# DreamCoder: Growing libraries of concepts with wake-sleep program induction

Kevin Ellis
Joint with: Lucas Morales, Mathias Sablé Meyer, Armando Solar-Lezama, Joshua B. Tenenbaum
Heavy inspiration from: Eyal Dechter

October 2018

MIT

# Human program induction everywhere

```
(MEMBER
 (LAMBDA (X L)
  (COND  ((NULL L) NIL)
         ((EQ X (FIRST L)) T)
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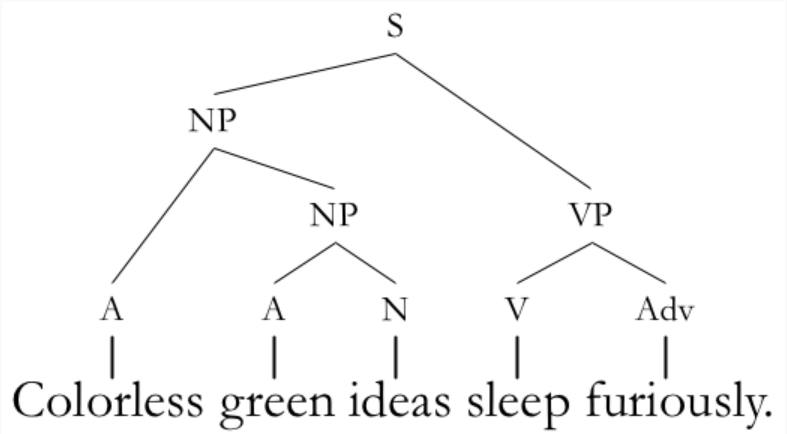


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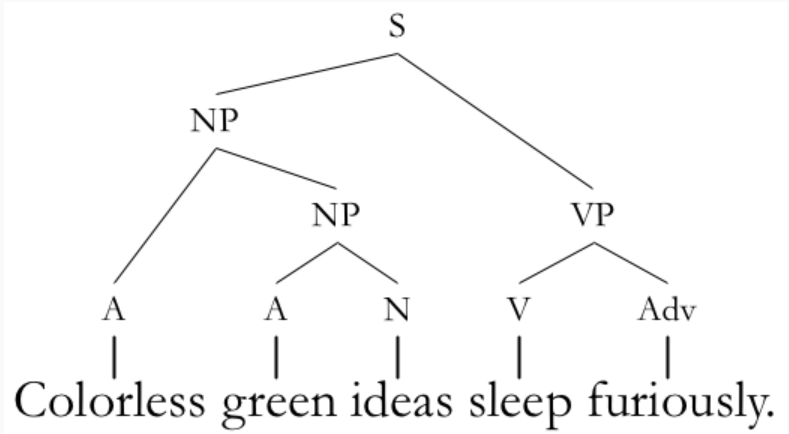


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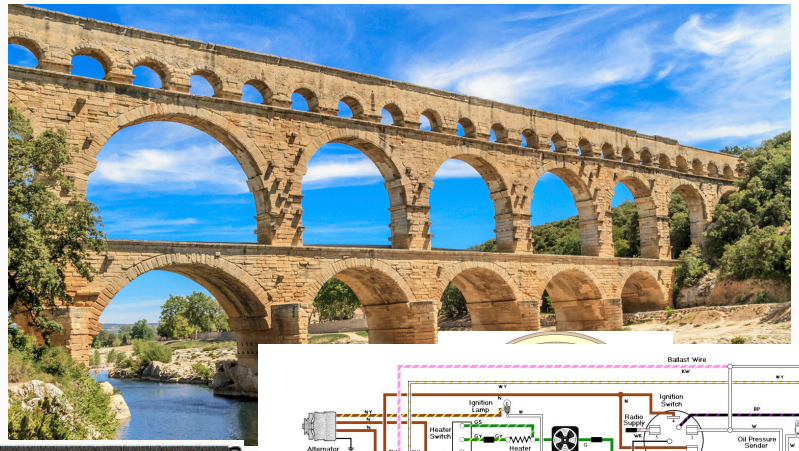

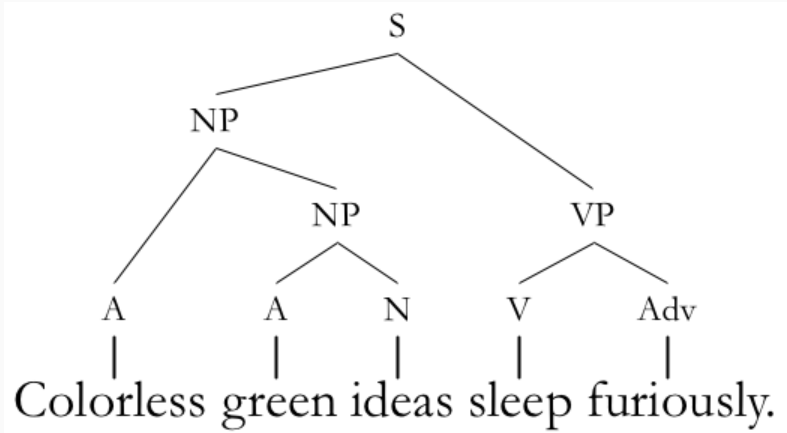


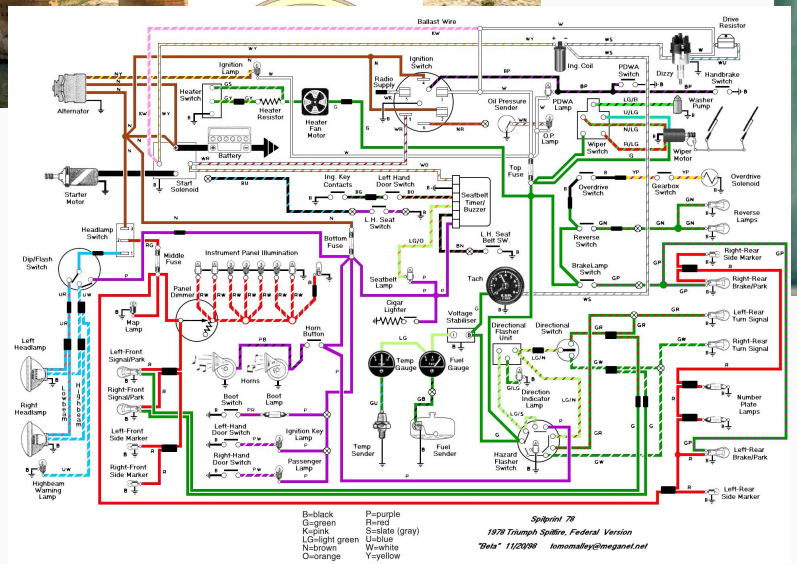

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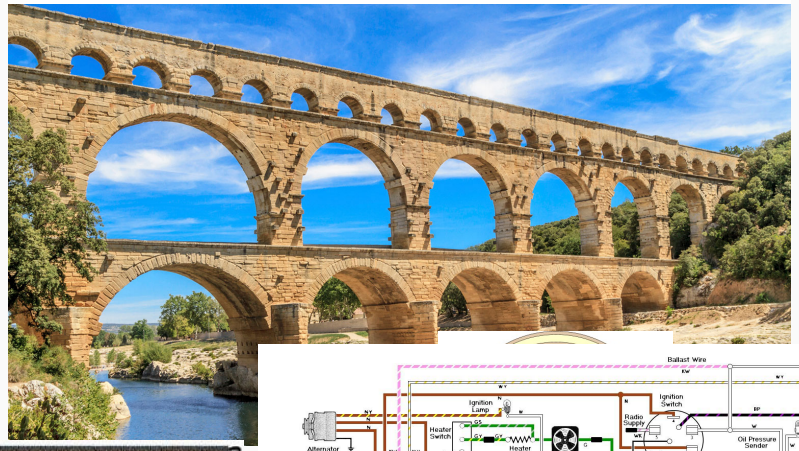

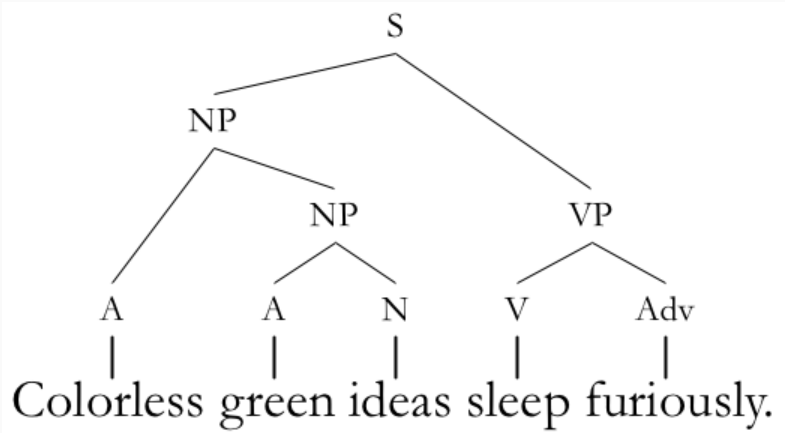


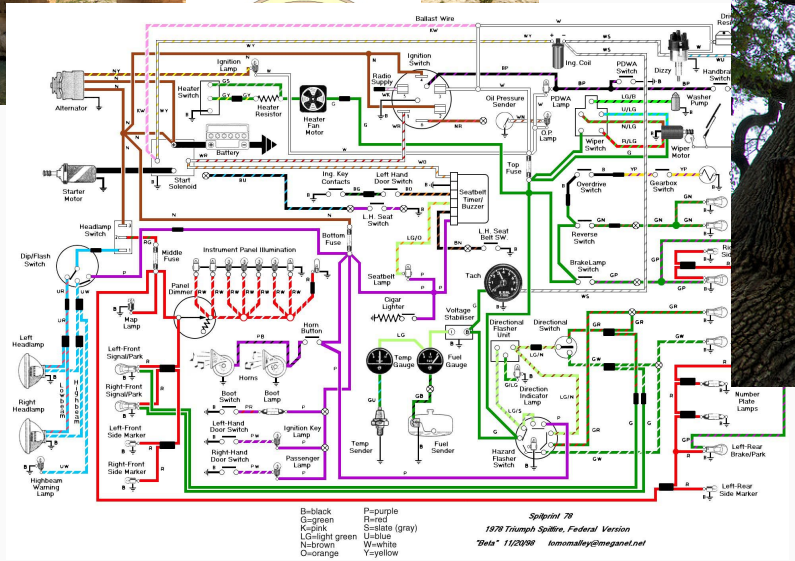

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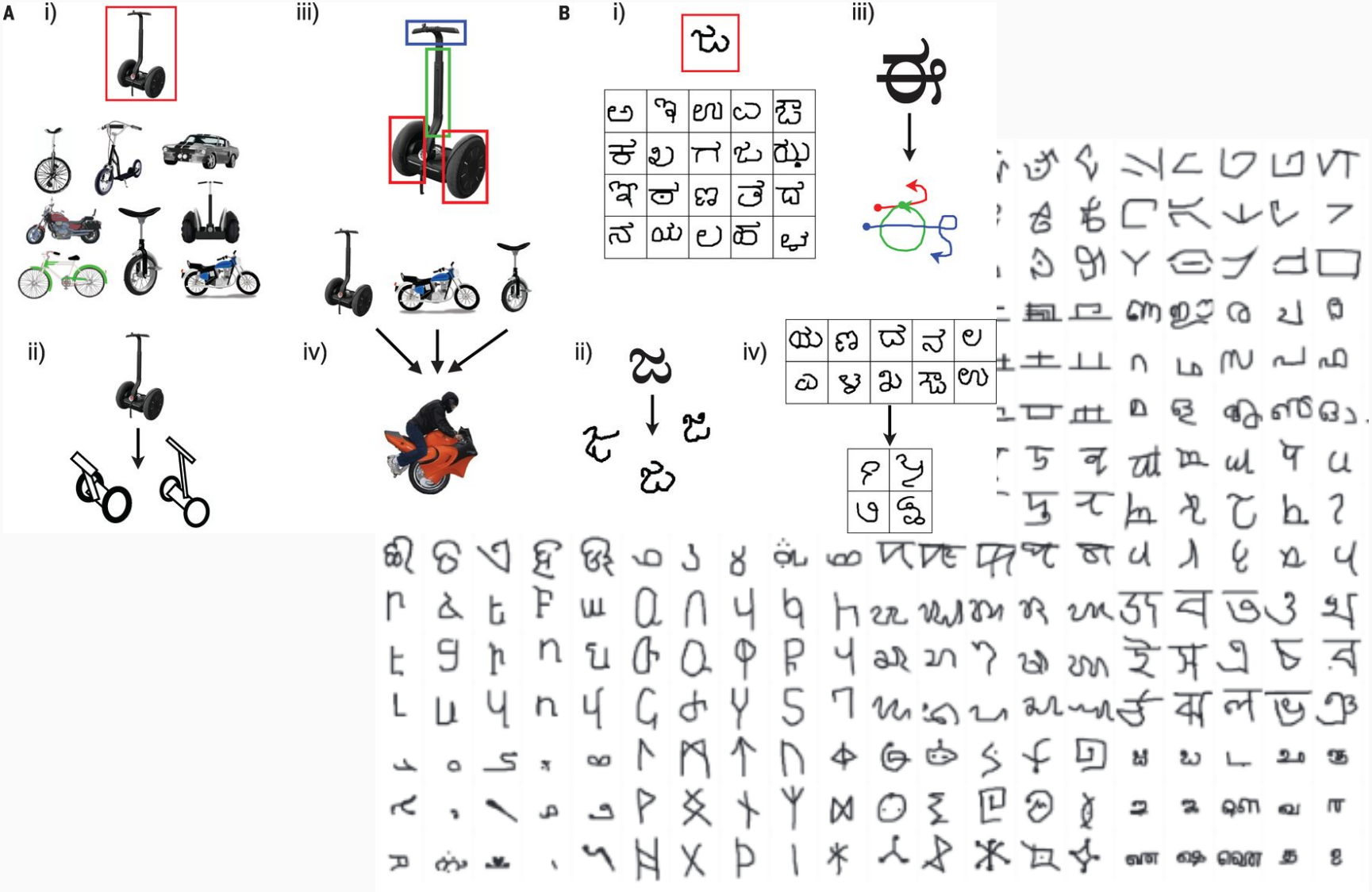


Lake et al 2015

# Growing a domain-specific language of thought

Goal: acquire domain-specific knowledge needed to induce a class of programs

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- Library of concepts (declarative knowledge)
- Inference strategy (procedural knowledge)

# DSL: Library of concepts

Tasks and Programs

```
[7 2 3]→[7 3]
[1 2 3 4]→[3 4]
[4 3 2 1]→[4 3]
f(ℓ) = (f1 ℓ (λ (x) (> x 2)))
```

