

# DREAMCODER: Bootstrapping Domain-Specific Languages for Neurally-Guided Bayesian Program Learning

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## 1. Introduction

Much of everyday human thinking and learning can be understood in terms of program induction: constructing a procedure that maps inputs to desired outputs, based on observing example input-output pairs. People can induce programs flexibly across many different domains, and remarkably, often from just one or a few examples. For instance, if shown that a text-editing program should map “Jane Morris Goodall” to “J. M. Goodall”, we can guess it maps “Richard Erskine Leakey” to “R. E. Leakey”; if instead the first input mapped to “Dr. Jane”, “Goodall, Jane”, or “Morris”, we might have guessed the latter should map to “Dr. Richard”, “Leakey, Richard”, or “Erskine”, respectively.

The FlashFill system (Gulwani, 2011) developed by Microsoft researchers and now embedded in Excel solves problems such as these and is probably the best known practical program-induction algorithm, but researchers in programming languages and AI have built successful program induction algorithms for many applications, such as handwriting recognition and generation (Lake et al., 2015), procedural graphics (Ellis et al., 2017), question answering (Johnson et al., 2017) and robot motion planning (Devlin et al., 2017a), to name just a few. These systems work in different ways, but most hinge upon a carefully engineered **Domain Specific Language (DSL)**. This is especially true for systems such as FlashFill that aim to induce a wide range of programs very quickly, in a few seconds or less. DSLs constrain the search over programs with strong prior knowledge in the form of a restricted set of programming primitives tuned to the needs of the domain: for text editing, these are operations like appending strings and splitting on characters.

In this work, we consider the problem of building agents that learn to solve program induction tasks, and also the problem of acquiring the prior knowledge necessary to quickly solve these tasks in a new domain. Representative problems in three domains are shown in Table 1. Our solution is an algo-

rithm that grows or bootstraps a DSL while jointly training a neural network to help write programs in the increasingly rich DSL. Because any computable learning problem can in principle be cast as program induction, it is important to delimit our focus. In contrast to computer assisted programming (Solar Lezama, 2008) or genetic programming (Koza, 1993), our goal is not to automate software engineering, to learn to synthesize large bodies of code, or to learn complex programs starting from scratch. Ours is a basic AI goal: capturing the human ability to learn to think flexibly and efficiently in new domains — to learn what you need to know about a domain so you don’t have to solve new problems starting from scratch. We are focused on the kinds of problems that humans can solve relatively quickly, once they acquire the relevant domain expertise. These correspond to tasks solved by short programs — if you have an expressive DSL. Even with a good DSL, program search may be intractable; so we train a neural network to guide the search procedure.

## 2. Overview of DREAMCODER

We take inspiration from several ways that skilled human programmers have learned to code: Skilled coders build libraries of reusable subroutines that are shared across related programming tasks, and can be composed to generate increasingly complex and powerful subroutines. In text editing, a good library should support routines for splitting on characters, but also specialize these routines to split on characters such as spaces or commas that are frequently used to delimit substrings. Skilled coders also learn to recognize what kinds of programming idioms and library routines would be useful for solving the task at hand, even if they cannot instantly work out the details. In text editing, one might learn that if outputs are consistently shorter than inputs, removing characters is likely to be part of the solution.

Our algorithm is called DREAMCODER because it is based on a novel kind of “wake-sleep” learning (c.f. (Hinton et al., 1995)), iterating between “wake” and “sleep” phases to achieve three goals: finding programs that solve tasks; creating a DSL by discovering and reusing domain-specific subroutines; and training a neural network that efficiently guides search for programs in the DSL. The learned DSL ef-

