

# **Growing domain-specific languages alongside neural program synthesizers via wake-sleep program learning**

---

Kevin Ellis

Collaborators: Catherine Wong, Maxwell Nye, Mathias Sablé-Meyer,  
Lucas Morales, Armando Solar-Lezama, Joshua B. Tenenbaum

2020

neurosym webinar

## The premise of program induction

1. Represent knowledge as programs: as symbolic code

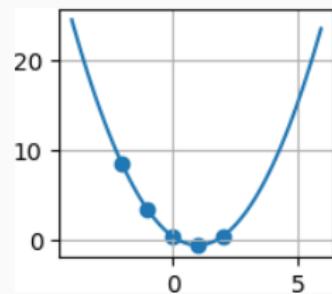
## The premise of program induction

1. Represent knowledge as programs: as symbolic code
2. Learning=adding to that body of knowledge=  
making new programs=program synthesis

# Why program induction?

# Why program induction?

strong generalization  
+data efficiency

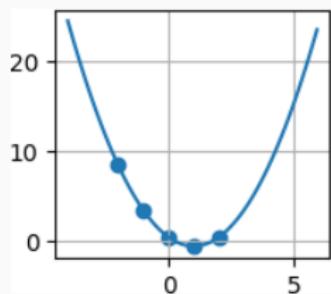


$$f(x) = (x-1)^{**2} - 0.5$$

# Why program induction?

strong generalization  
+data efficiency

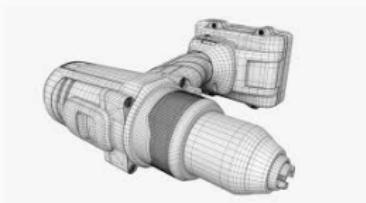
interpretability



$$f(x) = (x-1)^2 - 0.5$$

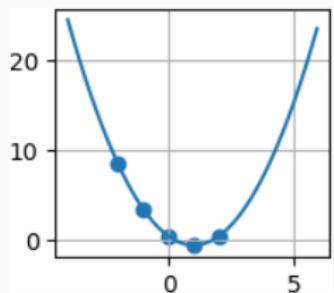


VS



# Why program induction?

strong generalization  
+data efficiency

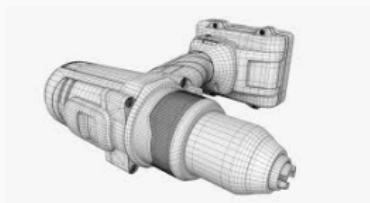


$$f(x) = (x-1)^{**2} - 0.5$$

interpretability



universal expressivity



## FlashFill (Gulwani 2012)

EXAMPLE 3 (Directory Name Extraction). Consider the following example taken from an excel online help forum.

Input $v_1$	Output
Company\Code\index.html	Company\Code\
Company\Docs\Spec\specs.doc	Company\Docs\Spec\

String Program:

$\text{SubStr}(v_1, \text{CPos}(0), \text{Pos}(\text{SlashTok}, \epsilon, -1))$

## FlashFill (Gulwani 2012)

EXAMPLE 3 (Directory Name Extraction). Consider the following example taken from an excel online help forum.

Input $v_1$	Output
Company\Code\index.html	Company\Code\
Company\Docs\Spec\specs.doc	Company\Docs\Spec\

String Program:

$\text{SubStr}(v_1, \text{CPos}(0), \text{Pos}(\text{SlashTok}, \epsilon, -1))$

## Szalinski (Nandi 2020)



(a) CAD model of ship's wheel

```
(Union  
  (Cylinder [1, 5, 5])  
  (Fold Union  
    (Tabulate (i 6)  
      (Rotate [0, 0, 60i]  
        (Translate [1, -0.5, 0]  
          (Cuboid [10, 1, 1]))))))
```

(b) Caddy program

# FlashFill (Gulwani 2012)

EXAMPLE 3 (Directory Name Extraction). Consider the following example taken from an excel online help forum.

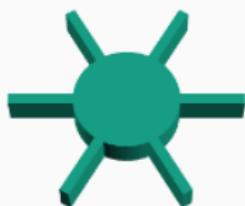
Input $v_1$	Output
Company\Code\index.html	Company\Code\
Company\Docs\Spec\specs.doc	Company\Docs\Spec\

String Program:

$\text{SubStr}(v_1, \text{CPos}(0), \text{Pos}(\text{SlashTok}, \epsilon, -1))$

String expr $P$	$\text{Switch}((b_1, e_1), \dots, (b_n, e_n))$
Bool $b$	$d_1 \vee \dots \vee d_n$
Conjunct $d$	$\pi_1 \wedge \dots \wedge \pi_n$
Predicate $\pi$	$\text{Match}(v_i, r, k) \mid \neg \text{Match}(v_i, r, k)$
Trace expr $e$	$\text{Concatenate}(f_1, \dots, f_n)$
Atomic expr $f$	$\text{SubStr}(v_i, p_1, p_2)$   $\text{ConstStr}(s)$   $\text{Loop}(\lambda w : e)$
Position $p$	$\text{CPos}(k) \mid \text{Pos}(r_1, r_2, c)$
Integer expr $c$	$k \mid k_1 w + k_2$
Regular Expression $r$	$\text{TokenSeq}(T_1, \dots, T_m)$
Token $T$	$C + \mid [\neg C] + \mid \text{SpecialToken}$

# Szalinski (Nandi 2020)



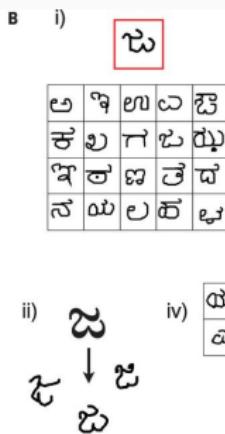
(a) CAD model of ship's wheel

```
(Union
  (Cylinder [1, 5, 5])
  (Fold Union
    (Tabulate (i 6)
      (Rotate [0, 0, 60i]
        (Translate [1, -0.5, 0]
          (Cuboid [10, 1, 1]))))))
```

(b) Caddy program

op	$\text{::= } + \mid - \mid \times \mid / \mid \text{num} \text{ ::= } \mathbb{R} \mid \langle \text{var} \rangle \mid \langle \text{num} \rangle \langle \text{op} \rangle \langle \text{num} \rangle$
vec2	$\text{::= } [\langle \text{num} \rangle, \langle \text{num} \rangle] \mid \text{vec3} \text{ ::= } [\langle \text{num} \rangle, \langle \text{num} \rangle, \langle \text{num} \rangle]$
affine	$\text{::= } \text{Translate} \mid \text{Rotate} \mid \text{Scale} \mid \text{TranslateSpherical}$
binop	$\text{::= } \text{Union} \mid \text{Difference} \mid \text{Intersection}$
cad	$\text{::= } (\text{Cuboid } \langle \text{vec3} \rangle) \mid (\text{Sphere } \langle \text{num} \rangle)$   $(\text{Cylinder } \langle \text{vec2} \rangle) \mid (\text{HexPrism } \langle \text{vec2} \rangle) \mid \dots$   $((\text{affine}) \langle \text{vec3} \rangle \langle \text{cad} \rangle)$   $((\text{binop}) \langle \text{cad} \rangle \langle \text{cad} \rangle)$   $(\text{Fold } \langle \text{binop} \rangle \langle \text{cad-list} \rangle)$
cad-list	$\text{::= } (\text{List } \langle \text{cad} \rangle^+)$   $(\text{Concat } \langle \text{cad-list} \rangle^+)$   $(\text{Tabulate } (\langle \text{var} \rangle \ Z^+)^+ \langle \text{cad} \rangle)$   $(\text{Map2 } \langle \text{affine} \rangle \langle \text{vec3-list} \rangle \langle \text{cad-list} \rangle)$
vec3-list	$\text{::= } (\text{List } \langle \text{vec3} \rangle^+)$   $(\text{Concat } \langle \text{vec3-list} \rangle^+)$   $(\text{Tabulate } (\langle \text{var} \rangle \ Z^+)^+ \langle \text{vec3} \rangle)$

# Program induction for learning and perception



이 책은 예술가 펠라스코의 그림을 바탕으로 한  
한국어로 번역된 책입니다.  
본 책은 예술가 펠라스코의 그림을 바탕으로 한  
한국어로 번역된 책입니다.  
본 책은 예술가 펠라스코의 그림을 바탕으로 한  
한국어로 번역된 책입니다.  
본 책은 예술가 펠라스코의 그림을 바탕으로 한  
한국어로 번역된 책입니다.  
본 책은 예술가 펠라스코의 그림을 바탕으로 한  
한국어로 번역된 책입니다.  
본 책은 예술가 펠라스코의 그림을 바탕으로 한  
한국어로 번역된 책입니다.

Program Induction and learning to learn  
learning a DSL  
learning to synthesize  
synergy between DSL+learned synthesizer

## Learning to write code

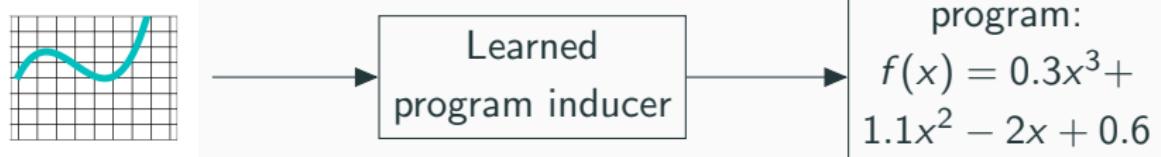
Goal: acquire domain-specific knowledge needed to induce a class of programs

- Library of abstractions (domain specific language)
- Inference strategy (synthesis algorithm)

# Learning to write code

Goal: acquire domain-specific knowledge needed to induce a class of programs

- Library of abstractions (domain specific language)
- Inference strategy (synthesis algorithm)



Concepts:  $x^3$ ,  $\alpha x + \beta$ , etc

Inference strategy: neurosymbolic search for programs

# Library learning

## Initial Primitives

: value  
:

map

fold func

if

cons

>

: value  
:

## Sample Problem: sort list

[9 2 7 1] → [1 2 7 9]

[3 8 9 4 2] → [2 3 4 8 9]

[6 2 2 3 8 5] → [2 2 3 5 6 8]

...

# Library learning

## Initial Primitives

:

⋮

map

fold

if

cons

>

⋮

⋮

## Sample Problem: sort list

[9 2 7 1] → [1 2 7 9]

[3 8 9 4 2] → [2 3 4 8 9]

[6 2 2 3 8 5] → [2 2 3 5 6 8]

⋮

# Library learning

## Initial Primitives

:

:

map

fold

if

cons

>

:

## Sample Problem: sort list

[9 2 7 1] → [1 2 7 9]

[3 8 9 4 2] → [2 3 4 8 9]

[6 2 2 3 8 5] → [2 2 3 5 6 8]

...

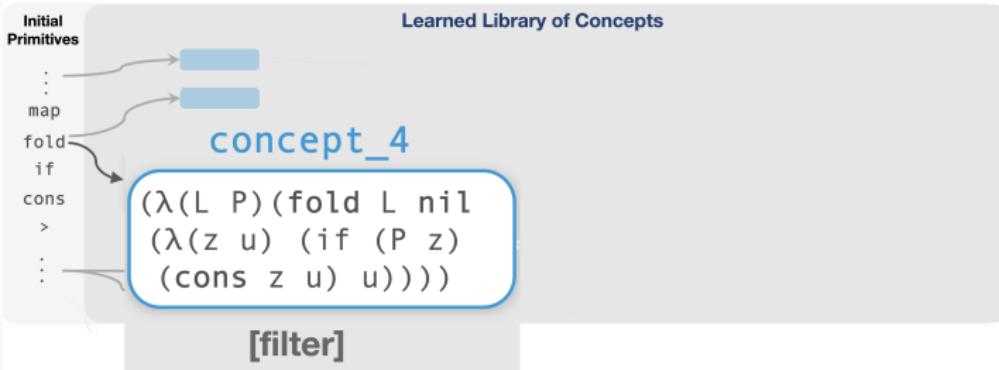
# Library learning



## Sample Problem: sort list

[9 2 7 1] → [1 2 7 9]  
[3 8 9 4 2] → [2 3 4 8 9]  
[6 2 2 3 8 5] → [2 2 3 5 6 8]  
...

# Library learning



Sample Problem: sort list

$[9 2 7 1] \rightarrow [1 2 7 9]$   
 $[3 8 9 4 2] \rightarrow [2 3 4 8 9]$   
 $[6 2 2 3 8 5] \rightarrow [2 2 3 5 6 8]$   
...

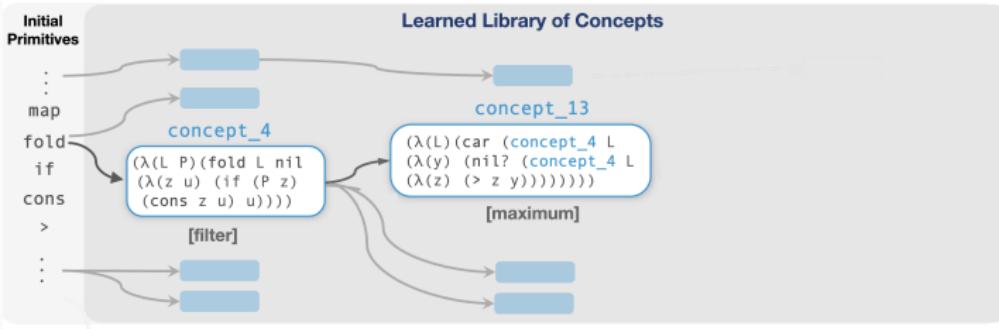
# Library learning



## Sample Problem: sort list

[9 2 7 1] → [1 2 7 9]  
[3 8 9 4 2] → [2 3 4 8 9]  
[6 2 2 3 8 5] → [2 2 3 5 6 8]  
...

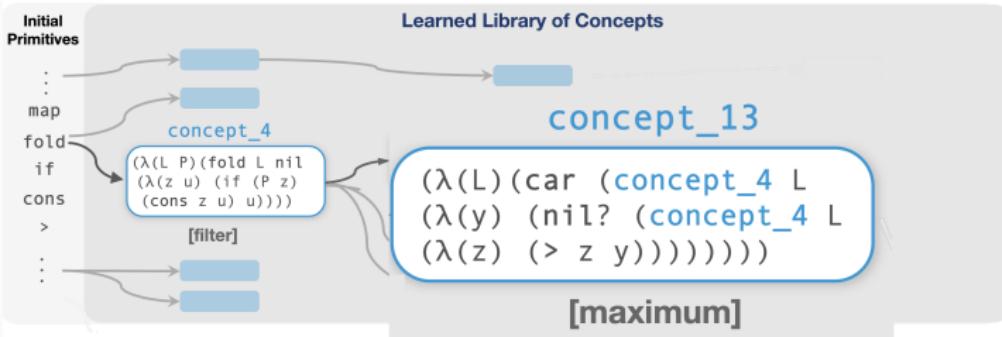
# Library learning



## Sample Problem: sort list

[9 2 7 1] → [1 2 7 9]  
[3 8 9 4 2] → [2 3 4 8 9]  
[6 2 2 3 8 5] → [2 2 3 5 6 8]  
...

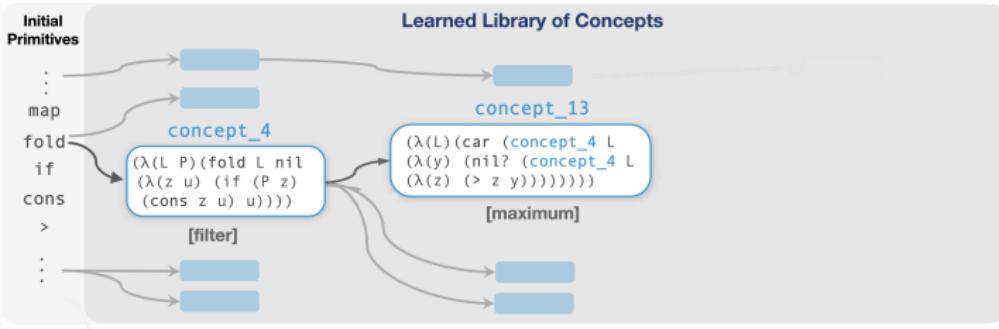
# Library learning



Sample Problem: sort list

[9 2 7 1] → [1 2 7 9]  
[3 8 9 4 2] → [2 3 4 8 9]  
[6 2 2 3 8 5] → [2 2 3 5 6 8]  
...

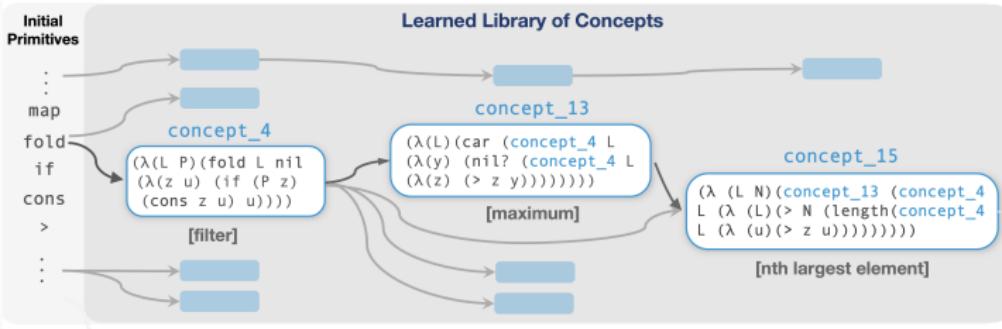
# Library learning



## Sample Problem: sort list

[9 2 7 1] → [1 2 7 9]  
[3 8 9 4 2] → [2 3 4 8 9]  
[6 2 2 3 8 5] → [2 2 3 5 6 8]  
...

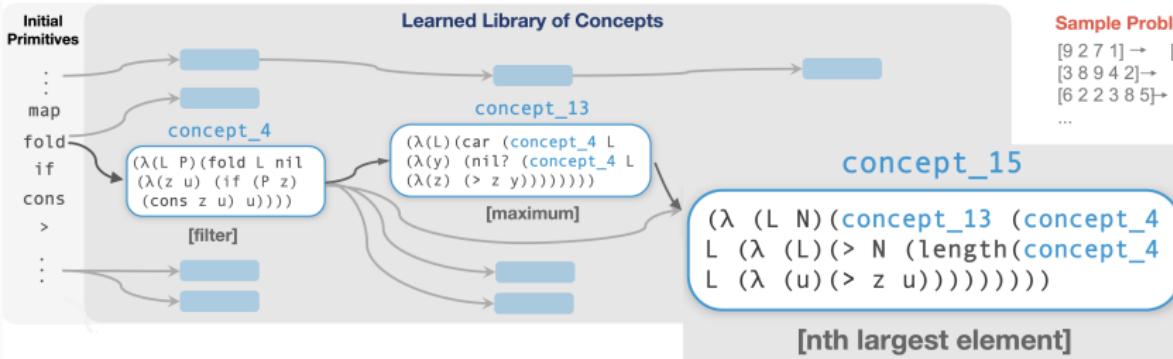
# Library learning



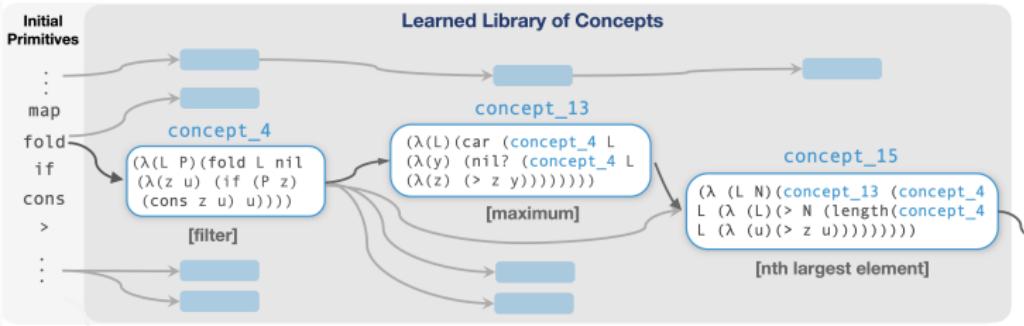
Sample Problem: sort list

[9 2 7 1] → [1 2 7 9]  
[3 8 9 4 2] → [2 3 4 8 9]  
[6 2 2 3 8 5] → [2 2 3 5 6 8]  
...

# Library learning



# Library learning



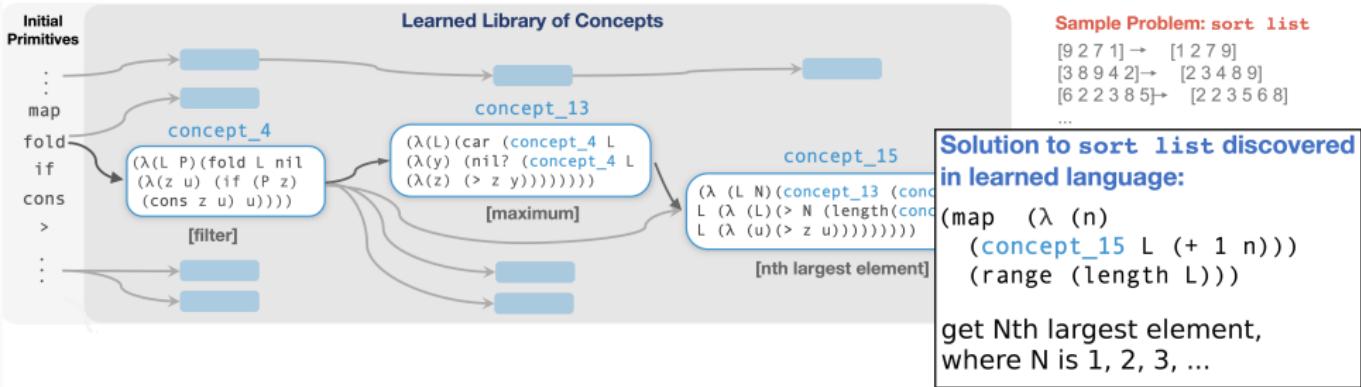
## Sample Problem: sort list

[9 2 7 1] → [1 2 7 9]  
[3 8 9 4 2] → [2 3 4 8 9]  
[6 2 2 3 8 5] → [2 2 3 5 6 8]  
...

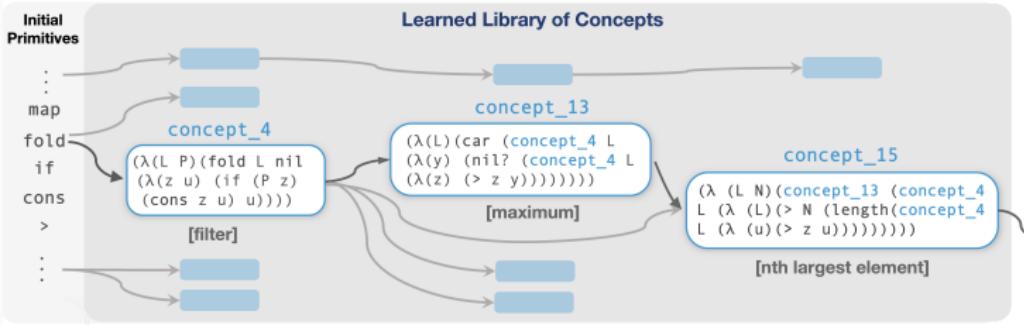
## Solution to sort list discovered in learned language:

```
(map (λ (n)
  (concept_15 L (+ 1 n)))
  (range (length L)))
```

# Library learning



# Library learning



## Sample Problem: sort list

[9 2 7 1] → [1 2 7 9]  
[3 8 9 4 2] → [2 3 4 8 9]  
[6 2 2 3 8 5] → [2 2 3 5 6 8]  
...

