

Program Induction: Bridging AI and program synthesis

Kevin Ellis

2020

MIT

What computational problems are solved by intelligence?

an endless range of problems

language



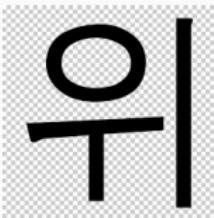
using new devices



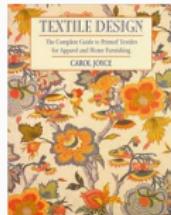
science



writing new characters



design



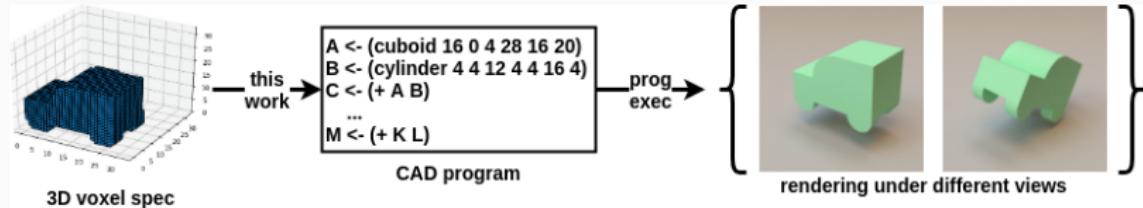
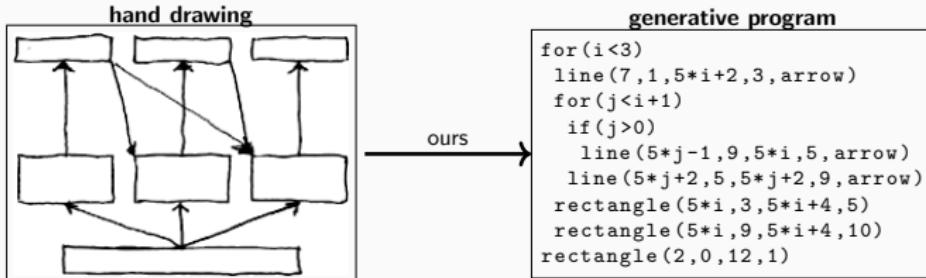
coding

```
(MEMBER  
(LAMBDA (X L)  
(COND ((NULL L) NIL)  
      ((EQ X (FIRST L)) T)  
      (T (MEMBER X (REST L)))))))
```

Allen, Anatomy of Lisp, 1975

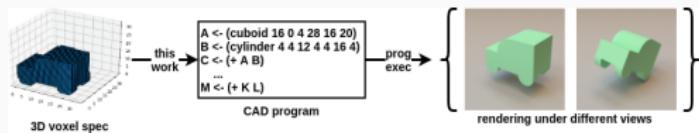
Perception, Learning-to-Learn, Synthesizing models

Theme #1: high-level scene understanding, pixels→programs



Perception, Learning-to-Learn, Synthesizing models

Theme #1: high-level scene understanding, pixels→programs



Theme #2: Learning to synthesize programs

List Processing

Sum List

$$[1 \ 2 \ 3] \rightarrow 6$$

$$[4 \ 6 \ 8 \ 1] \rightarrow 17$$

Double

$$[1 \ 2 \ 3 \ 4] \rightarrow [2 \ 4 \ 6 \ 8]$$

$$[6 \ 5 \ 1] \rightarrow [12 \ 10 \ 2]$$

Check Evens

$$[0 \ 2 \ 3] \rightarrow [T \ T \ F]$$

$$[2 \ 4 \ 9 \ 6] \rightarrow [T \ T \ F \ T]$$

Text Editing

Abbreviate

$$\text{Allen Newell} \rightarrow \text{A.N.}$$

$$\text{Herb Simon} \rightarrow \text{H.S.}$$

Drop Last Characters

$$\text{jabberwocky} \rightarrow \text{jabberw}$$

$$\text{copycat} \rightarrow \text{cop}$$

Regexes

Phone Numbers

$$(555) \ 867-5309$$

$$(650) \ 555-2368$$

LOGO Graphics

Currency

$$\$100.25$$

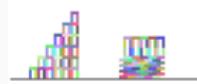
$$\$4.50$$

Dates

$$Y1775/0704$$

$$Y2000/0101$$

Block Towers



Symbolic Regression



Recursive Programming

Filter

$$[\textcolor{red}{\blacksquare} \textcolor{blue}{\blacksquare} \textcolor{red}{\blacksquare}] \rightarrow [\textcolor{blue}{\blacksquare} \textcolor{blue}{\blacksquare}]$$

$$[\textcolor{red}{\blacksquare} \textcolor{red}{\blacksquare} \textcolor{black}{\blacksquare} \textcolor{black}{\blacksquare}] \rightarrow [\textcolor{black}{\blacksquare} \textcolor{black}{\blacksquare} \textcolor{black}{\blacksquare}]$$

$$[\textcolor{red}{\blacksquare} \textcolor{black}{\blacksquare} \textcolor{black}{\blacksquare} \textcolor{red}{\blacksquare}] \rightarrow [\textcolor{black}{\blacksquare} \textcolor{black}{\blacksquare}]$$

Index List

$$0, [\textcolor{red}{\blacksquare} \textcolor{blue}{\blacksquare} \textcolor{red}{\blacksquare} \textcolor{blue}{\blacksquare}] \rightarrow \textcolor{red}{\blacksquare}$$

$$1, [\textcolor{red}{\blacksquare} \textcolor{blue}{\blacksquare} \textcolor{red}{\blacksquare} \textcolor{blue}{\blacksquare}] \rightarrow \textcolor{blue}{\blacksquare}$$

$$1, [\textcolor{red}{\blacksquare} \textcolor{blue}{\blacksquare} \textcolor{red}{\blacksquare} \textcolor{blue}{\blacksquare}] \rightarrow \textcolor{blue}{\blacksquare}$$

Physics

$$KE = \frac{1}{2} m |\vec{v}|^2$$

$$\vec{d} = \frac{1}{m} \sum_i \vec{F}_i$$

Length

$$[\textcolor{red}{\blacksquare} \textcolor{blue}{\blacksquare} \textcolor{red}{\blacksquare}] \rightarrow 4$$

$$[\textcolor{red}{\blacksquare} \textcolor{blue}{\blacksquare} \textcolor{black}{\blacksquare} \textcolor{black}{\blacksquare}] \rightarrow 6$$

$$[\textcolor{red}{\blacksquare} \textcolor{black}{\blacksquare}] \rightarrow 3$$

Every Other

$$[\textcolor{red}{\blacksquare} \textcolor{blue}{\blacksquare} \textcolor{red}{\blacksquare}] \rightarrow [\textcolor{red}{\blacksquare}]$$

$$[\textcolor{red}{\blacksquare} \textcolor{blue}{\blacksquare} \textcolor{black}{\blacksquare} \textcolor{black}{\blacksquare}] \rightarrow [\textcolor{black}{\blacksquare} \textcolor{black}{\blacksquare}]$$

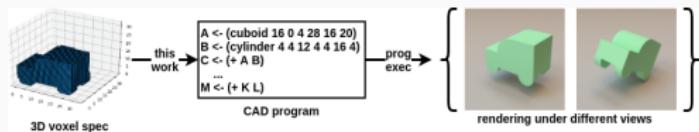
$$[\textcolor{red}{\blacksquare} \textcolor{black}{\blacksquare} \textcolor{black}{\blacksquare} \textcolor{red}{\blacksquare}] \rightarrow [\textcolor{black}{\blacksquare} \textcolor{black}{\blacksquare}]$$

$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^2} \vec{r}_1 \cdot \vec{r}_2$$

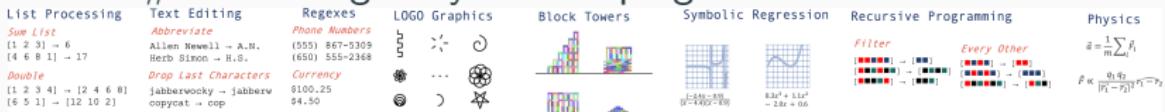
$$R_{total} = \left(\sum_i \frac{1}{R_i} \right)^{-1}$$

Perception, Learning-to-Learn, Synthesizing models

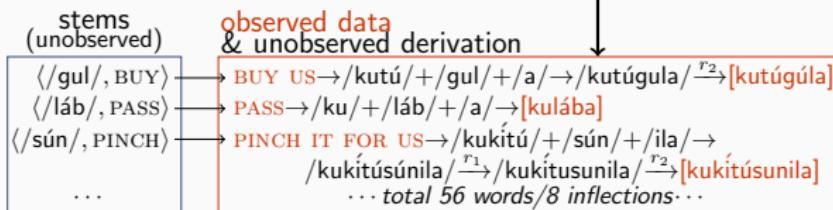
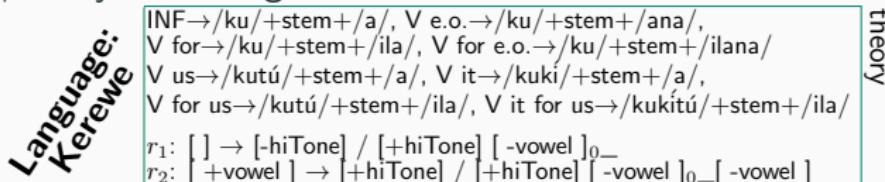
Theme #1: high-level scene understanding, pixels→programs



Theme #2: Learning to synthesize programs



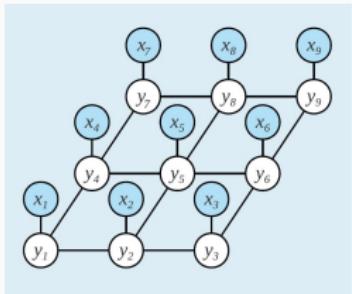
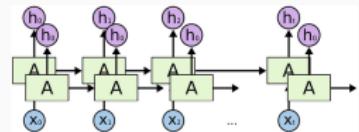
Theme #3: Synthesizing human-understandable models



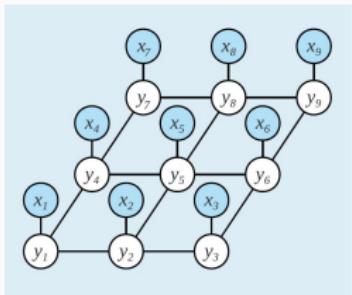
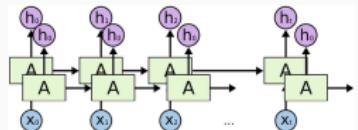
High-level, abstract visual abilities



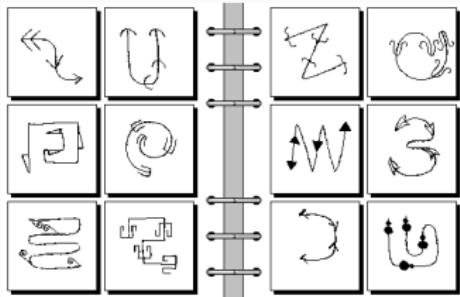
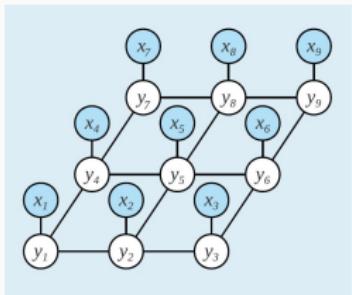
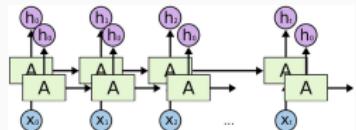
High-level, abstract visual abilities



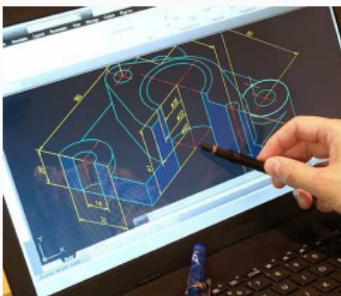
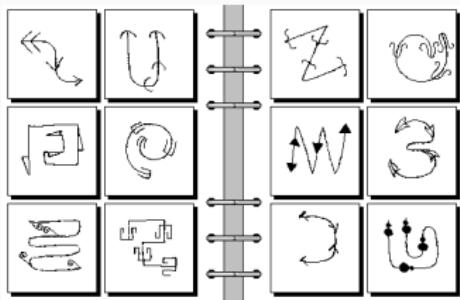
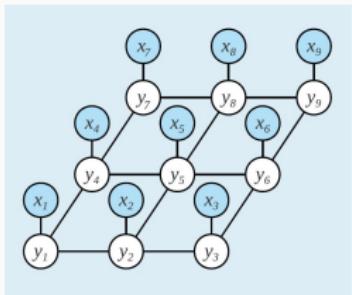
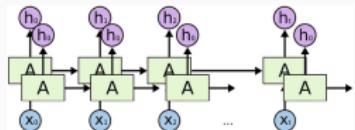
High-level, abstract visual abilities



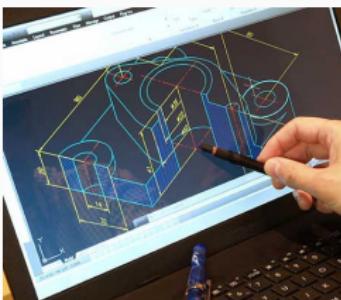
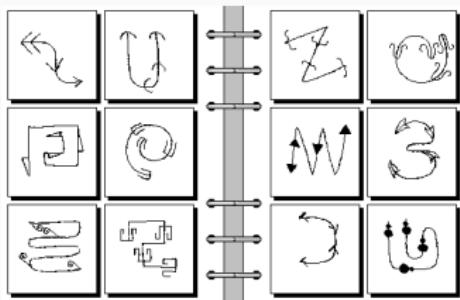
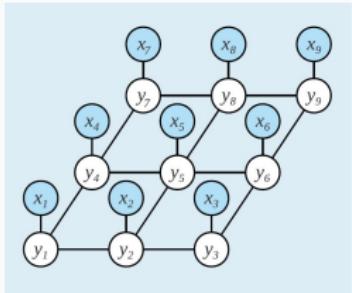
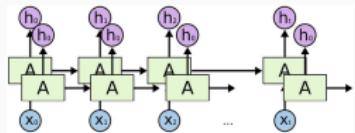
High-level, abstract visual abilities



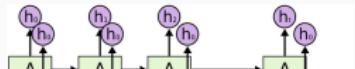
High-level, abstract visual abilities



High-level, abstract visual abilities



High-level, abstract visual abilities

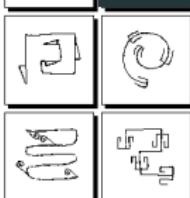


why?

impute missing objects, extrapolate percepts,
learn visual concepts ('arch', 'spiral', HMM),
assist graphic design, assist 3D modeling

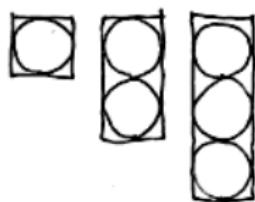
how?

