

# DreamCoder: Growing generalizable, interpretable knowledge with wake-sleep program learning

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Kevin Ellis

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Lucas Morales, Armando Solar-Lezama, Joshua B. Tenenbaum

2020

CAP NeurIPS workshop

## The premise of program induction

1. Represent knowledge as programs: as symbolic code

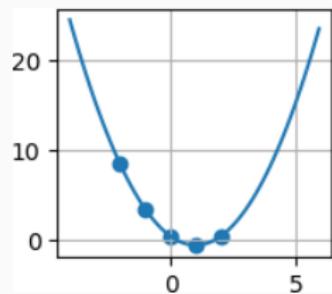
## The premise of program induction

1. Represent knowledge as programs: as symbolic code
2. Learning=adding to that body of knowledge=  
making new programs=program synthesis

# Why program induction?

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strong generalization  
+data efficiency

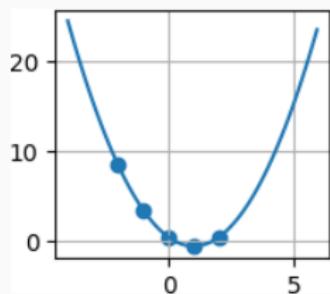


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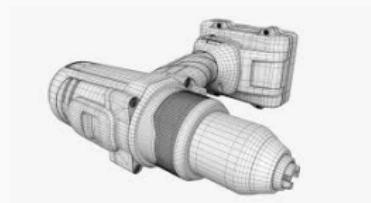
interpretability



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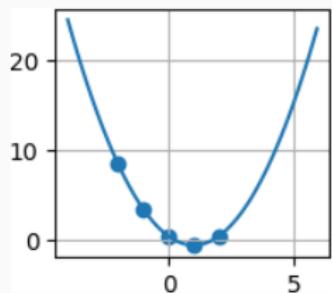


VS



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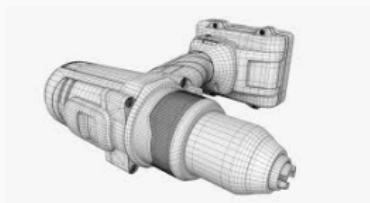


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interpretability



universal expressivity



## FlashFill (Gulwani 2012)

EXAMPLE 3 (Directory Name Extraction). Consider the following example taken from an excel online help forum.

Input $v_1$	Output
Company\Code\index.html	Company\Code\
Company\Docs\Spec\specs.doc	Company\Docs\Spec\

String Program:

$\text{SubStr}(v_1, \text{CPos}(0), \text{Pos}(\text{SlashTok}, \epsilon, -1))$

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## Szalinski (Nandi 2020)



(a) CAD model of ship's wheel

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(Union
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(b) Caddy program

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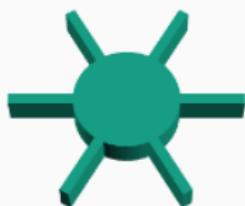
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String expr $P$	$\text{Switch}((b_1, e_1), \dots, (b_n, e_n))$
Bool $b$	$d_1 \vee \dots \vee d_n$
Conjunct $d$	$\pi_1 \wedge \dots \wedge \pi_n$
Predicate $\pi$	$\text{Match}(v_i, r, k) \mid \neg \text{Match}(v_i, r, k)$
Trace expr $e$	$\text{Concatenate}(f_1, \dots, f_n)$
Atomic expr $f$	$\text{SubStr}(v_i, p_1, p_2)$   $\text{ConstStr}(s)$   $\text{Loop}(\lambda w : e)$
Position $p$	$\text{CPos}(k) \mid \text{Pos}(r_1, r_2, c)$
Integer expr $c$	$k \mid k_1 w + k_2$
Regular Expression $r$	$\text{TokenSeq}(T_1, \dots, T_m)$
Token $T$	$C + \mid [\neg C] + \mid \text{SpecialToken}$

# Szalinski (Nandi 2020)



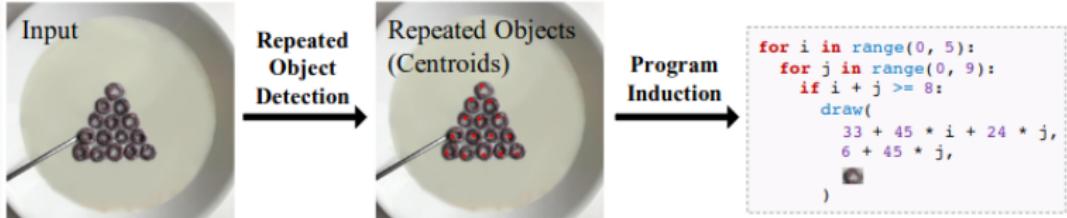
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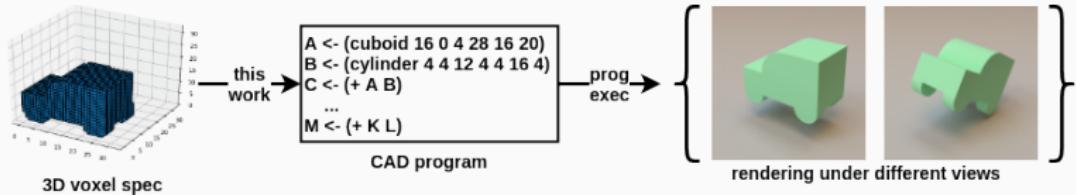
(b) Caddy program

op	$\text{::= } + \mid - \mid \times \mid / \mid \text{num} \text{ ::= } \mathbb{R} \mid \langle \text{var} \rangle \mid \langle \text{num} \rangle \langle \text{op} \rangle \langle \text{num} \rangle$
vec2	$\text{::= } [\langle \text{num} \rangle, \langle \text{num} \rangle] \mid \text{vec3} \text{ ::= } [\langle \text{num} \rangle, \langle \text{num} \rangle, \langle \text{num} \rangle]$
affine	$\text{::= } \text{Translate} \mid \text{Rotate} \mid \text{Scale} \mid \text{TranslateSpherical}$
binop	$\text{::= } \text{Union} \mid \text{Difference} \mid \text{Intersection}$
cad	$\text{::= } (\text{Cuboid } \langle \text{vec3} \rangle) \mid (\text{Sphere } \langle \text{num} \rangle)$   $(\text{Cylinder } \langle \text{vec2} \rangle) \mid (\text{HexPrism } \langle \text{vec2} \rangle) \mid \dots$   $((\text{affine}) \langle \text{vec3} \rangle \langle \text{cad} \rangle)$   $((\text{binop}) \langle \text{cad} \rangle \langle \text{cad} \rangle)$   $(\text{Fold } \langle \text{binop} \rangle \langle \text{cad-list} \rangle)$
cad-list	$\text{::= } (\text{List } \langle \text{cad} \rangle^+)$   $(\text{Concat } \langle \text{cad-list} \rangle^+)$   $(\text{Tabulate } (\langle \text{var} \rangle \ Z^+)^+ \langle \text{cad} \rangle)$   $(\text{Map2 } \langle \text{affine} \rangle \langle \text{vec3-list} \rangle \langle \text{cad-list} \rangle)$
vec3-list	$\text{::= } (\text{List } \langle \text{vec3} \rangle^+)$   $(\text{Concat } \langle \text{vec3-list} \rangle^+)$   $(\text{Tabulate } (\langle \text{var} \rangle \ Z^+)^+ \langle \text{vec3} \rangle)$

# Visual programs



Mao\*, Zhang\*, et al 2019



Ellis\*, Nye\*, Pu\*, Sosa\*, et al 2019



**for**  $i = 1..3$   
    **for**  $j = 1..1$   
        **draw**( $i^2$ ,  $j^1$ , )  
        \*\*\*

partial image  $x_{\text{part}}$

synthesized program  $P_{\text{part}}$

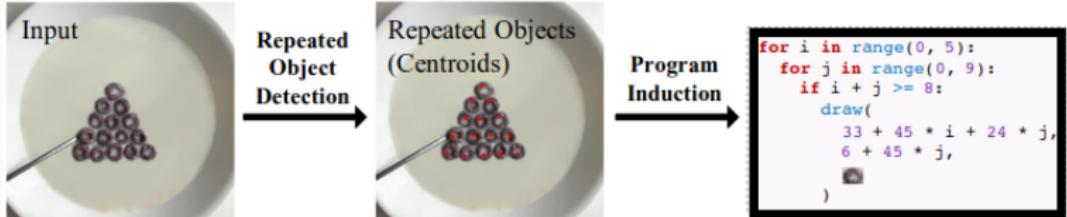


Draw("Top", "Circle", position, geometry)  
  
**for**( $i < 2$ , "translation", a)  
    **for**( $j < 2$ , "translation", b)  
        Draw("Leg", "Cub", position +  $i^a + j^b$ ,  
  
**for**( $i < 2$ , "translation", c)  
        Draw("Layer", "Rec", position +  $i^c$ , geomet

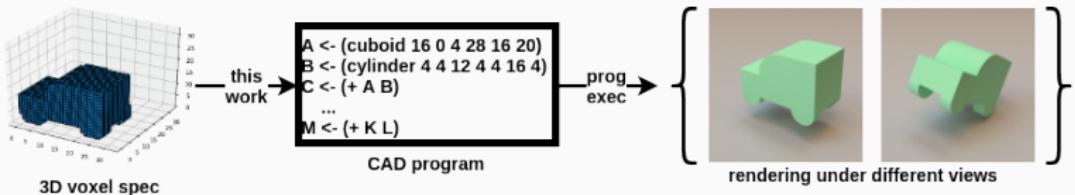
Young et al 2019

Tian et al 2019

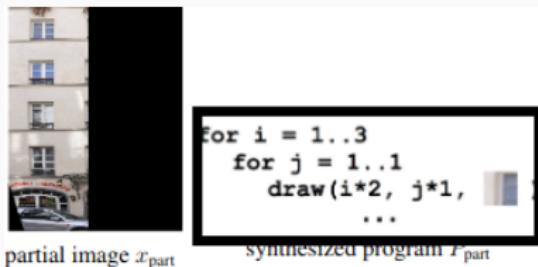
# Where does this language come from?



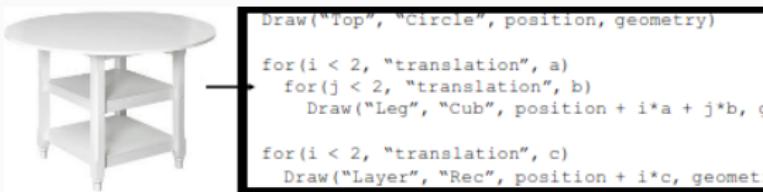
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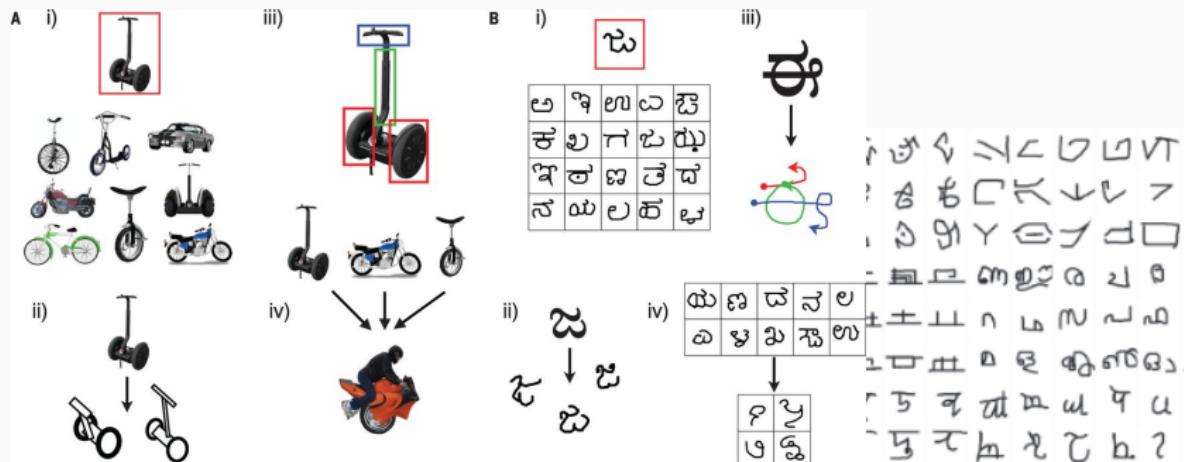


Young et al 2019



Tian et al 2019

## Human-level Program induction



Program Induction and learning to learn  
learning a DSL  
learning to synthesize  
synergy between DSL+learned synthesizer

## Learning to write code

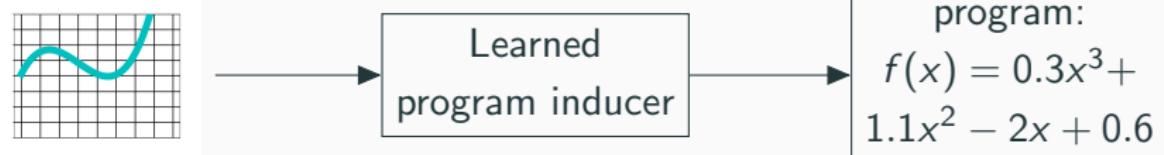
Goal: acquire domain-specific knowledge needed to induce a class of programs

- Library of abstractions (domain specific language)
- Inference strategy (synthesis algorithm)

# Learning to write code

Goal: acquire domain-specific knowledge needed to induce a class of programs

- Library of abstractions (domain specific language)
- Inference strategy (synthesis algorithm)



Concepts:  $x^3$ ,  $\alpha x + \beta$ , etc

Inference strategy: neurosymbolic search for programs

# Library learning

## Initial Primitives

: 

map

fold 

if

cons

>

: 

## Sample Problem: sort list

[9 2 7 1] → [1 2 7 9]

[3 8 9 4 2] → [2 3 4 8 9]

[6 2 2 3 8 5] → [2 2 3 5 6 8]

...

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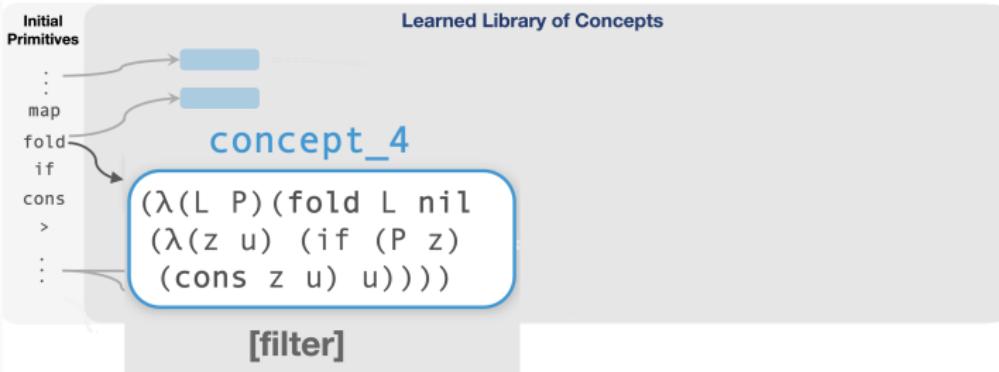
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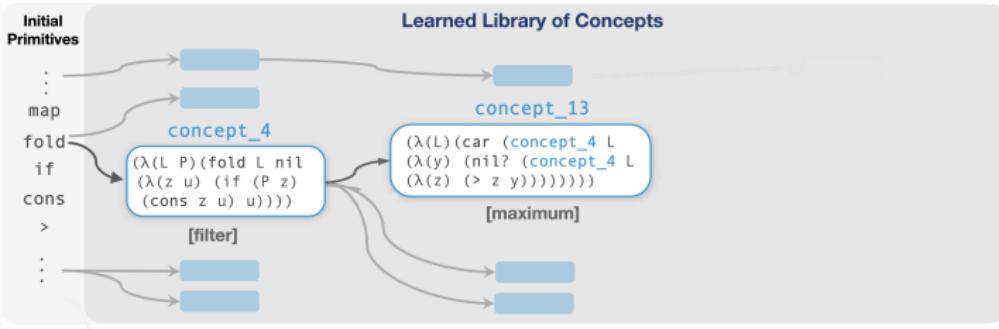
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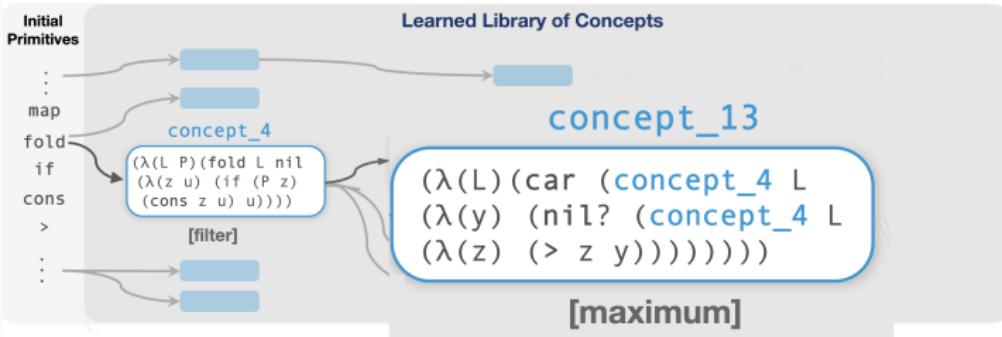
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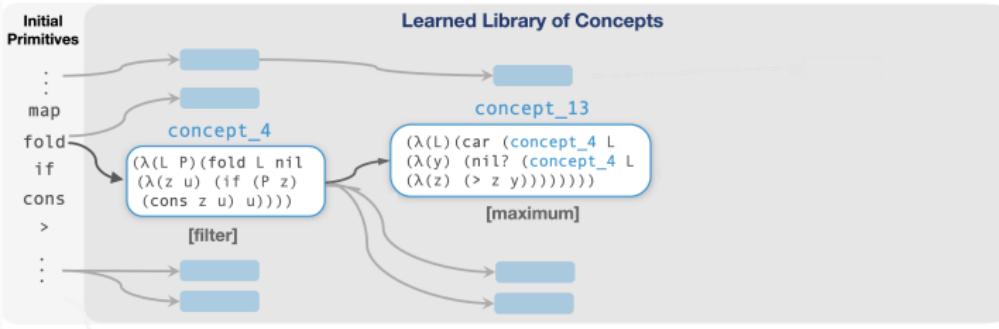
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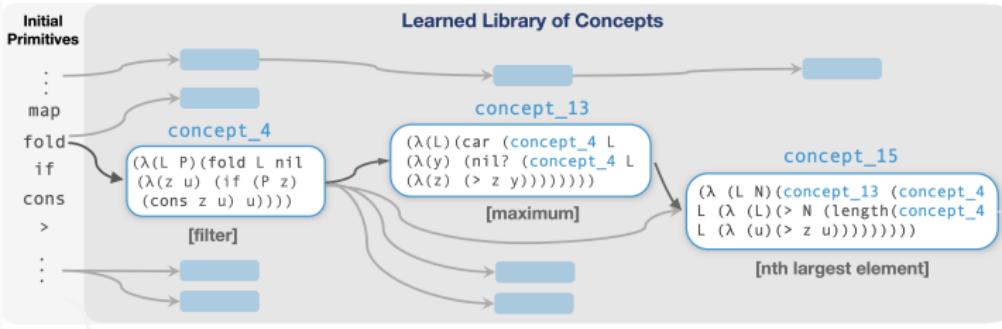
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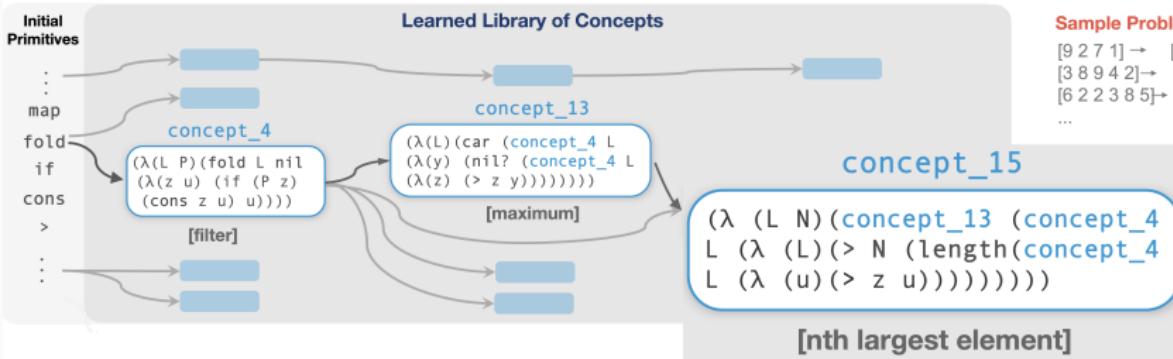
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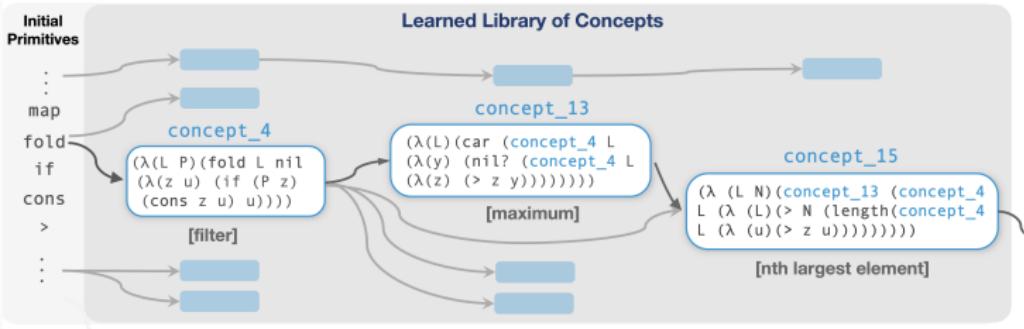
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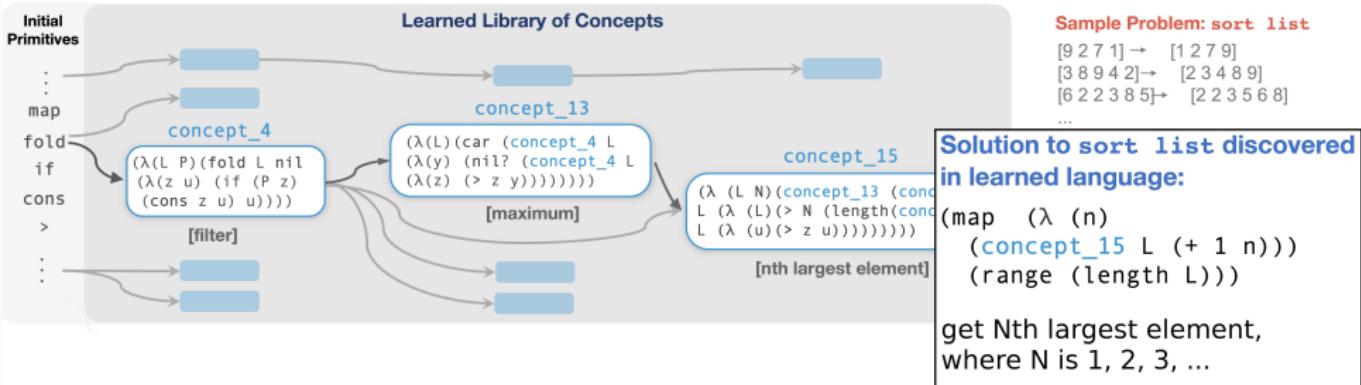
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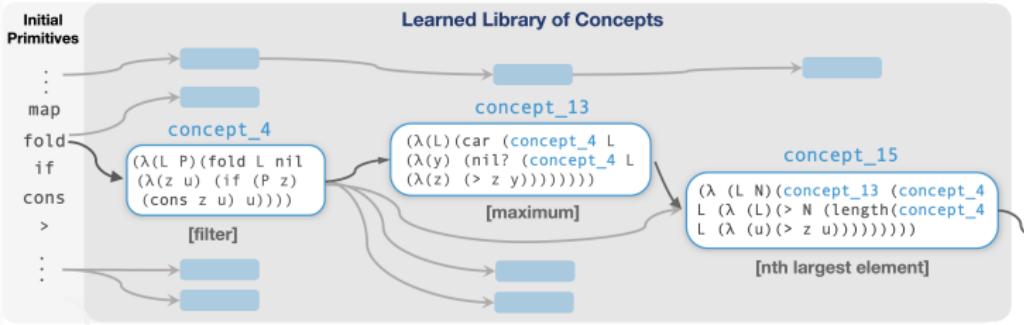
Solution to sort list discovered in learned language:

