

Growing libraries of concepts with wake-sleep program induction

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Joint with: Lucas Morales, Armando Solar-Lezama, Joshua B. Tenenbaum

Heavy inspiration from: Eyal Dechter

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MIT

The Language of Thought

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The Language of Thought

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The Language and Thought Series

Jerrold J. Katz

D. Terence Langendoen

George A. Miller

A FORMAL THEORY OF INDUCTIVE INFERENCE, Part I*†

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Engineering the language of thought

Universal Theory



Theory



Model



Data

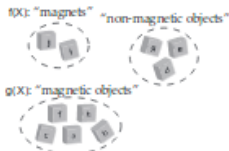
Magnetism

Core Predicates: $f(X)$, $g(X)$

Surface Predicates: $interacts(X,Y)$

Laws:

$interacts(X,Y) \leftarrow f(X) \wedge f(Y)$
 $interacts(X,Y) \leftarrow f(X) \wedge g(Y)$
 $interacts(X,Y) \leftarrow interacts(Y,X)$



Probabilistic Horn Clause Grammar

Taxonomy

Core Predicates: $s(X,Y)$, $t(X,Y)$

Surface Predicates: $has_a(X,Y)$, $is_a(X,Y)$

Laws:

$is_a(X,Y) \leftarrow s(X,Y)$
 $has_a(X,Y) \leftarrow t(X,Y)$
 $has_a(X,Y) \leftarrow is_a(X,Z) \wedge has_a(Z,Y)$
 $is_a(X,Y) \leftarrow is_a(X,Z) \wedge is_a(Z,Y)$



"a shark is a fish"
 "a bird can fly"
 "a canary can fly"
 "a salmon can breathe"
 ...

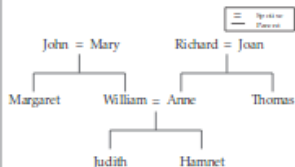
Kinship

Core Predicates: $u(X)$, $v(X,Y)$, $w(X,Y)$

Surface Predicates: $female(X)$, $child(X,Y)$, $parent(X,Y)$, $spouse(X,Y)$, $father(X,Y)$, ...

Laws:

$female(X) \leftarrow u(X)$
 $spouse(X,Y) \leftarrow v(X,Y)$
 $spouse(X,Y) \leftarrow v(Y,X)$
 $child(X,Y) \leftarrow w(X,Y)$
 $child(X,Y) \leftarrow child(X,Z) \wedge spouse(Z,Y)$
 $father(X,Y) \leftarrow \neg female(X) \wedge child(X,Y)$
 ...



"John is William's father"
 "John is Judith's grandfather"
 "Judith is Hamnet's sister"
 "Margaret is Judith's aunt"
 ...

Engineering the language of thought

A

i) 

iii) 

ii) 

iv) 

B

i) 

iii) 

ii) 

iv) 

Table 1: Unicycle Wheel and Axle Assembly

ಲ	ಇ	ಉ	ಊ	ಋ
ಕ	ಖ	ಗ	ಘ	ಙ
ಚ	ಛ	ಜ	ಝ	ಞ
ಟ	ಠ	ಡ	ಢ	ನ
ತ	ಥ	ದ	ಧ	ಪ
ಕ	ಖ	ಗ	ಘ	ಙ

Table 2: Bicycle Wheel and Axle Assembly

ಯ	ಋ	ಌ	಍	ಏ
ಒ	ಓ	ಔ	ಋ	ಌ
ಋ	ಌ	಍	ಏ	ಏ
ಏ	ಏ	ಏ	ಏ	ಏ
ಏ	ಏ	ಏ	ಏ	ಏ
ಏ	ಏ	ಏ	ಏ	ಏ

Table 3: Motorcycle Wheel and Axle Assembly

ಕ	ಖ	ಗ	ಘ	ಙ
ಚ	ಛ	ಜ	ಝ	ಞ
ಟ	ಠ	ಡ	ಢ	ನ
ತ	ಥ	ದ	ಧ	ಪ
ಕ	ಖ	ಗ	ಘ	ಙ
ಚ	ಛ	ಜ	ಝ	ಞ

Table 4: Unicycle Wheel and Axle Assembly

ಕ	ಖ	ಗ	ಘ	ಙ
ಚ	ಛ	ಜ	ಝ	ಞ
ಟ	ಠ	ಡ	ಢ	ನ
ತ	ಥ	ದ	ಧ	ಪ
ಕ	ಖ	ಗ	ಘ	ಙ
ಚ	ಛ	ಜ	ಝ	ಞ

Table 5: Bicycle Wheel and Axle Assembly

ಯ	ಋ	ಌ	಍	ಏ
ಒ	ಓ	ಔ	ಋ	ಌ
ಋ	ಌ	಍	ಏ	ಏ
ಏ	ಏ	ಏ	ಏ	ಏ
ಏ	ಏ	ಏ	ಏ	ಏ
ಏ	ಏ	ಏ	ಏ	ಏ

Table 6: Motorcycle Wheel and Axle Assembly

ಕ	ಖ	ಗ	ಘ	ಙ
ಚ	ಛ	ಜ	ಝ	ಞ
ಟ	ಠ	ಡ	ಢ	ನ
ತ	ಥ	ದ	ಧ	ಪ
ಕ	ಖ	ಗ	ಘ	ಙ
ಚ	ಛ	ಜ	ಝ	ಞ

Growing a domain-specific language of thought

Goal: acquire domain-specific knowledge needed to induce a class of programs

Growing a domain-specific language of thought

Goal: acquire domain-specific knowledge needed to induce a class of programs

- Library of concepts (declarative knowledge)
- Inference strategy (procedural knowledge)

DSL: Library of concepts

Tasks and Programs

```
[7 2 3] → [7 3]
[1 2 3 4] → [3 4]
[4 3 2 1] → [4 3]    [7 3] → False
f(ℓ) = (f1 ℓ (λ (x)    [3] → False
    (> x 2)))          [9 0 0] → True
                        [0] → True
                        [0 7 3] → True
                        f(ℓ) = (f3 ℓ 0)
[2 7 8 1] → 8
[3 19 14] → 19
f(ℓ) = (f2 ℓ)
```