## Solution to sort list discovered in learned language:

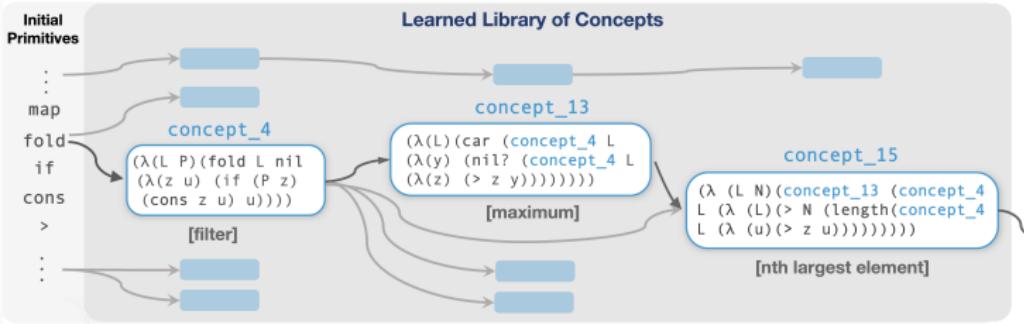
```
(map (λ (n)
  (concept_15 L (+ 1 n)))
  (range (length L)))
```

get Nth largest element,  
where N is 1, 2, 3, ...

## Solution rewritten in initial primitives:

```
(lambda (x) (map (lambda (y) (car (fold (fold x nil (lambda (z u) (if (gt? (+ y 1) (length
(fold x nil (lambda (v w) (if (gt? z v) (cons v w) w)))))) (cons z u) u))) nil (lambda (a b) (if
(nil? (fold (fold x nil (lambda (c d) (if (gt? (+ y 1) (length (fold x nil (lambda (e f) (if
(gt? c e) (cons e f) f)))))) (cons c d) d))) nil (lambda (g h) (if (gt? g a) (cons g h) h))) (cons a b) b)))) (range (length x))))
```

# Library learning



## Sample Problem: sort list

$[9 2 7 1] \rightarrow [1 2 7 9]$   
 $[3 8 9 4 2] \rightarrow [2 3 4 8 9]$   
 $[6 2 2 3 8 5] \rightarrow [2 2 3 5 6 8]$   
...

## Solution to sort list discovered in learned language:

```
(map (λ (n)
  (concept_15 L (+ 1 n)))
  (range (length L)))
```

get Nth largest element,  
where N is 1, 2, 3, ...

## Solution rewritten in initial primitives:

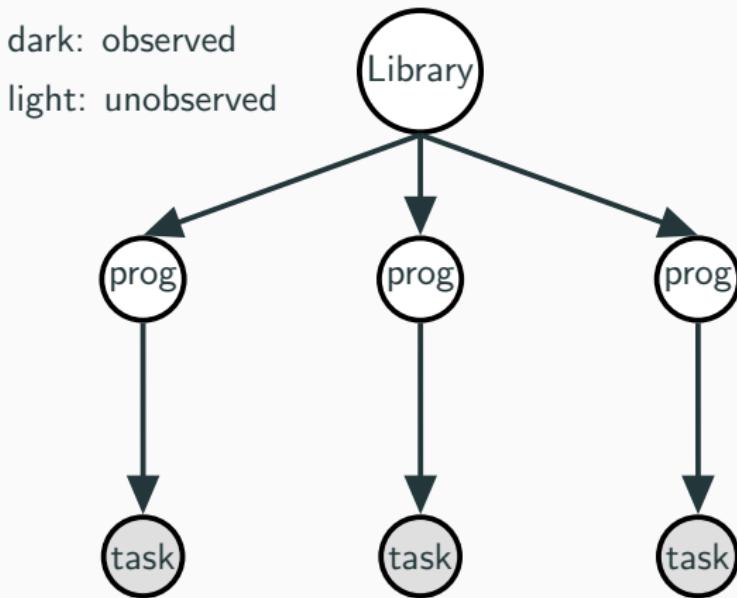
```
(lambda (x) (map (lambda (y) (car (fold (fold x nil (lambda (z u) (if (gt? (+ y 1) (length
(fold x nil (lambda (v w) (if (gt? z v) (cons v w) w)))))) (cons z u) u))) nil (lambda (a b) (if
(nil? (fold (fold x nil (lambda (c d) (if (gt? (+ y 1) (length (fold x nil (lambda (e f) (if
(gt? c e) (cons e f) f)))))) (cons c d) d))) nil (lambda (g h) (if (gt? g a) (cons g h) h))) (cons a b) b)))) (range (length x))))
```

induced sort program found in  $\leq 10\text{min}$ . Brute-force search  
without learned library would take  $\approx 10^{73}$  years

- **Wake:** Solve problems by writing programs
- **Sleep:** Improve library and neural recognition model:
  - **Abstraction sleep:** Improve library
  - **Dream sleep:** Improve neural recognition model

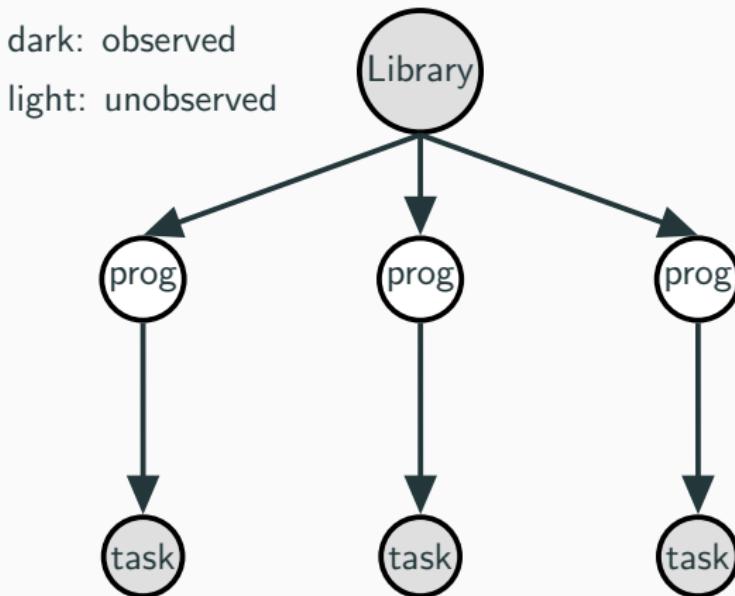
cf. Helmholtz machine, wake/sleep neural network training algorithms

# Library learning as Bayesian inference



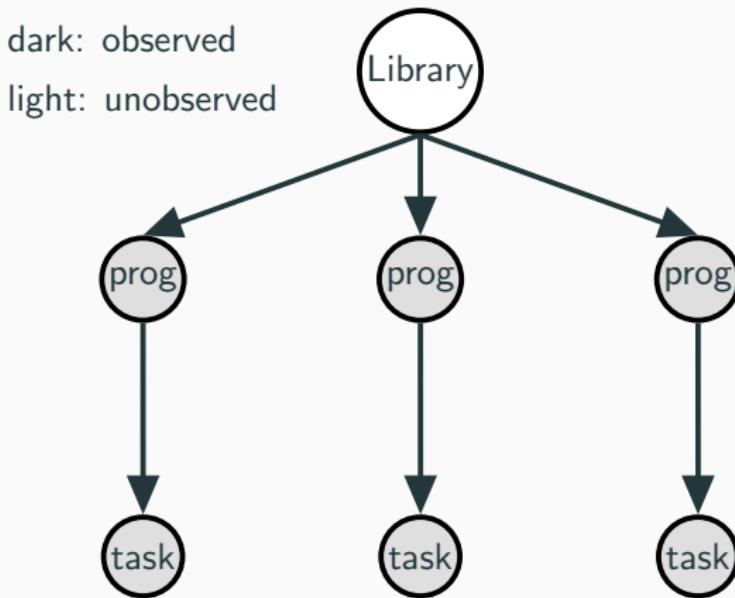
[Dechter et al, 2013] [Liang et al, 2010] [Lake et al, 2015]

# Library learning as Bayesian inference



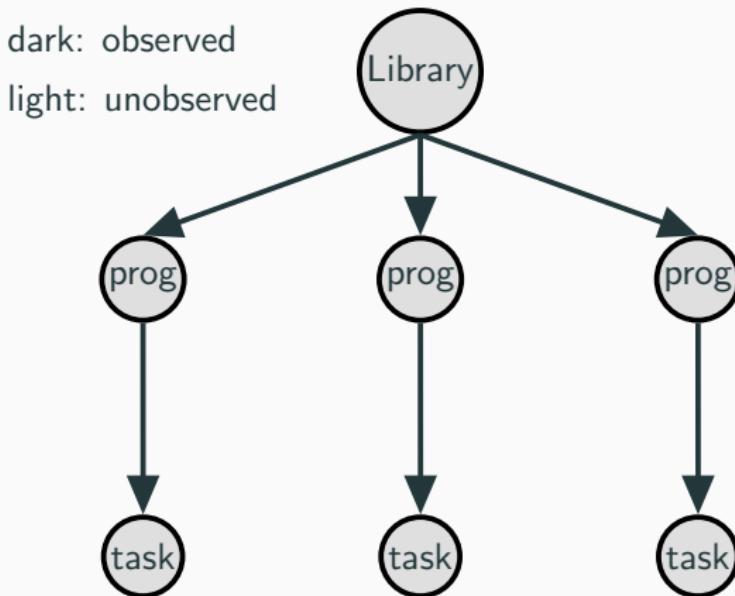
[Dechter et al, 2013] [Liang et al, 2010] [Lake et al, 2015]

# Library learning as Bayesian inference



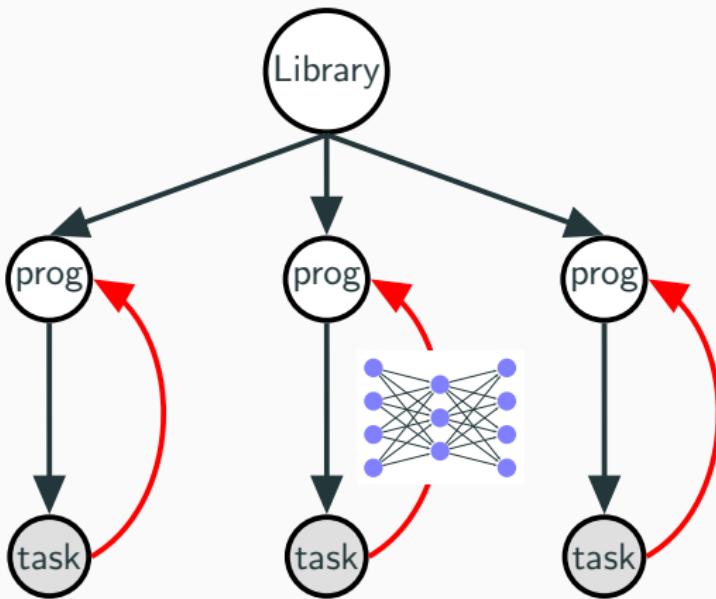
[Dechter et al, 2013] [Liang et al, 2010] [Lake et al, 2015]

# Library learning as Bayesian inference

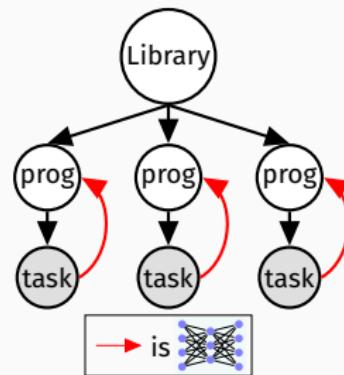


[Dechter et al, 2013] [Liang et al, 2010] [Lake et al, 2015]

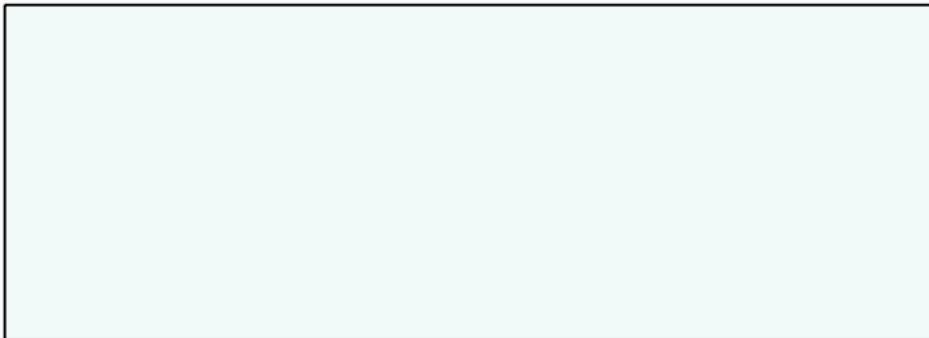
# Library learning as neurally-guided Bayesian inference



library learning via program analysis +  
new neural inference network for program synthesis +  
better program representation (Lisp+polymorphic types [Milner 1978])



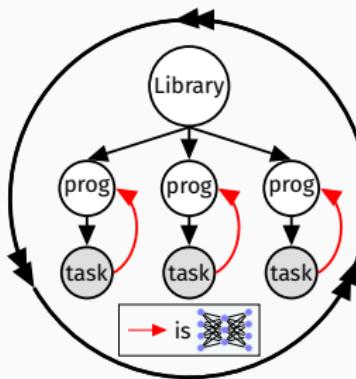
WAKE

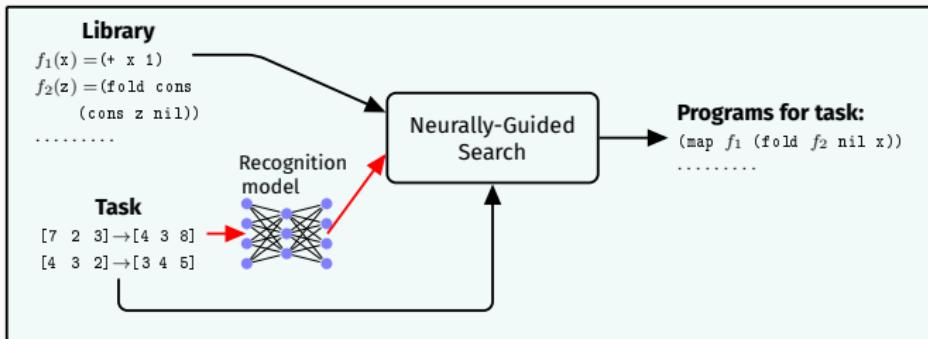
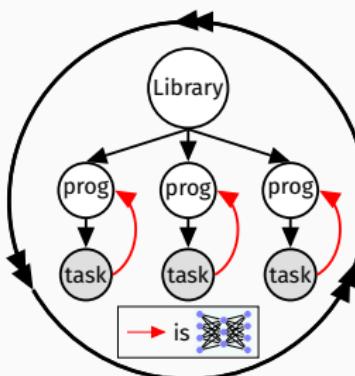


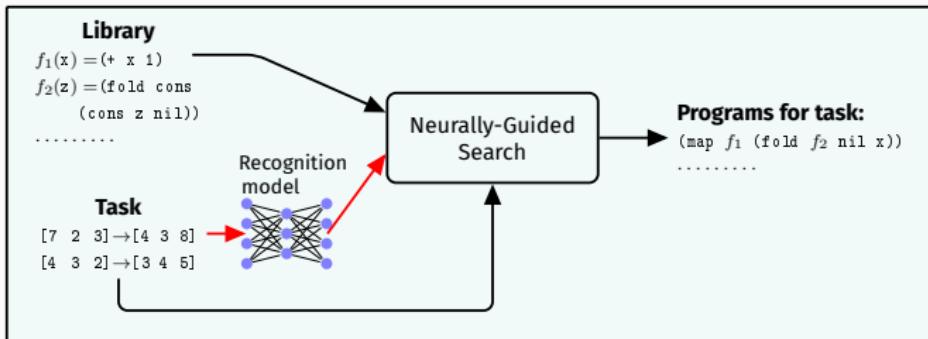
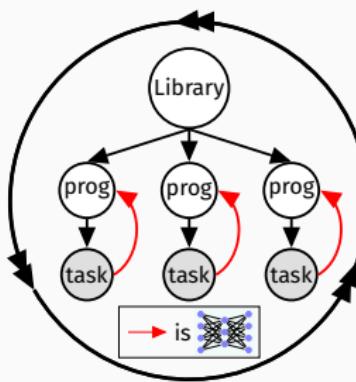
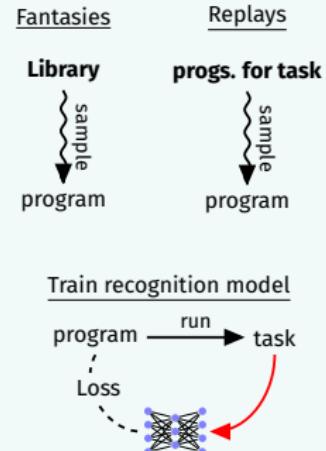
SLEEP: ABSTRACTION



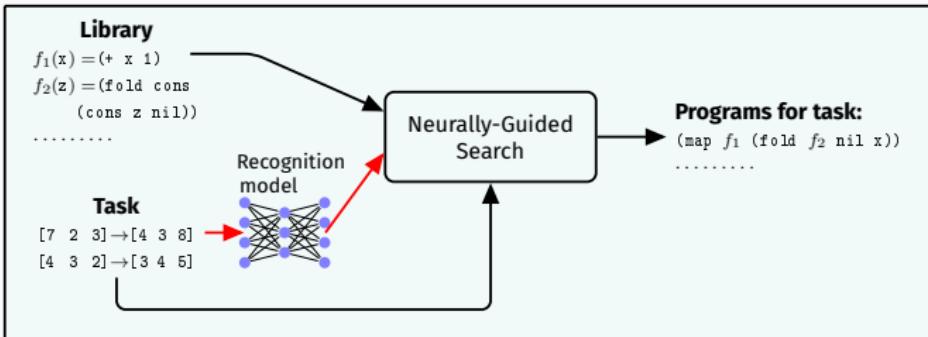
SLEEP: DREAMING



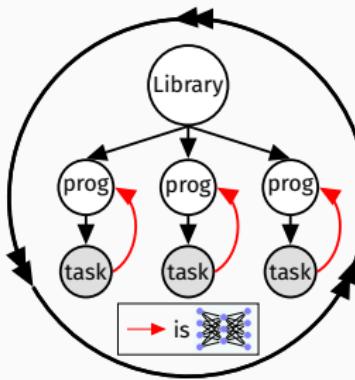
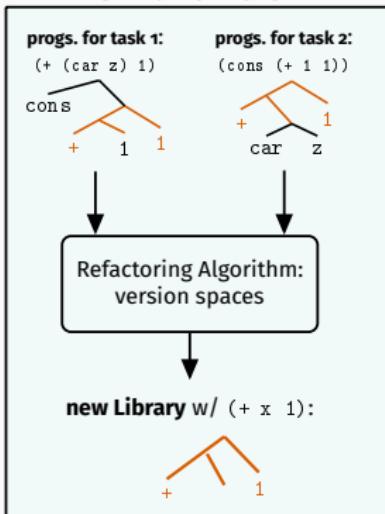
**WAKE****SLEEP: ABSTRACTION****SLEEP: DREAMING**

**WAKE****SLEEP: ABSTRACTION****SLEEP: DREAMING**

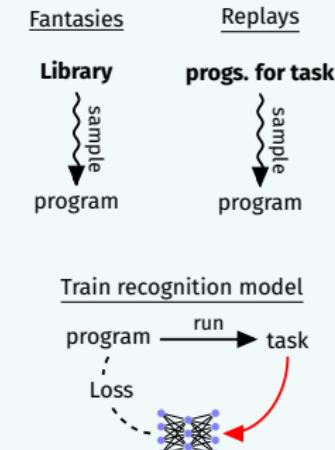
## WAKE



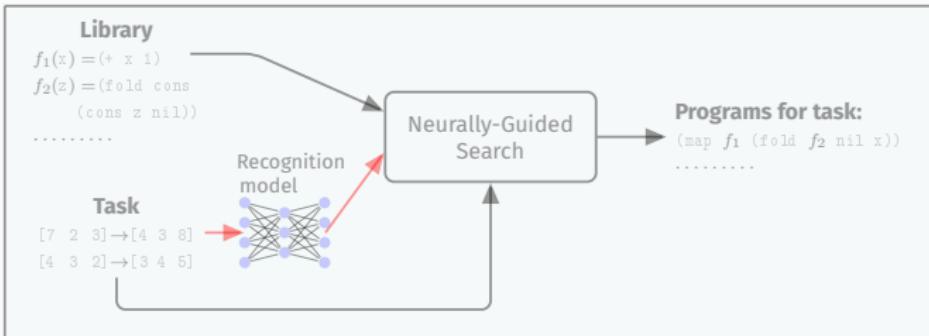
## SLEEP: ABSTRACTION



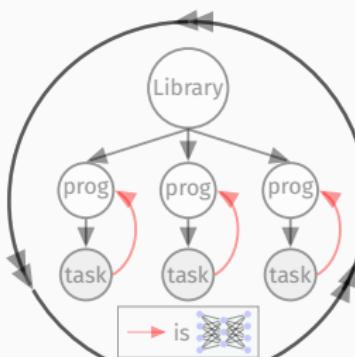
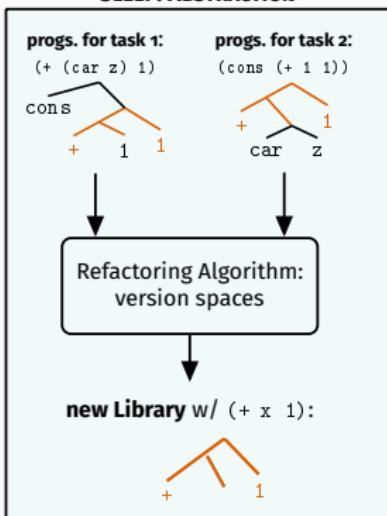
## SLEEP: DREAMING



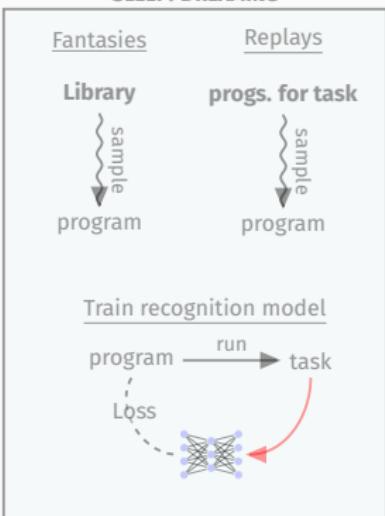
## WAKE



## SLEEP: ABSTRACTION



## SLEEP: DREAMING



Program Induction and learning to learn  
learning a DSL  
learning to synthesize  
synergy between DSL+learned synthesizer

# Abstraction Sleep: Growing the library via refactoring

$$5 + 5$$

## Abstraction Sleep: Growing the library via refactoring

$5 + 5$

(+ 5 5)

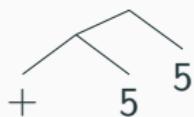
## Abstraction Sleep: Growing the library via refactoring

$5 + 5$

(+ 5 5)



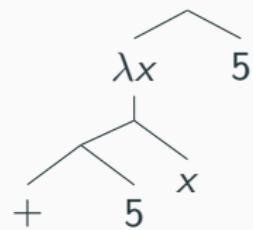
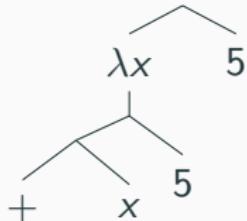
# Abstraction Sleep: Growing the library via refactoring



