Building Machines that Discover Generalizable, Interpretable Knowledge

Kevin Ellis

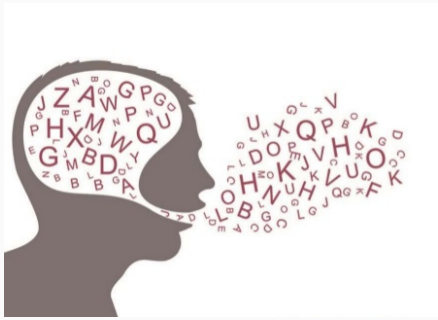
2020

MIT

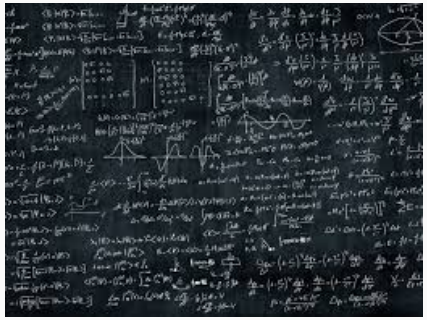
What computational problems are solved by intelligence?

an endless range of problems

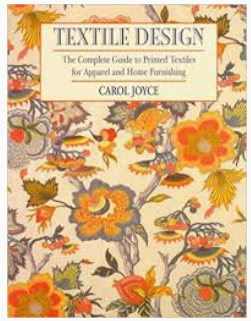
language



science

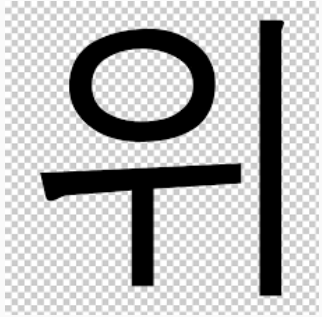


design



using new devices

writing new characters



coding

```
(MEMBER
 (LAMBDA (X L)
  (COND  ((NULL L) NIL)
         ((EQ X (FIRST L)) T)
         (T (MEMBER X (REST L))))))
```

Allen, Anatomy of Lisp, 1975

engineering

play

What computational frameworks can contribute to this picture?

Three AI traditions

What computational frameworks can contribute to this picture?

Three AI traditions

Symbolic


```
In[34]:= Solve[{(hw - hw^2) == Z}, h]
Out[34]= {}
```

```
In[33]:= Solve[{(hw-hw^2) ==Z},h]
```

Input interpretation:

solve | $h\,w - h\,w^2 = Z$ | for | h

Result:

$$h = \frac{Z}{w - w^2} \text{ and } w^2 \neq w$$

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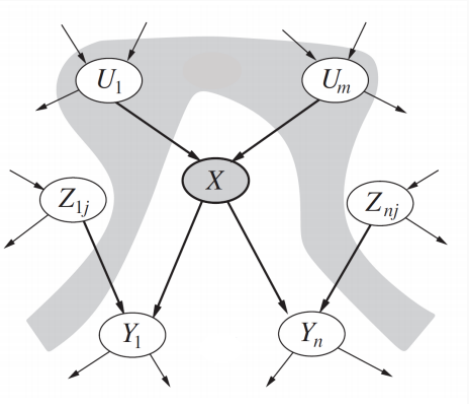

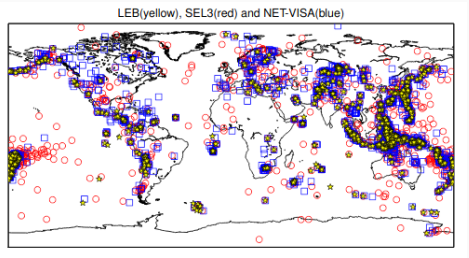

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h = Z / (w - w^2)  and  w^2 ≠ w
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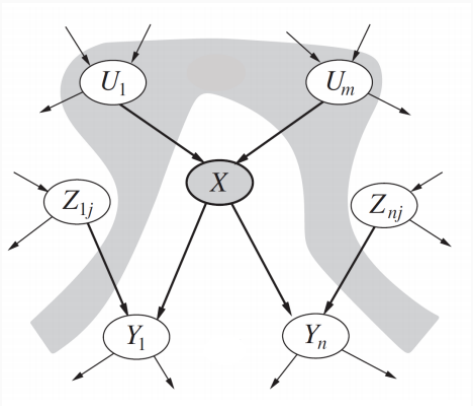
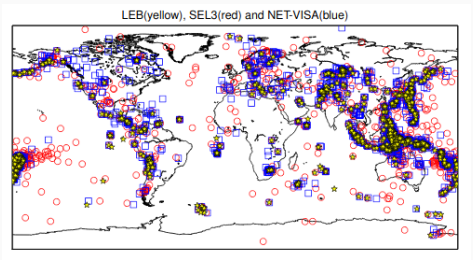
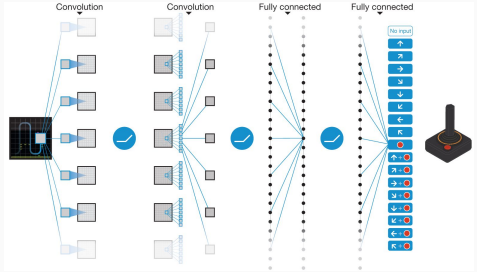
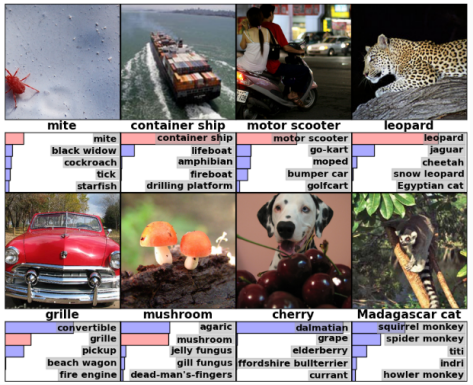
Probabilistic





Neural





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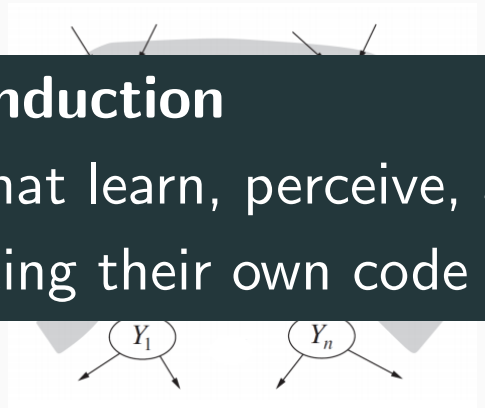

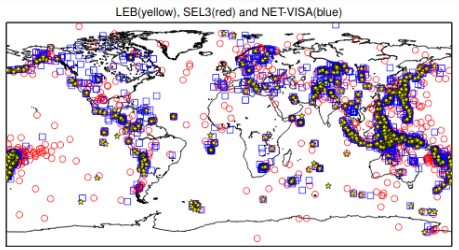

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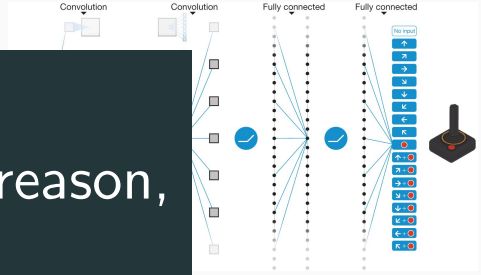
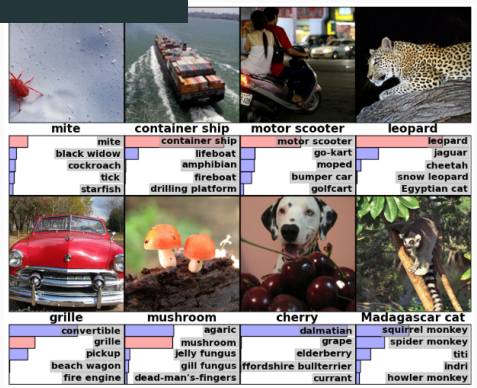


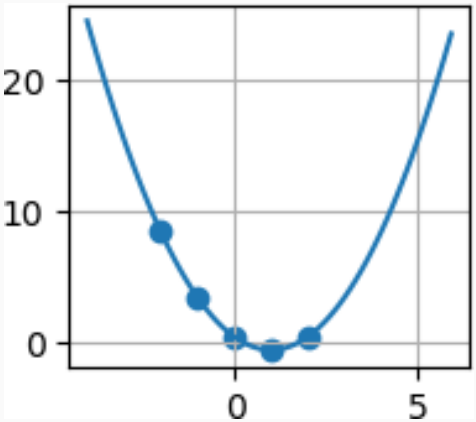


Program induction

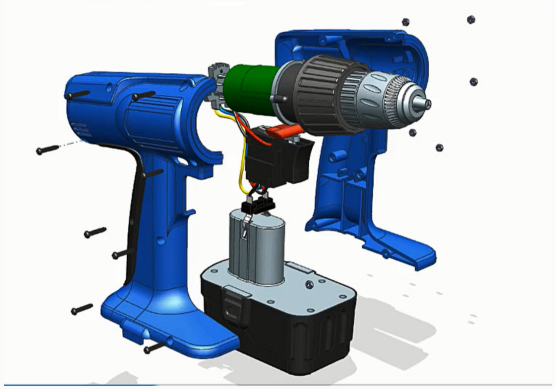
machines that learn, perceive, and reason, by writing their own code

Why program induction?

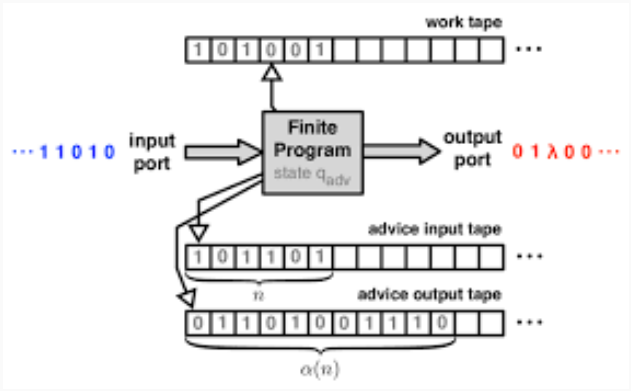
strong generalization
+data efficiency



interpretability



universal expressivity



Why didn't this old idea work?

Program induction goes back to the 1956 Dartmouth Workshop that founded the field of AI

A PROPOSAL FOR THE

DARTMOUTH SUMMER RESEARCH PROJECT

ON ARTIFICIAL INTELLIGENCE

J. McCarthy, Dartmouth College
M. L. Minsky, Harvard University
N. Rochester, I. B. M. Corporation
C. E. Shannon, Bell Telephone Laboratories

Why will it work this time?

better toolkits:

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- **probabilistic** methods for uncertainty and learning-to-learn

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- **neural** methods for guiding combinatorial search

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- **neural** methods for guiding combinatorial search
- **symbolic** methods, from the **programming languages** community
  - maturing **program synthesis** techniques
  - type systems, program analysis, constraint solving, ...

Why will it work this time?

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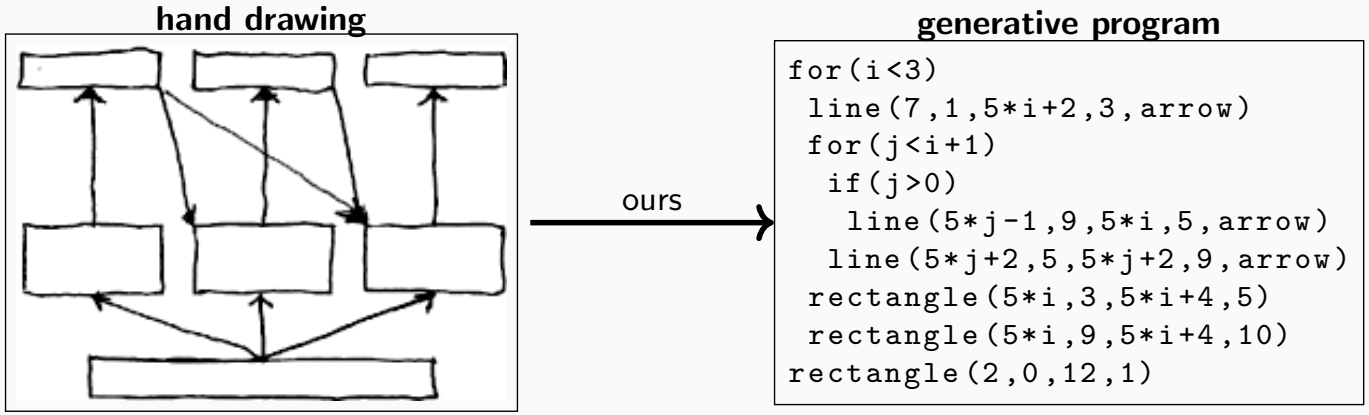
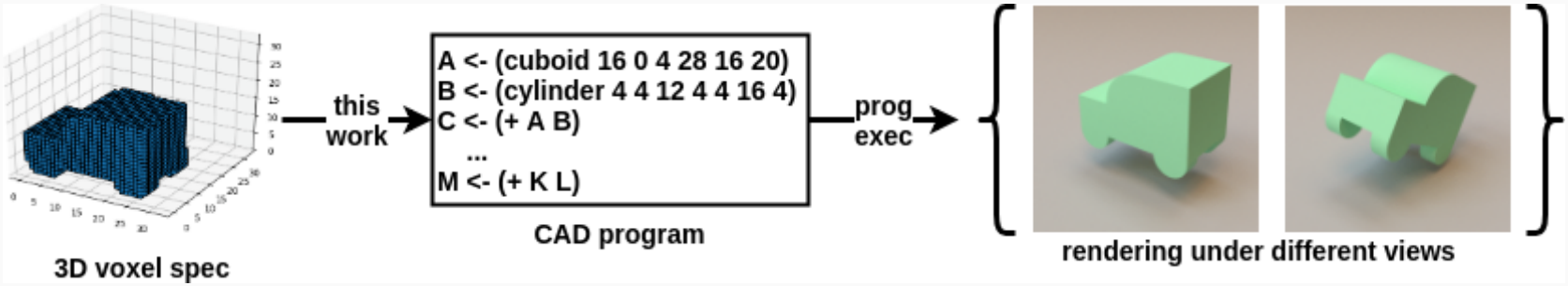
better problems:

- inverse CAD [Kulkarni et al. 2015]
- semantic parsing [Liang et al. 2011; Zettlemoyer et al. 2007]
- programming by examples [Gulwani 2011], computer-aided-programming [Solar-Lezama 2008]

