Inducing Domain Specific Languages for Bayesian Program Learning

Anonymous Authors[1]

Abstract

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1. Introduction

Automatically inducing programs from examples is a long-standing goal of artificial intelligence. Recent work in program induction show that deep networks can be very successful at learning to write certain kinds of programs (e.g., RobustFill: (Devlin et al., 2017b) and DeepCoder: (Balog et al., 2016)), as can symbolic search techniques (e.g., Metagol: (Muggleton et al., 2015) and FlashFill: (Gulwani, 2011)). However, the success of these approaches – both symbolic and neural – relies crucially upon a hand-engineered *Domain Specific Language* (DSL). The DSL is an inventory of restricted programming primitives, that imparts domain specific knowledge about the space of programs. To what extent can we dispense with hand-engineered DSLs?

This work proposes *learning* DSLs. We consider the problem of solving many related program induction tasks, starting with a Lisp-like representation. We introduce an algorithm, called the HELMHOLTZHACKER , which creates its own DSL by discovering and then reusing new useful programming idioms and subroutines.

Our algorithm is best understood as inference in a hierarchical Bayesian model (Figure 1(a): black lines): A DSL $\mathcal{D}$, equipped with a real-valued weight vector θ, give a distribution over programs from which a latent program is sampled, which then determines the agent's observations. Our goal will be both to infer the programs that best explain our observations, and at the same time induce a DSL that distills the commonalities across all of the programs that solve the tasks.

[1]Anonymous Institution, Anonymous City, Anonymous Region, Anonymous Country. Correspondence to: Anonymous Author <anon.email@domain.com>.

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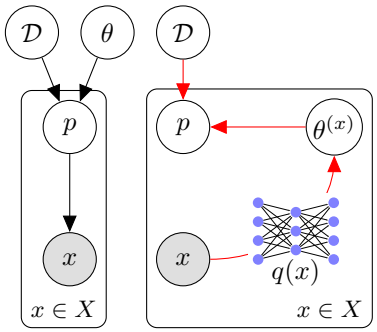


Figure 1: (a) DSL $\mathcal{D}$ generates programs p by sampling DSL primitives with probabilities θ (Algorithm 1). Observe tasks x (program inputs/outputs). (b) neural network $q(\cdot)$, the *recognition model*, regresses from x to a distribution over programs ($\theta^{(x)} = q(x)$). Black arrows correspond to the top-down generative model. Red arrows correspond to the bottom-up recognition model.

Inducing the DSL means learning the symbolic structure of a latent programming language, and so we are confronted with two distinct structure learning problems: the structure of the programs and the structure of the DSL.

Inference in the generative model in Figure 1(a) is difficult because of the vastness of the combinatorial space of all programs. To make inference tractable our model learns a bottom-up *recognition model* alongside the top-down generative model (Figure 1b). The recognition model q is a neural network that regresses from observations to a distribution over programs. The neural network q implements an amortized inference/compiled inference scheme (Le et al., 2017; Ritchie et al., 2016) for the generative model.

How should we perform inference for the model in 1(a)? We take the intuitive strategy of exploiting the conditional independence of the programs conditioned on the DSL, and so alternate between estimating the DSL $(\mathcal{D}, \theta)$ and estimating a posterior distribution over p by searching for programs written using $\mathcal{D}$. In practice searching for programs is difficult, so we use the recognition model to learn how to search efficiently.

Domain	Part of the learned DSL
Circuits	`(nand (nand a a) (nand b b))` *(an OR gate)*
Regression	`(+ (* real x) real)` *(a linear function of x)*
Strings	`(join '' (split c s))` *(delete occurrences of c in string s)*
Lists	

Table 1: Examples of structure found in DSLs learned by our algorithm. HELMHOLTZHACKER builds a new DSL by discovering and reusing useful fragments of code.

We apply HELMHOLTZHACKER to four domains: building Boolean circuits; symbolic regression; FlashFill-style (Gulwani, 2011) string processing problems; and Lisp-style functions on lists. For each of these we deliberately provide an impoverished set of programming primitives, and show that our algorithm discovers its own domain-specific vocabulary for expressing solutions in each of these domains (Table 1).

1.1. Related Work

Our work is far from the first to learn to learn programs, an idea that goes back to Solomonoff (Solomonoff, 1989):

Neural recognition models for program induction: Much recent work in the ML community has focused on engineering neural networks that regress from input/output examples to programs (Devlin et al., 2017b;a; Menon et al., 2013; Balog et al., 2016). This family of work is analogous to a lesioned version of HELMHOLTZHACKER where $(\mathcal{D}, \theta)$ is held fixed and q is learned.

Predicate Invention in ILP: The Inductive Logic Programming (ILP) community has long worked on the problem of inventing new primitive predicates (Lin et al., 2014; Muggleton et al., 2015), which is analogous to our model with θ held fixed and q ablated.

Inventing new subroutines for program induction: Several program induction algorithms, most prominently the EC algorithm (Dechter et al., 2013), take as their goal to learn new, reusable subroutines that are shared in a multitask setting. We find this work inspiring and motivating, and go beyond it along two dimensions: (1) we propose more sophisticated algorithms for inducing reusable structure, based on a formalism called *Fragment Grammars* (O'Donnell, 2015); and (2) we show how to combine these techniques with bottom-up neural recognition models. Other instances of this related idea are (Liang et al., 2010) and Schmidhuber's OOPS model (Schmidhuber, 2004).

Our work is an instance of *Bayesian Program Learning* (BPL; see (Lake et al., 2013; Dechter et al., 2013; Ellis et al., 2016; Liang et al., 2010)).

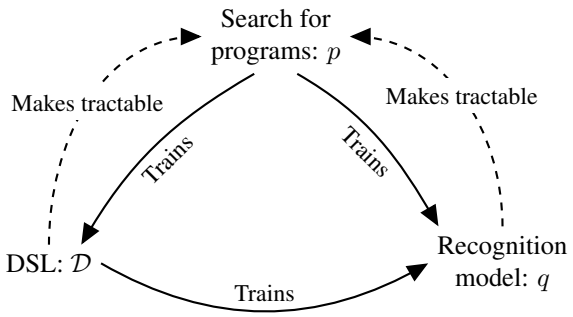


Figure 2: Our algorithm solves for programs, the DSL, and a recognition model. Each of these steps bootstrap off of the others.