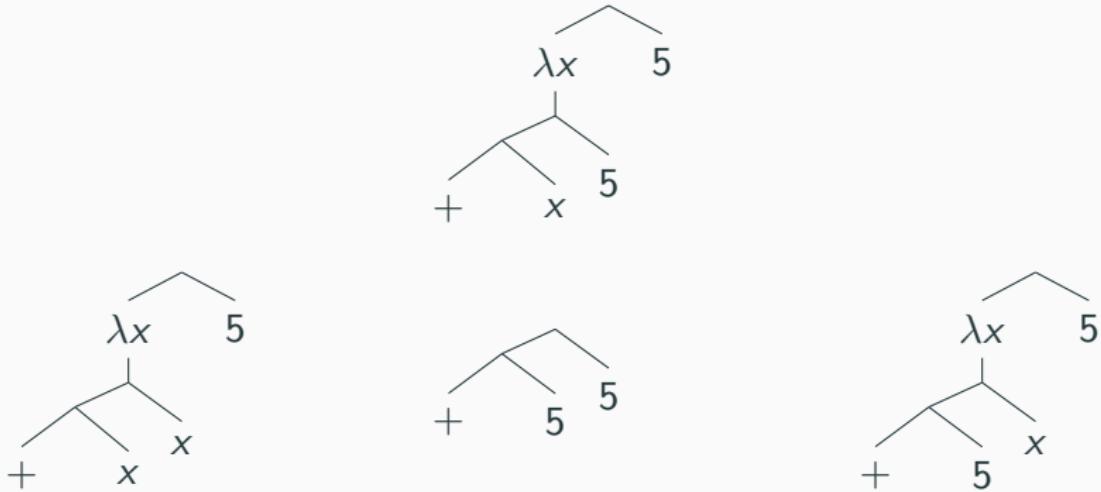
# Abstraction Sleep: Growing the library via refactoring



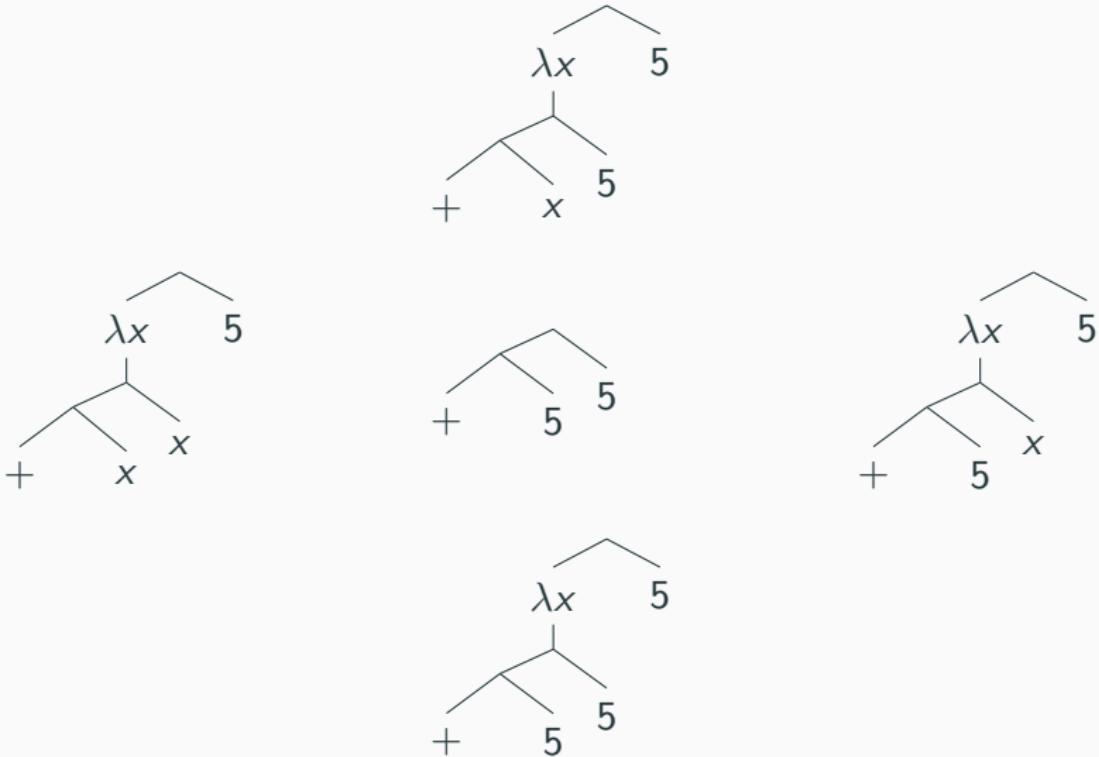
## Abstraction Sleep: Growing the library via refactoring



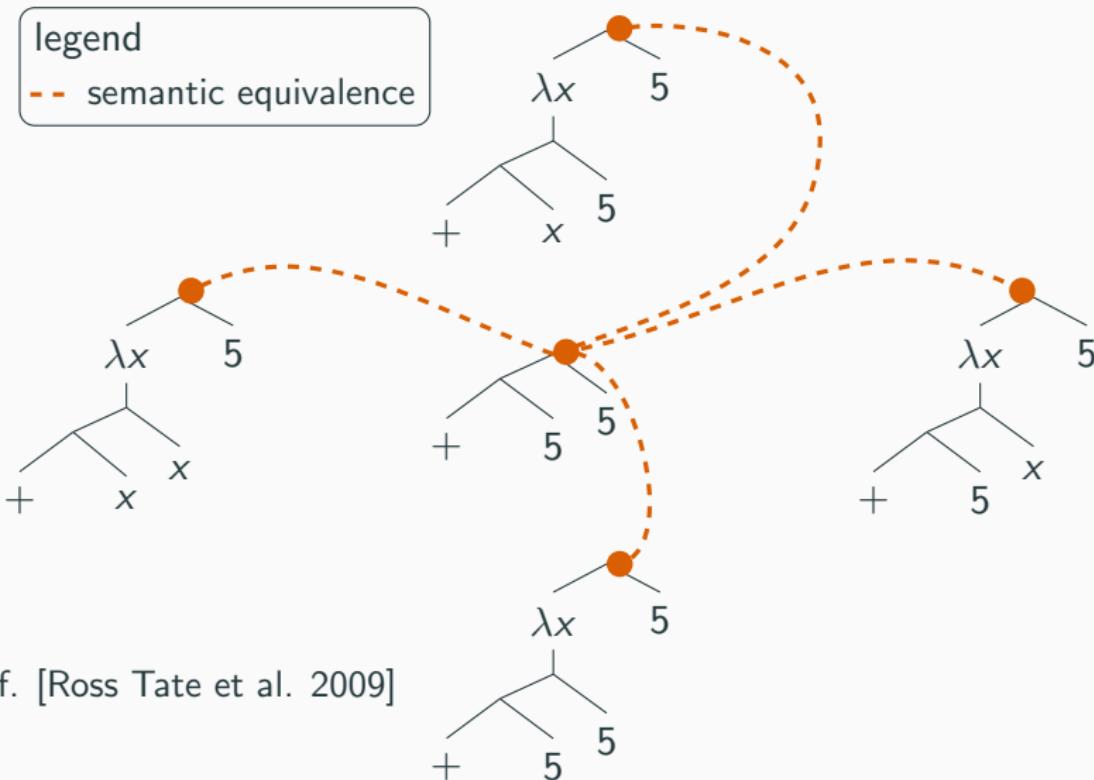
# Abstraction Sleep: Growing the library via refactoring



# Abstraction Sleep: Growing the library via refactoring



# Abstraction Sleep: Growing the library via refactoring



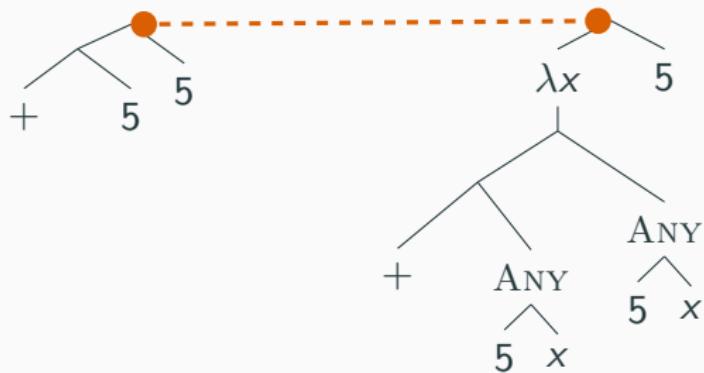
cf. [Ross Tate et al. 2009]

# Abstraction Sleep: Growing the library via refactoring

legend

semantic equivalence

ANY nondeterministic choice



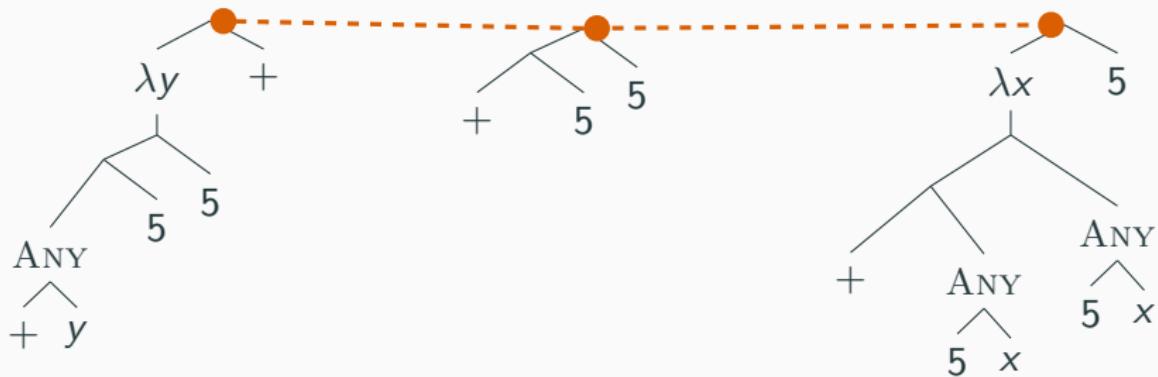
cf. [Gulwani 2012]

# Abstraction Sleep: Growing the library via refactoring

legend

semantic equivalence

ANY nondeterministic choice

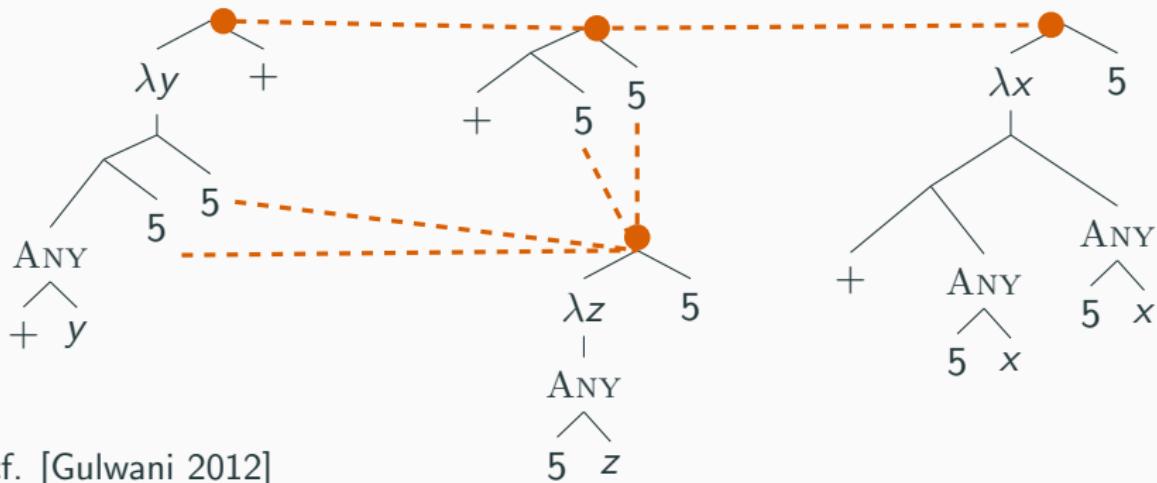


cf. [Gulwani 2012]

# Abstraction Sleep: Growing the library via refactoring

legend

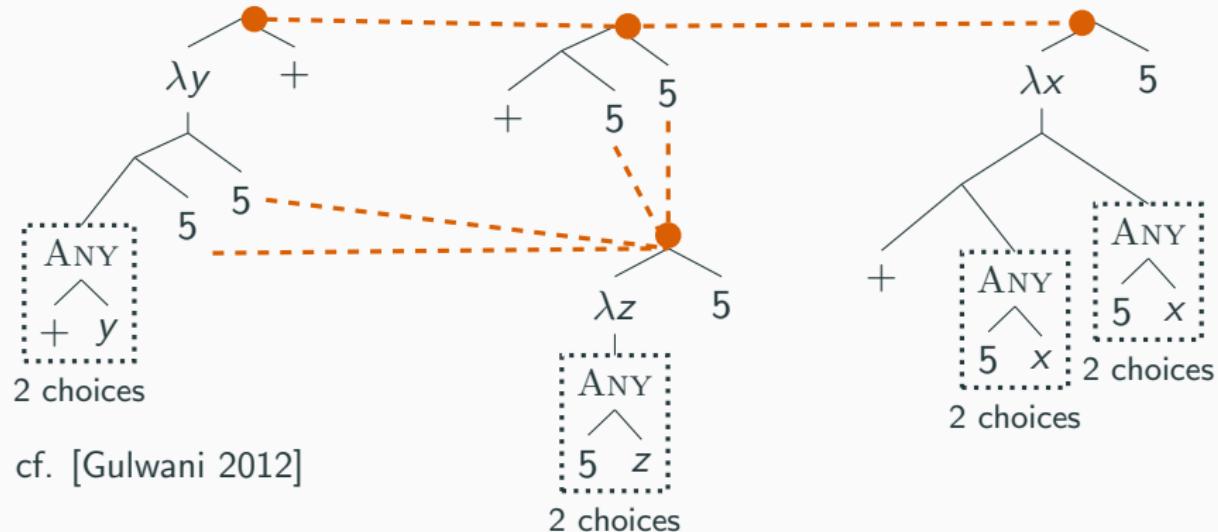
- semantic equivalence
- ANY nondeterministic choice



# Abstraction Sleep: Growing the library via refactoring

legend

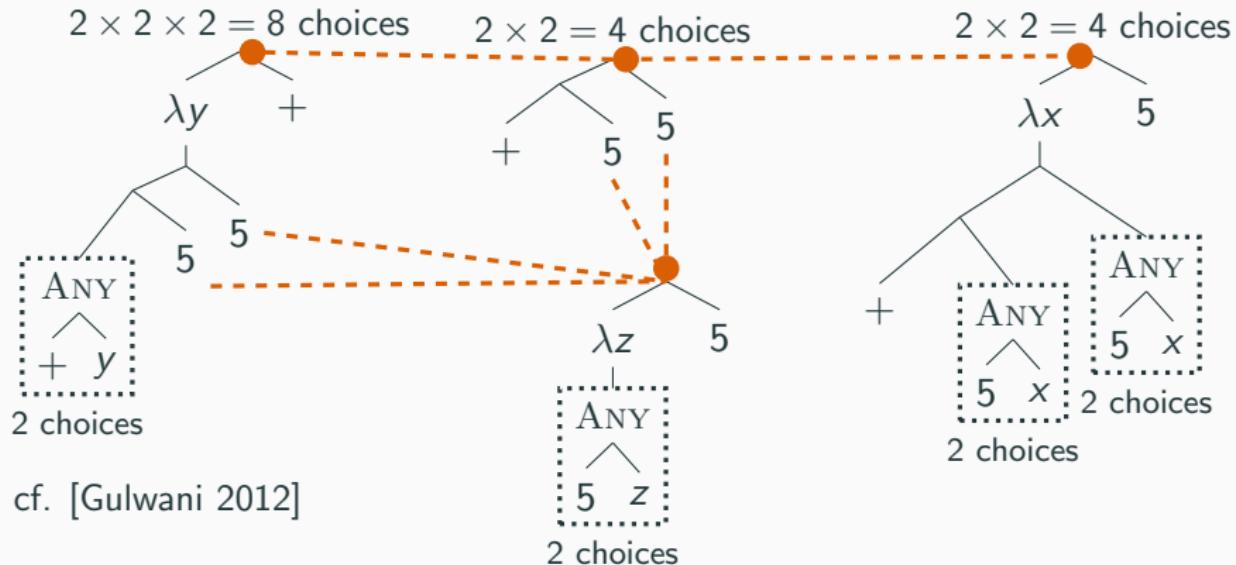
- semantic equivalence
- ANY nondeterministic choice

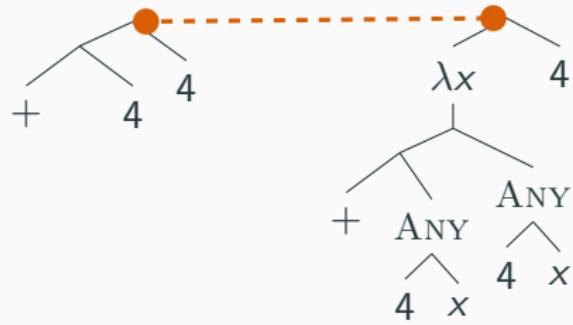
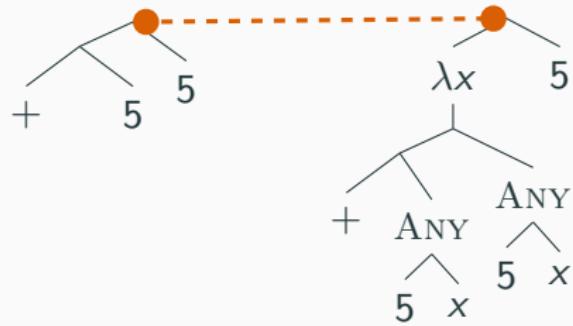


cf. [Gulwani 2012]

# Abstraction Sleep: Growing the library via refactoring

legend  
--- semantic equivalence  
ANY nondeterministic choice

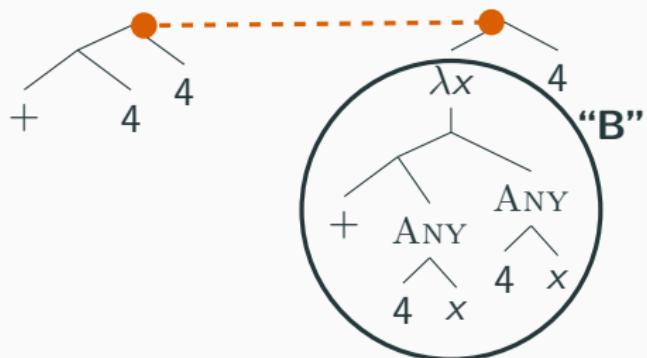
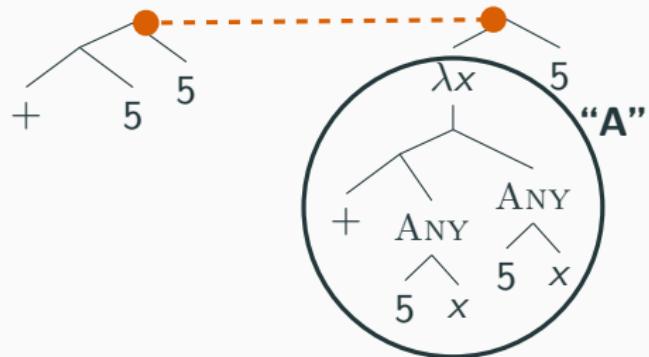




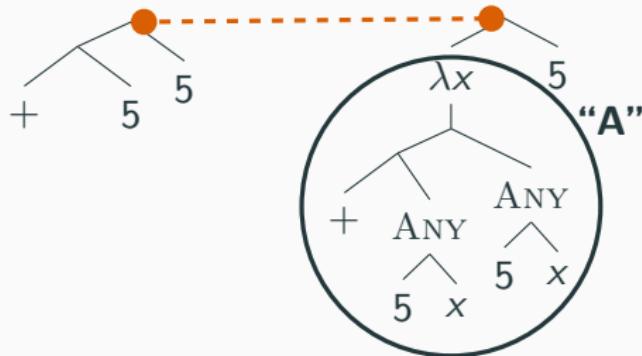
legend

— semantic equivalence

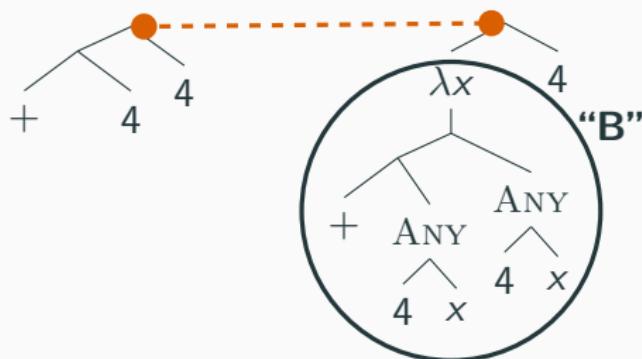
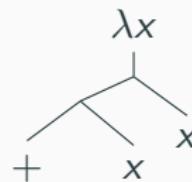
ANY nondeterministic choice



legend  
 - - - semantic equivalence  
 ANY nondeterministic choice



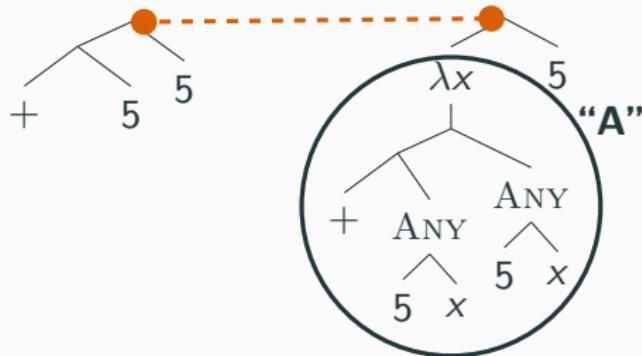
**A intersect B:**



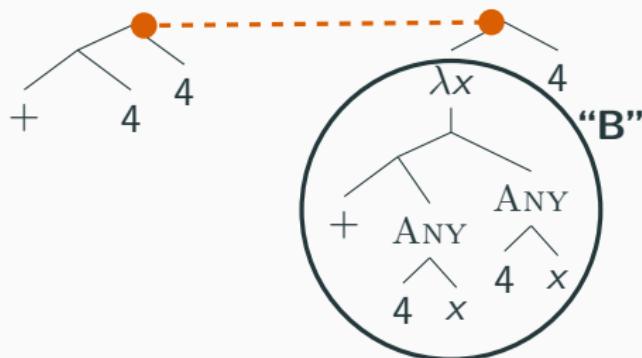
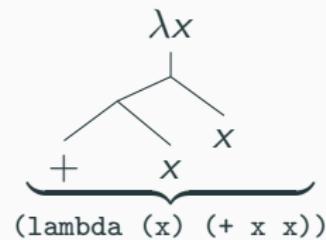
legend

— semantic equivalence

ANY nondeterministic choice



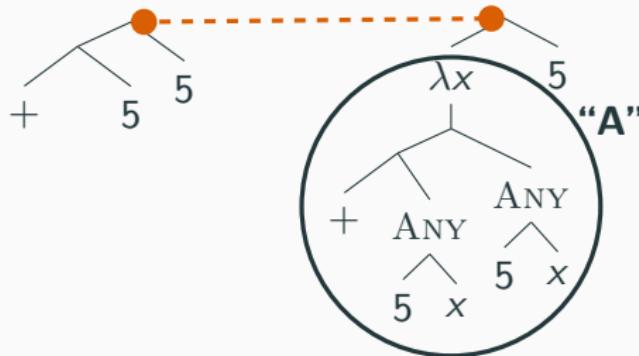
**A intersect B:**



legend

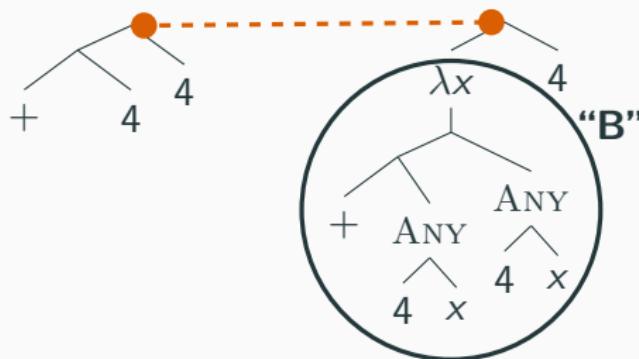
— semantic equivalence

ANY nondeterministic choice



**A intersect B:**

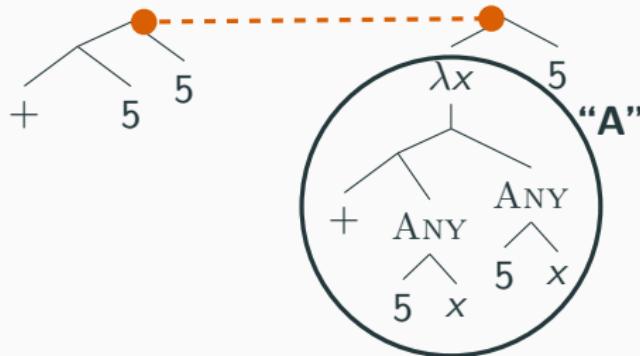
$\lambda x$   
 +      x  
 (lambda (x) (+ x x))  
 = double



legend

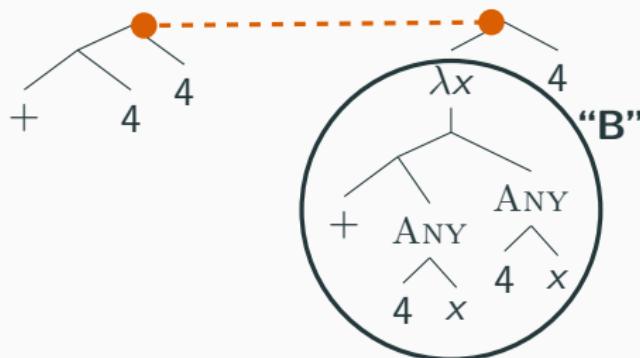
— dashed orange line semantic equivalence

