Program Induction:
Bridging AI and program synthesis

Kevin Ellis

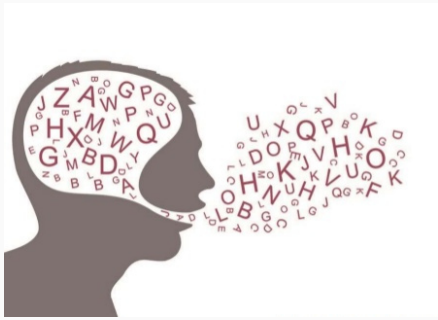
2020

MIT

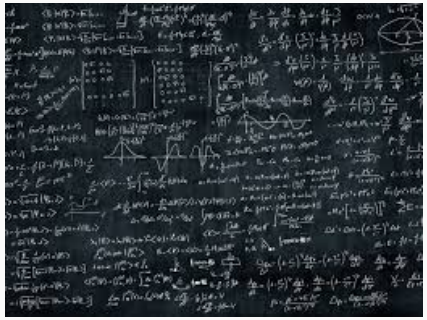
What computational problems are solved by intelligence?

an endless range of problems

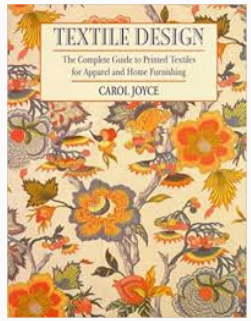
language



science

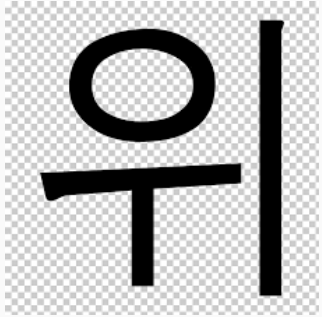


design



using new devices

writing new characters



coding

```
(MEMBER
 (LAMBDA (X L)
  (COND  ((NULL L) NIL)
         ((EQ X (FIRST L)) T)
         (T (MEMBER X (REST L))))))
```

Allen, Anatomy of Lisp, 1975

engineering

play

What computational problems are solved by intelligence?

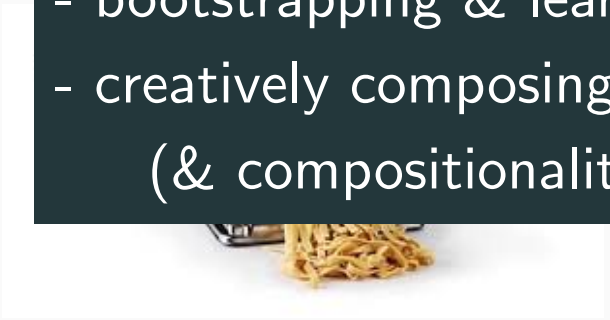
an endless range of problems

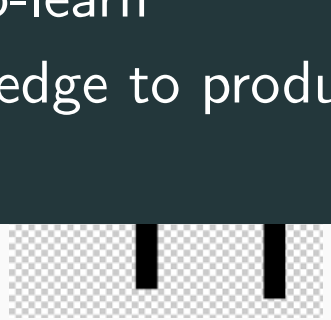
language

science

design

- learning from modest data/experience
- communicating & representing knowledge in understandable formats
- bootstrapping & learning-to-learn
- creatively composing knowledge to produce new concepts and artifacts (& compositionality)





engineering

play

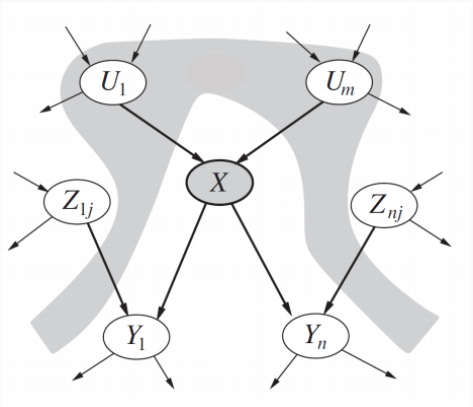
What computational frameworks can contribute to this picture?

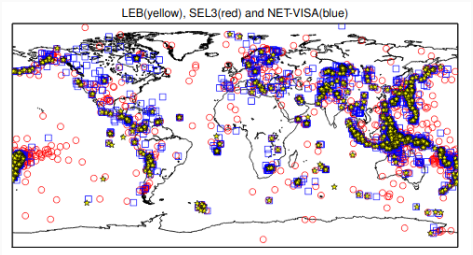
Three AI traditions

Symbolic

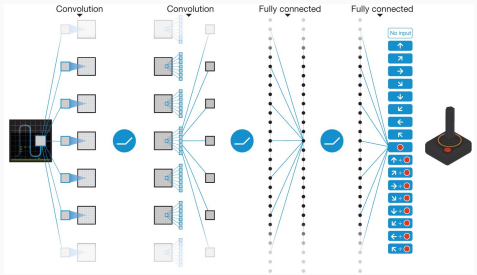

```
In[34]:= Solve[{(hw - hw^2) == Z}, h]
Out[34]= {}

In[33]:= Solve[{(hw-hw^2) ==Z},h]

Input interpretation:
solve | h w - h w^2 = Z | for | h

Result:
h = Z / (w - w^2)  and  w^2 ≠ w
```

Probabilistic





Neural



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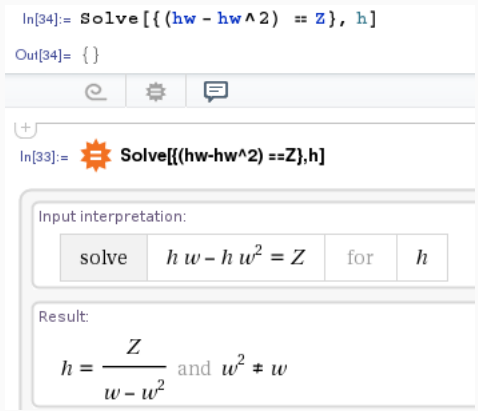
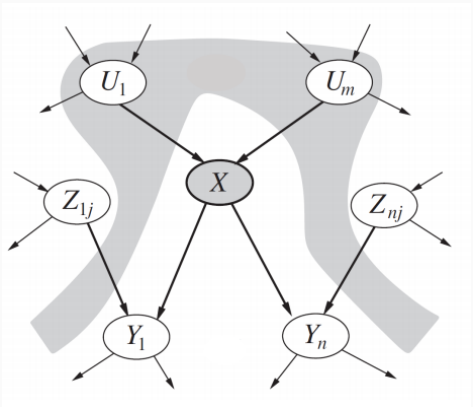
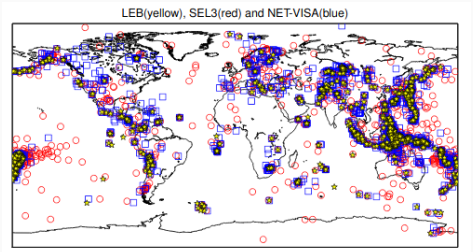
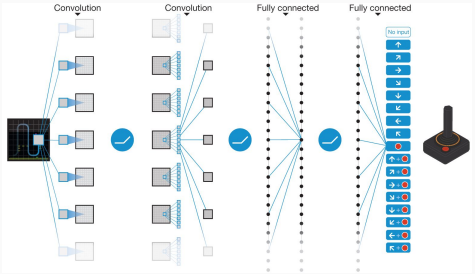


Probabilistic





Neural



Program induction: agents that learn by writing their own code

strong generalization

Why didn't this old idea work?

Program induction goes back to 1956 Dartmouth Workshop that founded the field of AI

A PROPOSAL FOR THE

DARTMOUTH SUMMER RESEARCH PROJECT

ON ARTIFICIAL INTELLIGENCE

J. McCarthy, Dartmouth College
M. L. Minsky, Harvard University
N. Rochester, I. B. M. Corporation
C. E. Shannon, Bell Telephone Laboratories

Why will it work this time?

better toolkits:

- **probabilistic** methods for uncertainty and learning-to-learn
- **neural** methods for guiding combinatorial search
- **symbolic** methods, from the **programming languages** community
  - maturing **program synthesis** techniques
  - type systems, program analysis, constraint solving, ...

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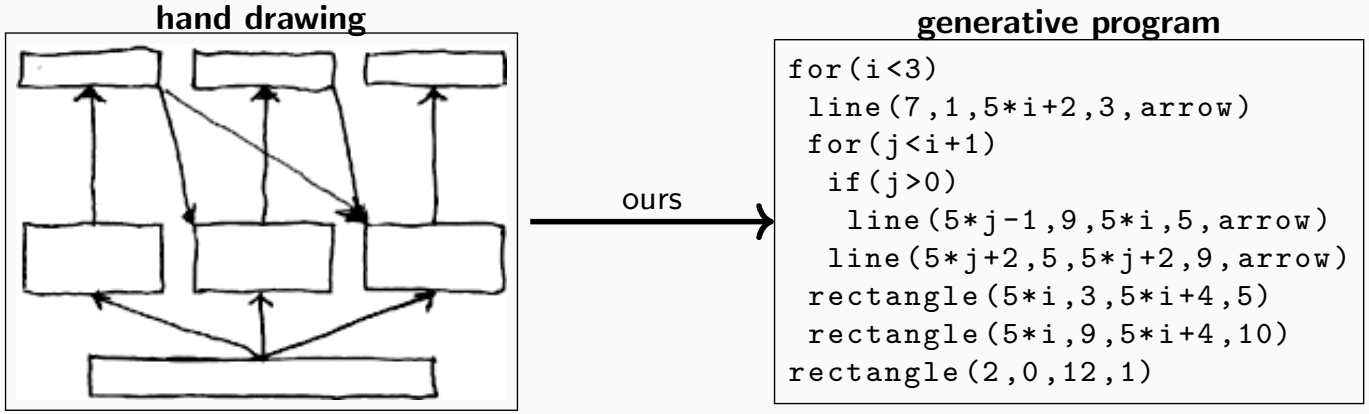
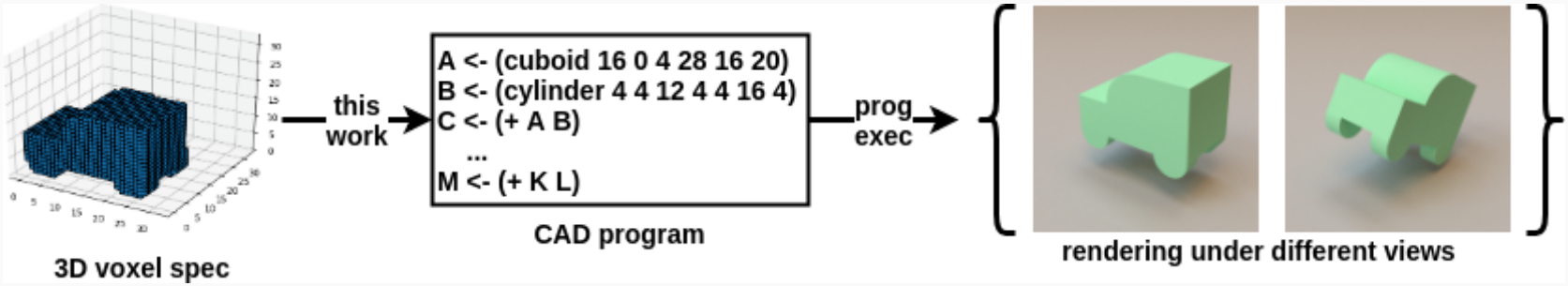
better problems:

- inverse perception, analysis-by-synthesis
- learning models
- learning to learn
- natural language→code
- programming by examples, computer-aided-programming

Perception, Synthesizing models, Learning-to-Learn

Theme #1: high-level scene understanding, pixels→programs





Perception, Synthesizing models, Learning-to-Learn

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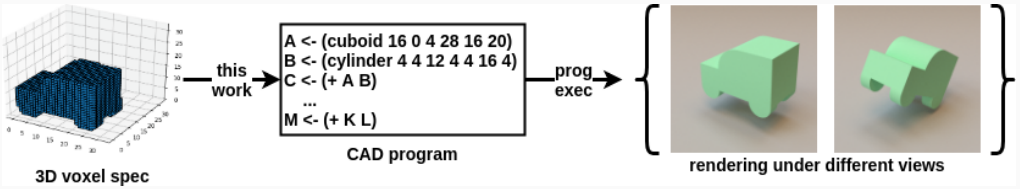
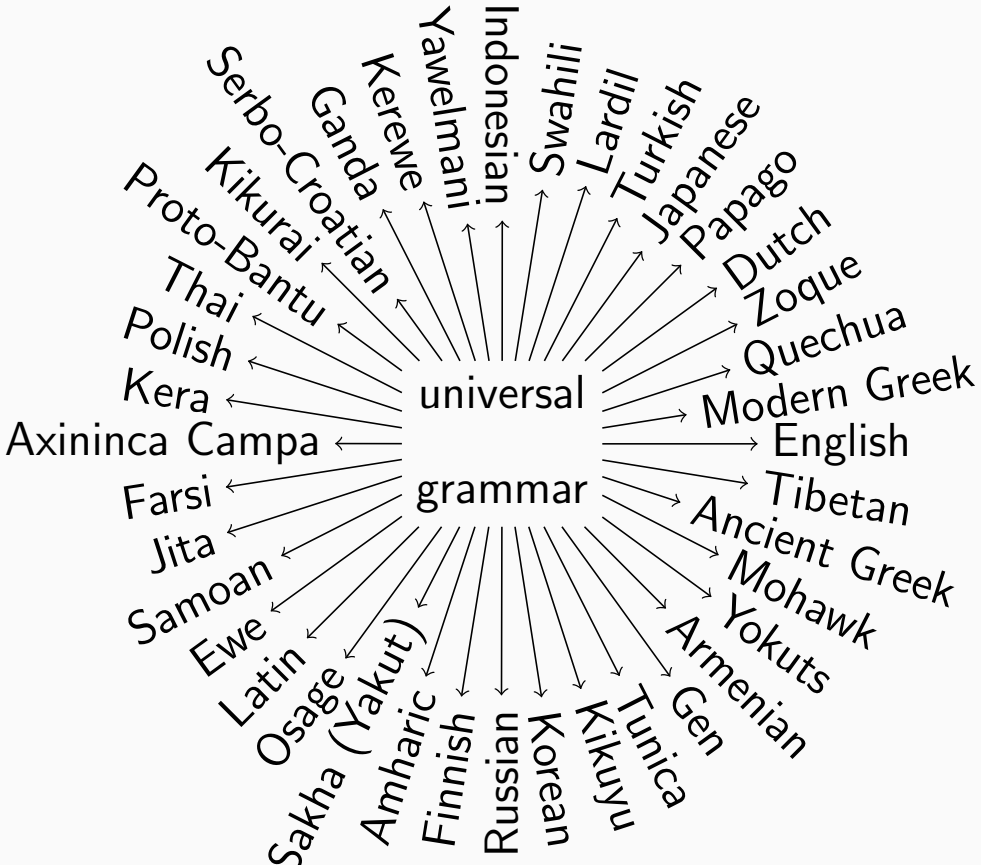
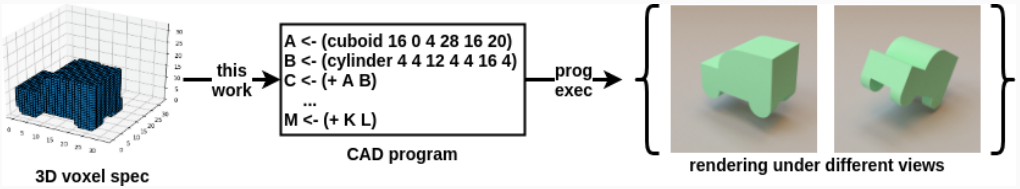


Theme #2: Synthesizing human-understandable models



Perception, Synthesizing models, Learning-to-Learn

Theme #1: high-level scene understanding, pixels→programs



Theme #2: Synthesizing interpretable models

Theme #3: Learning to synthesize programs

List Processing

Sum List

```
[1 2 3] → 6
[4 6 8 1] → 17
```

Double

```
[1 2 3 4] → [2 4 6 8]
[6 5 1] → [12 10 2]
```

Check Evens

```
[0 2 3] → [T T F]
[2 4 9 6] → [T T F T]
```

Text Editing

Abbreviate

```
Allen Newell → A.N.
Herb Simon → H.S.
```

Drop Last Characters

```
jabberwocky → jabberw
copycat → cop
```

Extract

```
see spot(run) → run
a (bee) see → bee
```

Regexes

Phone Numbers

```
(555) 867-5309
(650) 555-2368
```

Currency

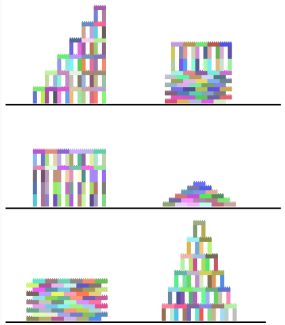
```
$100.25
$4.50
```

Dates

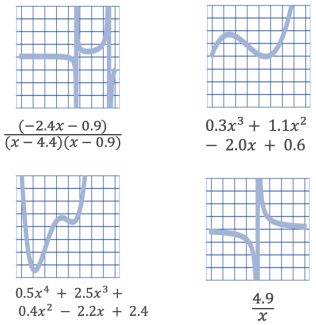
```
Y1775/0704
Y2000/0101
```

LOGO Graphics

Block Towers



Symbolic Regression

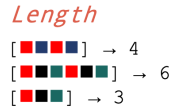


Recursive Programming

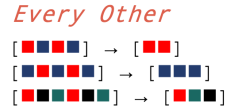
Filter

Index List

Length



Every Other



Physics

$$KE = \frac{1}{2}m|\vec{v}|^2$$

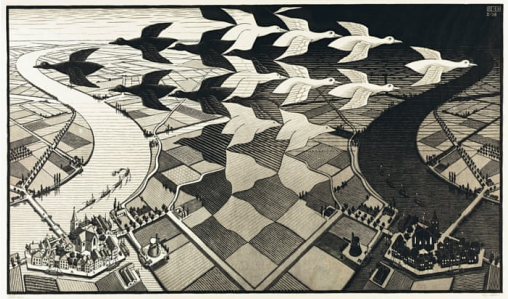
$$\vec{a} = \frac{1}{m}\sum_i \vec{F}_i$$

$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r_1} - \vec{r_2}|^2}\widehat{r_1 - r_2}$$

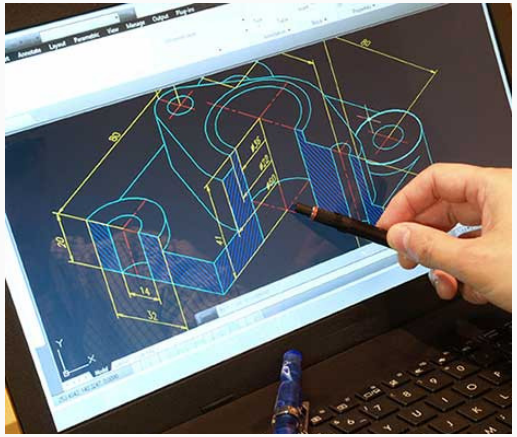
$$R_{total} = \left(\Sigma_i \frac{1}{R_i}\right)^{-1}$$

High-level, abstract visual abilities

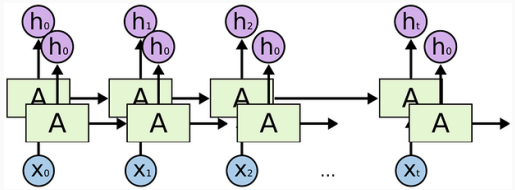
...in art

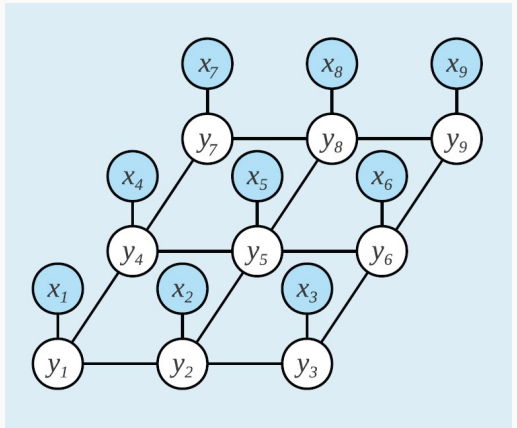


...in engineering



...in AI





High-level, abstract visual abilities

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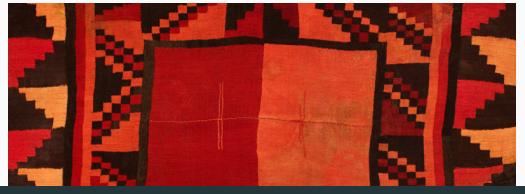
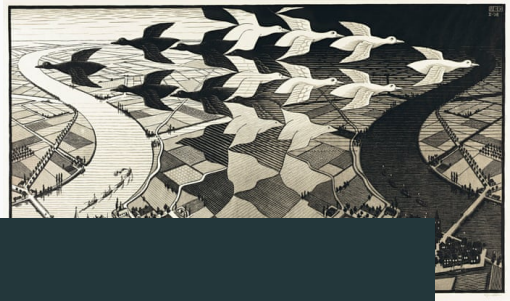




why?

impute missing objects, extrapolate percepts,

learn visual concepts ('arch', 'spiral', 'Ising model'),

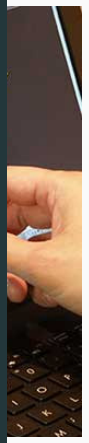
assist graphic design, assist 3D modeling

how?

Bayesian inference of graphics program conditioned on image,

+program synthesis

+learning



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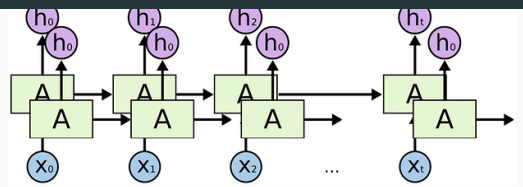
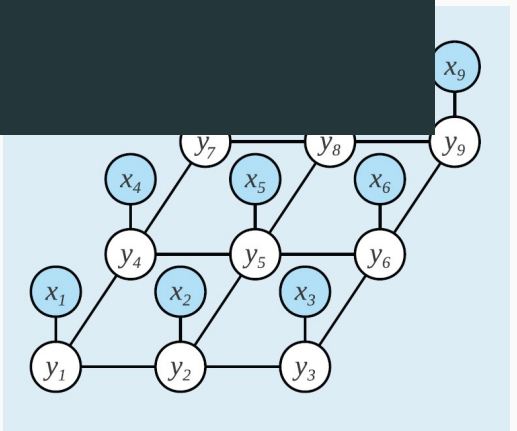
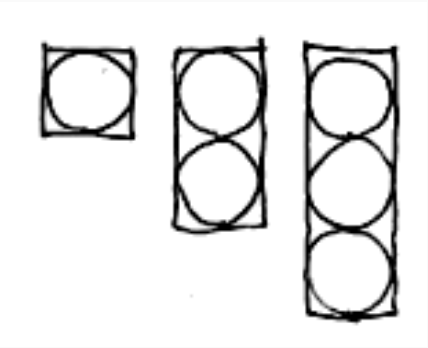




Learning to infer graphics programs from hand-drawn images

model infers program from drawing



```
for (0 <= i < 3)
 rectangle(3*i,-2*i+4,
           3*i+2,6)
 for (0 <= j < i + 1)
  circle(3*i+1,-2*j+5)
```

Ellis, Ritchie, Solar-Lezama, Tenenbaum. NeurIPS 2018.

Learning to infer graphics programs from hand-drawn images

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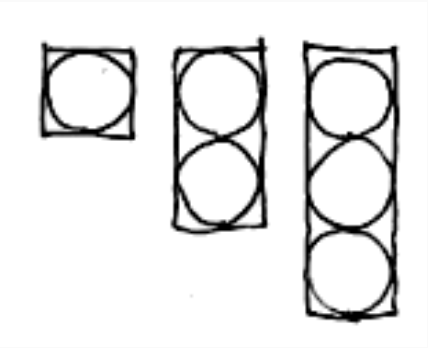
zero-shot generalization / extrapolation



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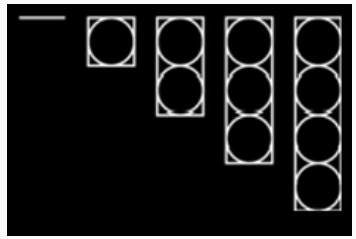

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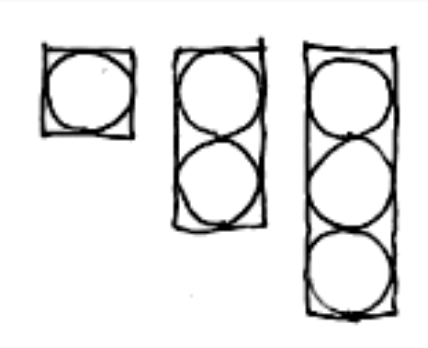

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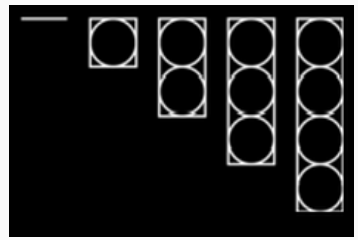
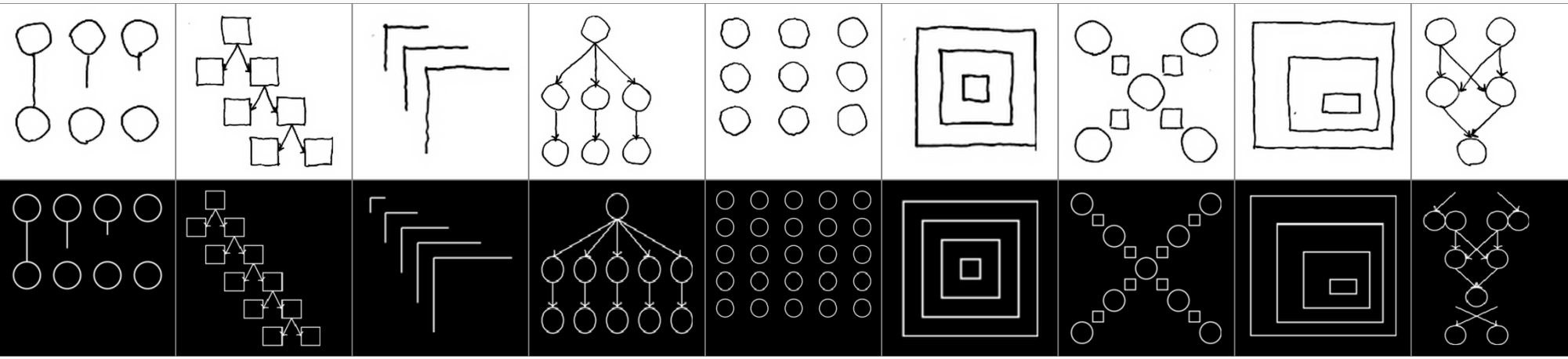
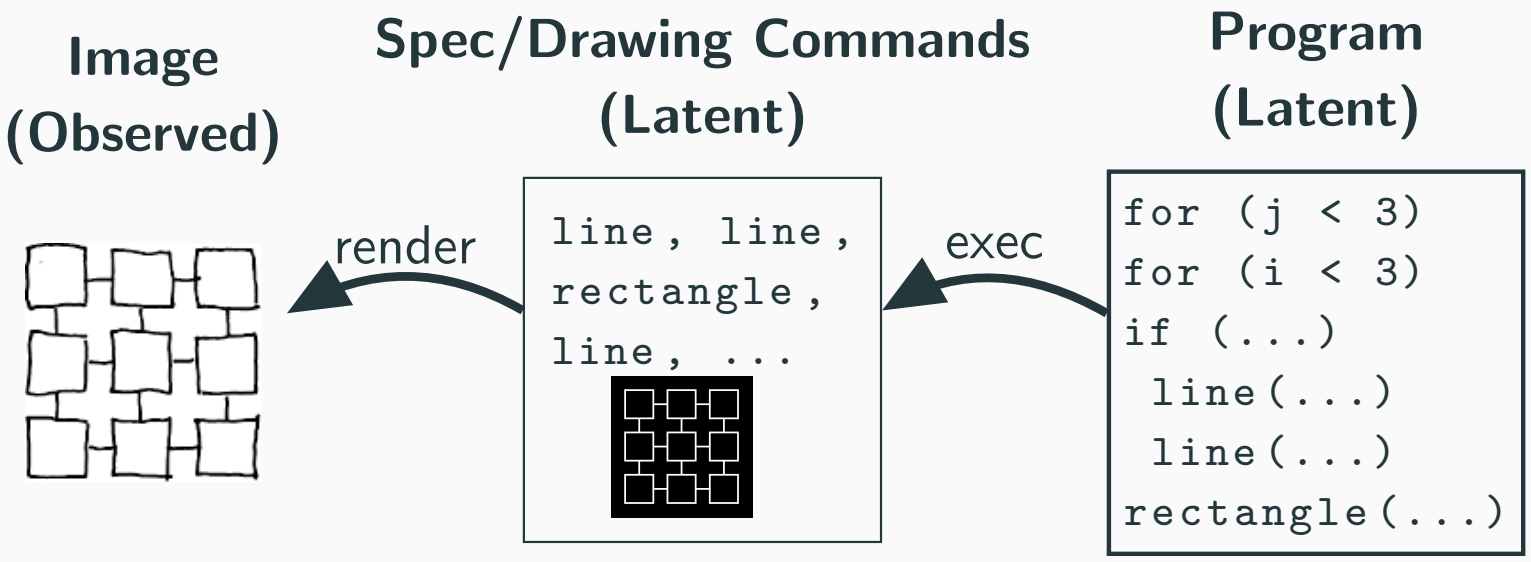




