Supplement to: Learning to Infer Graphics Programs from Hand-Drawn Images

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1 Correcting errors made by the neural network

The program synthesizer can help correct errors from the execution trace proposal network by favoring execution traces which lead to more concise or general programs. For example, one generally prefers figures with perfectly aligned objects over figures whose parts are slightly misaligned – and precise alignment lends itself to short programs. Similarly, figures often have repeated parts, which the program synthesizer might be able to model as a loop or reflectional symmetry. So, in considering several candidate traces proposed by the neural network, we might prefer traces whose best programs have desirable features such being short or having iterated structures.

Concretely, we implemented the following scheme: for an image I, the neurally guided sampling scheme of section 3 of the main paper samples a set of candidate traces, written $\mathcal{F}(I)$. Instead of predicting the most likely trace in $\mathcal{F}(I)$ according to the neural network, we can take into account the programs that best explain the traces. Writing $\hat{T}(I)$ for the trace the model predicts for image I,

$$\hat{T}(I) = \underset{T\in\mathcal{F}(I)}{\arg\max}\, L_{\text{learned}}(I|\text{render}(T)) \times \mathbb{P}_\theta[T|I] \times \mathbb{P}_\beta[\text{program}(T)] \tag{1}$$

where $\mathbb{P}_\beta[\cdot]$ is a prior probability distribution over programs parameterized by β. This is equivalent to doing MAP inference in a generative model where the program is first drawn from $\mathbb{P}_\beta[\cdot]$, then the program is executed deterministically, and then we observe a noisy version of the program's output, where $L_{\text{learned}}(I|\text{render}(\cdot)) \times \mathbb{P}_\theta[\cdot|I]$ is our observation model.

Given a corpus of graphics program synthesis problems with annotated ground truth traces (i.e. (I, T) pairs), we find a maximum likelihood estimate of β:

$$\beta^* = \underset{\beta}{\arg\max} \left[\log \frac{\mathbb{P}_\beta[\text{program}(T)] \times L_{\text{learned}}(I|\text{render}(T)) \times \mathbb{P}_\theta[T|I]}{\sum_{T'\in\mathcal{F}(I)} \mathbb{P}_\beta[\text{program}(T')] \times L_{\text{learned}}(I|\text{render}(T')) \times \mathbb{P}_\theta[T'|I]}\right] \tag{2}$$

where the expectation is taken both over the model predictions and the (I, T) pairs in the training corpus. We define $\mathbb{P}_\beta[\cdot]$ to be a log linear distribution $\propto \exp(\beta \cdot \phi(\text{program}))$, where $\phi(\cdot)$ is a feature extractor for programs. We extract a few basic features of a program, such as its size and how many loops it has, and use these features to help predict whether a trace is the correct explanation for an image.

We synthesized programs for the top 10 traces output by the deep network. Learning this prior over programs can help correct mistakes made by the neural network, and also occasionally introduces mistakes of its own; see Fig. 1 for a representative example of the kinds of corrections that it makes. On the whole it modestly improves our Top-1 accuracy from 63% to 67%. Recall that from Fig. 6 of the main paper that the best improvement in accuracy we could possibly get is 70% by looking at the top 10 traces.

2 Neural network architecture

2.1 Convolutional network

The convolutional network takes as input 2 256×256 images represented as a $2 \times 256 \times 256$ volume. These are passed through two layers of convolutions separated by ReLU nonlinearities and max pooling:

- Layer 1: 20 8×8 convolutions, 2 16×4 convolutions, 2 4×16 convolutions. Followed by 8×8 pooling with a stride size of 4.
- Layer 2: 10 8×8 convolutions. Followed by 4×4 pooling with a stride size of 4.

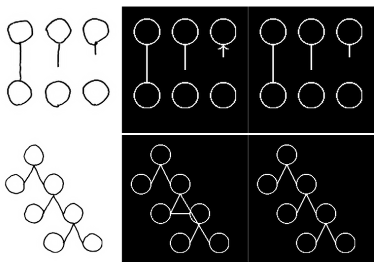


Figure 1: Left: hand drawing. Center: interpretation favored by the deep network. Right: interpretation favored after learning a prior over programs. Our learned prior favors shorter, simpler programs, thus (top example) continuing the pattern of not having an arrow is preferred, or (bottom example) continuing the binary search tree is preferred.

2.2 Autoregressive decoding of drawing commands

Given the image features f, we predict the first token (i.e., the name of the drawing command: `circle`, `rectangle`, `line`, or `STOP`) using logistic regression:

$$\mathbb{P}[t_1] \propto \exp\left(W_{t_1} f + b_{t_1}\right) \tag{3}$$

where W_{t_1} is a learned weight matrix and b_{t_1} is a learned bias vector.

Given an attention mechanism $a(\cdot|\cdot)$, subsequent tokens are predicted as:

$$\mathbb{P}[t_n | t_{1:(n-1)}] \propto \text{MLP}_{t_1,n}(a(f|t_{1:(n-1)}) \oplus \bigoplus_{j<n} \text{oneHot}(t_j)) \tag{4}$$

Thus each token of each drawing primitive has its own learned MLP. For predicting the coordinates of lines we found that using 32 hidden nodes with sigmoid activations worked well; for other tokens the MLP's are just logistic regression (no hidden nodes).

We use Spatial Transformer Networks [2] as our attention mechanism. The parameters of the spatial transform are predicted on the basis of previously predicted tokens. For example, in order to decide where to focus our attention when predicting the y coordinate of a circle, we condition upon both the identity of the drawing command (`circle`) and upon the value of the previously predicted x coordinate:

$$a(f|t_{1:(n-1)}) = \text{AffineTransform}(f, \text{MLP}_{t_1,n}(\bigoplus_{j<n} \text{oneHot}(t_j))) \tag{5}$$

So, we learn a different network for predicting special transforms *for each drawing command* (value of t_1) and also *for each token of the drawing command*. These networks ($\text{MLP}_{t_1,n}$ in equation 5) have no hidden layers and output the 6 entries of an affine transformation matrix; see [2] for more details.

Training takes a little bit less than a day on a Nvidia TitanX GPU. The network was trained on 10^5 synthetic examples.

2.3 Predicting continuous coordinates using Mixture Density Networks

Our main paper describes a neural model for parsing hand drawings into LaTeX, but comes with the restriction that all of the coordinates of all the drawing commands are snapped to a 16×16 grid. Now we describe a variant of our model that can predict continuous real valued coordinates.

The key obstacle to overcome is *multimodality*: if the network is unsure whether it should predict a coordinate value of a or b, we wish to predict a distribution that splits its mass between a and b, rather than predicting the single value $\frac{a+b}{2}$. In the model where everything is snapped to a discrete grid, supporting this multimodality falls out naturally from our parameterization of the network predictions (i.e., the network predicts the parameters of a multinomial distribution). To predict continuous real

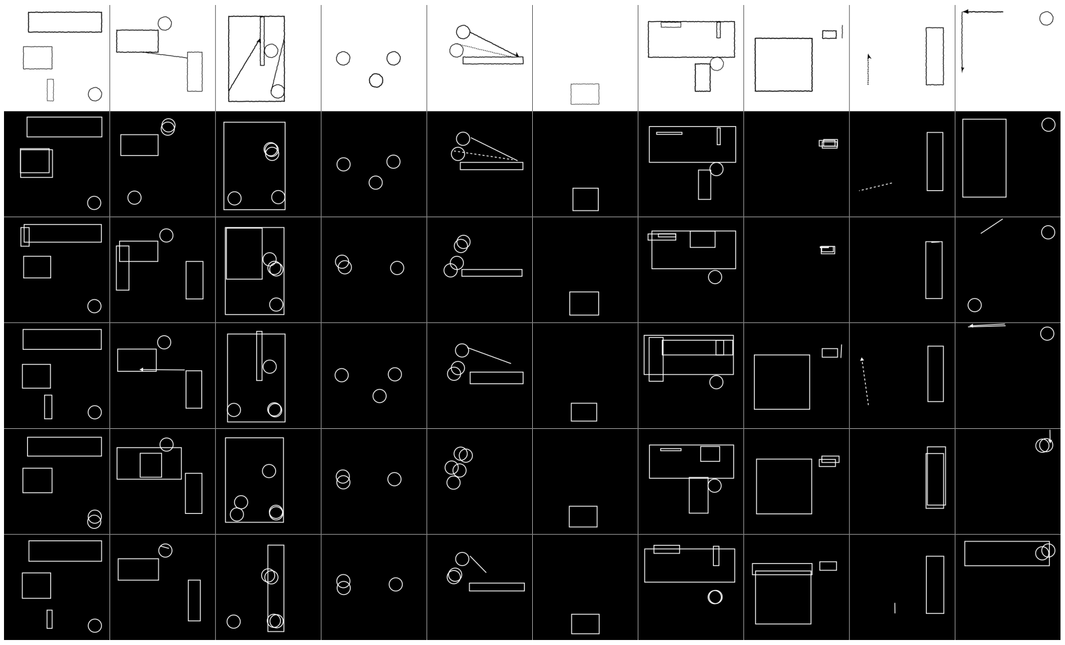


Figure 2: Left column: LaTeX output corrupted by noise process designed to resemble the variability is introduced by hand drawings. Right columns: samples from the MDN variant of the model.

valued coordinates we turn to Mixture Density Networks (MDN: [1]). In an MDN, a distribution over a real-valued random variable is parameterized using a mixture of Gaussians, and the deep network predicts the parameters of the Gaussian mixture.

Concretely, we modify our autoregressive drawing command decoder to predict distributions over real-valued coordinates as:

$$p(t_n|t_{1:(n-1)}) = \text{MDN}_{t_1,n}(a(f|t_{1:(n-1)}) \oplus \bigoplus_{j<n} t_j), \text{ if } n^{\text{th}} \text{ token is continuous} \tag{6}$$

Compare this equation with Equation 4 to see that we have only replaced a draw from a multinomial (whose parameters are predicted by an MLP) with a draw from a mixture of Gaussians (whose parameters are predicted by an MDN). Otherwise the two models are identical.

For an MDN i with K mixture components that takes as input x, we parameterize it's output distribution as:

$$\text{MDN}_i(x) = \sum_{1\leq k\leq K} \pi_{i,k}(x) \times \mathcal{N}(\mu_{i,k}(x), \sigma^2_{i,k}(x)) \tag{7}$$

$$\mu_{i,k}(x) = \text{sigmoid}(W^{\mu_{i,k}}x + b^{\mu_{i,k}}x) \tag{8}$$

$$\sigma^2_{i,k}(x) = \log\left(1 + \exp\left(W^{\sigma^2_{i,k}}x + b^{\sigma^2_{i,k}}x\right)\right) \tag{9}$$

$$\pi_{i,k}(x) = \frac{\exp\left(W^{\pi_{i,k}}x + b^{\pi_{i,k}}x\right)}{\sum_{k'} \exp\left(W^{\pi_{i,k'}}x + b^{\pi_{i,k'}}x\right)} \tag{10}$$

where W with a superscript is a learned weight matrix and b is a learned bias vector. Notice that Equation 8 normalizes the predicted means to be within $[0, 1]$: this is because we wish to constrain the predicted coordinates to lie within the plane $[0, 1]^2$. We used $K = 16$ mixture components.

Figure 2 shows the behavior of the MDN variant of our model on noisy synthetic drawings. Contrast these mediocre results with Figure 4 of the main paper to see that although our main model is essentially at ceiling for parsing scenes like these, the MDN variant fares poorly in comparison.

We evaluated the MDN version of our model on hand drawings by (1) performing a beam search in order to find the trace maximizing $\mathbb{P}[T|I]$, and (2) constraining the predicted coordinates to lie on the 16×16 grid. We constrain the predicted coordinates to lie on the grid by integrating the predicted densities about each grid location. So, using the probability density $p(t_n|t_{1:(n-1)})$ in Eq. 6, we compute a probability mass function $\mathbb{P}[t_n|t_{1:(n-1)}]$ as:

$$\mathbb{P}[t_n = t|t_{1:(n-1)}] = \int_{t-\frac{1}{2}}^{t+\frac{1}{2}} \mathrm{d}t_n \; p(t_n|t_{1:(n-1)}) \tag{11}$$

Even after using Eq. 11 to constrain the predicted coordinates to lie on the 16×16 grid, the MDN variant of the model only produces finished traces for 48/100 of the drawings. See Figure 3 to see the traces recovered by the beam search.

2.4 LSTM Baseline

We compared our deep network with a baseline that models the problem as a kind of image captioning. Given the target image, this baseline produces the program trace in one shot by using a CNN to extract features of the input which are passed to an LSTM which finally predicts the trace token-by-token. This general architecture is used in several successful neural models of image captioning (e.g., [3]).

Concretely, we kept the image feature extractor architecture (a CNN) as in our model, but only passed it one image as input (the target image to explain). Then, instead of using an autoregressive decoder to predict a single drawing command, we used an LSTM to predict a sequence of drawing commands token-by-token. This LSTM had 128 memory cells, and at each time step produced as output the next token in the sequence of drawing commands. It took as input both the image representation and its previously predicted token.

