`DreamCoder`: Growing libraries of concepts with wake-sleep program induction

Kevin Ellis
Joint with: Lucas Morales, Mathias Sablé Meyer, Armando Solar-Lezama, Joshua B. Tenenbaum
Heavy inspiration from: Eyal Dechter

October 2018

MIT

Human program induction everywhere

```
(MEMBER
 (LAMBDA (X L)
  (COND  ((NULL L) NIL)
         ((EQ X (FIRST L)) T)
         (T (MEMBER X (REST L))))))
```

Allen, Anatomy of Lisp, 1975

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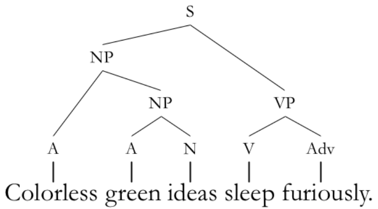

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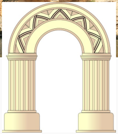

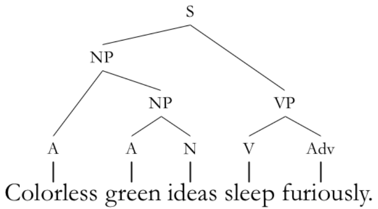

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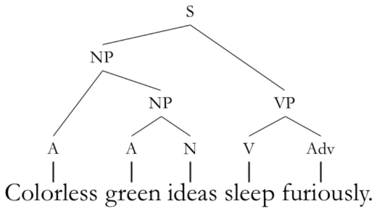
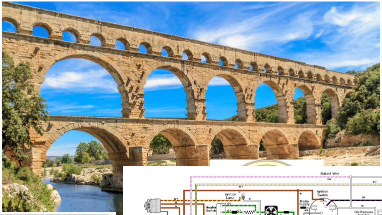
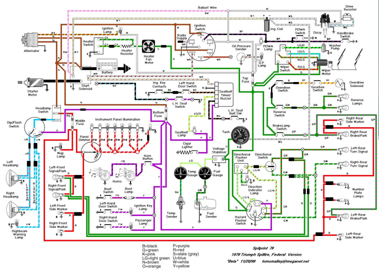
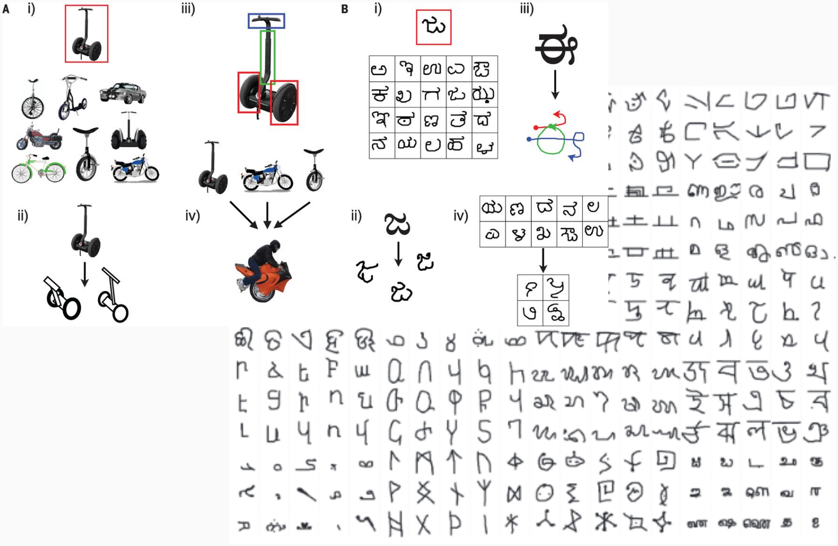






Human program induction everywhere



Lake et al 2015

Growing domain-specific knowledge

Goal: acquire domain-specific knowledge needed to induce a class of programs

- Library of concepts (declarative knowledge; generative model over programs)
- Inference strategy (procedural knowledge)

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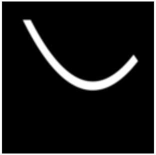

Learned program inducer

program: $f(x) = 0.7x^2 + 1.2$

Concepts: x^2, etc

Inference strategy: neurosymbolic search for programs

Growing domain-specific knowledge

Goal: acquire domain-specific knowledge needed to induce a class of programs

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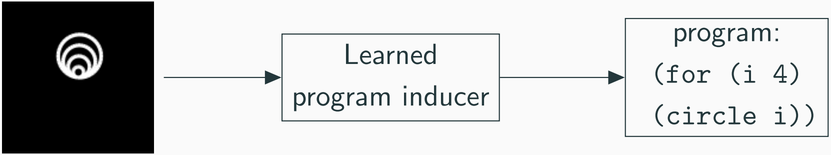

Concepts: `circle`, etc

Inference strategy: neurosymbolic search for programs

DSL: Library of concepts

Tasks and Programs	DSL
[7 2 3]→[7 3] [1 2 3 4]→[3 4] [4 3 2 1]→[4 3] $f(\ell) = (f_1\ \ell\ (\lambda\ (\texttt{x})$ $(\texttt{> x 2})))$ [7 3]→False [3]→False [9 0 0]→True [0]→True [0 7 3]→True $f(\ell) = (f_3\ \ell\ \texttt{0})$ [2 7 8 1]→8 [3 19 14]→19 $f(\ell) = (f_2\ \ell)$	$f_0(\ell,\texttt{r}) = (\texttt{foldr r}\ \ell\ \texttt{cons})$ (f_0: *Append lists* r *and* ℓ) $f_1(\ell,\texttt{p}) = (\texttt{foldr}\ \ell\ \texttt{nil}\ (\lambda\ (\texttt{x a})$ $(\texttt{if (p x) (cons x a) a})))$ (f_1: *Higher-order filter function*) $f_2(\ell) = (\texttt{foldr}\ \ell\ \texttt{0}\ (\lambda\ (\texttt{x a})$ $(\texttt{if (> a x) a x})))$ (f_2: *Maximum element in list* ℓ) $f_3(\ell,\texttt{k}) = (\texttt{foldr}\ \ell\ (\texttt{is-nil}\ \ell)$ $(\lambda\ (\texttt{x a})\ (\texttt{if a a (= k x)})))$ (f_2: *Whether* ℓ *contains* k)