How to infer graphics programs from hand-drawn images



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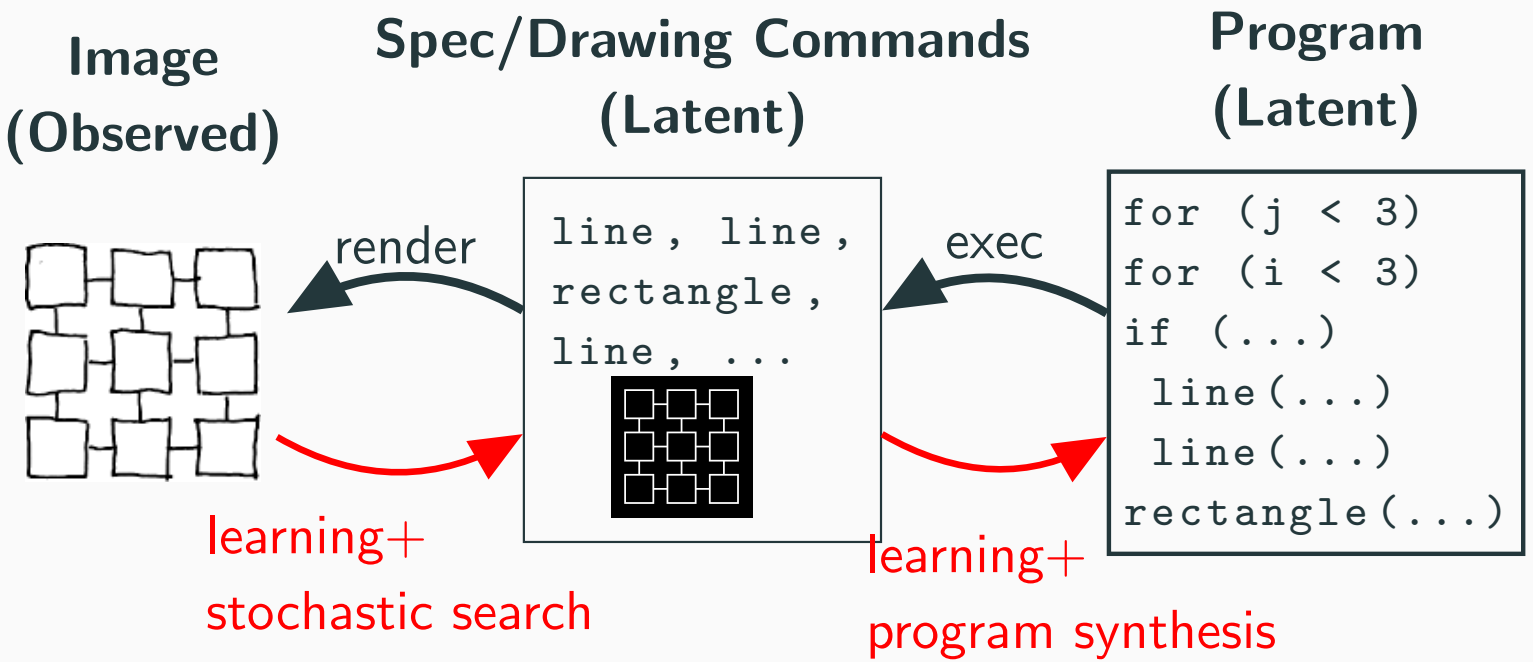
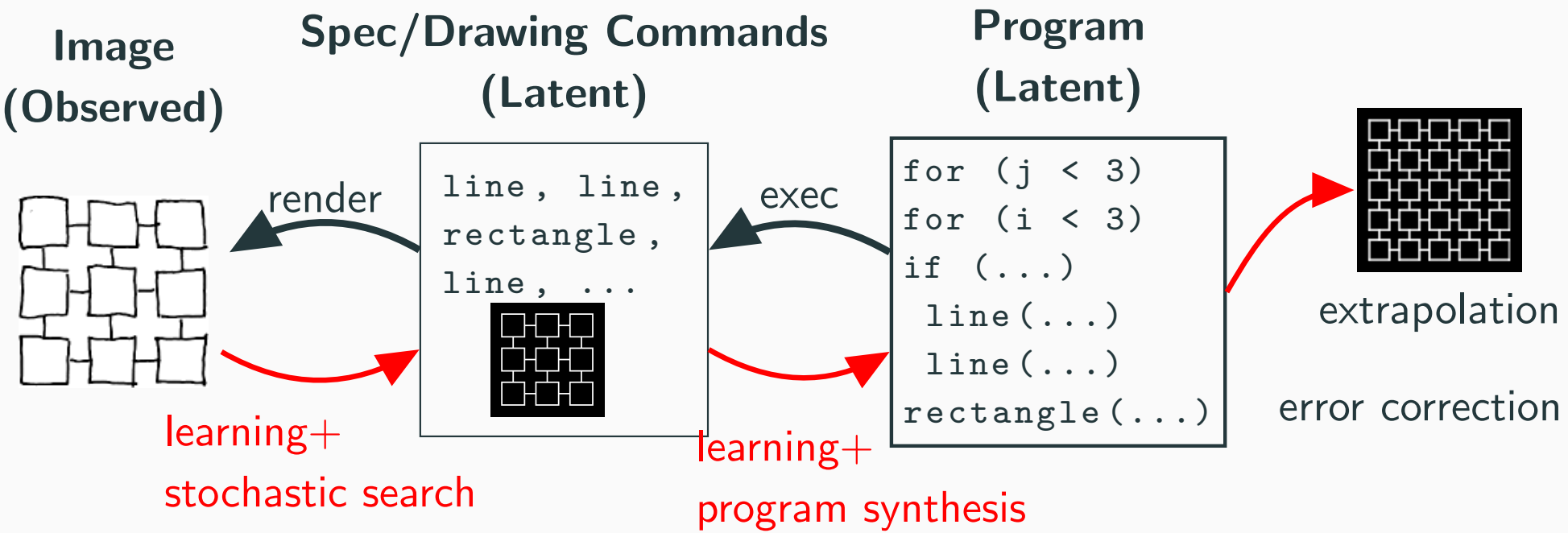


Image→Spec

Spec→Program

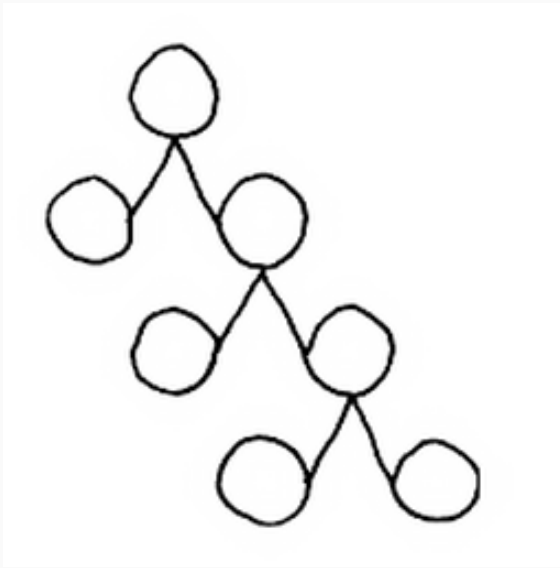
How to infer graphics programs from hand-drawn images



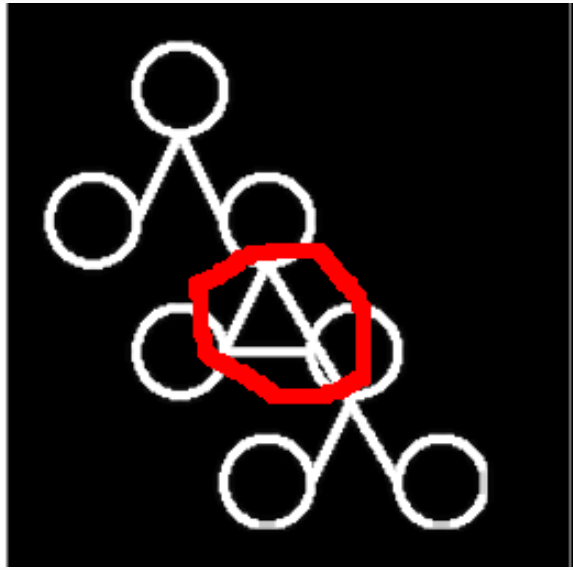
Image→Spec | Spec→Program | Applications

Top-down influences on perception

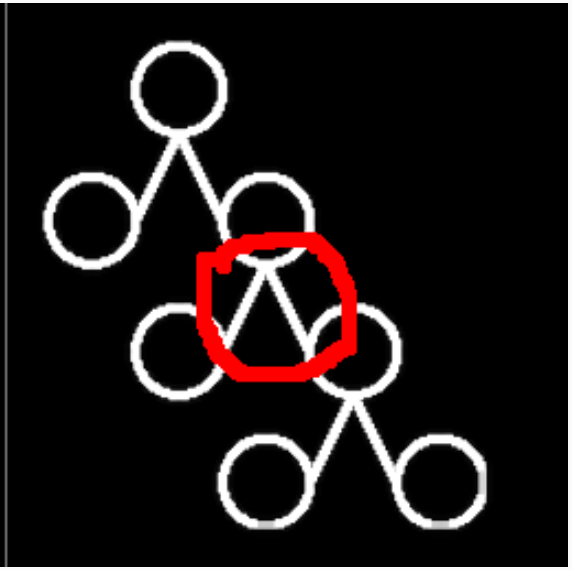
drawing



bottom-up neural net

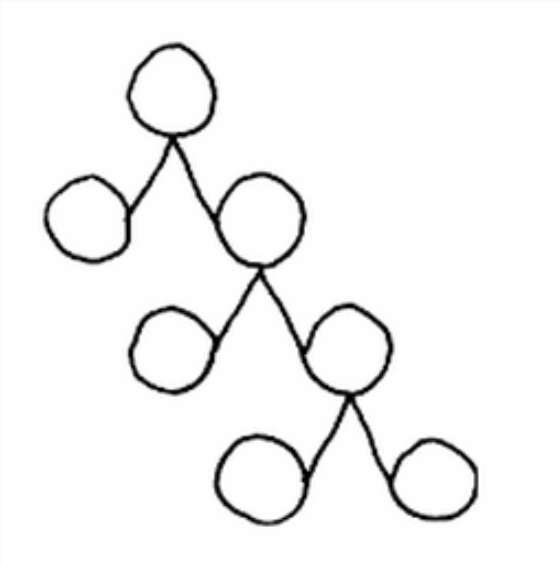


w/ top-down program bias

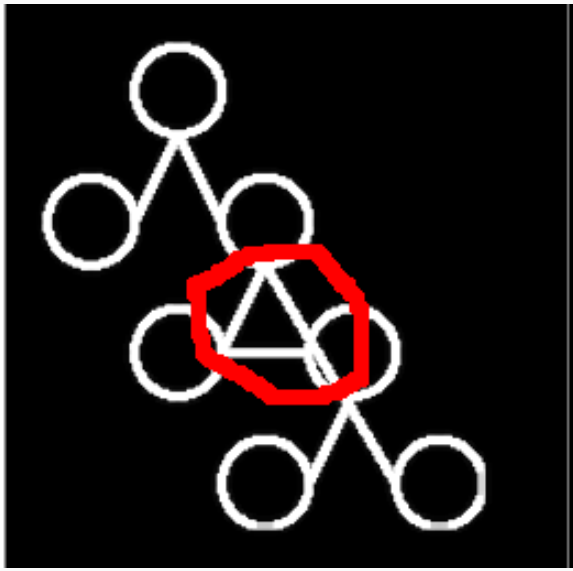


Top-down influences on perception

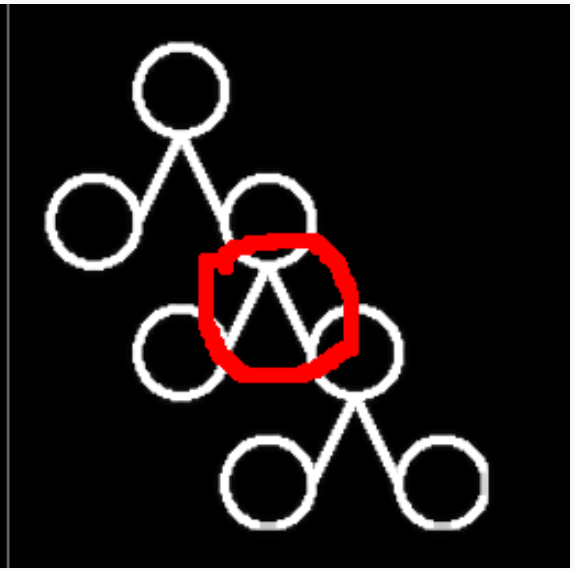
drawing

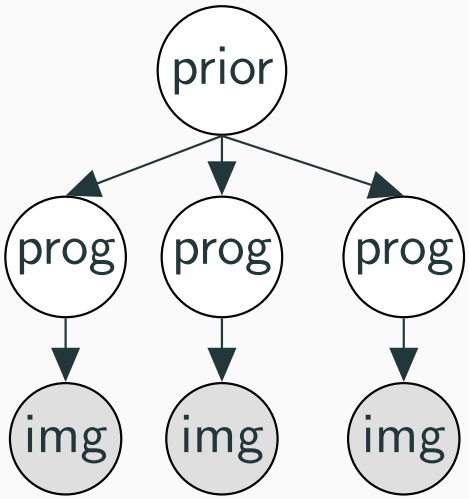


bottom-up neural net



w/ top-down program bias



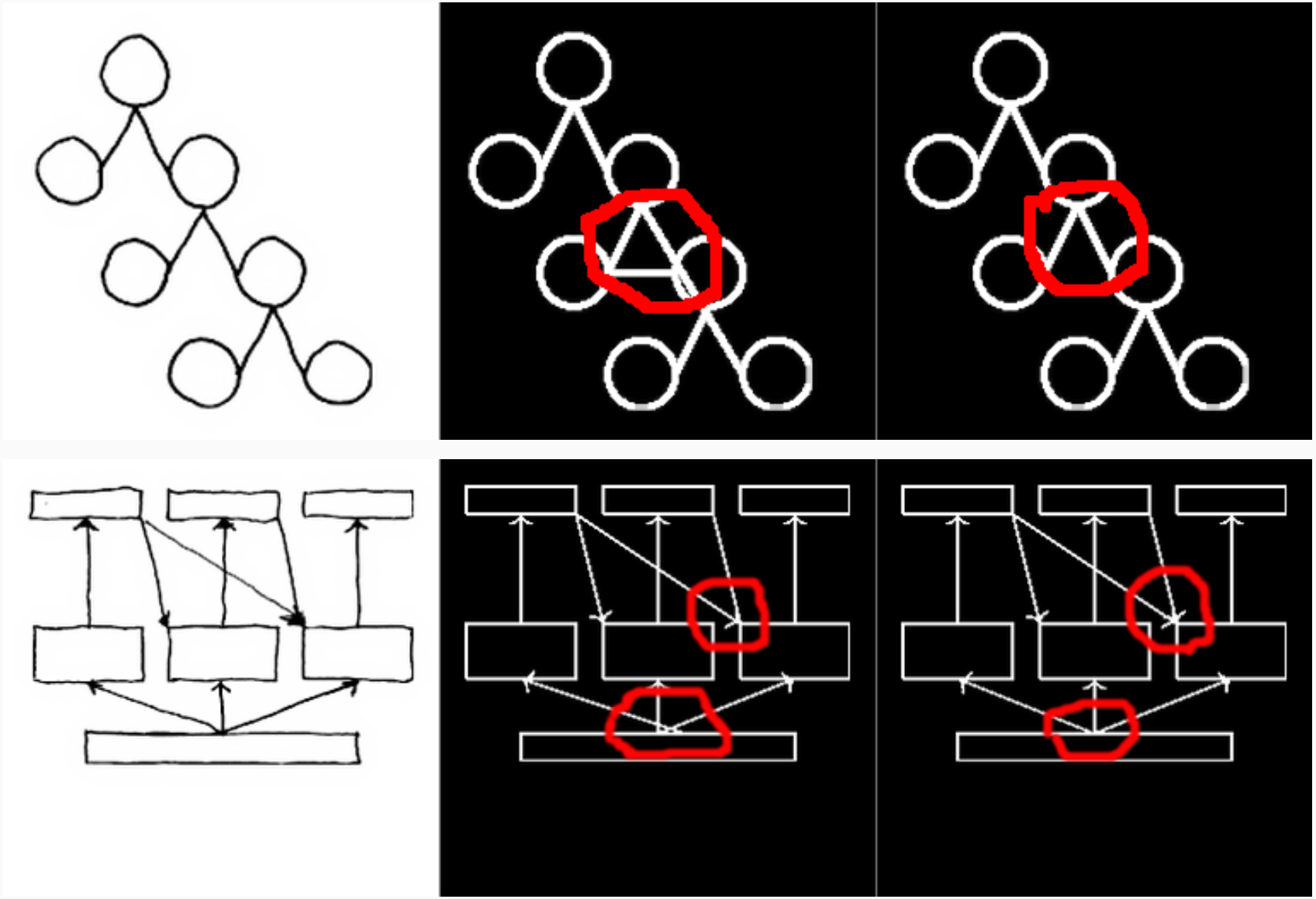


predicted program $=$

$\arg\max_{\text{progs}} \mathbb{P}\left[\text{img}|\text{prog}\right] \mathbb{P}\left[\text{prog}|\text{prior}\right]$

Top-down influences on perception

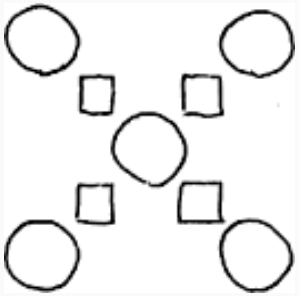
drawing | bottom-up neural net | w/ top-down program bias

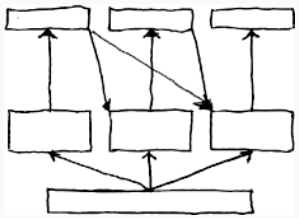


Example programs

Drawing	Program

	`for(i<3)`
	`  line(i,-1*i+6,`
	`  2*i+2,-1*i+6)`
	`  line(i,-2*i+4,i,-1*i+6)`

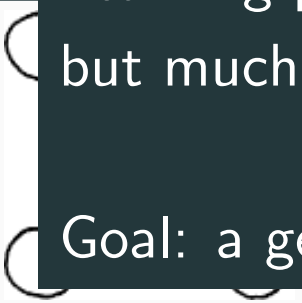
	`reflect(y=8)`
	`  for(i<3)`
	`  if(i>0)`
	`  rectangle(3*i-1,2,3*i,3)`
	`  circle(3*i+1,3*i+1)`

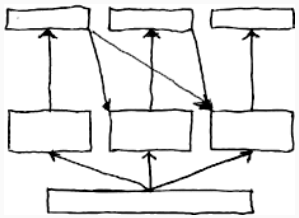
	`for(i<3)`
	`  line(7,1,5*i+2,3,arrow)`
	`  for(j<i+1)`
	`  if(j>0)`
	`  line(5*j-1,9,5*i,5,arrow)`
	`  line(5*j+2,5,5*j+2,9,arrow)`
	`  rectangle(5*i,3,5*i+4,5)`
	`  rectangle(5*i,9,5*i+4,10)`
	`rectangle(2,0,12,1)`

Example programs

Drawing	Program

	`for(i<3)`
	`  line(i,-1*i+6,`
	`  2*i+2,-1*i+6)`
	`  line(i,-2*i+4,i,-1*i+6)`

	`for(i<3)`
	`  line(7,1,5*i+2,3,arrow)`
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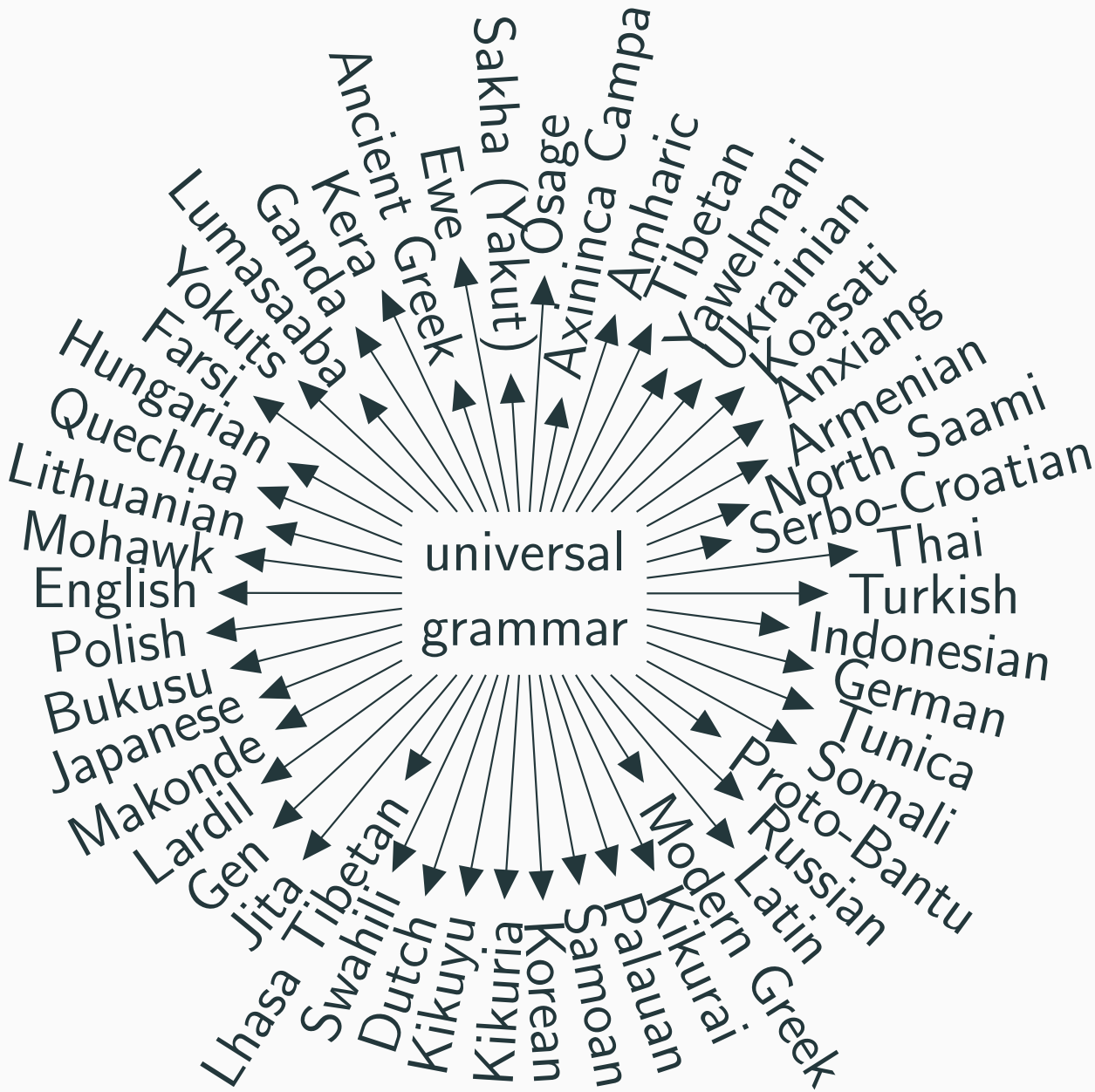
	`  line(5*j+2,5,5*j+2,9,arrow)`
	`  rectangle(5*i,3,5*i+4,5)`
	`  rectangle(5*i,9,5*i+4,10)`
	`rectangle(2,0,12,1)`

Learning played a role...

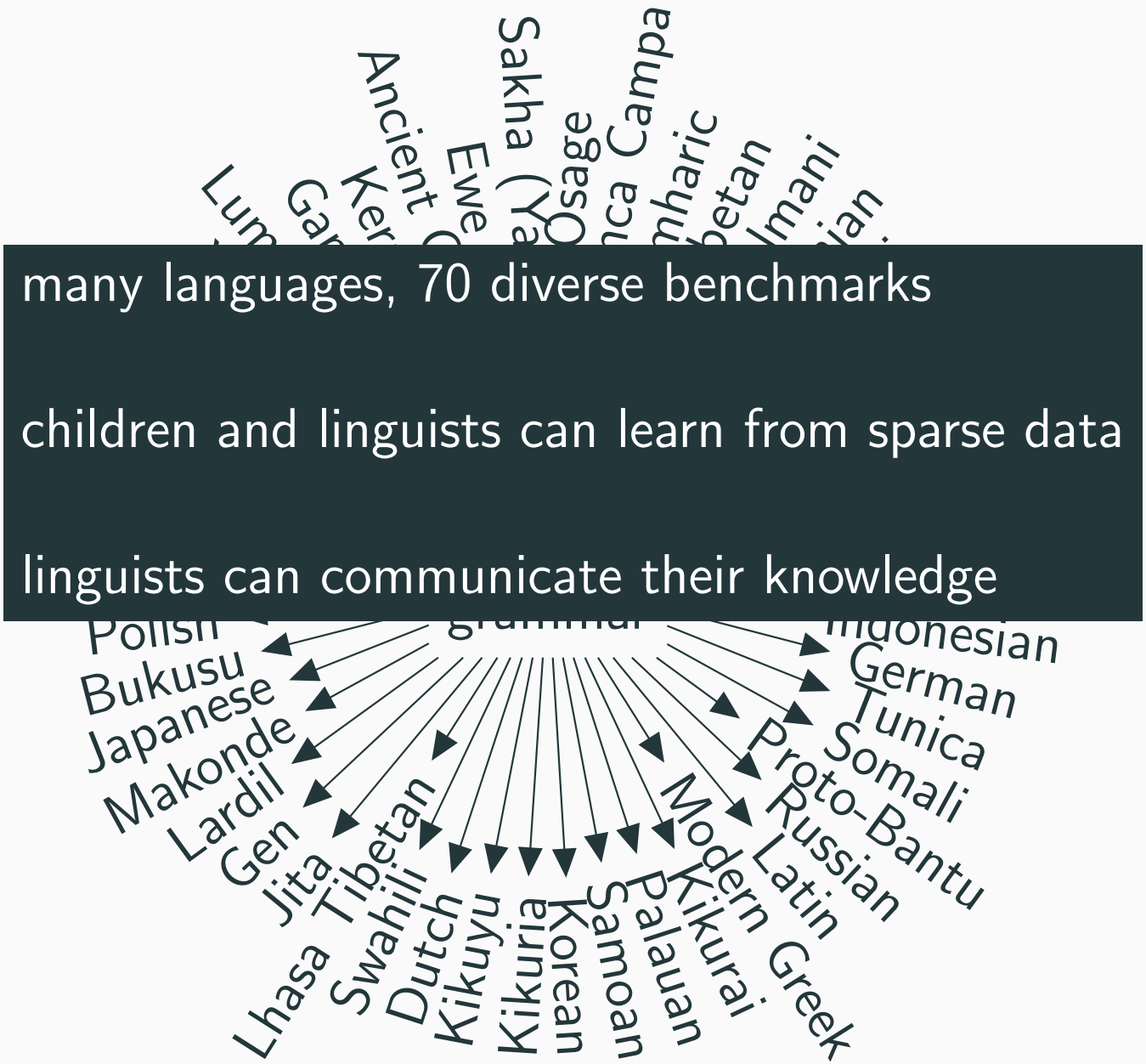
but much of this system is specific to 2-D graphics

Goal: a general algorithm for learning to synthesize programs

Synthesizing human-understandable models of language



Synthesizing human-understandable models of language



Few-shot language learning experiment

Farsi:

	singular	plural
“lip”	læb	læban
“gazelle”	ahu	ahuan
“mother”	valede	???

Few-shot language learning experiment

Farsi:

	singular	plural
“lip”	læb	læban
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Few-shot language learning experiment

Mandarin:

	adjective	adverb
“slow”	man	manmandə
“fast”	kuai	kuaikuaidə
“small”	xiao	???

Few-shot language learning experiment

Mandarin:

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Few-shot language learning experiment

Mandarin:

	adjective	adverb
“slow”	man	manmandə
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stem+stem+də

Few-shot language learning experiment

Serbo-Croatian:

	masculine	feminine
“rich”	bogat	bogata
“mild”	blag	blaga
“green”	zelen	???

Few-shot language learning experiment

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Serbo-Croatian:

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add “a” to stem to make feminine

Few-shot language learning experiment

Serbo-Croatian:

	masculine	feminine
“rich”	bogat	bogata
“mild”	blag	blaga
“green”	zelen	zelena
“clear”	???	yasna

add “a” to stem to make feminine

Few-shot language learning experiment

Serbo-Croatian:

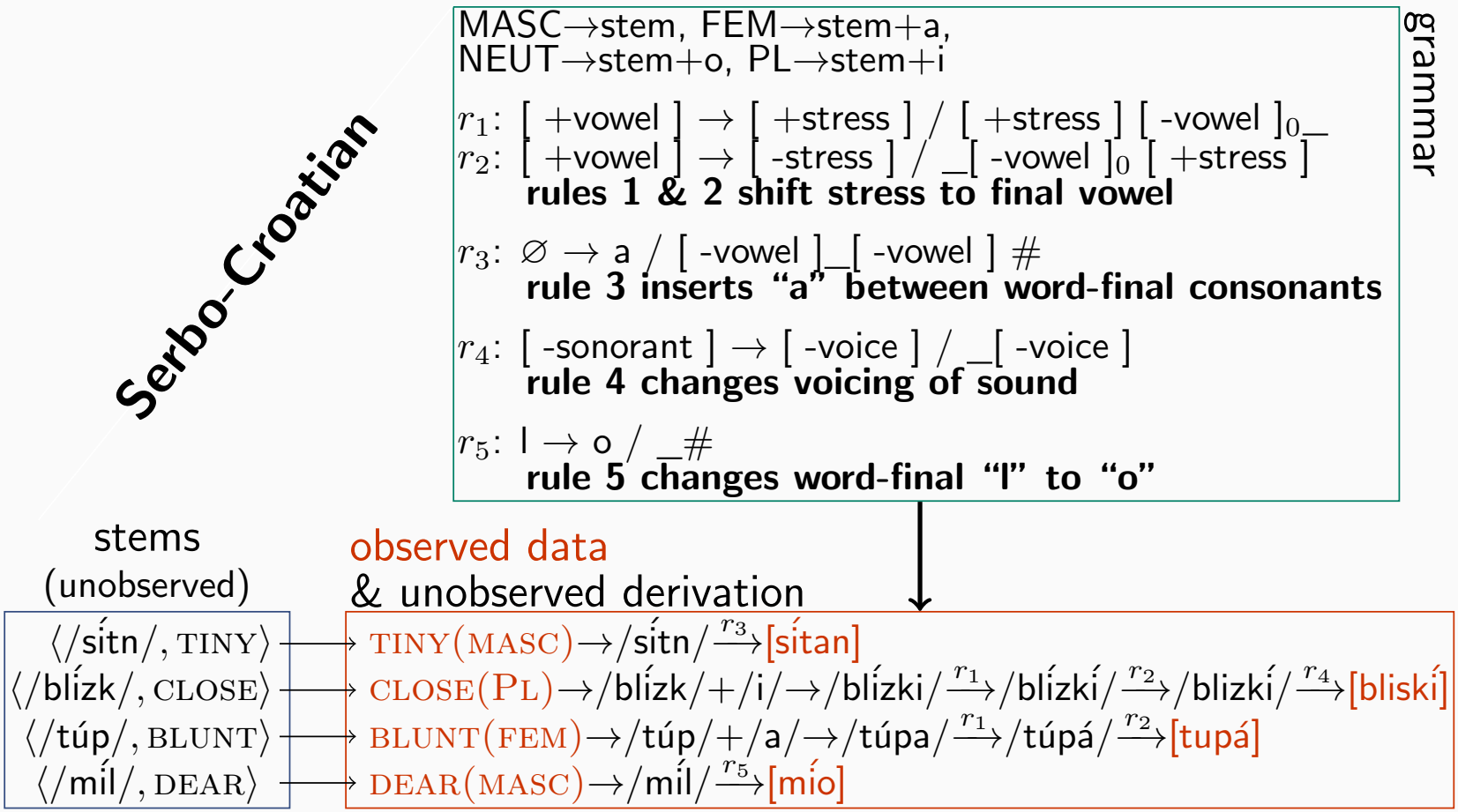
	masculine	feminine	stem (unobserved)
"rich"	bogat	bogata	bogat
"mild"	blag	blaga	blag
"green"	zelen	zelena	zelen
"clear"	**yasan**	yasna	yasn

add "a" to stem to make feminine

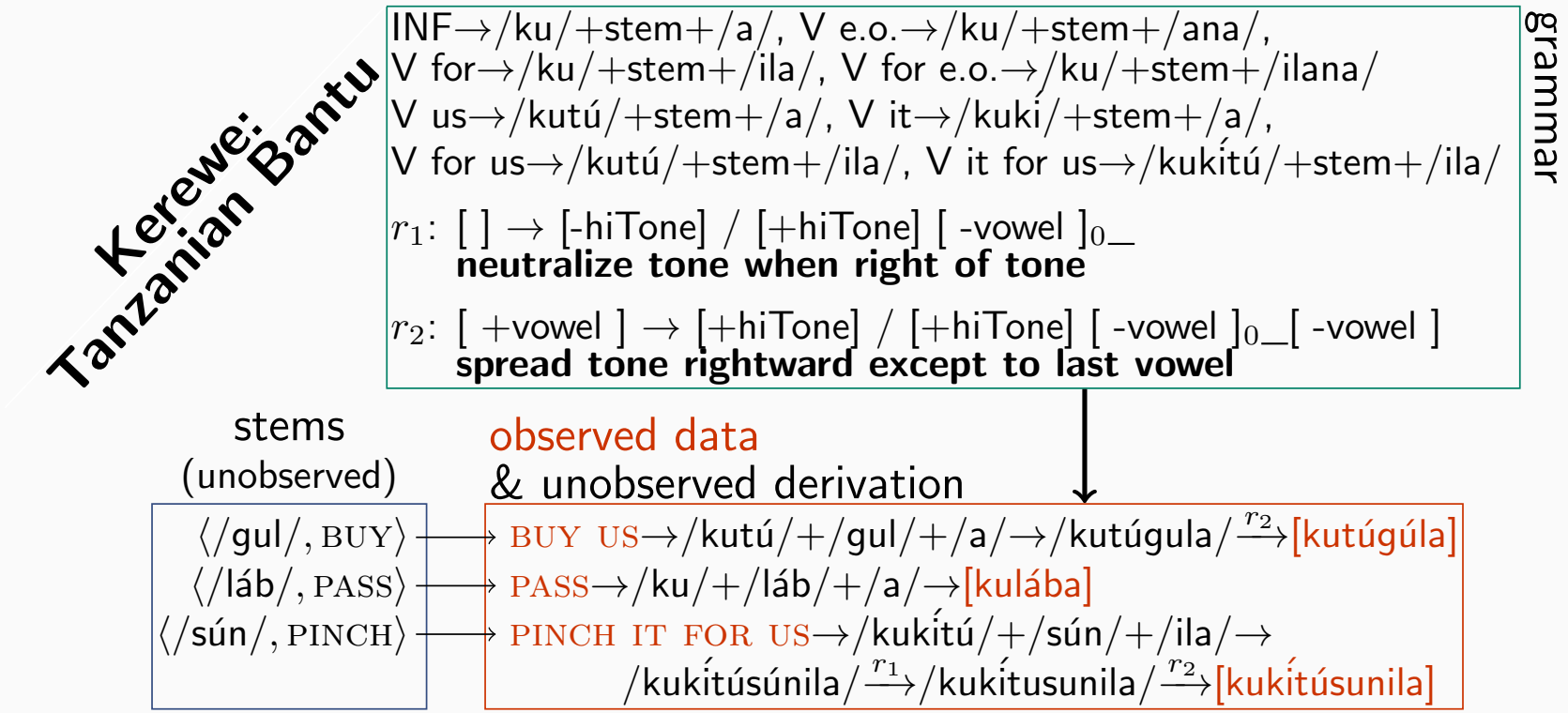
insert "a" between two word-final consonants