```
[7 3]→False
[3]→False
[9 0 0]→True
[0]→True
[0 7 3]→True
f(ℓ) = (f3 ℓ 0)
```

```
[2 7 8 1]→8
[3 19 14]→19
f(ℓ) = (f2 ℓ)
```

DSL

$f_0(\ell,\mathtt{r}) = (\mathtt{foldr\ r}\ \ell\ \mathtt{cons})$
($f_0$: *Append lists* r *and* $\ell$)

$f_1(\ell,\mathtt{p}) = (\mathtt{foldr}\ \ell\ \mathtt{nil}\ (\lambda\ (\mathtt{x\ a})\ (\mathtt{if\ (p\ x)\ (cons\ x\ a)\ a})))$
($f_1$: *Higher-order filter function*)

$f_2(\ell) = (\mathtt{foldr}\ \ell\ 0\ (\lambda\ (\mathtt{x\ a})\ (\mathtt{if\ (>\ a\ x)\ a\ x})))$
($f_2$: *Maximum element in list* $\ell$)

$f_3(\ell,\mathtt{k}) = (\mathtt{foldr}\ \ell\ (\mathtt{is\text{-}nil}\ \ell)\ (\lambda\ (\mathtt{x\ a})\ (\mathtt{if\ a\ a\ (=\ k\ x)}))))$
($f_2$: *Whether* $\ell$ *contains* k)

# DreamCoder

- **Wake:** Solve problems by writing programs
- **Sleep:** Improve DSL and neural recognition model:
  - **Sleep-G:** Improve DSL (**G**enerative model)
  - **Sleep-R:** Improve **R**ecognition model

Combines ideas from Wake-Sleep & Exploration-Compression algorithm by Eyal Dechter

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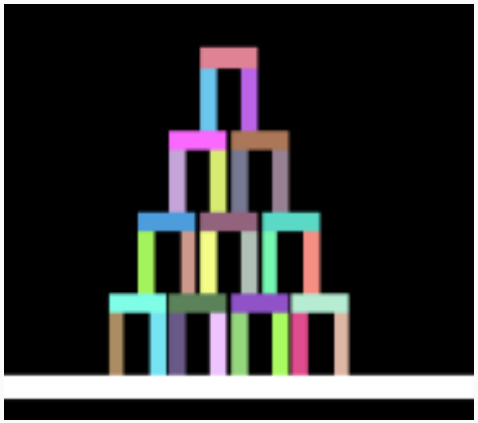
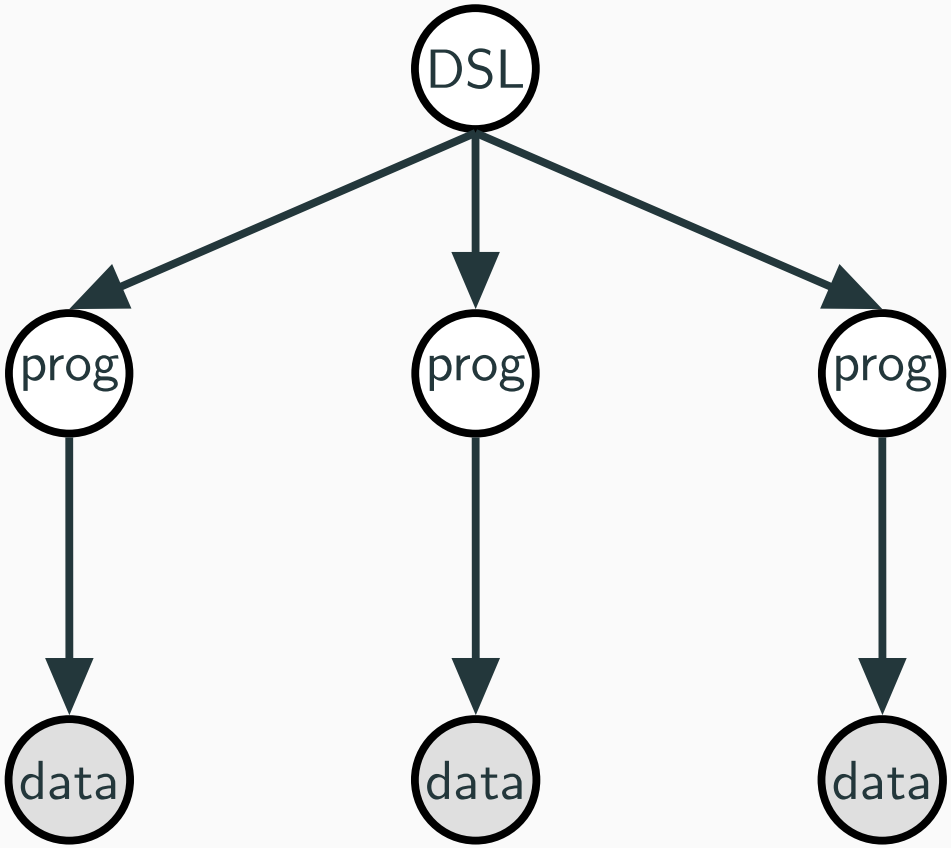
```
(let ((me 'whisper)
      (it 'into)
      (your 'ear))
  (let ((me (lisp it))
        (to (you)))
    (secretly)))
```

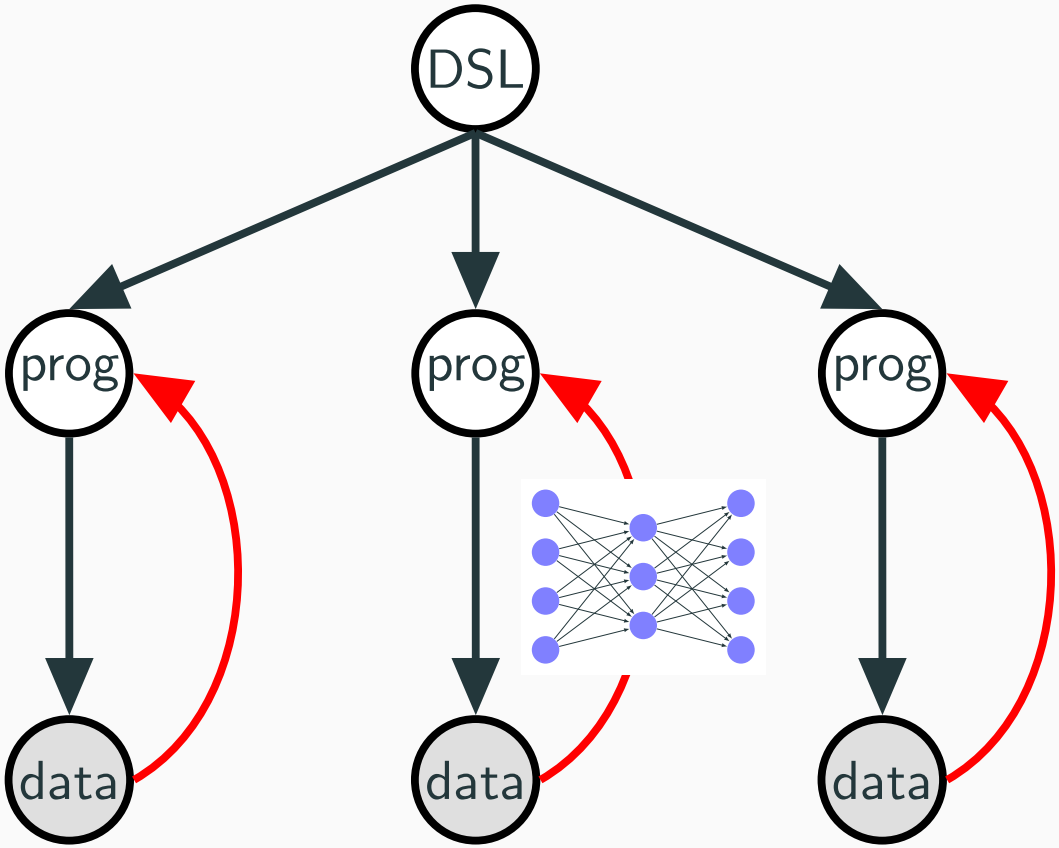


# DSL learning as Bayesian inference




**[Dechter et al., 2013]** [Liang et al, 2010]; [Lake et al, 2015]

# DSL learning as amortized Bayesian inference



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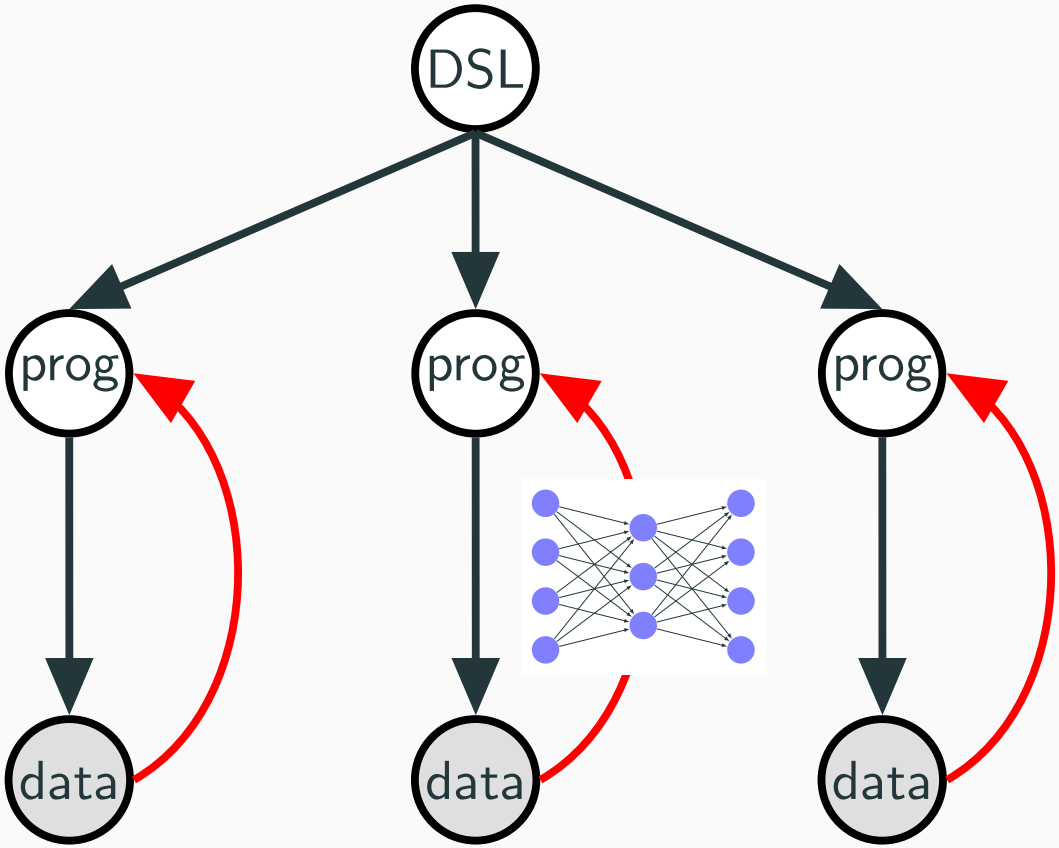
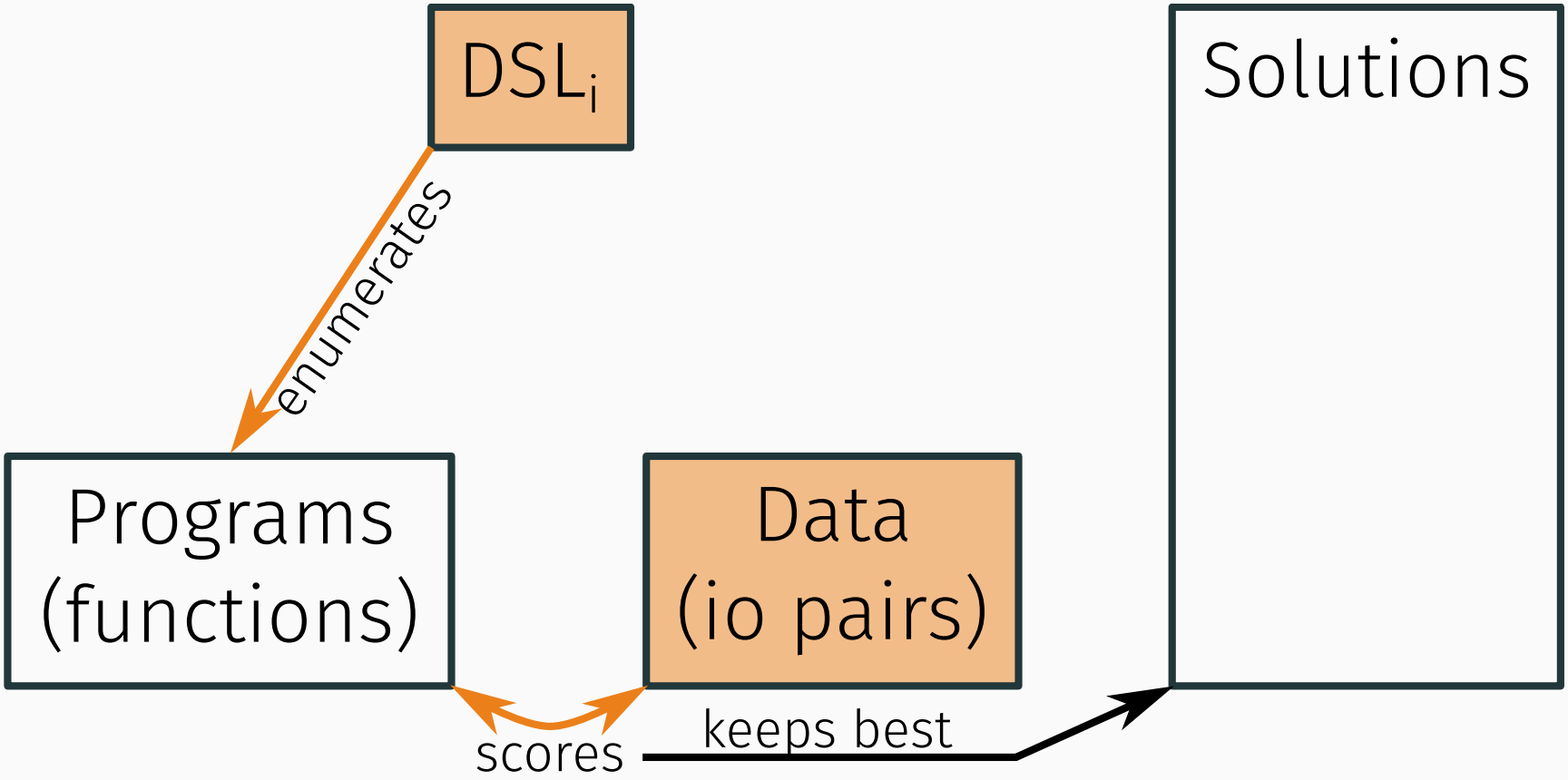



**New:** amortized inference +
better program representation (Lisp) +
better DSL inference