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(map (λ (n)
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# Library learning



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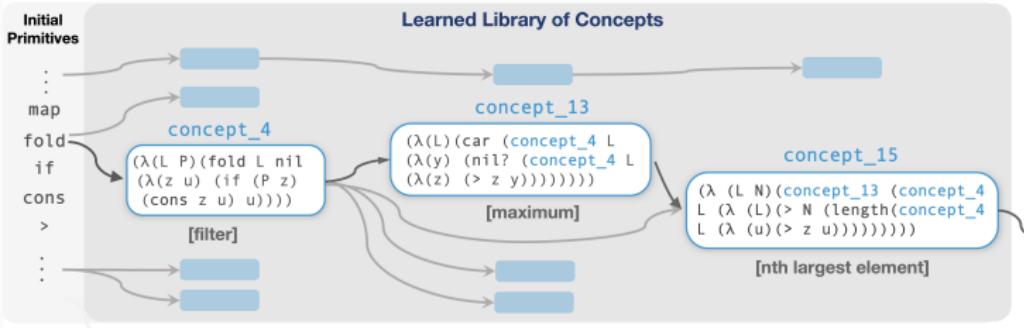
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get Nth largest element,  
where N is 1, 2, 3, ...

## Solution rewritten in initial primitives:

```
(lambda (x) (map (lambda (y) (car (fold (fold x nil (lambda (z u) (if (gt? (+ y 1) (length
(fold x nil (lambda (v w) (if (gt? z v) (cons v w) w)))) (cons z u) u))) nil (lambda (a b) (if
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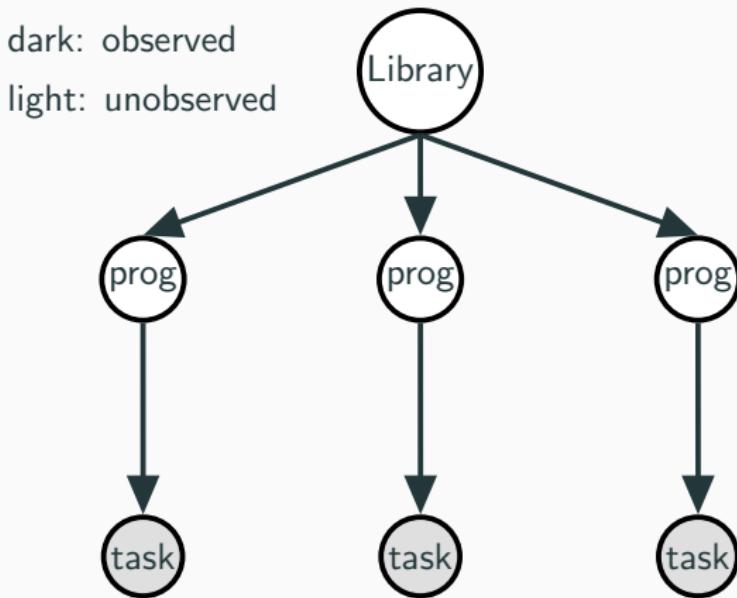
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induced sort program found in  $\leq 10\text{min}$ . Brute-force search  
without learned library would take  $\approx 10^{73}$  years

- **Wake:** Solve problems by writing programs
- **Sleep:** Improve library and neural recognition model:
  - **Abstraction sleep:** Improve library
  - **Dream sleep:** Improve neural recognition model

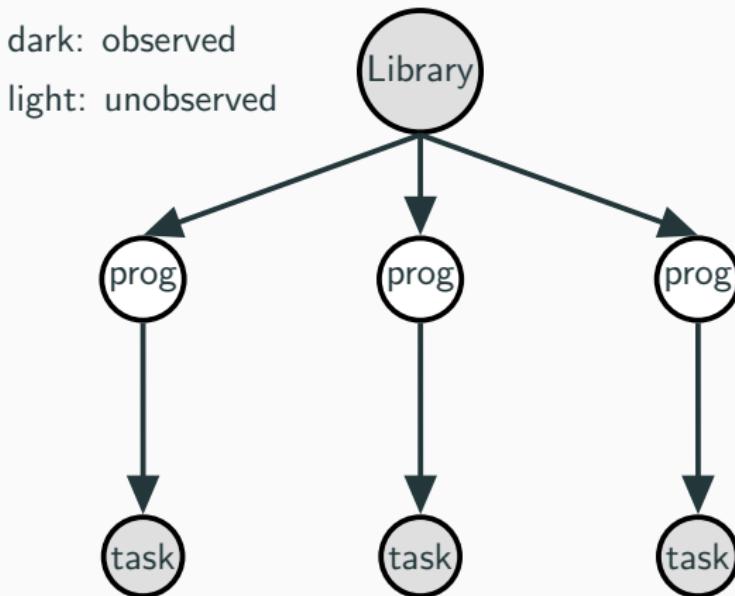
cf. Helmholtz machine, wake/sleep neural network training algorithms

# Library learning as Bayesian inference



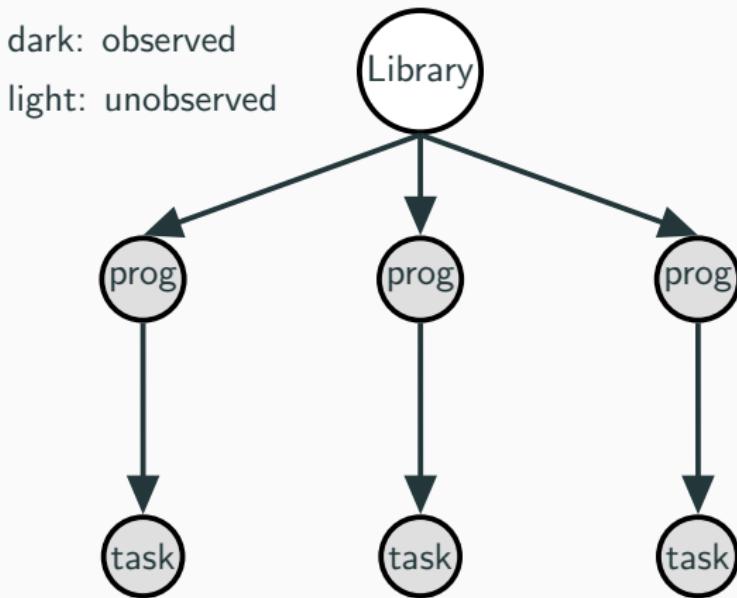
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# Library learning as Bayesian inference



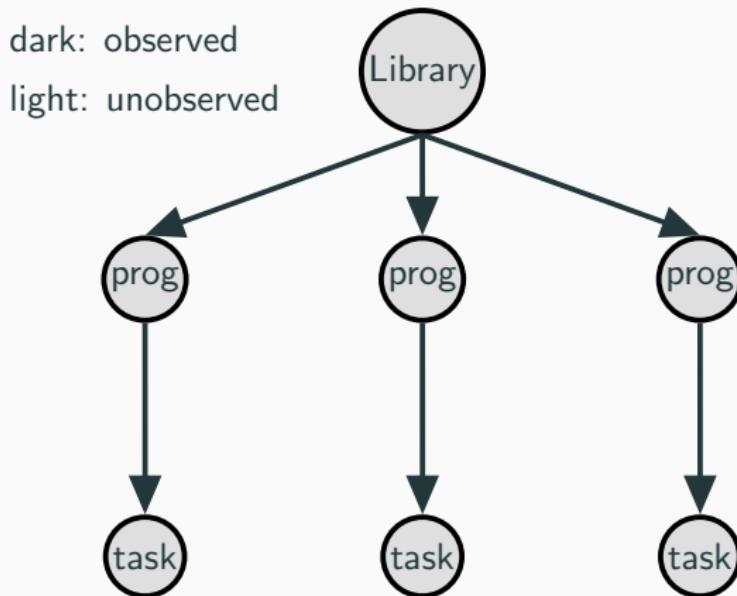
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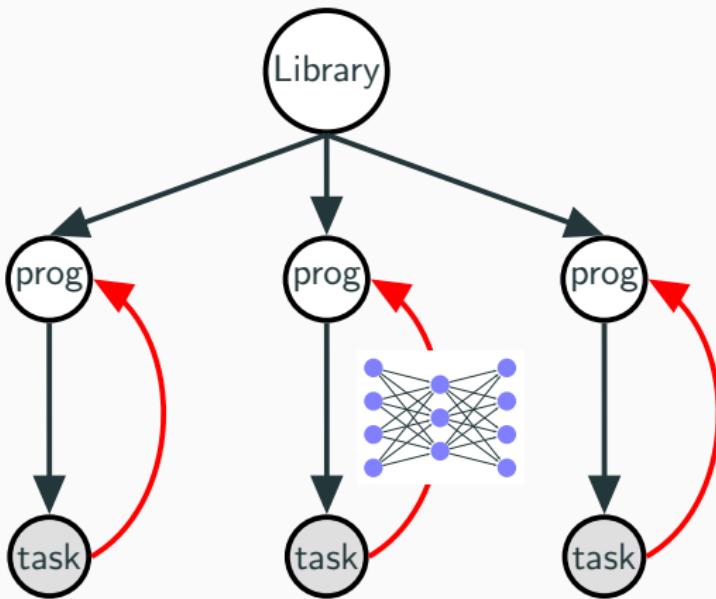
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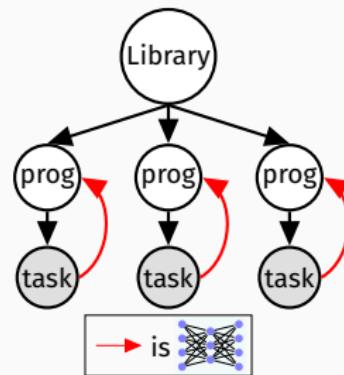


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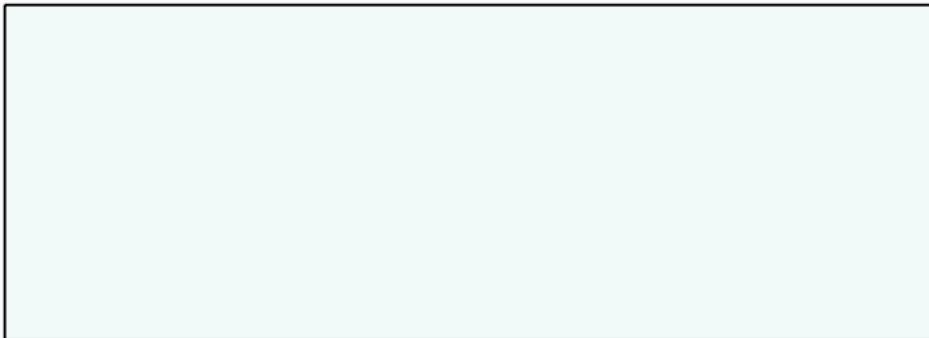
# Library learning as neurally-guided Bayesian inference



library learning via program analysis +  
new neural inference network for program synthesis +  
better program representation (Lisp+polymorphic types [Milner 1978])

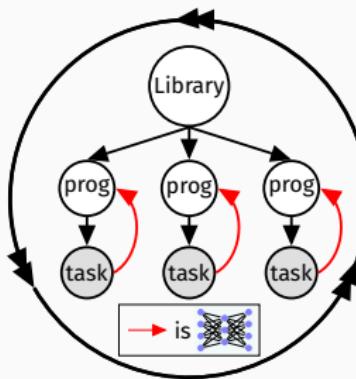


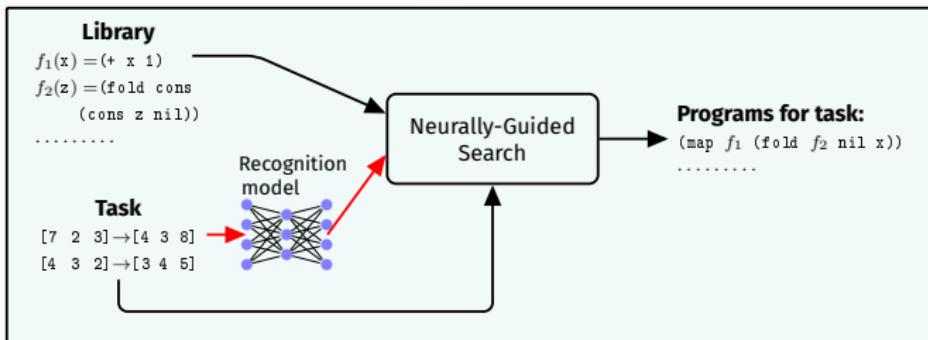
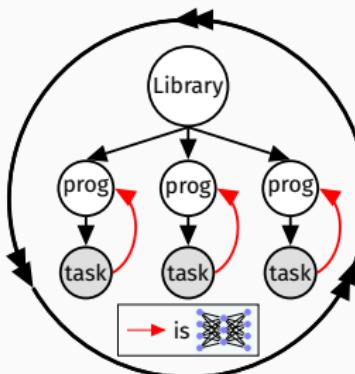
WAKE

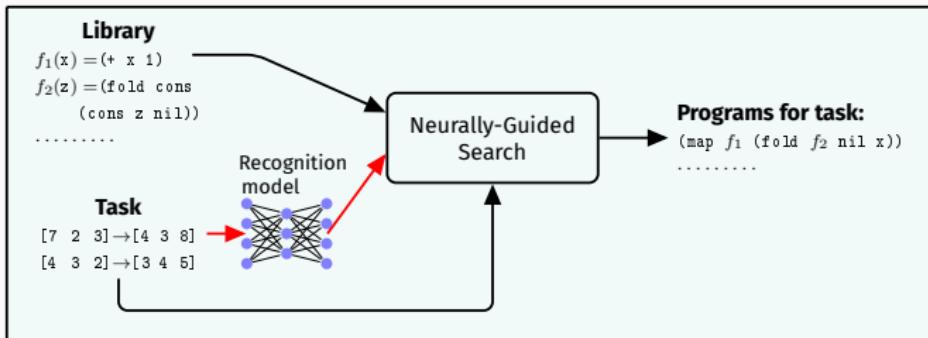
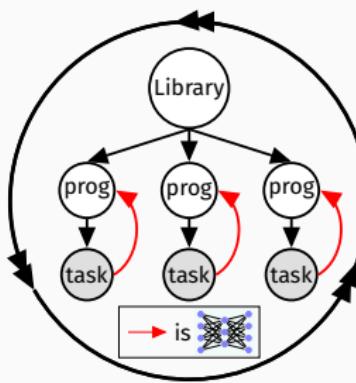
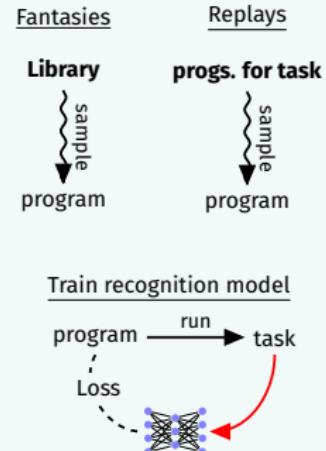


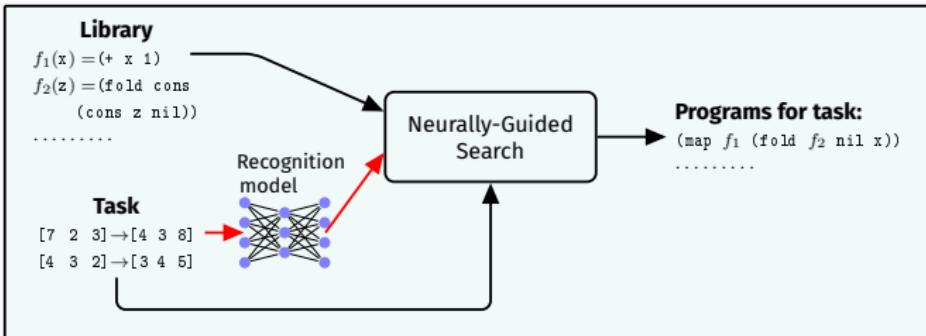
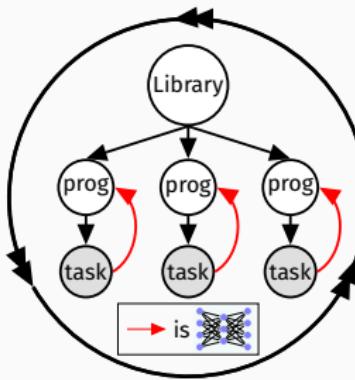
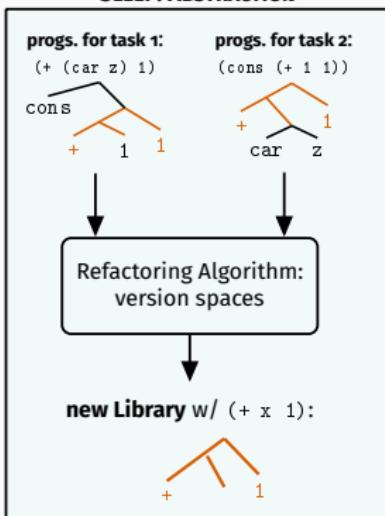
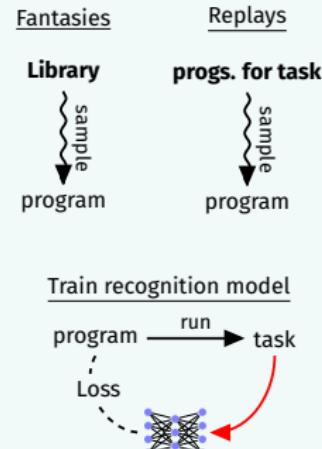
SLEEP: ABSTRACTION

SLEEP: DREAMING

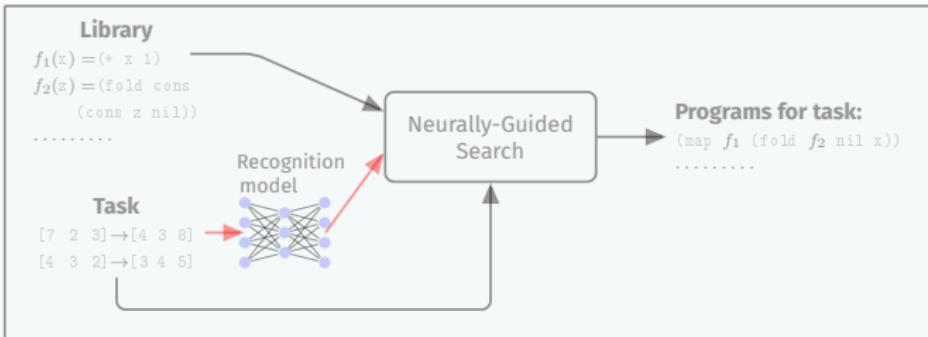


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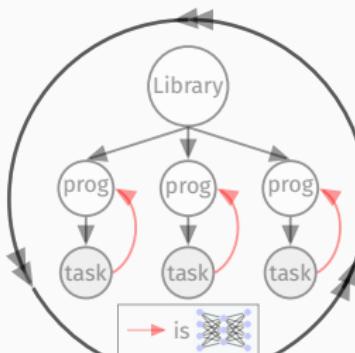
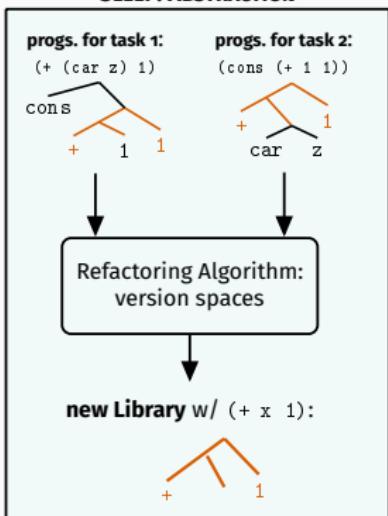
**WAKE****SLEEP: ABSTRACTION****SLEEP: DREAMING**

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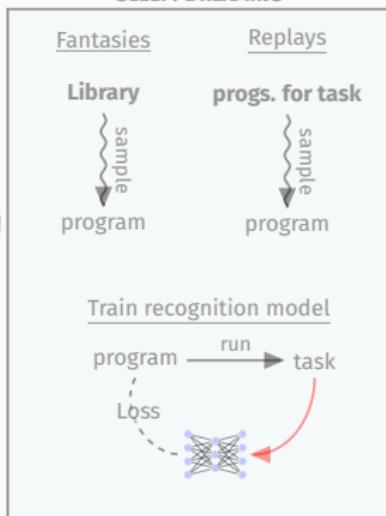
## WAKE



## SLEEP: ABSTRACTION



## SLEEP: DREAMING



Program Induction and learning to learn  
**learning a DSL**  
learning to synthesize  
synergy between DSL+learned synthesizer

# Abstraction Sleep: Growing the library via refactoring

**Task:**  $[1 \ 2 \ 3] \rightarrow [2 \ 4 \ 6]$   
 $[4 \ 3 \ 4] \rightarrow [8 \ 6 \ 8]$

**Task:**  $[1 \ 2 \ 3] \rightarrow [0 \ 1 \ 2]$   
 $[4 \ 3 \ 4] \rightarrow [3 \ 2 \ 3]$

# Abstraction Sleep: Growing the library via refactoring

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Wake: program search

