DreamCoder: Growing libraries of concepts with wake-sleep program induction

Kevin Ellis
Joint with: Lucas Morales, Mathias Sablé Meyer, Armando Solar-Lezama, Joshua B. Tenenbaum
Heavy inspiration from: Eyal Dechter

October 2018

MIT

Human program induction everywhere

```
(MEMBER
 (LAMBDA (X L)
  (COND  ((NULL L) NIL)
         ((EQ X (FIRST L)) T)
         (T (MEMBER X (REST L))))))
```

Allen, Anatomy of Lisp, 1975

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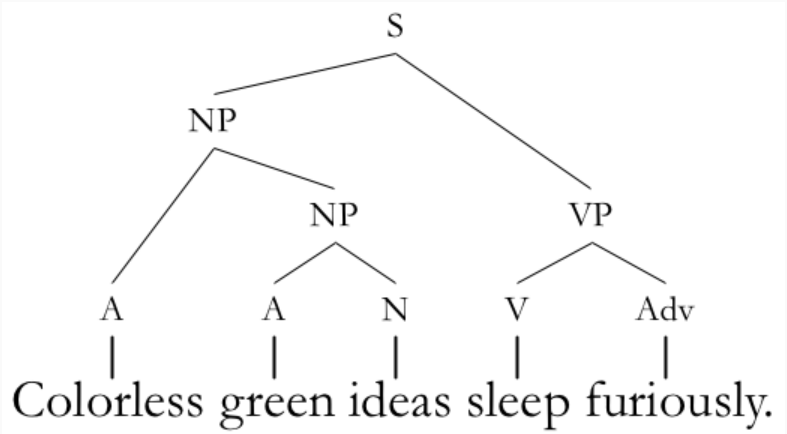

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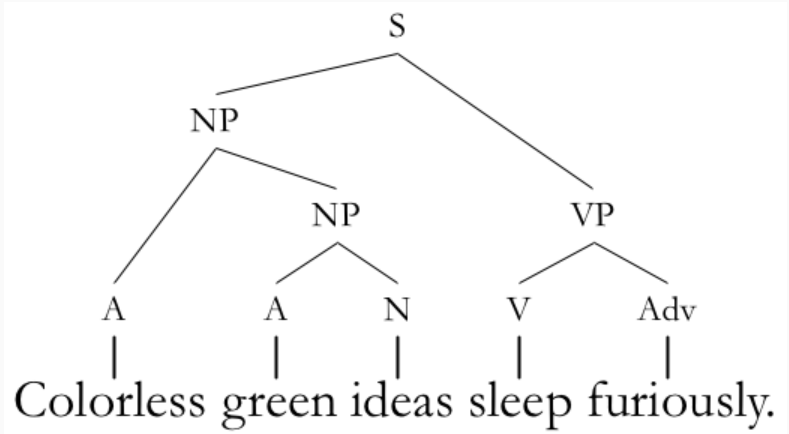

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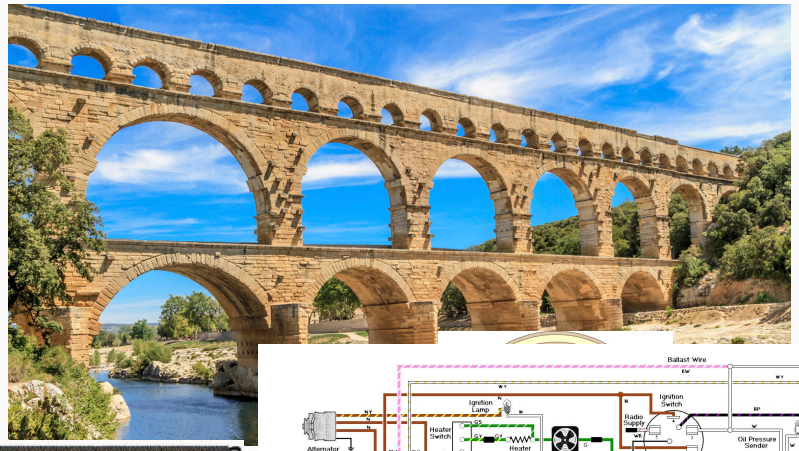

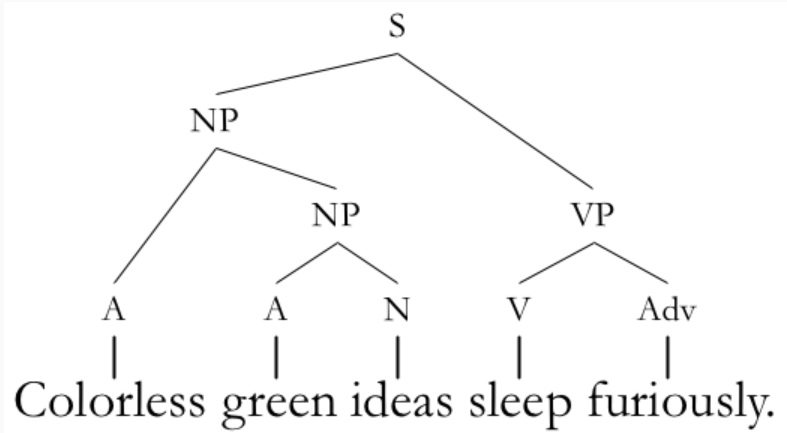

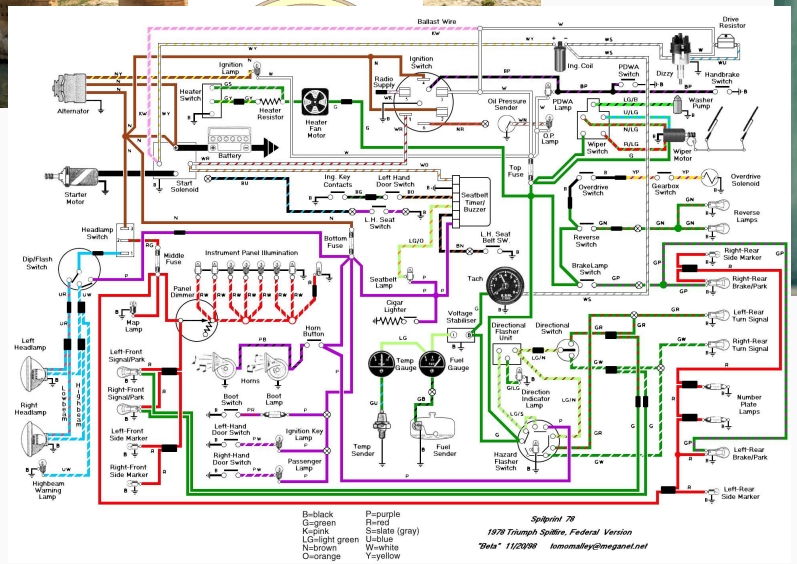

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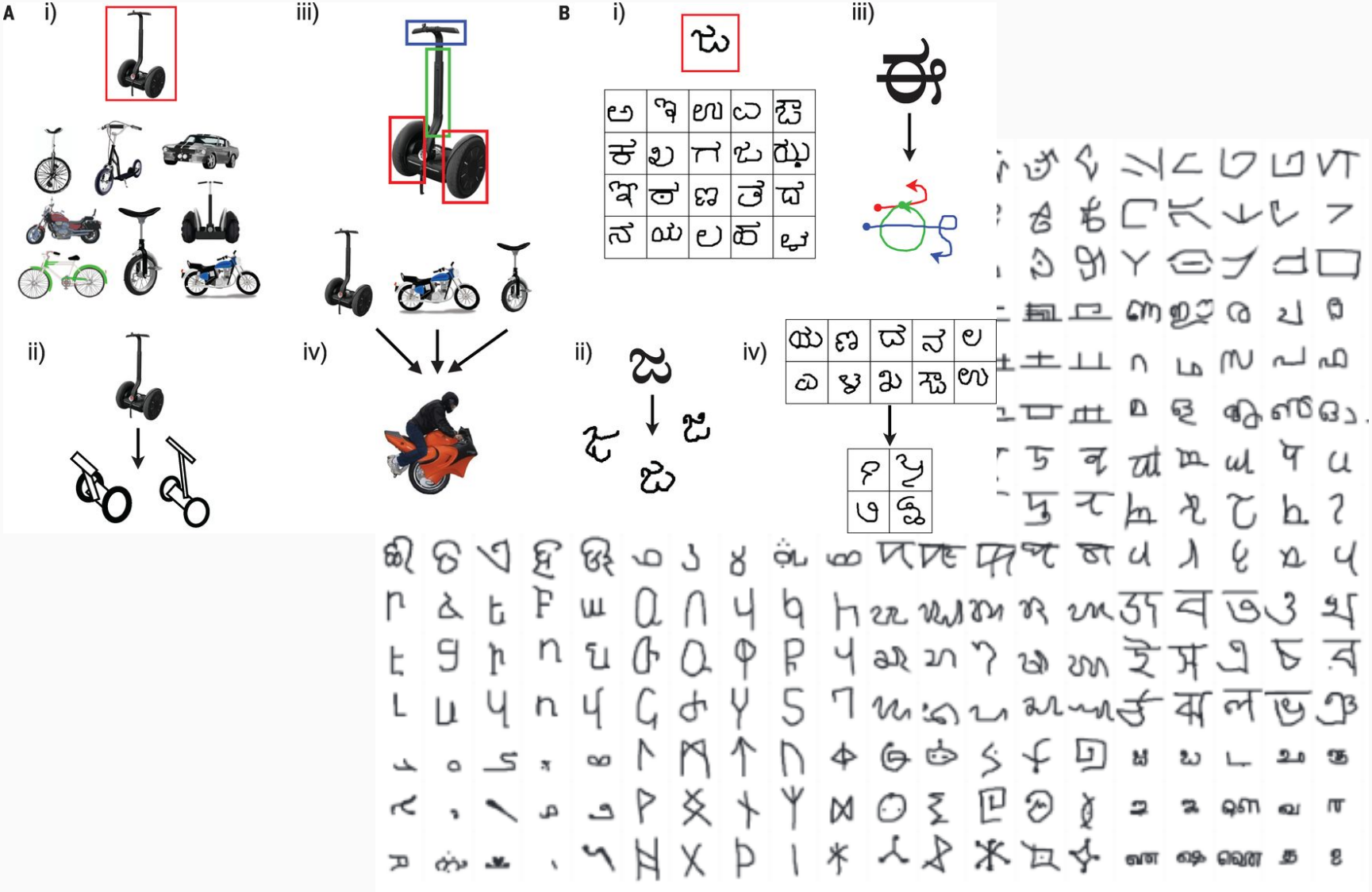

Lake et al 2015

Growing domain-specific knowledge

Goal: acquire domain-specific knowledge needed to induce a class of programs

- Library of concepts (declarative knowledge; generative model over programs)
- Inference strategy (procedural knowledge)

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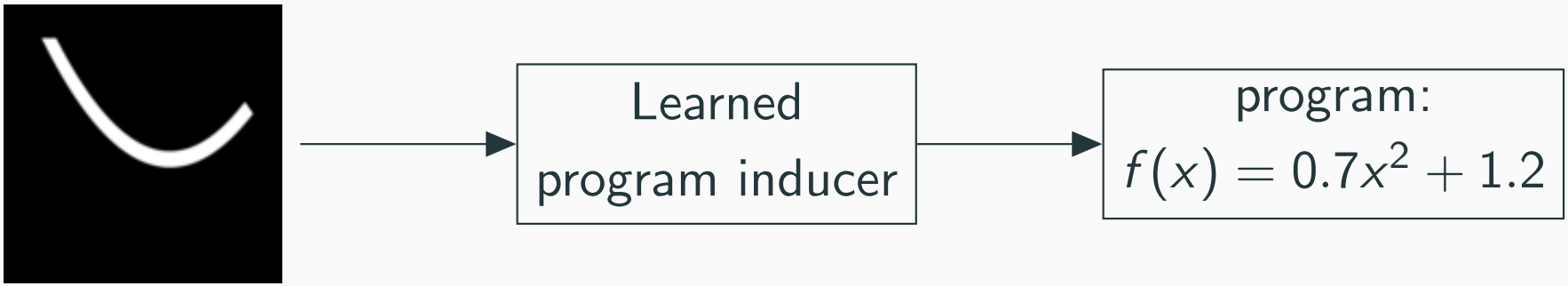

Concepts: x^2, etc

Inference strategy: neurosymbolic search for programs

Growing domain-specific knowledge

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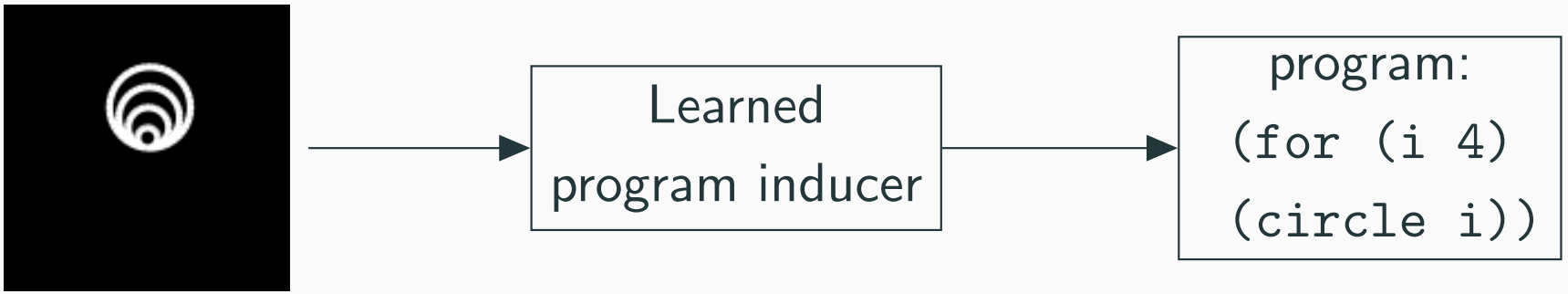

Concepts: `circle`, etc

Inference strategy: neurosymbolic search for programs

DSL: Library of concepts

Tasks and Programs

```
[7 2 3]→[7 3]
[1 2 3 4]→[3 4]
[4 3 2 1]→[4 3]
f(ℓ) = (f1 ℓ (λ (x) (> x 2)))
```

```
[7 3]→False
[3]→False
[9 0 0]→True
[0]→True
[0 7 3]→True
f(ℓ) = (f3 ℓ 0)
```

```
[2 7 8 1]→8
[3 19 14]→19
f(ℓ) = (f2 ℓ)
```

DSL

$f_0(\ell,\mathtt{r}) = (\mathtt{foldr\ r}\ \ell\ \mathtt{cons})$
(f_0: *Append lists* r *and* ℓ)

$f_1(\ell,\mathtt{p}) = (\mathtt{foldr}\ \ell\ \mathtt{nil}\ (\lambda\ (\mathtt{x\ a})\ (\mathtt{if\ (p\ x)\ (cons\ x\ a)\ a})))$
(f_1: *Higher-order filter function*)

$f_2(\ell) = (\mathtt{foldr}\ \ell\ 0\ (\lambda\ (\mathtt{x\ a})\ (\mathtt{if\ (>\ a\ x)\ a\ x})))$
(f_2: *Maximum element in list* ℓ)

$f_3(\ell,\mathtt{k}) = (\mathtt{foldr}\ \ell\ (\mathtt{is\text{-}nil}\ \ell)\ (\lambda\ (\mathtt{x\ a})\ (\mathtt{if\ a\ a\ (=\ k\ x)}))))$
(f_2: *Whether* ℓ *contains* k)

DreamCoder

- **Wake:** Solve problems by writing programs
- **Sleep:** Improve DSL and neural recognition model:
  - **Sleep-G:** Improve DSL (**G**enerative model)
  - **Sleep-R:** Improve **R**ecognition model

Combines ideas from Wake-Sleep & Exploration-Compression algorithm by Eyal Dechter