Perception, Synthesizing models, Learning-to-Learn

Theme: high-level visual understanding, pixels→programs





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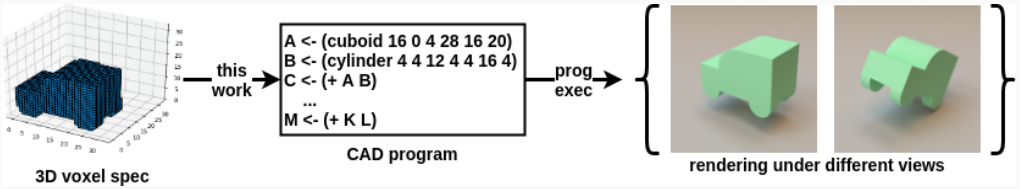
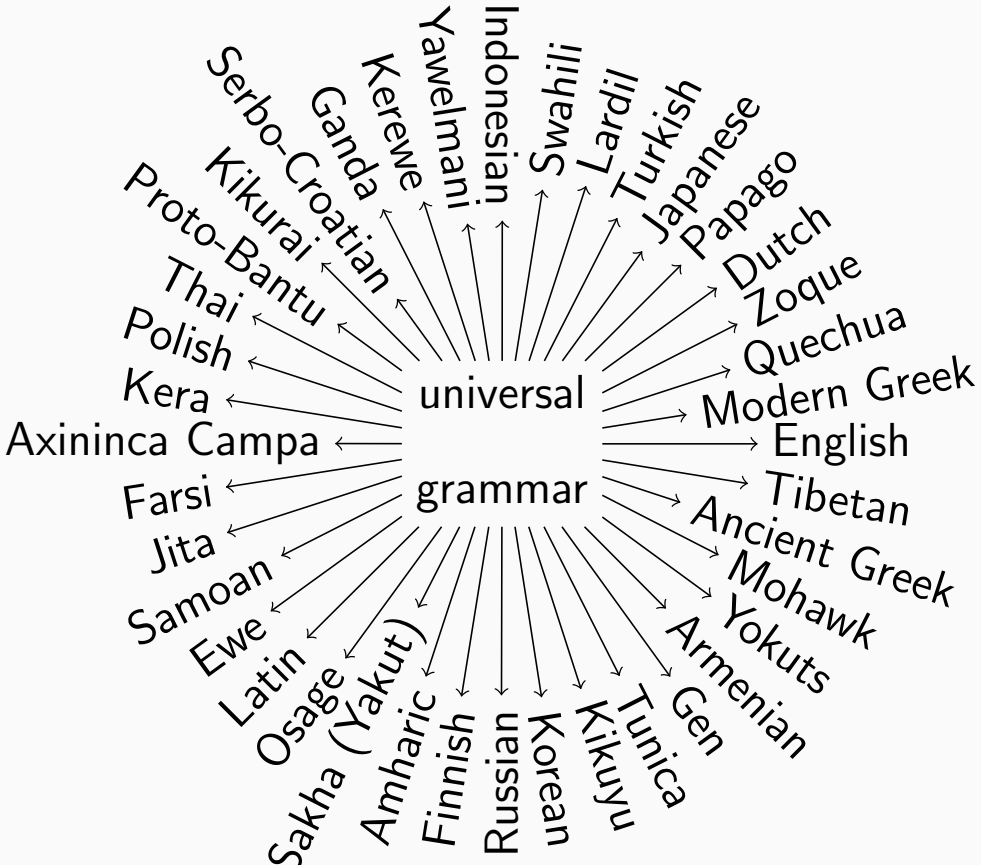


Theme: Synthesizing human-understandable models



Perception, Synthesizing models, Learning-to-Learn

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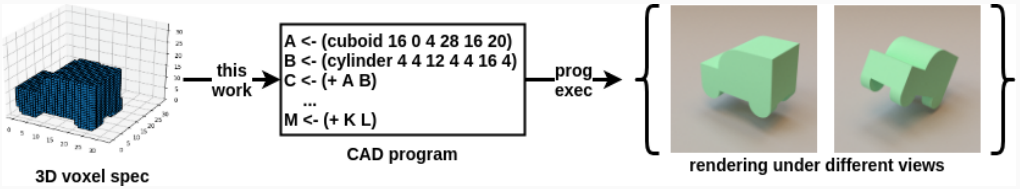


Theme: Synthesizing human-understandable models

Theme: Learning to synthesize programs

List Processing

Sum List

```
[1 2 3] → 6
[4 6 8 1] → 17
```

Double

```
[1 2 3 4] → [2 4 6 8]
[6 5 1] → [12 10 2]
```

Check Evens

```
[0 2 3] → [T T F]
[2 4 9 6] → [T T F T]
```

Text Editing

Abbreviate

```
Allen Newell → A.N.
Herb Simon → H.S.
```

Drop Last Characters

```
jabberwocky → jabberw
copycat → cop
```

Extract

```
see spot(run) → run
a (bee) see → bee
```

Regexes

Phone Numbers

```
(555) 867-5309
(650) 555-2368
```

Currency

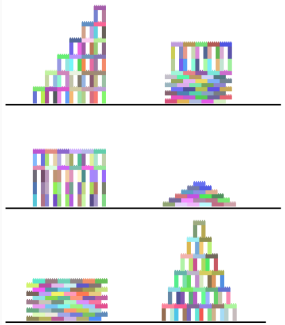
```
$100.25
$4.50
```

Dates

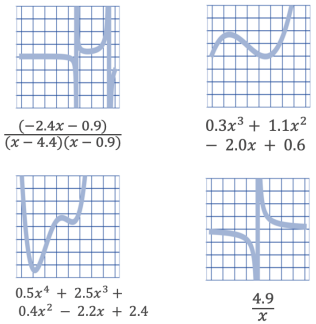
```
Y1775/0704
Y2000/0101
```

LOGO Graphics

Block Towers



Symbolic Regression



Recursive Programming

Filter

```
[■■■■■] → [■■■]
[■■■■■■] → [■■■■]
[■■■■■] → [■■■]
```

Length

```
[■■■■] → 4
[■■■■■■] → 6
[■■■] → 3
```

Index List

```
0, [■■■■■■] → ■
1, [■■■■■■] → ■
1, [■■■■■■■] → ■
```

Every Other

```
[■■■■■] → [■■■]
[■■■■■■] → [■■■■]
[■■■■■■■] → [■■■■]
```

Physics

$$KE = \frac{1}{2}m|\vec{v}|^2$$

$$\vec{a} = \frac{1}{m}\sum_i \vec{F}_i$$

$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r_1} - \vec{r_2}|^2}\widehat{r_1 - r_2}$$

$$R_{total} = \left(\Sigma_i \frac{1}{R_i}\right)^{-1}$$

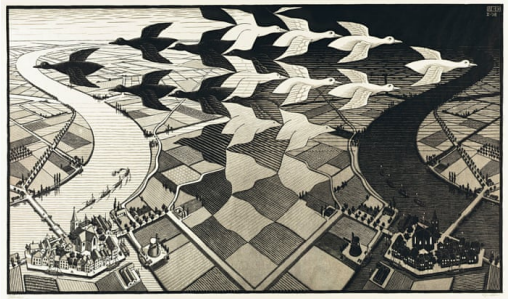
Program Induction and perception

learning to learn

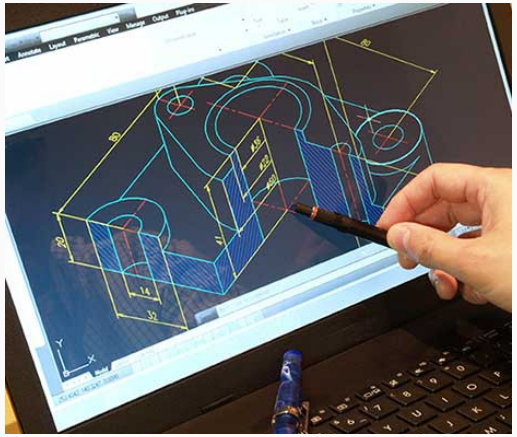
model discovery

High-level, abstract visual abilities

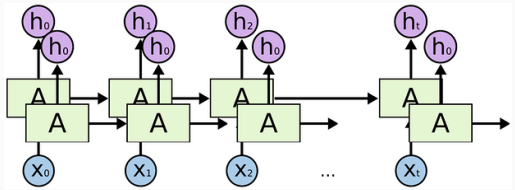
...in art

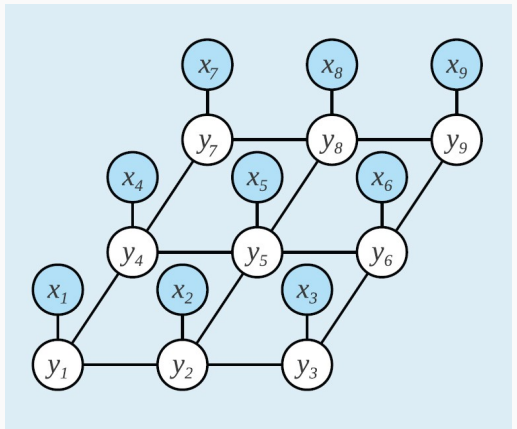


...in engineering



...in AI





High-level, abstract visual abilities

...in art

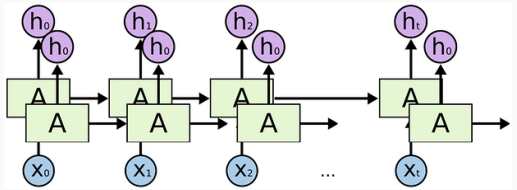
why?

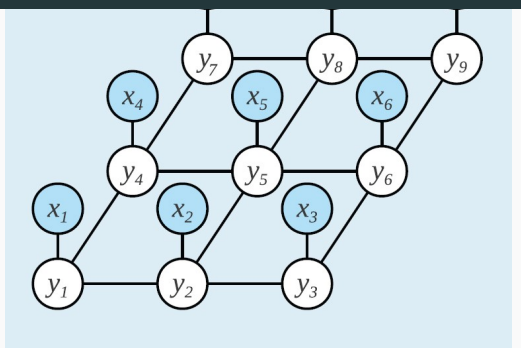
- impute missing objects, extrapolate percepts,
- learn visual concepts ('arch', 'spiral', 'Ising model'),
- assist graphic design, assist 3D modeling

how?

- machine learning +
- program synthesis techniques from programming languages community

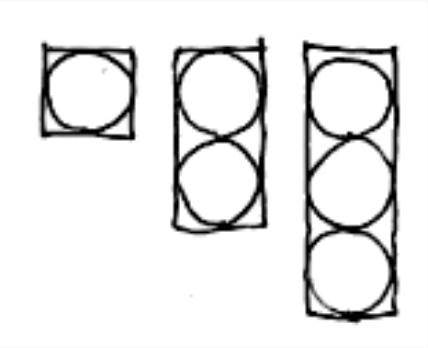
...in AI





Learning to infer graphics programs from hand-drawn images

model infers program from drawing



```
for (0 <= i < 3)
 rectangle(3*i,-2*i+4,
           3*i+2,6)
 for (0 <= j < i + 1)
  circle(3*i+1,-2*j+5)
```

Ellis, Ritchie, Solar-Lezama, Tenenbaum. NeurIPS 2018.

Learning to infer graphics programs from hand-drawn images

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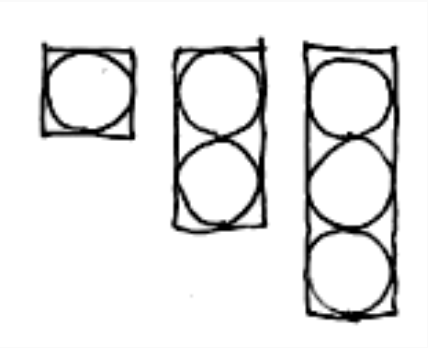
zero-shot generalization / extrapolation



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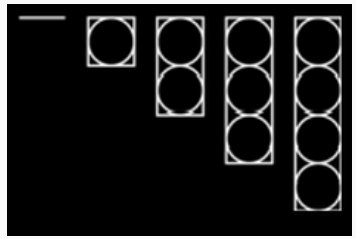

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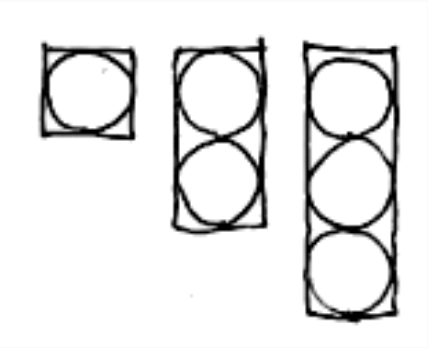

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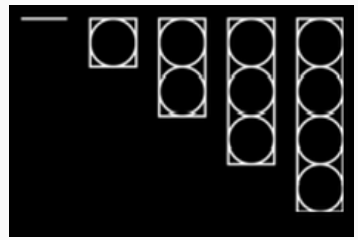
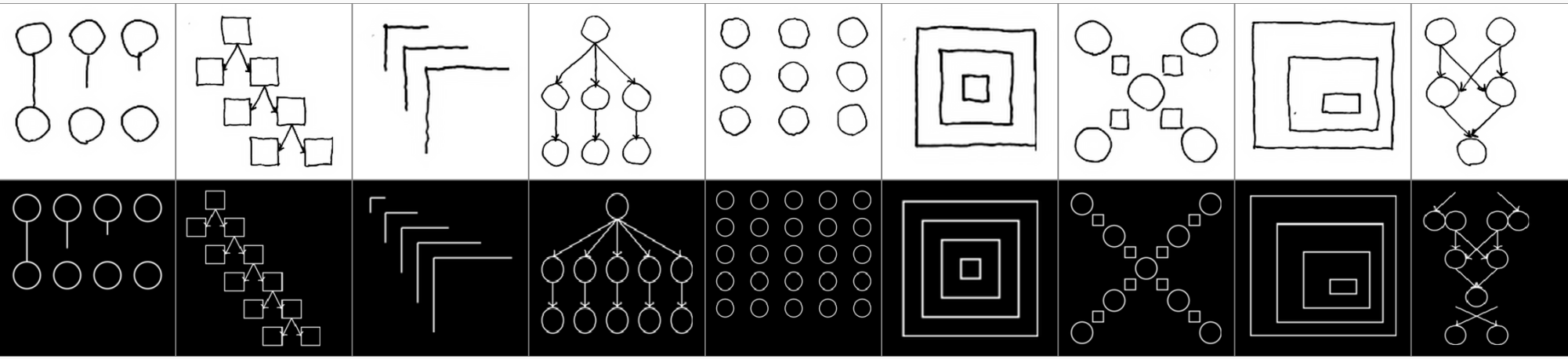
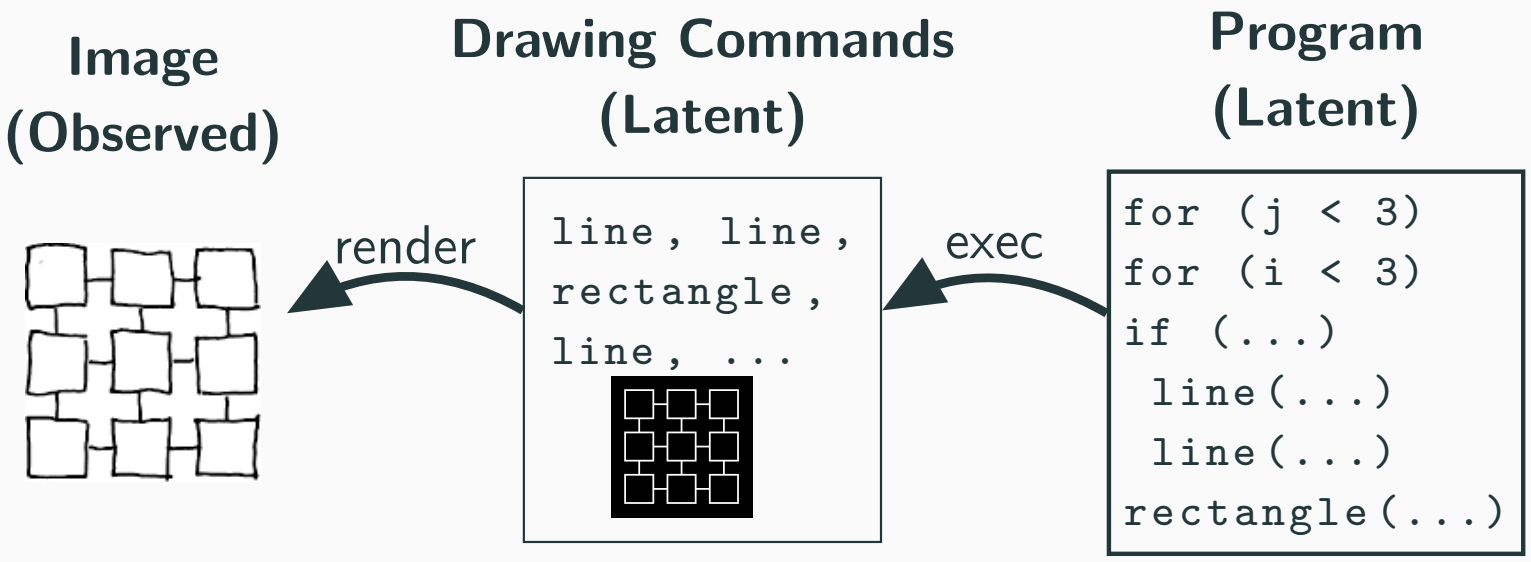