$\varnothing \rightarrow$ a / C_C#

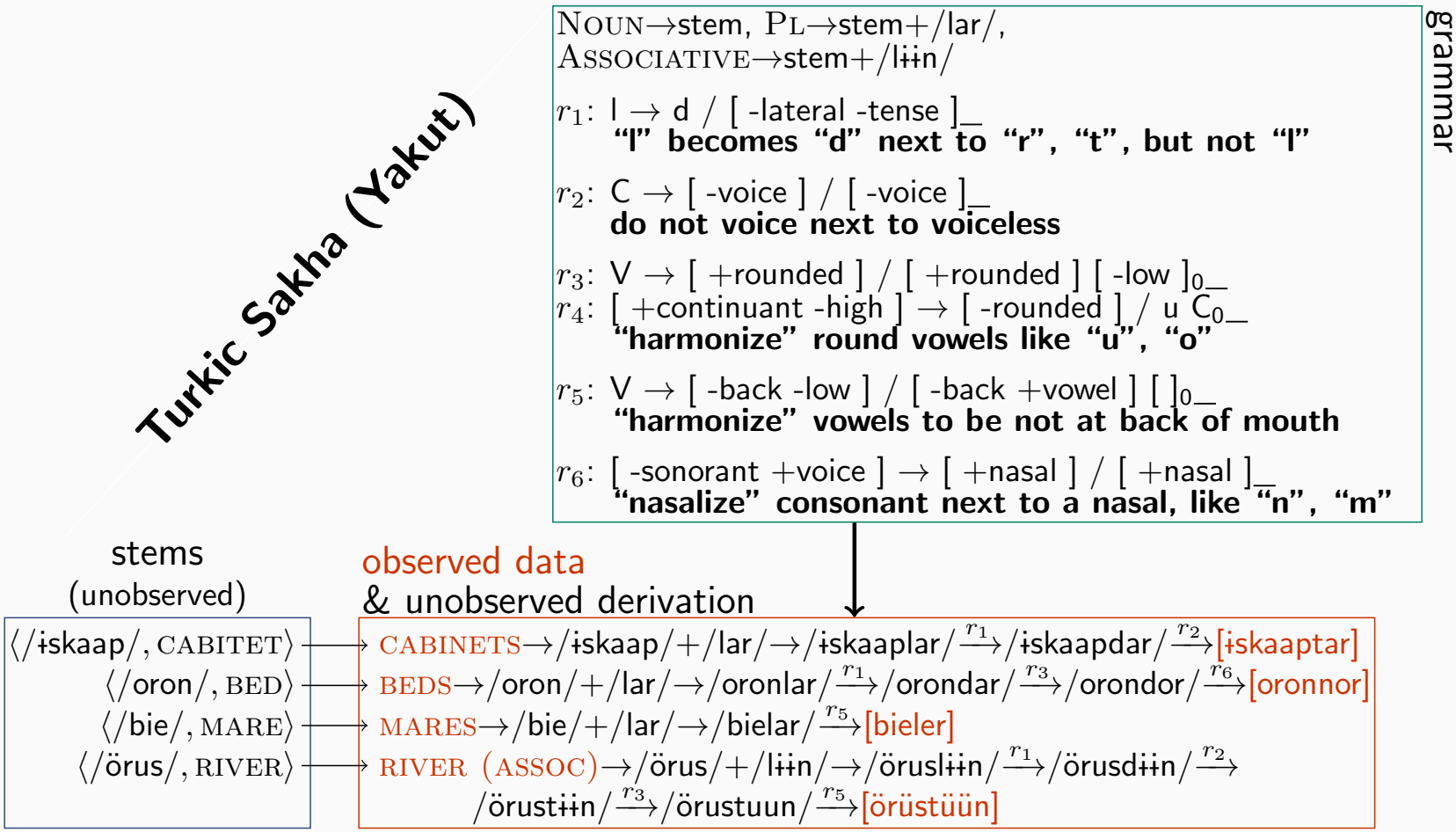
Diverse Linguistic Phenomena



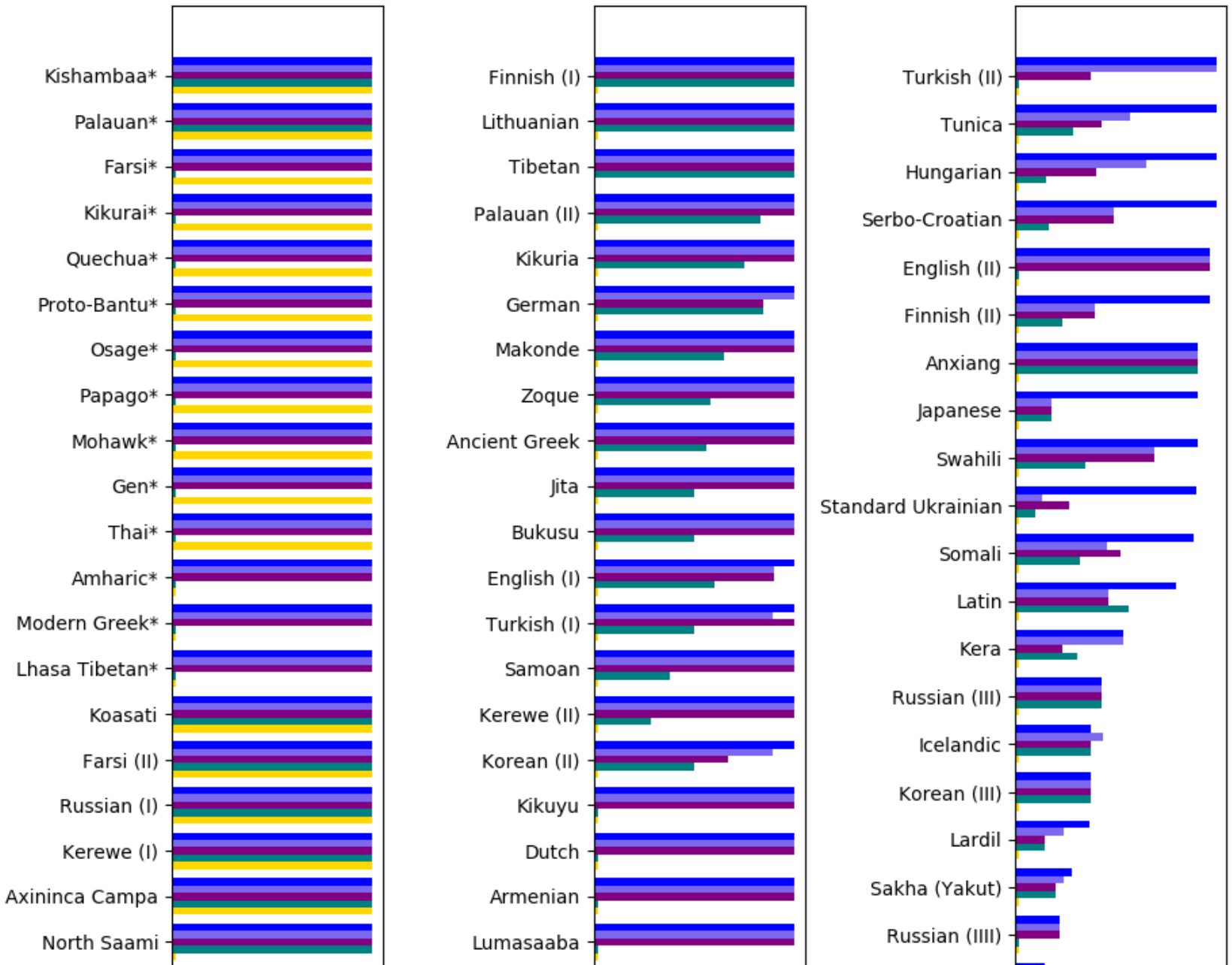
Languages with tones



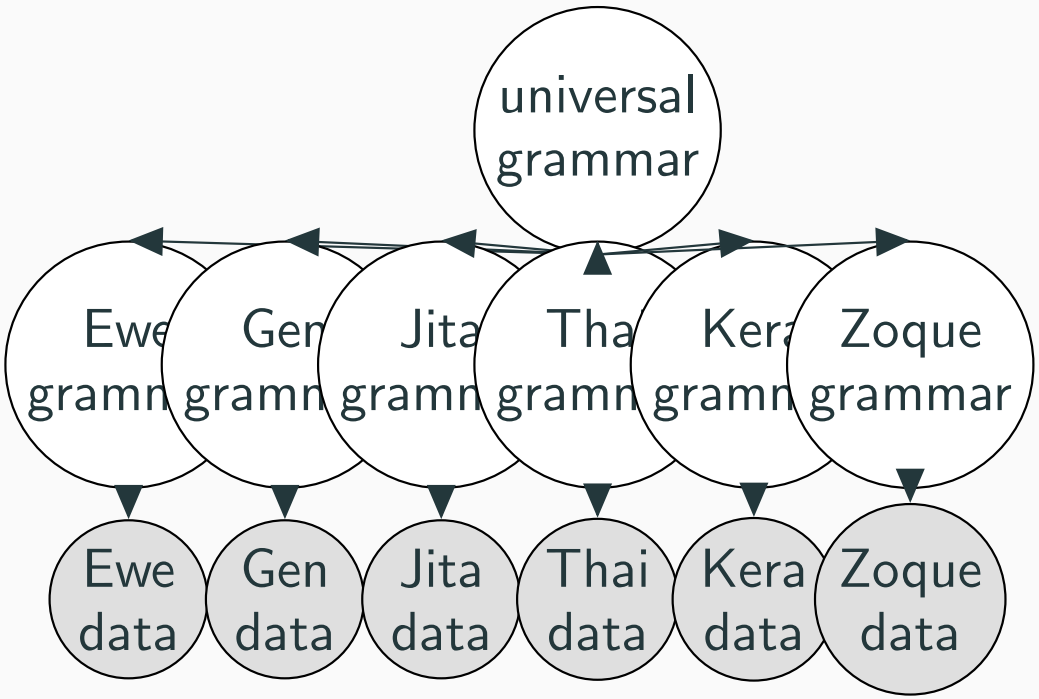
Vowel “harmony”



57 languages drawn from 70 datasets



Distilling higher-level knowledge



Distilling higher-level knowledge

Discovered universal grammar schema

consonant/vowel distinction

sounds$\leftarrow$ [-vowel]

sounds$\leftarrow$ [+vowel]

a set of sounds is commonly

all consonants,

or all vowels

w/o learned universal grammar

Tibetan: [-nasal] $\rightarrow \varnothing$ / #_

w/ learned universal grammar

[-vowel] $\rightarrow \varnothing$ / # _[-vowel]

Growing domain-specific knowledge

Goal: acquire domain-specific knowledge needed to induce a class of programs

- Library of concepts (declarative knowledge; domain specific language; generative model over programs)
- Inference strategy (procedural knowledge; synthesis algorithm)

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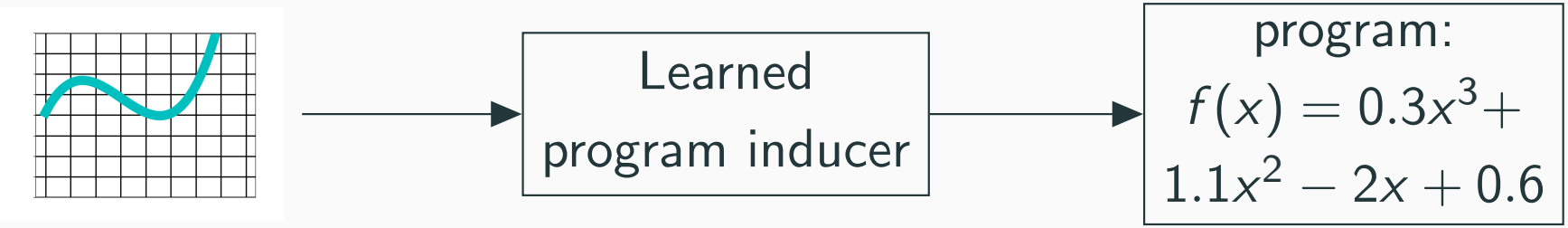

Concepts: x^3, $\alpha x + \beta$, etc

Inference strategy: neurosymbolic search for programs

Growing domain-specific knowledge

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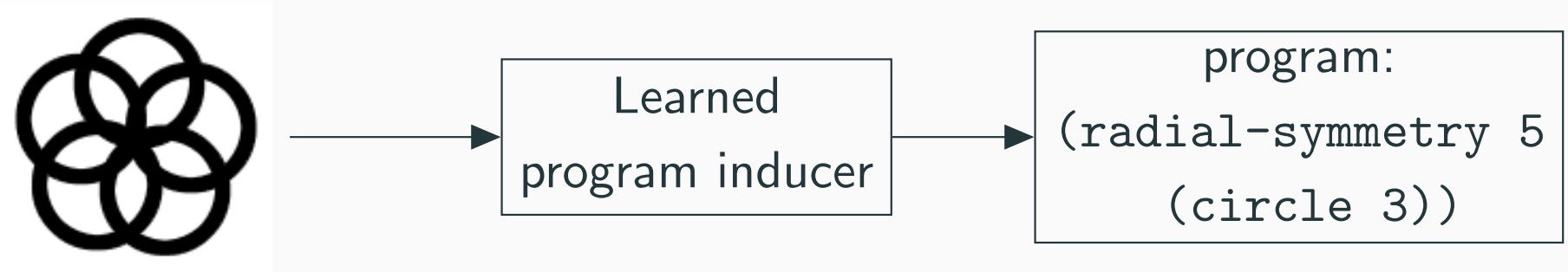
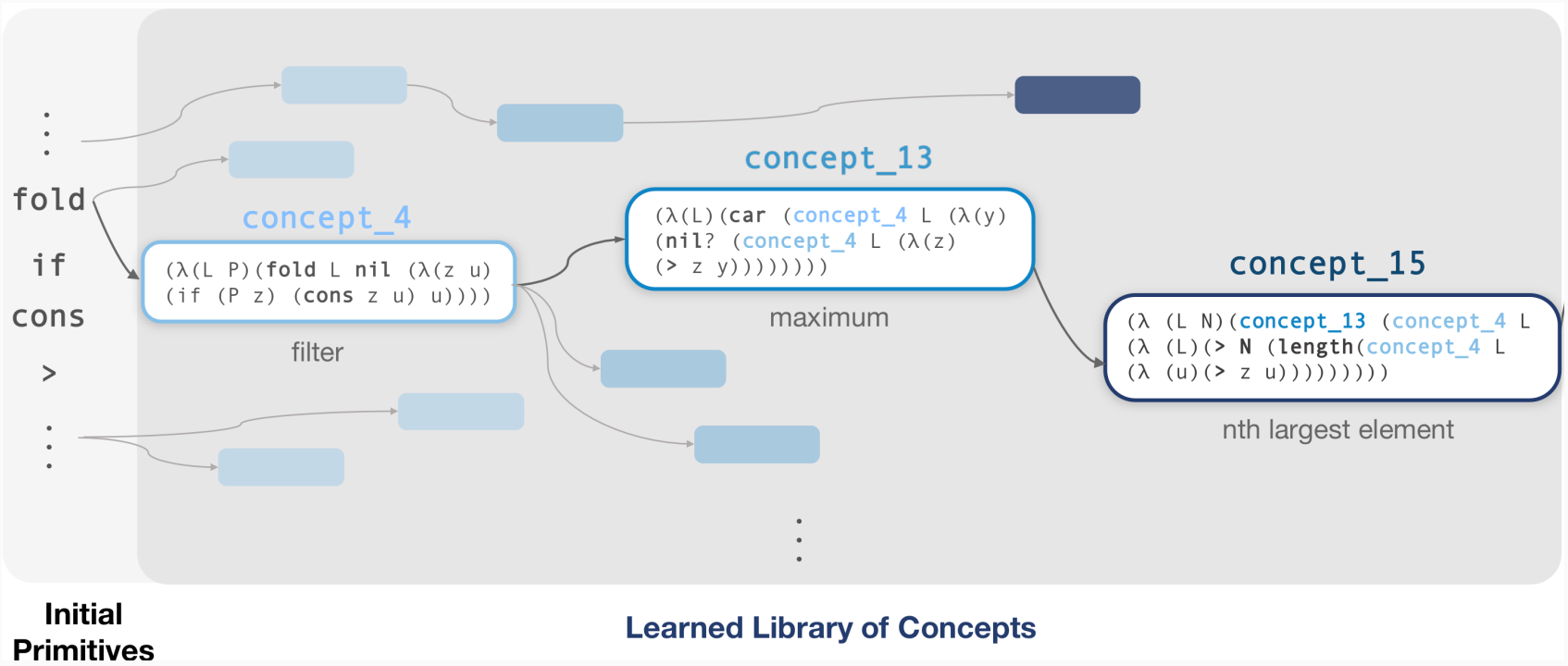


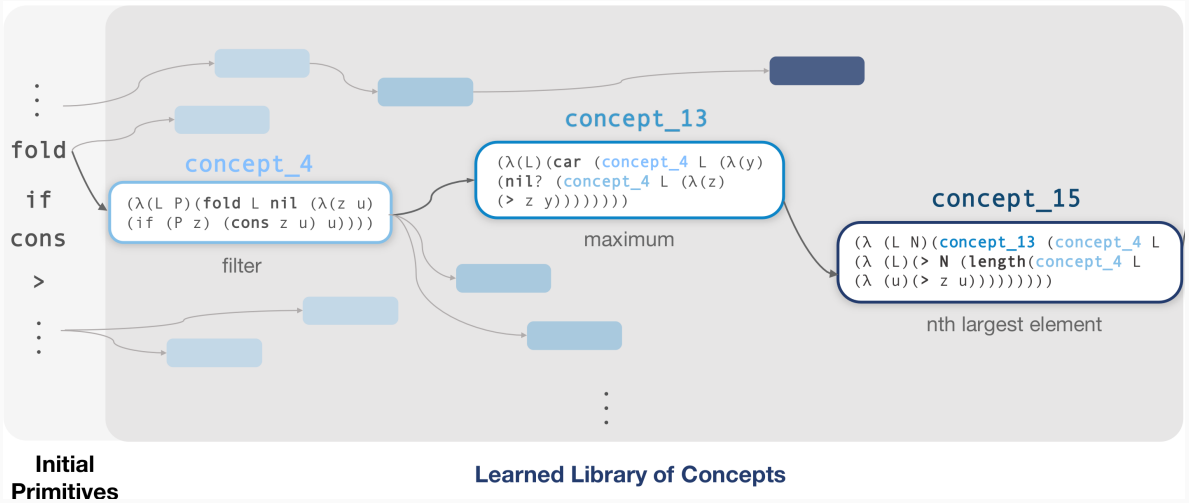
Concepts: `circle`, `radial-symmetry`, etc

Inference strategy: neurosymbolic search for programs

Library learning



Library learning



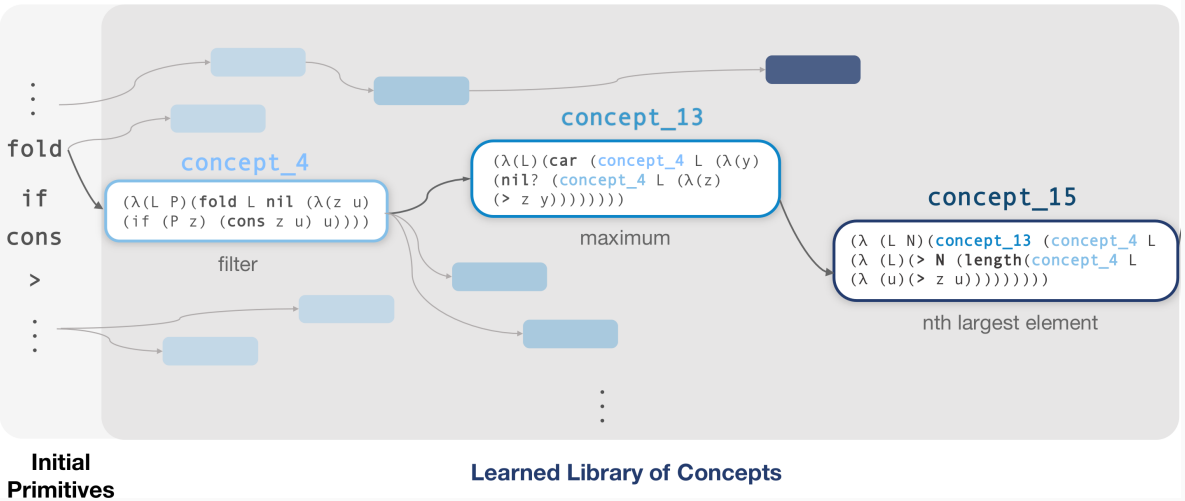
Problem: sort list

Solution:

```
(map (λ (n) (concept_15 L (+ 1 n))) (range (length L))))
```

get nth largest element where n = 1, 2, 3 ….length of list

Library learning



Problem: sort list

Solution:

```
(map (λ (n) (concept_15 L (+ 1 n))) (range (length L))))
```

get nth largest element where n = 1, 2, 3 ….length of list

Solution in initial primitives:

```
(λ (x) (map (λ (y) (car (fold (fold x nil (λ (z u) (if (gt?
(+ y 1) (length (fold x nil (λ (v w) (if (gt? z v) (cons v
w) w))))) (cons z u) u))) nil (λ (a b) (if (nil? (fold
(fold x nil (λ (c d) (if (gt? (+ y 1) (length (fold x nil
(λ (e f) (if (gt? c e) (cons e f) f))))) (cons c d) d)))
nil (λ (g h) (if (gt? g a) (cons g h) h)))) (cons a b)
b))))) (range (length x))))
```

Discovered Problem Solutions

DreamCoder

- **Wake:** Solve problems by writing programs
- **Sleep:** Improve DSL and neural recognition model:
  - **Abstraction sleep:** Improve library
  - **Dream sleep:** Improve neural inference model
- Combines ideas from Wake-Sleep & Exploration-Compression

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List Processing

Sum List

[1 2 3] → 6
[4 6 8 1] → 17

Double

[1 2 3 4] → [2 4 6 8]
[6 5 1] → [12 10 2]

Check Evens

[0 2 3] → [T T F]
[2 4 9 6] → [T T F T]

Text Editing

Abbreviate

Allen Newell → A.N.
Herb Simon → H.S.

Drop Last Characters

jabberwocky → jabberw
copycat → cop

Extract

see spot(run) → run
a (bee) see → bee

Regexes

Phone Numbers

(555) 867-5309
(650) 555-2368

Currency

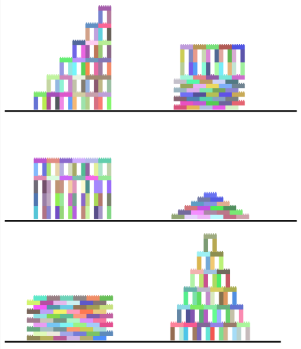
$100.25
$4.50

Dates

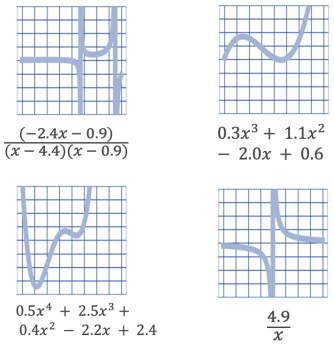
Y1775/0704
Y2000/0101

LOGO Graphics

Block Towers



Symbolic Regression



Recursive Programming

Filter

[■■■■■] → [■■]
[■■■■■■] → [■■■■]
[■■■■] → [■■■]

Index List

0, [■■■■■■] → ■
1, [■■■■■■] → ■
1, [■■■■■■■] → ■

Length

[■■■■] → 4
[■■■■■■] → 6
[■■■] → 3

Every Other

[■■■■] → [■■]
[■■■■■] → [■■■]
[■■■■■■] → [■■■]

Physics

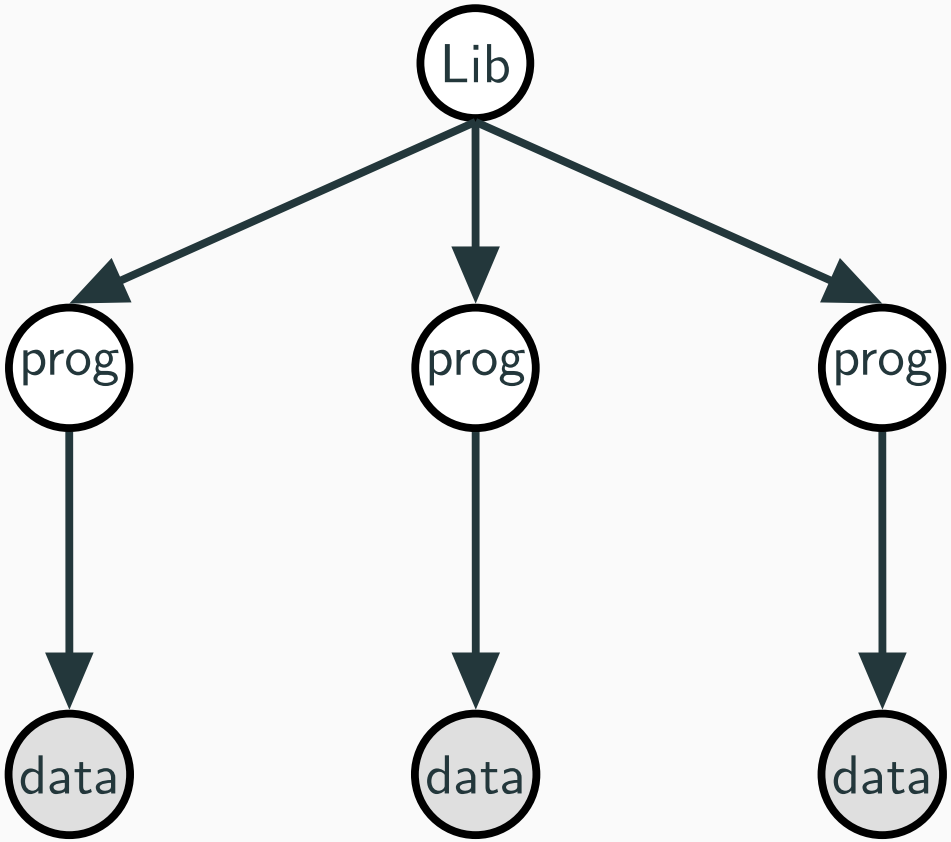
$$KE = \frac{1}{2}m|\vec{v}|^2$$

$$\vec{a} = \frac{1}{m}\sum_i \vec{F}_i$$

$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r_1} - \vec{r_2}|^2}\widehat{r_1 - r_2}$$

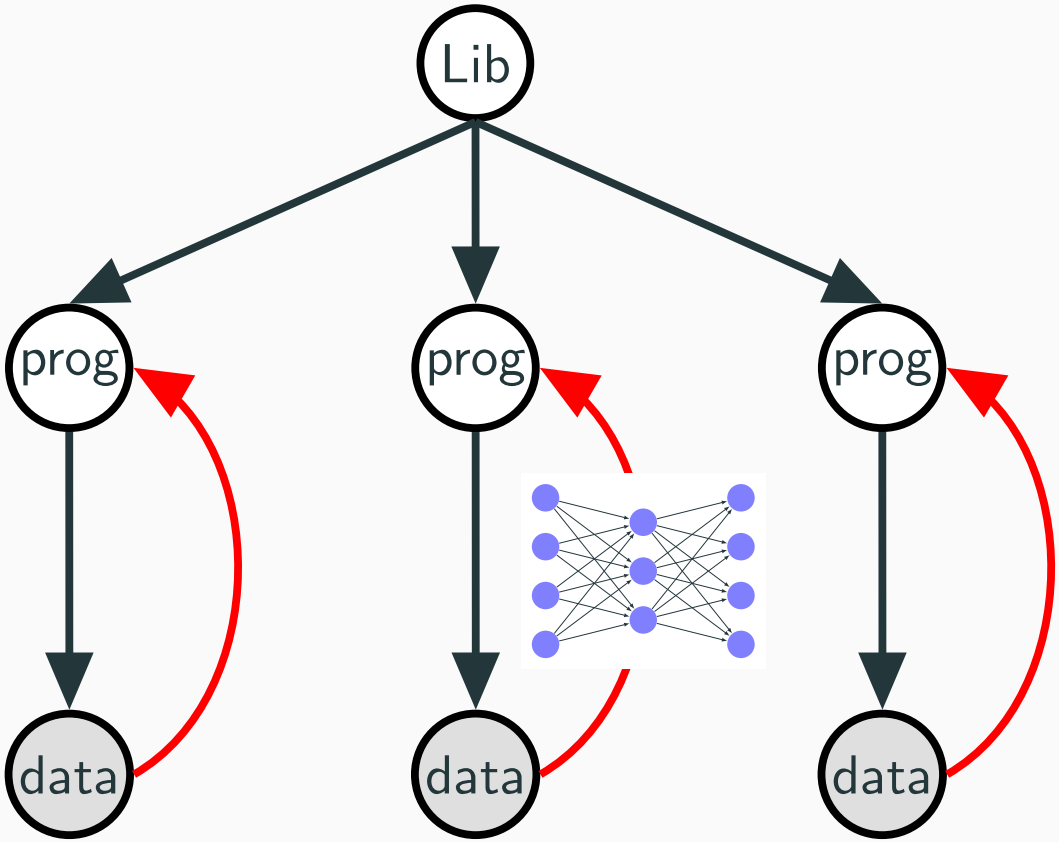
$$R_{total} = \left(\Sigma_i \frac{1}{R_i}\right)^{-1}$$