Bayesian inference of graphics program conditioned on image,
+program synthesis
+learning



Learning to infer graphics programs from hand-drawn images

model infers program from drawing



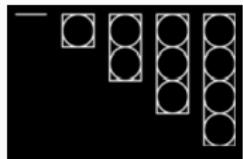
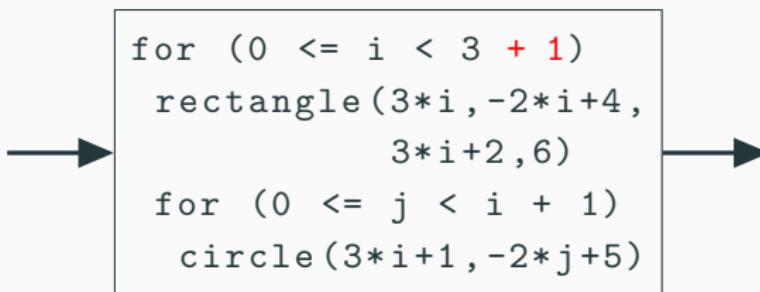
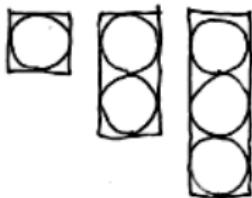
→

```
for (0 <= i < 3)
    rectangle(3*i, -2*i+4,
              3*i+2, 6)
    for (0 <= j < i + 1)
        circle(3*i+1, -2*j+5)
```

Learning to infer graphics programs from hand-drawn images

model infers program from drawing

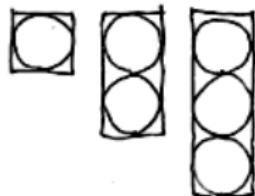
zero-shot generalization / extrapolation



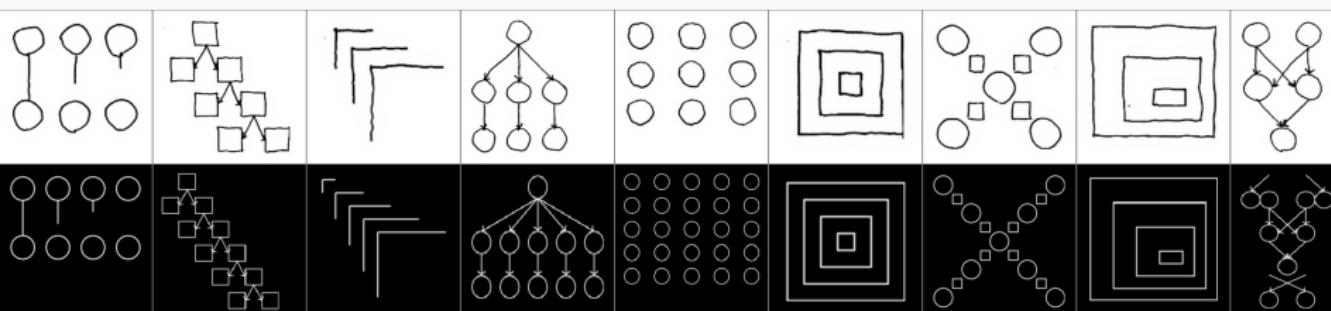
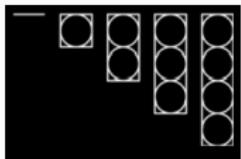
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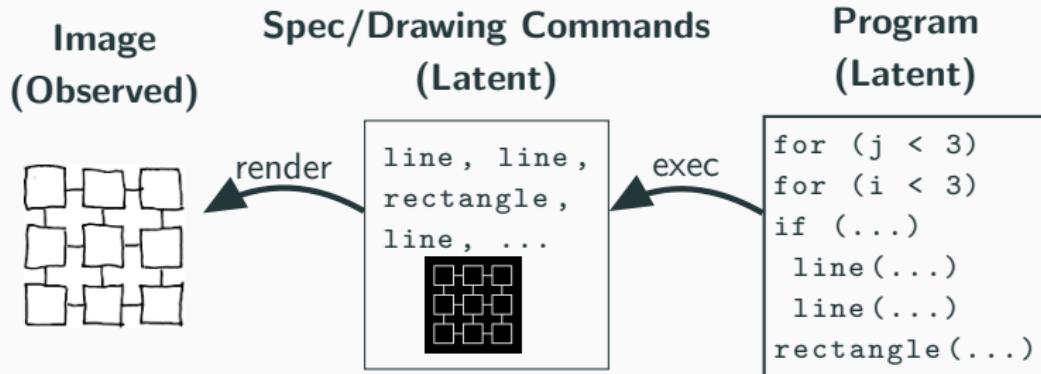
zero-shot generalization / extrapolation



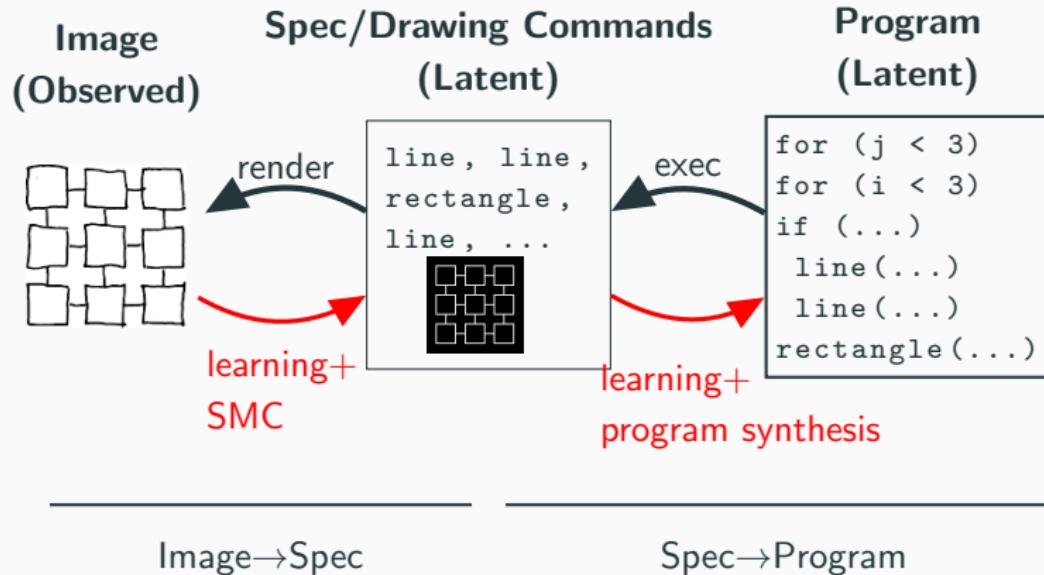
```
for (0 <= i < 3 + 1)
    rectangle(3*i, -2*i+4,
              3*i+2, 6)
    for (0 <= j < i + 1)
        circle(3*i+1, -2*j+5)
```



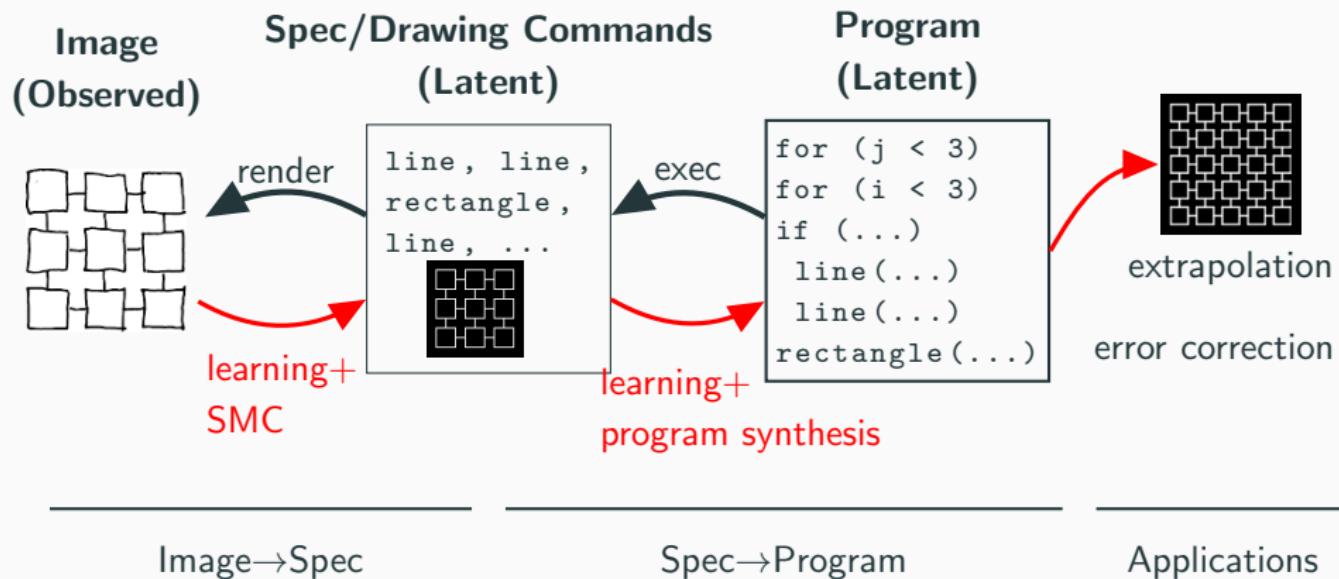
How to infer graphics programs from hand-drawn images



How to infer graphics programs from hand-drawn images



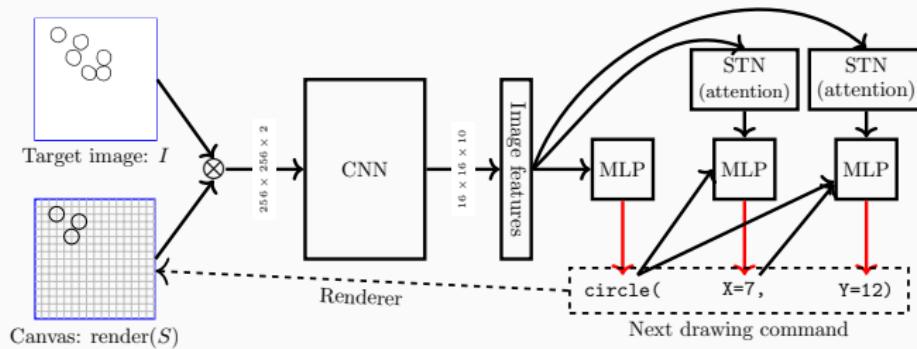
How to infer graphics programs from hand-drawn images



Parsing images into specs (\LaTeX TikZ commands)

Neurally Guided Procedural Modeling (Ritchie et al 2016) +

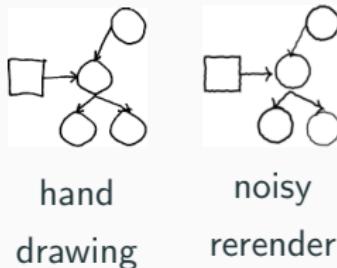
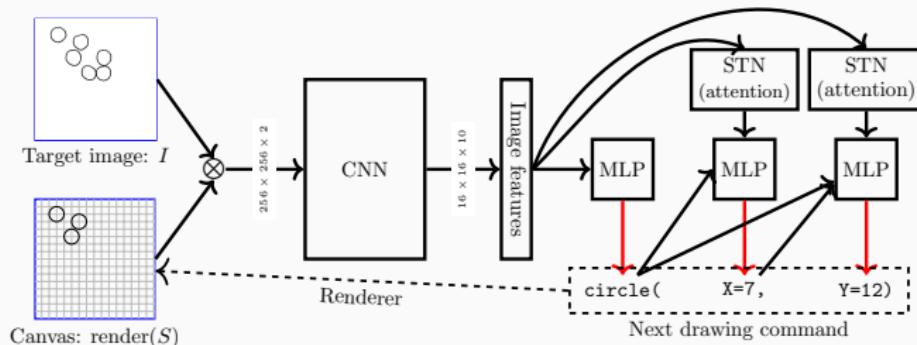
Attend, Infer, Repeat (Eslami et al 2016)