How to infer graphics programs from hand-drawn images



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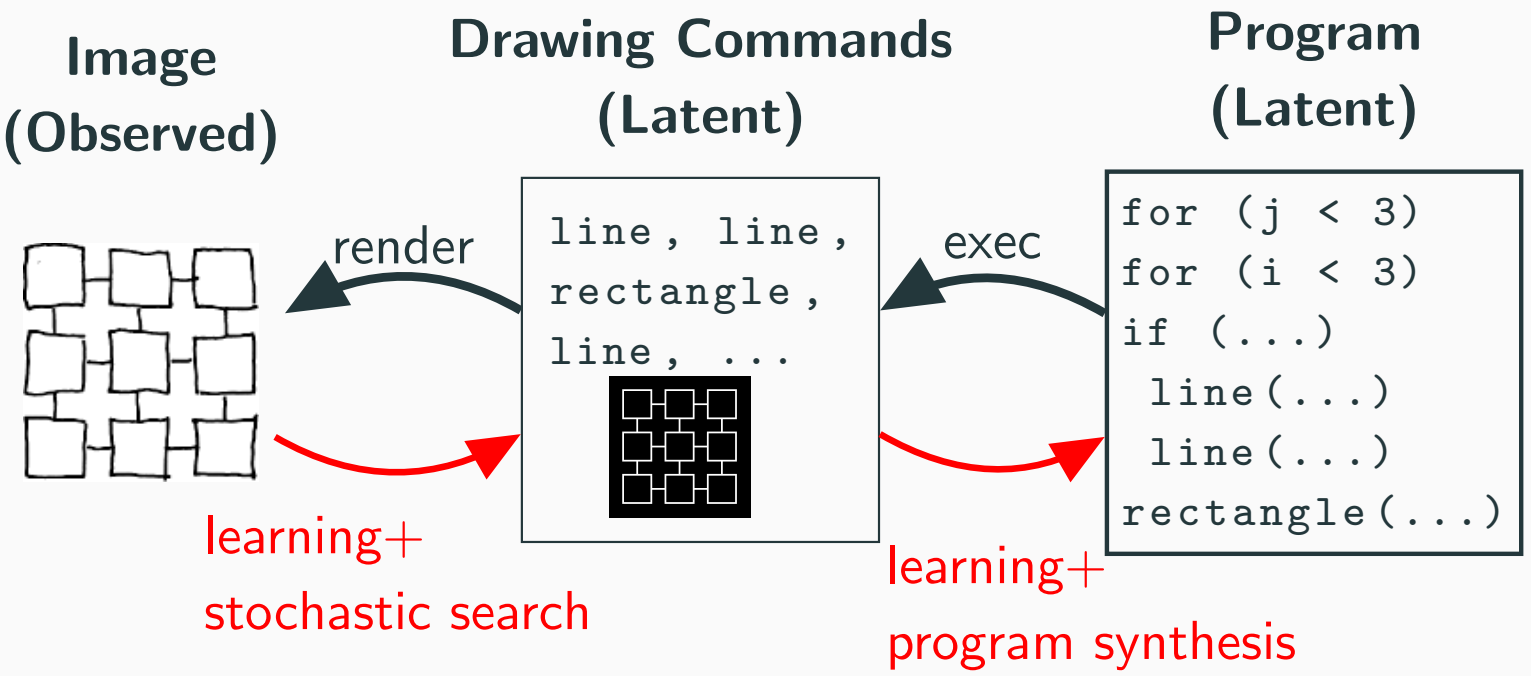
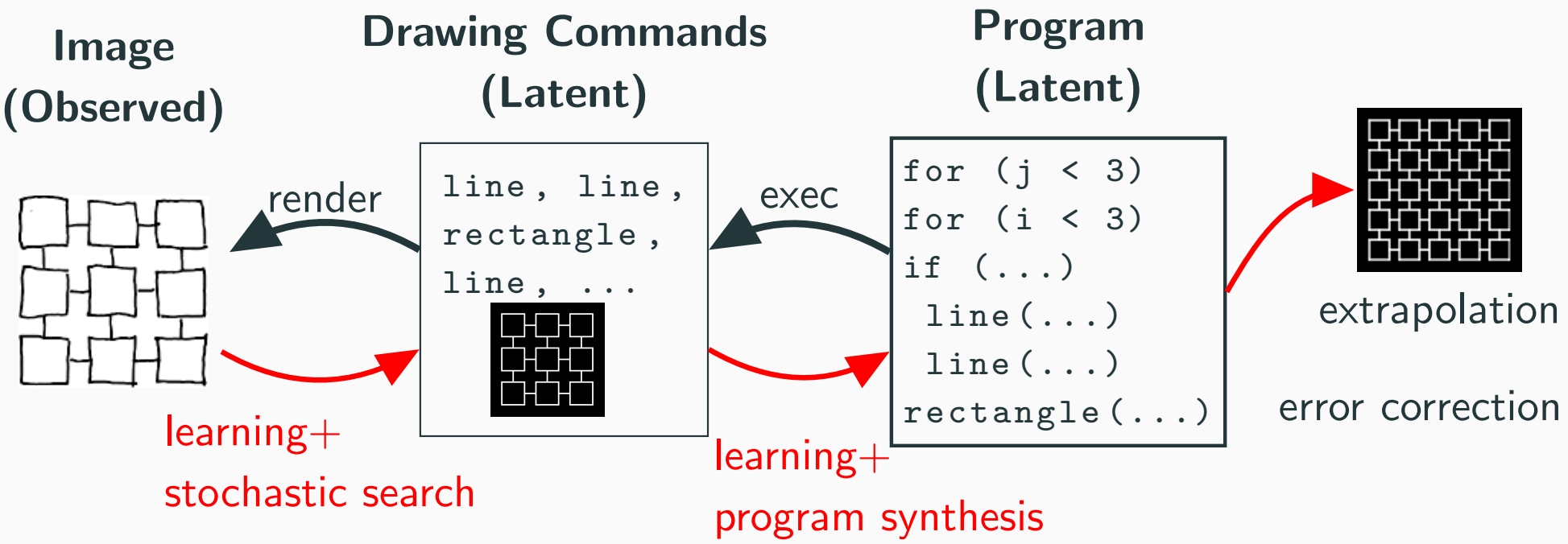
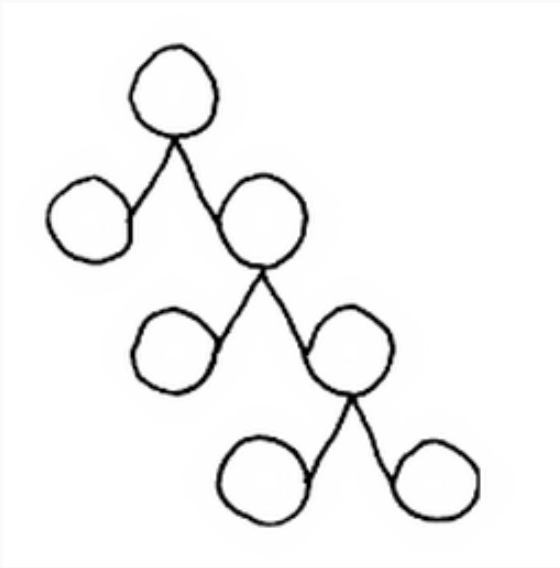


How to infer graphics programs from hand-drawn images

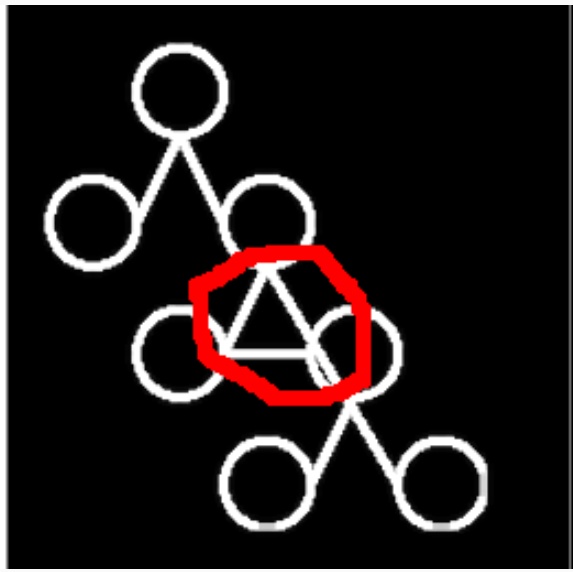


Top-down influences on perception

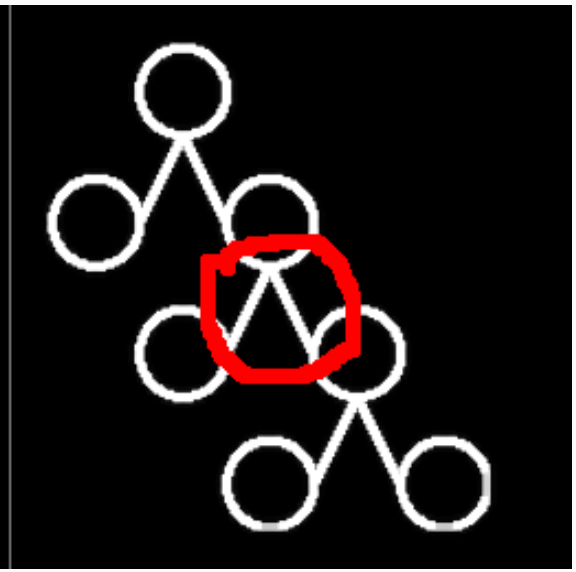
drawing



bottom-up neural net



w/ top-down program bias



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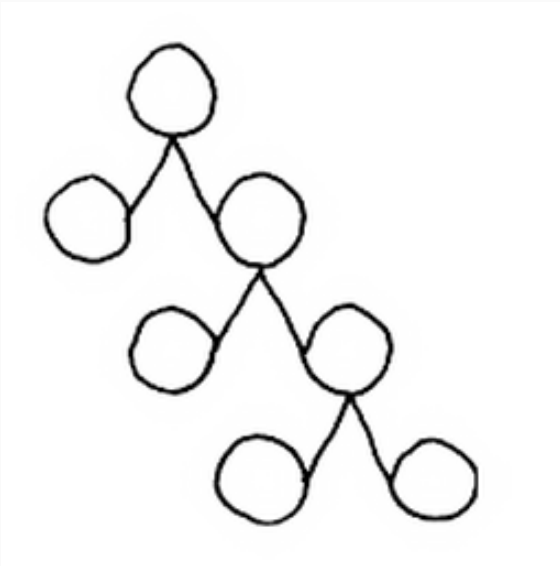
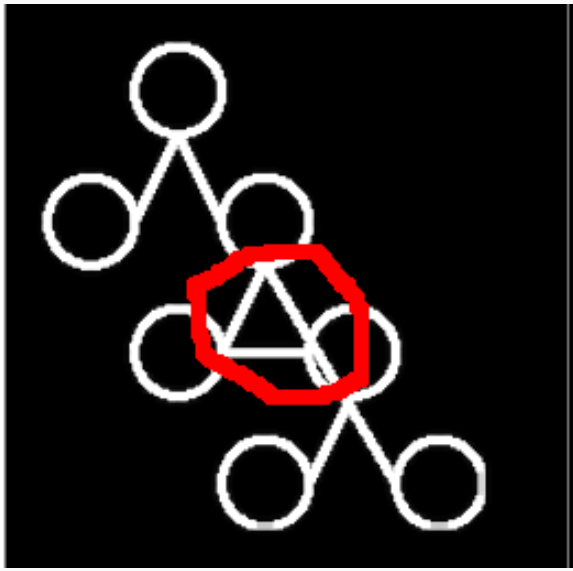


bottom-up neural net



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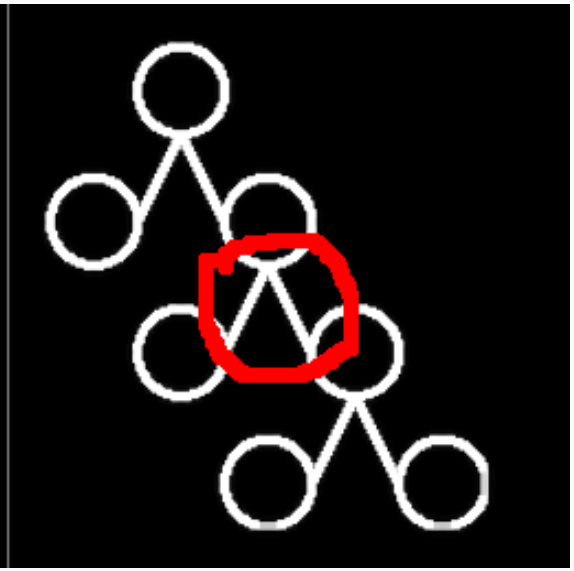
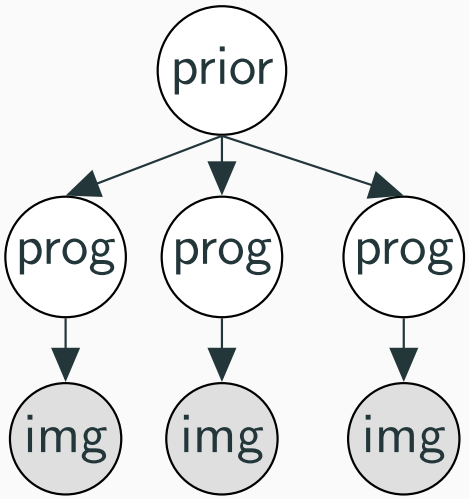