```
(Y (λ (r 1) (if (nil? 1) nil  
           (cons (+ (car 1) (car 1))  
                 (r (cdr 1)))))))
```

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(Y (λ (r 1) (if (nil? 1) nil  
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```

refactor

$(10^{14}$  refactorings)

```
((λ (f) (Y (λ (r 1) (if (nil? 1)  
                           nil  
                           (cons (f (car 1))  
                                 (r (cdr 1)))))))  
  (λ (z) (+ z z)))
```

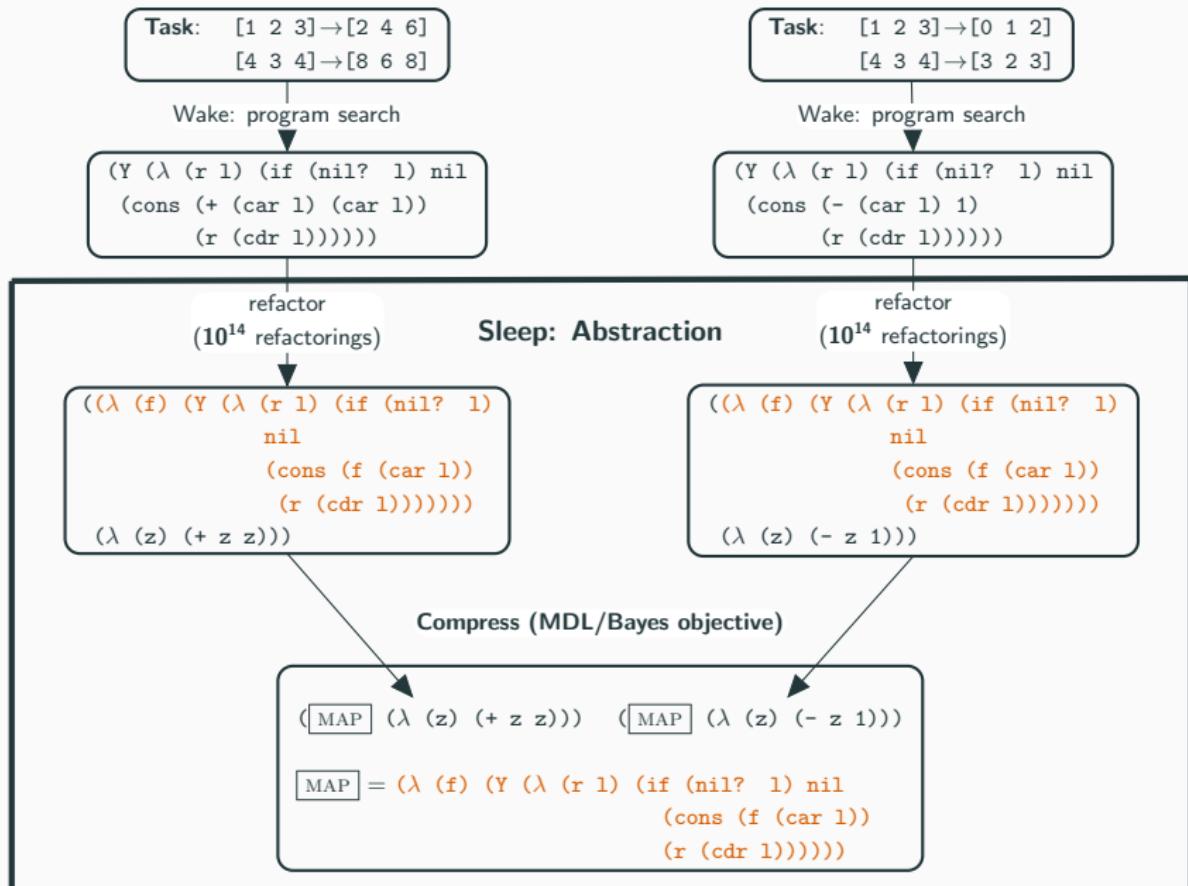
refactor

$(10^{14}$  refactorings)

```
((λ (f) (Y (λ (r 1) (if (nil? 1)  
                           nil  
                           (cons (f (car 1))  
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  (λ (z) (- z 1)))
```

## Sleep: Abstraction

# Abstraction Sleep: Growing the library via refactoring



# Abstraction Sleep: Growing the library via refactoring

Task:  $[1\ 2\ 3] \rightarrow [2\ 4\ 6]$   
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Task:  $[1\ 2\ 3] \rightarrow [0\ 1\ 2]$   
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Wake: program search

```
(Y (λ (r 1) (if (nil? 1) nil  
           (cons (- (car 1) 1)  
                  (r (cdr 1)))))))
```

these  $10^{14}$  refactorings represented in exponentially more efficient refactoring data structure:

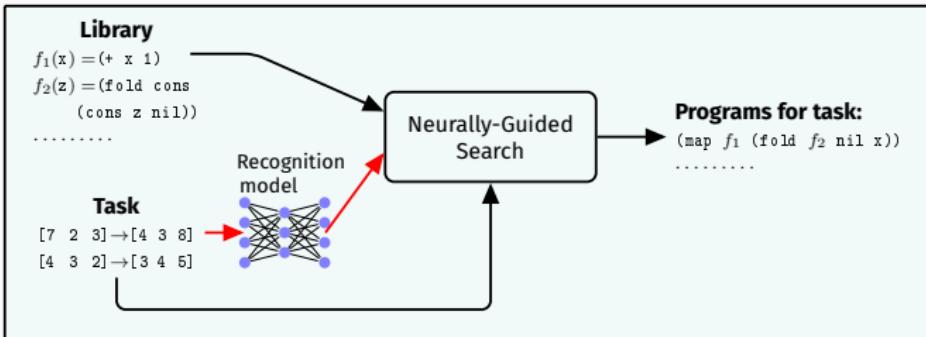
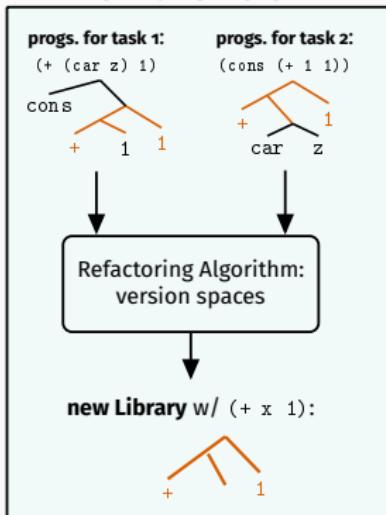
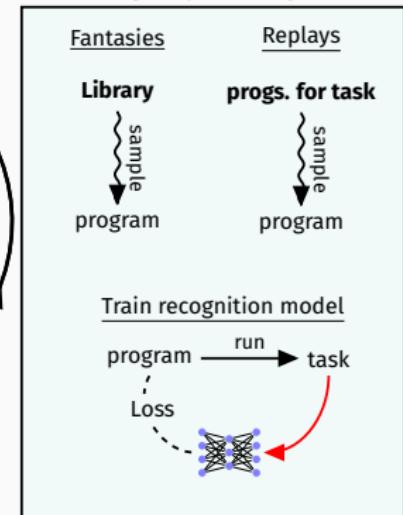
$(\lambda$  equivalence graphs+version spaces using  $10^6$  nodes,  
calculated in under 5min

c.f. [Tate et al 2009], [Gulwani 2012]

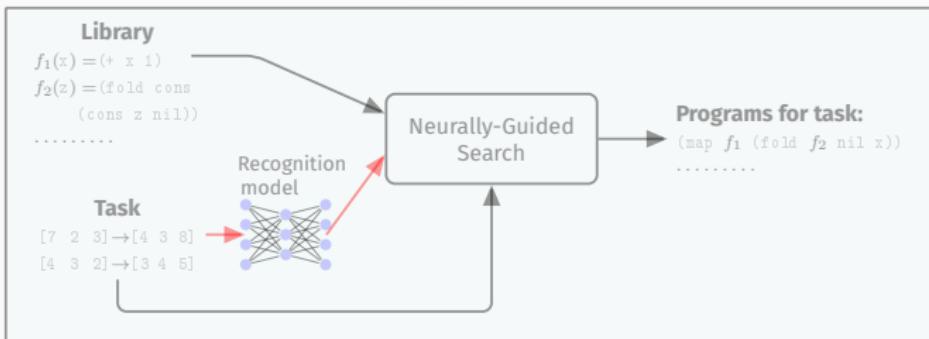
Compress (MDL/Bayes objective)