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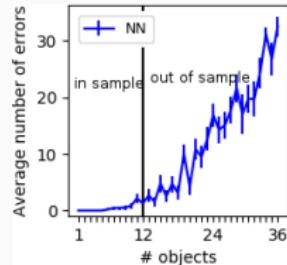
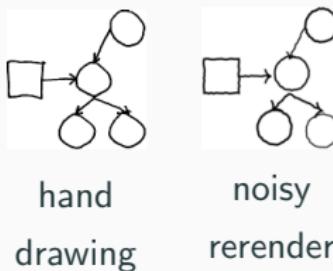
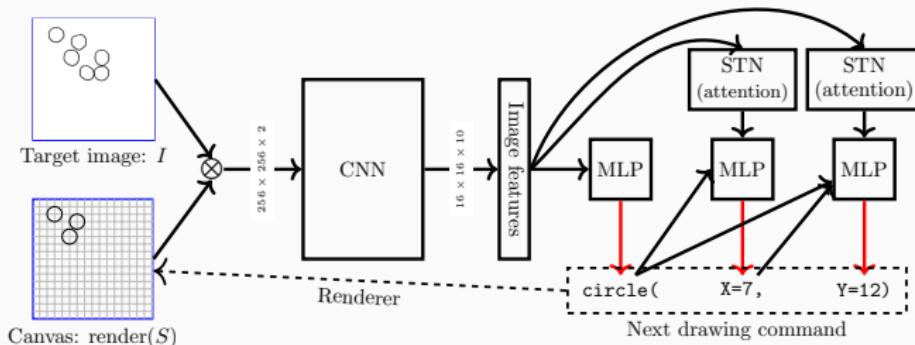
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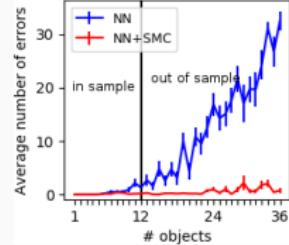
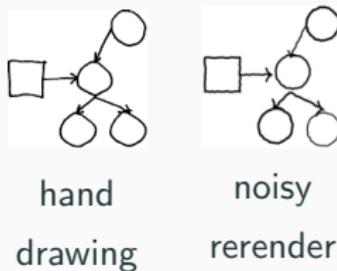
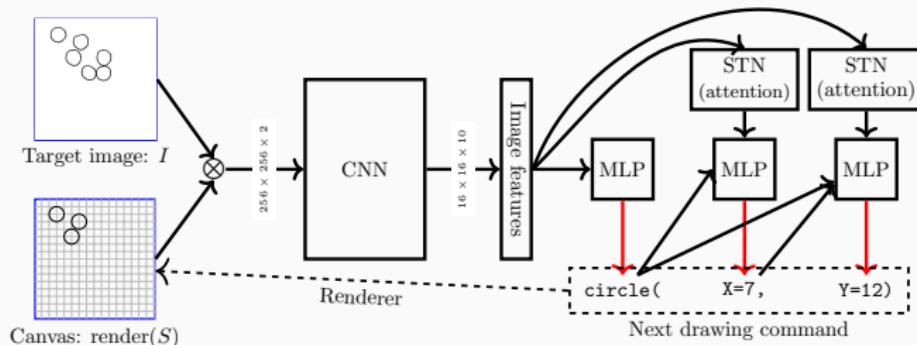
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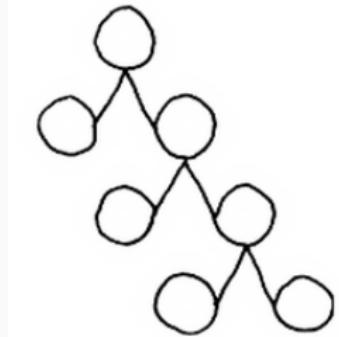
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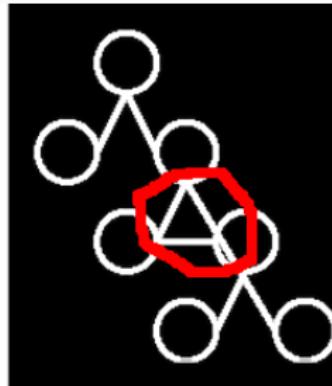


Top-down influences on perception

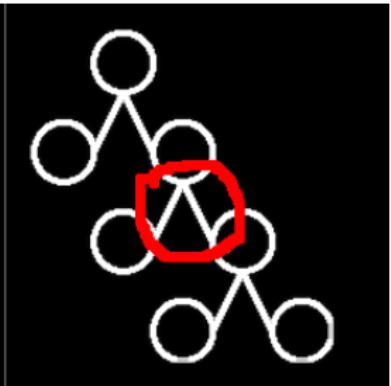
drawing



bottom-up neural net

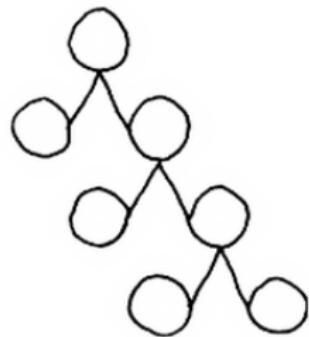


w/ top-down program bias

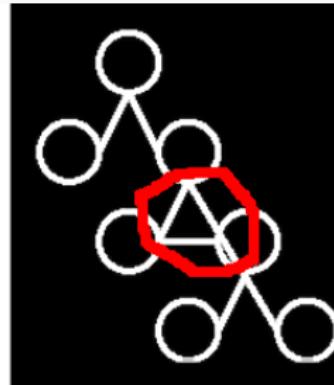


Top-down influences on perception

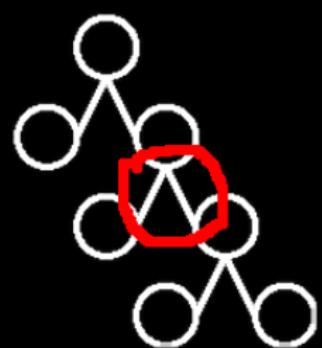
drawing



bottom-up neural net



w/ top-down program bias

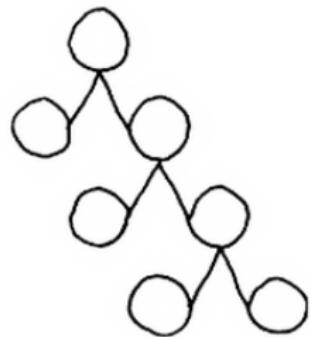


$$\hat{S}(I) = \arg \max_{S \in \mathcal{F}(I)} \mathbb{P}(I | \text{render}(S)) \times \mathbb{P}_\beta[\text{program}(S)]$$

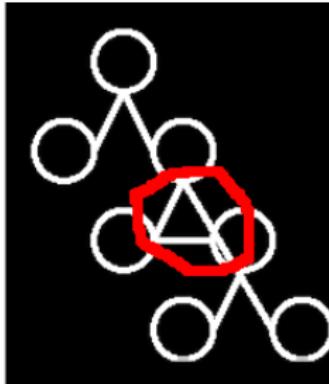
$$\beta^* = \arg \max_\beta \mathbb{E} \left[\log \frac{\mathbb{P}_\beta[\text{program}(S)] \times \mathbb{P}(I | \text{render}(S))}{\sum_{S' \in \mathcal{F}(I)} \mathbb{P}_\beta[\text{program}(S')] \times \mathbb{P}(I | \text{render}(S'))} \right]$$

Top-down influences on perception

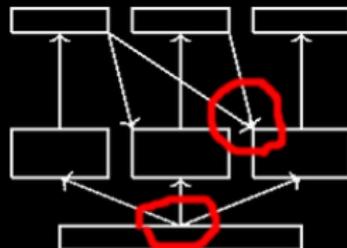
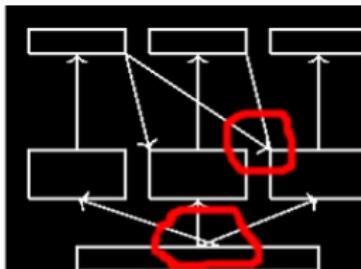
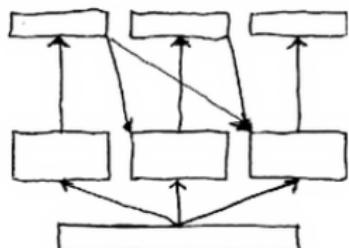
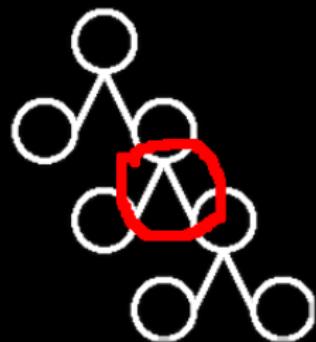
drawing



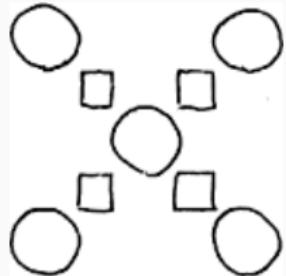
bottom-up neural net

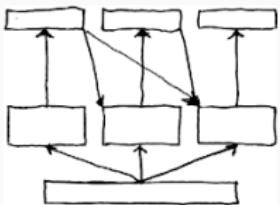


w/ top-down program bias



Example programs

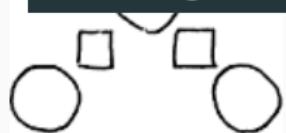
| Drawing | Spec | Program |
|---|---|---|
|  | <pre>Line(2,15, 4,15) Line(4,9, 4,13) Line(3,11, 3,14) Line(2,13, 2,15) Line(3,14, 6,14) Line(4,13, 8,13)</pre> | <pre>for(i<3) line(i,-1*i+6, 2*i+2,-1*i+6) line(i,-2*i+4,i,-1*i+6)</pre> |
|  | <pre>Circle(2,8) Rectangle(6,9, 7,10) Circle(8,8) Rectangle(6,12, 7,13) Rectangle(3,9, 4,10) ... etc.; 9 lines</pre> | <pre>reflect(y=8) for(i<3) if(i>0) rectangle(3*i-1,2,3*i,3) circle(3*i+1,3*i+1)</pre> |

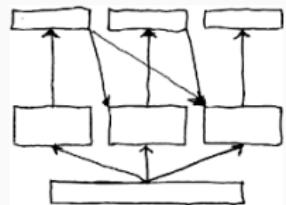
| | | |
|--|---|--|
|  | <pre>Line(3,10,3,14,arrow) Rectangle(11,8,15,10) Rectangle(11,14,15,15) Line(13,10,13,14,arrow) ... etc.; 16 lines</pre> | <pre>for(i<3) line(7,1,5*i+2,3,arrow) for(j<i+1) if(j>0) line(5*j-1,9,5*i,5,arrow) line(5*j+2,5,5*j+2,9,arrow) rectangle(5*i,3,5*i+4,5) rectangle(5*i,9,5*i+4,10) rectangle(2,0,12,1)</pre> |
|--|---|--|

Example programs

| Drawing | Spec | Program |
|---|---|---|
|  | <pre>Line(2,15, 4,15) Line(4,9, 4,13) Line(3,11, 3,14) Line(2,13, 2,15) Line(3,14, 6,14) Line(4,13, 8,13)</pre> | <pre>for(i<3) line(i,-1*i+6, 2*i+2,-1*i+6) line(i,-2*i+4,i,-1*i+6)</pre> |

Learning played a role...
but much of this system is specific to 2-D graphics

| | | |
|---|--|---|
|  | <pre>rectangle(8,12, 7,18) Rectangle(3,9, 4,10) ... etc.; 9 lines</pre> | <pre>rectangle(3*i-1,2,3*i,3) circle(3*i+1,3*i+1)</pre> |
|---|--|---|

| | | |
|--|---|--|
|  | <pre>Line(3,10,3,14,arrow) Rectangle(11,8,15,10) Rectangle(11,14,15,15) Line(13,10,13,14,arrow) ... etc.; 16 lines</pre> | <pre>for(i<3) line(7,1,5*i+2,3,arrow) for(j<i+1) if(j>0) line(5*j-1,9,5*i,5,arrow) line(5*j+2,5,5*j+2,9,arrow) rectangle(5*i,3,5*i+4,5) rectangle(5*i,9,5*i+4,10) rectangle(2,0,12,1)</pre> |
|--|---|--|

Growing domain-specific knowledge

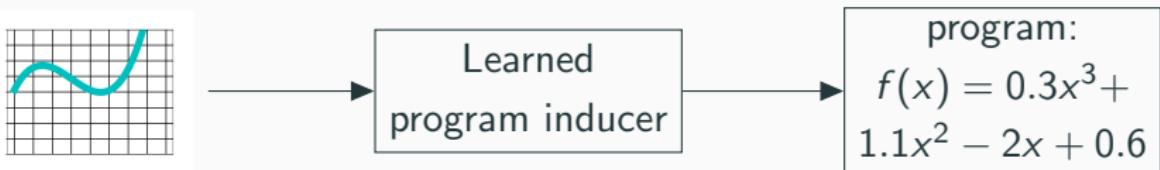
Goal: acquire domain-specific knowledge needed to induce a class of programs

- Library of concepts (declarative knowledge; domain specific language; generative model over programs)
- Inference strategy (procedural knowledge; synthesis algorithm)

Growing domain-specific knowledge

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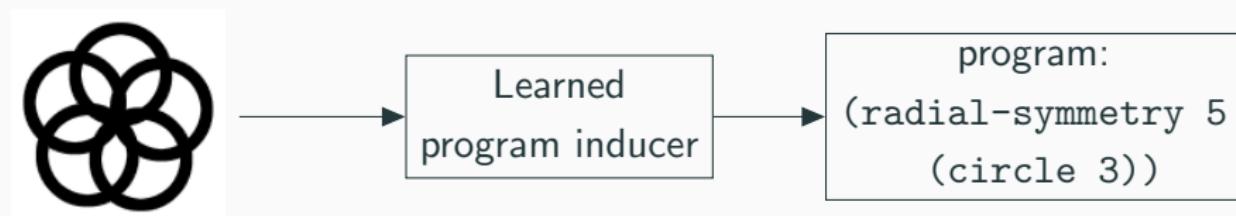
Concepts: $x^3, \alpha x + \beta$, etc