DreamCoder

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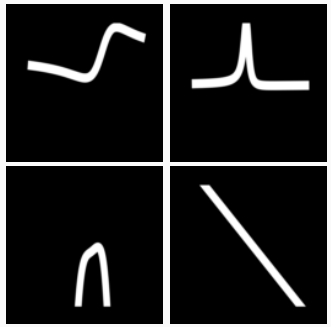
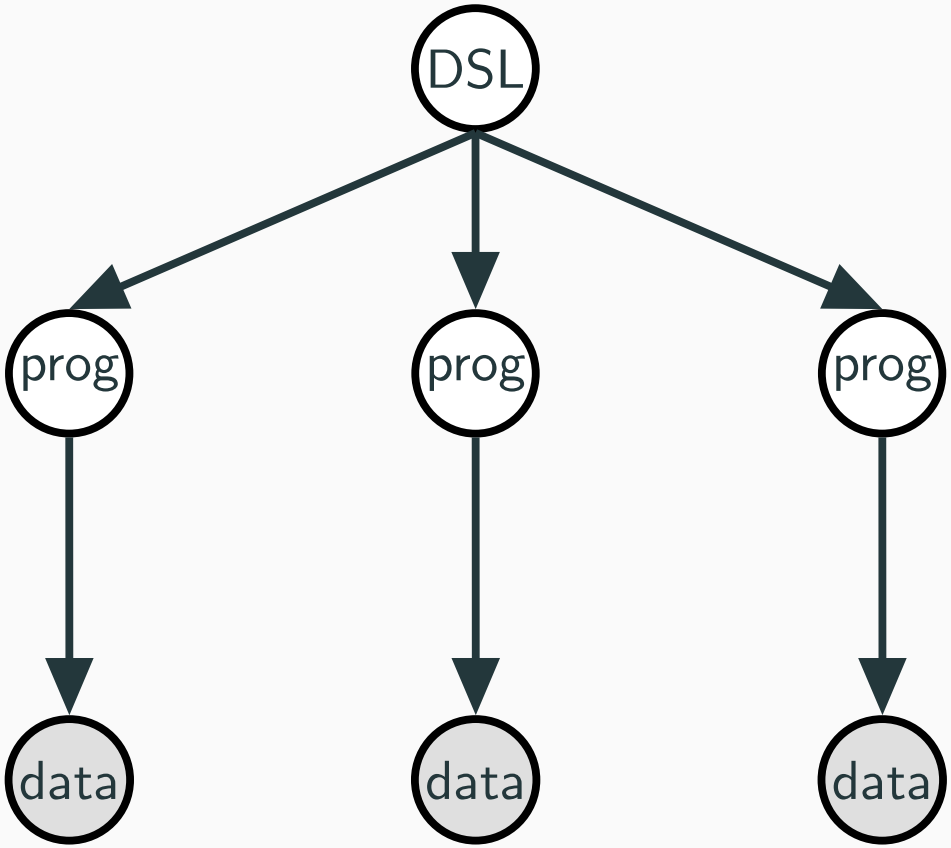
```
(let ((me 'whisper)
      (it 'into)
      (your 'ear))
  (let ((me (lisp it))
        (to (you)))
    (secretly)))
```



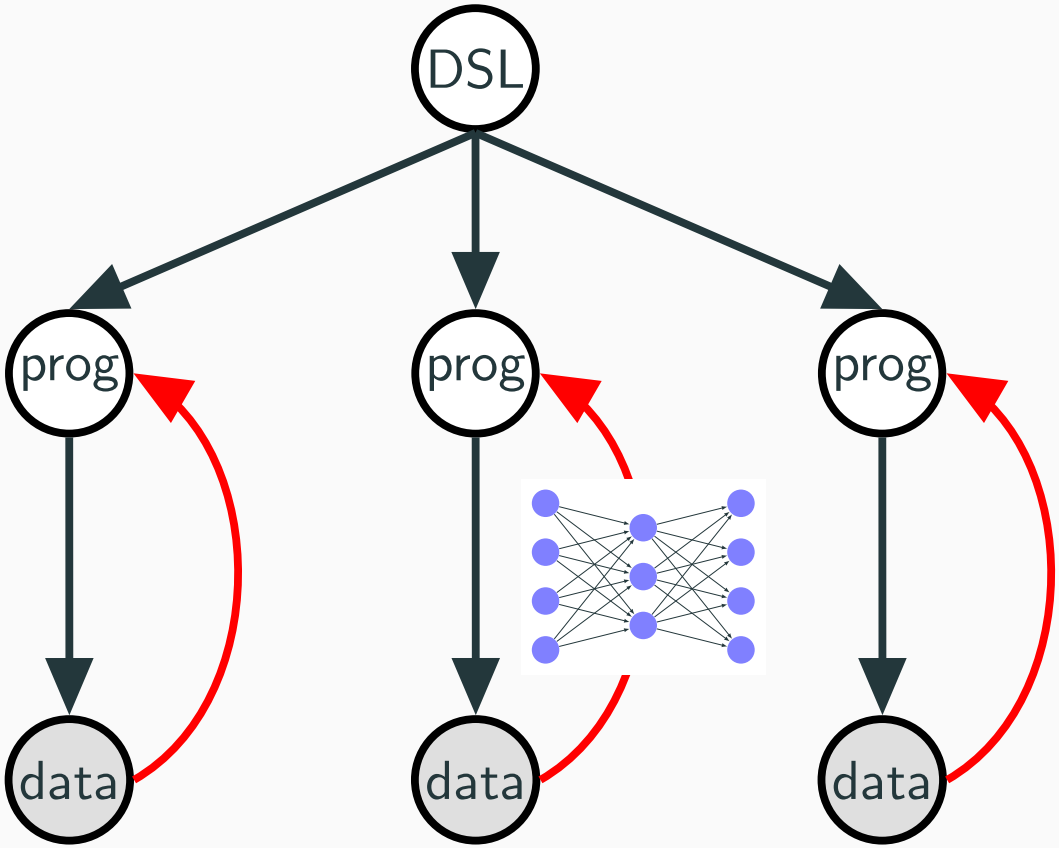
DSL learning as Bayesian inference



[Dechter et al., 2013] [Liang et al, 2010]; [Lake et al, 2015]

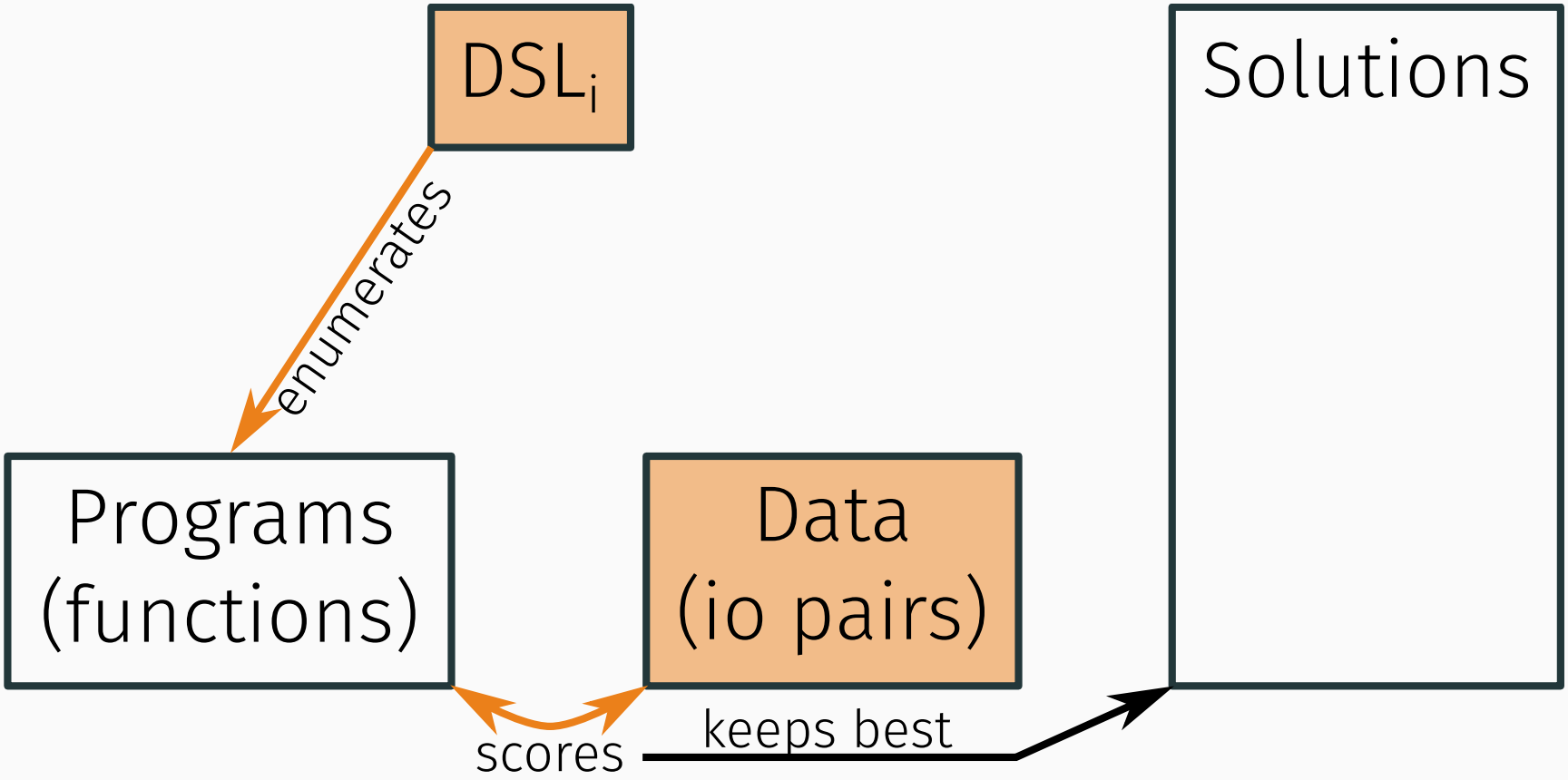
Dechter et al.: Exploration-Compression. Inspiration for DreamCoder.

DSL learning as amortized Bayesian inference



New: amortized inference +
better program representation (Lisp) +
better DSL inference

Wake — as in Exploration/Compression Algorithm



DSL

```
f0(l,r) = foldr r l cons
f1(l,p) = foldr r nil
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              (if (p x)
                (cons x a)
                a))
```
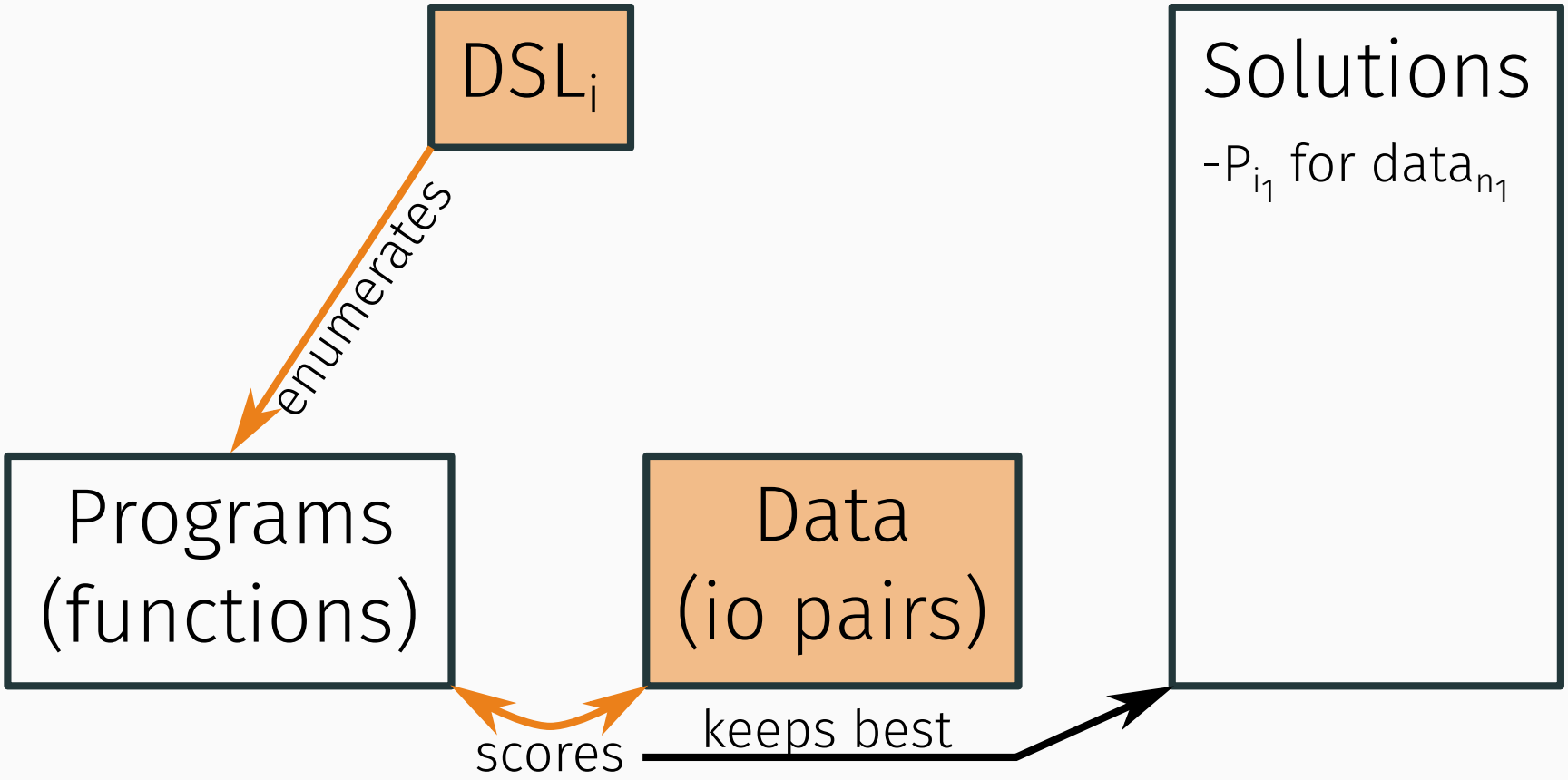

Programs

```
f(l) = (f0 l l)

f(l) = (f1 l (λx. > x 2))
```

Data

```
[7 2]→[7 2 7 2]
["a"]→["a" "a"]

[7 2 3]→[7 3]
[1 2 3 4]→[3 4]
[4 3 2 1]→[4 3]
```

Wake — as in Exploration/Compression Algorithm



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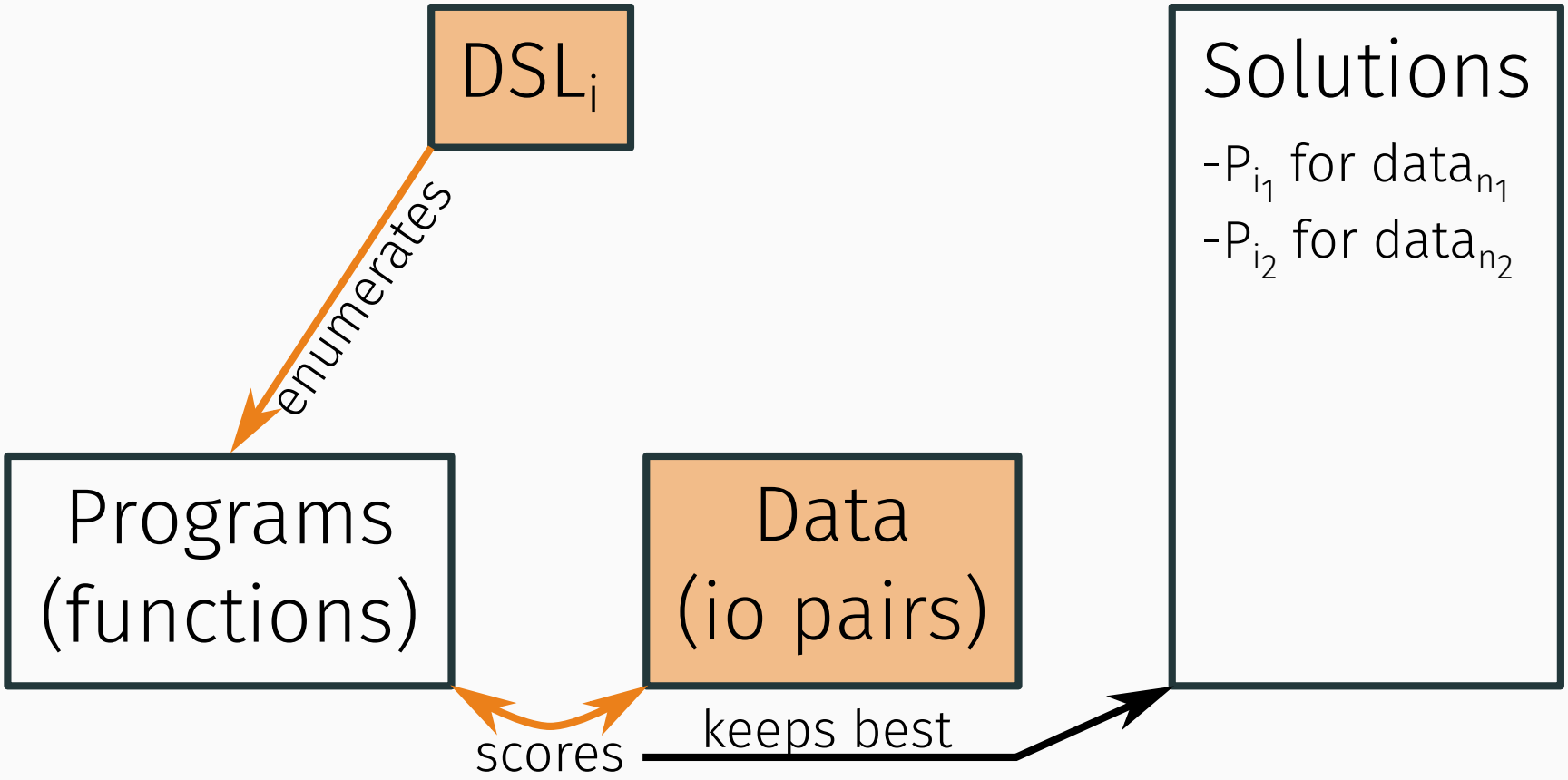