2. The HELMHOLTZHACKER Algorithm

Our goal is to induce a DSL and find good programs solving each of the tasks. Our strategy is to iterate through three steps: (1) searching for programs that solve the tasks, (2) learning a better neural recognition model – which we use to accelerate the search over programs – and (3) improving the DSL. The key observation here is that each of these three steps can bootstrap off each other:

Searching for programs: Our program search is informed by both the DSL and the recognition model. When these improve we find better programs.

Learning a recognition model: The recognition model is trained both on samples from the DSL and on programs found by the search procedure. As the DSL improves and as search finds more programs, the recognition model gets both more data to train on and better data.

Improving the DSL: We induce a DSL from the programs we have found so far which solve the tasks; as we solve more tasks, we can hone in on richer DSLs that more closely match the domain.

The Helmholtz machine (Dayan et al., 1995) introduced the idea of alternating between learning a generative model and training a recognition model on samples from the generative model. We apply this idea to program induction, motivating why we call our algorithm the HELMHOLTZHACKER . Figure 2 diagrams how these three steps feed off of each other.

Section 2.1 frames this 3-step procedure as a means of maximizing a lower bound on the posterior probability of the DSL given the tasks. Section 2.2 explains how we search for programs that solve the tasks; Section 2.3

explains how we train a neural network to accelerate the search over programs; and Section 2.4 explains how HELMHOLTZHACKER induces a DSL from programs.

2.1. Probabilistic Framing

HELMHOLTZHACKER takes as input a set of *tasks*, written X, each of which is a program induction problem. It has at its disposal a *likelihood model*, written $\mathbb{P}[x|p]$, which scores the likelihood of a task $x \in X$ given a program p. Its goal is to solve each of the tasks by writing a program, and also to infer a DSL $\mathcal{D}$.

We frame this problem as maximum a posteriori (MAP) inference in the generative model diagrammed in Figure 1. We wish to maximize the posterior probability of $\mathcal{D}$:

$$\mathbb{P}[\mathcal{D}|X] \propto \mathbb{P}[\mathcal{D}] \int \mathrm{d}\theta P(\theta|\mathcal{D}) \prod_{x \in X} \sum_{p} \mathbb{P}[x|p]\mathbb{P}[p|\mathcal{D},\theta]$$

In general this marginalization over θ is intractable, so we make an AIC-style approximation[1], $A \approx \log \mathbb{P}[\mathcal{D}|X]$:

$$A = \log \mathbb{P}[\mathcal{D}] + \arg\max_{\theta} \sum_{x \in X} \log \sum_{p} \mathbb{P}[x|p]\mathbb{P}[p|\mathcal{D},\theta] + \log P(\theta|\mathcal{D}) - ||\theta||_0 \quad (1)$$

If we had a $(\mathcal{D}, \theta)$ maximizing Eq. 1, then we could recover the most likely program for task x by maximizing $\mathbb{P}[x|p]\mathbb{P}[p|\mathcal{D},\theta]$. Through this lens we now take as our goal to maximize Eq. 1. But even *evaluating* Eq. 1 is intractable because it involves summing over the infinite set of all possible programs. In general, programs are hard-won: finding even a single program that explains a given observation presents a daunting combinatorial search problem. With this fact in mind, we will instead maximize the following tractable lower bound on Eq. 1, which we call J:

$$J = \log \mathbb{P}[\mathcal{D},\theta] + \sum_{x \in X} \log \sum_{p \in \mathcal{F}_x} \mathbb{P}[x|p]\mathbb{P}[p|\mathcal{D},\theta] \quad (2)$$

This lower bound depends on sets of programs, $\{\mathcal{F}_x\}_{x \in X}$:

Definition. A *frontier of task* x, written $\mathcal{F}_x$, is a finite set of programs where $\mathbb{P}[x|p] > 0$ for all $p \in \mathcal{F}_x$.

We maximize J by alternating maximization w.r.t. the frontiers and the DSL:

Program Search: Maxing J w.r.t. the frontiers. Here we want to find new programs to add to the frontiers so that J increases the most. Adding new programs to the frontiers means searching for new programs p for task x where $\mathbb{P}[x, p|\mathcal{D},\theta]$ is large.

DSL Induction: Maxing J w.r.t. the DSL. Here $\{\mathcal{F}_x\}_{x \in X}$ is held fixed so we can evaluate J. Now the problem is that of searching the discrete space of DSLs and finding one maximizing J.

Searching for programs is extremely difficult because of the large search space. We ease this difficulty by learning a neural recognition model:

Neural recognition model: tractably maxing J w.r.t. the frontiers. Here we train a neural network, q, to predict a distribution over programs conditioned on a task. The objective of q is to assign high probability to programs p where $\mathbb{P}[x, p|\mathcal{D},\theta]$ is large. With q in hand we can find programs for the frontier of a task $\mathcal{F}_x$ by searching for programs that maximize $q(p|x)$. The network q exploits the structure of the DSL $\mathcal{D}$: rather than directly predicting a distribution over p conditioned on x, it predicts a weight vector, $\theta^{(x)}$, and we define $q(p|x) \triangleq \mathbb{P}[p|\mathcal{D},\theta = q(x)]$. This approach implements an amortized inference scheme (Le et al., 2017; Ritchie et al., 2016) for the generative model in Figure 1.

2.2. Searching for Programs

Now our goal is to search for programs that solve the tasks. In this work we use the simple search strategy of enumerating programs from the DSL in decreasing order of their probability, and then checking if an enumerated program p assigns positive probability to a task ($\mathbb{P}[x|p] > 0$); if so, we include p in the frontier $\mathcal{F}_x$.