DreamCoder

- **Wake:** Solve problems by writing programs
- **Sleep:** Improve DSL and neural recognition model:
  - **Sleep-G:** Improve DSL (**G**enerative model)
  - **Sleep-R:** Improve **R**ecognition model

Combines ideas from Wake-Sleep & Exploration-Compression algorithm by Eyal Dechter

DreamCoder

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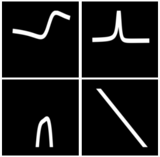
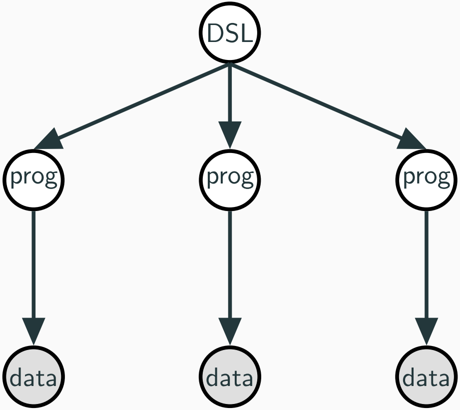
Combines ideas from Wake-Sleep & Exploration-Compression algorithm by Eyal Dechter

```
(let ((me 'whisper)
      (it 'into)
      (your 'ear))
  (let ((me (lisp it))
        (to (you)))
    (secretly)))
```



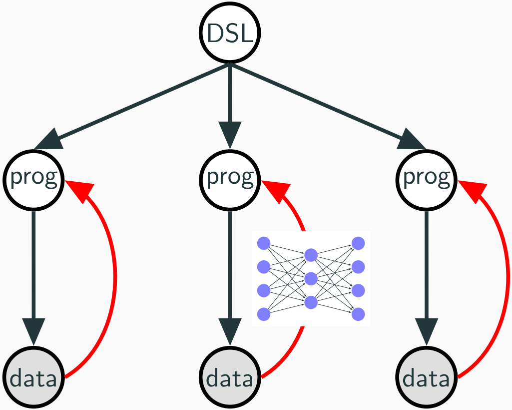
DSL learning as Bayesian inference



[Dechter et al., 2013] [Liang et al, 2010]; [Lake et al, 2015]

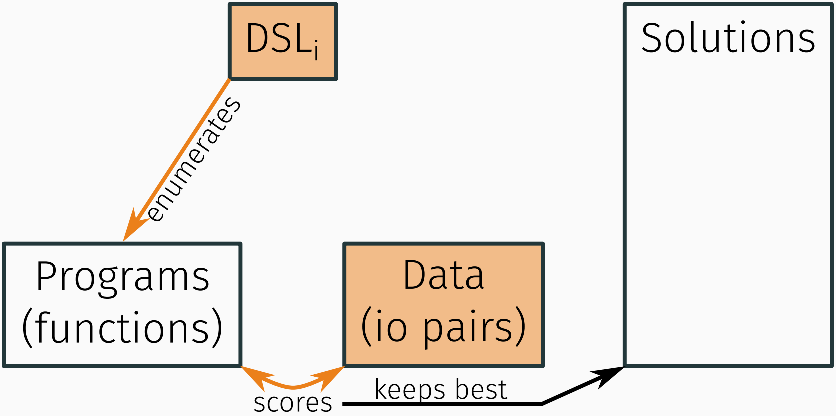
Dechter et al.: Exploration-Compression. Inspiration for DreamCoder.

DSL learning as amortized Bayesian inference



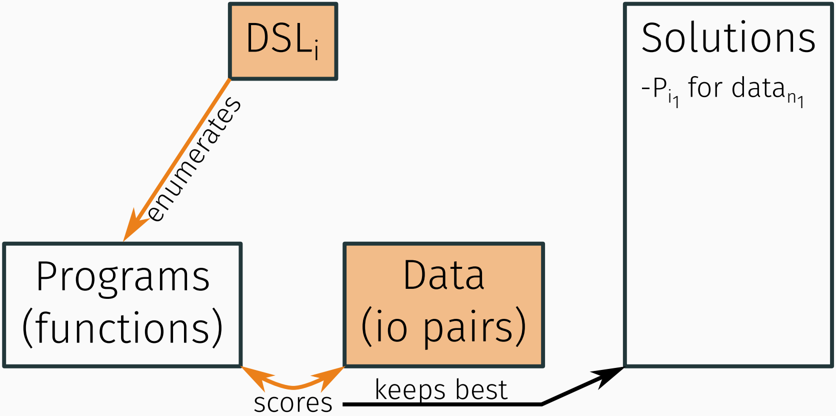
New: amortized inference +
better program representation (Lisp) +
better DSL inference

Wake — as in Exploration/Compression Algorithm



DSL	Programs	Data
f_0(l,r) = foldr r l cons	f(l) = (f_0 l l)	[7 2]→[7 2 7 2] ["a"]→["a" "a"]
f_1(l,p) = foldr r nil (λ (x a) (if (p x) (cons x a) a))	f(l) = (f_1 l (λx. > x 2))	[7 2 3]→[7 3] [1 2 3 4]→[3 4] [4 3 2 1]→[4 3]

Wake — as in Exploration/Compression Algorithm



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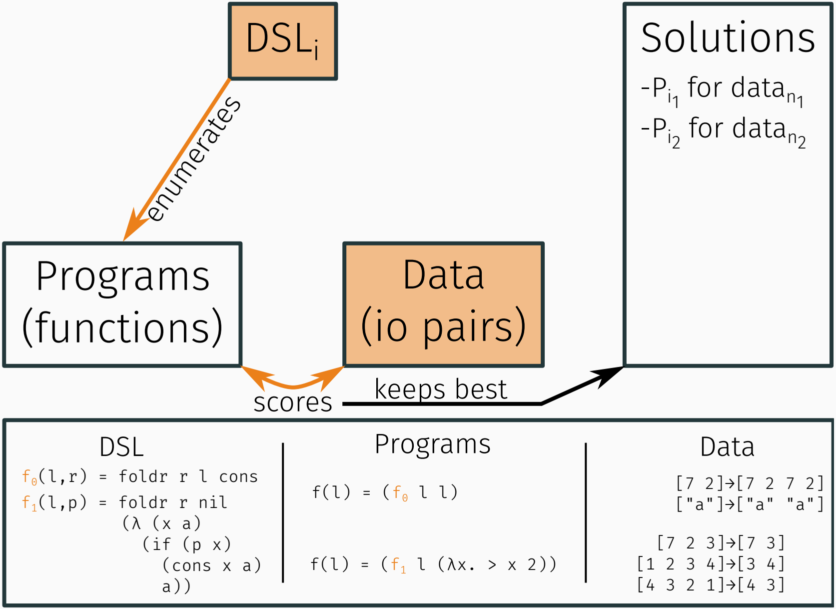