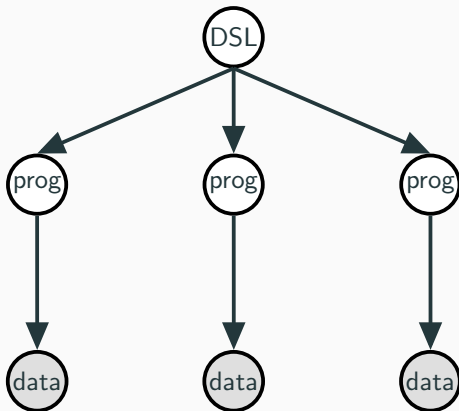
DSL

```
f0(ℓ, r) = (foldr r ℓ cons)
           (f0: Append lists r and ℓ)
f1(ℓ, p) = (foldr ℓ nil (λ (x a)
    (if (p x) (cons x a) a)))
           (f1: Higher-order filter function)
f2(ℓ) = (foldr ℓ 0 (λ (x a)
    (if (> a x) a x)))
           (f2: Maximum element in list ℓ)
f3(ℓ, k) = (foldr ℓ (is-nil ℓ)
    (λ (x a) (if a a (= k x))))
           (f3: Whether ℓ contains k)
```

- **Wake:** Solve problems by writing programs
- **Sleep:** Improve DSL and neural recognition model:
 - **Sleep-G:** Improve DSL (**G**enerative model)
 - **Sleep-R:** Improve **R**ecognition model

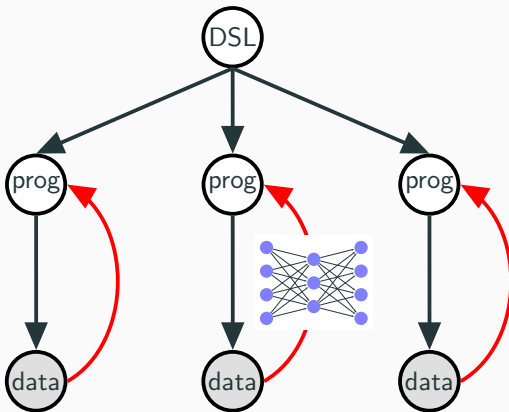
Combines ideas from Wake-Sleep & Exploration-Compression algorithm by Eyal Dechter

DSL learning as Bayesian inference

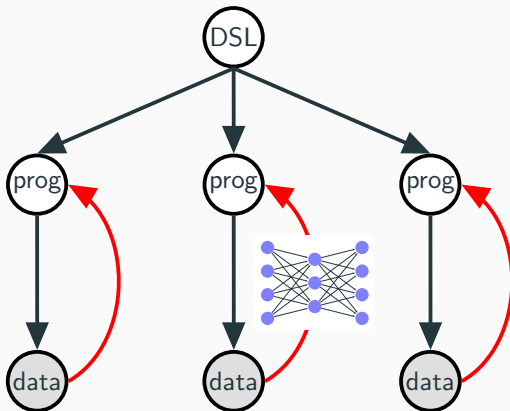


[Dechter et al., 2013] [Liang et al, 2010]; [Lake et al, 2015]

DSL learning as **amortized** Bayesian inference

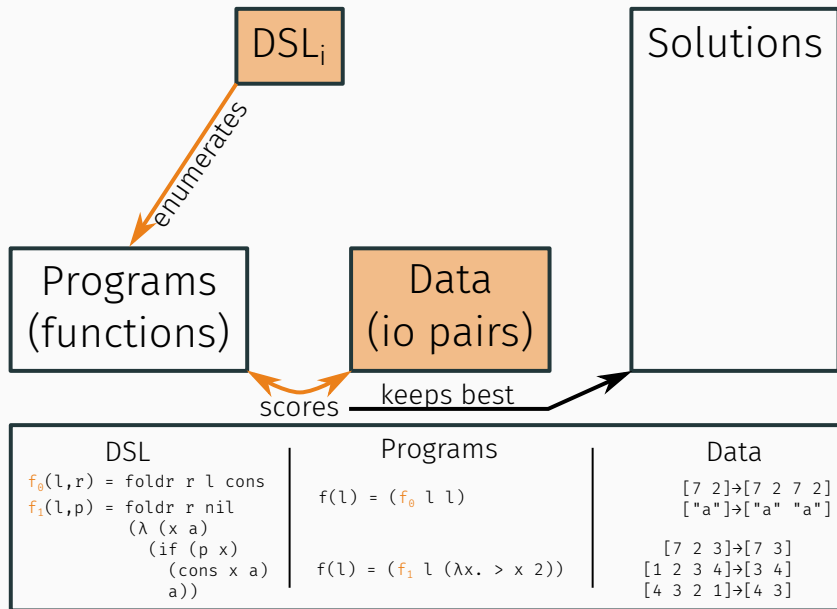


DSL learning as **amortized** Bayesian inference

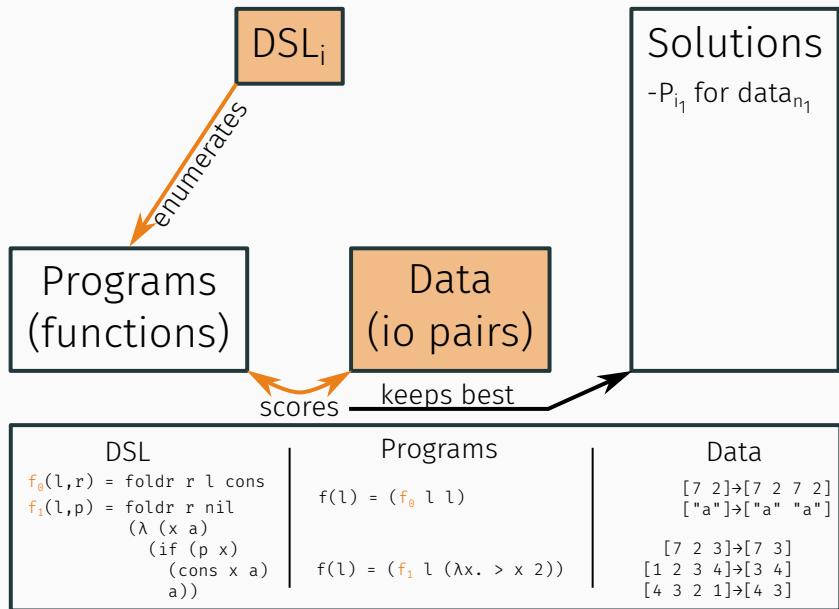


New: amortized inference +
better program representation (Lisp) +
better DSL inference