# Wake — as in Exploration/Compression Algorithm

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f0(l,r) = foldr r l cons
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**Programs**

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**Data**

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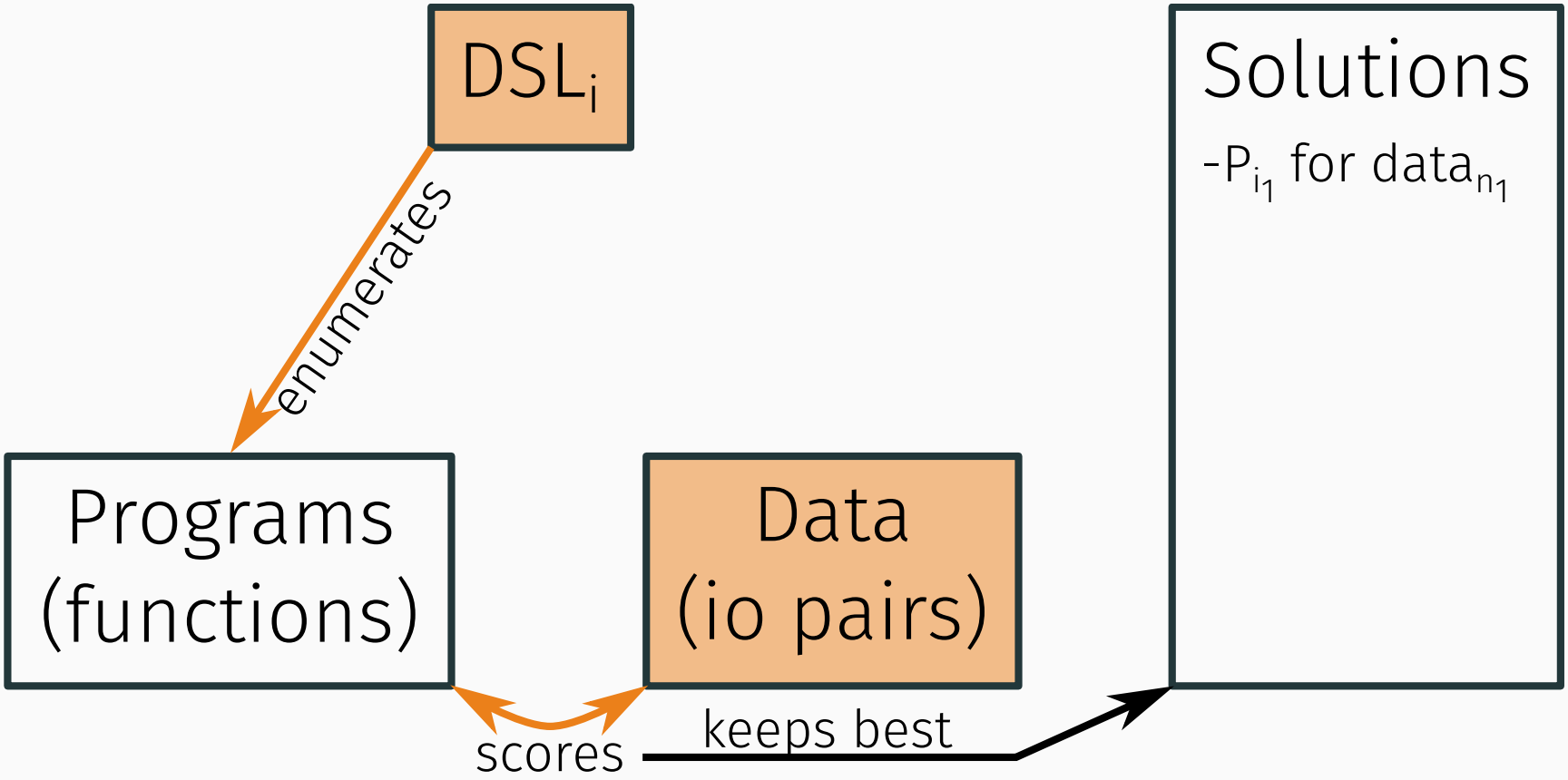


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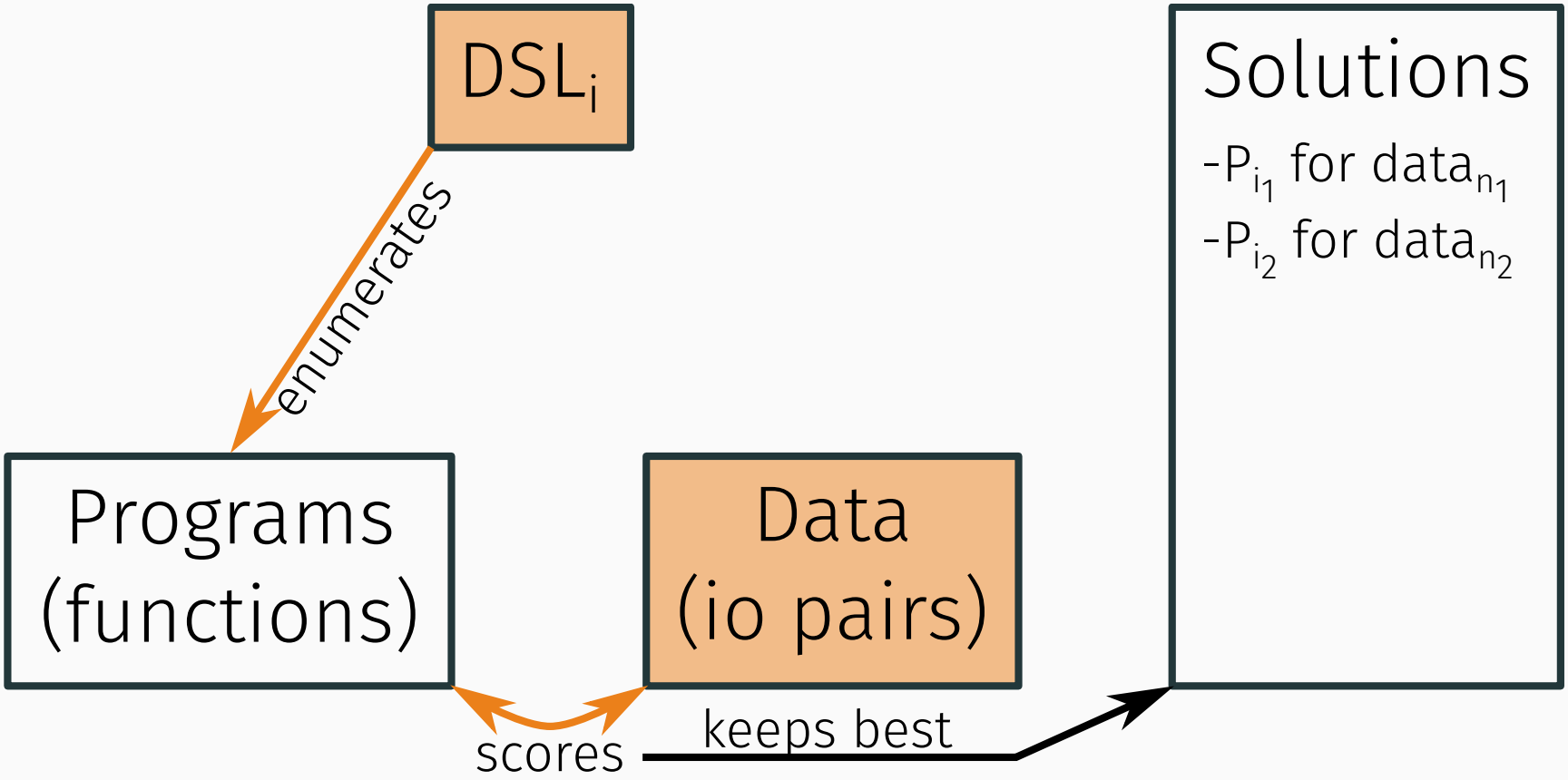


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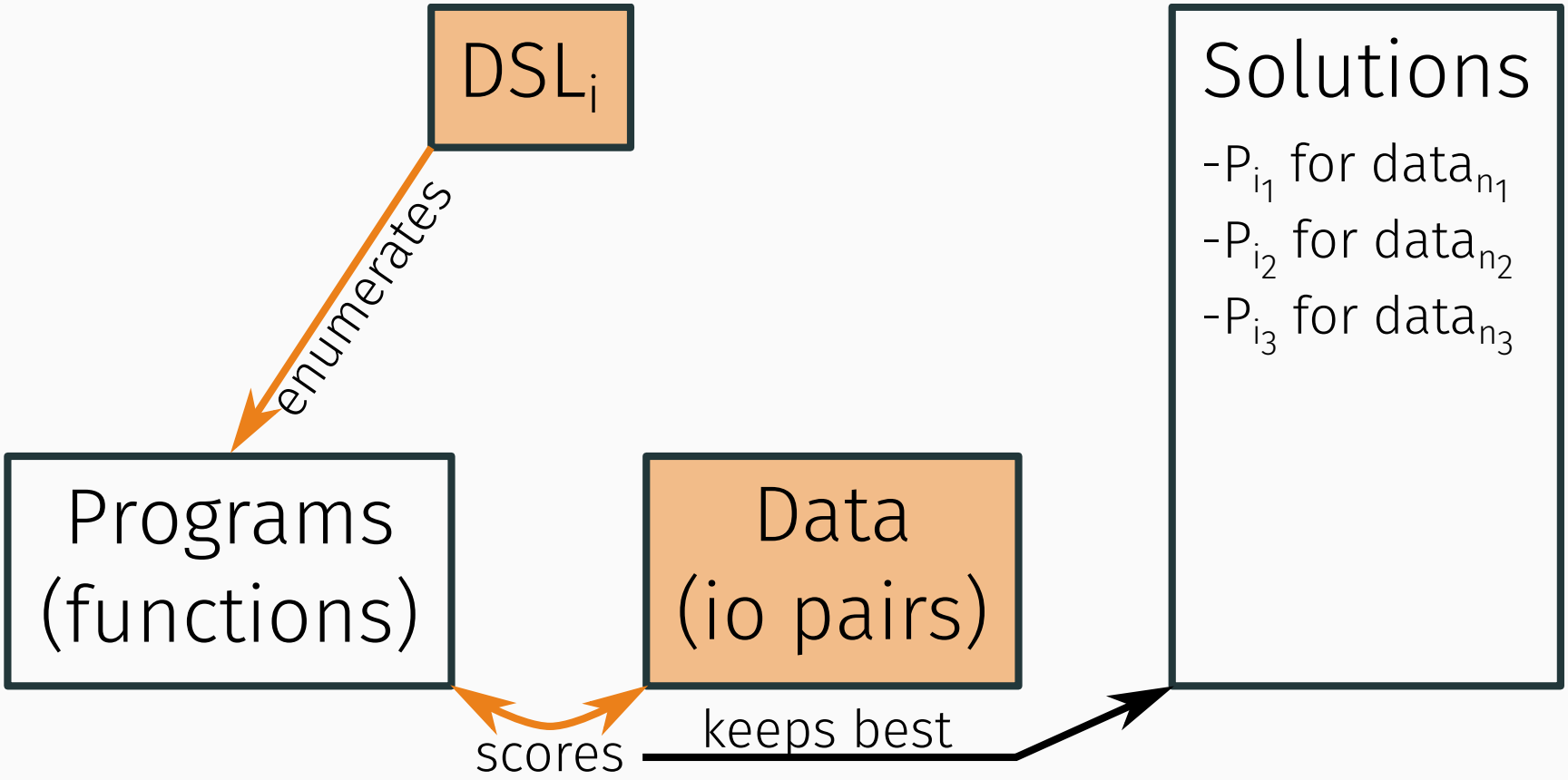


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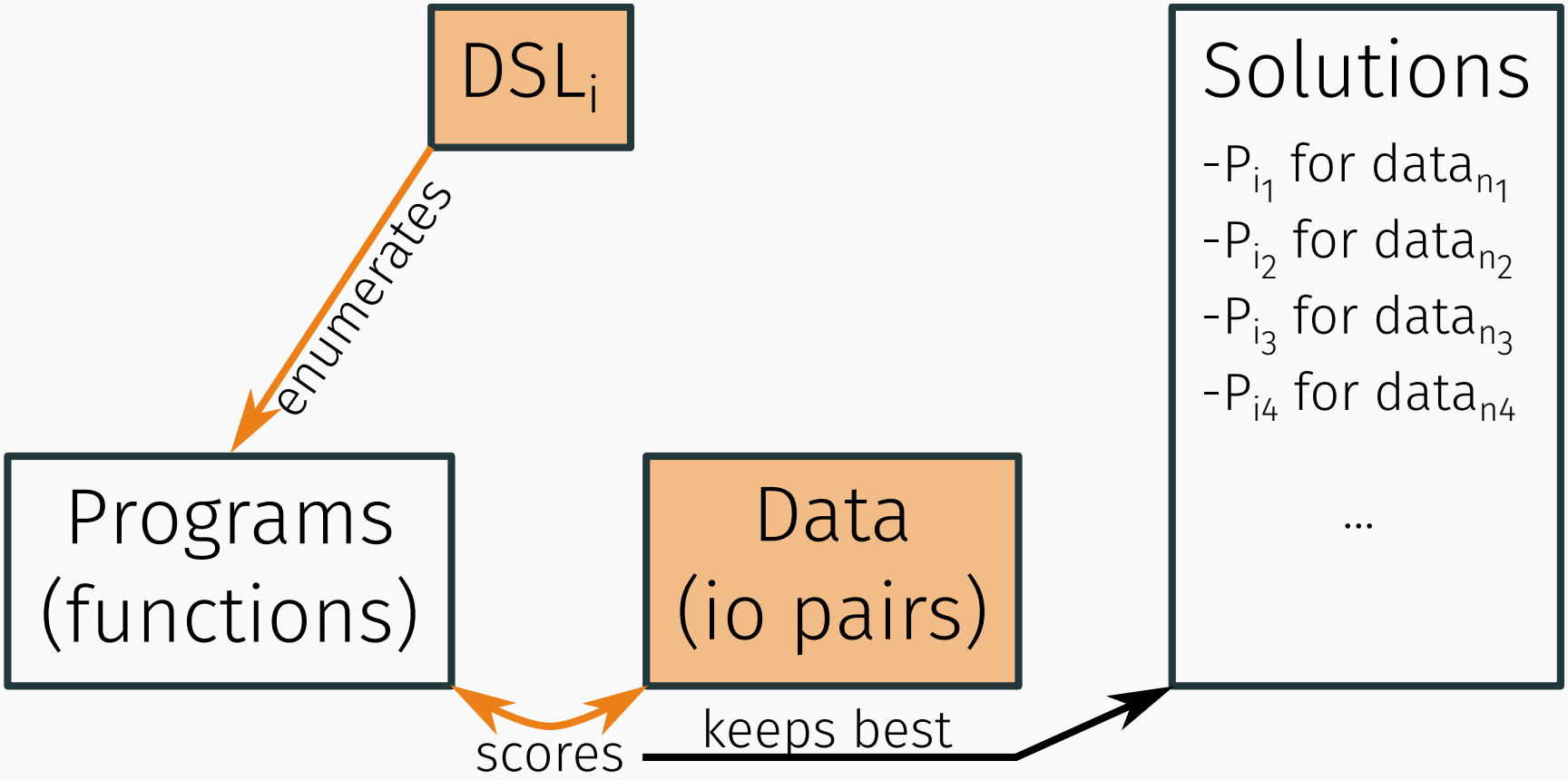

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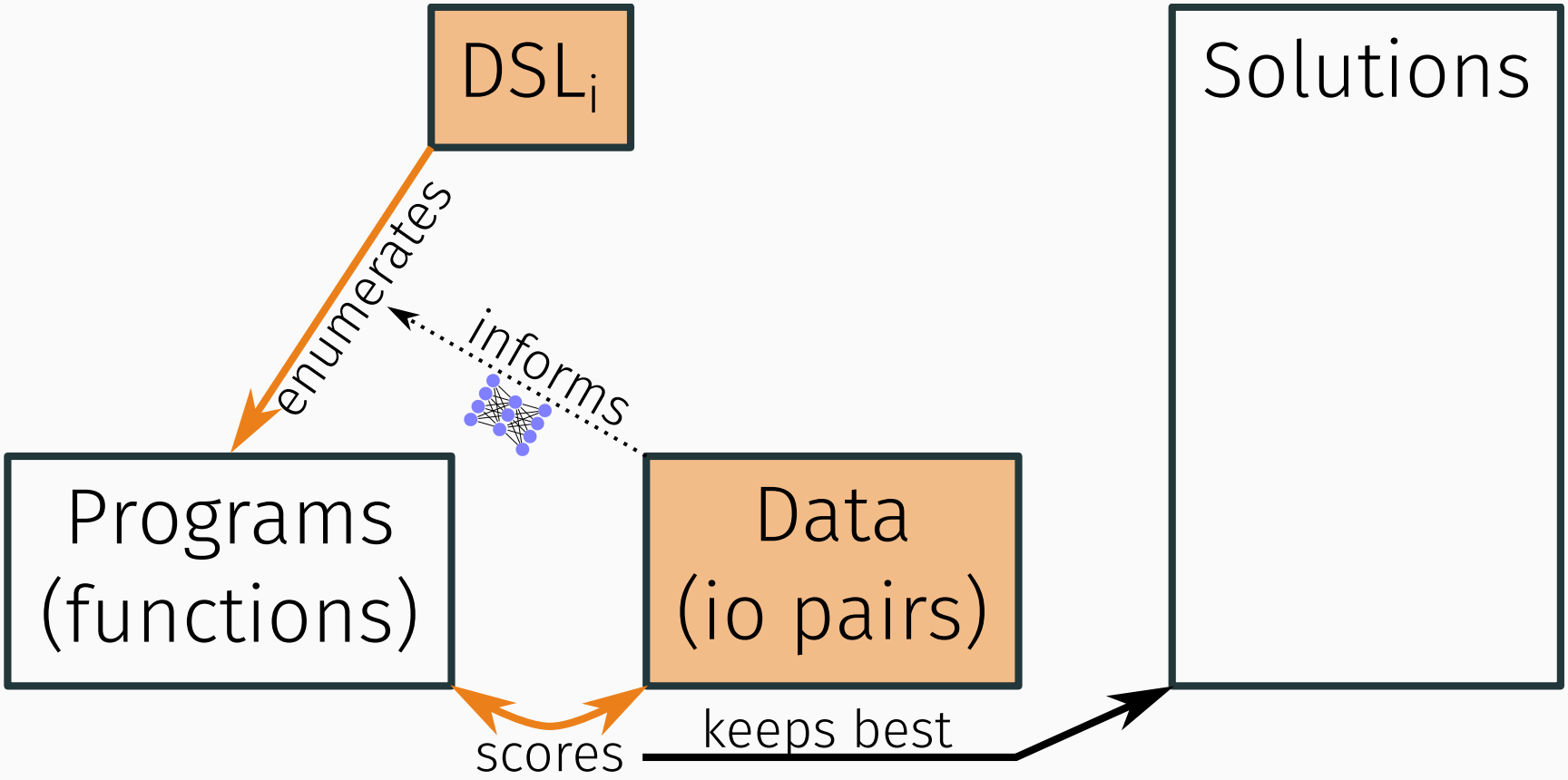


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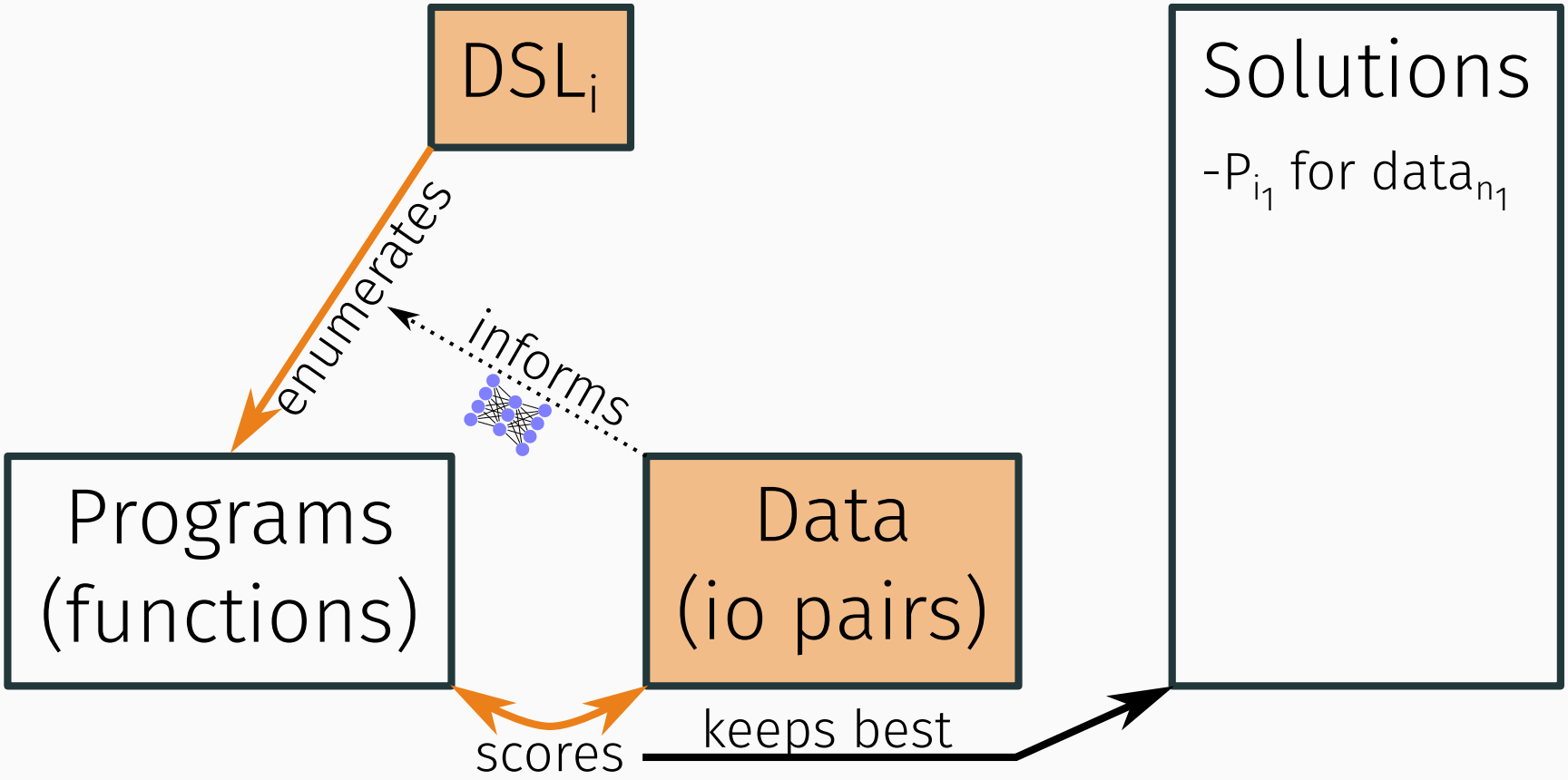


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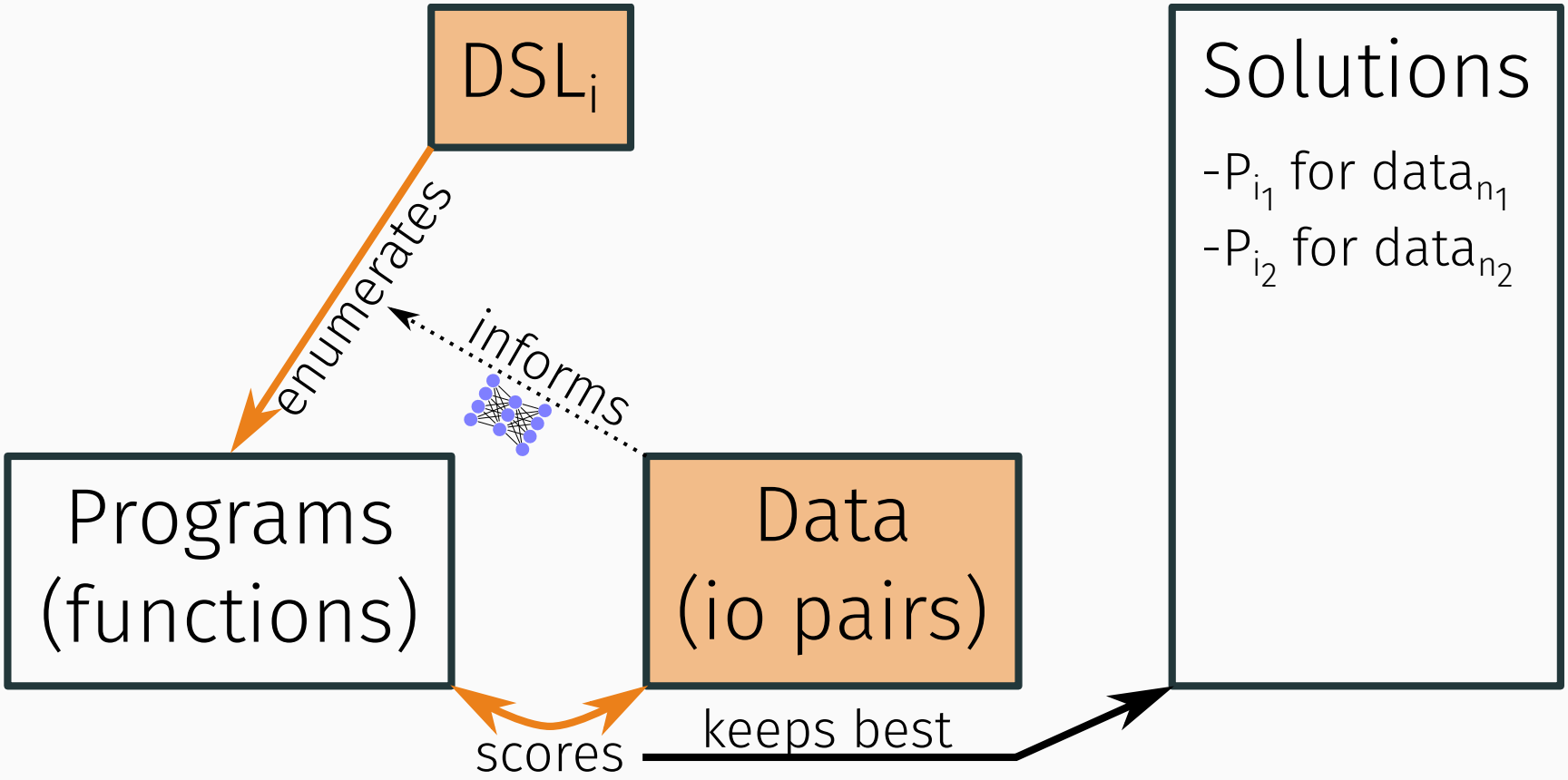

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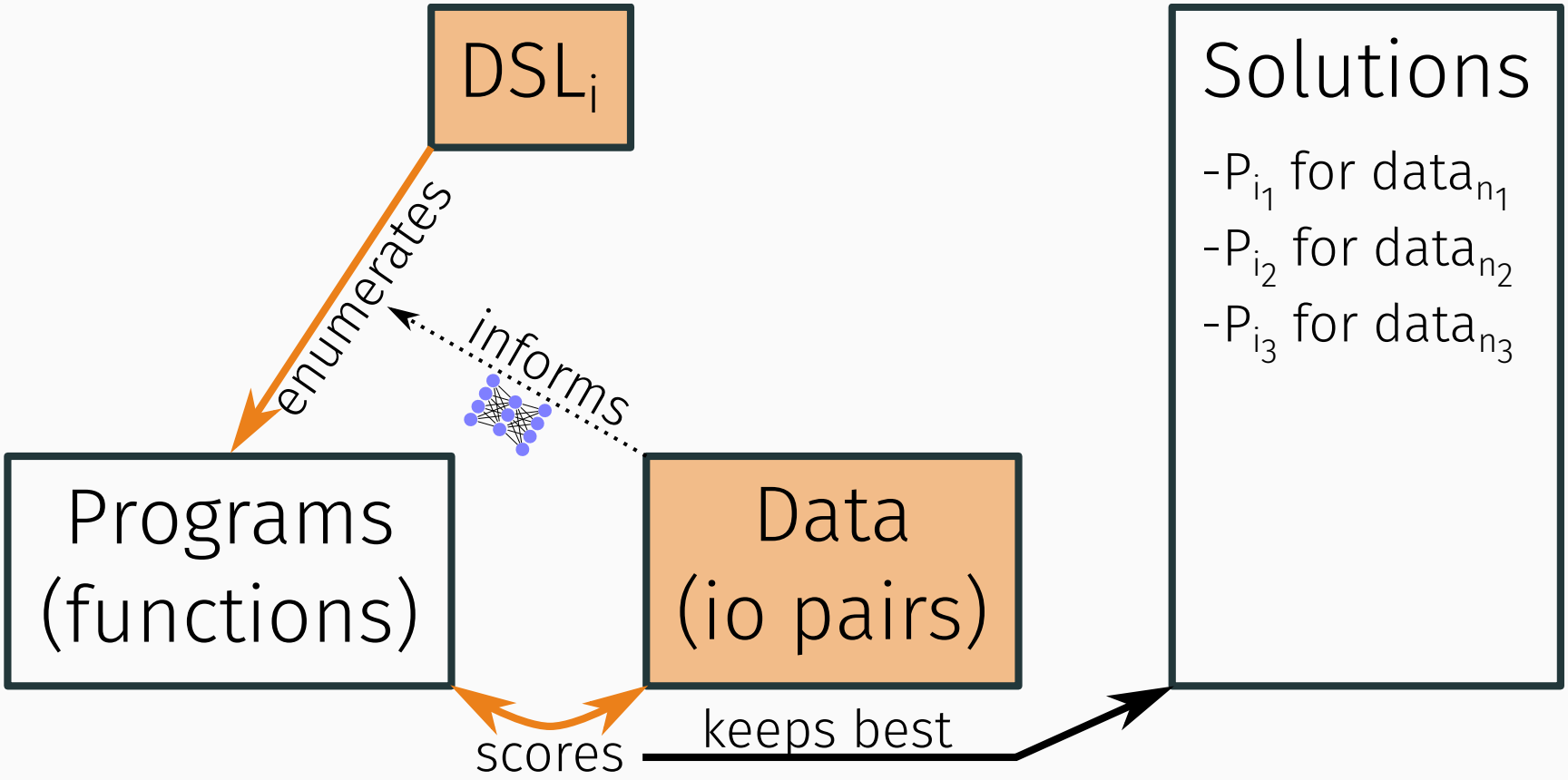

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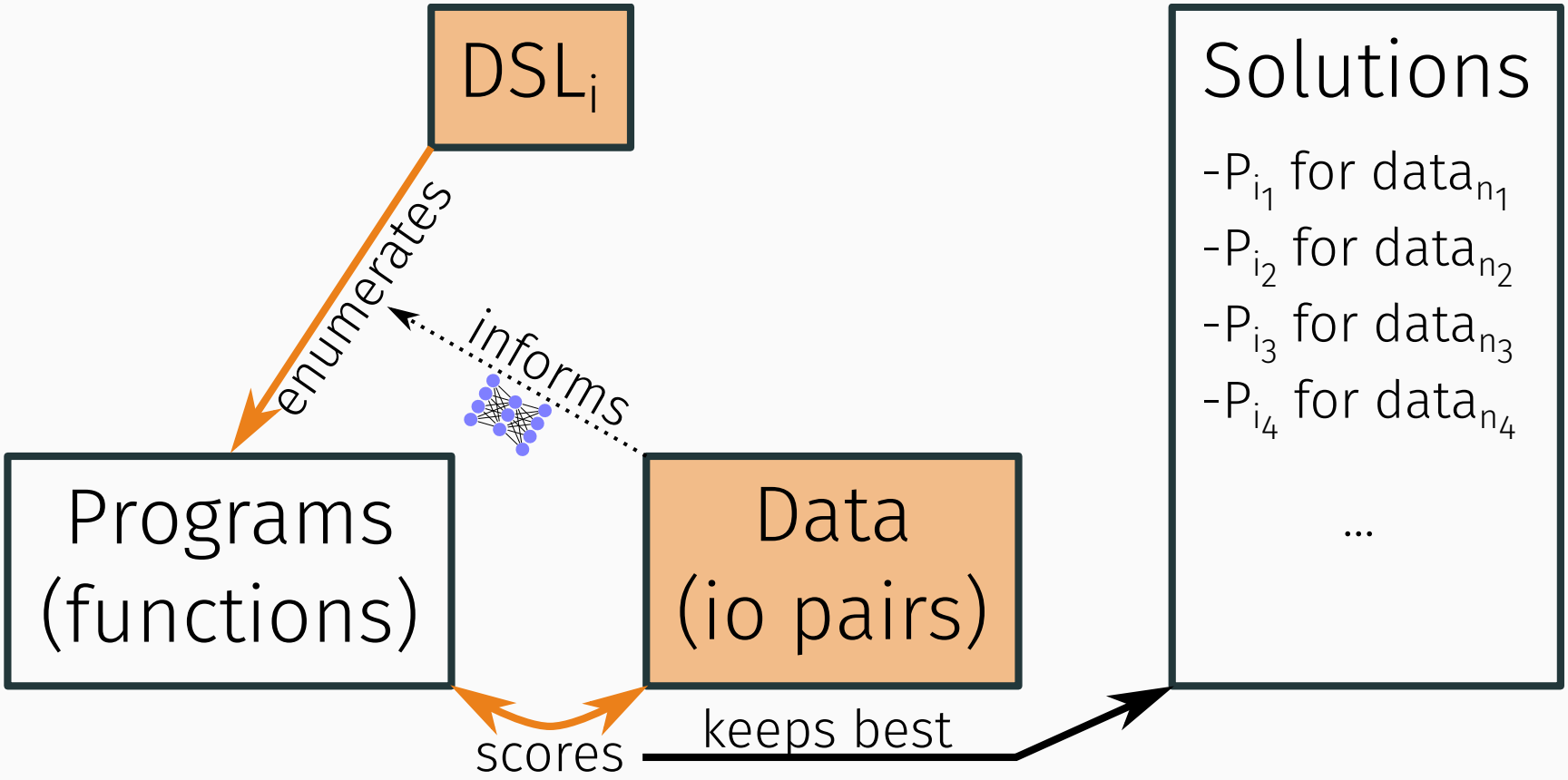
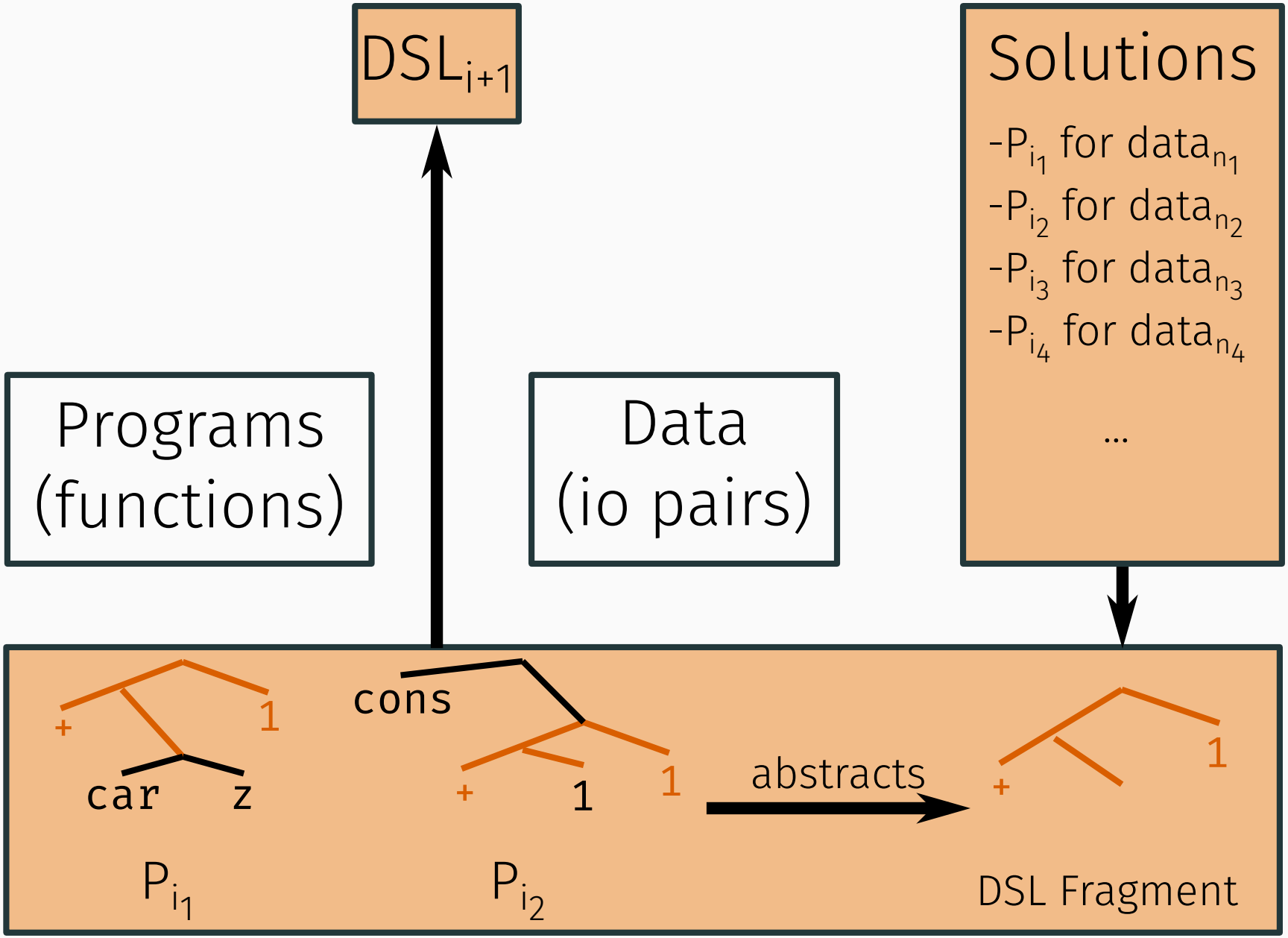


# DreamCoder — Sleep-G



# DreamCoder — Sleep-G (Refactoring)

**Learning higher-order `map` function**

| Task | Program |
|---|---|
| `(1 2 3)`→`(2 4 6)`<br>`(1 9 2)`→`(2 18 4)` | `(Y (λ (r l) (if (nil? l) nil`<br>`(cons (+ (car l) (car l))`<br>`(r (cdr l))))))` |
| `(1 2 3)`→`(2 3 4)`<br>`(1 9 2)`→`(2 10 3)` | `(Y (λ (r l) (if (nil? l) nil`<br>`(cons (+ (car l) 1)`<br>`(r (cdr l))))))` |

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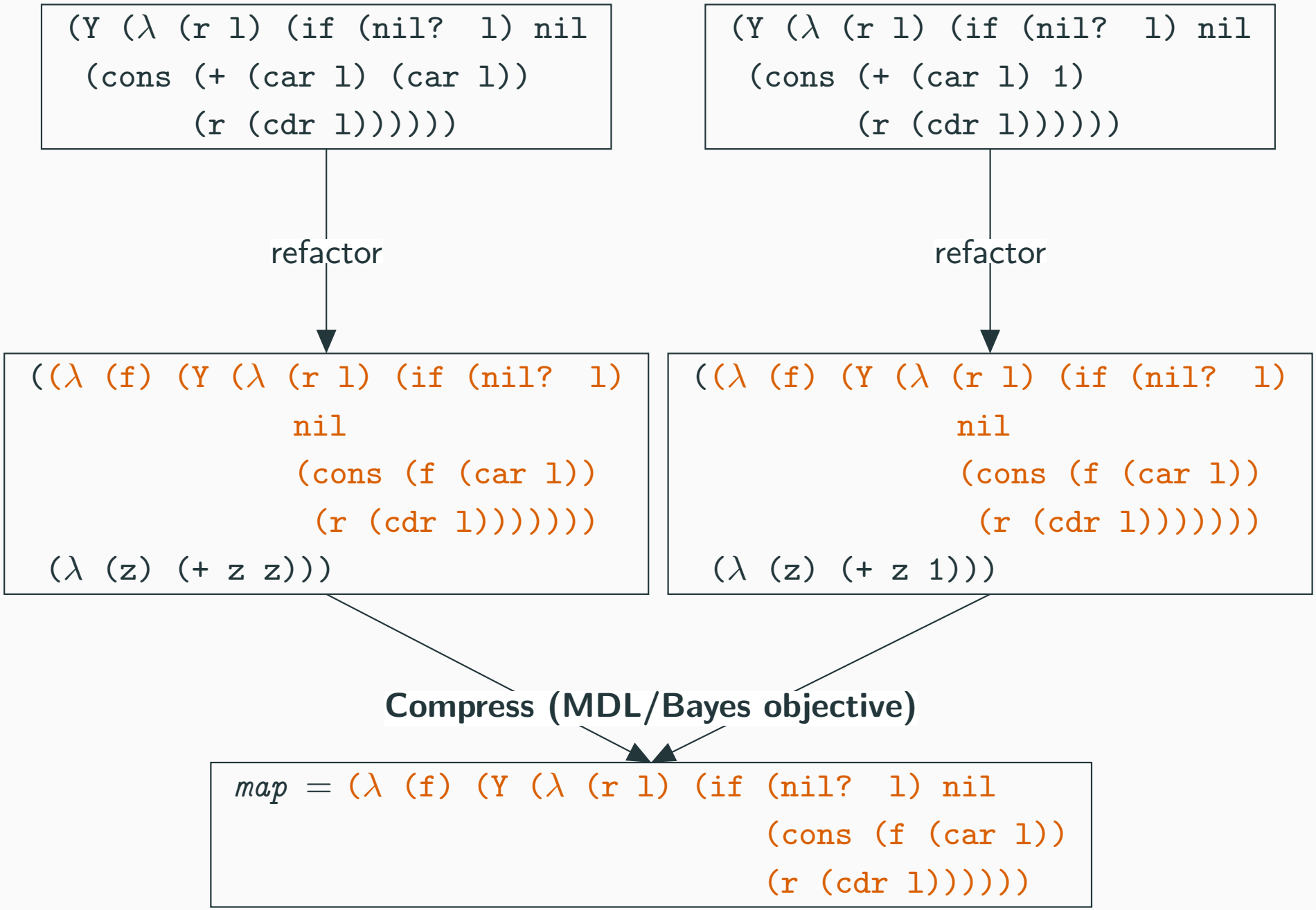
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refactor
refactor
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Compress (MDL/Bayes objective)