To make this concrete we need to define what programs actually are and what form $\mathbb{P}[p|\mathcal{D},\theta]$ takes. We represent programs as λ-calculus expressions. λ-calculus is a formalism for expressing functional programs that closely resembles the Lisp programming language. λ-calculus includes variables, function application, and the ability to create new functions. Throughout this paper we will write λ-calculus expressions in Lisp syntax. Our programs are all strongly typed. We use the Hindley-Milner polymorphic typing system (Pierce, 2002) which is used in functional languages like OCaml. Variables that range over types are always written using lowercase Greek letters and we write $\alpha \to \beta$ to mean a function that takes as input type α and returns something of type β. We use the notation $p : \tau$ to mean that the λ-calculus expression p has the type τ. For example, to describe the type of the identity function we would say `(lambda (x) x)`$: \alpha \to \alpha$. We say a type α *unifies* with τ if every expression $p : \alpha$ also satisfies $p : \tau$. Furthermore, the act of *unifying* a type α with τ is to introduce constraints on the type variables of α to ensure that α unifies with τ. See Section **??** for more detail on program representation. With this notation in hand we can now define DSLs:

Definition. A DSL $\mathcal{D}$ is a set of typed λ-calculus expres-

[1] Sec. 2.4 explains that $\mathcal{D}$ is a context-sensitive grammar. Conventional natural-language processing (NLP) approaches to using variational inference to lower bound the marginal over θ do not apply in our setting.

Algorithm 1 Generative model over programs

function samplePrimitiveFromDSL($\mathcal{D}, \theta, \tau$):
Input: DSL $\mathcal{D}$, weight vector θ, type τ
Output: a program whose type unifies with τ
return sample($\mathcal{D}, \theta, \varnothing, \tau$)

function sample($\mathcal{D}, \theta, \mathcal{E}, \tau$):
Input: DSL $\mathcal{D}$, weight vector θ, environment $\mathcal{E}$, type τ
Output: a program whose type unifies with τ
if $\tau = \alpha \to \beta$ **then**
  var $\leftarrow$ an unused variable name
  body $\sim$ sample($\mathcal{D}, \theta, \{\text{var} : \alpha\} \cup \mathcal{E}, \beta$)
  return `(lambda (var) body)`
end if
primitives $\leftarrow \{p | p : \tau' \in \mathcal{D} \cup \mathcal{E}$
  if τ can unify with yield(τ')$\}$
Draw $e \sim$ primitives, w.p. $\propto \theta_e$ if $e \in \mathcal{D}$
  w.p. $\propto \frac{\theta_{var}}{|\text{variables}|}$ if $e \in \mathcal{E}$
Unify τ with yield(τ').
$\{\alpha_k\}_{k \le K} \leftarrow$ argTypes(τ')
for $k = 1$ **to** K **do**
  $a_k \sim$ sample($\mathcal{D}, \theta, \mathcal{E}, \alpha_k$)
end for
return $(e\ a_1\ a_2\ \cdots\ a_K)$
where:

$$\text{yield}(\tau) = \begin{cases} \text{yield}(\beta) & \text{if } \tau = \alpha \to \beta \\ \tau & \text{otherwise.} \end{cases}$$

$$\text{argTypes}(\tau) = \begin{cases} [\alpha] + \text{argTypes}(\beta) & \text{if } \tau = \alpha \to \beta \\ [] & \text{otherwise.} \end{cases}$$

sions.

Definition. A weight vector θ for a DSL $\mathcal{D}$ is a vector of $|\mathcal{D}| + 1$ real numbers: one number for each DSL primitive $e : \tau \in \mathcal{D}$, written θ_e, and a weight controlling the probability of a variable occurring in a program, written θ_{var}.

Algorithm 1 is a procedure for drawing samples from $\mathbb{P}[p|\mathcal{D}, \theta]$. In practice, we enumerate programs rather than sampling them. Enumeration proceeds by a depth-first search over the random choices made by Algorithm 1; we wrap the depth-first search in iterative deepening to (approximately) build λ-calculus expressions in order of their probability.

Why enumerate, when the program synthesis community has invented many sophisticated algorithms that search for programs? (Solar Lezama, 2008; Schkufza et al., 2013; Feser et al., 2015; Osera & Zdancewic, 2015; Polozov & Gulwani, 2015). We have two reasons:

- A key point of our work is that learning the DSL, along with a neural recognition model, can make program induction tractable, even if the search algorithm is very simple.
- Enumeration is a general approach that can be applied to any program induction problem. Many of these more sophisticated approaches require special conditions on the space of of programs.

A main drawback of an enumerative search algorithm is that we have no efficient means of solving for arbitrary constants that might occur in the program. In Section 3.2, we will show how to find programs with real-valued constants by automatically differentiating through the program and setting the constants using gradient descent. In Section 3.3 we will show that the bottom-up neural recognition model can learn which discrete constants should be included in a program.

2.3. Learning a Neural Recognition Model

The purpose of the recognition model is to accelerate the search over programs. It does this by learning to predict which programs both assign high likelihood to a task, and at the same time have high prior probability under the prior $\mathbb{P}[\cdot|\mathcal{D}, \theta]$.

The recognition model q is a neural network that predicts, for each task $x \in X$, a weight vector $q(x) = \theta^{(x)} \in \mathbb{R}^{|\mathcal{D}|+1}$. Together with the DSL, this defines a distribution over programs, $\mathbb{P}[p|\mathcal{D}, \theta = q(x)]$. We abbreviate this distribution as $q(p|x)$. The crucial aspect of this framing is that the neural network leverages the structure of the learned DSL, and so is *not* responsible for generating programs wholesale.