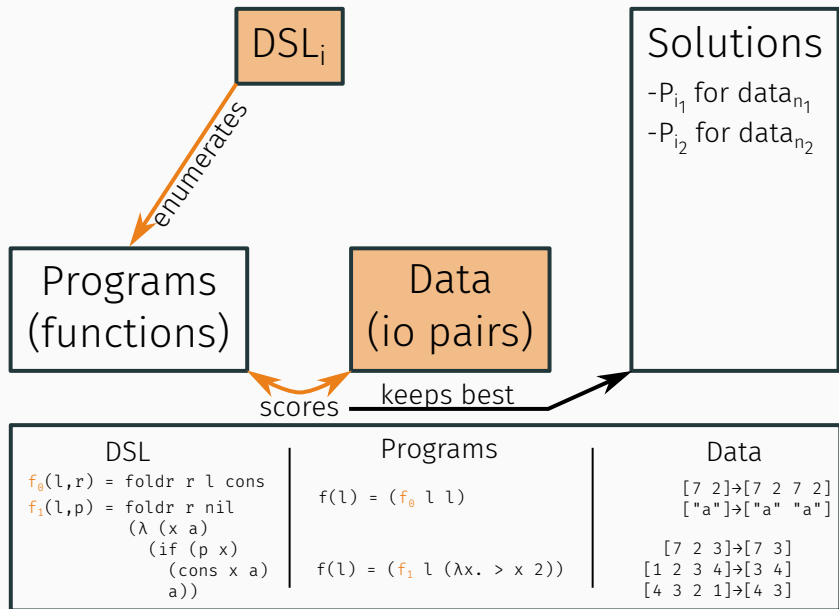
Wake — as in Exploration/Compression Algorithm



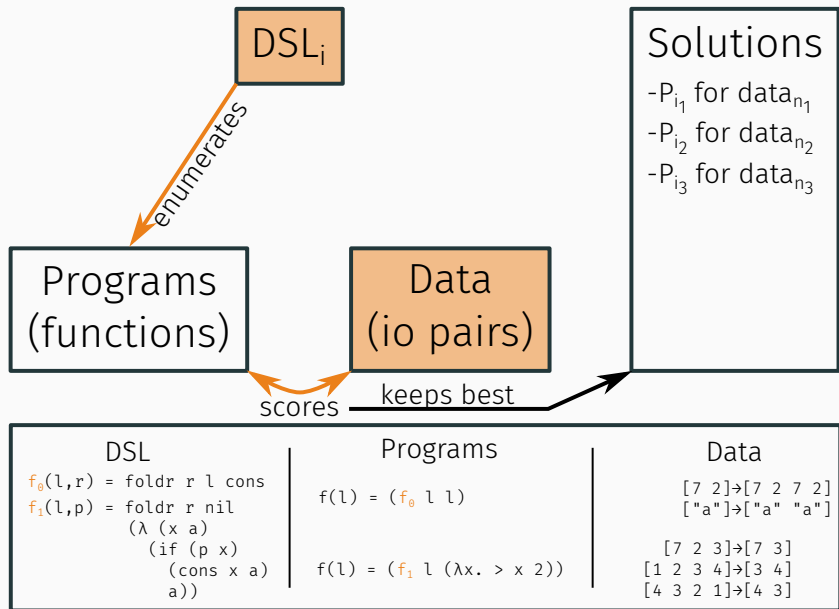
Wake — as in Exploration/Compression Algorithm



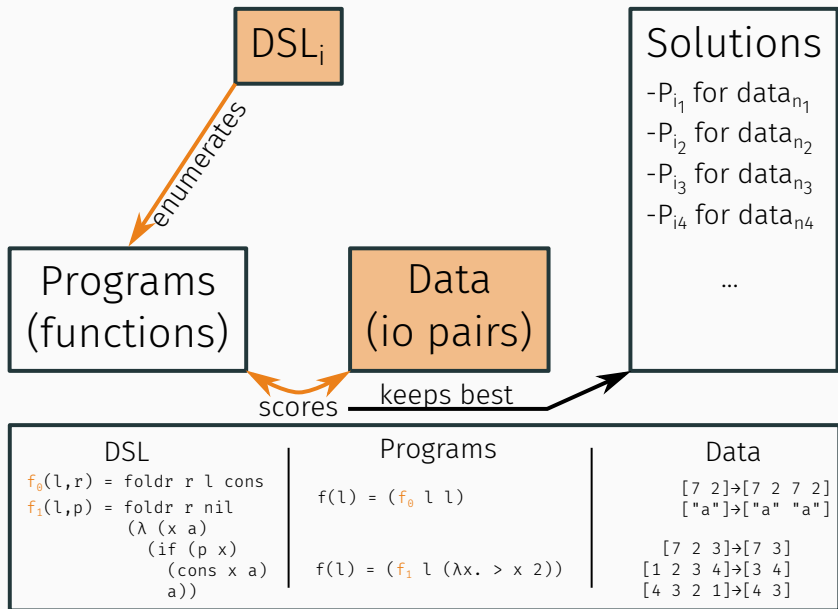
Wake — as in Exploration/Compression Algorithm