2.5 A learned likelihood surrogate

Our architecture for $L_{\text{learned}}(\text{render}(T_1)|\text{render}(T_2))$ has the same series of convolutions as the network that predicts the next drawing command. We train it to predict two scalars: $|T_1 - T_2|$ and $|T_2 - T_1|$. These predictions are made using linear regression from the image features followed by a ReLU nonlinearity; this nonlinearity makes sense because the predictions can never be negative but could be arbitrarily large positive numbers.

We train this network by sampling random synthetic scenes for T_1, and then perturbing them in small ways to produce T_2. We minimize the squared loss between the network's prediction and the ground truth symmetric differences. T_1 is rendered in a "simulated hand drawing" style which we describe next.

3 Simulating hand drawings

We introduce noise into the LaTeX rendering process by:

- Rescaling the image intensity by a factor chosen uniformly at random from $[0.5, 1.5]$
- Translating the image by ± 3 pixels chosen uniformly random
- Rendering the LaTeX using the `pencildraw` style, which adds random perturbations to the paths drawn by LaTeXin a way designed to resemble a pencil.
- Randomly perturbing the positions and sizes of primitive LaTeXdrawing commands

4 Likelihood surrogate for synthetic data

For synthetic data (e.g., LaTeX output) it is relatively straightforward to engineer an adequate distance measure between images, because it is possible for the system to discover drawing commands that exactly match the pixels in the target image. We use:

$$-\log L(I_1|I_2) = \sum_{1 \le x \le 256} \sum_{1 \le y \le 256} |I_1[x,y] - I_2[x,y]| \begin{cases} \alpha, \text{if } \mathrm{I}_1[x,y] > I_2[x,y] \\ \beta, \text{if } \mathrm{I}_1[x,y] < I_2[x,y] \\ 0, \text{if } \mathrm{I}_1[x,y] = I_2[x,y] \end{cases} \tag{12}$$

where α, β are constants that control the trade-off between preferring to explain the pixels in the image (at the expense of having extraneous pixels) and not predicting pixels where they don't exist

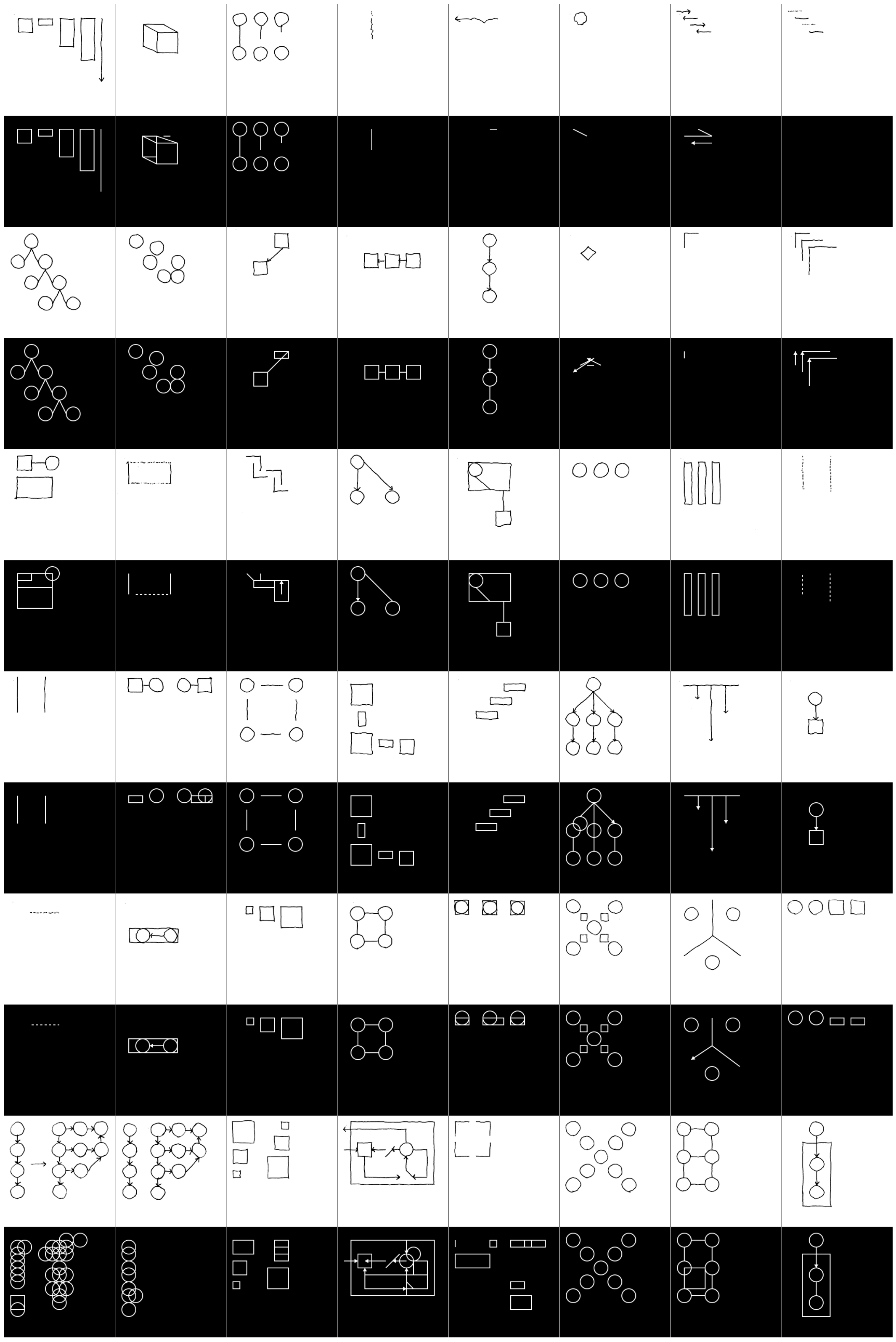


Figure 3: Parsing hand drawings using the MDN variant of our model. Black-on-white: hand drawings. White-on-black, below each hand drawing: the parse inferred by the model.

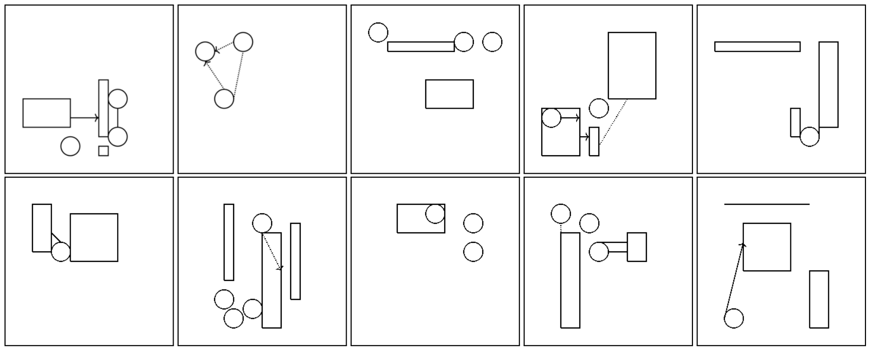


Figure 4: Example synthetic training data

(at the expense of leaving some pixels unexplained). Because our sampling procedure incrementally constructs the scene part-by-part, we want $\alpha > \beta$. That is, it is preferable to leave some pixels unexplained; for once a particle in SMC adds a drawing primitive to its trace that is not actually in the latent scene, it can never recover from this error. In our experiments on synthetic data we used $\alpha = 0.8$ and $\beta = 0.04$.

5 Generating synthetic training data

We generated synthetic training data for the neural network by sampling LaTeX code according to the following generative process: First, the number of objects in the scene are sampled uniformly from 1 to 8. For each object we uniformly sample its identity (circle, rectangle, or line). Then we sample the parameters of the circles, than the parameters of the rectangles, and finally the parameters of the lines; this has the effect of teaching the network to first draw the circles in the scene, then the rectangles, and finally the lines. We furthermore put the circle (respectively, rectangle and line) drawing commands in order by left-to-right, bottom-to-top; thus the training data enforces a canonical order in which to draw any scene.

To make the training data look more like naturally occurring figures, we put a Chinese restaurant process prior [4] over the values of the X and Y coordinates that occur in the execution trace. This encourages reuse of coordinate values, and so produces training data that tends to have parts that are nicely aligned.

In the synthetic training data we excluded any sampled scenes that had overlapping drawing commands. As shown in the main paper, the network is then able to generalize to scenes with, for example, intersecting lines or lines that penetrate a rectangle.

When sampling the endpoints of a line, we biased the sampling process so that it would be more likely to start an endpoint along one of the sides of a rectangle or at the boundary of a circle. If n is the number of points either along the side of a rectangle or at the boundary of a circle, we would sample an arbitrary endpoint with probability $\frac{2}{2+n}$ and sample one of the "attaching" endpoints with probability $\frac{1}{2+n}$.

See figure 4 for examples of the kinds of scenes that the network is trained on.

For readers wishing to generate their own synthetic training sets, we refer them to our source code at: `https://github.com/ellisk42/TikZ`.

6 The cost function for programs

We seek the minimum cost program which evaluates to (produces the drawing primitives in) an execution trace T:

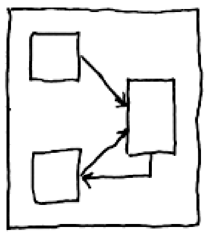
$$\text{program}(T) = \underset{\substack{p \in \text{DSL} \\ p \text{ evaluates to } T}}{\arg\min} \ \text{cost}(p) \tag{13}$$

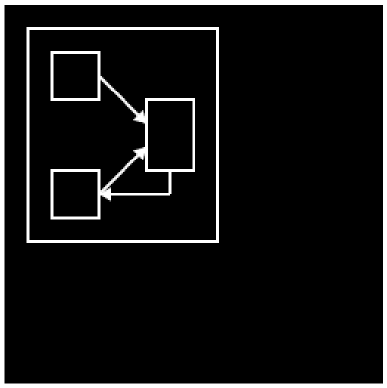
Programs incur a cost of 1 for each command (primitive drawing action, loop, or reflection). They incur a cost of $\frac{1}{3}$ for each unique coefficient they use in a linear transformation beyond the first coefficient. This encourages reuse of coefficients, which leads to code that has translational symmetry; rather than provide a translational symmetry operator as we did with reflection, we modify what is effectively a prior over the space of program so that it tends to produce programs that have this symmetry.

Programs also incur a cost of 1 for having loops of constant length 2; otherwise there is often no pressure from the cost function to explain a repetition of length 2 as being a reflection rather a loop.

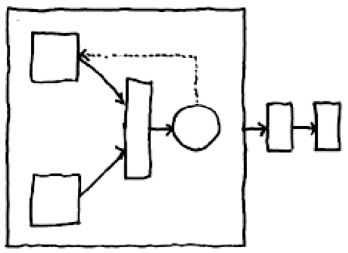
7 Full results on drawings data set

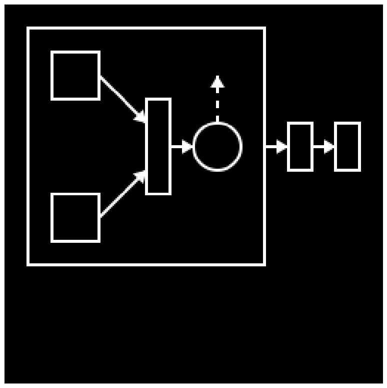
Below we show our full data set of drawings. The leftmost column is a hand drawing. The middle column is a rendering of the most likely trace discovered by the neurally guided SMC sampling scheme. The rightmost column is the program we synthesized from a ground truth execution trace of the drawing.