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	List Functions	Text Editing	Symbolic Regression
Programs & Tasks	$[7\ 2\ 3] \rightarrow [7\ 3]$ $[1\ 2\ 3\ 4] \rightarrow [3\ 4]$ $[4\ 3\ 2\ 1] \rightarrow [4\ 3]$ $f(\ell) = (f_1\ \ell\ (\lambda\ (x)\ (>\ x\ 2)))$	$+106\ 769\ 438 \rightarrow 106.769.438$ $+83\ 973\ 831 \rightarrow 83.973.831$ $f(s) = (f_0\ \text{"." " "-"}\ (f_0\ \text{"." " "-"}\ (cdr\ s)))$	  $f(x) = (f_1\ x)$ $f(x) = (f_6\ x)$
	$[7\ 3] \rightarrow \text{False}$ $[3] \rightarrow \text{False}$ $[9\ 0\ 0] \rightarrow \text{True}$ $[0] \rightarrow \text{True}$ $[0\ 7\ 3] \rightarrow \text{True}$ $f(\ell) = (f_3\ \ell\ 0)$	Temple Anna H $\rightarrow$ TAH Lara Gregori $\rightarrow$ LG $f(s) = (f_2\ s)$	  $f(x) = (f_4\ x)$ $f(x) = (f_3\ x)$
	$[2\ 7\ 8\ 1] \rightarrow 8$ $[3\ 19\ 14] \rightarrow 19$ $f(\ell) = (f_2\ \ell)$		
DSL	$f_0(\ell, r) = (\text{foldr}\ r\ \ell\ \text{cons})$ <i>(<math>f_0</math>: Append lists <math>r</math> and <math>\ell</math>)</i> $f_1(\ell, p) = (\text{foldr}\ \ell\ \text{nil}\ (\lambda\ (x\ a)\ (\text{if}\ (p\ x)\ (\text{cons}\ x\ a)\ a))))$ <i>(<math>f_1</math>: Higher-order filter function)</i> $f_2(\ell) = (\text{foldr}\ \ell\ 0\ (\lambda\ (x\ a)\ (\text{if}\ (>\ a\ x)\ a\ x))))$ <i>(<math>f_2</math>: Maximum element in list <math>\ell</math>)</i> $f_3(\ell, k) = (\text{foldr}\ \ell\ (\text{is-nil}\ \ell)\ (\lambda\ (x\ a)\ (\text{if}\ a\ a\ (= k\ x))))$ <i>(<math>f_3</math>: Whether <math>\ell</math> contains <math>k</math>)</i>	$f_0(s, a, b) = (\text{map}\ (\lambda\ (x)\ (\text{if}\ (= x\ a)\ b\ x))\ s)$ <i>(<math>f_0</math>: Performs character substitution)</i> $f_1(s, c) = (\text{foldr}\ s\ s\ (\lambda\ (x\ a)\ (\text{cdr}\ (\text{if}\ (= c\ x)\ s\ a))))$ <i>(<math>f_1</math>: Drop characters from <math>s</math> until <math>c</math> reached)</i> $f_2(s) = (\text{unfold}\ s\ \text{is-nil}\ \text{car}\ (\lambda\ (z)\ (f_1\ z\ \text{" "})))$ <i>(<math>f_2</math>: Abbreviates a sequence of words)</i> $f_3(a, b) = (\text{foldr}\ a\ b\ \text{cons})$ <i>(<math>f_3</math>: Concatenate strings <math>a</math> and <math>b</math>)</i>	$f_0(x) = (+\ x\ \text{real})$ $f_1(x) = (f_0\ (*\ \text{real}\ x))$ $f_2(x) = (f_1\ (*\ x\ (f_0\ x)))$ $f_3(x) = (f_0\ (*\ x\ (f_2\ x)))$ $f_4(x) = (f_0\ (*\ x\ (f_3\ x)))$ <i>(<math>f_4</math>: 4th order polynomial)</i> $f_5(x) = (/ \ \text{real}\ x)$ $f_6(x) = (f_4\ (f_0\ x))$ <i>(<math>f_6</math>: rational function)</i>