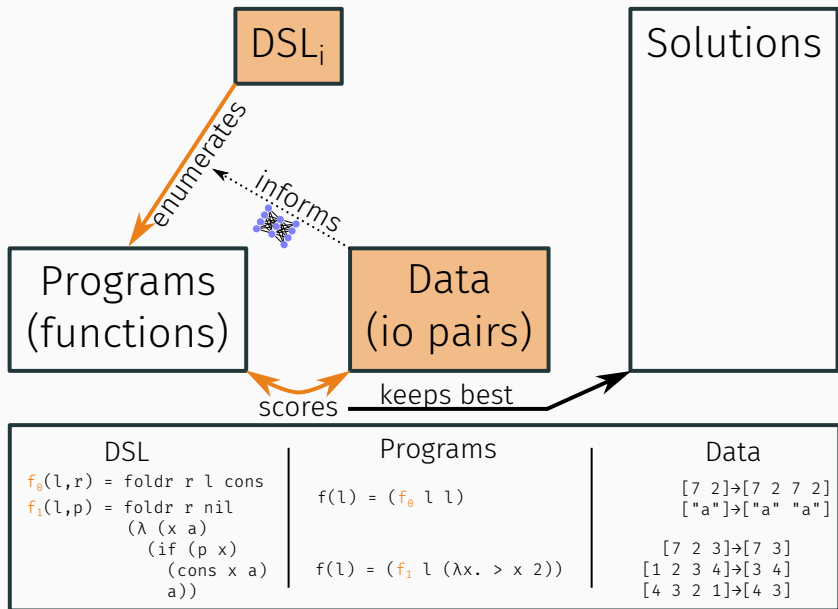


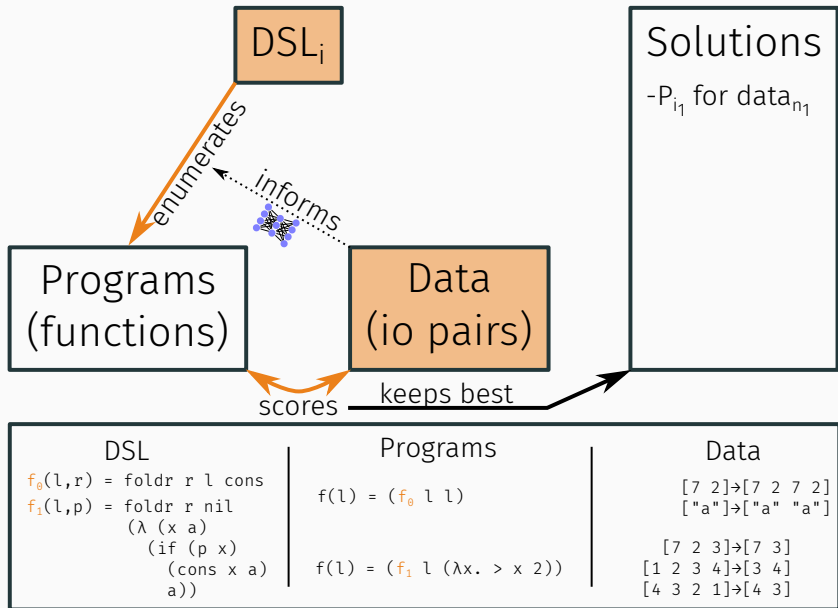
Wake — as in Exploration/Compression Algorithm

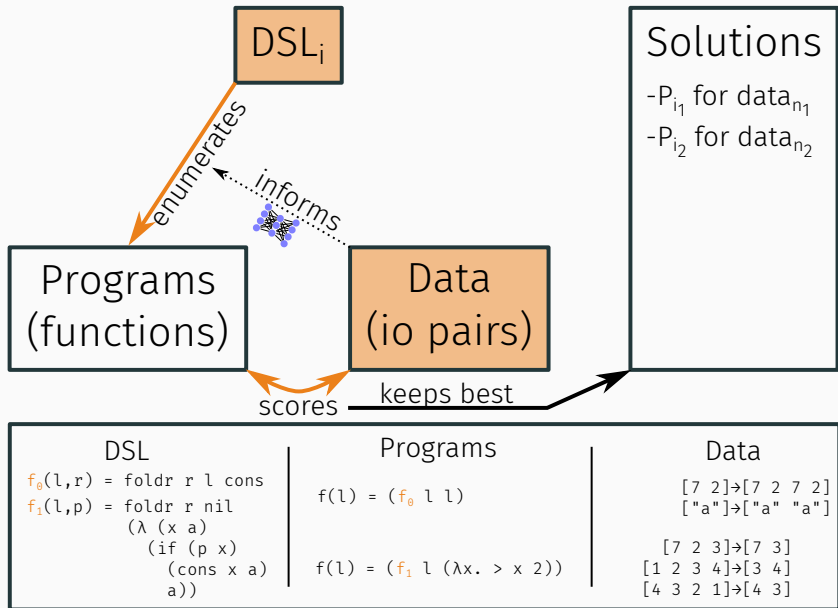


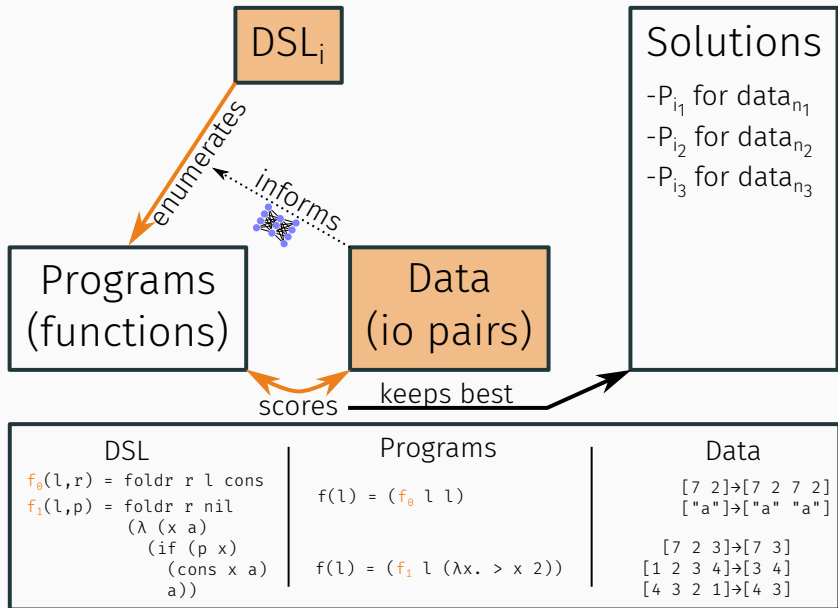
Wake — as in Exploration/Compression Algorithm