Library learning as Bayesian inference



[Dechter et al., 2013] [Liang et al, 2010]; [Lake et al, 2015]

Library learning as amortized Bayesian inference

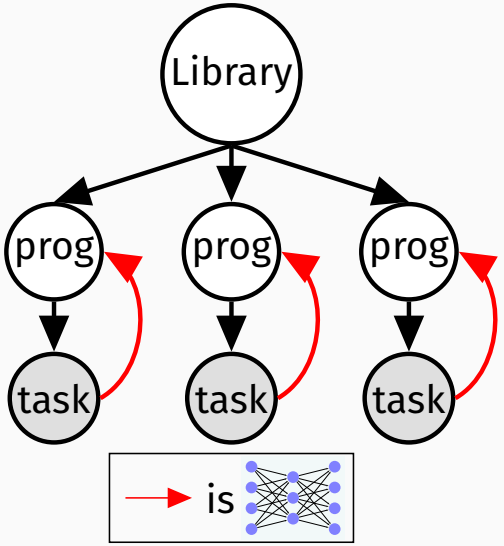


amortized inference +

better program representation (Lisp) +

library learning via program analysis +

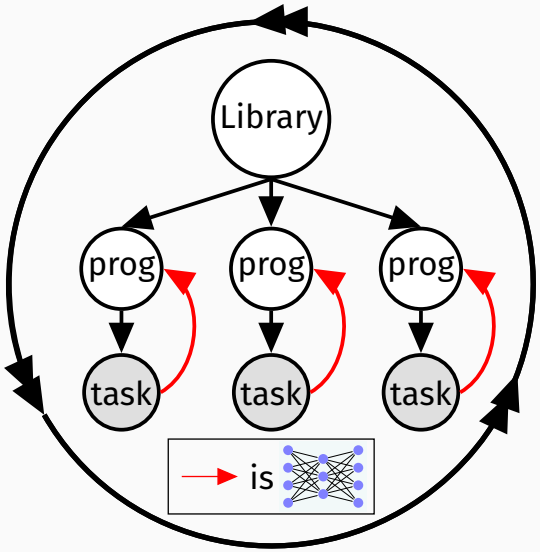
new neural inference network for program synthesis


Library
prog
prog
prog
task
task
task
is

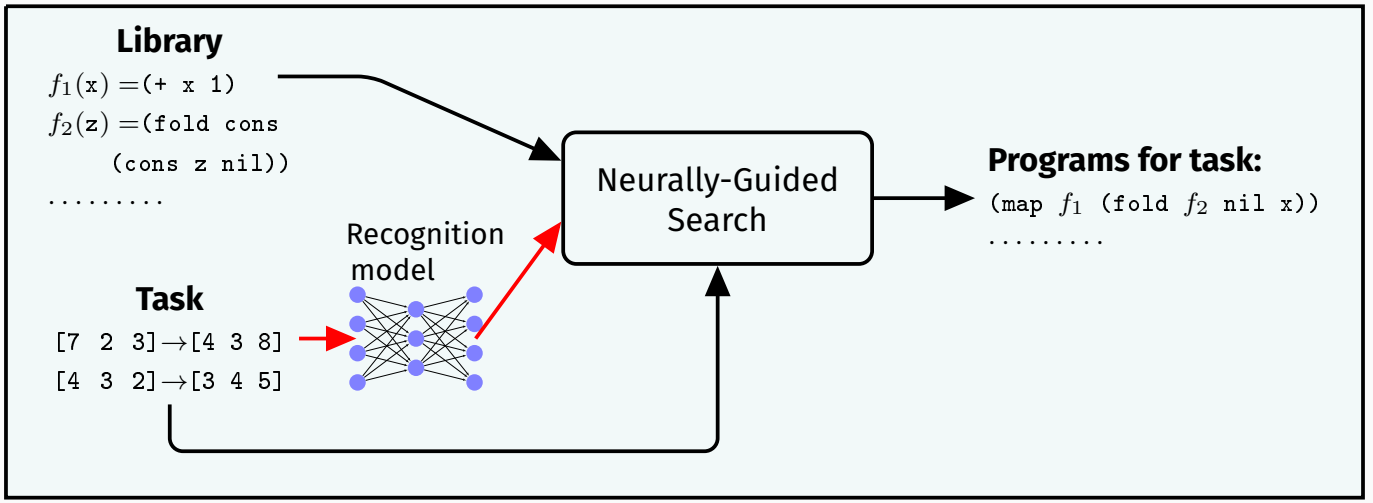
Wake

Sleep: Abstraction

Sleep: Dreaming

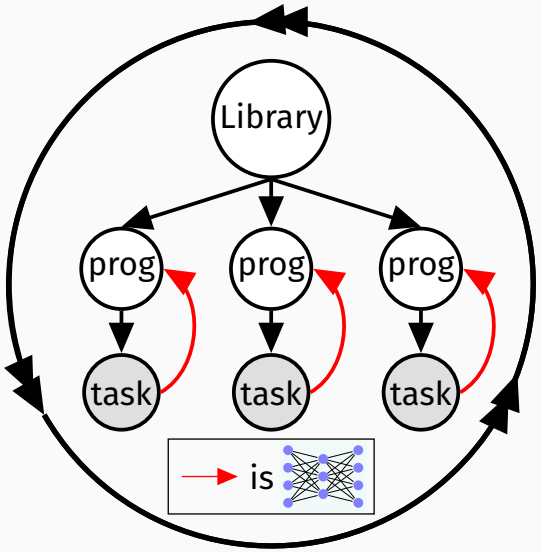


WAKE

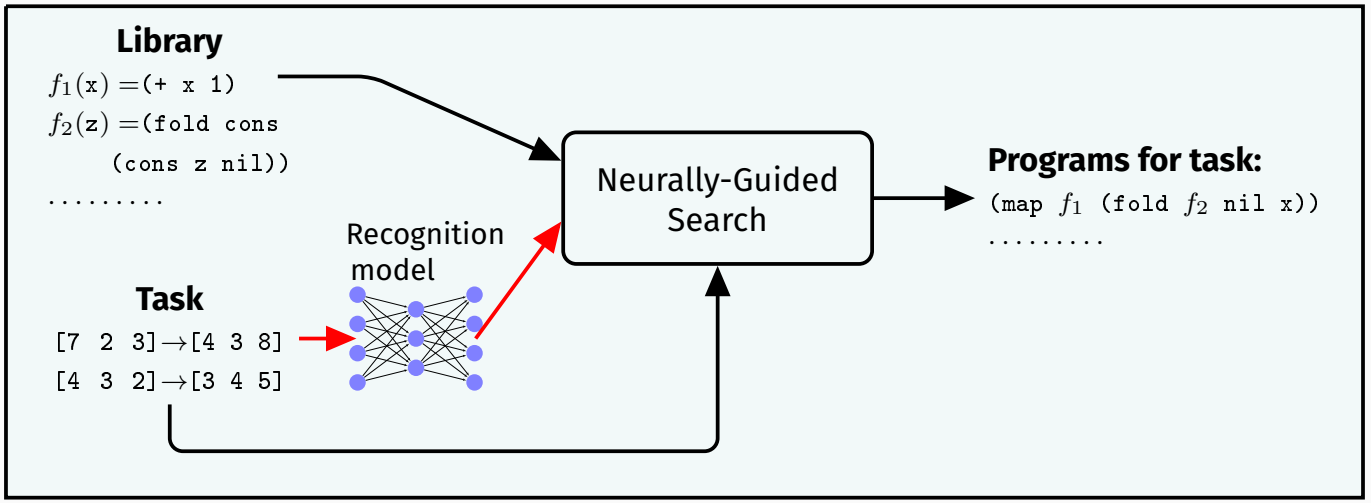


SLEEP: ABSTRACTION

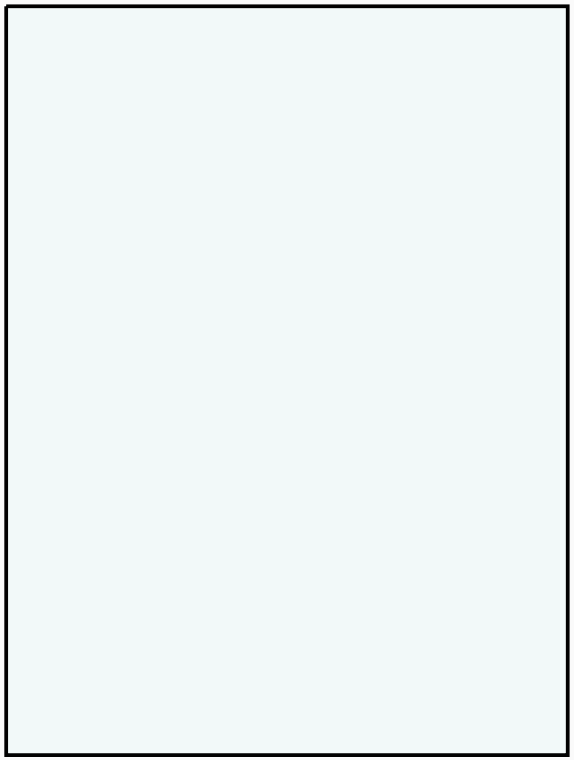
SLEEP: DREAMING

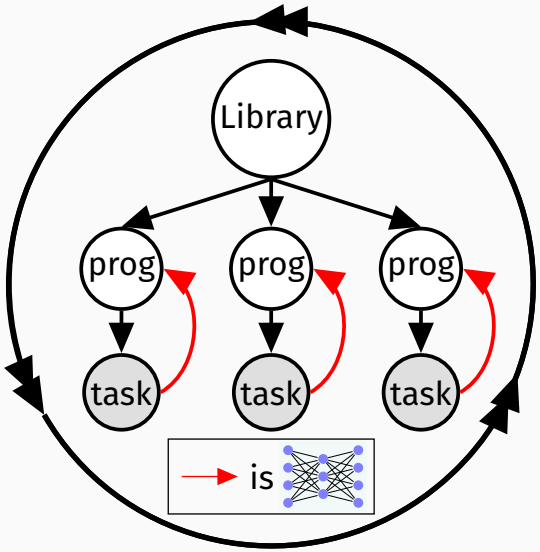


Wake

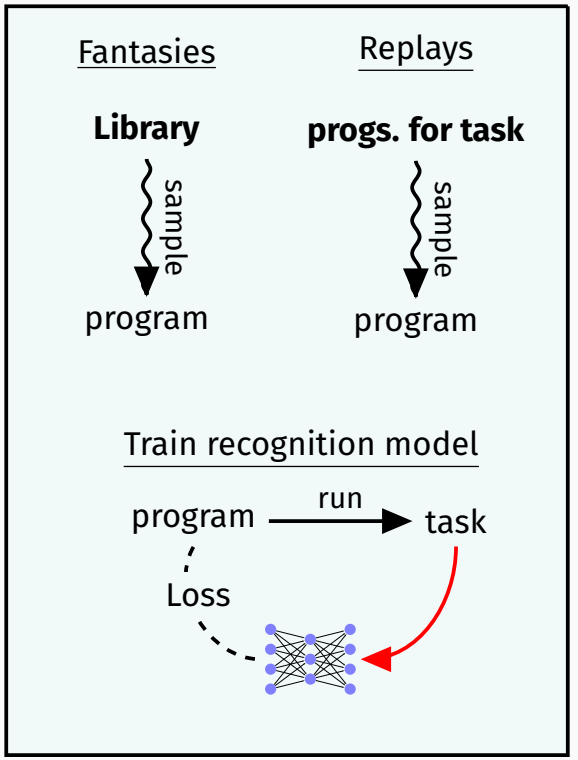


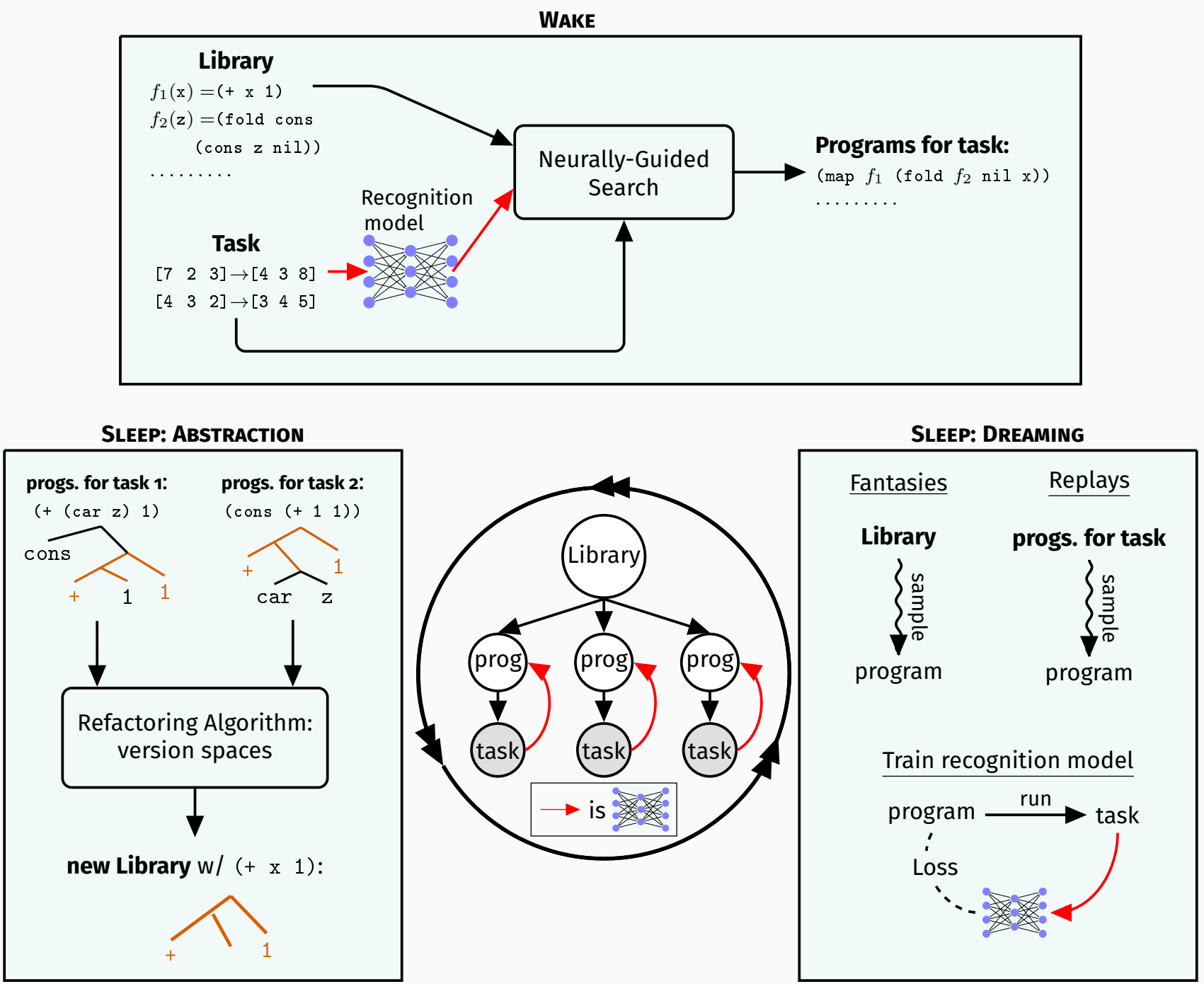
Sleep: Abstraction





Sleep: Dreaming




WAKE
Library
f_1(x) =(+ x 1)
f_2(z) =(fold cons
(cons z nil))
.....
Task
[7 2 3]→[4 3 8]
[4 3 2]→[3 4 5]
Recognition model
Neurally-Guided Search
Programs for task:
(map f_1 (fold f_2 nil x))
.....
SLEEP: ABSTRACTION
progs. for task 1:
(+ (car z) 1)
cons
+
1
1
progs. for task 2:
(cons (+ 1 1))
+
1
car
z
Refactoring Algorithm: version spaces
new Library w/ (+ x 1):
+
1
Library
prog
prog
prog
task
task
task
is
SLEEP: DREAMING
Fantasies
Replays
Library
progs. for task
sample
sample
program
program
Train recognition model
program
run
task
Loss