Table 1: Top: Tasks from three domains we apply our algorithm to, each followed by the programs DREAMCODER discovers for them. Bottom: Several examples from learned DSL. Notice that learned DSL primitives can call each other, and that DREAMCODER rediscovers higher-order functions like `filter` ( $f_1$  under List Functions)

fectively encodes a prior on programs likely to solve tasks in the domain, while the neural net looks at the example input-output pairs for a specific task and produces a “posterior” for programs likely to solve that specific task. The neural network thus functions as a **recognition model** supporting a form of approximate Bayesian program induction, jointly trained with a **generative model** for programs encoded in the DSL, in the spirit of the Helmholtz machine (Hinton et al., 1995). The recognition model ensures that searching for programs remains tractable even as the DSL (and hence the search space for programs) expands.

Concretely, our algorithm iterates through three steps. The **Wake** step takes a given set of **tasks**, typically several hundred, and searches for compact programs that solve these tasks guided by the current DSL and neural network. The **Sleep-G** step grows the DSL (or **Generative model**) which allows the agent to more compactly write programs in the domain; it modifies the structure of the DSL by discovering regularities across programs found during waking, compressing them to distill out common code fragments across successful programs. The **Sleep-R** step improves the search procedure by training a neural network (the **Recognition model**) to write programs in the current DSL, in the spirit of “amortized” or “compiled” inference (Le et al., 2017).

Prior work on program learning has largely assumed a fixed, hand-engineered DSL, both in classic symbolic program learning approaches (e.g., Metagol: (Muggleton et al., 2015), FlashFill: (Gulwani, 2011)), neural approaches (e.g., RobustFill: (Devlin et al., 2017b)), and hybrids of neural and symbolic methods (e.g., Neural-guided deductive search: (Kalyan et al., 2018), DeepCoder: (Balog et al.,

2016)). A notable exception is the EC algorithm (Dechter et al., 2013), which also learns a library of subroutines. We find EC motivating, and go beyond it and other prior work through the following contributions:

**Contributions.** (1) We show how to learn-to-learn programs in an expressive Lisp-like programming language, including conditionals, variables, and higher-order recursive functions; (2) We give an algorithm for learning DSLs, built on a formalism known as Fragment Grammars (O’Donnell, 2015); and (3) We give a hierarchical Bayesian framing of the problem that allows joint inference of the DSL and neural recognition model.

### 3. The DREAMCODER Algorithm

DREAMCODER takes as input a set of *tasks*, written  $X$ , each of which is a program synthesis problem. It has at its disposal a domain-specific *likelihood model*, written  $\mathbb{P}[x|p]$ , which scores the likelihood of a task  $x \in X$  given a program  $p$ . Its goal is to solve each of the tasks by writing a program, and also to infer a DSL, written  $\mathcal{D}$ , which is a generative model over programs, written  $\mathbb{P}[p|\mathcal{D}]$ . We frame our goal as maximum a posteriori (MAP) inference of  $\mathcal{D}$  given  $X$ :

$$\arg \max_{\mathcal{D}} \mathbb{P}[\mathcal{D}] \prod_{x \in X} \sum_p \mathbb{P}[x|p] \mathbb{P}[p|\mathcal{D}]$$

Even evaluating the above objective is intractable due to the sum over all programs, and so we instead marginalize over finite sets of programs, called **frontiers**, one for each task  $x \in X$ , written  $\mathcal{F}_x$ , giving the following lower bound on

the marginal, which we call  $\mathcal{L}$ :

$$\mathcal{L} \triangleq \mathbb{P}[\mathcal{D}] \prod_{x \in X} \sum_{p \in \mathcal{F}_x} \mathbb{P}[x|p] \mathbb{P}[p|\mathcal{D}] \quad (1)$$

We alternate maximization w.r.t.  $\{\mathcal{F}_x\}_{x \in X}$  (**Wake**) and  $\mathcal{D}$  (**Sleep-G**):

**Wake: Maxing  $\mathcal{L}$  w.r.t. the programs.** Here  $\mathcal{D}$  is fixed and we want to find new programs to add to the frontiers so that  $\mathcal{L}$  increases the most.  $\mathcal{L}$  most increases by finding programs where  $\mathbb{P}[x|p]\mathbb{P}[p|\mathcal{D}]$  is large. Here we use a simple and generic enumerative program synthesis algorithm.

