Building Machines that Discover Generalizable, Interpretable Knowledge

Kevin Ellis

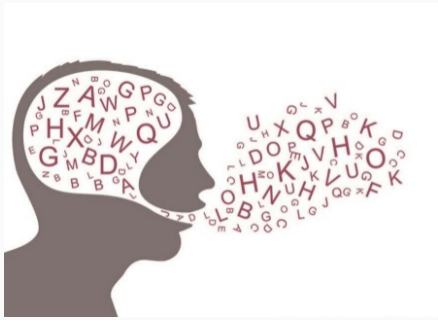
2022

Cornell University

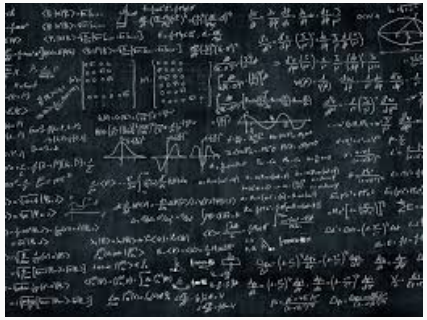
What computational problems are solved by intelligence?

an endless range of problems

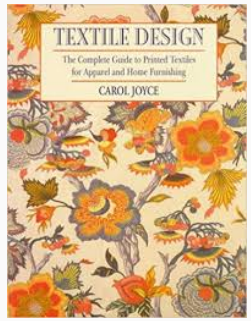
language



science

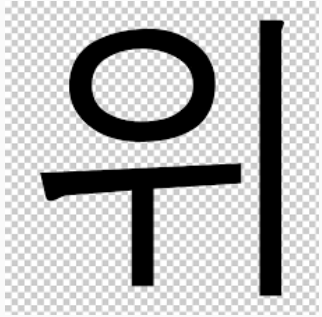


design



using new devices

writing new characters



coding

```
(MEMBER
 (LAMBDA (X L)
  (COND  ((NULL L) NIL)
         ((EQ X (FIRST L)) T)
         (T (MEMBER X (REST L))))))
```

Allen, Anatomy of Lisp, 1975

engineering

play

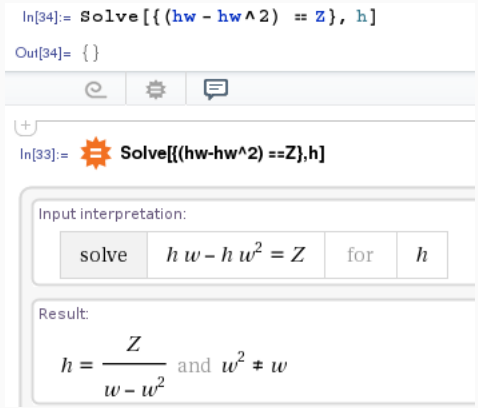
What computational frameworks can contribute to this picture?

Three AI traditions

What computational frameworks can contribute to this picture?

Three AI traditions

Symbolic



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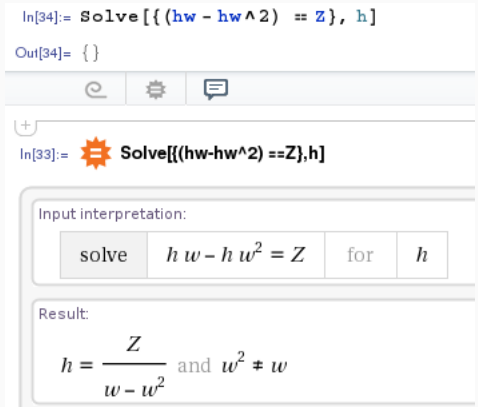
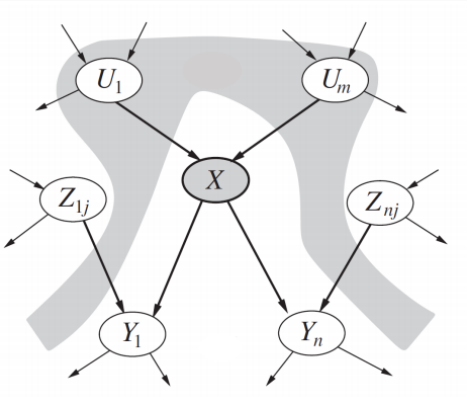
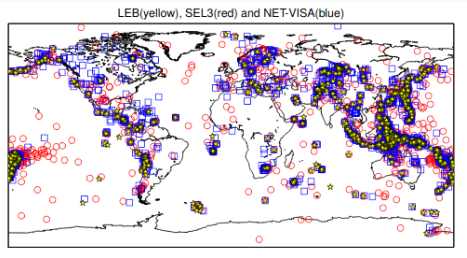
Three AI traditions

Symbolic



Probabilistic





What computational frameworks can contribute to this picture?

Three AI traditions

Symbolic

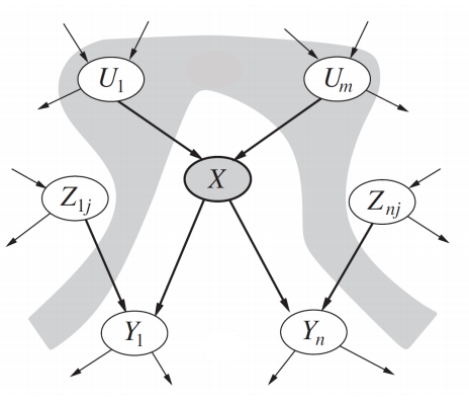

```
In[34]:= Solve[{(hw - hw^2) == Z}, h]
Out[34]= {}

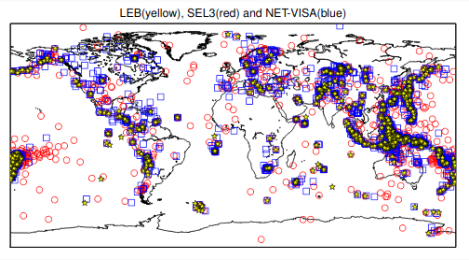
In[33]:= Solve[{(hw-hw^2) ==Z},h]

Input interpretation:
solve | h w − h w^2 = Z | for | h

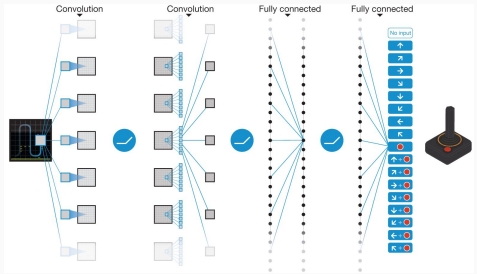
Result:
h = Z/(w − w^2)  and  w^2 ≠ w
```

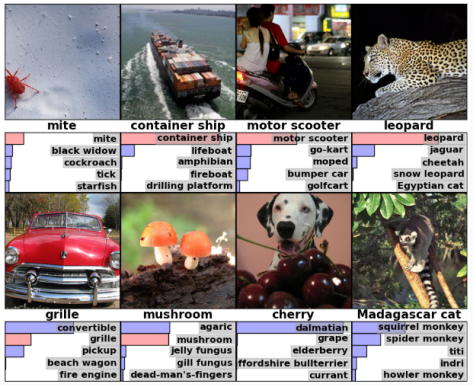
Probabilistic





Neural

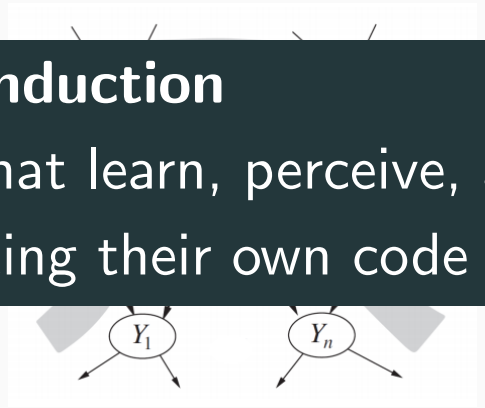


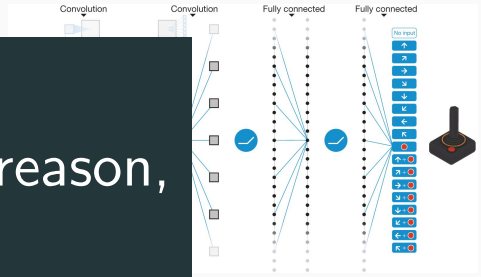


What computational frameworks can contribute to this picture?

Three AI traditions

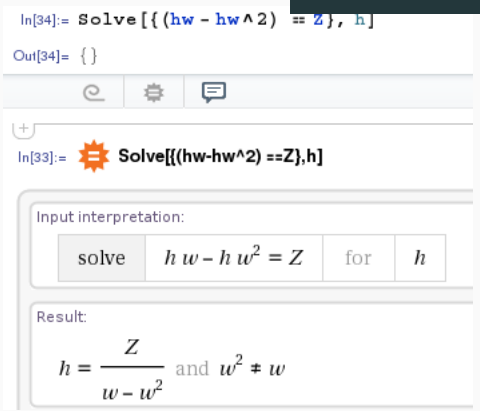
Symbolic | Probabilistic | Neural

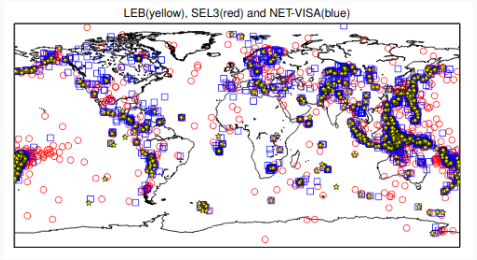


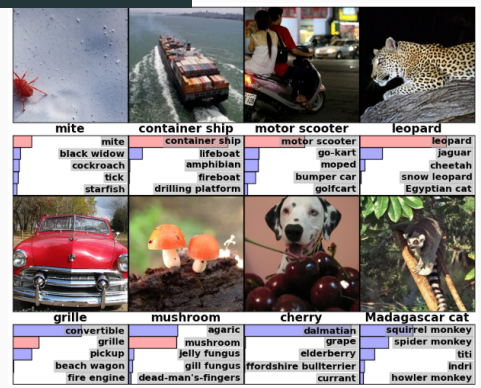


Program induction

machines that learn, perceive, and reason, by writing their own code





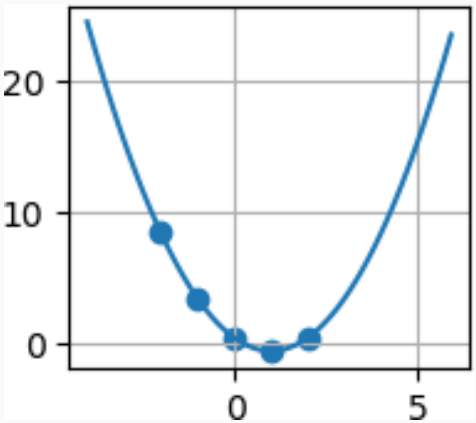


Why program induction?

Why program induction?

strong generalization

+data efficiency




```
f(x)=(x-1)**2 - 0.5
```

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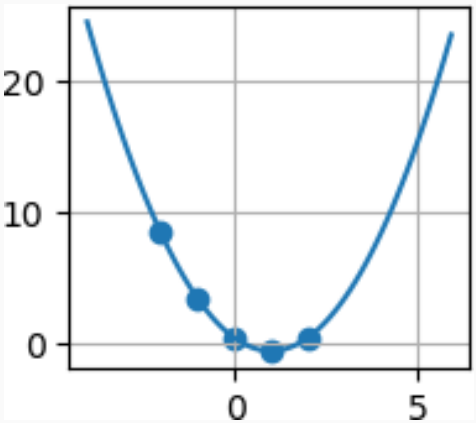
+data efficiency

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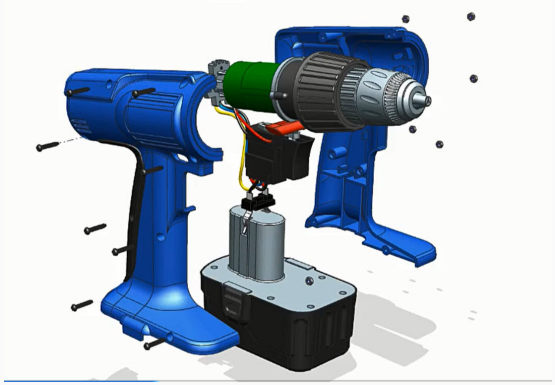
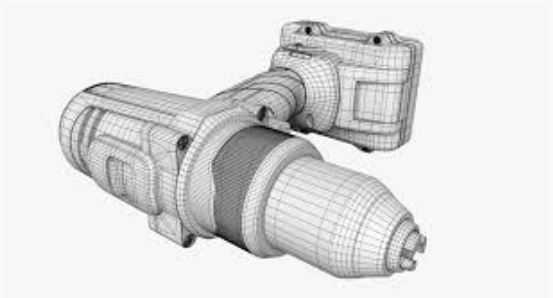
```

interpretability



vs



Why program induction?

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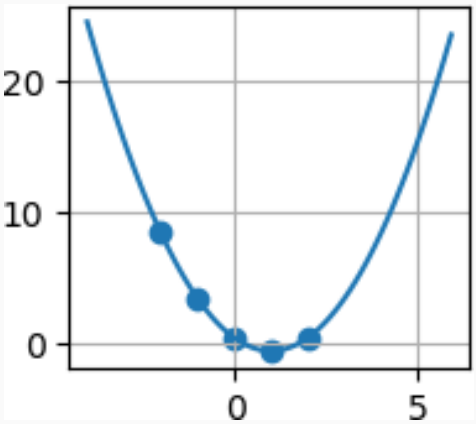


```
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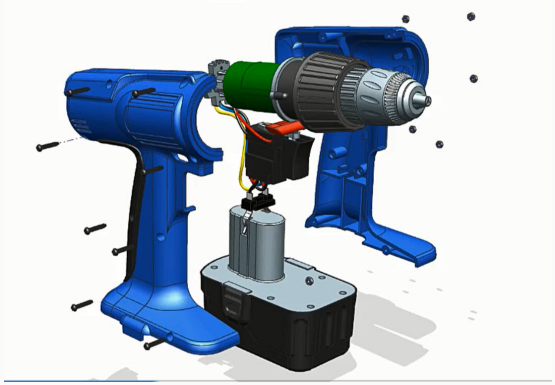
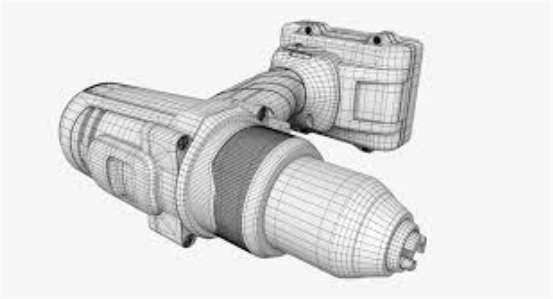
```

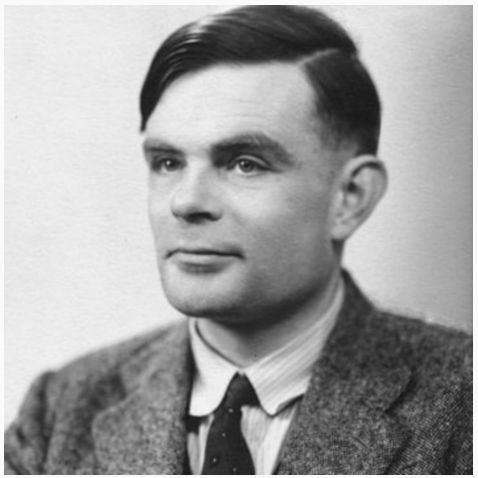
interpretability



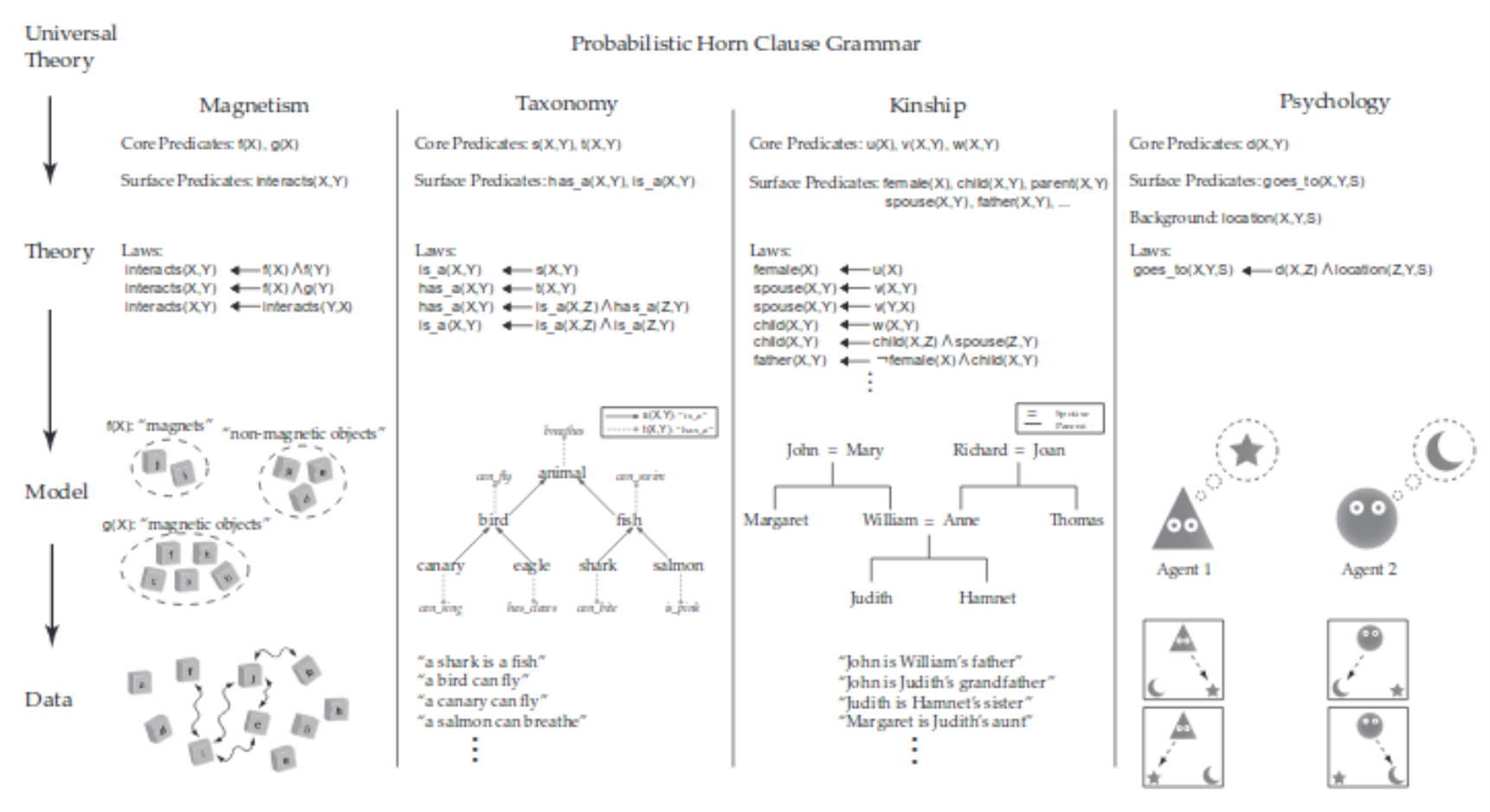
vs



universal expressivity

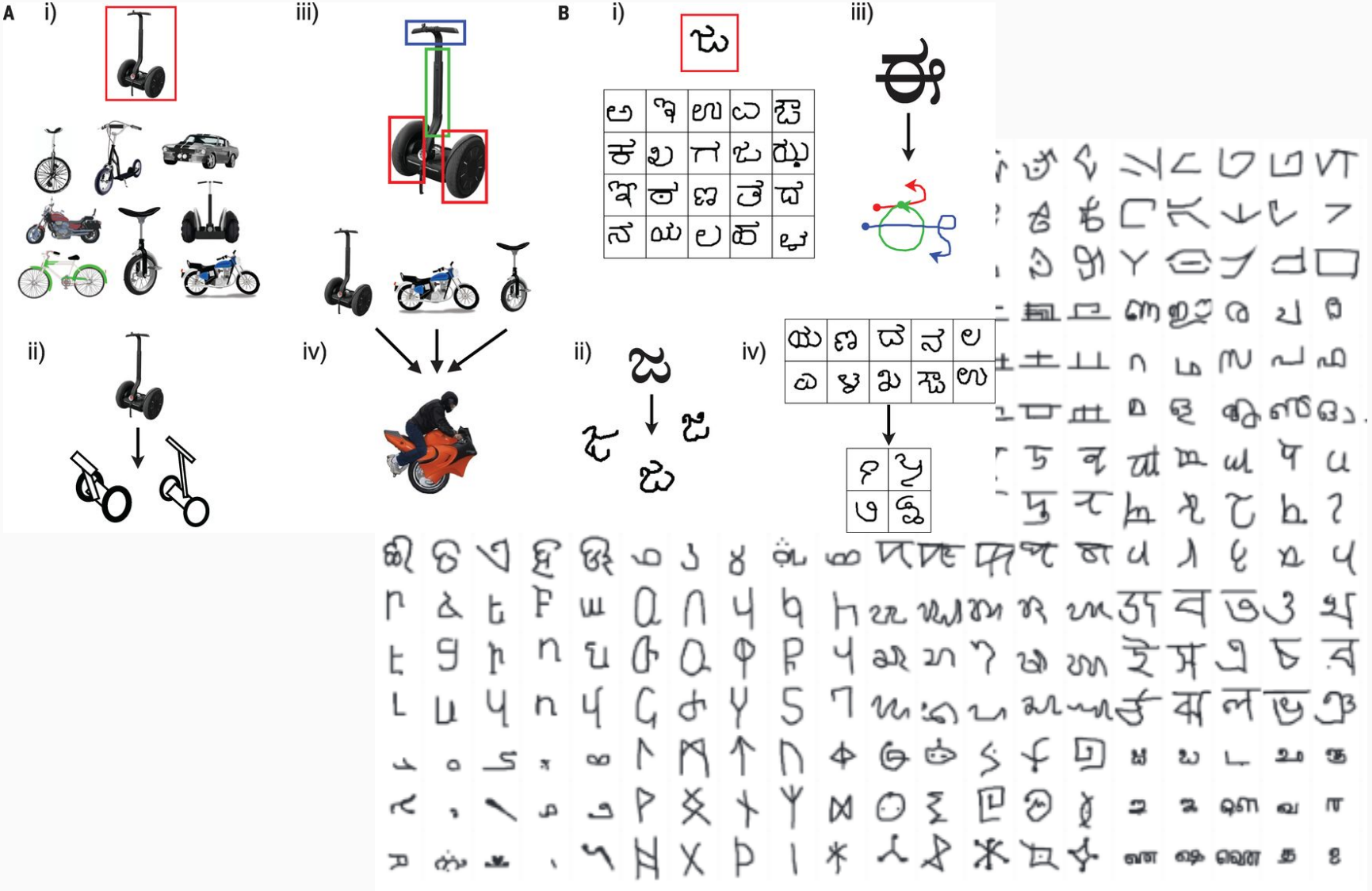


Human program induction



Ullman et al 2012

Human program induction



Lake et al 2015

Can we engineer program induction systems?

better toolkits: neural+probabilistic+symbolic, and knowing how to combine them

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maturing **program synthesis** techniques

Can we engineer program induction systems?

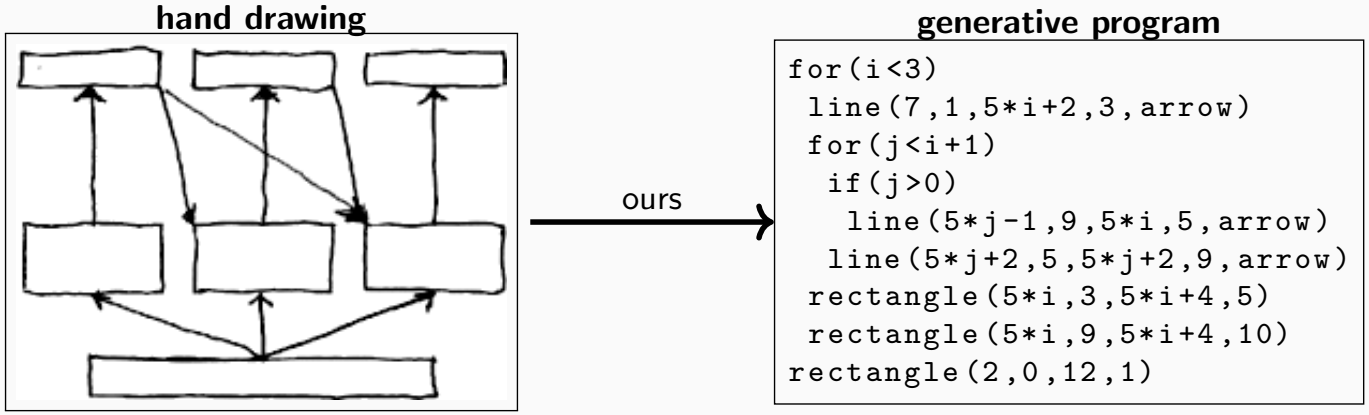
better toolkits: neural+probabilistic+symbolic, and knowing how to combine them

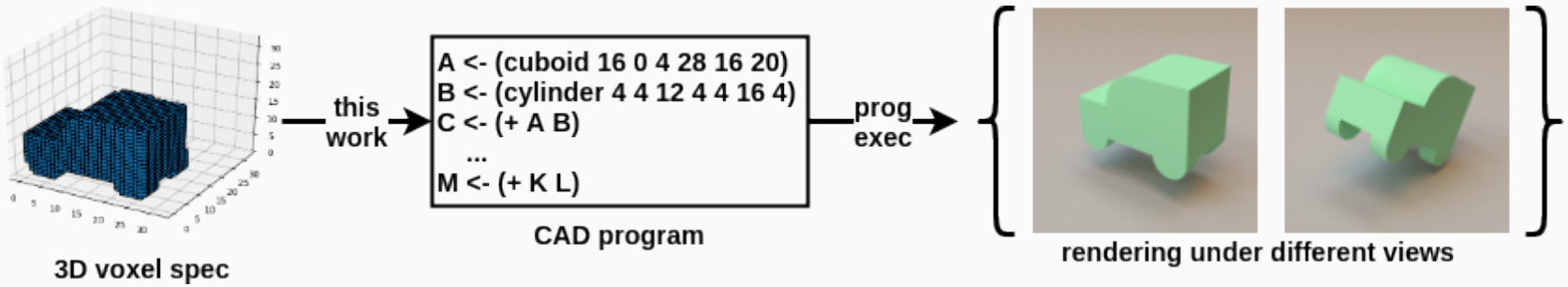
maturing **program synthesis** techniques

better compute+parallel algorithms

Perception, Synthesizing models, Learning-to-Learn

Theme #1: high-level visual understanding, pixels→programs

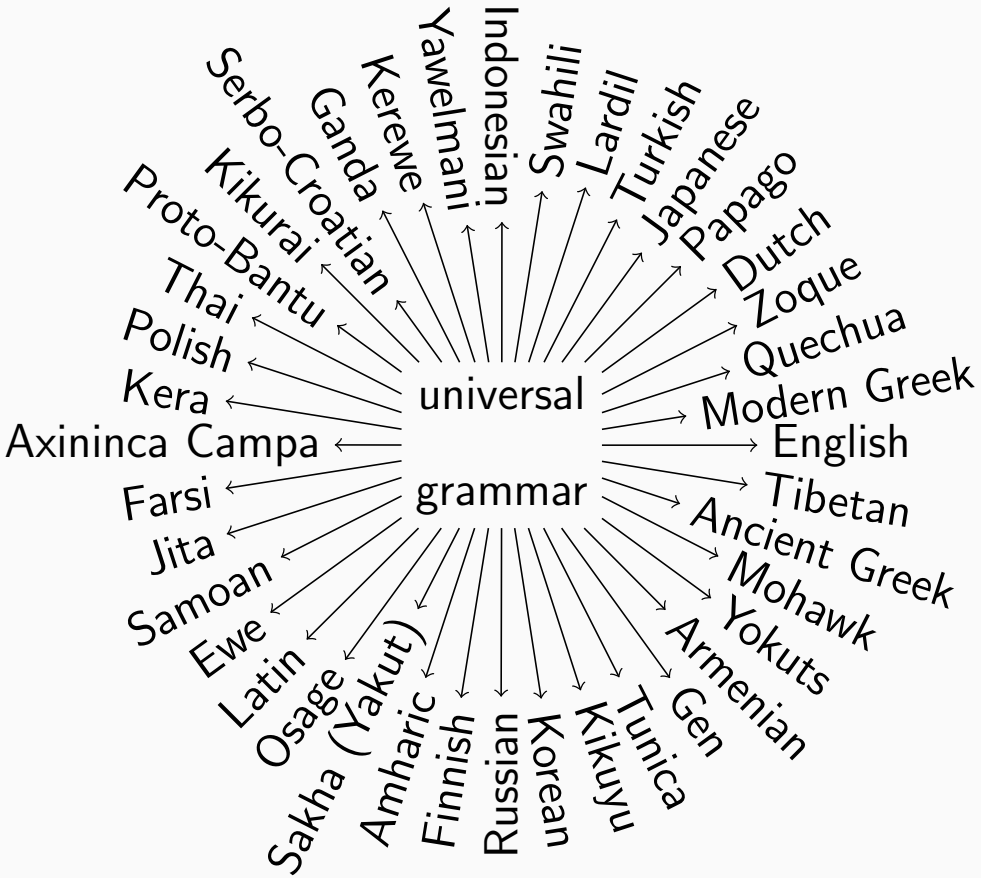




Perception, Synthesizing models, Learning-to-Learn

Theme #1: high-level visual understanding, pixels→programs

Theme #2: synthesizing human-understandable models



Perception, Synthesizing models, Learning-to-Learn

Theme #1: high-level visual understanding, pixels→programs

Theme #2: Synthesizing human-understandable models

Theme #3: learning to synthesize programs

List Processing

Sum List

```
[1 2 3] → 6
[4 6 8 1] → 17
```

Double

```
[1 2 3] → [2 4 6]
[4 5 1] → [8 10 2]
```

Text Editing

Abbreviate

```
Allen Newell → A.N.
Herb Simon → H.S.
```

Drop Last Three

```
shrdlu → shr
shakey → sha
```

Regexes

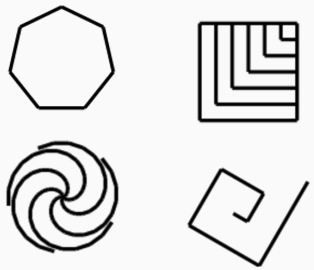
Phone numbers

```
(555) 867-5309
(650) 555-2368
```

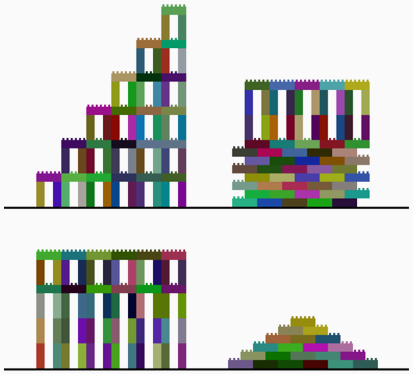
Currency

```
$100.25
$4.50
```

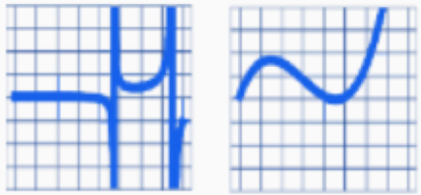
LOGO Graphics



Block Towers



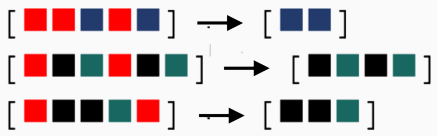
Symbolic Regression



$$y = f(x)$$

Recursive Programming

Filter Red



Physical Laws

$$\vec{a} = \frac{1}{m}\sum_i \vec{F}_i$$

$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r}|^2}\hat{r}$$

Program Induction and perception

model discovery

learning to learn

Vision is more than knowing what is where

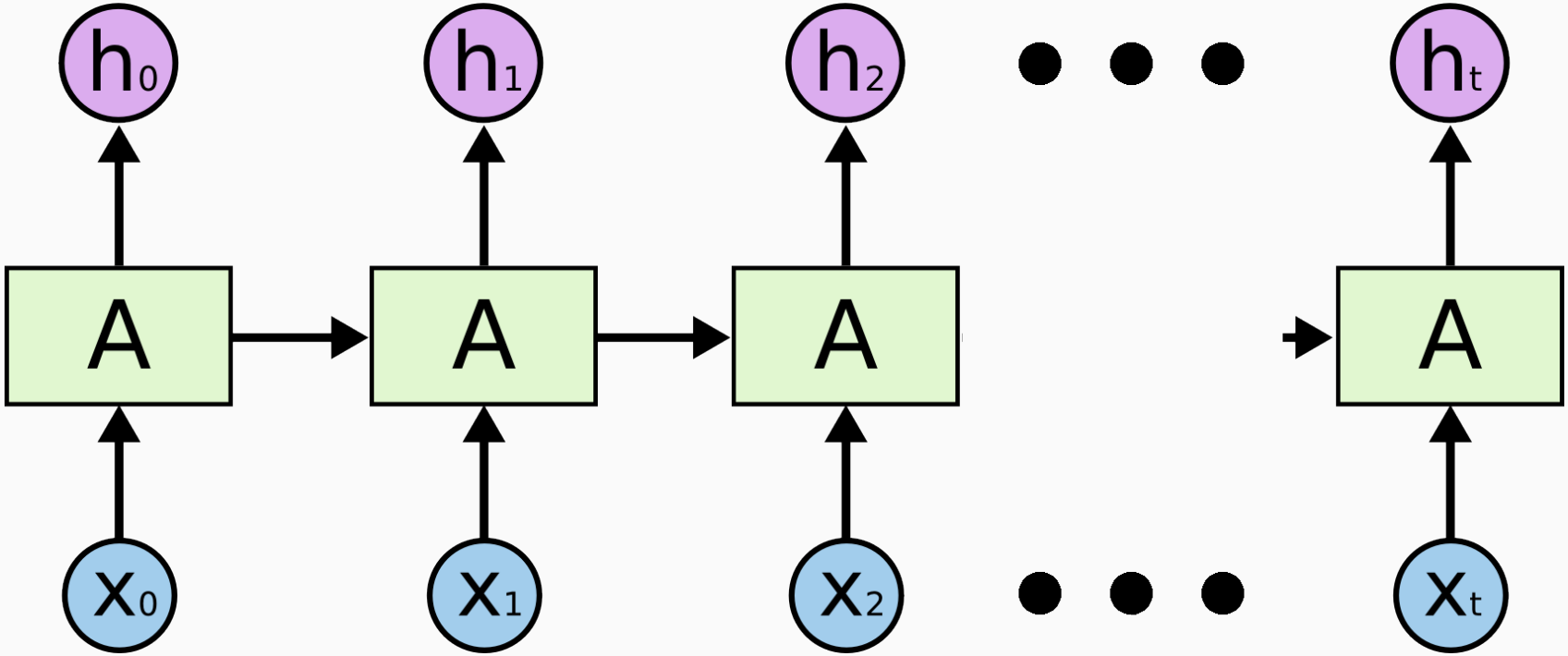
Can you visually extrapolate this aqueduct?

Vision is more than knowing what is where

Can you visually extrapolate this aqueduct?

Vision is more than knowing what is where

Can you infer what goes in the ellipses?



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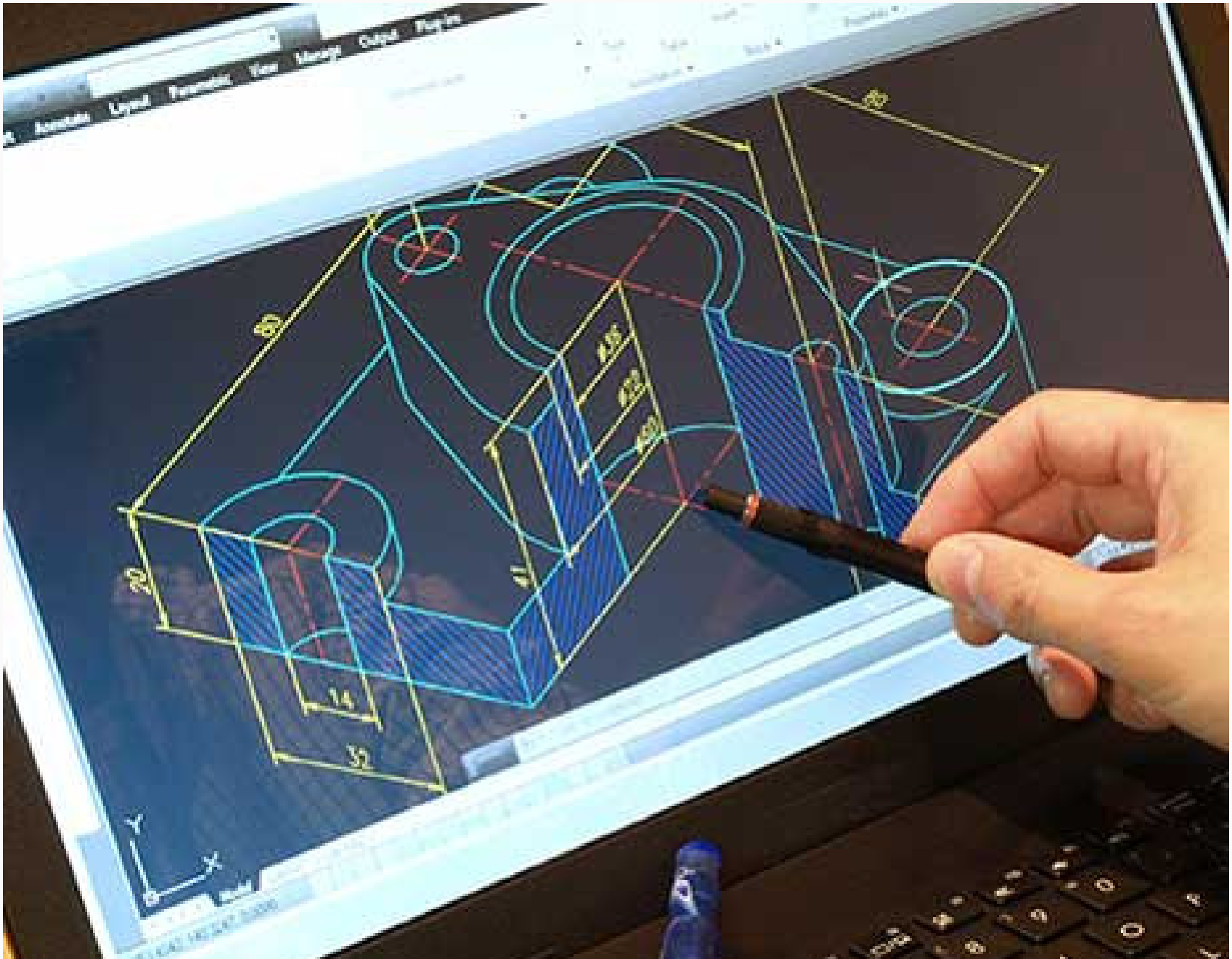
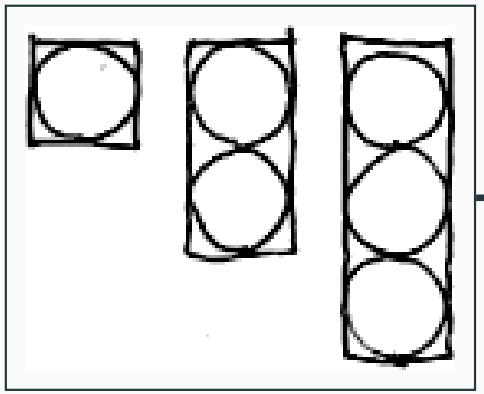


Learning to infer graphics programs from hand-drawn images

hand drawing



machine-generated program

```
for (0 <= i < 3)
 rectangle(3*i,-2*i+4,
           3*i+2,6)
 for (0 <= j < i + 1)
  circle(3*i+1,-2*j+5)
```

Dan Ritchie

Ellis, Ritchie, Solar-Lezama, Tenenbaum. NeurIPS 2018.

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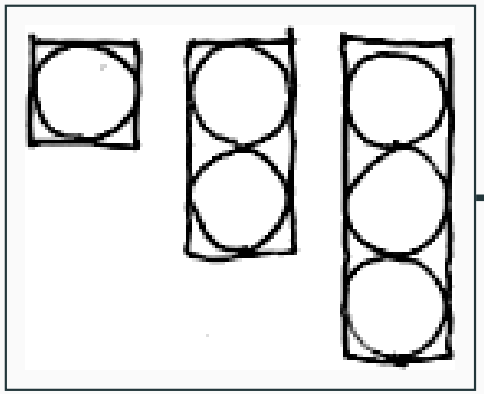

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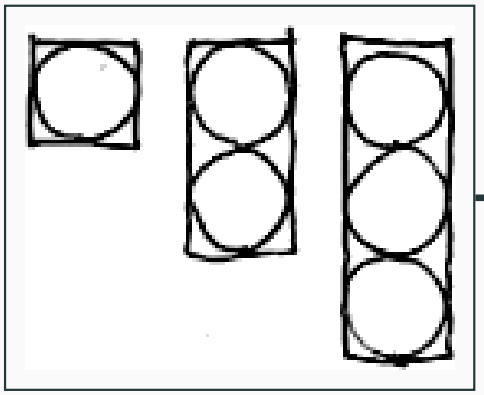

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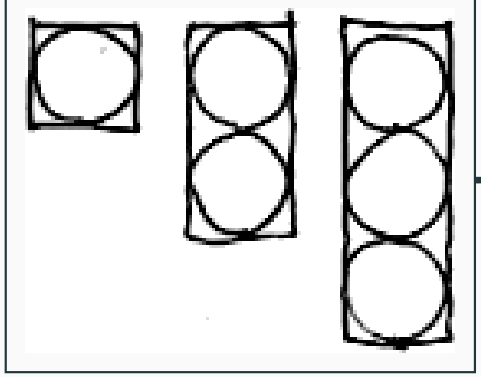

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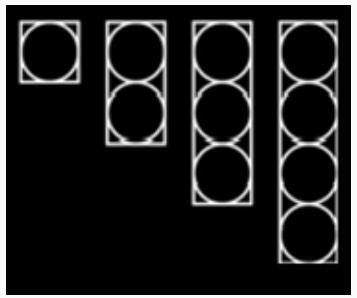
```

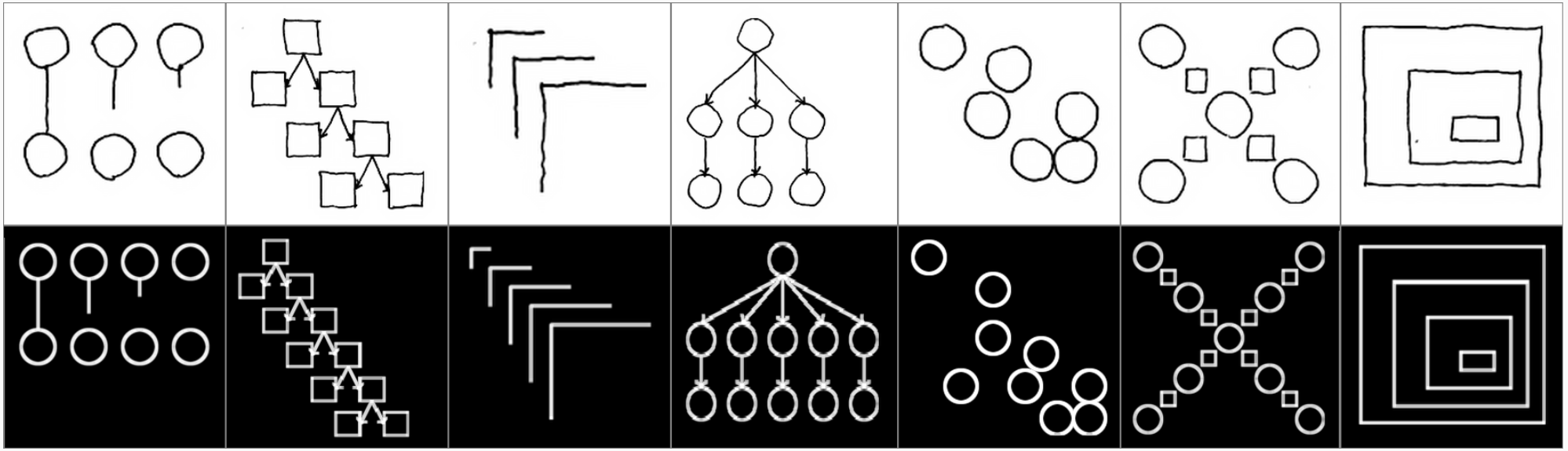
autogenerated extrapolation



one-shot generalization / extrapolation

Ellis, Ritchie, Solar-Lezama, Tenenbaum. NeurIPS 2018.

Extrapolation from a single image



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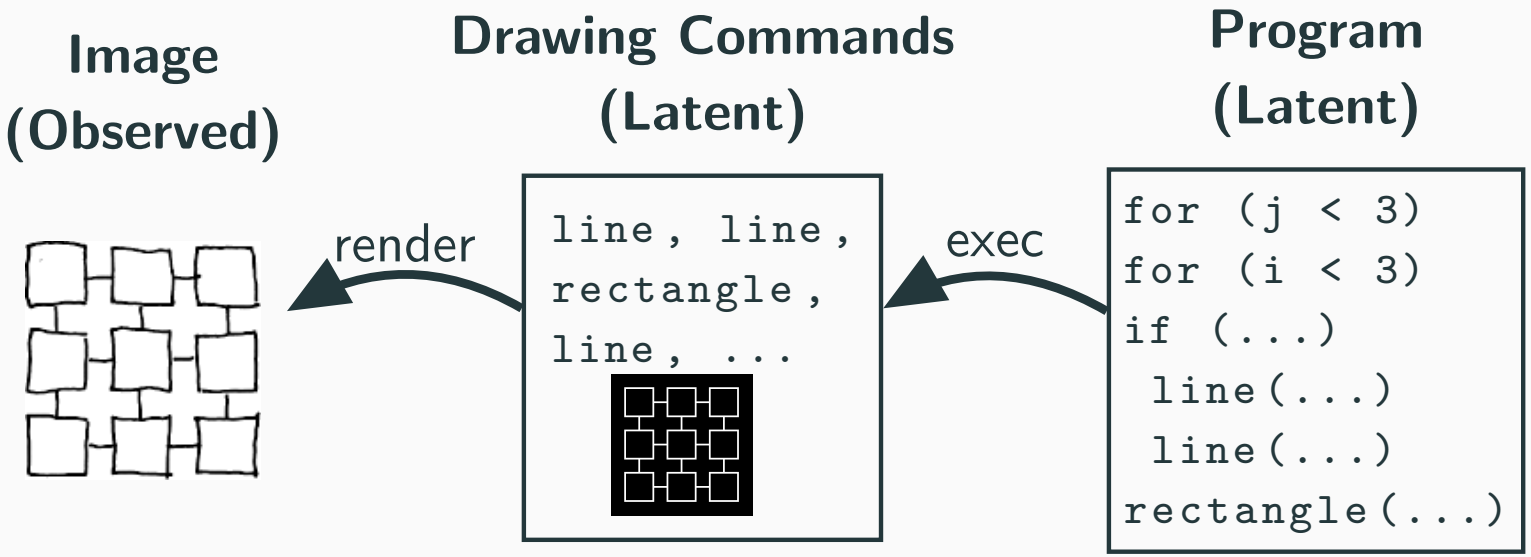
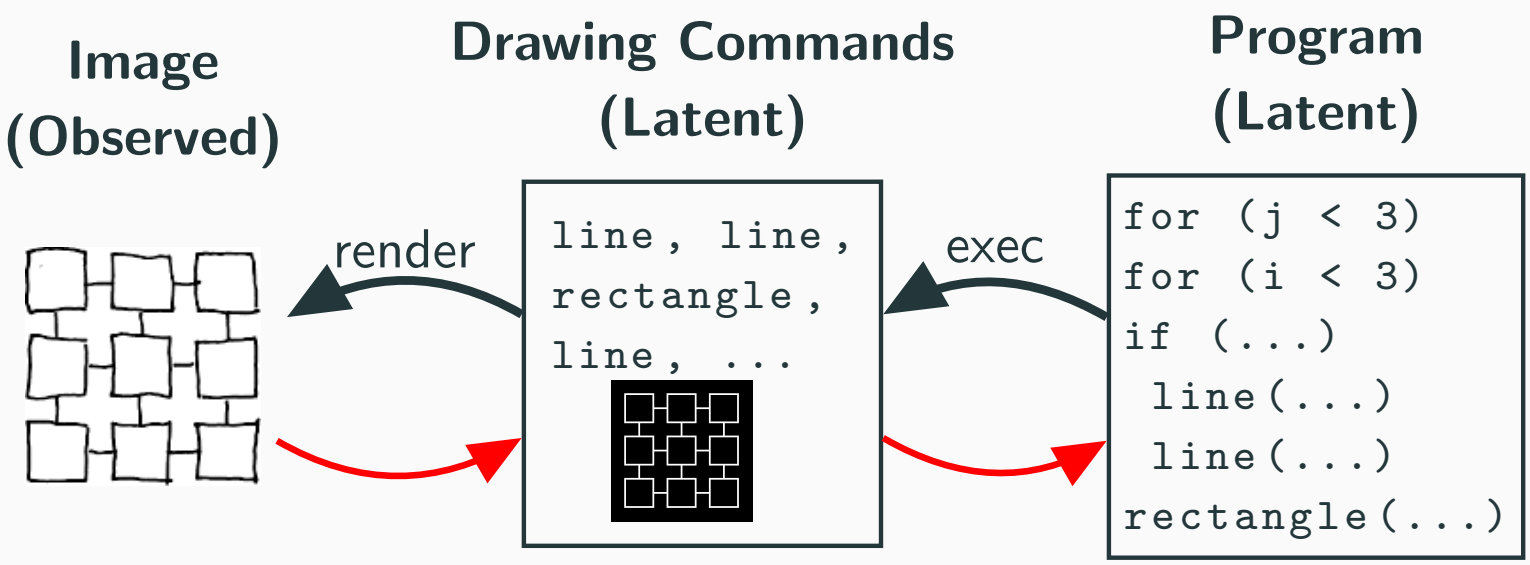


Ellis, Ritchie, Solar-Lezama, Tenenbaum. NeurIPS 2018.