version space

$5 + 5$

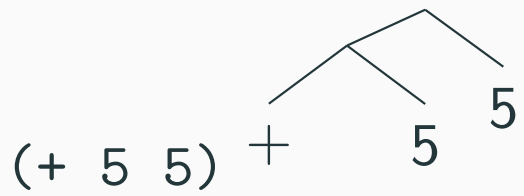
version space

$5 + 5$

```
(+ 5 5)
```

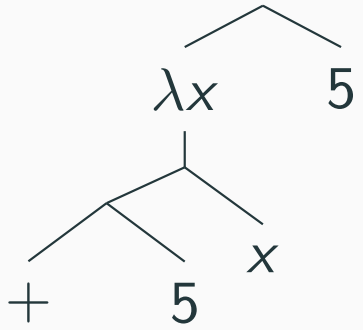
version space

$5 + 5$

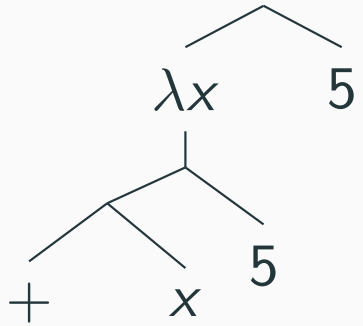


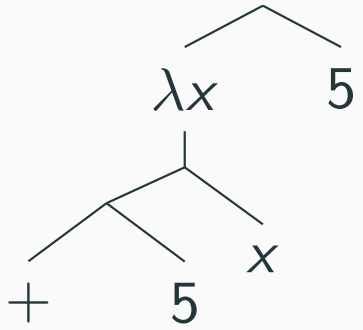
version space

version space

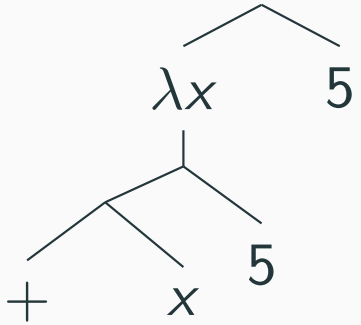


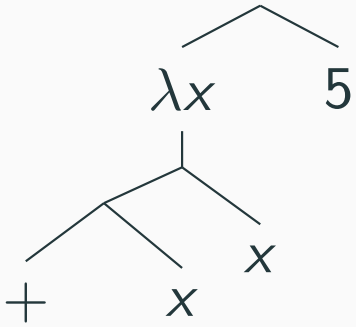
version space

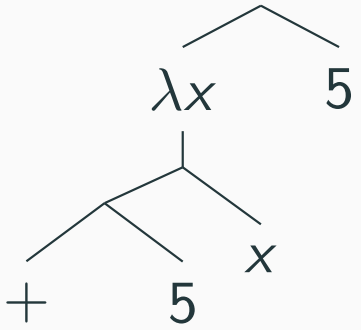




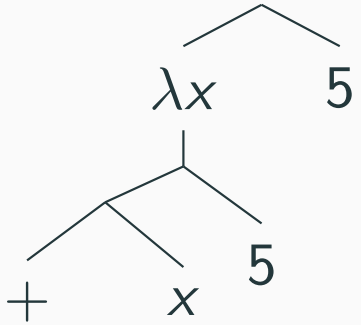
version space

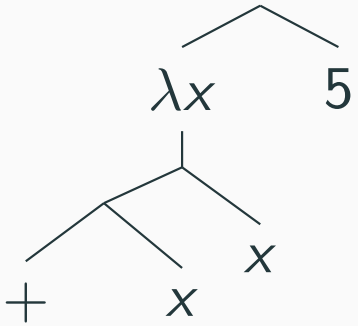


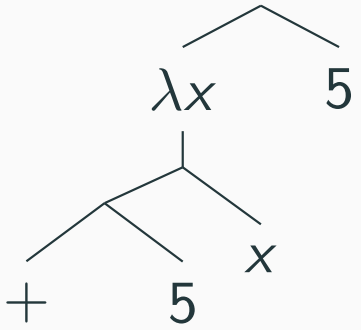


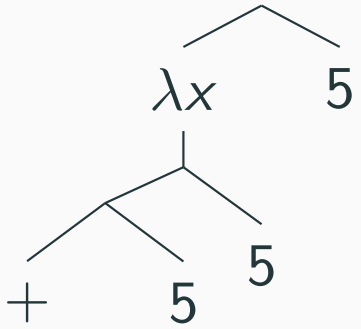


version space

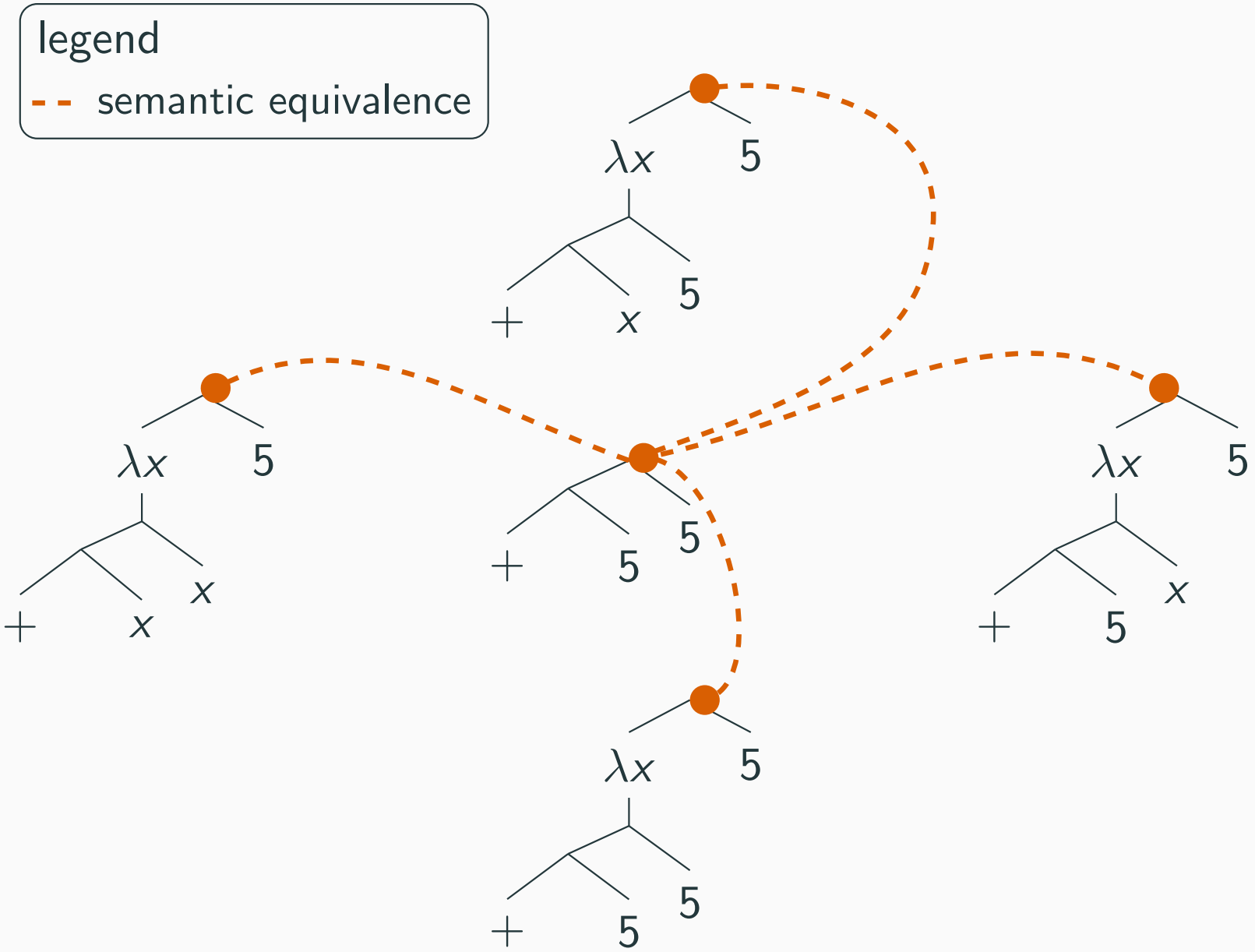








version space

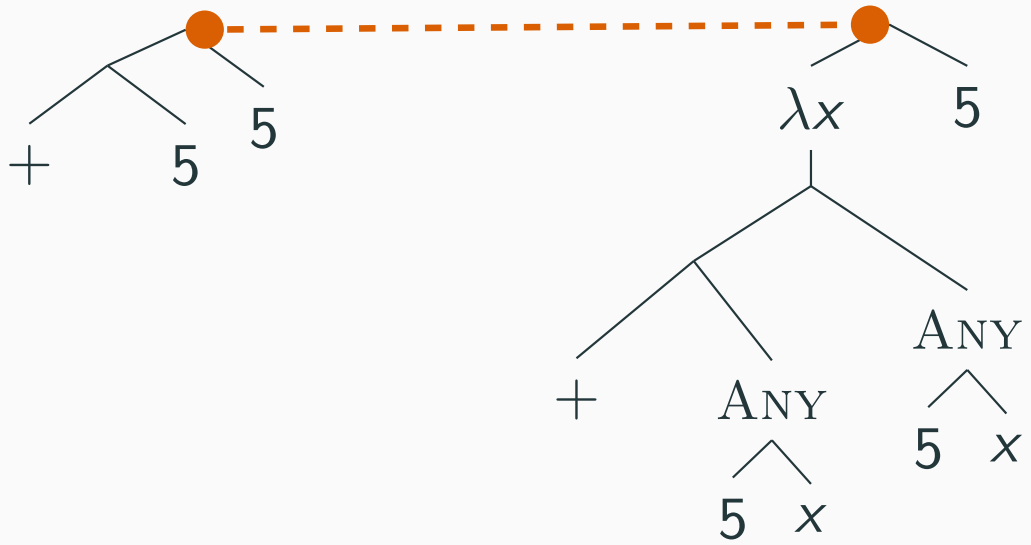


version space

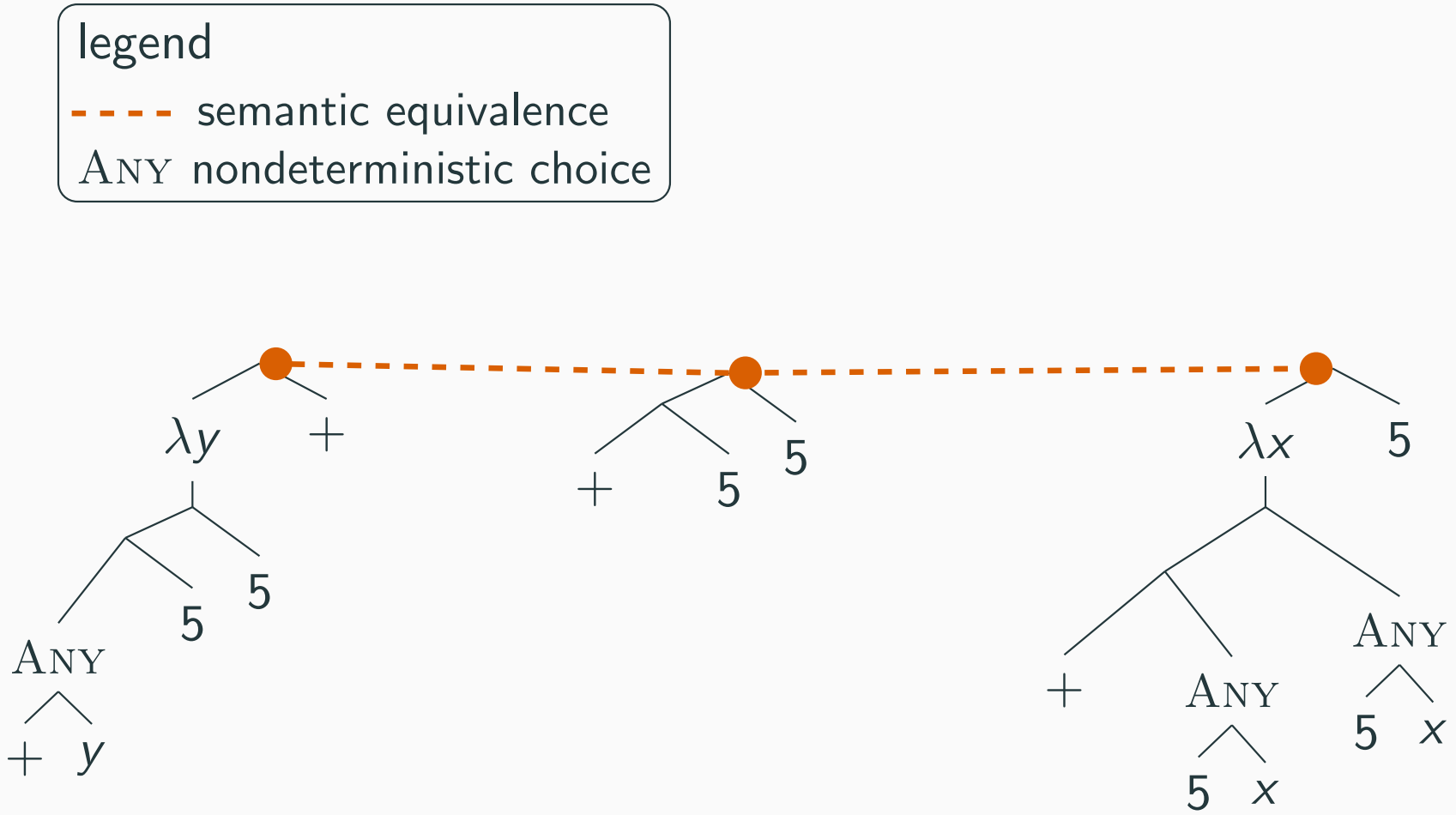
legend

- - - - semantic equivalence

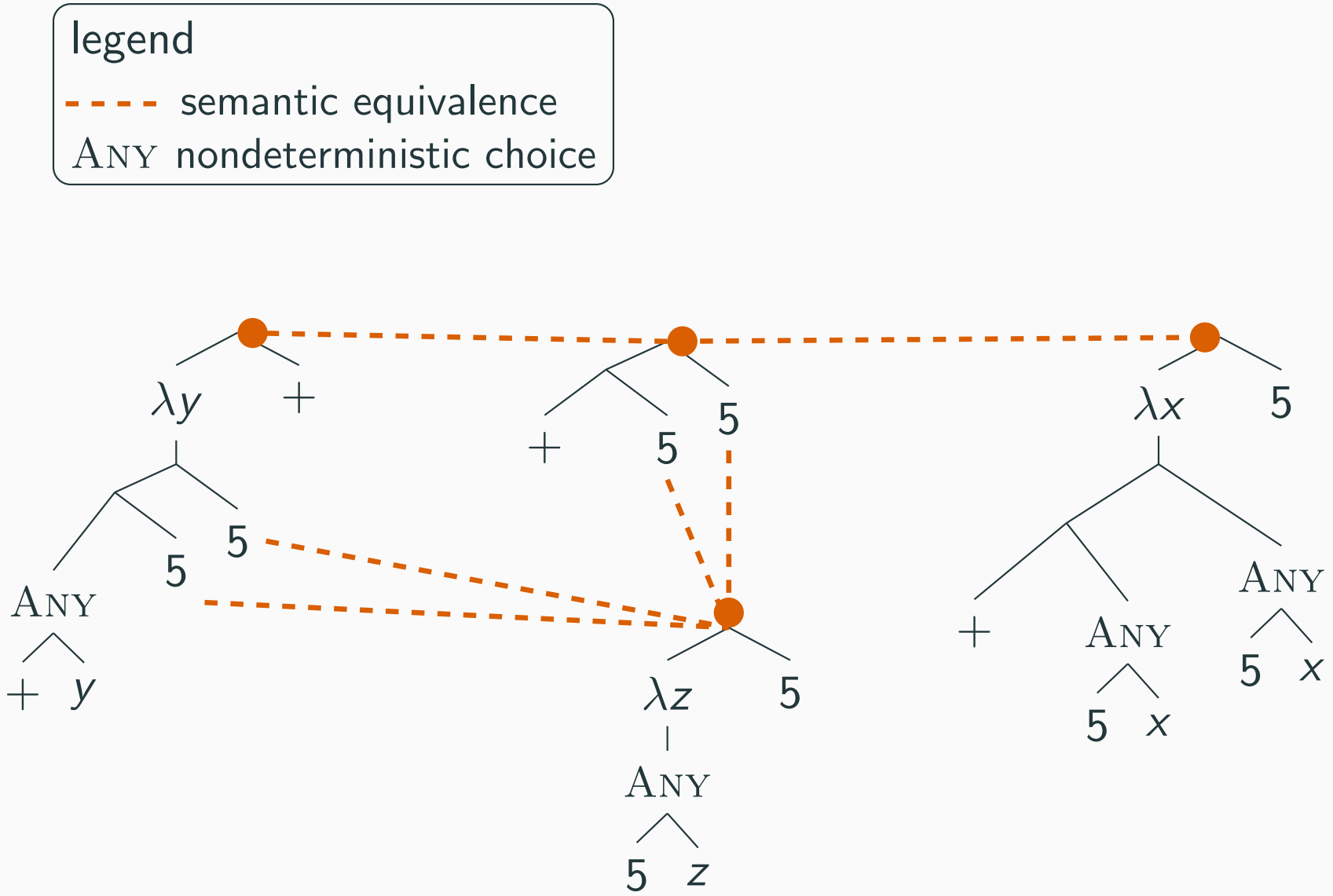
ANY nondeterministic choice



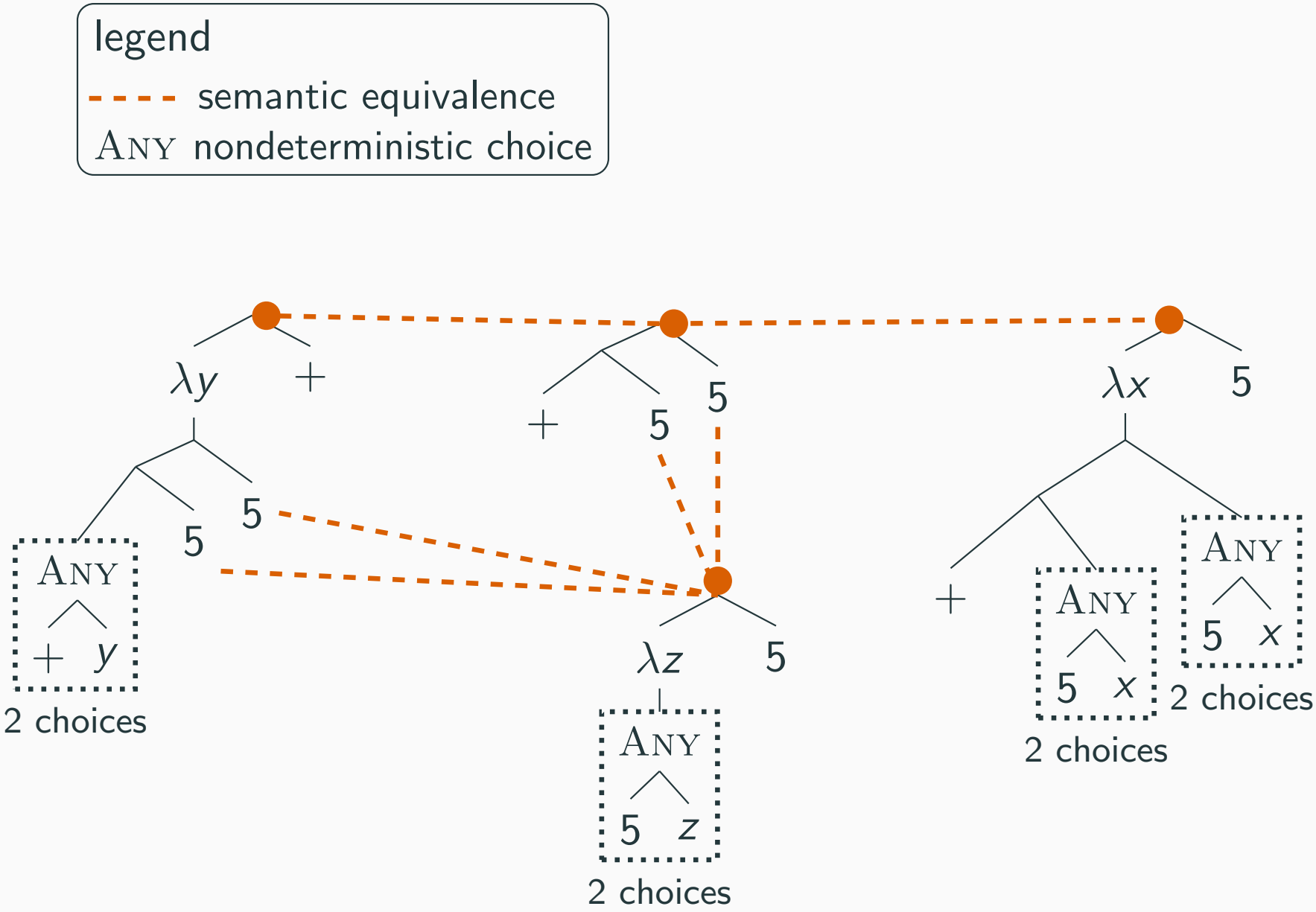
version space



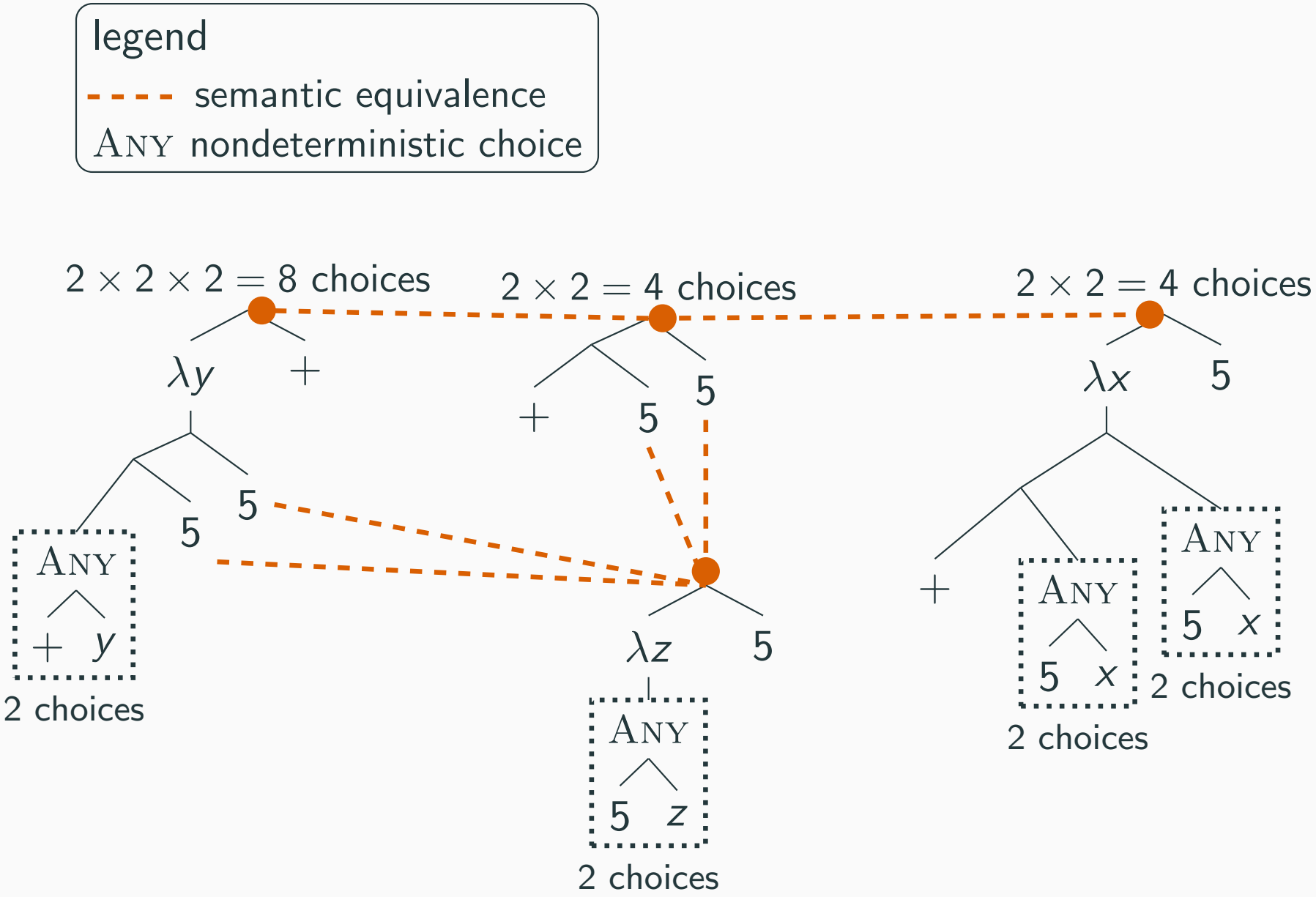
version space

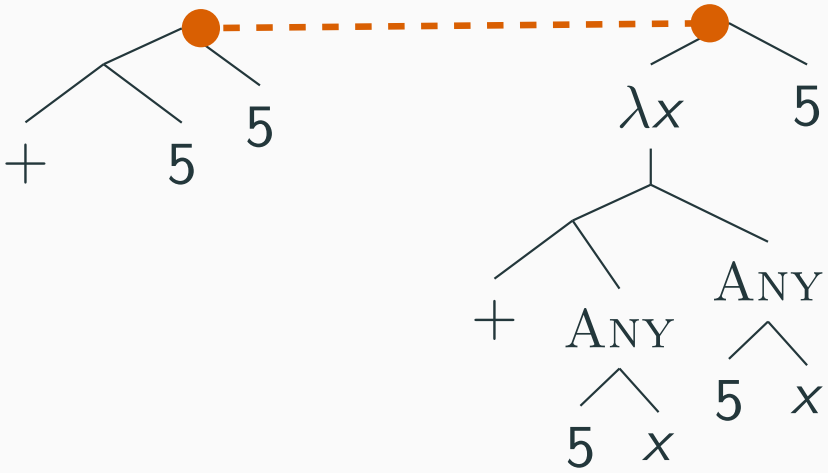


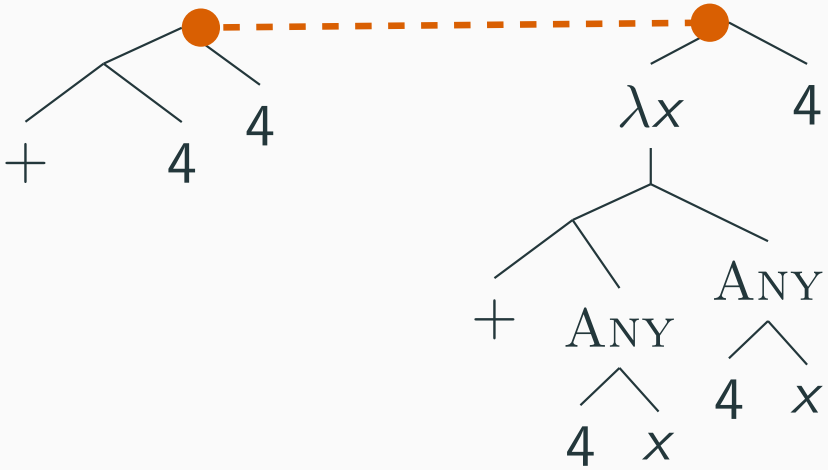
version space



version space



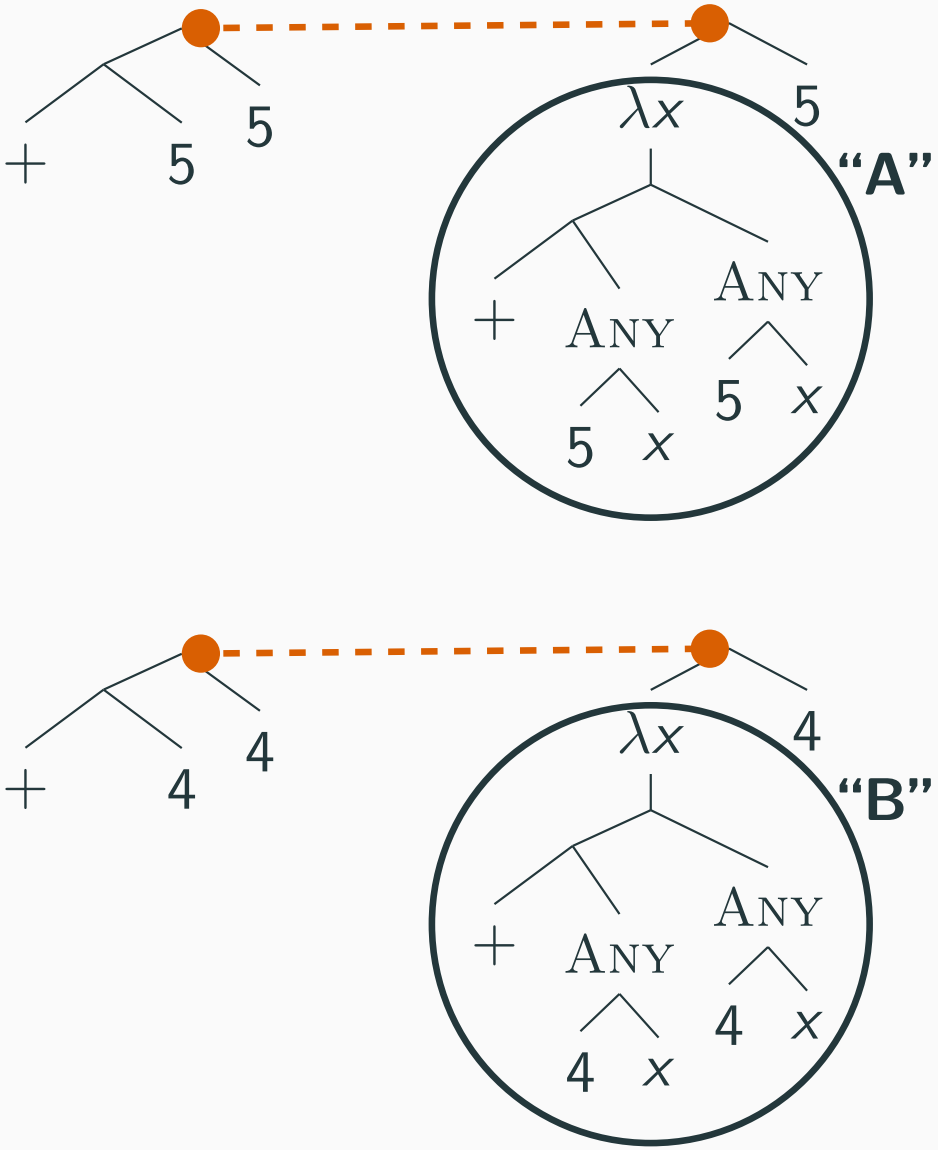


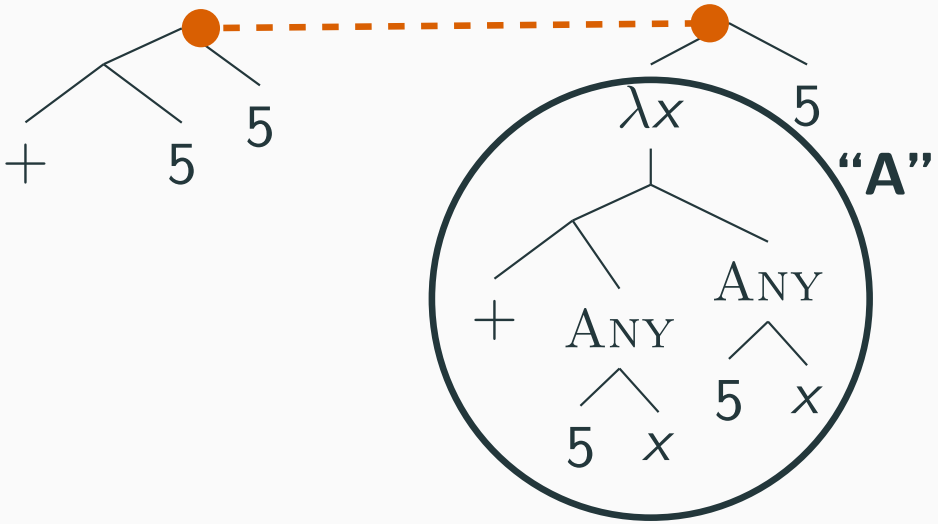


legend

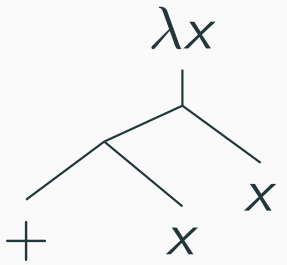
semantic equivalence

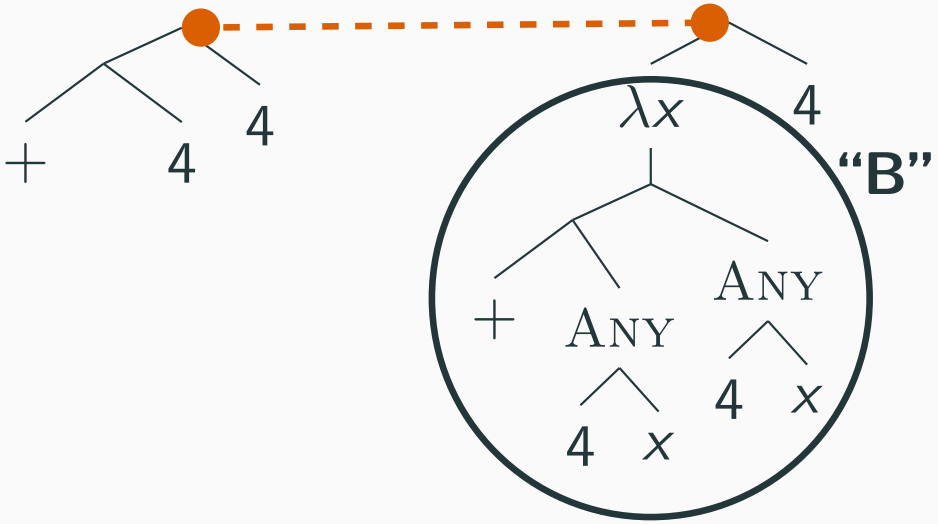
ANY nondeterministic choice


+
5
5
λx
5
“A”
+
ANY
ANY
5
x
5
x
+
4
4
λx
4
“B”
+
ANY
ANY
4
x
4
x
legend
semantic equivalence
ANY nondeterministic choice



A∩B is:

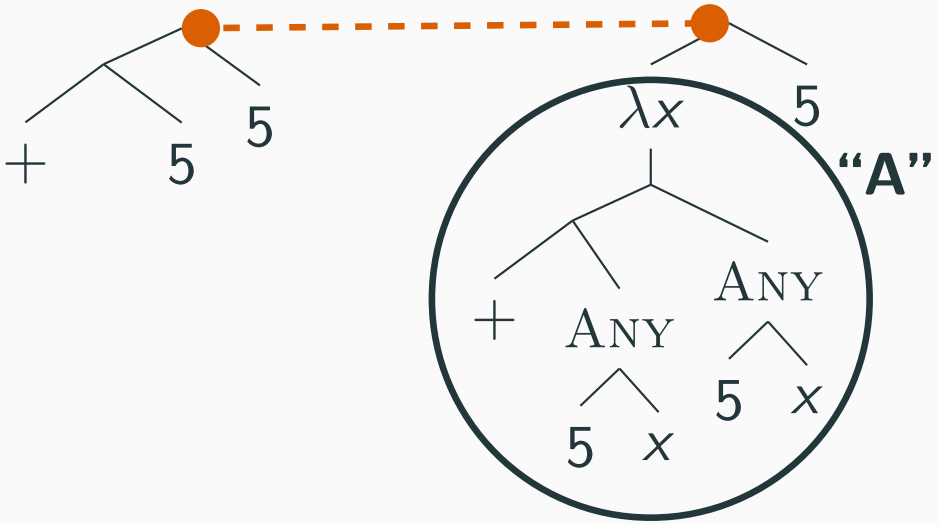




legend

- - - - semantic equivalence

ANY nondeterministic choice



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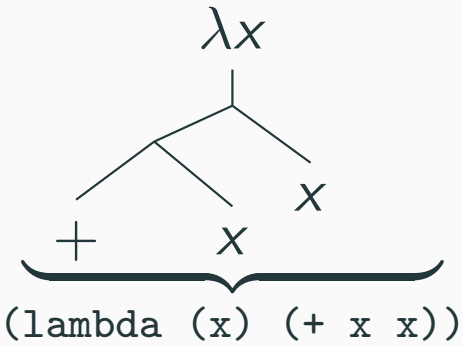
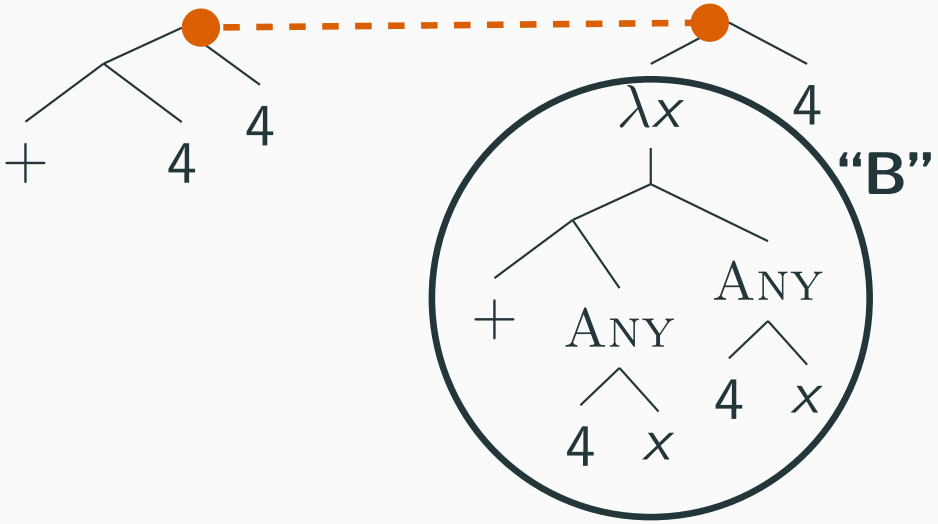
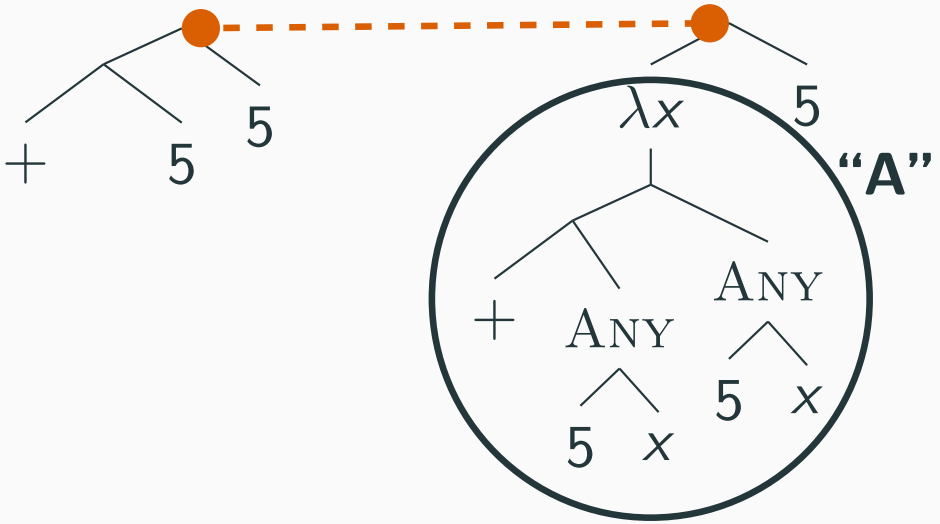




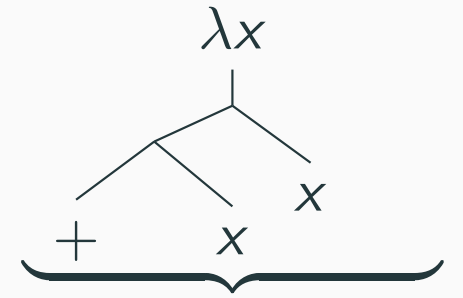
legend

- - - - semantic equivalence

ANY nondeterministic choice



A∩B is:



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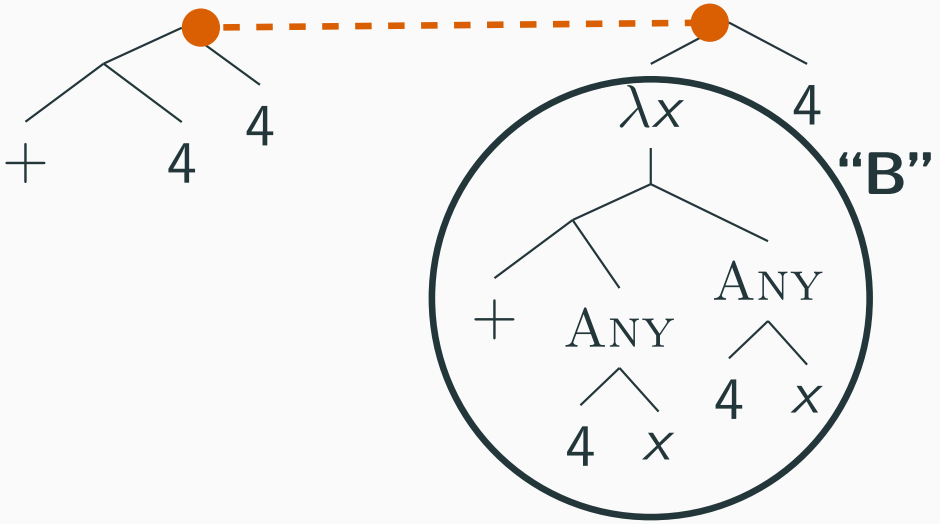
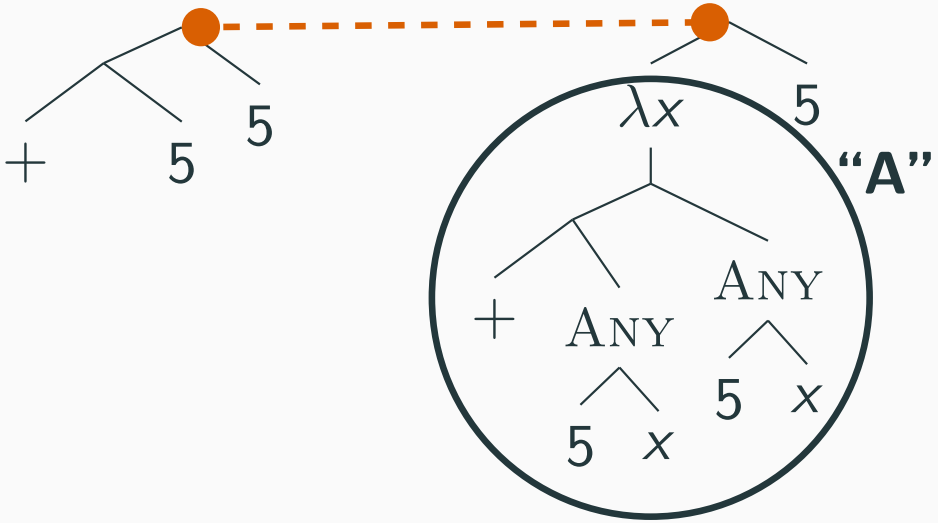


legend

- - - - semantic equivalence

ANY nondeterministic choice



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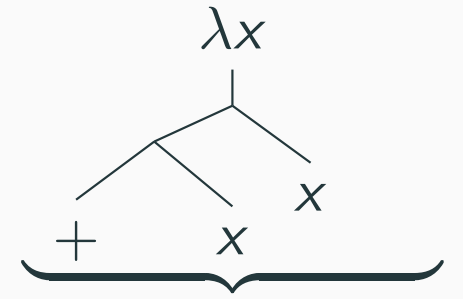
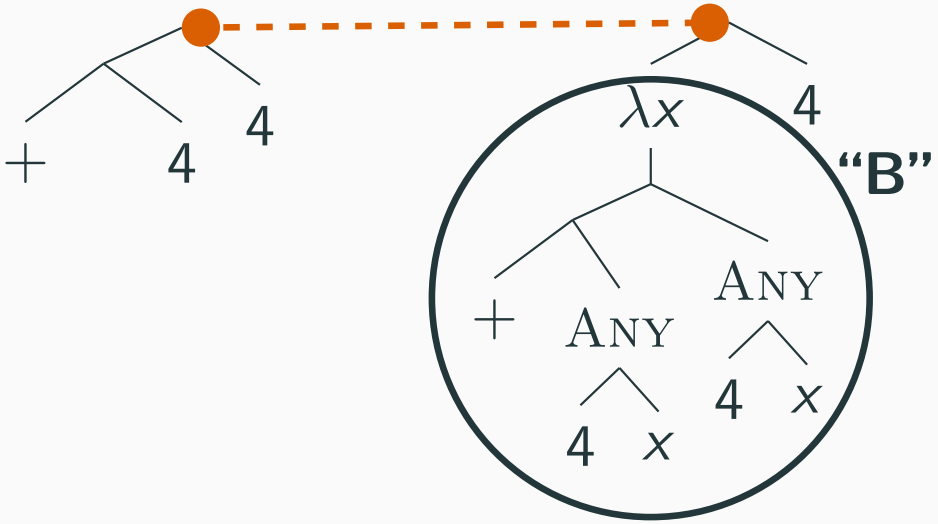


=double



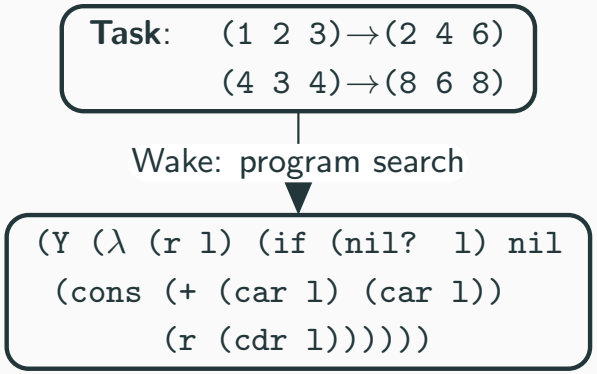
w/o double	w/ double
(+ 5 5)	(double 5)
(+ 4 4)	(double 4)
(+ 3 3)	(double 3)
	...