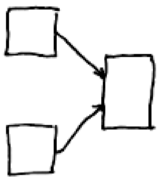
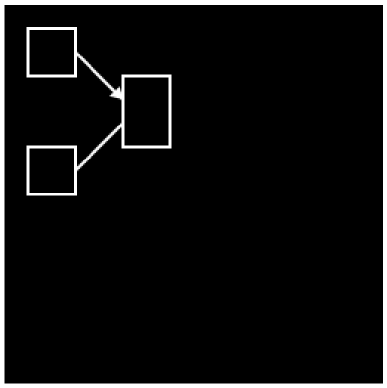


```
line(6,2,6,3,
arrow = False,solid = True);
line(6,2,3,2,
arrow = True,solid = True);
reflect(reflect(y = 9)){
line(3,2,5,4,
arrow = True,solid = True);
rectangle(0,0,8,9);
rectangle(5,3,7,6);
rectangle(1,1,3,3)
}
```

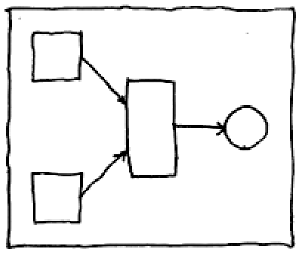
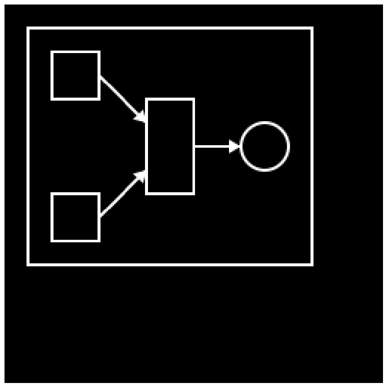




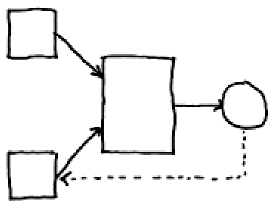
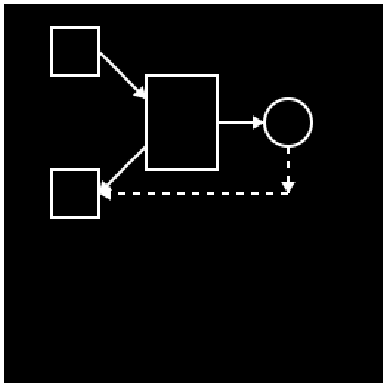
```
Solver timeout
```

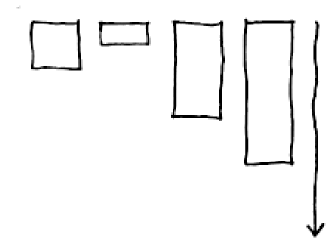
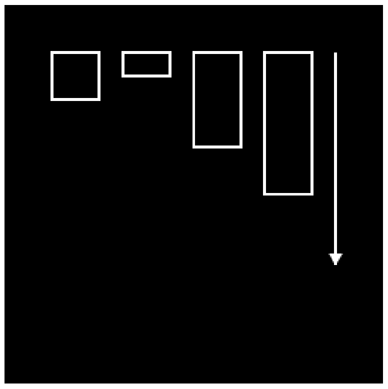
```
rectangle(4,2,6,5);
reflect(reflect(y = 7)){
line(2,6,4,4,
arrow = True,solid = True);
rectangle(0,0,2,2)
}
```

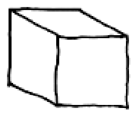
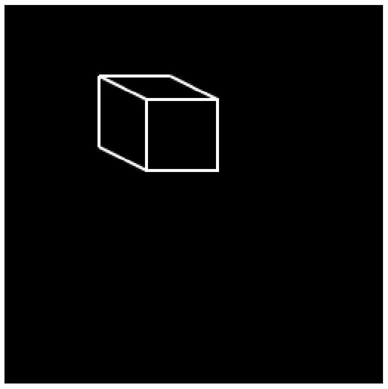
```
circle(10,5);
line(7,5,9,5,
arrow = True,solid = True);
rectangle(5,3,7,7);
rectangle(0,0,12,10);
reflect(reflect(y = 10)){
line(3,8,5,6,
arrow = True,solid = True);
rectangle(1,7,3,9)
}
```

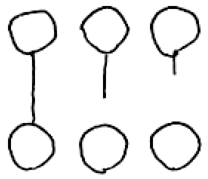
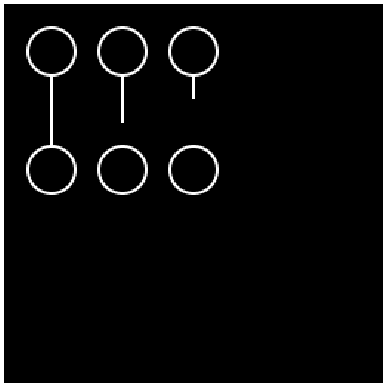
```
circle(10,4);
line(10,1,2,1,
arrow = True,solid = False);
line(10,1,10,3,
arrow = False,solid = False);
line(7,4,9,4,
arrow = True,solid = True);
reflect(reflect(y = 8)){
line(2,7,4,5,
arrow = True,solid = True);
rectangle(4,2,7,6);
rectangle(0,6,2,8)
}
```

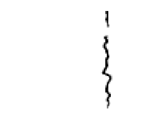
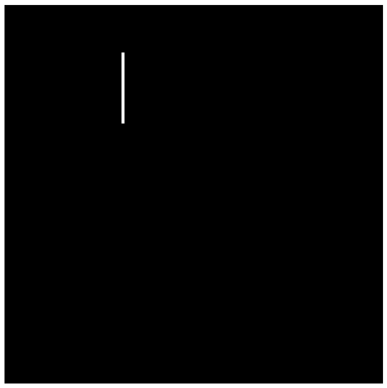
```
for (i < 3){
if (i > 0){
rectangle(-6*i + 12,2*i + 3,-6*i
rectangle(-6*i + 15,5*i + -2,-6*
}
line(12,9,12,0,
arrow = True,solid = True)
}
```

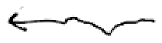
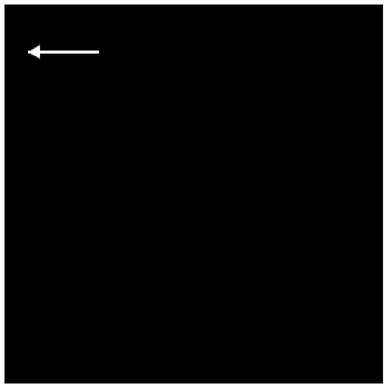
```
line(0,1,2,0,
arrow = False,solid = True);
for (i < 3){
if (i > 0){
line(3*i + -3,4,3*i + -1,3,
arrow = False,solid = True);
line(0,3*i + -2,3*i + -3,4,
arrow = False,solid = True)
}
rectangle(2,0,5,3)
}
```

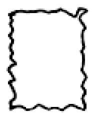
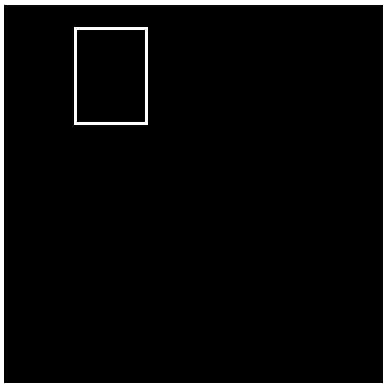
```
for (i < 3){
circle(-3*i + 7,1);
circle(-3*i + 7,6);
line(-3*i + 7,-1*i + 4,-3*i + 7,
arrow = False,solid = True)
}
```

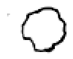
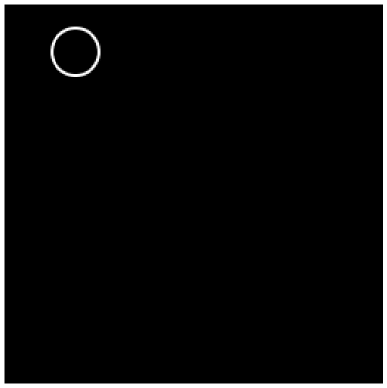
```
line(0,0,0,4,
arrow = False,solid = True)
```

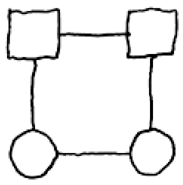
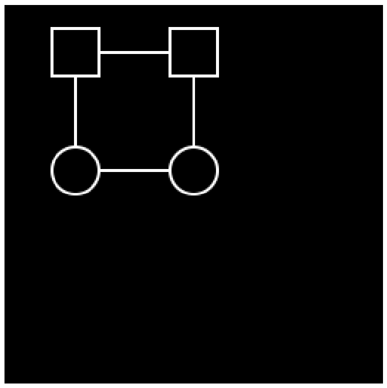
```
line(6,0,0,0,
arrow = True,solid = True)
```

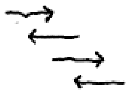
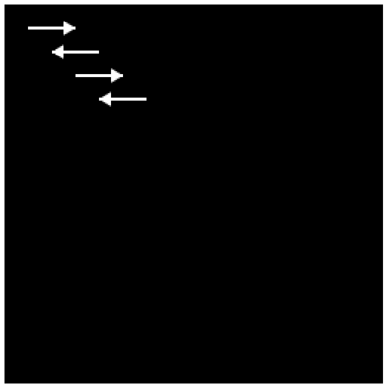
```
rectangle(0,0,3,4)
```

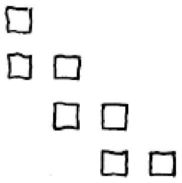
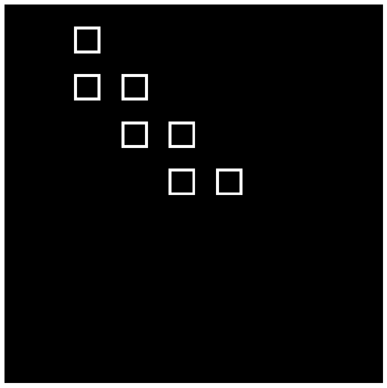
```
circle(1,1)
```

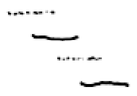
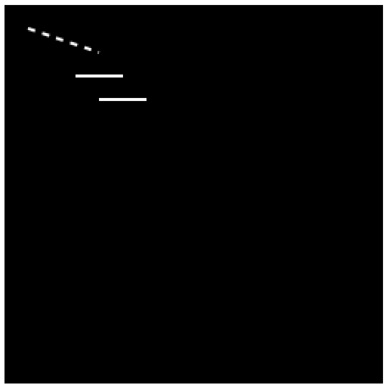
```
line(2,6,5,6,
arrow = False,solid = True);
reflect(reflect(x = 7)){
circle(6,1);
line(2,1,5,1,
arrow = False,solid = True);
line(1,2,1,5,
arrow = False,solid = True);
rectangle(5,5,7,7)
}
```

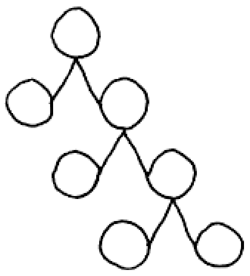
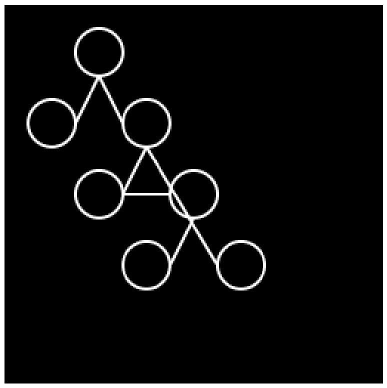
```
line(2,1,4,1,
arrow = True,solid = True);
line(3,2,1,2,
arrow = True,solid = True);
line(5,0,3,0,
arrow = True,solid = True);
line(0,3,2,3,
arrow = True,solid = True)
```

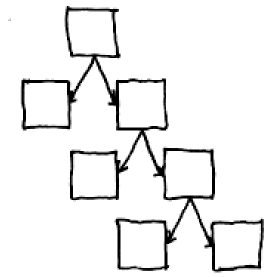
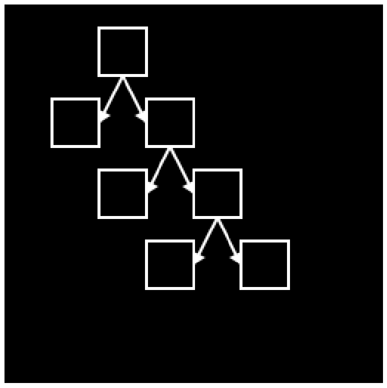
```
for (i < 4){
if (i > 0){
rectangle(-2*i + 6,2*i + -2,-2*i
}
rectangle(-2*i + 6,2*i,-2*i + 7,
}
```

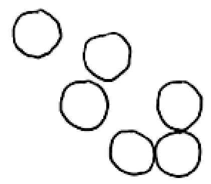
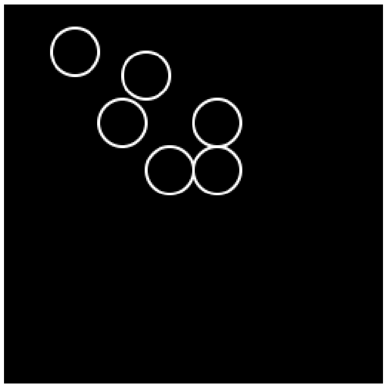
```
line(0,3,2,3,
arrow = False,solid = False);
line(2,1,4,1,
arrow = False,solid = False);
line(1,2,3,2,
arrow = False,solid = True);
line(3,0,5,0,
arrow = False,solid = True)
```

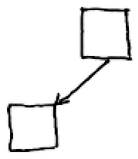
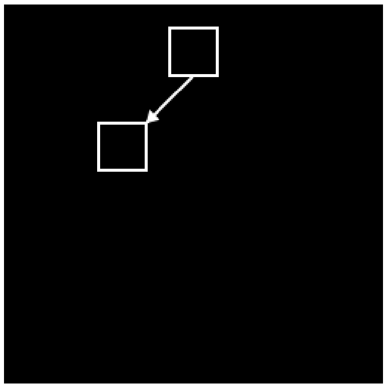
```
for (i < 4){
if (i > 0){
circle(-2*i + 7,3*i + -2);
line(-2*i + 9,3*i,-2*i + 10,3*i
arrow = False,solid = True);
line(-2*i + 8,3*i + -2,-2*i + 9,
arrow = False,solid = True)
}
circle(-2*i + 9,3*i + 1)
}
```