**Sleep-G: Maxing  $\mathcal{L}$  w.r.t. the DSL.** Here  $\{\mathcal{F}_x\}_{x \in X}$  is held fixed, and so we can evaluate  $\mathcal{L}$ . Now the problem is that of searching the discrete space of DSLs and finding one maximizing  $\mathcal{L}$ . To represent DSLs and search the space of DSLs, we use Fragment Grammars (O'Donnell, 2015).

Searching for programs is hard because of the large combinatorial search space. We ease this difficulty by training a neural recognition model,  $q(\cdot|\cdot)$ , during the **Sleep-R** phase:  $q$  is trained to approximate the posterior over programs,  $q(p|x) \approx \mathbb{P}[p|x, \mathcal{D}] \propto \mathbb{P}[x|p]\mathbb{P}[p|\mathcal{D}]$ , thus amortizing the cost of finding programs with high posterior probability.

**Sleep-R: tractably maxing  $\mathcal{L}$  w.r.t. the frontiers.** Here we train  $q(p|x)$  to assign high probability to programs  $p$  where  $\mathbb{P}[x|p]\mathbb{P}[p|\mathcal{D}]$  is large, because including those programs in the frontiers will most increase  $\mathcal{L}$ . Concretely, the loss for  $q(\cdot|x)$  is:

$$\mathbb{E}_X [\text{KL}(\mathbb{P}[\cdot|x, \mathcal{D}] || q(\cdot|x))]$$

which minimizes the KL between the distribution predicted by  $q$  and the true posterior over programs solving task  $x$ .

## 4. Programs that manipulate sequences

We apply DREAMCODER to list processing (Section 4.1) and text editing (Section 4.2). For both these domains we use a bidirectional GRU (Cho et al., 2014) for the recognition model, and initially provide the system with generic list processing primitives: `foldr`, `unfold`, `if`, `map`, `length`, `index`, `=`, `+`, `-`, `0`, `1`, `cons`, `car`, `cdr`, `nil`, and `is-nil`.

### 4.1. List Processing

Synthesizing programs that manipulate data structures is a widely studied problem in the programming languages community (Feser et al., 2015). We consider this problem within the context of learning functions that manipulate lists, and also perform arithmetic operations upon lists (Table 2). We created 236 human-interpretable list manipulation tasks, each with 15 input/output examples, evaluating on a 50/50 test/train split. The system composed 38 new subroutines, and rediscovered the higher-order function `filter`, yield-

Name	Input	Output
repeat-2	[7 0]	[7 0 7 0]
drop-3	[0 3 8 6 4]	[6 4]
rotate-2	[8 14 1 9]	[1 9 8 14]
count-head-in-tail	[1 2 1 1 3]	2
keep-mod-5	[5 9 14 6 3 0]	[5 0]
product	[7 1 6 2]	84

Table 2: Some tasks in our list function domain.

ing a more expressive DSL more closely matching the domain (Tbl. 1, left).

### 4.2. Text Editing

Synthesizing programs that edit text is a classic problem in the programming languages and AI literatures (Lau, 2001), and algorithms that learn text editing programs ship in Microsoft Excel (Gulwani, 2011). This prior work presumes a hand-engineered DSL. We show DREAMCODER can instead start out with generic sequence manipulation primitives and recover many of the higher-level building blocks that have made these other text editing systems successful.