How to infer graphics programs from hand-drawn images



neural network+
sequential monte carlo

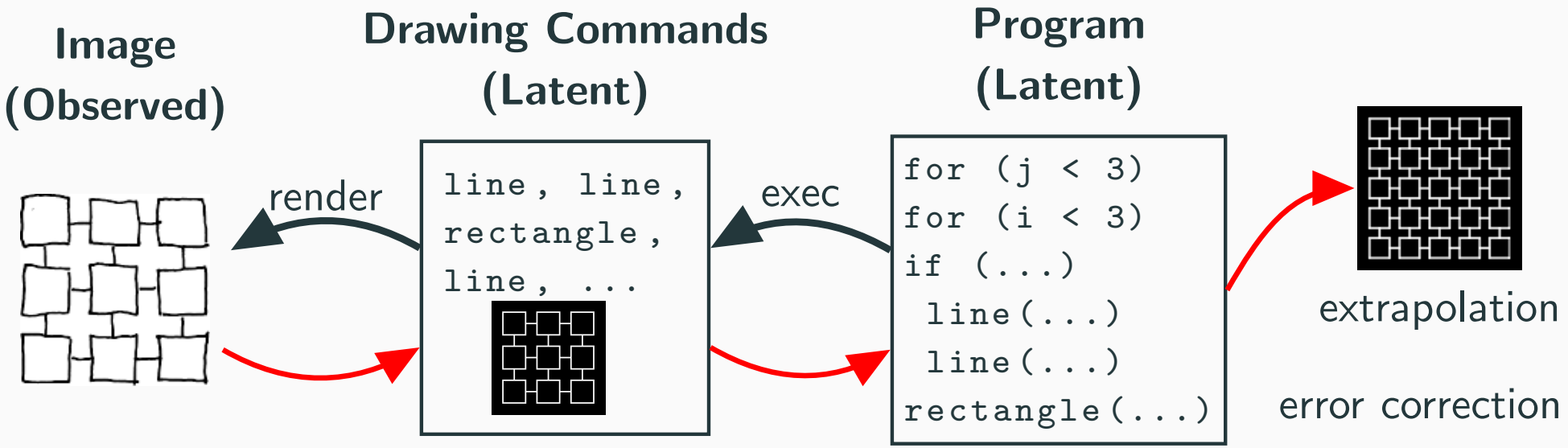
Image→Parse

learning+
constraint-based program synthesis

Parse→Program

Ellis, Ritchie, Solar-Lezama, Tenenbaum. NeurIPS 2018.

How to infer graphics programs from hand-drawn images



neural network+
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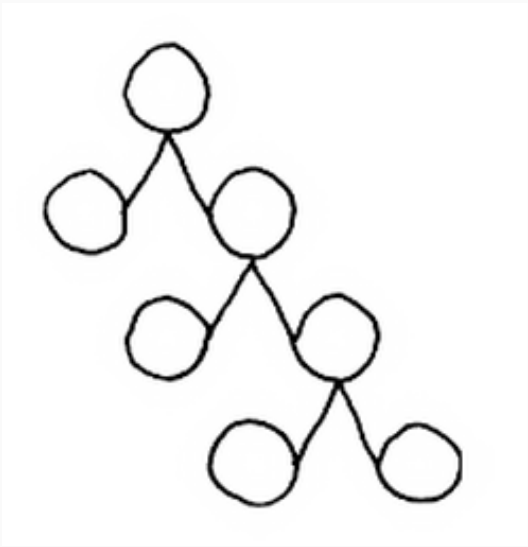
Ellis, Ritchie, Solar-Lezama, Tenenbaum. NeurIPS 2018.

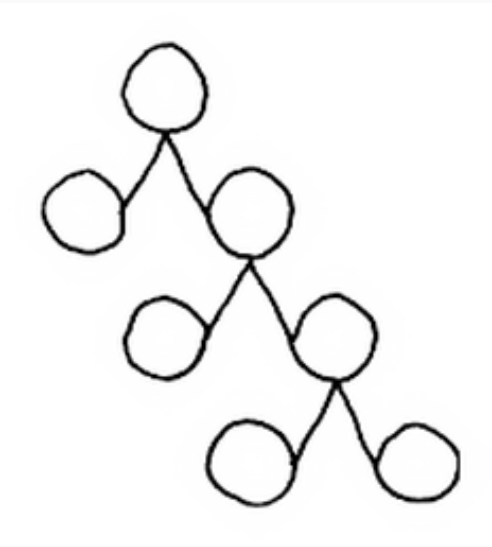
Top-down influences on perception

drawing

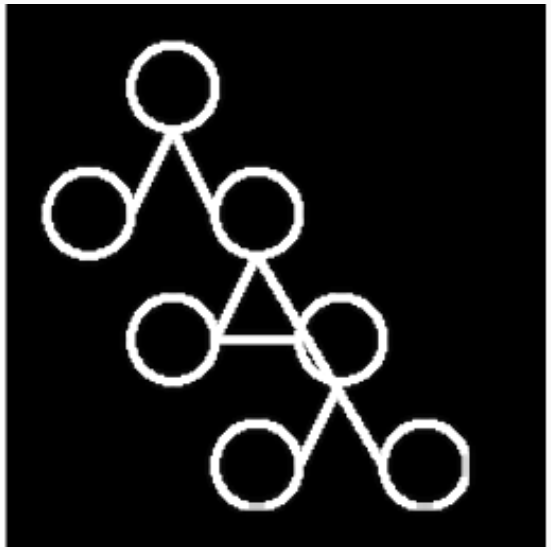


Top-down influences on perception

drawing

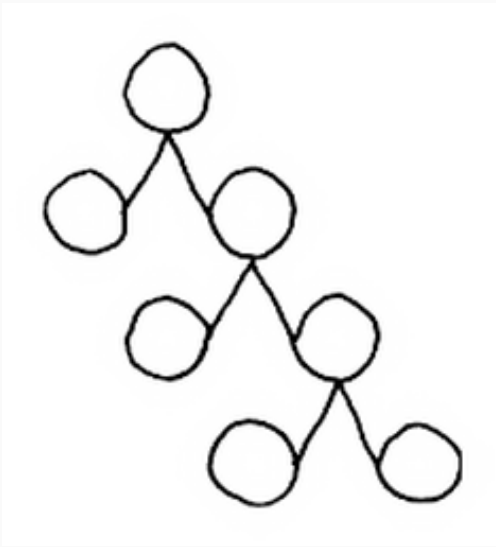


bottom-up neural net

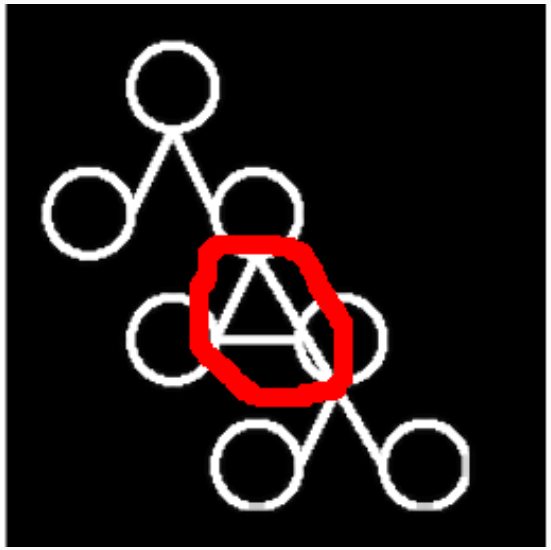


Top-down influences on perception

drawing

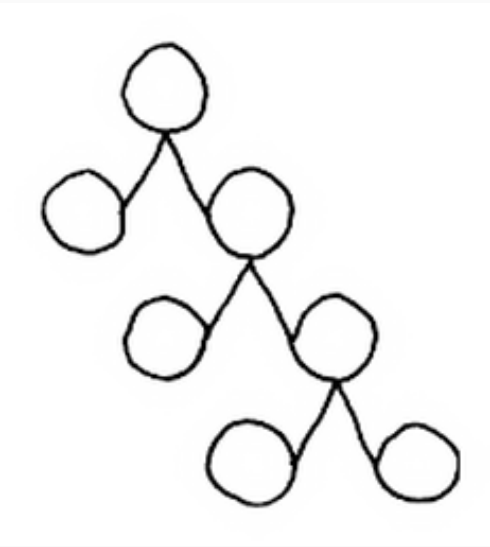


bottom-up neural net

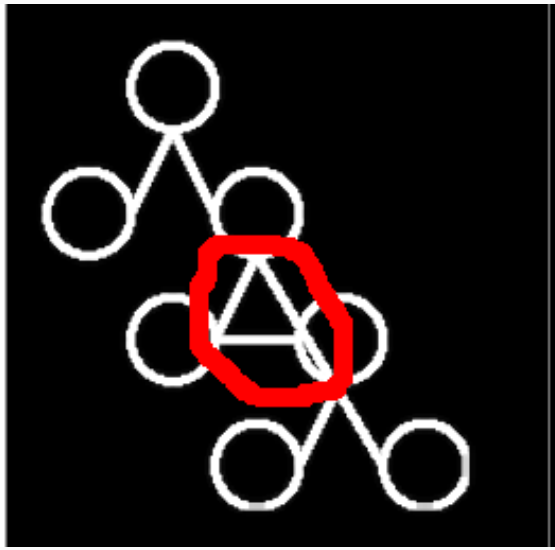


Top-down influences on perception

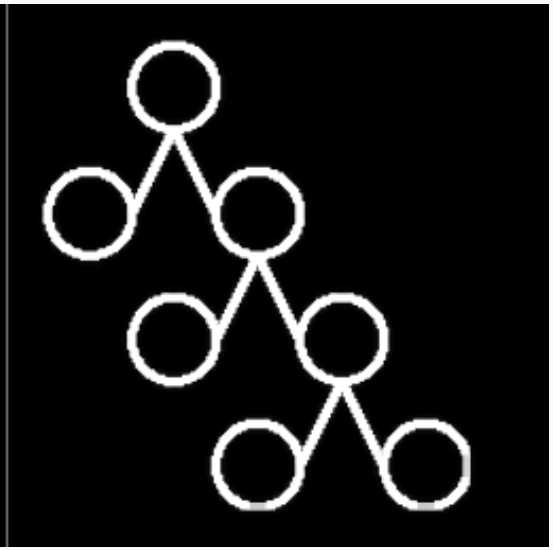
drawing



bottom-up neural net

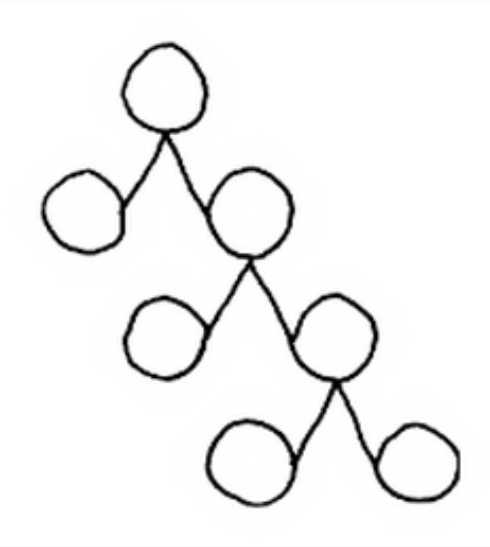


w/ top-down program bias

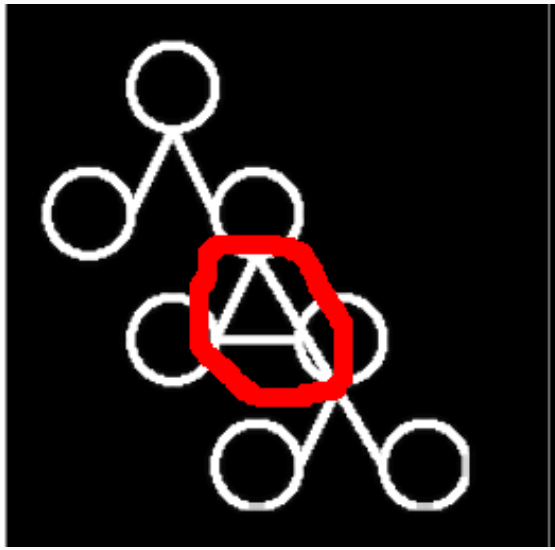


Top-down influences on perception

drawing



bottom-up neural net



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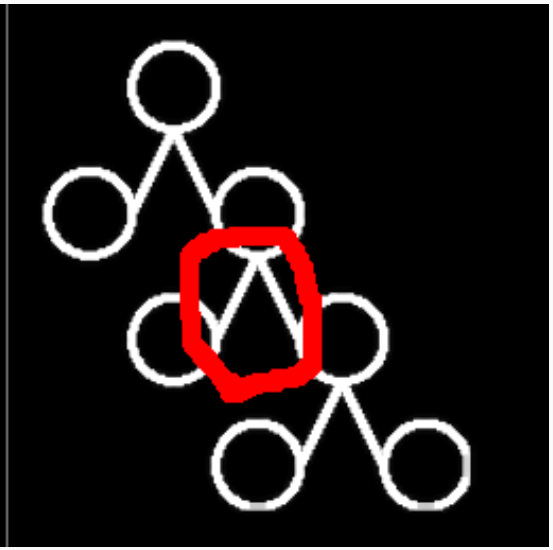
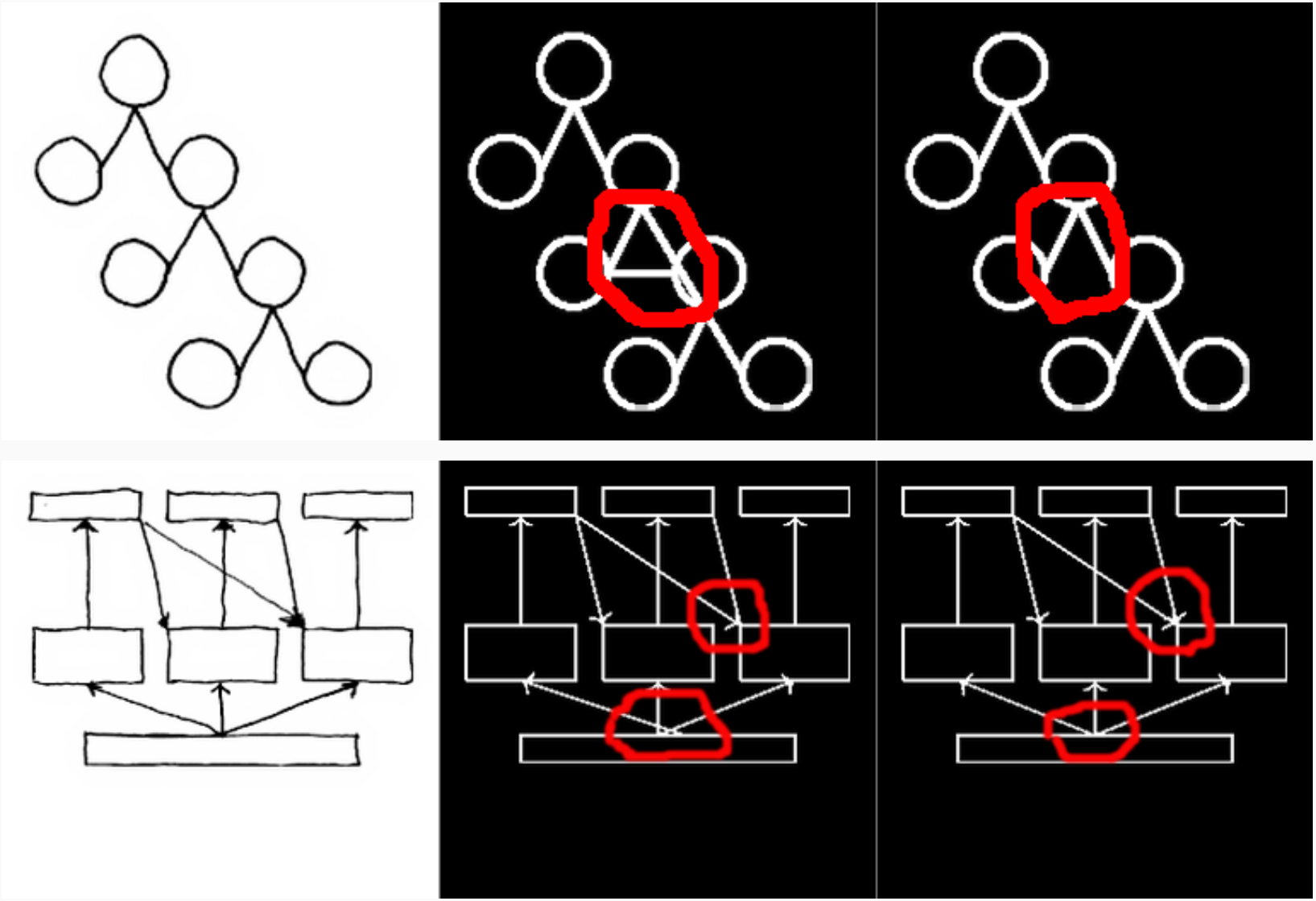
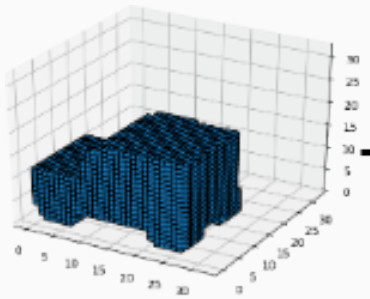


Top-down influences on perception

drawing | bottom-up neural net | w/ top-down program bias



3D program induction



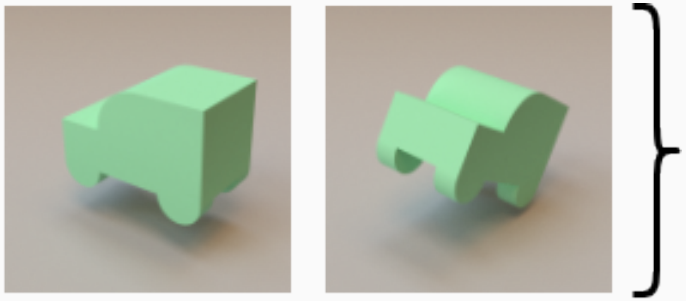
3D voxel spec

this work

```
A <- (cuboid 16 0 4 28 16 20)
B <- (cylinder 4 4 12 4 4 16 4)
C <- (+ A B)
...
M <- (+ K L)
```

CAD program

prog exec

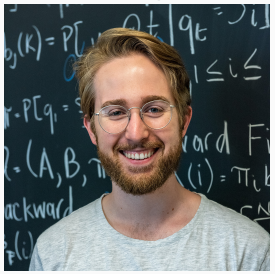


rendering under different views

Max Nye

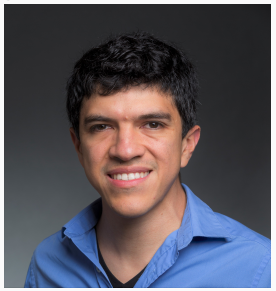
Evan Pu

Felix Sosa



Josh Tenenbaum

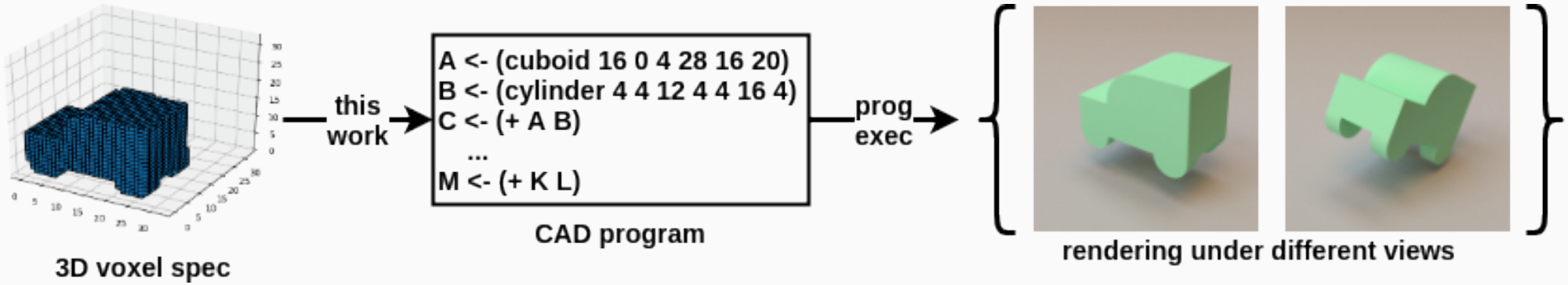
Armando Solar-Lezama



Ellis*, Nye*, Pu*, Sosa*, Tenenbaum, Solar-Lezama. NeurIPS 2019.

*equal contribution

3D program induction



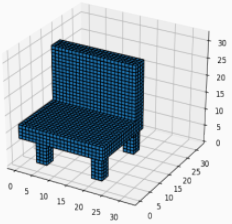
Challenge: combinatorial search!

Branching factor: > 1.3 million per line of code, ≈ 20 lines of code

search space size: $(1.3 \text{ million})^{20} \approx 10^{122}$ programs

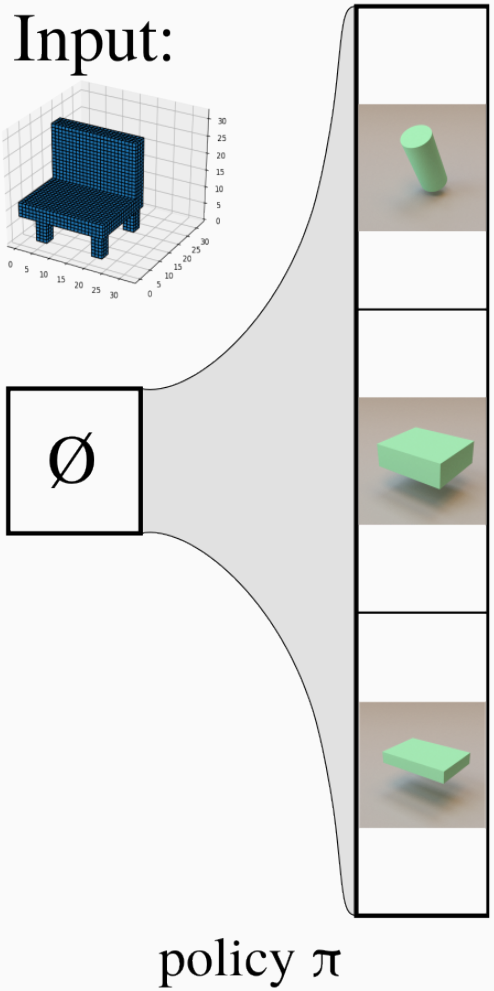
Solution: stochastic **tree search** + learn **policy** that writes code + learn **value** function that assesses execution of program so far; analogous to **AlphaGo** [Silver et al. 2016]

Input:



Ø

Solution: stochastic **tree search** + learn **policy** that writes code + learn **value** function that assesses execution of program so far; analogous to **AlphaGo** [Silver et al. 2016]



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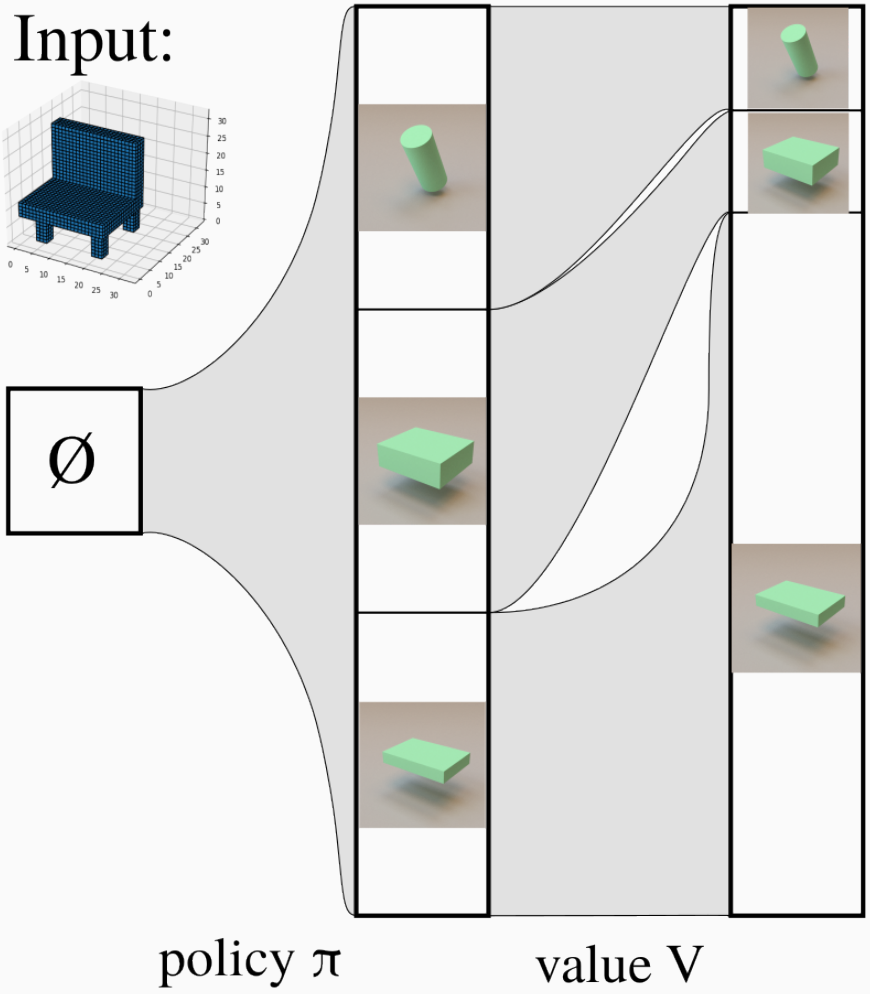

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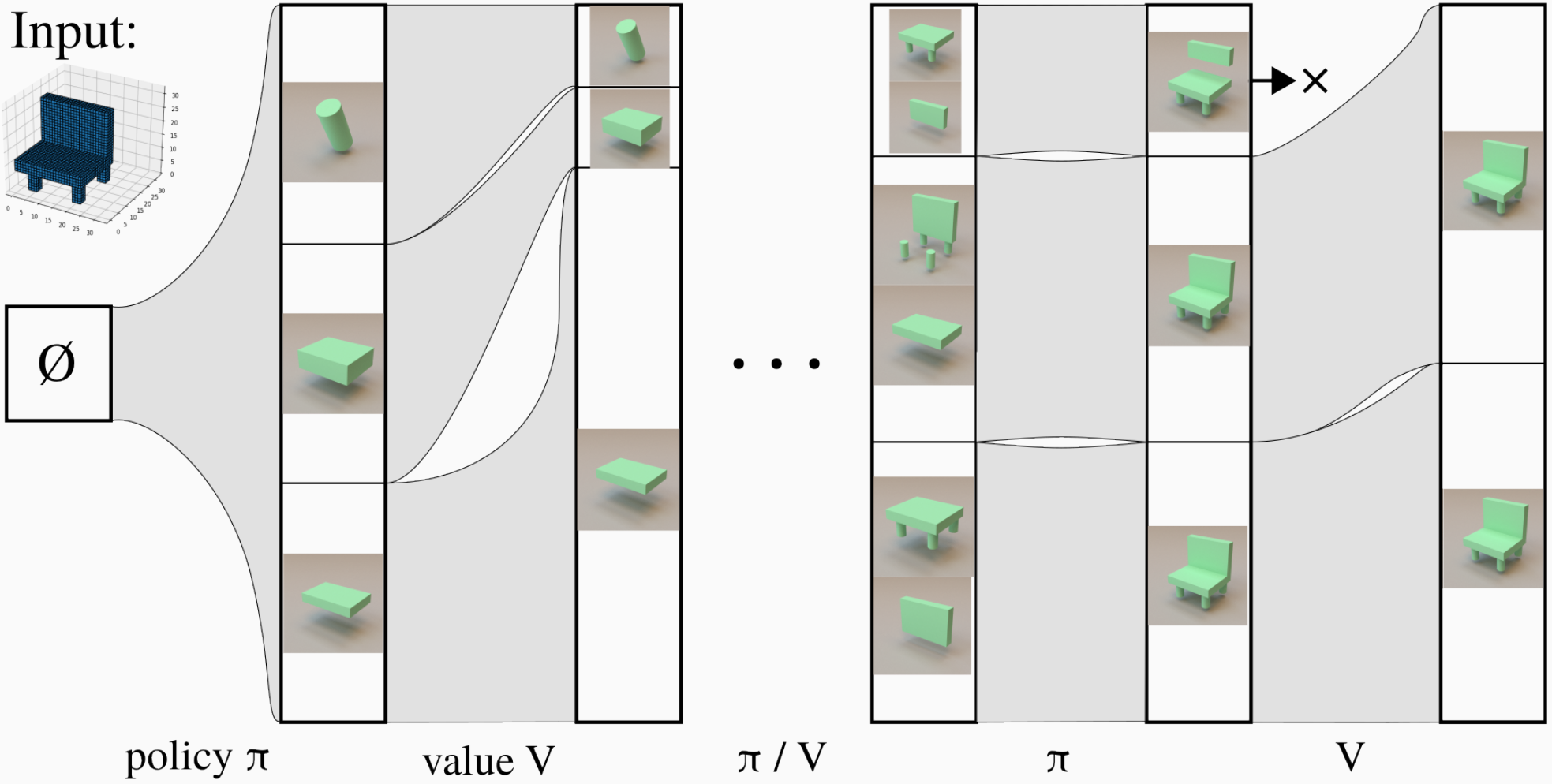
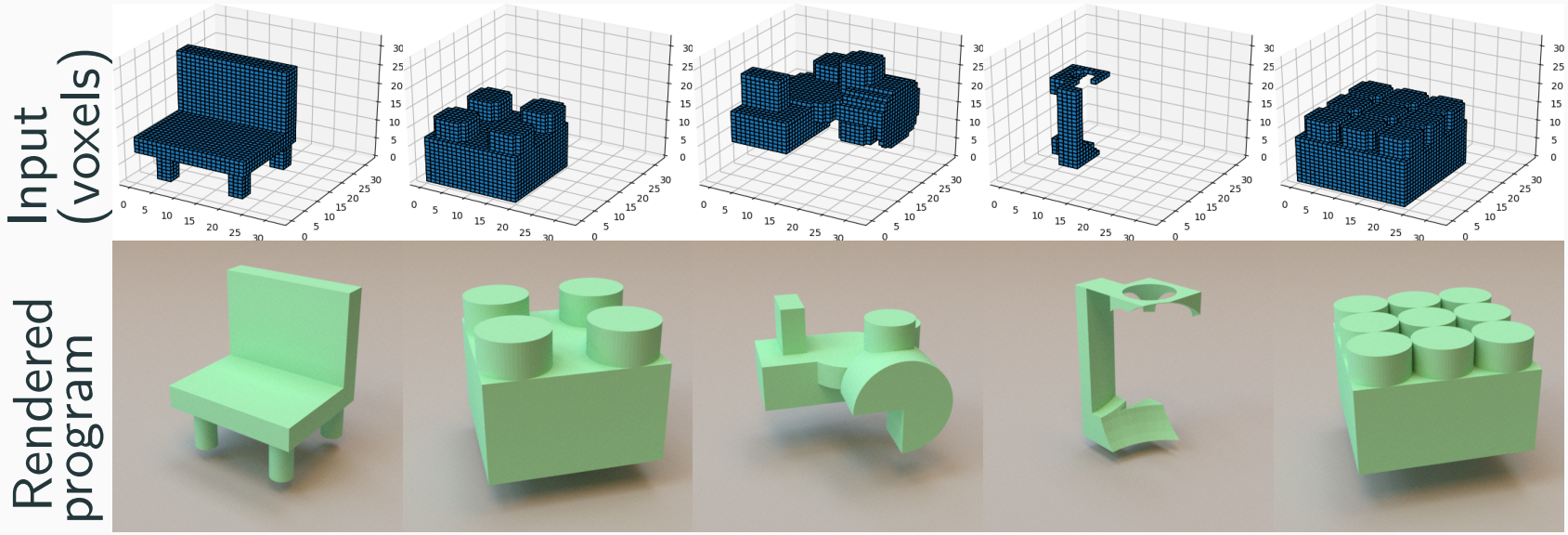
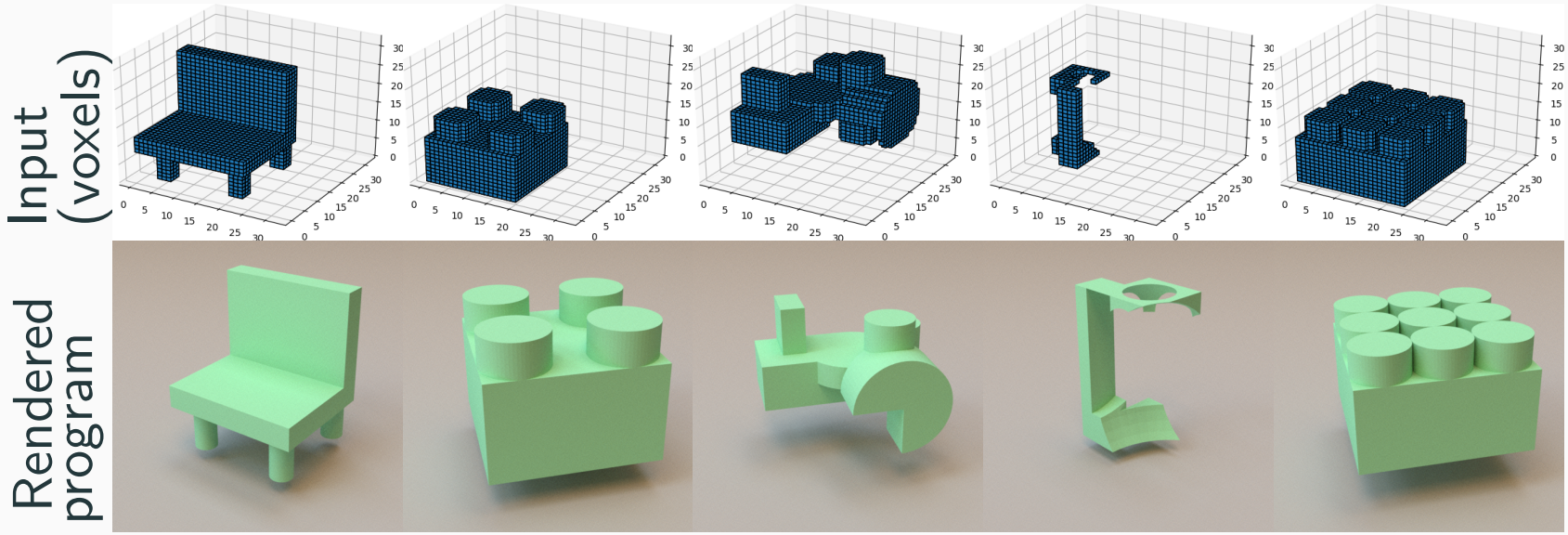


3D program induction



3D program induction



same architecture learns to synthesize text editing programs (FlashFill, Gulwani 2012)

Lessons

The inductive bias from a programming language gives extrapolation, or strong generalization

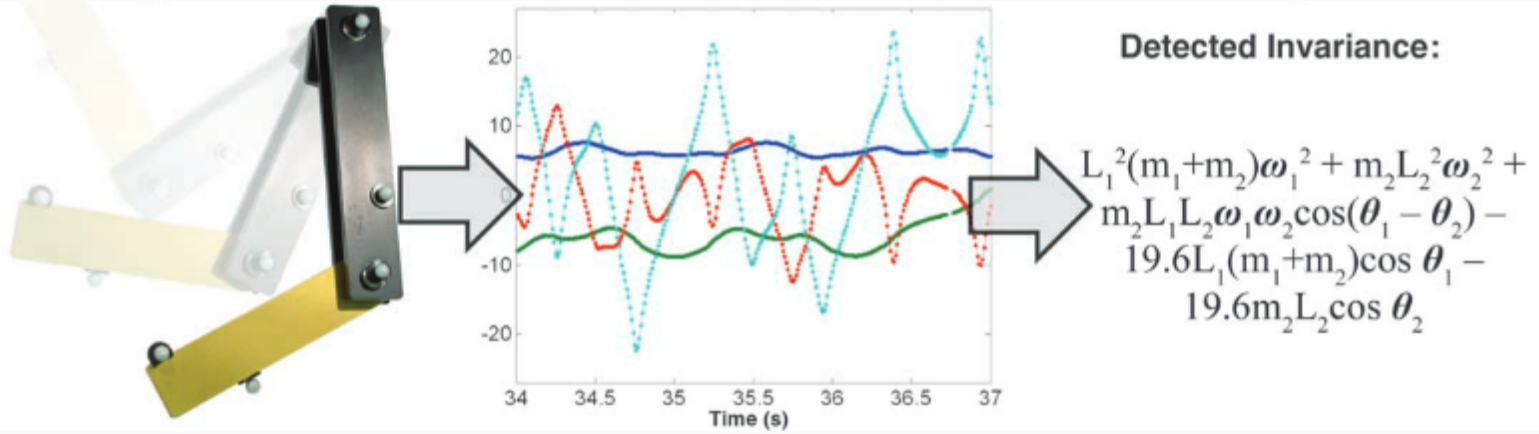
Combine the best of different techniques: neural nets for perception and pattern recognition, symbols for reasoning, Bayesian methods for uncertainty

Program Induction and perception

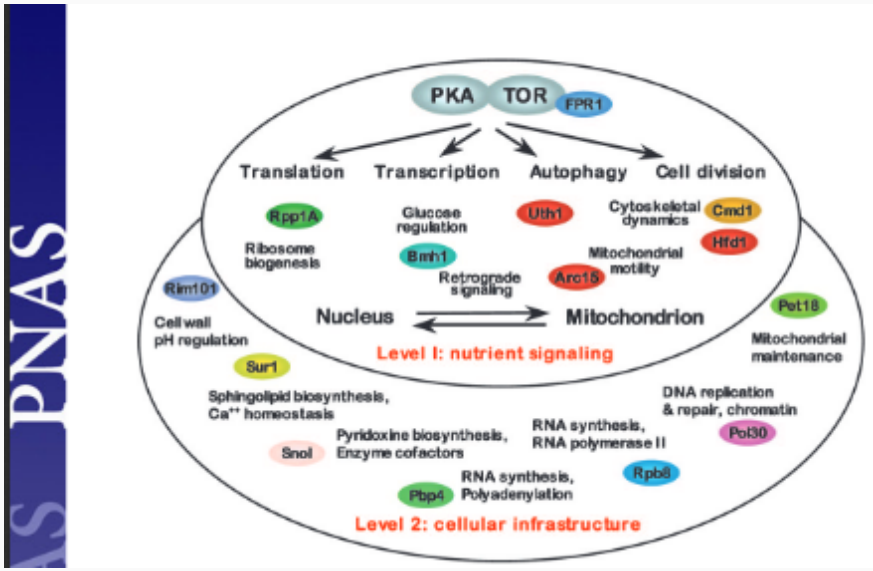
model discovery

learning to learn

Scientific discovery



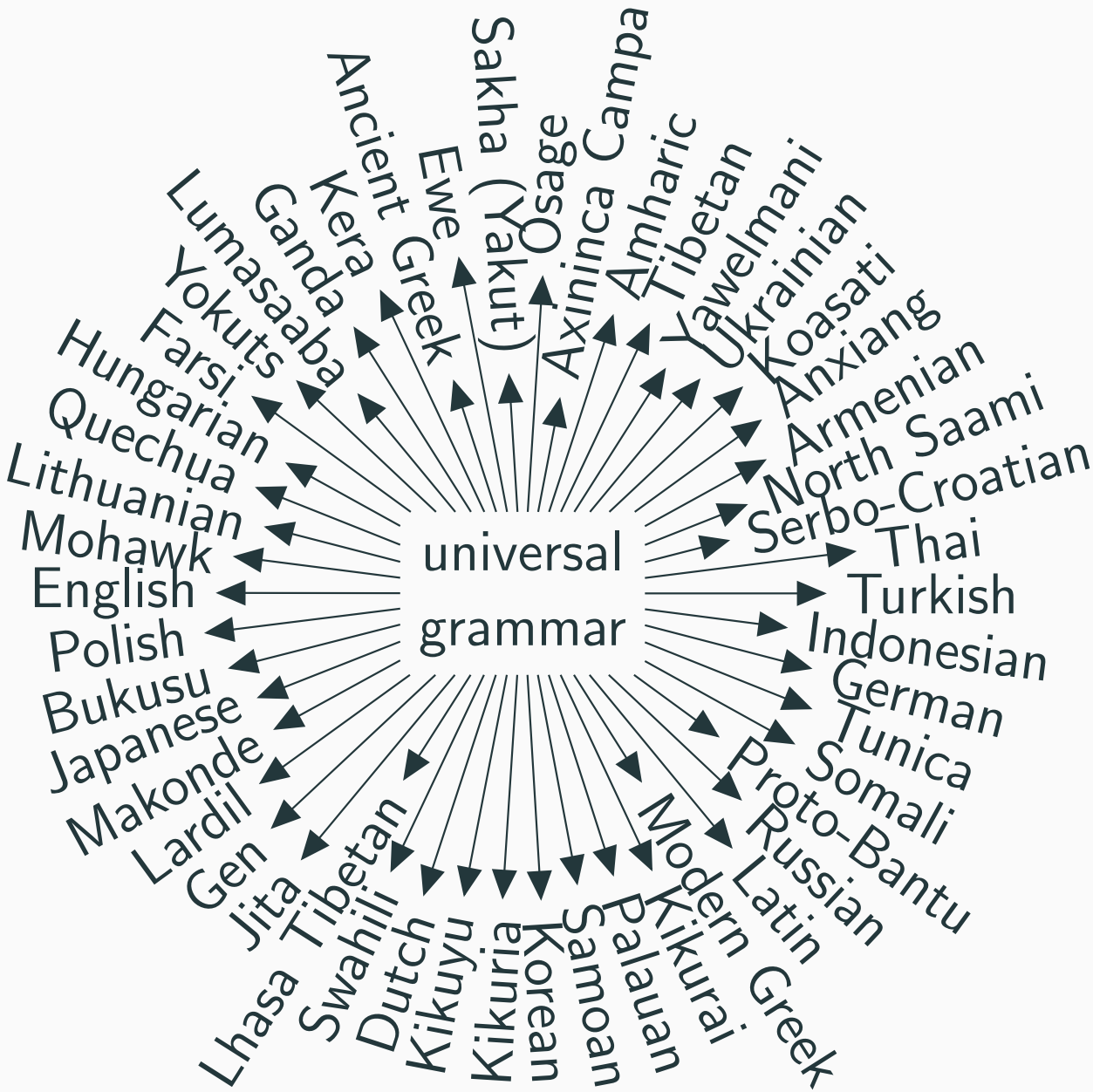
Schmidt & Lipson: "Distilling Free-Form Natural Laws from Experimental Data"



Lezon et al. 2006

inferring genetic interaction networks

Discovering human-understandable models of language



Tim O'Donnell

Few-shot language learning experiment

Mandarin:

	adjective	adverb
“slow”	man	manmandə
“small”	xiao	xiaoxiaodə
“fast”	kuai	???

Few-shot language learning experiment

Mandarin:

	adjective	adverb
“slow”	man	manmandə
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Few-shot language learning experiment

Mandarin:

	adjective	adverb
“slow”	man	manmandə
“small”	xiao	xiaoxiaodə
“fast”	kuai	kuaikuaidə

stem+stem+də

Few-shot language learning experiment

Serbo-Croatian:

	masculine	feminine
“rich”	bogat	bogata
“mild”	blag	blaga
“green”	zelen	???

Few-shot language learning experiment

Serbo-Croatian:

	masculine	feminine
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Few-shot language learning experiment

Serbo-Croatian:

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add “a” to stem to make feminine

Few-shot language learning experiment

Serbo-Croatian:

	masculine	feminine
"rich"	bogat	bogata
"mild"	blag	blaga
"green"	zelen	zelena
"clear"	???	yasna

add "a" to stem to make feminine

Few-shot language learning experiment

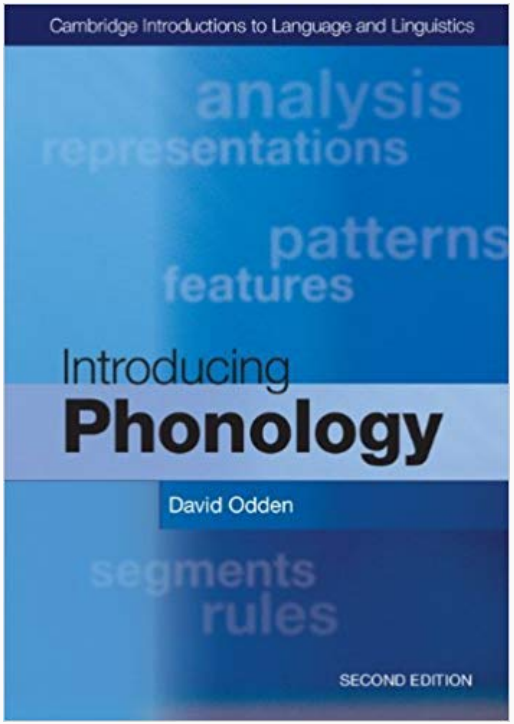
Serbo-Croatian:

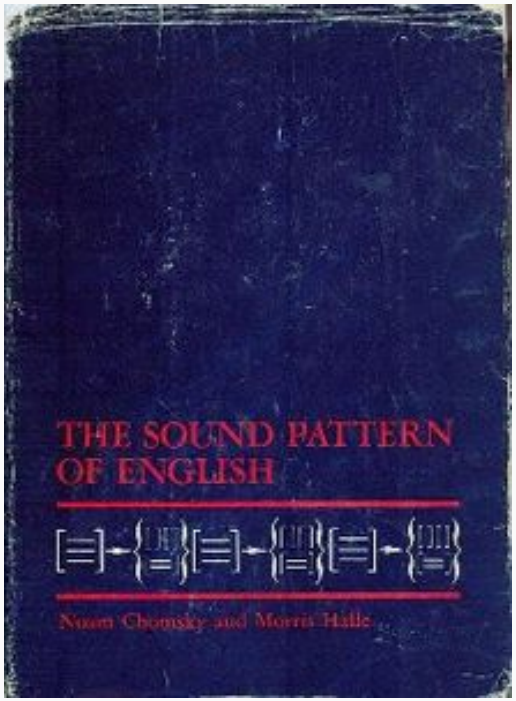
	masculine	feminine
"rich"	bogat	bogata
"mild"	blag	blaga
"green"	zelen	zelena
"clear"	**yasan**	yasna

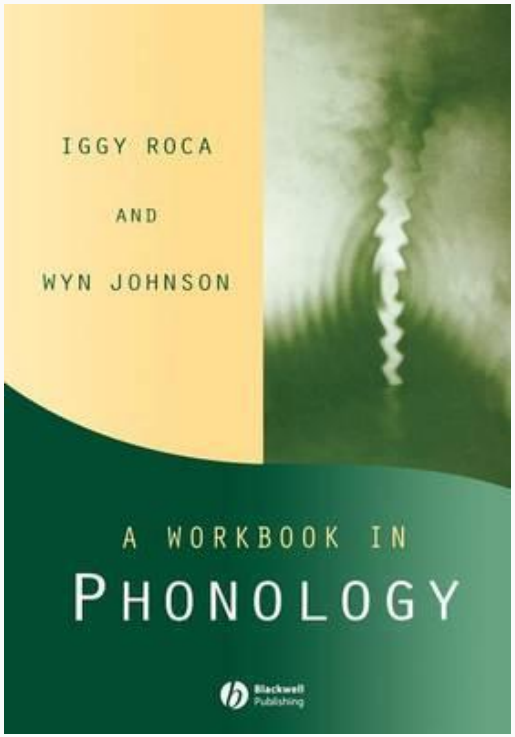
add "a" to stem to make feminine

insert "a" between two word-final consonants

$\varnothing \rightarrow$ a / C_C#


Cambridge Introductions to Language and Linguistics
analysis
representations
patterns
features
Introducing
Phonology
David Odden
segments
rules
SECOND EDITION


THE SOUND PATTERN OF ENGLISH
Noam Chomsky and Morris Halle


IGGY ROCA
AND
WYN JOHNSON
A WORKBOOK IN
PHONOLOGY
Blackwell Publishing

10 Sakha (Yakut)

Give a phonological analysis of the following case-marking paradigms of nouns in Sakha.