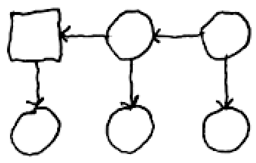
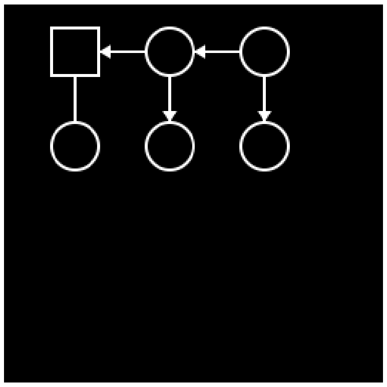
```
for (i < 4){
if (i > 0){
line(2*i + 1,-3*i + 12,2*i,-3*i
arrow = True,solid = True);
line(2*i + 1,-3*i + 12,2*i + 2,-
arrow = True,solid = True);
rectangle(2*i + -2,-3*i + 9,2*i,
}
rectangle(2*i + 2,-3*i + 9,2*i +
}
```

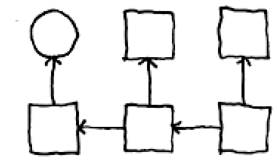
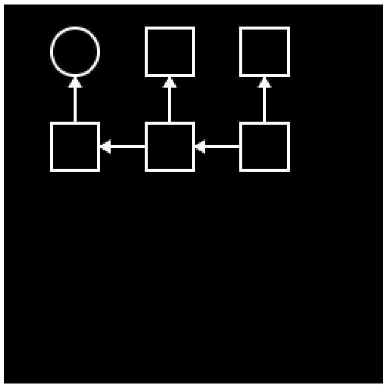
```
circle(5,1);
for (i < 3){
if (i > 0){
circle(7,2*i + -1);
circle(i + 2,2*i + 1)
}
circle(1,6)
}
```

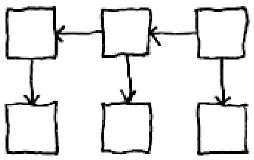
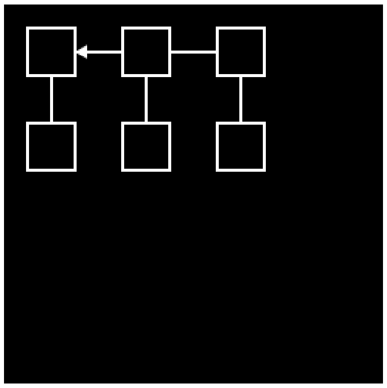
```
line(4,4,2,2,
arrow = True,solid = True);
rectangle(3,4,5,6);
rectangle(0,0,2,2)
```

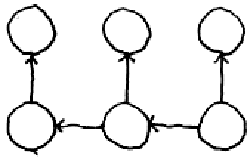
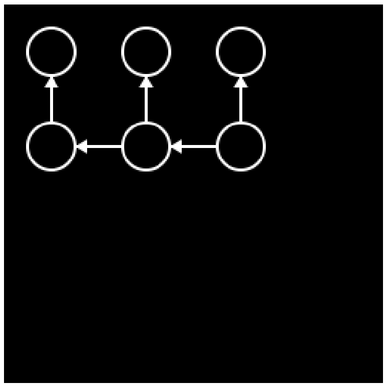
```
rectangle(0,4,2,6);
for (i < 3){
if (i > 0){
line(-4*i + 12,5,-4*i + 10,5,
arrow = True,solid = True);
for (j < i + 1){
circle(-4*j + 9,-4*i + 9)
}
}
line(-4*i + 9,4,-4*i + 9,2,
arrow = True,solid = True)
}
```

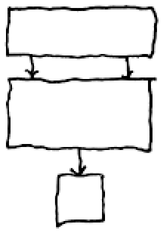
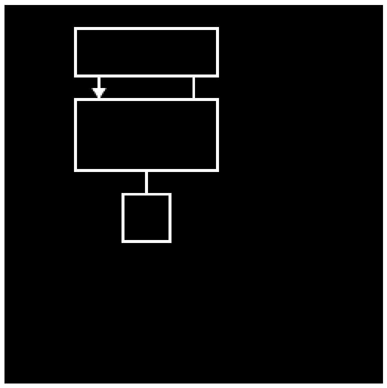
```
circle(1,5);
for (i < 3){
if (i > 0){
line(-4*i + 12,1,-4*i + 10,1,
arrow = True,solid = True);
rectangle(-4*i + 12,4,-4*i + 14,
}
line(-4*i + 9,2,-4*i + 9,4,
arrow = True,solid = True);
rectangle(-4*i + 8,0,-4*i + 10,2
}
```

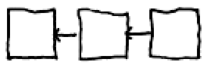
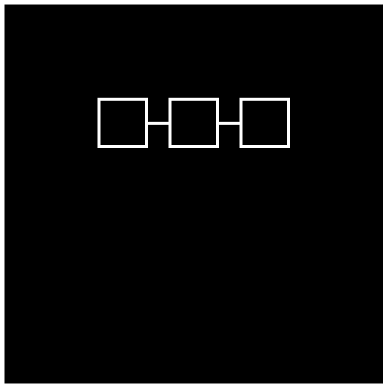
```
for (i < 3){
line(-4*i + 9,4,-4*i + 9,2,
arrow = True,solid = True);
for (j < 2){
line(-4*j + 8,5,-4*j + 6,5,
arrow = True,solid = True);
rectangle(-4*i + 8,4*j,-4*i + 10
}
}
```

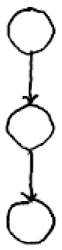
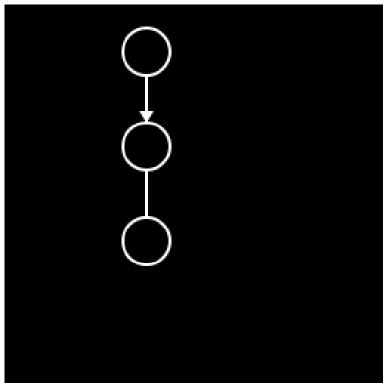
```
for (i < 3){
if (i > 0){
line(-4*i + 12,1,-4*i + 10,1,
arrow = True,solid = True)
}
circle(-4*i + 9,1);
circle(-4*i + 9,5);
line(-4*i + 9,2,-4*i + 9,4,
arrow = True,solid = True)
}
```

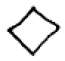
```
reflect(reflect(x = 6)){
for (i < 3){
if (i > 0){
line(-2*i + 7,-4*i + 11,-2*i + 7
arrow = True,solid = True);
rectangle(0,-4*i + 11,6,-3*i + 1
}
rectangle(2,0,4,2)
}
}
```

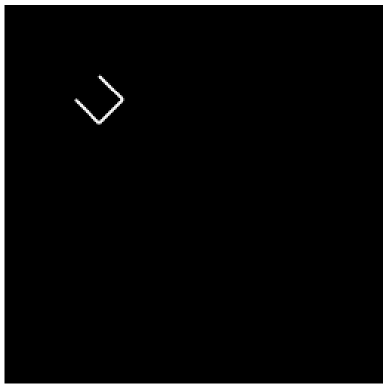



```
for (i < 3){
if (i > 0){
line(3*i,1,3*i + -1,1,
arrow = True,solid = True)
}
rectangle(3*i,0,3*i + 2,2)
}
```

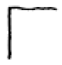



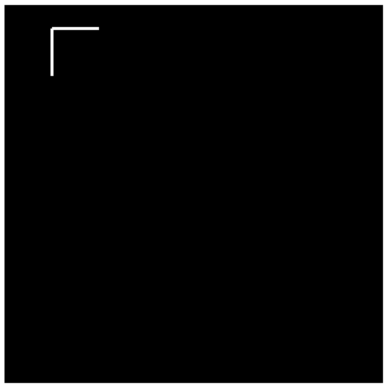
```
line(1,3,1,4,
arrow = False,solid = True);
for (i < 3){
if (i > 0){
line(1,-5*i + 13,1,-4*i + 10,
arrow = True,solid = True)
}
circle(1,-4*i + 9)
}
```



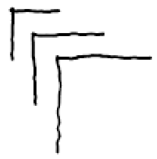


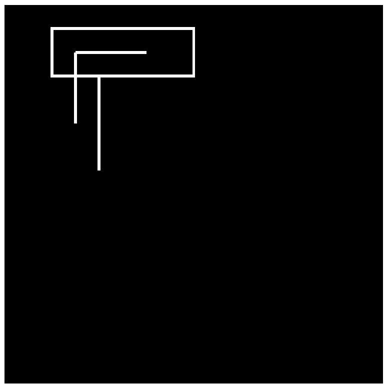
```
reflect(reflect(x = 2)){
line(0,1,1,2,
arrow = False,solid = True);
line(1,0,2,1,
arrow = False,solid = True)
}
```



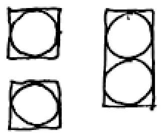
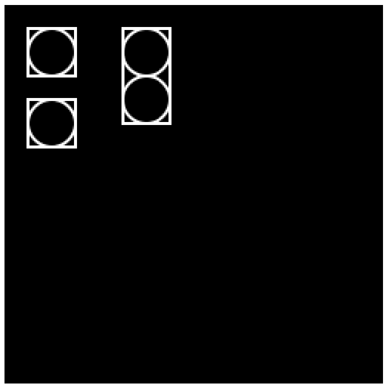


```
line(0,0,0,2,
arrow = False,solid = True);
line(0,2,2,2,
arrow = False,solid = True)
```

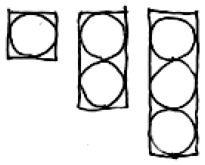
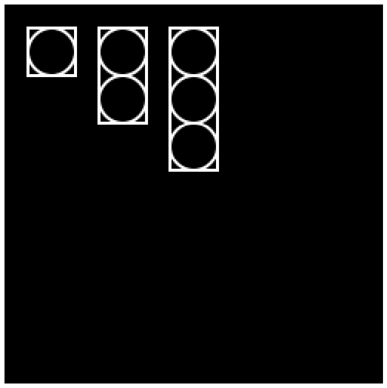




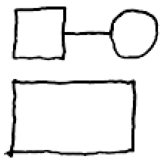
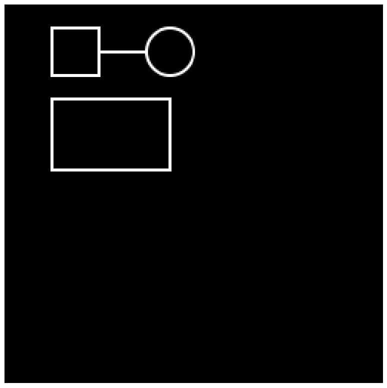
```
for (i < 3){
line(i,-1*i + 6,2*i + 2,-1*i + 6
arrow = False,solid = True);
line(i,-2*i + 4,i,-1*i + 6,
arrow = False,solid = True)
}
```

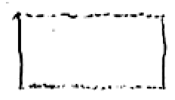
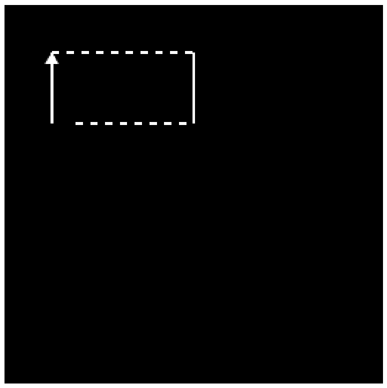
```
for (i < 3){
if (i > 0){
circle(1,-3*i + 7);
circle(5,-2*i + 6);
rectangle(0,-3*i + 6,2,-3*i + 8)
}
rectangle(4,1,6,5)
}
```

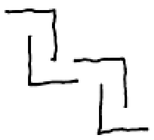
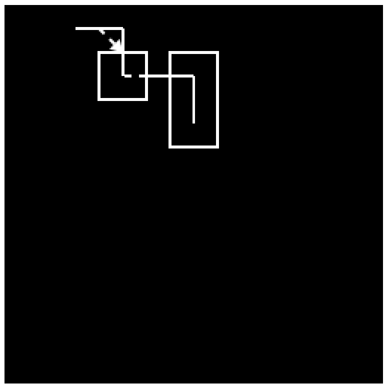
```
for (i < 3){
rectangle(3*i,-2*i + 4,3*i + 2,6
for (j < i + 1){
circle(3*i + 1,-2*j + 5)
}
}
```

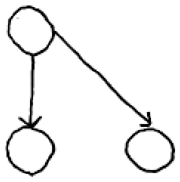
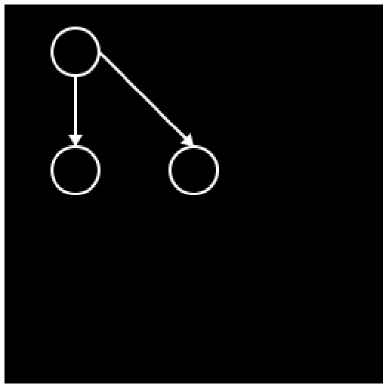
```
circle(5,5);
line(2,5,4,5,
arrow = False,solid = True);
rectangle(0,0,5,3);
rectangle(0,4,2,6)
```

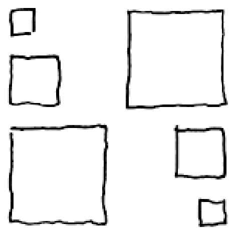
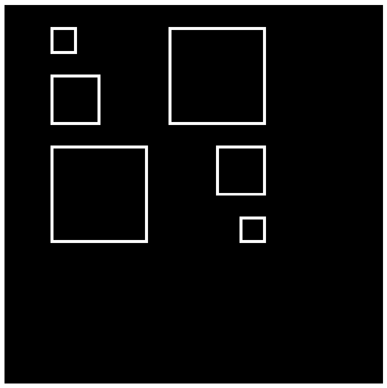
```
line(0,0,6,0,
arrow = False,solid = False);
reflect(reflect(x = 6)){
line(6,0,6,3,
arrow = False,solid = True);
line(0,3,6,3,
arrow = False,solid = False)
}
```