map = (λ (f) (Y (λ (r l) (if (nil? l) nil
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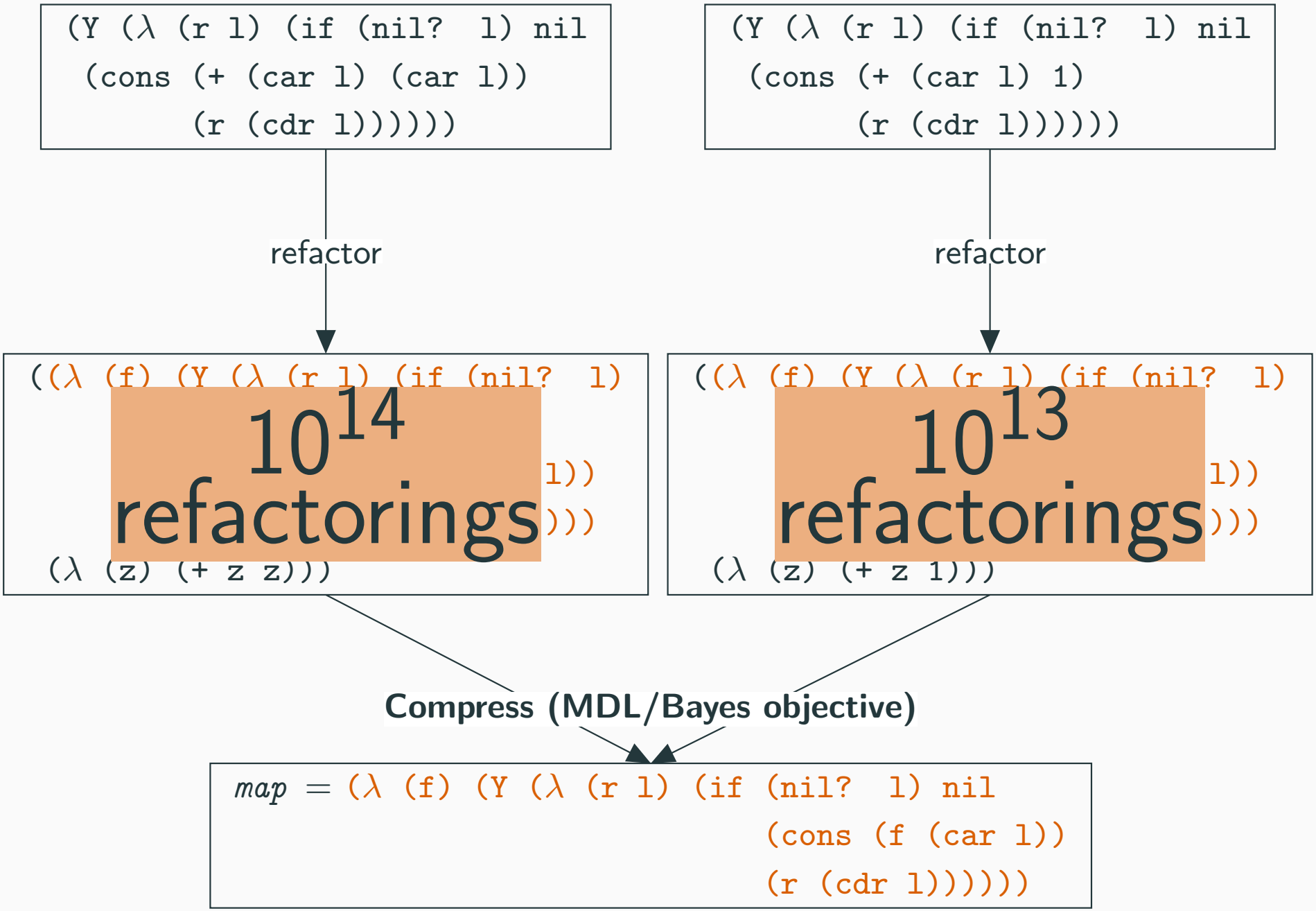
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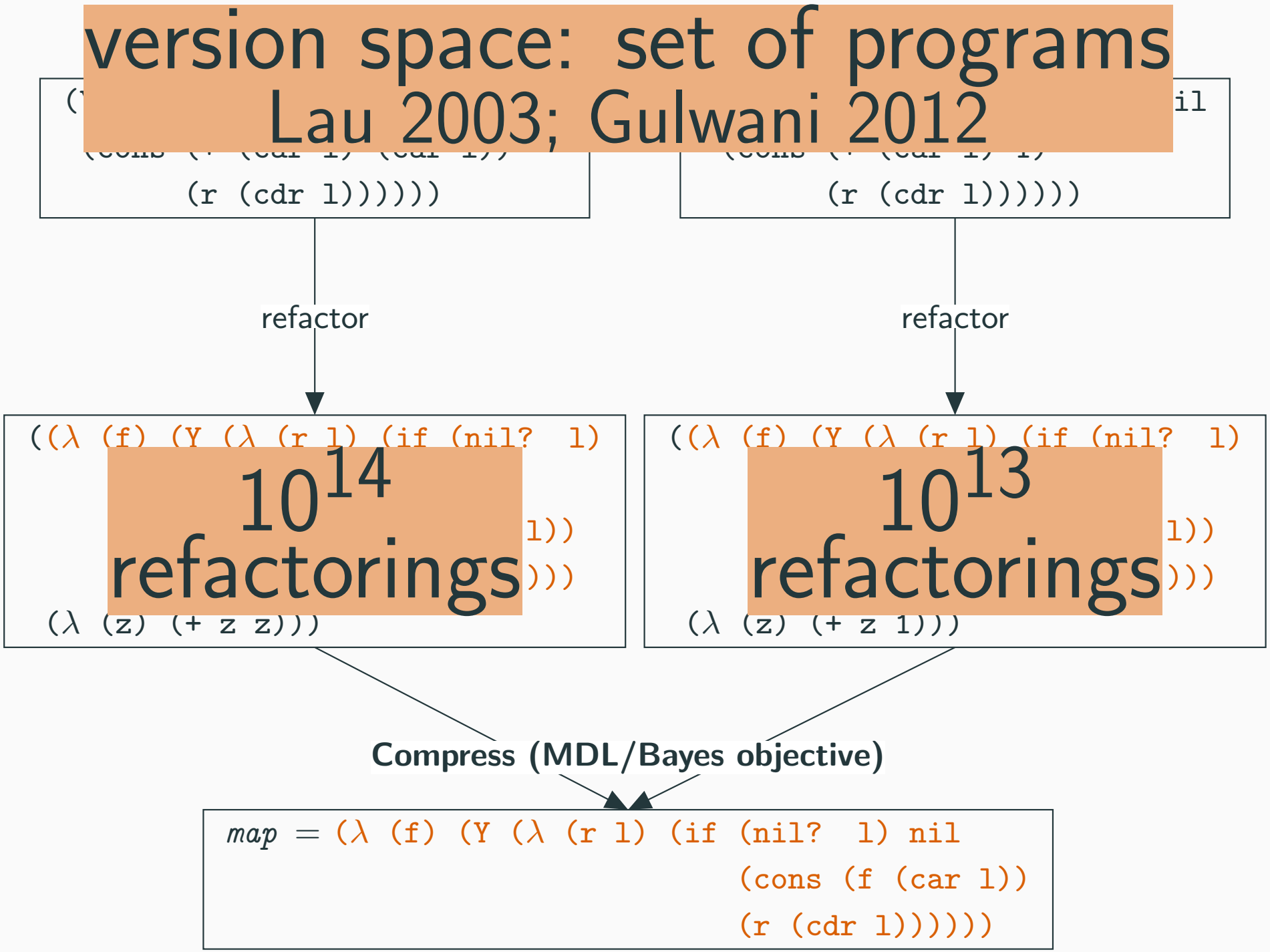
version space: set of programs
Lau 2003; Gulwani 2012
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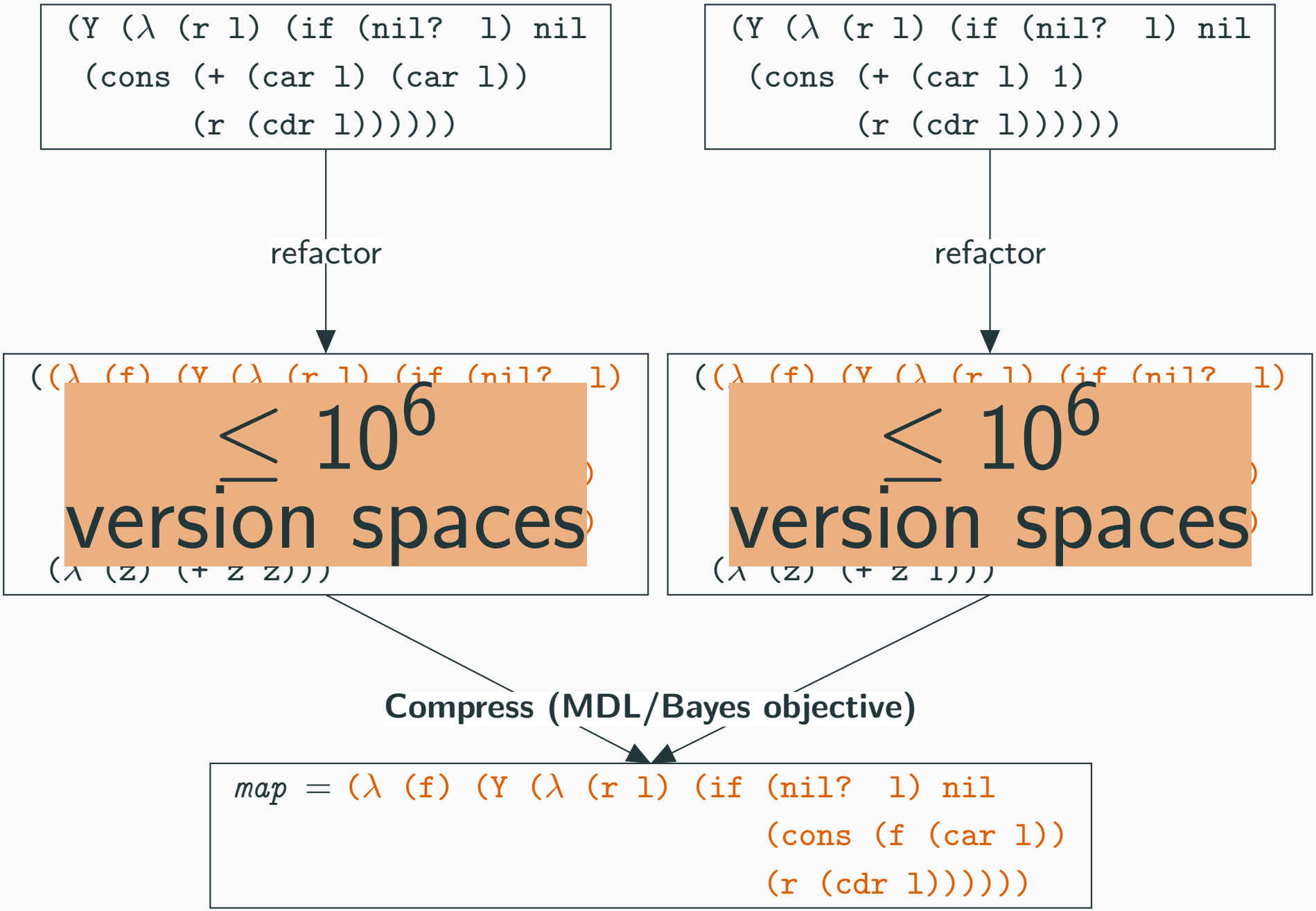
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refactor
refactor
≤ 10^6
version spaces
≤ 10^6
version spaces
Compress (MDL/Bayes objective)
map = (λ (f) (Y (λ (r l) (if (nil? l) nil
(cons (f (car l))
(r (cdr l))))))

# DreamCoder — Sleep-R (Experience Replay)

$DSL_{i+1}$

Solutions

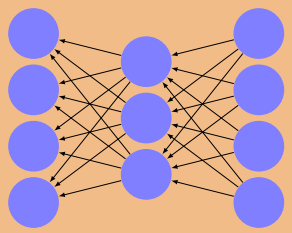
- $P_{i_1}$ for $data_{n_1}$
- $P_{i_2}$ for $data_{n_2}$
- $P_{i_3}$ for $data_{n_3}$
- $P_{i_4}$ for $data_{n_4}$

...

Programs (functions)

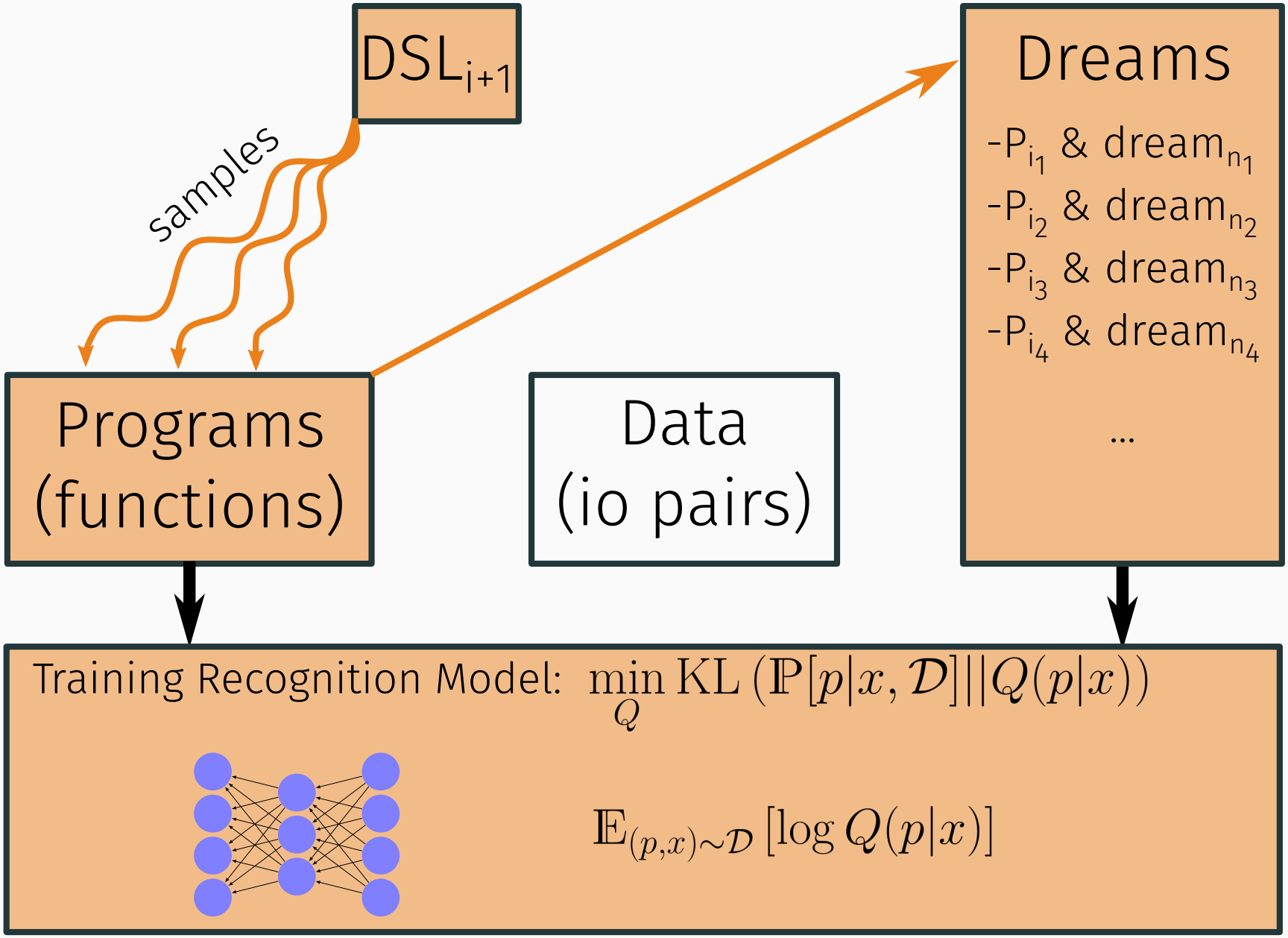
Data (io pairs)

Training Recognition Model: $\min_Q \mathrm{KL}\left(\mathbb{P}[p|x,\mathcal{D}]||Q(p|x)\right)$



$$\frac{\mathbb{P}\left[x_n, p_n|\mathcal{D}\right]}{\sum_{(x_n,p'_n)} \mathbb{P}\left[x_n, p_n|\mathcal{D}\right]} \log Q(p_n|x_n)$$

# DreamCoder — Sleep-R (Dreaming)



# List functions — Created & investigated by Lucas Morales

| Name | Input | Output |
|---|---|---|
| repeat-3 | [7 0] | [7 0 7 0 7 0] |
| drop-3 | [0 3 8 6 4] | [6 4] |
| rotate-2 | [8 14 1 9] | [1 9 8 14] |
| count-head-in-tail | [1 2 1 1 3] | 2 |
| keep-div-5 | [5 9 14 6 3 0] | [5 0] |
| product | [7 1 6 2] | 84 |

Discovers 38 concepts, including ‘filter’. With different tasks will also learn ‘map’, ‘fold’, ‘unfold’, etc. starting with 1950’s Lisp

# Text editing

In the style of FlashFill (Gulwani 2012)

**Text Editing**

+106 769-438→106.769.438
+83 973-831→83.973.831