Noun	*Plural*	*Associative*	
aya	ayalar	ayalɨɨn	'father'
paarta	paartalar	paartalɨɨn	'school desk'
tɨa	tɨalar	tɨalɨɨn	'forest'
kinige	kinigeler	kinigeliin	'book'
ǰie	ǰieler	ǰieliin	'house'
iye	iyeler	iyeliin	'mother'
kini	kiniler	kiniliin	'3rd person'
bie	bieler	bieliin	'mare'
oyo	oyolor	oyoluun	'child'
χopto	χoptolor	χoptoluun	'gull'
börö	börölör	börölüün	'wolf'
tɨal	tɨallar	tɨalɨɨn	'wind'
ɨal	ɨallar	ɨalɨɨn	'neighbor'
kuul	kuullar	kuulluun	'sack'
at	attar	attɨɨn	'horse'
balɨk	balɨktar	balɨktɨɨn	'fish'
ɨskaap	ɨskaaptar	ɨskaaptɨɨn	'cabinet'
oyus	oyustar	oyustuun	'bull'
kus	kustar	kustuun	'duck'
tünnük	tünnükter	tünnüktüün	'window'
sep	septer	septiin	'tool'
et	etter	ettiin	'meat'
örüs	örüster	örüstüün	'river'
tiis	tiister	tiistiin	'tooth'
soroχ	soroχtor	soroχtuun	'some person'
oχ	oχtor	oχtuun	'arrow'
oloppos	oloppostor	oloppostuun	'chair'
ötöχ	ötöχtör	ötöχtüün	'abandoned farm'
ubay	ubaydar	ubaydɨɨn	'elder brother'
saray	saraydar	saraydɨɨn	'barn'
tɨy	tɨydar	tɨydɨɨn	'foal'
atɨɨr	atɨɨrdar	atɨɨrdɨɨn	'stallion'
oyuur	oyuurdar	oyuurduun	'forest'
üčügey	üčügeyder	üčügeydiin	'good person'
eǰiy	eǰiyder	eǰiydiin	'elder sister'
tomtor	tomtordor	tomtorduun	'knob'
moyotoy	moyotoydor	moyotoyduun	'chipmunk'
kötör	kötördör	kötördüün	'bird'
bölköy	bölköydör	bölköydüün	'islet'
χatɨŋ	χatɨŋnar	χatɨŋnɨɨn	'birch'
aan	aannar	aannɨɨn	'door'
tɨŋ	tɨŋner	tɨŋnɨɨn	'squirrel'
sordoŋ	sordoŋnor	sordoŋnuun	'pike'
olom	olomnor	olomnuun	'ford'
oron	oronnor	oronnuun	'bed'
bödöŋ	bödöŋnör	bödöŋnüün	'strong one'

Noun	*Partitive*	*Comparative*	*Ablative*	
aya	ayata	ayataayar	ayattan	'father'
paarta	paartata	paartataayar	paartattan	'school desk'
tɨa	tɨata	tɨataayar	tɨattan	'forest'
kinige	kinigete	kinigeteeyer	kinigetten	'book'
ǰie	ǰiete	ǰieteeyer	ǰietten	'house'
iye	iyete	iyeteeyer	iyetten	'mother'
kini	kinite	kiniteeyer	kinitten	'3rd person'
bie	biete	bieteeyer	bietten	'mare'
oyo	oyoto	oyotooyor	oyotton	'child'
χopto	χoptoto	χoptotooyor	χoptotton	'gull'
börö	börötö	börötööyör	böröttön	'wolf'
tɨal	tɨalla	tɨallaayar	tɨaltan	'wind'
ɨal	ɨalla	ɨallaayar	ɨaltan	'neighbor'
kuul	kuulla	kuullaayar	kuultan	'sack'
moχsoyol	moχsoyollo	moχsoyollooyor	moχsoyolton	'falcon'
at	atta	attaayar	attan	'horse'
balɨk	balɨkta	balɨktaayar	balɨktan	'fish'
ɨskaap	ɨskaapta	ɨskaaptaayar	ɨskaaptan	'cabinet'
oyus	oyusta	oyustaayar	oyustan	'bull'
kus	kusta	kustaayar	kustan	'duck'
tünnük	tünnükte	tünnükteeyer	tünnükten	'window'

Turkic Sakha (Yakut)

observed data

	Singular	Plural
Bed	oron	oronnor
Mare	bie	bieler
Cabinet	ɨskaap	ɨskaaptar

138 total examples

Ellis, Albright, Solar-Lezama, Tenenbaum, O'Donnell. 2022.

Turkic Sakha (Yakut)

grammar (unobserved)

SINGULAR→stem
PLURAL→stem+lar

observed data

	SINGULAR	PLURAL
BED	oron	oronnor
MARE	bie	bieler
CABINET	ɨskaap	ɨskaaptar

138 total examples

Ellis, Albright, Solar-Lezama, Tenenbaum, O'Donnell. 2022.

Turkic Sakha (Yakut)

constraint–based synthesis [Solar-Lezama 2008]

test-driven synthesis [Perelman et al. 2016]

grammar (unobserved)

SINGULAR→stem
PLURAL→stem+lar

r_1: l → d / [-lateral -tense]_
"l" becomes "d" next to "r", "t", but not "l"

r_2: C → [-voice] / [-voice]_
do not voice next to voiceless

r_3: V → [+rounded] / [+rounded] [-low]$_0$_

r_4: [+continuant -high] → [-rounded] / u C$_0$_
"harmonize" round vowels like "u", "o"

r_5: V → [-back -low] / [-back +vowel] [ ]$_0$_
"harmonize" vowels to be not at back of mouth

r_6: [-sonorant +voice] → [+nasal] / [+nasal]_
"nasalize" consonant next to a nasal, like "m"

observed data

	SINGULAR	PLURAL
BED	oron	oronnor
MARE	bie	bieler
CABINET	ɨskaap	ɨskaaptar

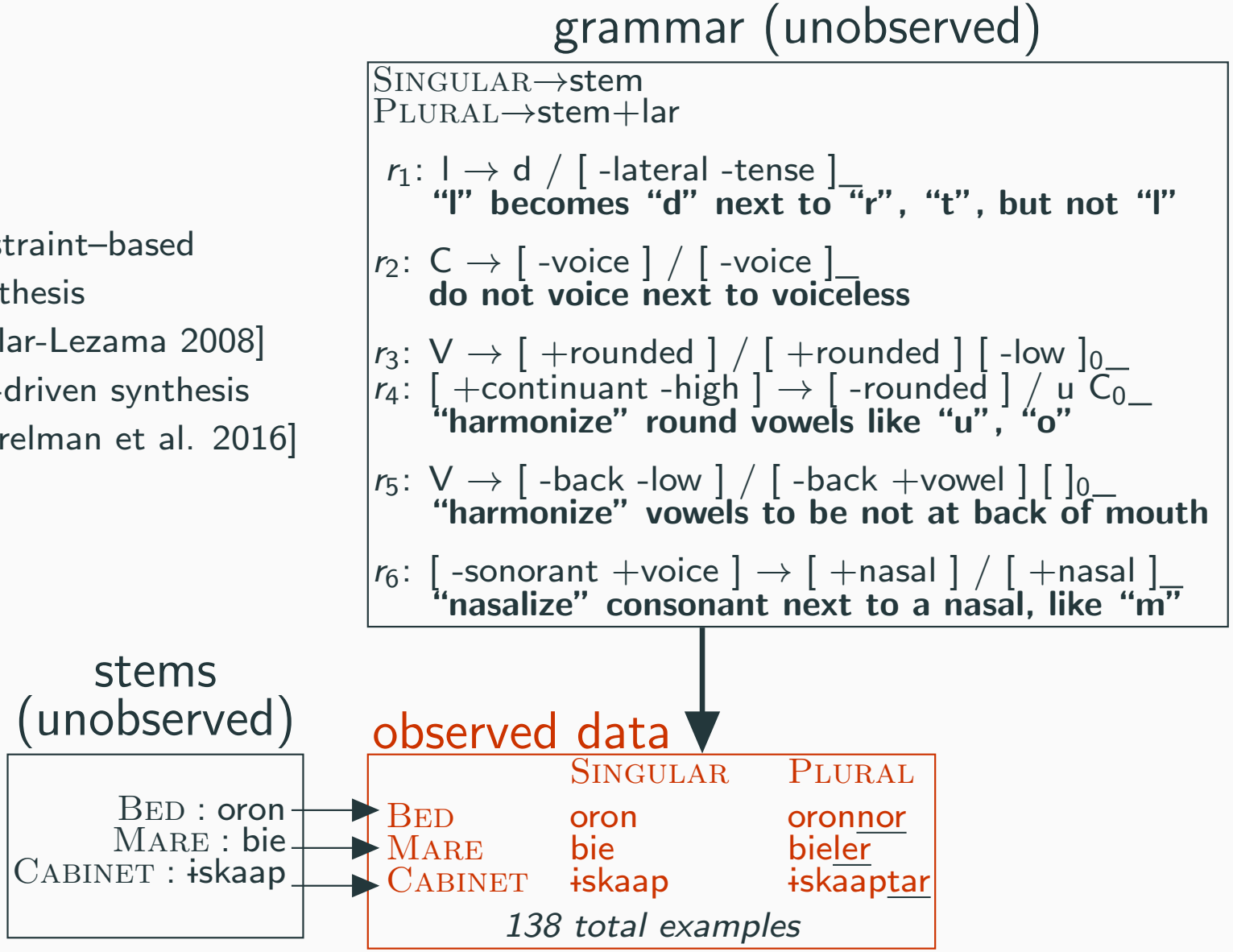
138 total examples

Ellis, Albright, Solar-Lezama, Tenenbaum, O'Donnell. 2022.

Turkic Sakha (Yakut)

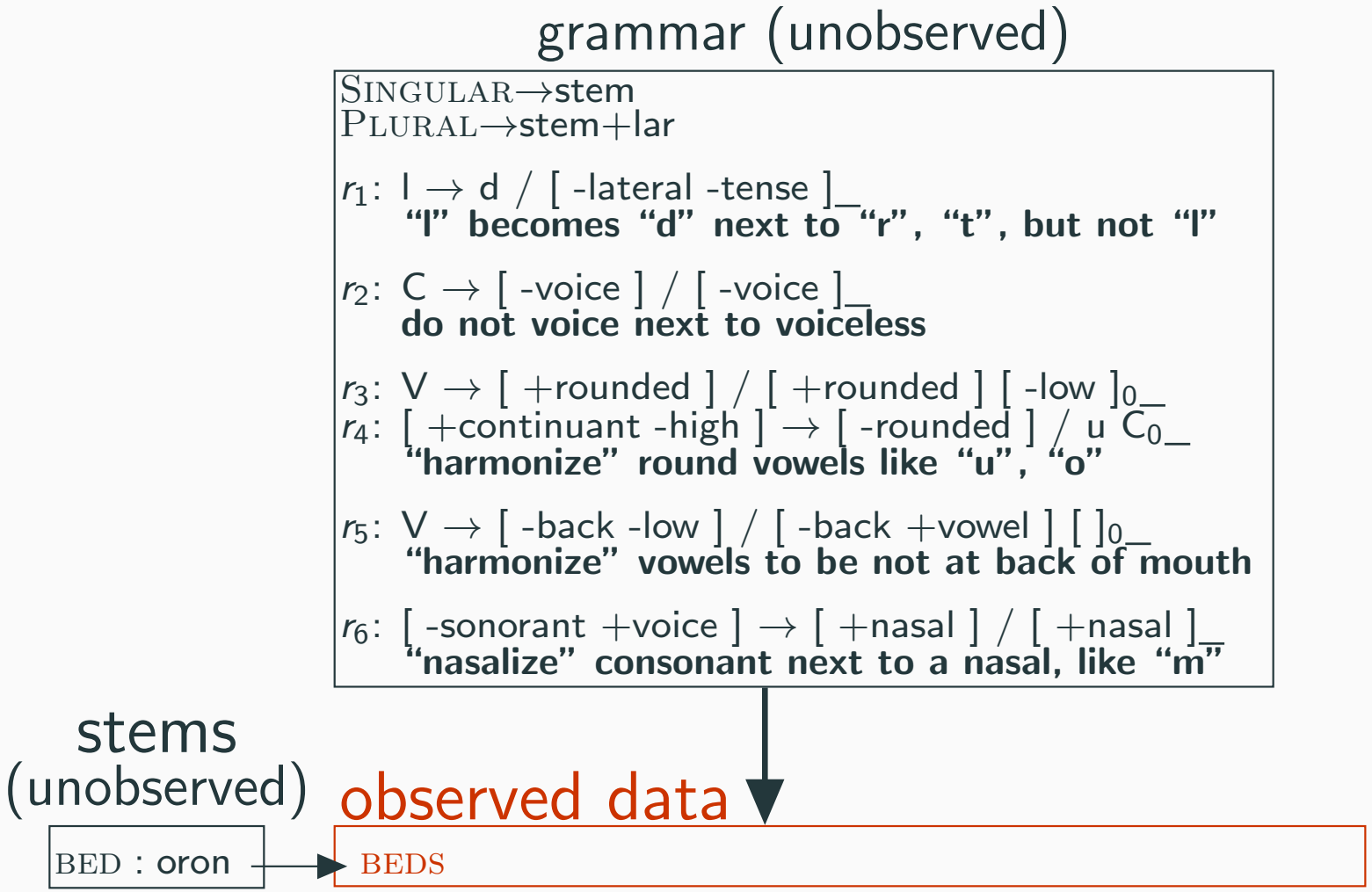
constraint–based synthesis [Solar-Lezama 2008]

test-driven synthesis [Perelman et al. 2016]



Ellis, Albright, Solar-Lezama, Tenenbaum, O'Donnell. 2022.

Turkic Sakha (Yakut)



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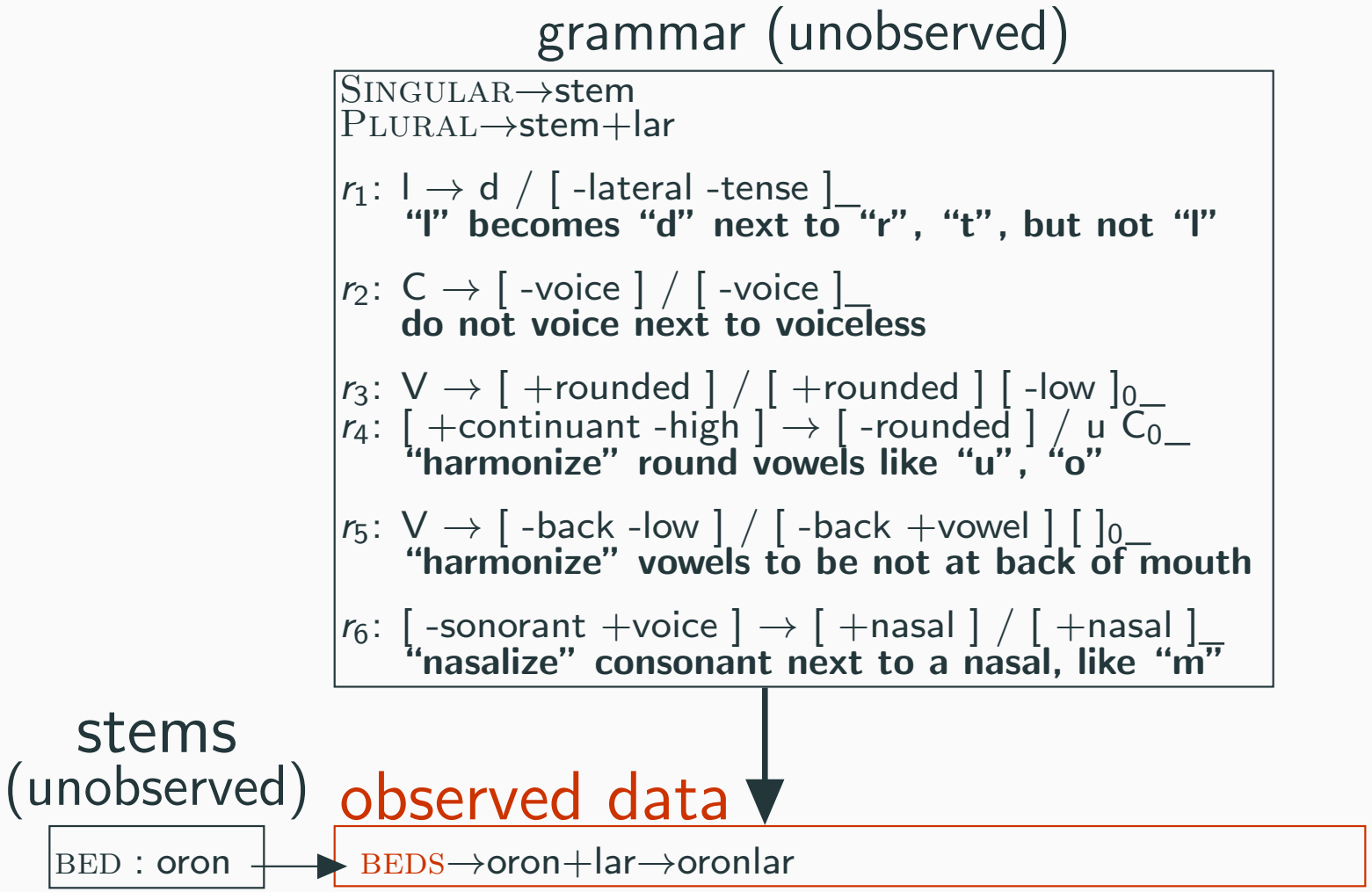

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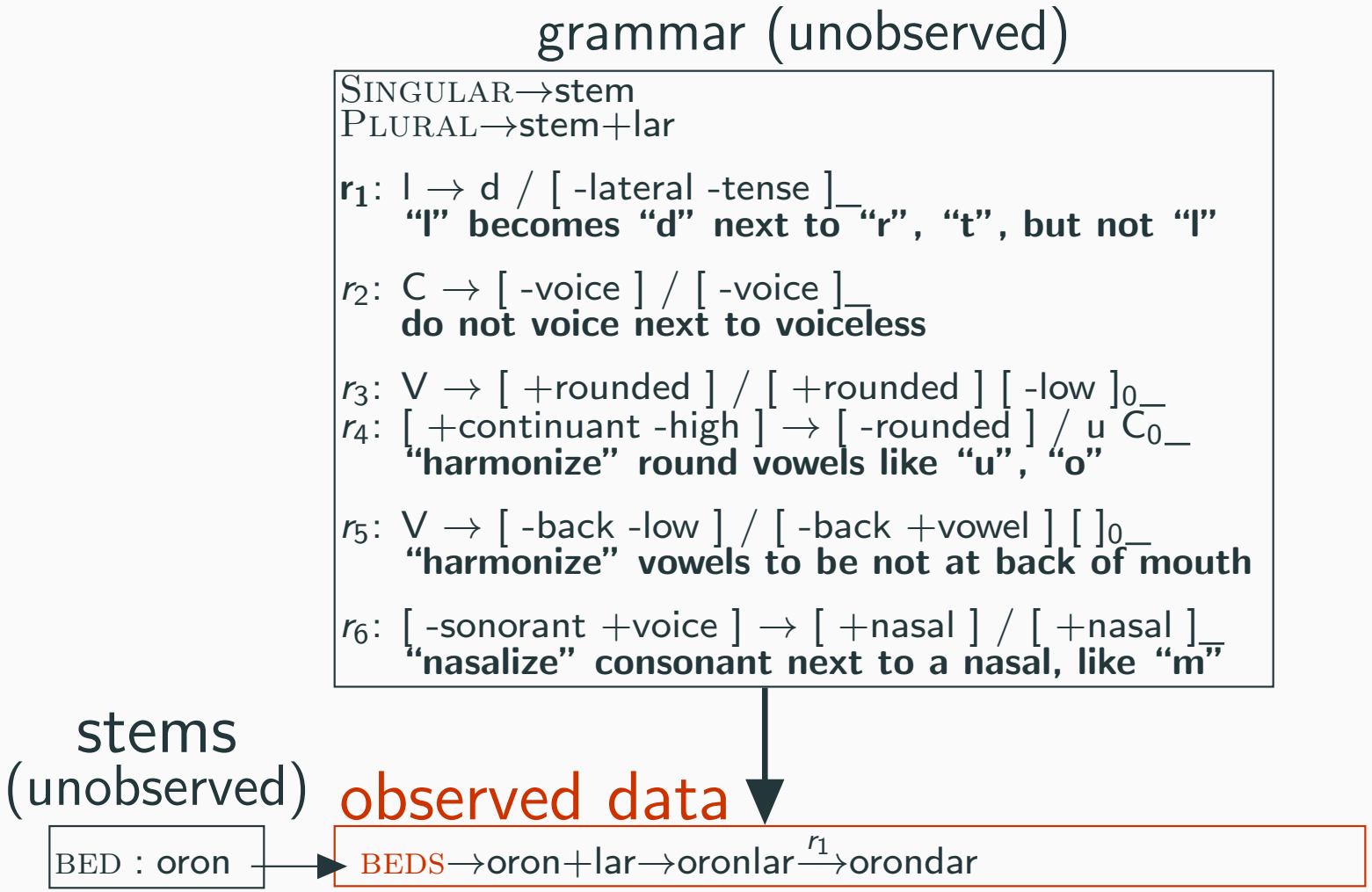

Ellis, Albright, Solar-Lezama, Tenenbaum, O'Donnell. 2022.

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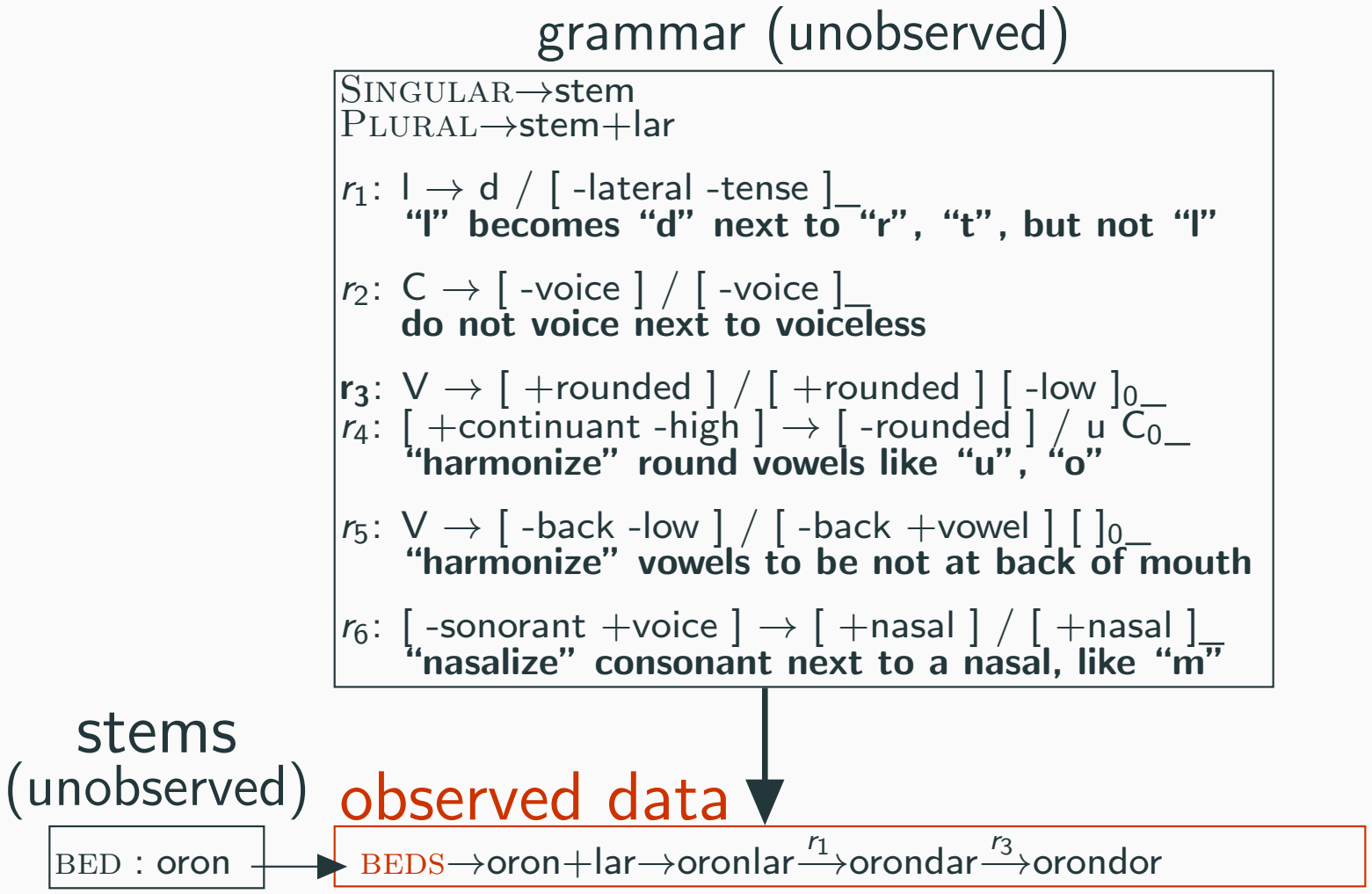

Ellis, Albright, Solar-Lezama, Tenenbaum, O'Donnell. 2022.

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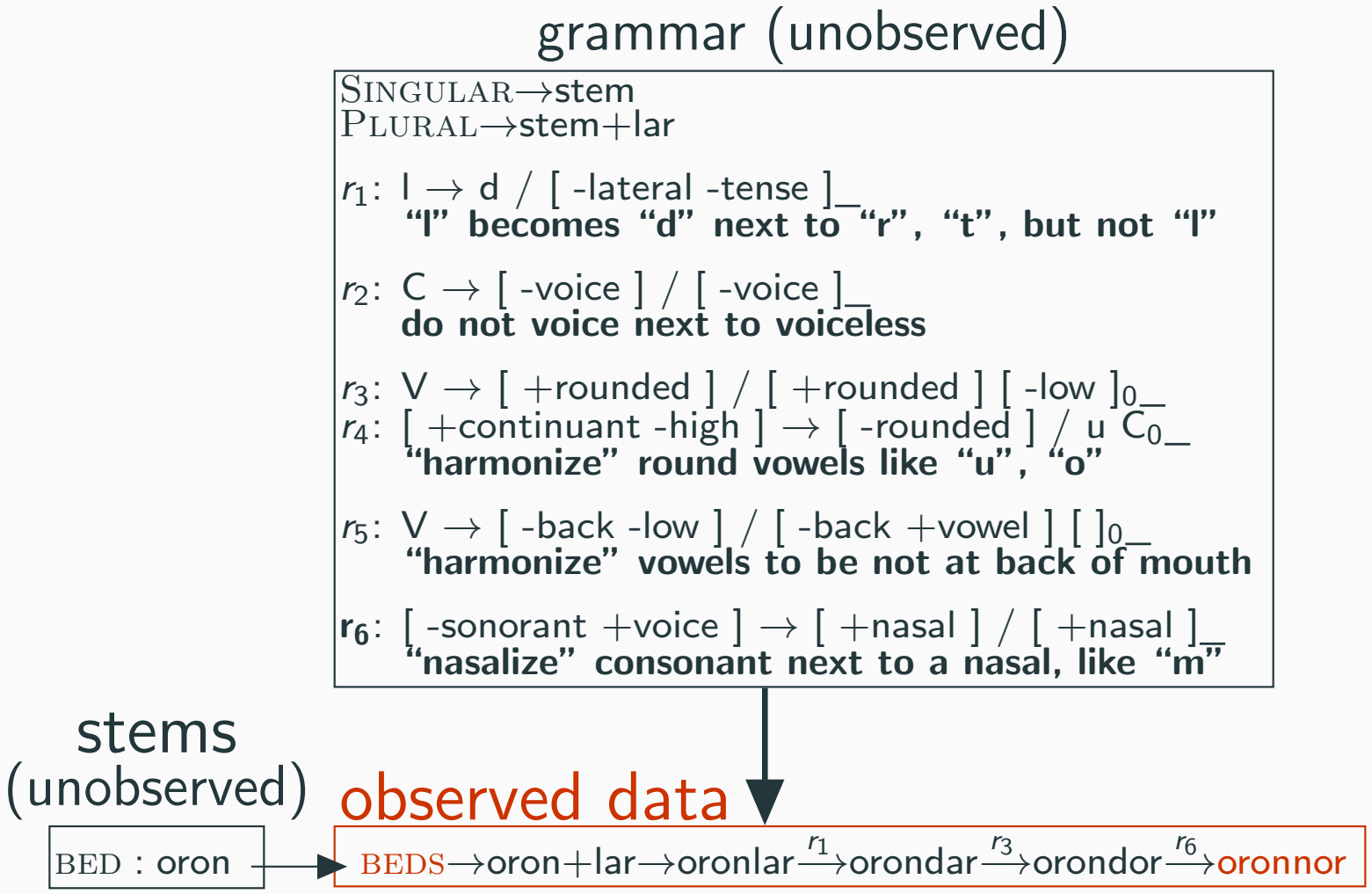

Ellis, Albright, Solar-Lezama, Tenenbaum, O'Donnell. 2022.

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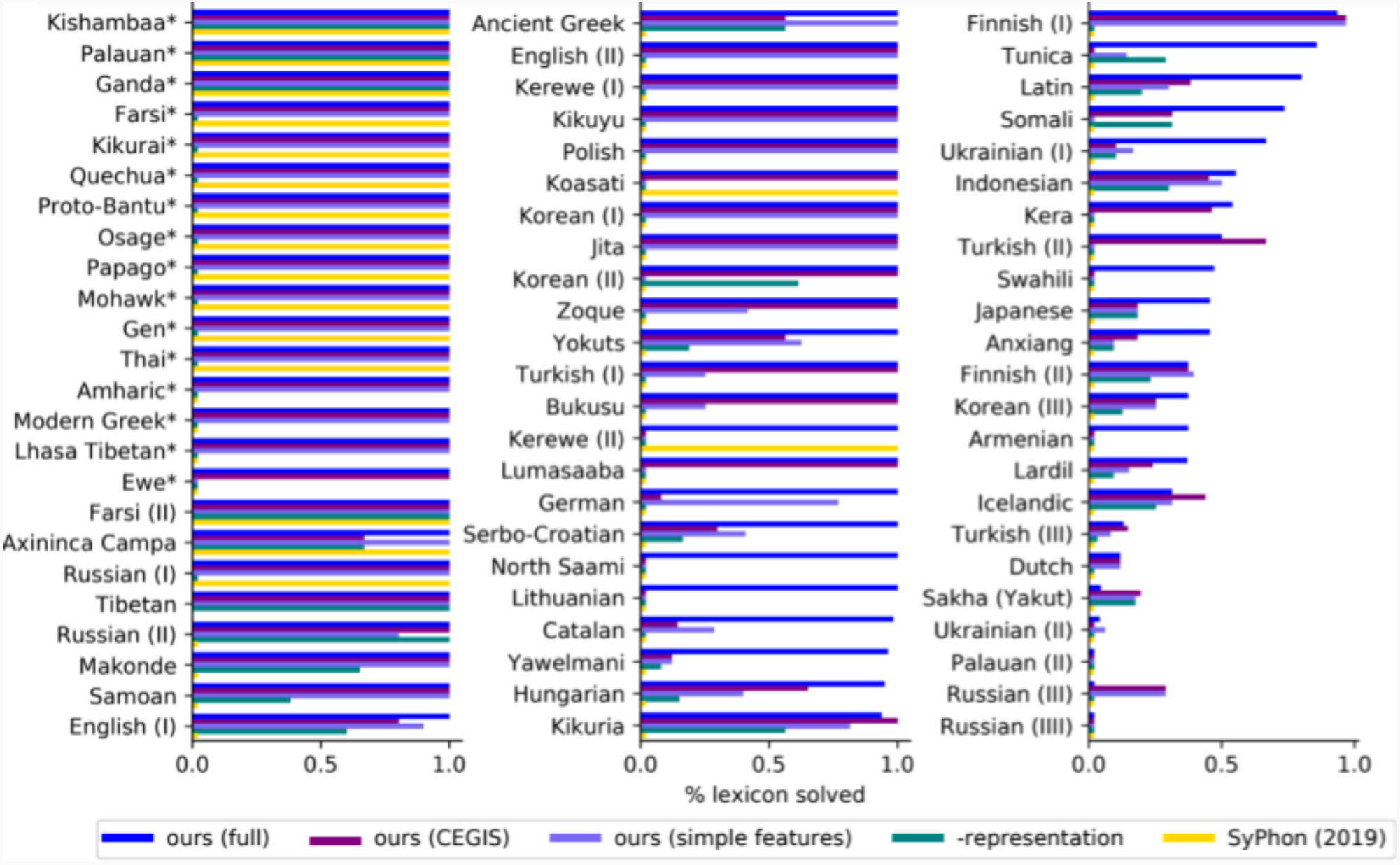
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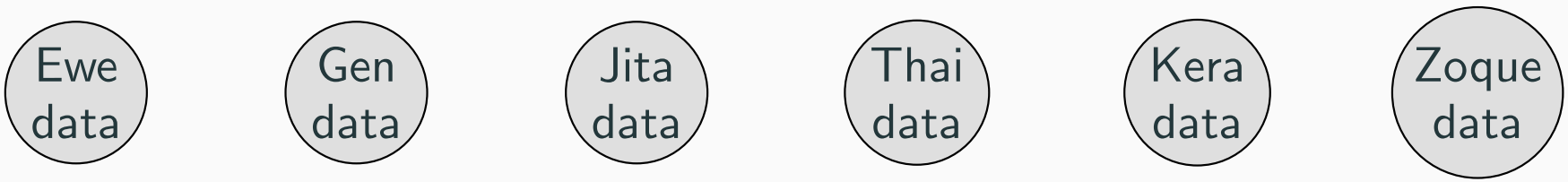
SyPhon (2019)

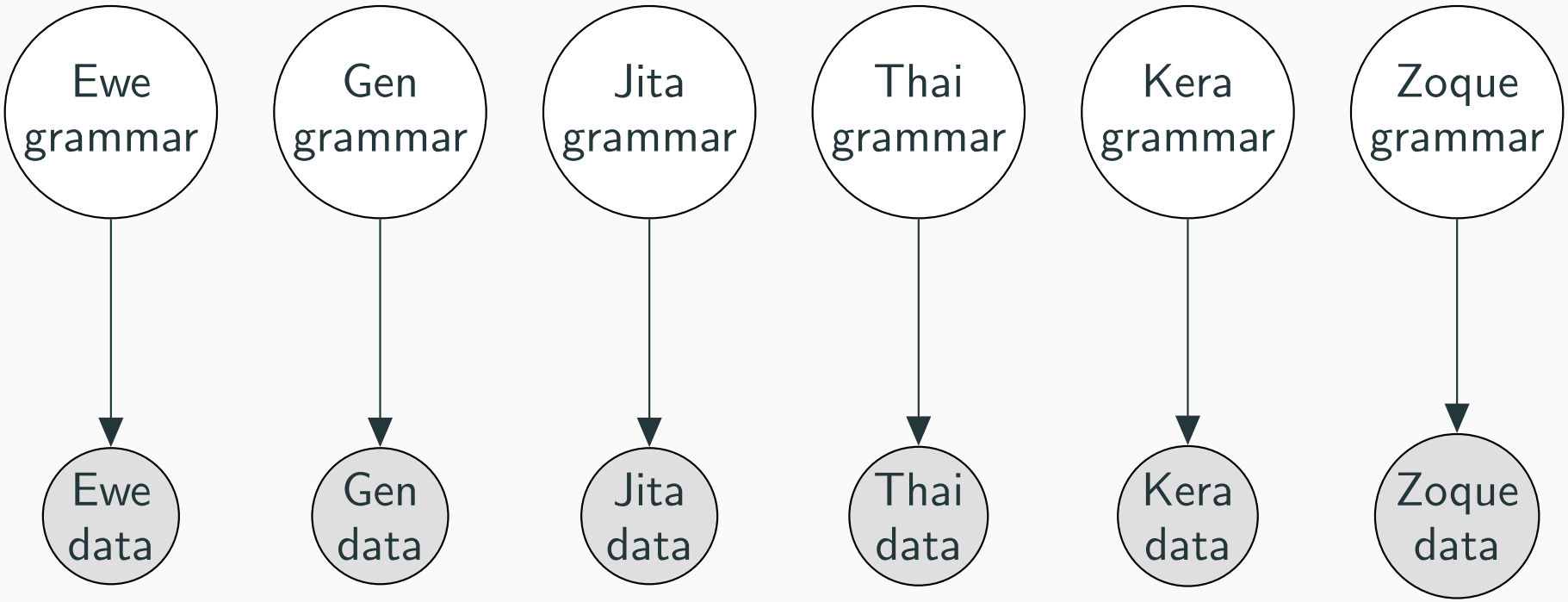
Distilling higher-level knowledge



Distilling higher-level knowledge

dark: we know it

light: we wanna know it



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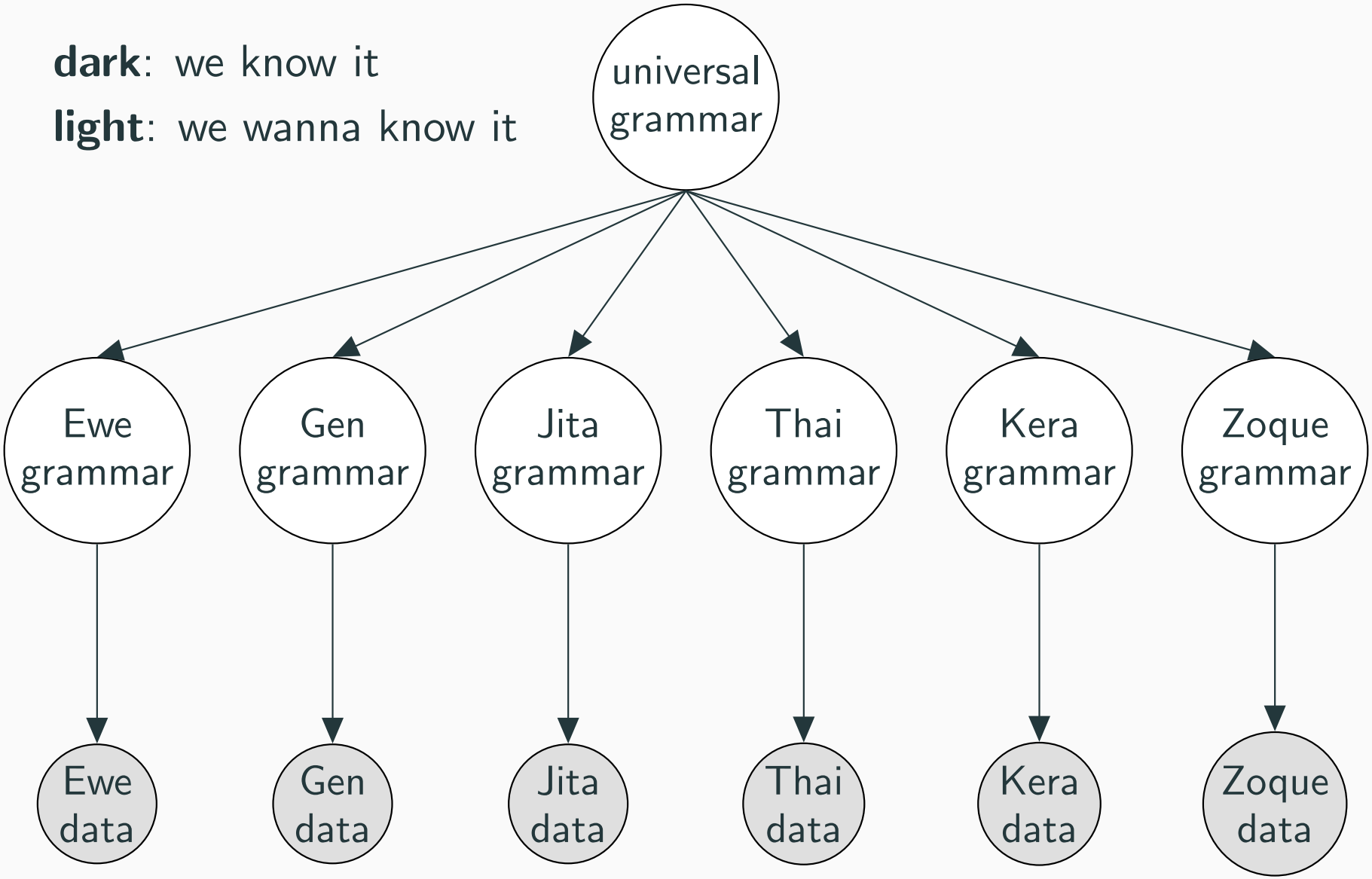

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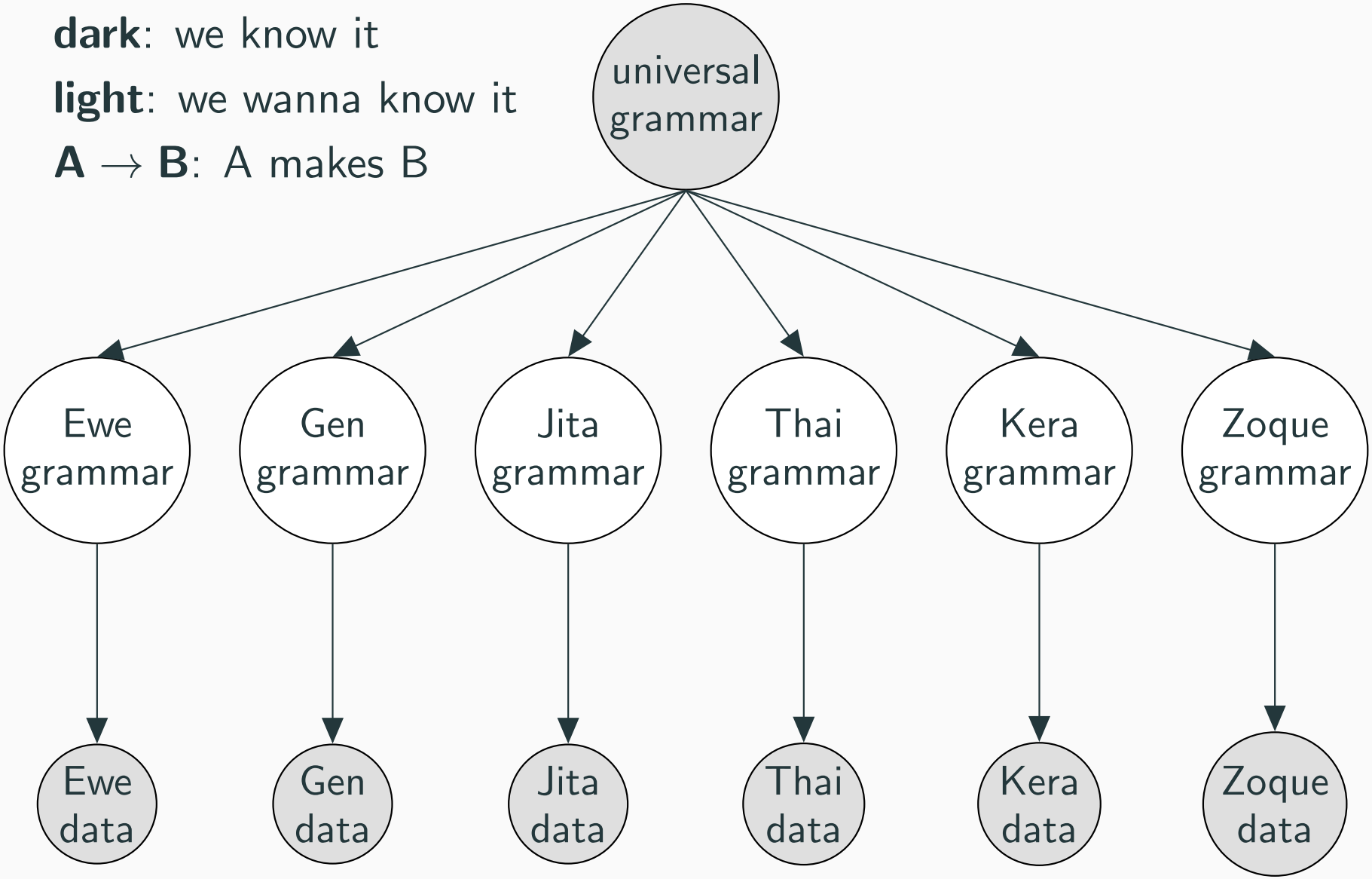
A $\rightarrow$ **B**: A makes B



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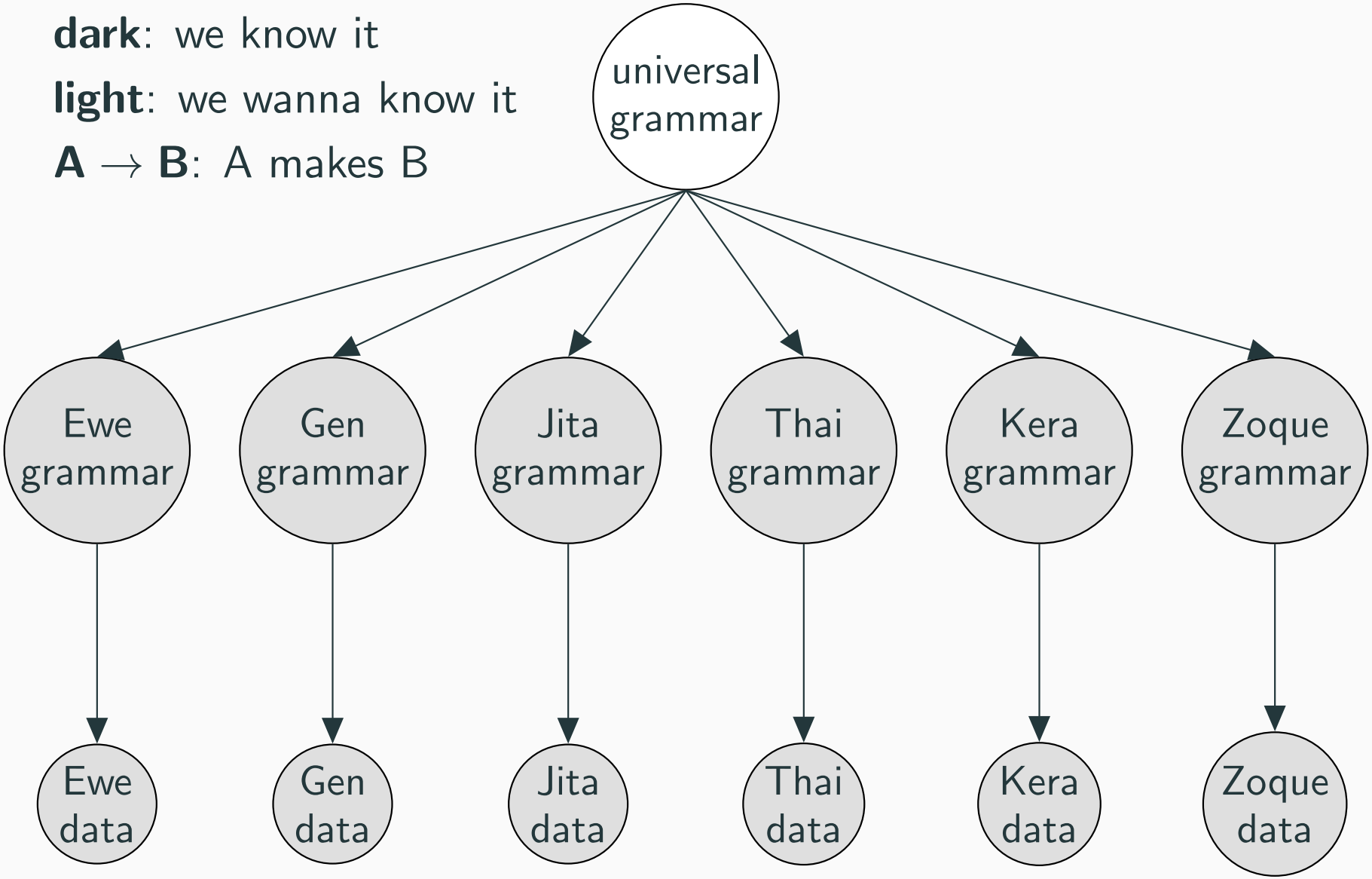