```
(MAP (λ (z) (+ z z))) (MAP (λ (z) (- z 1)))  
MAP = (λ (f) (Y (λ (r 1) (if (nil? 1) nil  
                           (cons (f (car 1))  
                                 (r (cdr 1)))))))
```

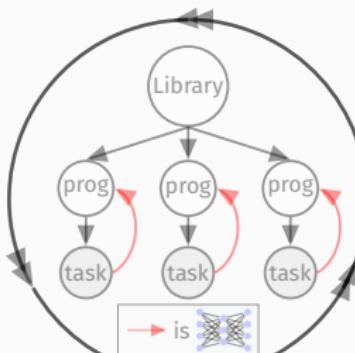
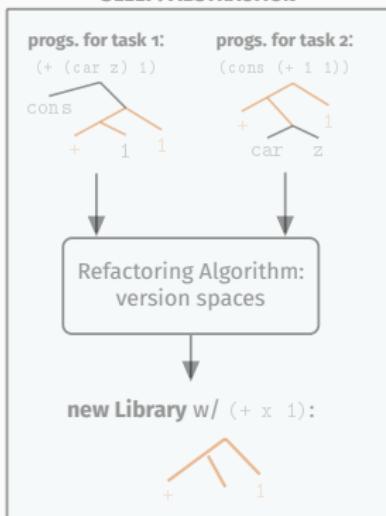
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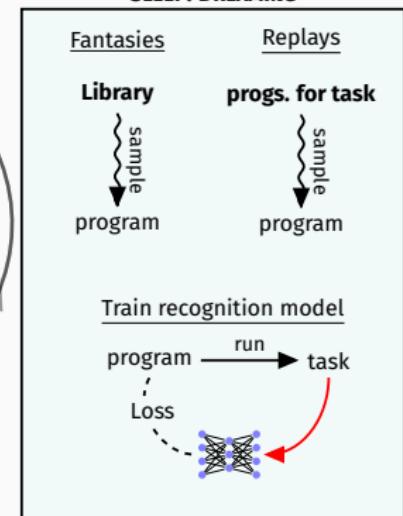
## WAKE



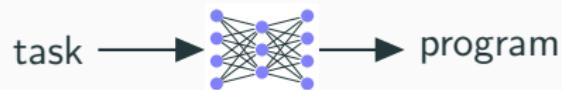
## SLEEP: ABSTRACTION



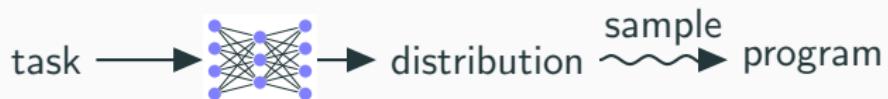
## SLEEP: DREAMING



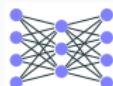
# Neural recognition model guides search



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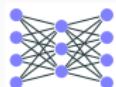


is a...

recurrent network (Devlin et al 2017)

unigram model (Menon et al 2013; Balog et al 2016)

# Neural recognition model guides search

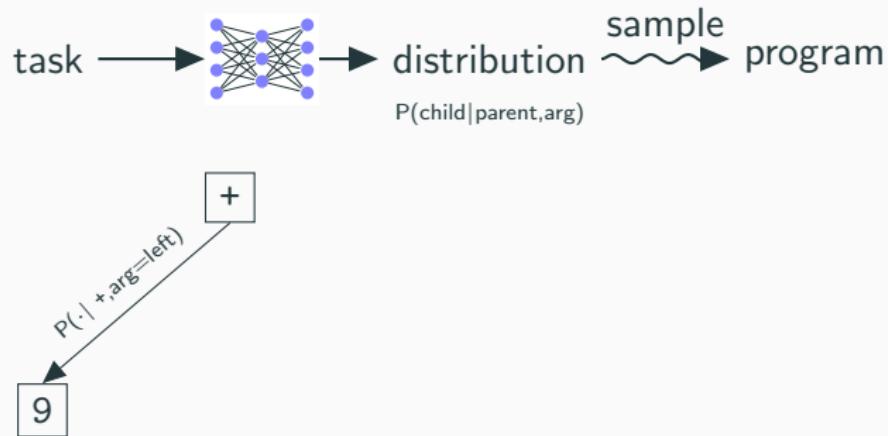


is a “**bigram**” model over syntax trees

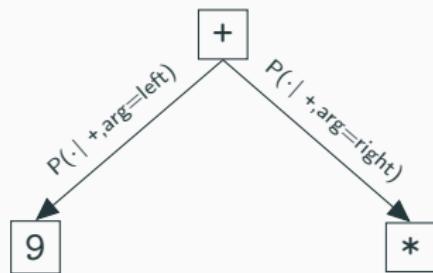
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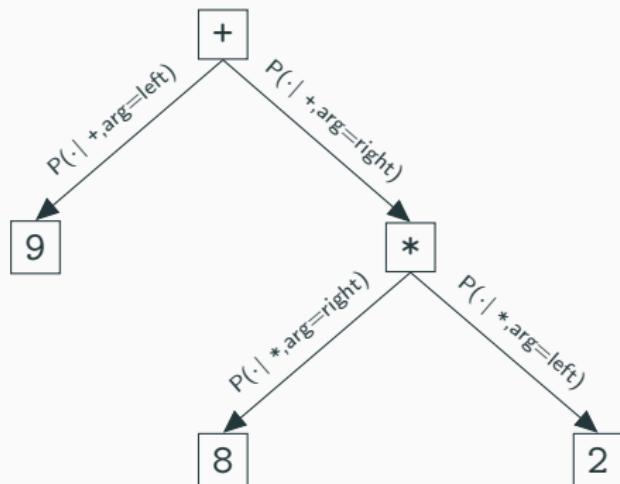
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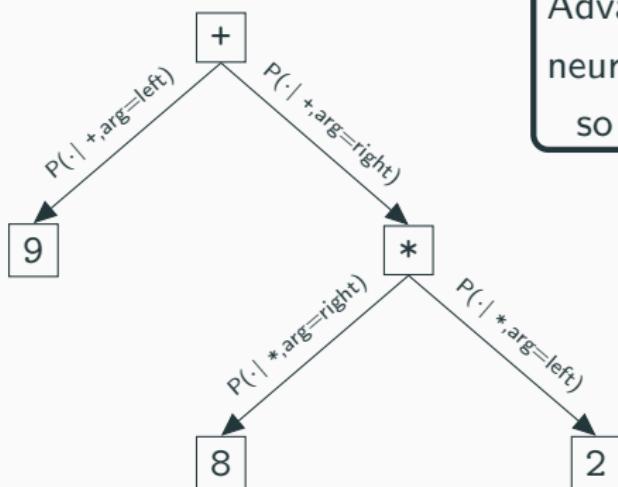
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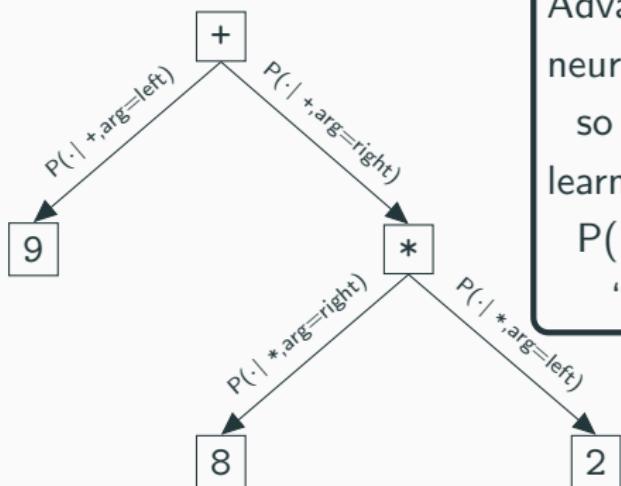


# Neural recognition model guides search



Advantages:  
neural net runs once per task,  
so CPU bottlenecks instead of GPU

# Neural recognition model guides search



Advantages:  
neural net runs once per task,  
so CPU bottlenecks instead of GPU  
learns to break syntactic symmetries:  
 $P(1|*,\text{arg}=left)=0.0$   
“do not multiply by one”

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learning a DSL  
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# DreamCoder Domains

## List Processing

### Sum List

[1 2 3] → 6

[4 6 8 1] → 17

### Double

[1 2 3] → [2 4 6]

[4 5 1] → [8 10 2]

## Text Editing

### Abbreviate

Allen Newell → A.N.

Herb Simon → H.S.

### Drop Last Three

shrdlu → shr

shakey → sha

## Regexes

### Phone numbers

(555) 867-5309

(650) 555-2368

### Currency

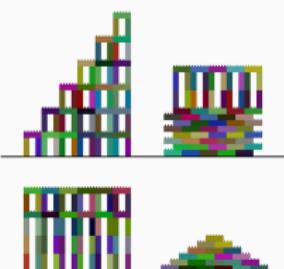
\$100.25

\$4.50

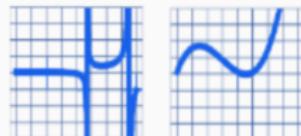
## LOGO Graphics



## Block Towers



## Symbolic Regression



$$y = f(x)$$

## Recursive Programming

### Filter Red

[■■■■■■■■] → [■■■■]

[■■■■■■■■■■] → [■■■■■■■■]

[■■■■■■■■■■] → [■■■■■■■■]

## Physical Laws

$$\vec{a} = \frac{1}{m} \sum_i \vec{F}_i$$

$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r}|^2} \hat{r}$$

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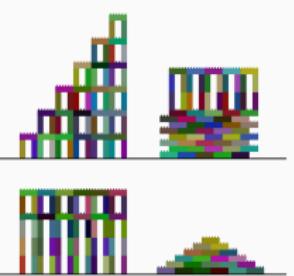
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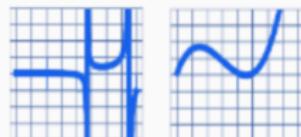
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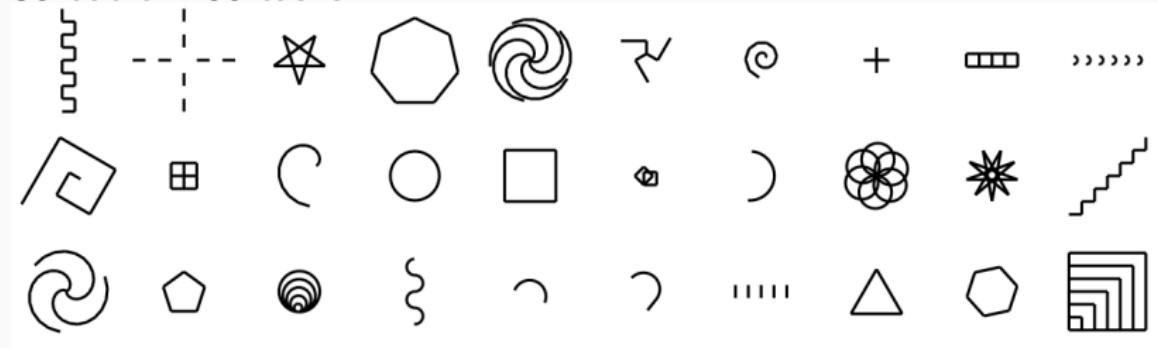
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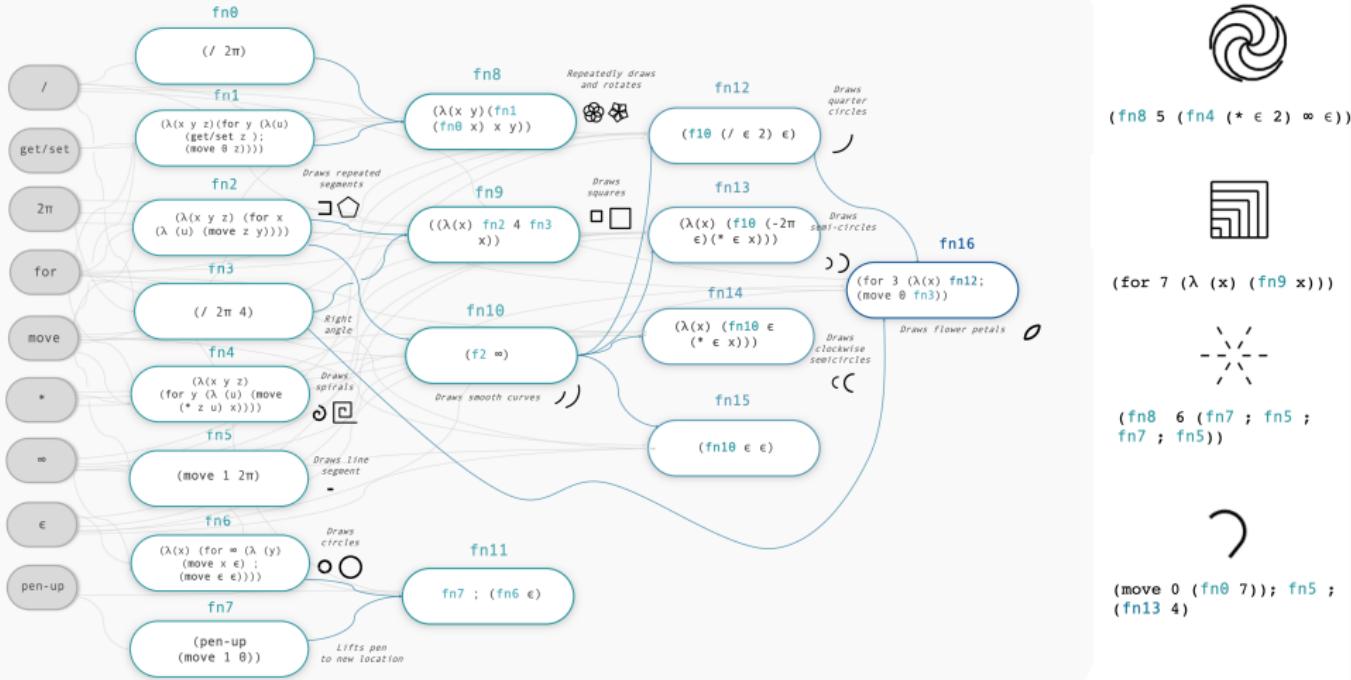
$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r}|^2} \hat{r}$$

# LOGO Turtle Graphics

30 out of 160 tasks



# LOGO Turtle Graphics – learning an interpretable library



(fn8 5 (fn4 (\* ε 2) ∞))



(for 7 (λ (x) (fn9 x)))

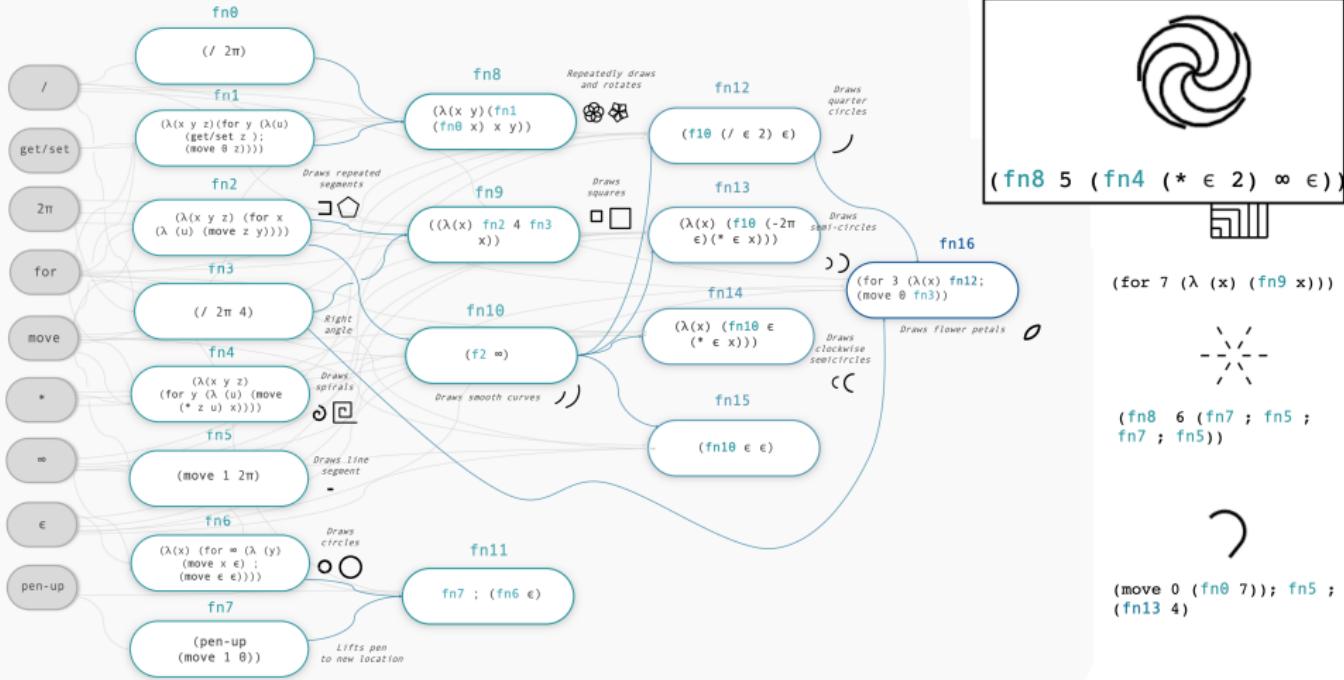


(fn8 6 (fn7 ; fn5 ; fn7 ; fn5))

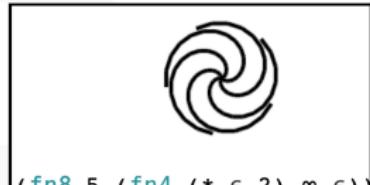
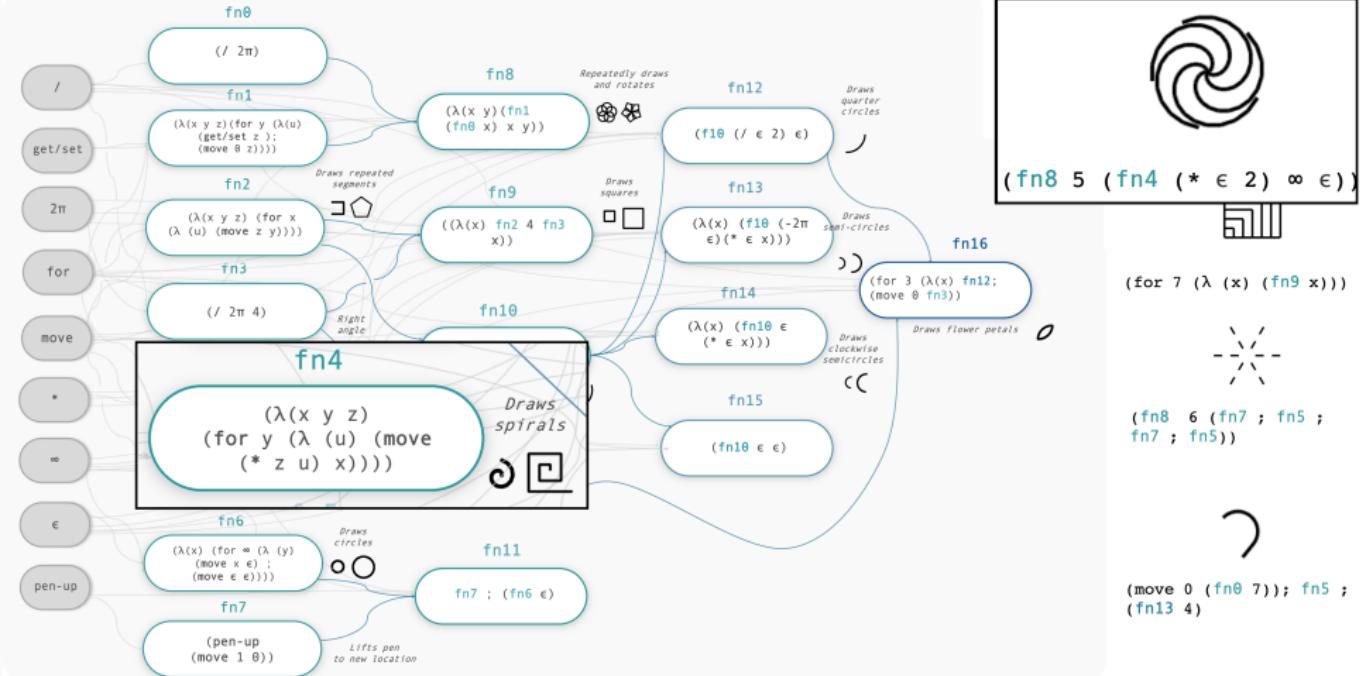


(move 0 (fn0 7)); fn5 ; (fn13 4)

# LOGO Turtle Graphics – learning an interpretable library



# LOGO Turtle Graphics – learning an interpretable library



$(\text{fn}8\ 5\ (\text{fn}4\ (*\ \infty\ 2)\ \infty\ \epsilon))$



$(\text{for } 7\ (\lambda(x)\ (\text{fn}9\ x)))$

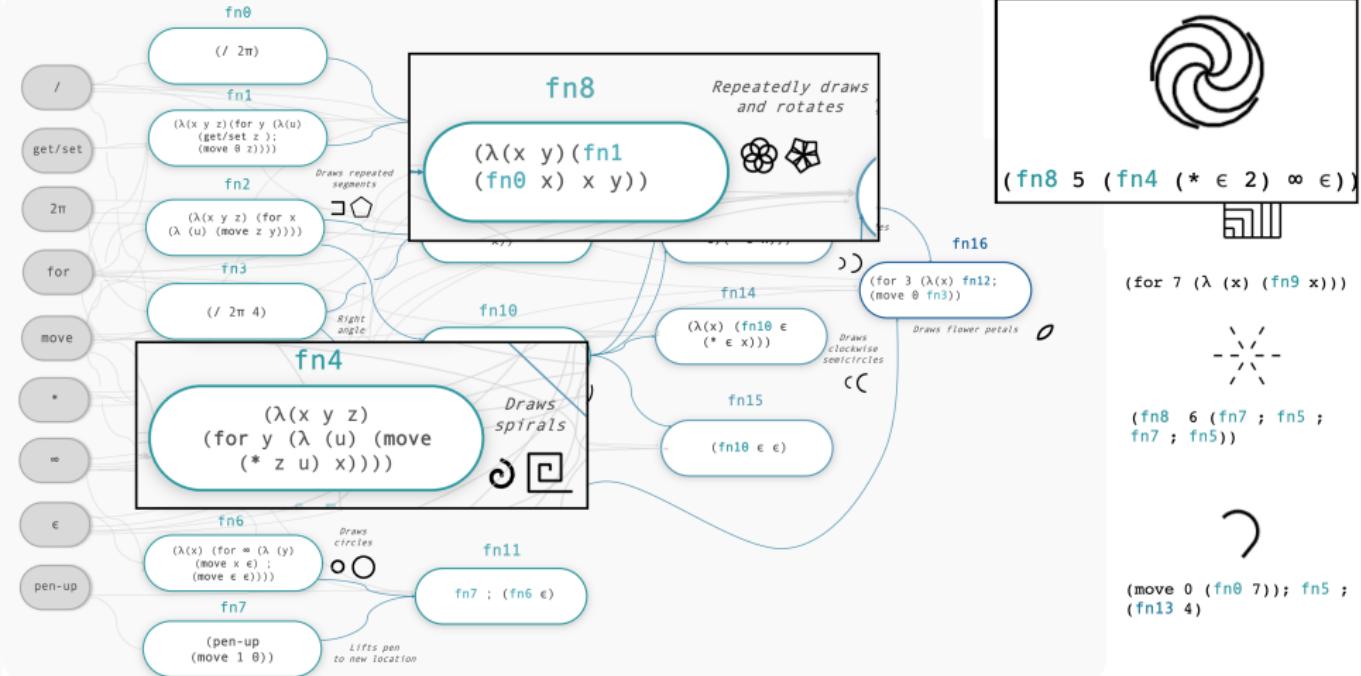


$(\text{fn}8\ 6\ (\text{fn}7\ ;\ \text{fn}5\ ;\ \text{fn}5))$

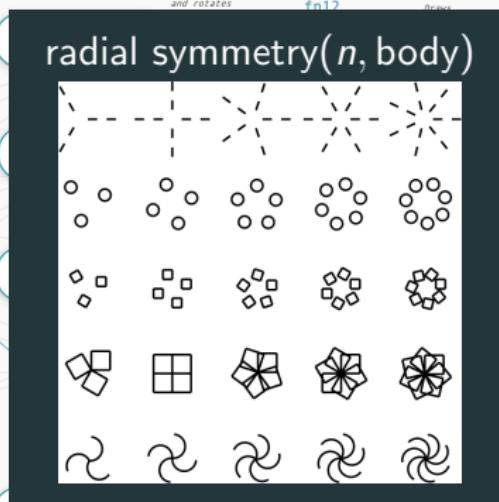
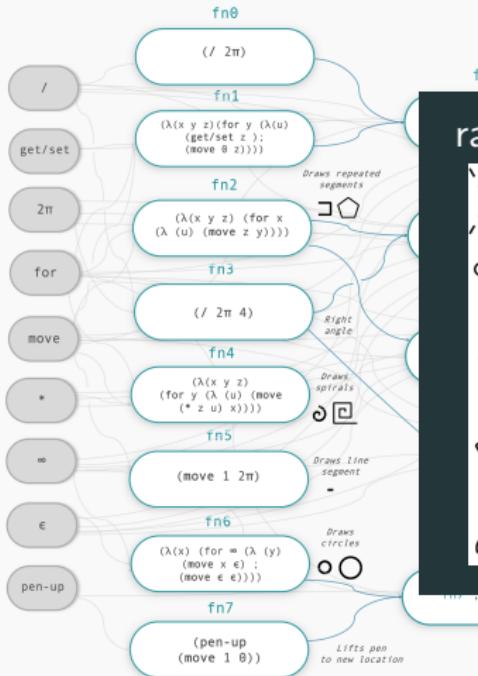


$(\text{move}\ 0\ (\text{fn}0\ 7));\ \text{fn}5\ ;\ (\text{fn}13\ 4)$

# LOGO Turtle Graphics – learning an interpretable library



LOGO Turtle Graphics – learning an interpretable library



(fn8 5 (fn4 (\* \ 2) \ \ ))