ANY nondeterministic choice



**A intersect B:**

$$\begin{array}{c}
 \lambda x \\
 / \quad \backslash \\
 + \quad x \\
 \underbrace{\qquad\qquad}_{(\text{lambda } (x) \ (\text{+ } x \ x))} \\
 = \text{double}
 \end{array}$$



w/o double	w/ double
(+ 5 5)	(double 5)
(+ 4 4)	(double 4)
(+ 3 3)	(double 3)
...	

legend

— semantic equivalence  
ANY nondeterministic choice

# Abstraction Sleep: Growing the library via refactoring

Task:  $[1\ 2\ 3] \rightarrow [2\ 4\ 6]$   
 $[4\ 3\ 4] \rightarrow [8\ 6\ 8]$

Wake: program search

```
(Y (λ (r 1) (if (nil? 1) nil  
           (cons (+ (car 1) (car 1))  
                 (r (cdr 1)))))))
```

Task:  $[1\ 2\ 3] \rightarrow [0\ 1\ 2]$   
 $[4\ 3\ 4] \rightarrow [3\ 2\ 3]$

Wake: program search

```
(Y (λ (r 1) (if (nil? 1) nil  
           (cons (- (car 1) 1)  
                 (r (cdr 1)))))))
```

# Abstraction Sleep: Growing the library via refactoring

Task:  $[1\ 2\ 3] \rightarrow [2\ 4\ 6]$   
 $[4\ 3\ 4] \rightarrow [8\ 6\ 8]$

Wake: program search

```
(Y (λ (r 1) (if (nil? 1) nil  
           (cons (+ (car 1) (car 1))  
                  (r (cdr 1)))))))
```

Task:  $[1\ 2\ 3] \rightarrow [0\ 1\ 2]$   
 $[4\ 3\ 4] \rightarrow [3\ 2\ 3]$

Wake: program search

```
(Y (λ (r 1) (if (nil? 1) nil  
           (cons (- (car 1) 1)  
                  (r (cdr 1)))))))
```

refactor

$(10^{14}$  refactorings)

```
((λ (f) (Y (λ (r 1) (if (nil? 1)  
                           nil  
                           (cons (f (car 1))  
                                 (r (cdr 1)))))))  
  (λ (z) (+ z z)))
```

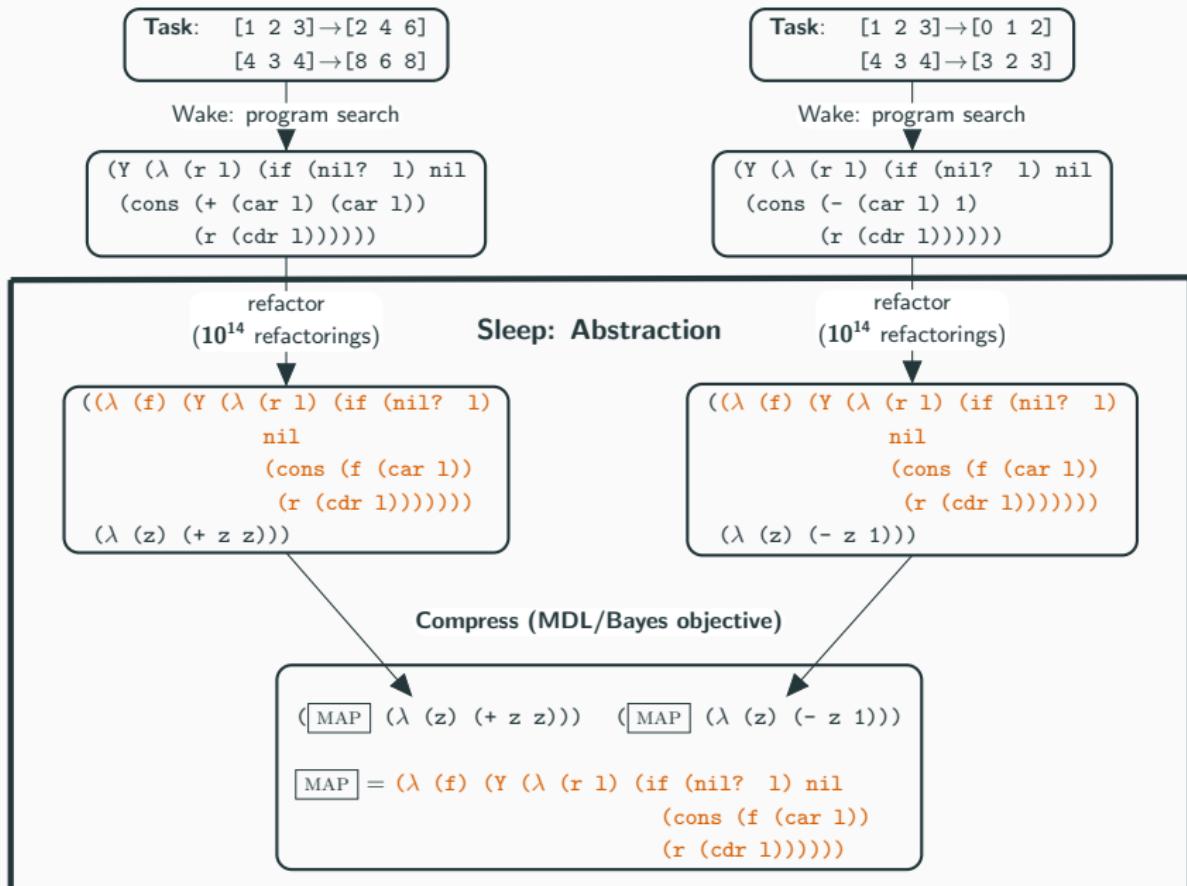
## Sleep: Abstraction

refactor

$(10^{14}$  refactorings)

```
((λ (f) (Y (λ (r 1) (if (nil? 1)  
                           nil  
                           (cons (f (car 1))  
                                 (r (cdr 1)))))))  
  (λ (z) (- z 1)))
```

# Abstraction Sleep: Growing the library via refactoring



# Abstraction Sleep: Growing the library via refactoring

Task:  $[1\ 2\ 3] \rightarrow [2\ 4\ 6]$   
 $[4\ 3\ 4] \rightarrow [8\ 6\ 8]$

Wake: program search

```
(Y (λ (r 1) (if (nil? 1) nil  
           (cons (+ (car 1) (car 1))  
                  (r (cdr 1)))))))
```

Task:  $[1\ 2\ 3] \rightarrow [0\ 1\ 2]$   
 $[4\ 3\ 4] \rightarrow [3\ 2\ 3]$

Wake: program search

```
(Y (λ (r 1) (if (nil? 1) nil  
           (cons (- (car 1) 1)  
                  (r (cdr 1)))))))
```

these  $10^{14}$  refactorings are represented in DreamCoder's exponentially more efficient refactoring data structure using  $10^6$  nodes, calculated in under 5min

$(\lambda (z) (+ z z))$

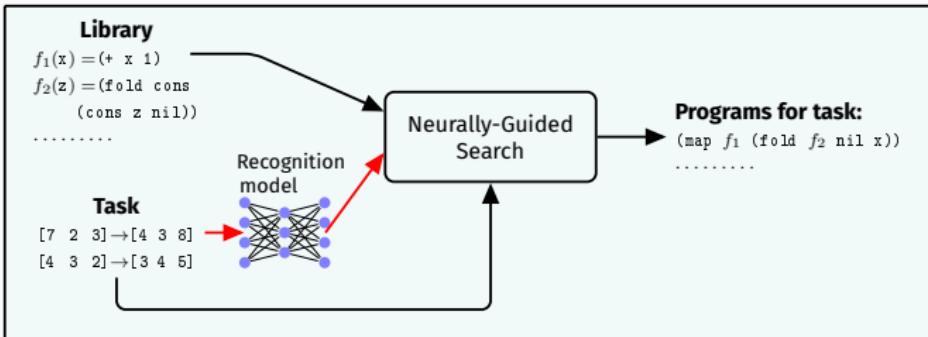
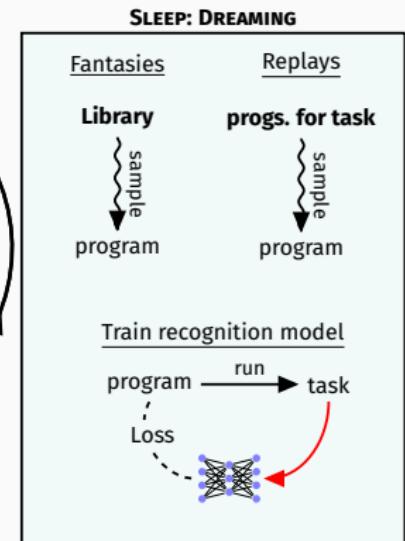
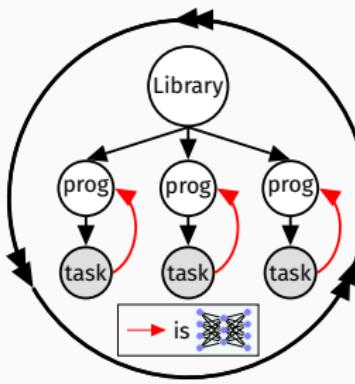
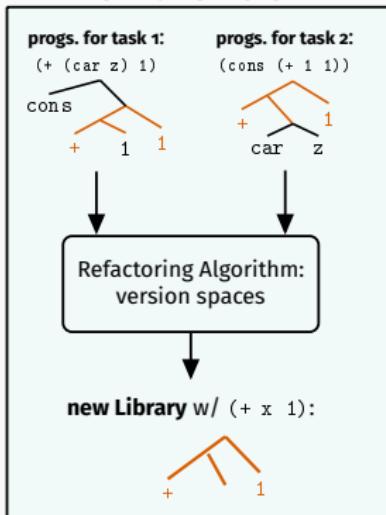
$(\lambda (z) (- z 1))$

Compress (MDL/Bayes objective)

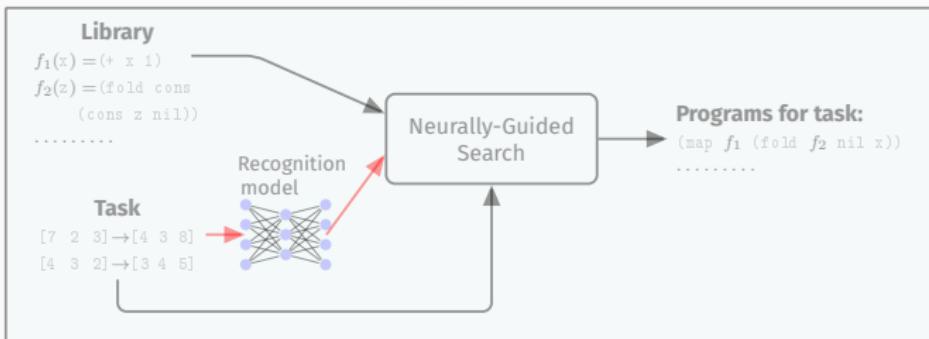
$(\boxed{\text{MAP}} (\lambda (z) (+ z z))) \quad (\boxed{\text{MAP}} (\lambda (z) (- z 1)))$

$\boxed{\text{MAP}} = (\lambda (f) (Y (\lambda (r 1) (if (nil? 1) nil  
 (cons (f (car 1))  
 (r (cdr 1)))))))$

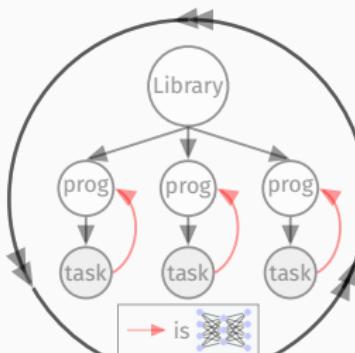
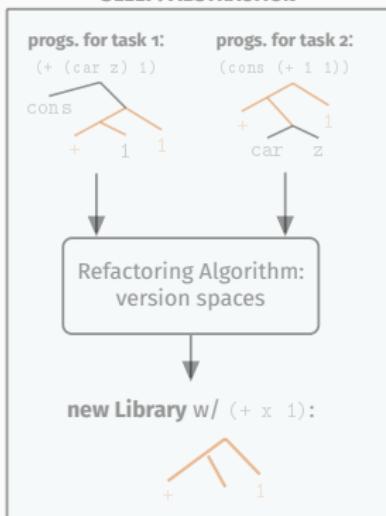
Program Induction and learning to learn  
learning a DSL  
**learning to synthesize**  
synergy between DSL+learned synthesizer

**WAKE****SLEEP: ABSTRACTION**

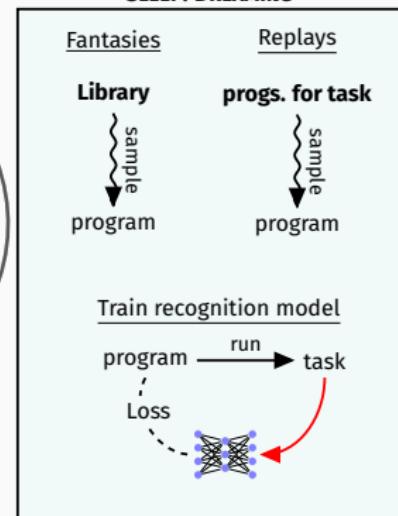
## WAKE



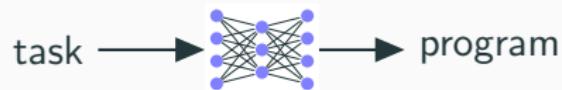
## SLEEP: ABSTRACTION



## SLEEP: DREAMING



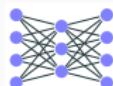
# Neural recognition model guides search



# Neural recognition model guides search



# Neural recognition model guides search

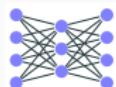


is a...

recurrent network (Devlin et al 2017)

unigram model (Menon et al 2013; Balog et al 2016)

# Neural recognition model guides search

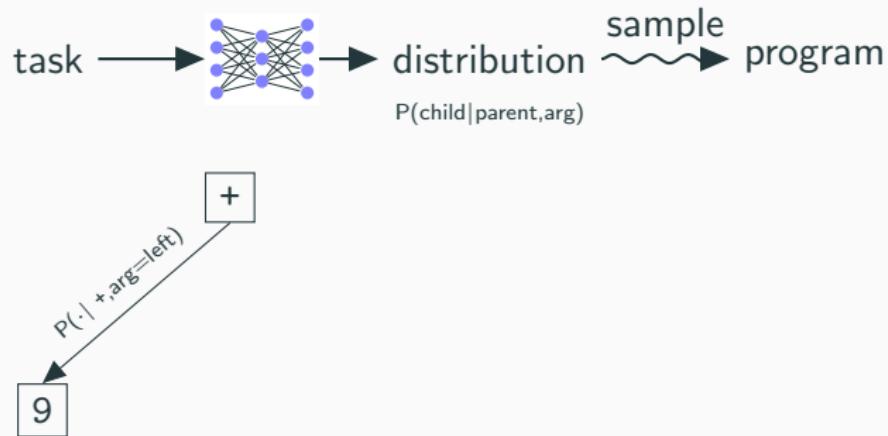


is a “**bigram**” model over syntax trees

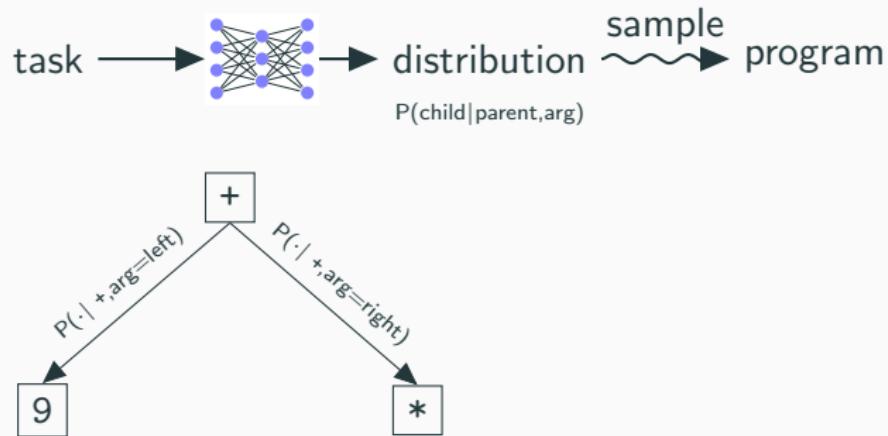
# Neural recognition model guides search



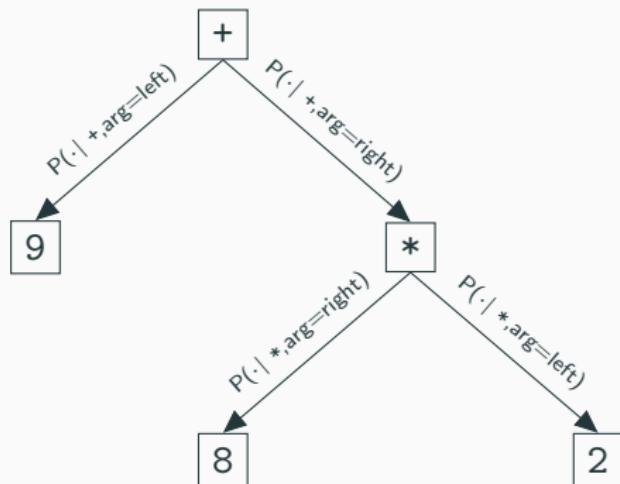
# Neural recognition model guides search



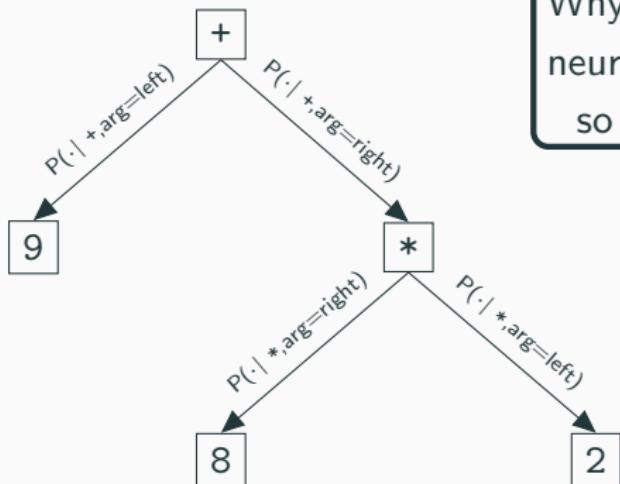
# Neural recognition model guides search



# Neural recognition model guides search

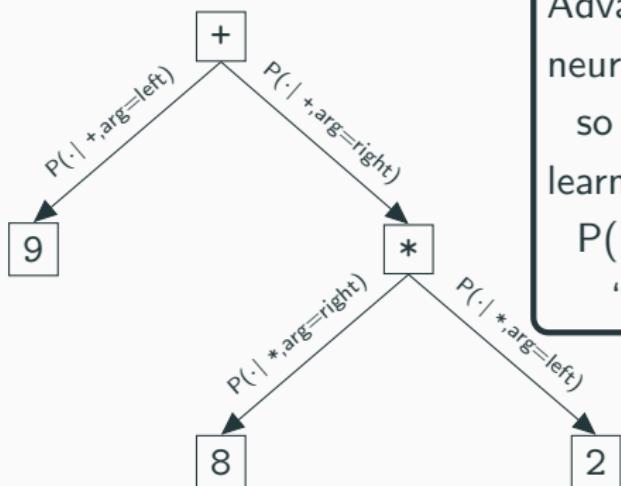


# Neural recognition model guides search



Why we do this:  
neural net runs once per task,  
so CPU bottlenecks instead of GPU

# Neural recognition model guides search



Advantages:  
neural net runs once per task,  
so CPU bottlenecks instead of GPU  
learns to break syntactic symmetries:  
 $P(1|*,\text{arg}=left)=0.0$   
“do not multiply by one”

Program Induction and learning to learn  
learning a DSL  
learning to synthesize  
synergy between DSL+learned synthesizer

# DreamCoder Domains

## List Processing

### Sum List

[1 2 3] → 6

[4 6 8 1] → 17

### Double

[1 2 3] → [2 4 6]

[4 5 1] → [8 10 2]

## Text Editing

### Abbreviate

Allen Newell → A.N.

Herb Simon → H.S.

### Drop Last Three

shrdlu → shr

shakey → sha

## Regexes

### Phone numbers

(555) 867-5309

(650) 555-2368

### Currency

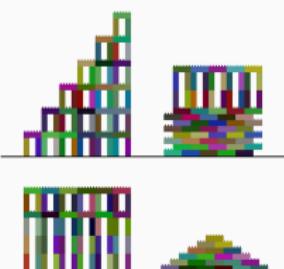
\$100.25

\$4.50

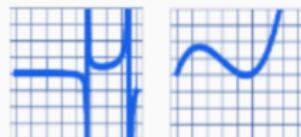
## LOGO Graphics



## Block Towers



## Symbolic Regression



$$y = f(x)$$

## Recursive Programming

### Filter Red

[■■■■■■■■] → [■■■■]

[■■■■■■■■■■] → [■■■■■■■■]

[■■■■■■■■■■] → [■■■■■■■■]

## Physical Laws

$$\vec{a} = \frac{1}{m} \sum_i \vec{F}_i$$

$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r}|^2} \hat{r}$$

# DreamCoder Domains

## List Processing

### Sum List

[1 2 3] → 6

[4 6 8 1] → 17

### Double

[1 2 3] → [2 4 6]

[4 5 1] → [8 10 2]

## Text Editing

### Abbreviate

Allen Newell → A.N.

Herb Simon → H.S.