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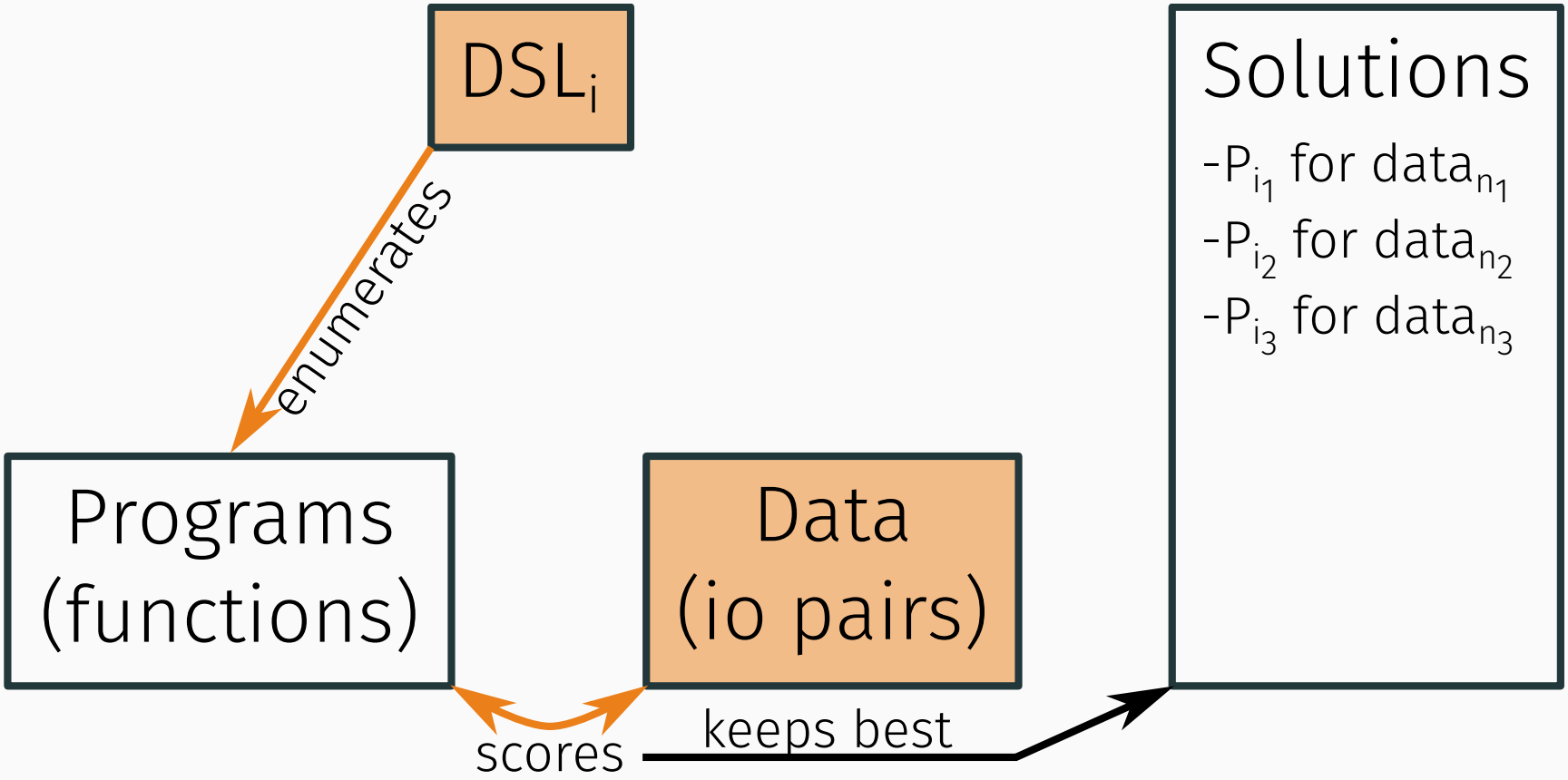

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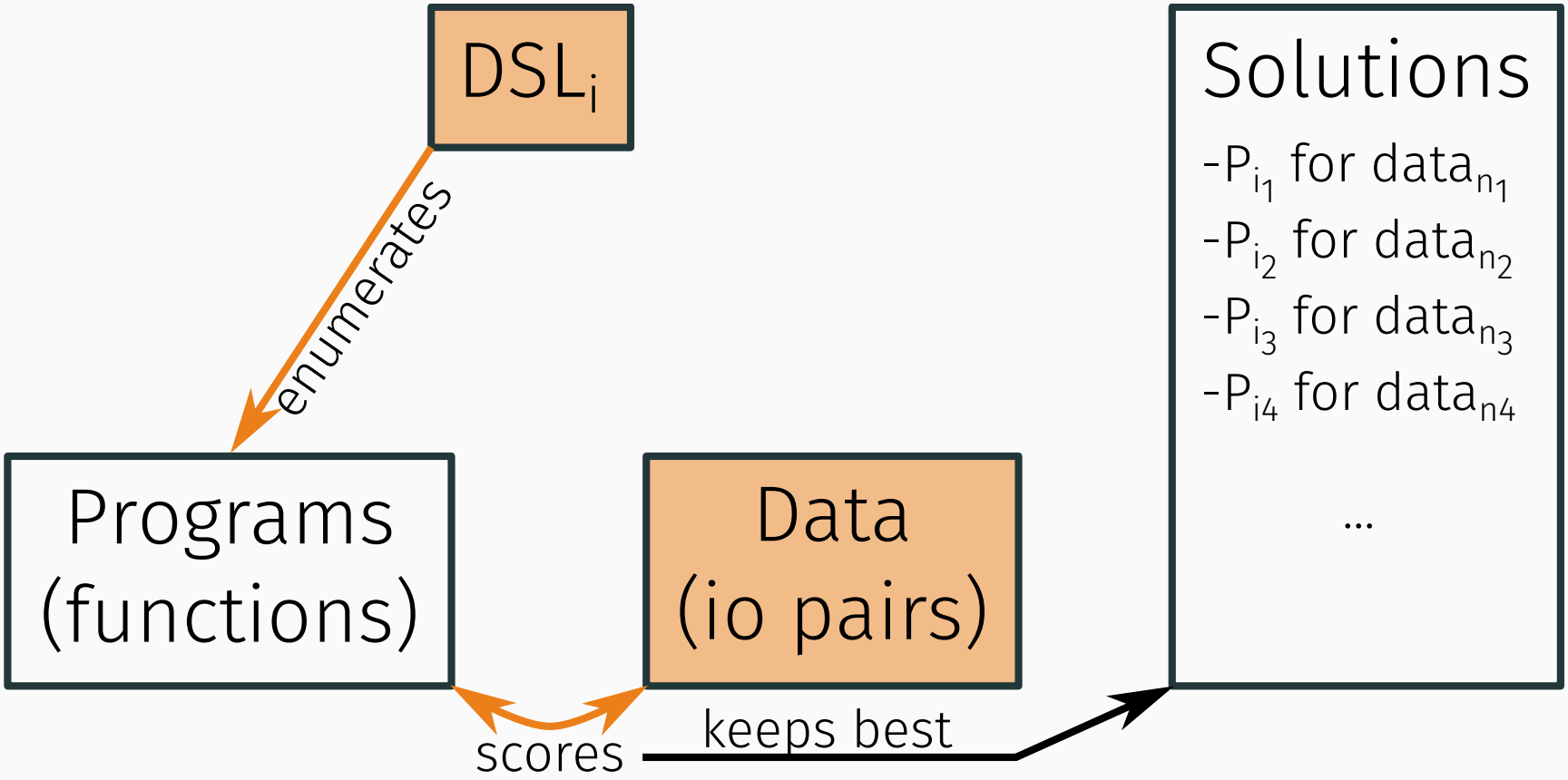
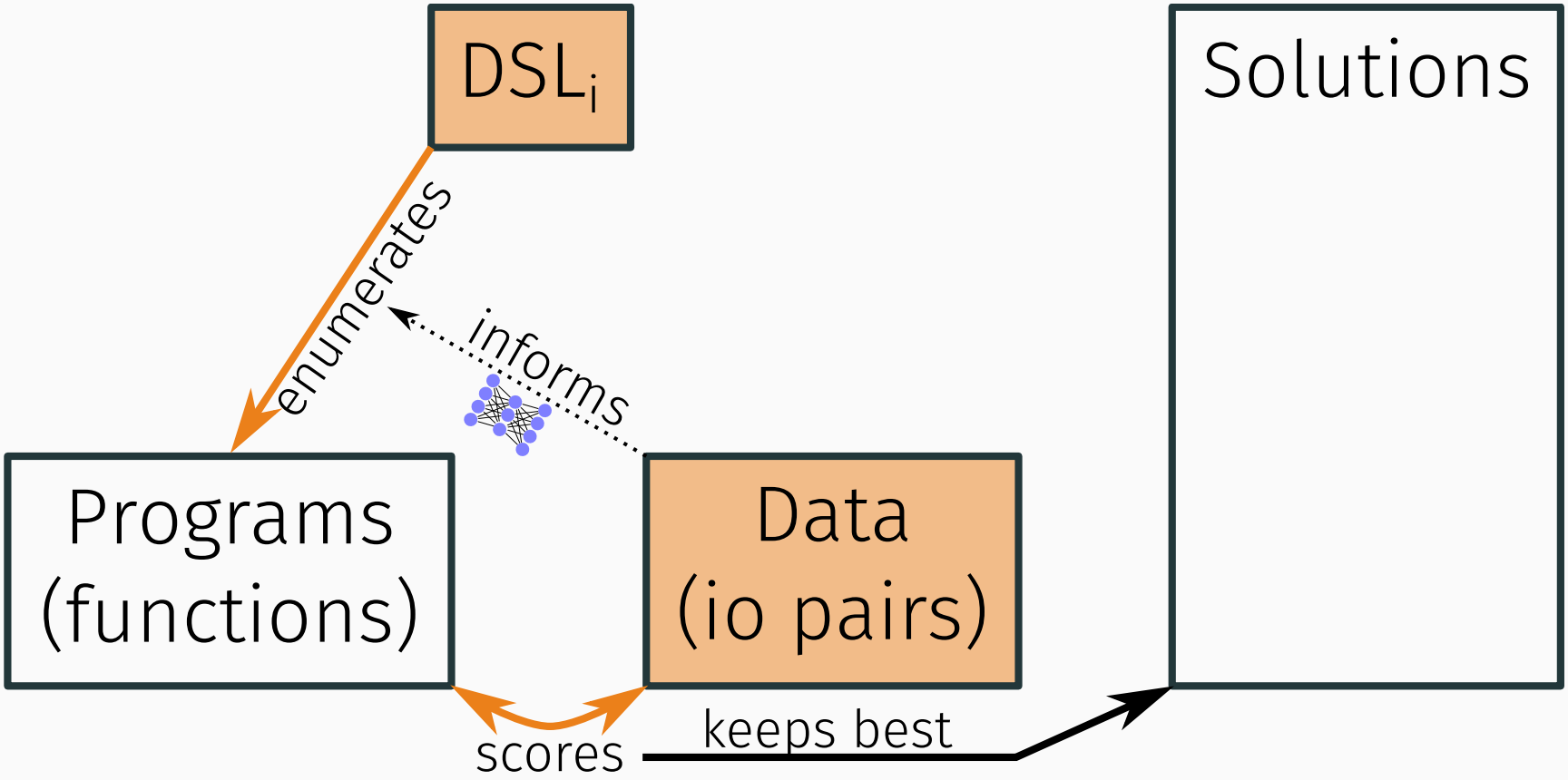


DreamCoder — Wake



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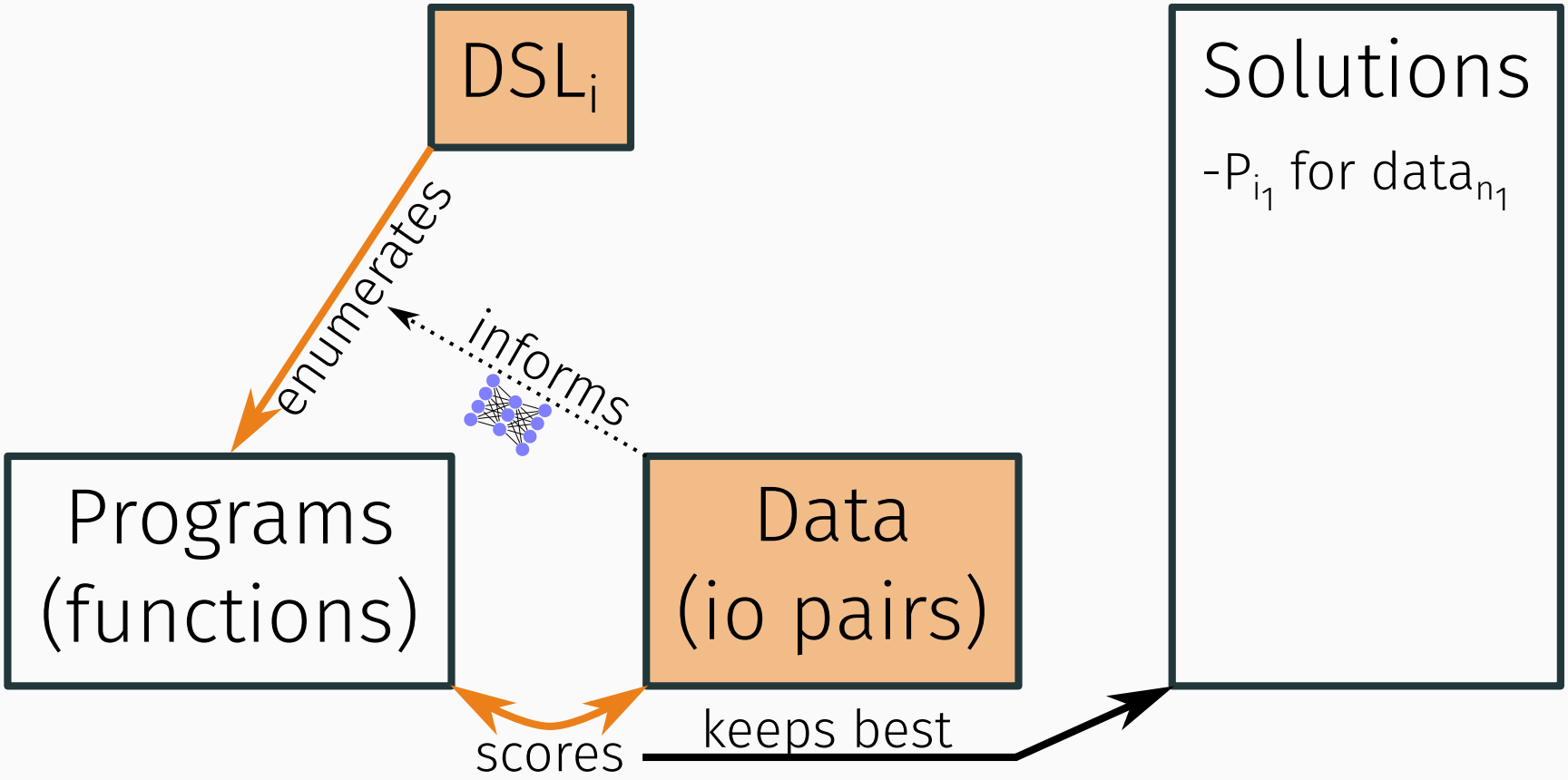