Inference strategy: neurosymbolic search for programs

Growing domain-specific knowledge

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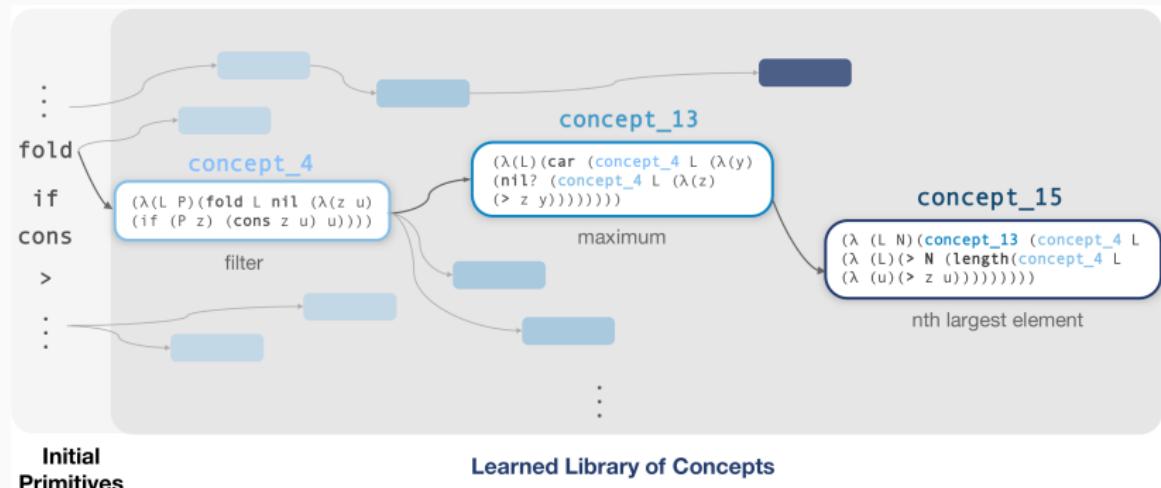
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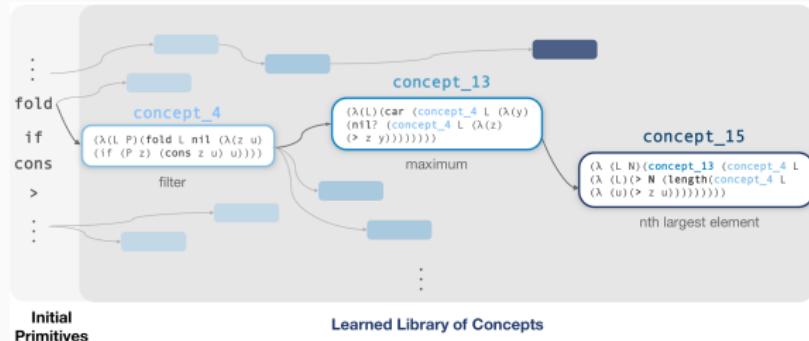
Concepts: circle, radial-symmetry, etc

Inference strategy: neurosymbolic search for programs

Library learning



Library learning

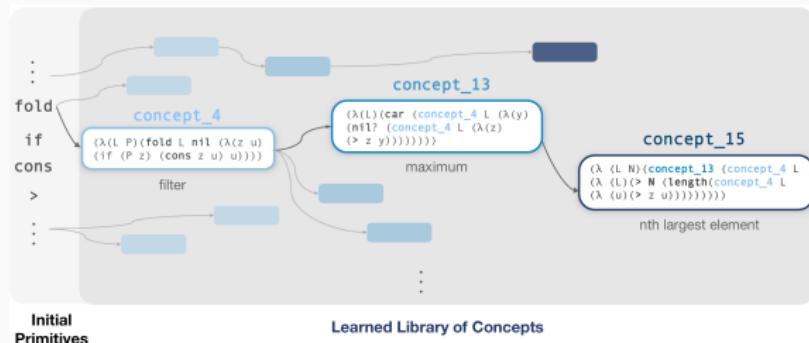


Problem: sort list

Solution:

```
(map (\n) (concept_15 L (+ 1 n))) (range (length L)))  
get nth largest element where n = 1, 2, 3 ....length of list
```

Library learning



Problem: sort list

Solution:

$(\text{map } (\lambda(n) (\text{concept}_15 L (+ 1 n))) (\text{range } (\text{length } L)))$
get nth largest element where $n = 1, 2, 3 \dots \text{length of list}$

Solution in initial primitives:

```
(λ(x) (map (λ(y) (car (fold (fold x nil (λ(z) u) (if (gt? (+ y 1) (length (fold x nil (λ(v) w) (if (gt? z v) (cons v w)))))) (cons z u) u))) nil (λ(a b) (if (nil? (fold (fold x nil (λ(c d) (if (gt? (+ y 1) (length (fold x nil (λ(e f) (if (gt? c e) (cons e f) f)))))) (cons c d) d))) nil (λ(g h) (if (gt? g a) (cons g h) h)))) (cons a b) b)))) (range (length x))))
```

Discovered Problem Solutions

DreamCoder

- **Wake:** Solve problems by writing programs
- **Sleep:** Improve DSL and neural recognition model:
 - **Abstraction sleep:** Improve library
 - **Dream sleep:** Improve neural inference model
- Combines ideas from Wake-Sleep & Exploration-Compression



DreamCoder

- **Wake:** Solve problems by writing programs
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 - **Abstraction sleep:** Improve library
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 $[1 \ 2 \ 3] \rightarrow 6$
 $[4 \ 6 \ 8 \ 1] \rightarrow 17$

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$[1 \ 2 \ 3 \ 4] \rightarrow [2 \ 4 \ 6 \ 8]$
 $[6 \ 5 \ 1] \rightarrow [12 \ 10 \ 2]$

Check Evens

$[0 \ 2 \ 3] \rightarrow [T \ T \ F]$
 $[2 \ 4 \ 9 \ 6] \rightarrow [T \ T \ F \ T]$

Text Editing

Abbreviate
Allen Newell → A.N.
Herb Simon → H.S.

Drop Last Characters
jabberwocky → jabberw
copycat → cop

Extract

see spot(run) → run
a (bee) see → bee

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Currency

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Dates

Y1775/07/04
Y2000/01/01

LOGO Graphics



Physics

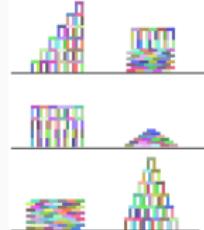
$$KE = \frac{1}{2}m|\vec{v}|^2$$

$$\bar{d} = \frac{1}{m} \sum_i \vec{F}_i$$

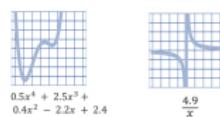
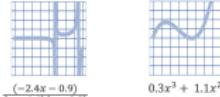
$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^2} \hat{r}_1 - \hat{r}_2$$

$$R_{total} = \left(\sum_i \frac{1}{R_i} \right)^{-1}$$

Block Towers



Symbolic Regression



Recursive Programming

Filter

$[■■■■■] \rightarrow [■■■]$
 $[■■■■■■■■] \rightarrow [■■■■■■]$
 $[■■■■■■■] \rightarrow [■■■■■]$

Length

$[■■■■■] \rightarrow 4$
 $[■■■■■■■■] \rightarrow 6$
 $[■■■■■■■] \rightarrow 3$

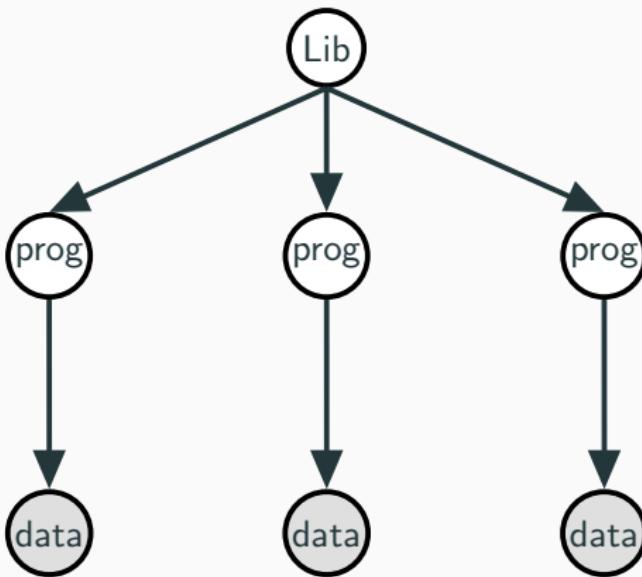
Index List

$0, [■■■■■■■■] \rightarrow ■$
 $1, [■■■■■■■■] \rightarrow ■■$
 $1, [■■■■■■■■] \rightarrow ■■■$

Every Other

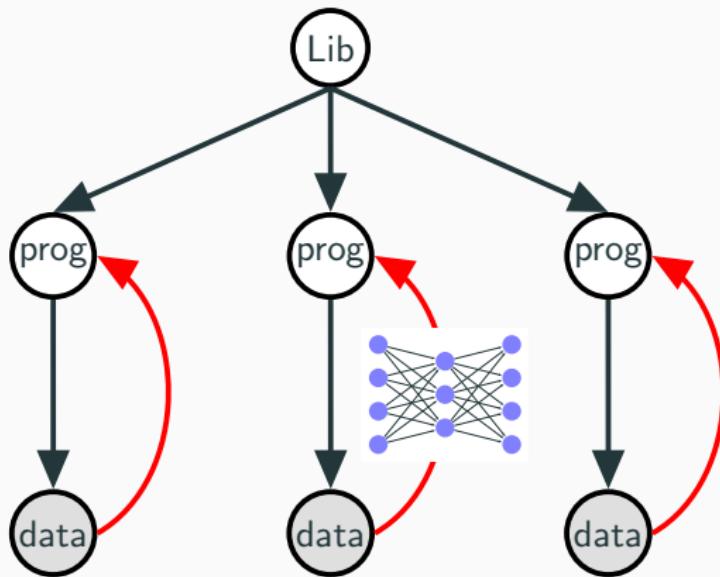
$[■■■■■■■■] \rightarrow [■■■■]$
 $[■■■■■■■■] \rightarrow [■■■■■■]$
 $[■■■■■■■■] \rightarrow [■■■■■■■]$

Library learning as Bayesian inference

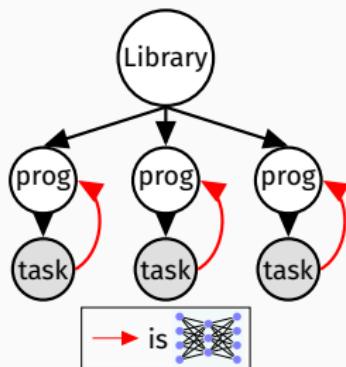


[Dechter et al., 2013] [Liang et al, 2010]; [Lake et al, 2015]

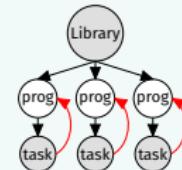
Library learning as amortized Bayesian inference



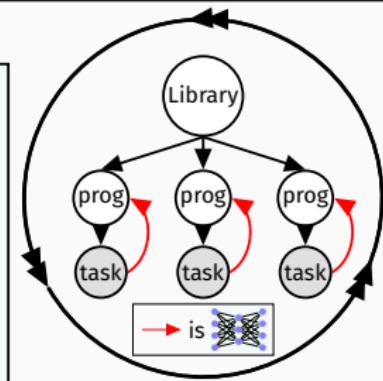
amortized inference +
better program representation (Lisp) +
library learning via program analysis +
new neural inference network for program synthesis