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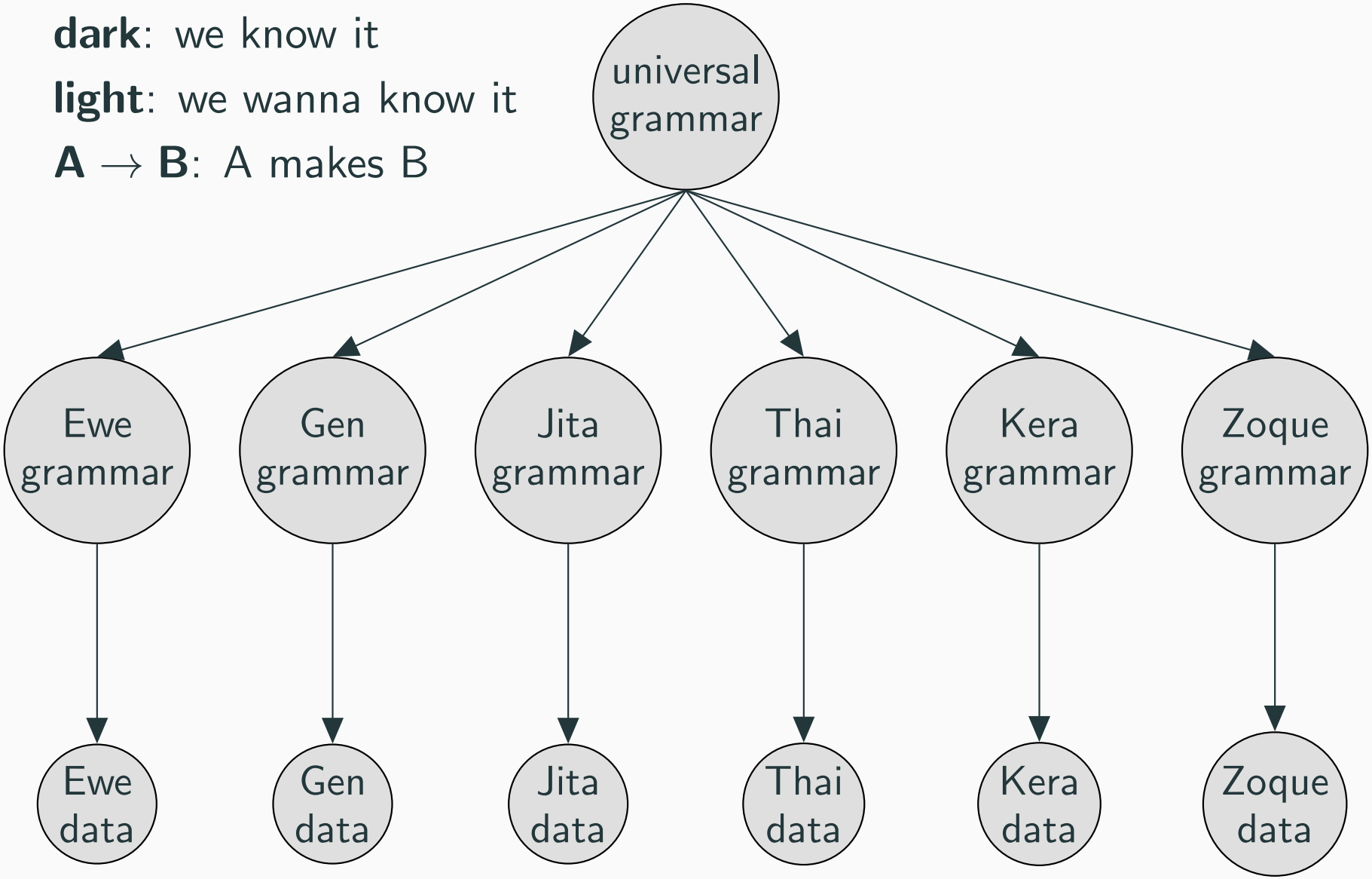

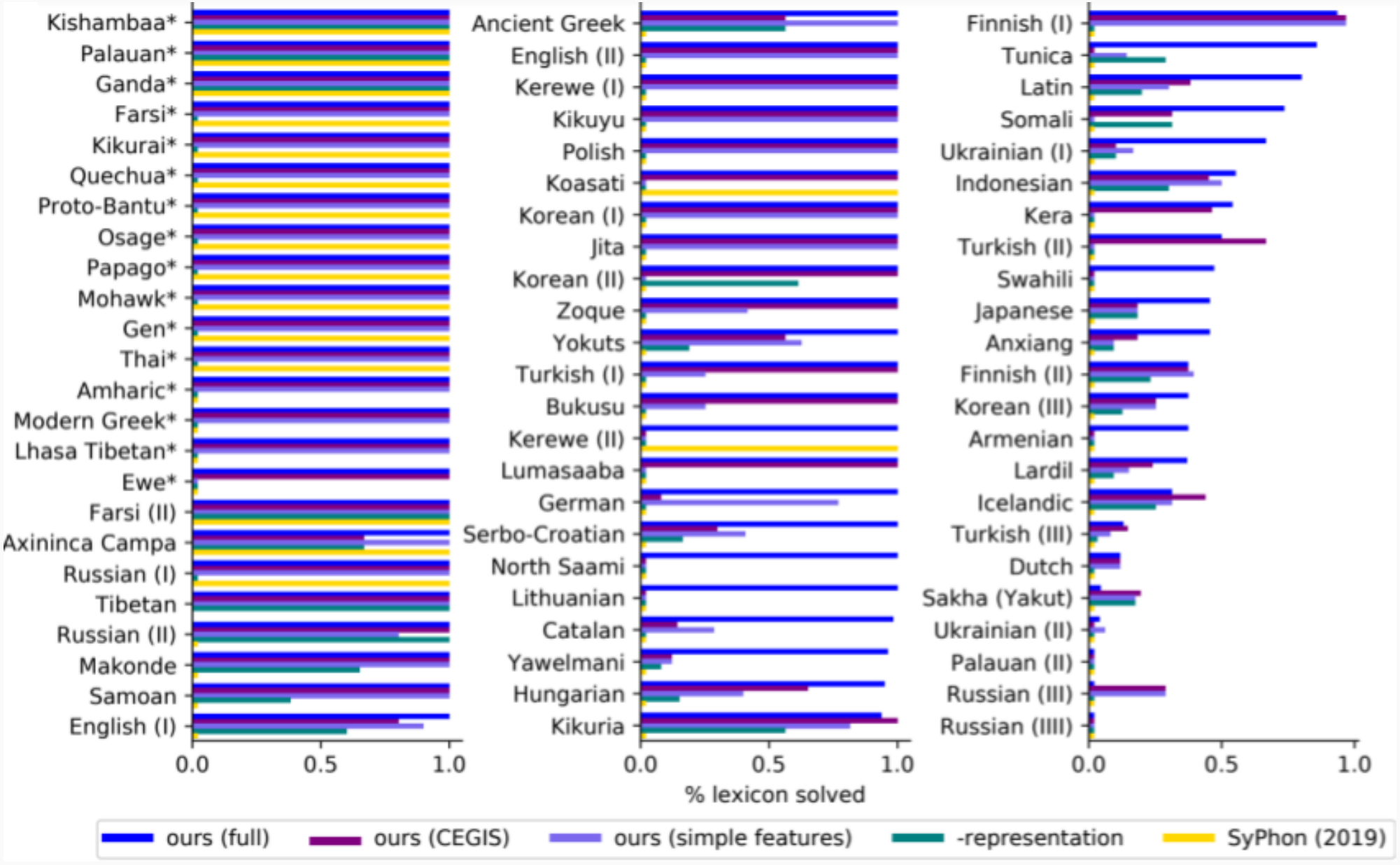
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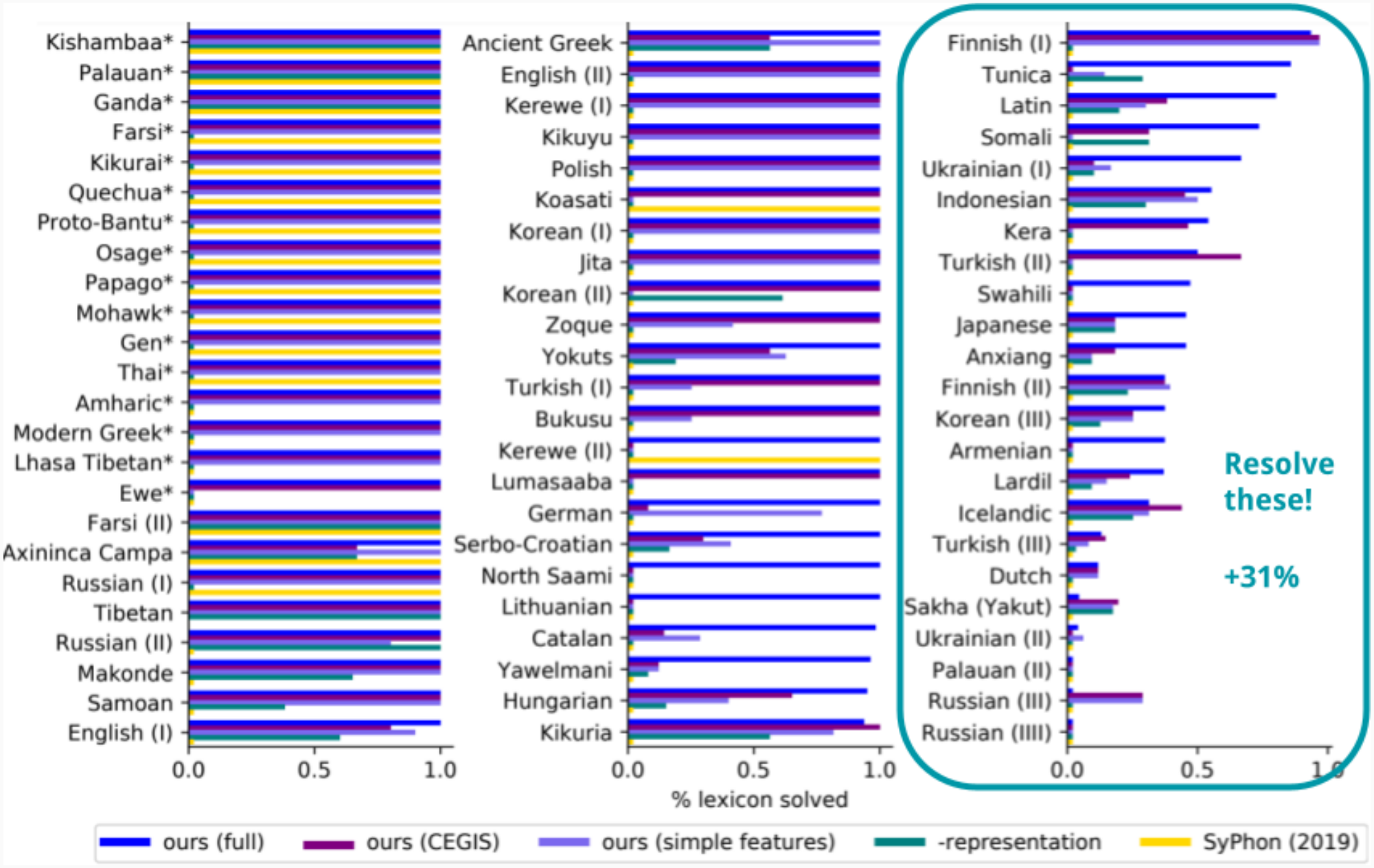
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Resolve these!
+31%
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Lessons

Higher-level knowledge matters ("universal grammar"). Get the basics of the representation correct

But *some* of this higher-level knowledge can be learned. You don't need millions of examples to learn it. But it's not a one-shot learning problem either

Program Induction and perception

model discovery

learning to learn

Learning to write code

Goal: acquire domain-specific knowledge needed to induce a class of programs

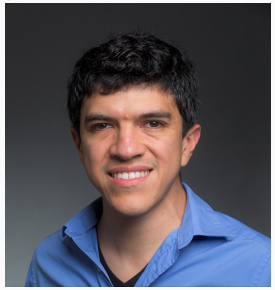
- Library of concepts (declarative knowledge; domain specific language)
- Inference strategy (procedural knowledge; synthesis algorithm)

Cathy Wong

Max Nye

Mathias Sable-Meyer

Armando Solar-Lezama



Josh Tenenbaum

Learning to write code

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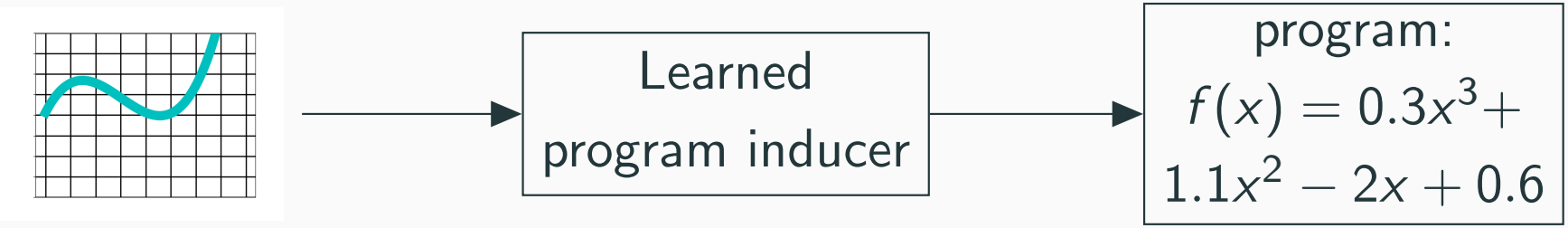

Concepts: x^3, $\alpha x + \beta$, etc

Inference strategy: neurosymbolic search for programs

Library learning

Initial Primitives

```
  :
  .
map
fold
 if
cons
  >
  .
  :
  .
```

Sample Problem: `sort list`

[9 2 7 1] → [1 2 7 9]
[3 8 9 4 2] → [2 3 4 8 9]
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...

Ellis, Morales, Sable-Meyer, Solar-Lezama, Tenenbaum. NeurIPS 2018.

Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. PLDI 2021.

Library learning

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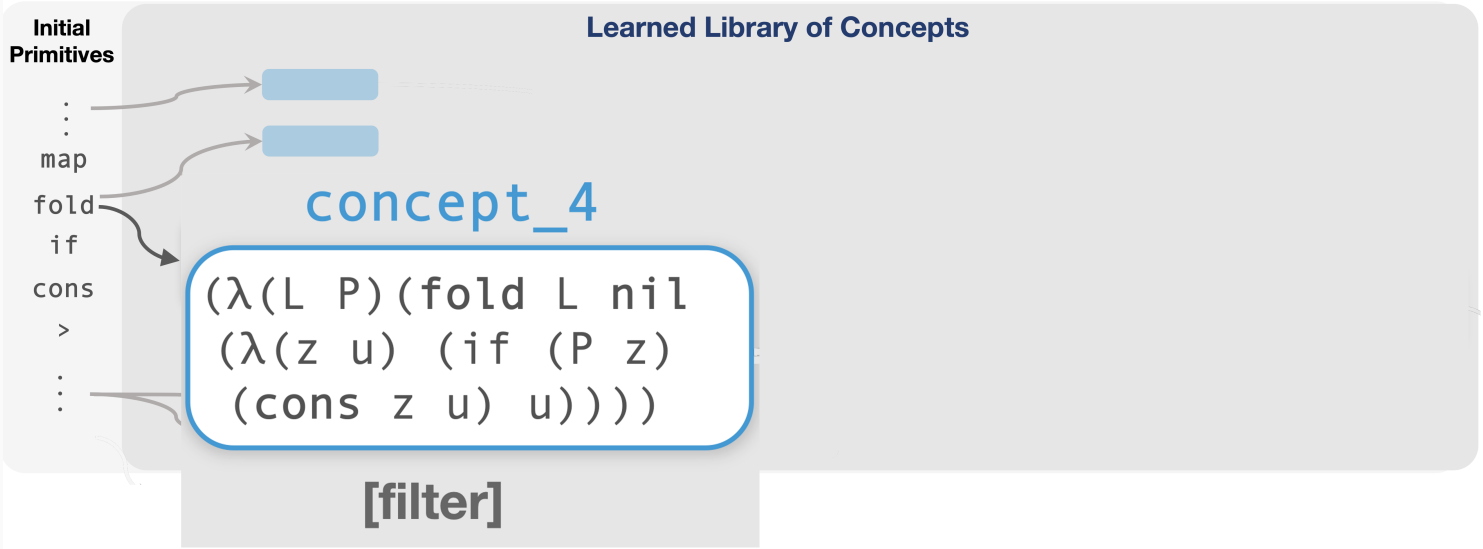

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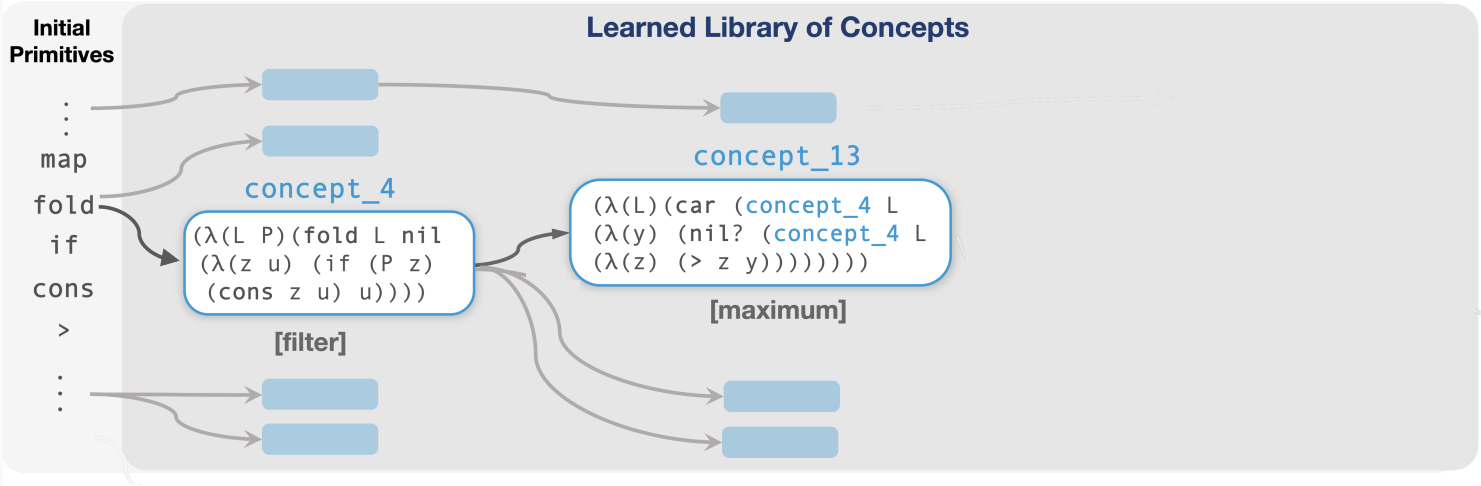

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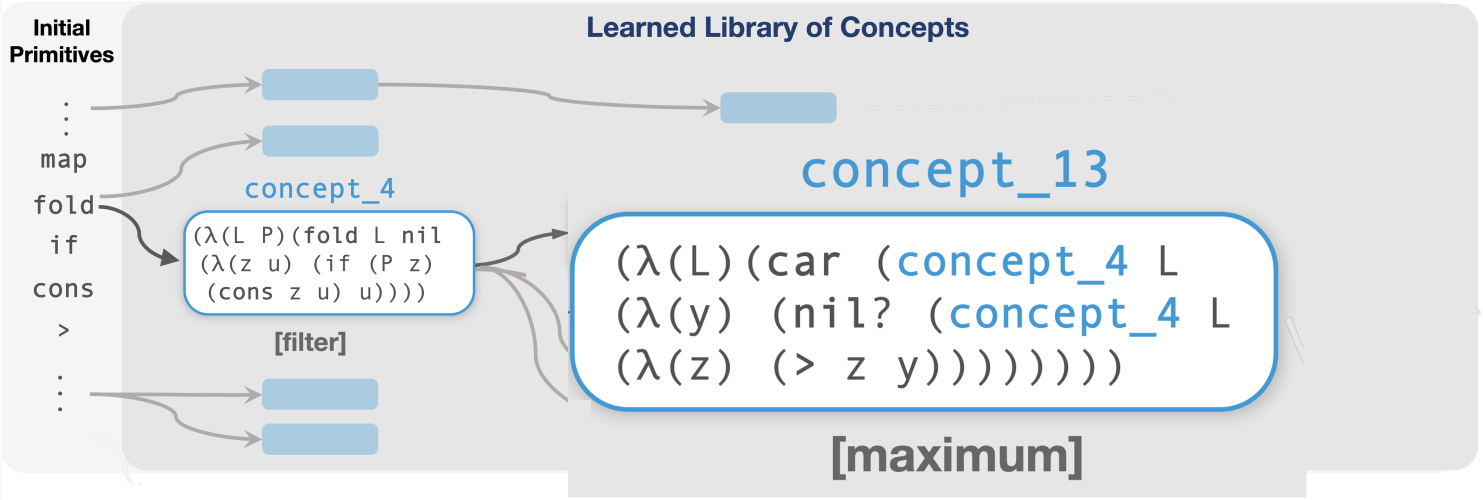

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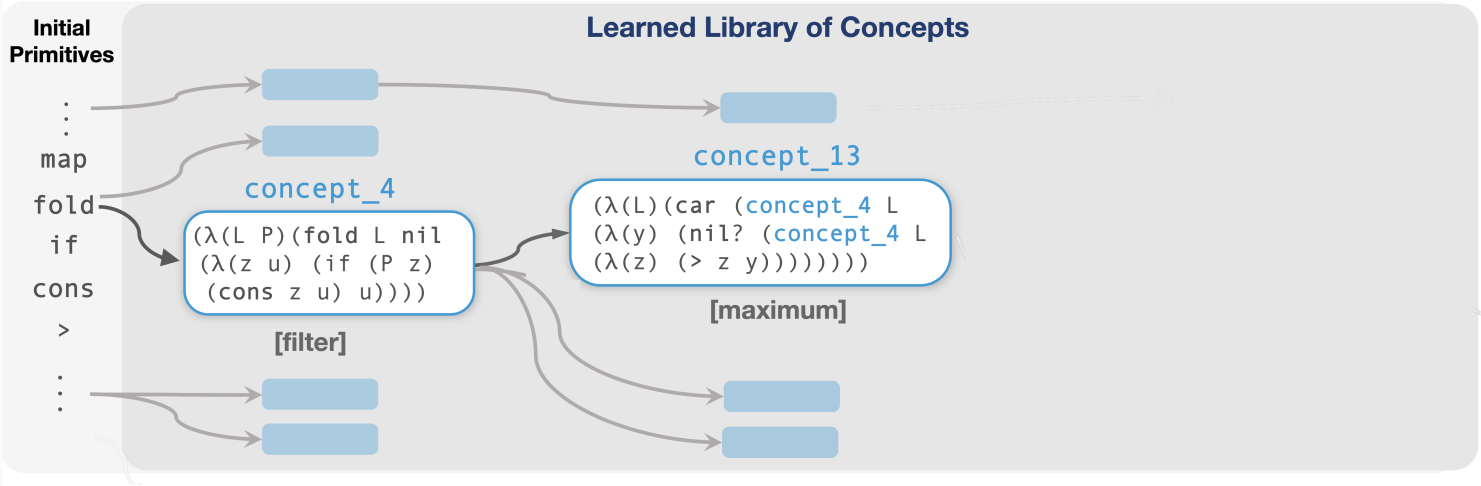

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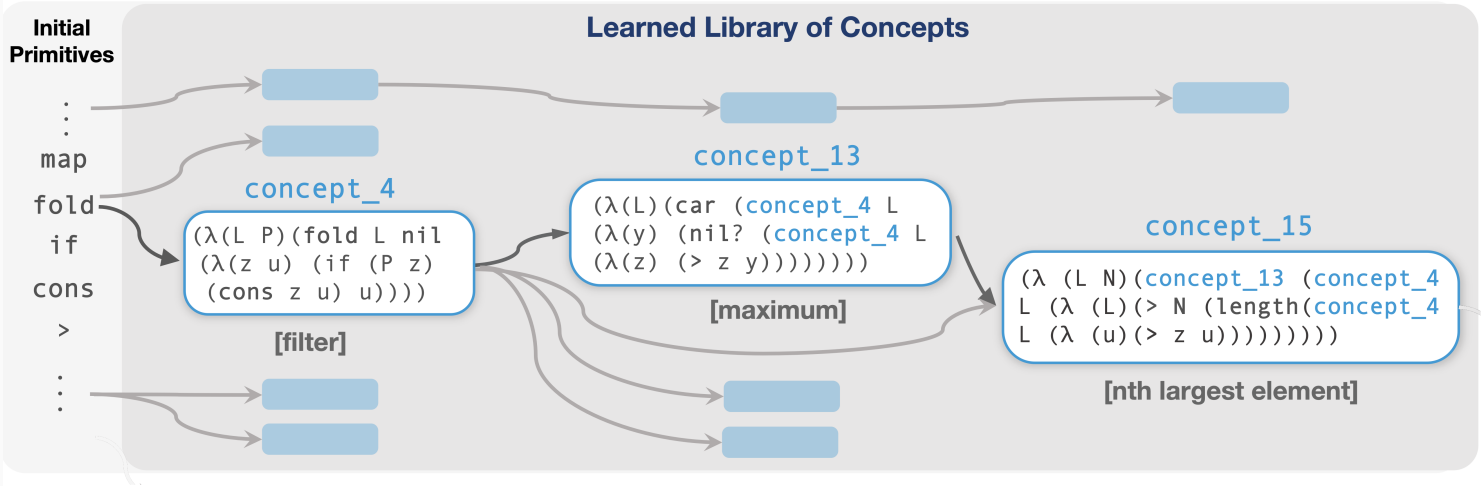

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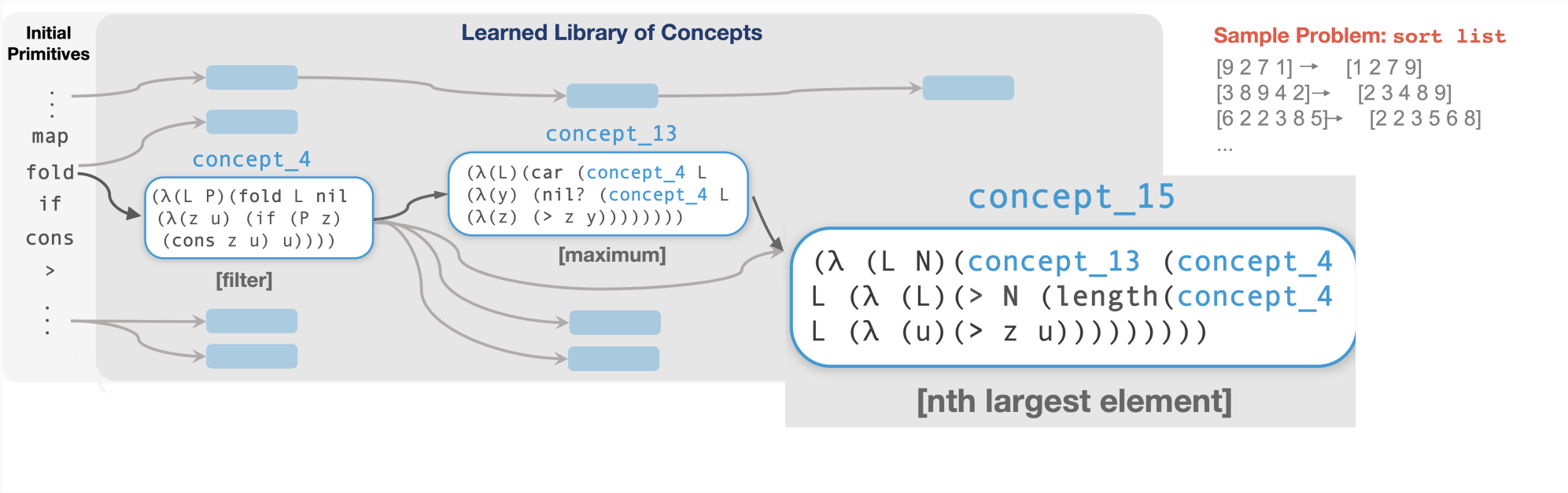

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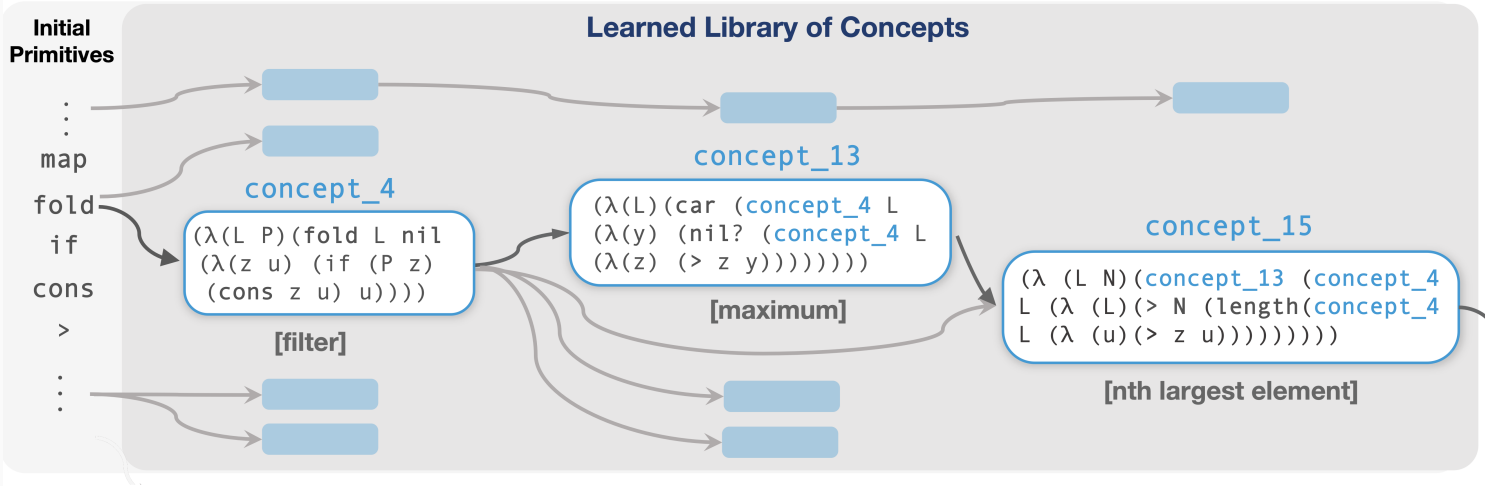

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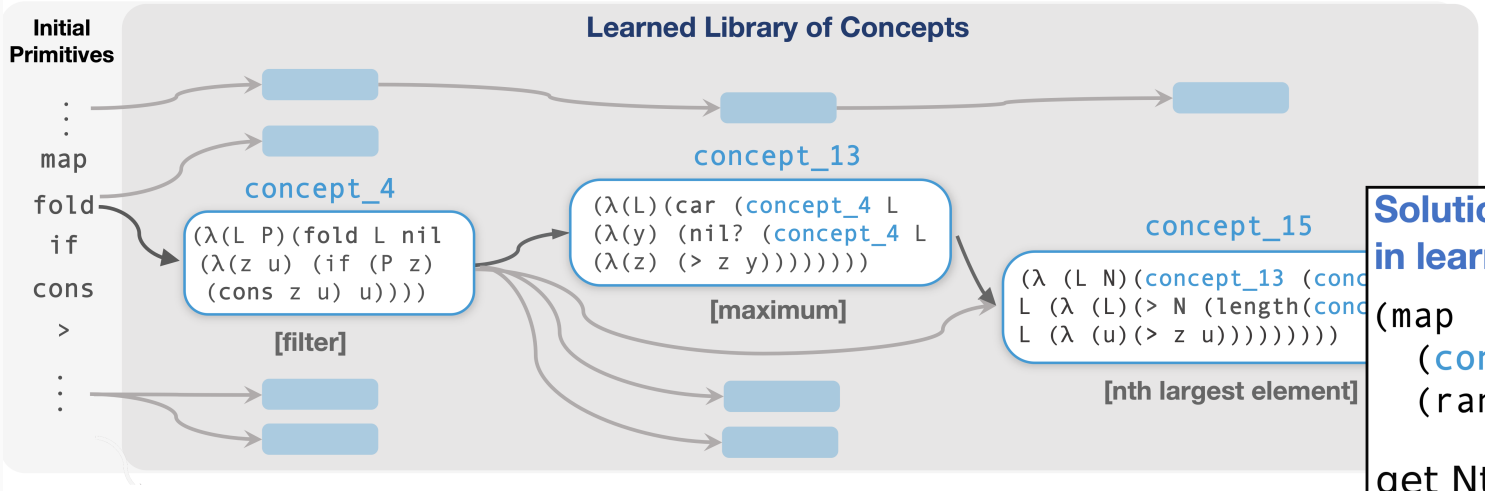
Solution to `sort list` discovered in learned language:

```
(map  (λ (n)
  (concept_15 L (+ 1 n)))
  (range (length L)))
```

Library learning



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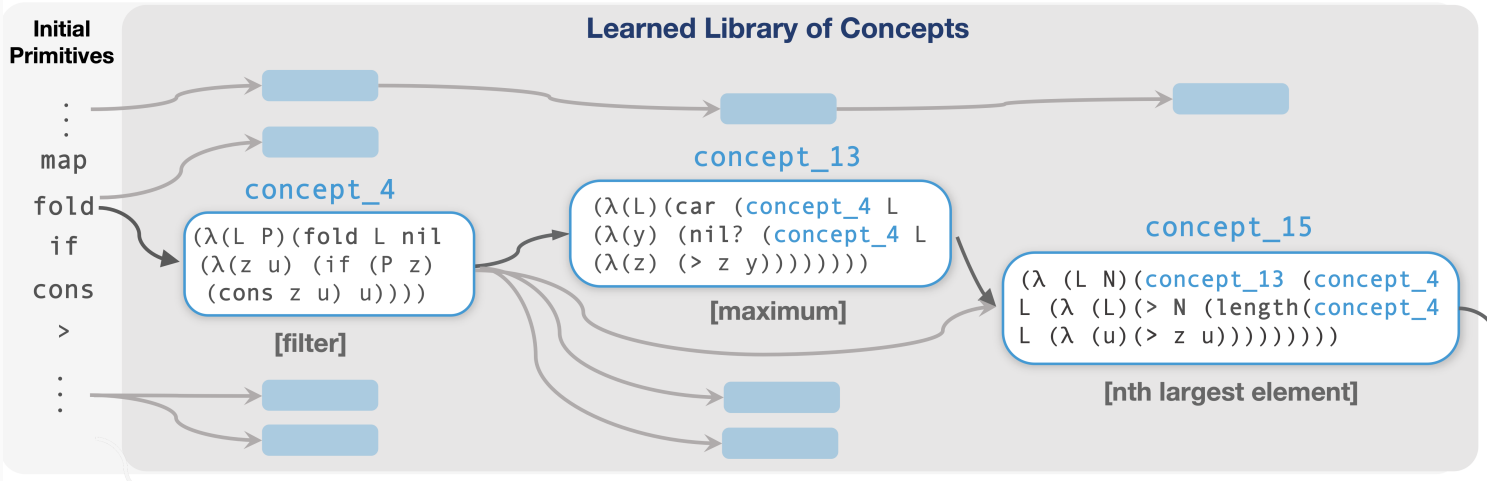
get Nth largest element, where N is 1, 2, 3, ...

Library learning



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```

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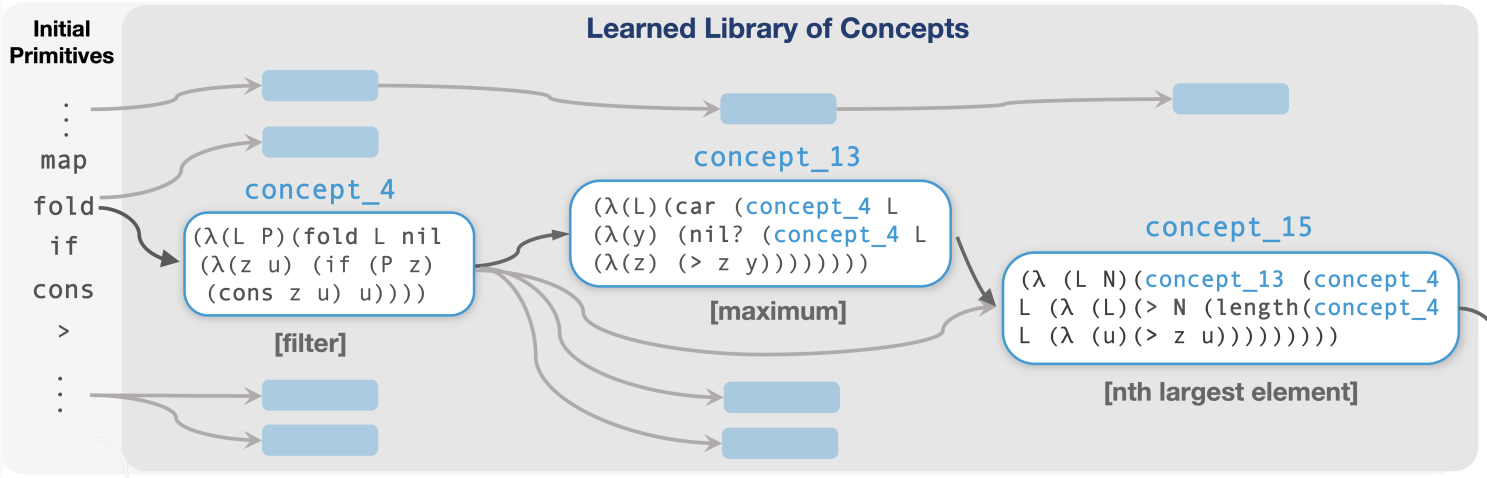
Solution rewritten in initial primitives:

```
(lambda (x) (map (lambda (y) (car (fold (fold x nil (lambda (z u) (if (gt?  (+ y 1) (length
(fold x nil (lambda (v w) (if (gt?  z v) (cons v w) w))))) (cons z u) u))) nil (lambda (a b) (if
(nil?  (fold (fold x nil (lambda (c d) (if (gt?  (+ y 1) (length (fold x nil (lambda (e f) (if
(gt?  c e) (cons e f) f))))) (cons c d) d))) nil (lambda (g h) (if (gt?  g a) (cons g h) h))))
(cons a b) b)))) (range (length x))))
```

Library learning



Sample Problem: `sort list`

[9 2 7 1] → [1 2 7 9]
[3 8 9 4 2] → [2 3 4 8 9]
[6 2 2 3 8 5] → [2 2 3 5 6 8]
...

Solution to `sort list` discovered in learned language:

```
(map  (λ (n)
  (concept_15 L (+ 1 n)))
  (range (length L)))
```

get Nth largest element, where N is 1, 2, 3, ...

Solution rewritten in initial primitives:

```
(lambda (x) (map (lambda (y) (car (fold (fold x nil (lambda (z u) (if (gt?  (+ y 1) (length
(fold x nil (lambda (v w) (if (gt?  z v) (cons v w) w))))) (cons z u) u))) nil (lambda (a b) (if
(nil?  (fold (fold x nil (lambda (c d) (if (gt?  (+ y 1) (length (fold x nil (lambda (e f) (if
(gt?  c e) (cons e f) f))))) (cons c d) d))) nil (lambda (g h) (if (gt?  g a) (cons g h) h))))
(cons a b) b)))) (range (length x))))
```

induced sort program found in $\leq$ 10min. Brute-force search without learned library would take $\approx 10^{73}$ years

DreamCoder

- **Wake:** Solve problems by writing programs
- **Sleep:** Improve library and neural recognition model:
  - **Abstraction sleep:** Improve library
  - **Dream sleep:** Improve neural recognition model

cf. Helmholtz machine, wake/sleep neural network training algorithms

DreamCoder

- **Wake:** Solve problems by writing programs
- **Sleep:** Improve library and neural recognition model:
  - **Abstraction sleep:** Improve library
  - **Dream sleep:** Improve neural recognition model

List Processing

Sum List

```
[1 2 3] → 6
[4 6 8 1] → 17
```

Double

```
[1 2 3] → [2 4 6]
[4 5 1] → [8 10 2]
```

Text Editing

Abbreviate

```
Allen Newell → A.N.
Herb Simon → H.S.
```

Drop Last Three

```
shrdlu → shr
shakey → sha
```

Regexes

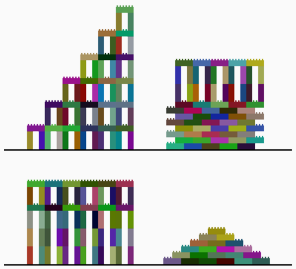
Phone numbers

```
(555) 867-5309
(650) 555-2368
```

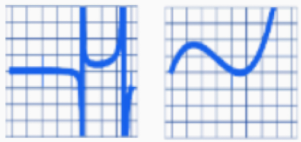
Currency

```
$100.25
$4.50
```

Block Towers



Symbolic Regression



$y = f(x)$

Recursive Programming

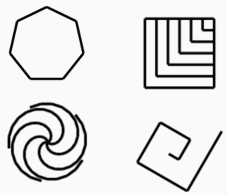
Filter Red

Physical Laws

$$\vec{a} = \frac{1}{m}\sum_i \vec{F}_i$$

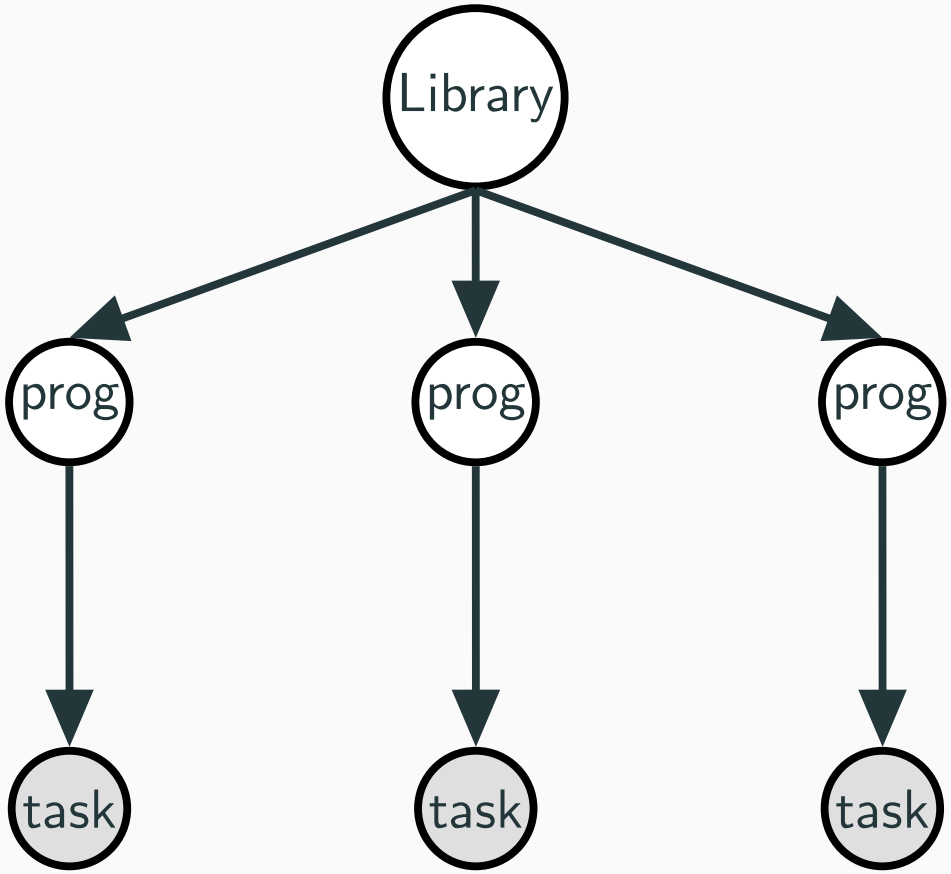
$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r}|^2}\hat{r}$$

LOGO Graphics



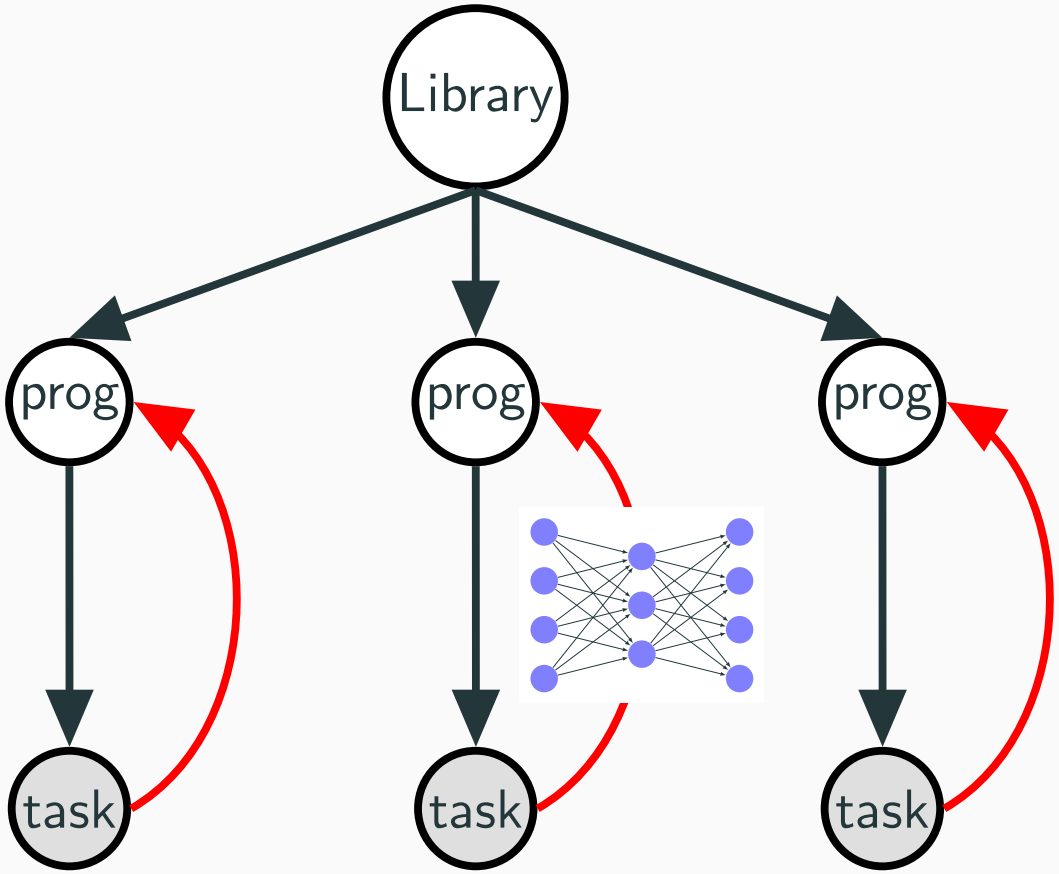
cf. Helmholtz machine, wake/sleep neural network training algorithms

Library learning as Bayesian inference



[Dechter et al, 2013] [Liang et al, 2010] [Lake et al, 2015]

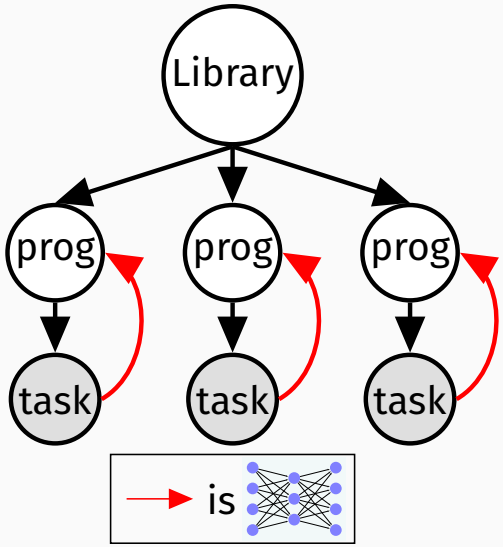
Library learning as neurally-guided Bayesian inference



library learning via program analysis +

new neural inference network for program synthesis +

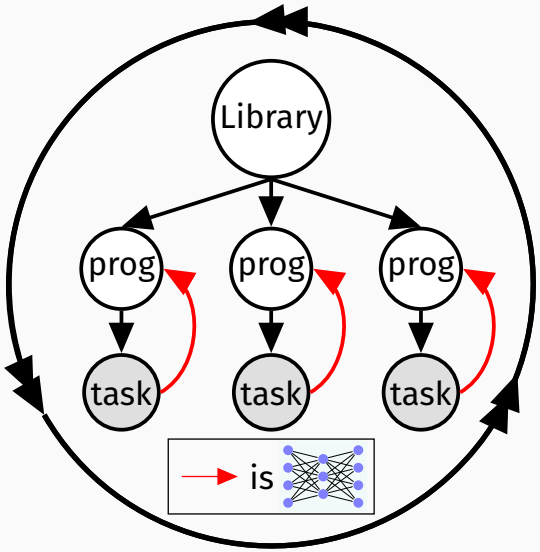
better program representation (Lisp+polymorphic types [Milner 1978])