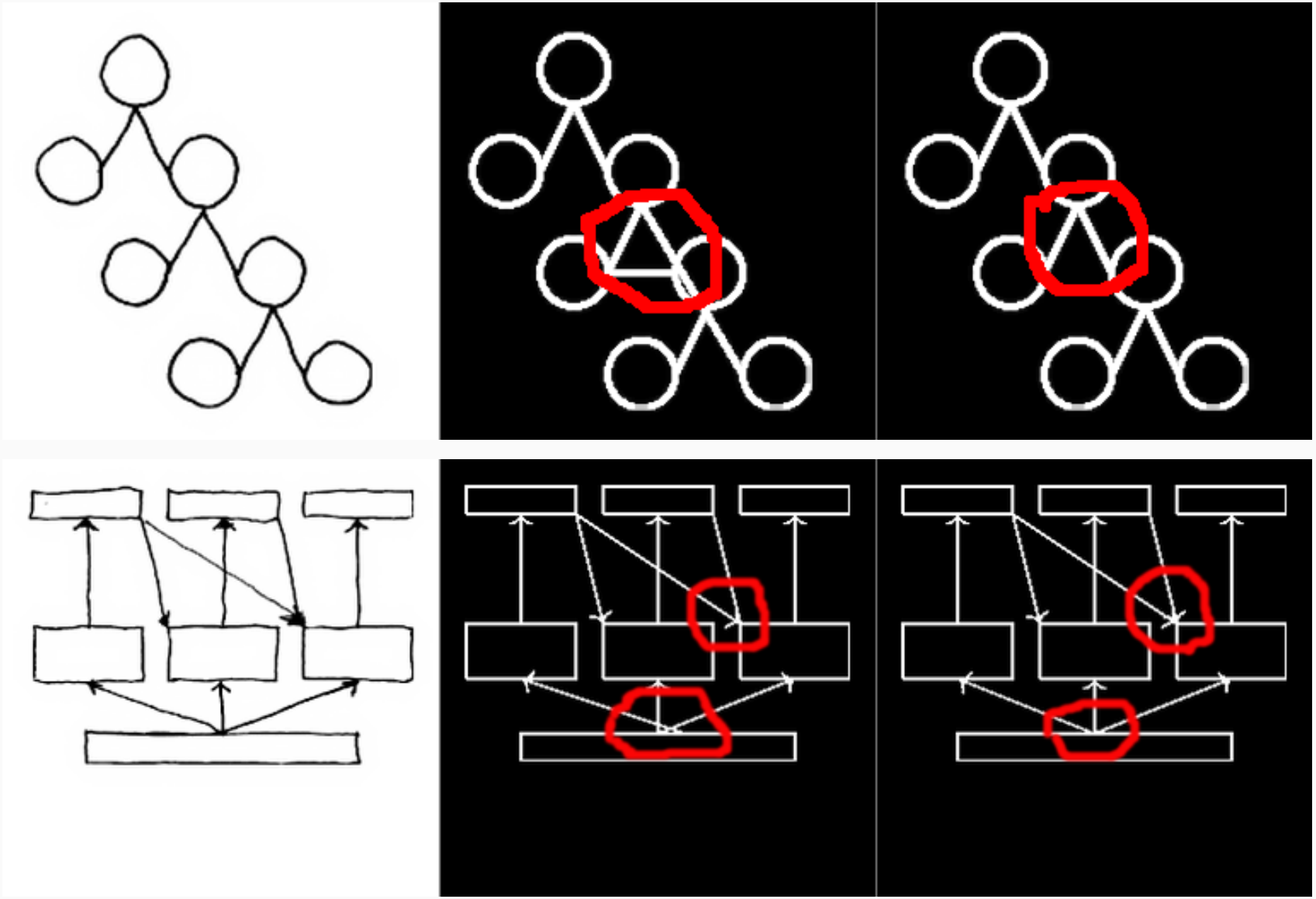


predicted program $=$

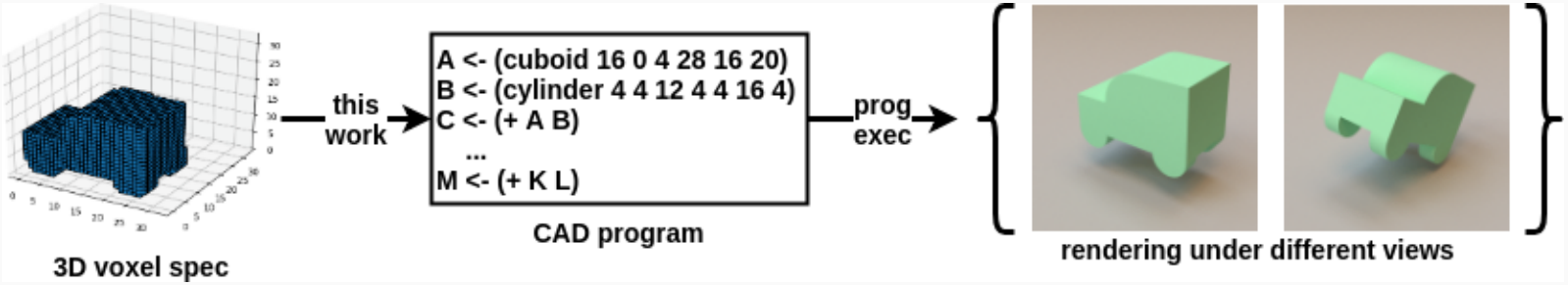
$\arg\max_{\text{progs}} \mathbb{P}\left[\text{img}|\text{prog}\right] \mathbb{P}\left[\text{prog}|\text{prior}\right]$

Top-down influences on perception

drawing | bottom-up neural net | w/ top-down program bias

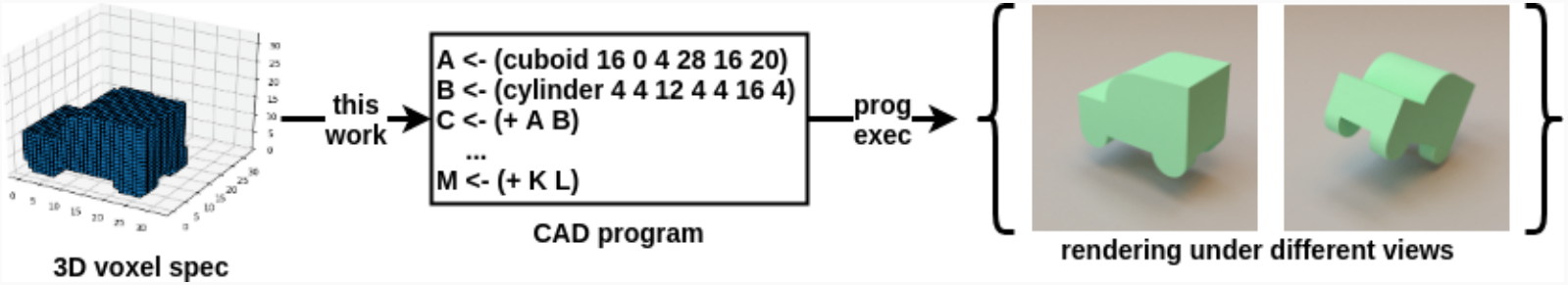


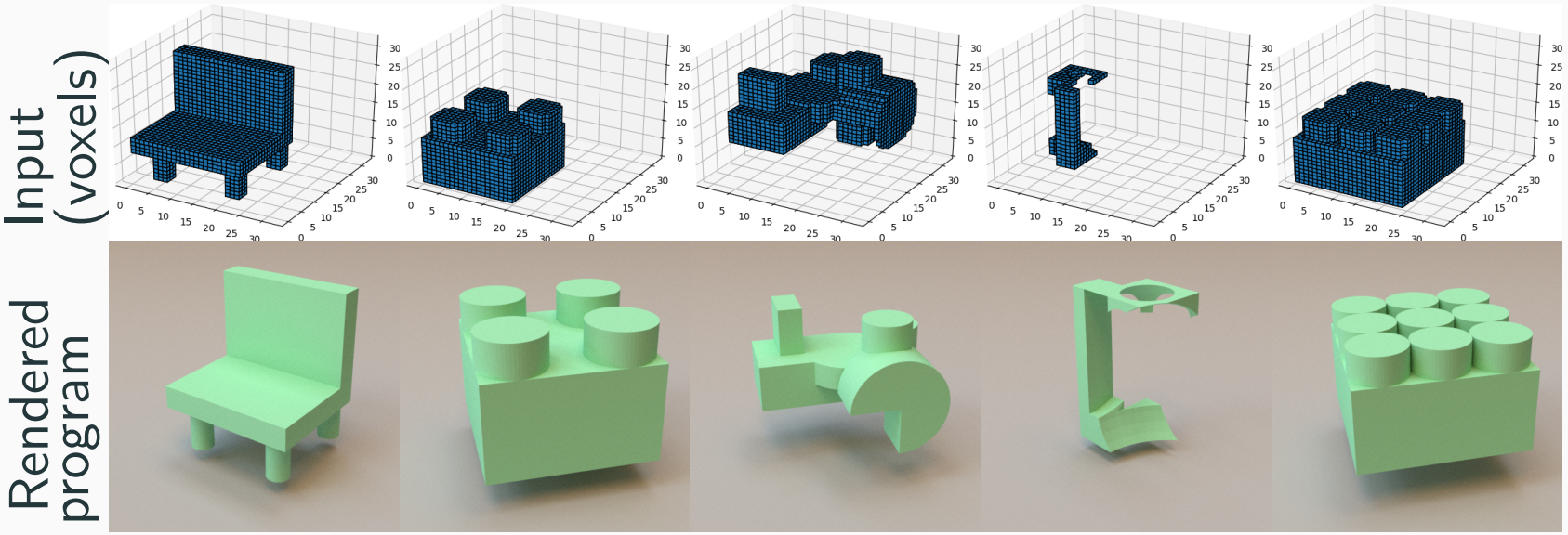
3D program induction



Ellis*, Nye*, Pu*, Sosa*, Tenenbaum, Solar-Lezama. NeurIPS 2019.
*equal contribution

3D program induction





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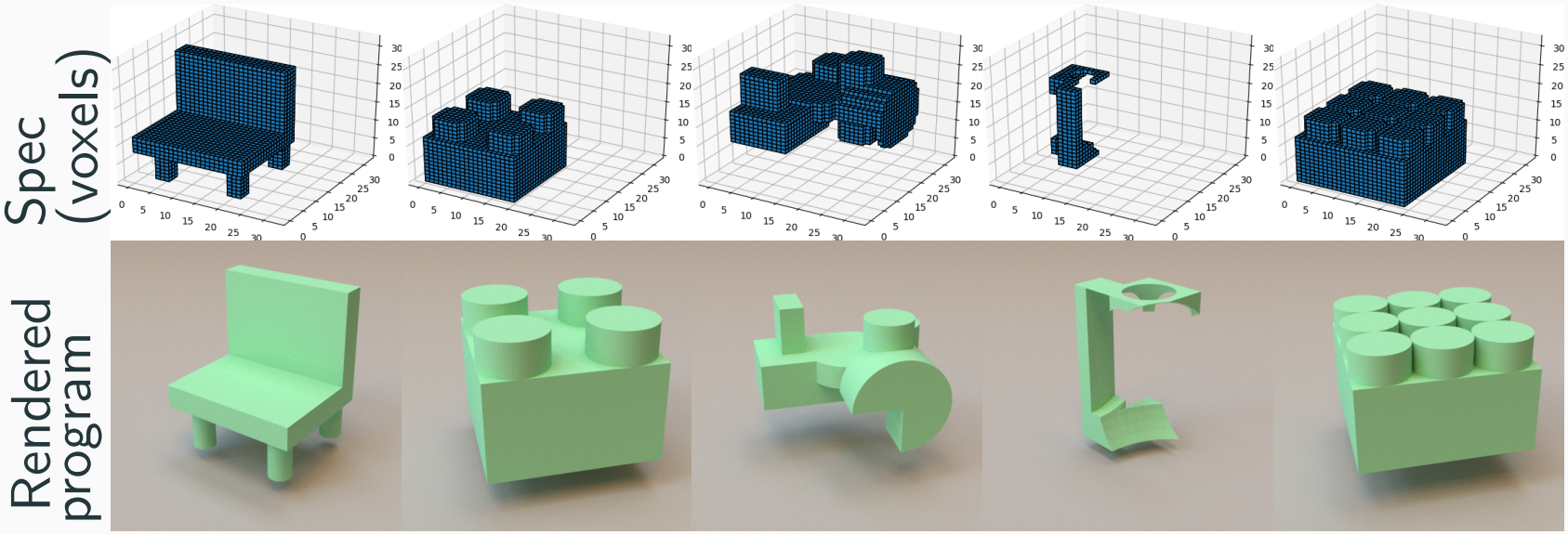

Challenge: combinatorics!

Branching factor: $>$ 1.3 million per line of code, $\approx$ 20 lines of code

Ellis*, Nye*, Pu*, Sosa*, Tenenbaum, Solar-Lezama. NeurIPS 2019.

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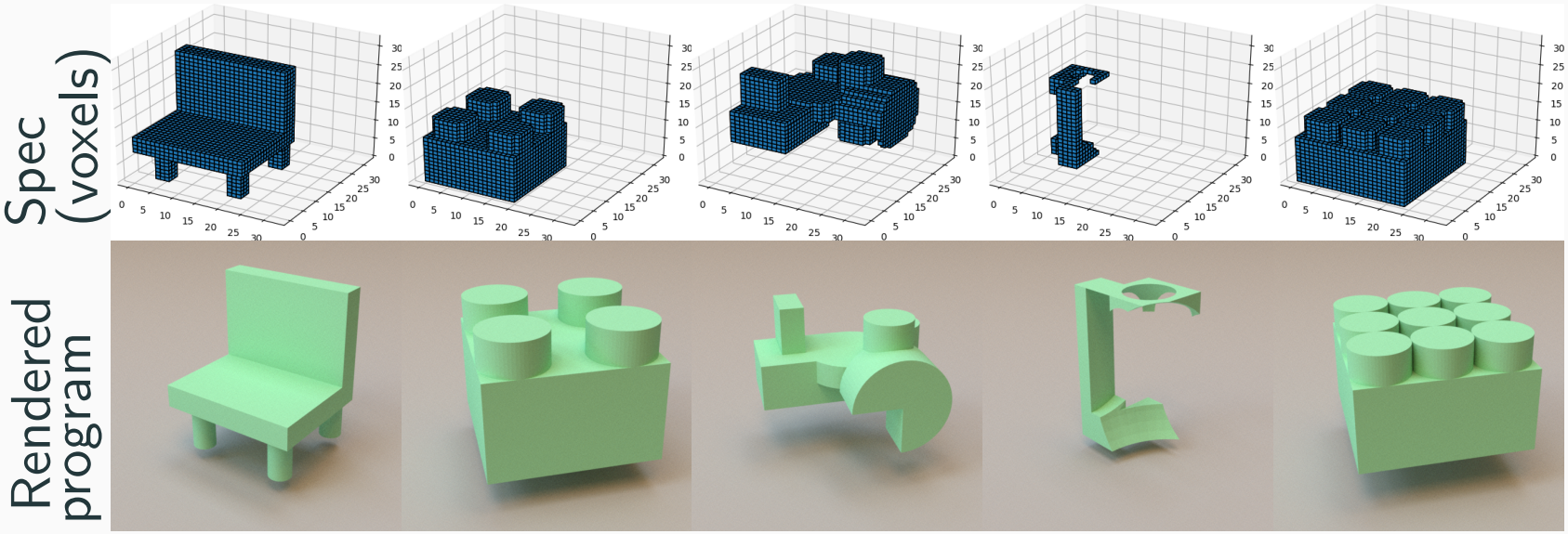

Challenge: combinatorics!

Branching factor: $>$ 1.3 million per line of code, $\approx$ 20 lines of code

Solution: stochastic **tree search** + learn **policy** that writes code + learn **value** function that assesses execution of program so far; analogous to AlphaGo

Ellis*, Nye*, Pu*, Sosa*, Tenenbaum, Solar-Lezama. NeurIPS 2019.