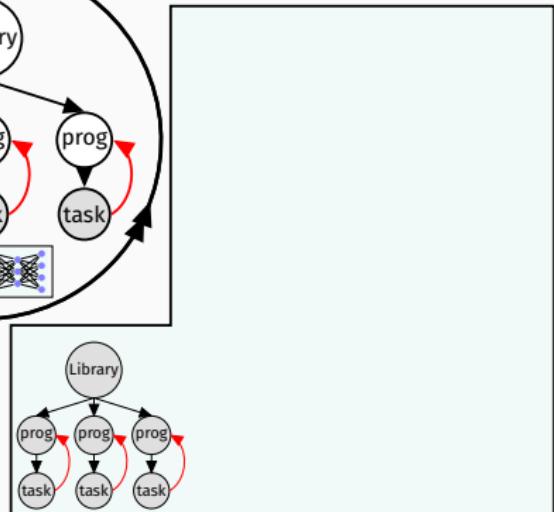
WAKE



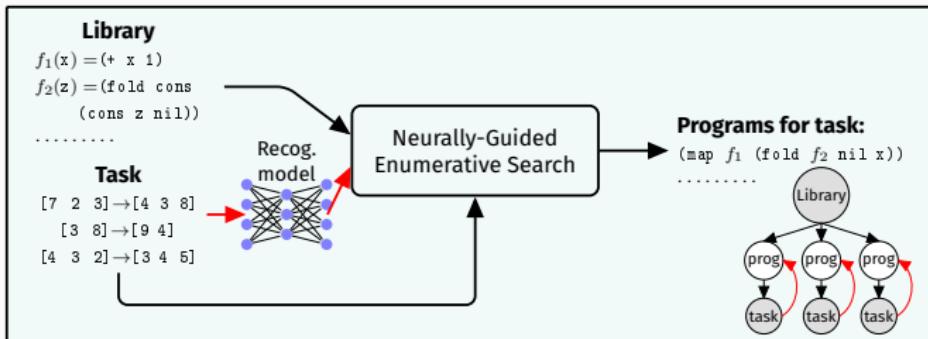
SLEEP: ABSTRACTION



SLEEP: DREAMING



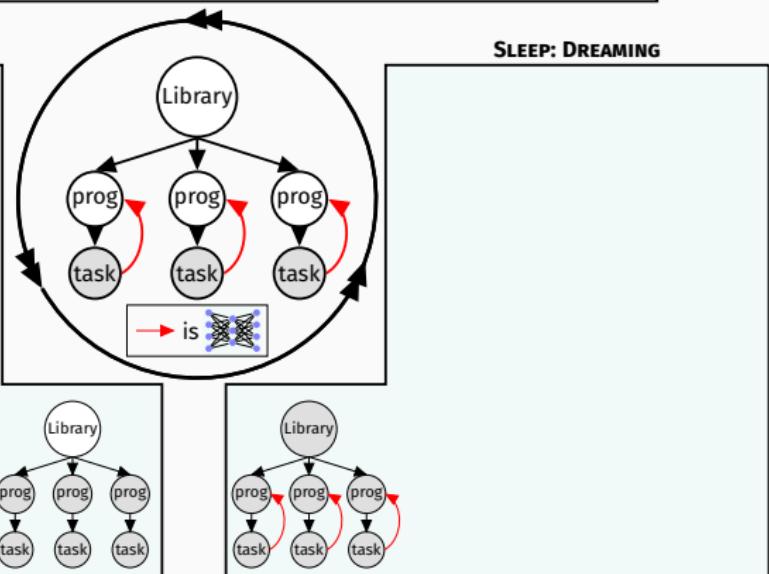
WAKE



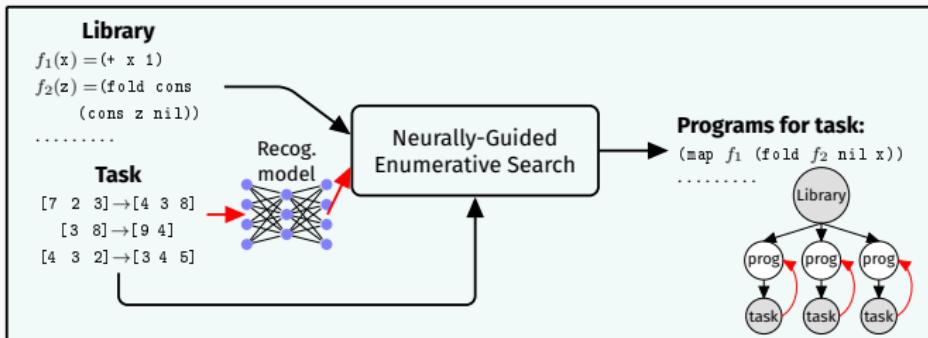
SLEEP: ABSTRACTION



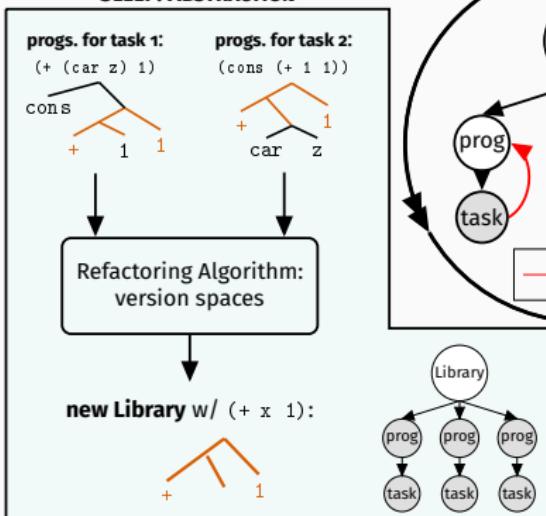
SLEEP: DREAMING



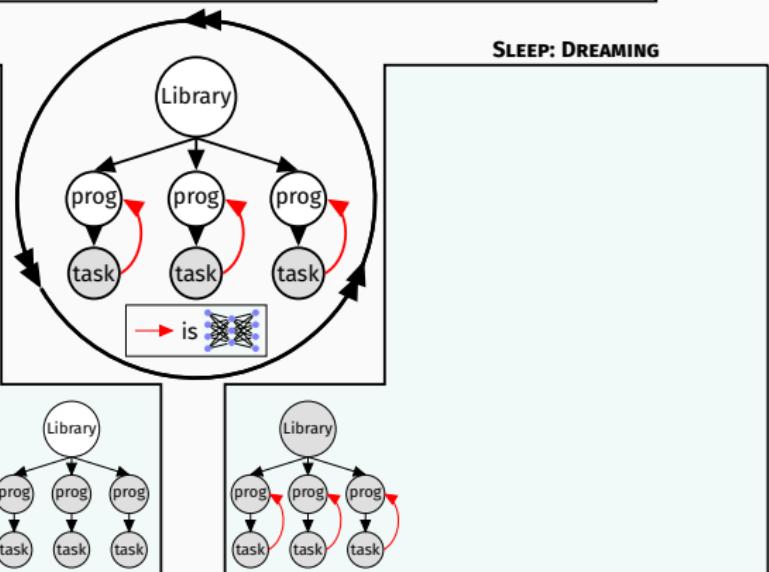
WAKE



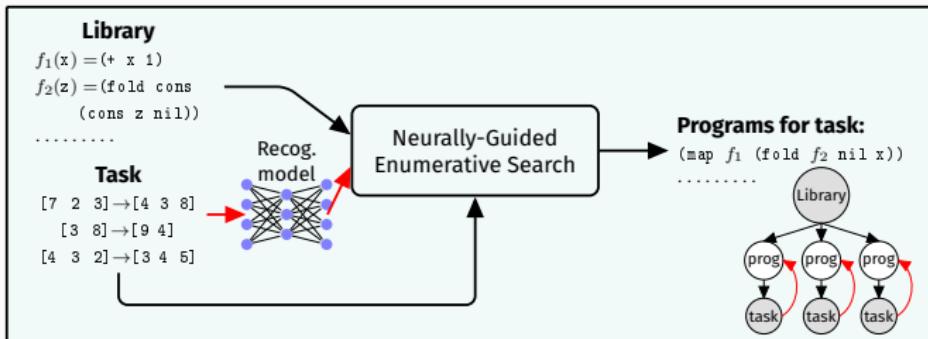
SLEEP: ABSTRACTION



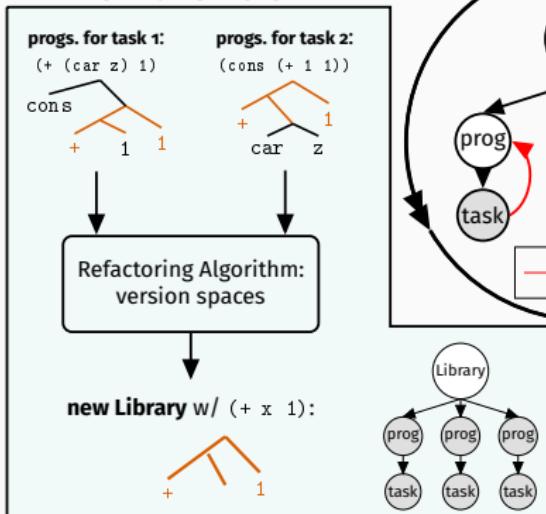
SLEEP: DREAMING



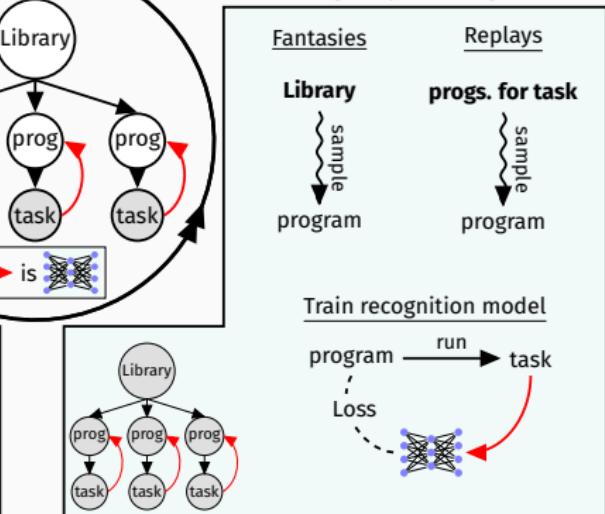
WAKE



SLEEP: ABSTRACTION



SLEEP: DREAMING



Abstraction Sleep: Growing the library via refactoring

Task: $(1\ 2\ 3) \rightarrow (2\ 4\ 6)$
 $(4\ 3\ 2) \rightarrow (8\ 6\ 8)$

Wake: program search

```
(Y (\lambda (r l) (if (nil? l) nil  
                      (cons (+ (car l) (car l))  
                            (r (cdr l)))))))
```

Task: $(1\ 2\ 3) \rightarrow (0\ 1\ 2)$
 $(4\ 3\ 2) \rightarrow (3\ 2\ 3)$

Wake: program search

```
(Y (\lambda (r l) (if (nil? l) nil  
                      (cons (- (car l) 1)  
                            (r (cdr l)))))))
```

Abstraction Sleep: Growing the library via refactoring

Task: $(1\ 2\ 3) \rightarrow (2\ 4\ 6)$
 $(4\ 3\ 2) \rightarrow (8\ 6\ 8)$

Wake: program search

```
(Y (λ (r 1) (if (nil? 1) nil  
           (cons (+ (car 1) (car 1))  
                  (r (cdr 1)))))))
```

Task: $(1\ 2\ 3) \rightarrow (0\ 1\ 2)$
 $(4\ 3\ 2) \rightarrow (3\ 2\ 3)$

Wake: program search

```
(Y (λ (r 1) (if (nil? 1) nil  
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                  (r (cdr 1)))))))
```

refactor
 $(10^{14}$ refactorings)

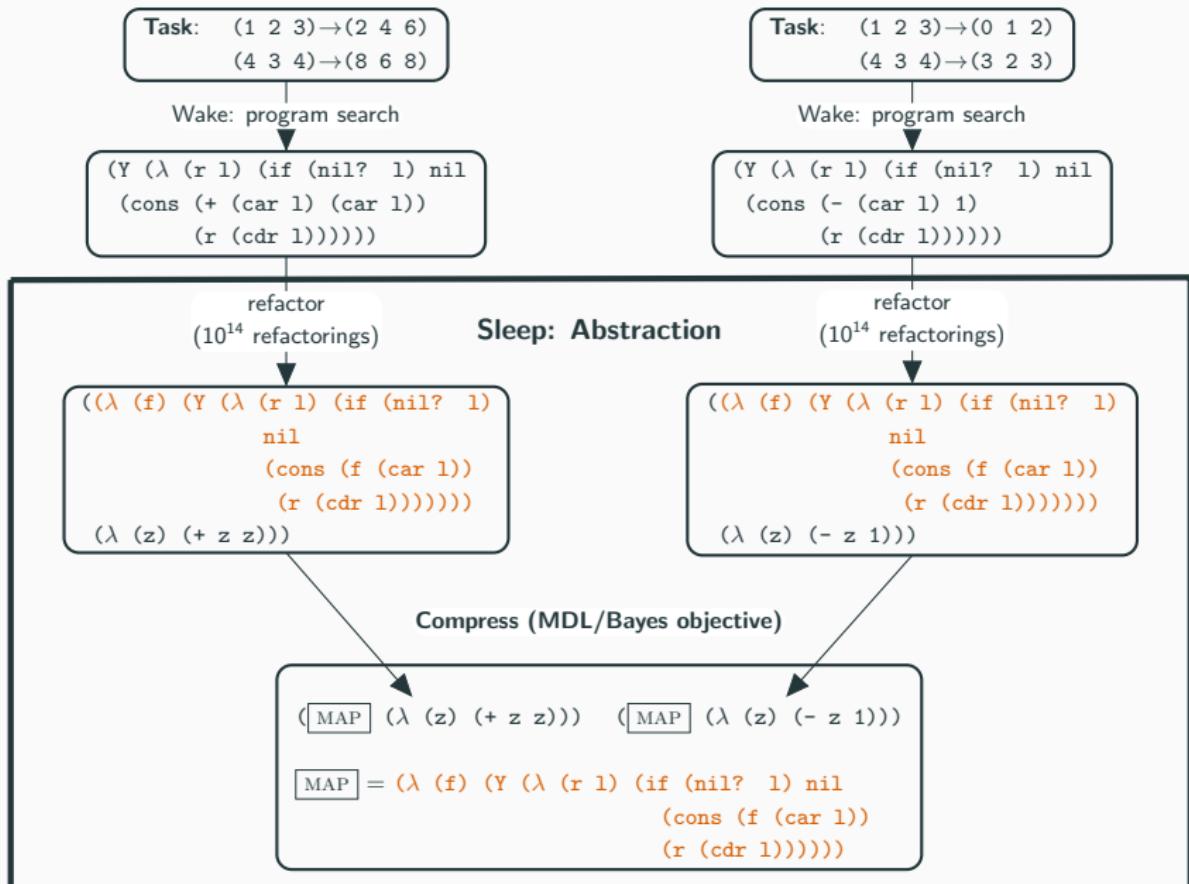
Sleep: Abstraction

refactor
 $(10^{14}$ refactorings)

```
((λ (f) (Y (λ (r 1) (if (nil? 1)  
                           nil  
                           (cons (f (car 1))  
                                 (r (cdr 1)))))))  
  (λ (z) (+ z z)))
```

```
((λ (f) (Y (λ (r 1) (if (nil? 1)  
                           nil  
                           (cons (f (car 1))  
                                 (r (cdr 1)))))))  
  (λ (z) (- z 1)))
```

Abstraction Sleep: Growing the library via refactoring



Version space algebra for refactoring

$\text{Expr} \rightarrow \text{var}$

- | $\lambda\text{var}.\text{Expr}$
- | $(\text{Expr } \text{Expr})$
- | primitive

Version space algebra for refactoring

$\text{Expr} \rightarrow \text{var}$

| $\lambda\text{var}.\text{Expr}$
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$\text{VS} \rightarrow \text{var}$

| $\lambda\text{var}.\text{VS}$
| ($\text{VS } \text{VS}$)
| primitive
| $\text{VS} \uplus \text{VS}$ *nondeterministic choice*
| Λ *choose any expression*
| \emptyset *choose no expression*