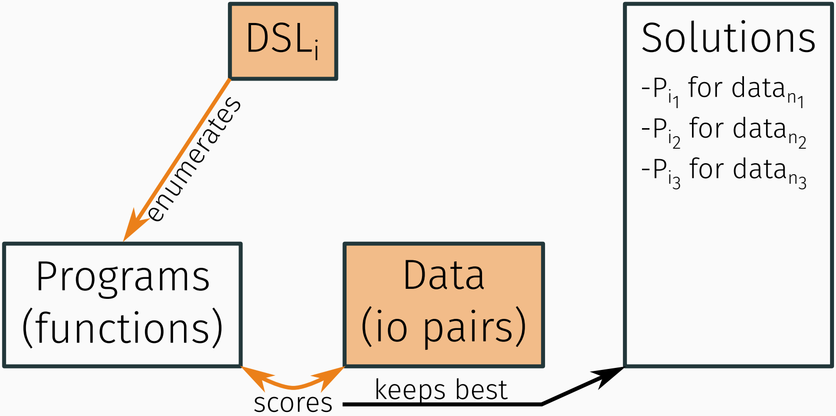

DSL

```
f0(l,r) = foldr r l cons
f1(l,p) = foldr r nil
           (λ (x a)
             (if (p x)
               (cons x a)
               a))
```

Programs

```
f(l) = (f0 l l)

f(l) = (f1 l (λx. > x 2))
```

Data

```
[7 2]→[7 2 7 2]
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[4 3 2 1]→[4 3]
```

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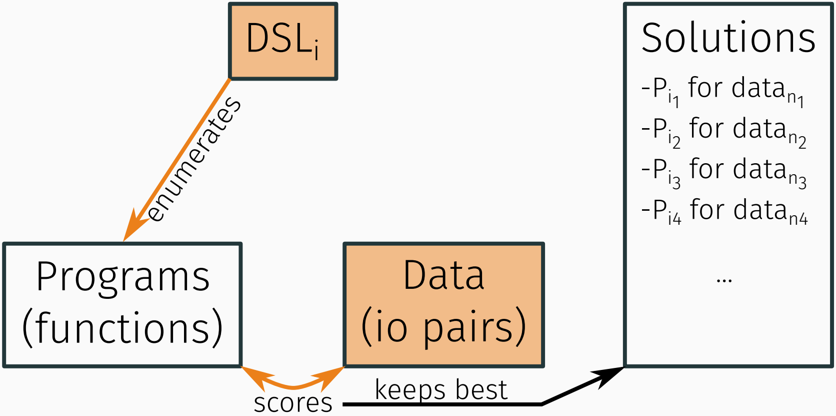

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Programs

```
f(l) = (f0 l l)

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```
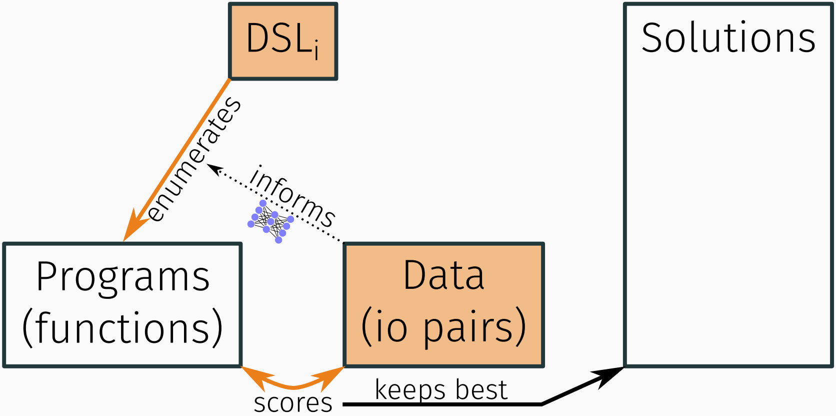

Data

```
[7 2]→[7 2 7 2]
["a"]→["a" "a"]

[7 2 3]→[7 3]
[1 2 3 4]→[3 4]
[4 3 2 1]→[4 3]
```

DreamCoder — Wake



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f0(l,r) = foldr r l cons
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```

Programs

```
f(l) = (f0 l l)

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```

Data

```
[7 2]→[7 2 7 2]
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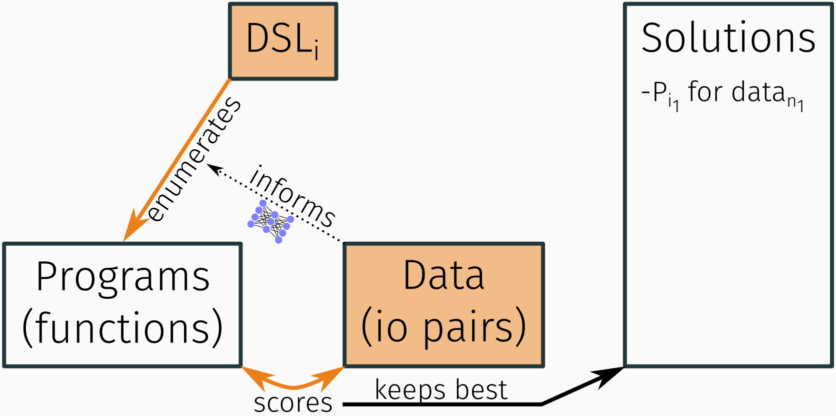

DSL

```
f0(l,r) = foldr r l cons
f1(l,p) = foldr r nil
        (λ (x a)
          (if (p x)
            (cons x a)
            a))
```

Programs

```
f(l) = (f0 l l)

f(l) = (f1 l (λx. > x 2))
```

Data

```
[7 2]→[7 2 7 2]
["a"]→["a" "a"]