*equal contribution

Lessons

The bias from a programming language gives extrapolation, or strong generalization, sometimes even without meta-learning

Mix-and-match techniques to play to their strengths: neural nets for perception, symbols for reasoning, Bayesian methods for uncertainty

Program Induction and perception

learning to learn

discovering models

Learning to write code

Goal: acquire domain-specific knowledge needed to induce a class of programs

- Library of concepts (declarative knowledge; domain specific language)
- Inference strategy (procedural knowledge; synthesis algorithm)

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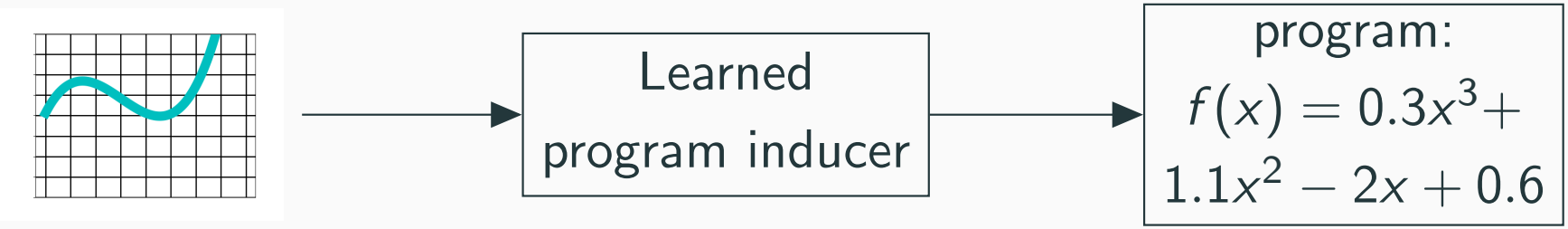

Concepts: x^3, $\alpha x + \beta$, etc

Inference strategy: neurosymbolic search for programs

Learning to write code

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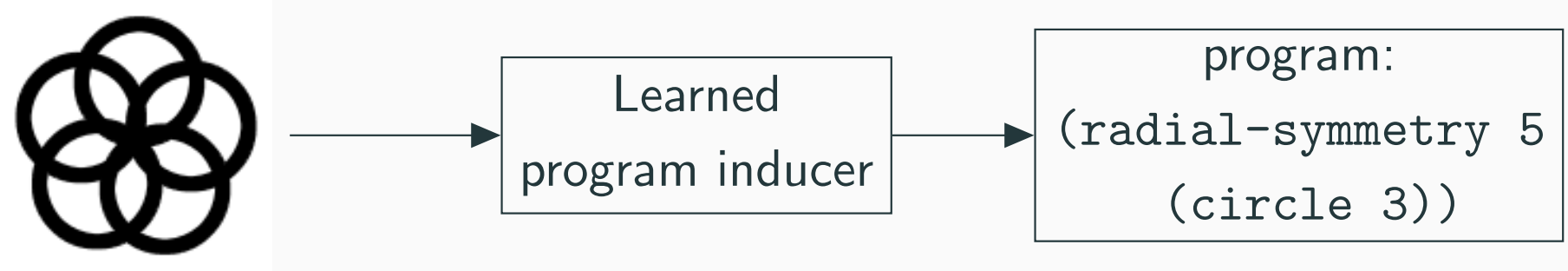


Concepts: `circle`, `radial-symmetry`, etc

Inference strategy: neurosymbolic search for programs

Library learning

Initial Primitives

map

fold

if

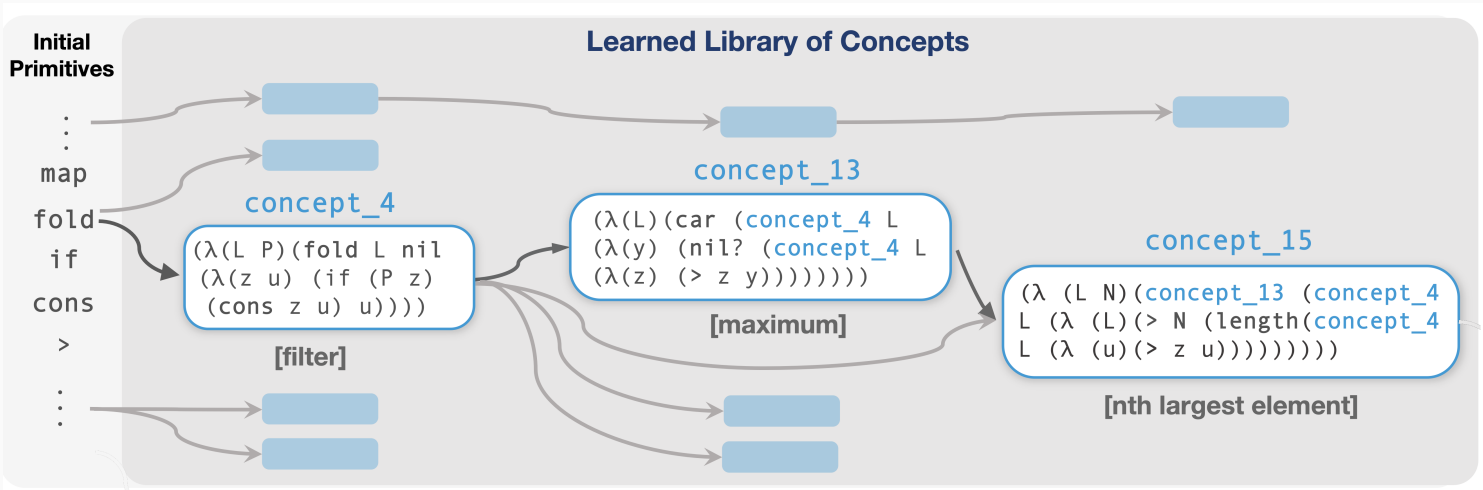
cons

>

Sample Problem: `sort list`

[9 2 7 1] → [1 2 7 9]
[3 8 9 4 2] → [2 3 4 8 9]
[6 2 2 3 8 5] → [2 2 3 5 6 8]
...

Library learning



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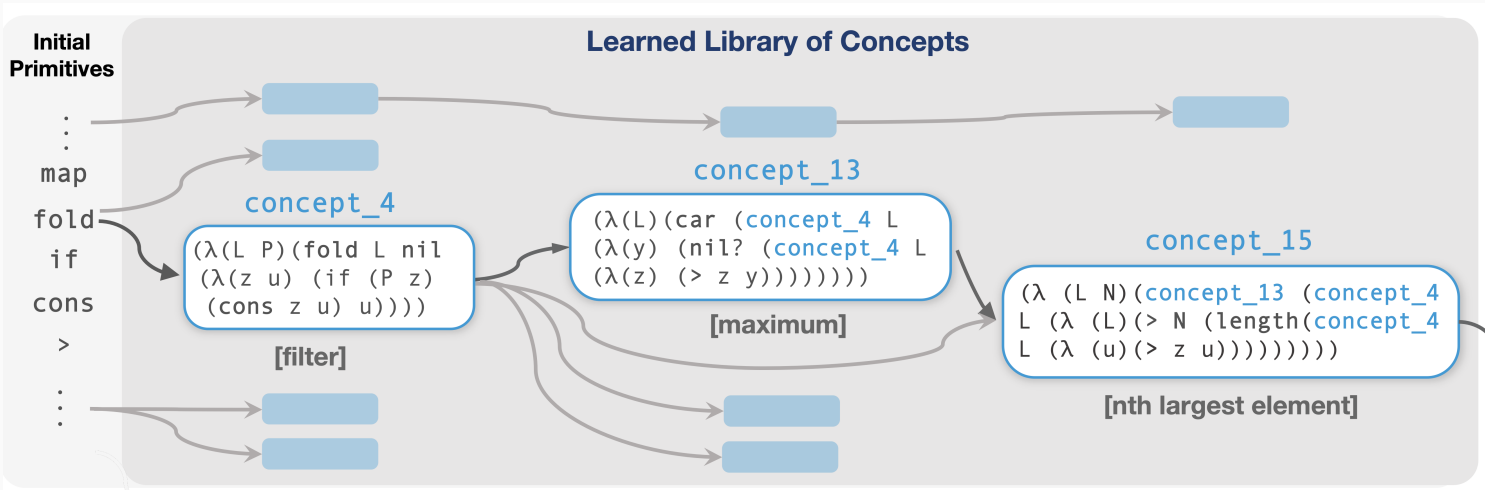

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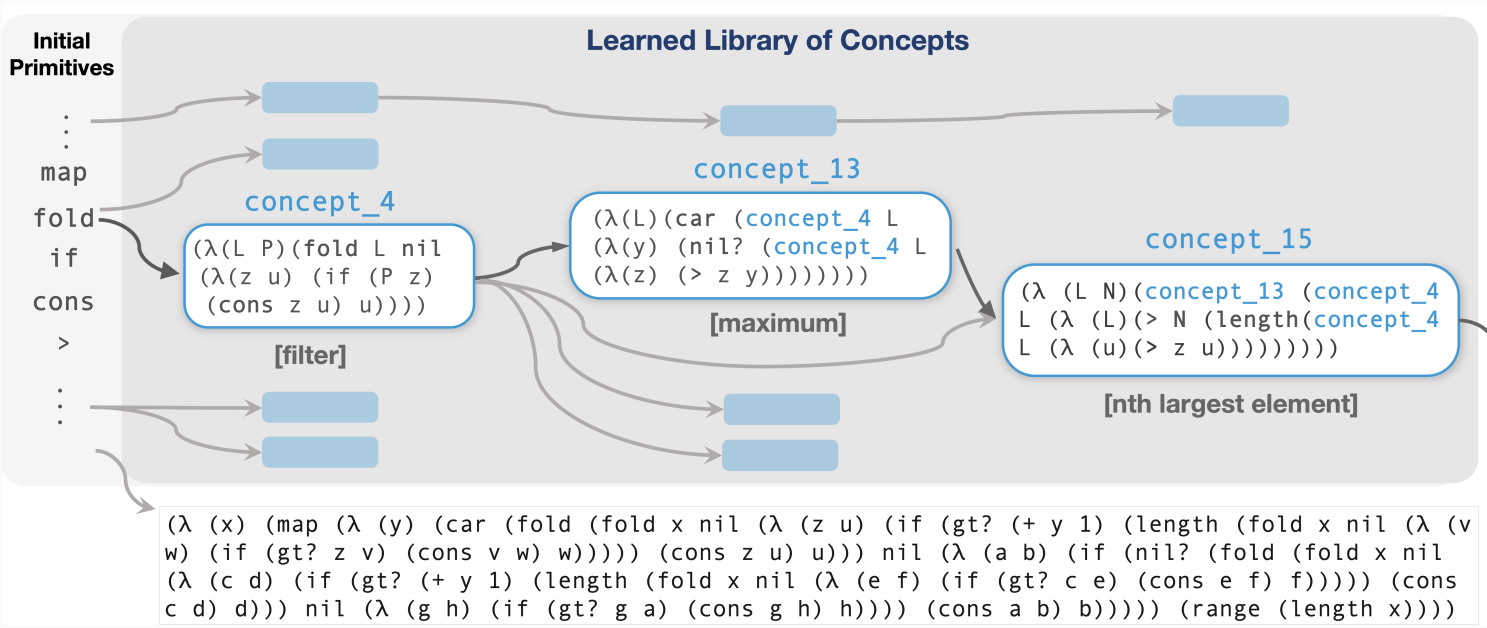
Solution to `sort list` discovered in learned language:

```
(map  (λ (n)
  (concept_15 L (+ 1 n)))
  (range (length L)))
```

Library learning



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Solution to `sort list` discovered in learned language:

```
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  (concept_15 L (+ 1 n)))
  (range (length L)))
```

Solution to `sort list` if expressed in initial primitives

brute-force search without learned library would take $\approx 10^{78}$ years to discover this solution

DreamCoder

- **Wake:** Solve problems by writing programs
- **Sleep:** Improve library and neural recognition model:
  - **Abstraction sleep:** Improve library
  - **Dream sleep:** Improve neural inference model

cf. Helmholtz machine, wake/sleep neural network training

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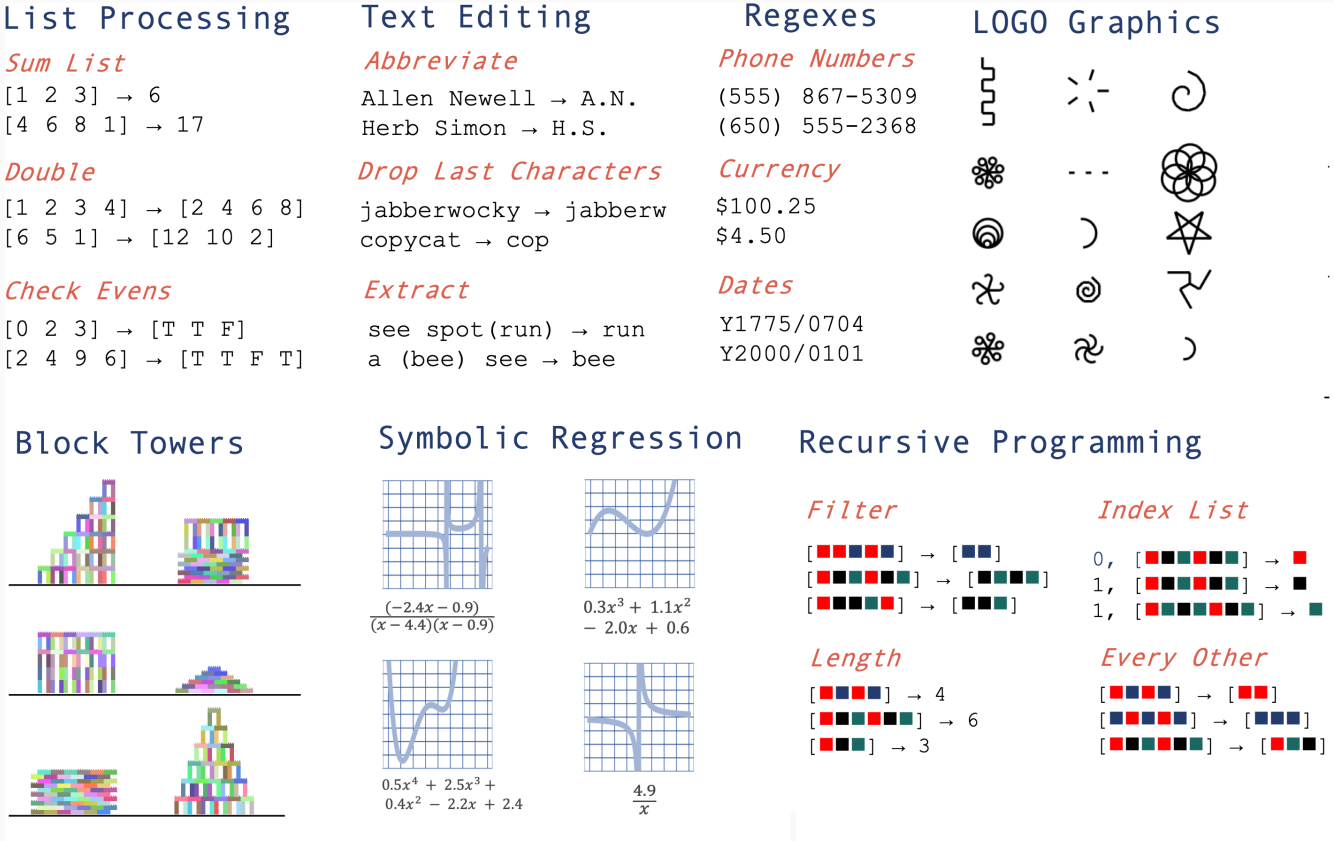
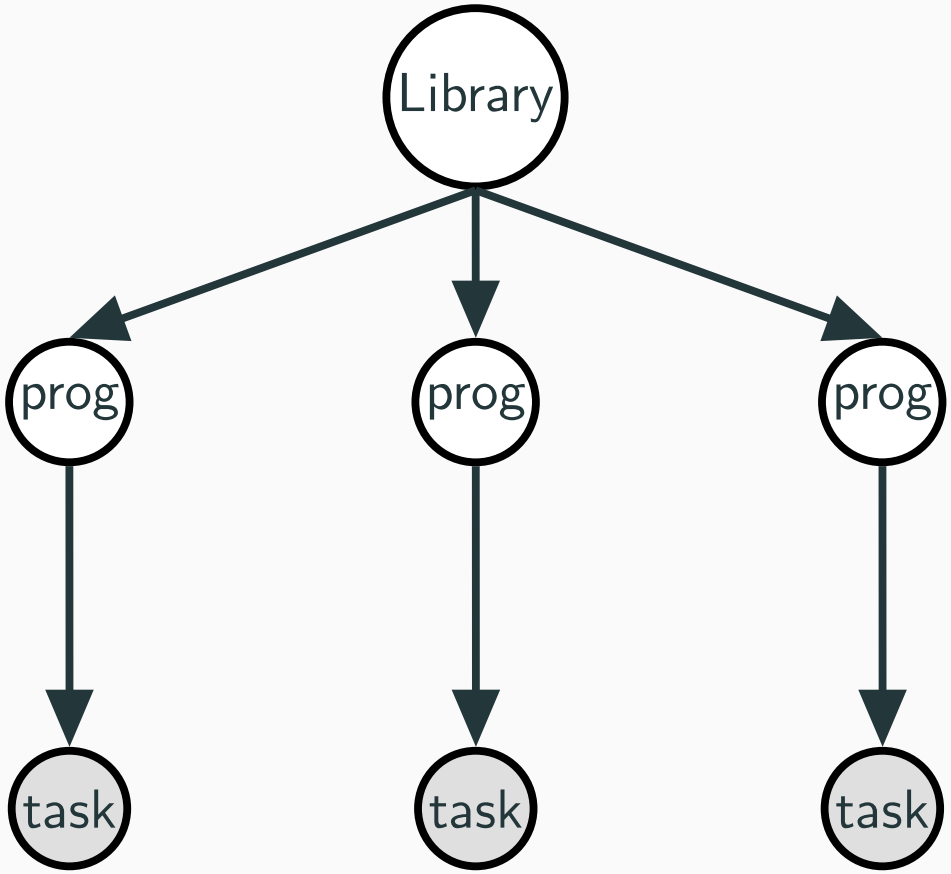