Version space algebra for refactoring

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| $\lambda \text{var}. \text{Expr}$
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| ($\text{VS } \text{VS}$)
| primitive
| $\text{VS} \uplus \text{VS}$ *nondeterministic choice*
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| \emptyset *choose no expression*

what version spaces mean:

$$[\![\text{var}]\!] = \{\text{var}\}$$

$$[\![v_1 \uplus v_2]\!] = \{e : v \in \{v_1, v_2\}, e \in [\![v]\!]\}$$

$$[\![\lambda x. v]\!] = \{\lambda x. e : e \in [\![v]\!]\}$$

$$[\![(v_1 v_2)]\!] = \{(e_1 e_2) : e_1 \in [\![v_1]\!], e_2 \in [\![v_2]\!]\}$$

$$[\![\emptyset]\!] = \emptyset$$

$$[\![\Lambda]\!] = \Lambda$$

Using version spaces

$VS \rightarrow var \mid \lambda var.VS \mid (VS\ VS) \mid primitive$

| $VS \cup VS$ *nondeterministic choice*

| Λ *choose any expression*

| \emptyset *choose no expression*

Using version spaces

$$\begin{aligned} \text{VS} \rightarrow & \text{ var } | \lambda \text{var}. \text{VS} | (\text{VS } \text{VS}) | \text{ primitive} \\ & | \text{VS} \sqcup \text{VS} \quad \textit{nondeterministic choice} \\ & | \Lambda \quad \textit{choose any expression} \\ & | \emptyset \quad \textit{choose no expression} \end{aligned}$$

exploit the fact that $e \in \llbracket v \rrbracket$ can be efficiently computed:

$$\text{REFACTOR}(v|\text{Lib}) = \begin{cases} e, & \text{if } e \in \text{Lib and } e \in \llbracket v \rrbracket \\ \text{REFACTOR}'(v|\text{Lib}), & \text{otherwise.} \end{cases}$$

$$\text{REFACTOR}'(v|\text{Lib}) = v, \text{ if } v \text{ is a primitive or variable}$$

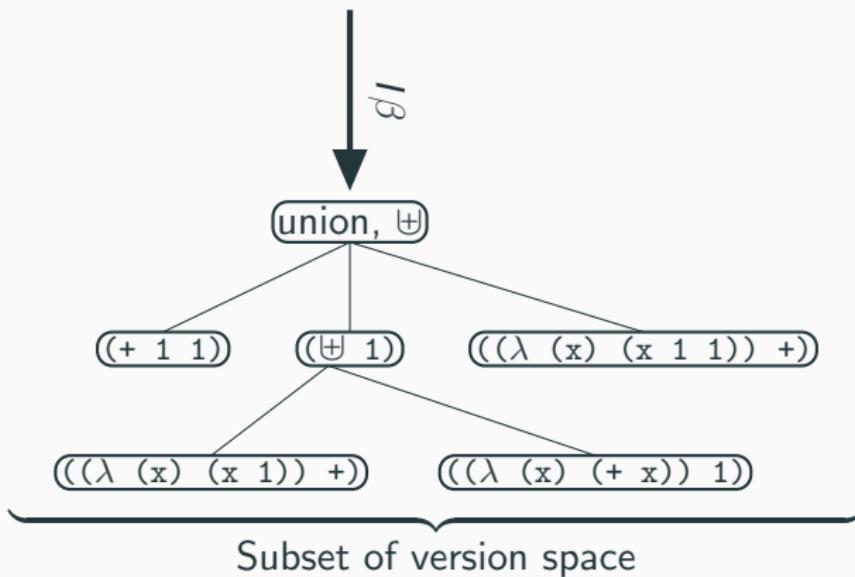
$$\text{REFACTOR}'(\lambda x. b|\text{Lib}) = \lambda x. \text{REFACTOR}(b|\text{Lib})$$

$$\text{REFACTOR}'(v_1 v_2|\text{Lib}) = (\text{REFACTOR}(v_1|\text{Lib}) \text{ REFACTOR}(v_2|\text{Lib}))$$

Using version spaces

$VS \rightarrow var \mid \lambda var.VS \mid (VS \vee VS) \mid primitive$
| $VS \sqcup VS$ *nondeterministic choice*
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Program: $(+ 1 1)$



Using version spaces

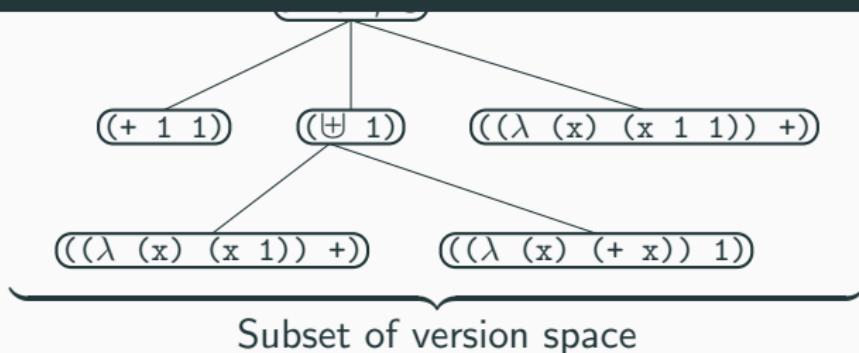
$VS \rightarrow var \mid \lambda var.VS \mid (VS \; VS) \mid primitive$
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completeness: $I\beta$ gets all the refactorings

let $v_2 = I\beta(v_1)$ and $e_1 \in \llbracket v_1 \rrbracket$. for any $e_2 \rightarrow e_1$ then $e_2 \in \llbracket v_2 \rrbracket$

consistency: $I\beta$ only gets valid refactorings

let $v_2 = I\beta(v_1)$ and $e_2 \in \llbracket v_2 \rrbracket$. then there is a $e_1 \in \llbracket v_1 \rrbracket$ where $e_2 \rightarrow e_1$



DreamCoder Domains

List Processing

Sum List

$[1 \ 2 \ 3] \rightarrow 6$

$[4 \ 6 \ 8 \ 1] \rightarrow 17$

Double

$[1 \ 2 \ 3 \ 4] \rightarrow [2 \ 4 \ 6 \ 8]$

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Check Evens

$[0 \ 2 \ 3] \rightarrow [T \ T \ F]$

$[2 \ 4 \ 9 \ 6] \rightarrow [T \ T \ F \ T]$

Text Editing

Abbreviate

Allen Newell → A.N.

Herb Simon → H.S.

Drop Last Characters

jabberwocky → jabberw

copycat → cop

Regexes

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Currency

\$100.25

\$4.50

Dates

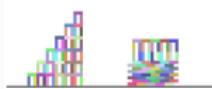
Y1775/0704

Y2000/0101

LOGO Graphics



Block Towers



Symbolic Regression



$$\frac{(-2.4x - 0.9)}{(x - 4.4)(x - 0.9)}$$

$$0.3x^3 + 1.1x^2 - 2.0x + 0.6$$



$$0.5x^4 + 2.5x^3 + 0.4x^2 - 2.2x + 2.4$$

$$\frac{4.9}{x}$$

Recursive Programming

Filter

[■■■■■] → [■■■]

[■■■■■■■] → [■■■■■■]

[■■■■■■] → [■■■■]

Length

[■■■■■] → 4

[■■■■■■■] → 6

[■■■■] → 3

Index List

0, [■■■■■■■■] → ■

1, [■■■■■■■■] → ■■

1, [■■■■■■■■] → ■■■

Every Other

[■■■■■] → [■■]

[■■■■■■■] → [■■■■■]

[■■■■■■■] → [■■■■■]

Physics

$$KE = \frac{1}{2} m |\vec{v}|^2$$

$$\vec{a} = \frac{1}{m} \sum_i \vec{F}_i$$

$$\vec{p} \propto \frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^2} \vec{r}_1 - \vec{r}_2$$

$$R_{total} = \left(\sum_i \frac{1}{R_i} \right)^{-1}$$

DreamCoder Domains

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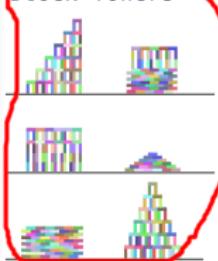
Y1775/0704

Y2000/0101

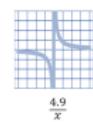
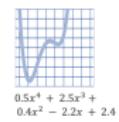
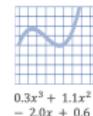
LOGO Graphics



Block Towers



Symbolic Regression



Recursive Programming

Filter

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[■■■■■] → [■■■■]
[■■■■] → [■■■]

Length

[■■■■] → 4
[■■■■■] → 6
[■■■] → 3

Index List

0, [■■■■■■] → ■
1, [■■■■■■] → ■■
1, [■■■■■■] → ■■■

Every Other

[■■■■■■] → [■■■]
[■■■■■■] → [■■■■]
[■■■■■■] → [■■■■■]

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$$KE = \frac{1}{2} m |\vec{v}|^2$$

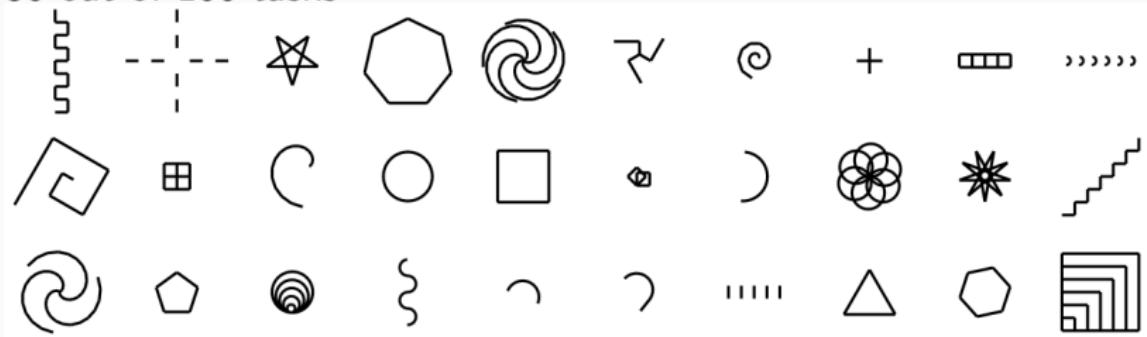
$$\vec{a} = \frac{1}{m} \sum_i \vec{F}_i$$

$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^2} \vec{r}_1 \vec{r}_2$$

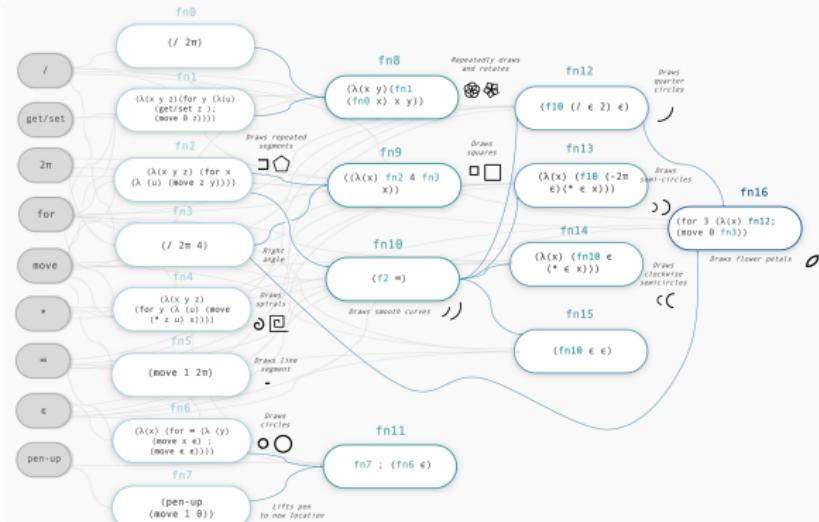
$$V_{total} = \left(\sum_i \frac{1}{R_i} \right)^{-1}$$

LOGO Graphics

30 out of 160 tasks



LOGO Graphics – learning interpretable library of concepts



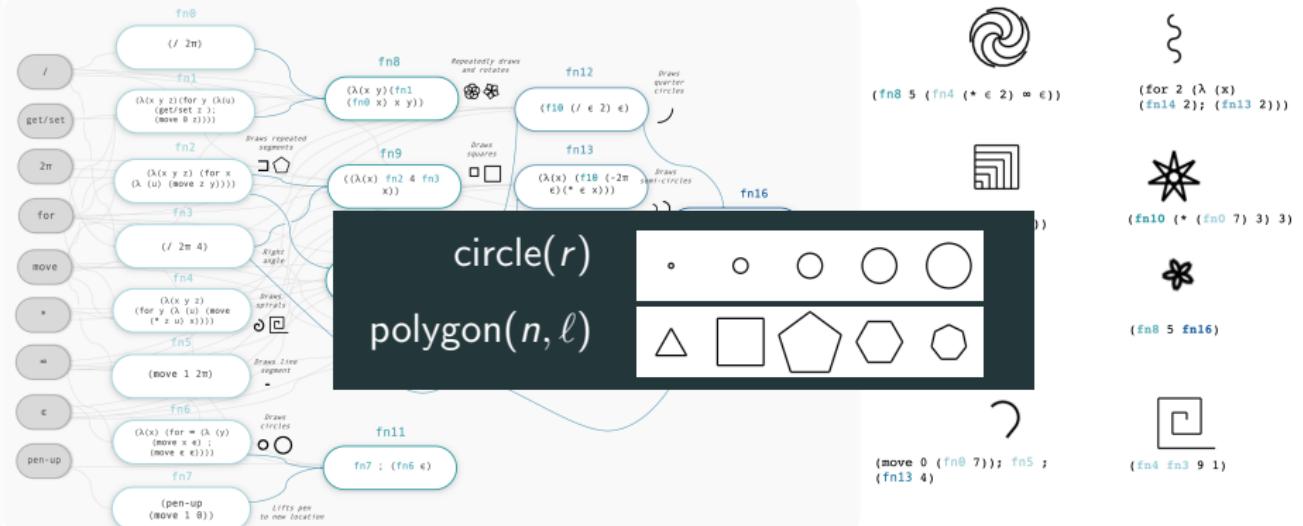
$\{\text{fn8} \cdot 5 \cdot (\text{fn4} \cdot (* \cdot 2) \cdot = \epsilon)\}$
 $\{\text{fn14} \cdot 2\}; \{\text{fn13} \cdot 2\})$