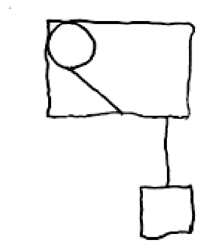
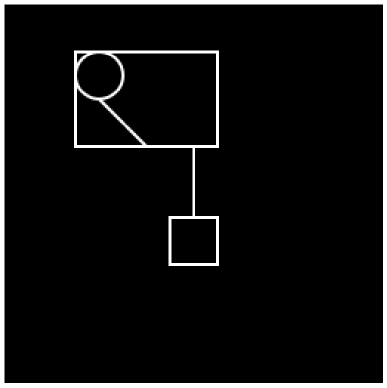
```
Solver timeout
```

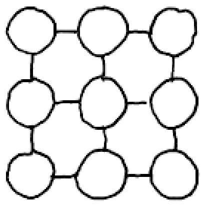
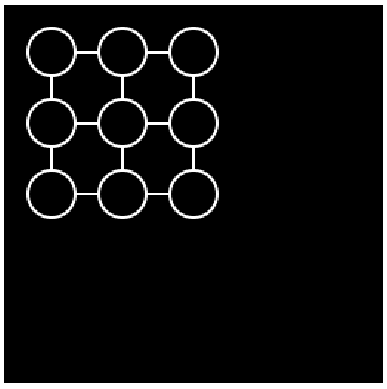
```
for (i < 3){
if (i > 0){
circle(-5*i + 11,1);
line(-1*i + 3,-1*i + 7,-5*i + 11
arrow = True,solid = True)
}
circle(1,6)
}
```

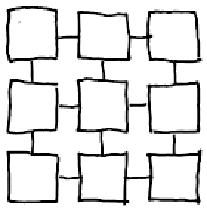
```
Solver timeout
```

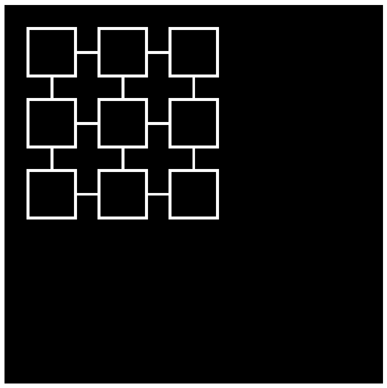



```
for (i < 3){
if (i > 0){
line(4*i + -3,-5*i + 12,2*i + 1,
arrow = False,solid = True);
rectangle(4*i + -4,-5*i + 10,6,-
}
circle(1,8)
}
```

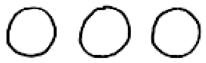



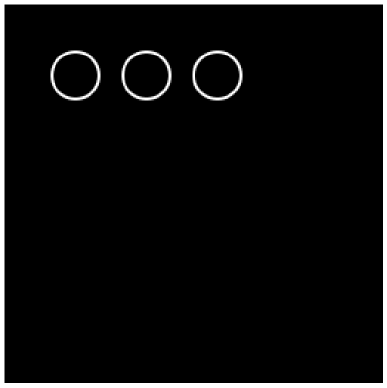
```
for (i < 3){
for (j < 3){
if (j > 0){
line(-3*j + 8,-3*i + 7,-3*j + 9,
arrow = False,solid = True);
line(-3*i + 7,-3*j + 8,-3*i + 7,
arrow = False,solid = True)
}
circle(-3*j + 7,-3*i + 7)
}
}
```



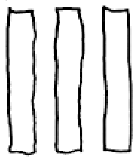


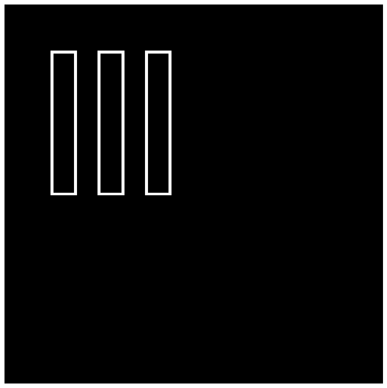
```
Solver timeout
```



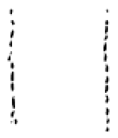
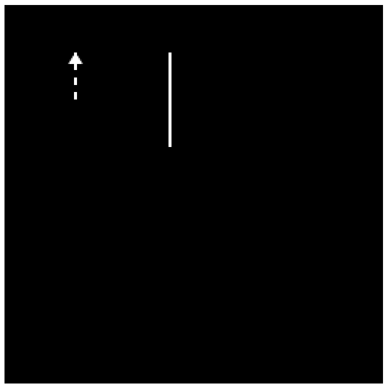


```
for (i < 3){
circle(-3*i + 7,1)
}
```

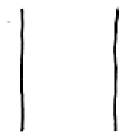
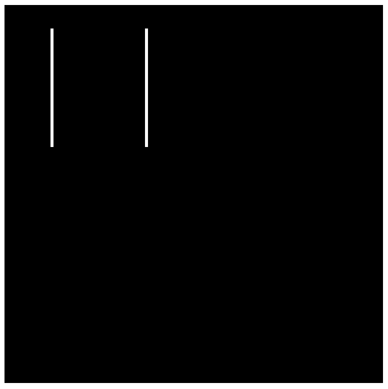




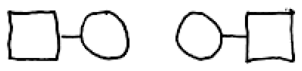
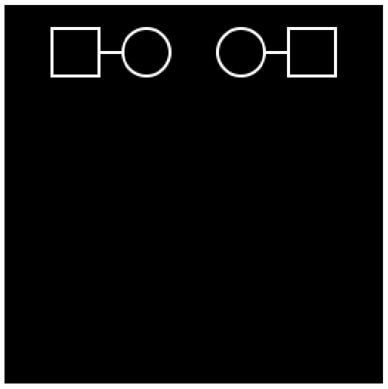
```
for (i < 3){
rectangle(-2*i + 4,0,-2*i + 5,6)
}
```

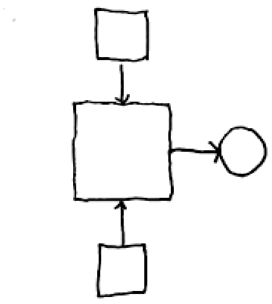
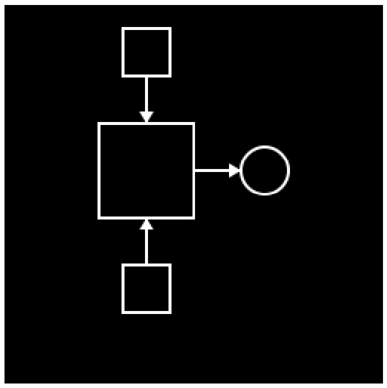
```
line(4,0,4,1,
arrow = False,solid = False);
line(0,0,0,5,
arrow = False,solid = False);
line(4,1,4,5,
arrow = False,solid = False)
```

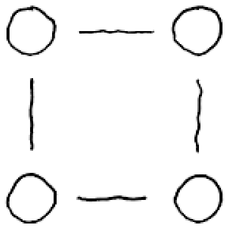
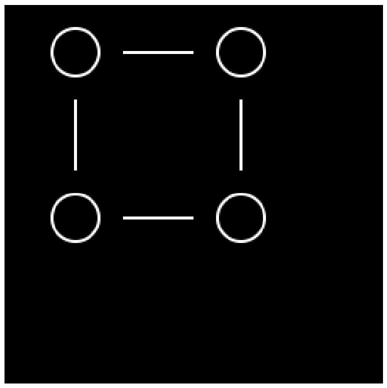
```
line(4,0,4,5,
arrow = False,solid = True);
line(0,0,0,5,
arrow = False,solid = True)
```

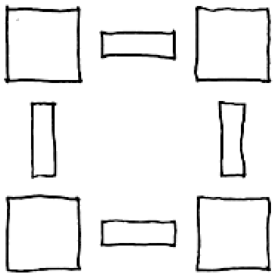
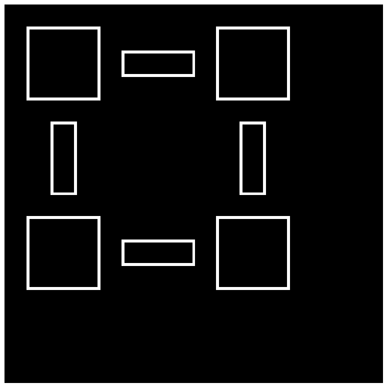
```
reflect(reflect(x = 12)){
circle(4,1);
line(9,1,10,1,
arrow = False,solid = True);
rectangle(0,0,2,2)
}
```

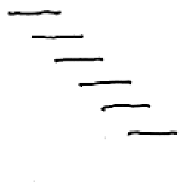
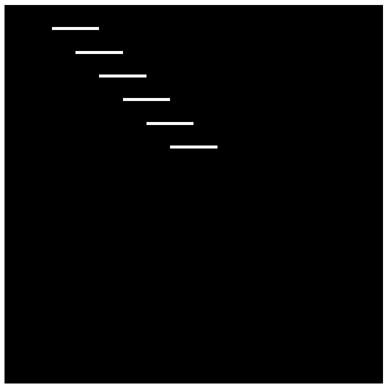
```
rectangle(0,4,4,8);
reflect(reflect(y = 12)){
circle(7,6);
line(2,2,2,4,
arrow = True,solid = True);
line(4,6,6,6,
arrow = True,solid = True);
rectangle(1,10,3,12)
}
```

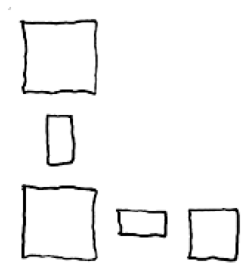
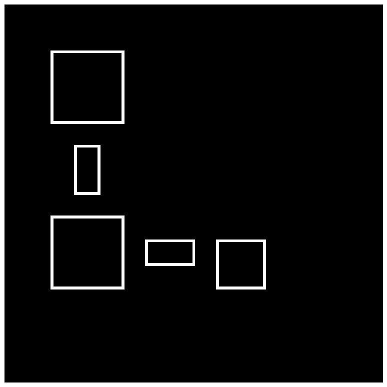
```
reflect(reflect(y = 9)){
line(3,8,6,8,
arrow = False,solid = True);
reflect(reflect(x = 9)){
circle(1,8);
line(1,3,1,6,
arrow = False,solid = True)
}
}
```

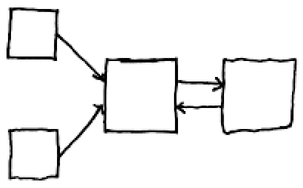
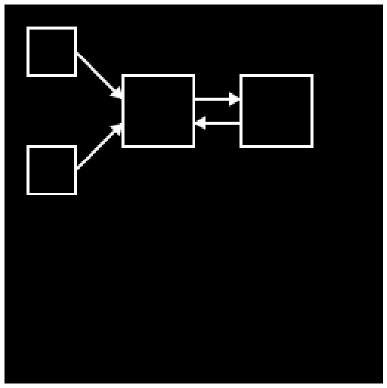
```
reflect(reflect(y = 11)){
rectangle(4,9,7,10);
reflect(reflect(x = 11)){
rectangle(1,4,2,7);
rectangle(8,8,11,11)
}
}
```

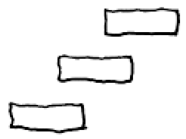
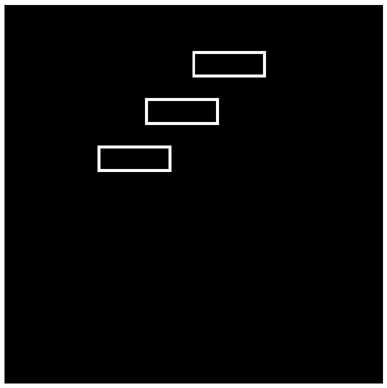
```
for (i < 4){
line(i,-1*i + 5,i + 2,-1*i + 5,
arrow = False,solid = True);
line(i + 2,-1*i + 3,i + 4,-1*i +
arrow = False,solid = True)
}
```

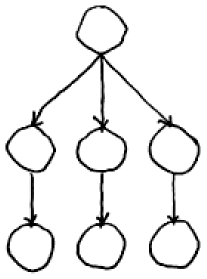
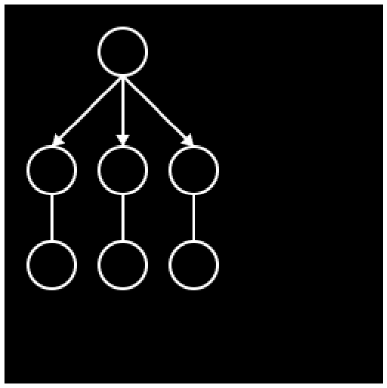
```
for (i < 3){
if (i > 0){
rectangle(3*i + 1,-1*i + 2,3*i +
rectangle(0,7*i + -7,3,7*i + -4)
}
rectangle(1,4,2,6)
}
```