```
f(s) = (f0 "." "-"
          (f0 "." " "
            (cdr s)))
```

Temple Anna H →TAH
Lara Gregori→LG

```
f(s) = (f2 s)
```

```
f0(s,a,b) = (map (λ (x)
              (if (= x a) b x)) s)
```
($f_0$: *Performs character substitution*)

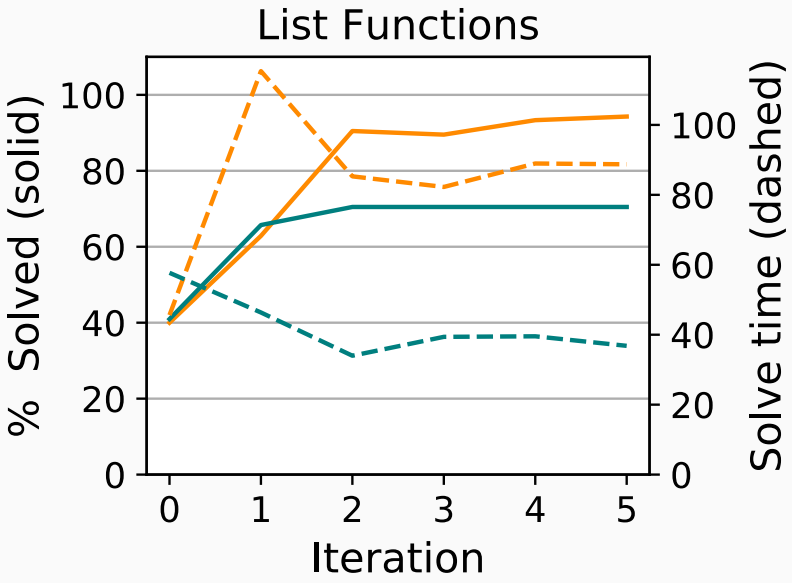
```
f1(s,c) = (foldr s s (λ (x a)
            (cdr (if (= c x) s a))))
```
($f_1$: *Drop characters from* s *until* c *reached*)

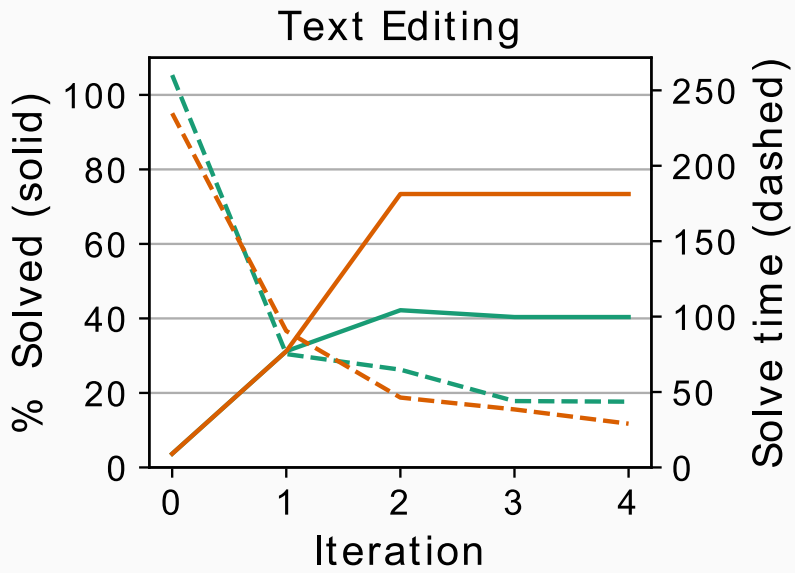
```
f2(s) = (unfold s is-nil car
          (λ (z) (f1 z " ")))
```
($f_2$: *Abbreviates a sequence of words*)

SyGuS problems: solves 3% before learning, vs 75% after learning.

Best prior work: 80%

# List functions & Text editing: Learning curves on hold out tasks




Learning curves for DreamCoder both with (in orange) and without (in teal) the recognition model. Solid lines: % holdout testing tasks solved w/ 10m timeout. Dashed lines: Average solve time, averaged only over tasks that are solved.

# Turtle graphics — Created & investigated by Mathias Sablé–Meyer

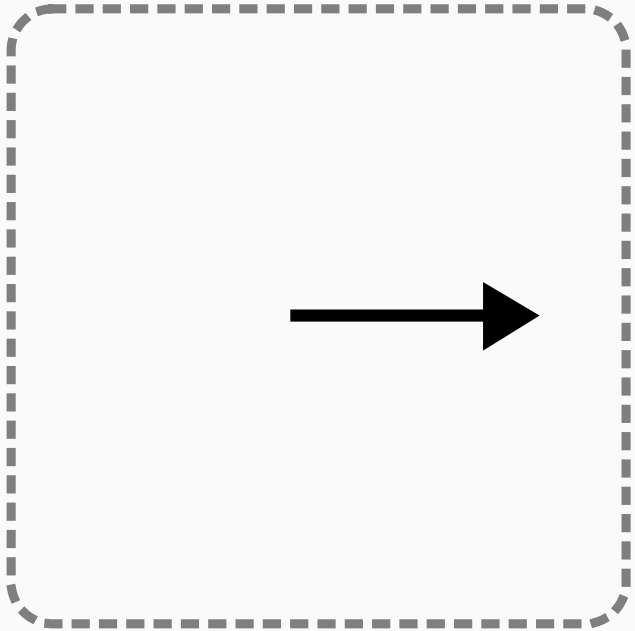
## DSL