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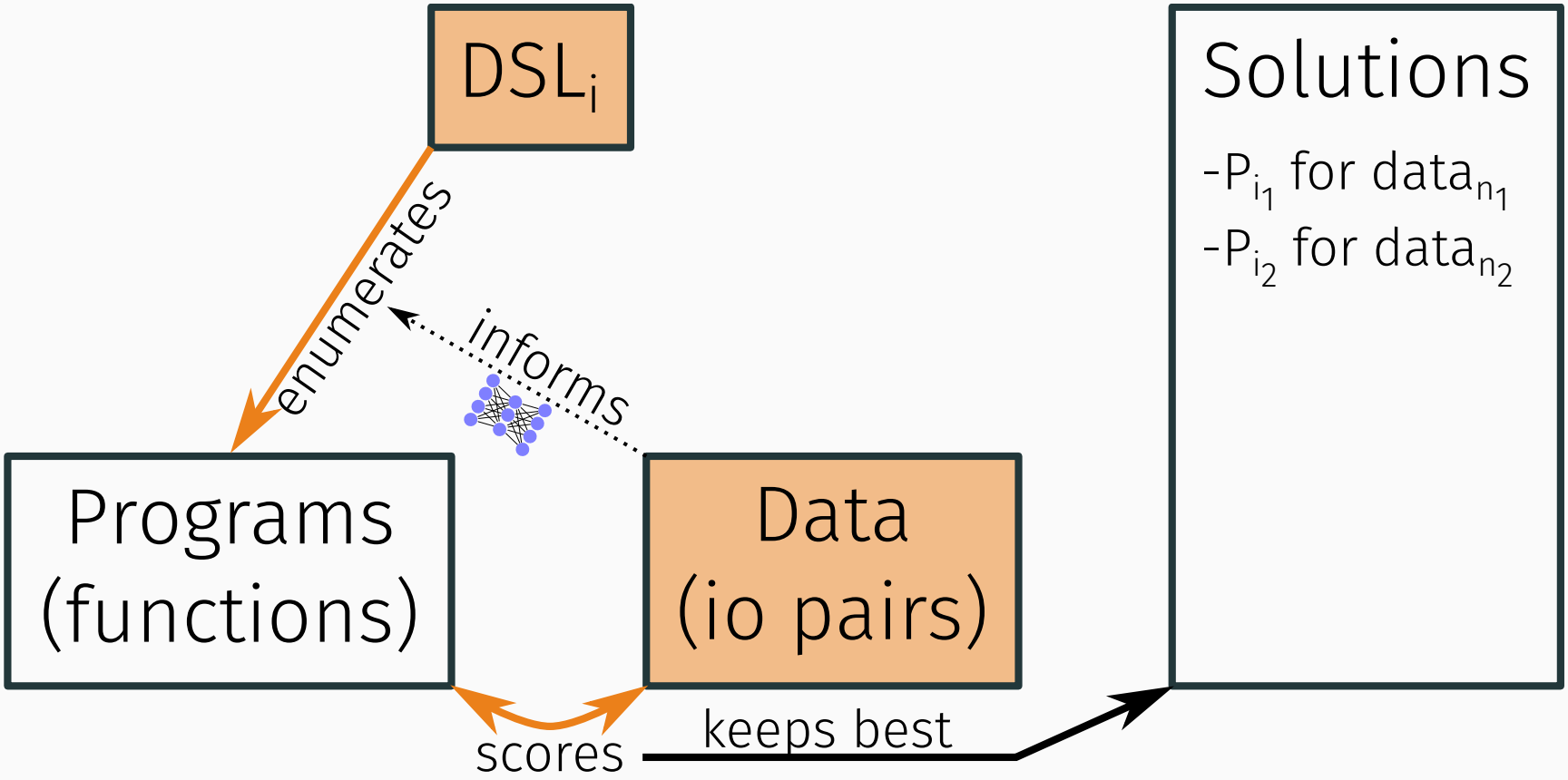

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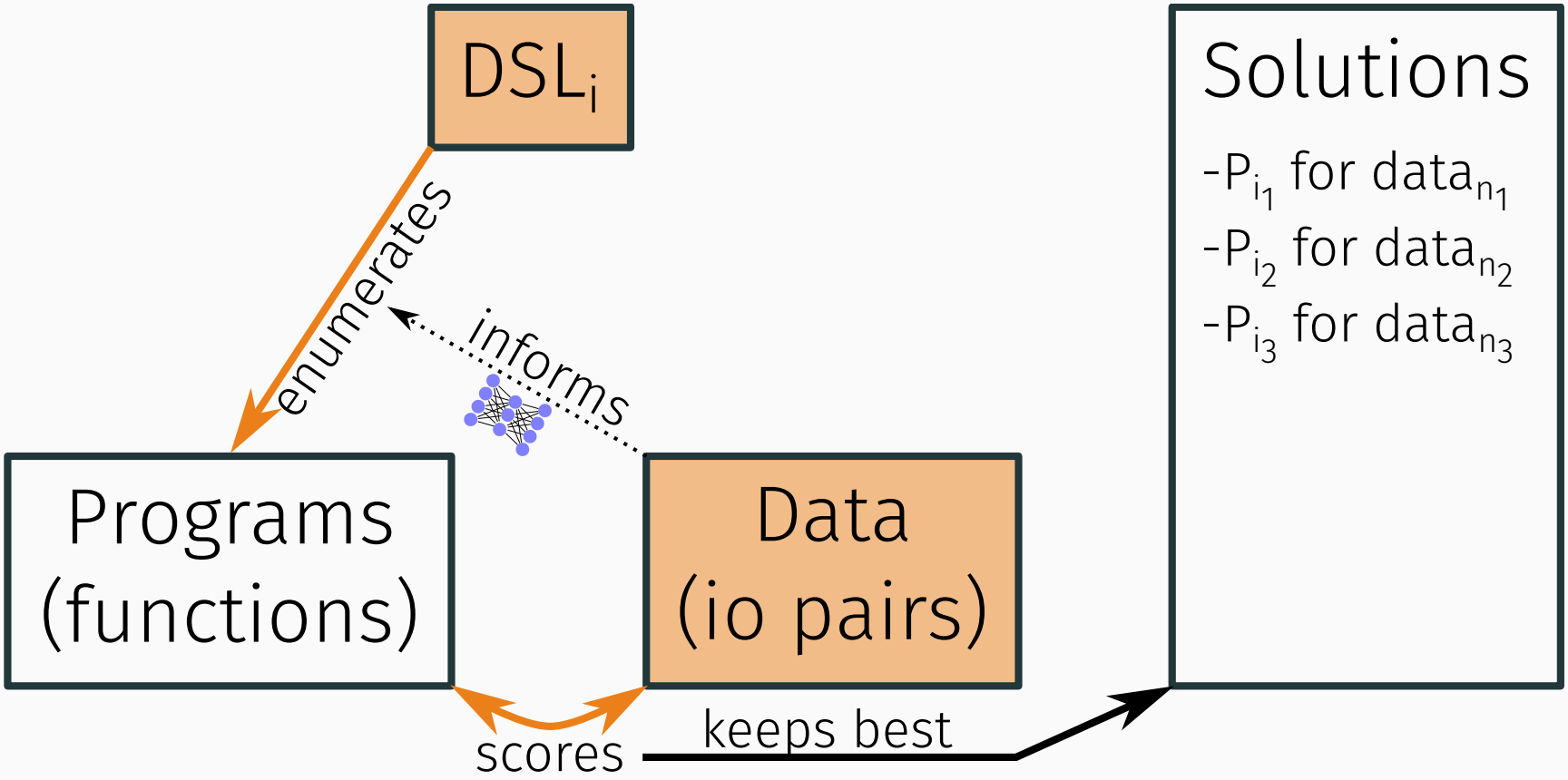

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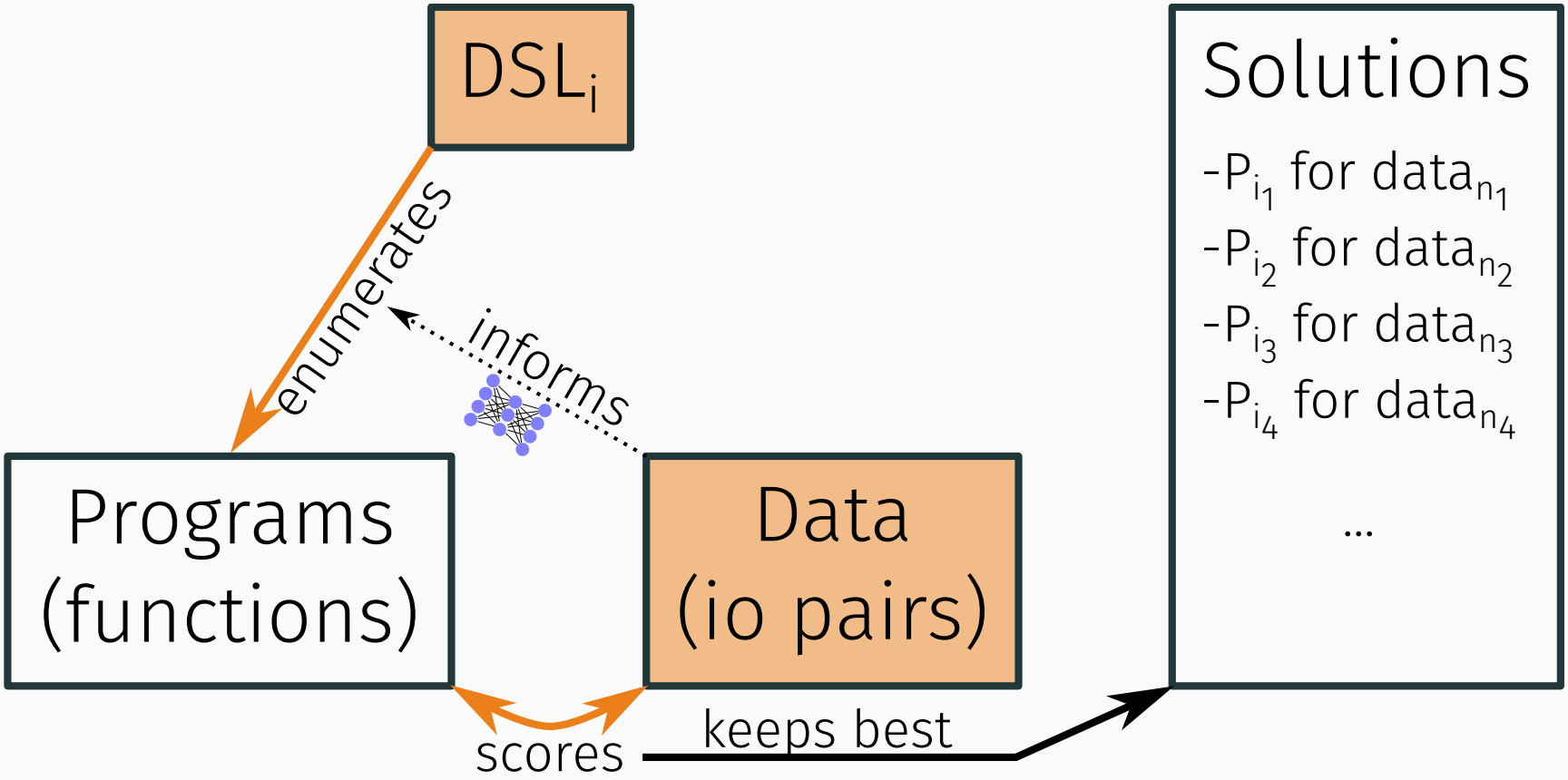
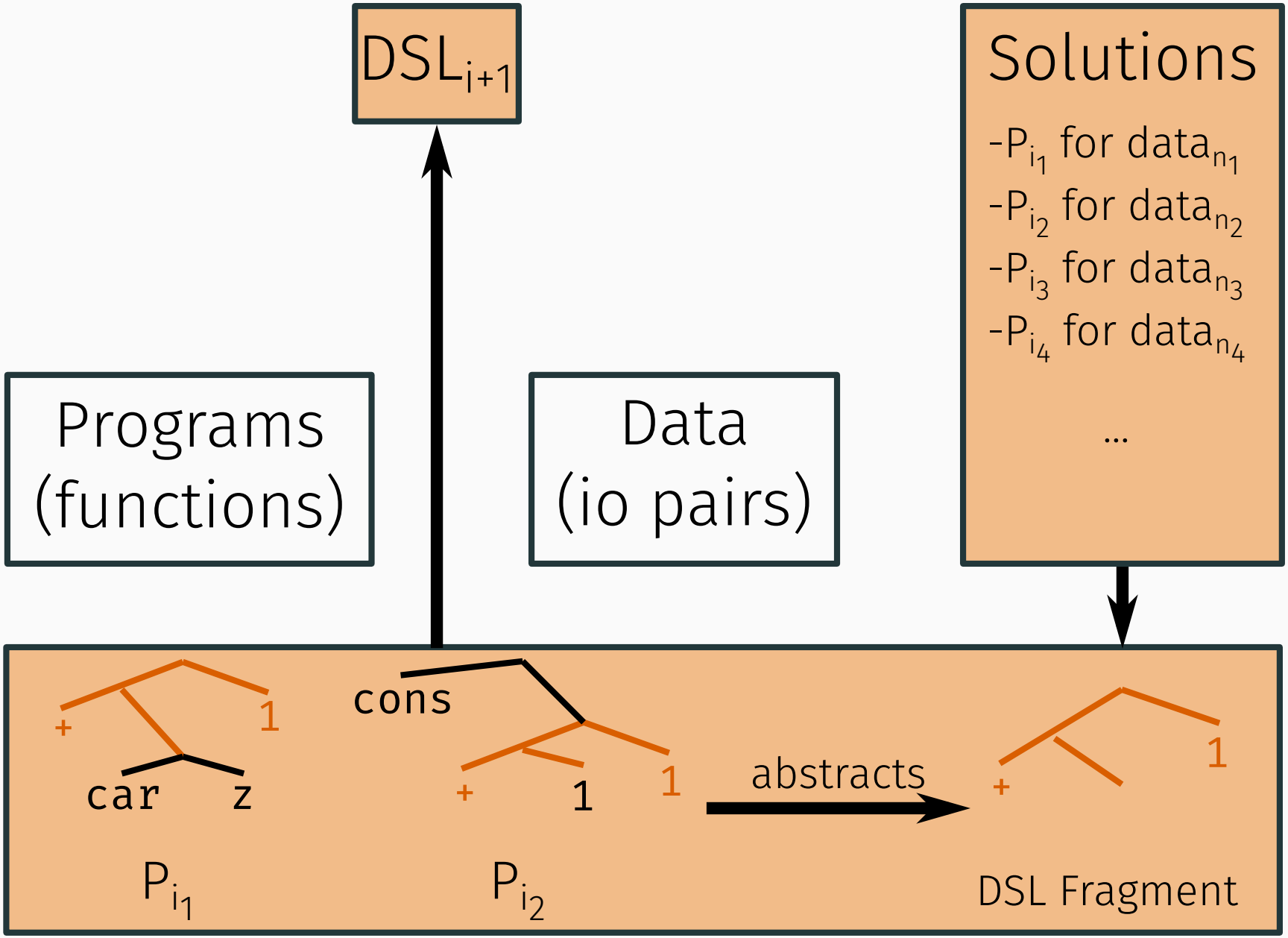


DreamCoder — Sleep-G



DreamCoder — Sleep-R (Experience Replay)

DSL_{i+1}

Solutions

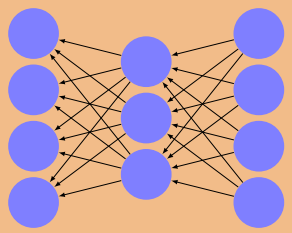
- P_{i_1} for $data_{n_1}$
- P_{i_2} for $data_{n_2}$
- P_{i_3} for $data_{n_3}$
- P_{i_4} for $data_{n_4}$

...