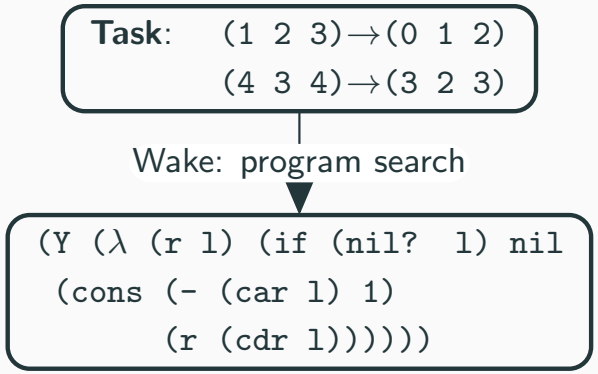
legend

- - - - semantic equivalence

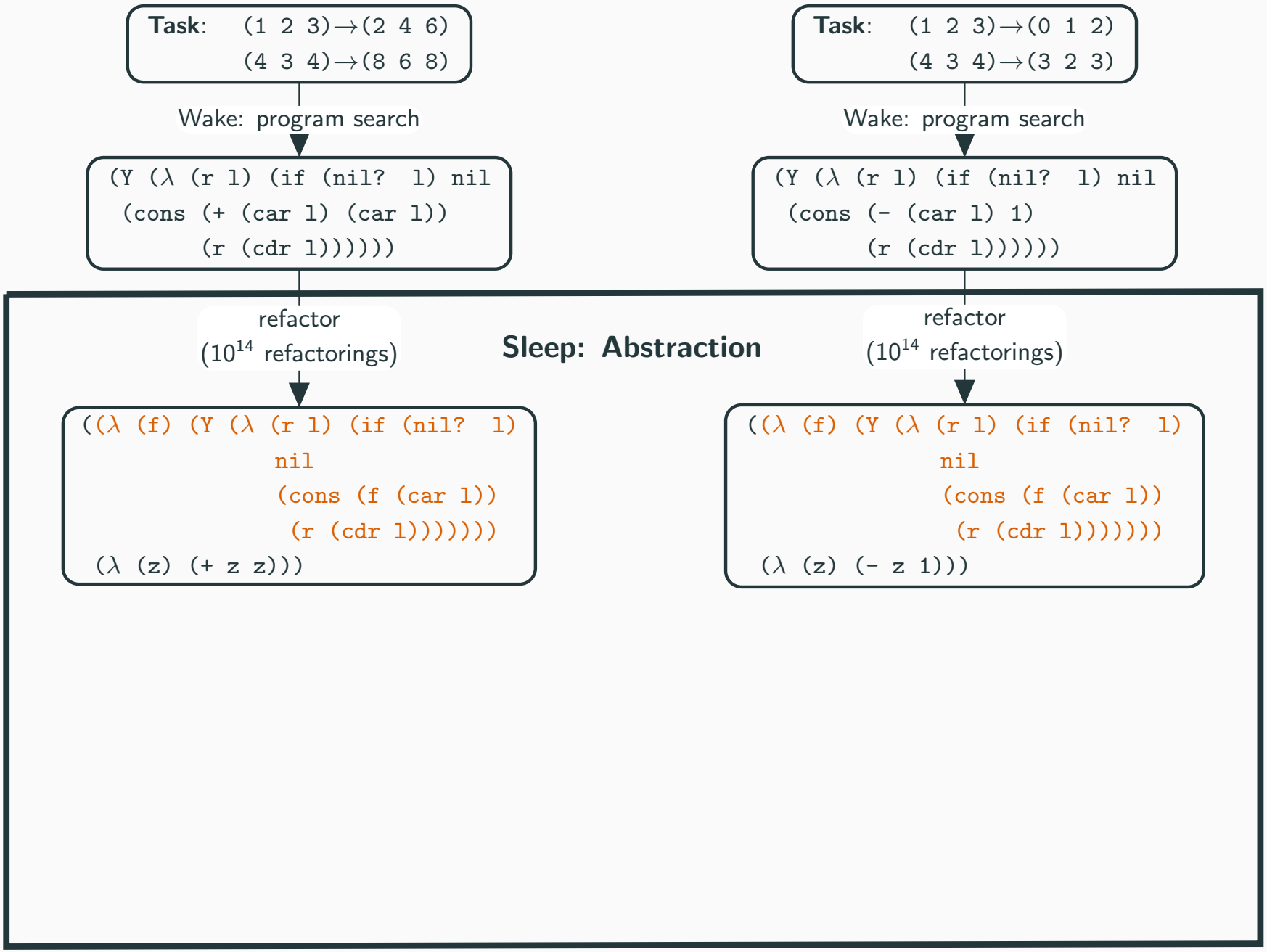
ANY nondeterministic choice

Abstraction Sleep: Growing the library via refactoring

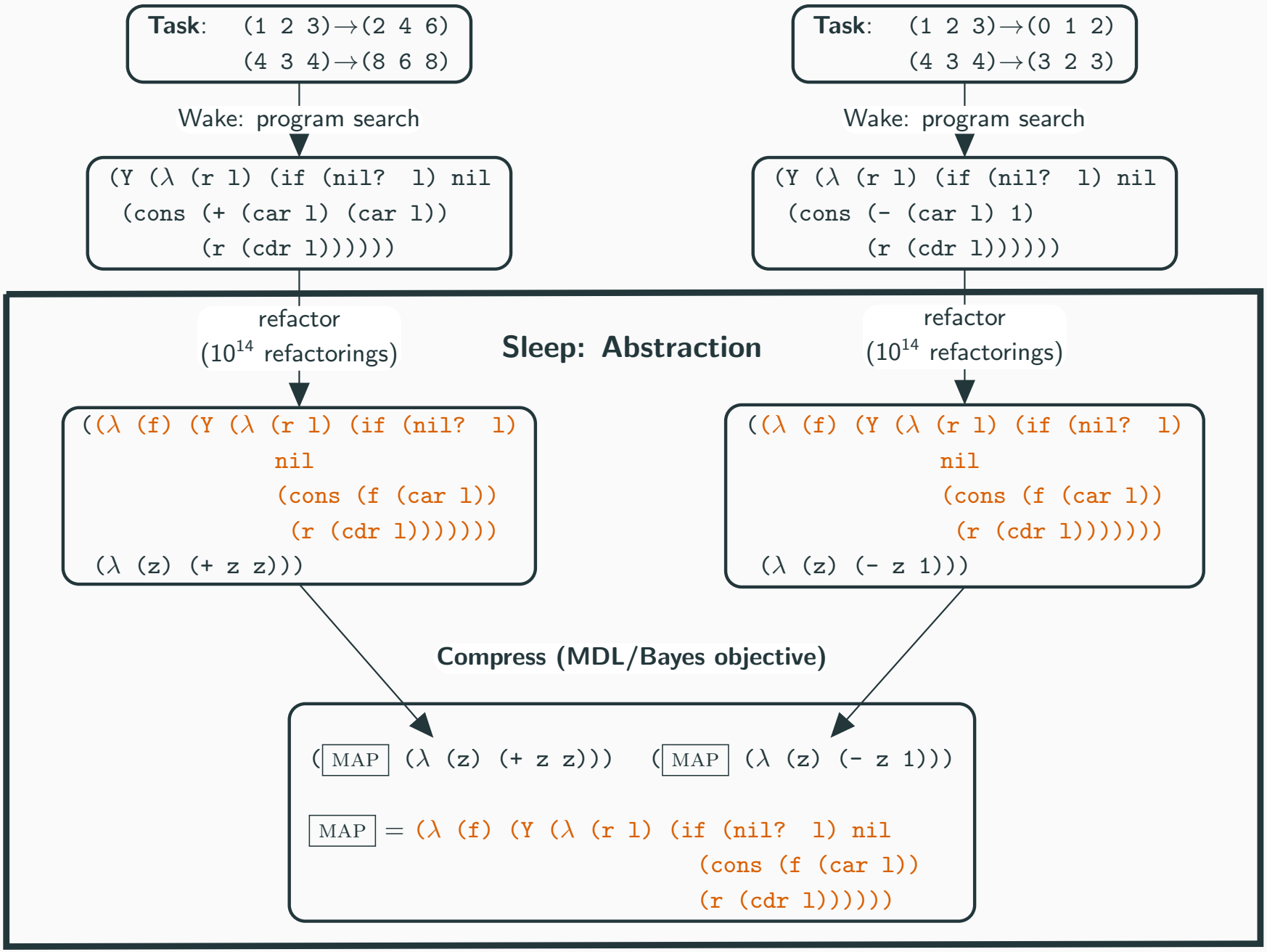




Abstraction Sleep: Growing the library via refactoring



Abstraction Sleep: Growing the library via refactoring



Version space algebra for refactoring

$$
\begin{aligned}
\text{Expr} \rightarrow\ & \text{var} \\
\mid\ & \lambda\text{var.Expr} \\
\mid\ & (\text{Expr Expr}) \\
\mid\ & \text{primitive}
\end{aligned}
$$

Version space algebra for refactoring

$$
\begin{array}{lll}
\text{Expr} \to \text{var} & \text{VS} \to \text{var} & \\
\quad \mid \lambda\text{var.Expr} & \quad \mid \lambda\text{var.VS} & \\
\quad \mid \text{(Expr Expr)} & \quad \mid \text{(VS VS)} & \\
\quad \mid \text{primitive} & \quad \mid \text{primitive} & \\
 & \quad \mid \text{VS} \uplus \text{VS} & \textit{nondeterministic choice} \\
 & \quad \mid \Lambda & \textit{choose any expression} \\
 & \quad \mid \varnothing & \textit{choose no expression}
\end{array}
$$

Version space algebra for refactoring

$$\begin{aligned}\text{Expr} \rightarrow\ & \text{var} \\ \mid\ & \lambda\text{var}.\text{Expr} \\ \mid\ & (\text{Expr}\ \text{Expr}) \\ \mid\ & \text{primitive}\end{aligned}$$

$$\begin{aligned}\text{VS} \rightarrow\ & \text{var} \\ \mid\ & \lambda\text{var}.\text{VS} \\ \mid\ & (\text{VS}\ \text{VS}) \\ \mid\ & \text{primitive} \\ \mid\ & \text{VS} \uplus \text{VS} \quad \textit{nondeterministic choice} \\ \mid\ & \Lambda \quad \textit{choose any expression} \\ \mid\ & \varnothing \quad \textit{choose no expression}\end{aligned}$$

what version spaces mean:

$$[\![\text{var}]\!] = \{\text{var}\}$$

$$[\![\lambda x.v]\!] = \{\lambda x.e : e \in [\![v]\!]\}$$

$$[\![\varnothing]\!] = \varnothing$$

$$[\![v_1 \uplus v_2]\!] = \{e : v \in \{v_1, v_2\}, e \in [\![v]\!]\}$$

$$[\![(v_1\, v_2)]\!] = \{(e_1\, e_2) : e_1 \in [\![v_1]\!], e_2 \in [\![v_2]\!]\}$$

$$[\![\Lambda]\!] = \Lambda$$

Using version spaces

$$
\begin{aligned}
\mathrm{VS} \rightarrow\ & \mathrm{var} \mid \lambda \mathrm{var}.\mathrm{VS} \mid (\mathrm{VS}\ \mathrm{VS}) \mid \mathrm{primitive} \\
\mid\ & \mathrm{VS} \uplus \mathrm{VS} && \textit{nondeterministic choice} \\
\mid\ & \Lambda && \textit{choose any expression} \\
\mid\ & \varnothing && \textit{choose no expression}
\end{aligned}
$$

Using version spaces

$$\begin{aligned}
\text{VS} \rightarrow\ & \text{var} \mid \lambda\text{var.VS} \mid (\text{VS VS}) \mid \text{primitive} \\
\mid\ & \text{VS} \uplus \text{VS} && \textit{nondeterministic choice} \\
\mid\ & \Lambda && \textit{choose any expression} \\
\mid\ & \varnothing && \textit{choose no expression}
\end{aligned}$$

exploit the fact that $e \in [\![v]\!]$ can be efficiently computed:

$$\textsc{refactor}(v|\text{Lib}) = \begin{cases} e, \text{ if } e \in \text{Lib and } e \in [\![v]\!] \\ \textsc{refactor}'(v|\text{Lib}), \text{ otherwise.} \end{cases}$$

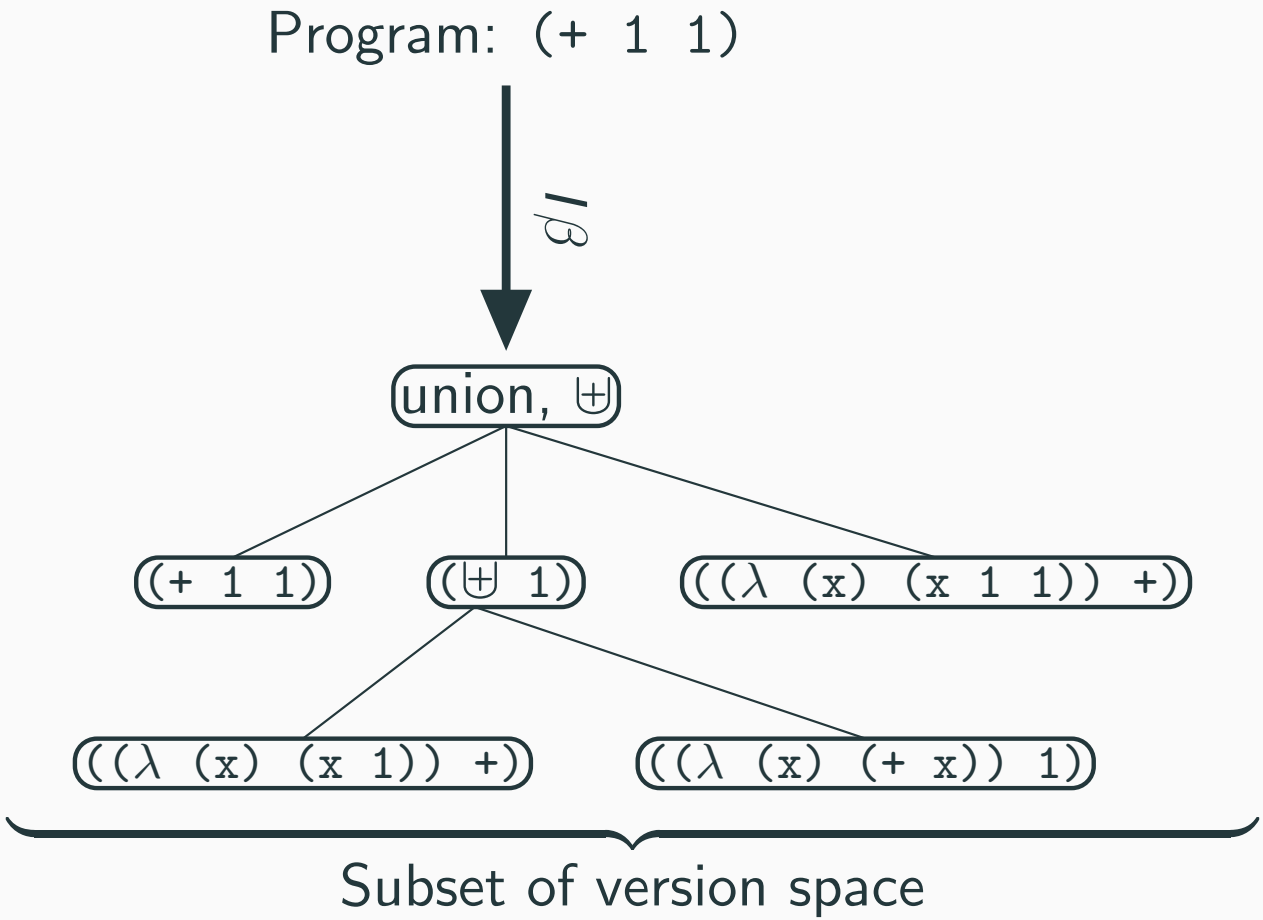
$$\textsc{refactor}'(v|\text{Lib}) = v, \text{ if } v \text{ is a primitive or variable}$$

$$\textsc{refactor}'(\lambda x.b|\text{Lib}) = \lambda x.\textsc{refactor}(b|\text{Lib})$$

$$\textsc{refactor}'(v_1\, v_2|\text{Lib}) = (\textsc{refactor}(v_1|\text{Lib})\ \ \textsc{refactor}(v_2|\text{Lib}))$$

Using version spaces

$$\begin{aligned} \text{VS} \rightarrow\ & \text{var} \mid \lambda\text{var.VS} \mid (\text{VS VS}) \mid \text{primitive} \\ \mid\ & \text{VS} \uplus \text{VS} && \textit{nondeterministic choice} \\ \mid\ & \Lambda && \textit{choose any expression} \\ \mid\ & \varnothing && \textit{choose no expression} \end{aligned}$$



Using version spaces

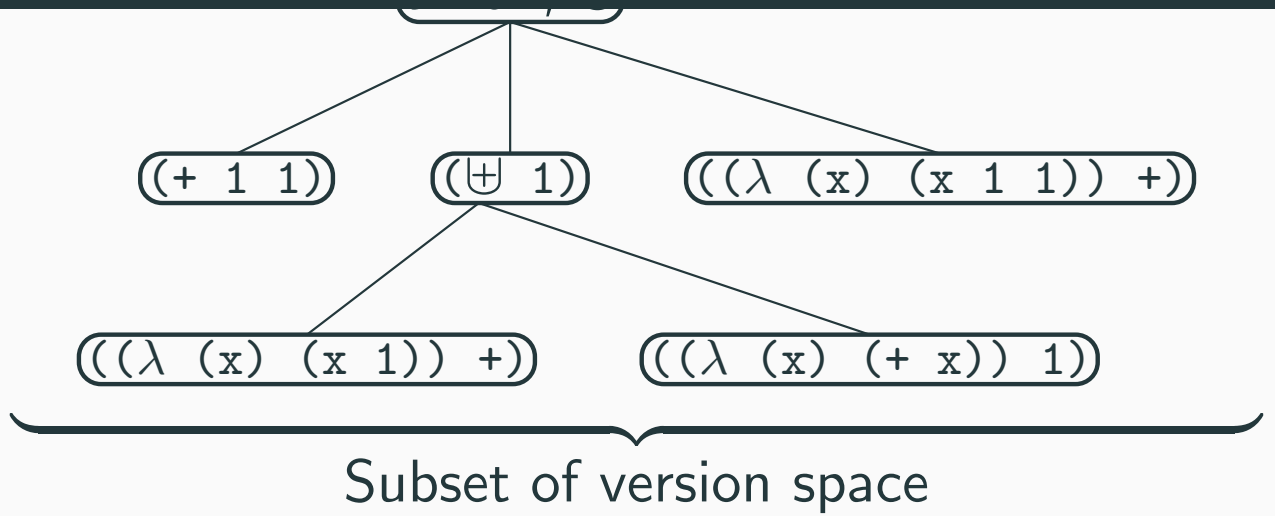
$$\begin{aligned} \text{VS} \rightarrow\ & \text{var} \mid \lambda\text{var.VS} \mid (\text{VS VS}) \mid \text{primitive} \\ \mid\ & \text{VS} \uplus \text{VS} && \textit{nondeterministic choice} \\ \mid\ & \Lambda && \textit{choose any expression} \\ \mid\ & \varnothing && \textit{choose no expression} \end{aligned}$$

completeness: $I\beta$ gets all the refactorings

let $v_2 = I\beta(v_1)$ and $e_1 \in [\![v_1]\!]$. for any $e_2 \longrightarrow e_1$ then $e_2 \in [\![v_2]\!]$

consistency: $I\beta$ only gets valid refactorings

let $v_2 = I\beta(v_1)$ and $e_2 \in [\![v_2]\!]$. then there is a $e_1 \in [\![v_1]\!]$ where $e_2 \longrightarrow e_1$



DreamCoder Domains

List Processing

Sum List

```
[1 2 3] → 6
[4 6 8 1] → 17
```

Double

```
[1 2 3 4] → [2 4 6 8]
[6 5 1] → [12 10 2]
```

Check Evens

```
[0 2 3] → [T T F]
[2 4 9 6] → [T T F T]
```

Text Editing

Abbreviate

```
Allen Newell → A.N.
Herb Simon → H.S.
```

Drop Last Characters

```
jabberwocky → jabberw
copycat → cop
```

Extract

```
see spot(run) → run
a (bee) see → bee
```

Regexes

Phone Numbers

```
(555) 867-5309
(650) 555-2368
```

Currency

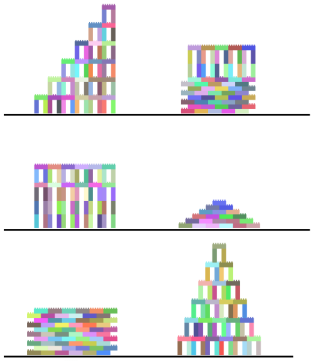
```
$100.25
$4.50
```

Dates

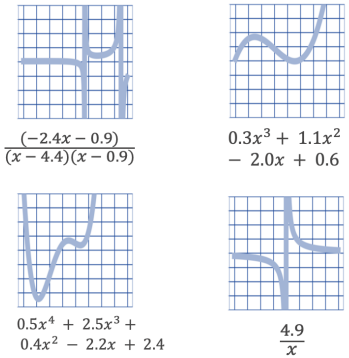
```
Y1775/0704
Y2000/0101
```

LOGO Graphics

Block Towers

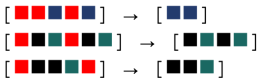


Symbolic Regression



Recursive Programming

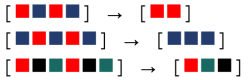
Filter



Index List

Length

Every Other



Physics

$$KE = \frac{1}{2}m|\vec{v}|^2$$

$$\vec{a} = \frac{1}{m}\sum_i \vec{F}_i$$

$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r_1} - \vec{r_2}|^2}\widehat{r_1 - r_2}$$

$$R_{total} = \left(\Sigma_i \frac{1}{R_i}\right)^{-1}$$

DreamCoder Domains

List Processing

Sum List

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[4 6 8 1] → 17

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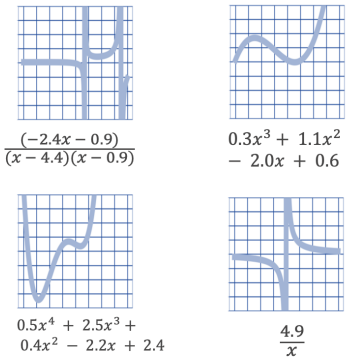
LOGO Graphics

Block Towers

Symbolic Regression



Recursive Programming

Filter

[ ] → [ ]
[ ] → [ ]
[ ] → [ ]

Index List

0, [ ] →
1, [ ] →
1, [ ] →

Length

[ ] → 4
[ ] → 6
[ ] → 3

Every Other

[ ] → [ ]
[ ] → [ ]
[ ] → [ ]

Physics

$$KE = \frac{1}{2}m|\vec{v}|^2$$

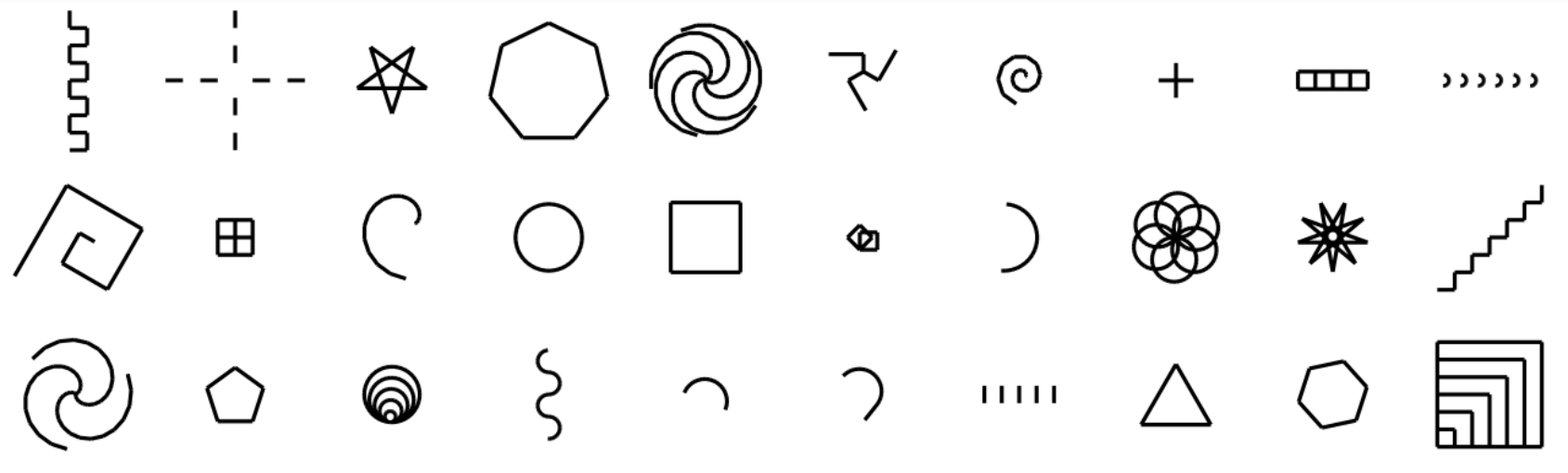
$$\vec{a} = \frac{1}{m}\sum_i \vec{F}_i$$

$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r_1} - \vec{r_2}|^2}\widehat{r_1 - r_2}$$

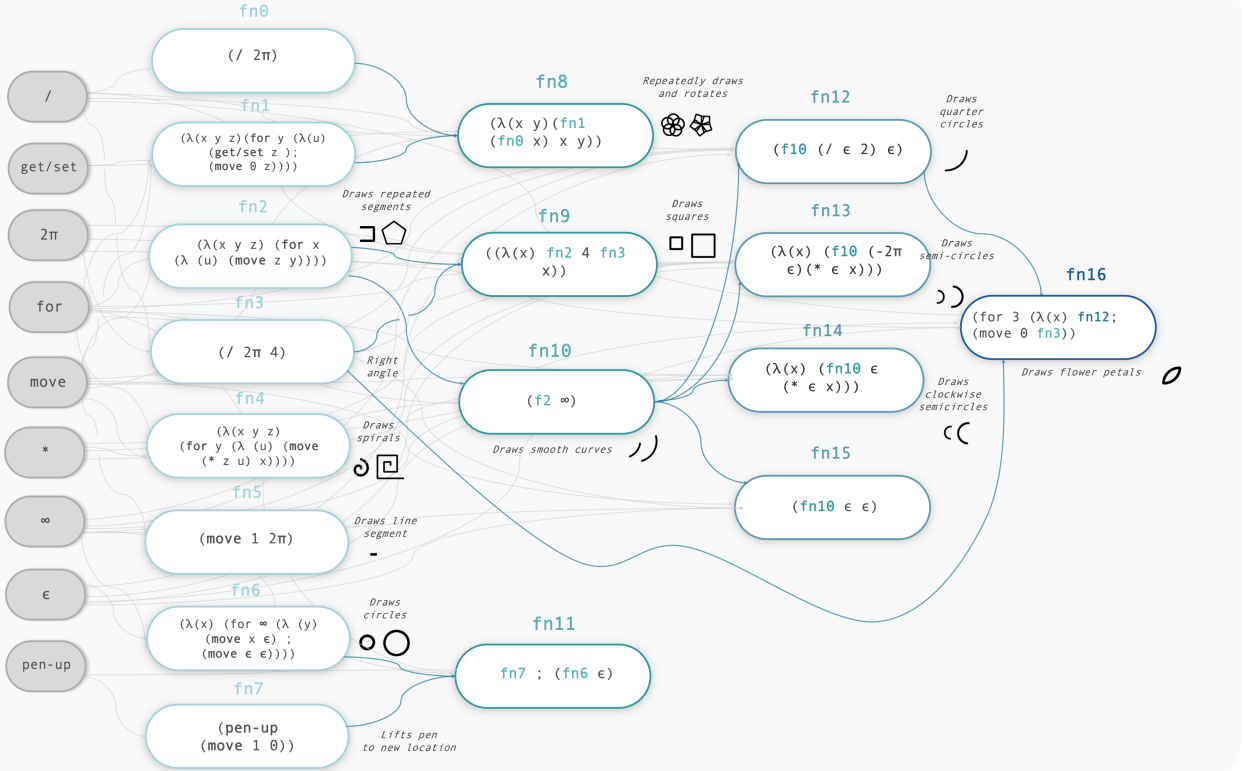
$$R_{total} = \left(\Sigma_i \frac{1}{R_i}\right)^{-1}$$

LOGO Graphics

30 out of 160 tasks



LOGO Graphics – learning interpretable library of concepts



```
(fn8 5 (fn4 (* ϵ 2) ∞ ϵ))
```

```
(for 2 (λ (x)
(fn14 2); (fn13 2)))
```

```
(for 7 (λ (x) (fn9 x)))
```

```
(fn10 (* (fn0 7) 3) 3)
```

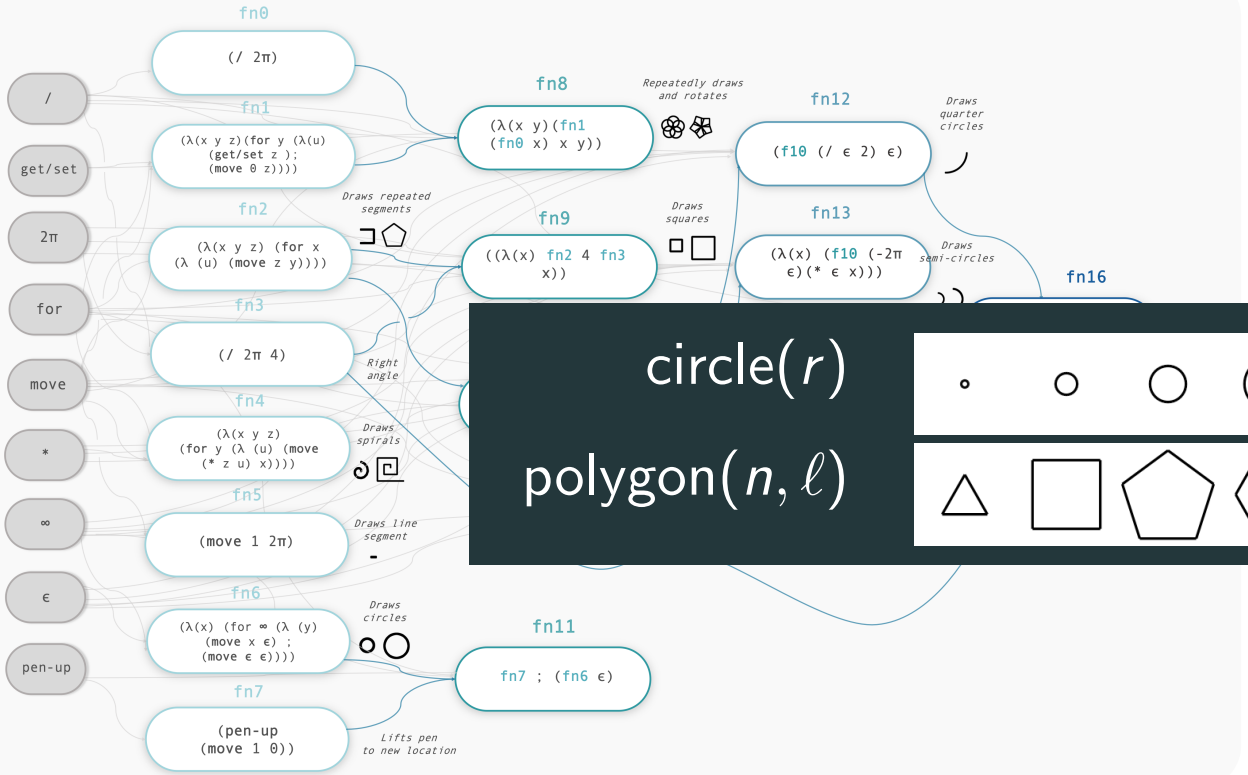
```
(fn8  6 (fn7 ; fn5 ;
fn7 ; fn5))
```