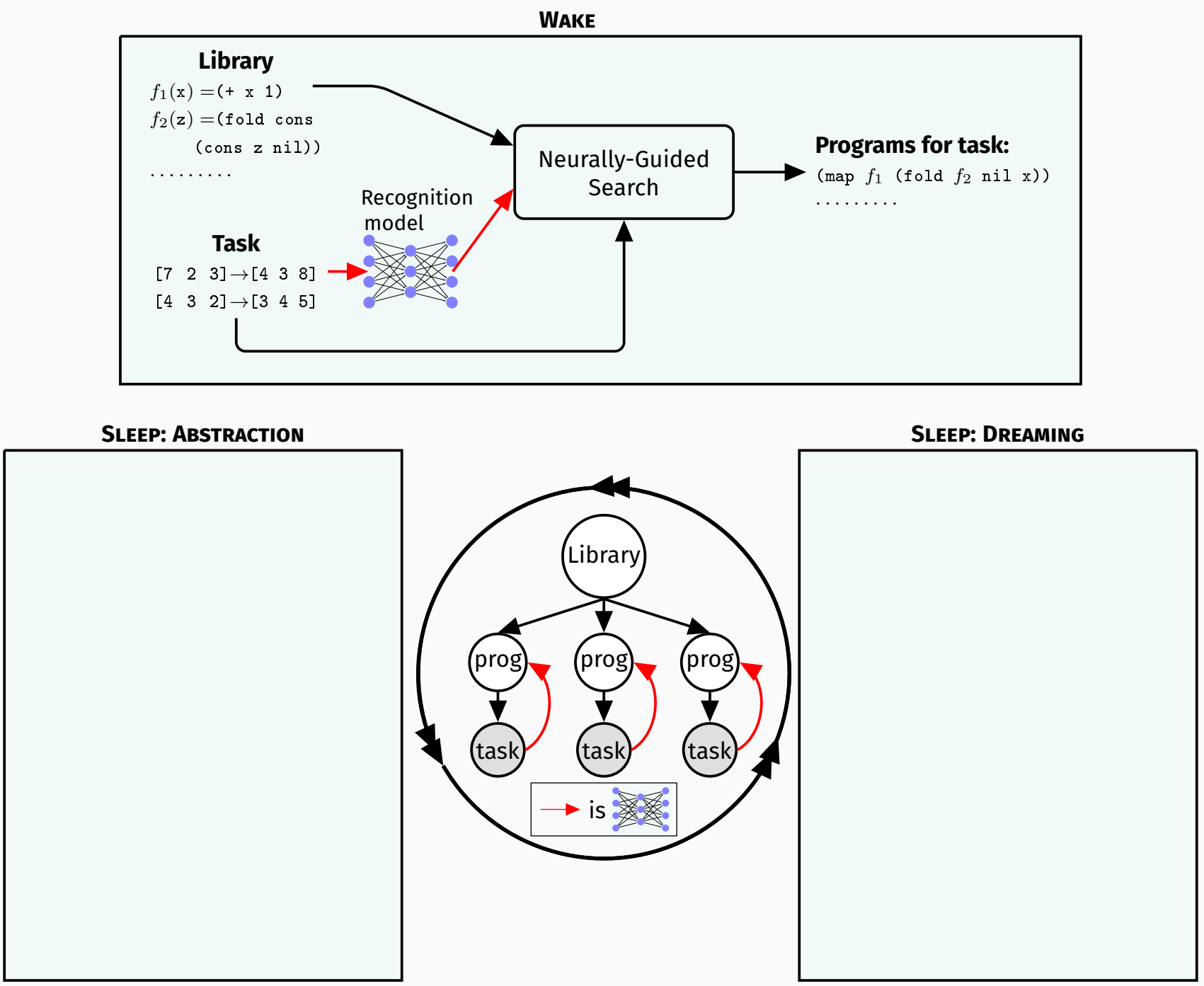

Library
prog
prog
prog
task
task
task
is

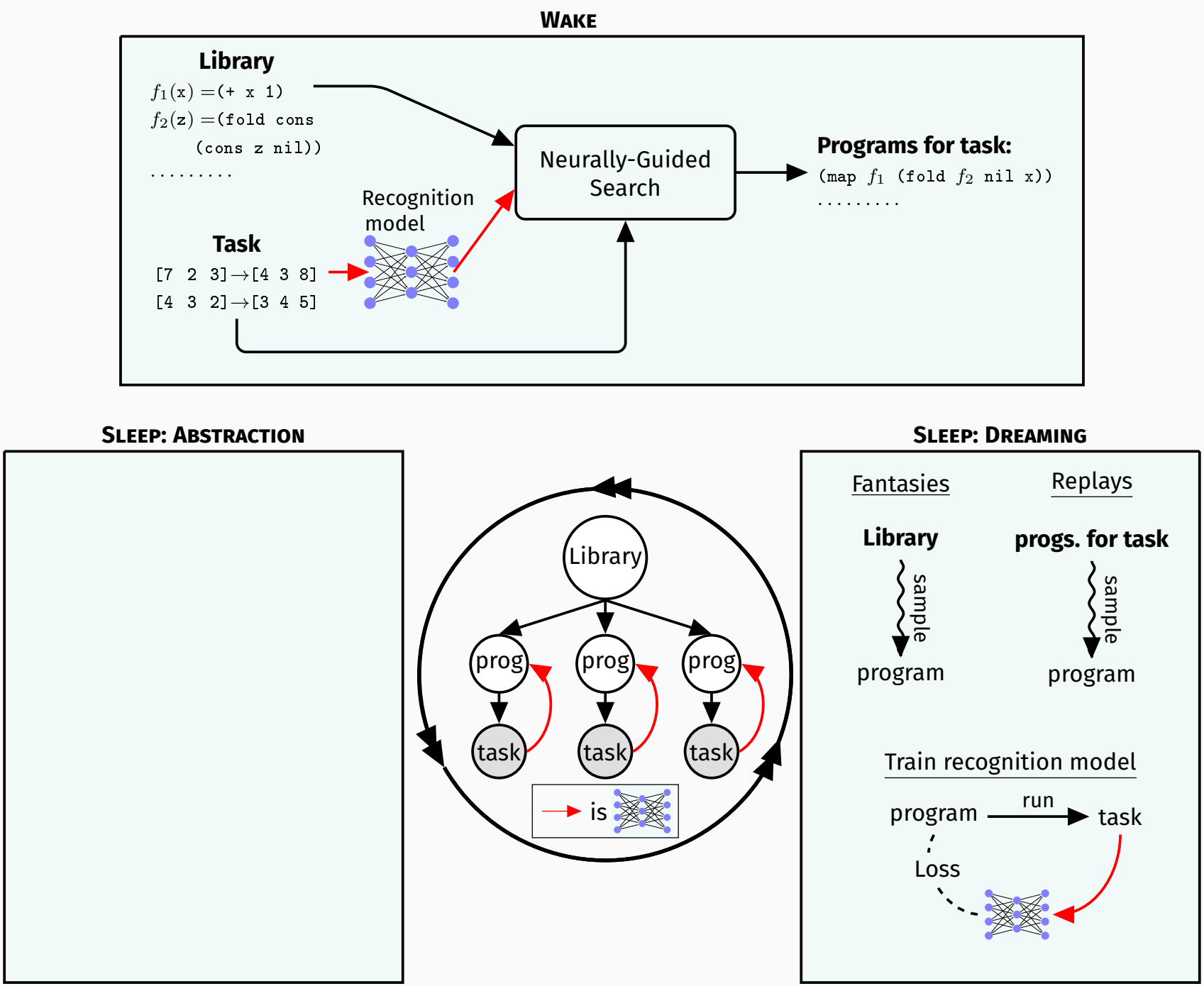
Wake

Sleep: Abstraction

Sleep: Dreaming




WAKE
Library
f_1(x) =(+ x 1)
f_2(z) =(fold cons
(cons z nil))
.....
Task
[7 2 3]→[4 3 8]
[4 3 2]→[3 4 5]
Recognition model
Neurally-Guided Search
Programs for task:
(map f_1 (fold f_2 nil x))
.....
SLEEP: ABSTRACTION
SLEEP: DREAMING
Library
prog
prog
prog
task
task
task
is


WAKE
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.....
Recognition
model
Task
[7 2 3]→[4 3 8]
[4 3 2]→[3 4 5]
Neurally-Guided
Search
Programs for task:
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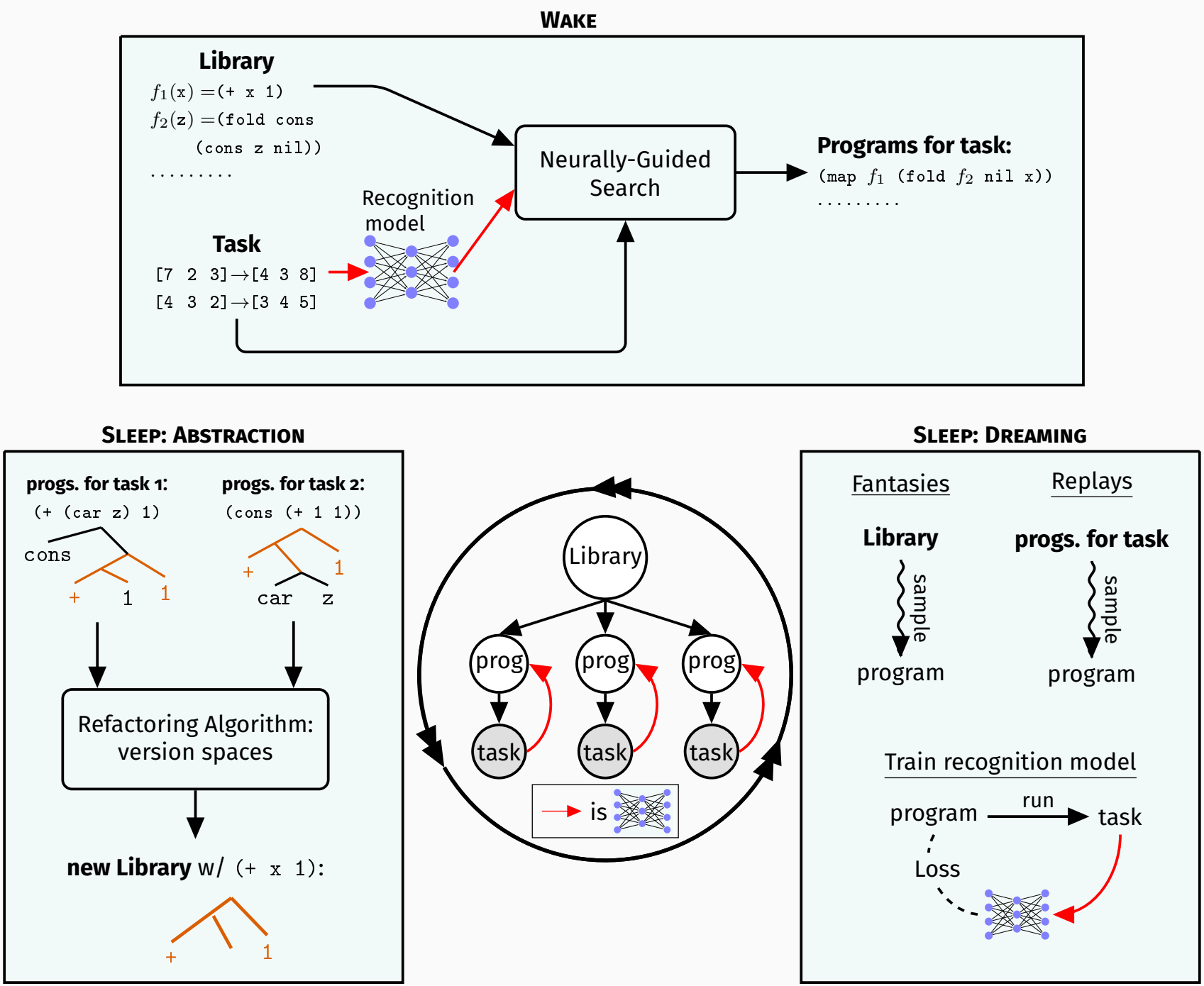
.....
SLEEP: ABSTRACTION
SLEEP: DREAMING
Library
prog
prog
prog
task
task
task
is
Fantasies
Replays
Library
progs. for task
sample
sample
program
program
Train recognition model
program
run
task
Loss


WAKE
Library
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Task
[7 2 3]→[4 3 8]
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Recognition model
Neurally-Guided Search
Programs for task:
(map f_1 (fold f_2 nil x))
.....
SLEEP: ABSTRACTION
progs. for task 1:
(+ (car z) 1)
cons
+
1
1
progs. for task 2:
(cons (+ 1 1))
+
1
car
z
Refactoring Algorithm: version spaces
new Library w/ (+ x 1):
+
1
Library
prog
prog
prog
task
task
task
is
SLEEP: DREAMING
Fantasies
Replays
Library
progs. for task
sample
sample
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program
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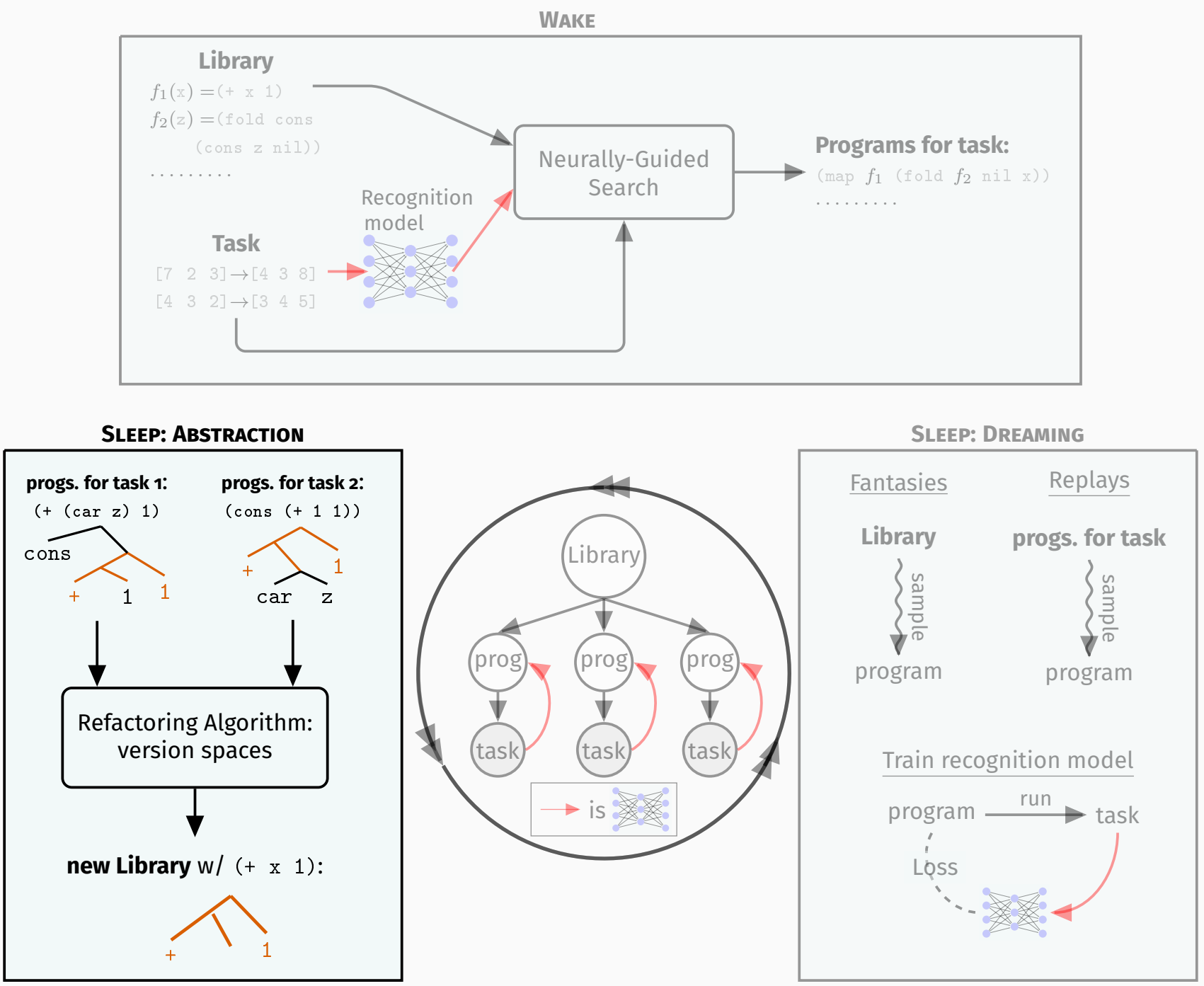
Loss


WAKE
Library
$f_1(x) = (+\ x\ 1)$
$f_2(z) = (\text{fold cons } (\text{cons } z \text{ nil}))$
.....
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Task
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task
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SLEEP: DREAMING
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Train recognition model
program
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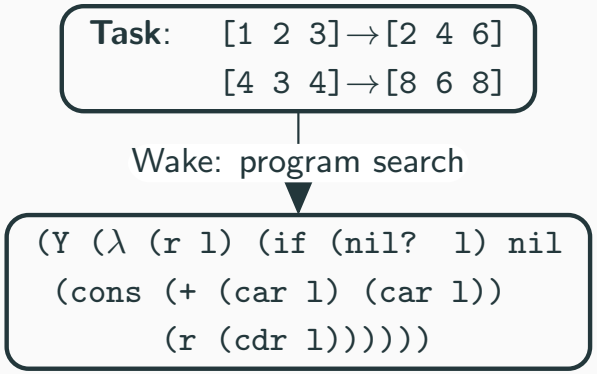
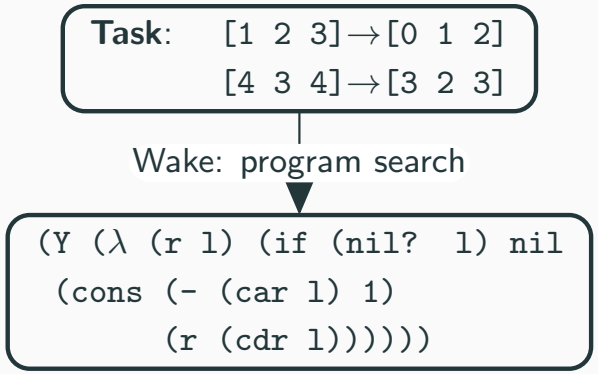
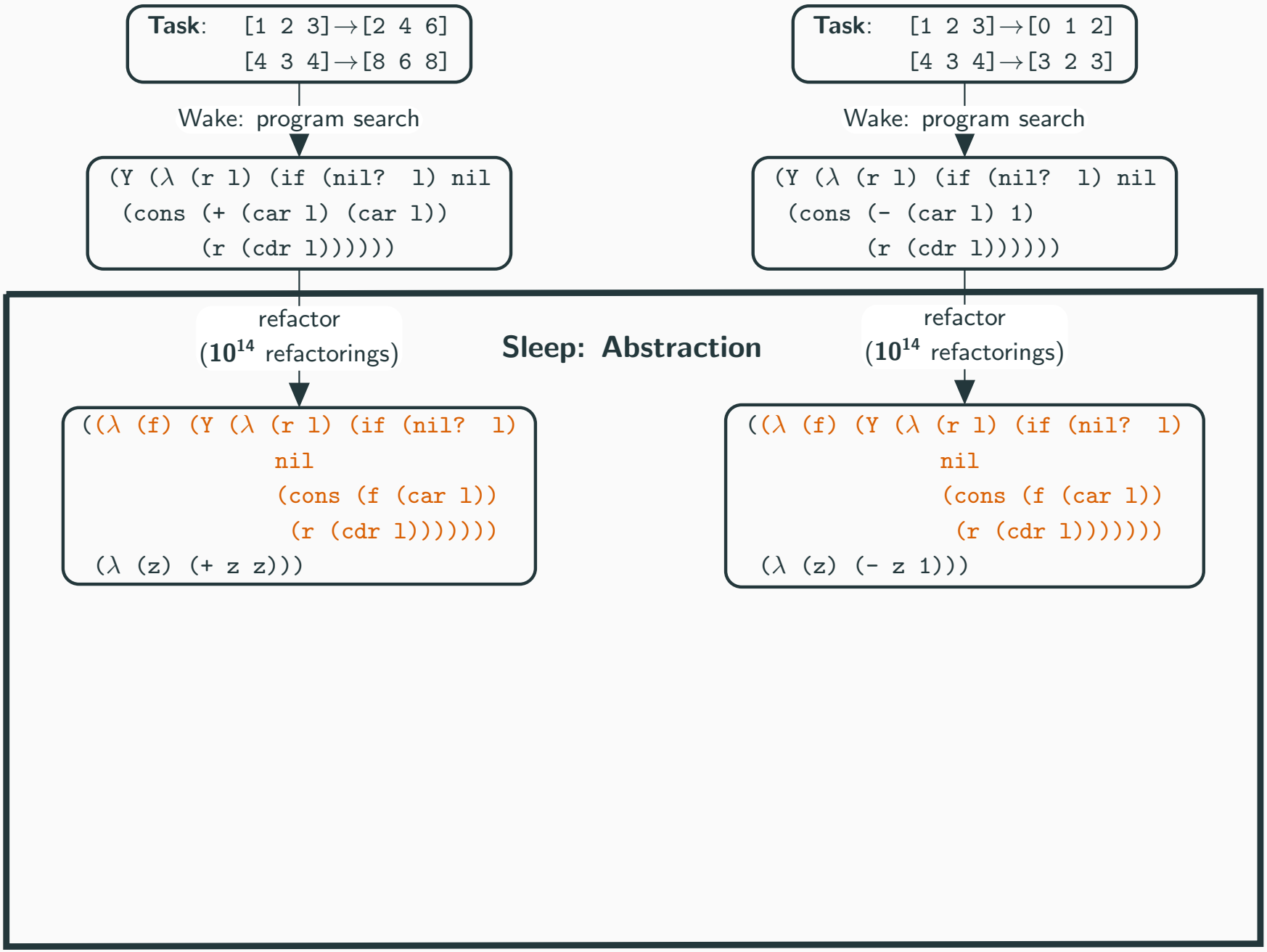
Loss

Abstraction Sleep: Growing the library via refactoring

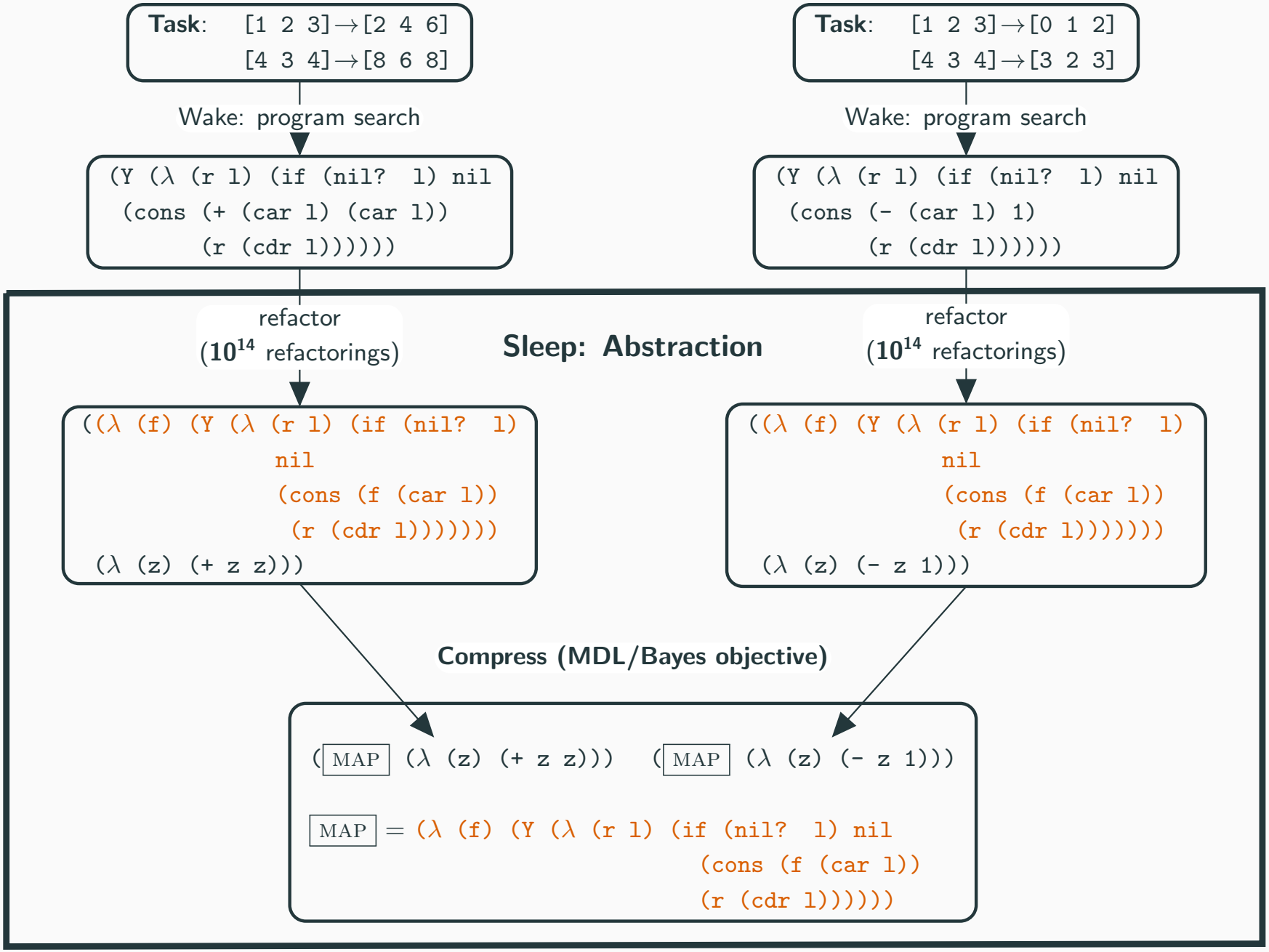




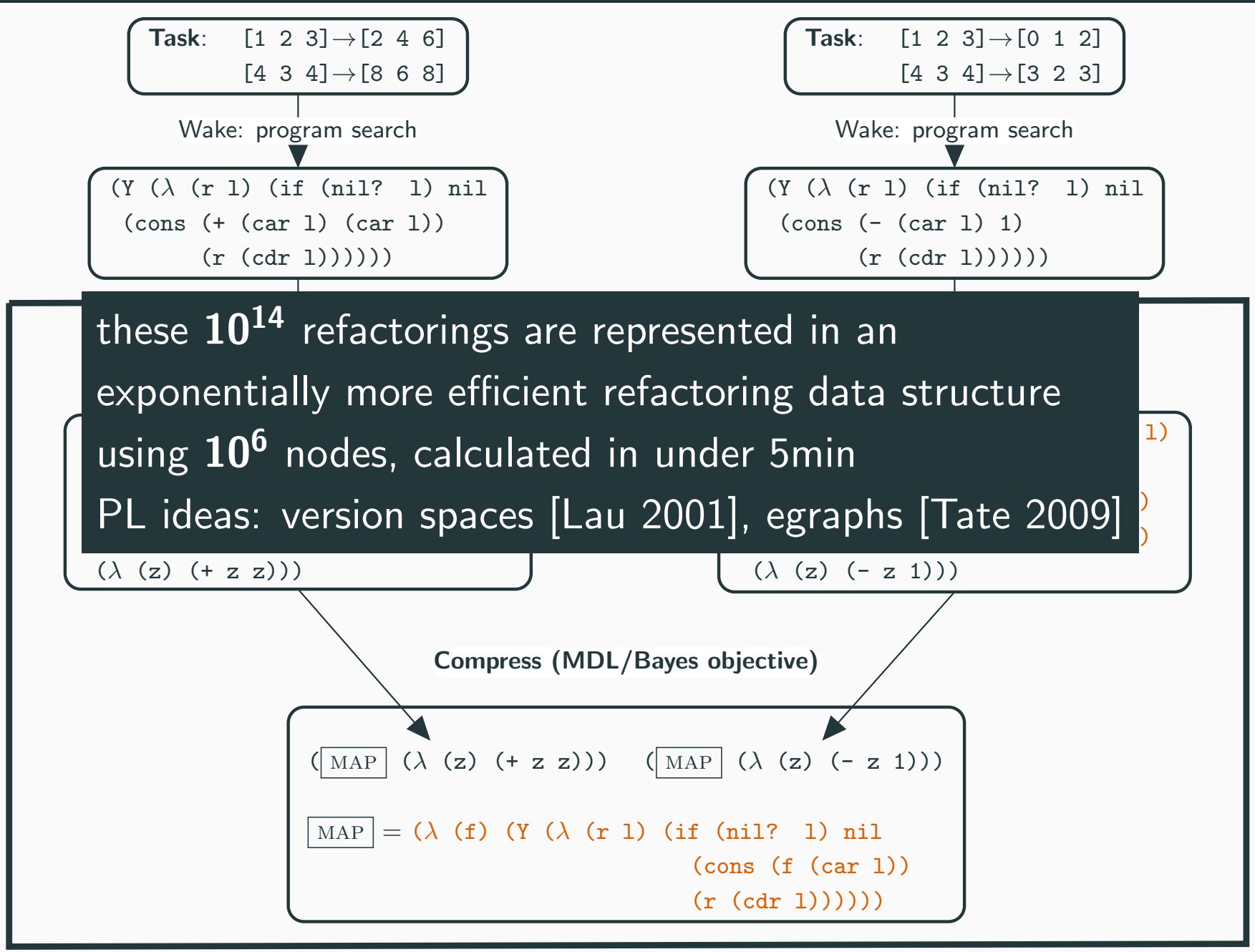
Abstraction Sleep: Growing the library via refactoring



Abstraction Sleep: Growing the library via refactoring



Abstraction Sleep: Growing the library via refactoring



DreamCoder Domains

List Processing

Sum List

```
[1 2 3] → 6
[4 6 8 1] → 17
```

Double

```
[1 2 3] → [2 4 6]
[4 5 1] → [8 10 2]
```

Text Editing

Abbreviate

```
Allen Newell → A.N.
Herb Simon → H.S.
```

Drop Last Three

```
shrdlu → shr
shakey → sha
```

Regexes

Phone numbers

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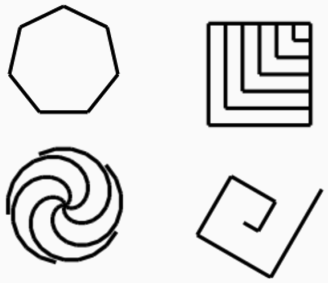
```

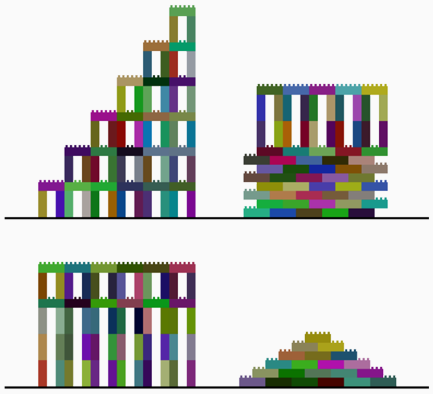
Currency

```
$100.25
$4.50
```

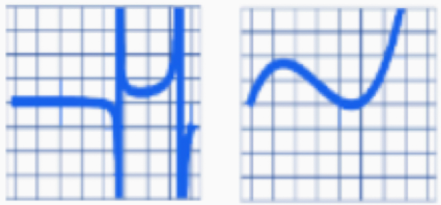
LOGO Graphics



Block Towers



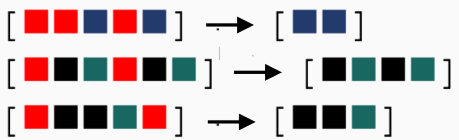
Symbolic Regression



$$y = f(x)$$

Recursive Programming

Filter Red



Physical Laws

$$\vec{a} = \frac{1}{m}\sum_i \vec{F}_i$$

$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r}|^2}\hat{r}$$

DreamCoder Domains

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`[1 2 3]` → `6`
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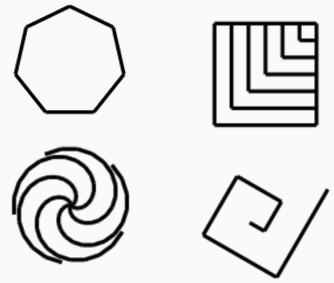

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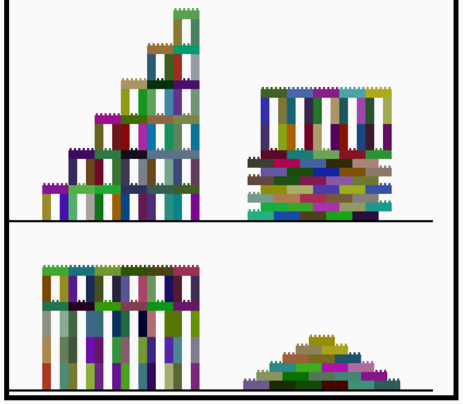
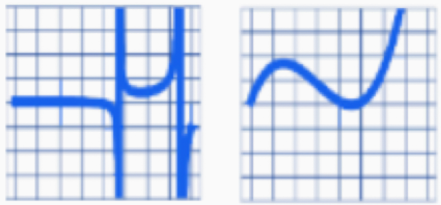


Symbolic Regression



$$y = f(x)$$

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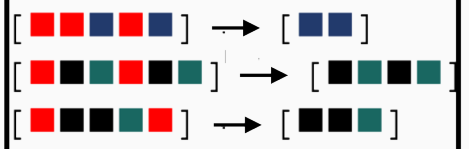
Filter Red



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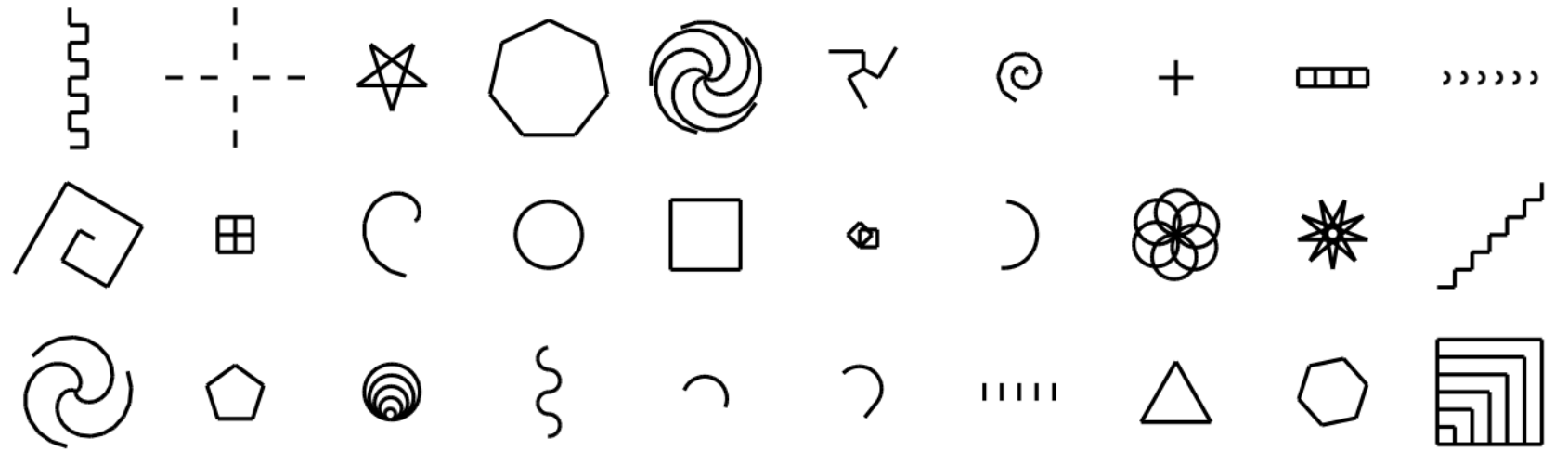
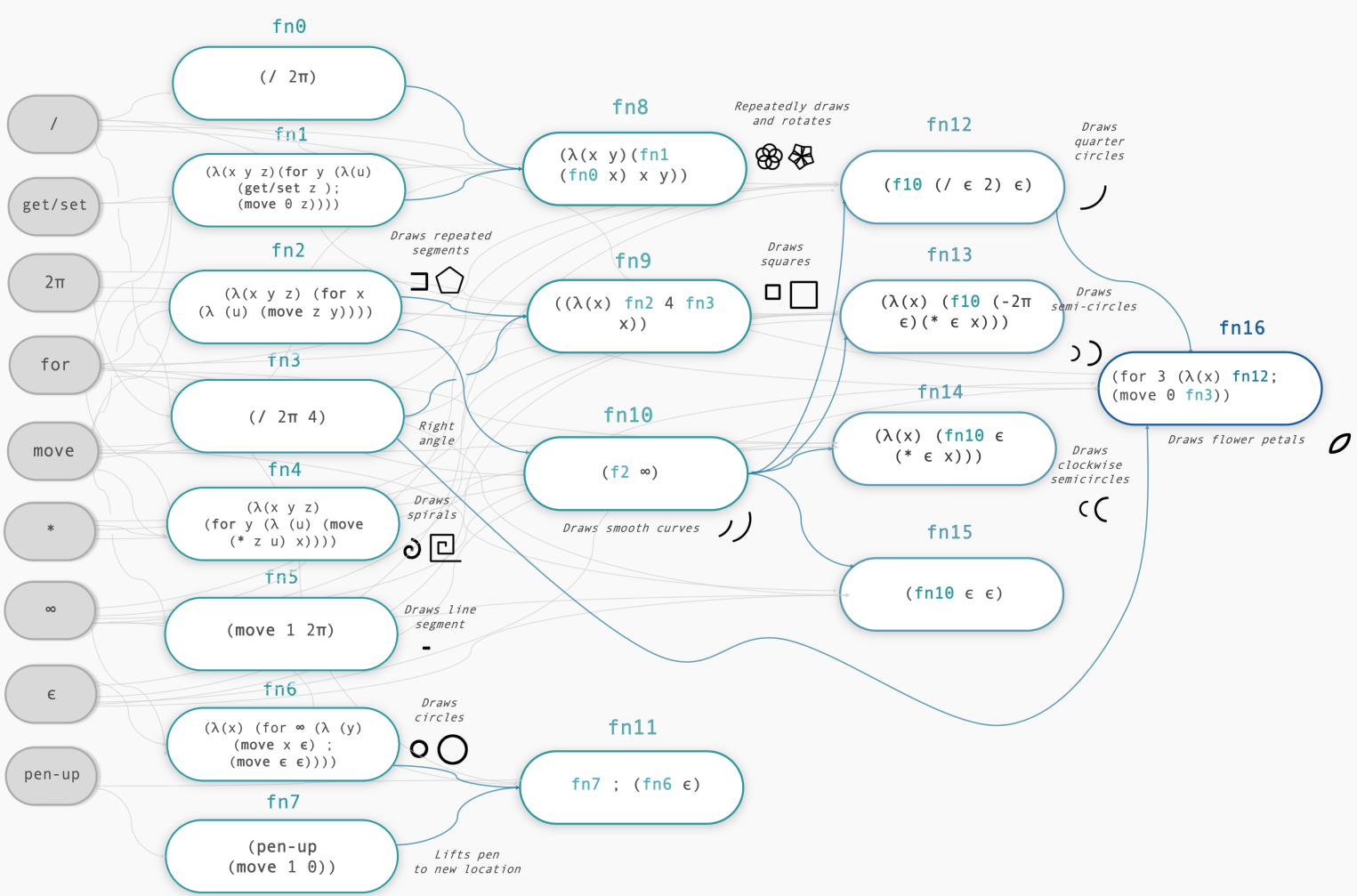
LOGO Turtle Graphics

30 out of 160 tasks

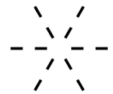


LOGO Turtle Graphics – learning an interpretable library



```
(fn8 5 (fn4 (* ϵ 2) ∞ ϵ))
```

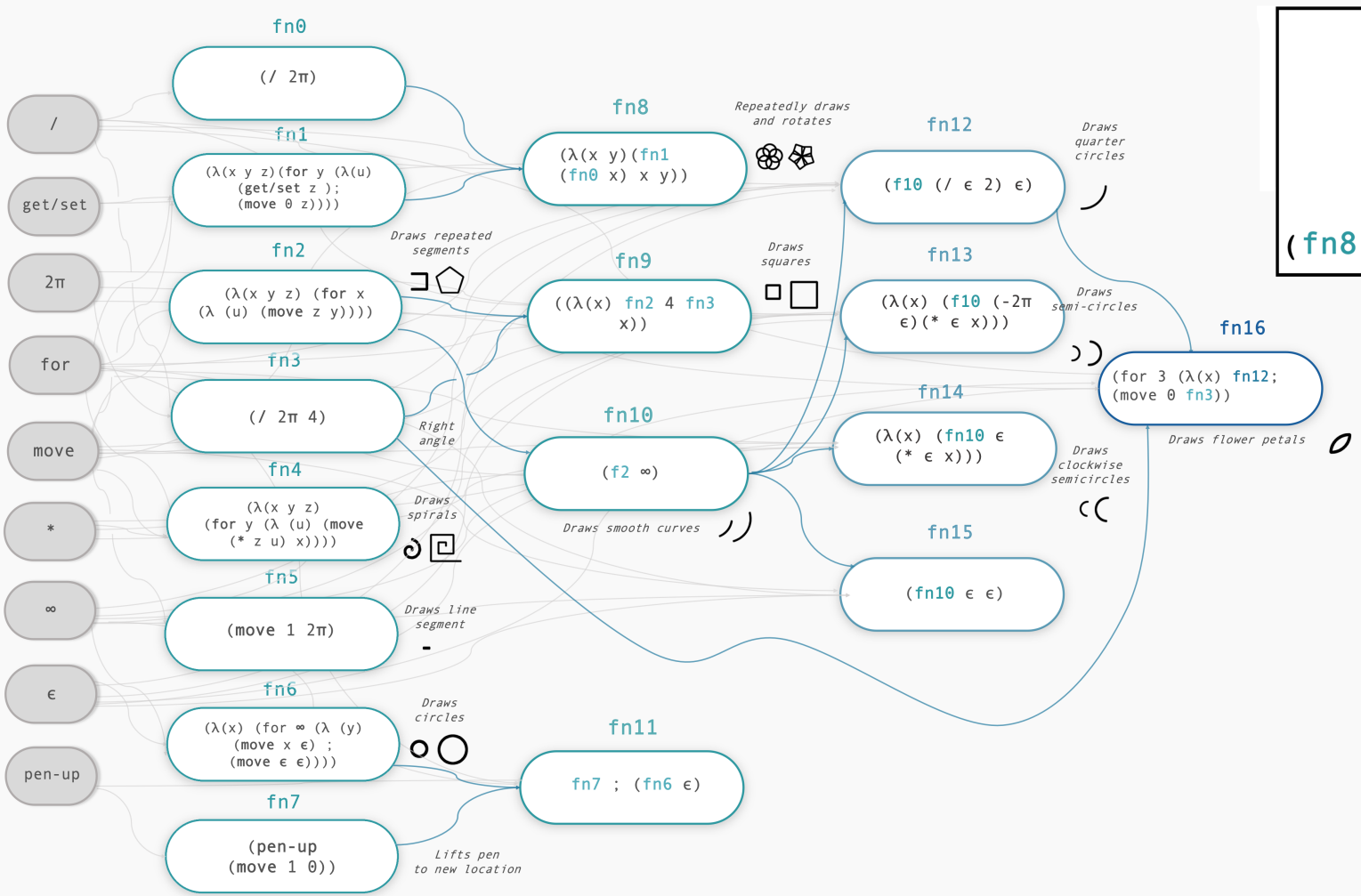
```
(for 7 (λ (x) (fn9 x)))
```

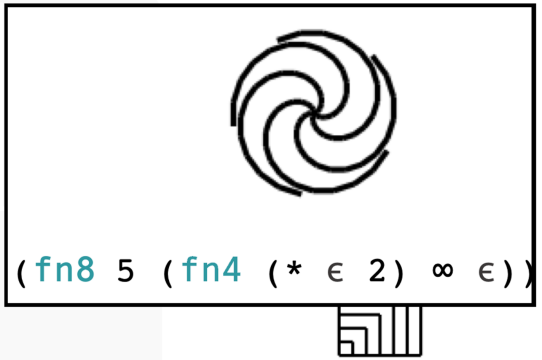


```
(fn8  6 (fn7 ; fn5 ;
fn7 ; fn5))
```

```
(move 0 (fn0 7)); fn5 ;
(fn13 4)
```

LOGO Turtle Graphics – learning an interpretable library






```
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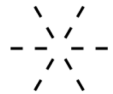