```
OP ::= FW x | RT x | UP | DOWN | SET state
```

## Tasks

```
task :  image
```

```
FW 1
```



# Turtle graphics — Created & investigated by Mathias Sablé–Meyer

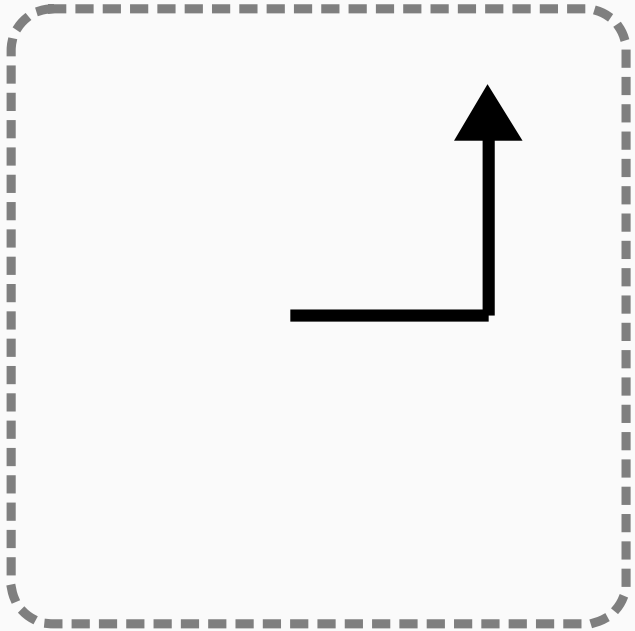
## DSL

```
OP ::= FW x | RT x | UP | DOWN | SET state
```

## Tasks

```
task :  image
```

```
FW 1
RT π/2
FW 1
```



# Turtle graphics — Created & investigated by Mathias Sablé–Meyer

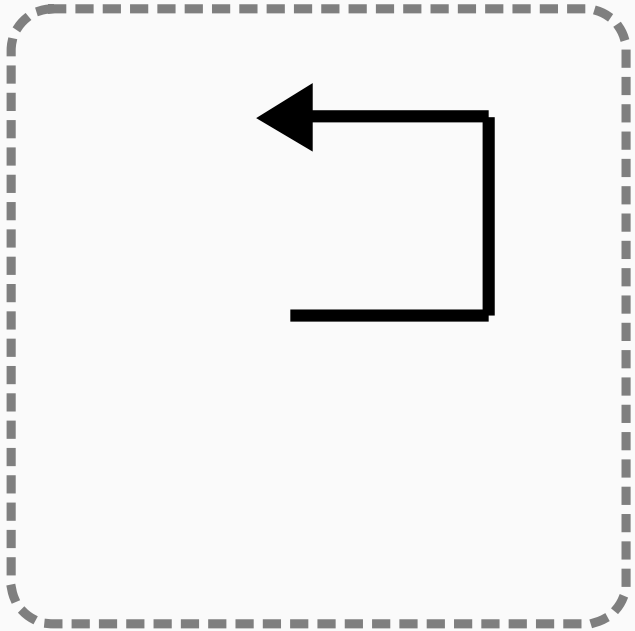
## DSL

```
OP ::= FW x | RT x | UP | DOWN | SET state
```

## Tasks

`task : image`

```
FW 1
RT π/2
FW 1
RT π/2
FW 1
```



# Turtle graphics — Created & investigated by Mathias Sablé–Meyer

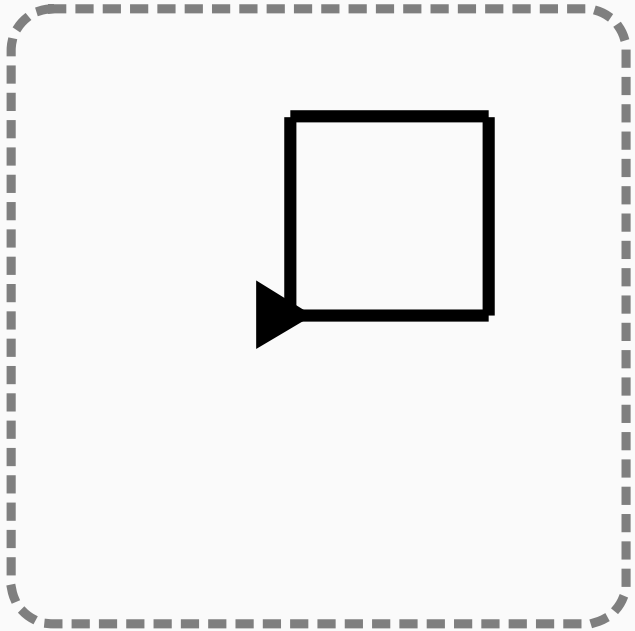
## DSL

```
OP ::= FW x | RT x | UP | DOWN | SET state
```

## Tasks

```
task :  image
```

```
for i in range(4)
> FW 1
> RT π/2
```



# Turtle graphics — Created & investigated by Mathias Sablé–Meyer

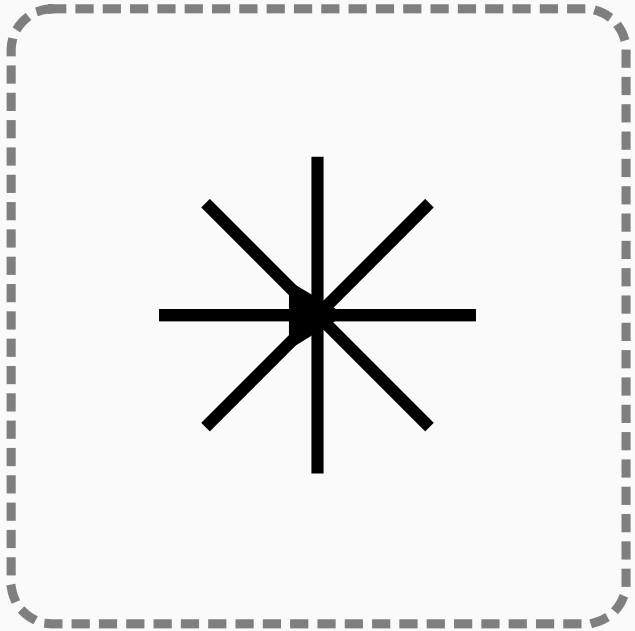
## DSL

```
OP ::= FW x | RT x | UP | DOWN | SET state
```

## Tasks

```
task :  image
```

```
for i in range(8)
> FW 1
> SET origin
> RT 2π/8
```



# Turtle graphics — Created & investigated by Mathias Sablé–Meyer

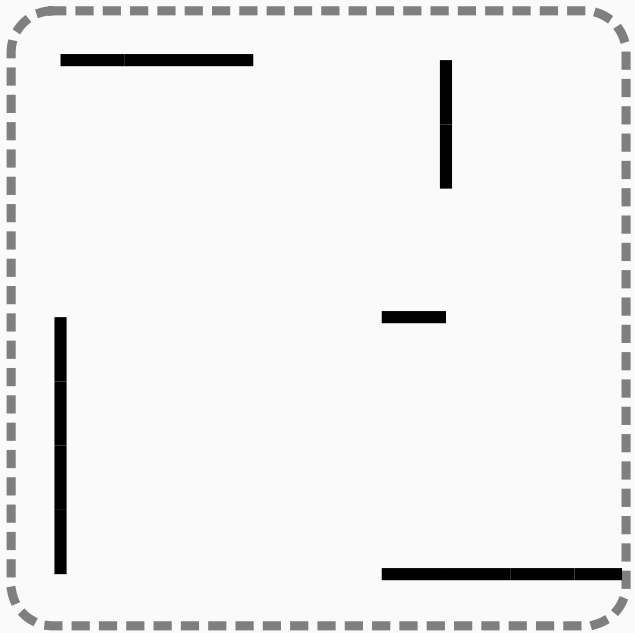
## DSL

```
OP ::= FW x | RT x | UP | DOWN | SET state
```

## Tasks

```
task :  image
```

```
for i in range(8)
> PU
> FW i/2
> PD
> FW i/2
> RT π/2
```



# Turtle graphics — Created & investigated by Mathias Sablé–Meyer

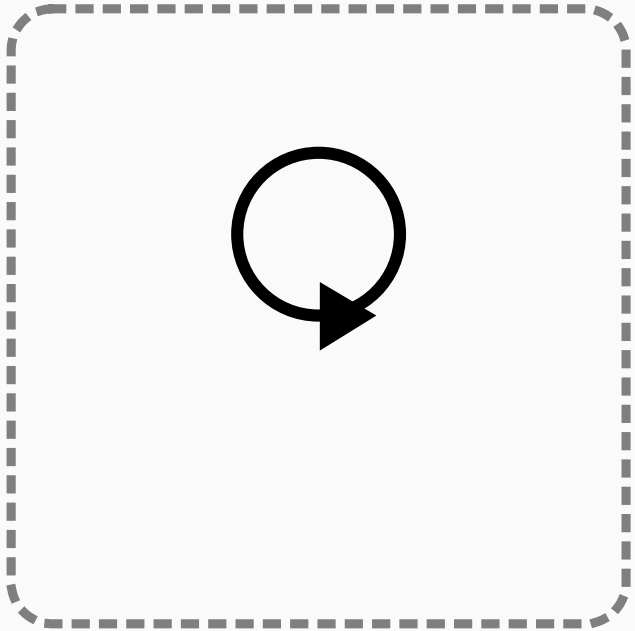
## DSL

```
OP ::= FW x | RT x | UP | DOWN | SET state
```

## Tasks

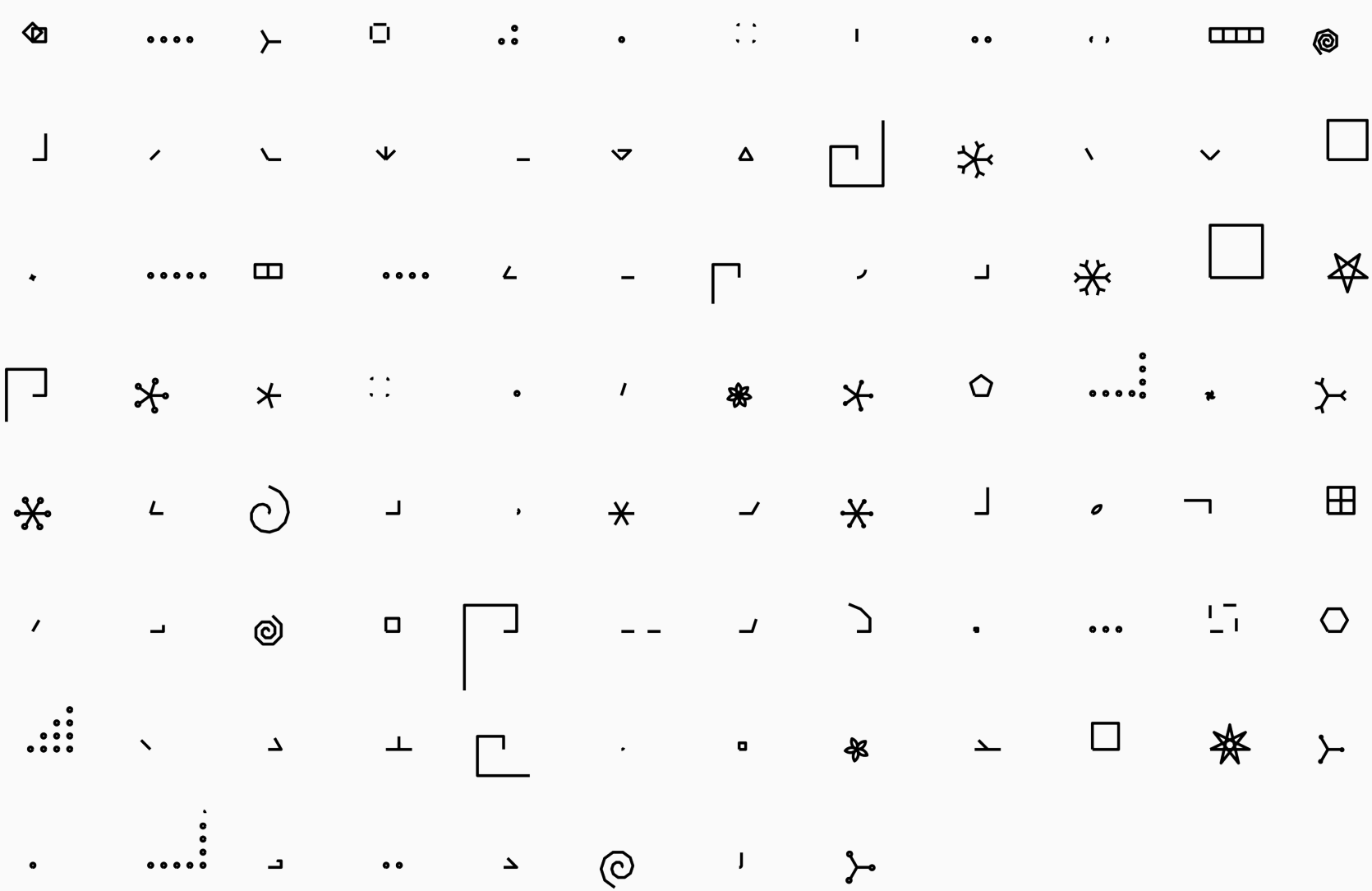
```
task :  image
```

```
for i in range(∞)
> FW ε
> RT ε
```

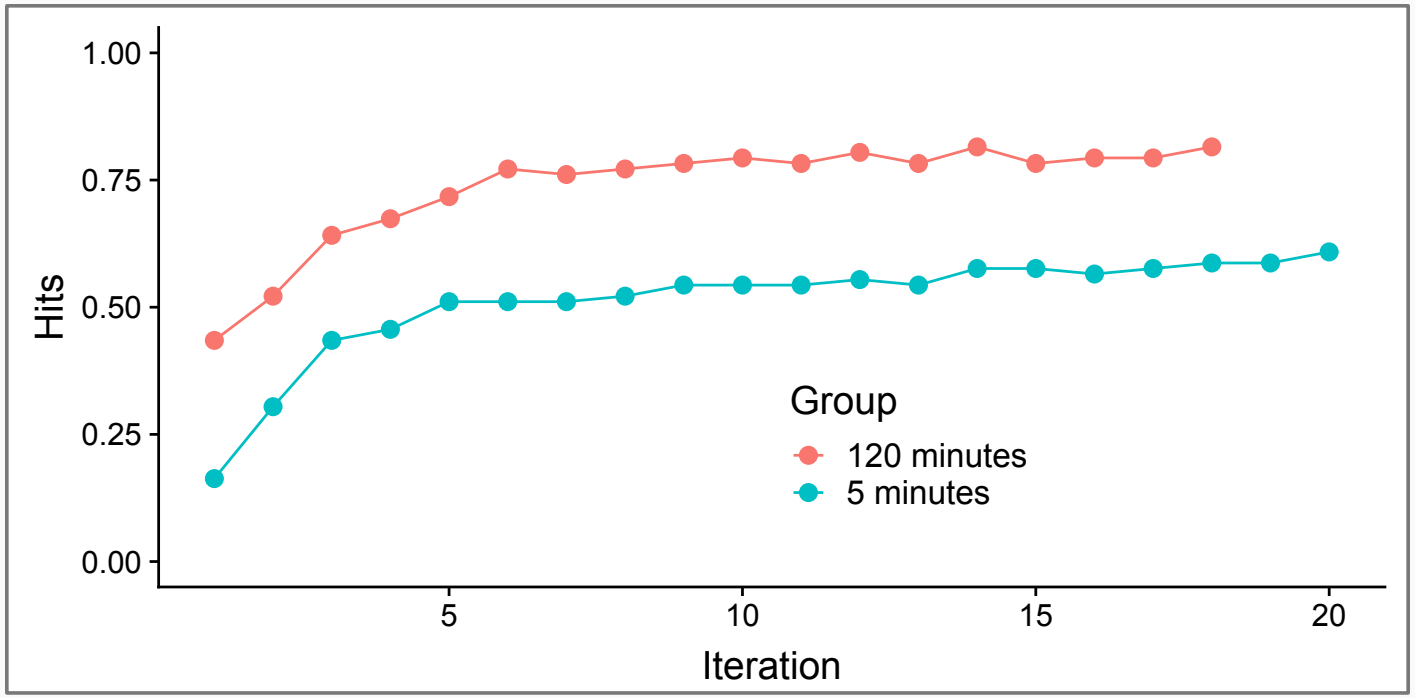