We trained our system on 109 synthetic tasks, with 4 input/output examples each. After three iterations, it assembles a DSL containing a dozen new functions (Fig. 1, center) solving all the training tasks. But, how well does the learned DSL generalized to real text-editing scenarios? We tested, but did not train, on the 108 text editing problems from the SyGuS (Alur et al., 2016) program synthesis competition. Before any learning, DREAMCODER solves 3.7% of the problems with an average search time of 235 seconds. After learning, it solves 74.1%, and does so much faster, solving them in an average of 29 seconds. As of the 2017 SyGuS competition, the best-performing algorithm solves 79.6% of the problems. But, SyGuS comes with a different hand-engineered DSL *for each text editing problem*. Here we learned a single DSL that applied generically to all of the tasks, and perform comparably to the best prior work.

## 5. Symbolic Regression: Programs from visual input

We apply DREAMCODER to symbolic regression problems. Here, the agent observes points along the curve of a function, and must write a program that fits those points. We initially equip our learner with addition, multiplication, and division, and task it with solving 100 symbolic regression problems, each either a polynomial or rational function. The recognition model is a convolutional network that observes an image of the target function's graph (Fig. 1) — visually, different kinds of polynomials and rational functions

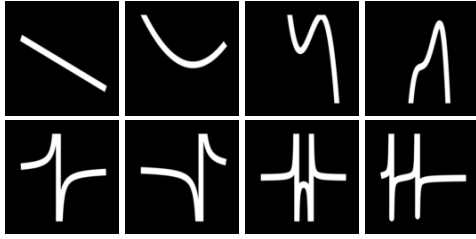


Figure 1: Recognition model input for symbolic regression. DSL learns subroutines for polynomials (top row) and rational functions (bottom) while the recognition model jointly learns to look at a graph of the function (above) and predict which learned subroutines best explain the observation.

produce different kinds of graphs, and so the recognition model can learn to look at a graph and predict what kind of function best explains it. These programs can contain real numbers, which we set with gradient descent, and penalize using real numbers by incorporating a BIC penalty into the likelihood model  $\mathbb{P}[x|p]$ .

DREAMCODER learns a DSL containing 13 new functions, most of which are templates for polynomials of different orders or ratios of polynomials. It also learns to find programs that minimize the number of continuous degrees of freedom. For example, it learns to represent linear functions with the program `(* real (+ x real))`, which has two continuous degrees of freedom, and represents quartic functions using the discovered DSL primitive  $f_4$  in the rightmost column of Fig. 1 which has five continuous parameters. This phenomenon arises from our Bayesian framing.

## 6. Learning from Scratch

A long-standing dream within the program induction community is “learning from scratch”: starting with a *minimal* Turing-complete programming language, and then learning to solve a wide swath of induction problems (Solomonoff, 1964). All existing systems, including ours, fall far short of this dream, and it is unclear (and we believe unlikely) that this dream could ever be realized in its fullness. How far can our system push in this direction?

“Learning from scratch” is subjective, but a reasonable starting point is the set of primitives provided in 1959 Lisp (McCarthy, 1960): these include conditionals, recursion, arithmetic, and the list operators `cons`, `car`, `cdr`, and `nil`. A sensible and modest goal is to start with 1959 Lisp primitives, and then recover a DSL that more closely resembles modern functional languages like Haskell and OCaml. Recall (Sec. 4) that we initially provided our system with modern functional programming routines like `map` and `fold`.