We want a recognition model that closely approximates the true posteriors over programs, and so aim to minimize the following KL-divergence:

$$\mathbb{E}\left[\text{KL}\left(\mathbb{P}[p|x, \mathcal{D}, \theta] || q(p|x)\right)\right]$$

which is equivalent to maximizing

$$\mathbb{E}\left[\sum_p \mathbb{P}[p|x, \mathcal{D}, \theta] \log q(p|x)\right]$$

where the expectation is taken over tasks. One could take this expectation over the empirical distribution of the observations, like how an autoencoder is trained (Hinton & Salakhutdinov, 2006); or, one could take this expectation over samples from the generative model, like how a Helmholtz machine is trained (Dayan et al., 1995). We found it useful to maximize both an autoencoder-style objective (written $\mathcal{L}_{\text{AE}}$) and a Helmholtz-style objective ($\mathcal{L}_{\text{HM}}$), giving the HELMHOLTZHACKER objective for a recognition

model, $\mathcal{L}_{\text{RM}}$:

$$\mathcal{L}_{\text{RM}} = \mathcal{L}_{\text{AE}} + \mathcal{L}_{\text{HM}} \tag{3}$$
$$\mathcal{L}_{\text{HM}} = \mathbb{E}_{p\sim(\mathcal{D},\theta)}\left[\log q(p|x)\right],\ p \text{ evaluates to } x$$
$$\mathcal{L}_{\text{AE}} = \mathbb{E}_{x\sim X}\left[\sum_{p\in\mathcal{F}_x} \frac{\mathbb{P}\left[x,p|\mathcal{D},\theta\right]}{\sum_{p'\in\mathcal{F}_x}\mathbb{P}\left[x,p|\mathcal{D},\theta\right]}\log q(p|x)\right]$$

Evaluating $\mathcal{L}_{\text{HM}}$ involves sampling programs from the current DSL, running them to get their outputs, and then training q to regress from the outputs to the program. If these programs map inputs to outputs, then we need some way of sampling these inputs as well. Our solution to this problem is to sample the inputs from the empirical observed distribution of inputs in X.

2.4. Inducing the DSL from the Frontiers

The purpose of the DSL is to offer a set of abstractions that allow an agent to easily express solutions to the tasks at hand. In the HELMHOLTZHACKER algorithm we infer the DSL from a collection of frontiers. Intuitively, we want the algorithm to look at the programs in the frontiers and generalize beyond them; not only so the DSL can better express the current solutions, but also so that the DSL might expose new abstractions which will later be used to discover even better programs.

Exact maximization of J (Eq.2) w.r.t. $(\mathcal{D},\theta)$ is intractable, so we take a more heuristic approach. The strategy is to search locally through the space of DSLs, proposing small local changes to $\mathcal{D}$ until J fails to increase. The search moves work by introducing new λ-expressions into the DSL. We propose these new expressions by extracting subexpressions from programs already in the frontier. These extracted subexpressions are fragments of the original programs, and can introduce new variables (Figure 3), which then become new functions in the DSL.

Closely related to Fragment Grammars (O'Donnell, 2015) and Tree-Substitution Grammars (Cohn et al., 2010), but context-sensitive

We define a prior distribution over DSLs which penalizes the sizes of the λ-calculus expressions in the DSL, and put a Dirichlet prior over the weight vector:

$$\mathbb{P}[\mathcal{D}] \propto \exp\left(\lambda \sum_{p\in\mathcal{D}} \text{size}(p)\right)$$
$$P(\theta|\mathcal{D}) = \text{Dir}(\theta|\alpha)$$

where $\text{size}(p)$ measures the size of the syntax tree of program p, λ is a hyperparameter that acts as a regularizer on the size of the DSL, and α is a concentration parameter controlling the smoothness of the prior over θ. Algorithm 2 specifies the DSL induction algorithm.

Algorithm 2 DSL Induction Algorithm

Input: Set of frontiers $\{\mathcal{F}_x\}$
Hyperparameters: Pseudocounts α, regularization parameter λ
Output: DSL $\mathcal{D}$, weight vector θ
Define $L(\mathcal{D},\theta) = \prod_x \sum_{z\in\mathcal{F}_x}\mathbb{P}[z|\mathcal{D},\theta]$
Define $\theta^*(\mathcal{D}) = \arg\max_\theta \text{Dir}(\theta|\alpha)L(\mathcal{D},\theta)$
Define $\text{score}(\mathcal{D}) = \log\mathbb{P}[\mathcal{D}] + L(\mathcal{D},\theta^*) - ||\theta||_0$
$\mathcal{D} \leftarrow$ every primitive in $\{\mathcal{F}_x\}$
while true **do**
  $\text{N} \leftarrow \{\mathcal{D}\cup\{s\}|x\in X, z\in\mathcal{F}_x, s \text{ subexpression of } z\}$
  $\mathcal{D}' \leftarrow \arg\max_{\mathcal{D}'\in N}\text{score}(\mathcal{D}')$
  if $\text{score}(\mathcal{D}') < \text{score}(\mathcal{D})$ **return** $\mathcal{D},\theta^*(\mathcal{D})$
  $\mathcal{D} \leftarrow \mathcal{D}'$
end while

Additionally, for each proposed $\mathcal{D}$ we have to reestimate θ. Although this problem may seem very similar to estimating the parameters of a probabilistic context free grammar (PCFG: see ()), for which we have effective approaches like the Inside/Outside algorithm (Lafferty, 2000), $\mathcal{D}$ is actually context-sensitive due to the presence of variables in the programs and also due to the polymorphic typing system. In the Appendix we derive a tractable MAP estimator for θ.

2.5. Implementing the HELMHOLTZHACKER

Algorithm 3 describes how we combine the program search, recognition model training, and DSL induction. We additionally make the following implementation details: (1) On the first iteration we do *not* train the recognition model on samples from the generative model because, on the first iteration, we have not yet learned a new DSL, so we instead trained the network to only maximize $\mathcal{L}_{\text{AE}}$; (2) Because the frontiers can grow very large, we only keep around the top 10^4 programs p in each frontier $\mathcal{F}_x$ with the highest $\mathbb{P}[x,p|\mathcal{D},\theta]$; (3) During both DSL induction and neural net training, we calculate $\text{score}(\mathcal{D}')$ and $\mathcal{L}_{\text{RM}}$ by only summing over the top K programs in $\mathcal{F}_x$. We found that $K=2$ sufficed. (4) For added robustness, at each iteration we enumerate programs from both the generative model and the recognition model.