[7 2 3]→[7 3]
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```
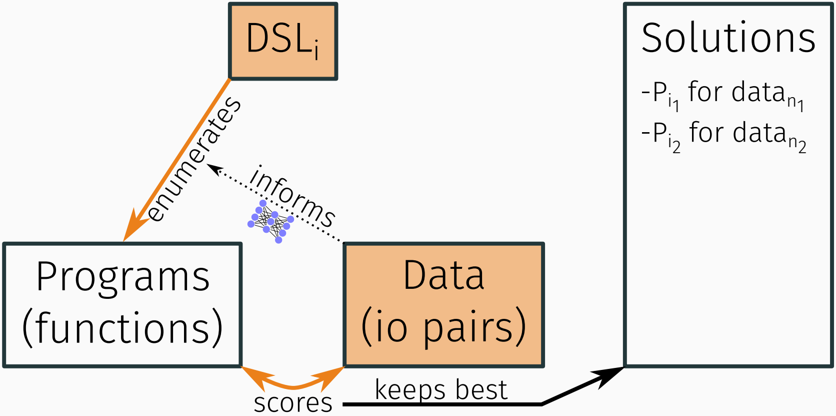

DreamCoder — Wake



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```
f0(l,r) = foldr r l cons
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```

Programs

```
f(l) = (f0 l l)

f(l) = (f1 l (λx. > x 2))
```

Data

```
[7 2]→[7 2 7 2]
["a"]→["a" "a"]

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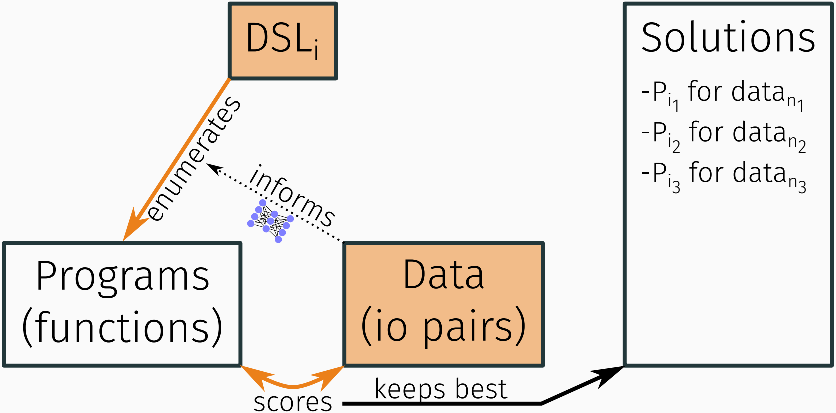

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Programs

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[7 2]→[7 2 7 2]
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[7 2 3]→[7 3]
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```
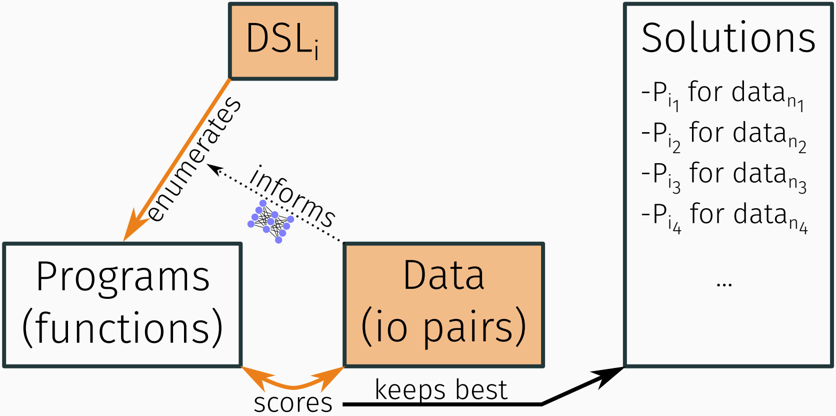

DreamCoder — Wake



DSL

```
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f1(l,p) = foldr r nil
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             (if (p x)
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               a))
```
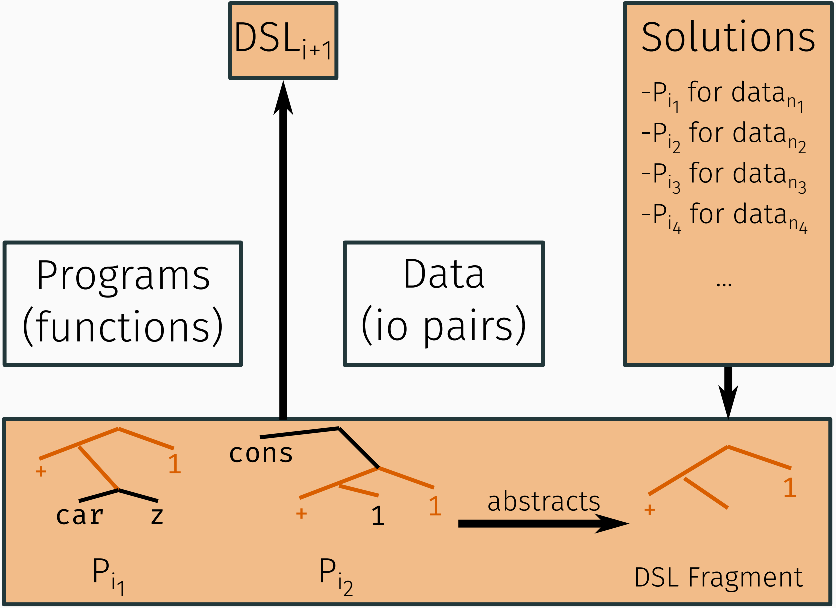

Programs

```
f(l) = (f0 l l)

f(l) = (f1 l (λx. > x 2))
```

Data

```
[7 2]→[7 2 7 2]
["a"]→["a" "a"]