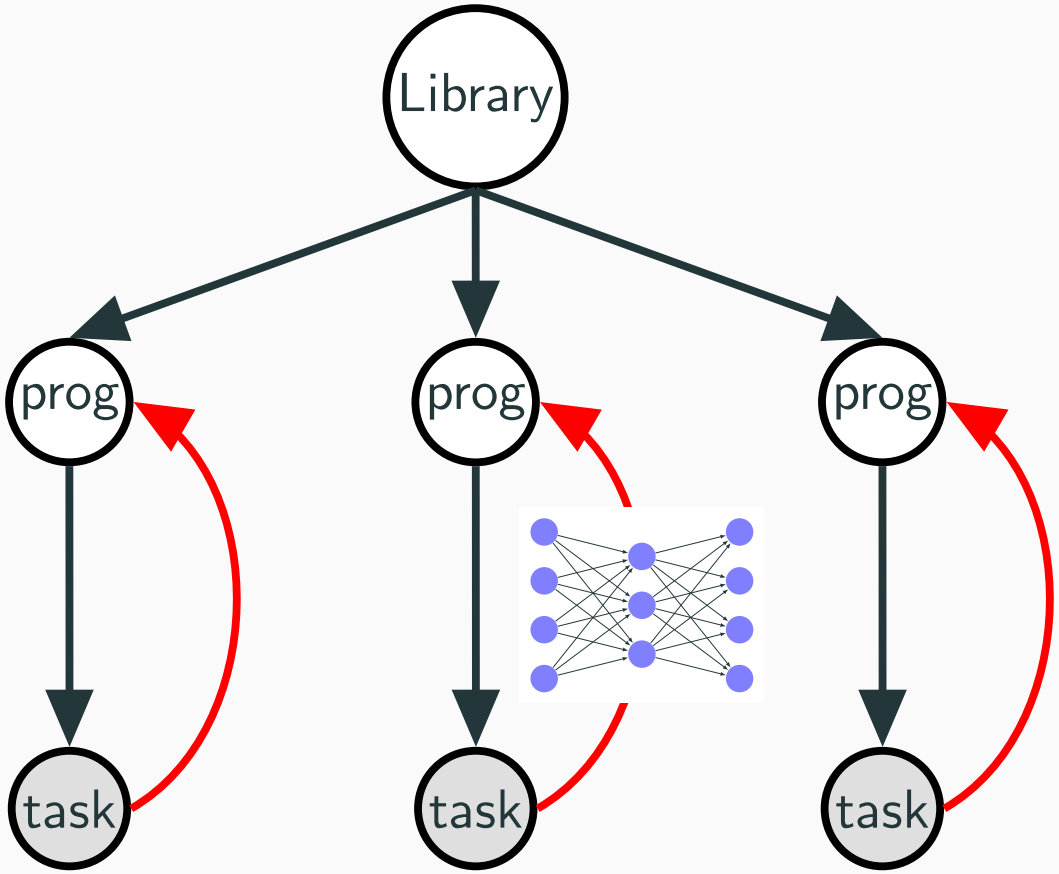
cf. Helmholtz machine, wake/sleep neural network training

Library learning as Bayesian inference



[Dechter et al., 2013] [Liang et al, 2010]; [Lake et al, 2015]

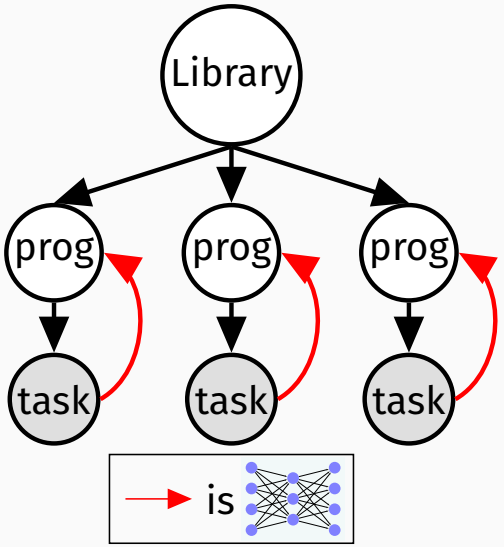
Library learning as neurally-guided Bayesian inference



library learning via program analysis +

new neural inference network for program synthesis +

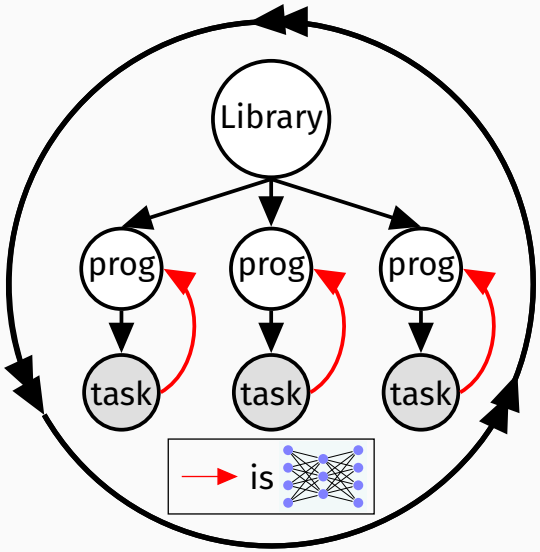
better program representation (Lisp+polymorphic types [Milner 1978])