3. Experiments

3.1. Boolean circuits

As a toy warm-up domain, we consider the problem of learning to build the Boolean circuits out of logic gates. Now, if we equipped our agent with a full repertoire of circuit primitives – XOR gates, multiplexers, etc. – this problem

Domain	Example programs in frontiers	Example proposed subexpression
Boolean circuits	`(lambda x (nand x x))` `(lambda x (lambda y (nand x (nand y y))))`	`(nand z z)`
String editing	`(lambda s (+ ',' (index 0 (split ',' s))))` `(lambda s (index 0 (split ' ' s)))`	`(index 0 (split z s))`

Figure 3: The DSL induction algorithm works by proposing subexpressions of programs to add to the DSL. These subexpressions are taken from programs in the frontiers (middle column), and can introduce new variables (`z` in the right column).

Algorithm 3 The HELMHOLTZHACKER Algorithm

Input: Initial DSL $\mathcal{D}$, set of tasks X, iterations I
Hyperparameters: Maximum frontier size F
Output: DSL $\mathcal{D}$, weight vector θ, recognition model $q(\cdot)$
Initialize $\theta \leftarrow$ uniform
for $i = 1$ **to** I **do**
  $\mathcal{F}_x^{(\theta)} \leftarrow \{z | z \in \text{enumerate}(\mathcal{D}, \theta, F) \text{ if } \mathbb{P}[x|z] > 0\}$
  $q_i \leftarrow$ train recognition model to maximize $\mathcal{L}_{\text{RM}}$
  $\mathcal{F}_x^{(q)} \leftarrow \{z | z \in \text{enumerate}(\mathcal{D}, q(x), F) \text{ if } \mathbb{P}[x|z] > 0\}$
  $\mathcal{D}, \theta \leftarrow$ induceDSL$(\{\mathcal{F}_x^{(\theta)} \cup \mathcal{F}_x^{(q)}\}_{x \in X})$
end for
return $\mathcal{D}, \theta, q$

Primitives	`nand`
Observation x	N input/output examples: $\{(i_n, o_n)\}_{n \le N}$
Likelihood $\mathbb{P}[x\|p]$	$\prod_n \mathbb{1}[p(i_n) = o_n]$
Subset of Learned DSL	

Table 2

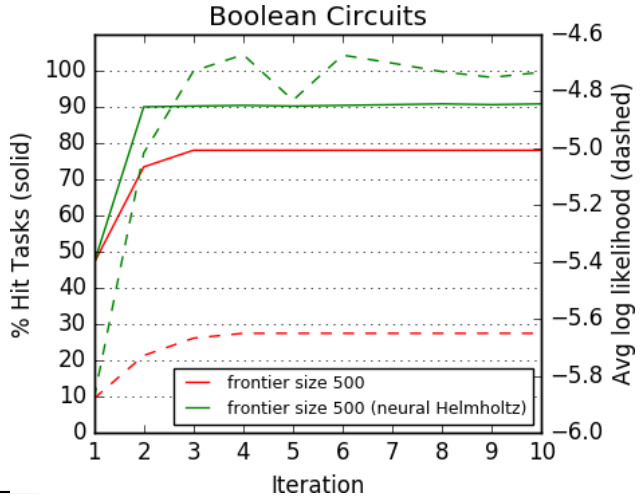


Figure 4: HELMHOLTZHACKER learning to build Boolean circuits. We start out with only NAND gates and learn DSL that contains ...

would be easy, and we could use sophisticated symbolic search algorithms like SMT solvers to synthesize circuits. Instead, we gave HELMHOLTZHACKER only a NAND gate, and tasked it with building a set of 500 random circuits made out of AND, NOT, and OR gates. This experimental setup was introduced in the original EC algorithm (Dechter et al., 2013). For this domain our recognition model is a simple multilayer perceptron whose inputs are the input/output examples. Figure 4 shows our algorithm learning to build circuits; Table 2

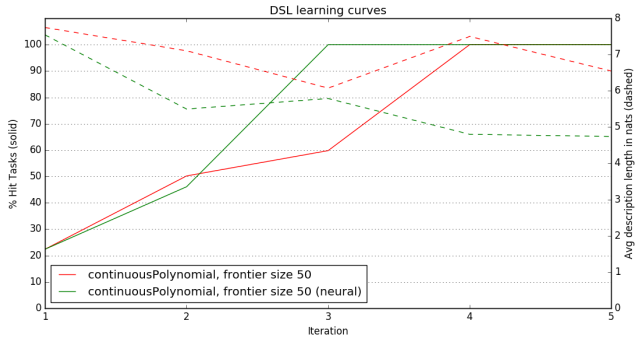
3.2. Symbolic Regression

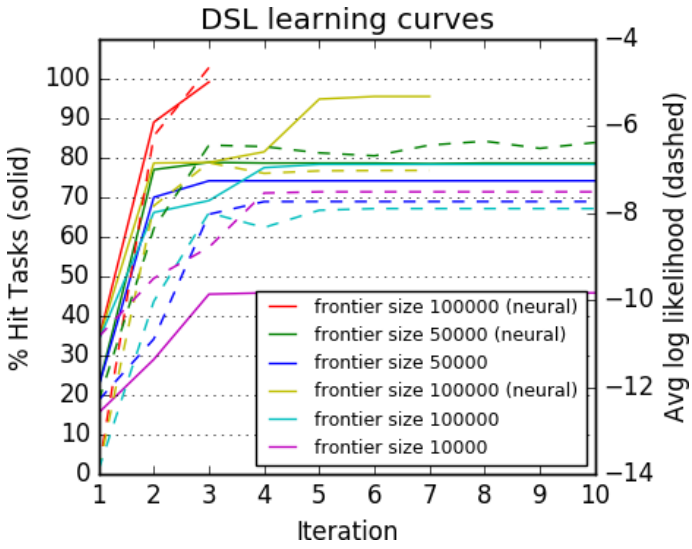
We show how to use HELMHOLTZHACKER to infer programs containing both discrete structure and continuous parameters. The high-level idea is to synthesize programs with unspecified-real-valued parameters, and to fit those parameters using gradient descent. Concretely, we ask the algorithm to solve a set of 1000 symbolic regression problems, each a polynomial of degree 0, 1, or 2, where our observations x take the form of N input/output examples, which we write as $x = \{(i_n, o_n)\}_{n \le N}$. For example, one task is to infer a program calculating $3x + 2$, and the observations are the input-output examples $\{(-1, -1), (0, 2), (1, 5)\}$.