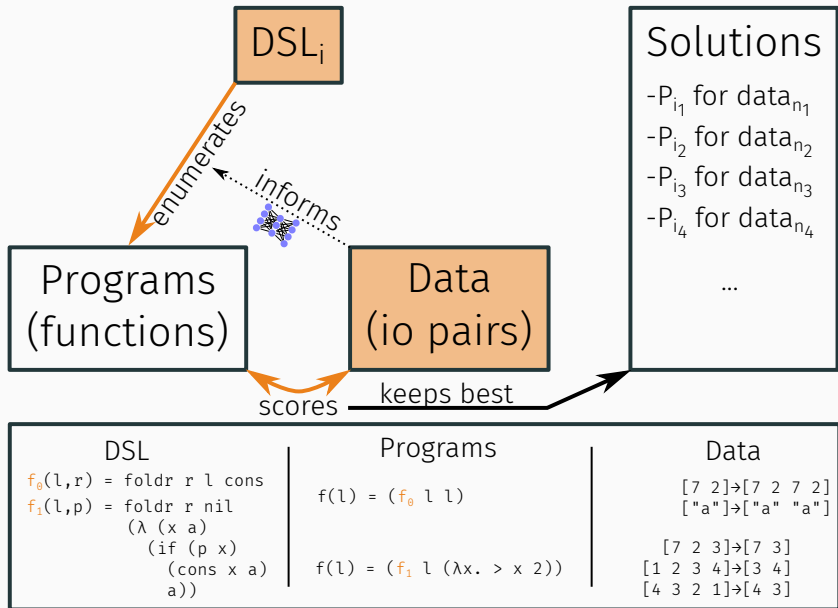




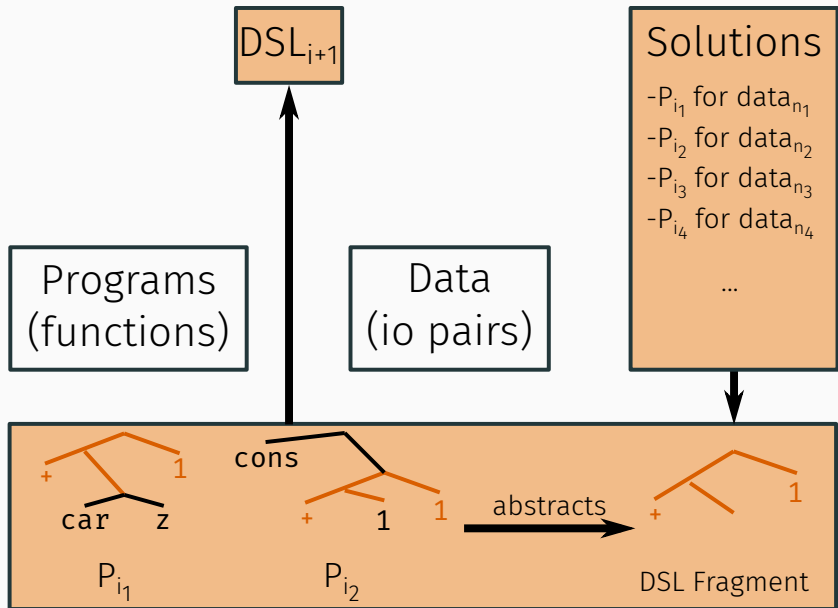




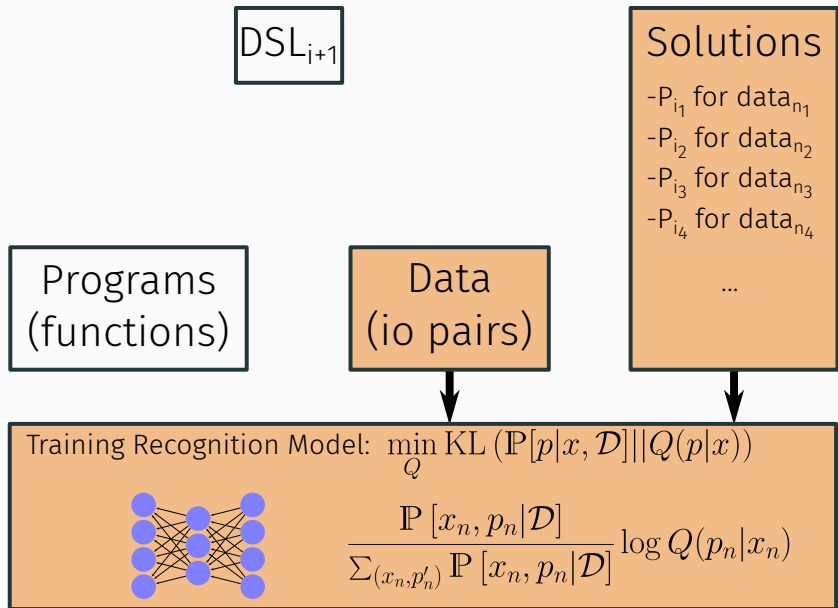




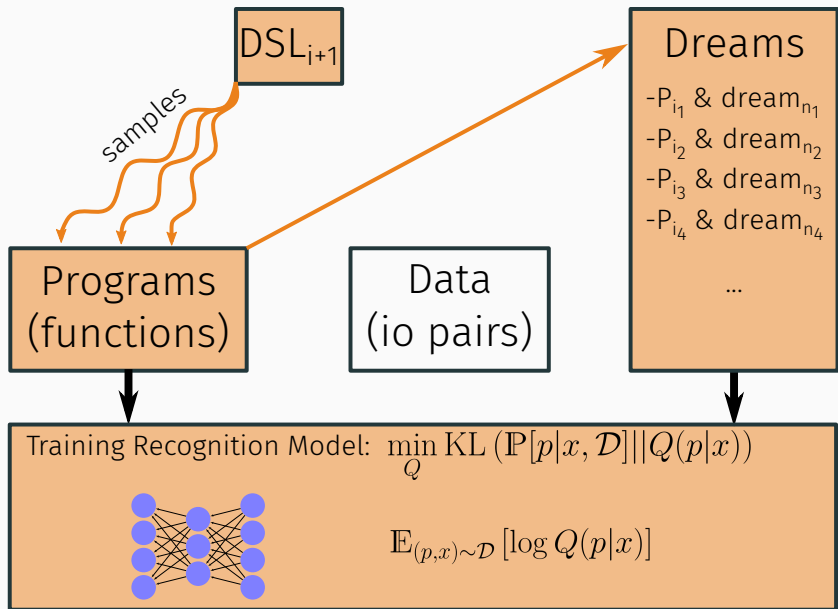
DreamCoder — Sleep-G



DreamCoder — Sleep-R (Experience Replay)



DreamCoder — Sleep-R (Dreaming)

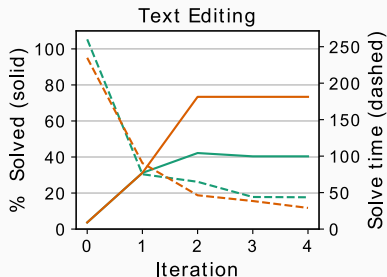
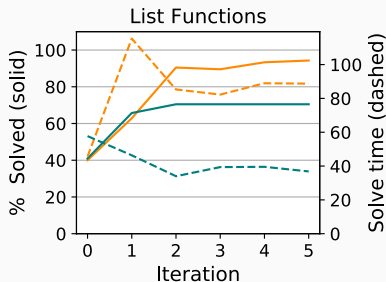


List functions — Created & investigated by Lucas Morales

Name	Input	Output
repeat-3	[7 0]	[7 0 7 0 7 0]
drop-3	[0 3 8 6 4]	[6 4]
rotate-2	[8 14 1 9]	[1 9 8 14]
count-head-in-tail	[1 2 1 1 3]	2
keep-div-5	[5 9 14 6 3 0]	[5 0]
product	[7 1 6 2]	84

Discovers 38 concepts, including 'filter'. With suitable curriculum can also learn 'map', 'fold', etc.

