{D}]}_{\text{Sleep-G}}$$

$$Q^* = \underbrace{\arg\min_{Q} \text{KL}\left(\mathbb{P}[p|x,\mathcal{D}]\,||\,Q(p|x)\right)}_{\text{Sleep-R}}$$

x_n: program synthesis problem. p_n: (latent) program solving x_n. $Q(p|x)$: (learned) neural recognition model. $\mathcal{D}$: (latent) DSL.