[7 2 3]→[7 3]
[1 2 3 4]→[3 4]
[4 3 2 1]→[4 3]
```

DreamCoder — Sleep-G



DreamCoder — Sleep-R (Experience Replay)

DSL_{i+1}

Solutions

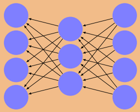
- P_{i_1} for data_{n_1}
- P_{i_2} for data_{n_2}
- P_{i_3} for data_{n_3}
- P_{i_4} for data_{n_4}

...

Programs (functions)

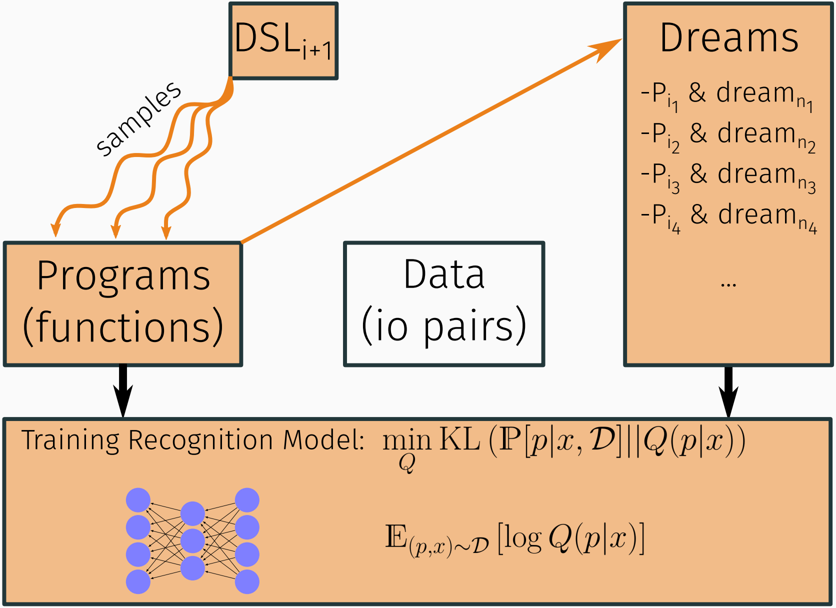
Data (io pairs)

Training Recognition Model: $\min_{Q} \mathrm{KL}\left(\mathbb{P}[p|x,\mathcal{D}]||Q(p|x)\right)$



$$\frac{\mathbb{P}\left[x_n, p_n|\mathcal{D}\right]}{\sum_{(x_n,p'_n)} \mathbb{P}\left[x_n, p_n|\mathcal{D}\right]} \log Q(p_n|x_n)$$

DreamCoder — Sleep-R (Dreaming)



List functions — Created & investigated by Lucas Morales

Name	Input	Output
repeat-3	[7 0]	[7 0 7 0 7 0]
drop-3	[0 3 8 6 4]	[6 4]
rotate-2	[8 14 1 9]	[1 9 8 14]
count-head-in-tail	[1 2 1 1 3]	2
keep-div-5	[5 9 14 6 3 0]	[5 0]
product	[7 1 6 2]	84

Discovers 38 concepts, including ‘filter’.

Text editing

In the style of FlashFill (Gulwani 2012)

Text Editing

+106 769-438→106.769.438
+83 973-831→83.973.831

f(s) = (f_0 "." "-"
  (f_0 "." " "
   (cdr s)))

Temple Anna H →TAH
Lara Gregori→LG

f(s) = (f_2 s)

f_0(s,a,b) = (map (λ (x)
  (if (= x a) b x)) s)
(f_0: *Performs character substitution*)

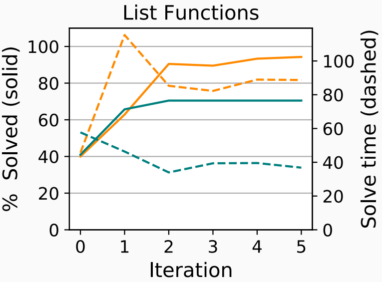
f_1(s,c) = (foldr s s (λ (x a)
  (cdr (if (= c x) s a))))
(f_1: *Drop characters from* s *until* c *reached*)

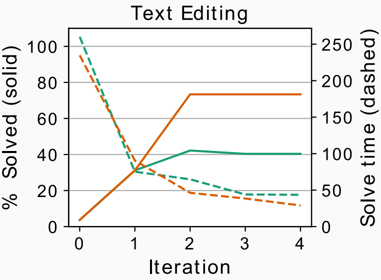
f_2(s) = (unfold s is-nil car
  (λ (z) (f_1 z " ")))
(f_2: *Abbreviates a sequence of words*)

SyGuS problems: solves 3% before learning, vs 75% after learning.

Best prior work: 80%

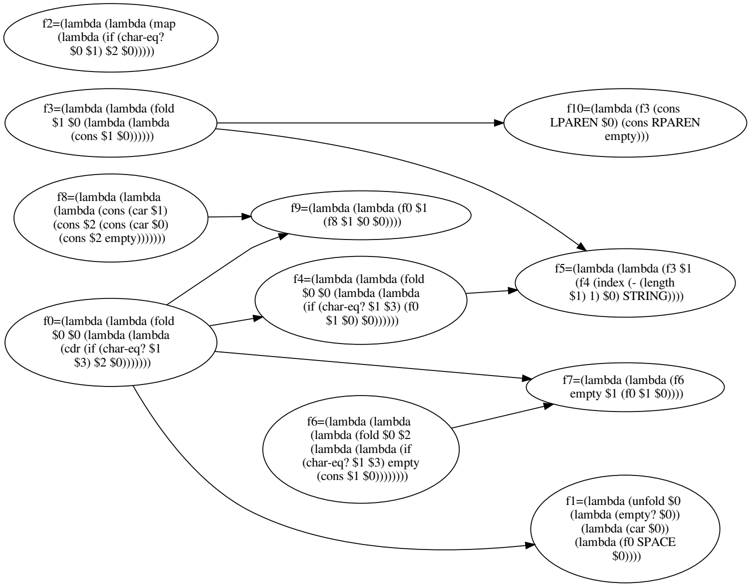
List functions & Text editing: Learning curves on hold out tasks





Learning curves for DreamCoder both with (in orange) and without (in teal) the recognition model. Solid lines: % holdout testing tasks solved w/ 10m timeout. Dashed lines: Average solve time, averaged only over tasks that are solved.

Learned text processing DSL



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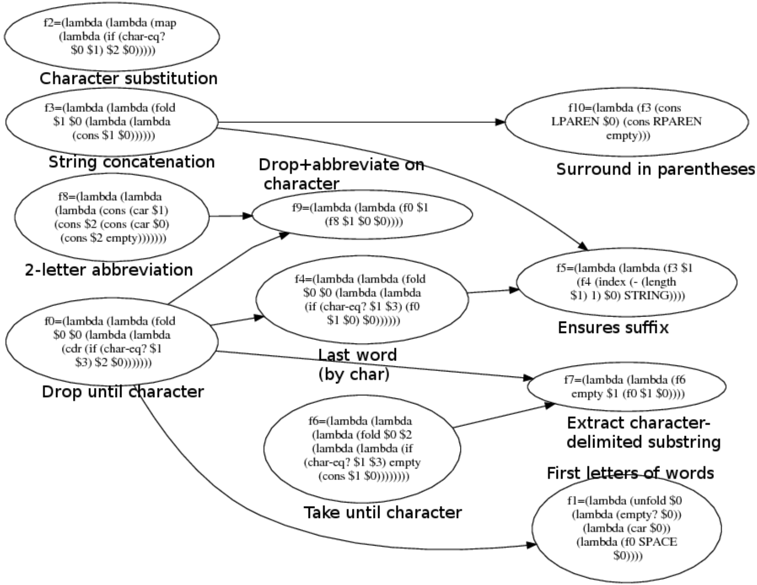


Learning the fundamentals of programming

Programs & Tasks	DSL

`[2 1 4]`$\rightarrow$`[2 1 4 0]`
`[9 8]`$\rightarrow$`[9 8 0]`
$f(\ell) =$ `(`f_2 `cons` ℓ `(cons 0 nil))`
	f_0`(p,f,n,x)` $=$ `(if (p x) nil (cons (f x) (`f_0` (n x))))`
	(f_0: *unfold*)

`[2 5 6 0 6]`$\rightarrow$`19`
`[9 2 7 6 3]`$\rightarrow$`27`
$f(\ell) =$ `(`f_2 `+` ℓ `0)`
	f_1`(i,l)` $=$ `(if (= i 0) (car l) (`f_1` (- i 1) (cdr l))))`
	(f_1: *index*)
	f_2`(f,l,x)` $=$ `(if (empty? l) x (f (car l) (`f_2` (cdr l))))`
	(f_2: *fold*)

`[4 2 6 4]`$\rightarrow$`[8 4 12 8]`
`[2 3 0 7]`$\rightarrow$`[4 6 0 14]`
$f(\ell) =$ `(`f_3 `(`λ `(x) (+ x x))` ℓ`)`
	f_3`(f,l)` $=$ `(`f_2` nil l (`λ` (x a) (cons (f x) a)))`
	(f_3: *map*)
	$f_4(\ell) =$ `(if (empty?` ℓ`) 0 (+ 1 (`f_4` (cdr` ℓ`))))`
	(f_4: *length*)

`[1 5 2 9]`$\rightarrow$`[1 2]`
`[3 8 1 3 1 2]`$\rightarrow$`[3 1 1]`
$f(\ell) =$ `(`f_0 `empty? car (`λ `(l) (cdr (cdr l)))` ℓ`)`
	$f_5(n) =$ `(`f_0 `(=` n`) (`λ` (x) x) (+ 1) 0)`
	(f_5: *range*)

McCarthy 1959 Lisp $\longrightarrow$ Modern functional programming

22 tasks. 64 CPUs. 93 hours.

Symbolic regression from visual input

Symbolic Regression

$f(\mathrm{x}) = (f_1\ \mathtt{x})$

$f(\mathrm{x}) = (f_6\ \mathtt{x})$

$f(\mathrm{x}) = (f_4\ \mathtt{x})$

$f(\mathrm{x}) = (f_3\ \mathtt{x})$

$f_0(\mathrm{x}) = \mathtt{(+\ x\ real)}$

$f_1(\mathrm{x}) = \mathtt{(}f_0\ \mathtt{(\star\ real\ x))}$

$f_2(\mathrm{x}) = \mathtt{(}f_1\ \mathtt{(\star\ x\ (}f_0\ \mathtt{x)))}$

$f_3(\mathrm{x}) = \mathtt{(}f_0\ \mathtt{(\star\ x\ (}f_2\ \mathtt{x)))}$

$f_4(\mathrm{x}) = \mathtt{(}f_0\ \mathtt{(\star\ x\ (}f_3\ \mathtt{x)))}$

(f_4: 4th order polynomial)

$f_5(\mathrm{x}) = \mathtt{(/\ real\ x)}$

$f_6(\mathrm{x}) = \mathtt{(}f_5\ \mathtt{(}f_0\ \mathtt{x))}$

(f_6: rational function)

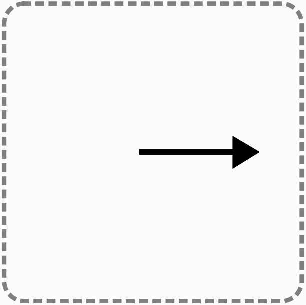
DSL

`OP ::= FW x | RT x | UP | DOWN | SET state`

Tasks

`task : image`