Library
prog
prog
prog
task
task
task
is

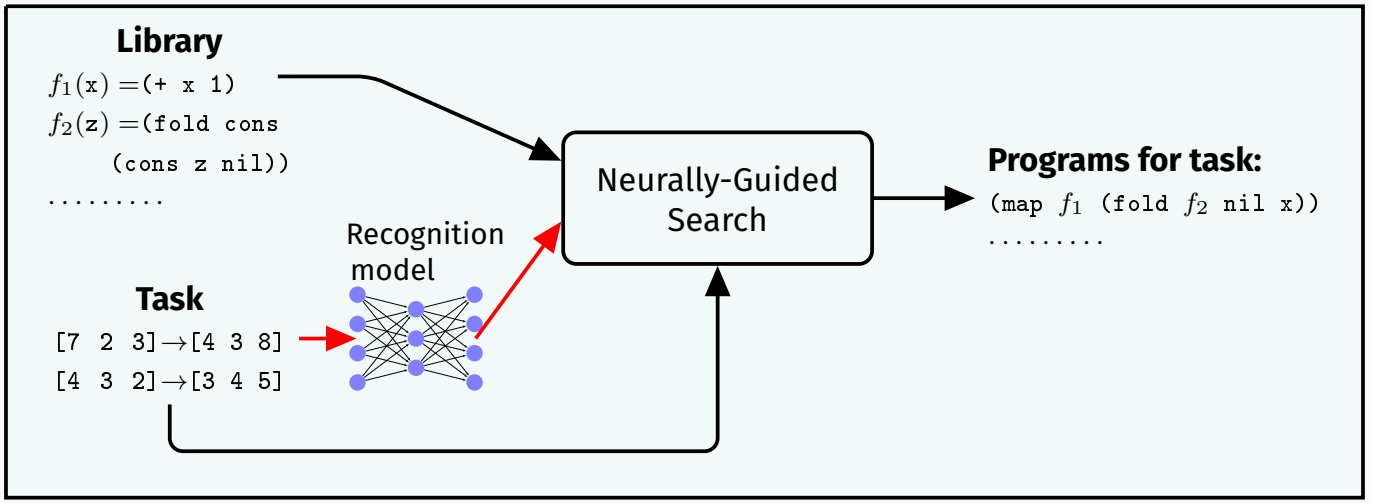
Wake

Sleep: Abstraction

Sleep: Dreaming

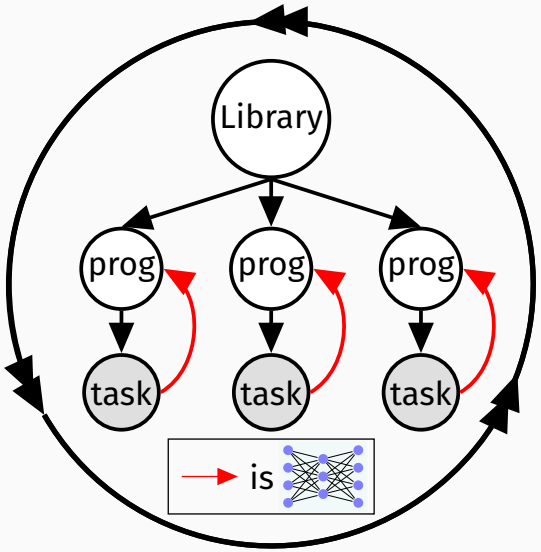


WAKE

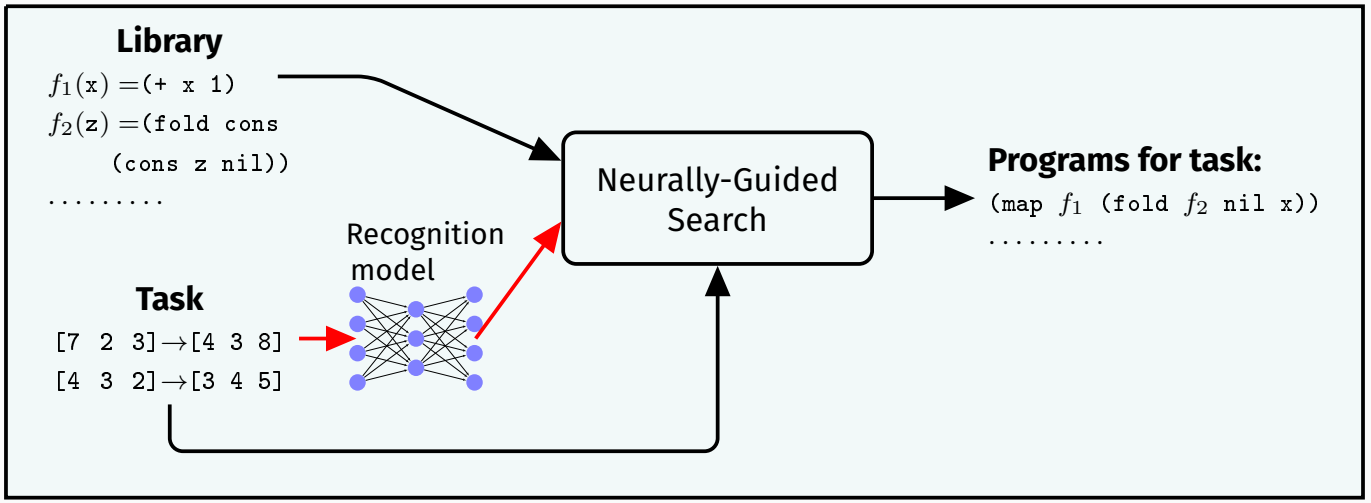


SLEEP: ABSTRACTION

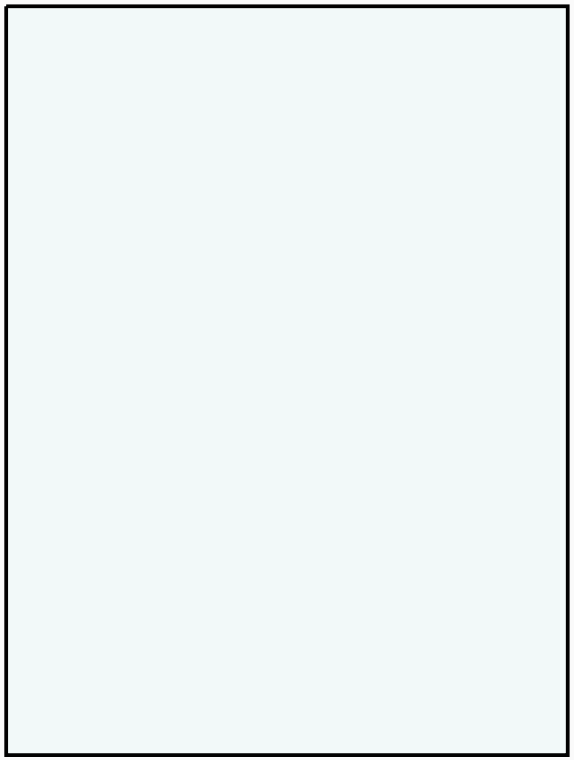
SLEEP: DREAMING

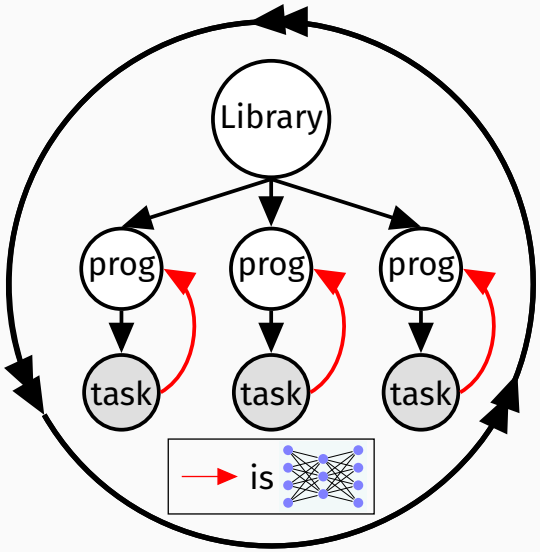


Wake

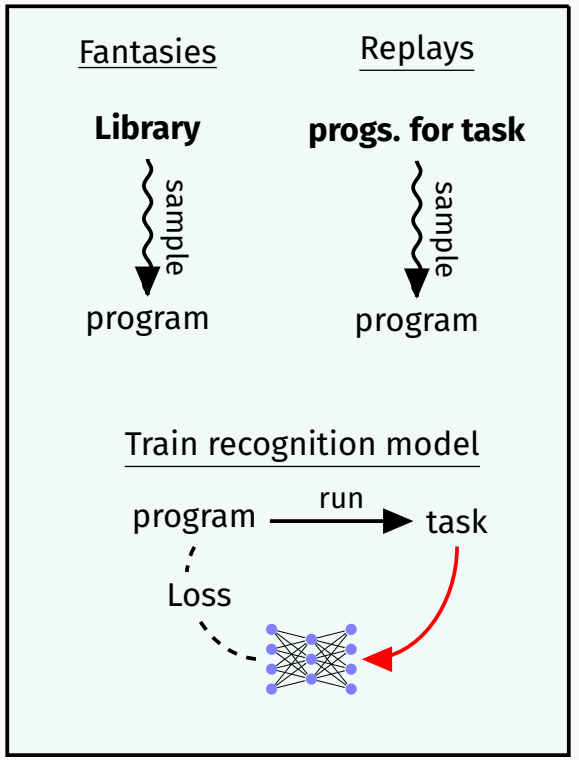


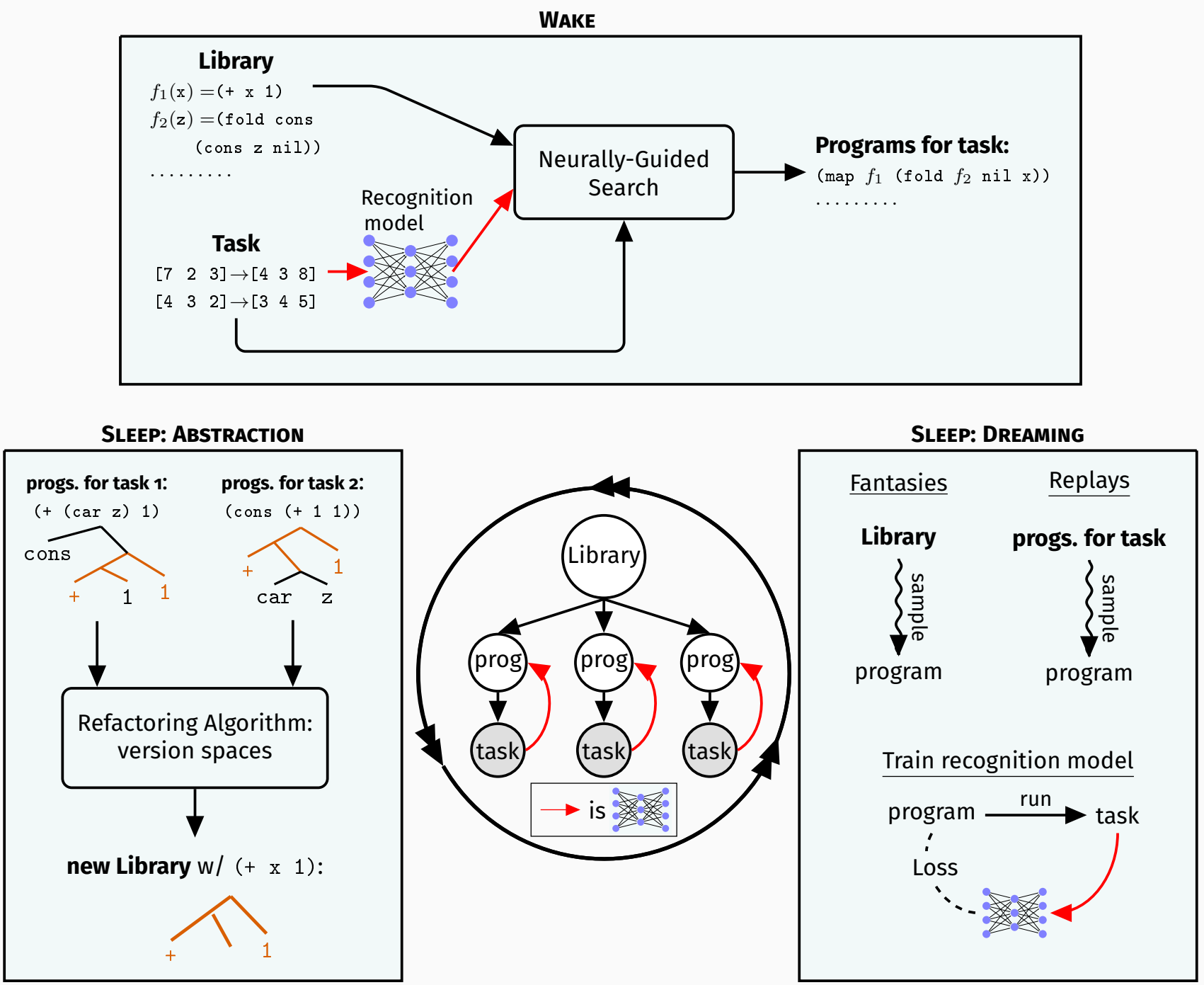
Sleep: Abstraction





Sleep: Dreaming




WAKE
Library
f_1(x) =(+ x 1)
f_2(z) =(fold cons
(cons z nil))
.....
Task
[7 2 3]→[4 3 8]
[4 3 2]→[3 4 5]
Recognition model
Neurally-Guided Search
Programs for task:
(map f_1 (fold f_2 nil x))
.....
SLEEP: ABSTRACTION
progs. for task 1:
(+ (car z) 1)
progs. for task 2:
(cons (+ 1 1))
cons
+
1
1
+
1
car
z
Refactoring Algorithm: version spaces
new Library w/ (+ x 1):
+
1
Library
prog
prog
prog
task
task
task
is
SLEEP: DREAMING
Fantasies
Replays
Library
progs. for task
sample
sample
program
program
Train recognition model
program
run
task
Loss

Abstraction Sleep: Growing the library via refactoring

$5 + 5$

Abstraction Sleep: Growing the library via refactoring

$5 + 5$

```
(+ 5 5)
```

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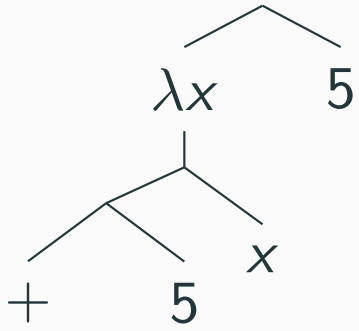

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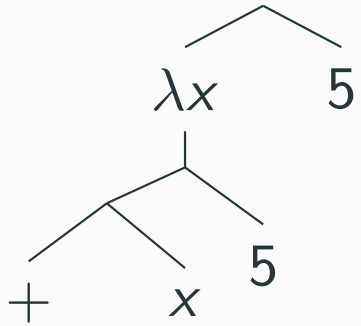

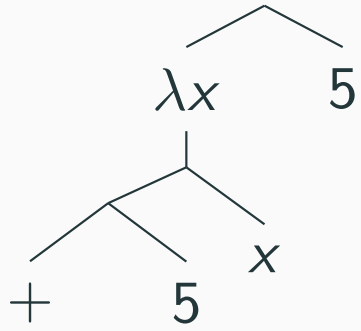

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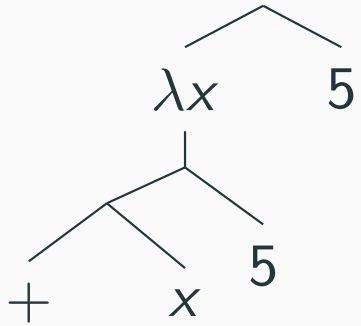

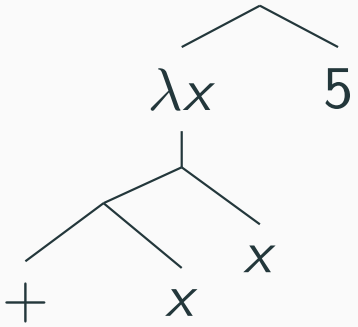

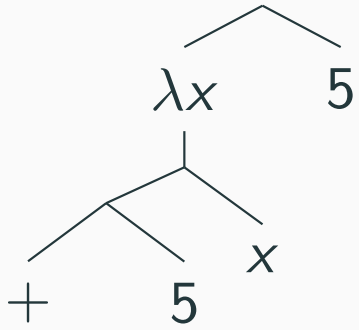

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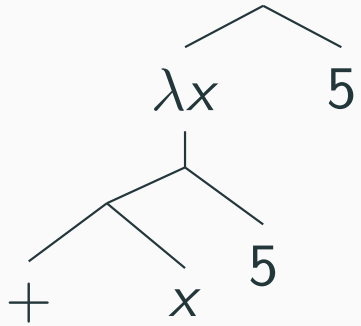
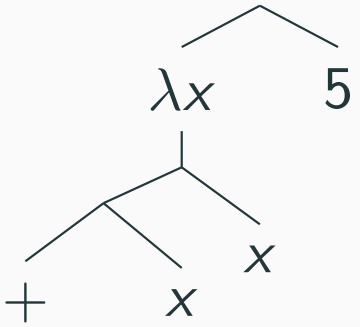
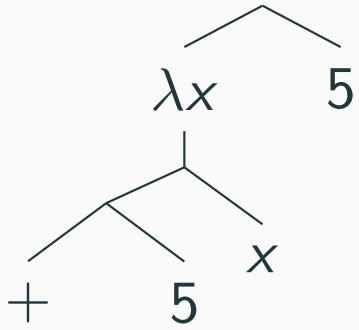
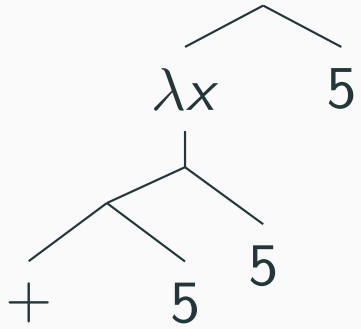
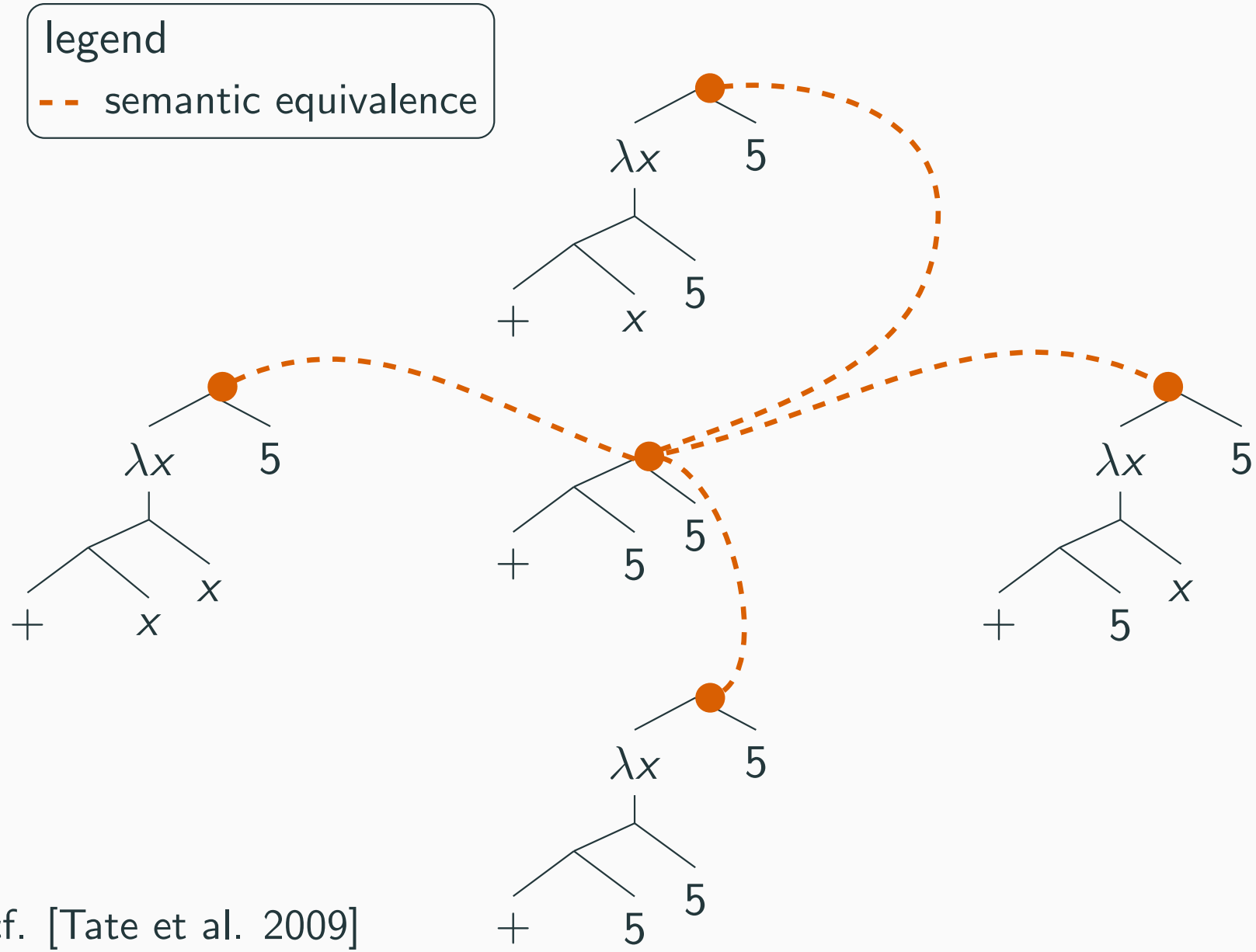








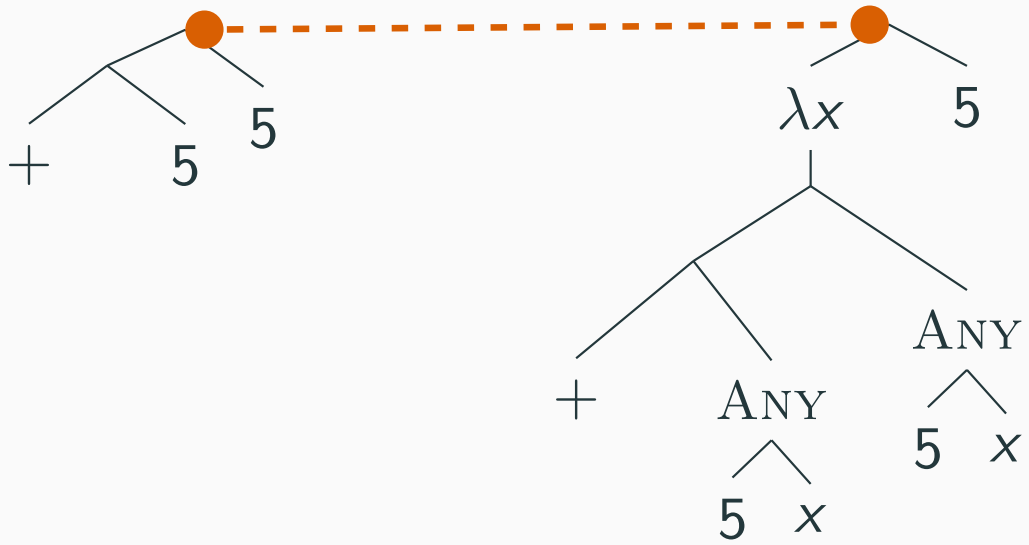
Abstraction Sleep: Growing the library via refactoring



cf. [Tate et al. 2009]

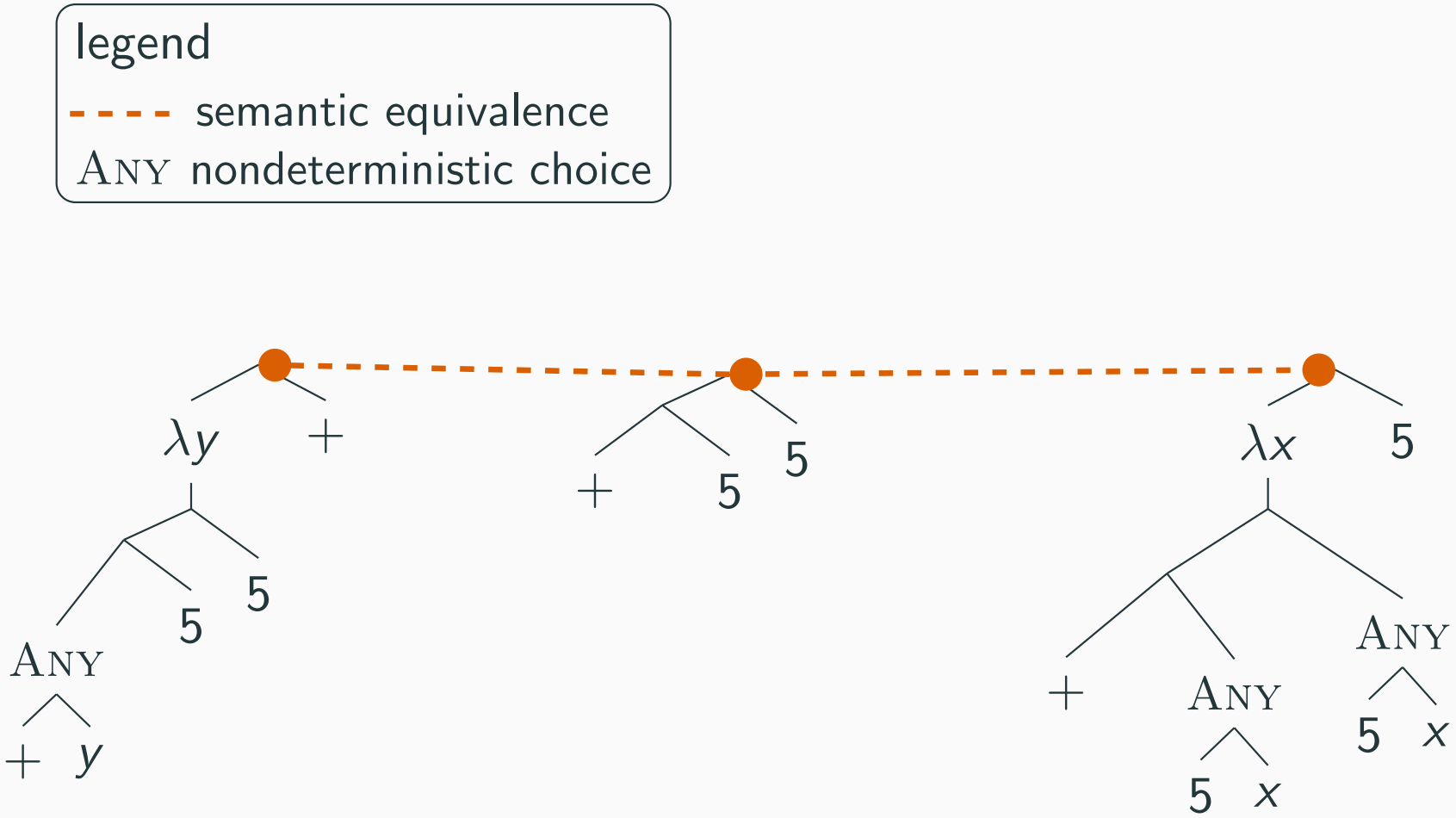
Abstraction Sleep: Growing the library via refactoring