Programs (functions)

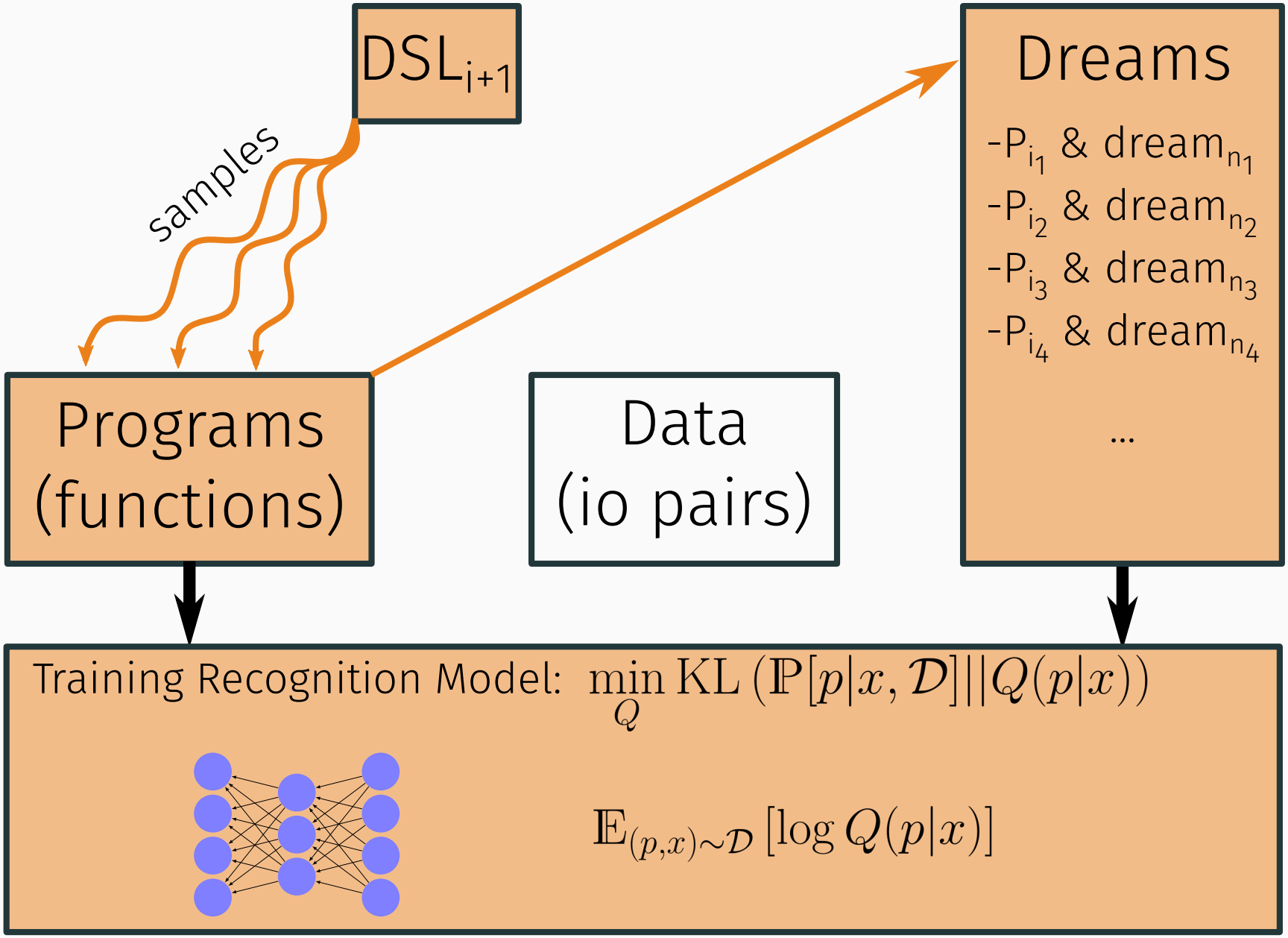
Data (io pairs)

Training Recognition Model: $\min_Q \mathrm{KL}\left(\mathbb{P}[p|x,\mathcal{D}]||Q(p|x)\right)$



$$\frac{\mathbb{P}\left[x_n, p_n|\mathcal{D}\right]}{\sum_{(x_n,p'_n)} \mathbb{P}\left[x_n, p_n|\mathcal{D}\right]} \log Q(p_n|x_n)$$

DreamCoder — Sleep-R (Dreaming)



List functions — Created & investigated by Lucas Morales

Name	Input	Output
repeat-3	[7 0]	[7 0 7 0 7 0]
drop-3	[0 3 8 6 4]	[6 4]
rotate-2	[8 14 1 9]	[1 9 8 14]
count-head-in-tail	[1 2 1 1 3]	2
keep-div-5	[5 9 14 6 3 0]	[5 0]
product	[7 1 6 2]	84

Discovers 38 concepts, including ‘filter’.

Text editing

In the style of FlashFill (Gulwani 2012)

Text Editing

+106 769-438→106.769.438
+83 973-831→83.973.831

```
f(s) = (f0 "." "-"
          (f0 "." " "
            (cdr s)))
```

Temple Anna H →TAH
Lara Gregori→LG

```
f(s) = (f2 s)
```

```
f0(s,a,b) = (map (λ (x)
              (if (= x a) b x)) s)
```
(f_0: *Performs character substitution*)

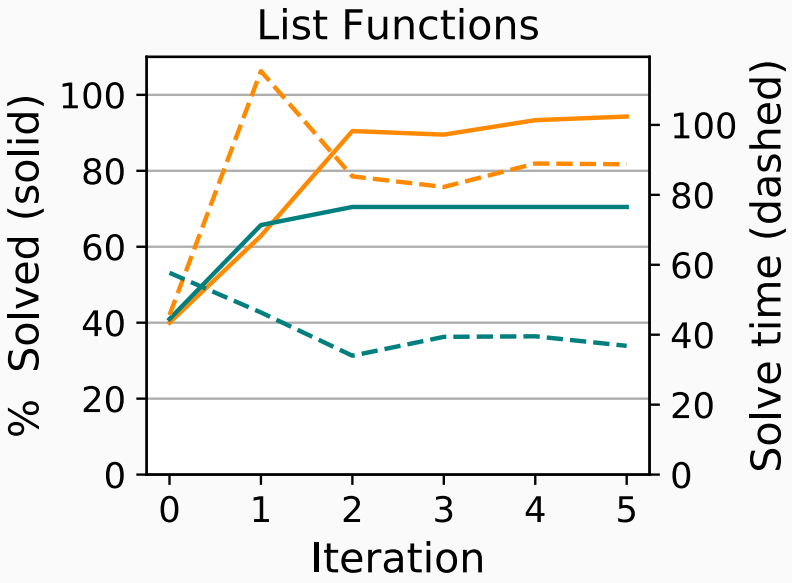
```
f1(s,c) = (foldr s s (λ (x a)
            (cdr (if (= c x) s a))))
```
(f_1: *Drop characters from* s *until* c *reached*)

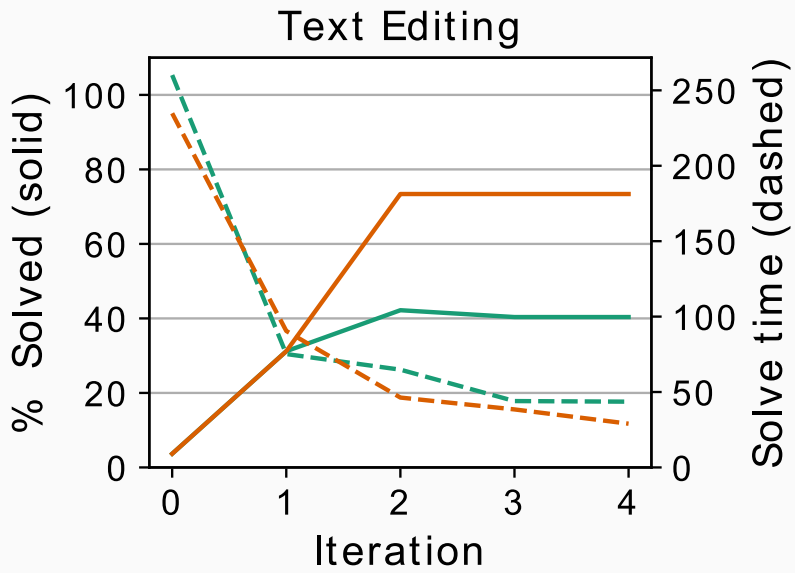
```
f2(s) = (unfold s is-nil car
          (λ (z) (f1 z " ")))
```
(f_2: *Abbreviates a sequence of words*)

SyGuS problems: solves 3% before learning, vs 75% after learning.

Best prior work: 80%

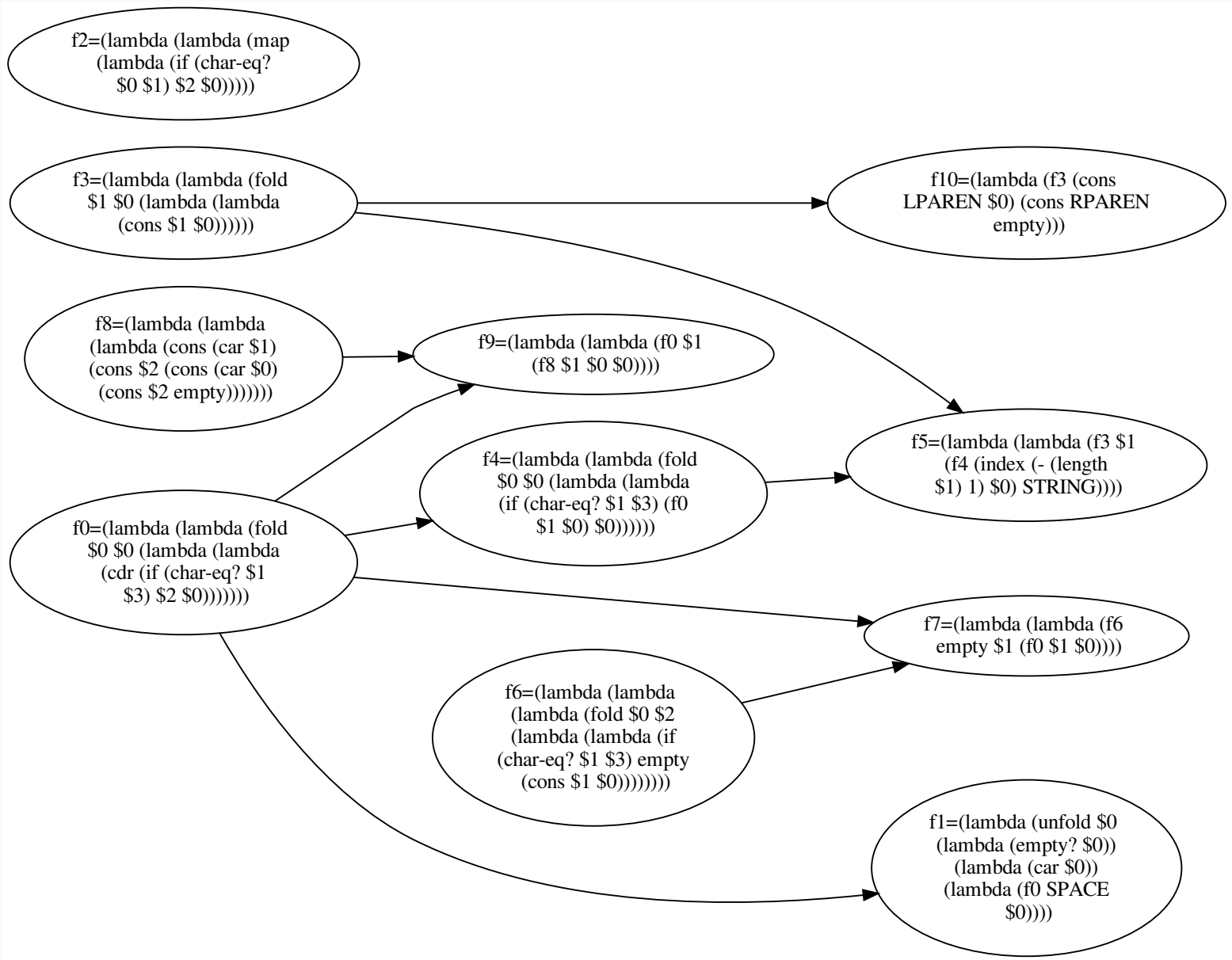
List functions & Text editing: Learning curves on hold out tasks





Learning curves for DreamCoder both with (in orange) and without (in teal) the recognition model. Solid lines: % holdout testing tasks solved w/ 10m timeout. Dashed lines: Average solve time, averaged only over tasks that are solved.

Learned text processing DSL



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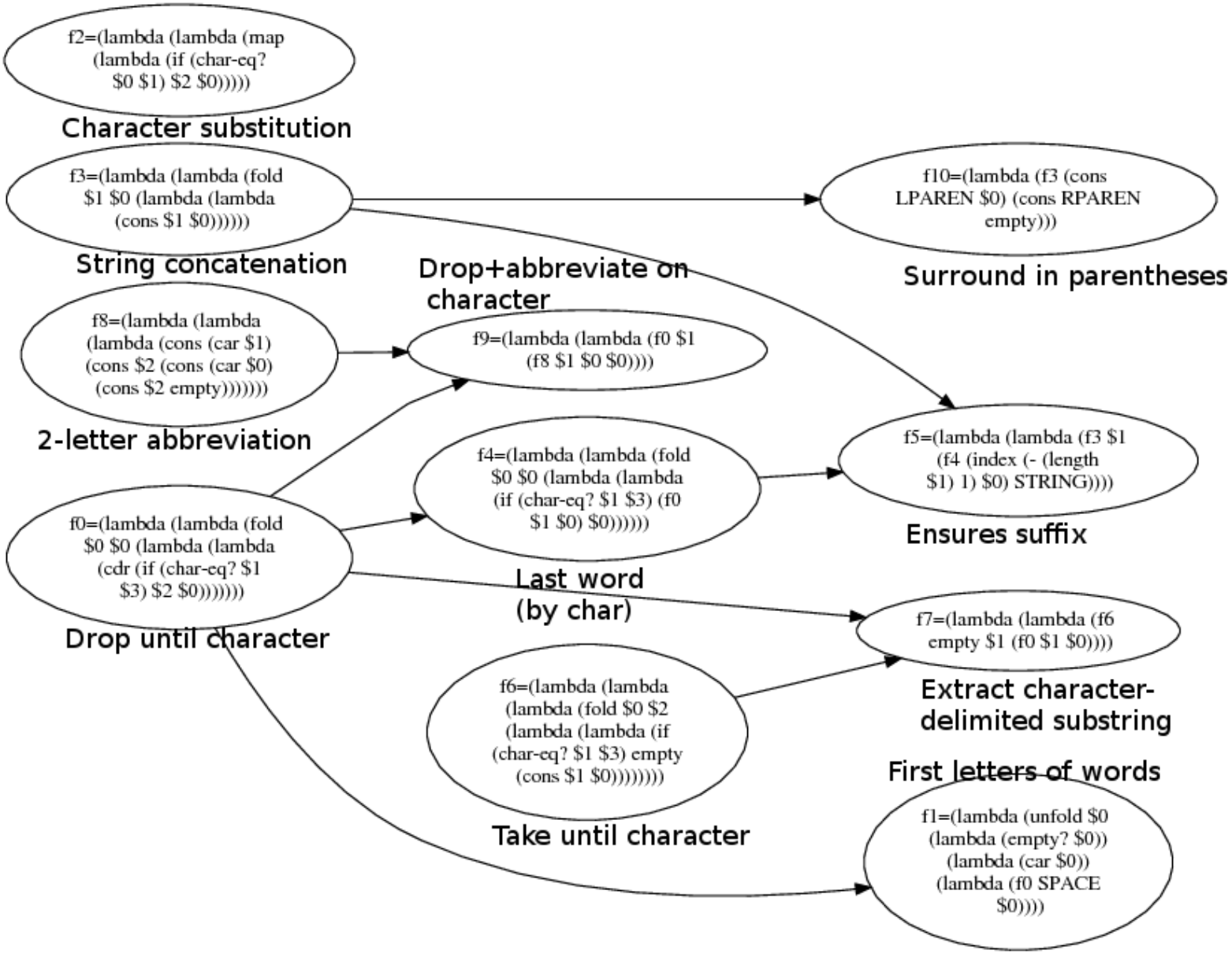


Learning the fundamentals of programming

Programs & Tasks	DSL

`[2 1 4]→[2 1 4 0]`
`[9 8]→[9 8 0]`
$f(\ell)$ = (f₂ cons ℓ (cons 0 nil))
	f_0(p,f,n,x) = (if (p x) nil
	(cons (f x) (f_0 (n x))))
	(f_0: *unfold*)

`[2 5 6 0 6]→19`
`[9 2 7 6 3]→27`
$f(\ell)$ = (f₂ + ℓ 0)
	f_1(i,l) = (if (= i 0) (car l)
	(f_1 (- i 1) (cdr l))))
	(f_1: *index*)

`[4 2 6 4]→[8 4 12 8]`
`[2 3 0 7]→[4 6 0 14]`
$f(\ell)$ = (f₃ (λ (x) (+ x x)) ℓ)
	f_2(f,l,x) = (if (empty? l) x
	(f (car l) (f_2 (cdr l))))
	(f_2: *fold*)

`[1 5 2 9]→[1 2]`
`[3 8 1 3 1 2]→[3 1 1]`
$f(\ell)$ = (f₀ empty? car
(λ (l) (cdr (cdr l))) ℓ)
	f_3(f,l) = (f_2 nil l (λ (x a) (cons (f x) a)))
	(f_3: *map*)
	$f_4(\ell)$ = (if (empty? ℓ) 0 (+ 1 (f_4 (cdr ℓ)))))
	(f_4: *length*)
	$f_5(n)$ = (f_0 (= n) (λ (x) x) (+ 1) 0)
	(f_5: *range*)

McCarthy 1959 Lisp $\longrightarrow$ Modern functional programming

22 tasks. 64 CPUs. 93 hours.