```
FW 1
```



Turtle graphics — Created & investigated by Mathias Sablé–Meyer

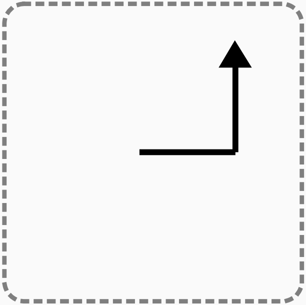
DSL

`OP ::= FW x | RT x | UP | DOWN | SET state`

Tasks

`task : image`

```
FW 1
RT π/2
FW 1
```



Turtle graphics — Created & investigated by Mathias Sablé–Meyer

DSL

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Tasks

`task :  image`

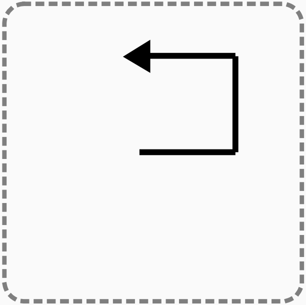
FW 1

RT $\frac{\pi}{2}$

FW 1

RT $\frac{\pi}{2}$

FW 1



Turtle graphics — Created & investigated by Mathias Sablé–Meyer

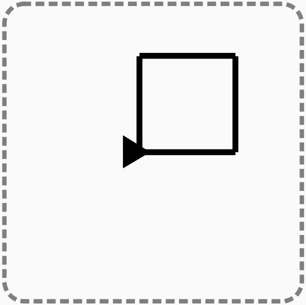
DSL

`OP ::= FW x | RT x | UP | DOWN | SET state`

Tasks

`task :  image`