### Drop Last Three

shrdlu → shr

shakey → sha

## Regexes

### Phone numbers

(555) 867-5309

(650) 555-2368

### Currency

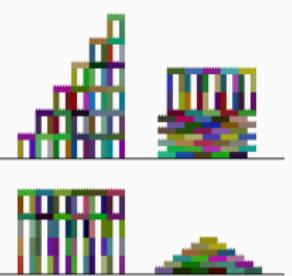
\$100.25

\$4.50

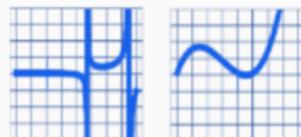
## LOGO Graphics



## Block Towers



## Symbolic Regression



$$y = f(x)$$

## Recursive Programming

### Filter Red

[■■■■■■■■] → [■■■■■■■]

[■■■■■■■■■■] → [■■■■■■■■■]

[■■■■■■■■■■■] → [■■■■■■■■■]

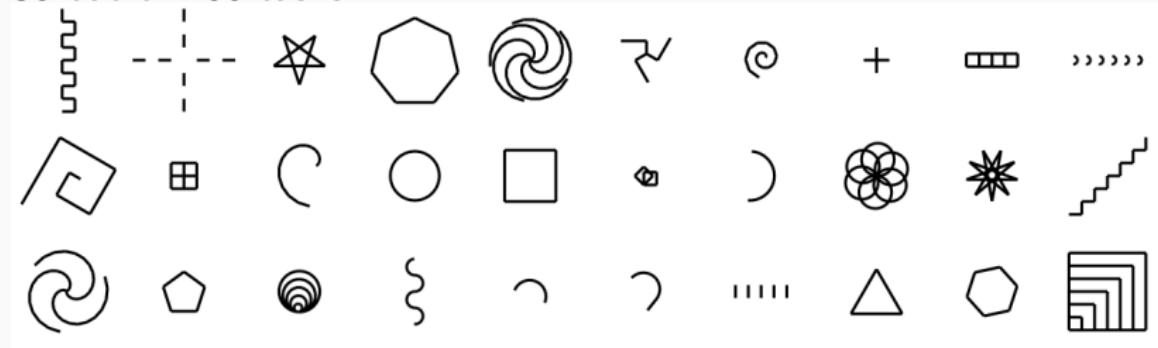
## Physical Laws

$$\vec{a} = \frac{1}{m} \sum_i \vec{F}_i$$

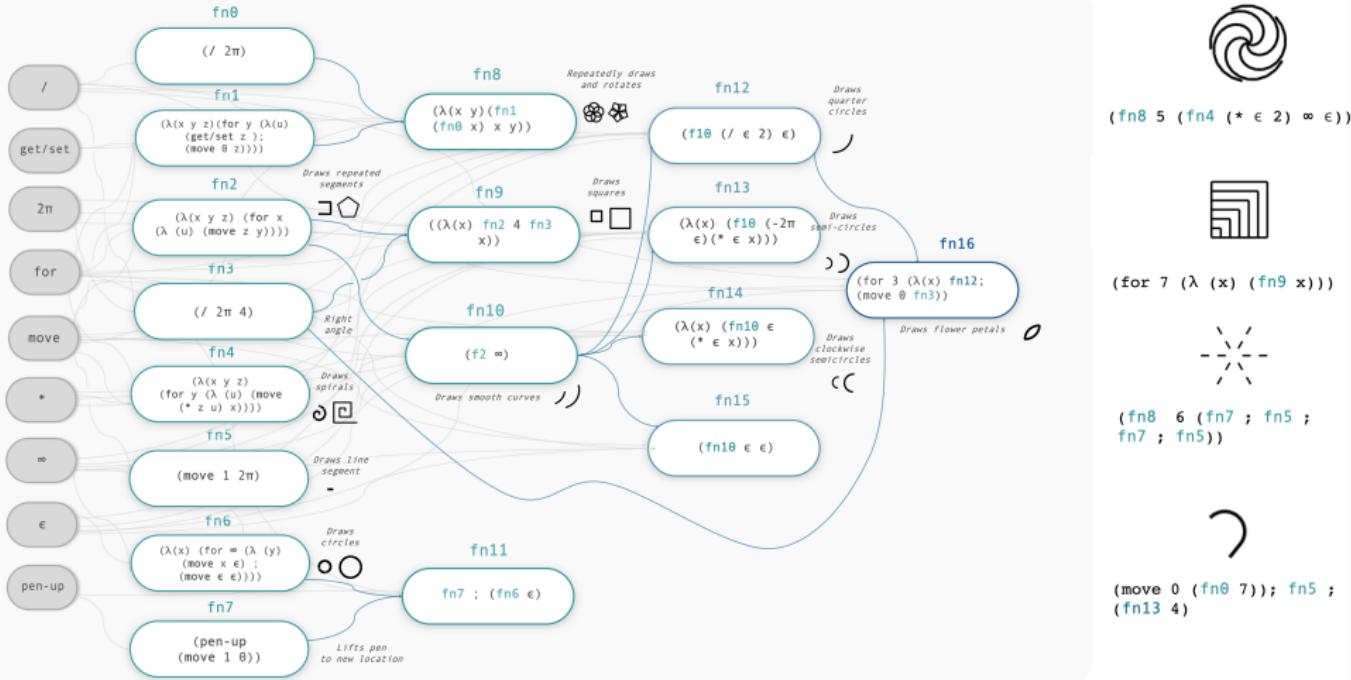
$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r}|^2} \hat{r}$$

# LOGO Turtle Graphics

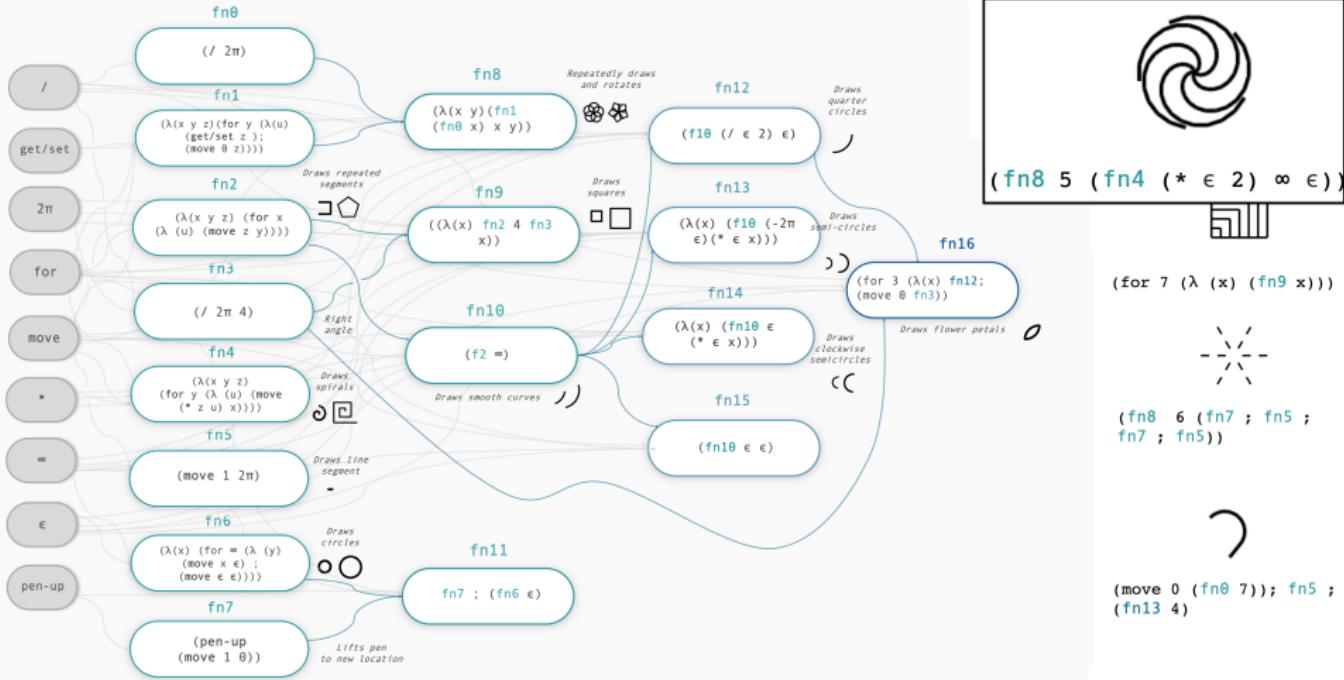
30 out of 160 tasks



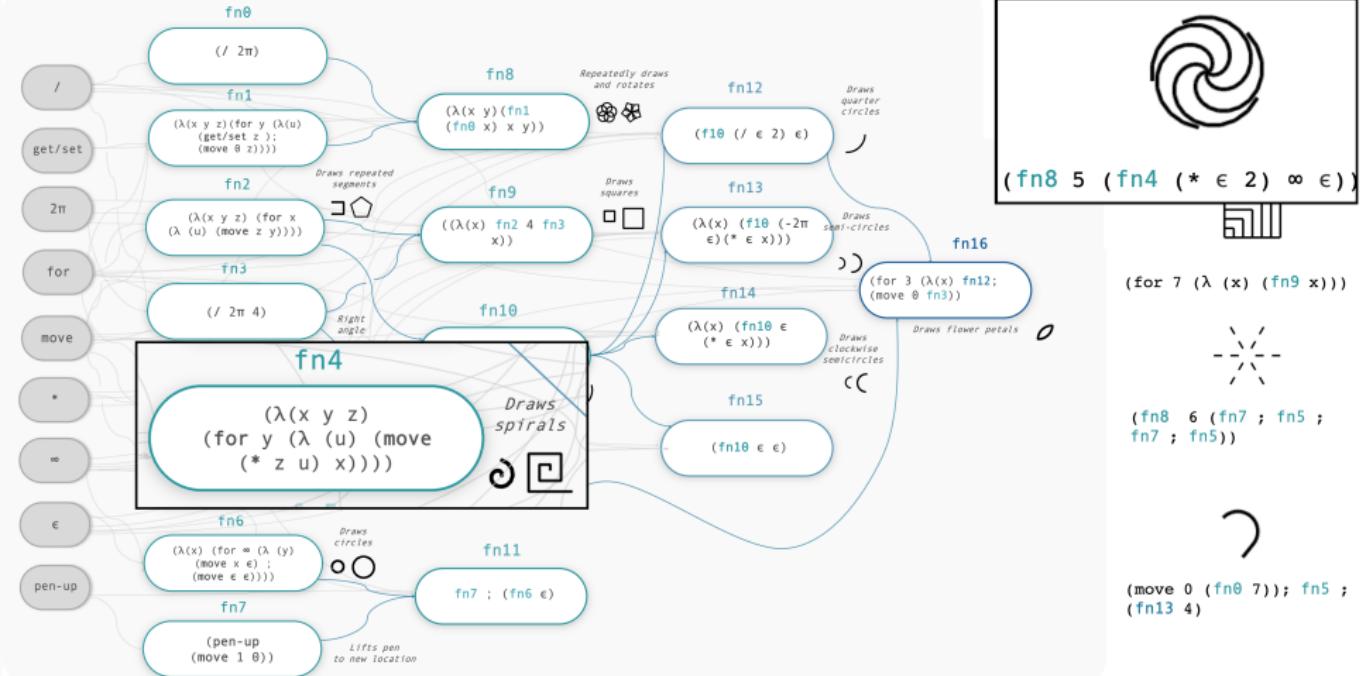
# LOGO Turtle Graphics – learning an interpretable library



# LOGO Turtle Graphics – learning an interpretable library



# LOGO Turtle Graphics – learning an interpretable library



$(\text{for } 7\ (\lambda(x)\ (\text{fn}9\ x)))$

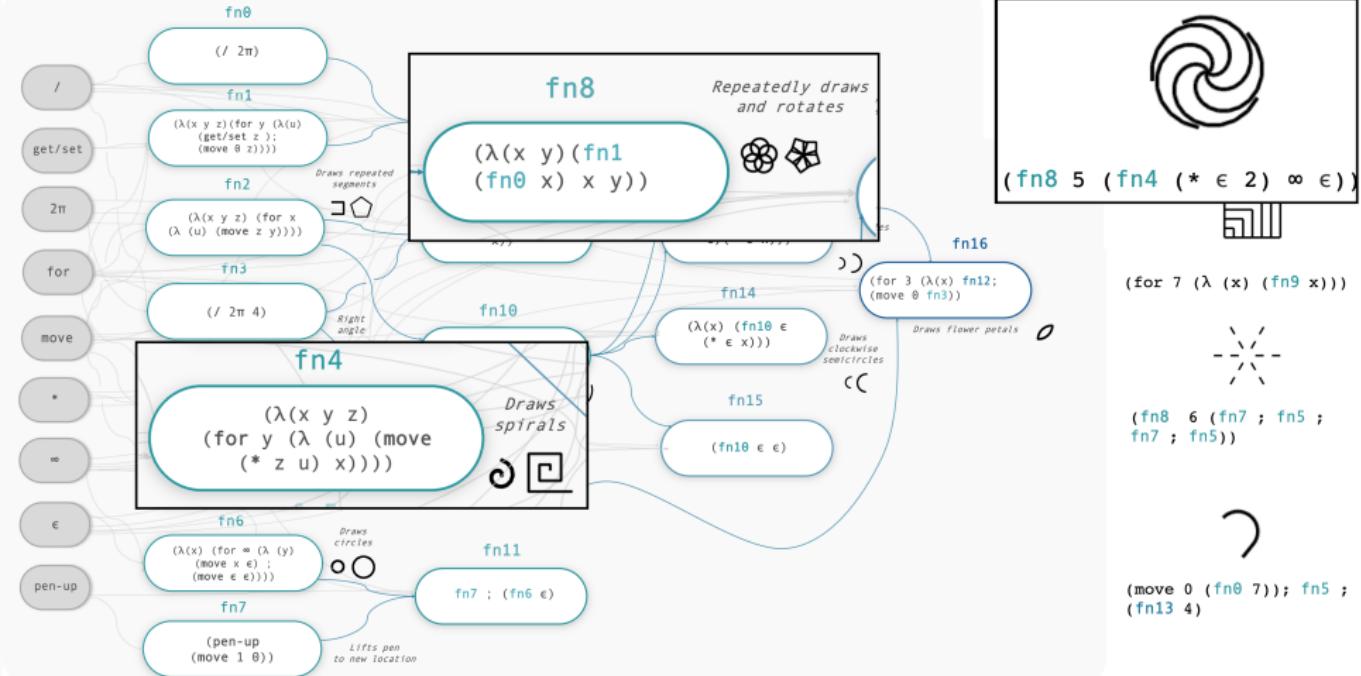


$(\text{fn}8\ 6\ (\text{fn}7\ ;\ \text{fn}5\ ;\ \text{fn}7\ ;\ \text{fn}5))$

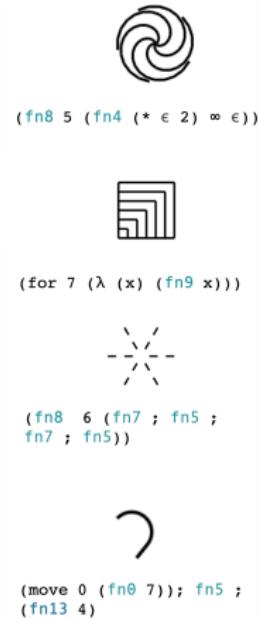
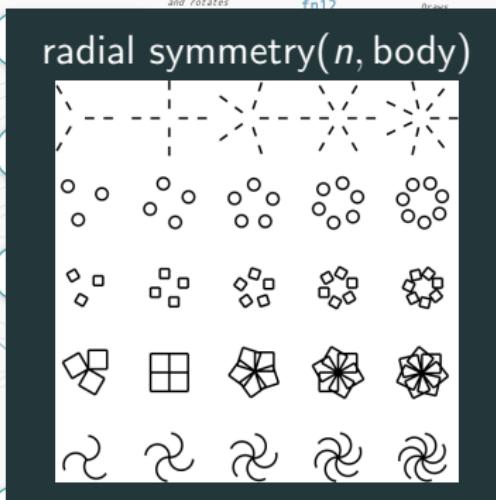
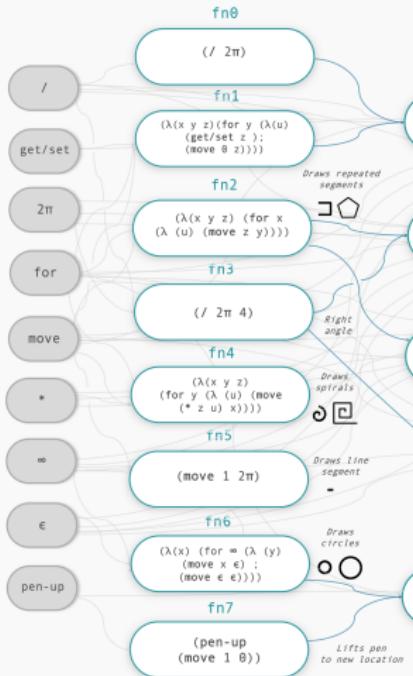


$(\text{move}\ 0\ (\text{fn}0\ 7));\ \text{fn}5\ ;\ (\text{fn}13\ 4)$

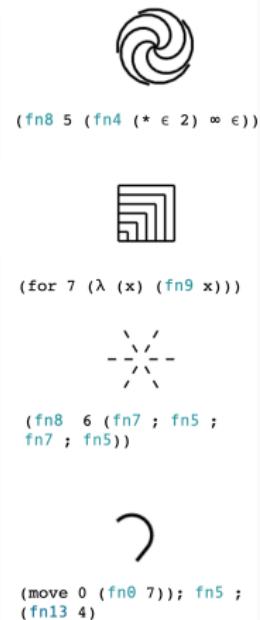
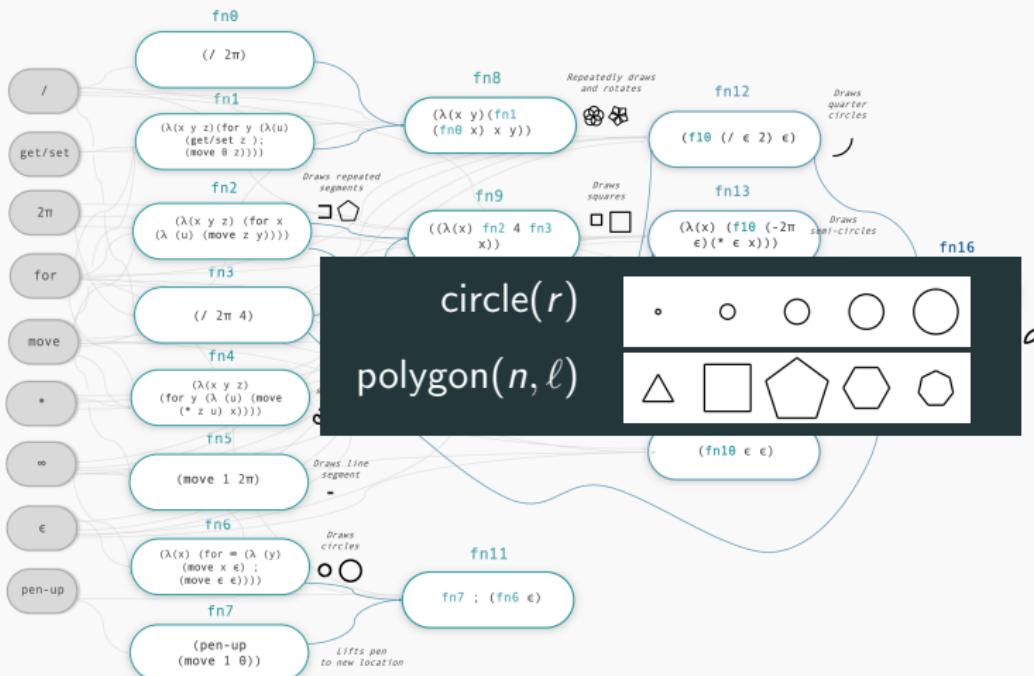
# LOGO Turtle Graphics – learning an interpretable library



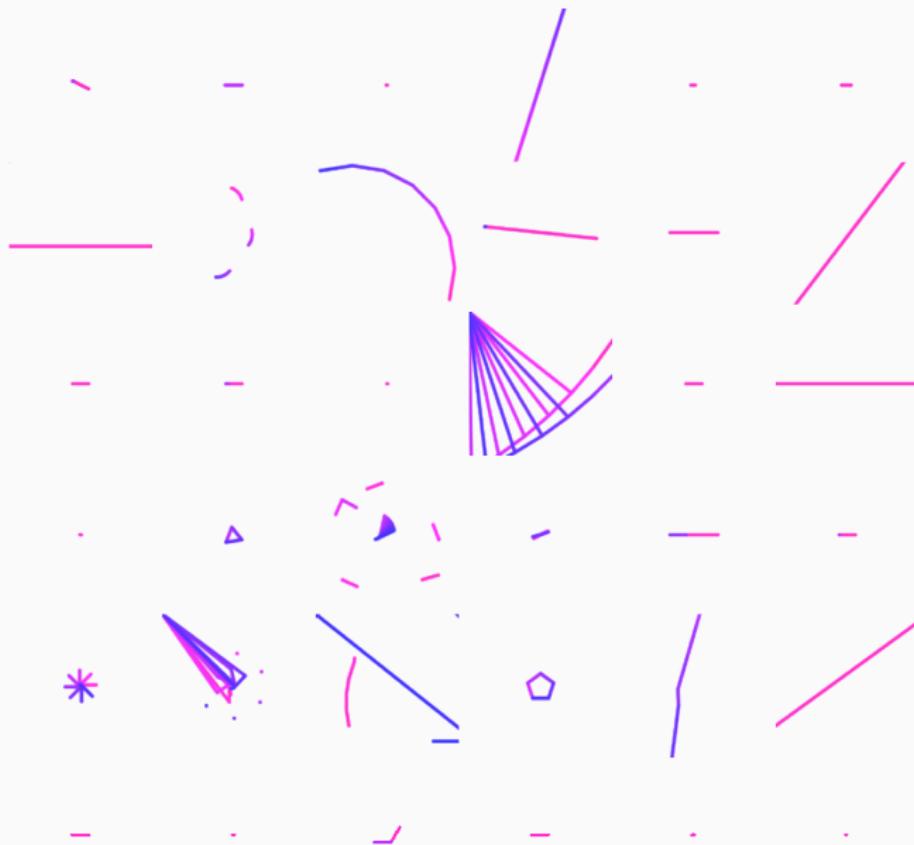
# LOGO Turtle Graphics – learning an interpretable library



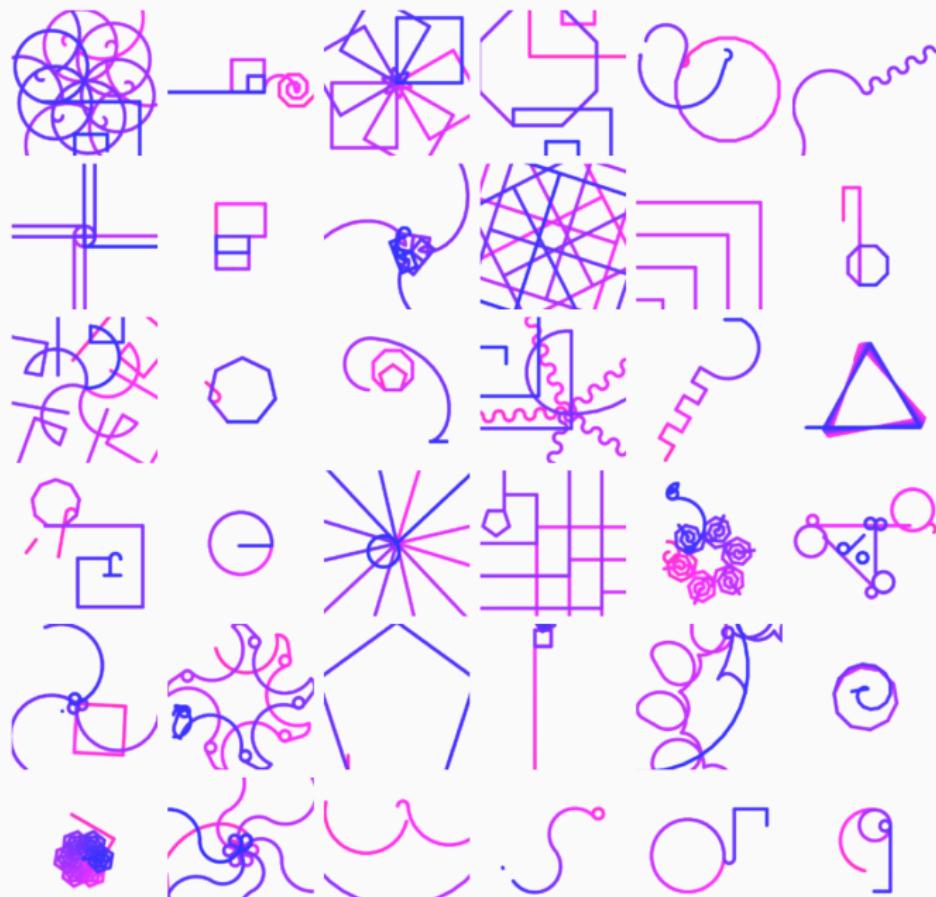
# LOGO Turtle Graphics – learning an interpretable library