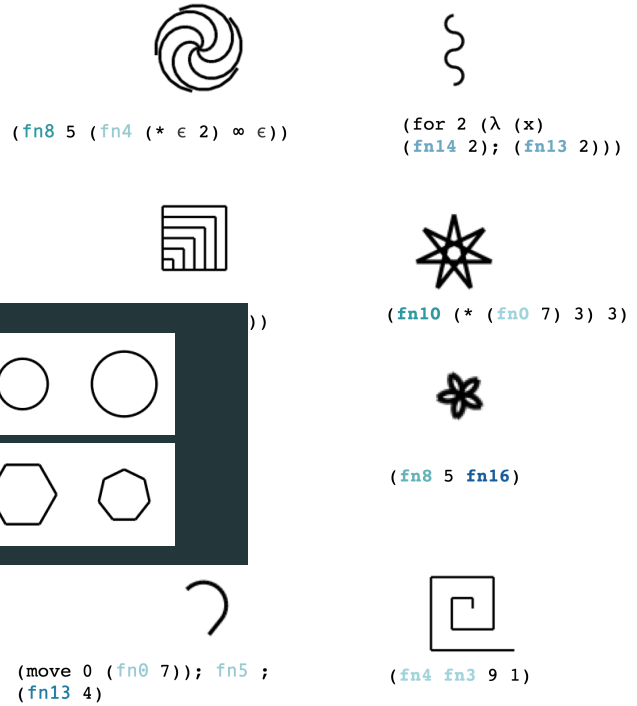
```
(fn8 5 fn16)
```

```
(move 0 (fn0 7)); fn5 ;
(fn13 4)
```

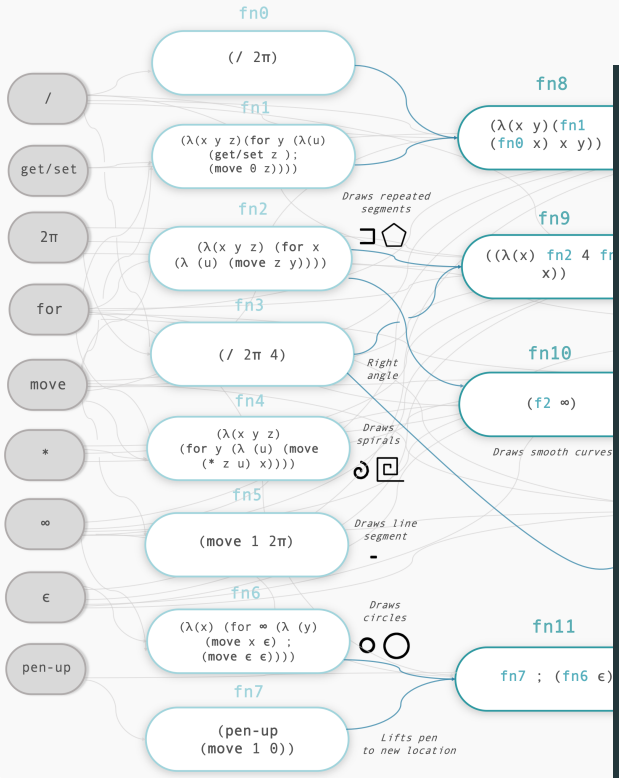
```
(fn4 fn3 9 1)
```

LOGO Graphics – learning interpretable library of concepts

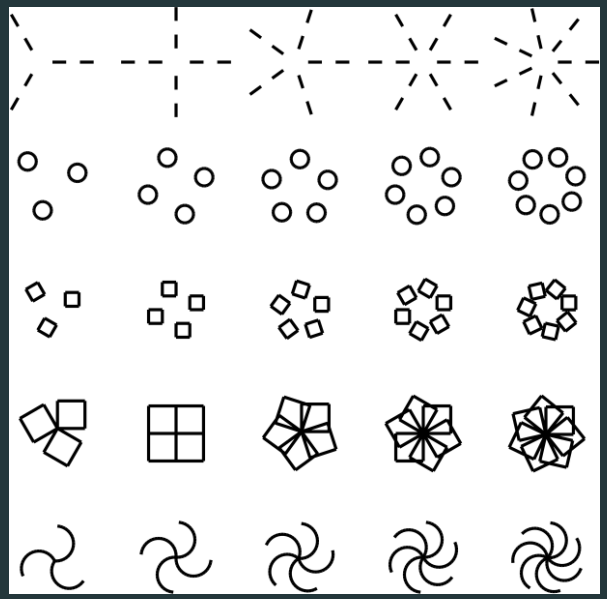


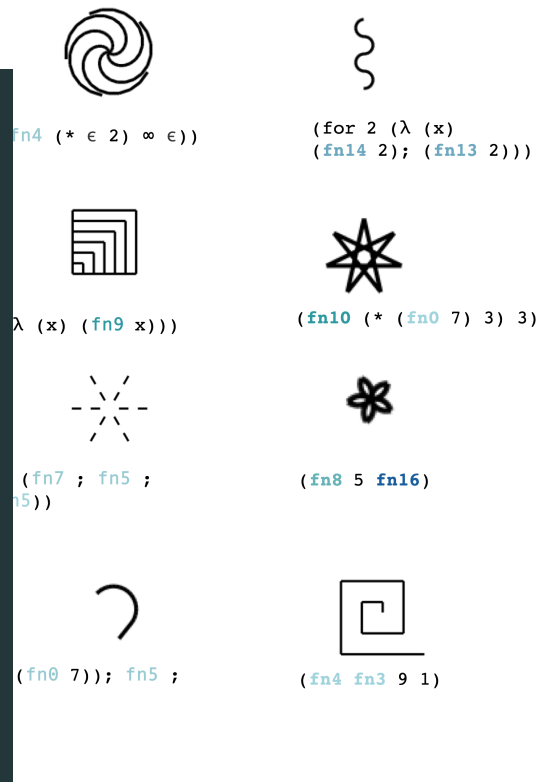


LOGO Graphics – learning interpretable library of concepts



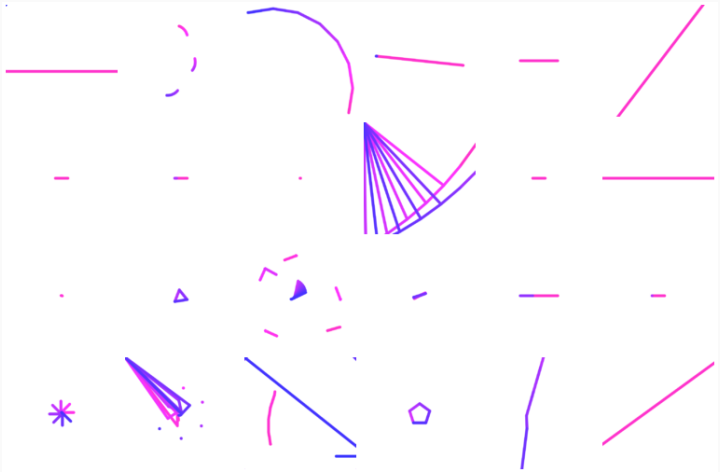
radial symmetry(n, body)



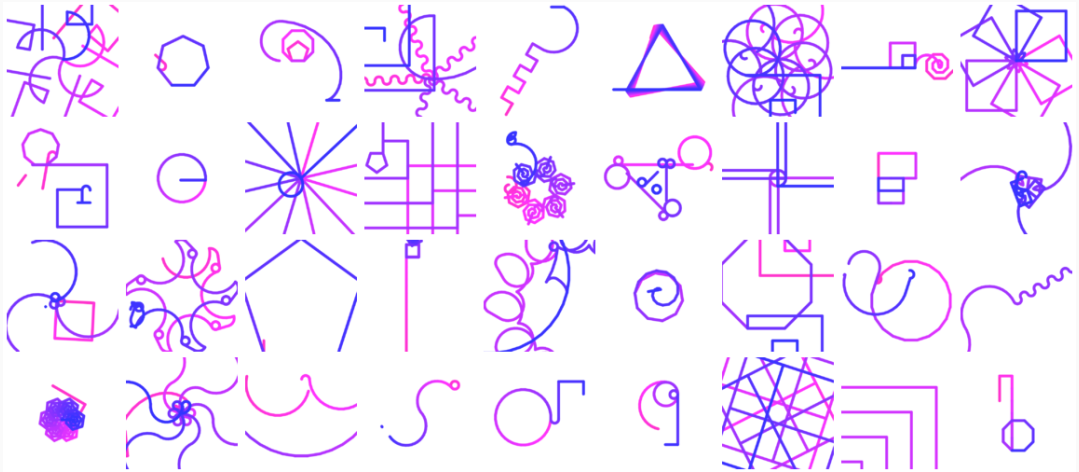


what does DreamCoder dream of?

before learning

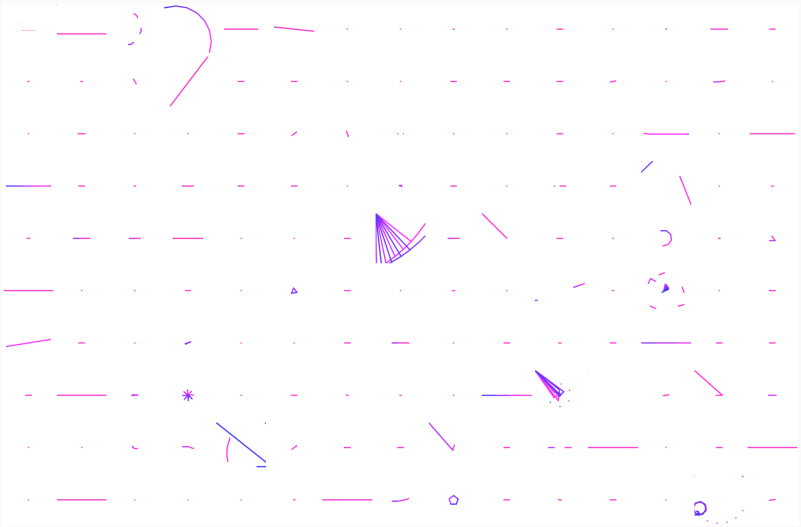


after learning

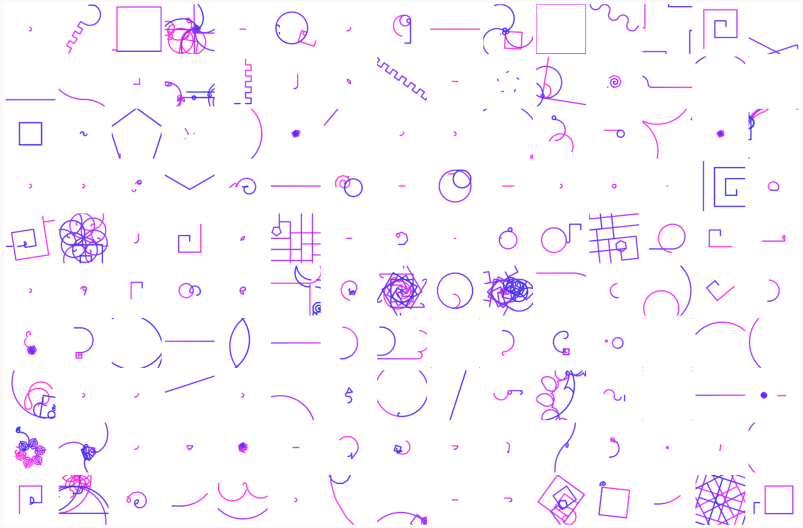


what does DreamCoder dream of?

before learning

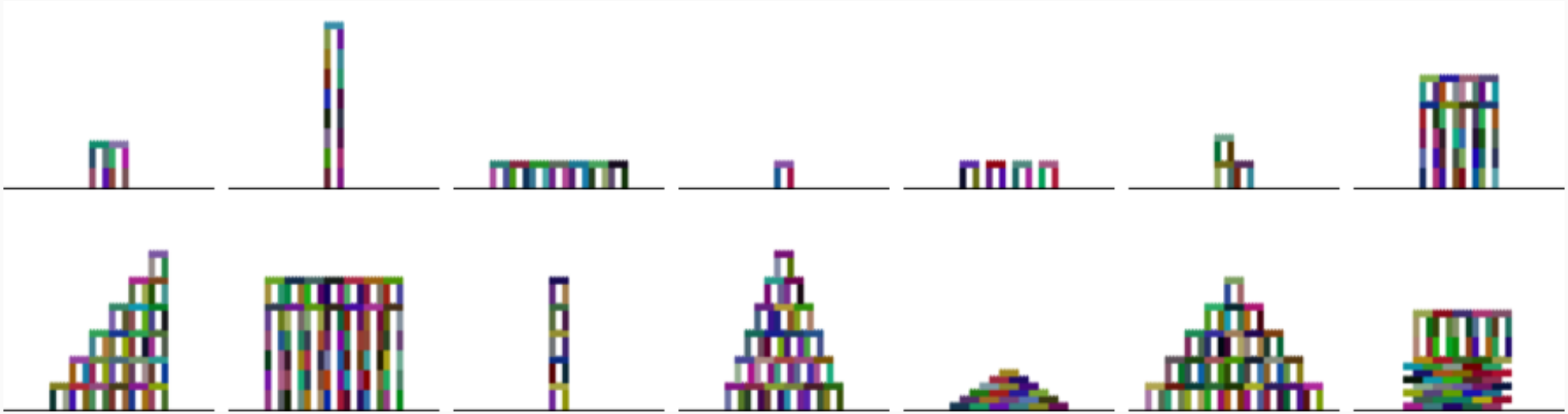


after learning



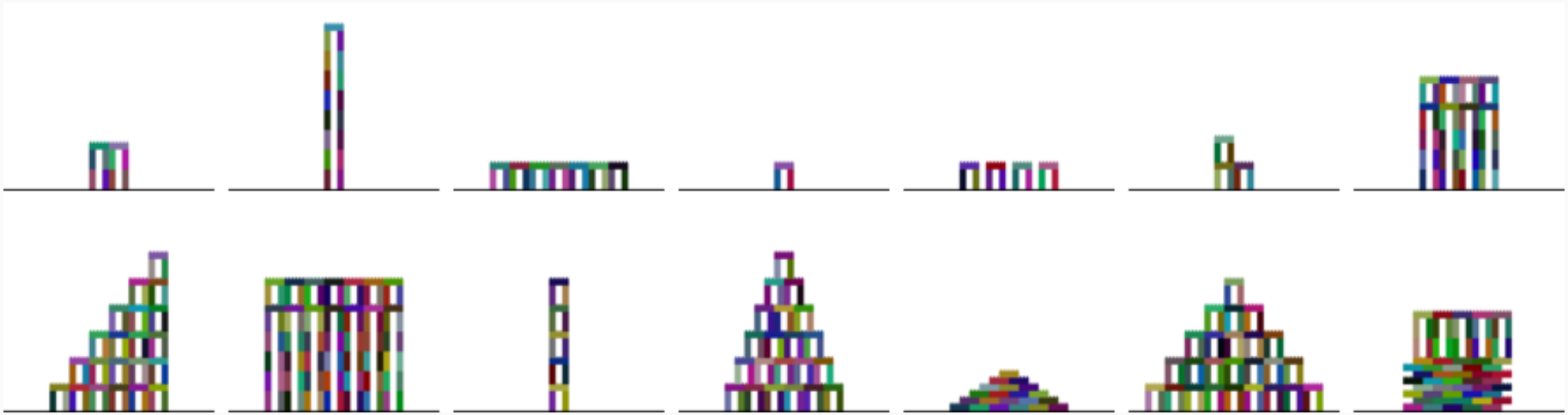
Planning to build towers

example tasks (112 total)



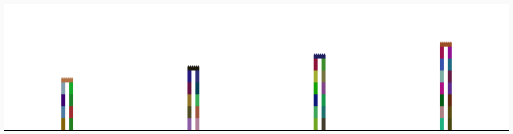
Planning to build towers

example tasks (112 total)

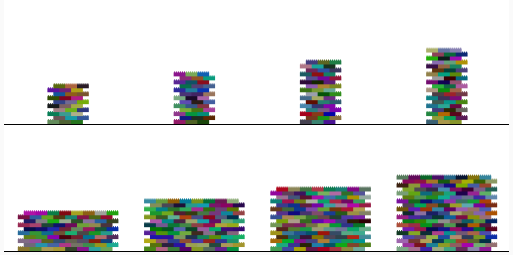


learned library routines ($\approx$ 20 total)

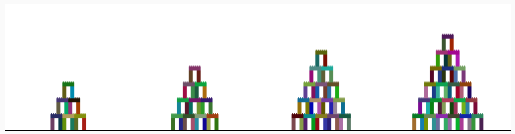
arch(h)



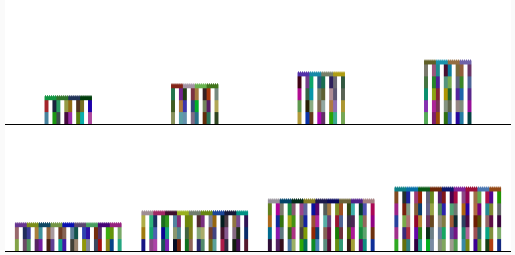
wall(w, h)



pyramid(h)



bridge(w, h)



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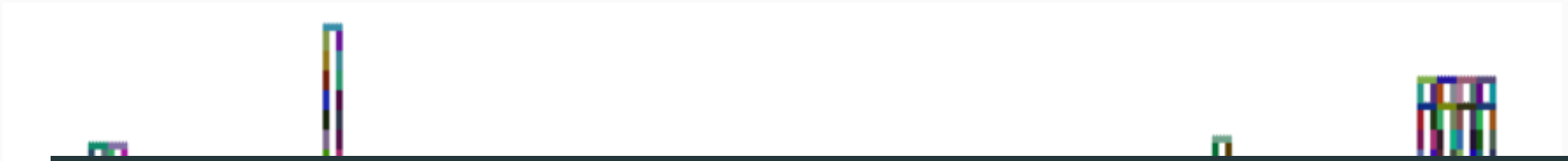
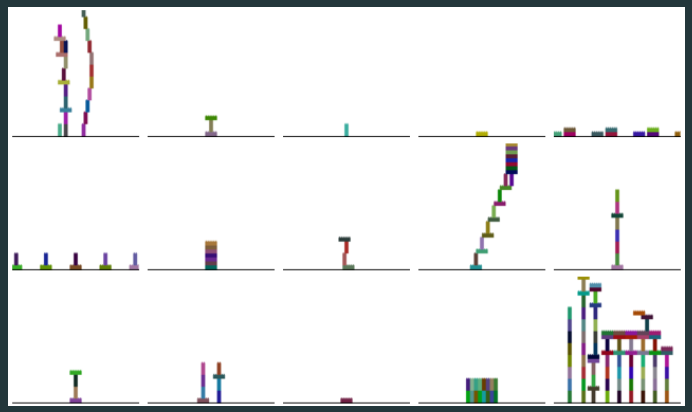
example tasks (112 total)

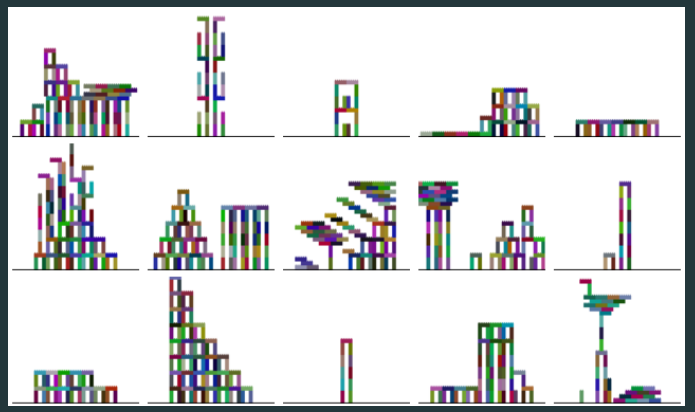


lea

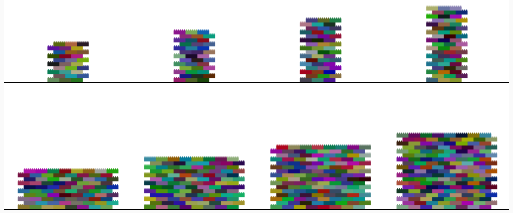
dreams before learning



dreams after learning



wall(w, h)



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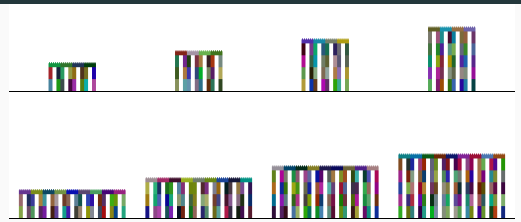
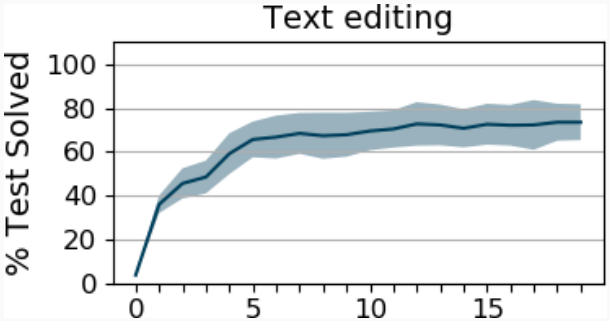
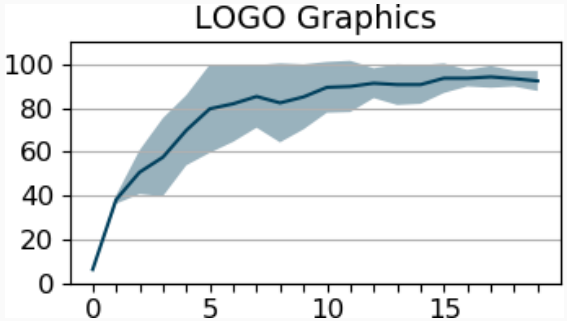
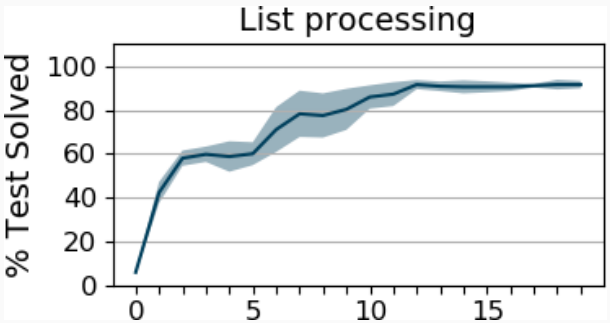
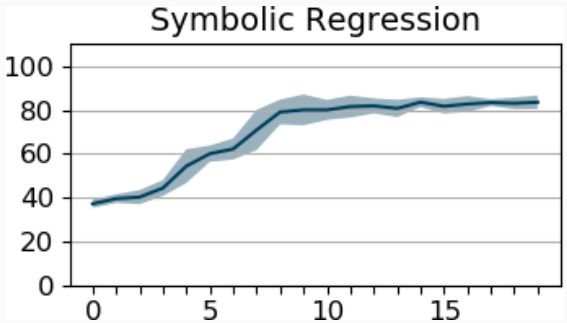


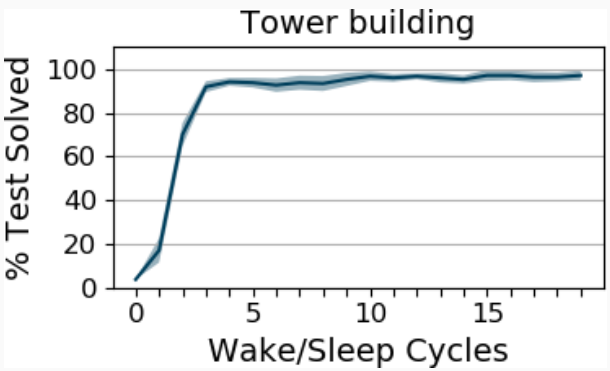
synergy between dreaming and library learning

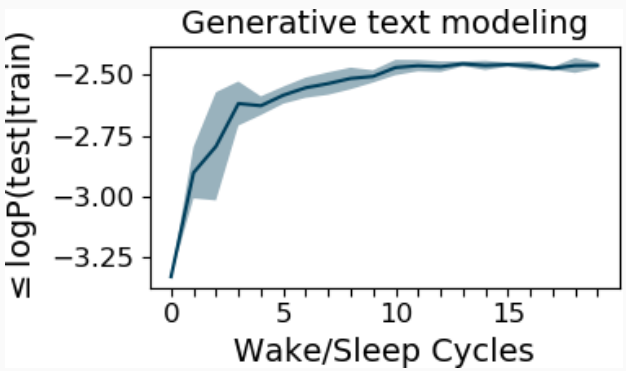


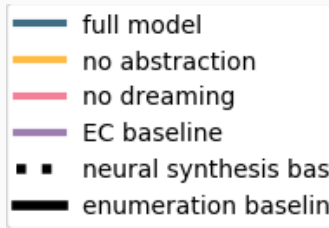




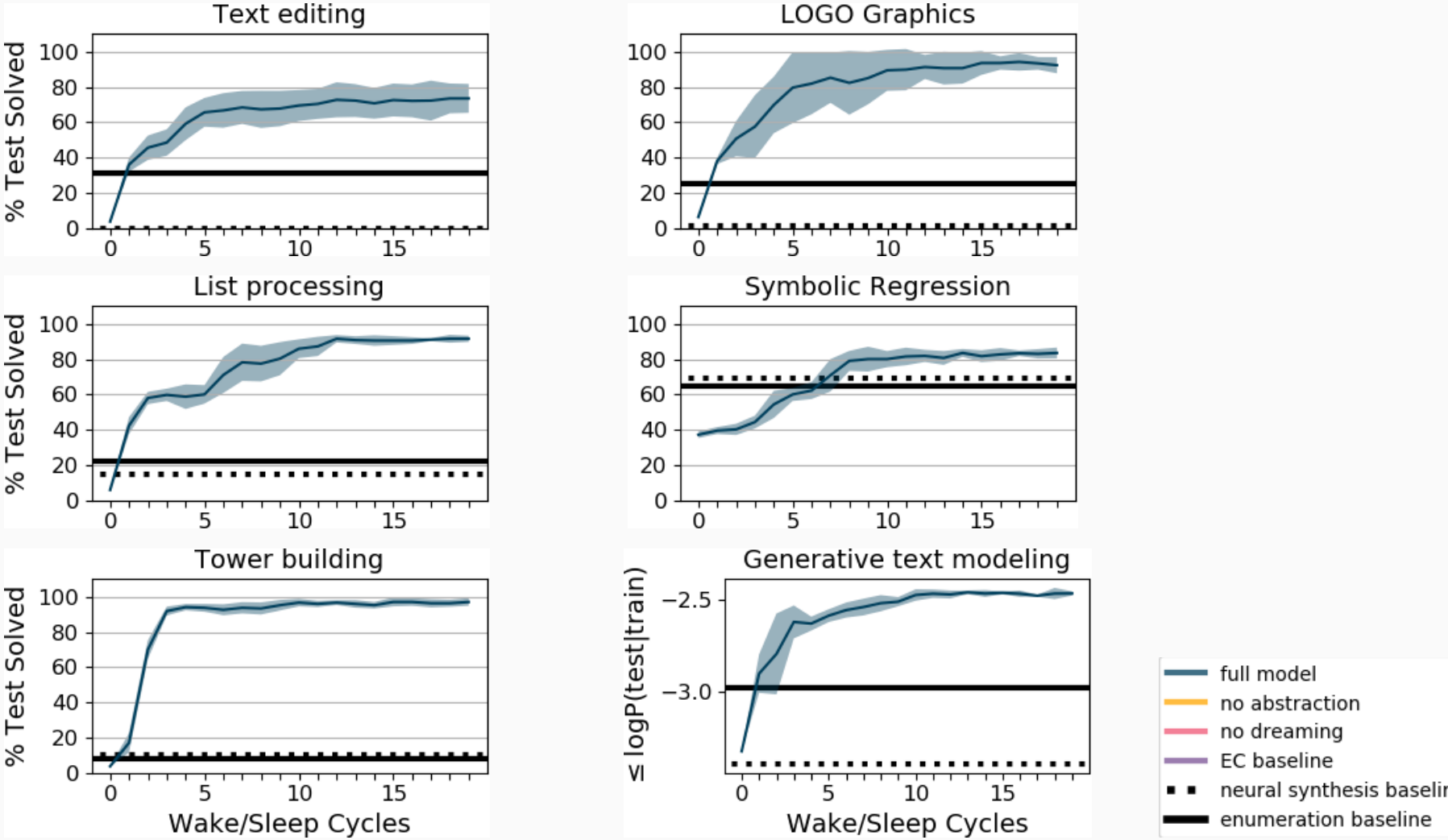








synergy between dreaming and library learning



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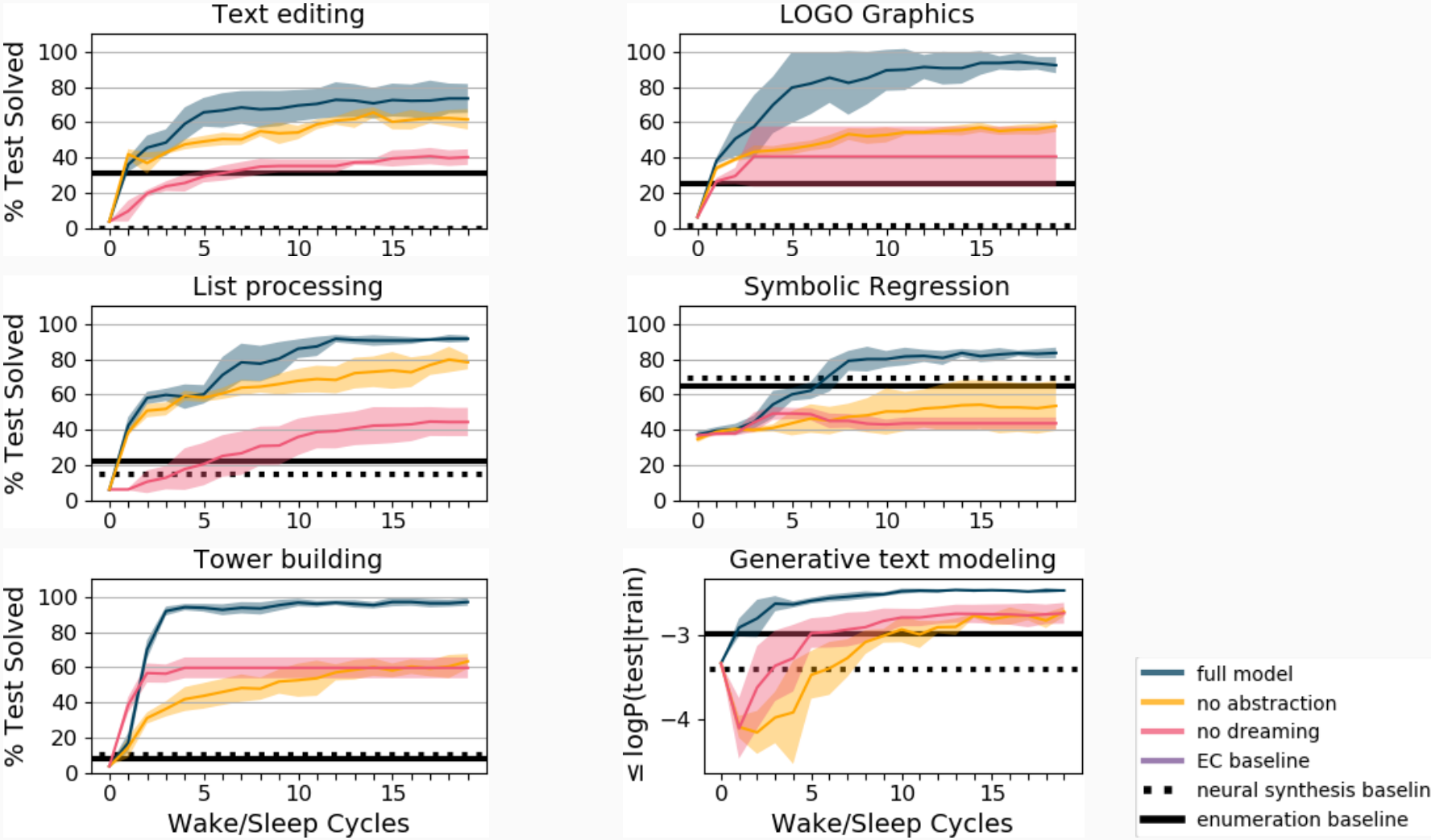