```
for i in range(4)
> FW 1
> RT π/2
```



Turtle graphics — Created & investigated by Mathias Sablé–Meyer

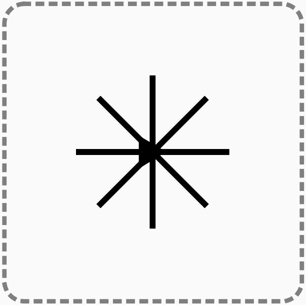
DSL

`OP ::= FW x | RT x | UP | DOWN | SET state`

Tasks

`task :  image`

```
for i in range(8)
> FW 1
> SET origin
> RT 2π/8
```



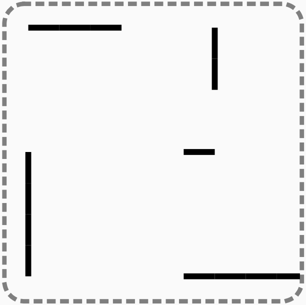
DSL

`OP ::= FW x | RT x | UP | DOWN | SET state`

Tasks

`task :  image`

```
for i in range(8)
> PU
> FW i/2
> PD
> FW i/2
> RT π/2
```



Turtle graphics — Created & investigated by Mathias Sablé–Meyer

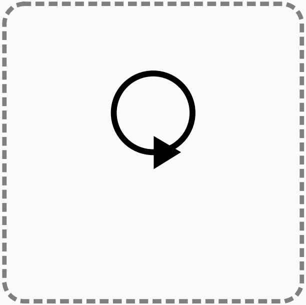
DSL

`OP ::= FW x | RT x | UP | DOWN | SET state`

Tasks

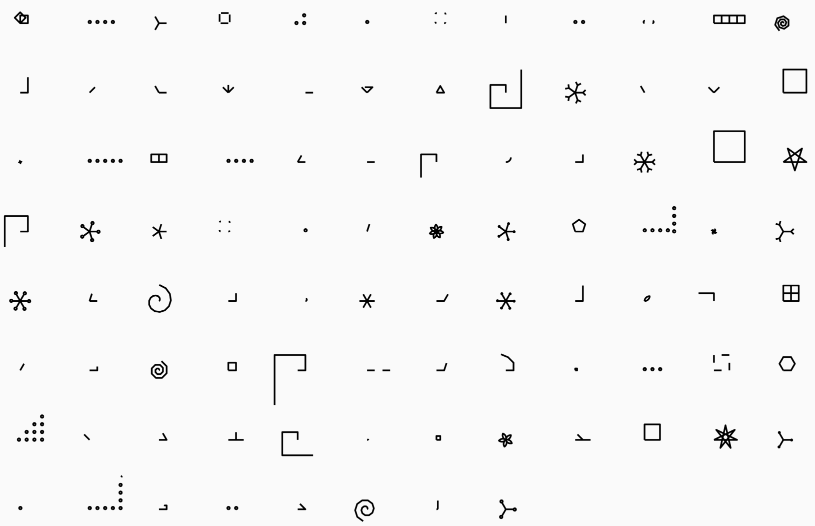
`task :  image`