Symbolic regression from visual input

Symbolic Regression

$f(x) = (f_1\ x)$

$f(x) = (f_6\ x)$

$f(x) = (f_4\ x)$

$f(x) = (f_3\ x)$

$f_0(x) = (+\ x\ real)$
$f_1(x) = (f_0\ (*\ real\ x))$
$f_2(x) = (f_1\ (*\ x\ (f_0\ x)))$
$f_3(x) = (f_0\ (*\ x\ (f_2\ x)))$
$f_4(x) = (f_0\ (*\ x\ (f_3\ x)))$
(f_4: 4th order polynomial)
$f_5(x) = (/\ real\ x)$
$f_6(x) = (f_5\ (f_0\ x))$
(f_6: rational function)

Turtle graphics — Created & investigated by Mathias Sablé–Meyer

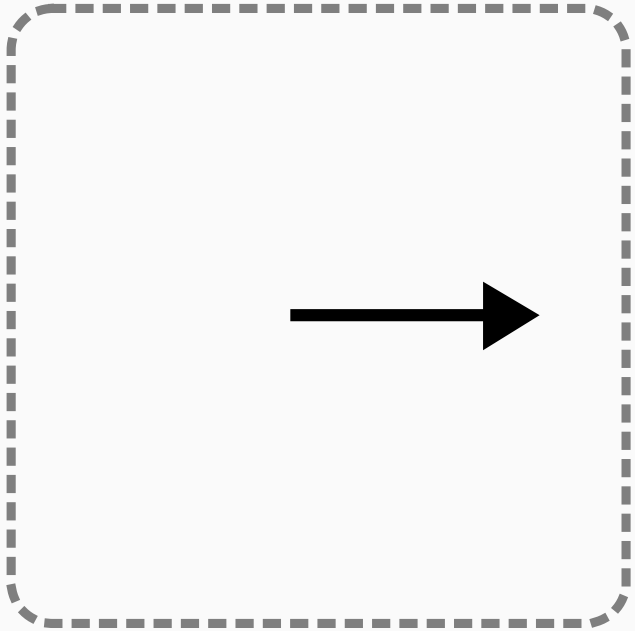
DSL

```
OP ::= FW x | RT x | UP | DOWN | SET state
```

Tasks

```
task :  image
```

```
FW 1
```



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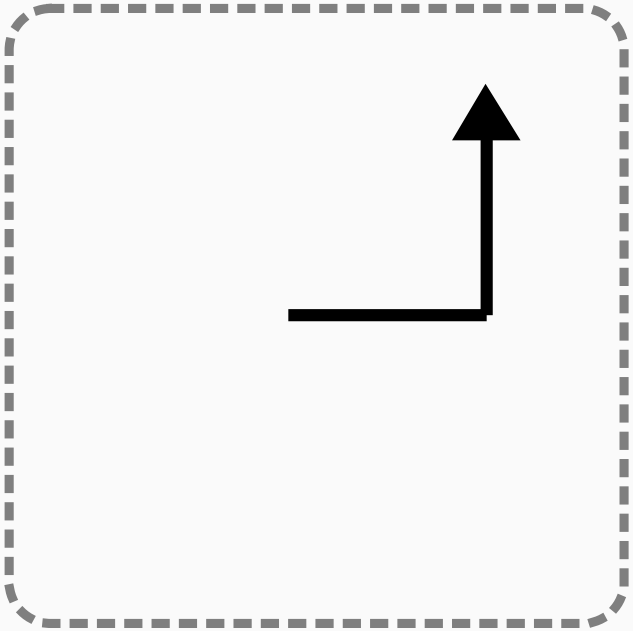
DSL

```
OP ::= FW x | RT x | UP | DOWN | SET state
```

Tasks

```
task :  image
```

```
FW 1
RT π/2
FW 1
```



Turtle graphics — Created & investigated by Mathias Sablé–Meyer

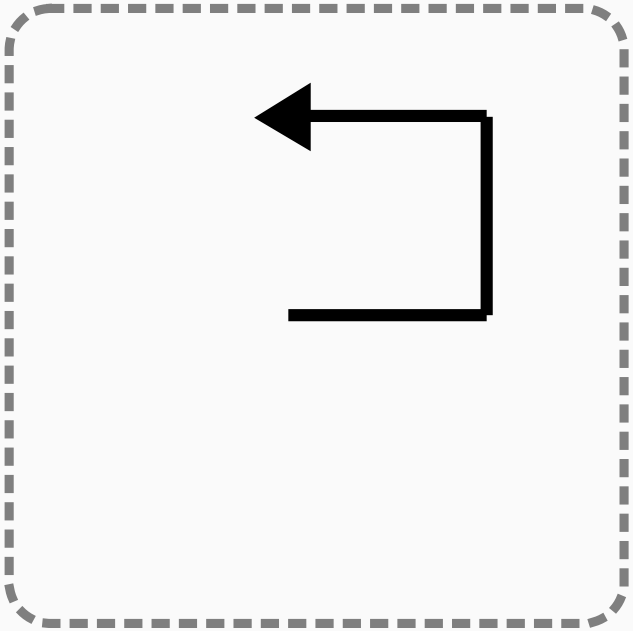
DSL

```
OP ::= FW x | RT x | UP | DOWN | SET state
```

Tasks

```
task :  image
```

```
FW 1
RT π/2
FW 1
RT π/2
FW 1
```



Turtle graphics — Created & investigated by Mathias Sablé–Meyer

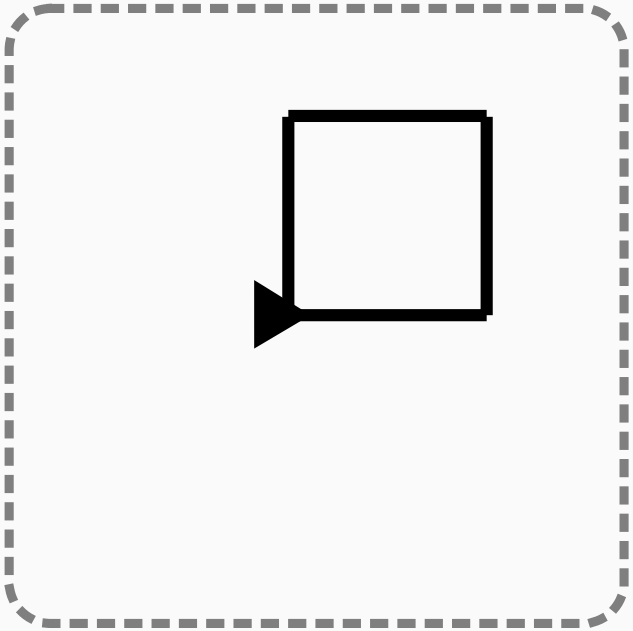
DSL

```
OP ::= FW x | RT x | UP | DOWN | SET state
```

Tasks

```
task :  image
```

```
for i in range(4)
> FW 1
> RT π/2
```



Turtle graphics — Created & investigated by Mathias Sablé–Meyer

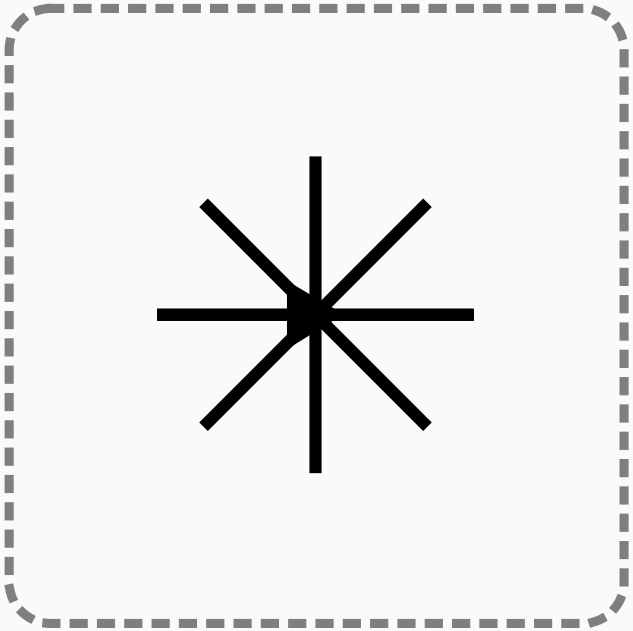
DSL

```
OP ::= FW x | RT x | UP | DOWN | SET state
```

Tasks

```
task :  image
```

```
for i in range(8)
> FW 1
> SET origin
> RT 2π/8
```



Turtle graphics — Created & investigated by Mathias Sablé–Meyer

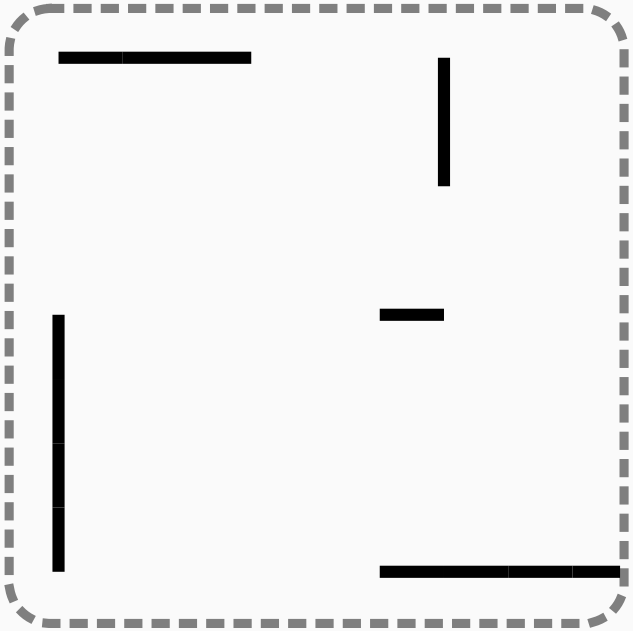
DSL

```
OP ::= FW x | RT x | UP | DOWN | SET state
```

Tasks

```
task :  image
```

```
for i in range(8)
> PU
> FW i/2
> PD
> FW i/2
> RT π/2
```



Turtle graphics — Created & investigated by Mathias Sablé–Meyer

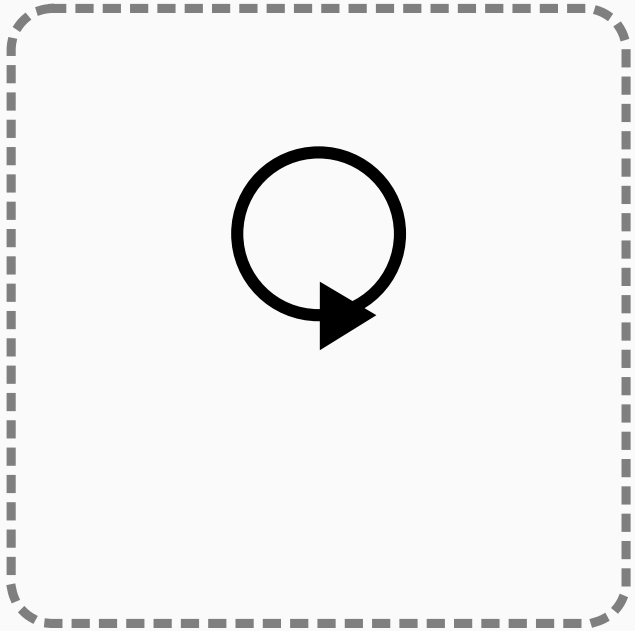
DSL

```
OP ::= FW x | RT x | UP | DOWN | SET state
```

Tasks

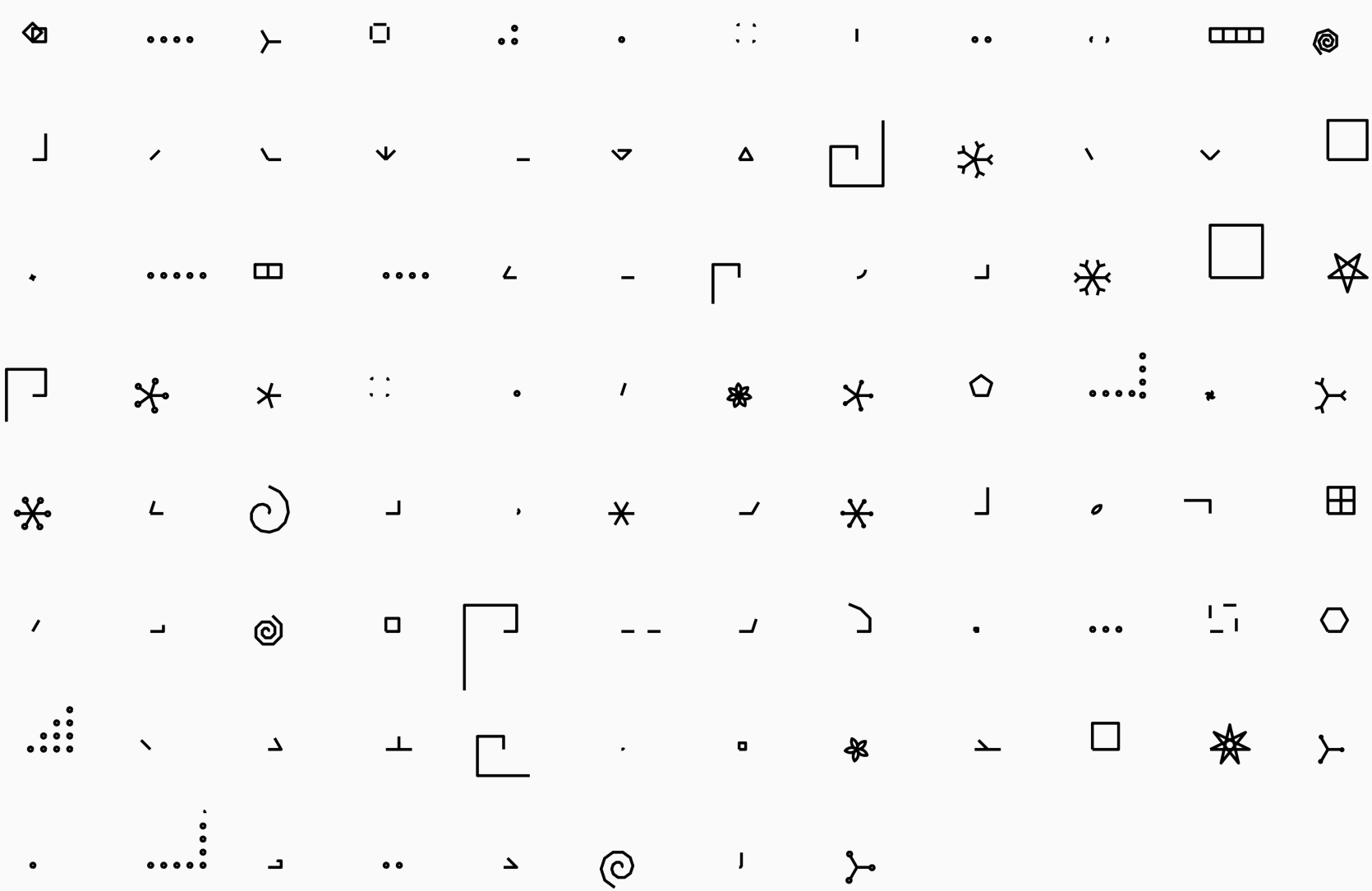
```
task :  image
```

```
for i in range(∞)
> FW ε
> RT ε
```