# What does DreamCoder dream of? (before learning)

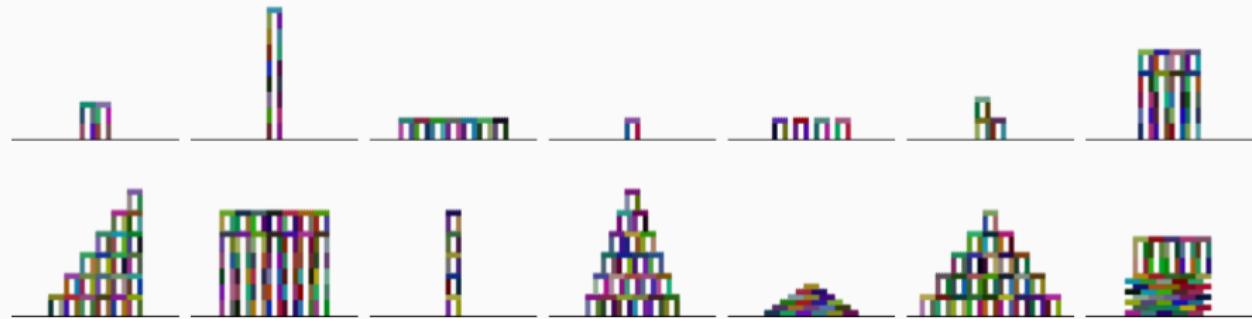


# What does DreamCoder dream of? (after learning)



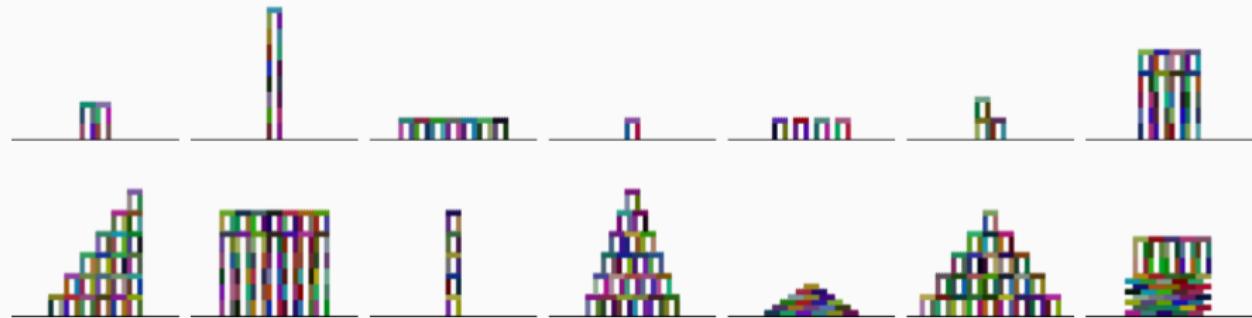
# Planning to build towers

example tasks (112 total)



# Planning to build towers

example tasks (112 total)

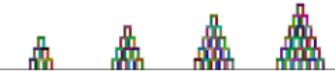


learned library routines ( $\approx 20$  total)

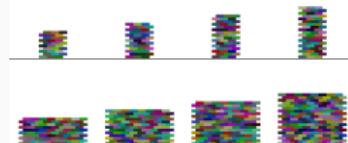
$\text{arch}(h)$



$\text{pyramid}(h)$



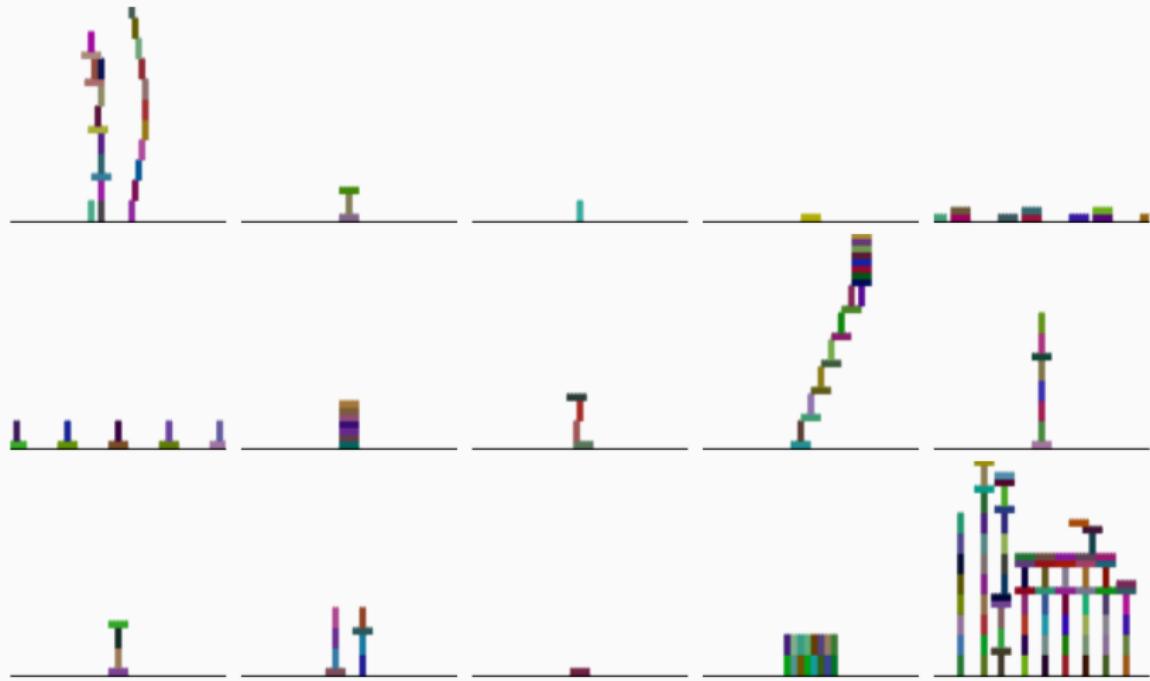
$\text{wall}(w, h)$



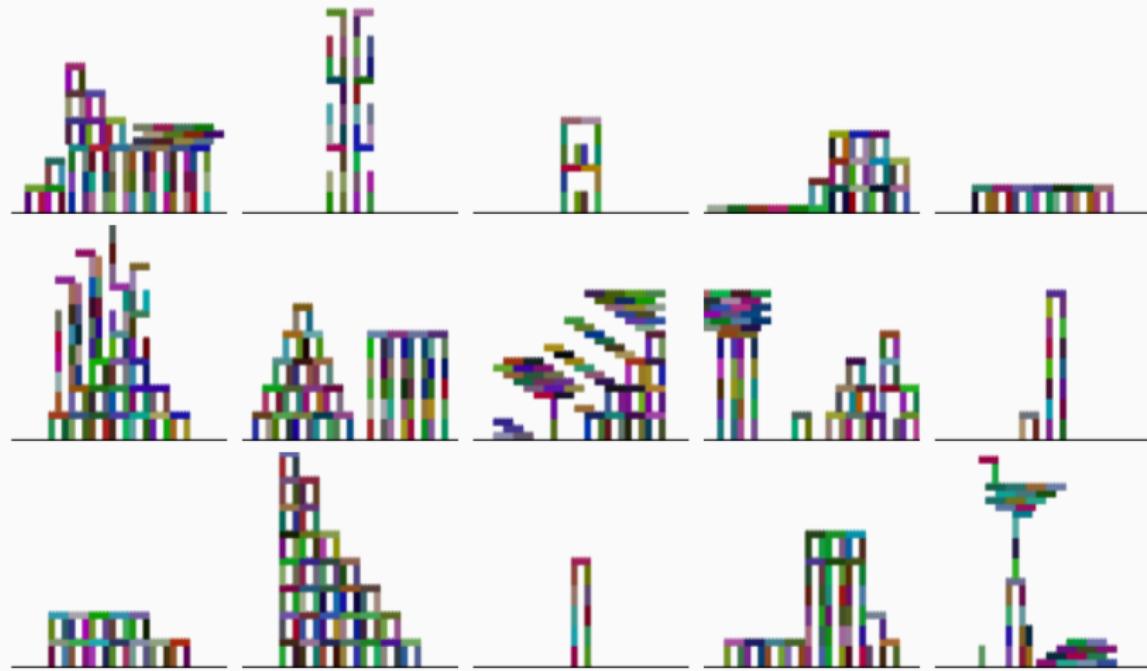
$\text{bridge}(w, h)$



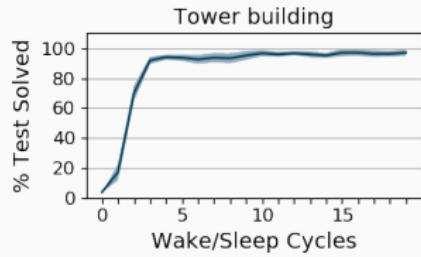
## Dreams before learning



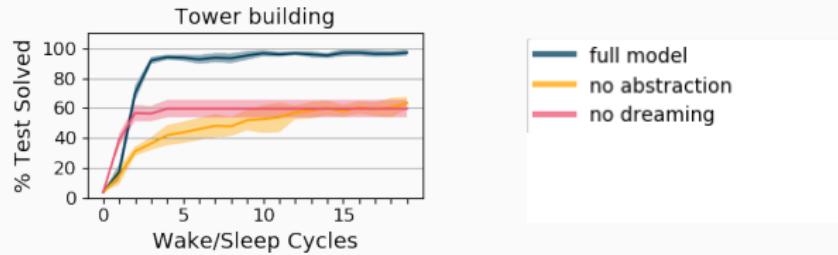
## Dreams after learning



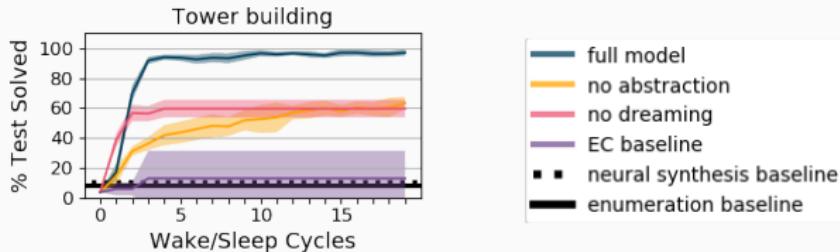
# Learning dynamics



# Learning dynamics

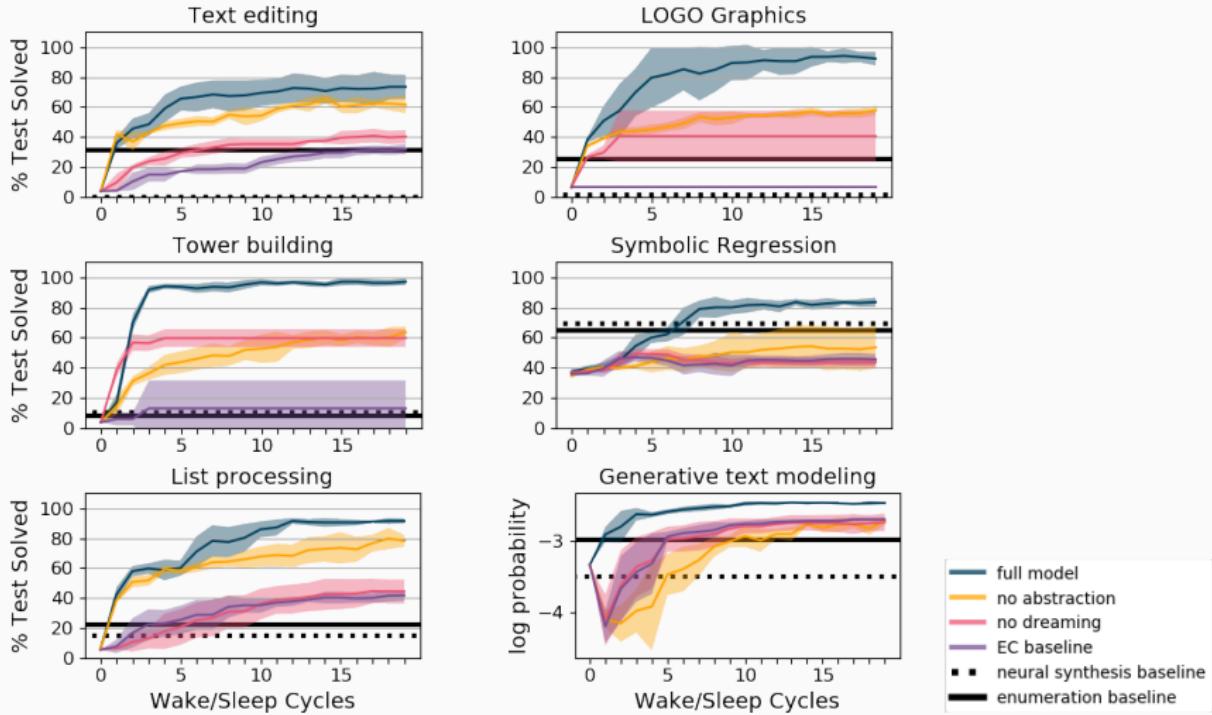


# Learning dynamics



baselines: Exploration-Compression, EC [Dechter et al. 2013]  
neural program synthesis, RobustFill [Devlin et al. 2017]  
24 hours of brute-force enumeration

# Learning dynamics



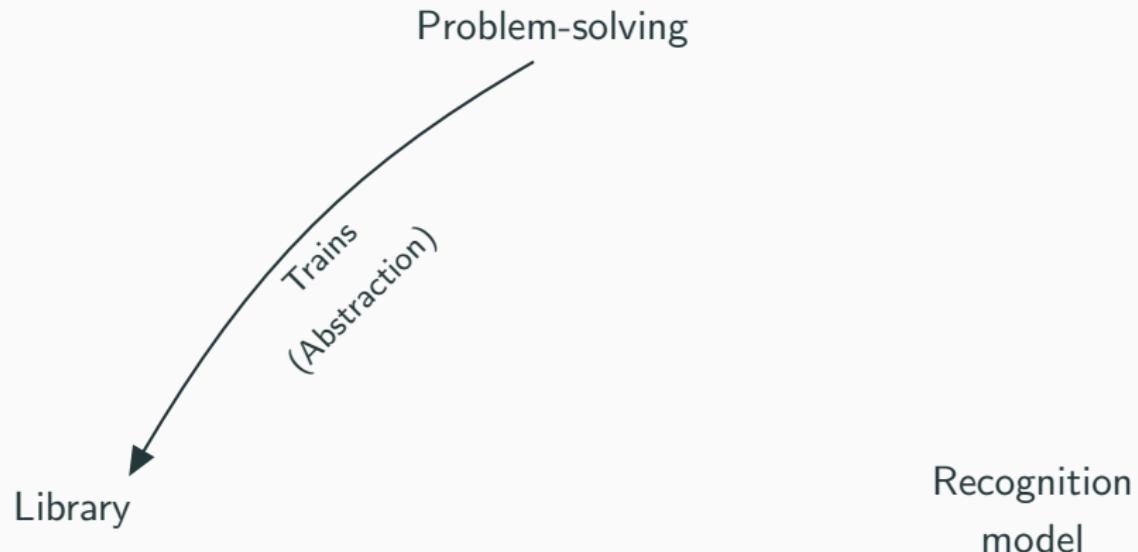
# Synergy between dreaming and library learning

Problem-solving

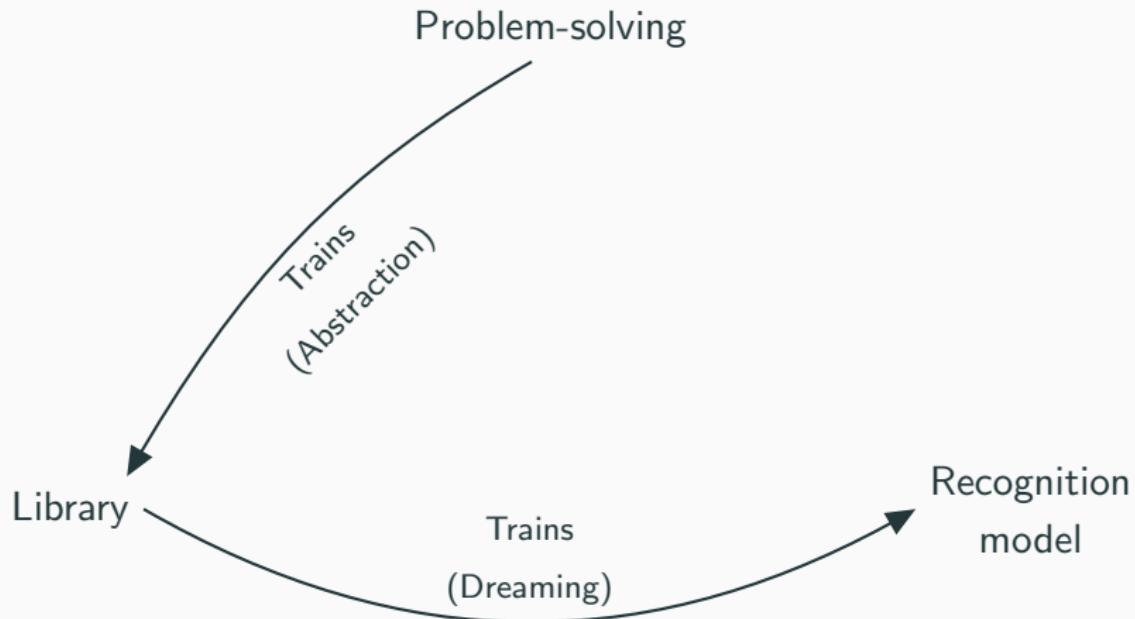
Library

Recognition  
model

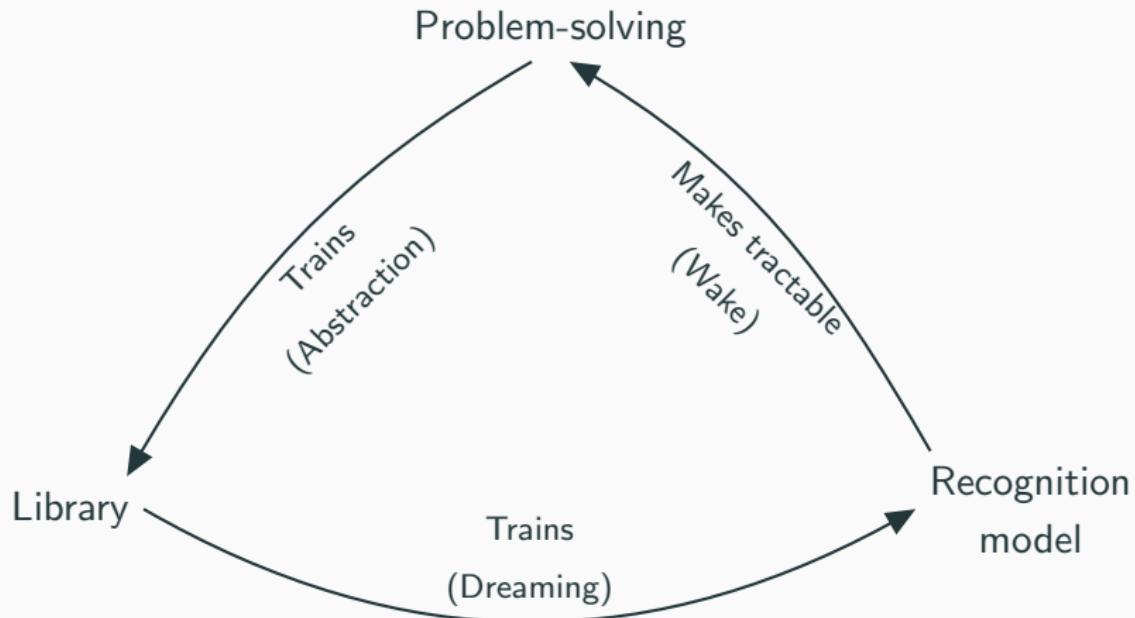
# Synergy between dreaming and library learning



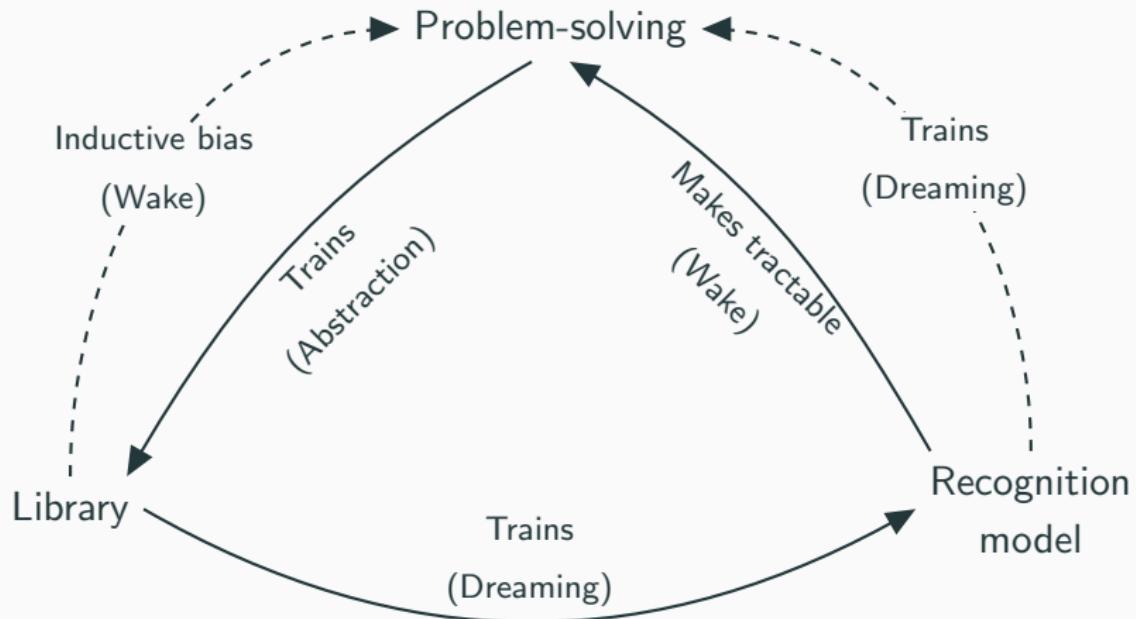
# Synergy between dreaming and library learning



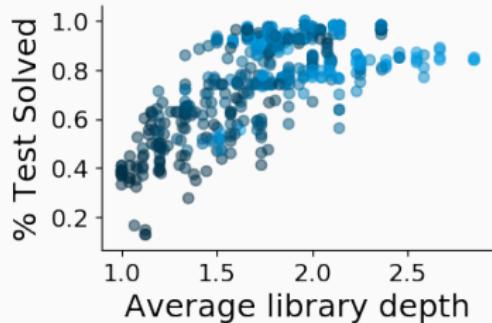
# Synergy between dreaming and library learning



# Synergy between dreaming and library learning



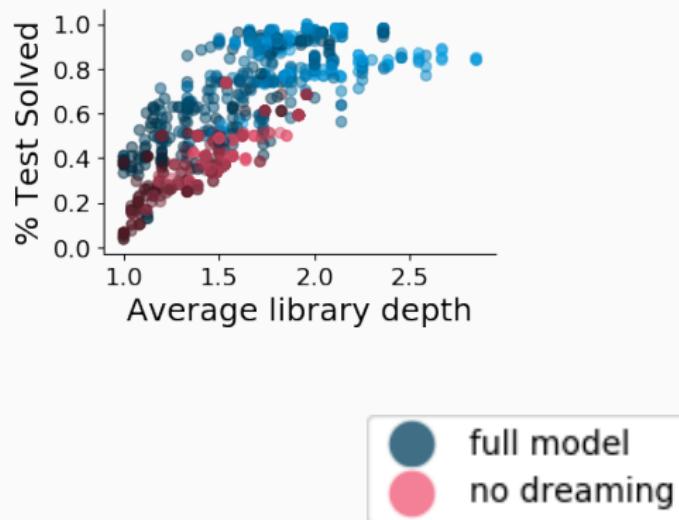
# Evidence for dreaming bootstrapping better libraries