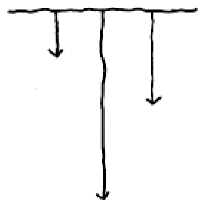
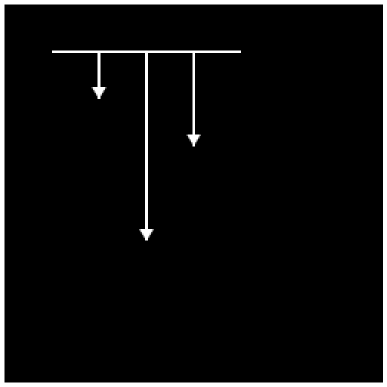
```
Solver timeout
```

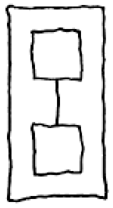
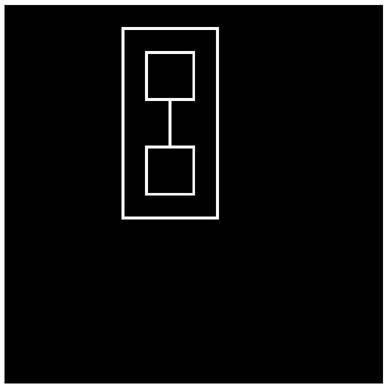
```
for (i < 3){
rectangle(-2*i + 4,-2*i + 4,-2*i
}
```

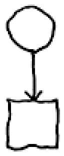
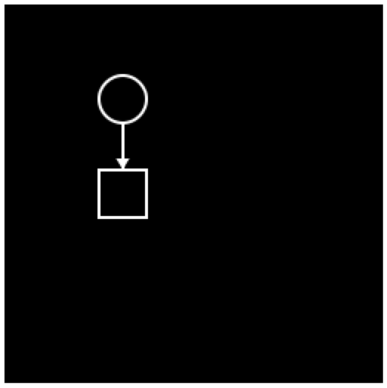
```
circle(4,10);
for (i < 3){
circle(-3*i + 7,5);
circle(-3*i + 7,1);
line(-3*i + 7,4,-3*i + 7,2,
arrow = True,solid = True);
line(4,9,-3*i + 7,6,
arrow = True,solid = True)
}
```

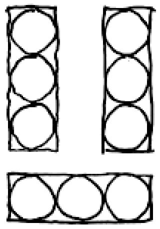
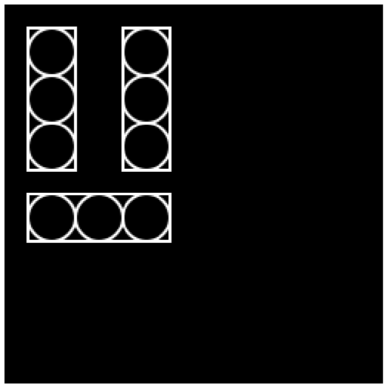
```
line(2,8,2,6,
arrow = True,solid = True);
line(6,8,6,4,
arrow = True,solid = True);
line(4,8,4,0,
arrow = True,solid = True);
line(0,8,8,8,
arrow = False,solid = True)
```

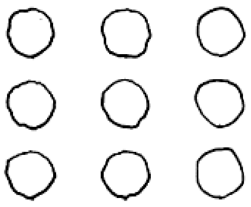
```
line(2,3,2,5,
arrow = False,solid = True);
rectangle(1,1,3,3);
rectangle(1,5,3,7);
rectangle(0,0,4,8)
```

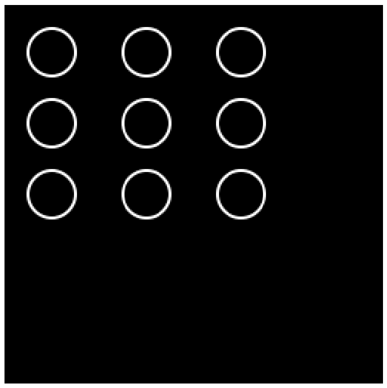



```
circle(1,5);
line(1,4,1,2,
arrow = True,solid = True);
rectangle(0,0,2,2)
```

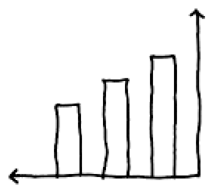



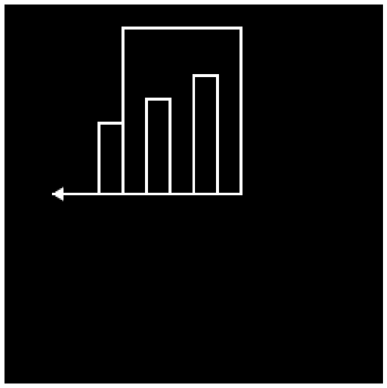
```
rectangle(0,0,6,2);
reflect(reflect(x = 6)){
rectangle(0,3,2,9);
for (i < 3){
circle(5,2*i + 4);
circle(2*i + 1,1)
}
}
```



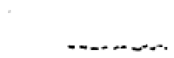


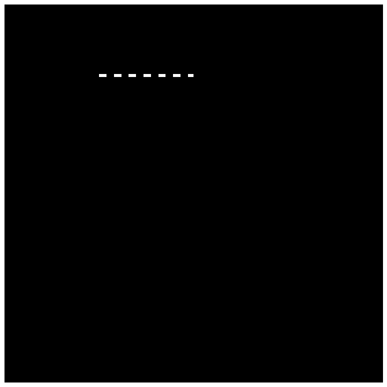
```
for (i < 3){
for (j < 3){
circle(-4*j + 9,-3*i + 7)
}
}
```



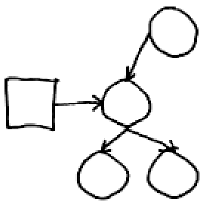
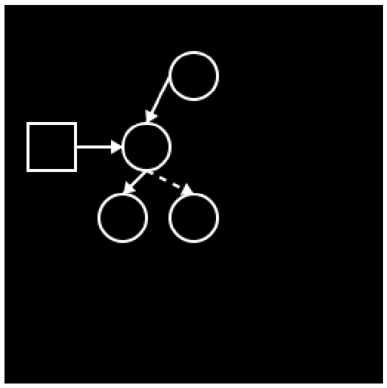


```
for (i < 3){
if (i > 0){
line(8,0,8*i + -8,7*i + -7,
arrow = True,solid = True)
}
rectangle(2*i + 2,0,2*i + 3,i +
}
```

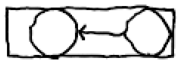
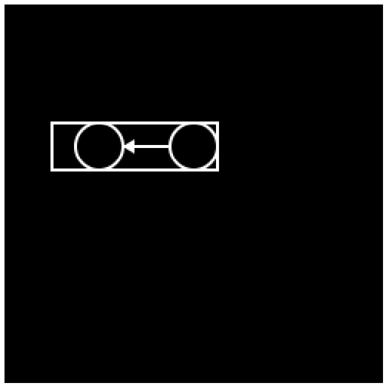




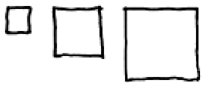
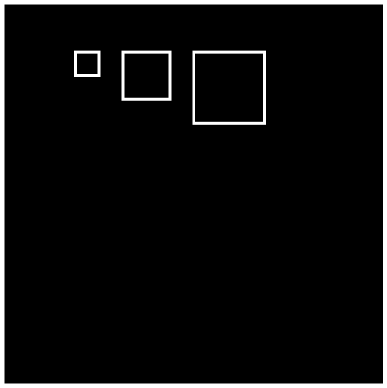
```
line(4,0,0,0,
arrow = False,solid = False)
```

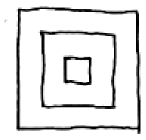
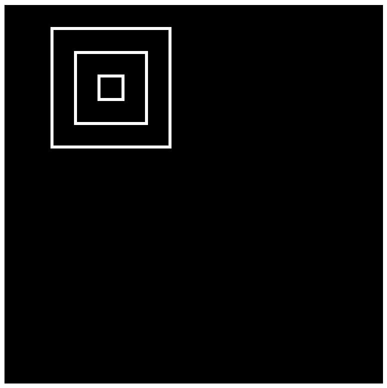
```
Solver timeout
```

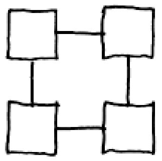
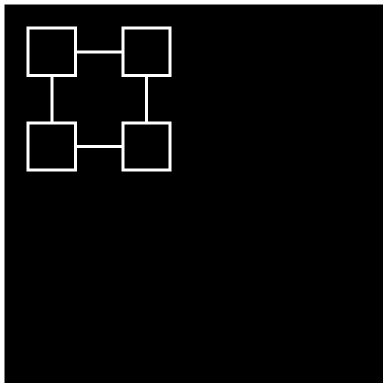
```
circle(2,1);
circle(6,1);
line(5,1,3,1,
arrow = True,solid = True);
rectangle(0,0,7,2)
```

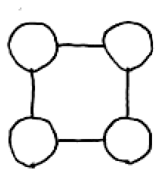
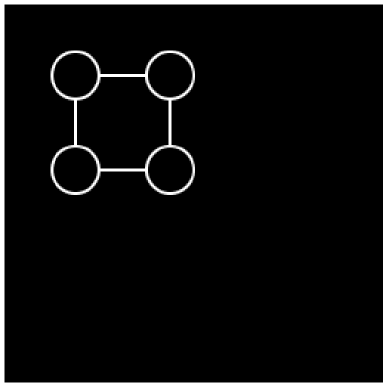
```
rectangle(5,0,8,3);
rectangle(2,1,4,3);
rectangle(0,2,1,3)
```

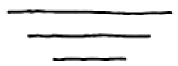
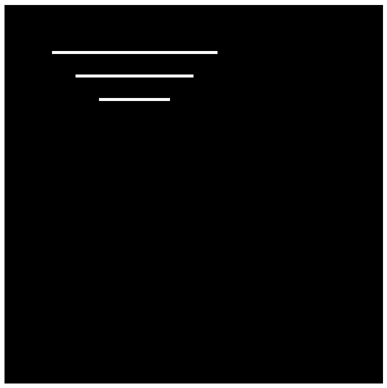
```
for (i < 3){
rectangle(-1*i + 2,-1*i + 2,i +
}
```

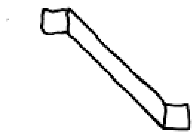
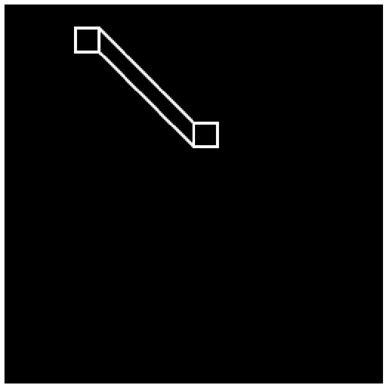
```
reflect(reflect(x = 6)){
line(5,2,5,4,
arrow = False,solid = True);
reflect(reflect(y = 6)){
line(2,1,4,1,
arrow = False,solid = True);
rectangle(4,4,6,6)
}
}
```

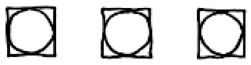
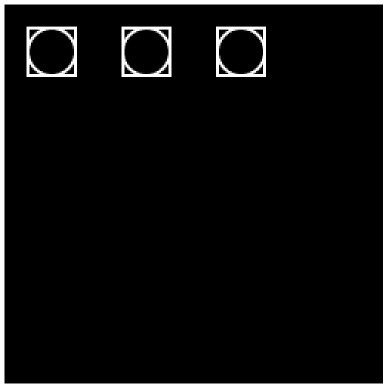
```
reflect(reflect(y = 6)){
line(2,5,4,5,
arrow = False,solid = True);
reflect(reflect(x = 6)){
circle(5,5);
line(1,2,1,4,
arrow = False,solid = True)
}
}
```

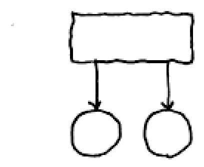
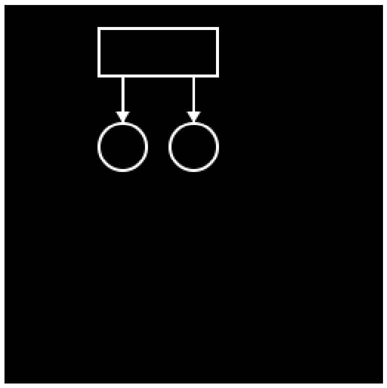
```
for (i < 3){
line(i,-1*i + 2,-1*i + 7,-1*i +
arrow = False,solid = True)
}
```

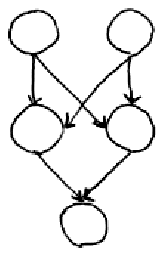
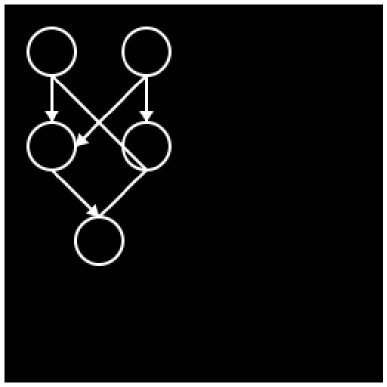
```
line(1,4,5,0,
arrow = False,solid = True);
line(1,5,5,1,
arrow = False,solid = True);
rectangle(5,0,6,1);
rectangle(0,4,1,5)
```