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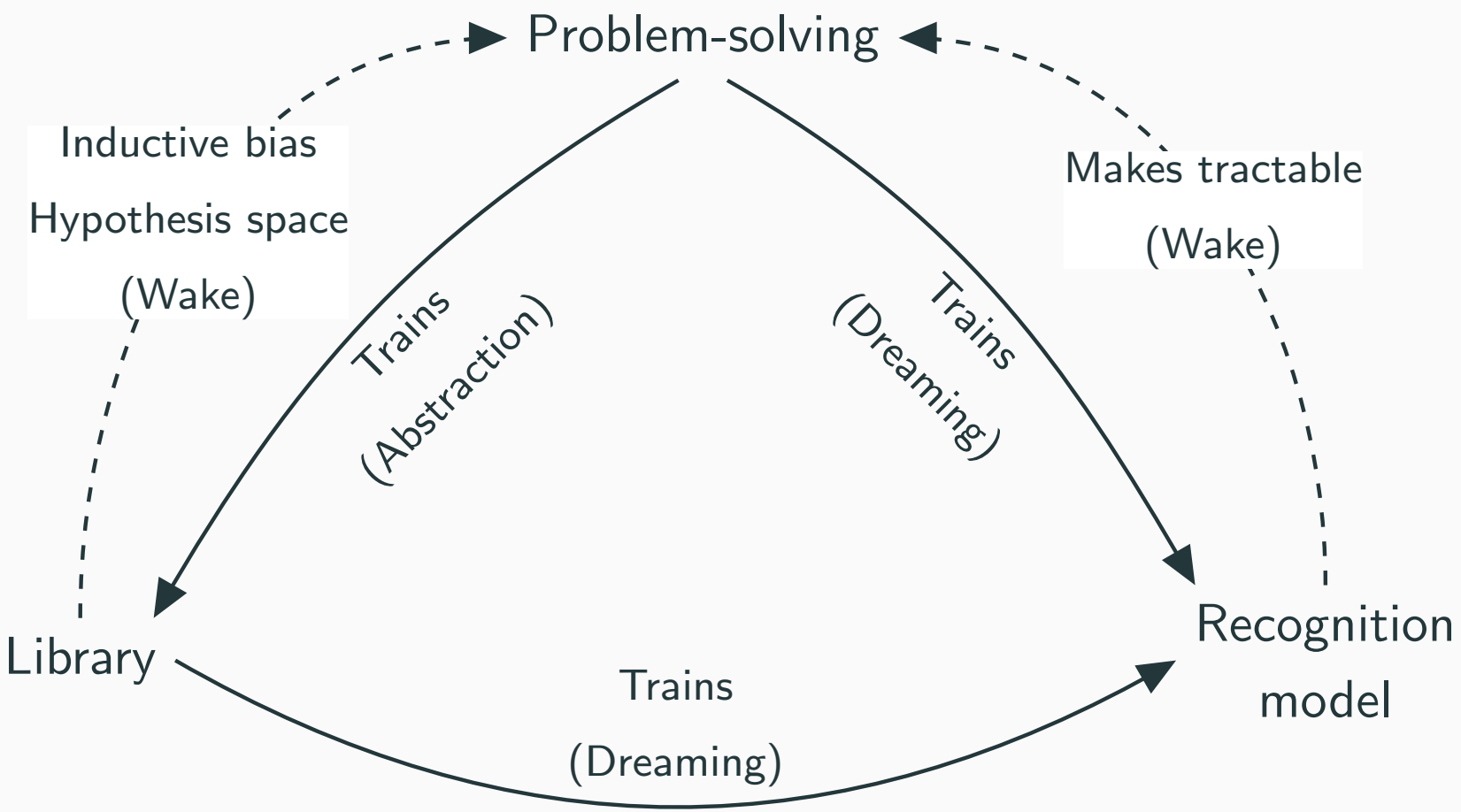
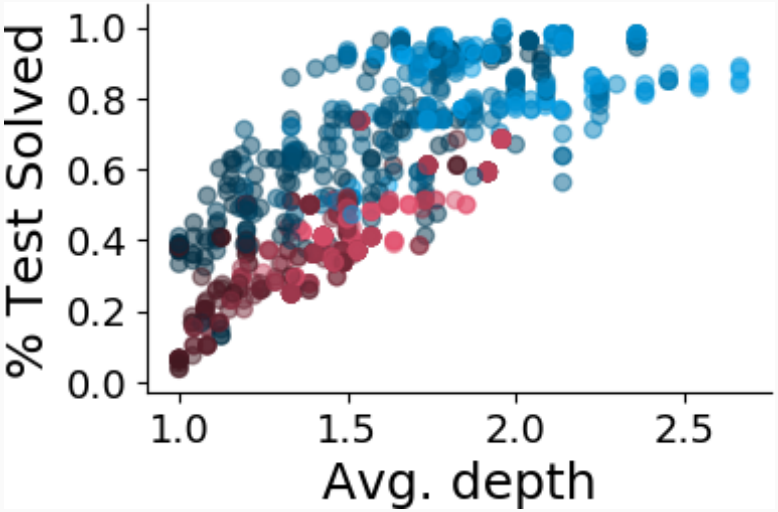
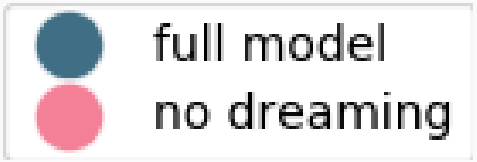


Evidence for dreaming bootstrapping better libraries

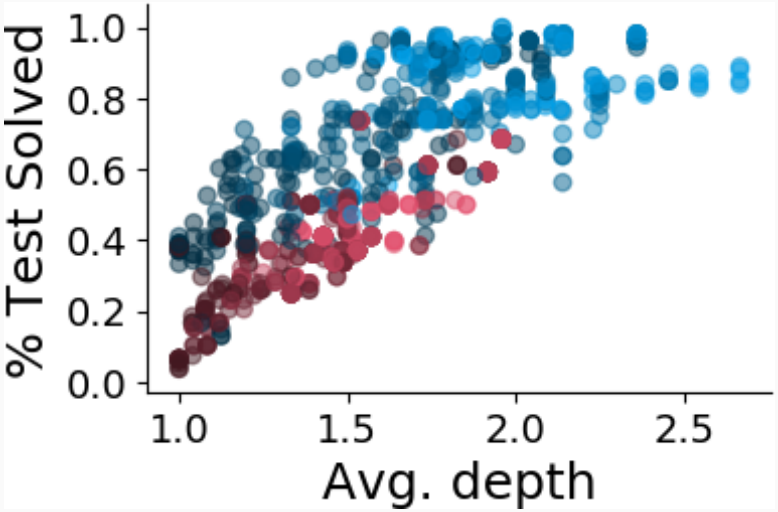


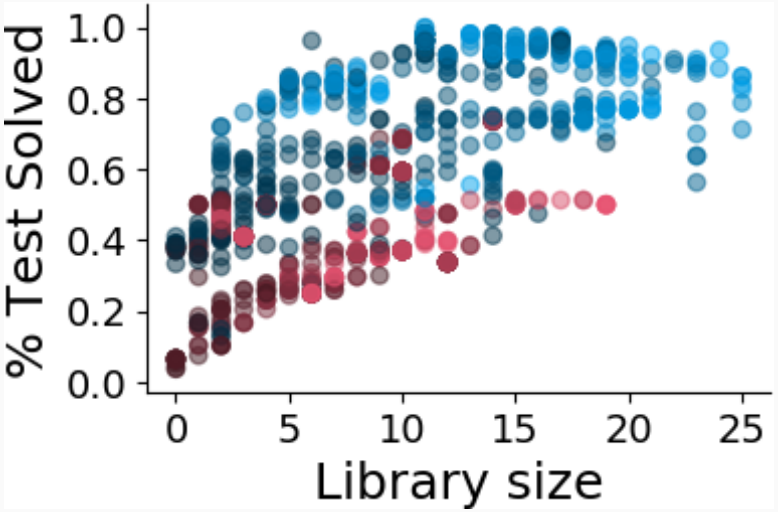


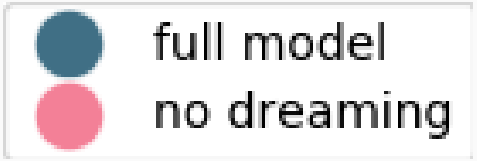
Darker: Early in learning

Lighter: Later in learning

Evidence for dreaming bootstrapping better libraries





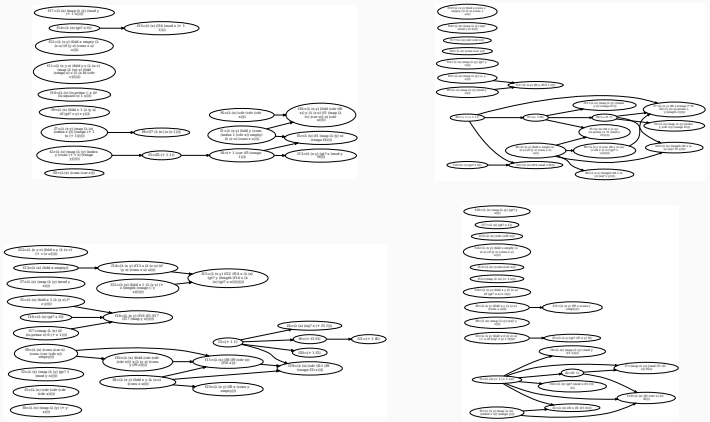


Darker: Early in learning

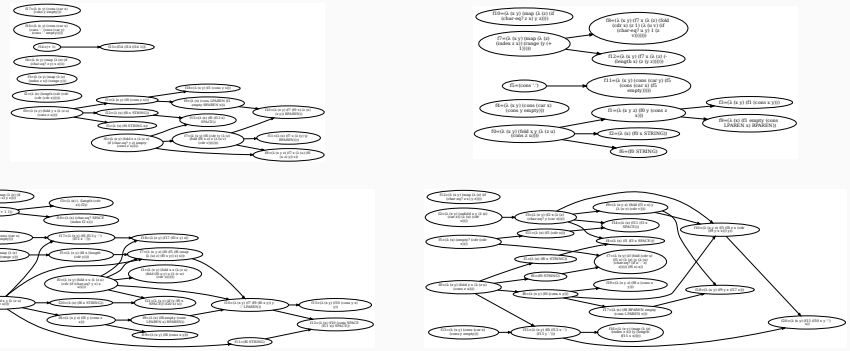
Lighter: Later in learning

Variability in learned library

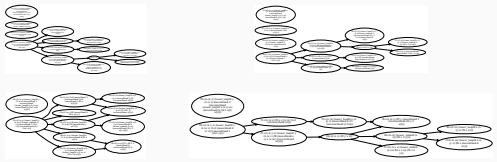
List processing



Text editing

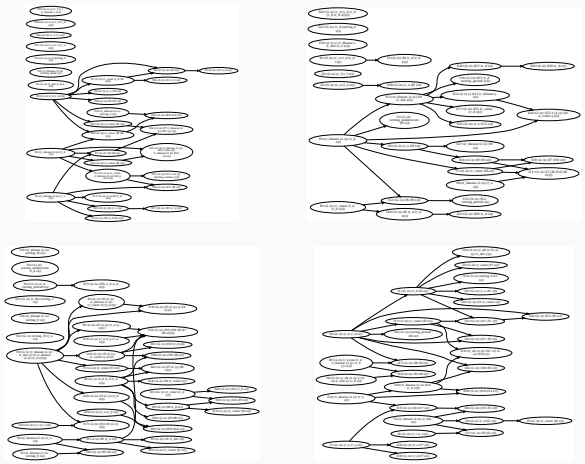


Tower building

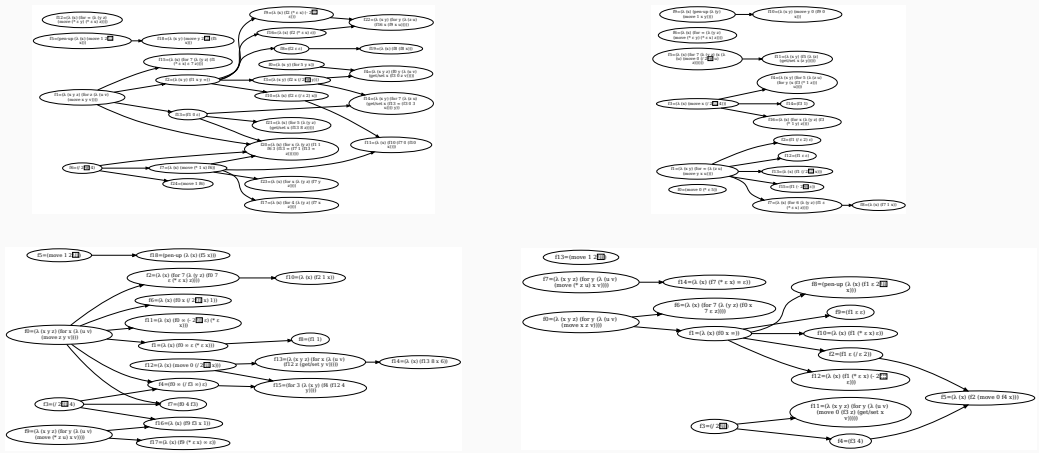


Symbolic regression

Generative regex



LOGO graphics



From learning libraries to learning languages

these experiments study how DreamCoder grows from a "beginner" state to "expert":

- "beginner:" basic domain-specific procedures, only easiest problems have short solutions
- "expert:" learned library allowing hardest problems to have short meaningful solutions

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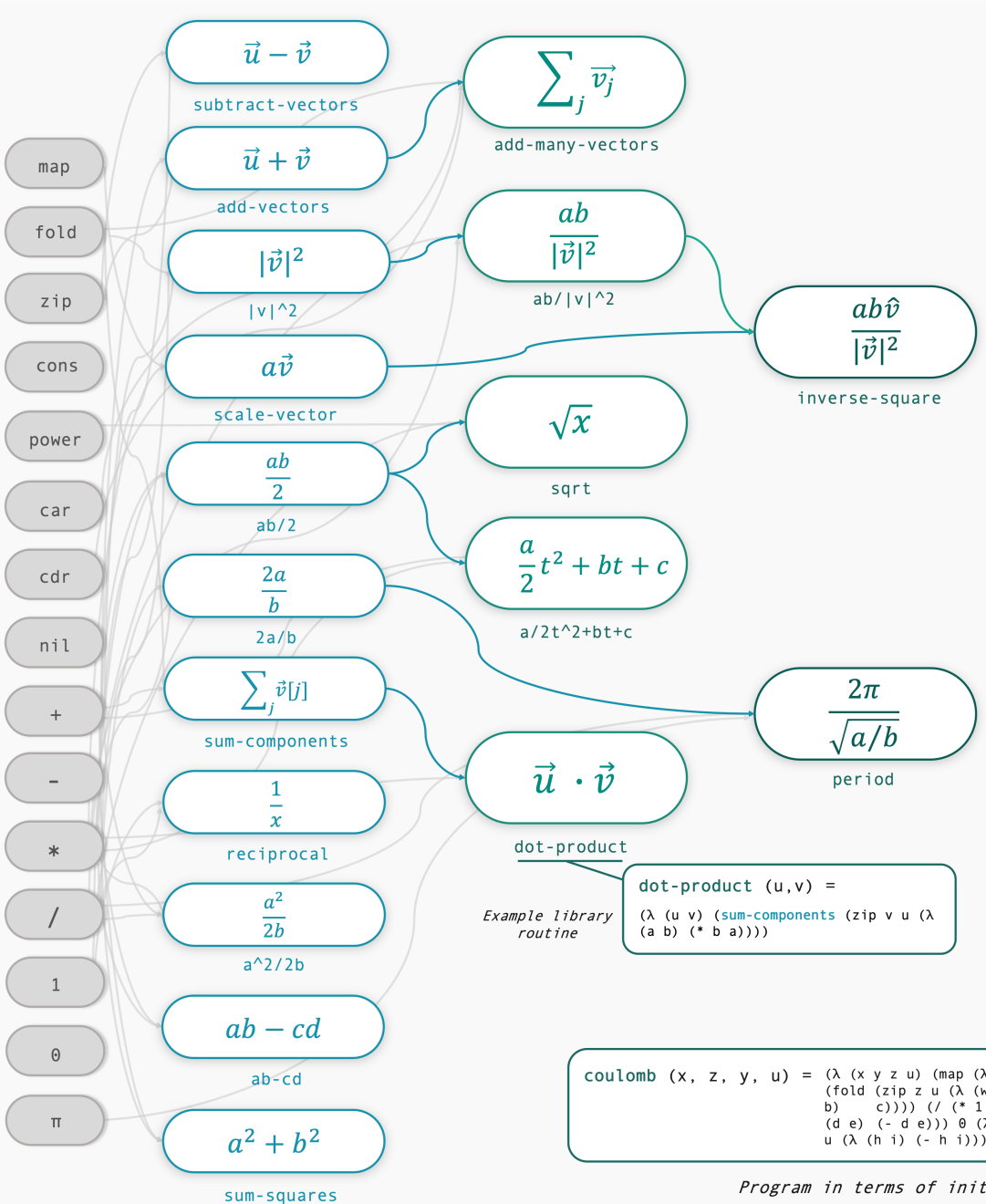
go beyond: start with generic arithmetic & control flow, learn fundamentals of domain

- Physics from recursive higher-order functions
- Recursive higher-order language from 1959 proto-Lisp & Y-combinator

Rediscovering vector algebra



Electric Field to Charge Flux

$$\vec{E}=\rho\vec{J}$$

```
(scale-vector p J)
```

Newton's Second Law

$$\vec{a}=\frac{1}{m}\sum_i \vec{F}_i$$

```
(scale-vector (reciprocal m)
(add-many-vectors Fs))
```

Parallel Resistors

$$R_{total}=\left(\Sigma_i \frac{1}{R_i}\right)^{-1}$$

```
(reciprocal (sum-components (map
(λ(r) (reciprocal r)) Rs)))
```

Kinetic Energy

$$KE=\frac{1}{2}m|\vec{v}|^2$$

```
(ab/2 m (|v|^2 v))
```

Energy in a Capacitor

$$U=\frac{1}{2}CV^2$$

```
(a^2/2b  v (reciprocal c))
```

Ballistic Motion

$$x(t)=\frac{a}{2}t^2+v_0t+x_0$$

```
(a/2t^2+bt+c c b a t)
```

Force in a Magnetic Field

$$|\vec{F}|=q|\vec{v}\times\vec{B}|$$

```
(* q (ab-cd v_x b_y v_y b_x))
```

Work

$$U=\vec{F}\cdot\vec{d}$$

```
(dot-product F d)
```

Coulomb's Law

$$\vec{F}\propto\frac{q_1\,q_2}{|\vec{r_1}-\vec{r_2}|^2}\widehat{r_1-r_2}$$

```
(inverse-square q_1 q_2
(subtract-vectors r_1 r_2))
```

Center of Mass

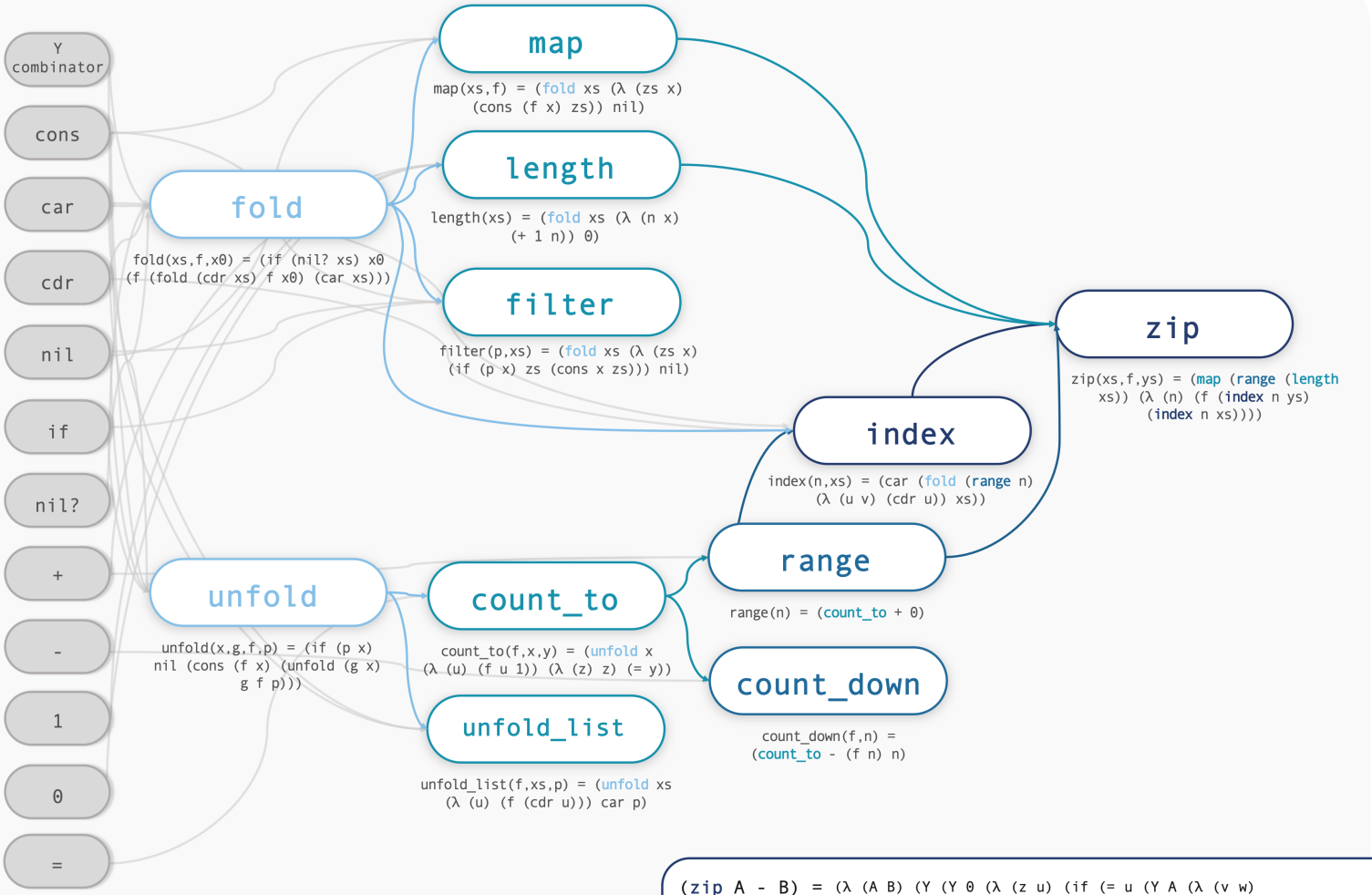
$$X_{CM}=\frac{\sum_i m_i x_i}{\sum_i m_i}$$

```
(/ (dot-product x m) (sum-
components m))
```

```
coulomb (x, z, y, u) = (λ (x y z u) (map (λ (v) (* (/ (* (power (/ (* x x)
(fold (zip z u (λ (w a) (- w a))) 0 (λ (b c) (+ (* b
b)    c)))) (/ (* 1 1) (+ 1 1))) y) (fold (zip z u (λ
(d e) (- d e))) 0 (λ (f g) (+ (* f f) g)))) v)) (zip z
u (λ (h i) (- h i)))))
```

Program in terms of initial library

Rediscovering origami programming



Stutter

[■■] → [■■■■]
[■■■] → [■■■■■■]

```
(fold A (λ (u v) (cons
v (cons v u))) nil)
```

Index List

0, [■■■■■] → ■
1, [■■■■■] → ■

```
(index n A)
```

Take Every Other

[■■■■] → [■■]
[■■■■■■] → [■■■]

```
(unfold_list cdr A nil?)
```

List Lengths

[[■■■], [■]] → [3 1]
[[■■], [], [■]] → [2 0 1]

```
(map A length)
```

Differences

[1 8 2], [0 5 1] → [1 3 1]
[2 3 6], [1 2 4] → [1 1 2]

```
( zip A - B )
```

```
(zip A - B) = (λ (A B) (Y (Y 0 (λ (z u) (if (= u (Y A (λ (v w)
(if (nil? w) 0 (+ 1 (v (cdr w)))))))) nil (cons u (z (+ u 1))))))
(λ (a b) (if (nil? b) nil (cons (- (car (Y (Y 0 (λ (c d) (if (= d
(car b)) nil (cons d (c (+ d 1)))))))) (λ (e f) (if (nil? f) A (cdr
(e (cdr f)))))))) (car (Y (Y 0 (λ (g h) (if (= h (car b)) nil (cons
h (g (+ h 1)))))))) (λ (i j) (if (nil? j) B (cdr (i (cdr j))))))))
(a (cdr b)))))))
```

Program in terms of initial library

From learning libraries to using interpreters

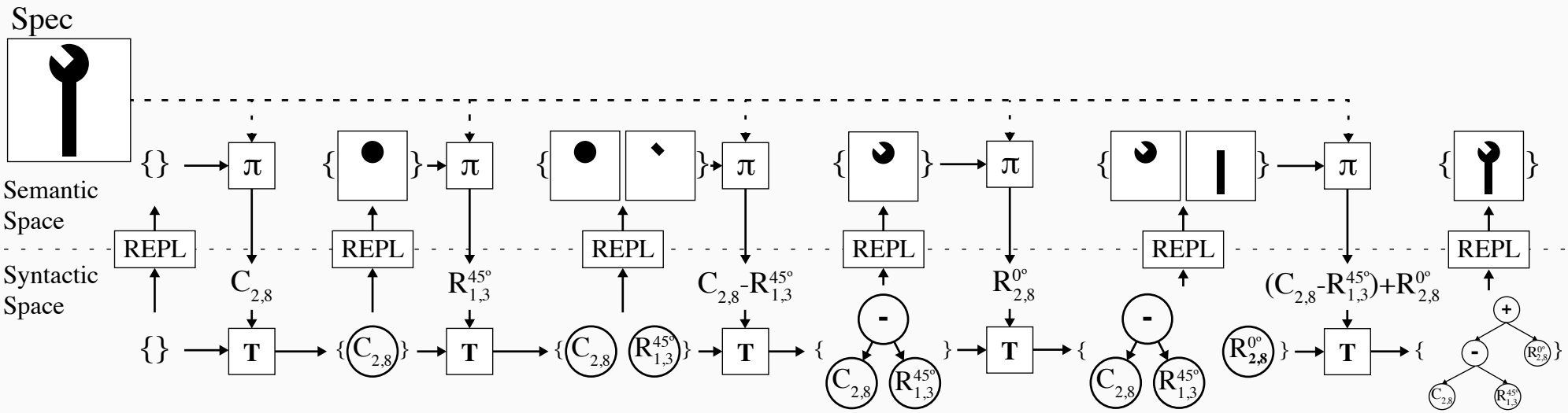
DreamCoder: library building as inspiration

Software engineers: build libraries, use interpreters, version control, debuggers, ...

Ellis*, Nye*, Pu*, Sosa*, Tenenbaum, Solar-Lezama. NeurIPS 2019.

*equal contribution

Synthesis with a REPL



REPL: Bridges syntax and semantics

π: policy, writes code conditioned on REPL state

T: Markov Decision Process transition function

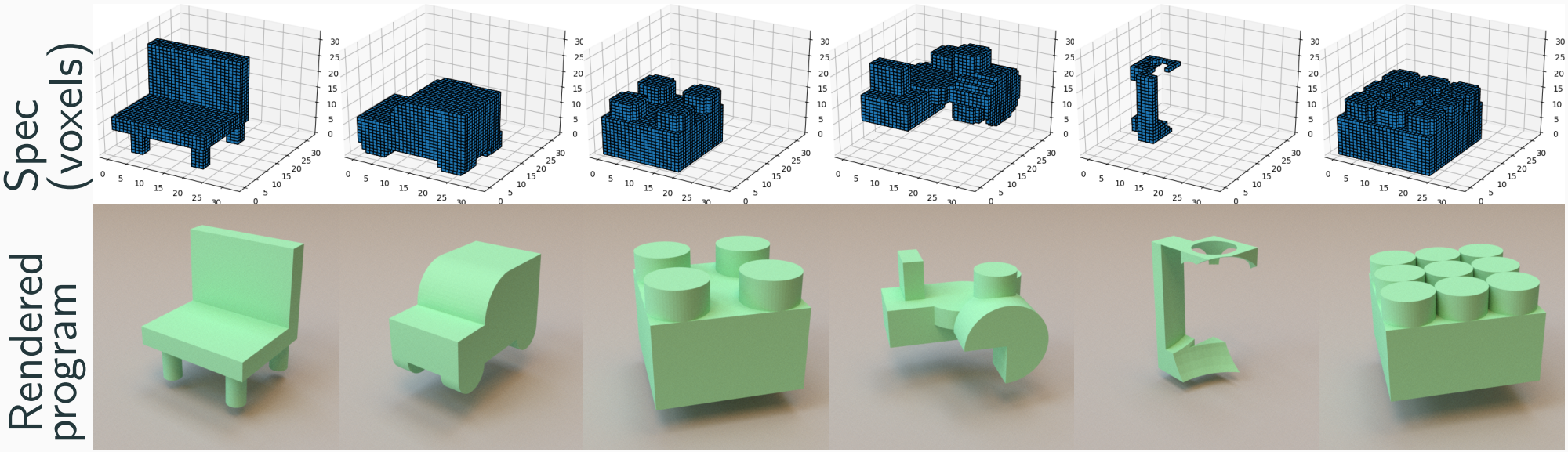
Spec: Image to draw

Ellis*, Nye*, Pu*, Sosa*, Tenenbaum, Solar-Lezama. NeurIPS 2019.
*equal contribution

Scaling to long programs

Branching factor: > 1.3 million per line of code

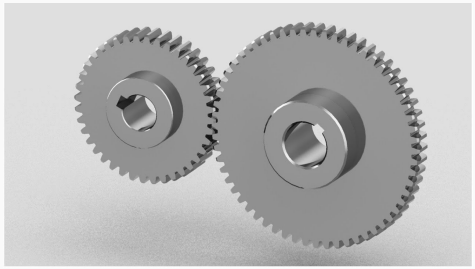
Successfully synthesizes >20-line programs in seconds (>100 tokens)



Programmatic models of the physical world

hinge

gear



doorknob