```
(for 7 (λ (x) (fn9 x))))
```

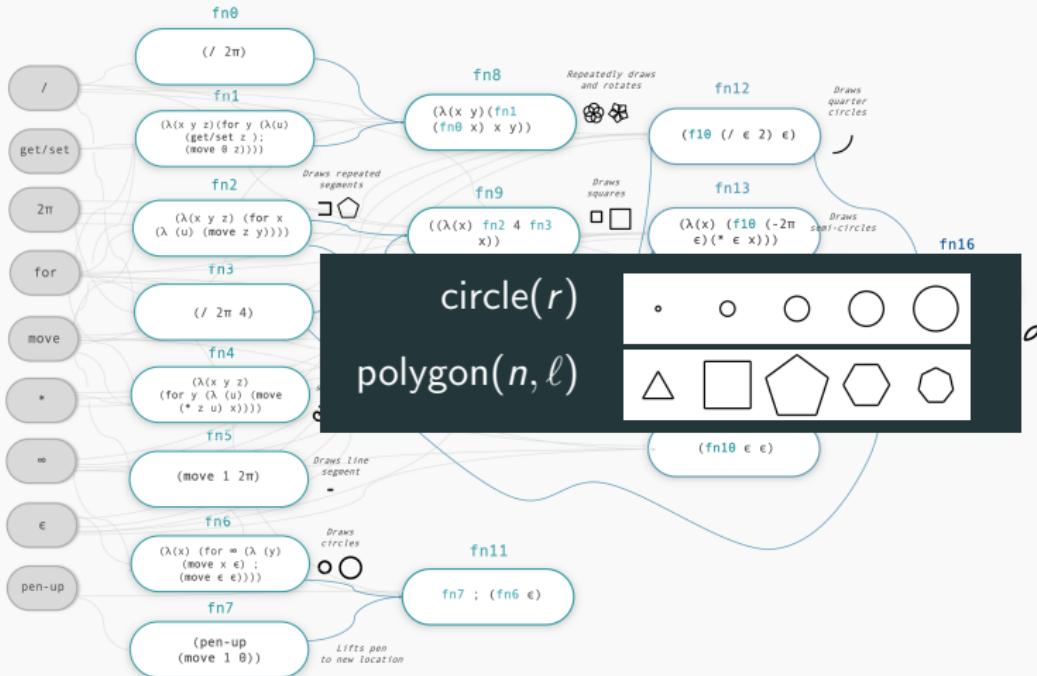


(fn8 6 (fn7 ; fn5 ;  
fn7 ; fn5)))