$\{\text{fn10} \cdot (* \cdot (\text{fn9} \cdot 7) \cdot 3) \cdot 3\}$

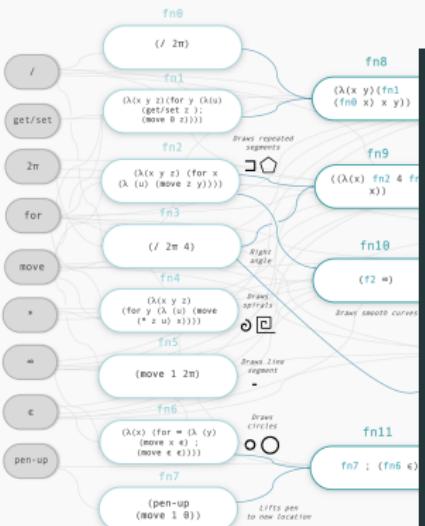
$\{\text{fn8} \cdot 6 \cdot (\text{fn7} \cdot ; \text{fn5} \cdot ; \text{fn7} \cdot ; \text{fn5}))\}$
 $\{\text{fn4} \cdot \text{fn3} \cdot 9 \cdot 1\}$

$\{\text{move 0} \cdot (\text{fn0} \cdot 7); \text{fn5} \cdot ; \text{fn13} \cdot 4\}$

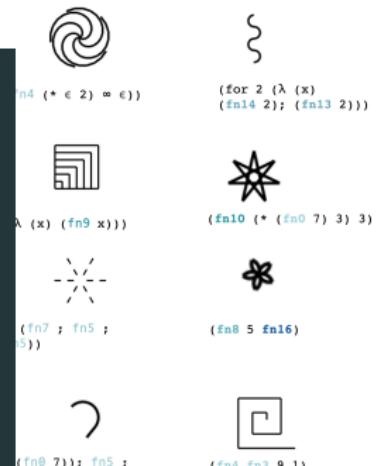
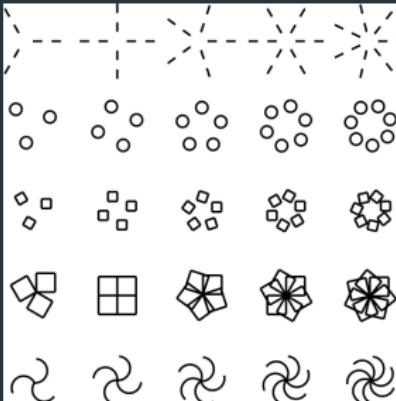
LOGO Graphics – learning interpretable library of concepts



LOGO Graphics – learning interpretable library of concepts



radial symmetry(*n, body*)

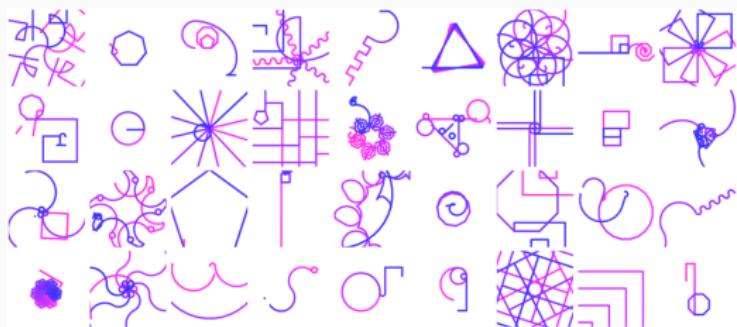


what does DreamCoder dream of?

before learning

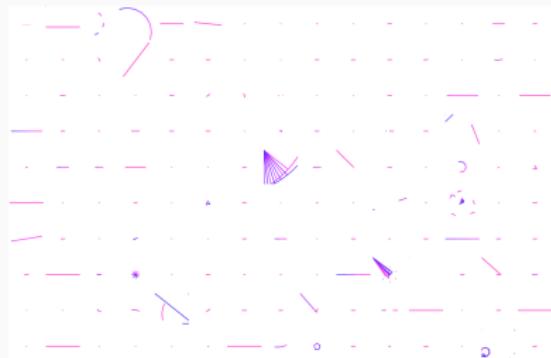


after learning

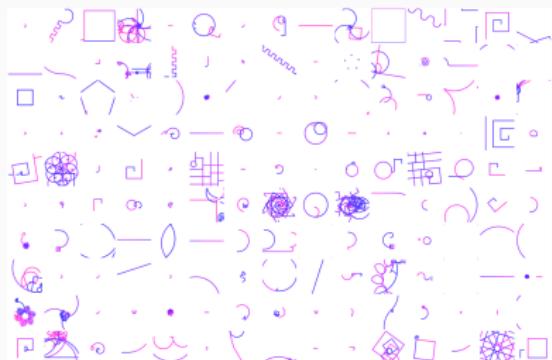


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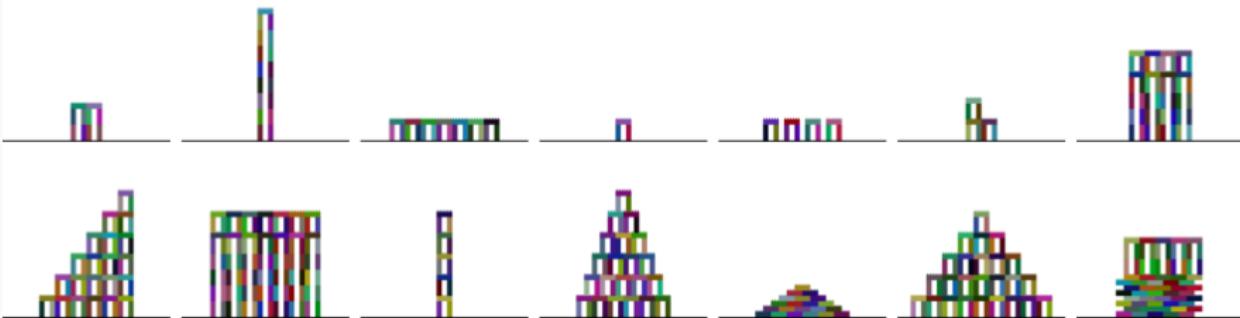


after learning



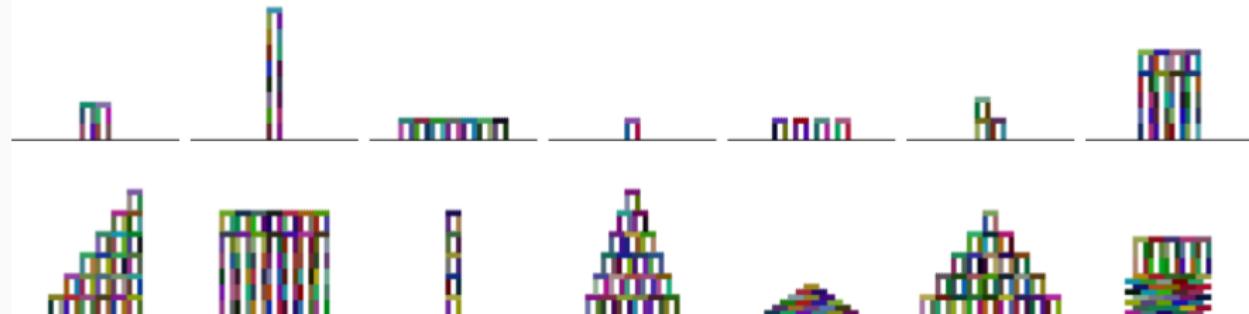
Planning to build towers

example tasks (112 total)

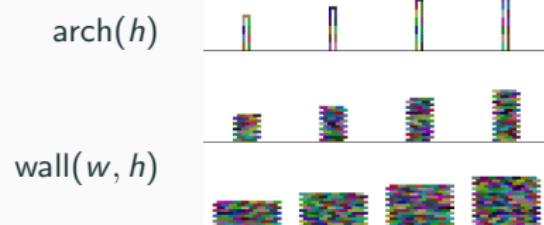


Planning to build towers

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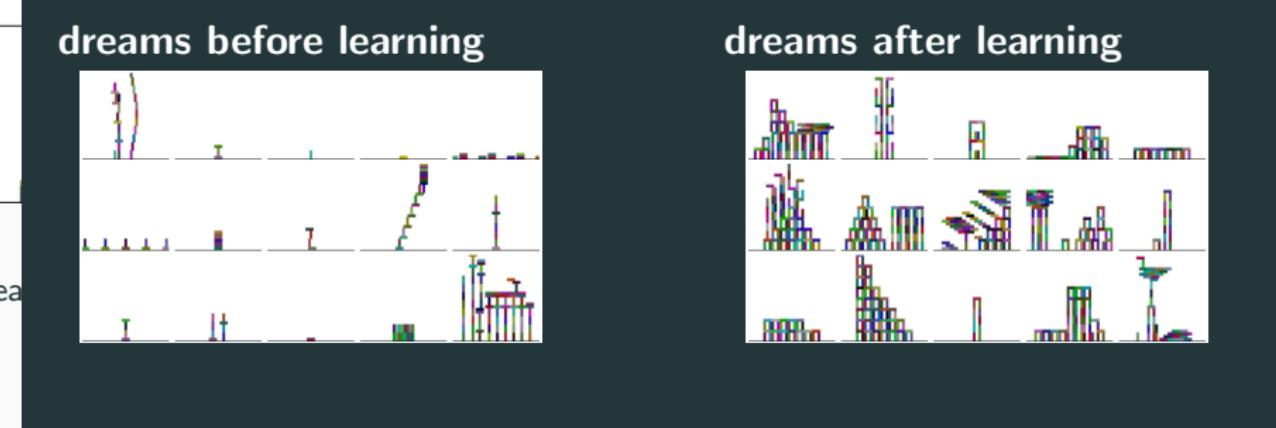


learned library routines (≈ 20 total)

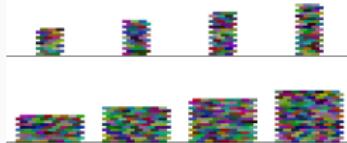


Planning to build towers

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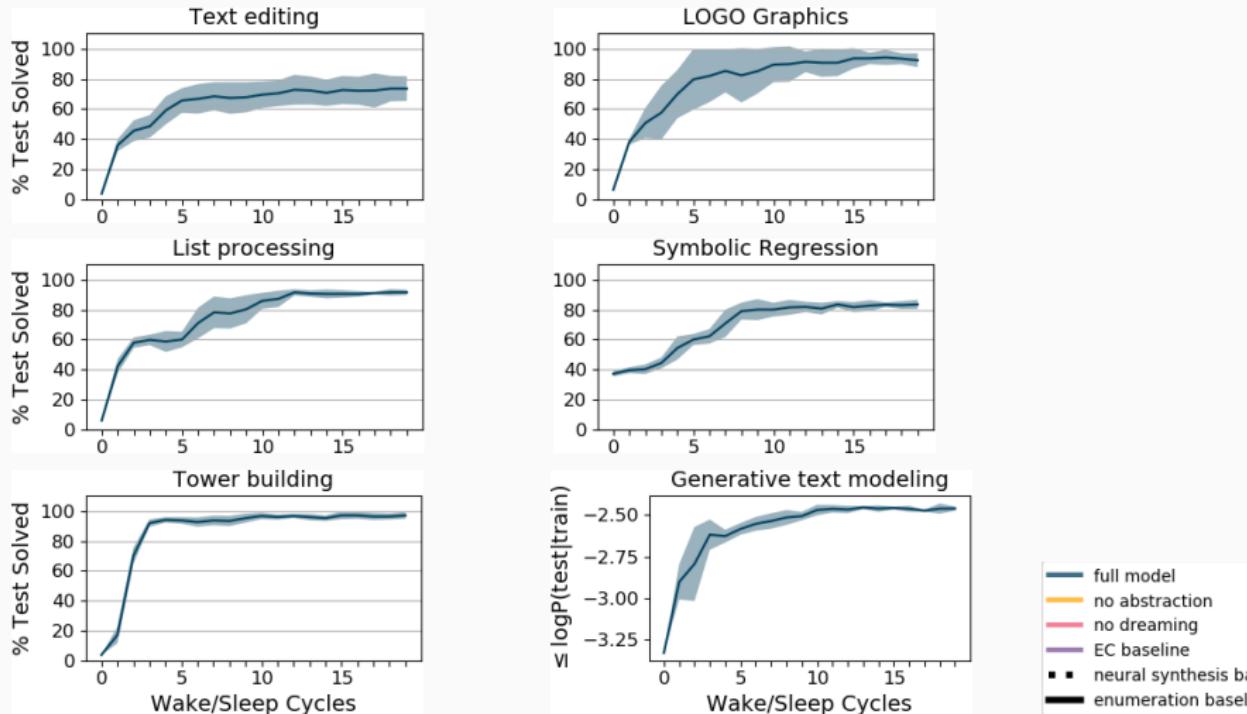
wall(w, h)