```
for i in range(∞)
> FW ε
> RT ε
```

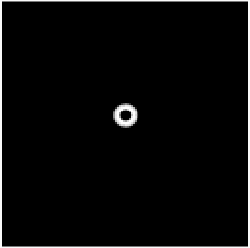


`NUM ::= 1 | π | ∞ | ε | + | - | * | /`

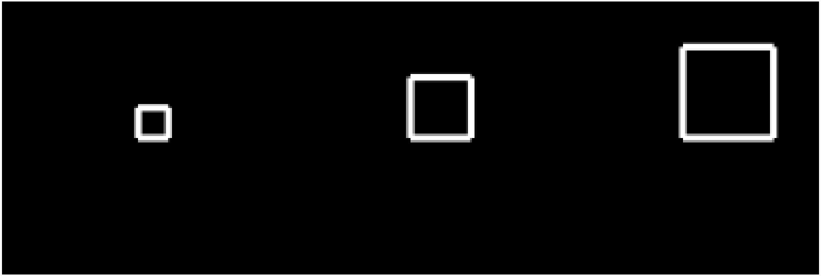
Turtle graphics — Training tasks



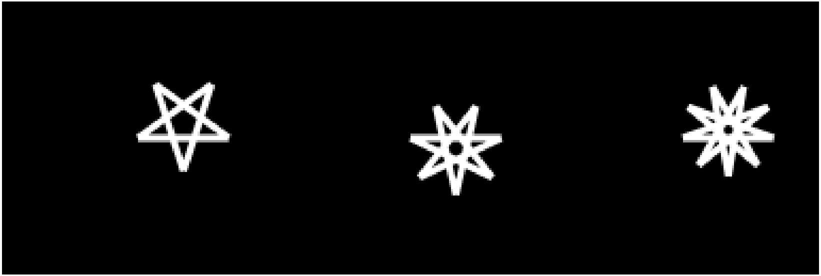
Turtle graphics — Illustration of learned DSL



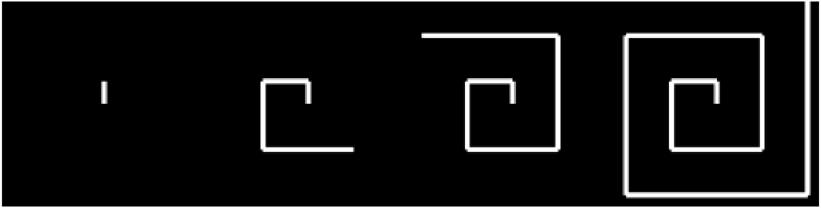
Turtle graphics — Illustration of learned DSL



Turtle graphics — Illustration of learned DSL



Turtle graphics — Illustration of learned DSL

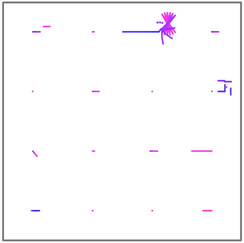


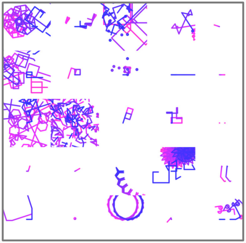
Turtle graphics — Illustration of learned DSL

Turtle graphics — Illustration of learned DSL

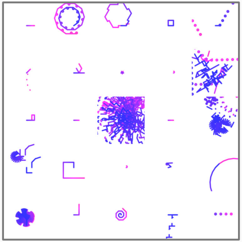
Turtle graphics — Dreams

Before training



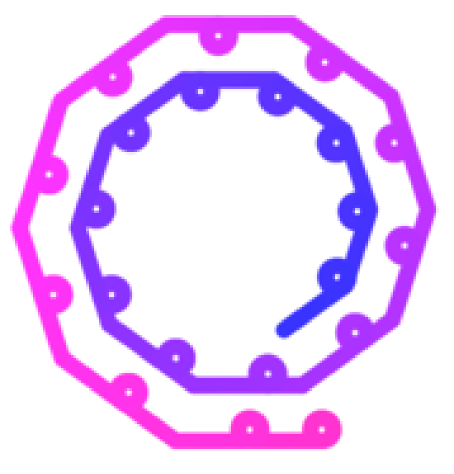


Plateau 5 minutes

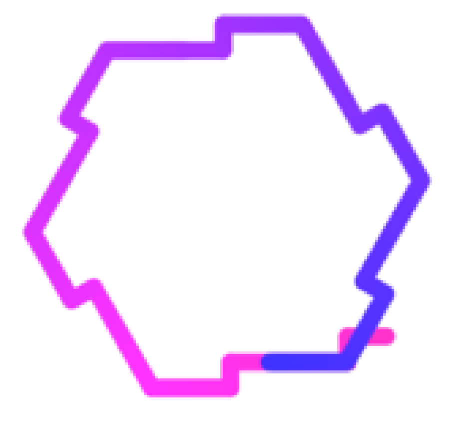


Plateau 2 hours

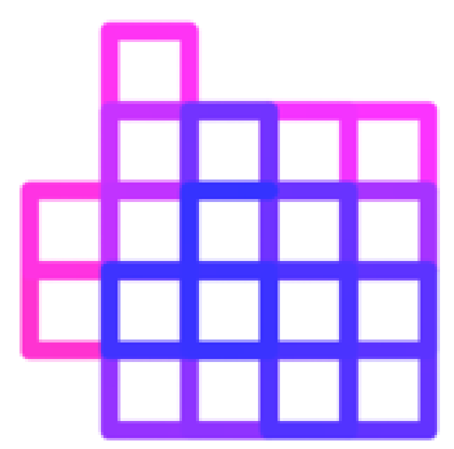
Turtle graphics — Dreaming from learned generative model



Turtle graphics — Dreaming from learned generative model

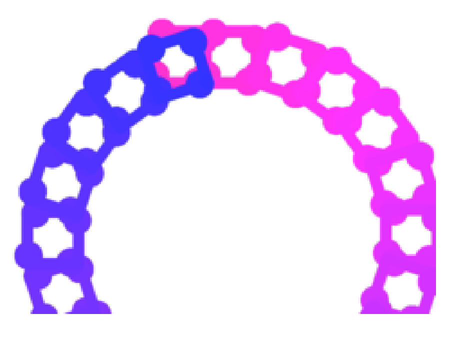


Turtle graphics — Dreaming from learned generative model

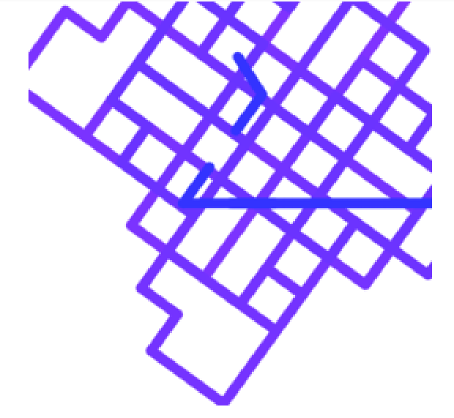


Turtle graphics — Dreaming from learned generative model

Turtle graphics — Dreaming from learned generative model



Turtle graphics — Dreaming from learned generative model



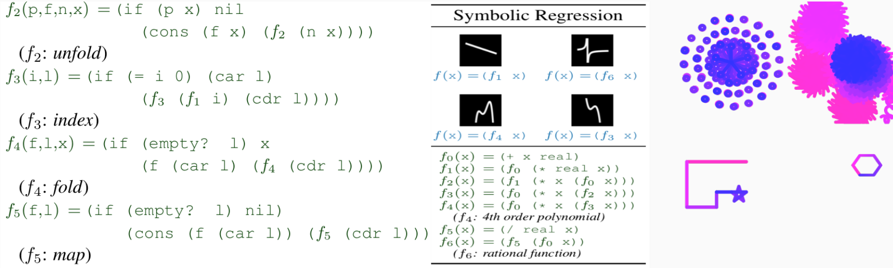
Turtle graphics — Dreaming from learned generative model

Turtle graphics — Dreaming from learned generative model

More human-like machine intelligence

- Acquiring a domain-specific representation (DSL)
- Learning to use that representation (recognition model)

DreamCoder: an algorithm for jointly realizing these goals

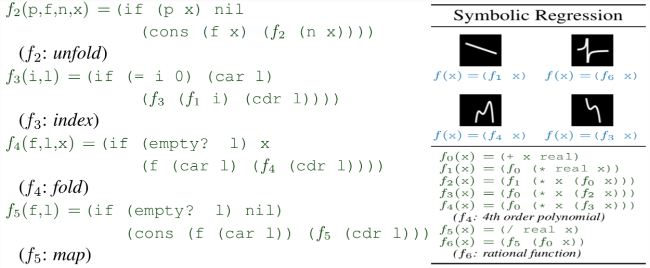


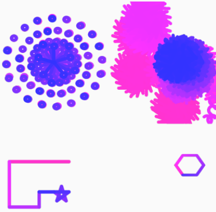
Vision

More human-like machine intelligence

- Acquiring a domain-specific representation (DSL)
- Learning to use that representation (recognition model)

DreamCoder: an algorithm for jointly realizing these goals





The End.