We ran the following experiment: DREAMCODER was provided with a subset of the 1959 Lisp primitives, and tasked

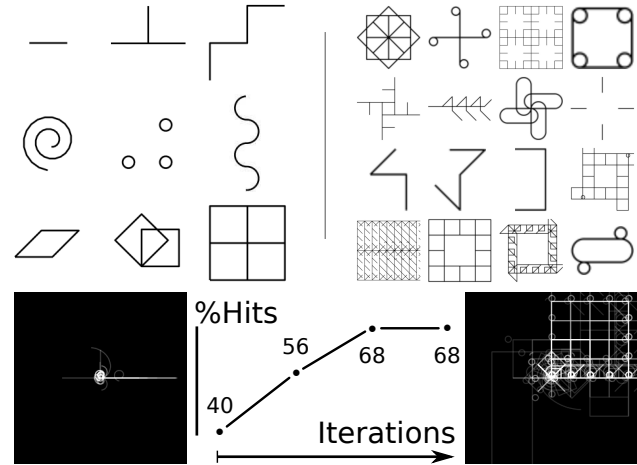


Figure 2: Top left: Example training tasks. Top right: samples from the learned DSL. Bottom: % holdout testing tasks solved (middle); on the sides are averaged samples from the DSL before any training (left) and after last iteration (right).

with solving 22 functional programming exercises. After running for 93 hours on 64 CPUs, our algorithm solves these exercises, along the way assembling a DSL containing 9 subroutines, recovering a modern repertoire of functional programming idioms and subroutines, including `map`, `fold`, `zip`, `unfold`, `index`, `length`, and simple arithmetic operations like incrementing, decrementing, and building lists of natural numbers between an interval. We did not use the recognition model for this experiment: a bottom-up pattern recognizer is of little use for acquiring this abstract knowledge from less than two dozen problems.

To be clear, we believe that program learners should *not* start from scratch, but instead should start from a rich, domain-agnostic basis like those embodied in the standard libraries of modern programming languages. What this experiment shows is that DREAMCODER doesn’t *need* to start from a rich basis, and can in principle recover many of the amenities of modern programming systems, provided it is given enough computational power and a suitable spectrum of programming exercises.

## 7. Learning Generative Models

We apply DREAMCODER to learning generative models for images (Figure 2) and text (Figure ??). For images, we learn programs in a LOGO/Turtle-like representation, and the task is to look at an image and explain it in terms of a graphics program. For text, we learn probabilistic regular expressions – a simple probabilistic program for which inference is always tractable – and the task is to infer a regex from a set of strings.

## 8. Quantitative Results

## 9. Discussion

We contribute an algorithm, DREAMCODER, that learns to program by bootstrapping a DSL with new domain-specific primitives that the algorithm itself discovers, together with a neural recognition model that learns how to efficiently deploy the DSL on new tasks. We believe this integration of top-down symbolic representations and bottom-up neural networks — both of them learned — helps make program induction systems more generally useful for AI. Many directions remain open. Two immediate goals are to integrate more sophisticated neural recognition models (Devlin et al., 2017b) and program synthesizers (Solar Lezama, 2008), which may improve performance in some domains over the generic methods used here. Another direction is to explore DSL meta-learning: can we find a *single* universal primitive set that could effectively bootstrap DSLs for new domains, including the three domains considered, but also many others?

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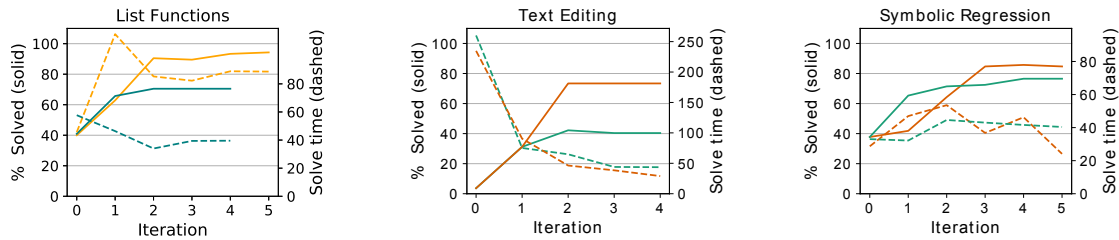


Figure 3: Learning curves for DREAMCODER both with (in orange) and without (in teal) the recognition model. Solid lines: % holdout testing tasks solved. Dashed lines: Average solve time, averaged only over those tasks that are solved.

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