List functions & Text editing: Learning curves on hold out tasks



Learning curves for DreamCoder both with (in orange) and without (in teal) the recognition model. Solid lines: % holdout testing tasks solved w/ 10m timeout. Dashed lines: Average solve time, averaged only over tasks that are solved.

DSL

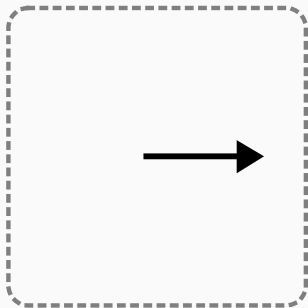
OP ::= FW x | RT x | UP | DOWN | SET state

Tasks

task : image



FW 1



DSL

OP ::= FW x | RT x | UP | DOWN | SET state

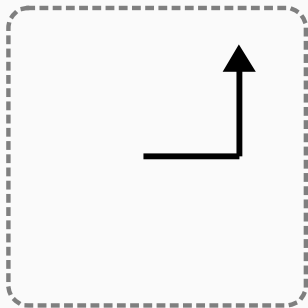
Tasks

task : image

```
FW 1
```

```
RT  $\frac{\pi}{2}$ 
```

```
FW 1
```



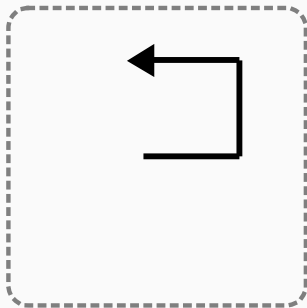
DSL

OP ::= FW x | RT x | UP | DOWN | SET state

Tasks

task : image

```
FW 1  
RT  $\frac{\pi}{2}$   
FW 1  
RT  $\frac{\pi}{2}$   
FW 1
```



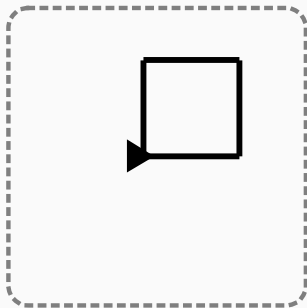
DSL

OP ::= FW x | RT x | UP | DOWN | SET state

Tasks

task : image

```
for i in range(4)
> FW 1
> RT  $\frac{\pi}{2}$ 
```



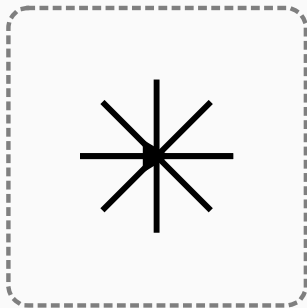
DSL

OP ::= FW x | RT x | UP | DOWN | SET state

Tasks

task : image

```
for i in range(8)
> FW 1
> SET origin
> RT  $\frac{2\pi}{8}$ 
```



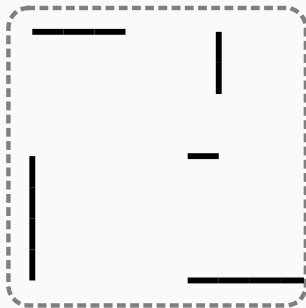
DSL

OP ::= FW x | RT x | UP | DOWN | SET state

Tasks

task : image

```
for i in range(8)
> PU
> FW  $\frac{i}{2}$ 
> PD
> FW  $\frac{i}{2}$ 
> RT  $\frac{\pi}{2}$ 
```



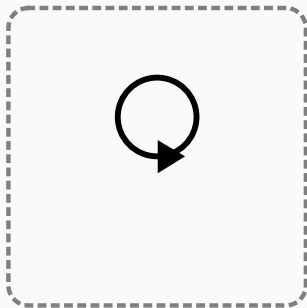
DSL

OP ::= FW x | RT x | UP | DOWN | SET state

Tasks

task : image

```
for i in range( $\infty$ )  
> FW  $\varepsilon$   
> RT  $\varepsilon$ 
```



DSL

`OP ::= FW x | RT x | UP | DOWN | SET state`

Tasks

`task : image`

```
for i in range(5 × ∞)
> FW i × ε
> RT ε
```



DSL

OP ::= FW x | RT x | UP | DOWN | SET state

Tasks

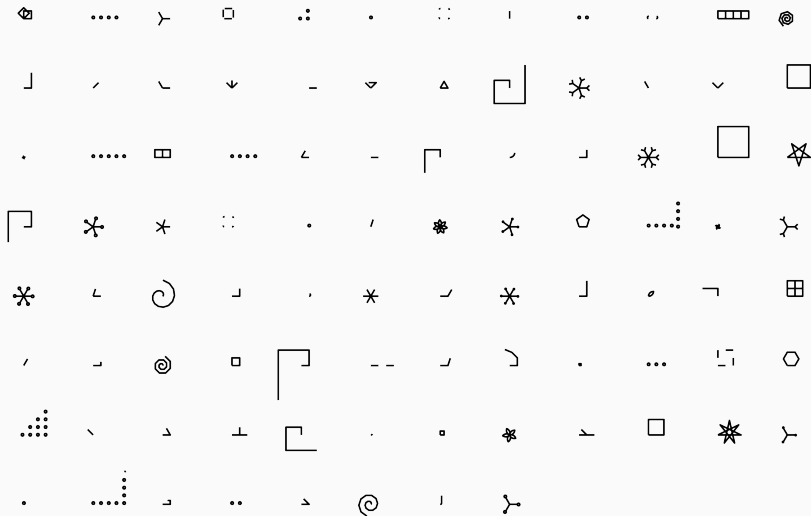
task : image

```
for i in range(5 × ∞)  
> FW i × ε  
> RT ε
```