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fn7 ; fn5))
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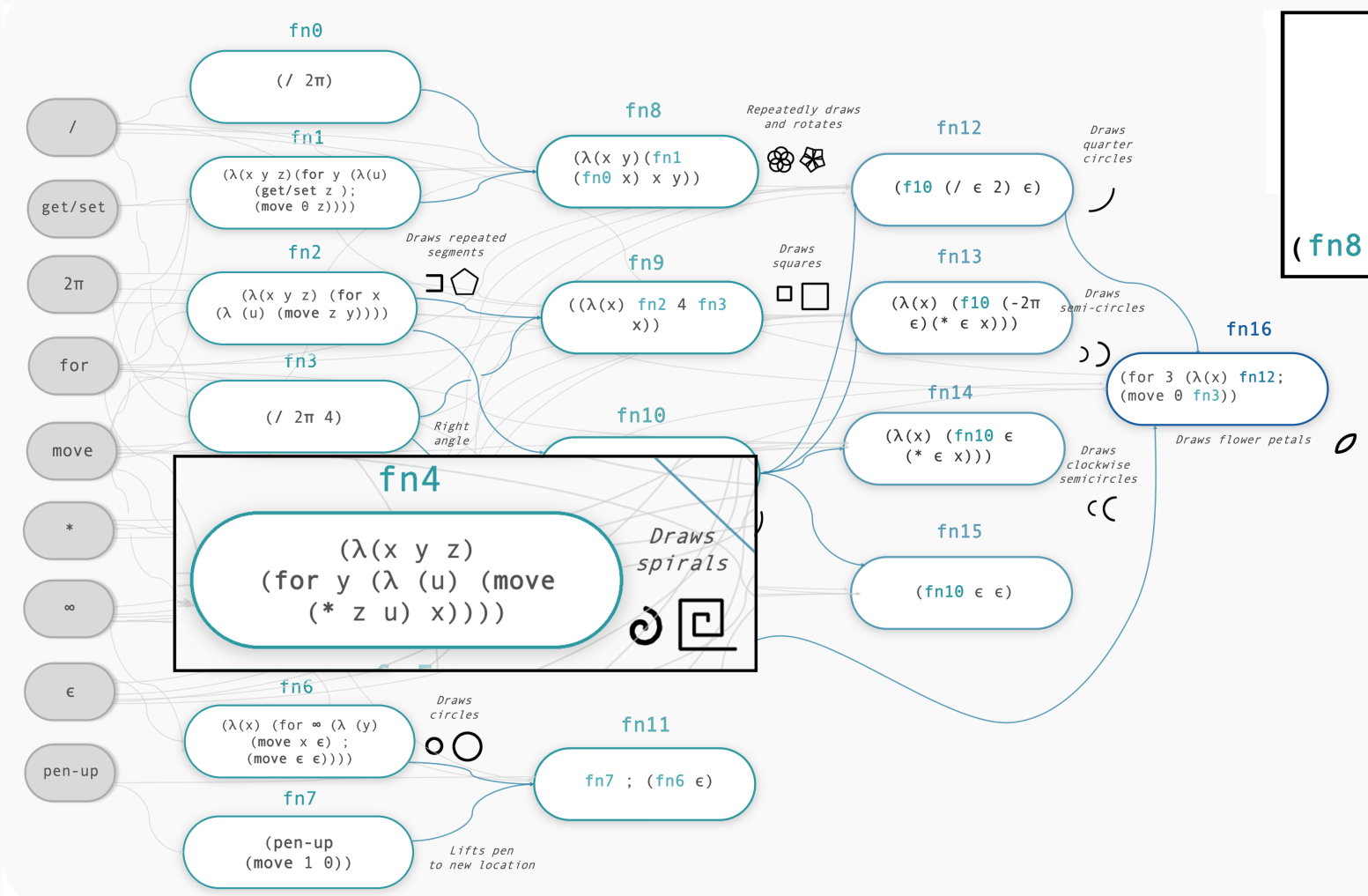
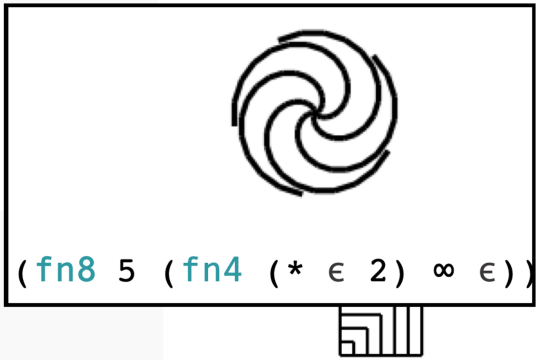
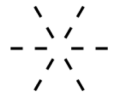
(fn13 4)
```

LOGO Turtle Graphics – learning an interpretable library





(for 7 (λ (x) (fn9 x)))



(fn8 6 (fn7 ; fn5 ;
fn7 ; fn5))

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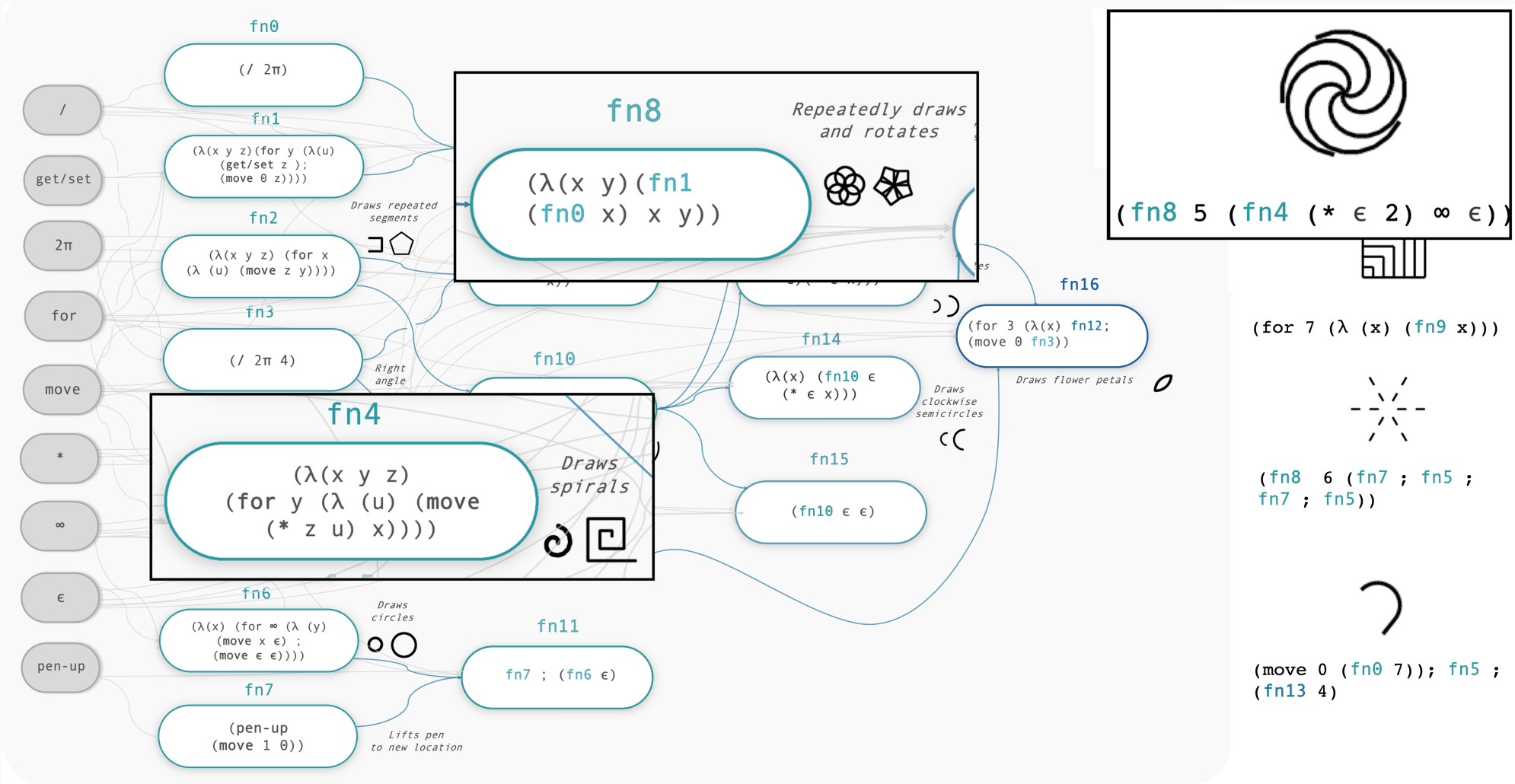
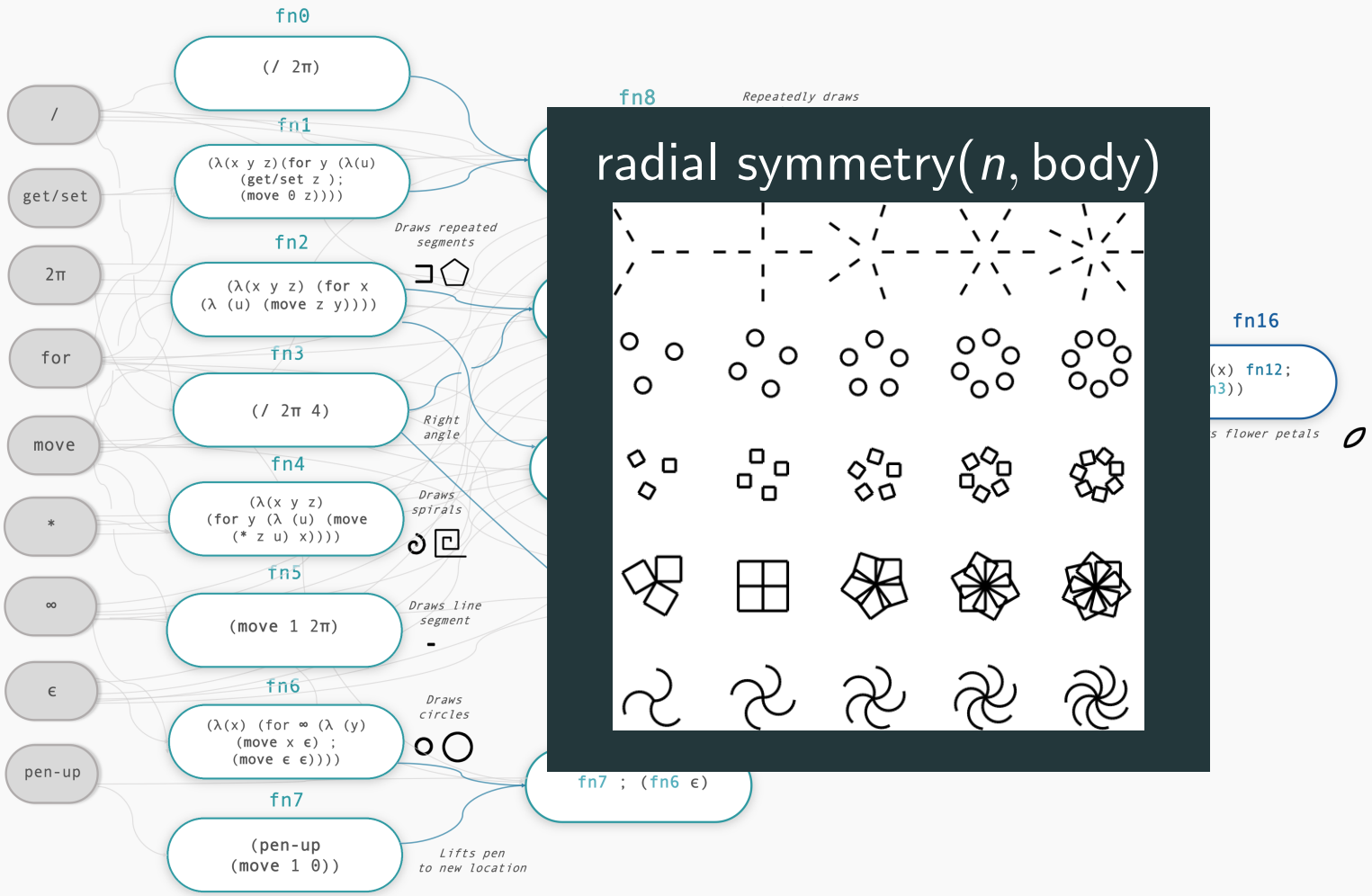
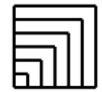
LOGO Turtle Graphics – learning an interpretable library



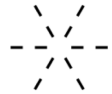
LOGO Turtle Graphics – learning an interpretable library



```
(fn8 5 (fn4 (* ϵ 2) ∞ ϵ))
```



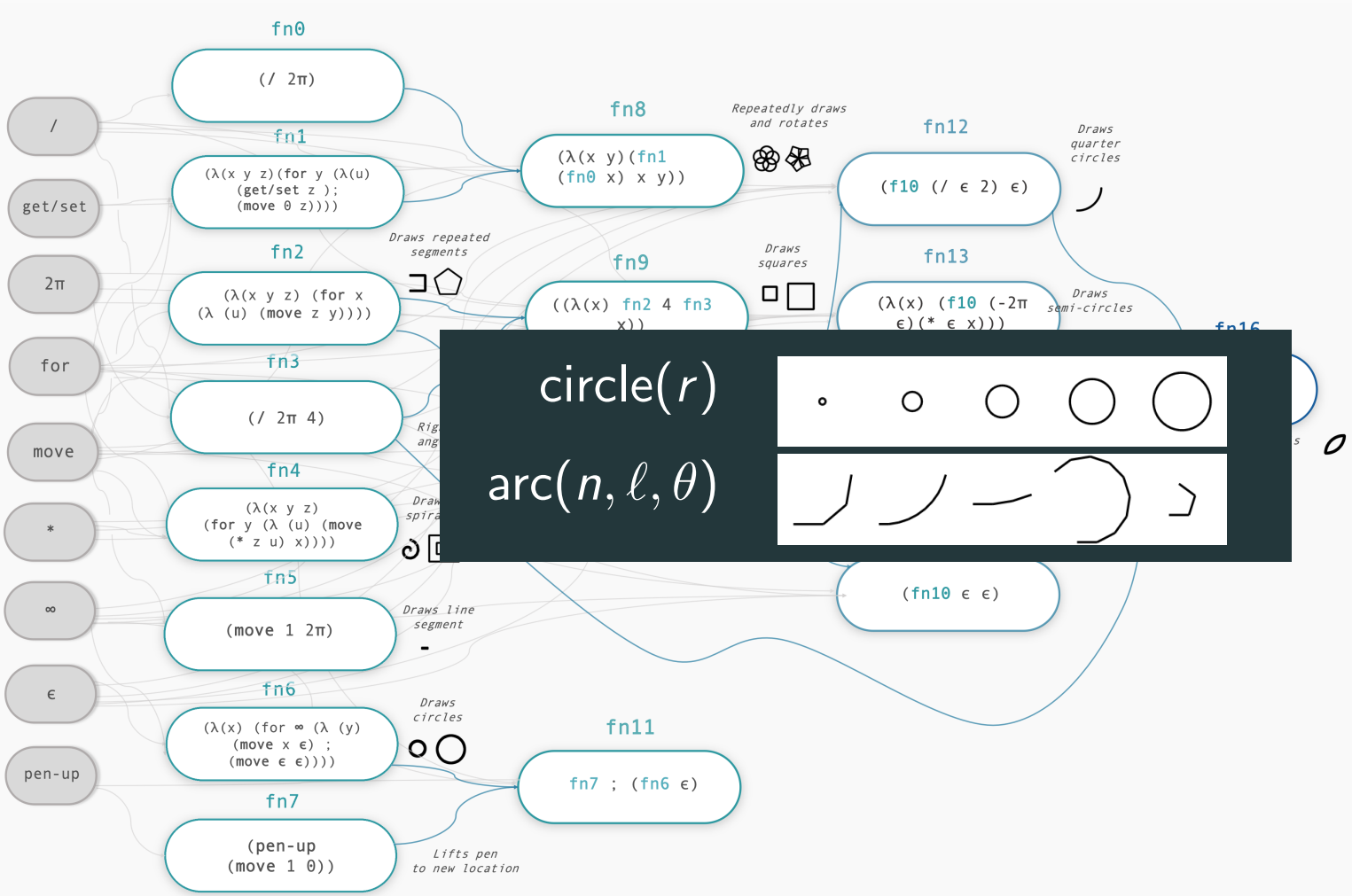
```
(for 7 (λ (x) (fn9 x)))
```



```
(fn8  6 (fn7 ; fn5 ;
fn7 ; fn5))
```

```
(move 0 (fn0 7)); fn5 ;
(fn13 4)
```

LOGO Turtle Graphics – learning an interpretable library



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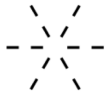

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(fn8  6 (fn7 ; fn5 ;
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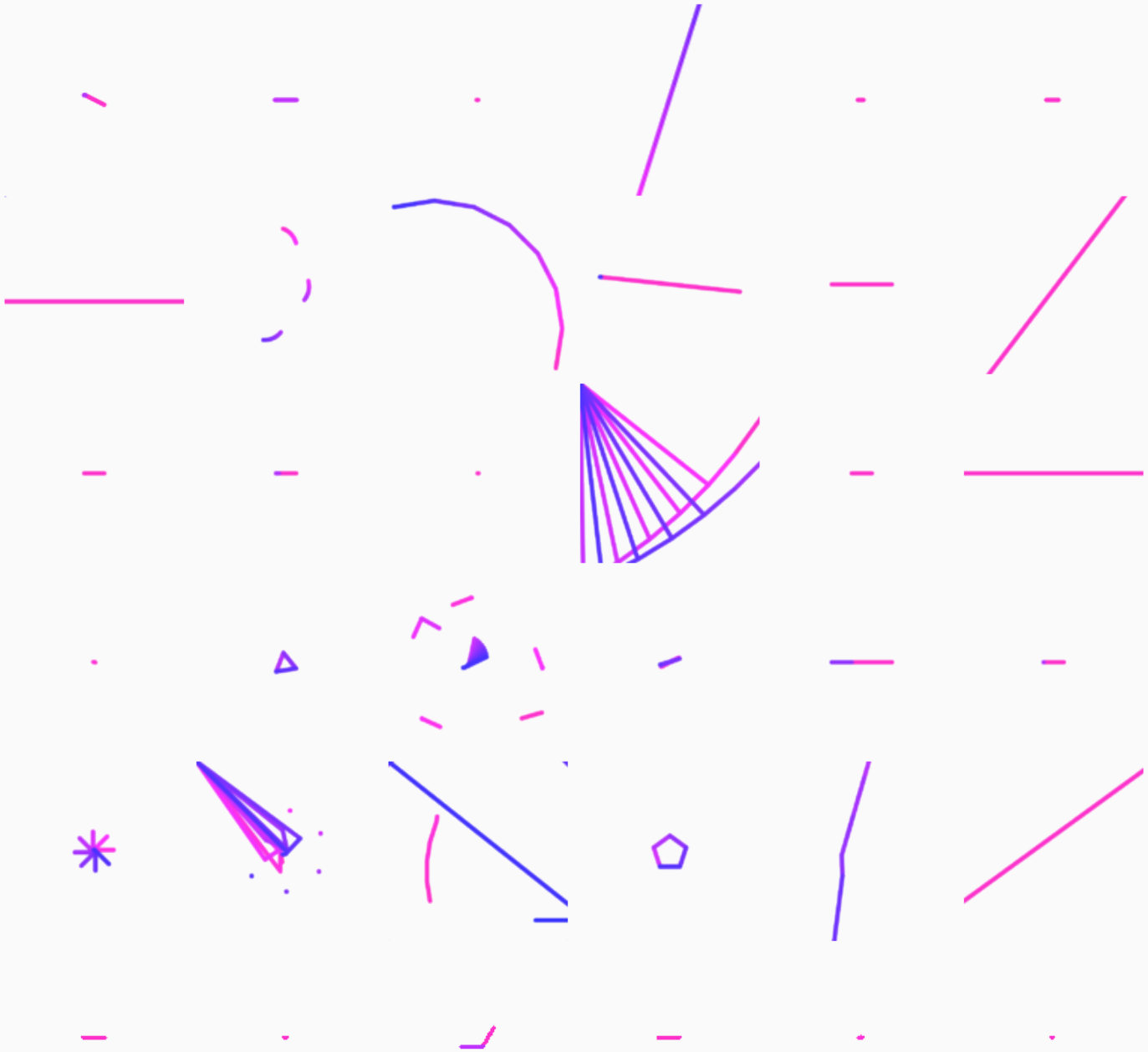
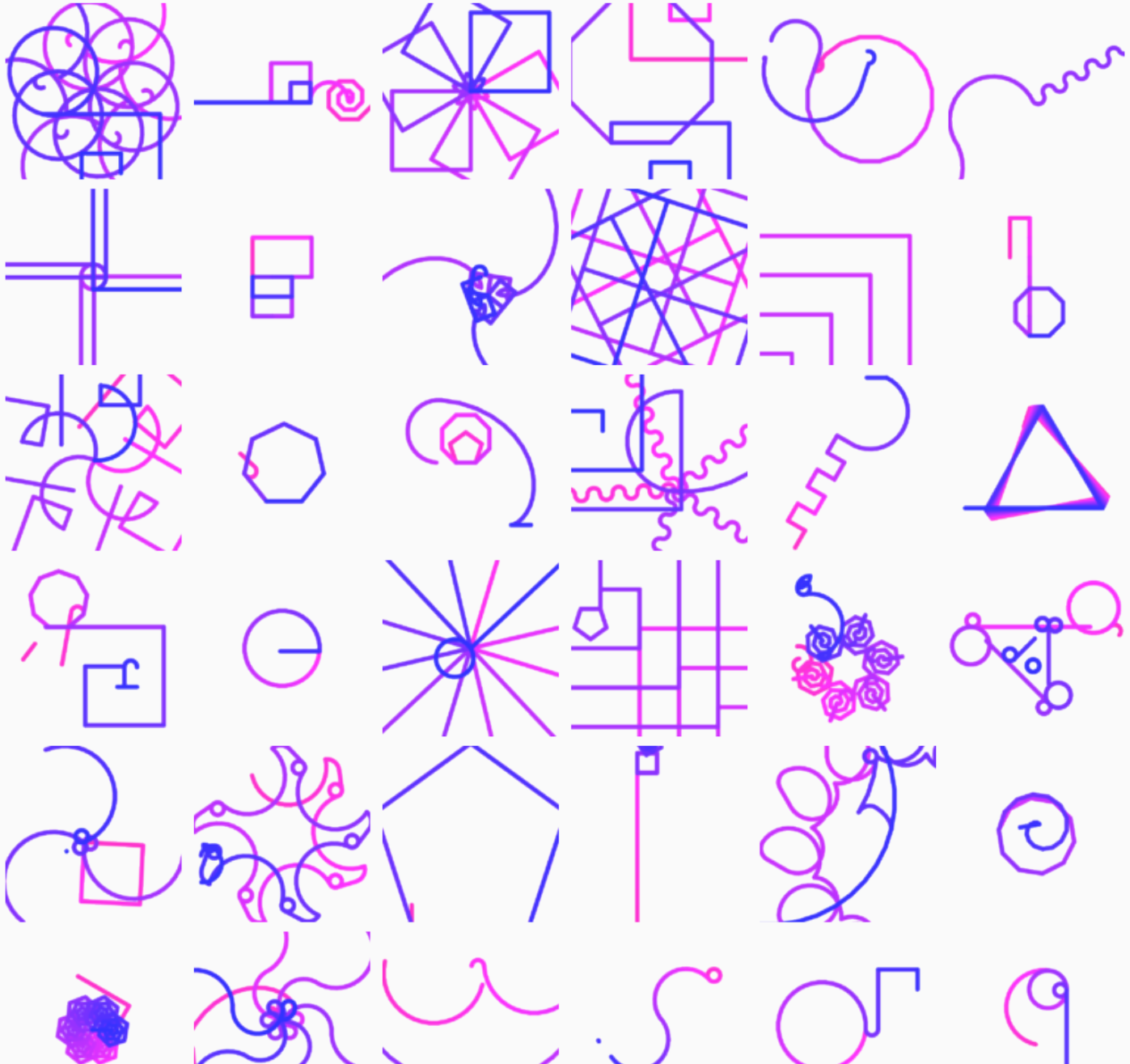
```

What does DreamCoder dream of? (before learning)

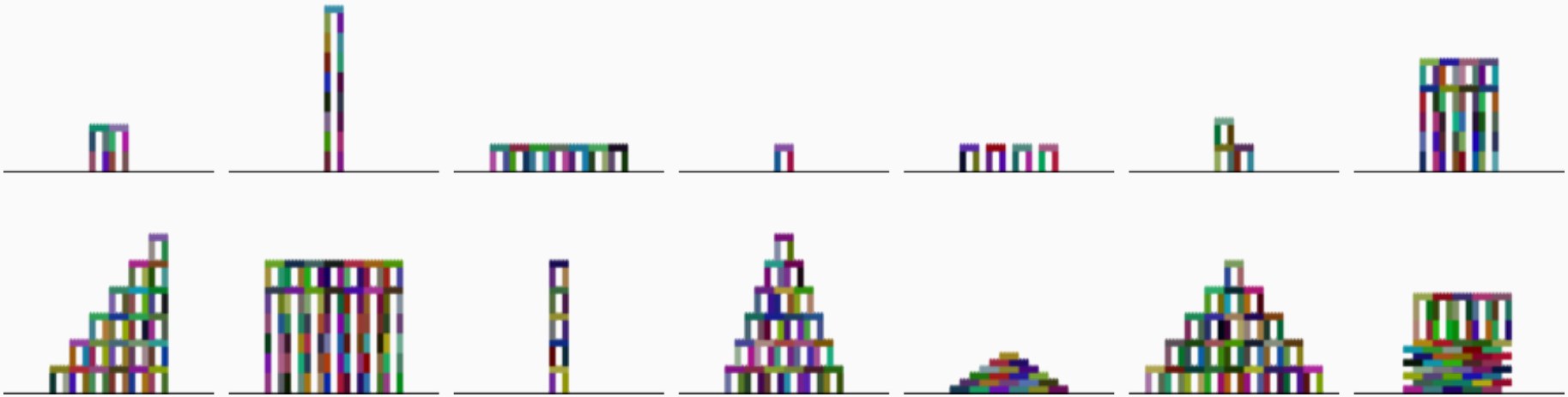


What does DreamCoder dream of? (after learning)



Planning to build towers

example tasks (112 total)



Planning to build towers

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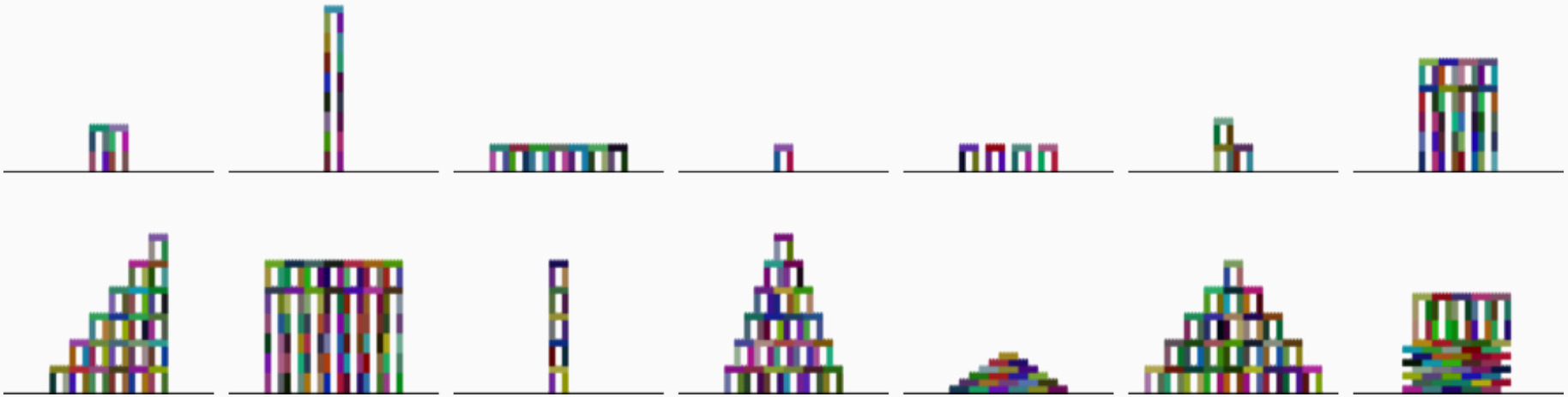
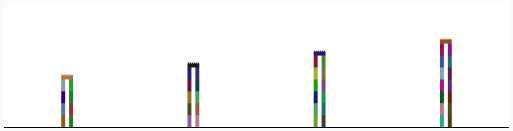
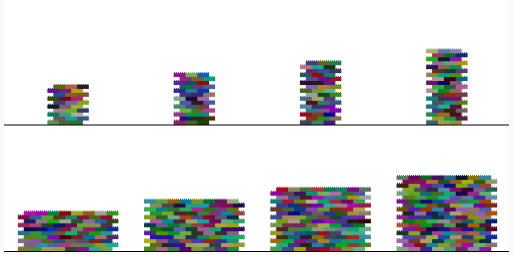


learned library routines ($\approx$ 20 total)

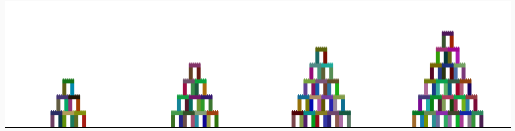
arch(h)



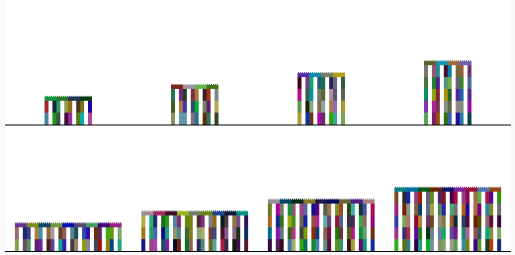
wall(w, h)



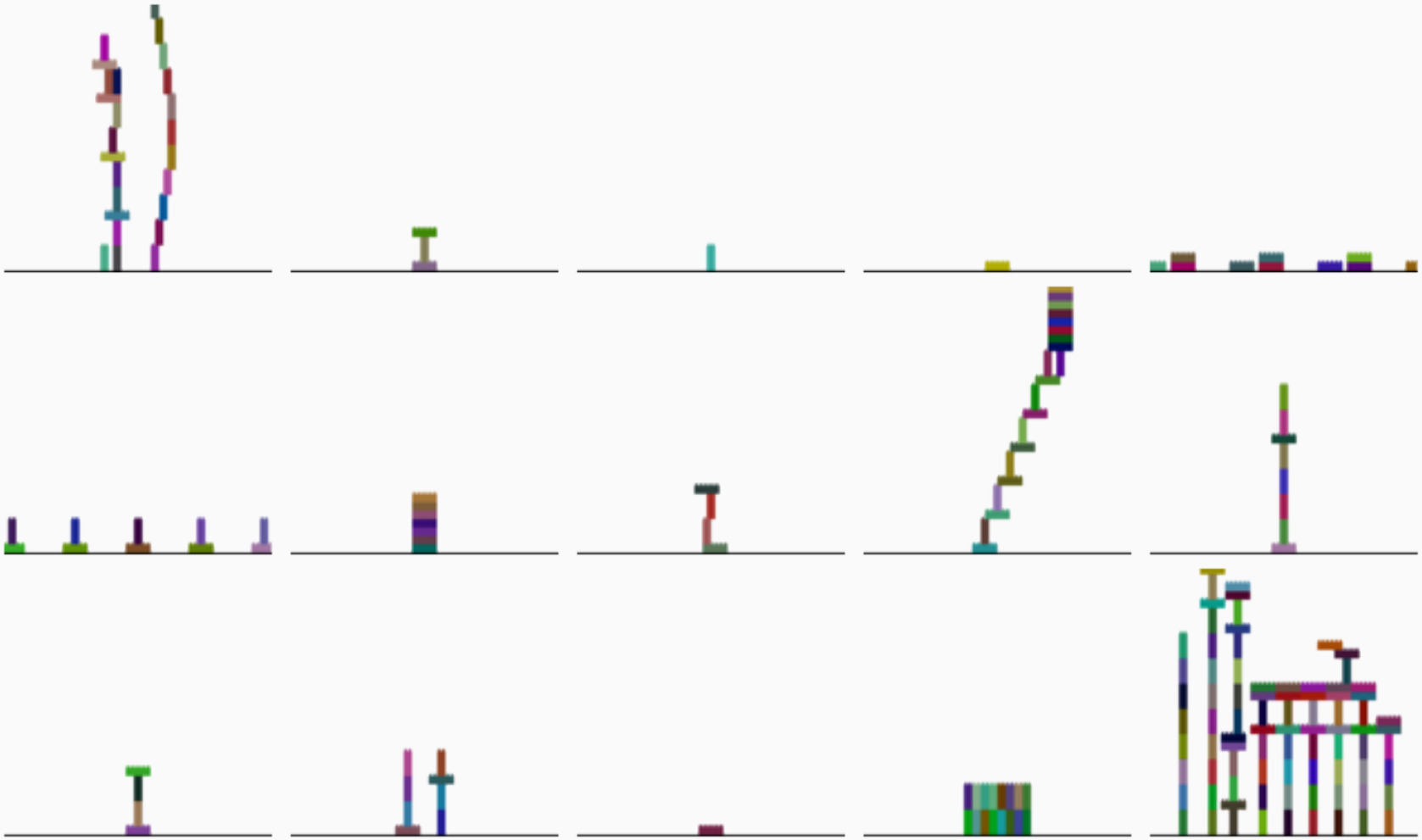
pyramid(h)



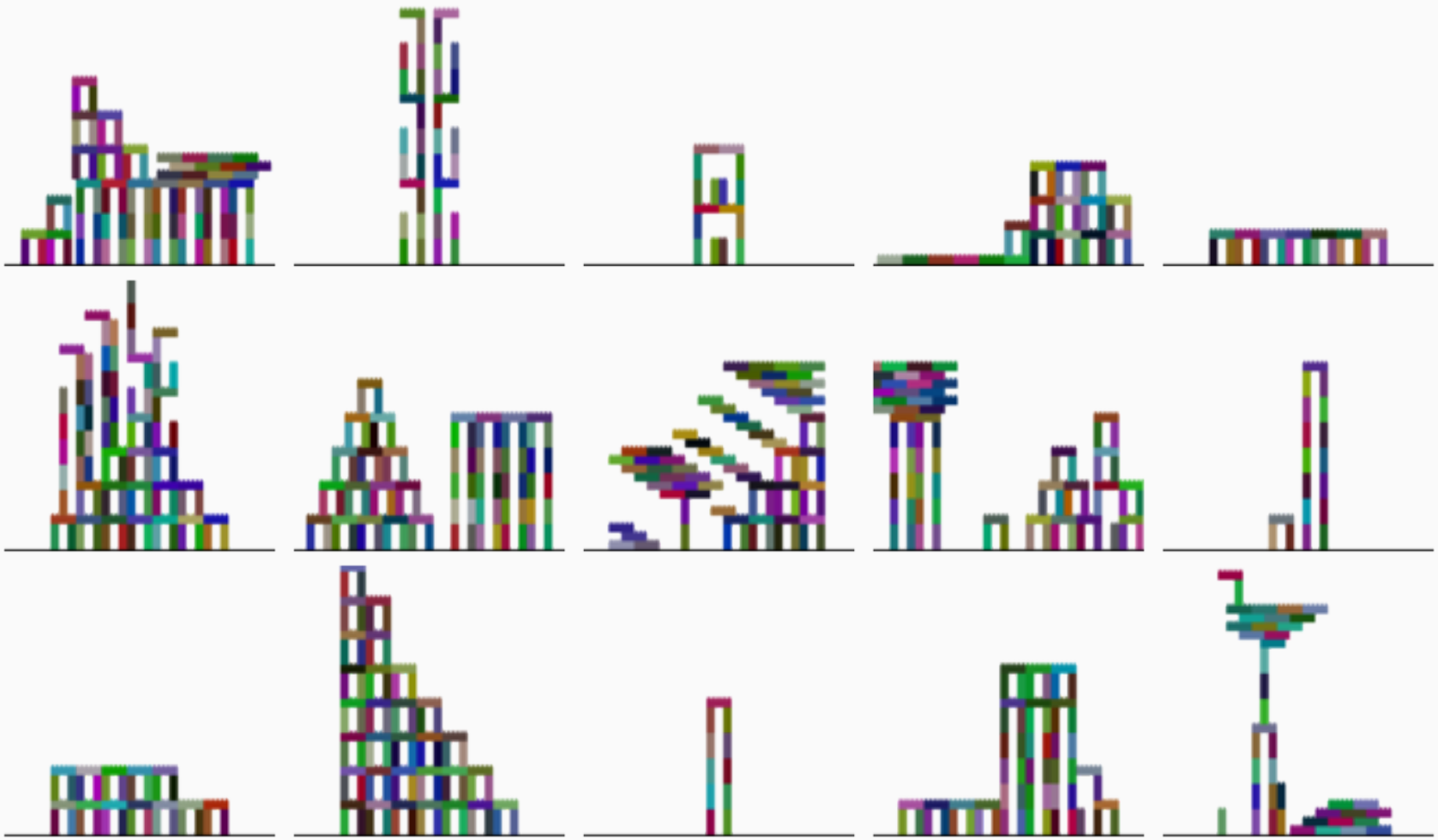
bridge(w, h)



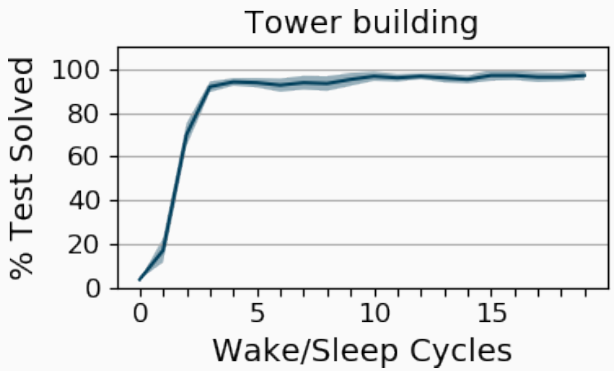
Dreams before learning



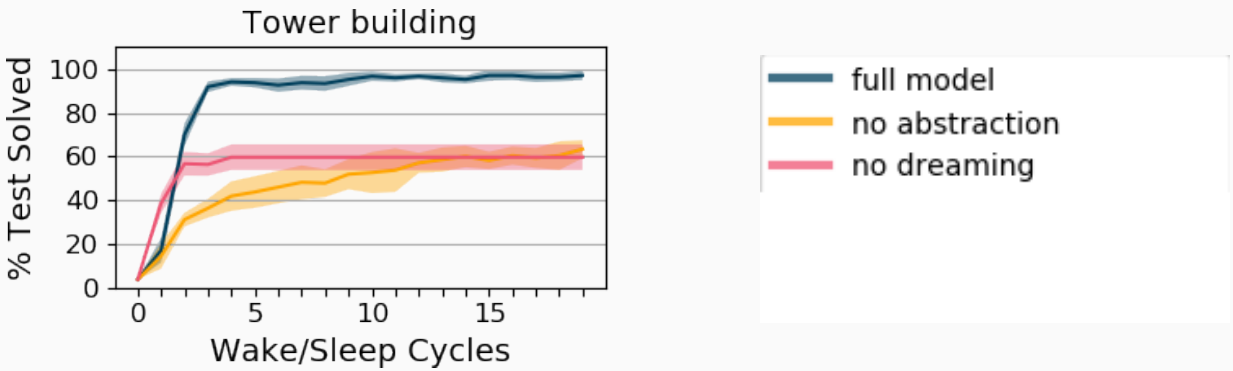
Dreams after learning



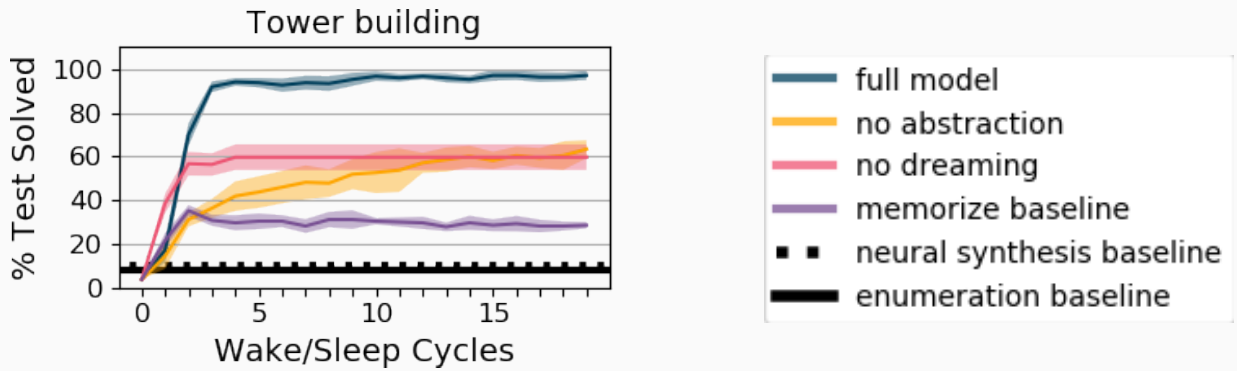
Learning dynamics



Learning dynamics

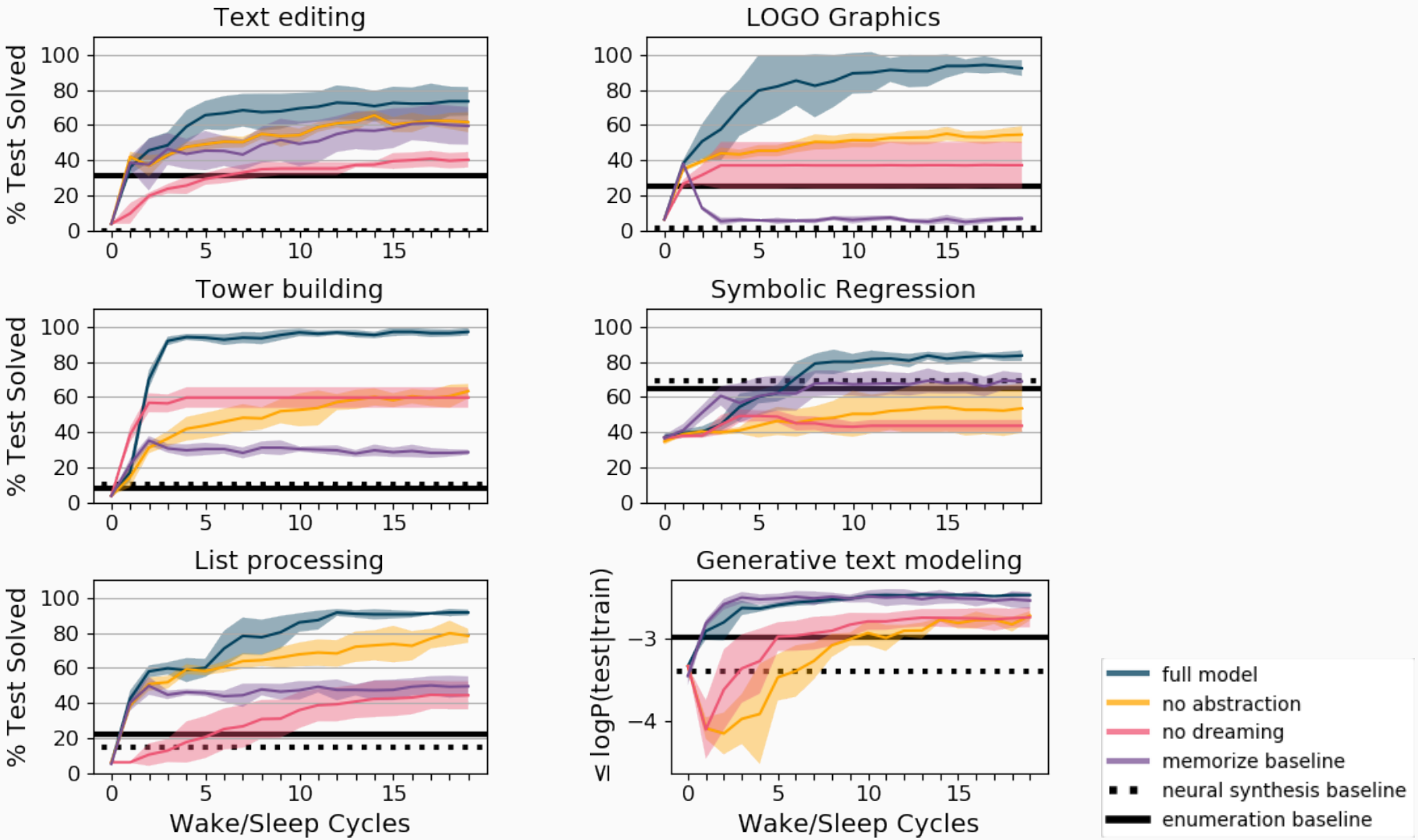


Learning dynamics



baselines: memorize programs rather than refactor them [Cropper 2019]
neural program synthesis, RobustFill [Devlin et al. 2017]
24 hours of brute-force enumeration

Learning dynamics



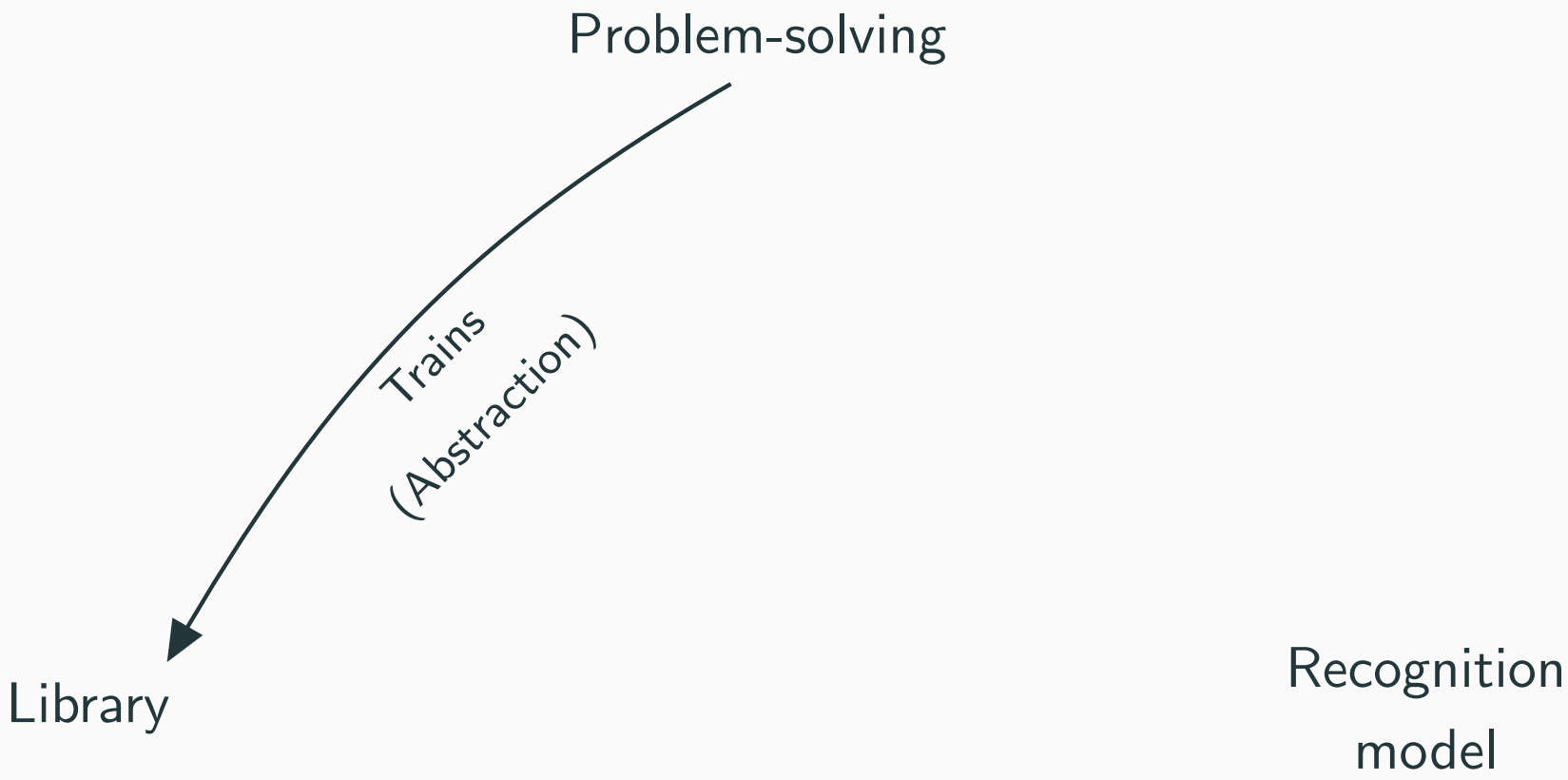
Synergy between dreaming and library learning

Problem-solving

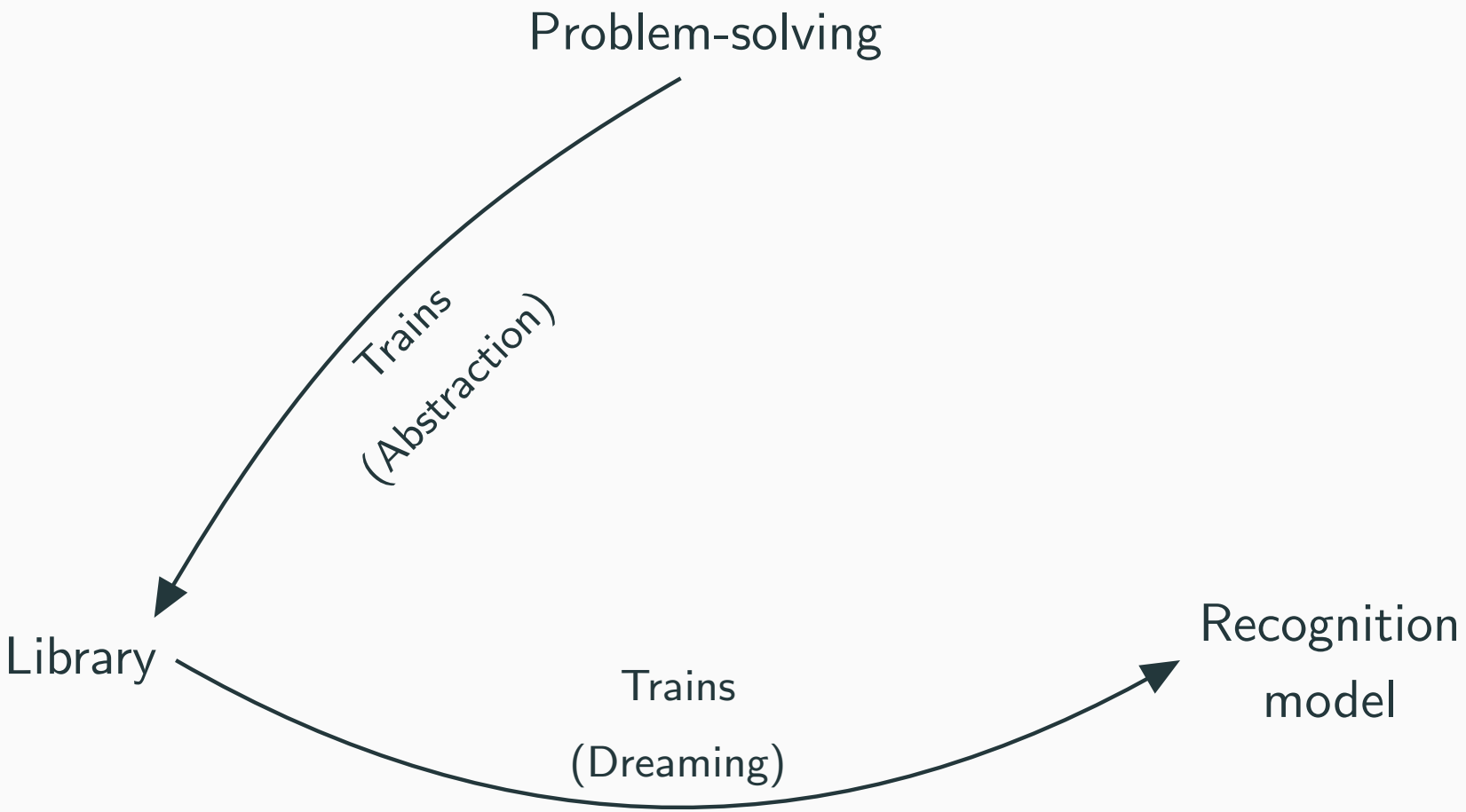
Library

Recognition model

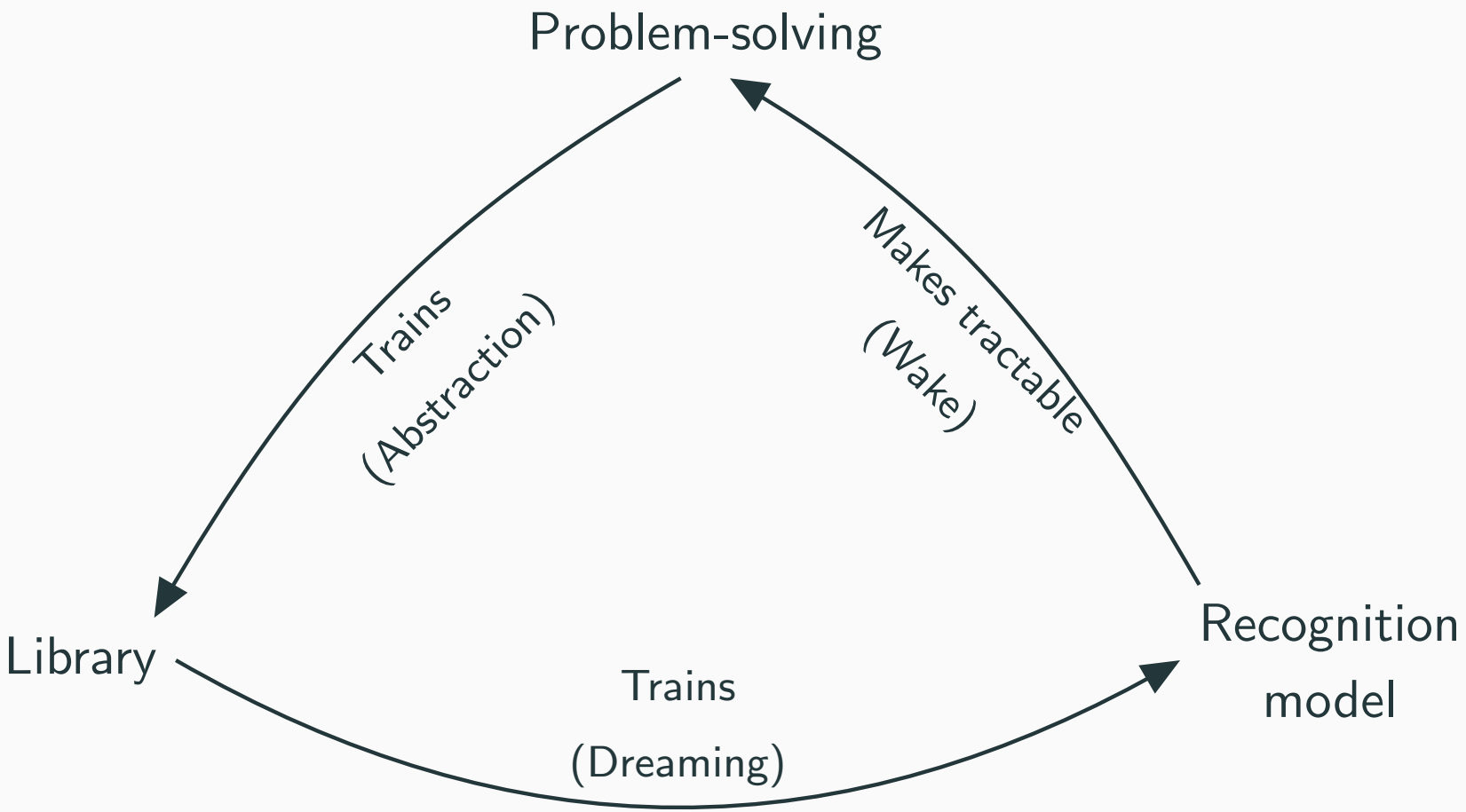
Synergy between dreaming and library learning