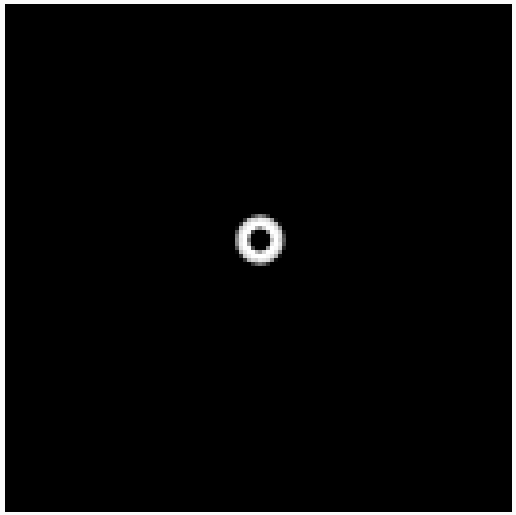


```
NUM ::= 1 | π | ∞ | ε | + | - | * | /
```

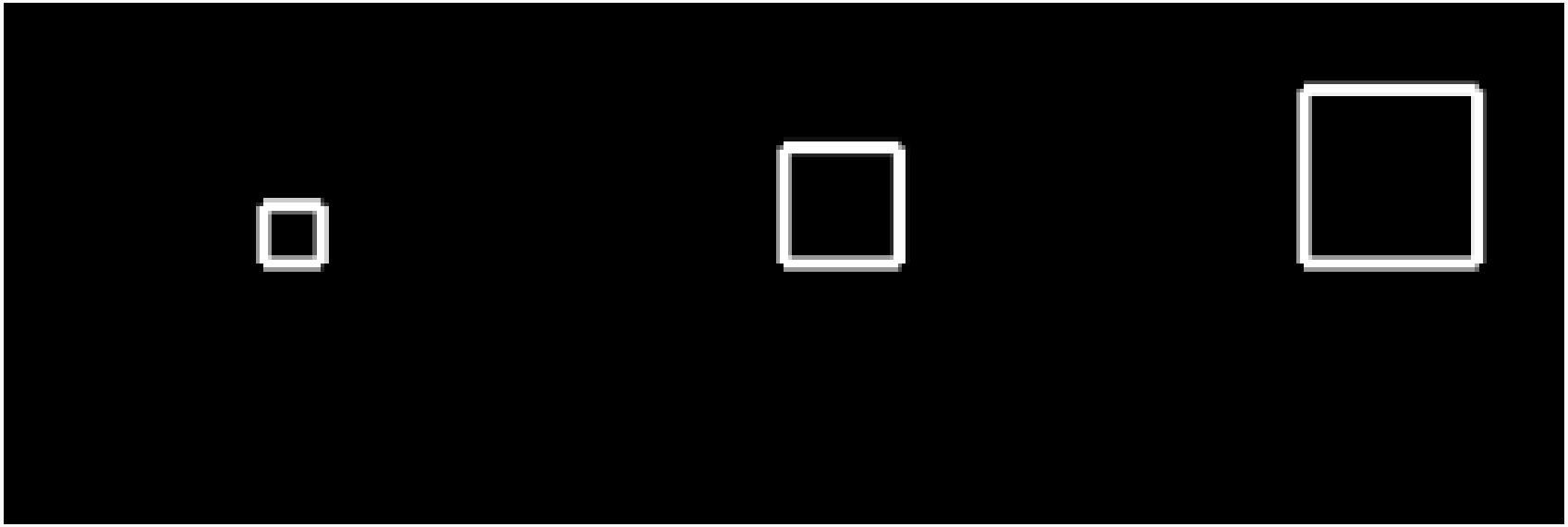
Turtle graphics — Training tasks



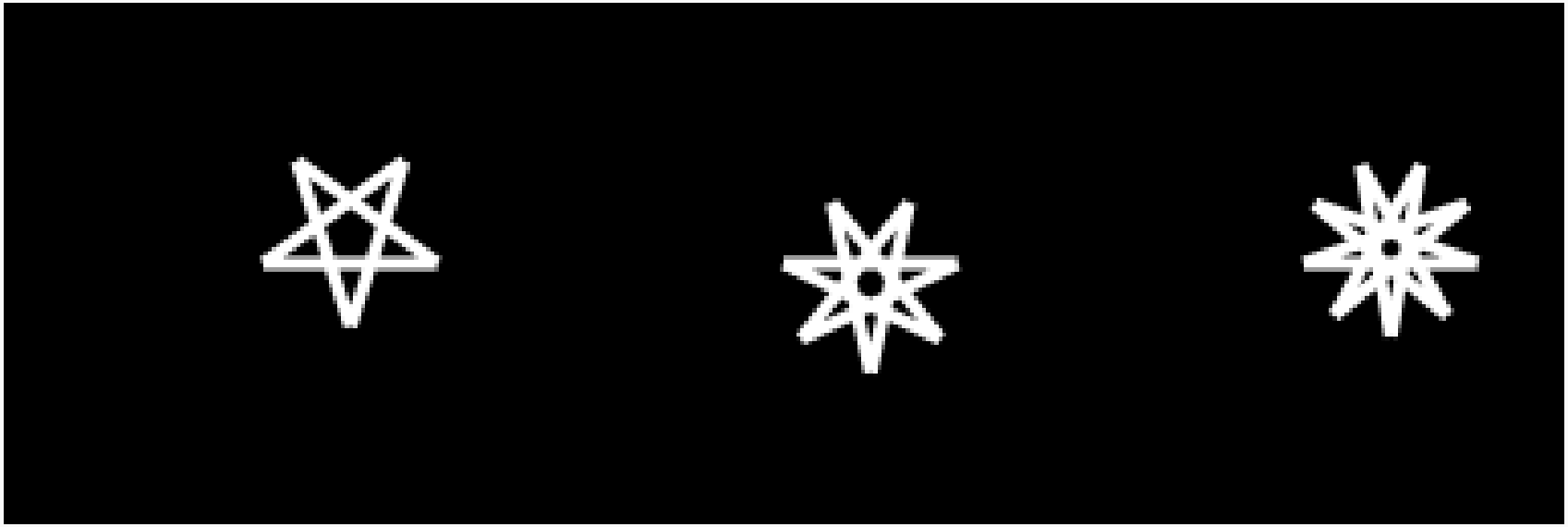
Turtle graphics — Illustration of learned DSL



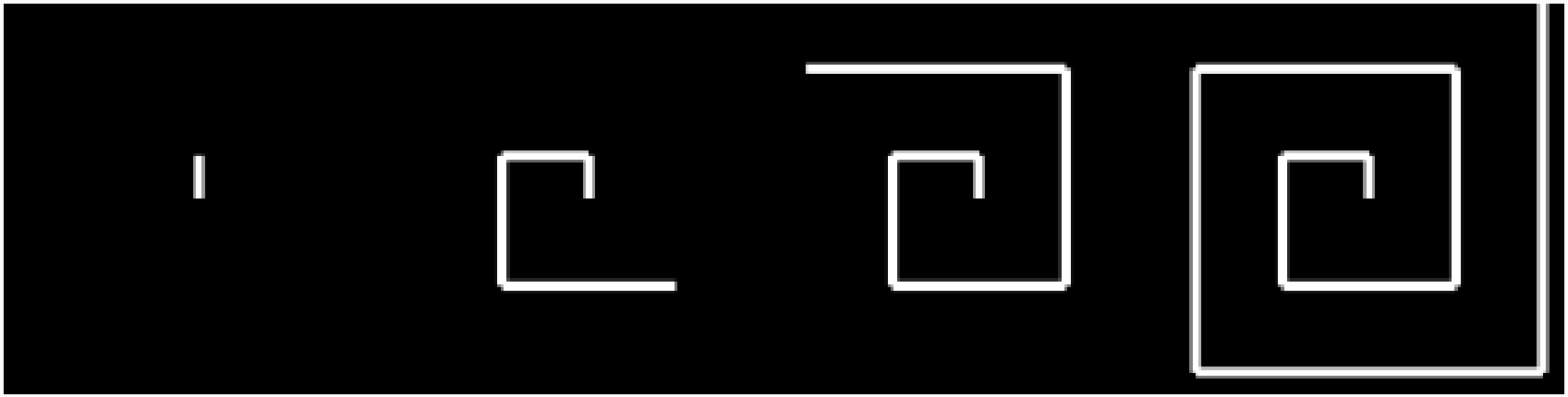
Turtle graphics — Illustration of learned DSL



Turtle graphics — Illustration of learned DSL

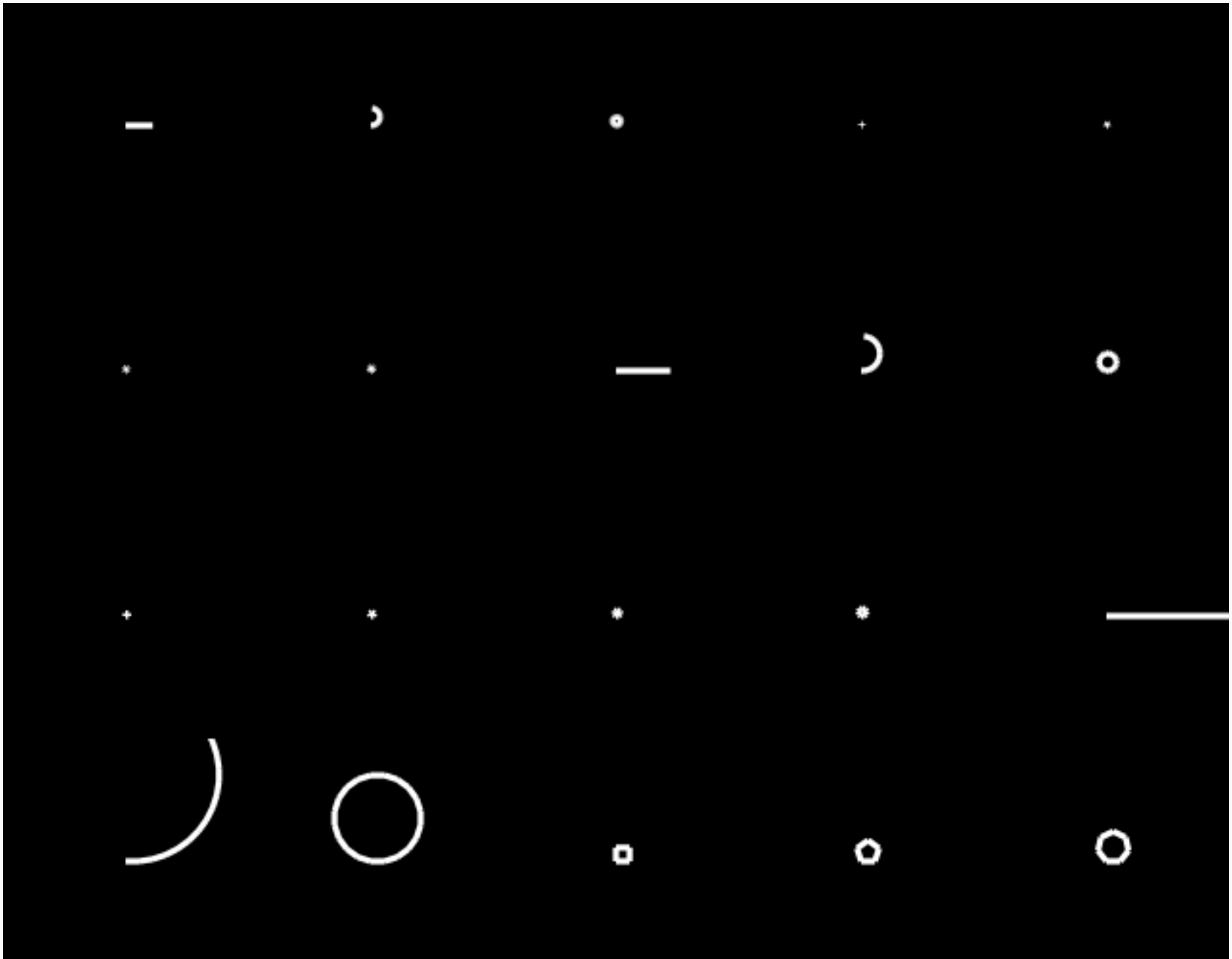


Turtle graphics — Illustration of learned DSL



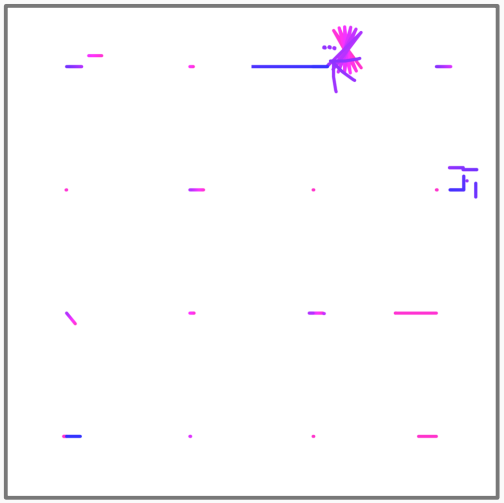
Turtle graphics — Illustration of learned DSL

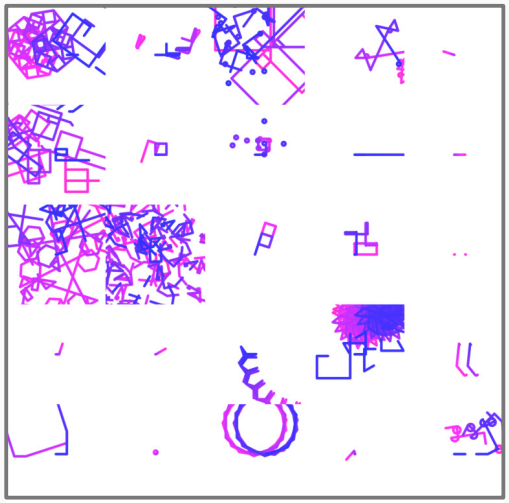
Turtle graphics — Illustration of learned DSL