We initially equip our DSL learner with addition and multiplication, along with the possibility of introducing real-valued parameters, which we write as $\mathcal{R}$. We define the likelihood of an observation x by assuming a Gaussian noise model for the input/output examples and integrate over the real-valued parameters, which we collectively write as $\vec{\mathcal{R}}$:

$$\log \mathbb{P}\left[\{(i_n, o_n)\}|p\right] = \log \int \mathrm{d}\vec{\mathcal{R}}\; P_{\vec{\mathcal{R}}}(\vec{\mathcal{R}}) \prod_{n \le N} \mathcal{N}(p(i_n, \vec{\mathcal{R}})|o_n)$$

where $\mathcal{N}(\cdot|\cdot)$ is the normal density and $P_{\vec{\mathcal{R}}}(\cdot)$ is a prior over $\vec{\mathcal{R}}$. We approximate this marginal using the BIC (Bishop,





2006):

$$\log \mathbb{P}[x|p] \approx \sum_{n \leq N} \log \mathcal{N}(p(i_n, \vec{\mathcal{R}}^*)|o_n) - \frac{D \log N}{2}$$

where $\vec{\mathcal{R}}^*$ is an assignment to $\vec{\mathcal{R}}$ found by performing gradient ascent on the likelihood of the observations w.r.t. $\vec{\mathcal{R}}$.

What DSL does HELMHOLTZHACKER learn? The learned DSL contains templates for quadratic and linear functions, which lets the algorithm quickly hone in on the kinds of functions that are most appropriate to this domain. Examining the programs themselves, one finds that the algorithm discovers representations for each of the polynomials that minimizes the number of continuous degrees of freedom: for example, it represents the polynomial $8x^2 + 8x$

Primitives	$+, \times : \mathbb{R} \to \mathbb{R} \to \mathbb{R}$ $\mathcal{R} : \mathbb{R}$ (real valued parameter)	
Observation x	N input/output examples: $\{(i_n, o_n)\}_{n \leq N}$	
Likelihood $\mathbb{P}[x\|p]$	$\propto \exp(-D \log N) \prod_{n \leq N} \mathcal{N}(p(i_n)\|o_n)$	
Subset of Learned DSL	$\lambda x.\mathcal{R} \times x + \mathcal{R}$ $\lambda x.\mathcal{R} + x$ $\lambda x.x \times$ (linear x) $\lambda x.$increment (quadratic$_0$ x)	linear increment quadratic$_0$ quadratic

3.3. String editing

Primitives	`0`, `+1`,`-1`
	`split`, `join`, `map`, `length`, `slice`
	`append`, `chr->str`
	`lowercase`, `uppercase`, `capitalize`
	`""`, `","`, `" "`, `"<"`, `">"`	

Observation x	N input/output examples: $\{(i_n, o_n)\}_{n \leq N}$	
Likelihood $\mathbb{P}[x\|p]$	$\prod_n \mathbb{1}[p(i_n) = o_n]$	
Subset of Learned DSL		

name	*input*	*output*
add-3	[1 2 3 4]	[4 5 6 7]
append-4	[7 0 2]	[7 0 2 4]
len	[3 5 12 1]	4
range	3	[1 2 3]
has-2	[4 5 7 4]	`false`
has-4	[4 5 7 4]	`true`
repeat-2	[7 0]	[7 0 7 0]
drop-3	[0 3 8 6 4]	[6 4]

Table 3: Examples from the domain of list functions.

3.4. List functions

Our list function domain consists of tasks which are solved by functions that take as input an integer or a list of integers, and have as output either a Boolean, an integer, or a list of integers. Examples of these functions are in Table 3. For each function, we create a task x by generating 15 input/output examples used for testing whether a program produces the correct output. Supplying many examples reduces ambiguity in the task's function, ensuring solutions achieve the desired concept.

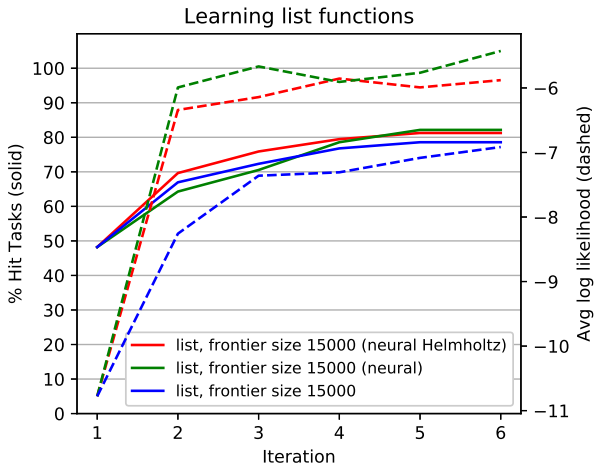
We supply HELMHOLTZHACKER with the DSL outlined in Table 4.

We found that using a less sophisticated but equally-capable DSL made common patterns, such as summation, unlikely and unlearnable in a small enumeration bound.