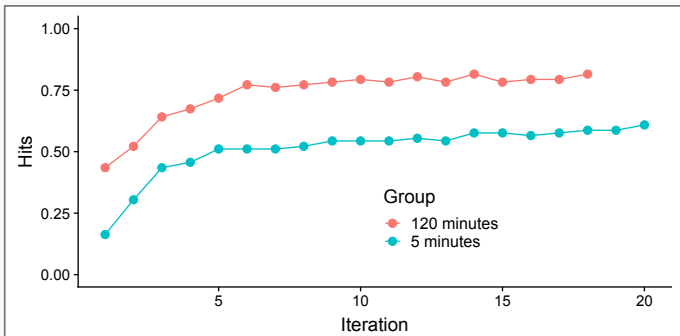


NUM ::= 1 | π | ∞ | ε | + | - | * | /

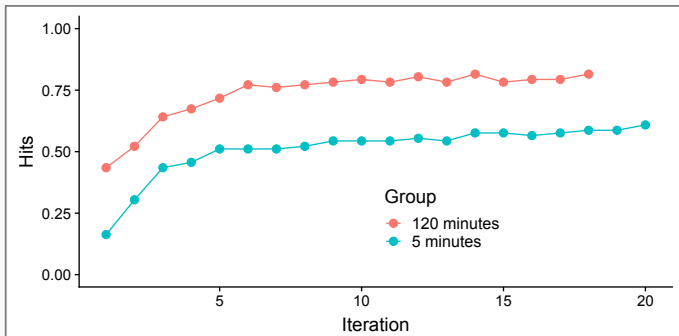
Turtle graphics — Training tasks



Turtle graphics — Learning curves



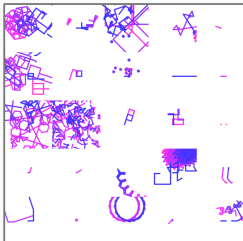
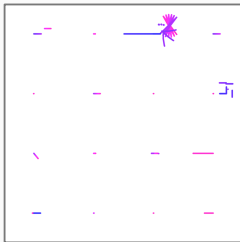
Turtle graphics — Learning curves



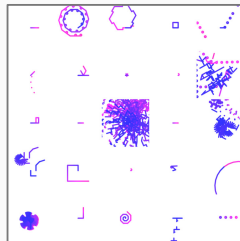
- $\frac{\pi}{2}$ and $\frac{\pi}{4}$ from π , 2, + and /
- A line of length n followed with a right angle
- Loops of length n that uses the number n inside.
- Unit line then teleport back to origin
- ...