```
(move 0 (fn0 7)); fn5 ;  
(fn13 4)
```

# LOGO Turtle Graphics – learning an interpretable library



$(\text{fn8}\ 5\ (\text{fn4}\ (*\ \in\ 2)\ \infty))$



$(\text{for } 7\ (\lambda(x) (\text{fn9}\ x)))$

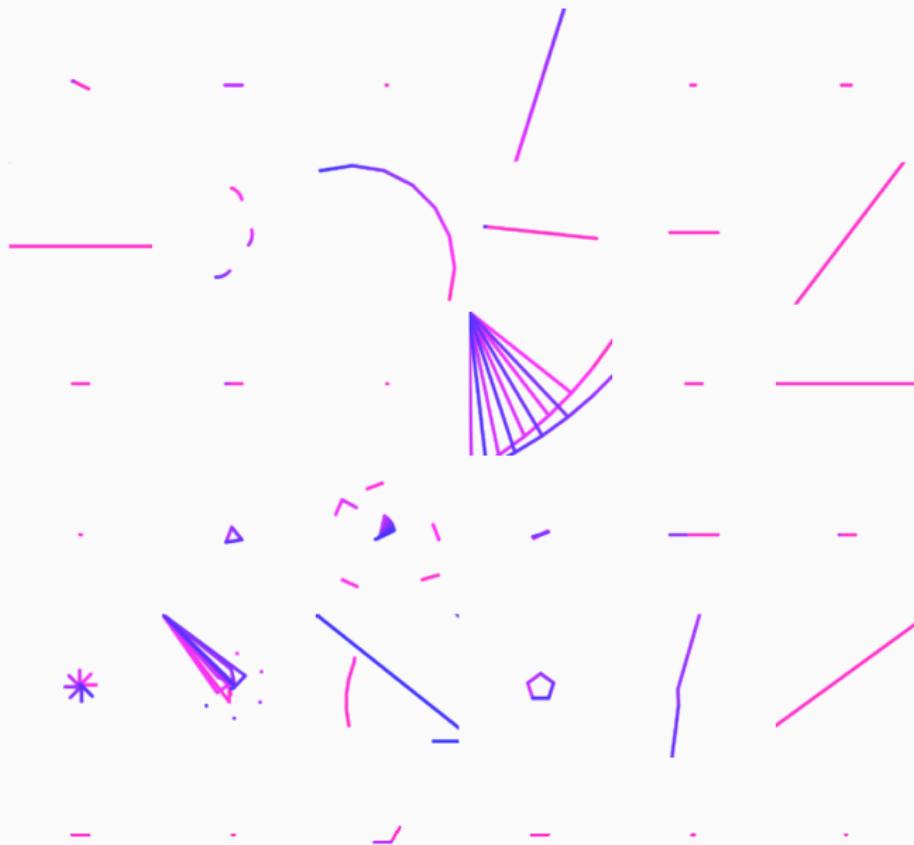


$(\text{fn8}\ 6\ (\text{fn7}\ ;\ \text{fn5}\ ;\ \text{fn7}\ ;\ \text{fn5}))$

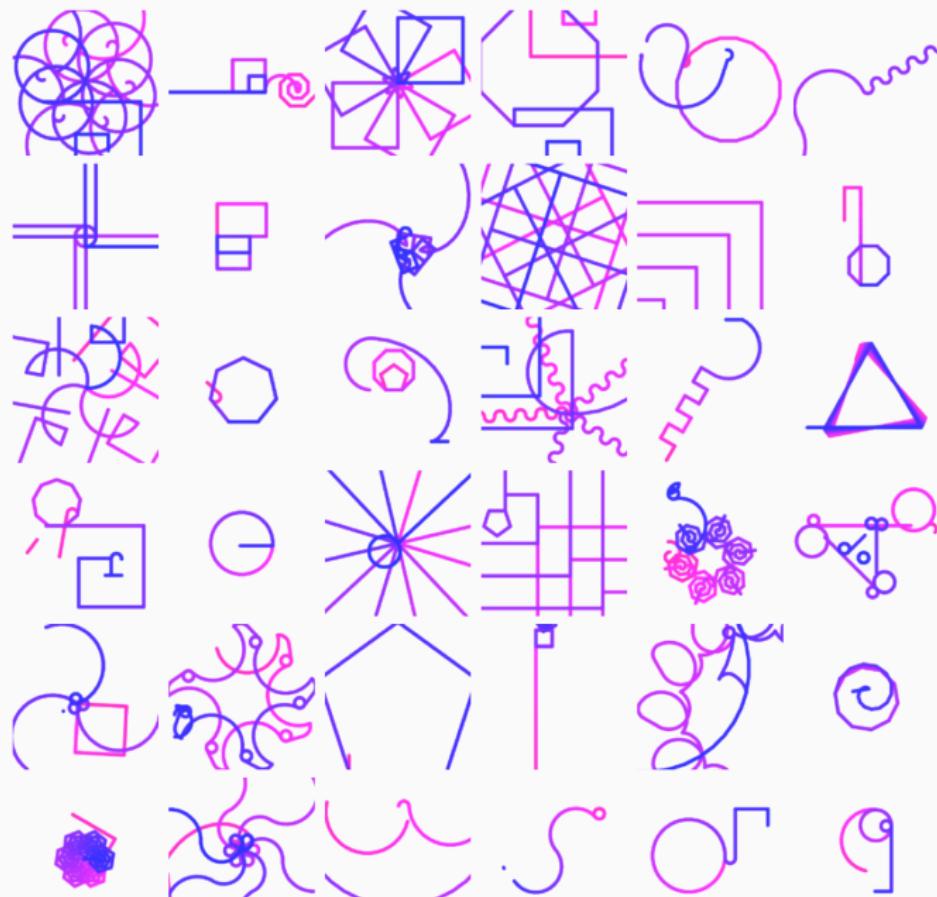


$(\text{move } 0\ (\text{fn0}\ 7));\ \text{fn5}\ ;\ (\text{fn13}\ 4)$

# What does DreamCoder dream of? (before learning)

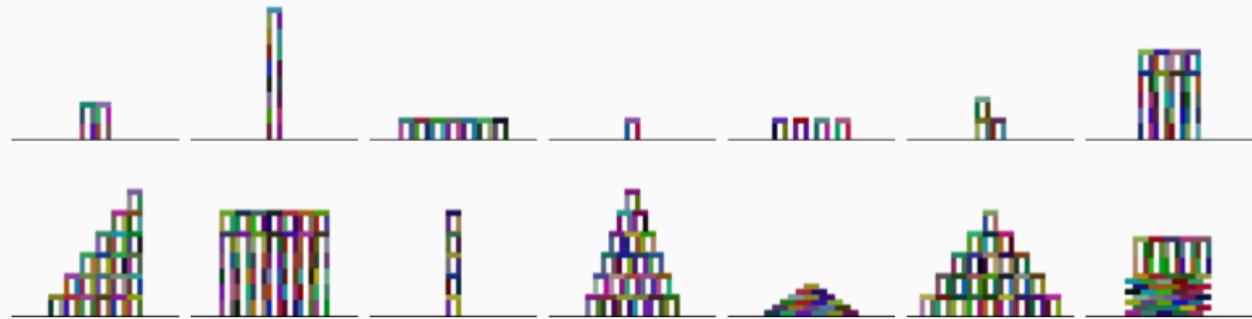


# What does DreamCoder dream of? (after learning)



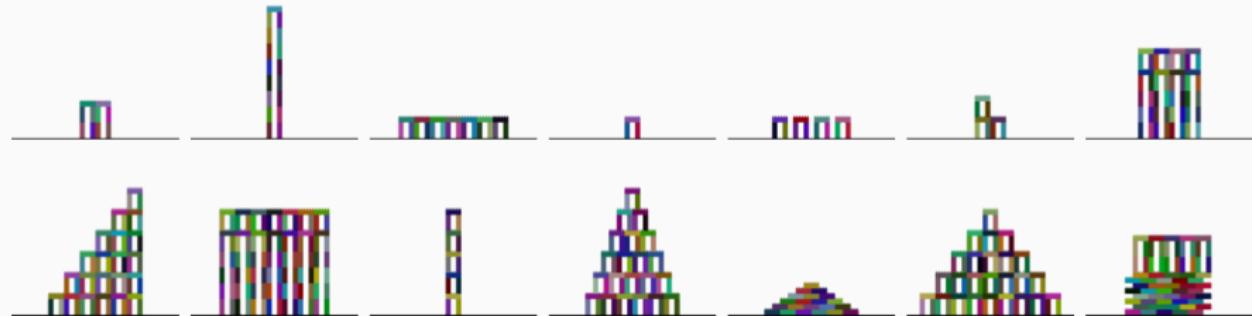
# Planning to build towers

example tasks (112 total)



# Planning to build towers

example tasks (112 total)

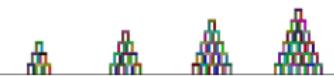


learned library routines ( $\approx 20$  total)

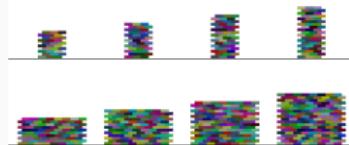
$\text{arch}(h)$



$\text{pyramid}(h)$



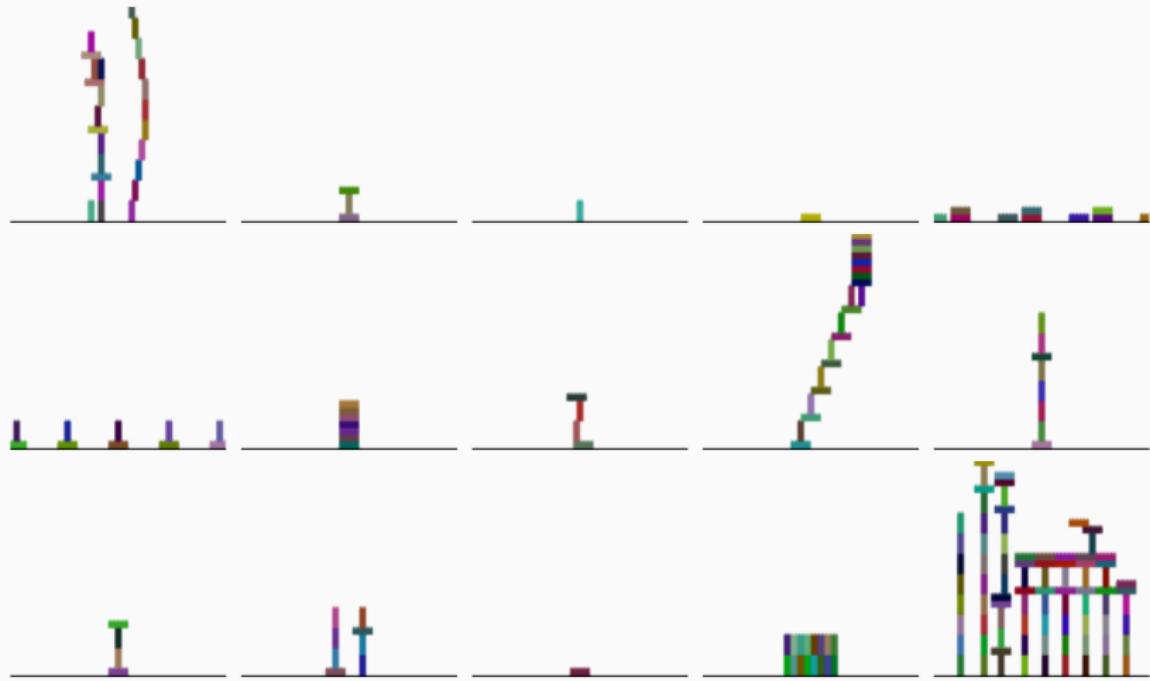
$\text{wall}(w, h)$



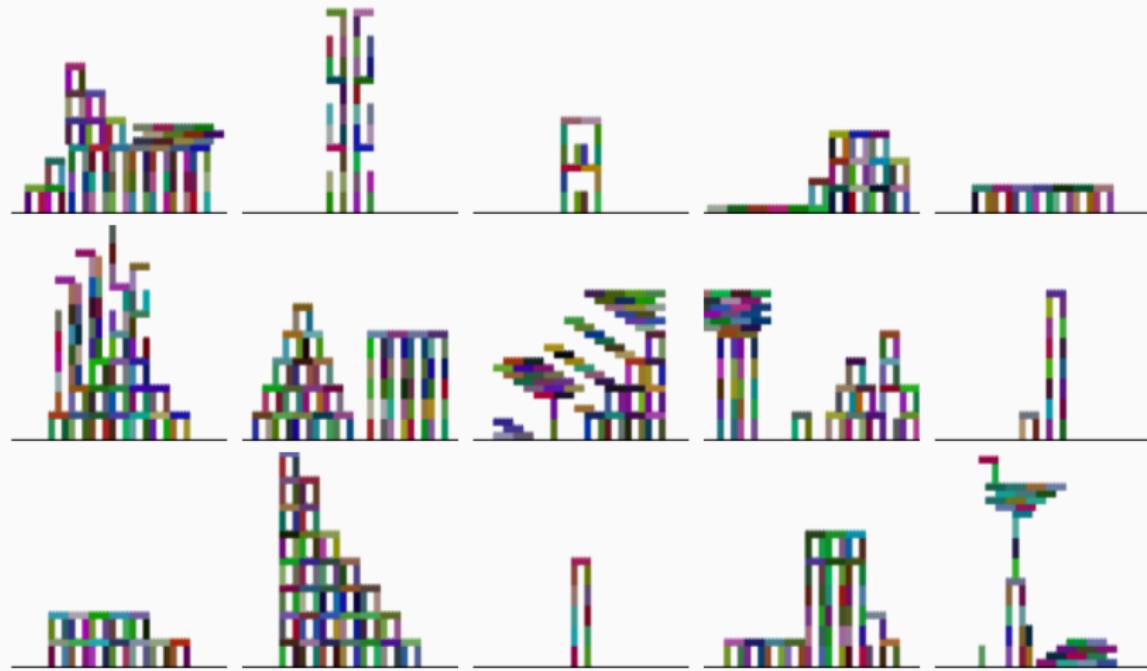
$\text{bridge}(w, h)$



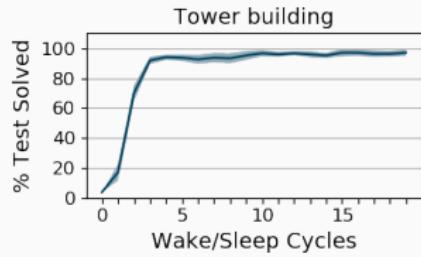
## Dreams before learning



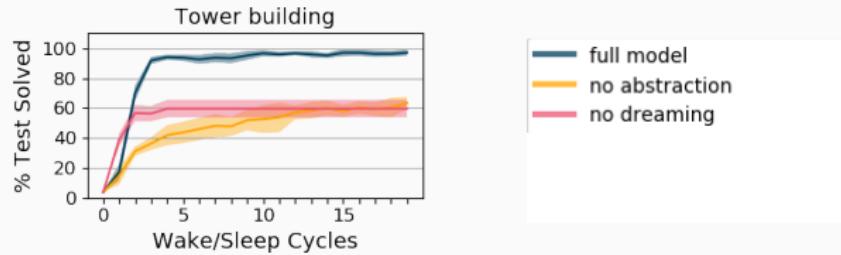
## Dreams after learning



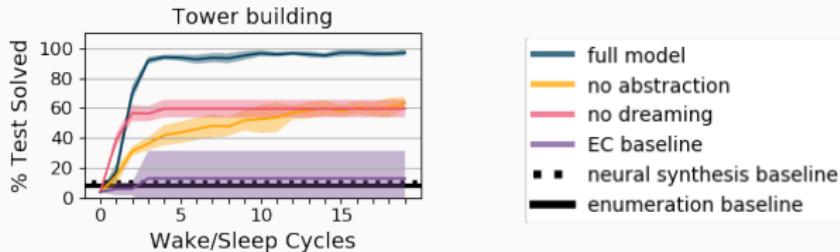
# Learning dynamics



# Learning dynamics

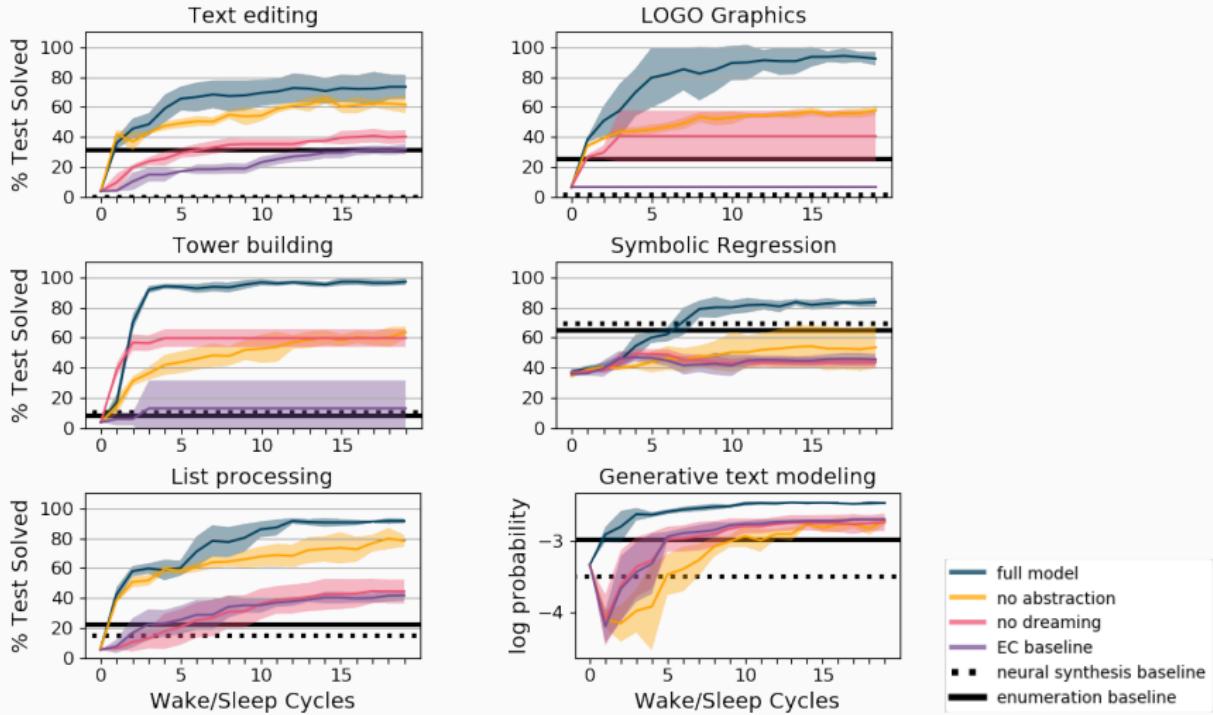


# Learning dynamics



baselines: Exploration-Compression, EC [Dechter et al. 2013]  
neural program synthesis, RobustFill [Devlin et al. 2017]  
24 hours of brute-force enumeration

# Learning dynamics



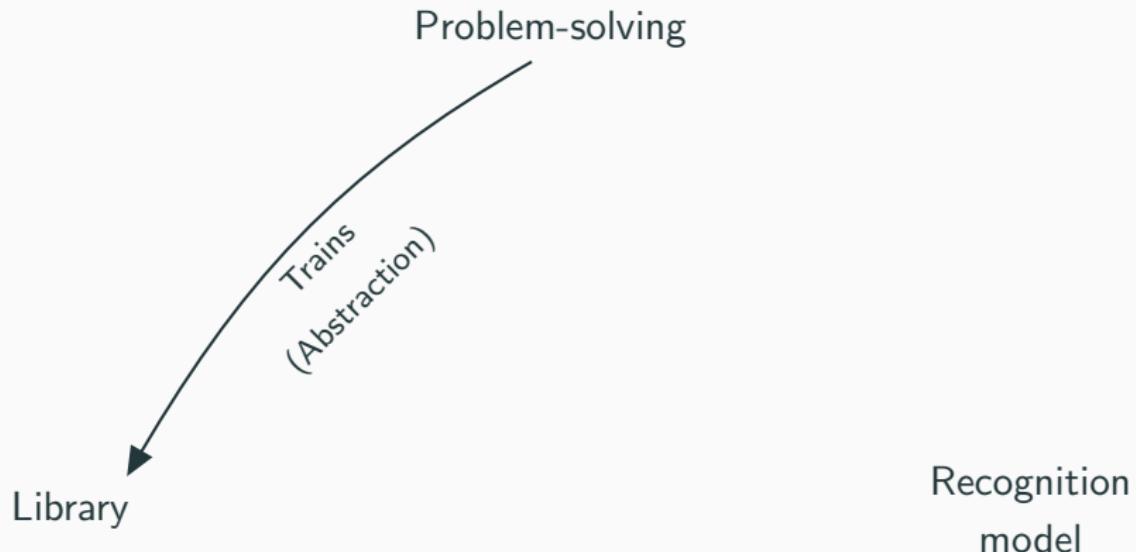
# Synergy between recognition model and library learning

Problem-solving

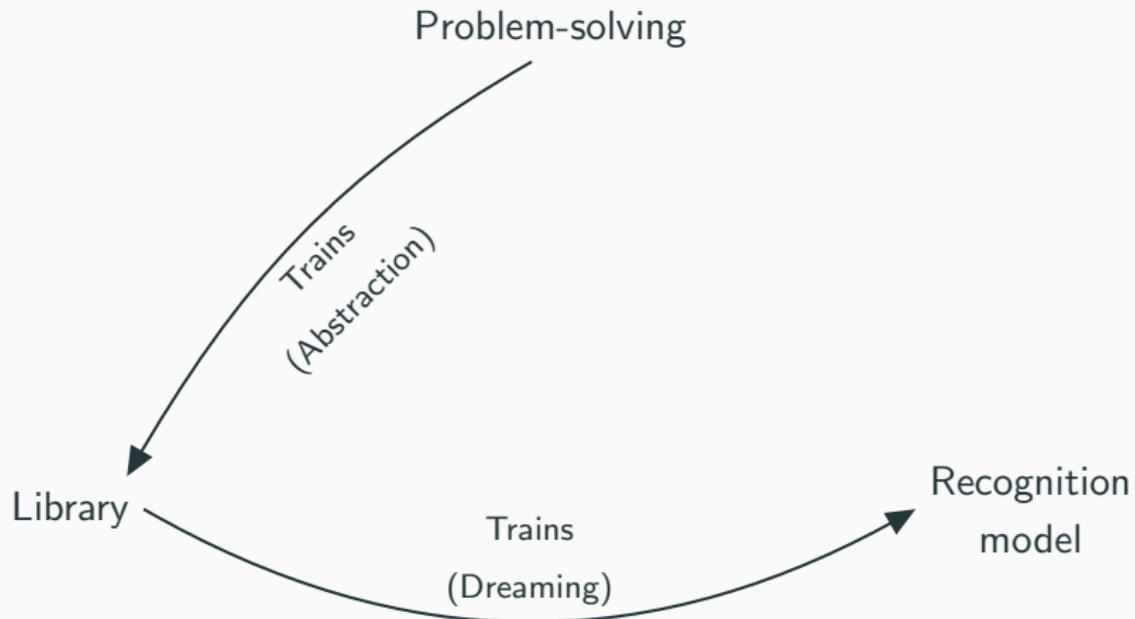
Library

Recognition  
model

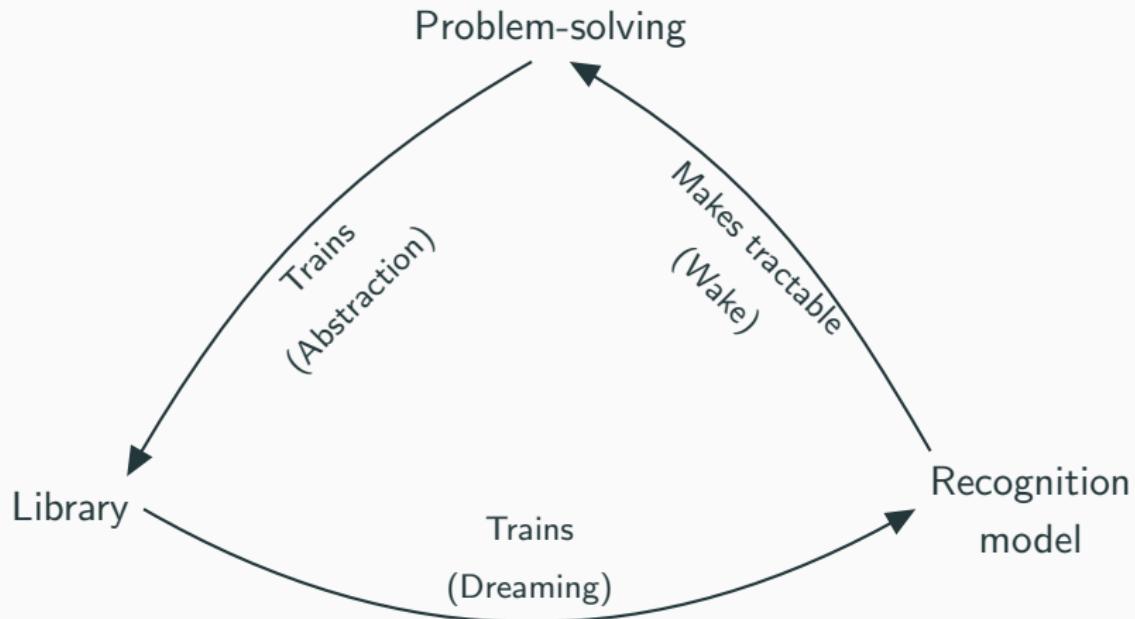
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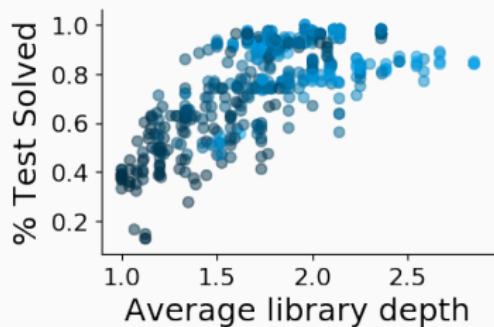
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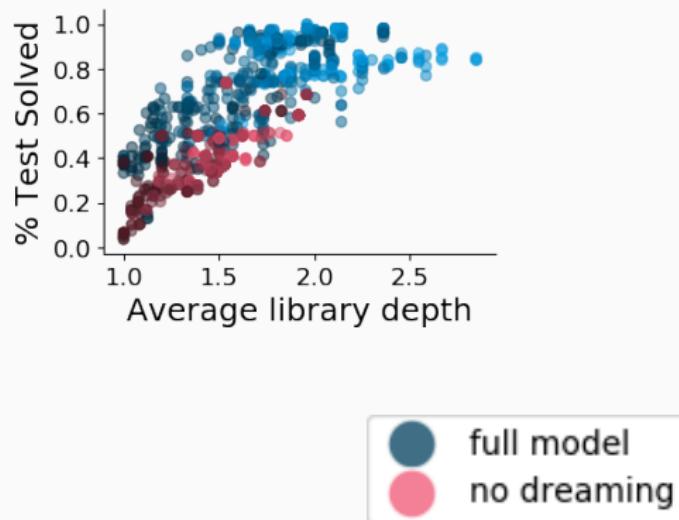
# Evidence for dreaming bootstrapping better libraries



Darker: Early in learning

Brighter: Later in learning

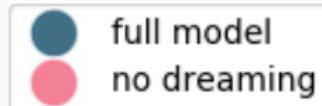
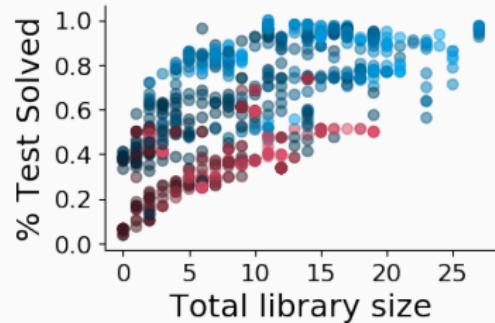
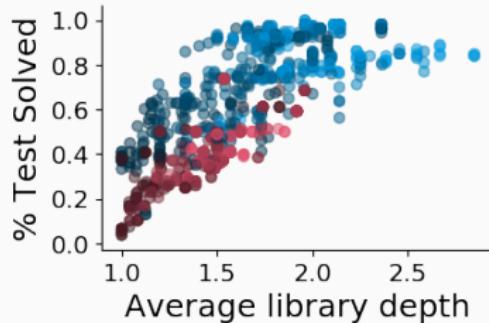
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From learning libraries,  
to learning languages

# From learning libraries, to learning languages

modern functional programming → physics

From learning libraries,  
to learning languages

1950's Lisp → modern functional programming → physics

# Physics Formula Sheet

## Mechanics

$x = x_0 + v_{x0}t + \frac{1}{2}a_xt^2$	$a_t = \frac{v^2}{r}$	$ \vec{F}_{\text{spring}}  = k x $
$v = v_0 + at$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$	$\text{PE}_{\text{spring}} = \frac{1}{2}kx^2$
$v_s^2 - v_{s0}^2 = 2a(x - x_0)$	$\omega = \omega_0 + \alpha t$	$T_{\text{spring}} = 2\pi\sqrt{\frac{m}{k}}$
$\bar{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{\text{net}}}{m}$	$T = \frac{2\pi}{\omega} = \frac{1}{f}$	$T_{\text{pendulum}} = 2\pi\sqrt{\frac{L}{g}}$
$ \vec{F}_{\text{friction}}  \leq \mu  \vec{F}_{\text{Normal}} $	$v = f\lambda$	
$\bar{p} = m\bar{v}$	$x = A\cos(2\pi ft)$	$ \vec{F}_{\text{gravity}}  = G \frac{m_1 m_2}{r^2}$
$\Delta \bar{p} = \vec{F} \Delta t$	$\bar{a} = \frac{\sum \vec{F}}{I} = \frac{\vec{F}_{\text{net}}}{I}$	$ \vec{F}_{\text{gravity}}  = m\bar{g}$
$KE = \frac{1}{2}mv^2$	$\vec{r} = r \times F$	$\text{PE}_{\text{gravity}} = -G \frac{m_1 m_2}{r}$
$\Delta PE = mg\Delta y$	$L = I\omega$	$p = \frac{m}{V}$
$\Delta E = W = Fd\cos\theta$	$\Delta L = \tau \Delta t$	$KE = \frac{1}{2}I\omega^2$

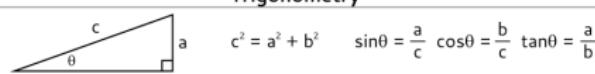