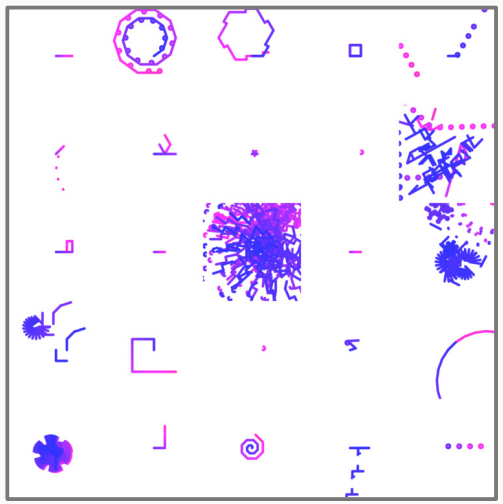
Turtle graphics — Dreams

Before training



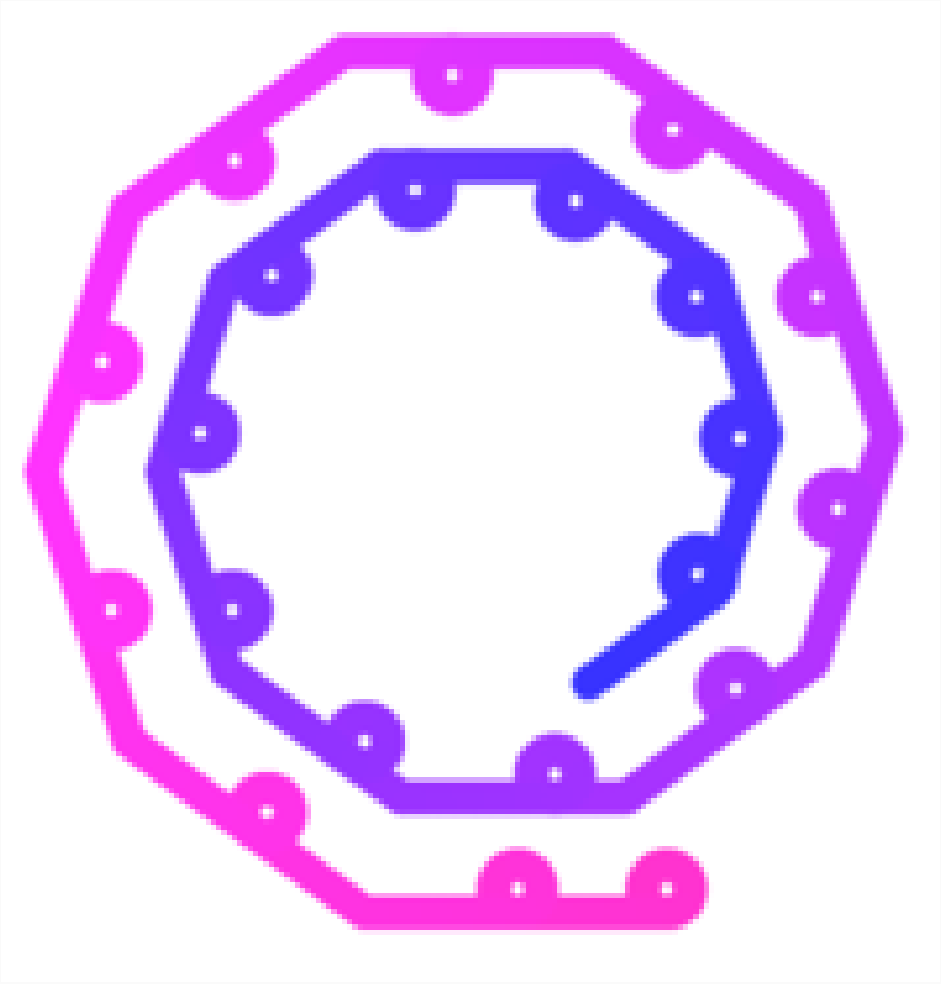


Plateau 5 minutes

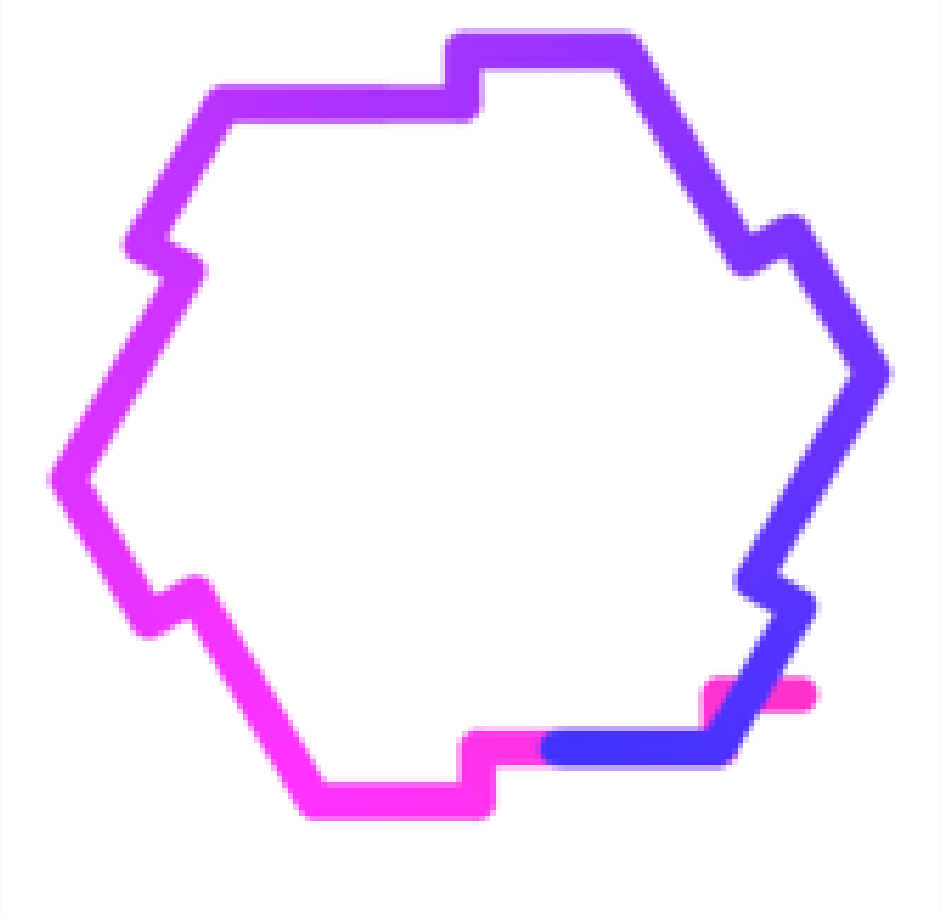


Plateau 2 hours

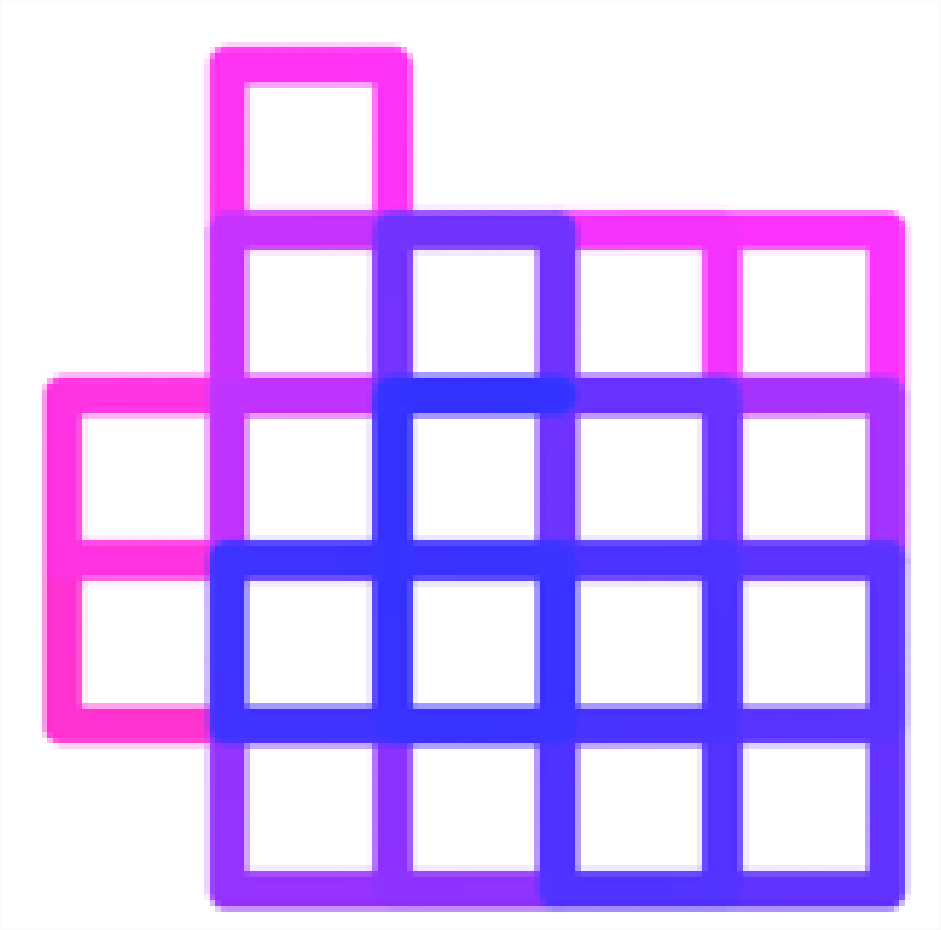
Turtle graphics — Dreaming from learned generative model



Turtle graphics — Dreaming from learned generative model

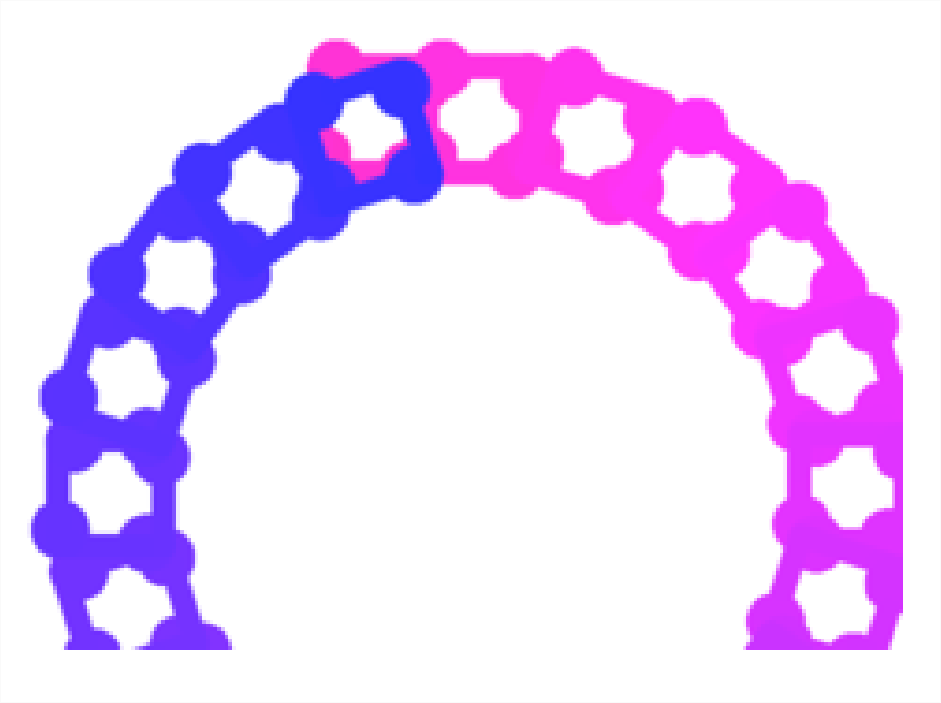


Turtle graphics — Dreaming from learned generative model

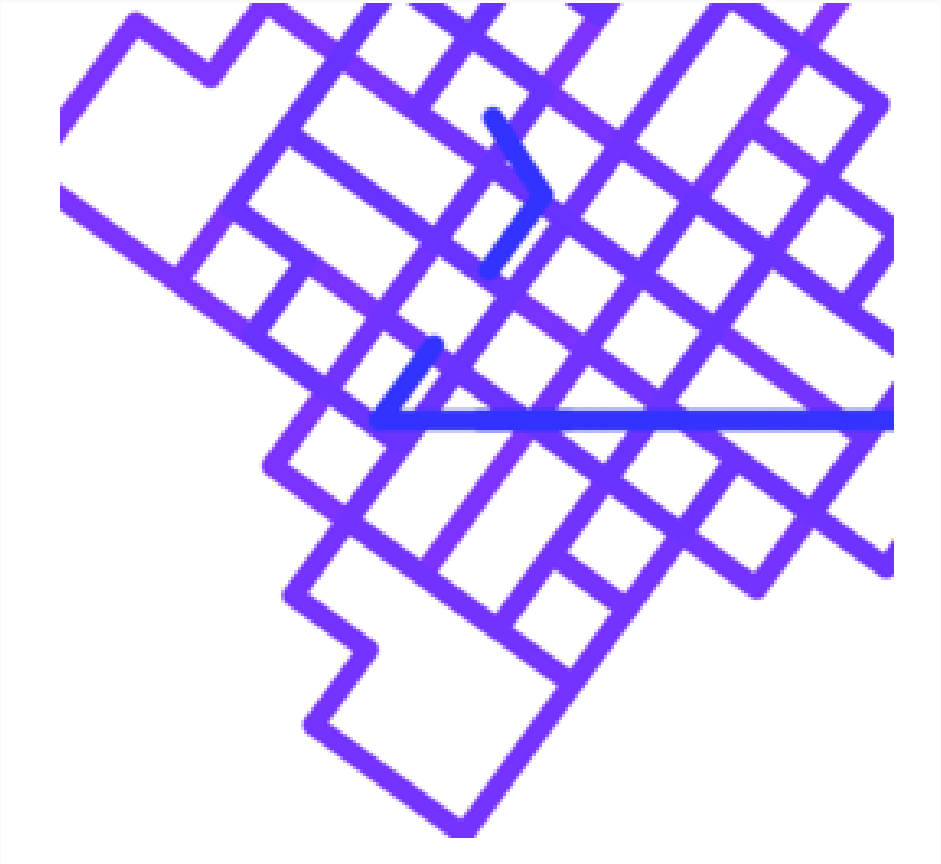


Turtle graphics — Dreaming from learned generative model

Turtle graphics — Dreaming from learned generative model



Turtle graphics — Dreaming from learned generative model



Turtle graphics — Dreaming from learned generative model

Turtle graphics — Dreaming from learned generative model

Vision

More human-like machine intelligence

- Acquiring a domain-specific representation (DSL)
- Learning to use that representation (recognition model)

DreamCoder: an algorithm for jointly realizing these goals

f_2(p,f,n,x) = (if (p x) nil
(cons (f x) (f_2 (n x))))
(f_2: *unfold*)
f_3(i,l) = (if (= i 0) (car l)
(f_3 (f_1 i) (cdr l))))
(f_3: *index*)
f_4(f,l,x) = (if (empty? l) x
(f (car l) (f_4 (cdr l))))
(f_4: *fold*)
f_5(f,l) = (if (empty? l) nil)
(cons (f (car l)) (f_5 (cdr l)))
(f_5: *map*)

Symbolic Regression

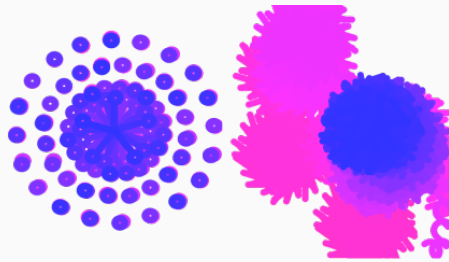

$f(x) = (f_1\ x)$


$f(x) = (f_6\ x)$


$f(x) = (f_4\ x)$


$f(x) = (f_3\ x)$

$f_0(x) = (+\ x\ real)$
$f_1(x) = (f_0\ (*\ real\ x))$
$f_2(x) = (f_1\ (*\ x\ (f_0\ x)))$
$f_3(x) = (f_0\ (*\ x\ (f_2\ x)))$
$f_4(x) = (f_0\ (*\ x\ (f_3\ x)))$
(f_4: *4th order polynomial*)
$f_5(x) = (/\ real\ x)$
$f_6(x) = (f_5\ (f_0\ x))$
(f_6: *rational function*)



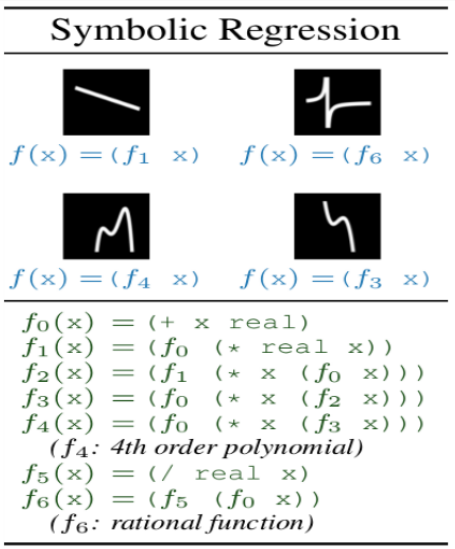
Vision

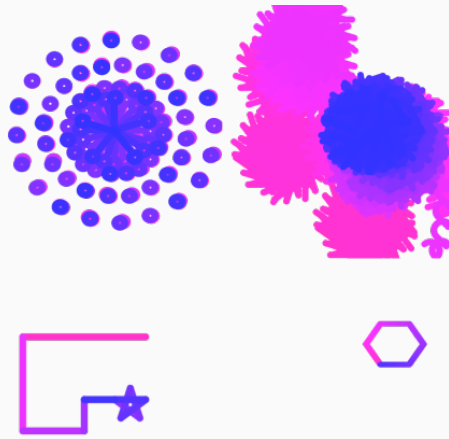
More human-like machine intelligence

- Acquiring a domain-specific representation (DSL)
- Learning to use that representation (recognition model)

DreamCoder: an algorithm for jointly realizing these goals

```
f2(p,f,n,x) = (if (p x) nil
                  (cons (f x) (f2 (n x))))
  (f2: unfold)
f3(i,l) = (if (= i 0) (car l)
              (f3 (f1 i) (cdr l))))
  (f3: index)
f4(f,l,x) = (if (empty? l) x
                (f (car l) (f4 (cdr l))))
  (f4: fold)
f5(f,l) = (if (empty? l) nil)
              (cons (f (car l)) (f5 (cdr l)))
  (f5: map)
```





The End.