Turtle graphics — Dreams

Before training

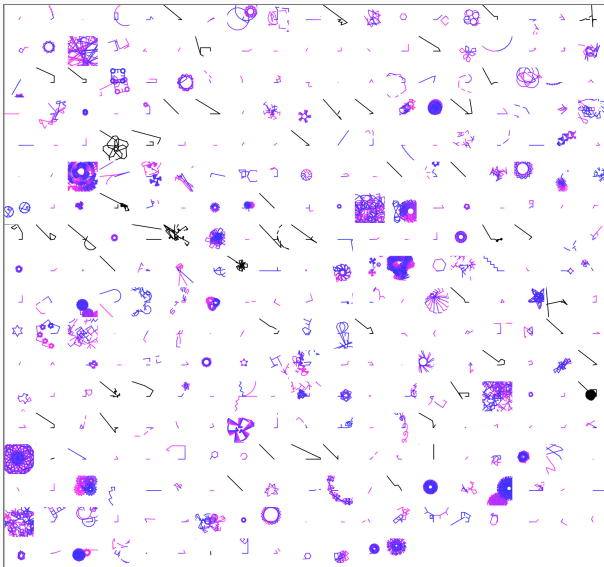


Plateau 5 minutes

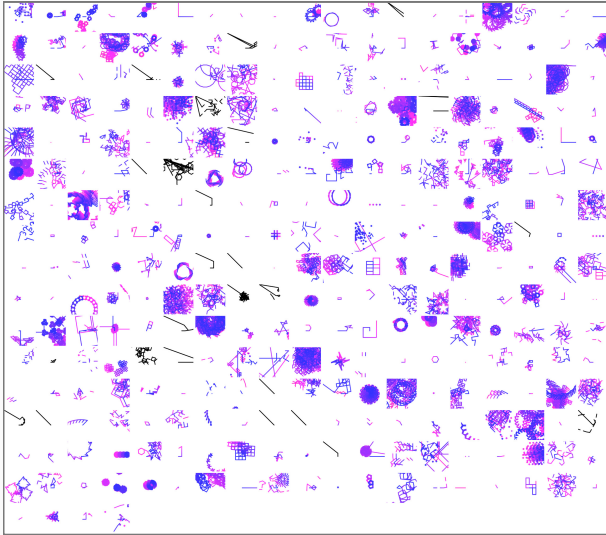


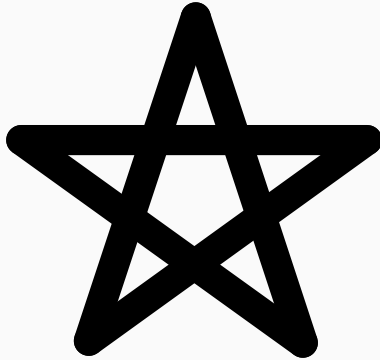
Plateau 2 hours

Turtle graphics — More dreams, 5 minutes, 1st iteration

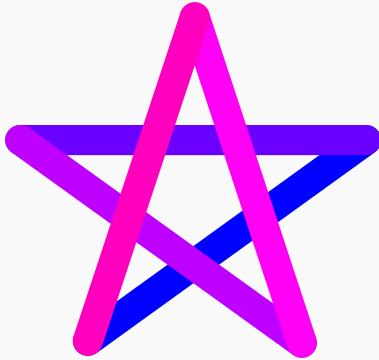


Turtle graphics — More dreams, 5 minutes, last iteration

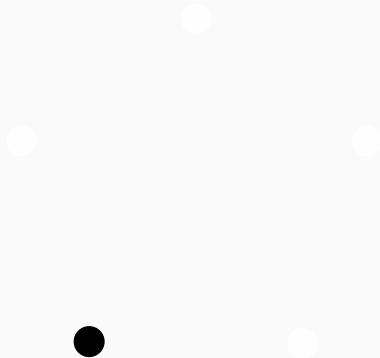




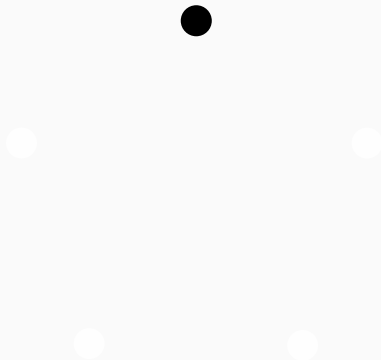
Turtle graphics — All the way down



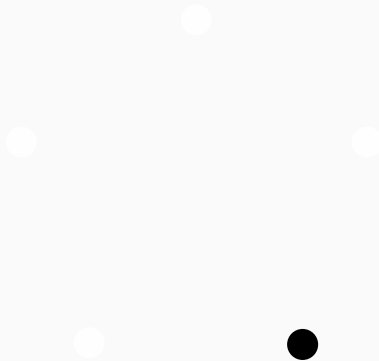
Turtle graphics — All the way down



Turtle graphics — All the way down



Turtle graphics — All the way down



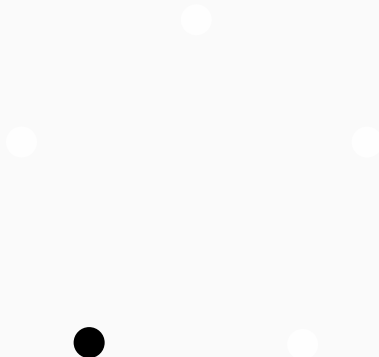
Turtle graphics — All the way down



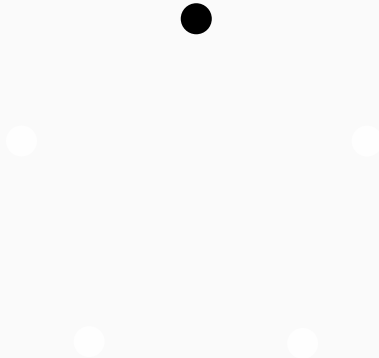
Turtle graphics — All the way down



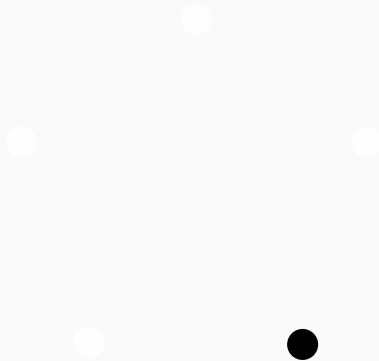
Turtle graphics — All the way down



Turtle graphics — All the way down



Turtle graphics — All the way down



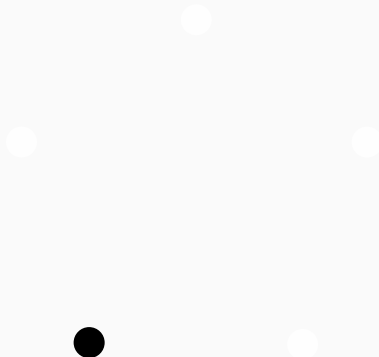
Turtle graphics — All the way down



Turtle graphics — All the way down



Turtle graphics — All the way down



Turtle graphics — All the way down



Turtle graphics — All the way down



Turtle graphics — All the way down



Turtle graphics — All the way down



Turtle graphics — All the way down



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Turtle graphics — All the way down



Turtle graphics — All the way down



Turtle graphics — All the way down



Turtle graphics — All the way down



Turtle graphics — All the way down



Turtle graphics — All the way down



Turtle graphics — All the way down



Turtle graphics — All the way down



Turtle graphics — All the way down



More human-like machine intelligence

Flexibly adapting to new problem domains: acquiring a domain-specific language-of-thought (DSL); learning how to use the language of thought (recognition model)

```
f2(p,f,n,x) = (if (p x) nil
                  (cons (f x) (f2 (n x))))

(f2: unfold)

f3(i,l) = (if (= i 0) (car l)
              (f3 (f1 i) (cdr l)))





(f3: index)

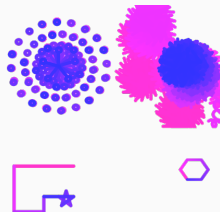
f4(f,l,x) = (if (empty? l) x
                (f (car l) (f4 (cdr l))))

(f4: fold)

f5(f,l) = (if (empty? l) nil
              (cons (f (car l)) (f5 (cdr l))))

(f5: map)
```

Symbolic Regression	
	
$f(x) = (f_1 \ x)$	$f(x) = (f_6 \ x)$
	
$f(x) = (f_4 \ x)$	$f(x) = (f_3 \ x)$
$f_0(x) = (+ \ x \ \text{real})$ $f_1(x) = (f_0 \ (* \ \text{real} \ x))$ $f_2(x) = (f_1 \ (* \ x \ (f_0 \ x)))$ $f_3(x) = (f_0 \ (* \ x \ (f_2 \ x)))$ $f_4(x) = (f_0 \ (* \ x \ (f_3 \ x)))$ (f4: 4th order polynomial)	
$f_5(x) = (/ \ \text{real} \ x)$ $f_6(x) = (f_5 \ (f_0 \ x))$ (f6: rational function)	



More human-like machine intelligence

Flexibly adapting to new problem domains: acquiring a domain-specific language-of-thought (DSL); learning how to use the language of thought (recognition model)

```
f2(p,f,n,x) = (if (p x) nil
                  (cons (f x) (f2 (n x))))

(f2: unfold)

f3(i,l) = (if (= i 0) (car l)
              (f3 (f1 i) (cdr l)))





(f3: index)

f4(f,l,x) = (if (empty? l) x
                (f (car l) (f4 (cdr l))))

(f4: fold)

f5(f,l) = (if (empty? l) nil
              (cons (f (car l)) (f5 (cdr l))))

(f5: map)
```

Symbolic Regression	
	
$f(x) = (f_1 \ x)$	$f(x) = (f_6 \ x)$
	
$f(x) = (f_4 \ x)$	$f(x) = (f_3 \ x)$
$f_0(x) = (+ \ x \ \text{real})$	
$f_1(x) = (f_0 \ (* \ \text{real} \ x))$	
$f_2(x) = (f_1 \ (* \ x \ (f_0 \ x)))$	
$f_3(x) = (f_0 \ (* \ x \ (f_2 \ x)))$	
$f_4(x) = (f_0 \ (* \ x \ (f_3 \ x)))$	
(f4: <i>4th order polynomial</i>)	
$f_5(x) = (/ \ \text{real} \ x)$	
$f_6(x) = (f_5 \ (f_0 \ x))$	
(f6: <i>rational function</i>)	



The End.