## Electricity

$ \vec{F}_E  = k \left  \frac{q_1 q_2}{r^2} \right $	$\Delta V = IR$	$R = \frac{\rho l}{A}$
$I = \frac{\Delta q}{\Delta t}$		$P = I\Delta V$
$R_{\text{series}} = R_1 + R_2 + \dots + R_n$	$\frac{1}{R_{\text{parallel}}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}$	

## Geometry

Rectangle	$A = bh$	Rectangular Solid	$V = lwh$	Triangle	$A = \frac{1}{2}bh$
Circle	$A = \pi r^2$	Cylinder	$V = \pi r^2 l$	Sphere	$V = \frac{4}{3}\pi r^3$
	$C = 2\pi r$		$S = 2\pi rl + 2\pi r^2$		$S = 4\pi r^2$

## Trigonometry



## Variables

a = acceleration  
 A = amplitude  
 A = Area  
 b = base length  
 C = circumference  
 d = distance  
 E = energy  
 f = frequency  
 F = force  
 h = height  
 I = current  
 I = rotational inertia  
 KE = kinetic energy  
 k = spring constant  
 L = angular momentum  
 l = length  
 m = mass  
 P = power  
 p = momentum  
 q = charge  
 r = radius  
 R = resistance  
 S = surface area  
 T = period  
 t = time  
 PE = potential energy  
 V = electric potential  
 V = volume  
 v = velocity  
 w = width  
 W = work  
 x = position  
 y = height  
 $\alpha$  = angular acceleration  
 $\lambda$  = wavelength  
 $\mu$  = coefficient of friction

# Growing languages for vector algebra and physics

## Initial Primitives

map  
zip

cons

empty

cdr

power

fold

car

+

-

\*

/

0

1

$\pi$

## Physics Equations

### Newton's Second Law

$$\vec{a} = \frac{1}{m} \sum_l \vec{F}_l$$

### Parallel Resistors

$$R_{total} = \left( \sum_i \frac{1}{R_i} \right)^{-1}$$

### Work

$$U = \vec{F} \cdot \vec{d}$$

### Force in a Magnetic Field

$$|\vec{F}| = q |\vec{v} \times \vec{B}|$$

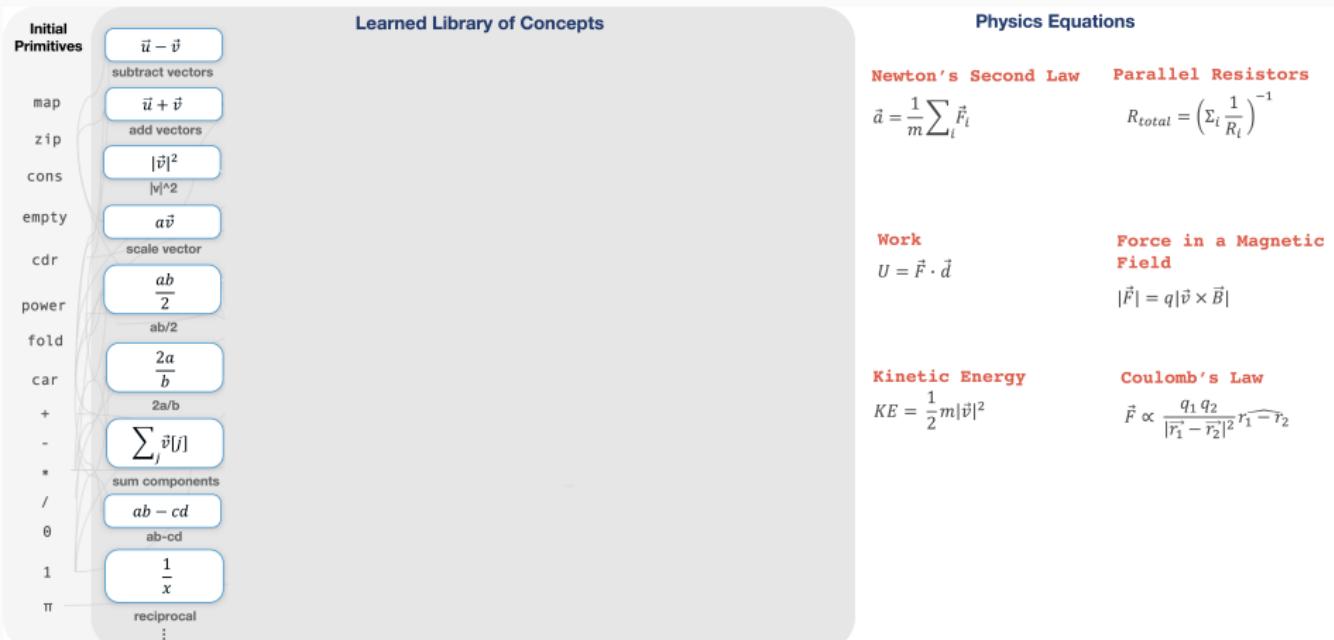
### Kinetic Energy

$$KE = \frac{1}{2} m |\vec{v}|^2$$

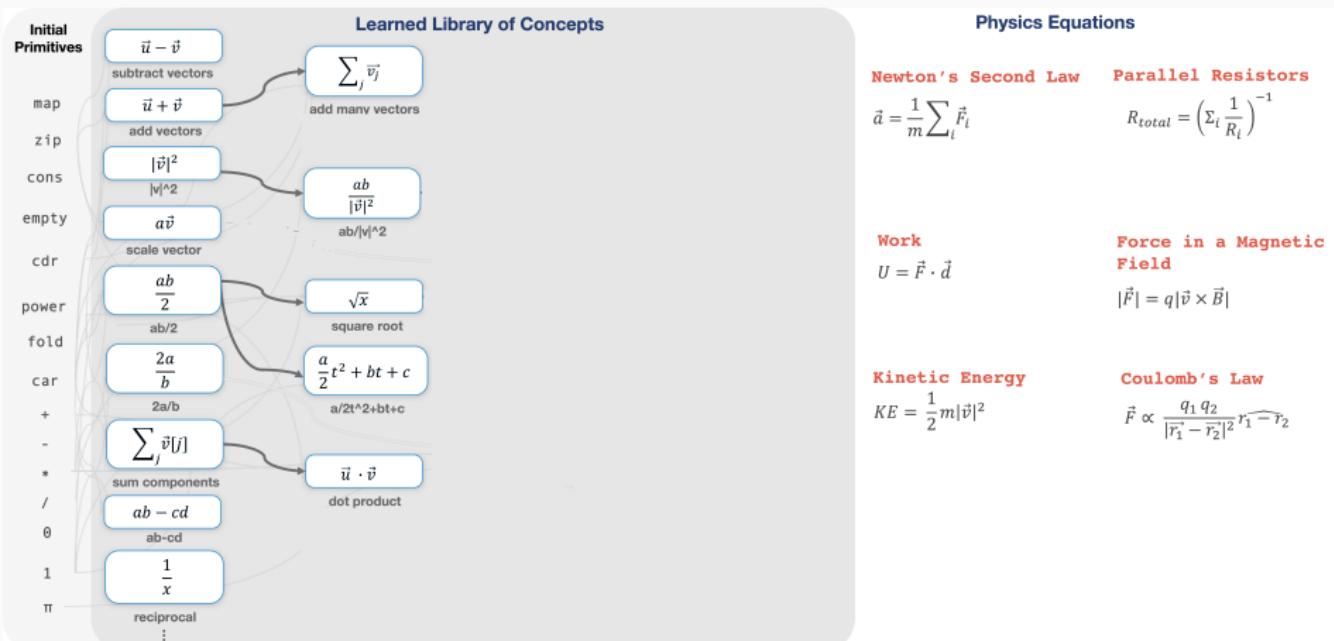
### Coulomb's Law

$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^2} \hat{r}_1 - \hat{r}_2$$

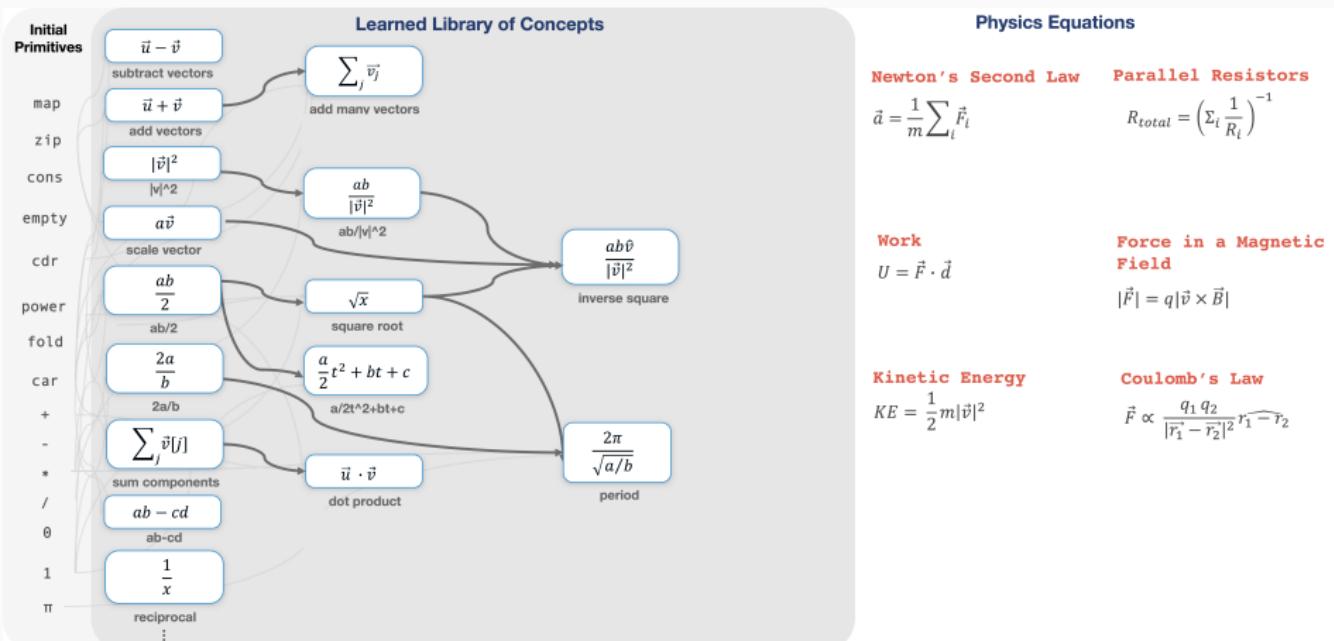
# Growing languages for vector algebra and physics



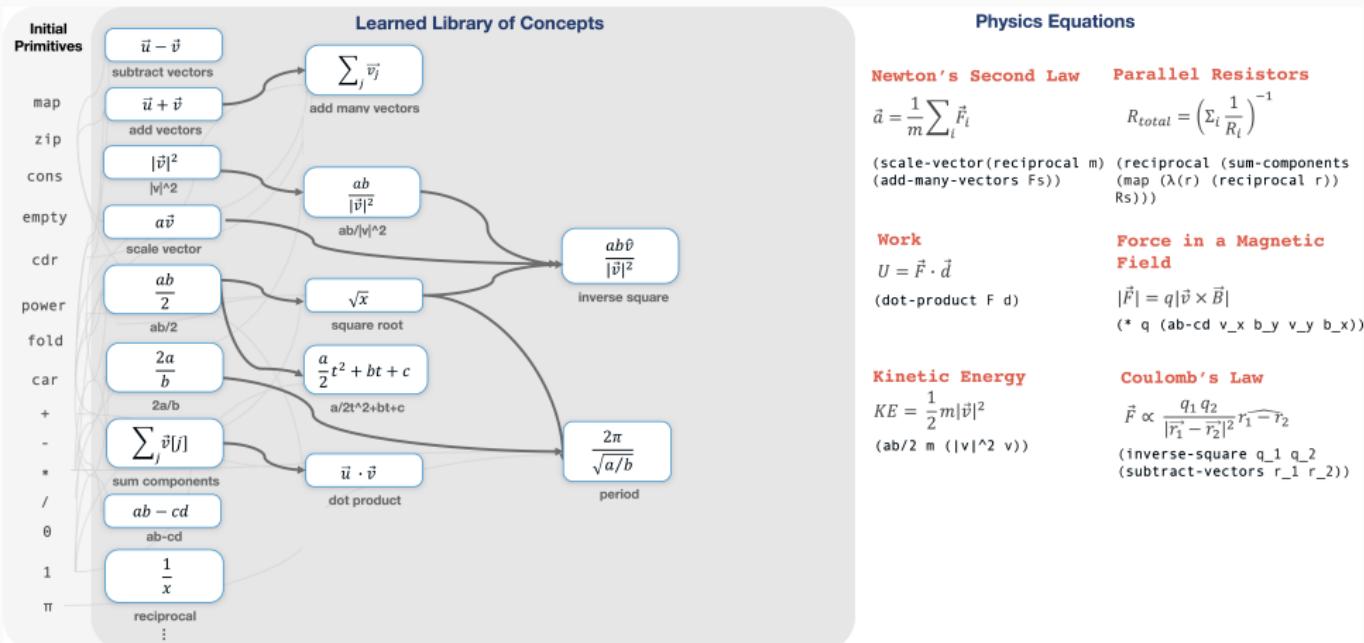
# Growing languages for vector algebra and physics



# Growing languages for vector algebra and physics



# Growing languages for vector algebra and physics



# Growing languages for vector algebra and physics

Initial  
Primitives

$\vec{u} - \vec{v}$   
subtract vectors

map

$\vec{u} + \vec{v}$   
add vectors

zip

cons

empty

cdr

power

fold

car

+

-

\*

/

0

1

$\pi$

Learned Library of Concepts

$$\sum_j \vec{v}_j$$

add many vectors

$$|\vec{v}|^2$$

$$|v|^2$$

$$ab$$

$$|v|^2$$

$$ab\hat{v}$$

$$|v|^2$$

$$\sqrt{x}$$

$$\text{square root}$$

$$a/2t^2 + bt + c$$

$$a/2t^2 + bt + c$$

$$\sum \vec{v}[j]$$

$$\text{sum components}$$

$$ab - cd$$

$$ab - cd$$

$$\frac{1}{x}$$

$$\text{reciprocal}$$

$$\vec{u} \cdot \vec{v}$$

dot product

$$\frac{2\pi}{\sqrt{a/b}}$$

period

Physics Equations

Newton's Second Law

$$\vec{a} = \frac{1}{m} \sum_l \vec{F}_l$$

Parallel Resistors

$$R_{total} = \left( \sum_i \frac{1}{R_i} \right)^{-1}$$

(scale-vector(reciprocal m) (reciprocal (sum-components (add-many-vectors Fs)))

(map (\lambda(r) (reciprocal r)) Rs)))

Work

$$U = \vec{F} \cdot \vec{d}$$

(dot-product F d)

Force in a Magnetic Field

$$|\vec{F}| = q|\vec{v} \times \vec{B}|$$

(\* q (ab-cd v\_x b\_y v\_y b\_x))

Kinetic Energy

$$KE = \frac{1}{2} m |\vec{v}|^2$$

(ab/2 m (|v|^2 v))

Coulomb's Law

$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^2} \widehat{\vec{r}_1 - \vec{r}_2}$$

(inverse-square q\_1 q\_2  
(subtract-vectors r\_1 r\_2))

(\lambda (x y z u) (map (\lambda (v) (\* (/ (\* (power (/ (\* x) (fold (zip z u (\lambda (w a) (- w a)))) \theta (\lambda (b c) (+ (\* b b) c)))) (/ (\* 1 1) (+ 1 1)) y) (fold (zip z u (\lambda (d e) (- d e))) \theta (\lambda (f g) (+ (\* f f) g)))) v)) (zip z u (\lambda (h i) (- h i)))))