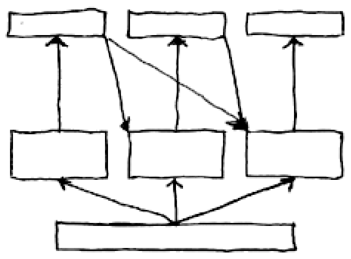
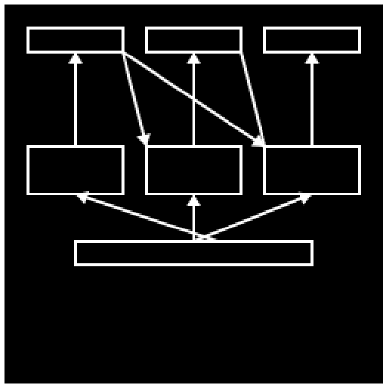
```
for (i < 3){
circle(4*i + 1,1);
rectangle(4*i,0,4*i + 2,2)
}
```

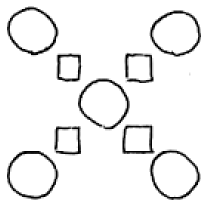
```
reflect(reflect(x = 5)){
circle(1,1);
line(4,4,4,2,
arrow = True,solid = True);
rectangle(0,4,5,6)
}
```

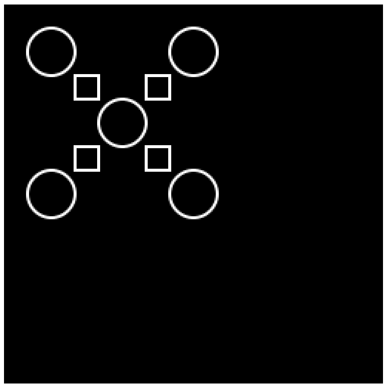



```
circle(3,1);
reflect(reflect(x = 6)){
for (i < 3){
if (i > 0){
circle(1,-4*i + 13);
line(5,-4*i + 12,-2*i + 7,-4*i +
arrow = True,solid = True)
}
line(1,8,4,5,
arrow = True,solid = True)
}
}
```

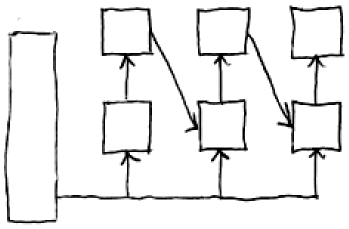



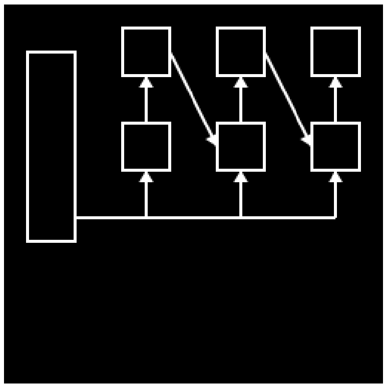
```
Solver timeout
```



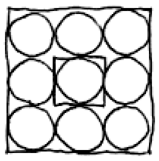


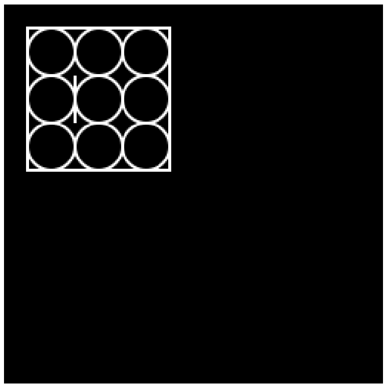
```
reflect(reflect(y = 8)){
for (i < 3){
if (i > 0){
rectangle(3*i + -1,2,3*i,3)
}
circle(3*i + 1,3*i + 1)
}
}
```



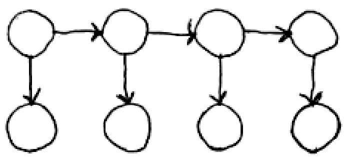
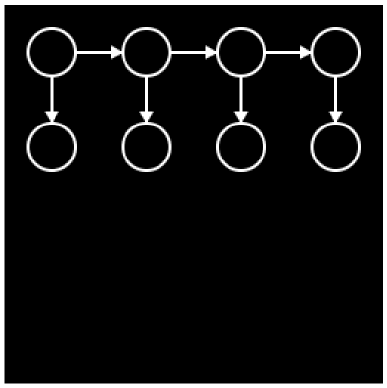


```
Solver timeout
```

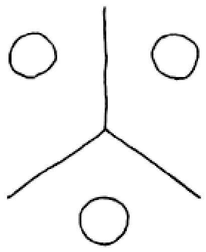
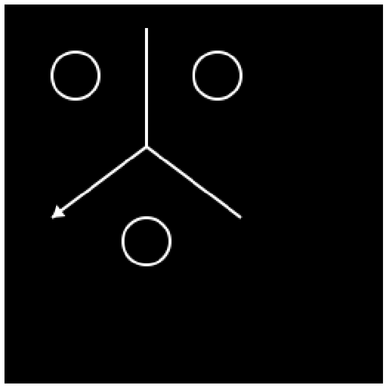




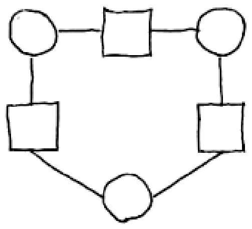
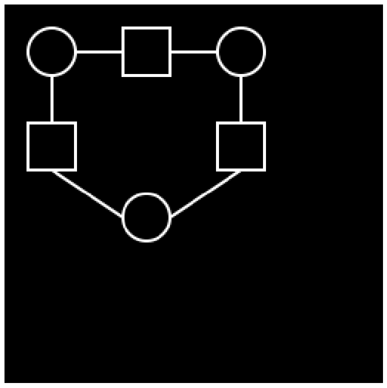
```
for (i < 3){
if (i > 0){
rectangle(-2*i + 4,-2*i + 4,2*i
}
for (j < 3){
circle(-2*i + 5,2*j + 1)
}
}
```

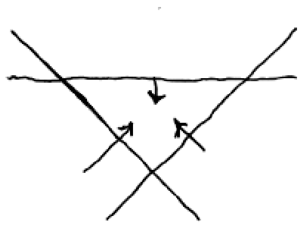
```
for (i < 4){
line(-4*i + 13,4,-4*i + 13,2,
arrow = True,solid = True);
for (j < 3){
if (j > 0){
circle(-4*i + 13,4*j + -3)
}
line(-4*j + 10,5,-4*j + 12,5,
arrow = True,solid = True)
}
}
```

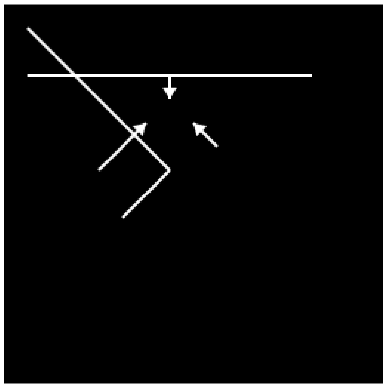



```
circle(4,1);
reflect(reflect(x = 8)){
circle(1,8);
line(4,5,8,2,
arrow = False,solid = False);
line(4,5,4,10,
arrow = False,solid = True)
}
```

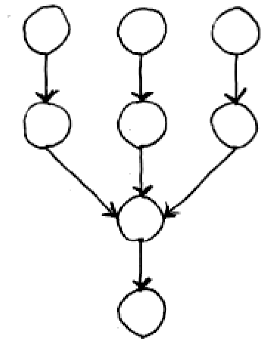



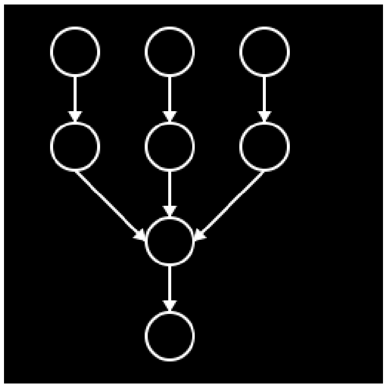
```
Solver timeout
```



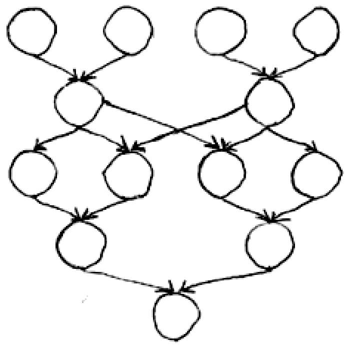


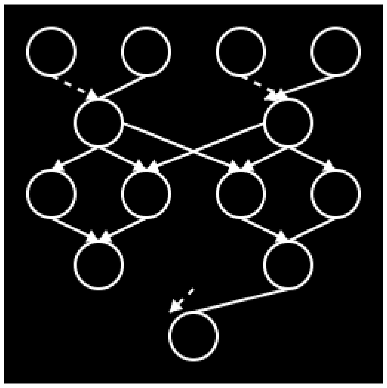
```
line(0,6,12,6,
arrow = False,solid = True);
line(6,6,6,5,
arrow = True,solid = True);
line(8,3,7,4,
arrow = True,solid = True);
line(3,2,5,4,
arrow = True,solid = True);
reflect(reflect(x = 12)){
line(4,0,12,8,
arrow = False,solid = True)
}
```



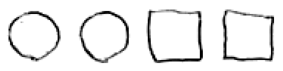


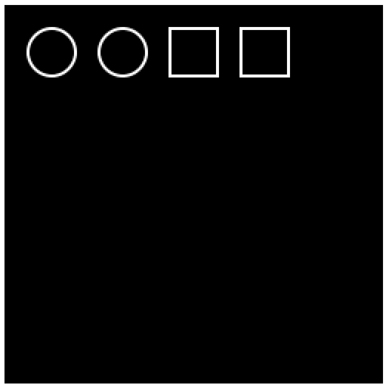
```
Solver timeout
```



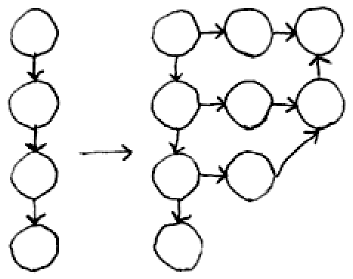


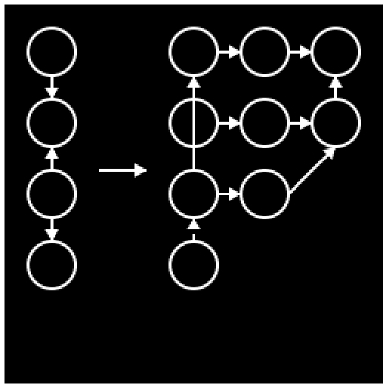
```
Solver timeout
```



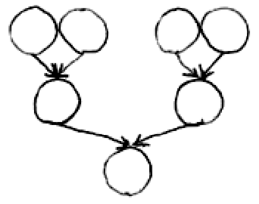


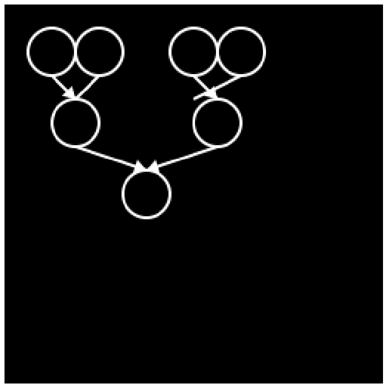
```
circle(1,1);
circle(4,1);
rectangle(9,0,11,2);
rectangle(6,0,8,2)
```



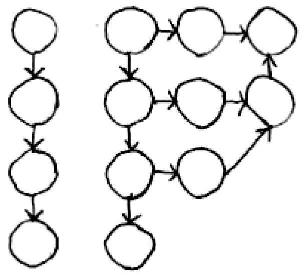


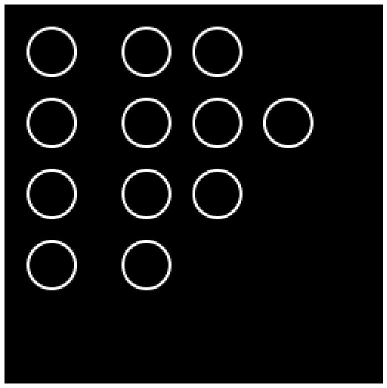
```
Solver timeout
```



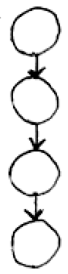
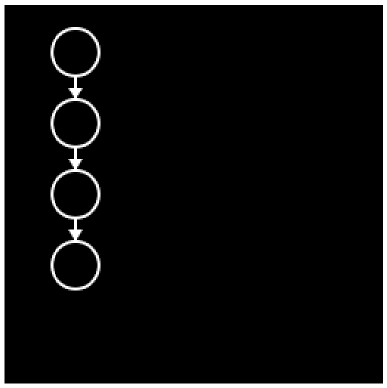


```
Solver timeout
```