4. Model

name	*type*
empty	$[\alpha]$
singleton	$\alpha \rightarrow [\alpha]$
concat	$[\alpha] \rightarrow [\alpha] \rightarrow [\alpha]$
map	$(\alpha \rightarrow \beta) \rightarrow [\alpha] \rightarrow [\beta]$
reduce	$(\beta \rightarrow \alpha \rightarrow \beta) \rightarrow \beta \rightarrow [\alpha] \rightarrow \beta$
true	bool
not	bool → bool
and	bool → bool → bool
or	bool → bool → bool
0, . . ., 9	int
+	int → int → int
*	int → int → int
negate	int → int
mod	int → int → int
eq?	int → int → bool
gt?	int → int → bool
is-prime	int → bool
is-square	int → bool
range	int → [int]
sort	[int] → [int]
sum	[int] → int
reverse	$[\alpha] \rightarrow [\alpha]$
all	$(\alpha \rightarrow$ bool$) \rightarrow [\alpha] \rightarrow$ bool
any	$(\alpha \rightarrow$ bool$) \rightarrow [\alpha] \rightarrow$ bool
index	int → $[\alpha] \rightarrow \alpha$
filter	$(\alpha \rightarrow$ bool$) \rightarrow [\alpha] \rightarrow [\alpha]$
slice	int → int → $[\alpha] \rightarrow [\alpha]$

Table 4: DSL for the domain of list function.



5. Program Representation

We choose to represent programs using λ-calculus (Pierce, 2002). A λ-calculus expression is either:
A *primitive*, like the number 5 or the function sum.
A *variable*, like x, y, z
A *λ-abstraction*, which creates a new function. λ-abstractions have a variable and a body. The body is a λ-calculus expression. Abstractions are written as λvar.body.
An *application* of a function to an argument. Both the function and the argument are λ-calculus expressions. The application of the function f to the argument x is written as $f\ x$.

For example, the function which squares the logarithm of a number is $\lambda x.\texttt{square}(\texttt{log}\ x)$, and the identity function $f(x) = x$ is $\lambda x.x$. The λ-calculus serves as a spartan but expressive Turing complete program representation, and distills the essential features of functional languages like Lisp.

However, many λ-calculus expressions correspond to ill-typed programs, such as the program that takes the logarithm of the Boolean true (i.e., log true) or which applies the number five to the identity function (i.e., 5 $(\lambda x.x)$). We use a well-established typing system for λ-calculus called *Hindley-Milner typing* (Pierce, 2002), which is used in programming languages like OCaml. The purpose of the typing system is to ensure that our programs never call a function with a type it is not expecting (like trying to take the logarithm of true). Hindley-Milner has two important features:
Feature 1: It supports *parametric polymorphism*: meaning that types can have variables in them, called *type variables*. Lowercase Greek letters are conventionally used for type variables. For example, the type of the identity function is $\alpha \rightarrow \alpha$, meaning it takes something of type α and return something of type α. A function that returns the first element of a list has the type $\texttt{list}(\alpha) \rightarrow \alpha$. Type variables are not the same has variables introduced by λ-abstractions.
Feature 2: Remarkably, there is a simple algorithm for automatically inferring the polymorphic Hindley-Milner type of a λ-calculus expression (Damas & Milner, 1982). A detailed exposition of Hindley-Milner is beyond the scope of this work.

6. Estimating θ

We write $c(e, p)$ to mean the number of times that primitive e was used in program p; $R(p)$ to mean the sequence of types input to sample in Alg.1. Jensen's inequality gives an

intuitive lower bound on the likelihood of a program p:

$$\begin{aligned}\log \mathbb{P}[p|\theta] &\stackrel{+}{=} \sum_{e\in\mathcal{D}} c(e,p)\log\theta_e - \sum_{\tau\in R(p)} \log \sum_{\substack{e:\tau'\in\mathcal{D}\\ \text{unify}(\tau,\tau')}} \theta_e \\ &\stackrel{+}{\geq} \sum_{e\in\mathcal{D}} c(e,p)\log\theta_e - c(p)\log \sum_{\tau\in R(p)} \sum_{\substack{e:\tau'\in\mathcal{D}\\ \text{unify}(\tau,\tau')}} \theta_e \\ &= \sum_{e\in\mathcal{D}} c(e,p)\log\theta_e - c(p)\log \sum_{e\in\mathcal{D}} r(e,p)\theta_e\end{aligned}$$

where $c(p) = \sum_{e\in\mathcal{D}} c(e,p)$ and $r(e:\tau',p) = \sum_{\tau\in R(p)} \mathbb{1}[\text{canUnify}(\tau,\tau')]$.

Differentiate with respect to θ_e and set to zero

$$\frac{c(x)}{\theta^{(x)}} = N\frac{a(x)}{\sum_y a(y)\theta_y} \tag{4}$$

This equality holds if $\theta^{(x)} = c(x)/a(x)$:

$$\frac{c(x)}{\theta_x} = a(x). \tag{5}$$

$$N\frac{a(x)}{\sum_y a(y)\theta_y} = N\frac{a(x)}{\sum_y c(y)} = N\frac{a(x)}{N} = a(x). \tag{6}$$

If this equality holds then $\theta_x \propto c(x)/a(x)$:

$$\theta_x = \frac{c(x)}{a(x)} \times \underbrace{\frac{\sum_y a(y)\theta_y}{N}}_{\text{Independent of } x}. \tag{7}$$

Now what we are actually after is the parameters that maximize the joint log probability of the data+parameters, which I will write J:

$$J = L + \log \mathrm{D}(\theta|\alpha) \tag{8}$$

$$\stackrel{+}{\geq} \sum_x c(x)\log\theta_x - N\log\sum_x a(x)\theta_x + \sum_x(\alpha_x - 1)\log\theta_x \tag{9}$$

$$= \sum_x (c(x) + \alpha_x - 1)\log\theta_x - N\log\sum_x a(x)\theta_x \tag{10}$$

So you add the pseudocounts to the *counts* ($c(x)$), but not to the *possible counts* ($a(x)$).

References

Balog, Matej, Gaunt, Alexander L, Brockschmidt, Marc, Nowozin, Sebastian, and Tarlow, Daniel. Deepcoder: Learning to write programs. *arXiv preprint arXiv:1611.01989*, 2016.