Darker: Early in learning

Brighter: Later in learning

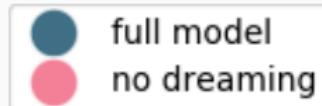
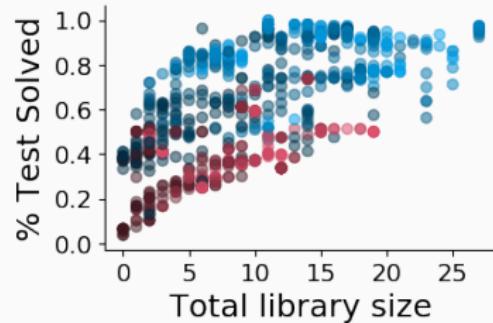
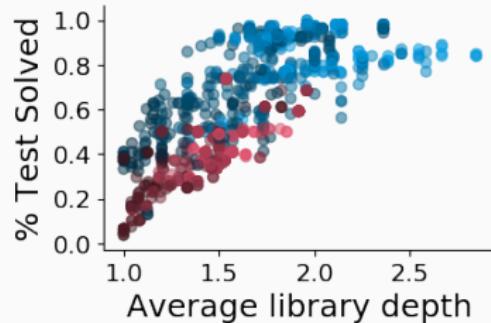
# Evidence for dreaming bootstrapping better libraries



Darker: Early in learning

Brighter: Later in learning

# Evidence for dreaming bootstrapping better libraries



Darker: Early in learning

Brighter: Later in learning

From learning libraries,  
to learning languages

From learning libraries,  
to learning languages

modern functional programming → physics

From learning libraries,  
to learning languages

1950's Lisp → modern functional programming → physics

# Physics Formula Sheet

## Mechanics

$x = x_0 + v_{x0}t + \frac{1}{2}a_xt^2$	$a_t = \frac{v^2}{r}$	$ \vec{F}_{\text{spring}}  = k x $
$v = v_0 + at$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$	$\text{PE}_{\text{spring}} = \frac{1}{2}kx^2$
$v_s^2 - v_{s0}^2 = 2a(x - x_0)$	$\omega = \omega_0 + \alpha t$	$T_{\text{spring}} = 2\pi\sqrt{\frac{m}{k}}$
$\bar{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{\text{net}}}{m}$	$T = \frac{2\pi}{\omega} = \frac{1}{f}$	$T_{\text{pendulum}} = 2\pi\sqrt{\frac{L}{g}}$
$ \vec{F}_{\text{friction}}  \leq \mu  \vec{F}_{\text{Normal}} $	$v = f\lambda$	
$\bar{p} = m\bar{v}$	$x = A\cos(2\pi ft)$	$ \vec{F}_{\text{gravity}}  = G \frac{m_1 m_2}{r^2}$
$\Delta \bar{p} = \vec{F} \Delta t$	$\bar{a} = \frac{\sum \vec{F}}{I} = \frac{\vec{F}_{\text{net}}}{I}$	$ \vec{F}_{\text{gravity}}  = m\bar{g}$
$KE = \frac{1}{2}mv^2$	$\vec{r} = r \times F$	$\text{PE}_{\text{gravity}} = -G \frac{m_1 m_2}{r}$
$\Delta PE = mg\Delta y$	$L = I\omega$	$p = \frac{m}{V}$
$\Delta E = W = Fd\cos\theta$	$\Delta L = \tau \Delta t$	$KE = \frac{1}{2}I\omega^2$

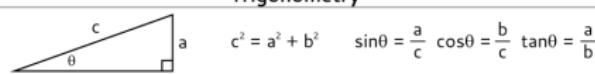
## Electricity

$ \vec{F}_E  = k \left  \frac{q_1 q_2}{r^2} \right $	$\Delta V = IR$	$R = \frac{\rho l}{A}$
$I = \frac{\Delta q}{\Delta t}$		$P = I\Delta V$
$R_{\text{series}} = R_1 + R_2 + \dots + R_n$	$\frac{1}{R_{\text{parallel}}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}$	

## Geometry

Rectangle	$A = bh$	Rectangular Solid	$V = lwh$	Triangle	$A = \frac{1}{2}bh$
Circle	$A = \pi r^2$	Cylinder	$V = \pi r^2 l$	Sphere	$V = \frac{4}{3}\pi r^3$
	$C = 2\pi r$		$S = 2\pi rl + 2\pi r^2$		$S = 4\pi r^2$

## Trigonometry



## Variables

a = acceleration  
 A = amplitude  
 A = Area  
 b = base length  
 C = circumference  
 d = distance  
 E = energy  
 f = frequency  
 F = force  
 h = height  
 I = current  
 I = rotational inertia  
 KE = kinetic energy  
 k = spring constant  
 L = angular momentum  
 l = length  
 m = mass  
 P = power  
 p = momentum  
 q = charge  
 r = radius  
 R = resistance  
 S = surface area  
 T = period  
 t = time  
 PE = potential energy  
 V = electric potential  
 V = volume  
 v = velocity  
 w = width  
 W = work  
 x = position  
 y = height  
 $\alpha$  = angular acceleration  
 $\lambda$  = wavelength  
 $\mu$  = coefficient of friction

# Growing languages for vector algebra and physics

## Initial Primitives

map  
zip

cons

empty

cdr

power

fold

car

+

-

\*

/

0

1

$\pi$

## Physics Equations

### Newton's Second Law

$$\vec{a} = \frac{1}{m} \sum_l \vec{F}_l$$

### Parallel Resistors

$$R_{total} = \left( \sum_i \frac{1}{R_i} \right)^{-1}$$

### Work

$$U = \vec{F} \cdot \vec{d}$$

### Force in a Magnetic Field

$$|\vec{F}| = q |\vec{v} \times \vec{B}|$$

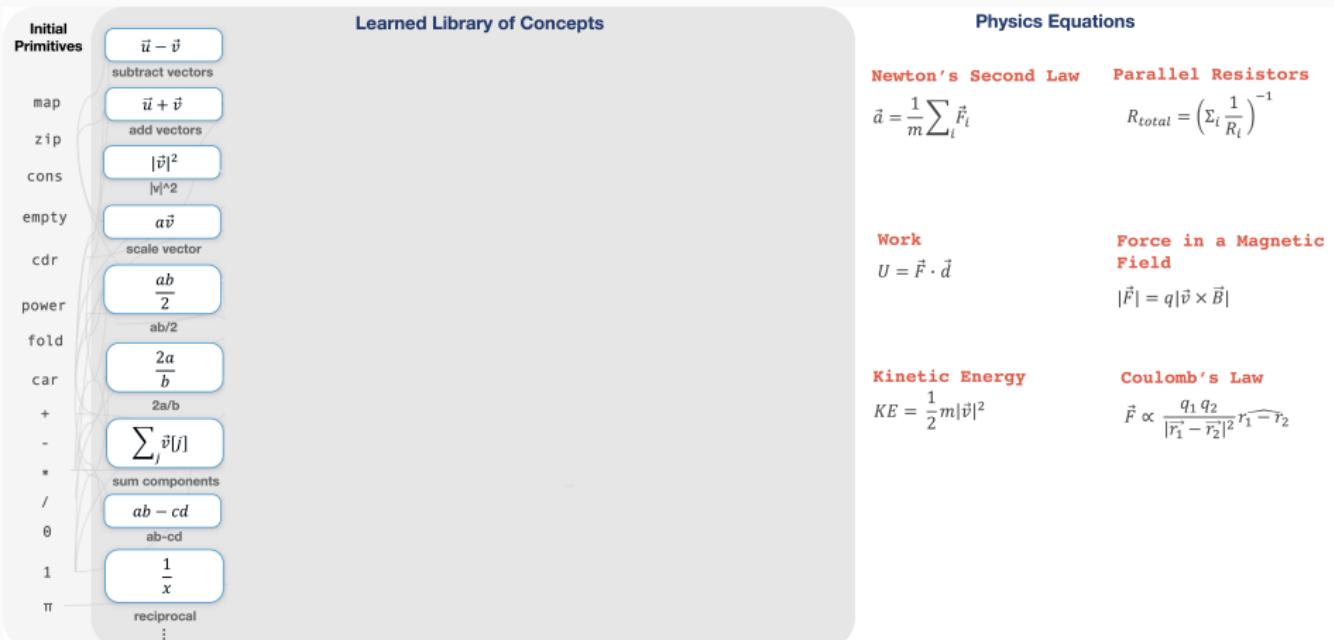
### Kinetic Energy

$$KE = \frac{1}{2} m |\vec{v}|^2$$

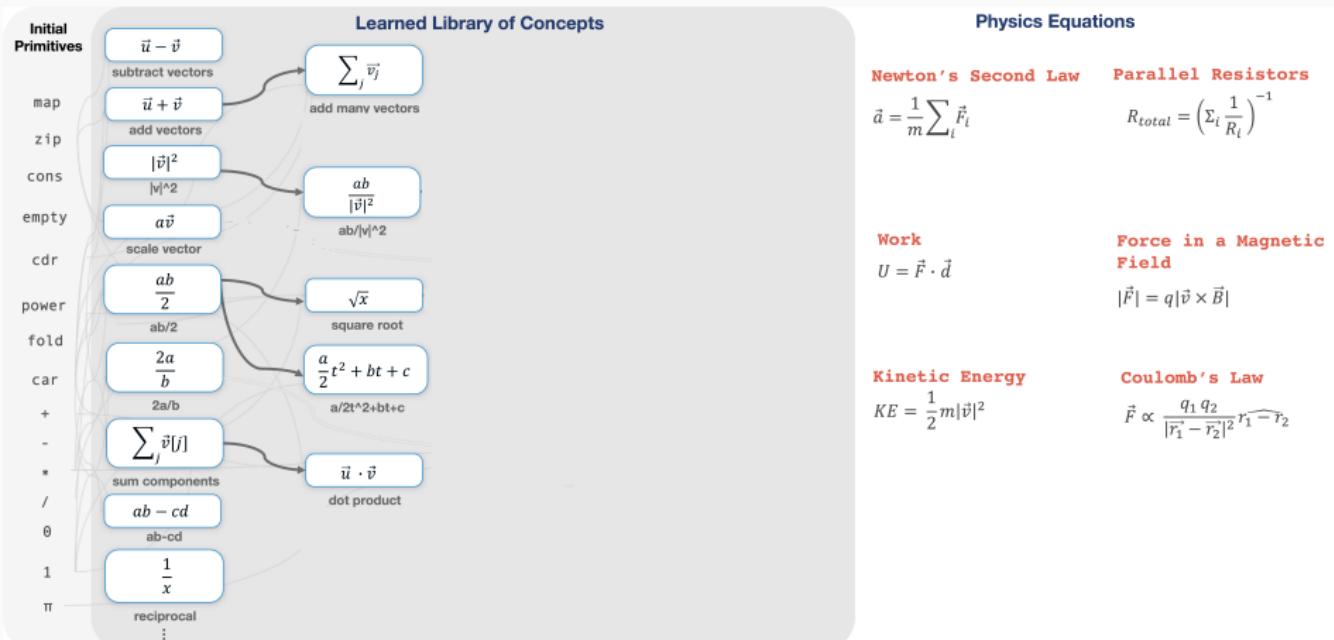
### Coulomb's Law

$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^2} \hat{r}_1 - \hat{r}_2$$

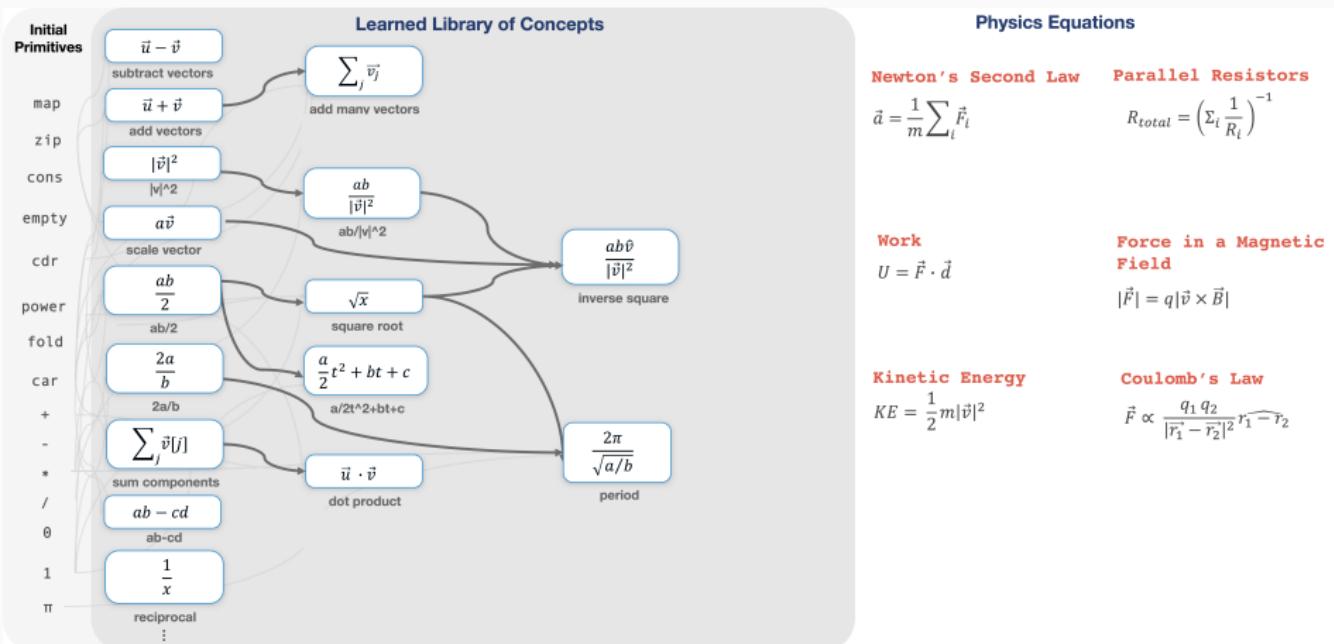
# Growing languages for vector algebra and physics



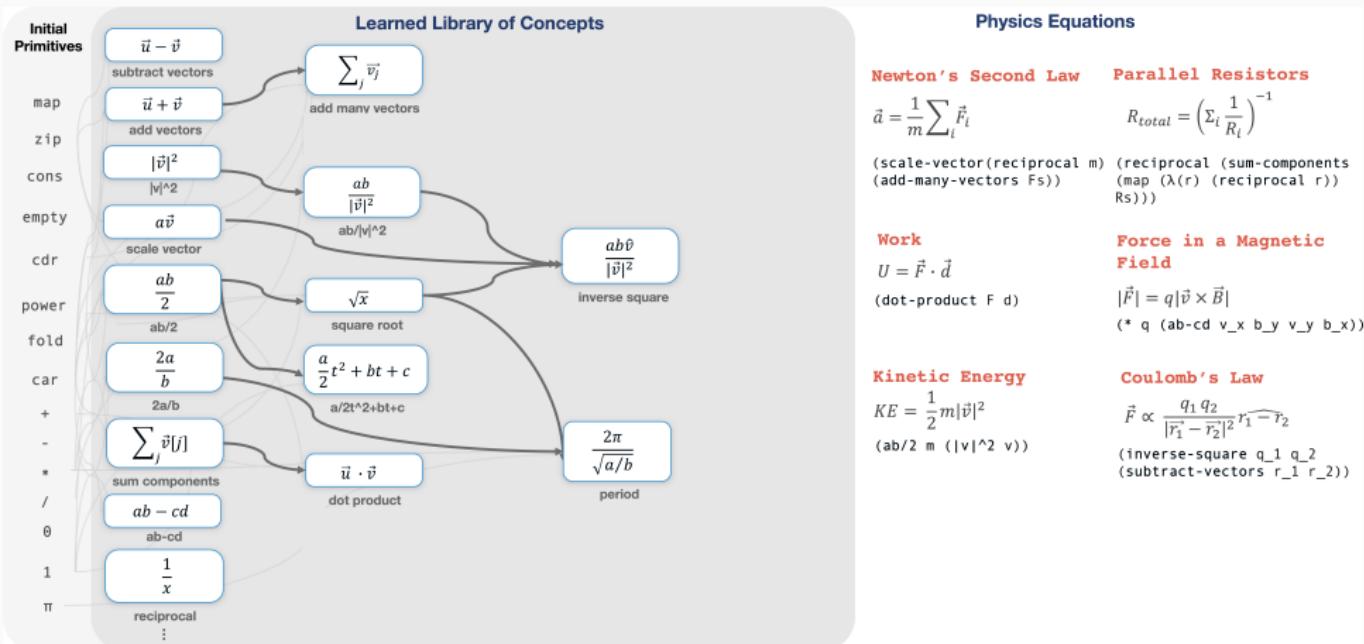
# Growing languages for vector algebra and physics



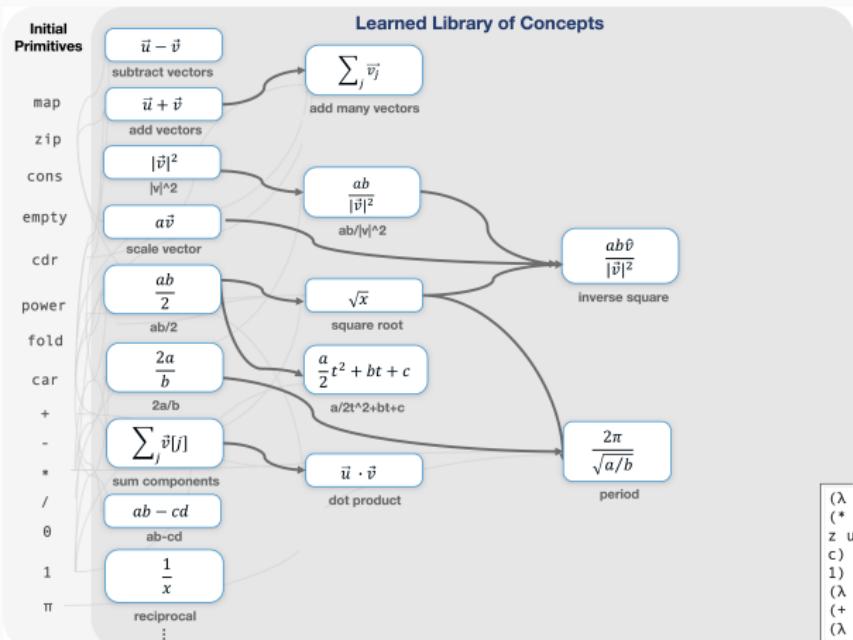
# Growing languages for vector algebra and physics



# Growing languages for vector algebra and physics



# Growing languages for vector algebra and physics



Physics Equations	
<b>Newton's Second Law</b>	<b>Parallel Resistors</b>
$\vec{d} = \frac{1}{m} \sum_i \vec{F}_i$	$R_{total} = \left( \sum_i \frac{1}{R_i} \right)^{-1}$
(scale-vector(reciprocal m) (add-many-vectors Fs))	(reciprocal (sum-components (map (lambda(r) (reciprocal r)) Rs)))
<b>Work</b>	<b>Force in a Magnetic Field</b>
$U = \vec{F} \cdot \vec{d}$	$ \vec{F}  = q  \vec{v} \times \vec{B} $
(dot-product F d)	(* q (ab-cd v_x b_y v_y b_x))
<b>Kinetic Energy</b>	<b>Coulomb's Law</b>
$KE = \frac{1}{2} m  \vec{v} ^2$	$\vec{F} \propto \frac{q_1 q_2}{ \vec{r}_1 - \vec{r}_2 ^2} \hat{r}_1 - \vec{r}_2$
(ab/2 m ( v ^2 v))	(inverse-square (q_1 q_2 (subtract-vectors r_1 r_2)))
(x y z u) (map (lambda(v) (* (/ (power (/ (* x x) (fold (zip (lambda(w a) (- w a))) theta (lambda(b (+* b b) c)))) (/ (* 1 (+* 1 1)) y) (fold (zip z u (d e) (- d e))) theta (lambda(f g) (+* f f) g)))) v)) (zip z u (h i) (- h i))))	Solution to Coulomb's Law if expressed in initial primitives

# Growing a language for recursive programming

## Initial Primitives

Y  
combinator  
cons  
car  
cdr  
nil  
if  
nil?  
+  
-  
0  
1  
=

## Recursive Programming Algorithms

### Stutter

[ ] → [ ]  
[ ] → [ ]

### Take every other

[ ] → [ ]  
[ ] → [ ]

### List lengths

[ , []] → [3 1]  
[[ ], [], []] → [2 0 1]

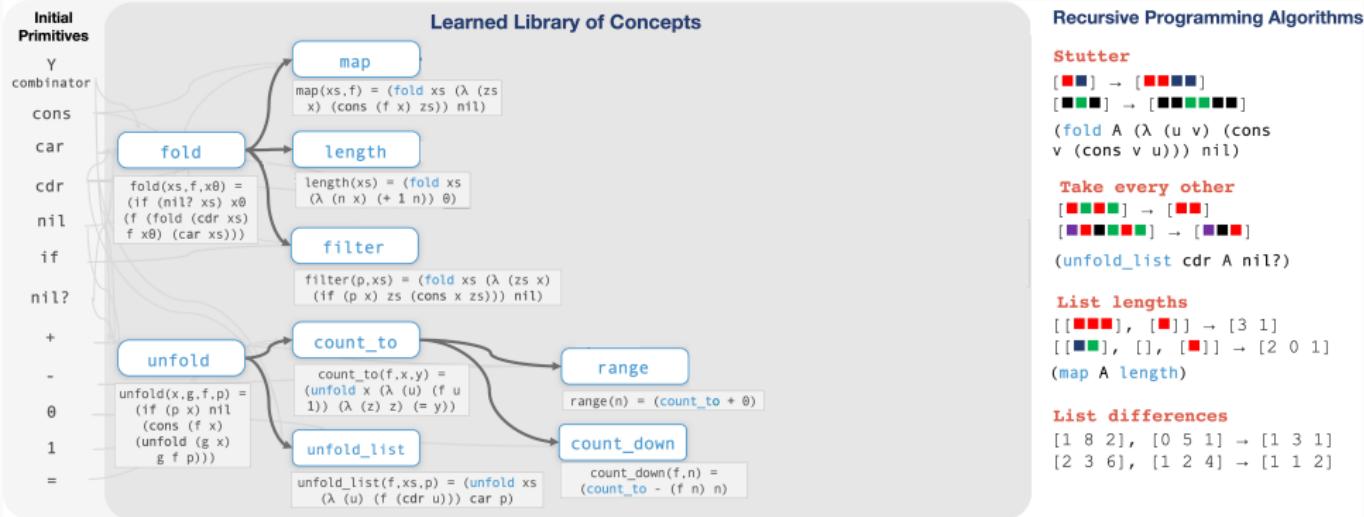
### List differences

[1 8 2], [0 5 1] → [1 3 1]  
[2 3 6], [1 2 4] → [1 1 2]

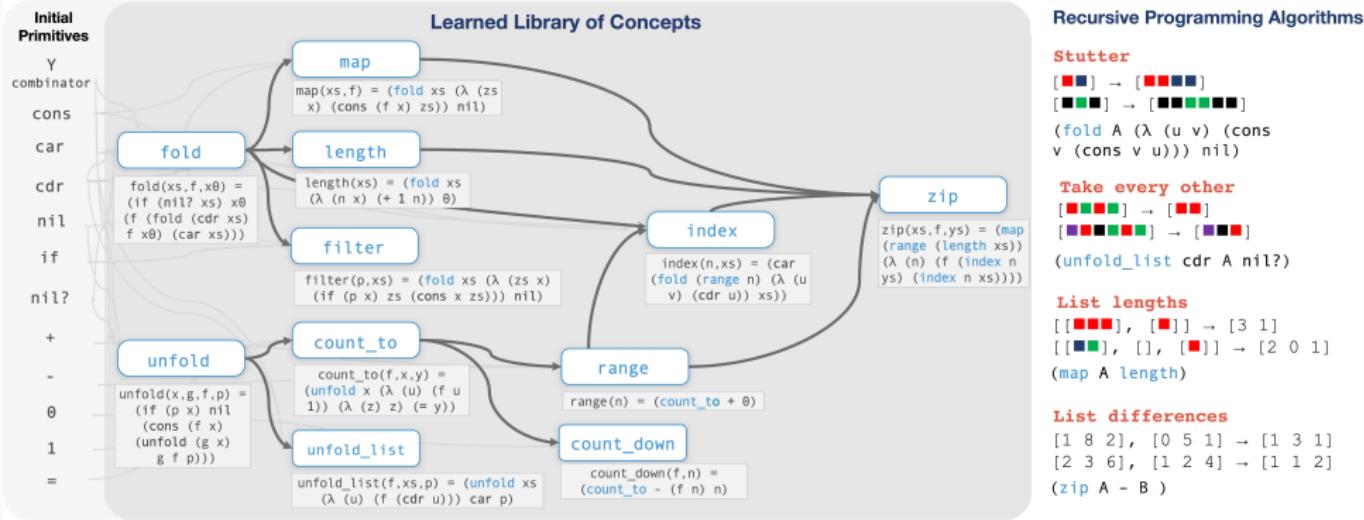
# Growing a language for recursive programming