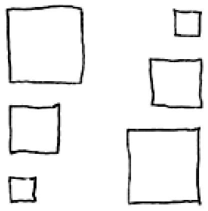
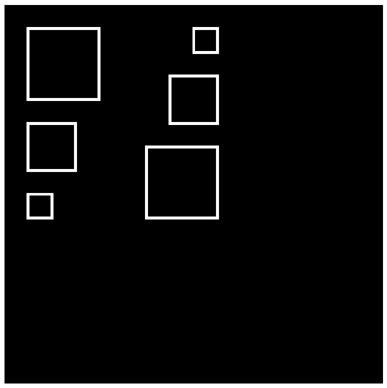




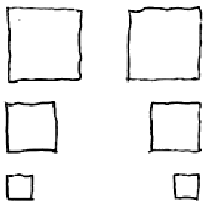
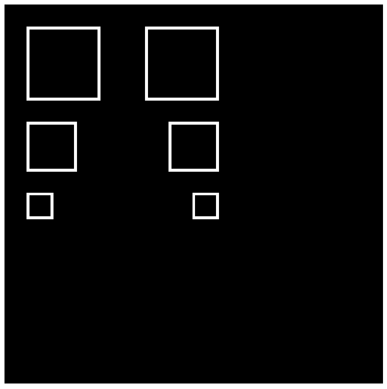
```
Solver timeout
```

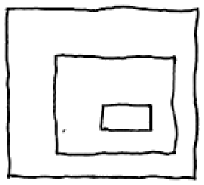
```
for (i < 4){
if (i > 0){
line(1,-3*i + 12,1,-3*i + 11,
arrow = True,solid = True)
}
circle(1,-3*i + 10)
}
```

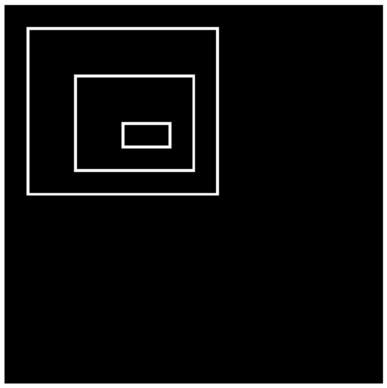



```
Solver timeout
```

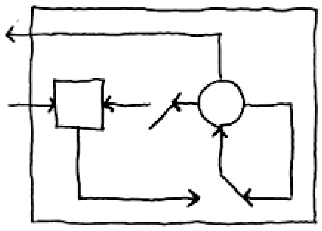



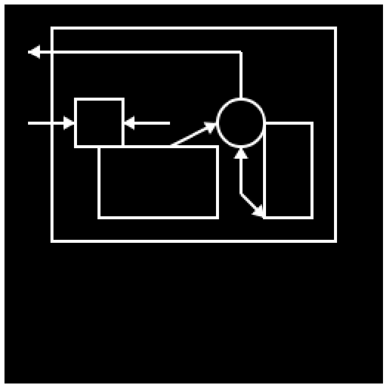
```
reflect(reflect(x = 8)){
rectangle(0,5,3,8);
rectangle(0,2,2,4);
rectangle(0,0,1,1)
}
```



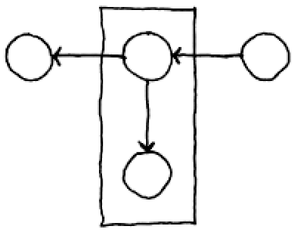


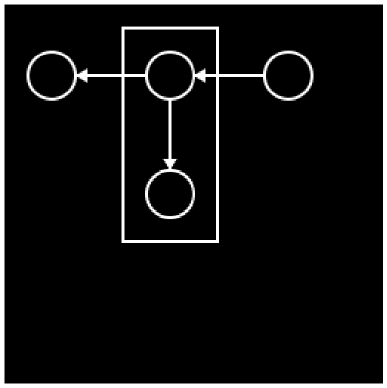
```
for (i < 3){
rectangle(-2*i + 4,-1*i + 2,i +
}
```



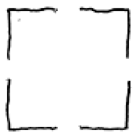


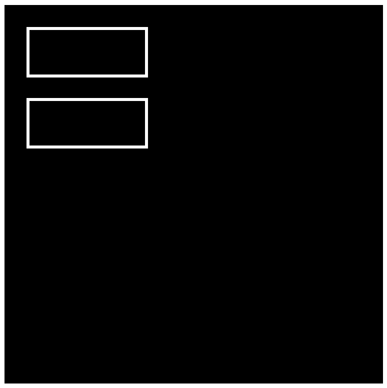
```
Solver timeout
```



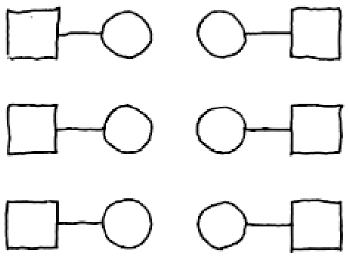


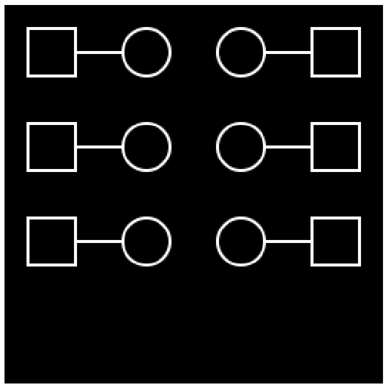
```
line(6,6,6,3,
arrow = True,solid = True);
for (i < 3){
if (i > 0){
circle(-5*i + 16,7);
circle(-5*i + 11,5*i + -3);
line(-5*i + 15,7,-5*i + 12,7,
arrow = True,solid = True)
}
rectangle(4,0,8,9)
}
```



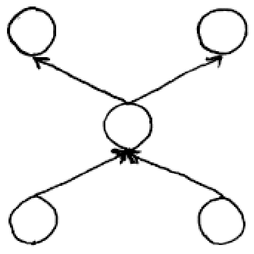


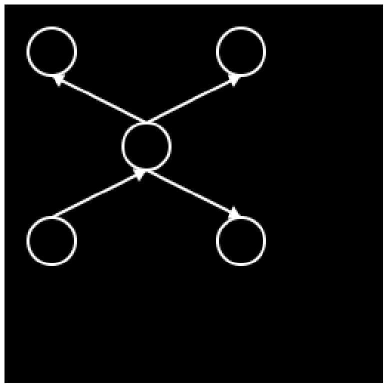
```
reflect(reflect(x = 5)){
reflect(reflect(y = 5)){
line(5,3,5,5,
arrow = False,solid = True);
line(3,5,5,5,
arrow = False,solid = True)
}
}
```



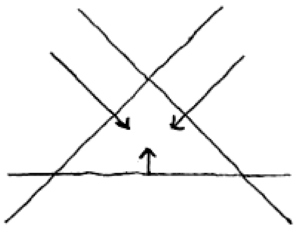
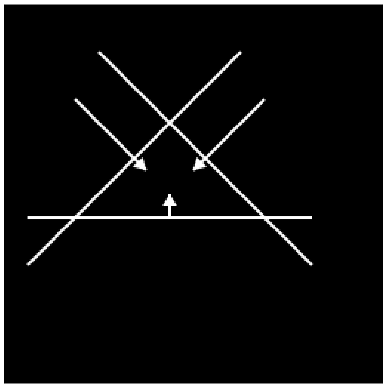


```
reflect(reflect(x = 14)){
for (i < 3){
circle(9,-4*i + 9);
line(10,-4*i + 9,12,-4*i + 9,
arrow = False,solid = True);
rectangle(0,-4*i + 8,2,-4*i + 10
}
}
```

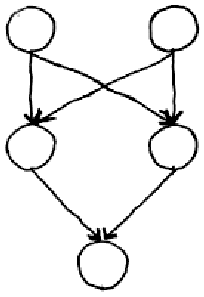
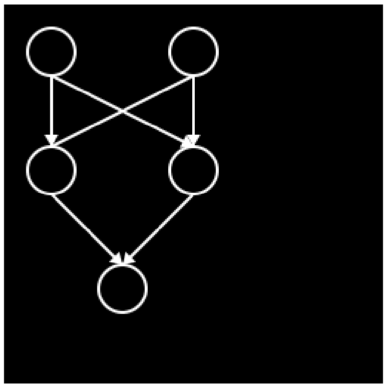




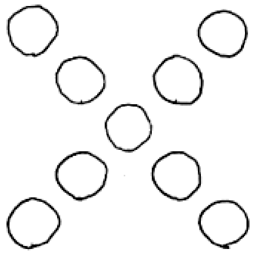
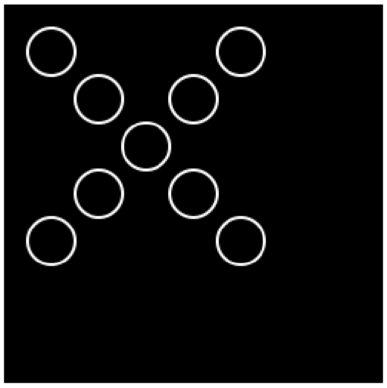
```
reflect(reflect(x = 10)){
for (i < 3){
if (i > 0){
line(4*i + -3,4*i + -2,4*i + 1,4
arrow = True,solid = True)
}
circle(4*i + 1,4*i + 1)
}
}
```

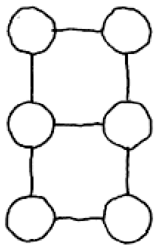
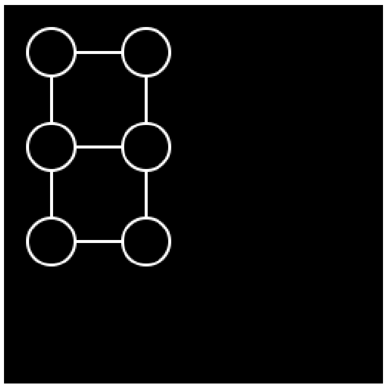
```
line(0,2,12,2,
arrow = False,solid = True);
line(6,2,6,3,
arrow = True,solid = True);
reflect(reflect(x = 12)){
line(0,0,9,9,
arrow = False,solid = True);
line(10,7,7,4,
arrow = True,solid = True)
}
```

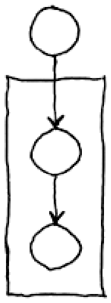
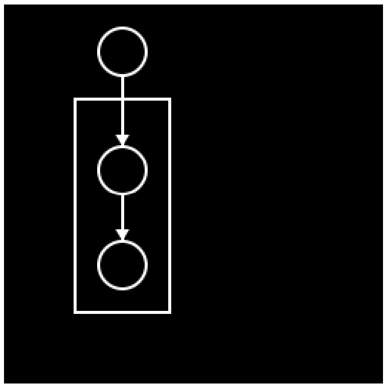
```
reflect(reflect(x = 8)){
circle(4,1);
for (i < 3){
if (i > 0){
circle(7,-5*i + 16);
line(-6*i + 13,10,7,7,
arrow = True,solid = True)
}
line(1,5,4,2,
arrow = True,solid = True)
}
}
```

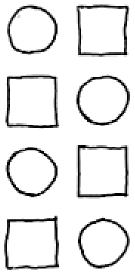
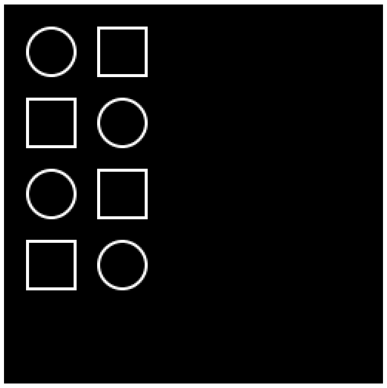
```
reflect(reflect(x = 10)){
circle(1,1);
for (i < 4){
circle(-2*i + 7,2*i + 3)
}
}
```

```
reflect(reflect(x = 6)){
for (i < 3){
if (i > 0){
line(1,-4*i + 10,1,-4*i + 12,
arrow = False,solid = True)
}
circle(5,-4*i + 9);
line(2,-4*i + 9,4,-4*i + 9,
arrow = False,solid = True)
}
}
```

```
rectangle(0,0,4,9);
for (i < 3){
if (i > 0){
circle(2,-4*i + 10);
line(2,-5*i + 15,2,-4*i + 11,
arrow = True,solid = True)
}
circle(2,11)
}
```

```
for (i < 2){
circle(4,-6*i + 7);
circle(1,-6*i + 10);
rectangle(0,-6*i + 6,2,-6*i + 8)
rectangle(3,-6*i + 9,5,-6*i + 11
}
```

References

[1] Christopher M. Bishop. Mixture Density Networks. Technical report, 1994.

[2] Max Jaderberg, Karen Simonyan, Andrew Zisserman, et al. Spatial transformer networks. In *Advances in Neural Information Processing Systems*, pages 2017–2025, 2015.

[3] Oriol Vinyals, Alexander Toshev, Samy Bengio, and Dumitru Erhan. Show and tell: A neural image caption generator. In *Proceedings of the IEEE conference on computer vision and pattern recognition*, pages 3156–3164, 2015.

[4] Samuel J Gershman and David M Blei. A tutorial on bayesian nonparametric models. *Journal of Mathematical Psychology*, 56(1):1–12, 2012.