```
NUM ::= 1 | π | ∞ | ε | + | - | * | /
```

# Turtle graphics — Training tasks



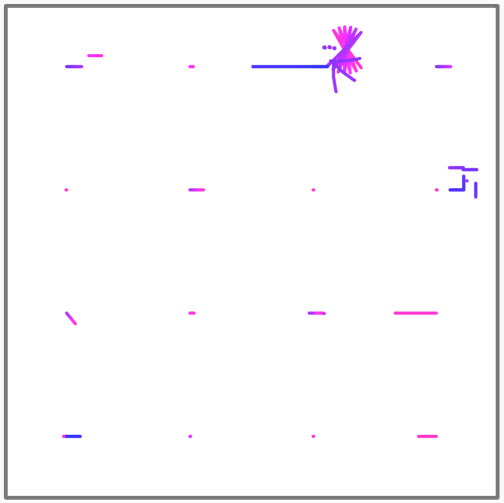
# Turtle graphics — Learning curves

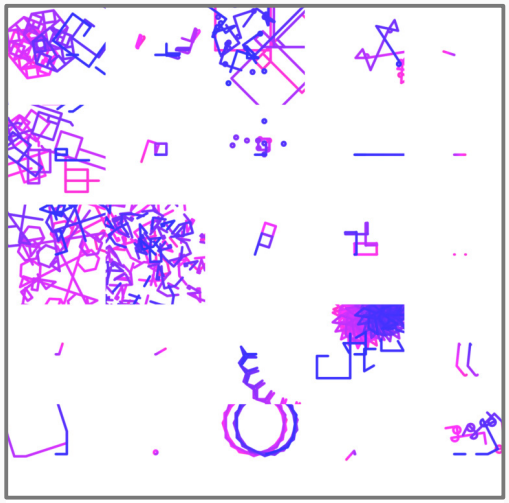



- $\frac{\pi}{2}$ and $\frac{\pi}{4}$ from $\pi$, 2, $+$ and $/$
- A line of length `n` followed with a right angle
- Loops of length `n` that uses the number `n` inside.
- Unit line then teleport back to origin
- ...

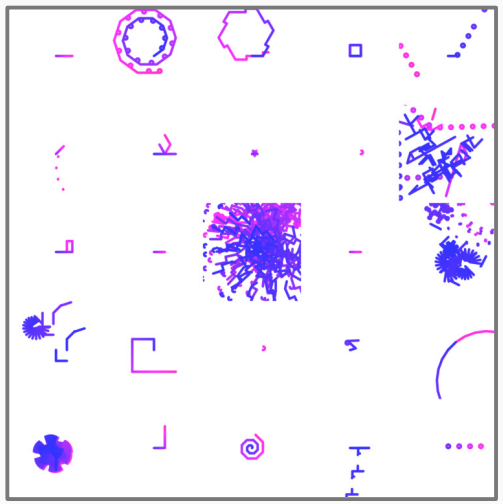
# Turtle graphics — Dreams

Before training



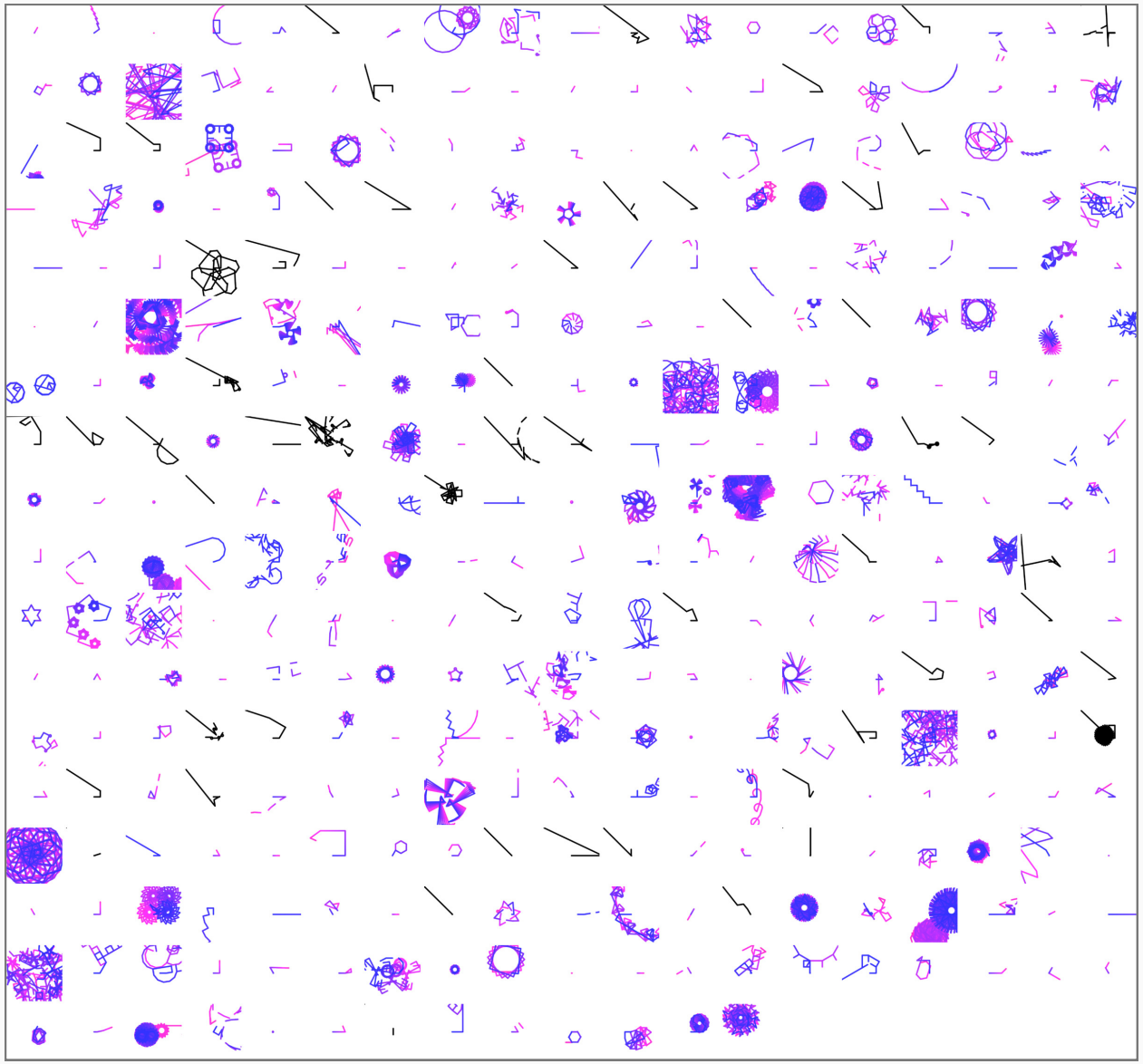


Plateau 5 minutes

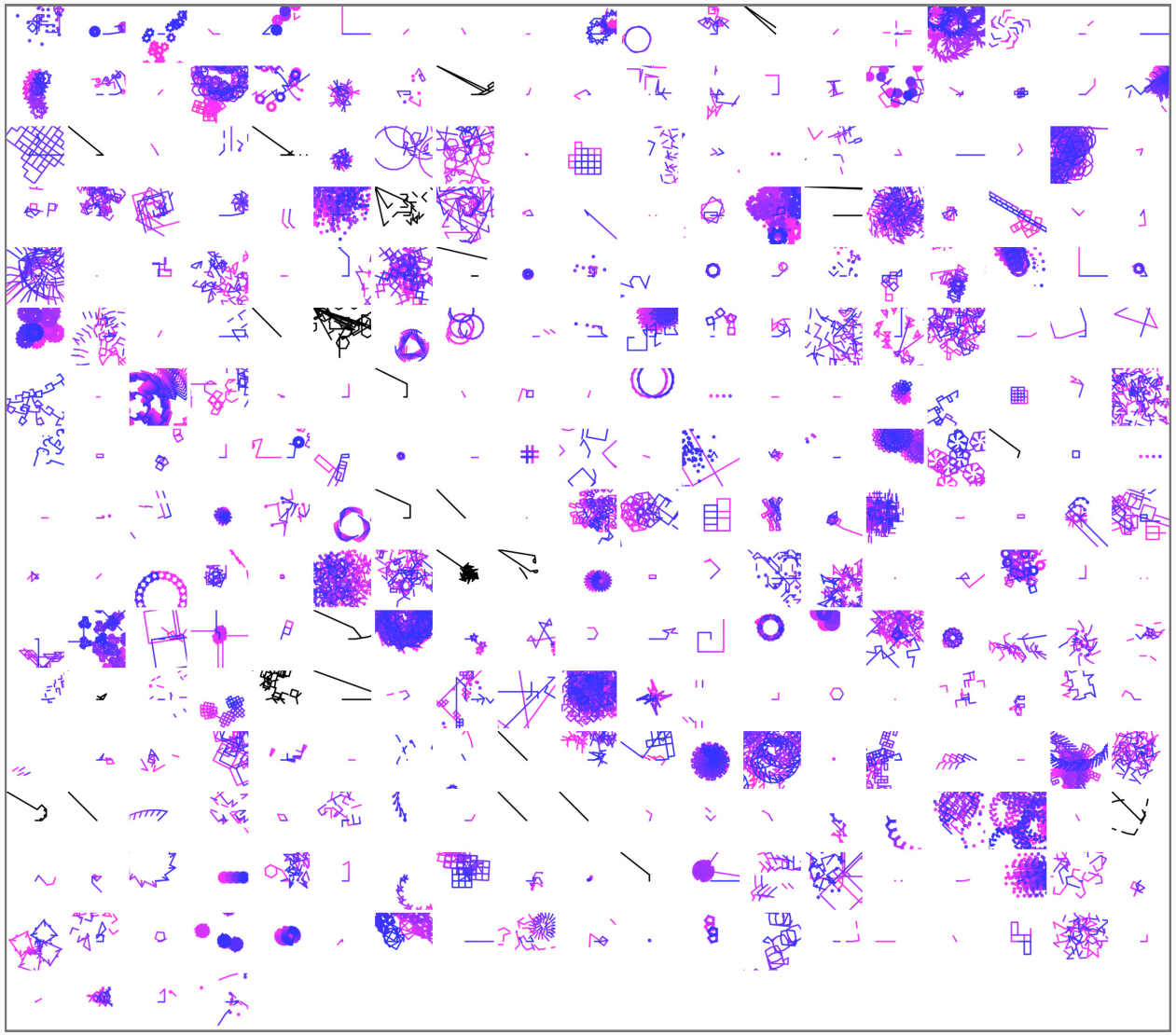


Plateau 2 hours

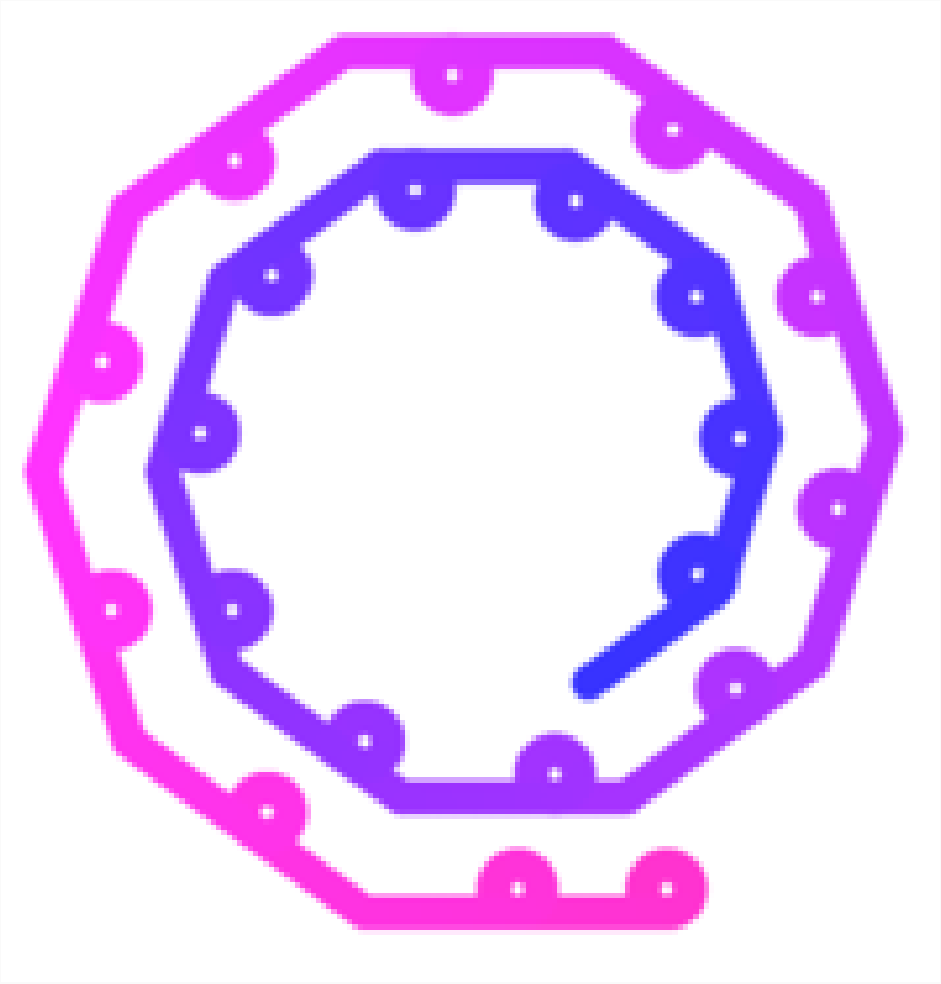
# Turtle graphics — More dreams, 5 minutes, 1st iteration



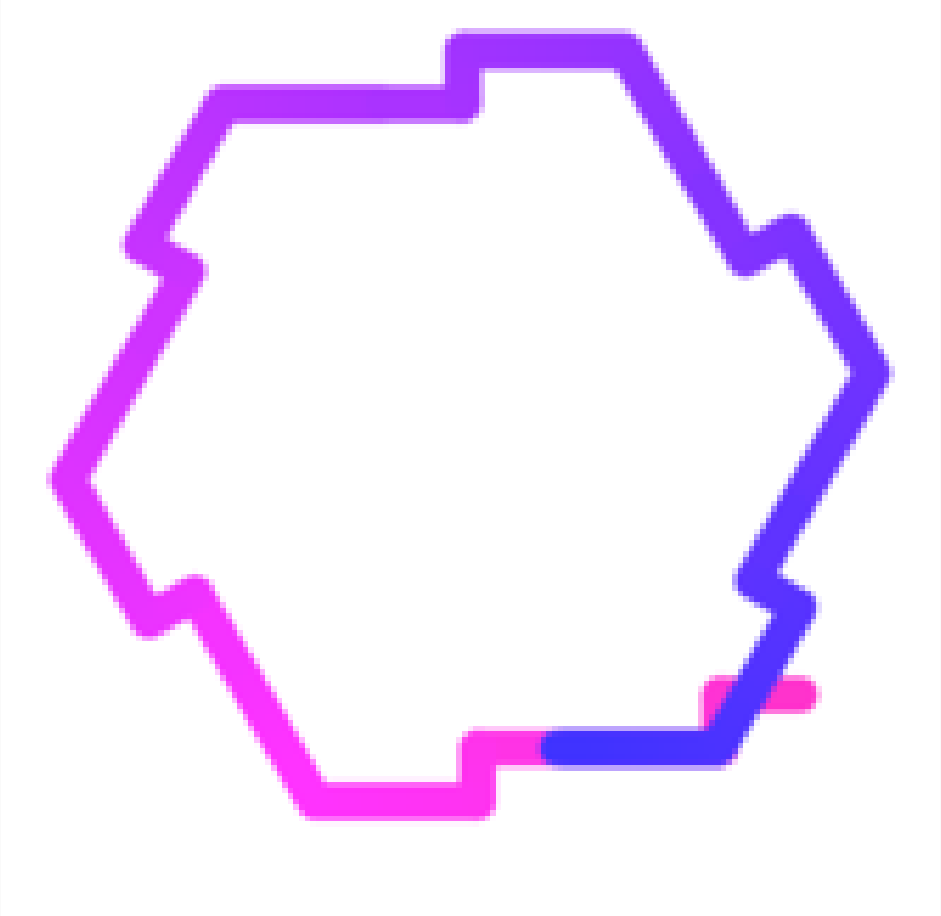
# Turtle graphics — More dreams, 5 minutes, last iteration



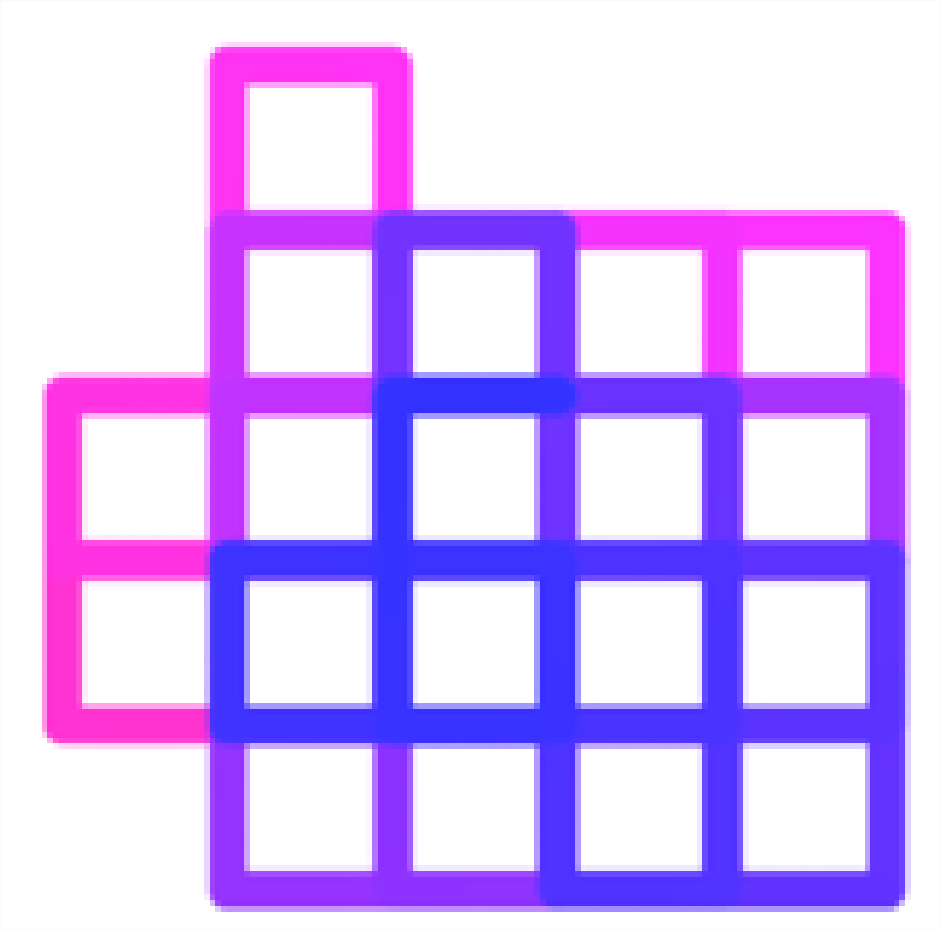
# Turtle graphics — Dreaming from learned generative model



# Turtle graphics — Dreaming from learned generative model

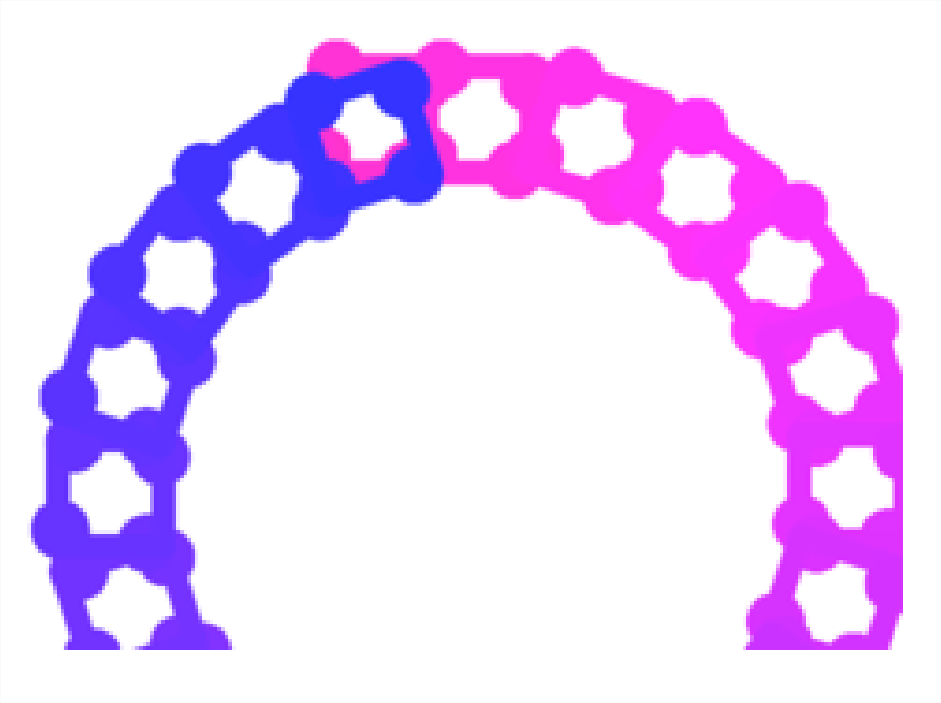


# Turtle graphics — Dreaming from learned generative model

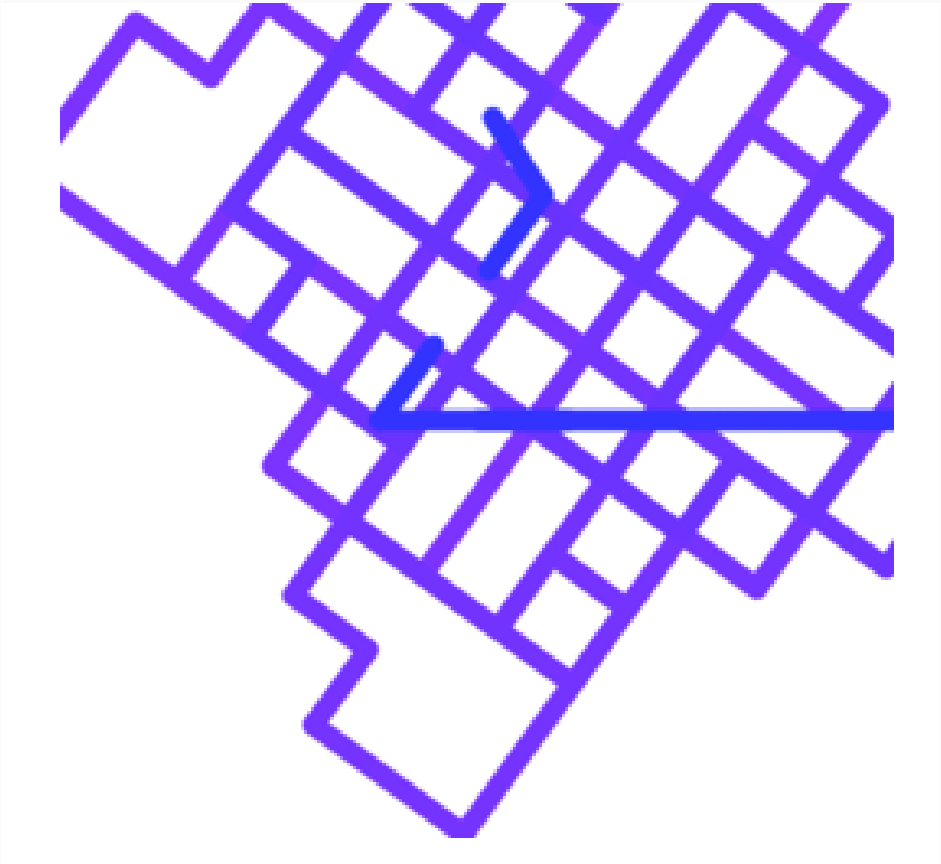


# Turtle graphics — Dreaming from learned generative model

# Turtle graphics — Dreaming from learned generative model



# Turtle graphics — Dreaming from learned generative model



# Turtle graphics — Dreaming from learned generative model

# Turtle graphics — Dreaming from learned generative model

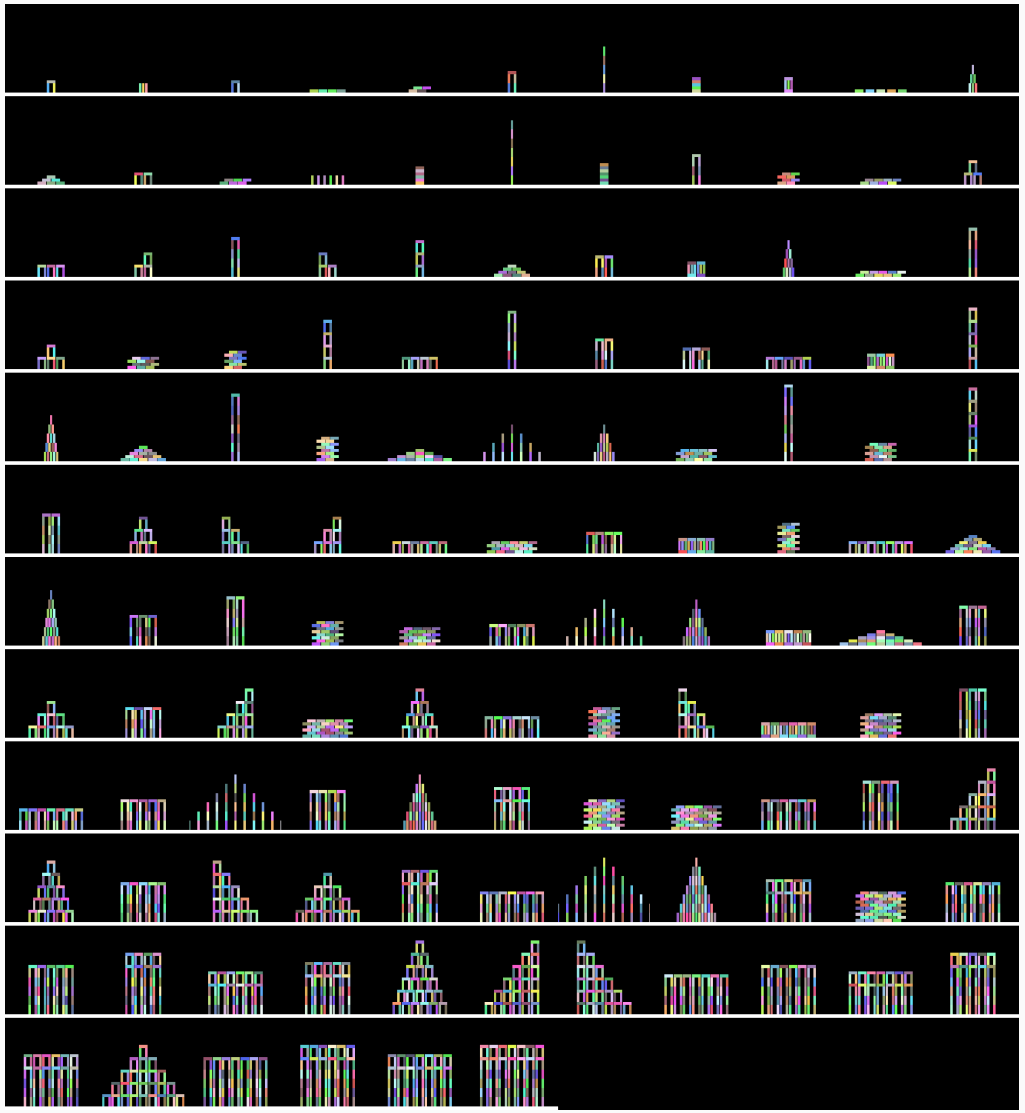
# Tower building in blocks world

Control a hand that puts down blocks

(turtle: control a pen that puts down ink)

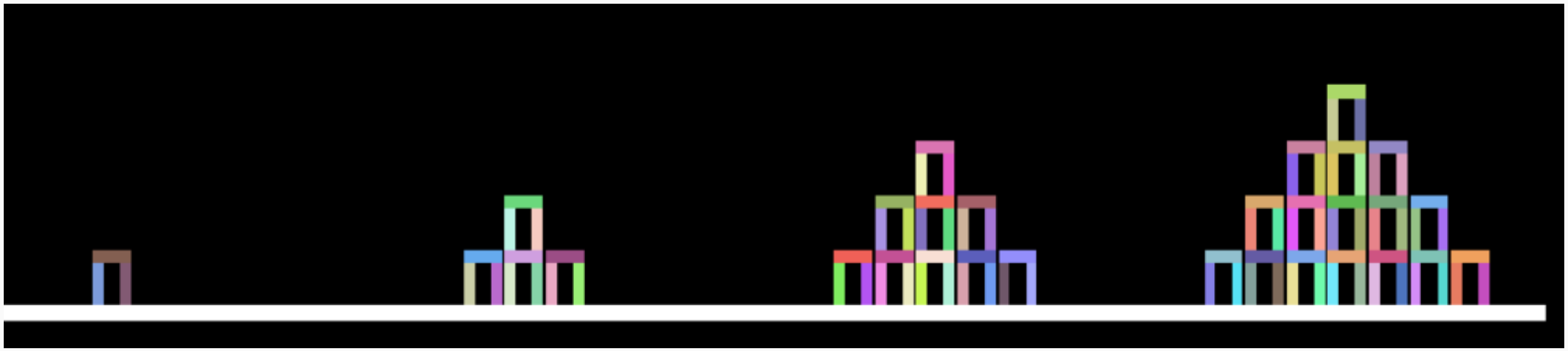
# Tower building in blocks world

Control a hand that puts down blocks
(turtle: control a pen that puts down ink)



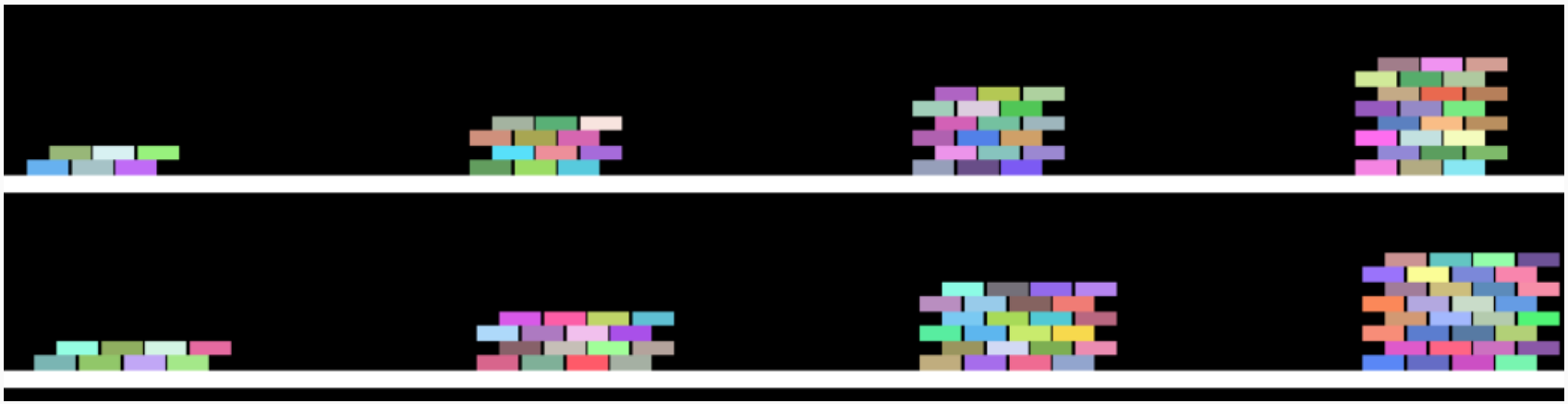
# Tower building in blocks world: Learned concepts

Parametric planning primitives. Example pyramid concept:



# Tower building in blocks world: Learned concepts

Parametric planning primitives. Example brickwall concept:



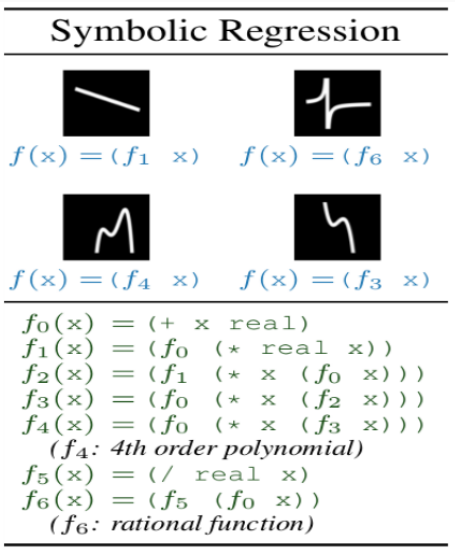
# Vision

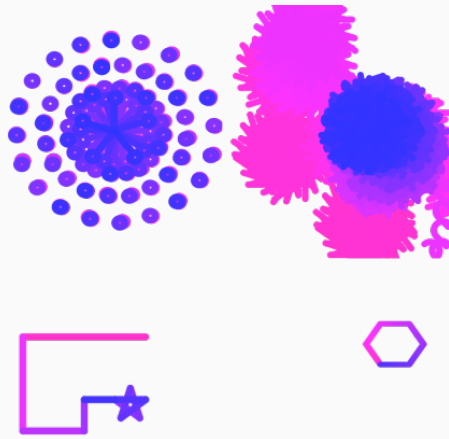
More human-like machine intelligence

- Acquiring a domain-specific representation (DSL)
- Learning to write programs (recognition model)

DreamCoder: an algorithm for jointly realizing these goals