Solution to Coulomb's Law if expressed in initial primitives

# Growing a language for recursive programming

## Initial Primitives

Y  
combinator  
cons  
car  
cdr  
nil  
if  
nil?  
+  
-  
0  
1  
=

## Recursive Programming Algorithms

### Stutter

[ ] → [ ]  
[ ] → [ ]

### Take every other

[ ] → [ ]  
[ ] → [ ]

### List lengths

[ , []] → [3 1]  
[[ ], [], []] → [2 0 1]

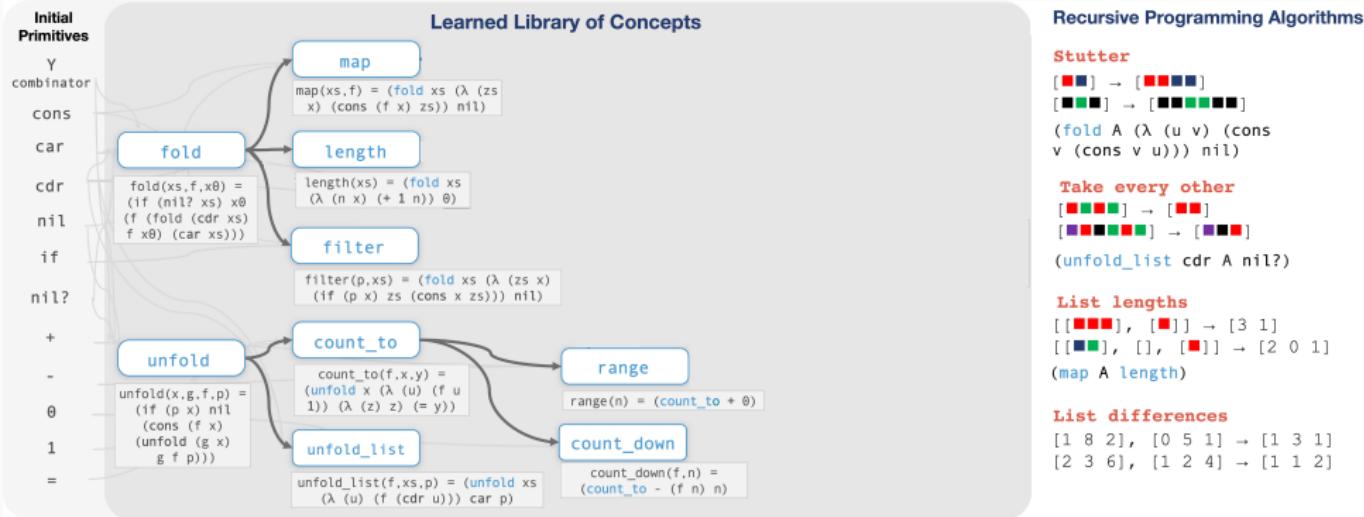
### List differences

[1 8 2], [0 5 1] → [1 3 1]  
[2 3 6], [1 2 4] → [1 1 2]

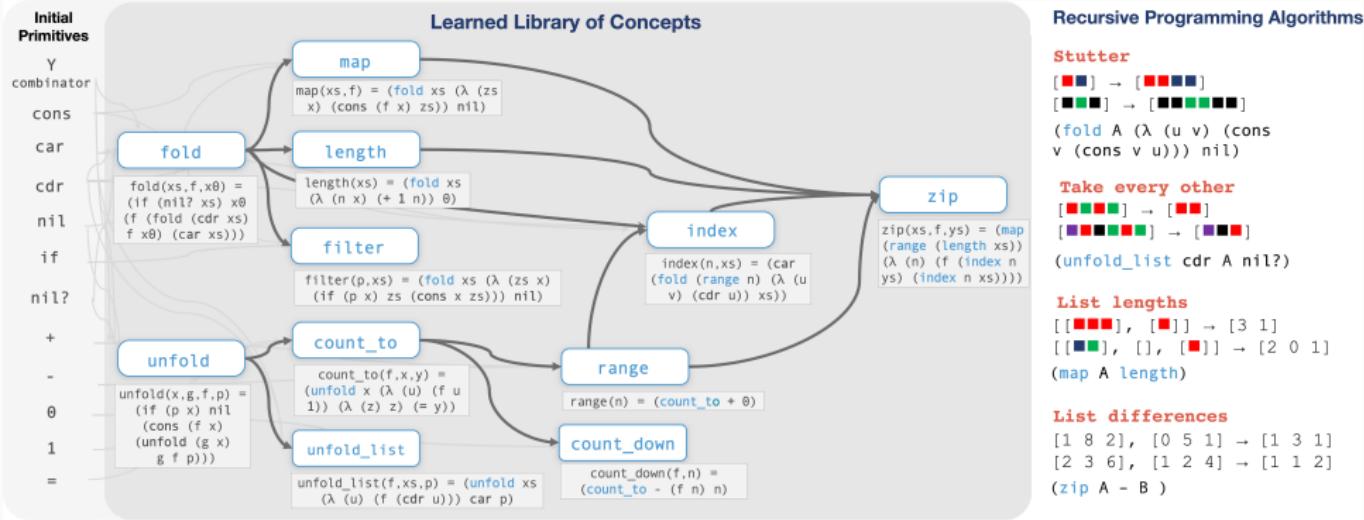
# Growing a language for recursive programming



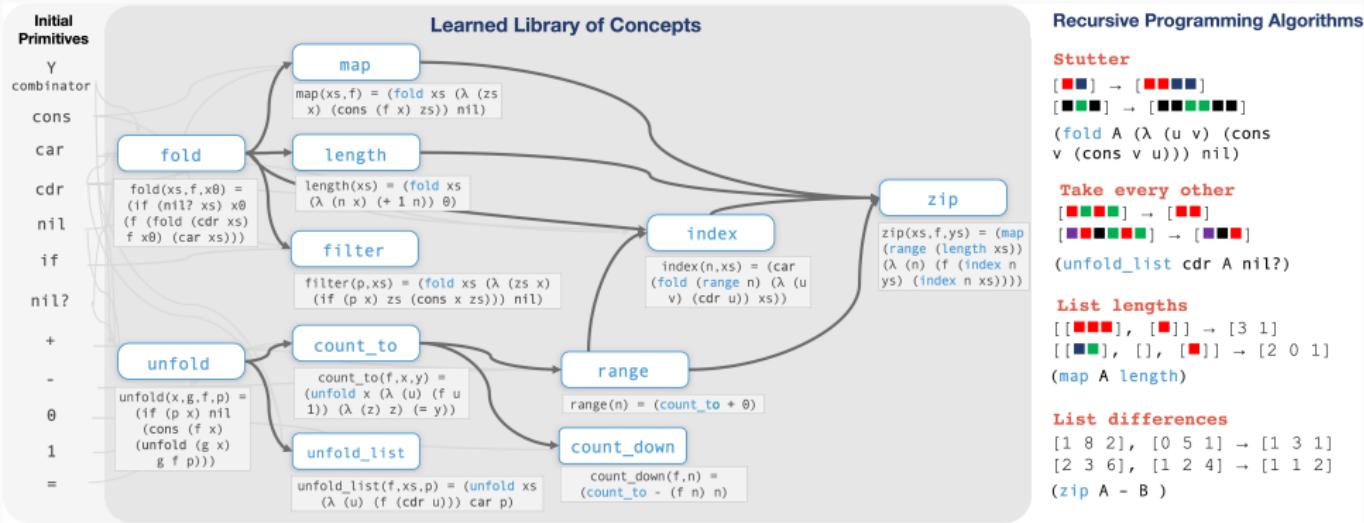
# Growing a language for recursive programming



# Growing a language for recursive programming

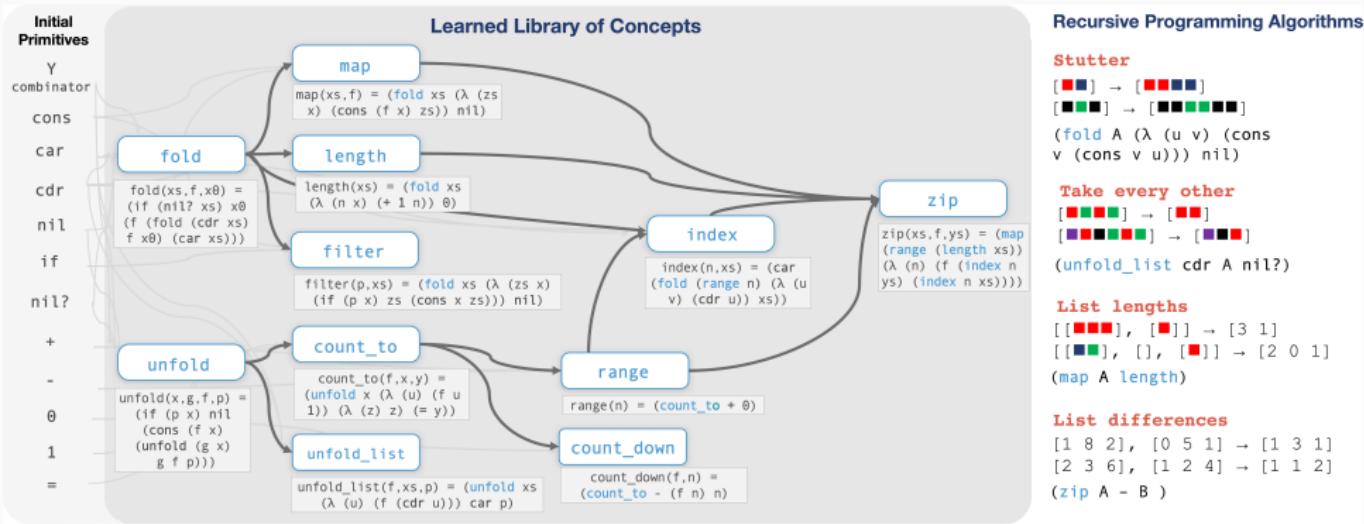


# Growing a language for recursive programming

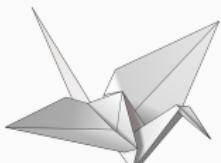


Origami Programming: Jeremy Gibbons, 2003

# Growing a language for recursive programming



1 year of compute. 5 days on 64 CPUs.



Origami Programming: Jeremy Gibbons, 2003

## Lessons

Library learning interacts synergistically with neural synthesis:  
bootstrapping, more than sum of parts

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on experience to make it more powerful *and* more human  
understandable

Learning-from-scratch is possible in principle. Don't do it. But  
program induction makes it convenient to build in what we know  
how to build in, and then learn on top of that

**Program Induction and learning to learn**  
learning a DSL  
learning to synthesize  
**next steps**

## Some unresolved questions

Not everything is crisp and symbolic. How do we learn DSLs for  
neurosymbolic hybrid programs?  
(see Memoised Wake-Sleep, Hewitt et al 2020)

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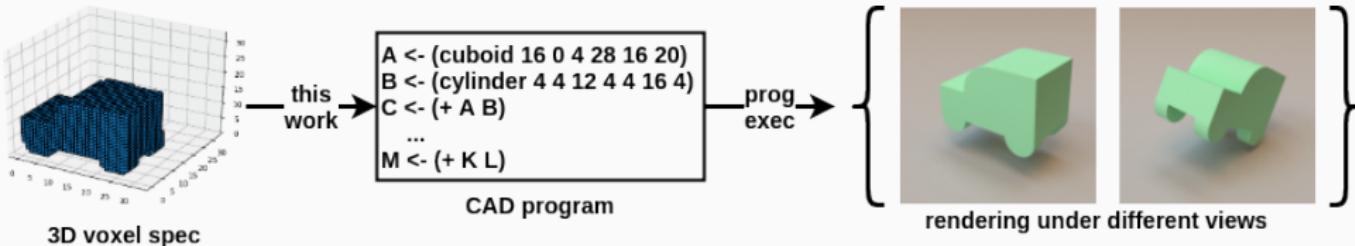
Combinatorial search is still hard

Library  $\neq$  Language:

How do we learn data structures, or discover new types, and thereby *actually* learn DSLs?

the end.

# 3D program induction

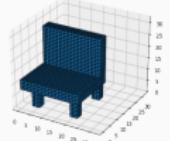


Challenge: combinatorial search!

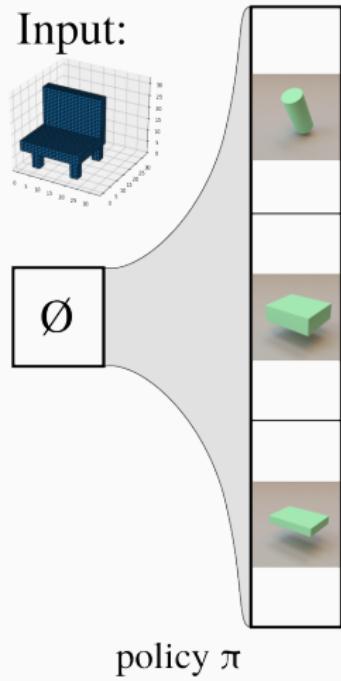
Branching factor:  $> 1.3$  million per line of code,  $\approx 20$  lines of code  
search space size:  $(1.3 \text{ million})^{20} \approx 10^{122}$  programs

Solution: stochastic **tree search** + learn **policy** that writes code  
+ learn **value** function that assesses execution of program so far;  
analogous to **AlphaGo** [Silver et al. 2016]

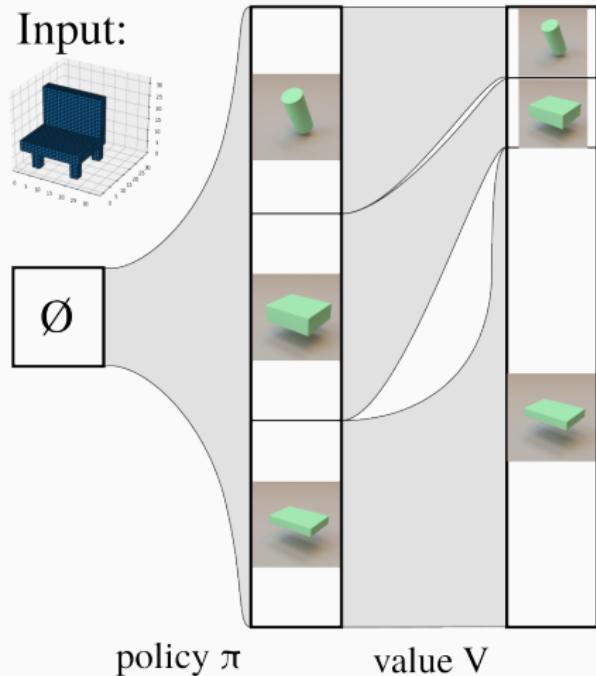
Input:



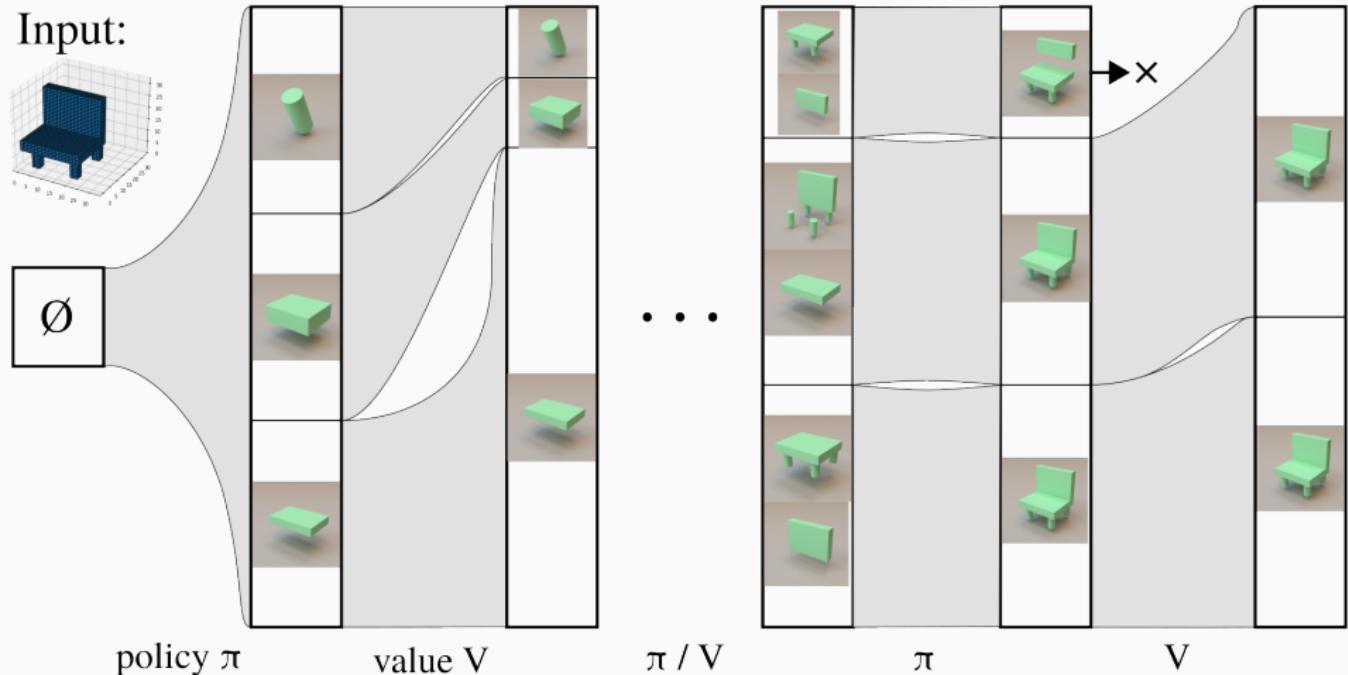
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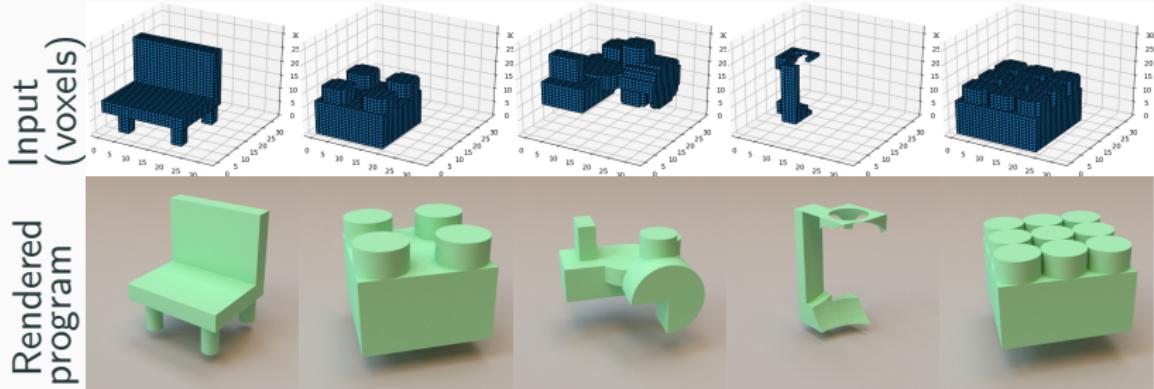
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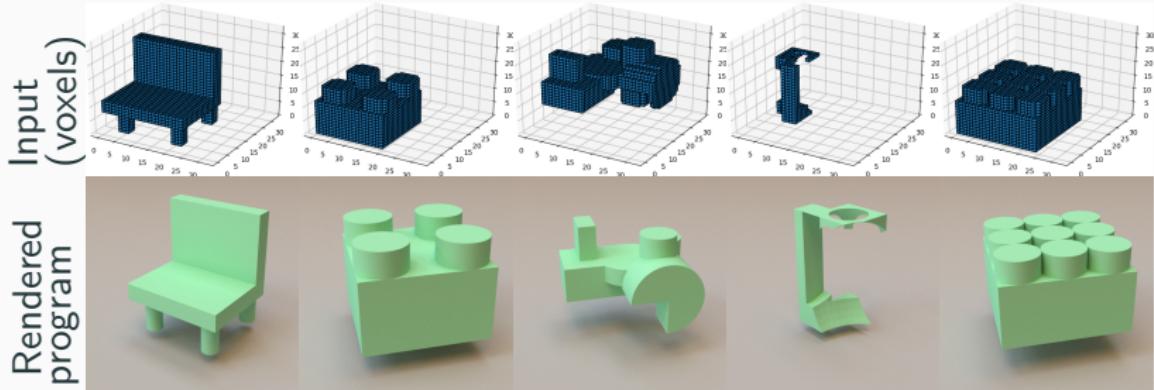
# 3D program induction



Ellis\*, Nye\*, Pu\*, Sosa\*, Tenenbaum, Solar-Lezama. NeurIPS 2019.

\*equal contribution

# 3D program induction



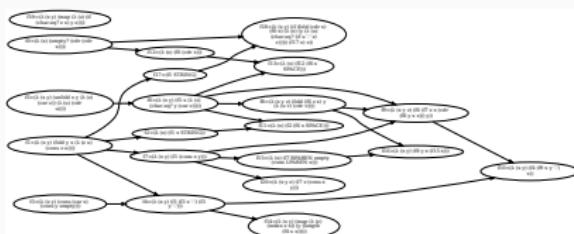
same architecture learns to synthesize text editing programs  
(FlashFill, Gulwani 2012)

Ellis\*, Nye\*, Pu\*, Sosa\*, Tenenbaum, Solar-Lezama. NeurIPS 2019.

\*equal contribution

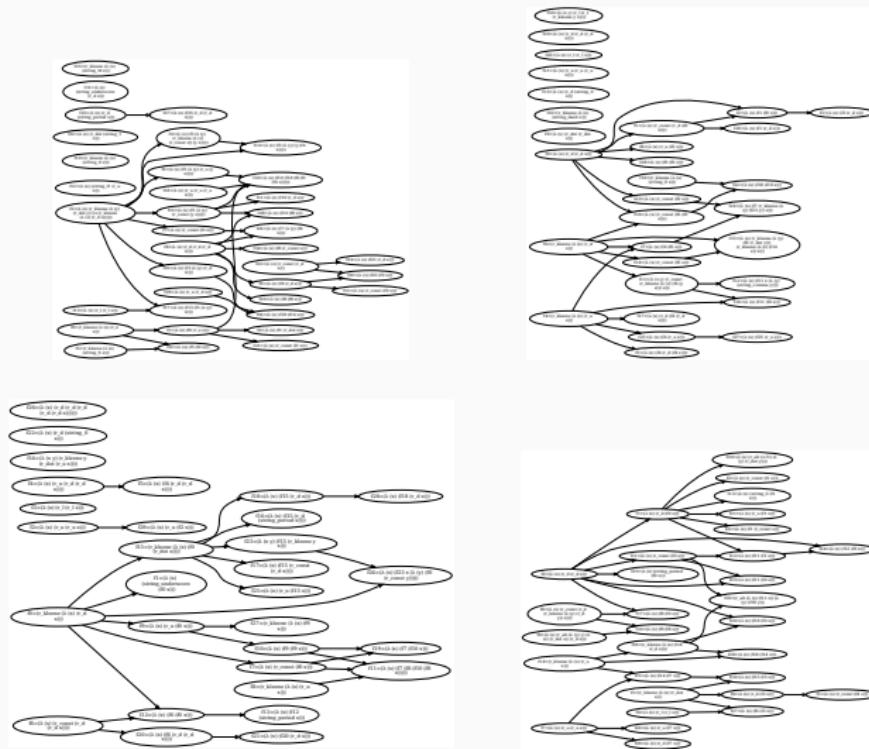
# Library structure: Text Editing

DreamCoder learns libraries for FlashFill-style text editing [Gulwani 2012]

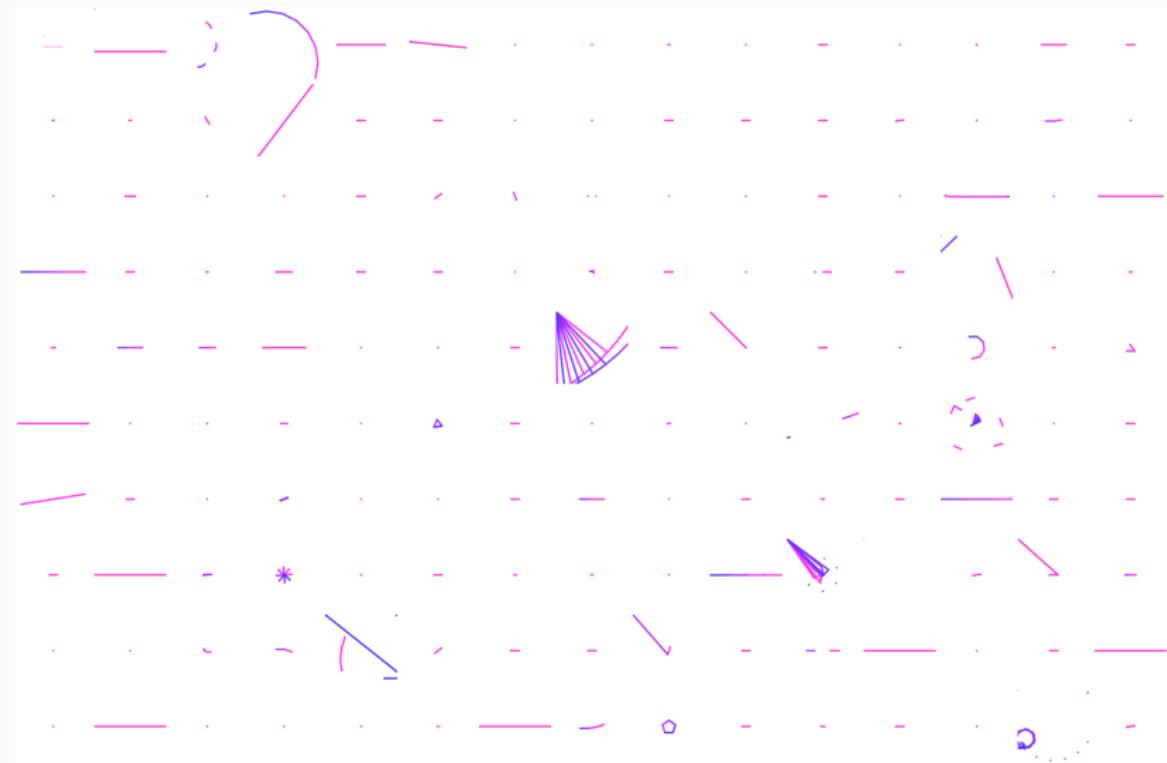


# Library structure: Generating Text

## Libraries for probabilistic generative models over text: data from crawling web for CSV files



# 150 random dreams before learning



# 150 random dreams after learning