Bishop, Christopher M. *Pattern Recognition and Machine Learning (Information Science and Statistics)*. Springer-Verlag New York, Inc., Secaucus, NJ, USA, 2006. ISBN 0387310738.

Cohn, Trevor, Blunsom, Phil, and Goldwater, Sharon. Inducing tree-substitution grammars. *Journal of Machine Learning Research*, 11(Nov):3053–3096, 2010.

Damas, Luis and Milner, Robin. Principal type-schemes for functional programs. In *Proceedings of the 9th ACM SIGPLAN-SIGACT symposium on Principles of programming languages*, pp. 207–212. ACM, 1982.

Dayan, Peter, Hinton, Geoffrey E, Neal, Radford M, and Zemel, Richard S. The helmholtz machine. *Neural computation*, 7(5):889–904, 1995.

Dechter, Eyal, Malmaud, Jon, Adams, Ryan P., and Tenenbaum, Joshua B. Bootstrap learning via modular concept discovery. In *IJCAI*, pp. 1302–1309. AAAI Press, 2013. ISBN 978-1-57735-633-2. URL http://dl.acm.org/citation.cfm?id=2540128.2540316.

Devlin, Jacob, Bunel, Rudy R, Singh, Rishabh, Hausknecht, Matthew, and Kohli, Pushmeet. Neural program meta-induction. In *Advances in Neural Information Processing Systems*, pp. 2077–2085, 2017a.

Devlin, Jacob, Uesato, Jonathan, Bhupatiraju, Surya, Singh, Rishabh, Mohamed, Abdel-rahman, and Kohli, Pushmeet. Robustfill: Neural program learning under noisy i/o. *arXiv preprint arXiv:1703.07469*, 2017b.

Ellis, Kevin, Solar-Lezama, Armando, and Tenenbaum, Josh. Sampling for bayesian program learning. In *Advances in Neural Information Processing Systems*, 2016.

Feser, John K, Chaudhuri, Swarat, and Dillig, Isil. Synthesizing data structure transformations from input-output examples. In *ACM SIGPLAN Notices*, volume 50, pp. 229–239. ACM, 2015.

Gulwani, Sumit. Automating string processing in spreadsheets using input-output examples. In *ACM SIGPLAN Notices*, volume 46, pp. 317–330. ACM, 2011.

Hinton, Geoffrey E and Salakhutdinov, Ruslan R. Reducing the dimensionality of data with neural networks. *science*, 313(5786):504–507, 2006.

Lafferty, J.D. *A Derivation of the Inside-outside Algorithm from the EM Algorithm*. Research report. IBM T.J. Watson Research Center, 2000. URL http://books.google.com/books?id=aOKAGwAACAAJ.

Lake, Brenden M, Salakhutdinov, Ruslan R, and Tenenbaum, Josh. One-shot learning by inverting a compositional causal process. In *Advances in neural information processing systems*, pp. 2526–2534, 2013.

Le, Tuan Anh, Baydin, Atlm Gne, and Wood, Frank. Inference Compilation and Universal Probabilistic Programming. In *20th International Conference on Artificial Intelligence and Statistics, April 20–22, 2017, Fort Lauderdale, FL, USA (To Appear)*, 2017.

Liang, Percy, Jordan, Michael I., and Klein, Dan. Learning programs: A hierarchical bayesian approach. In Fürnkranz, Johannes and Joachims, Thorsten (eds.), *ICML*, pp. 639–646. Omnipress, 2010. ISBN 978-1-60558-907-7.

Lin, Dianhuan, Dechter, Eyal, Ellis, Kevin, Tenenbaum, Joshua B., and Muggleton, Stephen. Bias reformulation for one-shot function induction. In *ECAI 2014*, pp. 525–530, 2014. doi: 10.3233/978-1-61499-419-0-525. URL http://dx.doi.org/10.3233/978-1-61499-419-0-525.

Menon, Aditya, Tamuz, Omer, Gulwani, Sumit, Lampson, Butler, and Kalai, Adam. A machine learning framework for programming by example. In *ICML*, pp. 187–195, 2013.

Muggleton, Stephen H, Lin, Dianhuan, and Tamaddoni-Nezhad, Alireza. Meta-interpretive learning of higher-order dyadic datalog: Predicate invention revisited. *Machine Learning*, 100(1):49–73, 2015.

O'Donnell, Timothy J. *Productivity and Reuse in Language: A Theory of Linguistic Computation and Storage*. The MIT Press, 2015.

Osera, Peter-Michael and Zdancewic, Steve. Type-and-example-directed program synthesis. In *ACM SIGPLAN Notices*, volume 50, pp. 619–630. ACM, 2015.

Pierce, Benjamin C. *Types and programming languages*. MIT Press, 2002. ISBN 978-0-262-16209-8.

Polozov, Oleksandr and Gulwani, Sumit. Flashmeta: A framework for inductive program synthesis. *ACM SIGPLAN Notices*, 50(10):107–126, 2015.

Ritchie, Daniel, Horsfall, Paul, and Goodman, Noah D. Deep amortized inference for probabilistic programs. *arXiv preprint arXiv:1610.05735*, 2016.

Schkufza, Eric, Sharma, Rahul, and Aiken, Alex. Stochastic superoptimization. In *ACM SIGARCH Computer Architecture News*, volume 41, pp. 305–316. ACM, 2013.

Schmidhuber, Jürgen. Optimal ordered problem solver. *Machine Learning*, 54(3):211–254, 2004.

Solar Lezama, Armando. *Program Synthesis By Sketching*. PhD thesis, EECS Department, University of California, Berkeley, Dec 2008. URL http://www.eecs.berkeley.edu/Pubs/TechRpts/2008/EECS-2008-177.html.

Solomonoff, Ray J. A system for incremental learning based on algorithmic probability. Sixth Israeli Conference on Artificial Intelligence, Computer Vision and Pattern Recognition, 1989.