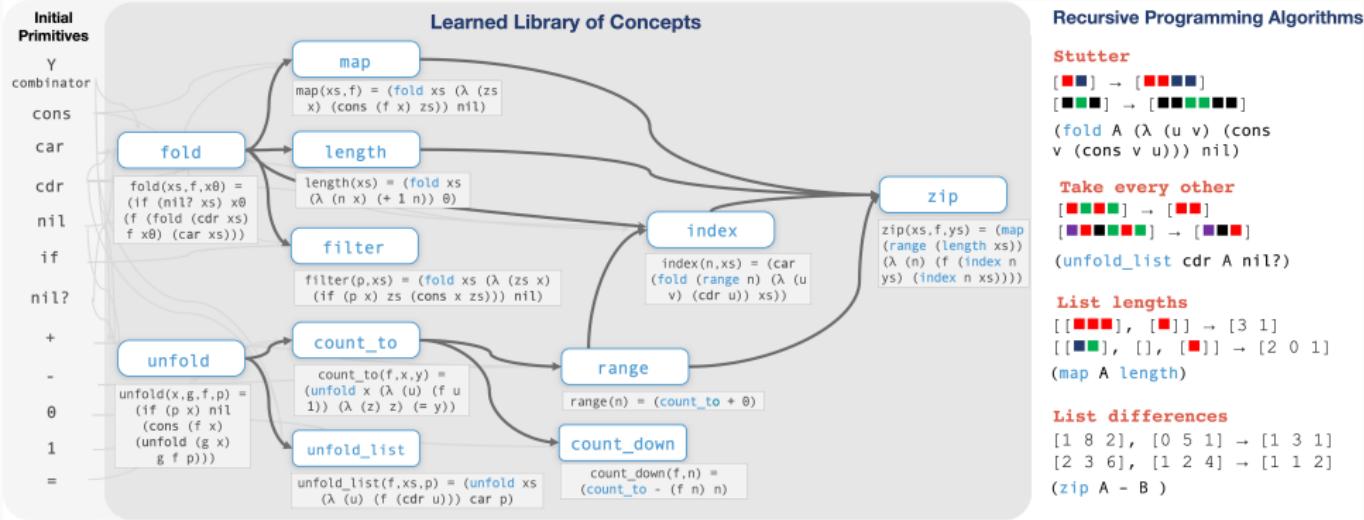
# Growing a language for recursive programming



# Growing a language for recursive programming

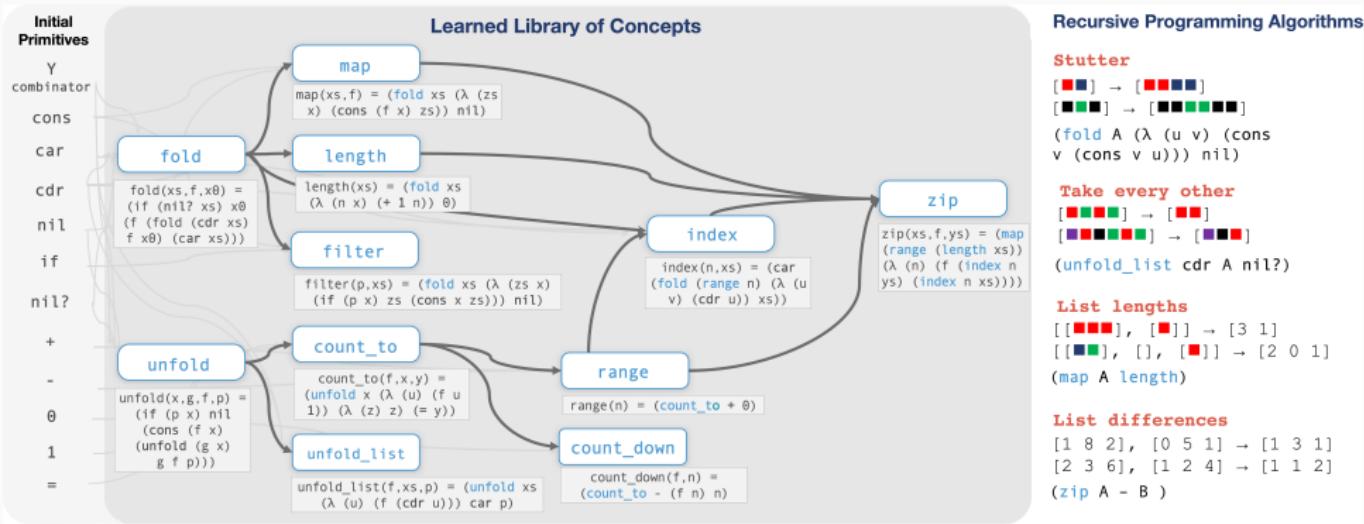


# Growing a language for recursive programming



Origami Programming: Jeremy Gibbons, 2003

# Growing a language for recursive programming



1 year of compute. 5 days on 64 CPUs.



Origami Programming: Jeremy Gibbons, 2003

## Lessons

Symbols aren't necessarily interpretable. Flexibly grow the language based on experience to make it more powerful *and* more human understandable

Learning-from-scratch is possible in principle. Don't do it. But program induction makes it convenient to build in what we know how to build in, and then learn and adapt on top of that

Program Induction and learning to learn  
learning a DSL  
**learning to synthesize**  
synergy between DSL+learned synthesizer

## Some unresolved questions

Not everything is crisp and symbolic. How do we learn DSLs for neurosymbolic hybrid programs?  
(see Memoised Wake-Sleep, Hewitt et al 2020)

## Some unresolved questions

Not everything is crisp and symbolic. How do we learn DSLs for neurosymbolic hybrid programs?  
(see Memoised Wake-Sleep, Hewitt et al 2020)

Search is still hard  
(see “REPL”: Ellis, Nye, Pu, Sosa et al 2019)

## Some unresolved questions

Not everything is crisp and symbolic. How do we learn DSLs for neurosymbolic hybrid programs?  
(see Memoised Wake-Sleep, Hewitt et al 2020)

Search is still hard  
(see “REPL”: Ellis, Nye, Pu, Sosa et al 2019)

Library $\neq$ Language:  
How do we learn data structures, or discover new types, and thereby *actually* learn DSLs?

# Collaborators

Josh  
Tenenbaum



Armando  
Solar-Lezama



Max Nye



Cathy Wong



Mathias Sable-Meyer

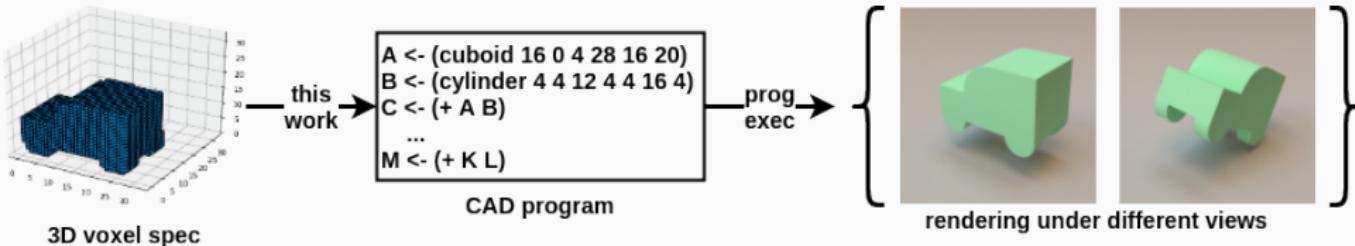


Lucas Morales



thank  
you

# 3D program induction

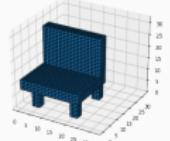


Challenge: combinatorial search!

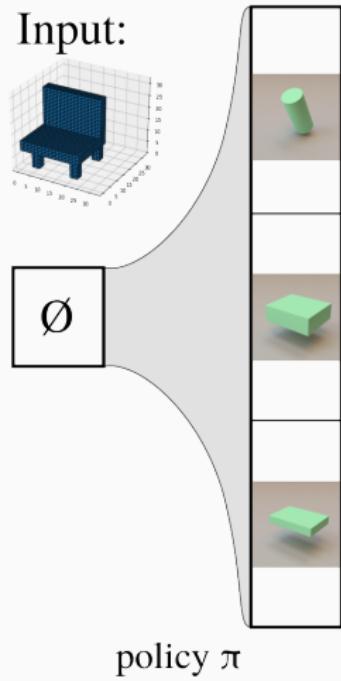
Branching factor:  $> 1.3$  million per line of code,  $\approx 20$  lines of code  
search space size:  $(1.3 \text{ million})^{20} \approx 10^{122}$  programs

Solution: stochastic **tree search** + learn **policy** that writes code  
+ learn **value** function that assesses execution of program so far;  
analogous to **AlphaGo** [Silver et al. 2016]

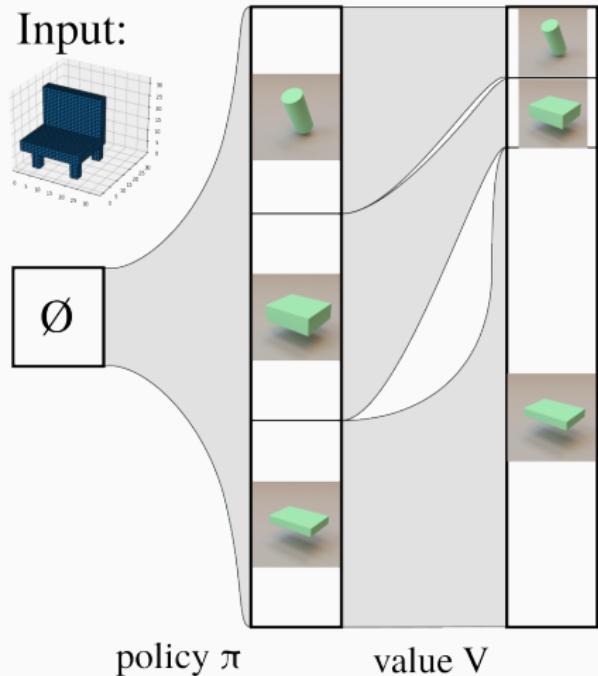
Input:



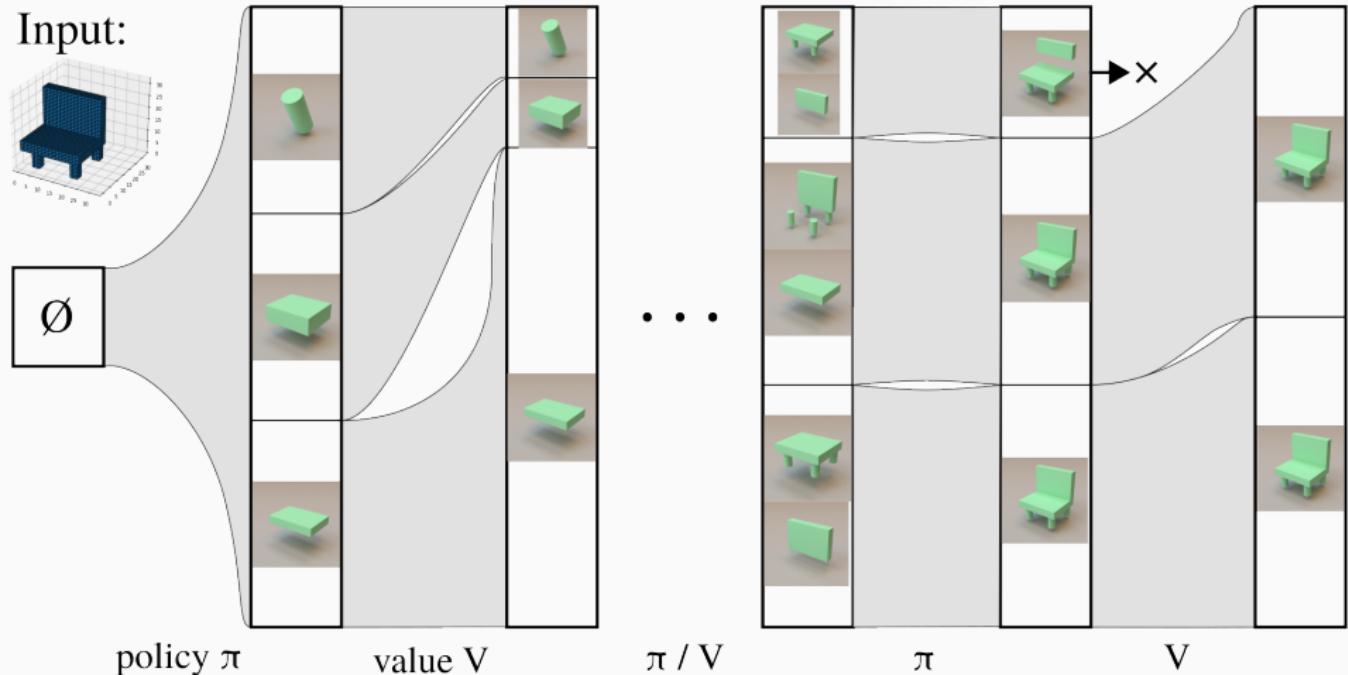
Solution: stochastic **tree search** + learn **policy** that writes code  
+ learn **value** function that assesses execution of program so far;  
analogous to **AlphaGo** [Silver et al. 2016]



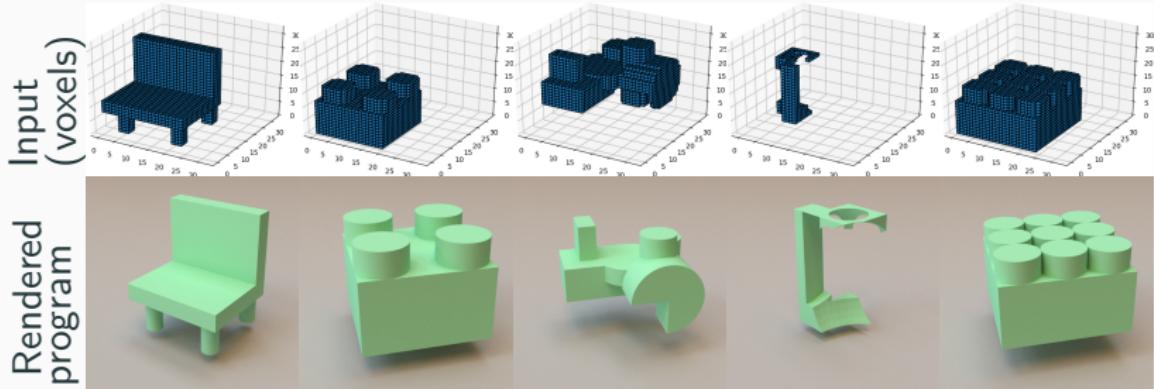
Solution: stochastic **tree search** + learn **policy** that writes code  
+ learn **value** function that assesses execution of program so far;  
analogous to **AlphaGo** [Silver et al. 2016]



Solution: stochastic **tree search** + learn **policy** that writes code  
+ learn **value** function that assesses execution of program so far;  
analogous to **AlphaGo** [Silver et al. 2016]



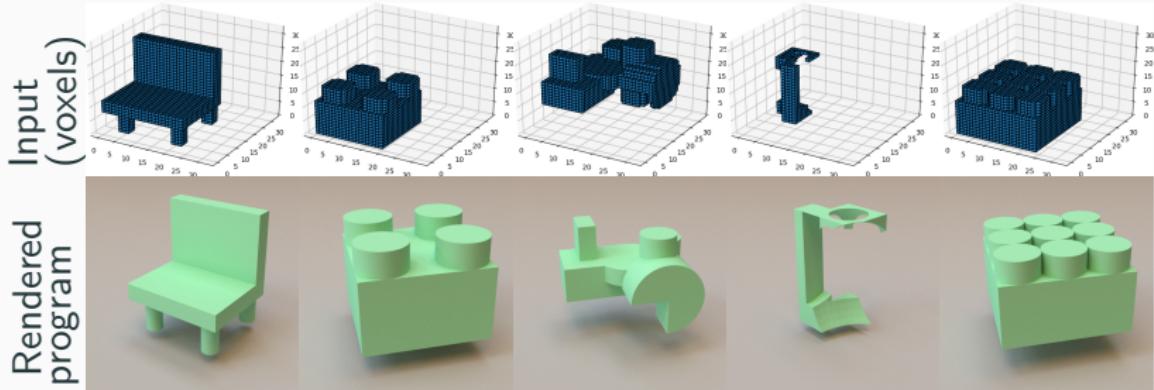
# 3D program induction



Ellis\*, Nye\*, Pu\*, Sosa\*, Tenenbaum, Solar-Lezama. NeurIPS 2019.

\*equal contribution

# 3D program induction



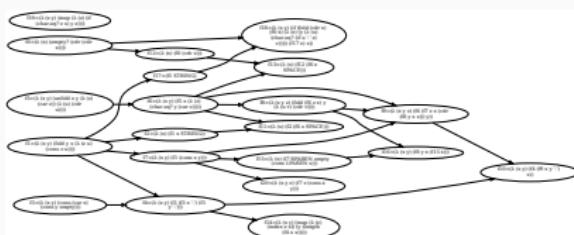
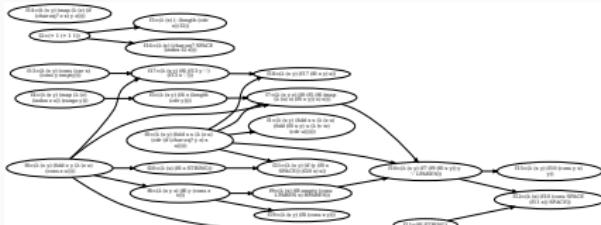
same architecture learns to synthesize text editing programs  
(FlashFill, Gulwani 2012)

Ellis\*, Nye\*, Pu\*, Sosa\*, Tenenbaum, Solar-Lezama. NeurIPS 2019.

\*equal contribution

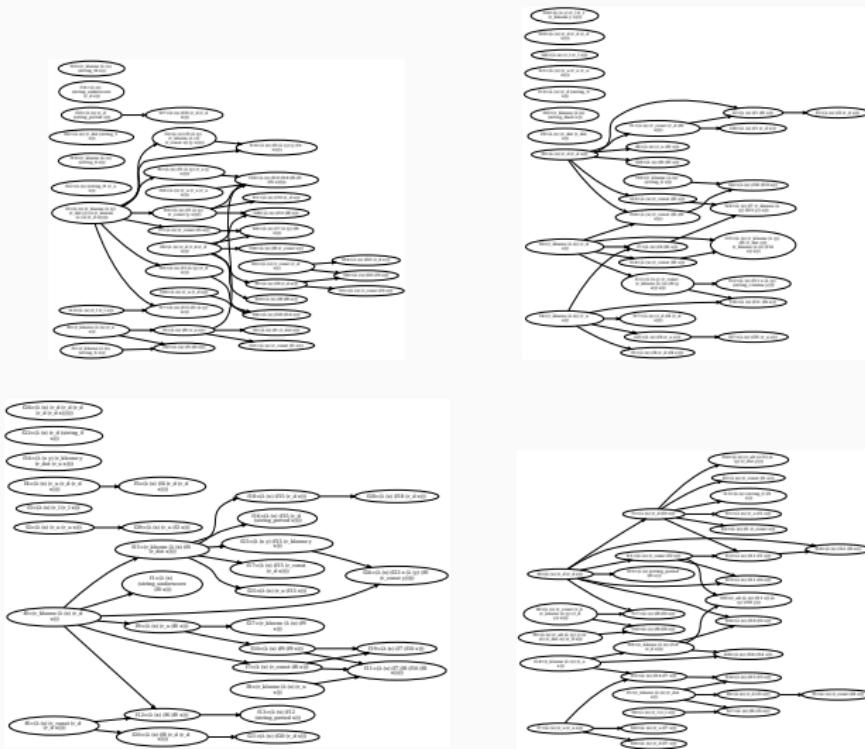
# Library structure: Text Editing

DreamCoder learns libraries for FlashFill-style text editing [Gulwani 2012]

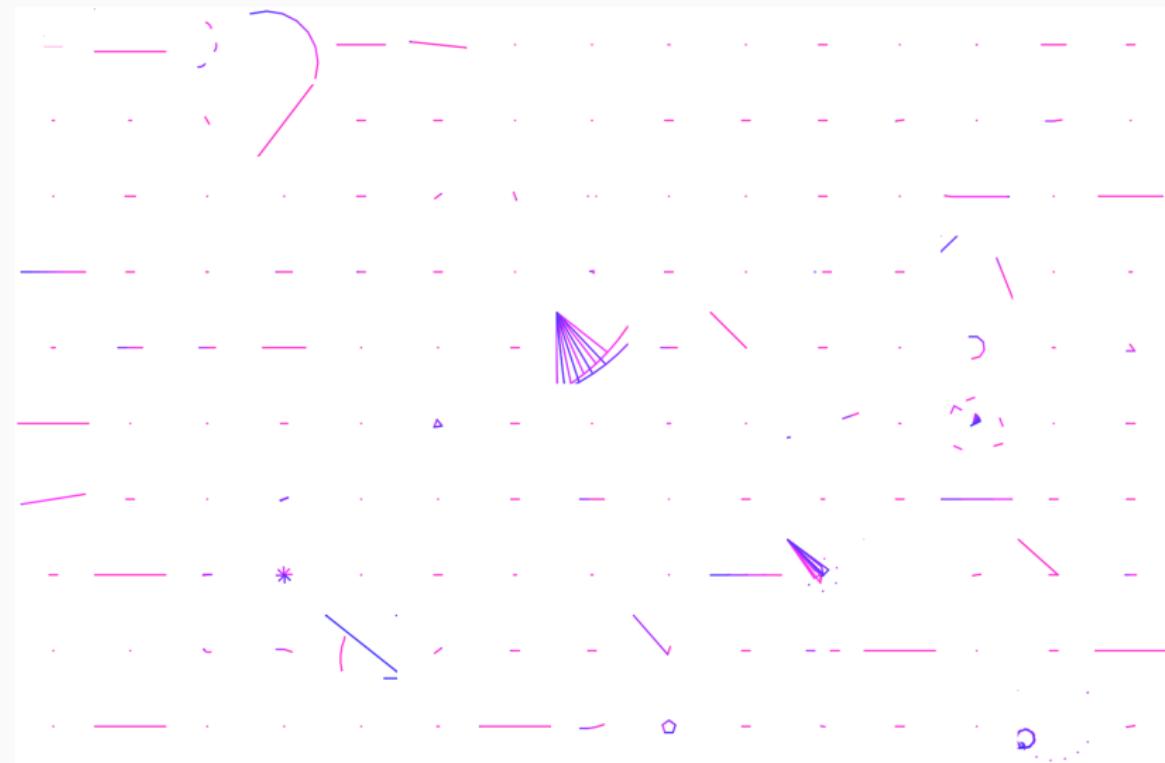


# Library structure: Generating Text

Libraries for probabilistic generative models over text:  
data from crawling web for CSV files



# 150 random dreams before learning



# 150 random dreams after learning