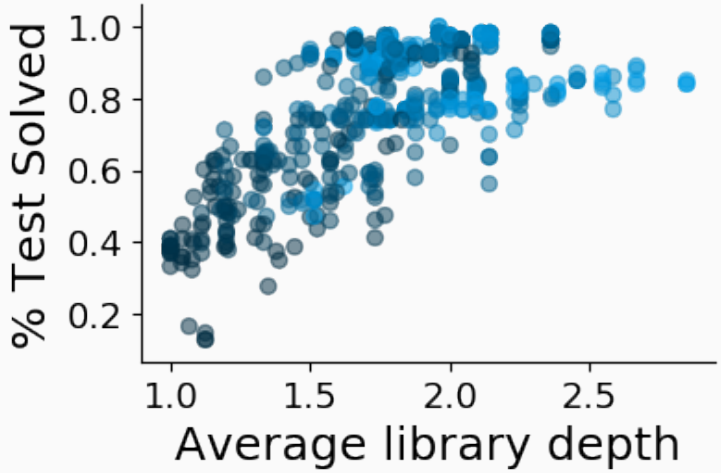
Synergy between dreaming and library learning



Synergy between dreaming and library learning



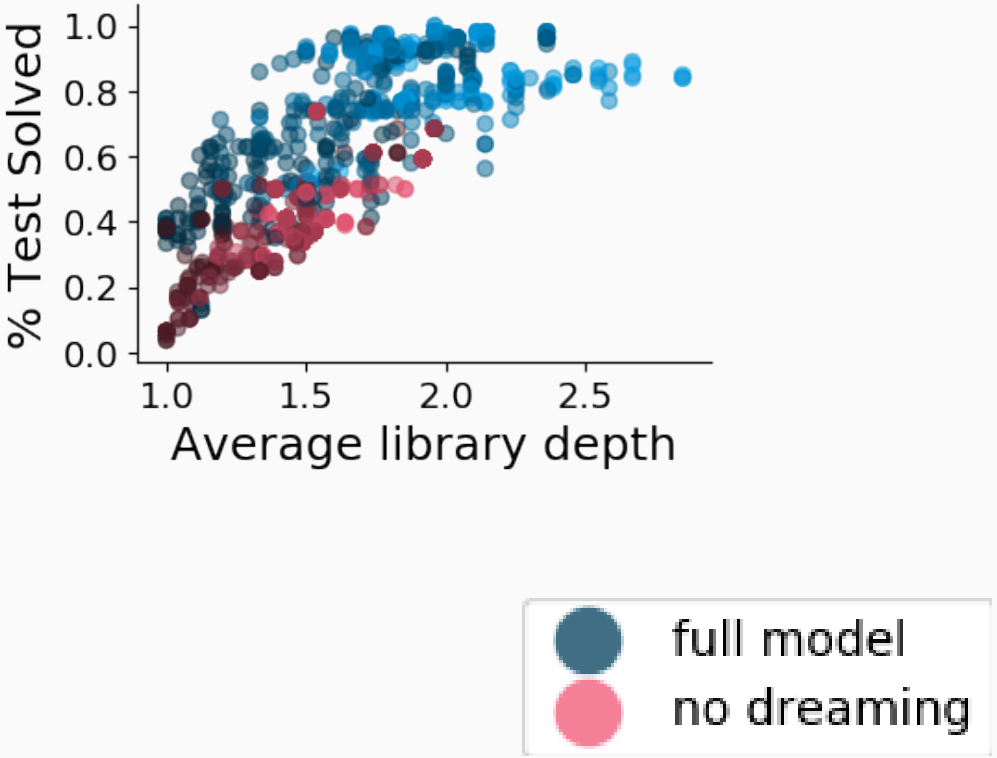
Evidence for dreaming bootstrapping better libraries



Darker: Early in learning

Brighter: Later in learning

Evidence for dreaming bootstrapping better libraries



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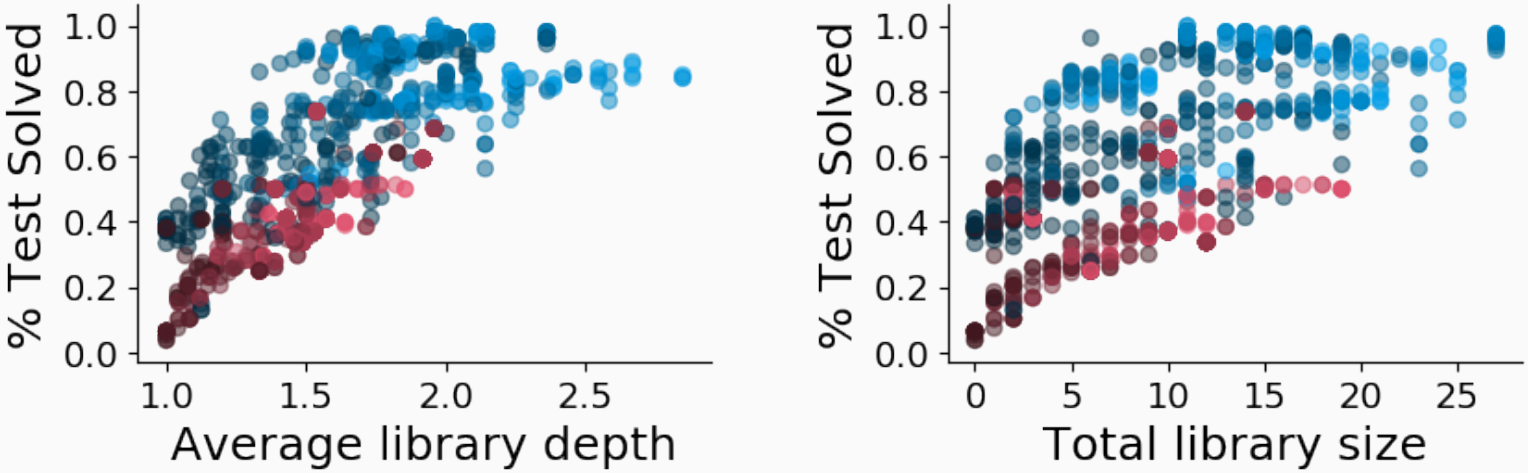
Evidence for dreaming bootstrapping better libraries



Darker: Early in learning

Brighter: Later in learning

From learning libraries, to learning languages

From learning libraries, to learning languages

modern functional programming $\rightarrow$ physics

From learning libraries, to learning languages

1950's Lisp $\rightarrow$ modern functional programming $\rightarrow$ physics

Physics Formula Sheet

Mechanics

$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$

$v = v_0 + at$

$v_x^2 - v_{x0}^2 = 2a(x - x_0)$

$\vec{a} = \frac{\Sigma \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$

$|\vec{F}_{friction}| \leq \mu |\vec{F}_{Normal}|$

$\vec{p} = m\vec{v}$

$\Delta \vec{p} = \vec{F}\Delta t$

$KE = \frac{1}{2}mv^2$

$\Delta PE = mg\Delta y$

$\Delta E = W = Fd\cos\theta$

$a_c = \frac{v^2}{r}$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$\omega = \omega_0 + \alpha t$

$T = \frac{2\pi}{\omega} = \frac{1}{f}$

$\nu = f\lambda$

$x = A\cos(2\pi f t)$

$\vec{\alpha} = \frac{\Sigma \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$

$\vec{\tau} = r \times F$

$L = I\,\omega$

$\Delta L = \tau \Delta t$

$|\vec{F}_{spring}| = k|\vec{x}|$

$PE_{spring} = \frac{1}{2}kx^2$

$T_{spring} = 2\pi\sqrt{\frac{m}{k}}$

$T_{pendulum} = 2\pi\sqrt{\frac{\ell}{g}}$

$|\vec{F}_{gravity}| = G\frac{m_1 m_2}{r^2}$

$|\vec{F}_{gravity}| = m\vec{g}$

$PE_{gravity} = -G\frac{m_1 m_2}{r}$

$\rho = \frac{m}{V}$

$KE = \frac{1}{2}I\,\omega^2$

Electricity

$|\vec{F}_E| = k\left|\frac{q_1 q_2}{r^2}\right|$

$\Delta V = IR$

$R = \frac{\rho \ell}{A}$

$I = \frac{\Delta q}{\Delta t}$

$P = I\Delta V$

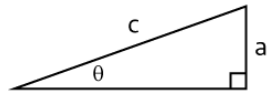
$R_{series} = R_1 + R_2 + \ldots + R_n$

$\frac{1}{R_{Parallel}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$

Geometry

Rectangle	$A = bh$	Rectangular Solid	$V = \ell wh$	Triangle	$A = \frac{1}{2}bh$
Circle	$A = \pi r^2$ $C = 2\pi r$	Cylinder	$V = \pi r^2 \ell$ $S = 2\pi r\ell + 2\pi r^2$	Sphere	$V = \frac{4}{3}\pi r^3$ $S = 4\pi r^2$

Trigonometry



$c^2 = a^2 + b^2$

$\sin\theta = \frac{a}{c}$ $\cos\theta = \frac{b}{c}$ $\tan\theta = \frac{a}{b}$

Variables

a = acceleration
A = amplitude
A = Area
b = base length
C = circumference
d = distance
E = energy
f = frequency
F = force
h = height
I = current
I = rotational inertia
KE = kinetic energy
k = spring constant
L = angular momentum
ℓ = length
m = mass
P = power
p = momentum
q = charge
r = radius
R = resistance
S = surface area
T = period
t = time
PE = potential energy
V = electric potential
V = volume
v = velocity
w = width
W = work
x = position
y = height
α = angular acceleration
λ = wavelength
μ = coefficient of friction

Growing languages for vector algebra and physics

Initial Primitives

```
map
zip
cons
empty
cdr
power
fold
car
+
-
*
/
0
1
π
```

Physics Equations

Newton's Second Law

$$\vec{a} = \frac{1}{m}\sum_i \vec{F}_i$$

Parallel Resistors

$$R_{total} = \left(\Sigma_i \frac{1}{R_i}\right)^{-1}$$

Work

$$U = \vec{F} \cdot \vec{d}$$

Force in a Magnetic Field

$$|\vec{F}| = q|\vec{v} \times \vec{B}|$$

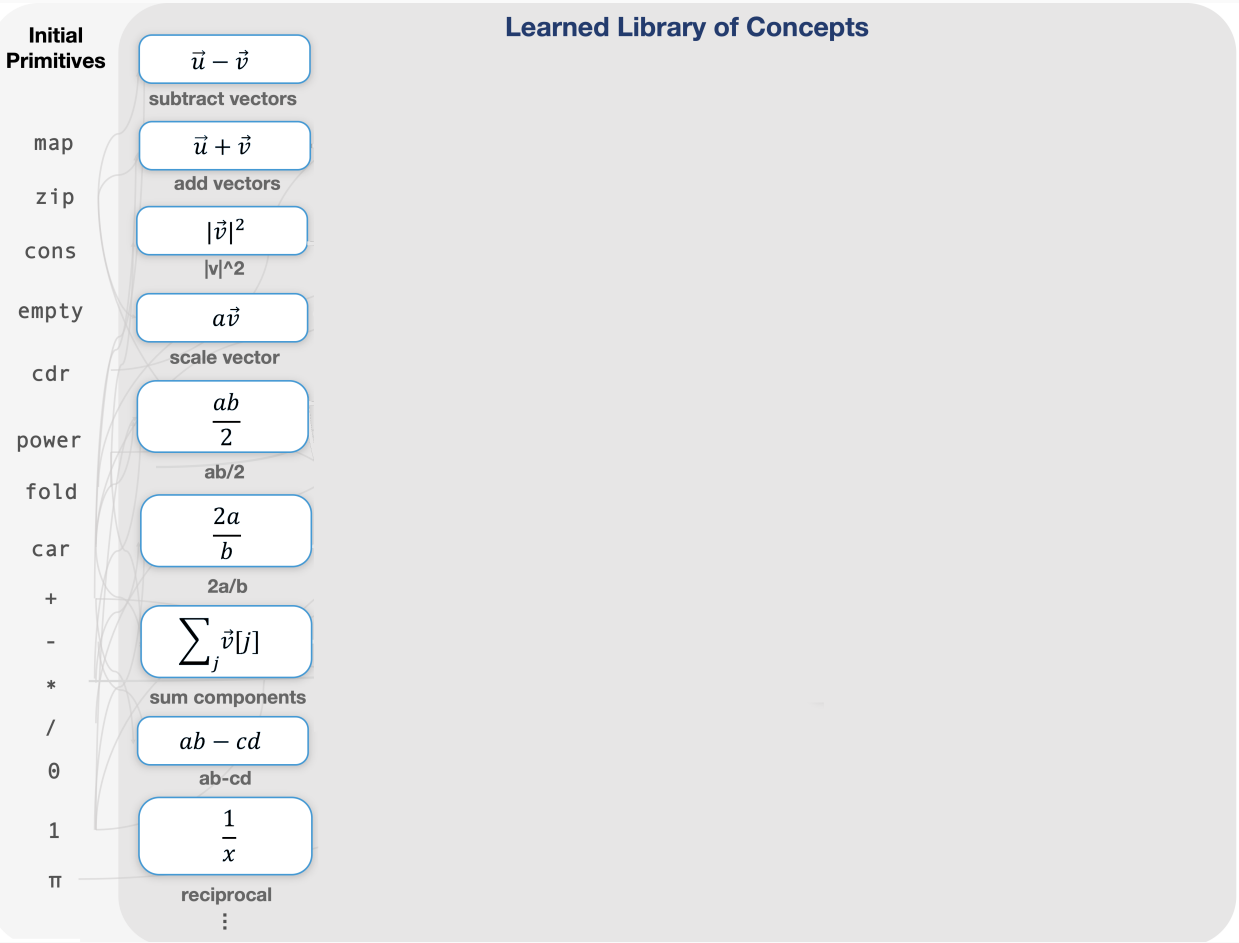
Kinetic Energy

$$KE = \frac{1}{2}m|\vec{v}|^2$$

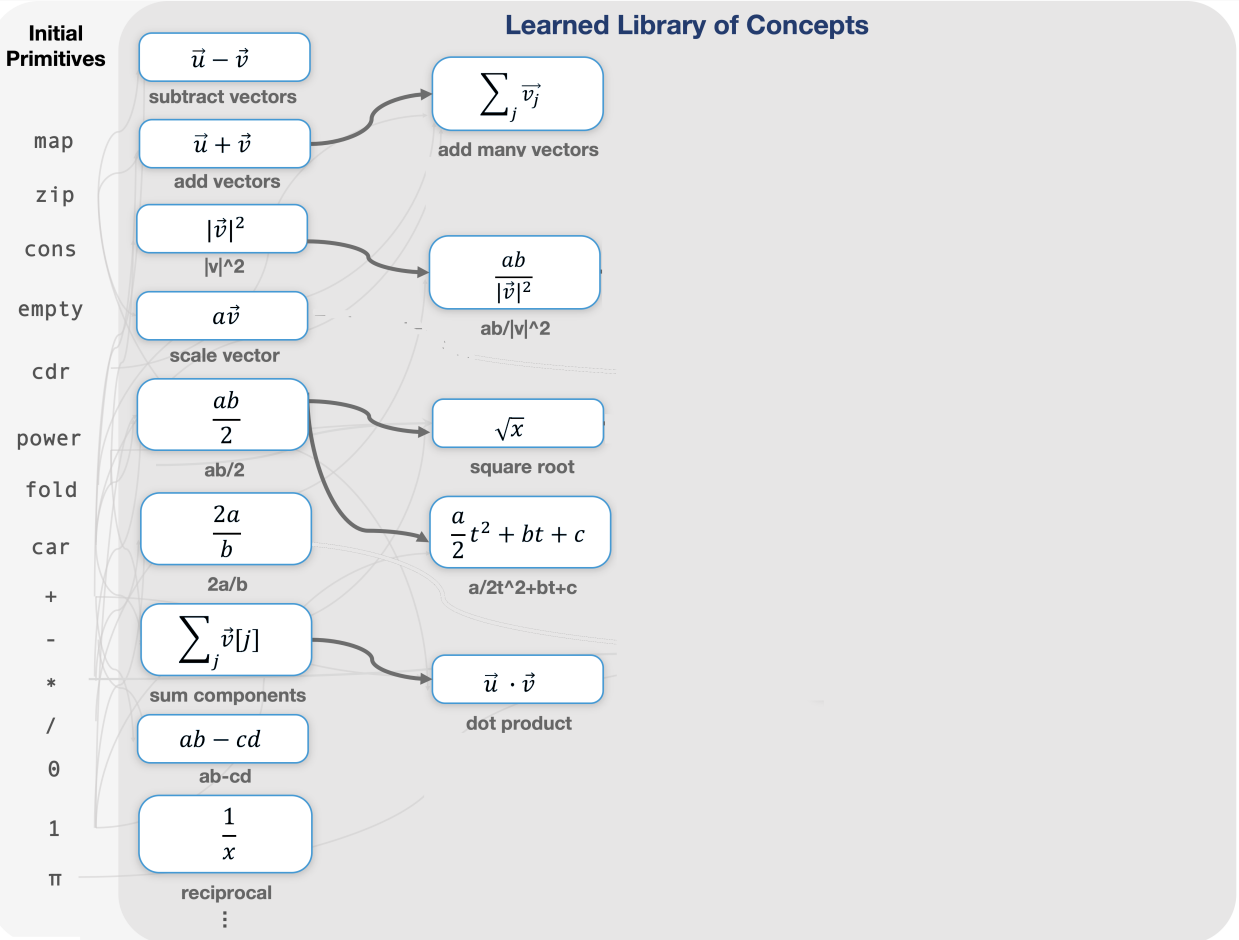
Coulomb's Law

$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r_1} - \vec{r_2}|^2}\widehat{r_1 - r_2}$$

Growing languages for vector algebra and physics



Growing languages for vector algebra and physics



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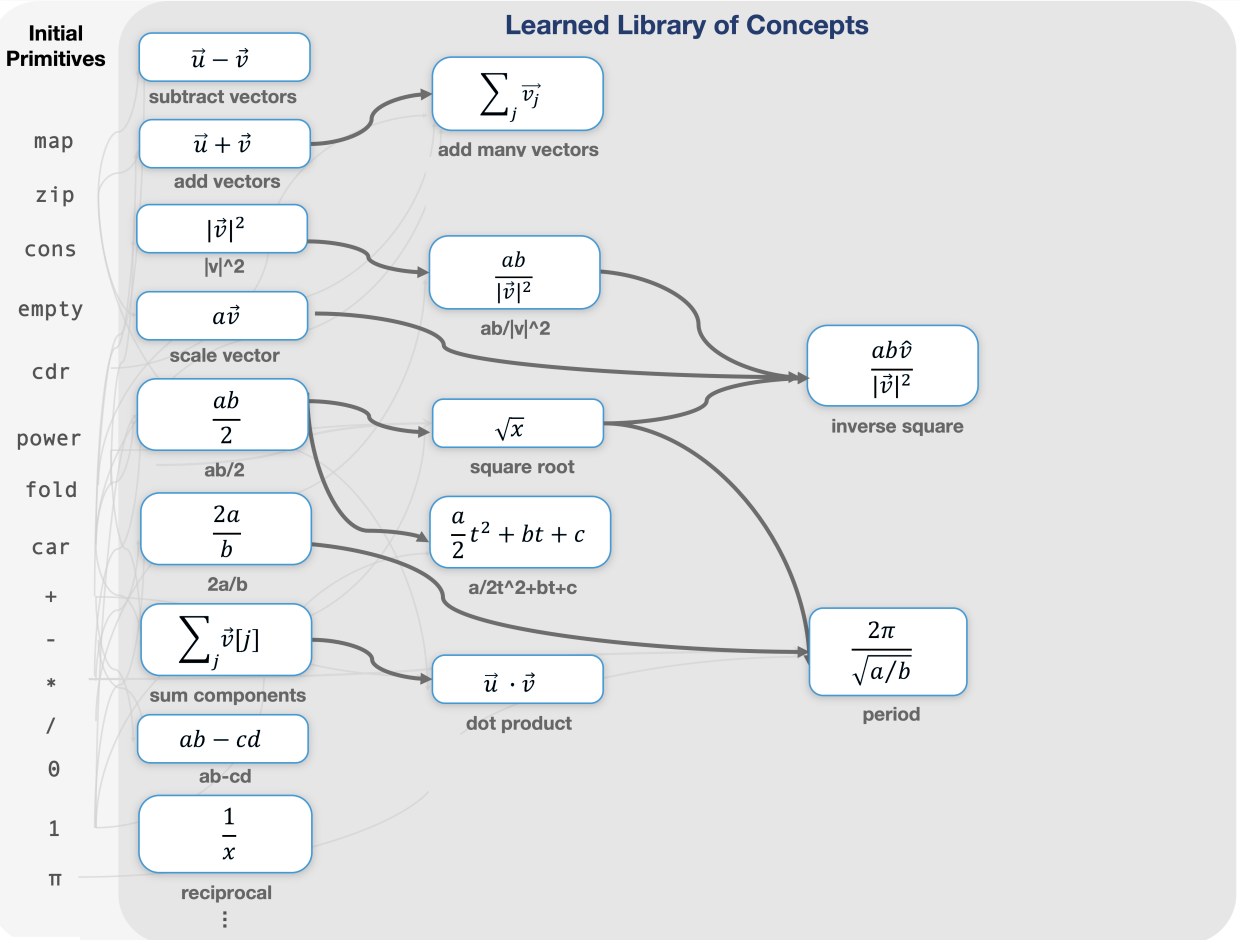

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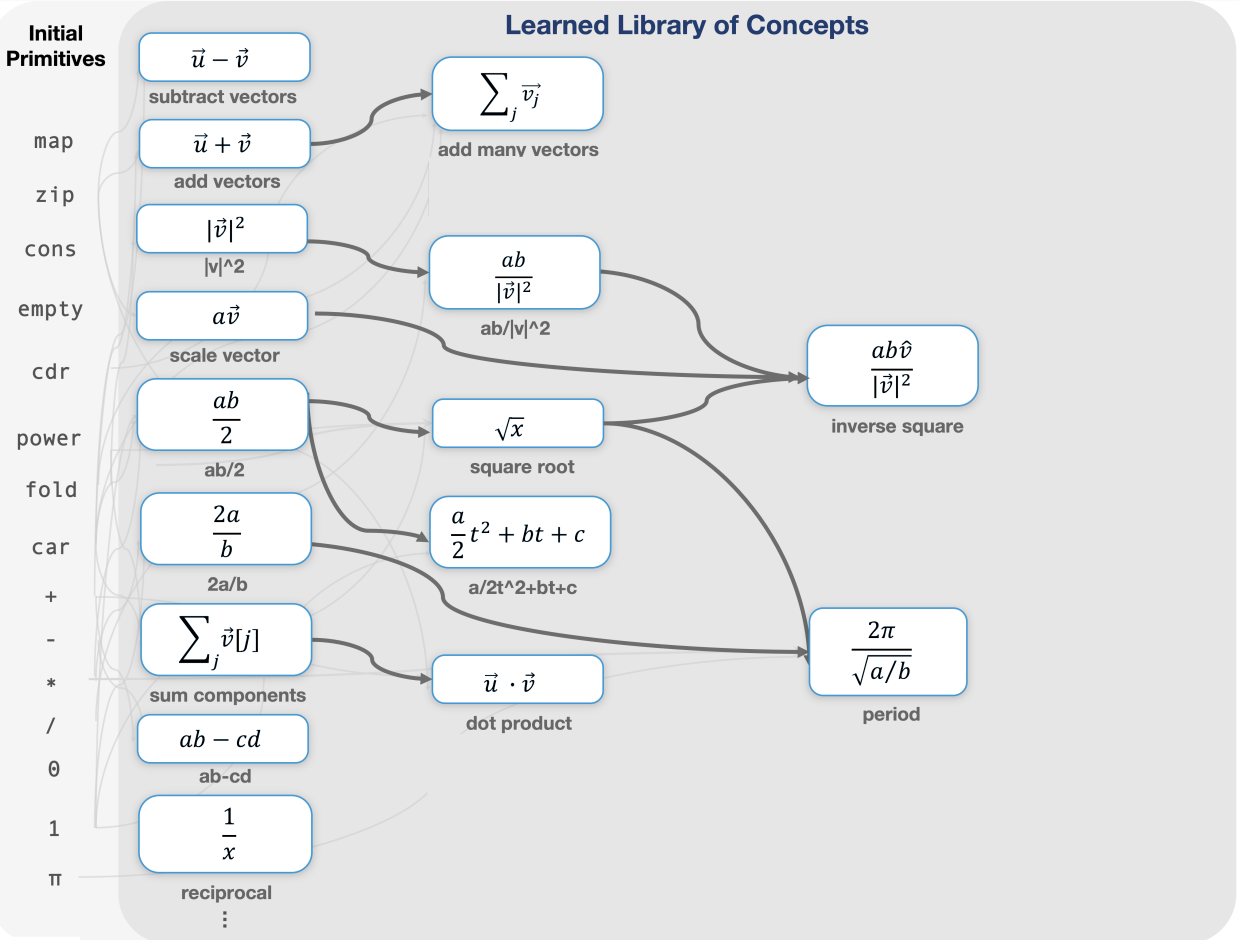


Physics Equations

Newton's Second Law

$$\vec{a} = \frac{1}{m}\sum_i \vec{F}_i$$

```
(scale-vector(reciprocal m)
(add-many-vectors Fs))
```

Parallel Resistors

$$R_{total} = \left(\Sigma_i \frac{1}{R_i}\right)^{-1}$$

```
(reciprocal (sum-components
(map (λ(r) (reciprocal r))
Rs)))
```

Work

$$U = \vec{F} \cdot \vec{d}$$

```
(dot-product F d)
```

Force in a Magnetic Field

$$|\vec{F}| = q|\vec{v} \times \vec{B}|$$

```
(* q (ab-cd v_x b_y v_y b_x))
```

Kinetic Energy

$$KE = \frac{1}{2}m|\vec{v}|^2$$

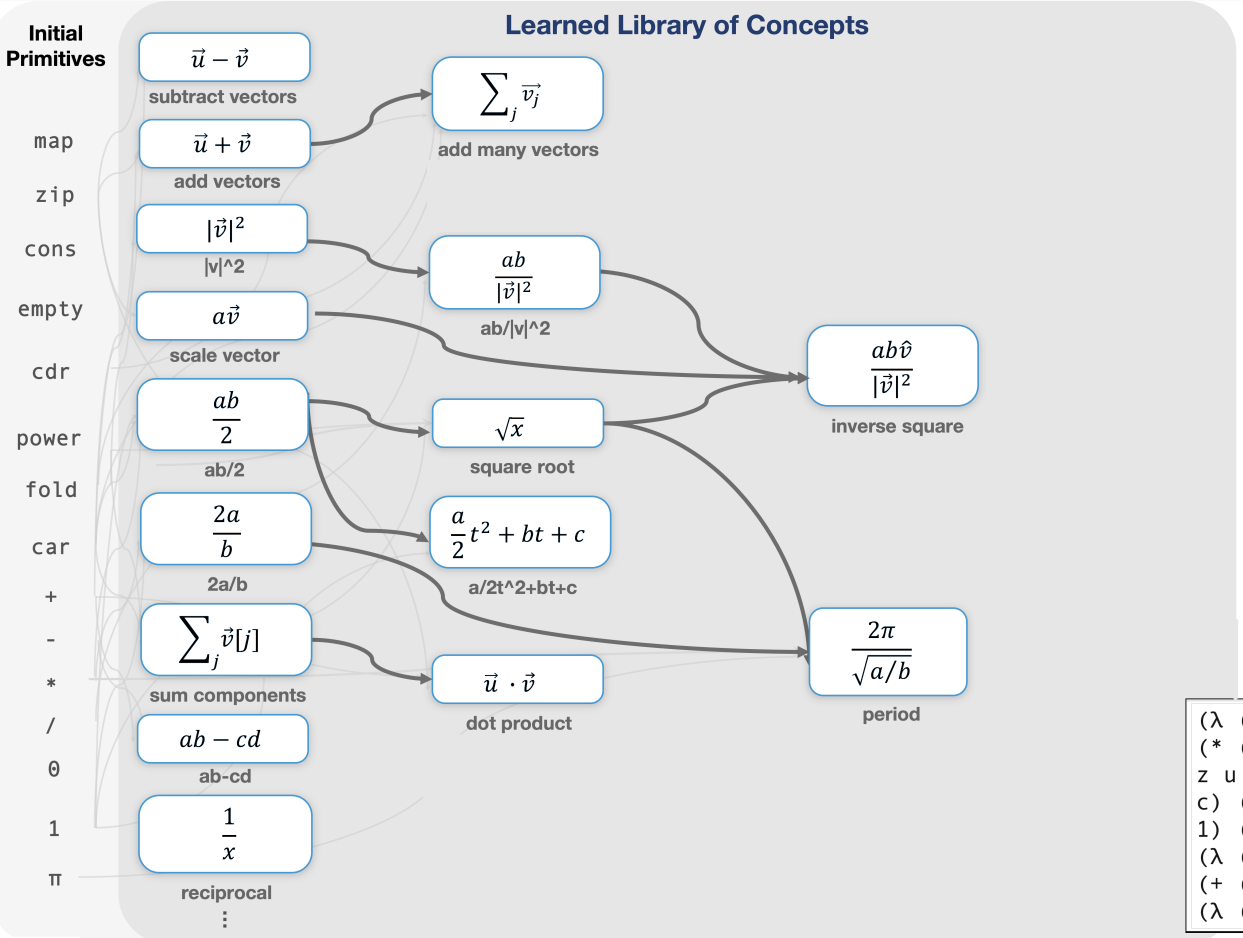
```
(ab/2 m (|v|^2 v))
```

Coulomb's Law

$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r_1} - \vec{r_2}|^2}\widehat{r_1 - r_2}$$

```
(inverse-square q_1 q_2
(subtract-vectors r_1 r_2))
```

Growing languages for vector algebra and physics



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```
(inverse-square q_1 q_2
(subtract-vectors r_1 r_2))
```

```
(λ (x y z u) (map (λ (v) (* (/
(* (power (/ (* x x) (fold (zip
z u (λ (w a) (- w a))) 0 (λ (b
c) (+ (* b b)    c)))) (/ (* 1
1) (+ 1 1))) y) (fold (zip z u
(λ (d e)  (- d e))) 0 (λ (f g)
(+ (* f f) g)))) v)) (zip z u
(λ (h i) (- h i)))))
```

Solution to `Coulomb's Law` if expressed in initial primitives

Growing a language for recursive programming

Initial Primitives

- Y combinator
- cons
- car
- cdr
- nil
- if
- nil?
- +
- -
- 0
- 1
- =

Recursive Programming Algorithms

Stutter

[■■] → [■■■■]
[■■■] → [■■■■■■]

Take every other

[■■■■] → [■■]
[■■■■■■] → [■■■]

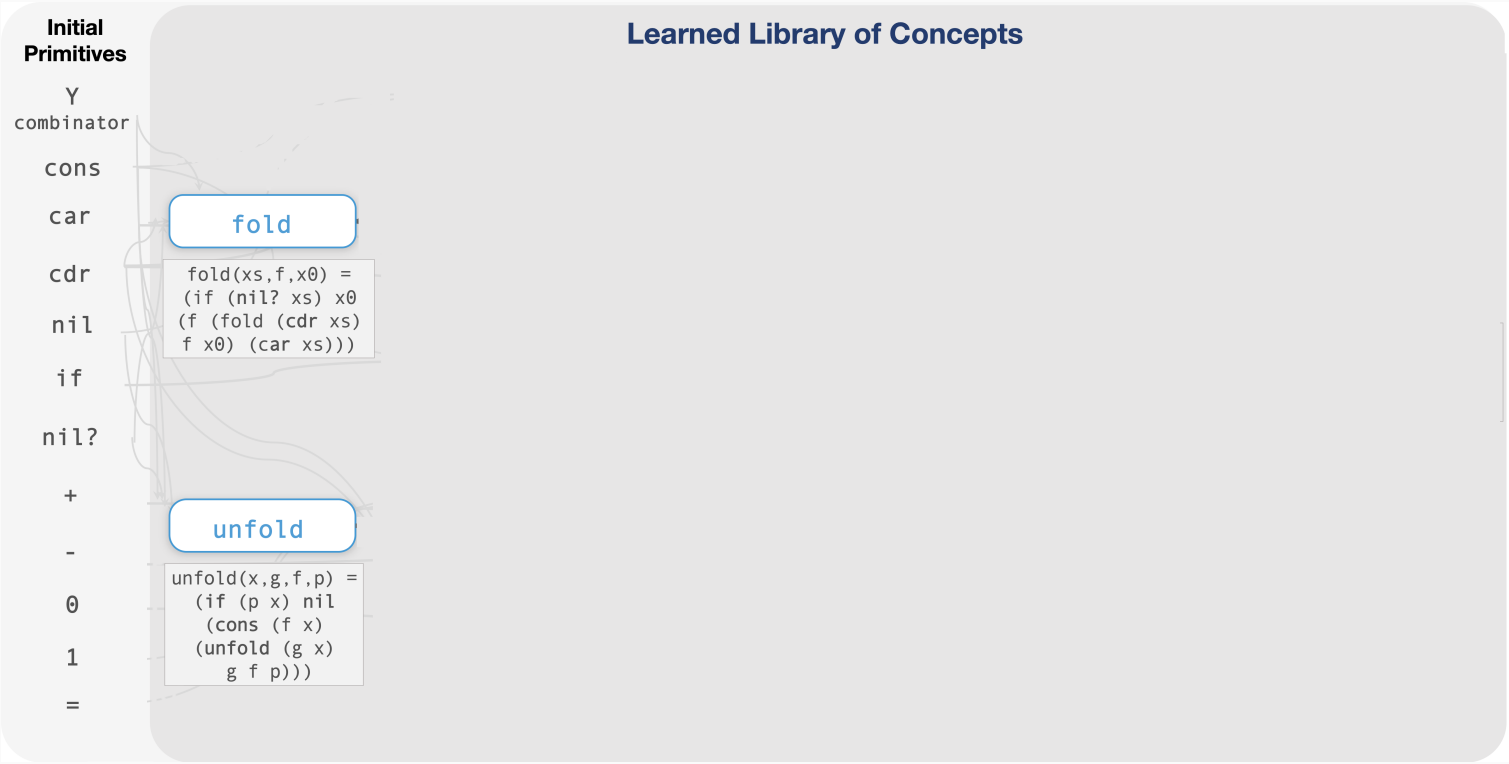
List lengths

[[■■■], [■]] → [3 1]
[[■■], [], [■]] → [2 0 1]

List differences

[1 8 2], [0 5 1] → [1 3 1]
[2 3 6], [1 2 4] → [1 1 2]

Growing a language for recursive programming



Recursive Programming Algorithms

Stutter

[■■] → [■■■■]
[■■■] → [■■■■■■]

```
(fold A (λ (u v) (cons
v (cons v u))) nil)
```

Take every other

[■■■■] → [■■]
[■■■■■■] → [■■■]

List lengths

[[■■■], [■]] → [3 1]
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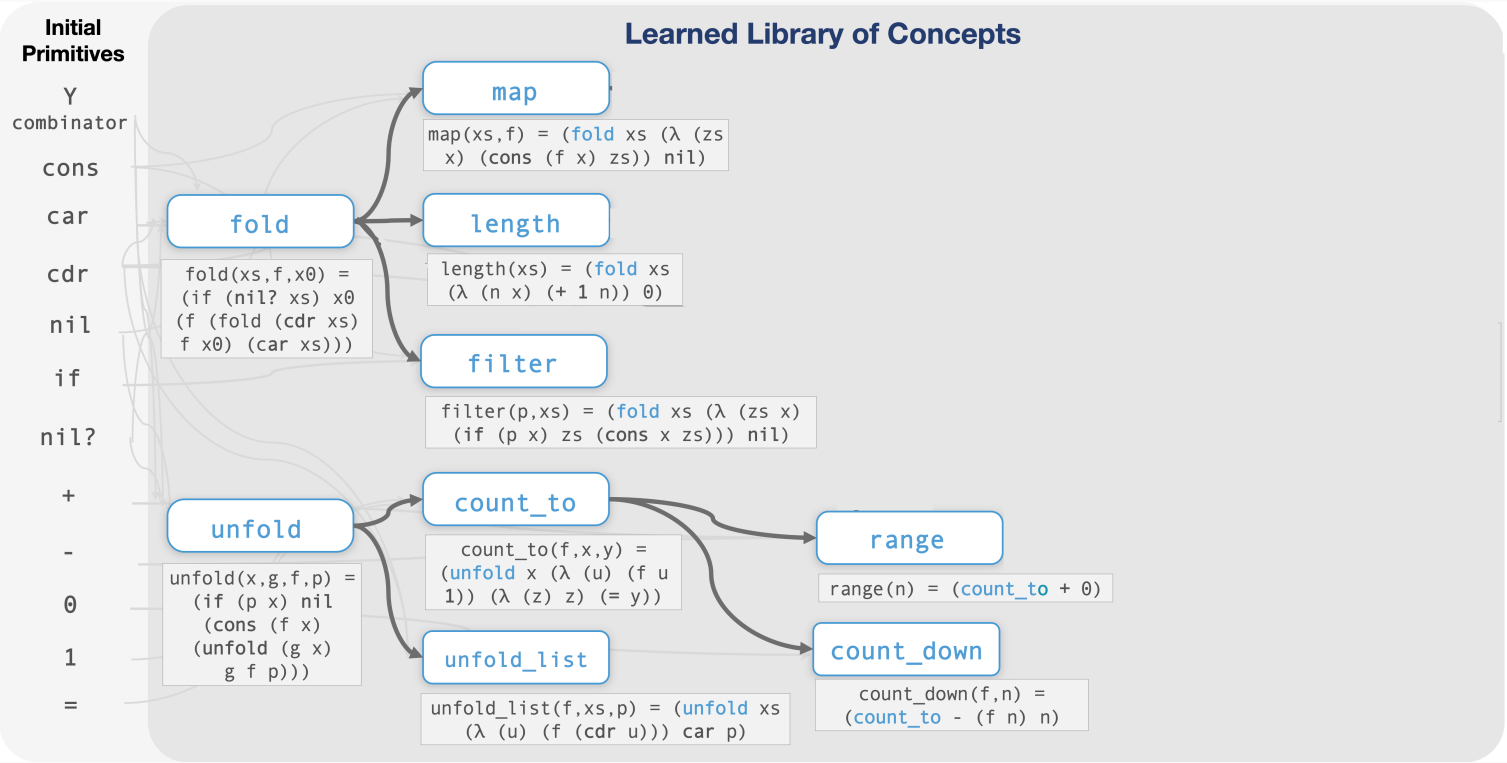

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v (cons v u))) nil)
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Take every other

[■■■■] → [■■]
[■■■■■■] → [■■■]

```
(unfold_list cdr A nil?)
```

List lengths

[[■■■], [■]] → [3 1]
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```
(map A length)
```

List differences

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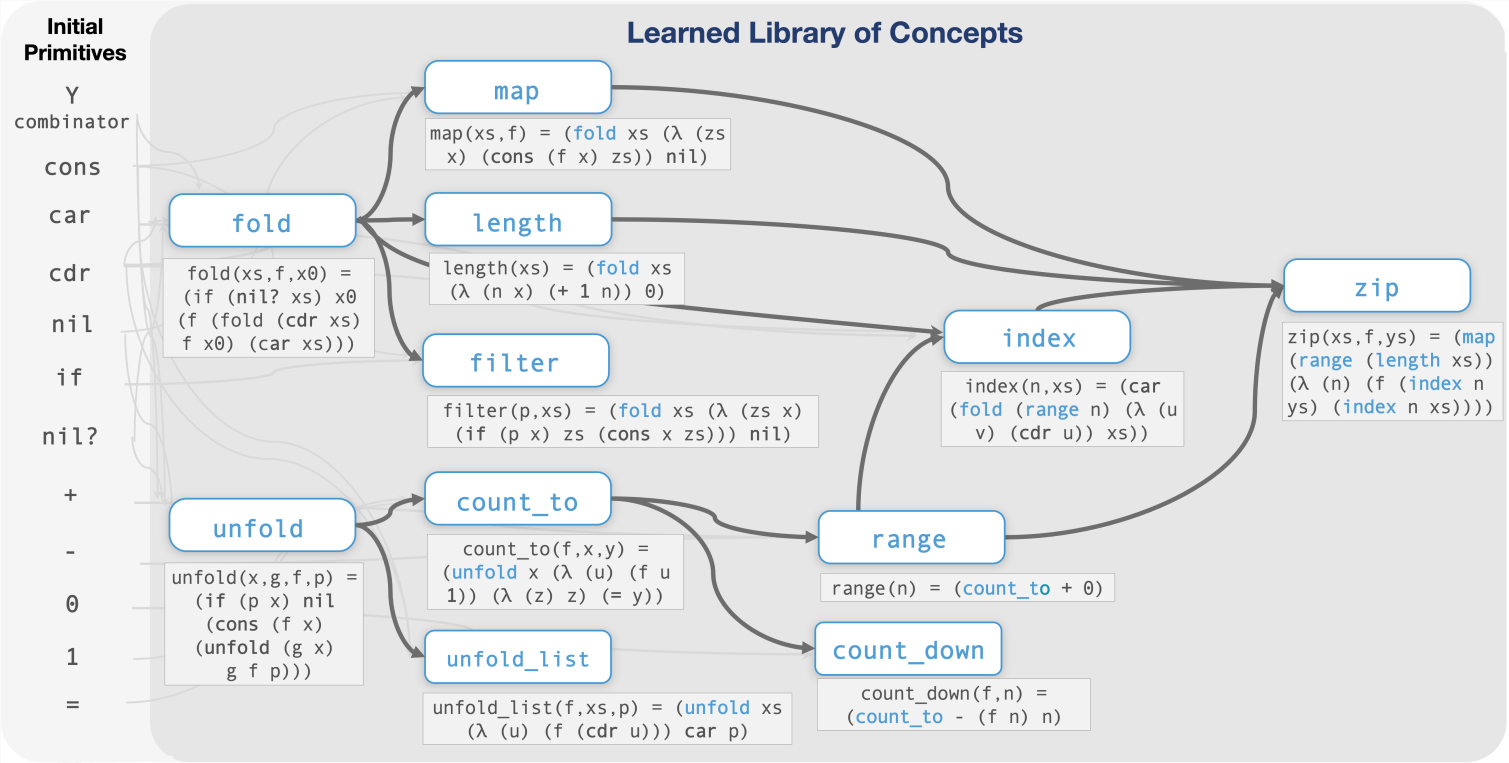
Growing a language for recursive programming



Recursive Programming Algorithms

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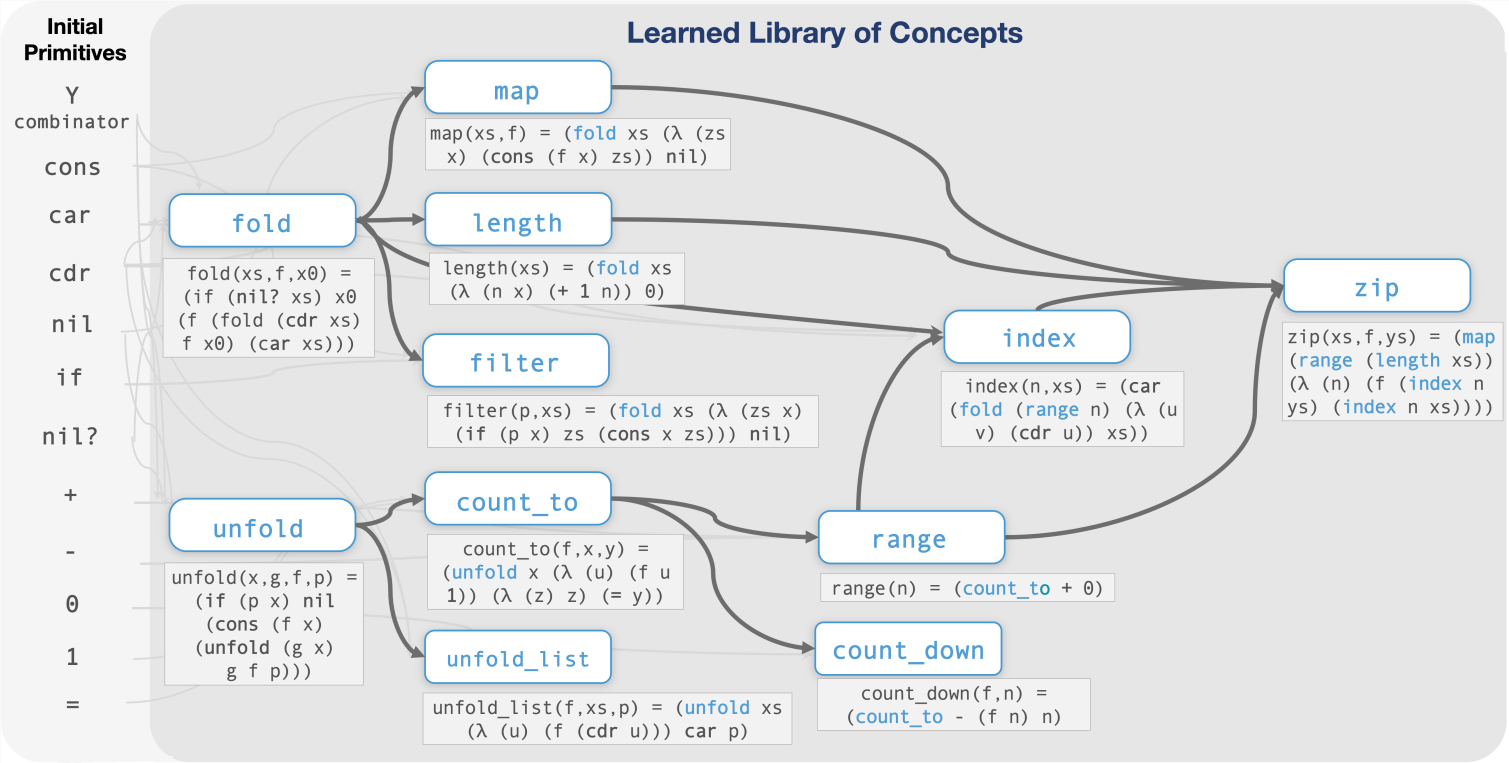
```

List differences

[1 8 2], [0 5 1] → [1 3 1]
[2 3 6], [1 2 4] → [1 1 2]

```
(zip A - B )
```

Growing a language for recursive programming



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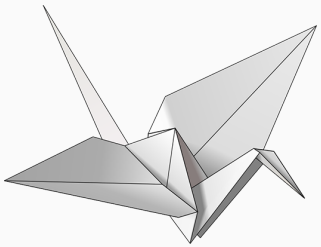
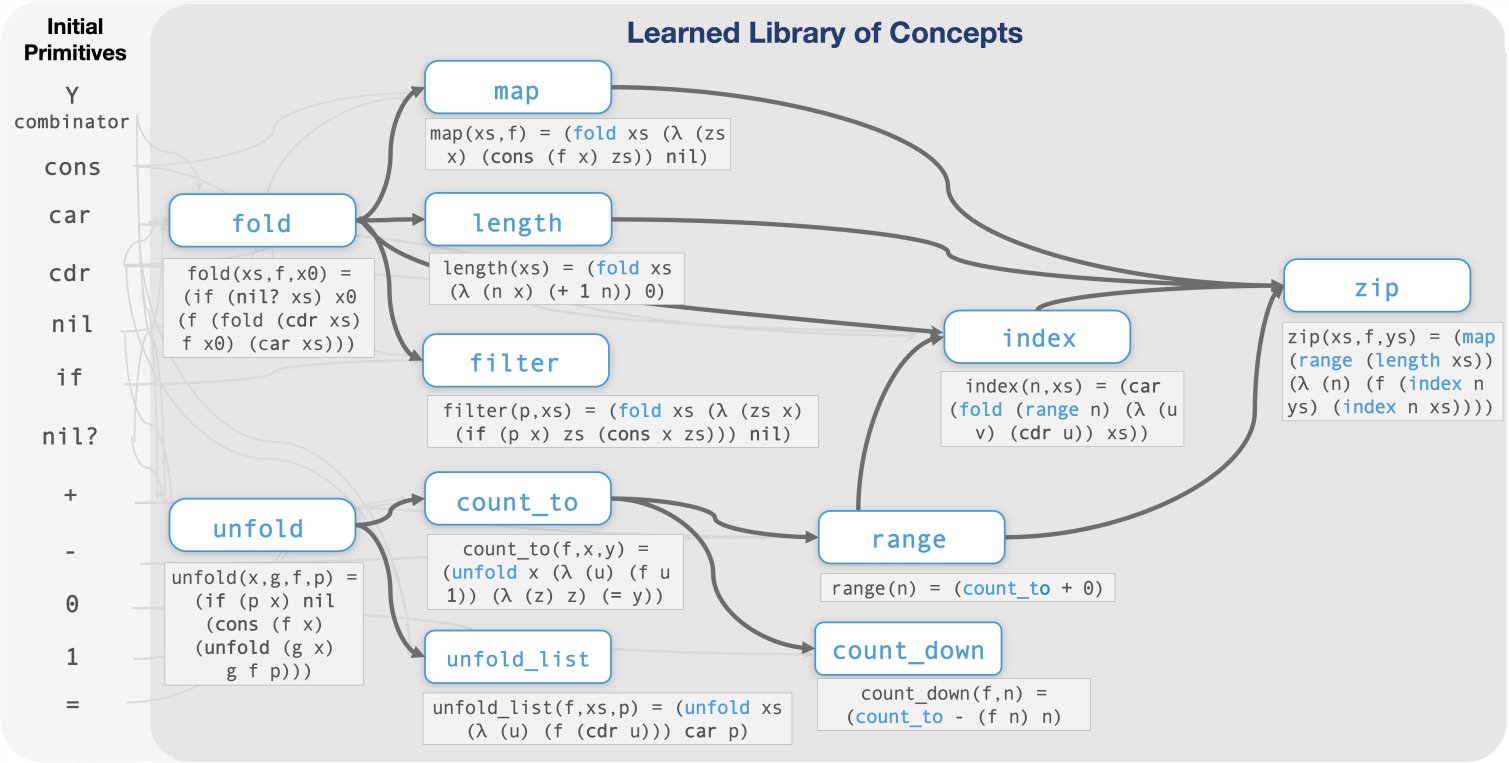


Origami Programming: Jeremy Gibbons, 2003

Growing a language for recursive programming



Recursive Programming Algorithms

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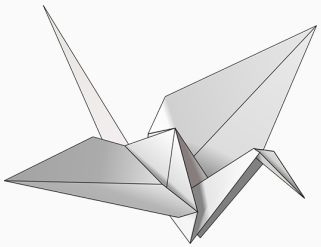
```

List differences

[1 8 2], [0 5 1] → [1 3 1]
[2 3 6], [1 2 4] → [1 1 2]

```
(zip A - B )
```

1 year of compute. 5 days on 64 CPUs.