```
f2(p,f,n,x) = (if (p x) nil
                  (cons (f x) (f2 (n x))))
  (f2: unfold)
f3(i,l) = (if (= i 0) (car l)
              (f3 (f1 i) (cdr l))))
  (f3: index)
f4(f,l,x) = (if (empty? l) x
                (f (car l) (f4 (cdr l))))
  (f4: fold)
f5(f,l) = (if (empty? l) nil)
              (cons (f (car l)) (f5 (cdr l)))
  (f5: map)
```



# Vision

More human-like machine intelligence

- Acquiring a domain-specific representation (DSL)
- Learning to write programs (recognition model)

DreamCoder: an algorithm for jointly realizing these goals

$f_2$(p,f,n,x) = (if (p x) nil
(cons (f x) ($f_2$ (n x))))
($f_2$: *unfold*)
$f_3$(i,l) = (if (= i 0) (car l)
($f_3$ ($f_1$ i) (cdr l))))
($f_3$: *index*)
$f_4$(f,l,x) = (if (empty? l) x
(f (car l) ($f_4$ (cdr l))))
($f_4$: *fold*)
$f_5$(f,l) = (if (empty? l) nil)
(cons (f (car l)) ($f_5$ (cdr l)))
($f_5$: *map*)

**Symbolic Regression**

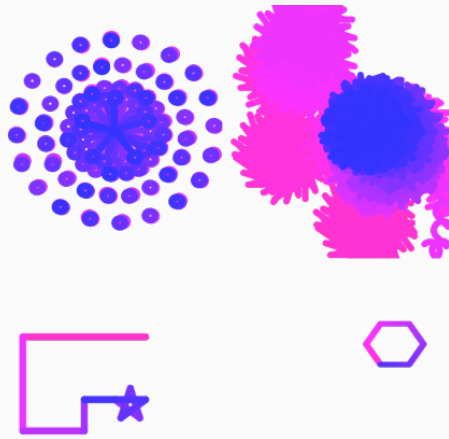

$f(x) = (f_1\ x)$


$f(x) = (f_6\ x)$


$f(x) = (f_4\ x)$


$f(x) = (f_3\ x)$

$f_0(x) = (+\ x\ \text{real})$
$f_1(x) = (f_0\ (*\ \text{real}\ x))$
$f_2(x) = (f_1\ (*\ x\ (f_0\ x)))$
$f_3(x) = (f_0\ (*\ x\ (f_2\ x)))$
$f_4(x) = (f_0\ (*\ x\ (f_3\ x)))$
($f_4$: *4th order polynomial*)
$f_5(x) = (/\ \text{real}\ x)$
$f_6(x) = (f_5\ (f_0\ x))$
($f_6$: *rational function*)



# The End.