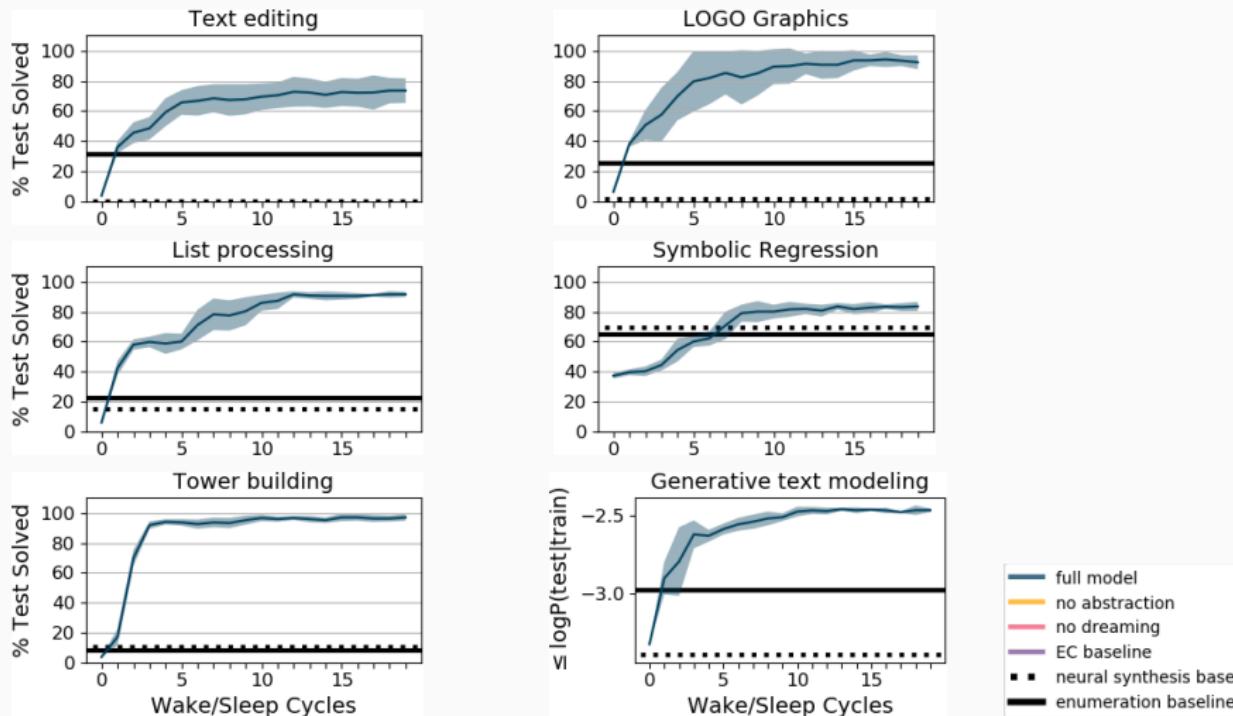
bridge(w, h)



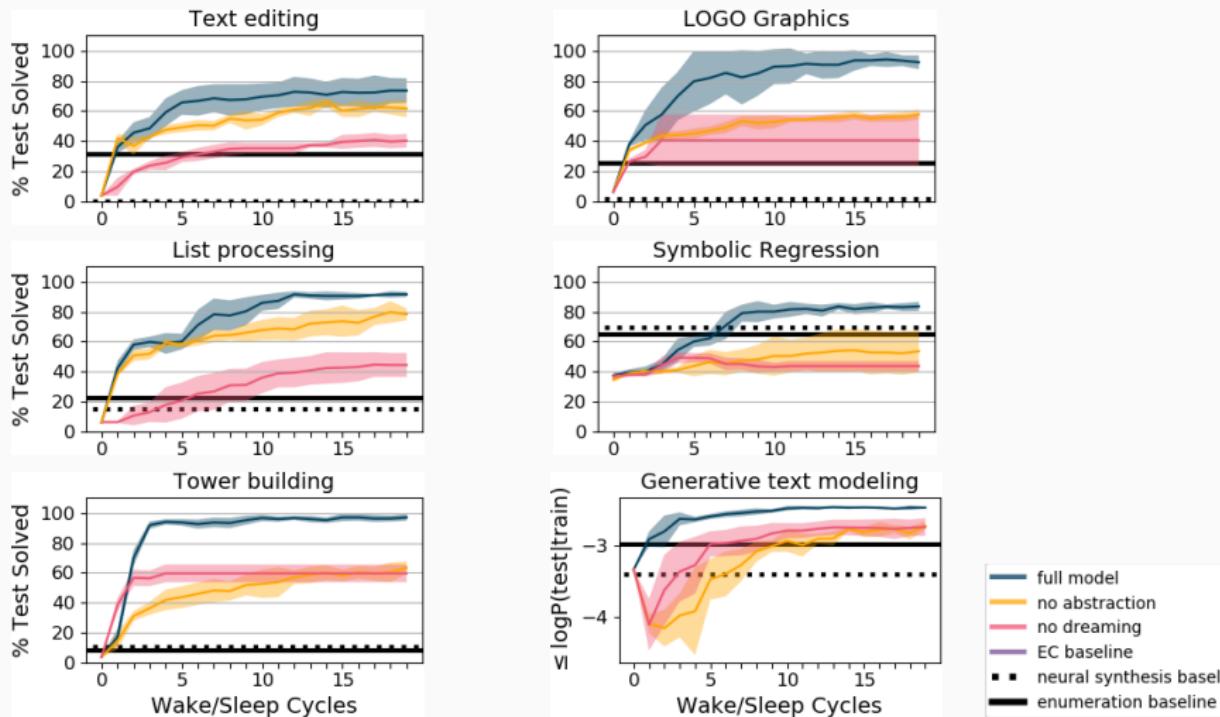
synergy between dreaming and library learning



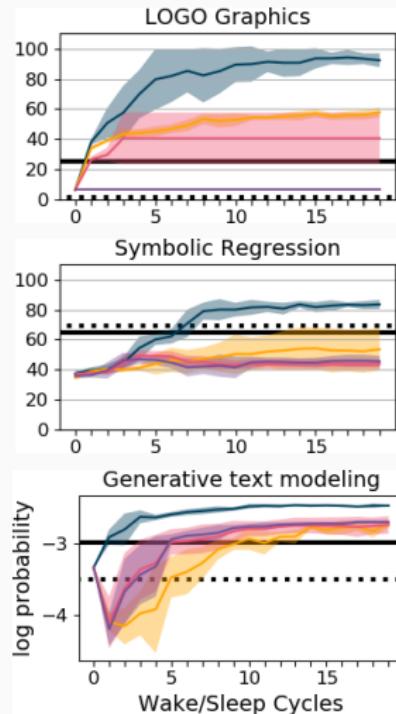
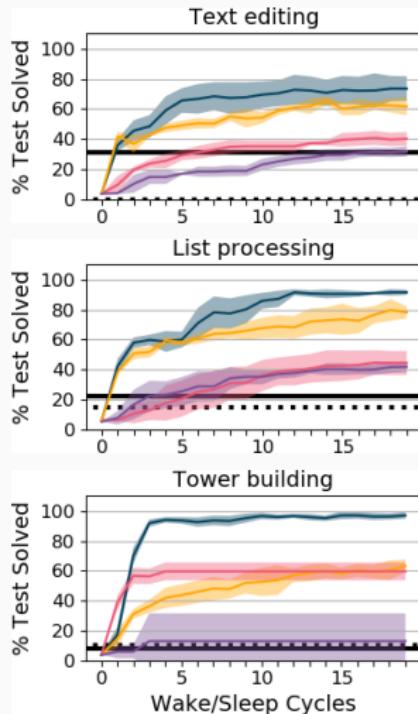
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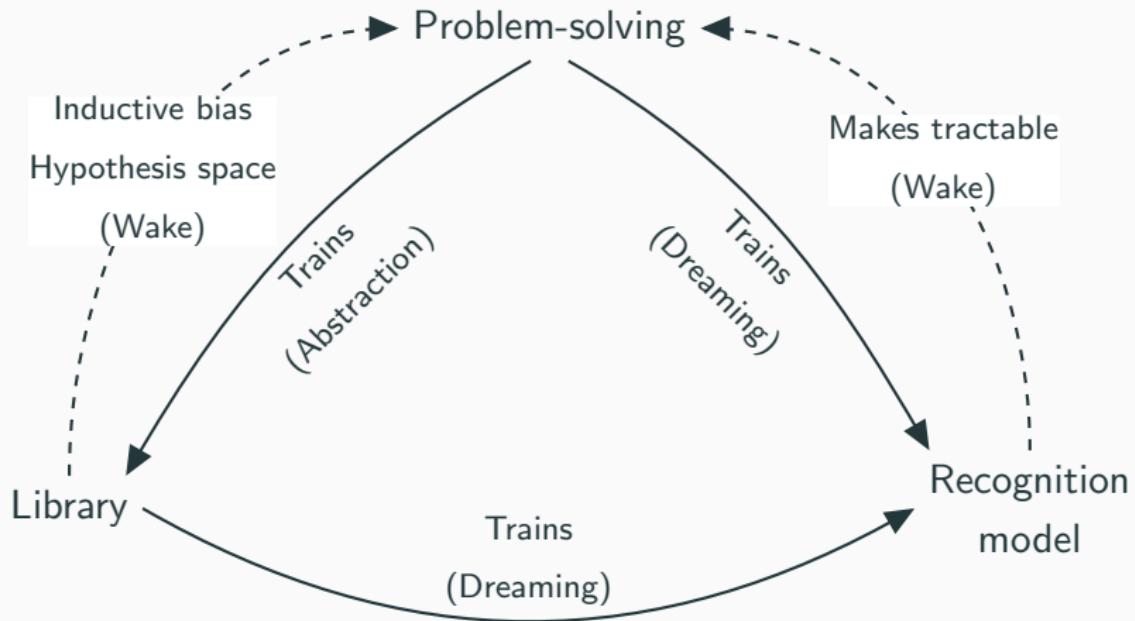
synergy between dreaming and library learning



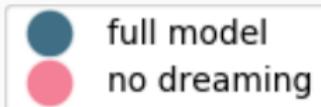
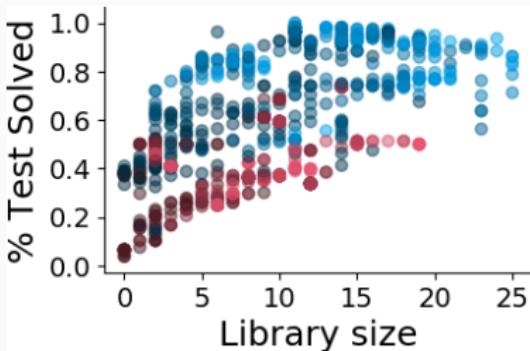
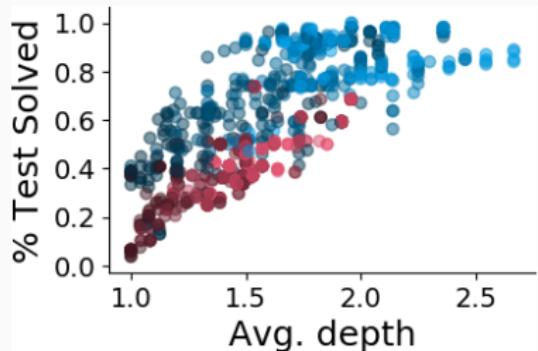
synergy between dreaming and library learning



synergy between dreaming and library learning



Evidence for dreaming bootstrapping better libraries



Dark→Light: Early in learning→Later in learning

Vision

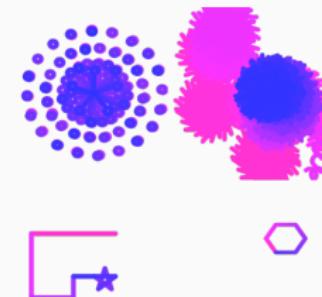
More human-like machine intelligence

- Acquiring a domain-specific representation (DSL)
- Learning to use that representation (recognition model)

DreamCoder: an algorithm for jointly realizing these goals

```
f2(p,f,n,x) = (if (p x) nil  
                  (cons (f x) (f2 (n x))))  
(f2: unfold)  
f3(i,l) = (if (= i 0) (car l)  
              (f3 (f1 i) (cdr l)))  
(f3: index)  
f4(f,l,x) = (if (empty? l) x  
                  (f (car l) (f4 (cdr l))))  
(f4: fold)  
f5(f,l) = (if (empty? l) nil  
              (cons (f (car l)) (f5 (cdr l))))  
(f5: map)
```

| Symbolic Regression | |
|--|---|
| | $f(x) = (f_1 \times)$ |
| | $f(x) = (f_6 \times)$ |
| | $f(x) = (f_4 \times)$ |
| | $f(x) = (f_3 \times)$ |
| $f_0(x) = (+ \times \text{real})$ | |
| $f_1(x) = (f_0 (\star \text{real} \times))$ | |
| $f_2(x) = (f_1 (\star x (f_0 \times)))$ | |
| $f_3(x) = (f_0 (\star x (f_2 \times)))$ | |
| $f_4(x) = (f_0 (\star x (f_3 \times)))$ | |
| <i>(f₄: 4th order polynomial)</i> | |
| $f_5(x) = (/ \text{real} \times)$ | |
| $f_6(x) = (f_5 (f_0 \times))$ | |
| | <i>(f₆: rational function)</i> |



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| $f_1(x) = (f_0 (\star \text{real} \times))$ | |
| $f_2(x) = (f_1 (\star x (f_0 \times)))$ | |
| $f_3(x) = (f_0 (\star x (f_2 \times)))$ | |
| $f_4(x) = (f_0 (\star x (f_3 \times)))$ | <i>(f4: 4th order polynomial)</i> |
| $f_5(x) = (/ \text{real} \times)$ | |
| $f_6(x) = (f_5 (f_0 \times))$ | |
| | <i>(f6: rational function)</i> |