Origami Programming: Jeremy Gibbons, 2003

Lessons

Symbols aren't necessarily interpretable. Flexibly grow the language based on experience to make it more powerful *and* more human understandable

Learning-from-scratch is possible in principle. Don't do it. But program induction makes it convenient to build in what we know how to build in, and then learn and adapt on top of that

Program Induction and perception

learning to learn

model discovery

the future

This work:

a toolkit for program induction,

addressing combinatorial program search via learning, integrating techniques for machine learning, AI, and programming languages

Where should we aim?

What we want for the future of machine learning

Strong generalization

What we want for the future of machine learning

Strong generalization

Bootstrapping, learning-to-learn, representation learning

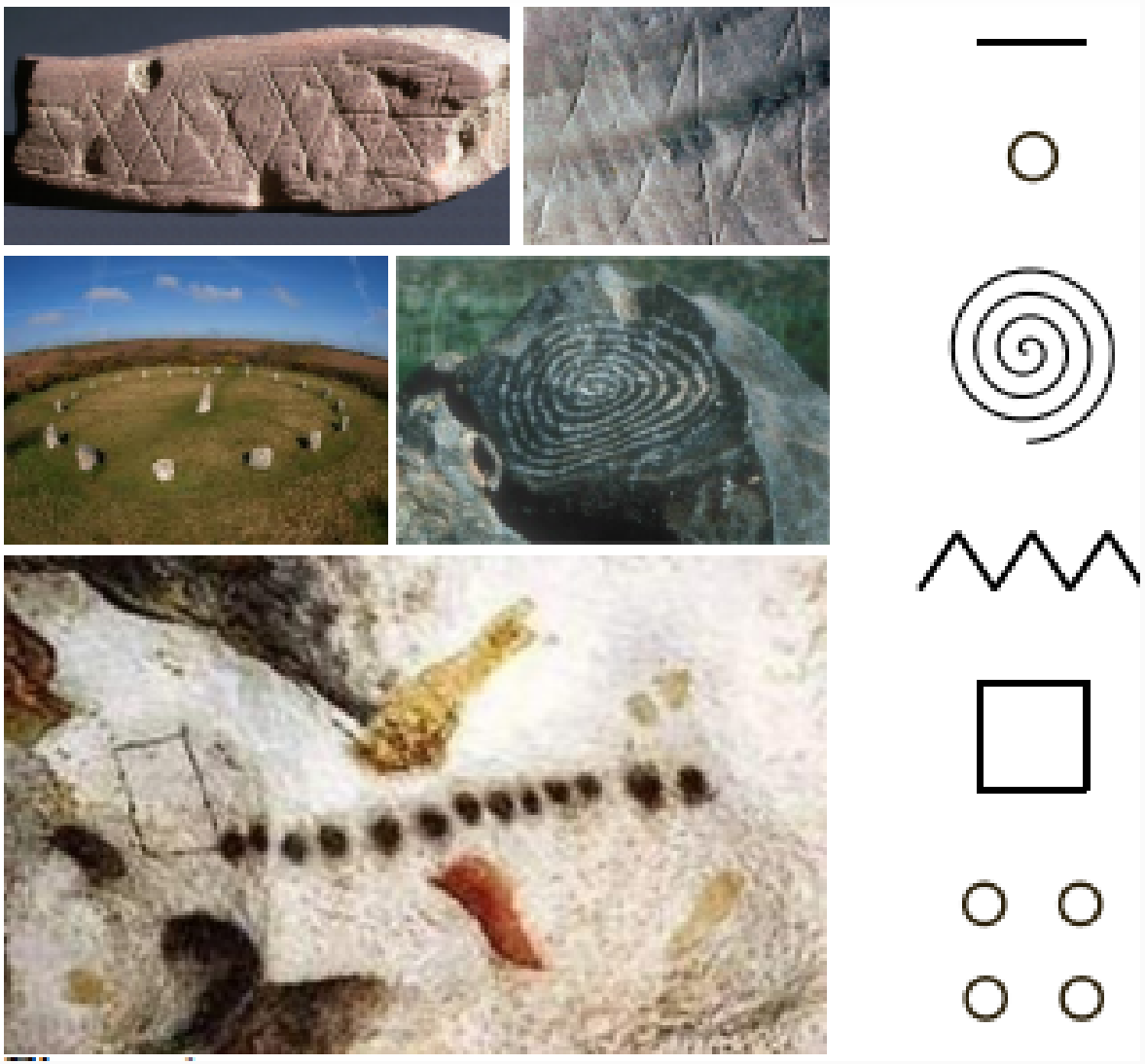
What we want for the future of machine learning

Strong generalization

Bootstrapping, learning-to-learn, representation learning

Discovering knowledge that humans can understand and build on

A language of thought for the mental representation of geometric shapes



Mathias Sable-Meyer, Ellis, Tenenbaum, Dehaene. 2022

A language of thought for the mental representation of geometric shapes

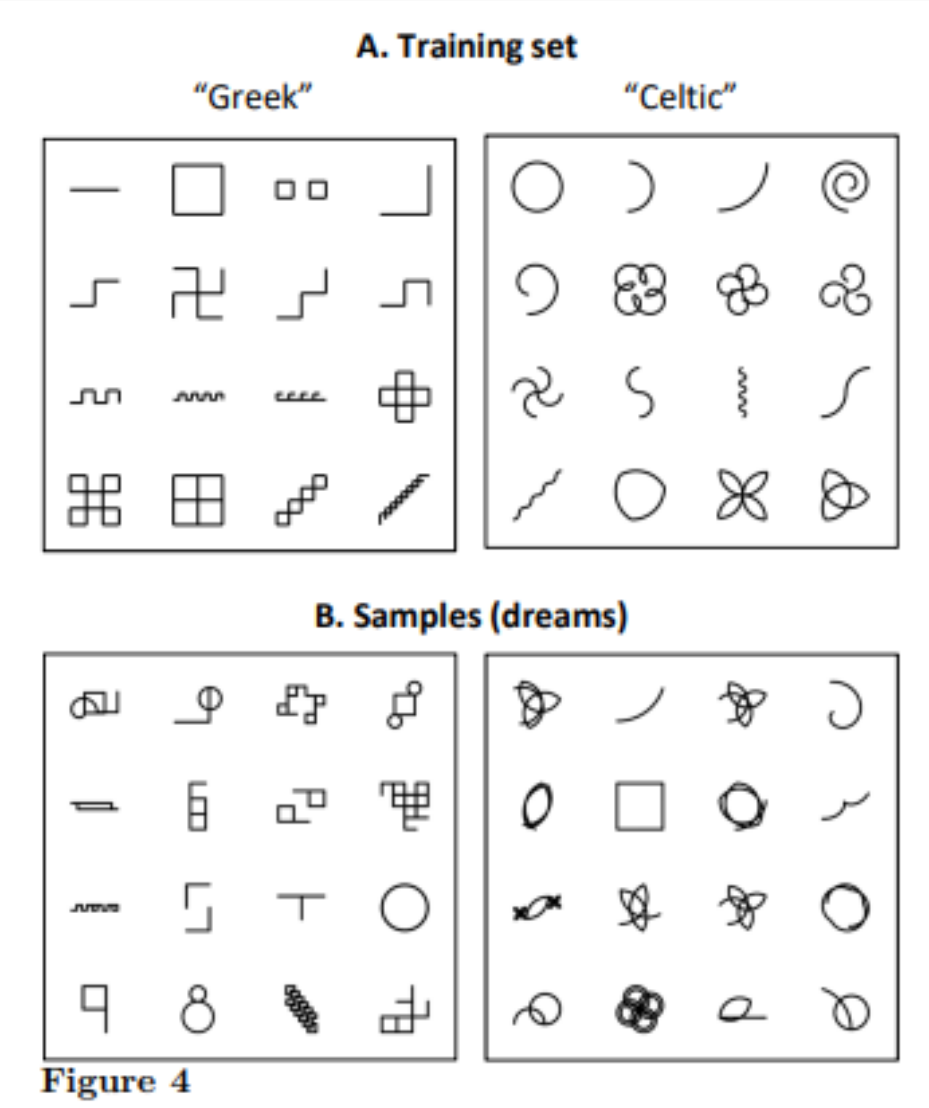


Figure 4

Mathias Sable-Meyer, Ellis, Tenenbaum, Dehaene. 2022

A language of thought for the mental representation of geometric shapes

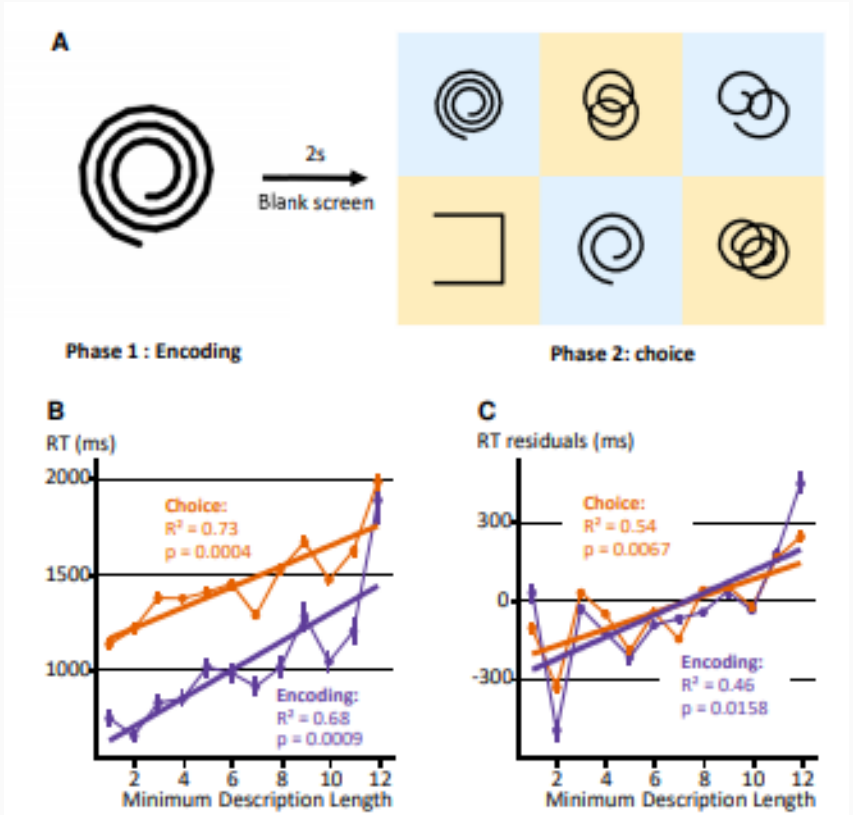


Figure 5

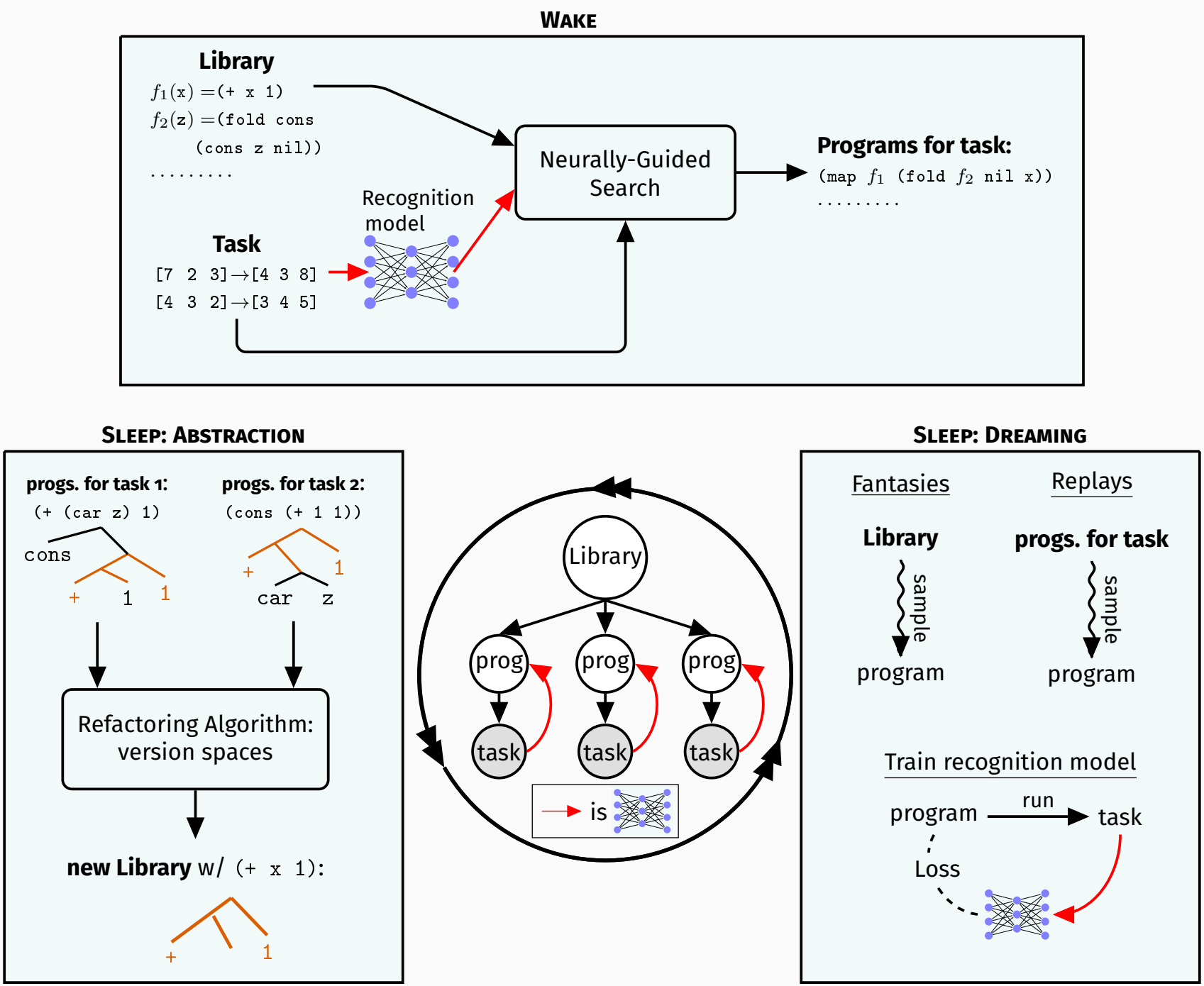
Program structure predicts:

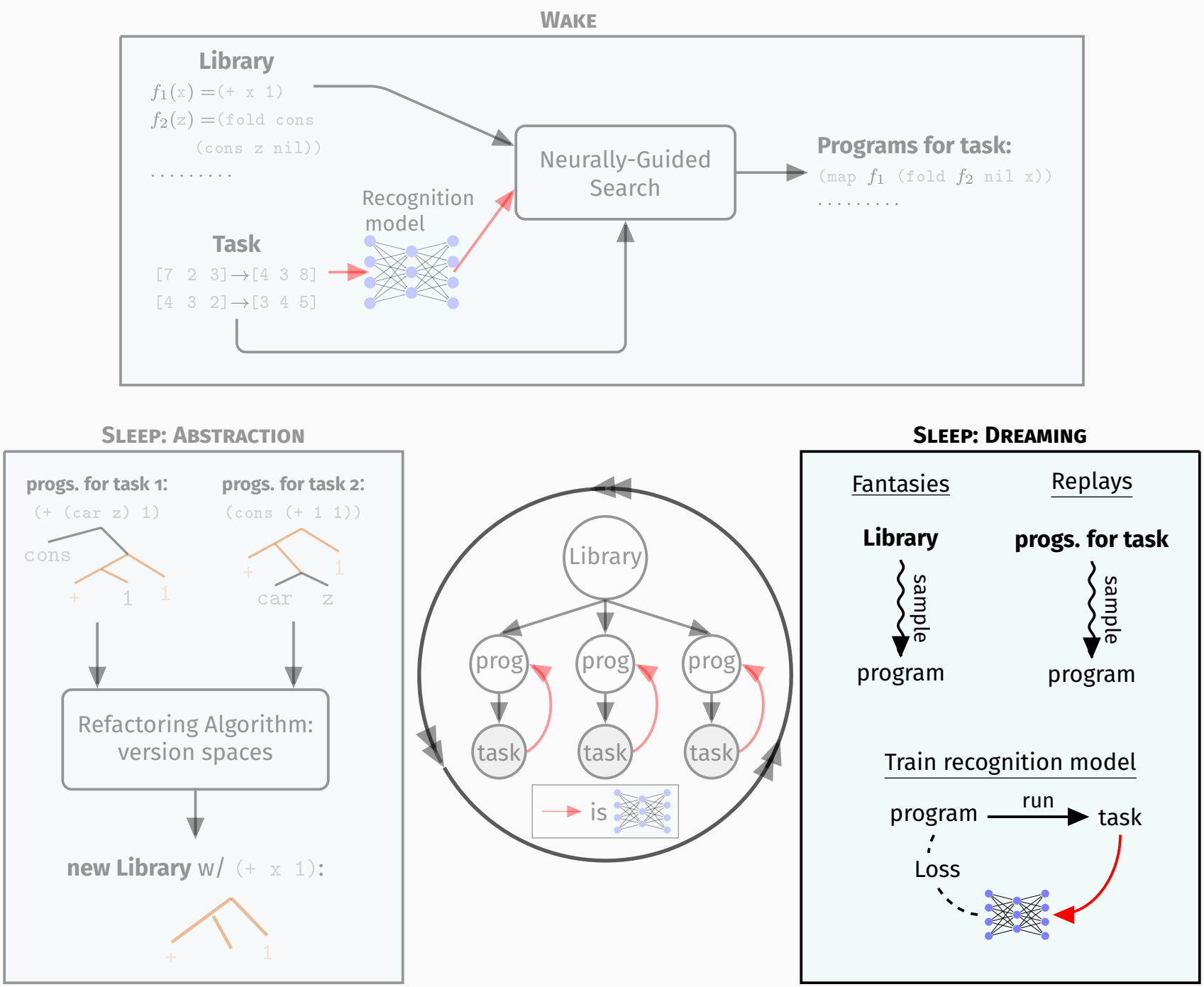
human shape similarity judgments

human processing time

Mathias Sable-Meyer, Ellis, Tenenbaum, Dehaene. 2022

the end.


WAKE
Library
f_1(x) =(+ x 1)
f_2(z) =(fold cons
(cons z nil))
.....
Task
[7 2 3]→[4 3 8]
[4 3 2]→[3 4 5]
Recognition model
Neurally-Guided Search
Programs for task:
(map f_1 (fold f_2 nil x))
.....
SLEEP: ABSTRACTION
progs. for task 1:
(+ (car z) 1)
cons
+
1
1
progs. for task 2:
(cons (+ 1 1))
+
1
car
z
Refactoring Algorithm: version spaces
new Library w/ (+ x 1):
+
1
Library
prog
prog
prog
task
task
task
is
SLEEP: DREAMING
Fantasies
Replays
Library
progs. for task
sample
sample
program
program
Train recognition model
program
run
task
Loss


WAKE
Library
$f_1(x) = (+\ x\ 1)$
$f_2(z) = (\text{fold cons } (\text{cons } z \text{ nil}))$
.....
Recognition model
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Programs for task:
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.....
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progs. for task 1:
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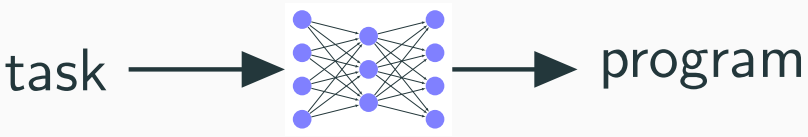
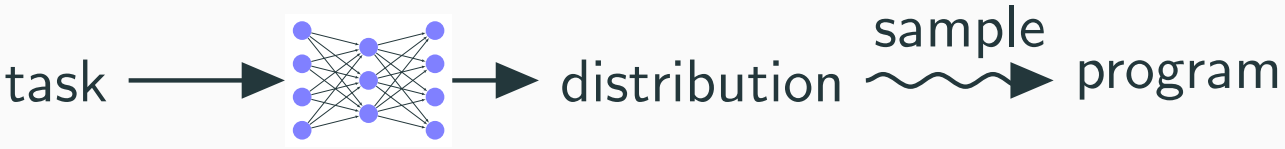
progs. for task 2:
(cons (+ 1 1))
cons
+
1
1
car
z
Refactoring Algorithm: version spaces
new Library w/ (+ x 1):
Library
prog
task
is
SLEEP: DREAMING
Fantasies
Replays
Library
progs. for task
sample
program
Train recognition model
run
Loss

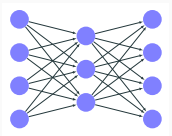
Neural recognition model guides search



Neural recognition model guides search



Neural recognition model guides search

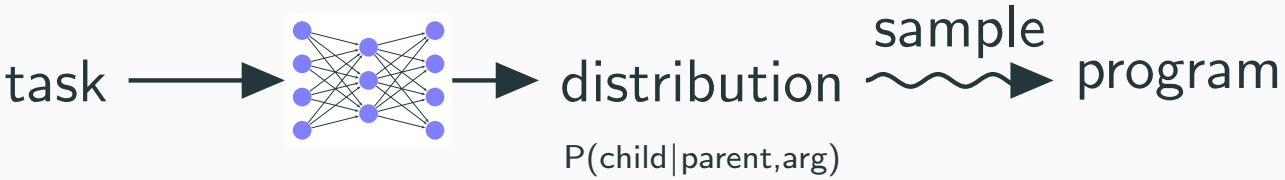


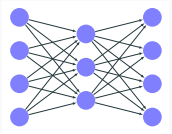
is a...

recurrent network (Devlin et al 2017)

unigram model (Menon et al 2013; Balog et al 2016)

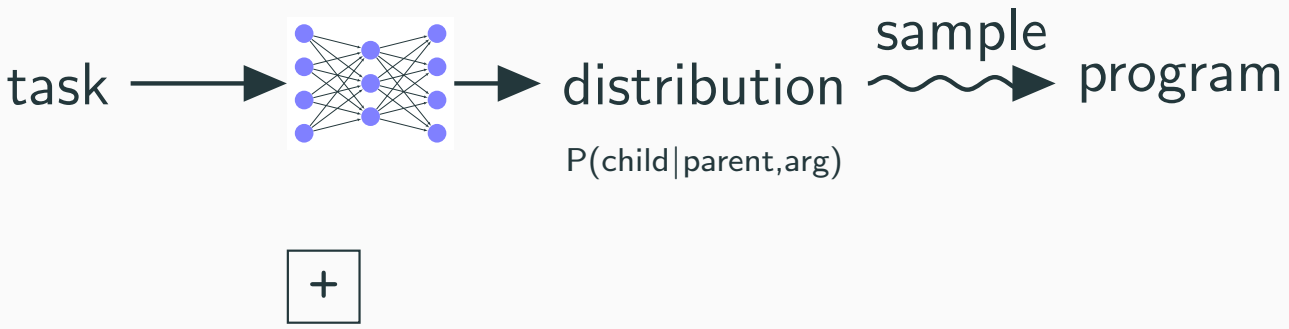
Neural recognition model guides search



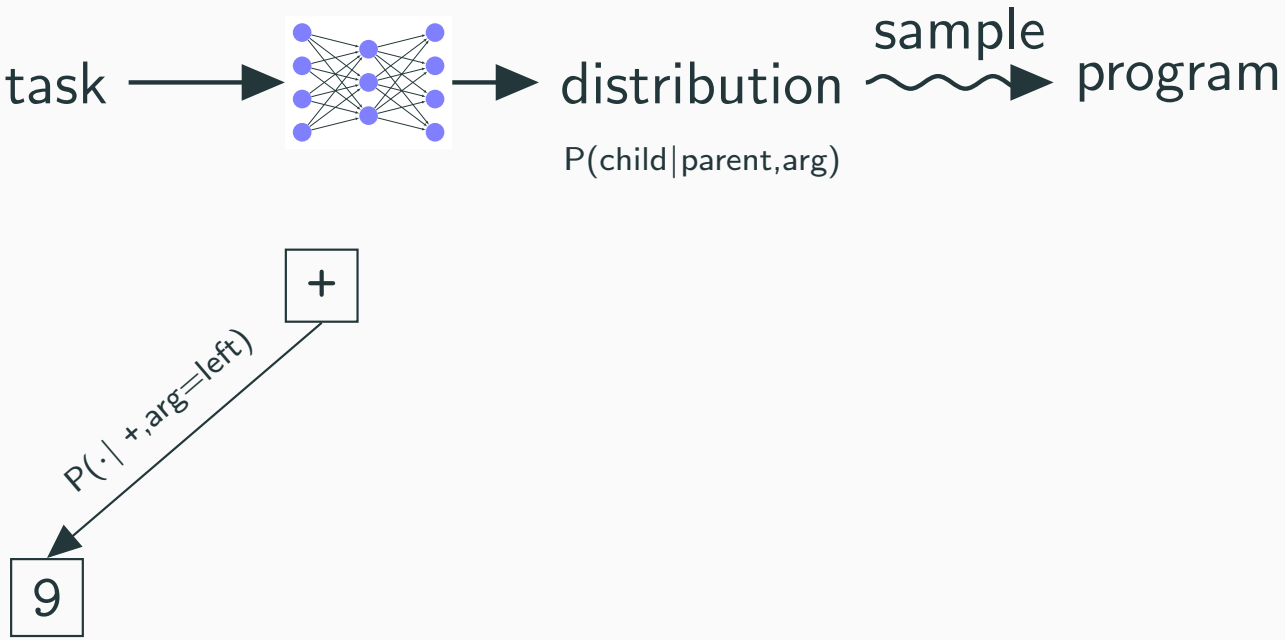


is a **"bigram" model over syntax trees**

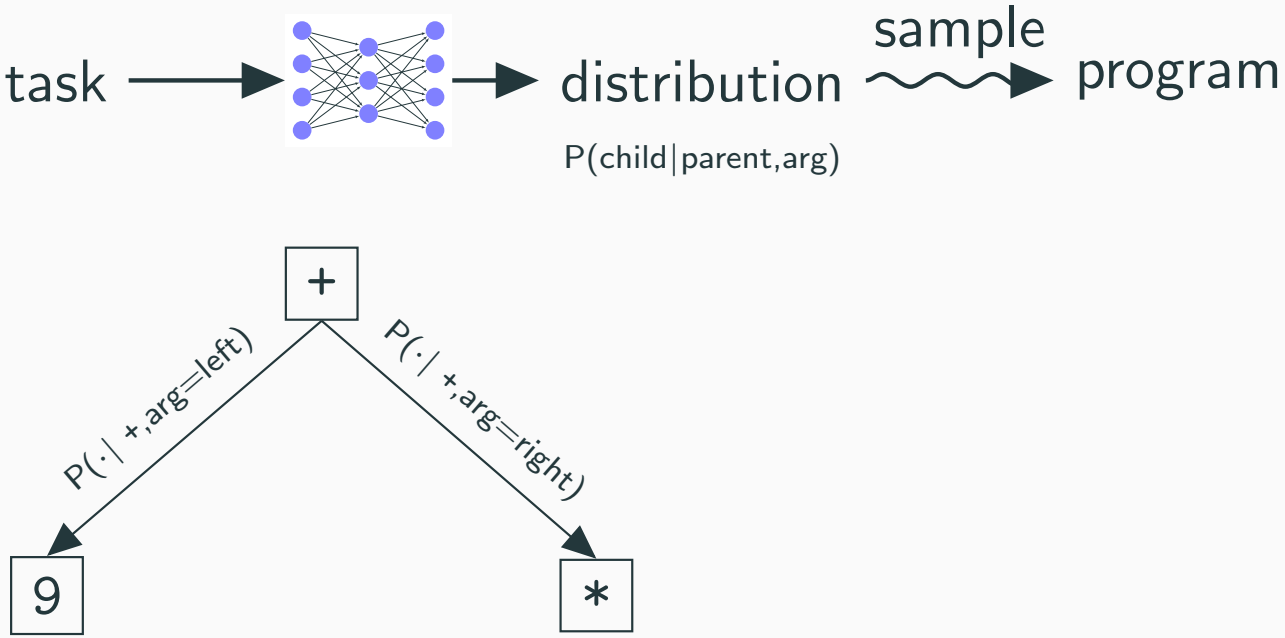
Neural recognition model guides search



Neural recognition model guides search



Neural recognition model guides search



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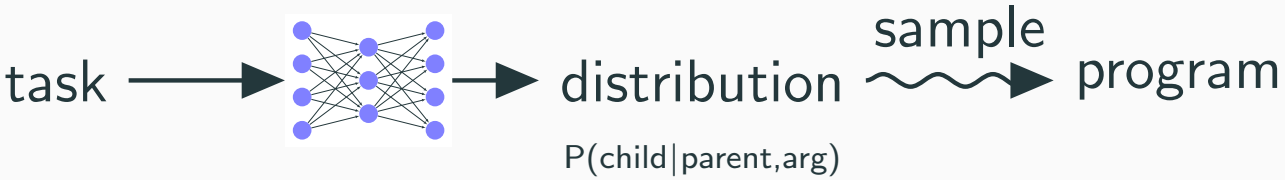

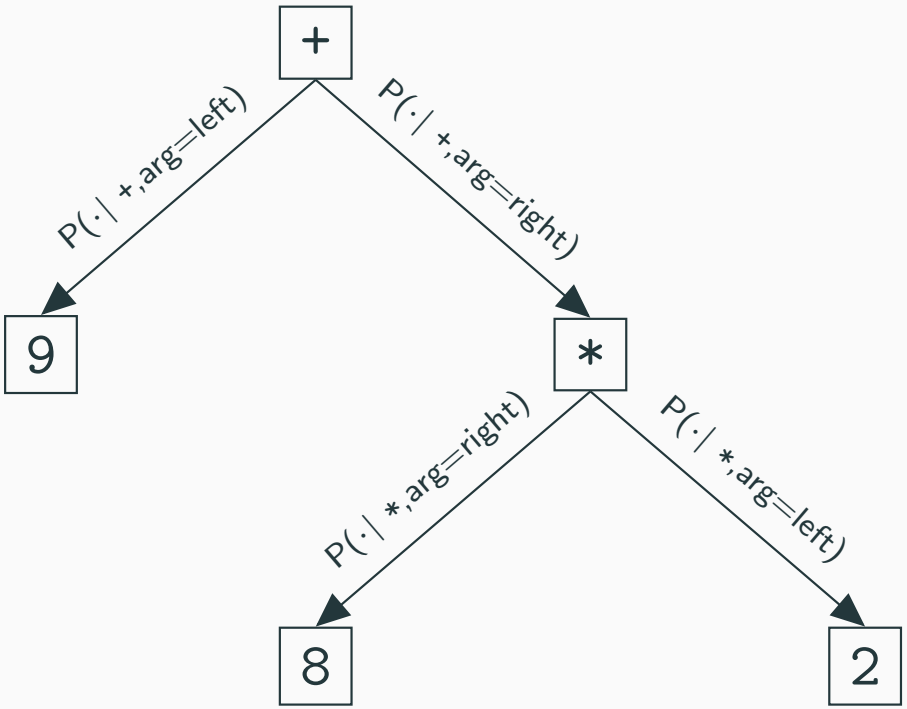

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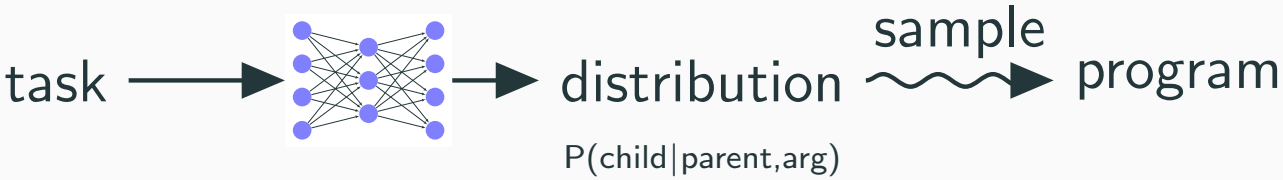
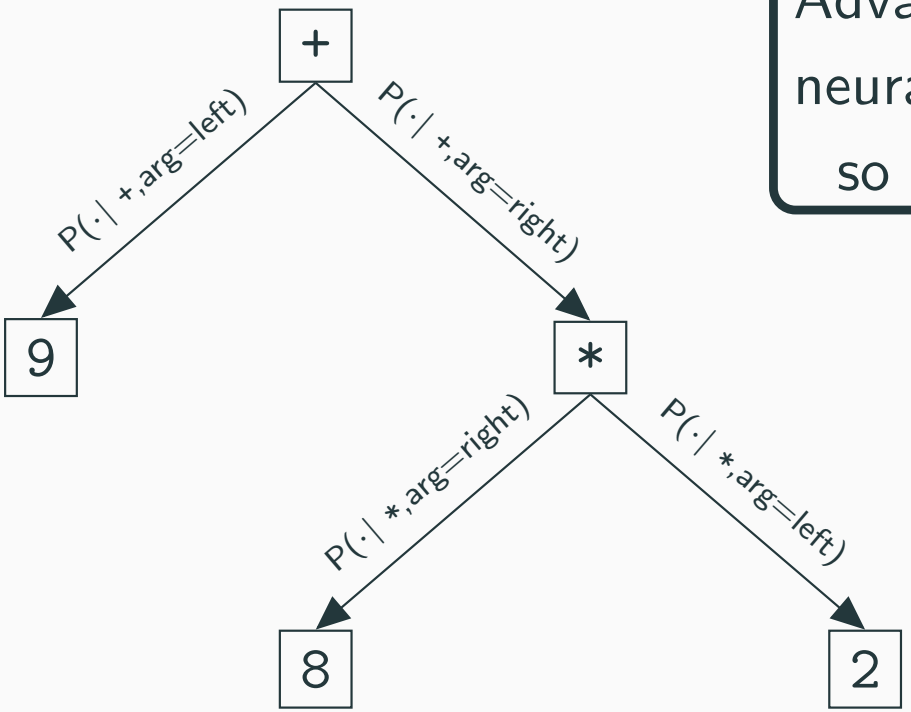


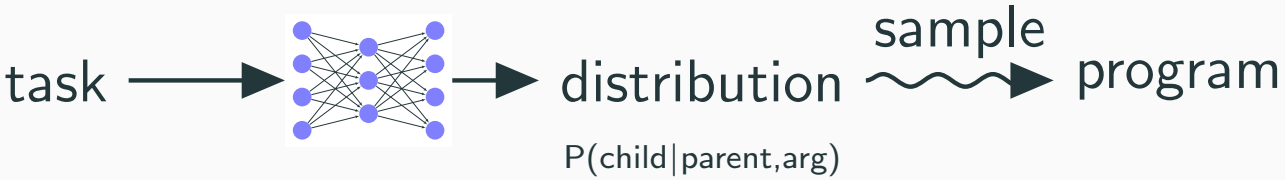


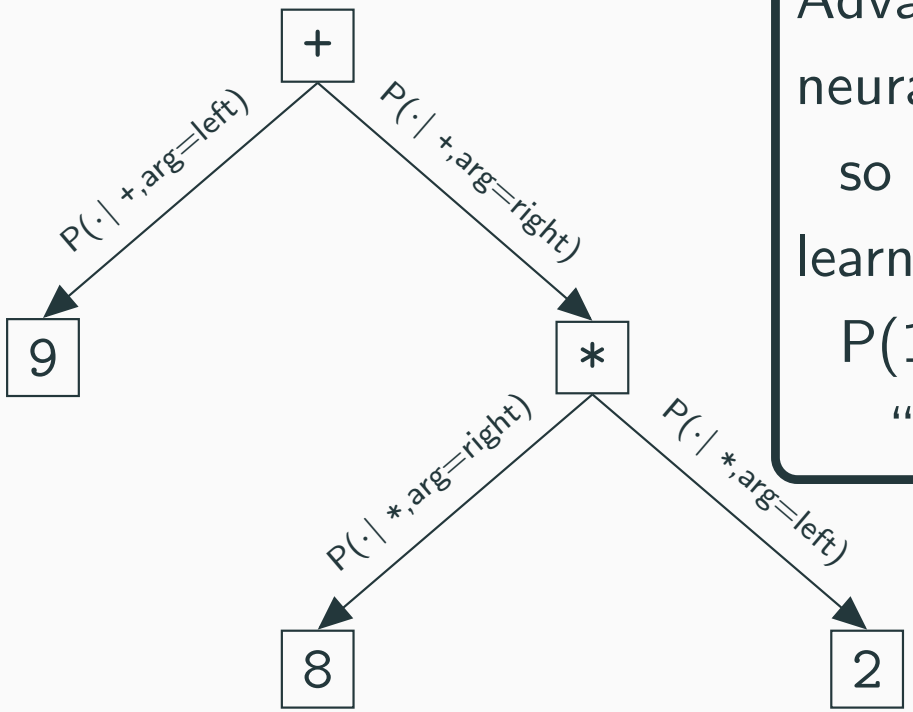
Advantages:

neural net runs once per task,

so CPU bottlenecks instead of GPU

Neural recognition model guides search





Advantages:

neural net runs once per task,
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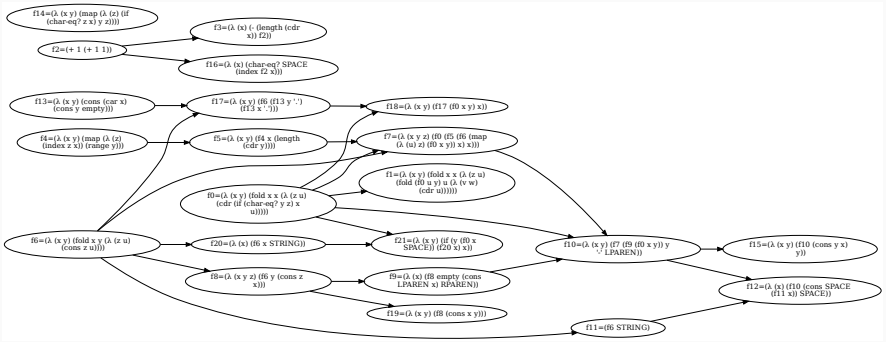
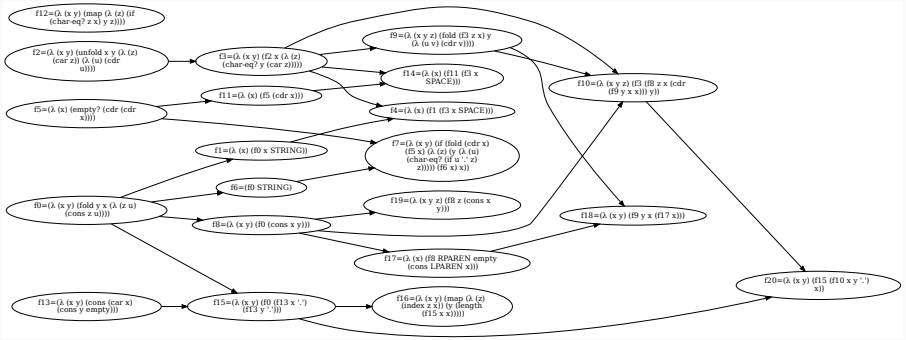
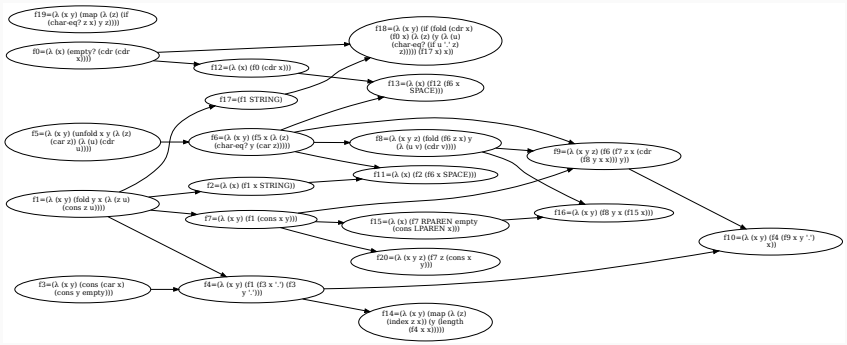
learns to break syntactic symmetries:
$P(1|*,\text{arg}=\text{left})=0.0$
"do not multiply by one"

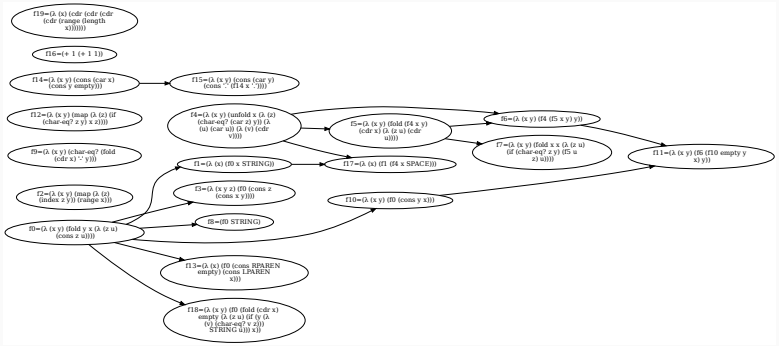
Library structure: Text Editing

DreamCoder learns libraries for FlashFill-style text editing [Gulwani 2012]





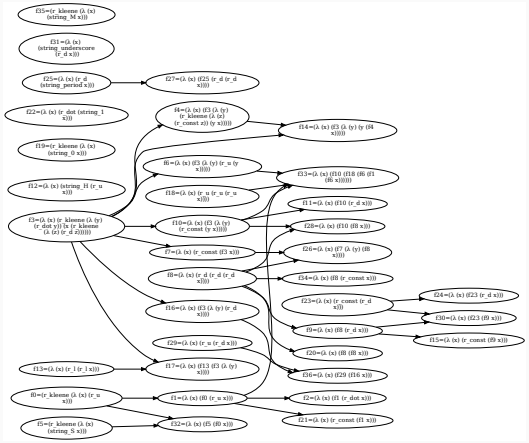


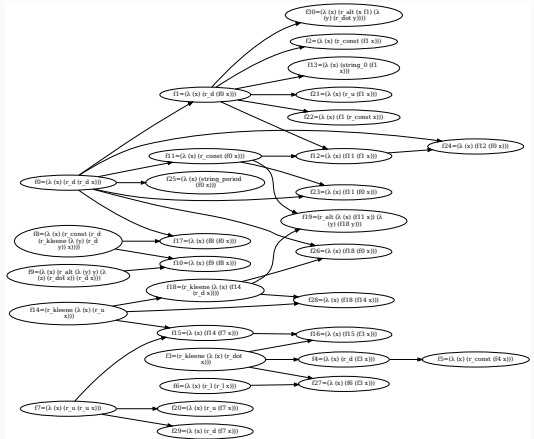


Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. PLDI 2021.

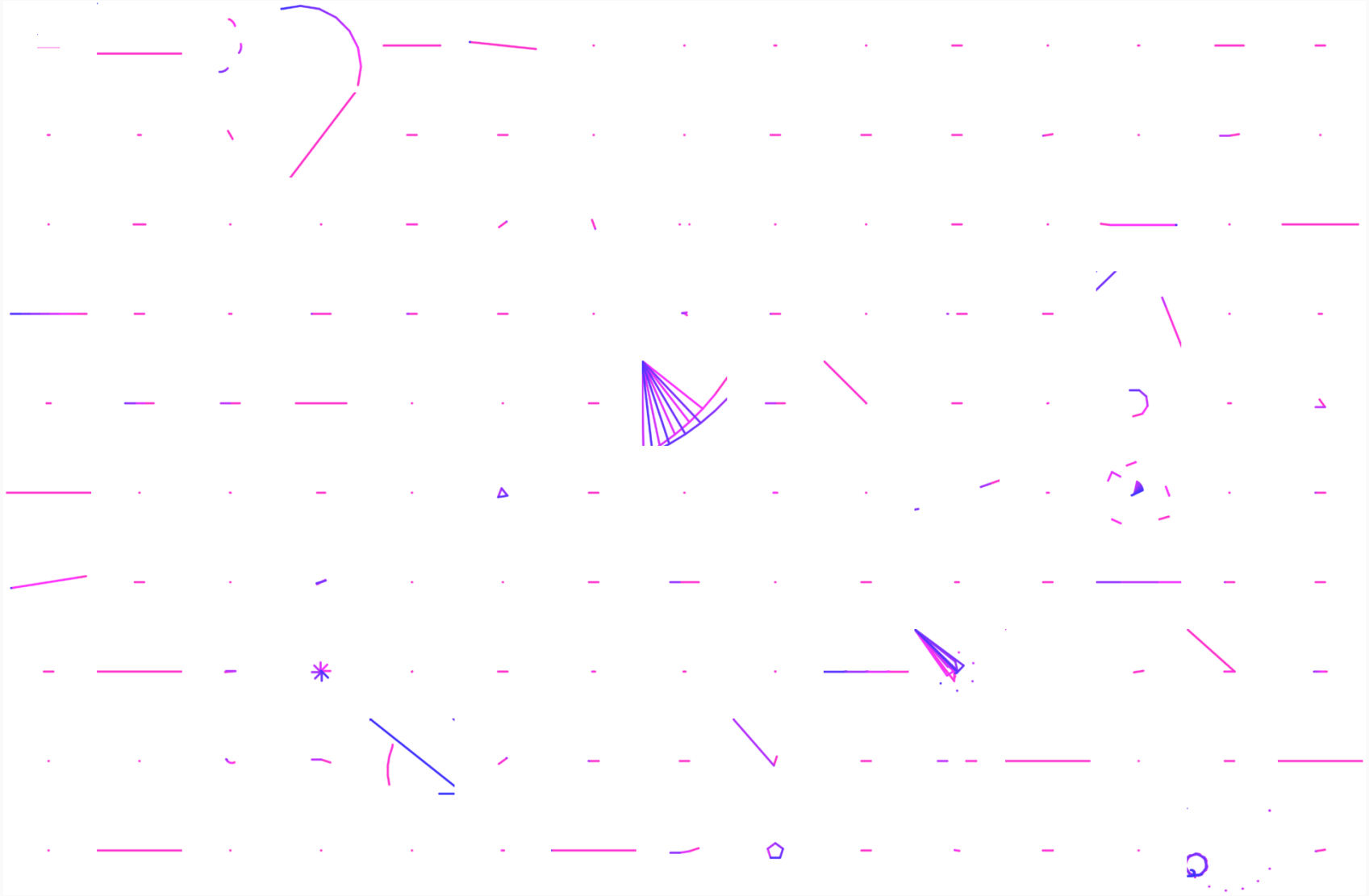
Library structure: Generating Text

Libraries for probabilistic generative models over text:
data from crawling web for CSV files





150 random dreams before learning



150 random dreams after learning