legend
- - - - semantic equivalence
ANY nondeterministic choice



cf. Tate et al. 2009
Gulwani 2012
Liang et al. 2010

Abstraction Sleep: Growing the library via refactoring

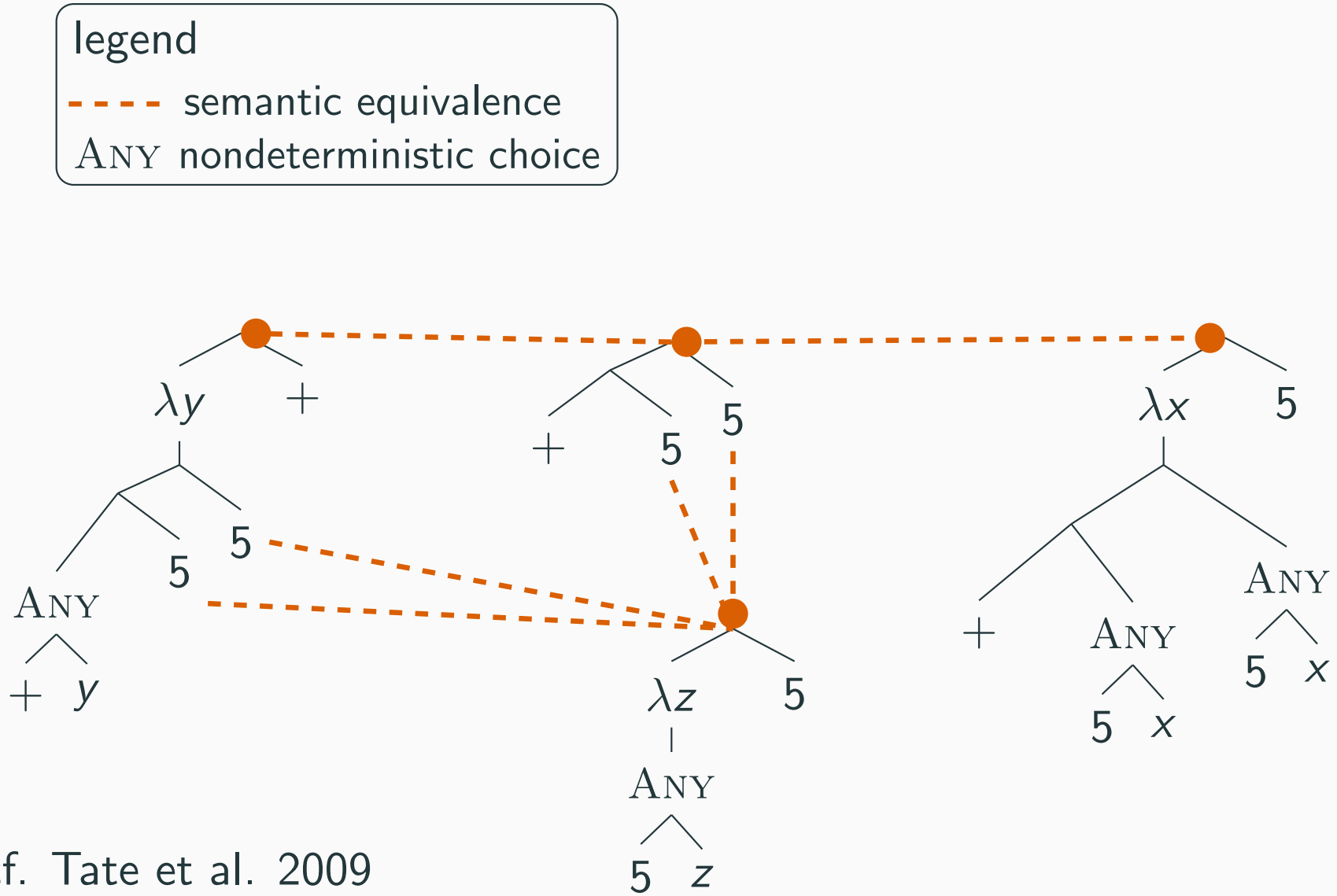


cf. Tate et al. 2009

Gulwani 2012

Liang et al. 2010

Abstraction Sleep: Growing the library via refactoring

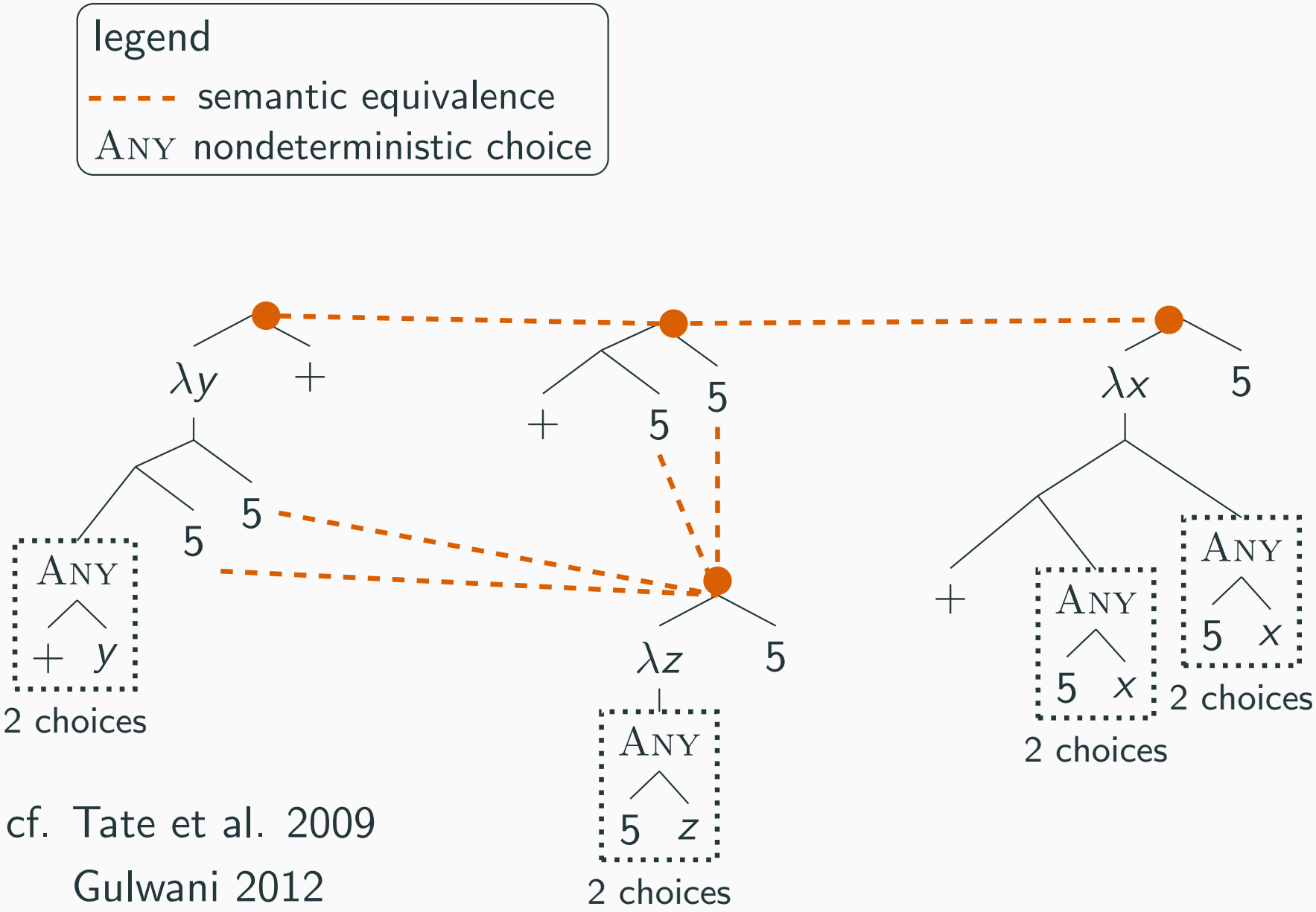


cf. Tate et al. 2009

Gulwani 2012

Liang et al. 2010

Abstraction Sleep: Growing the library via refactoring



cf. Tate et al. 2009
Gulwani 2012
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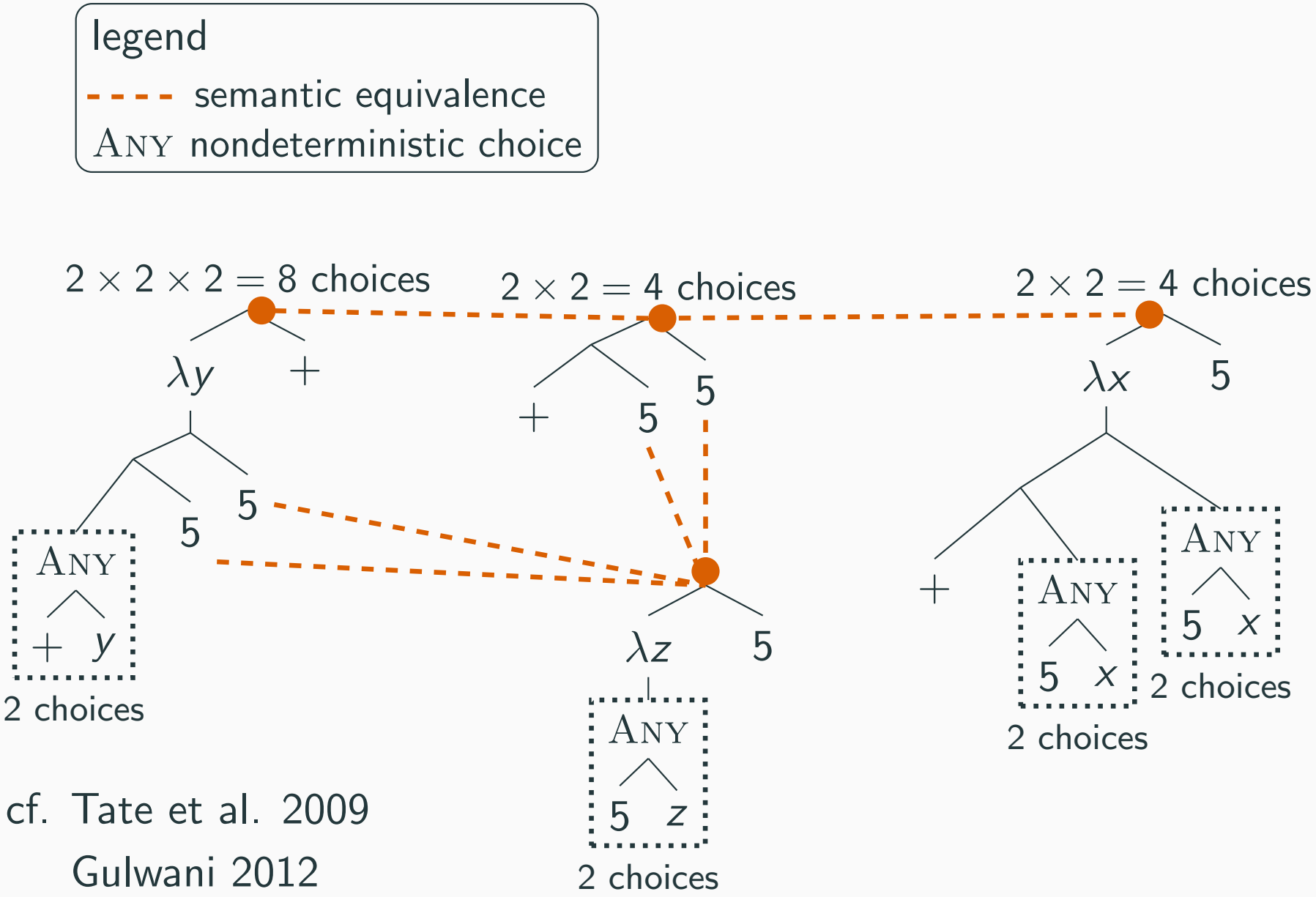
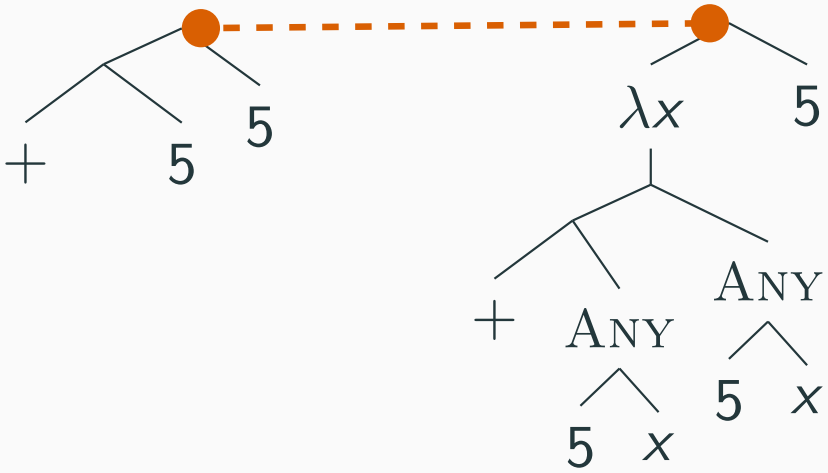
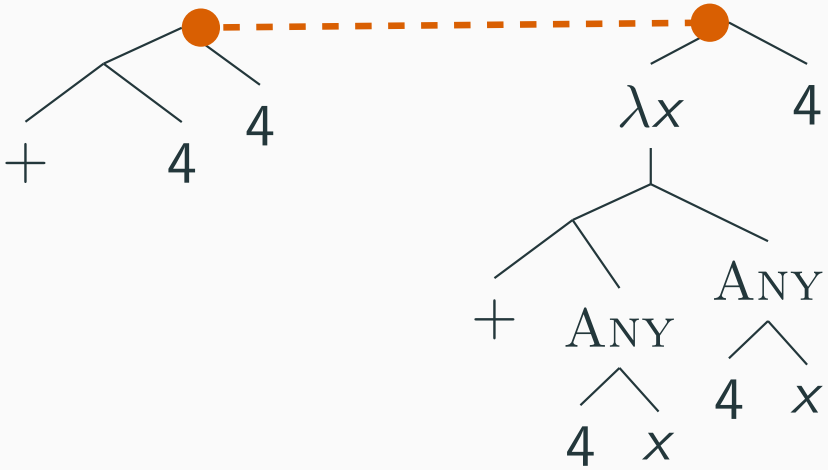
Abstraction Sleep: Growing the library via refactoring



cf. Tate et al. 2009
Gulwani 2012
Liang et al. 2010

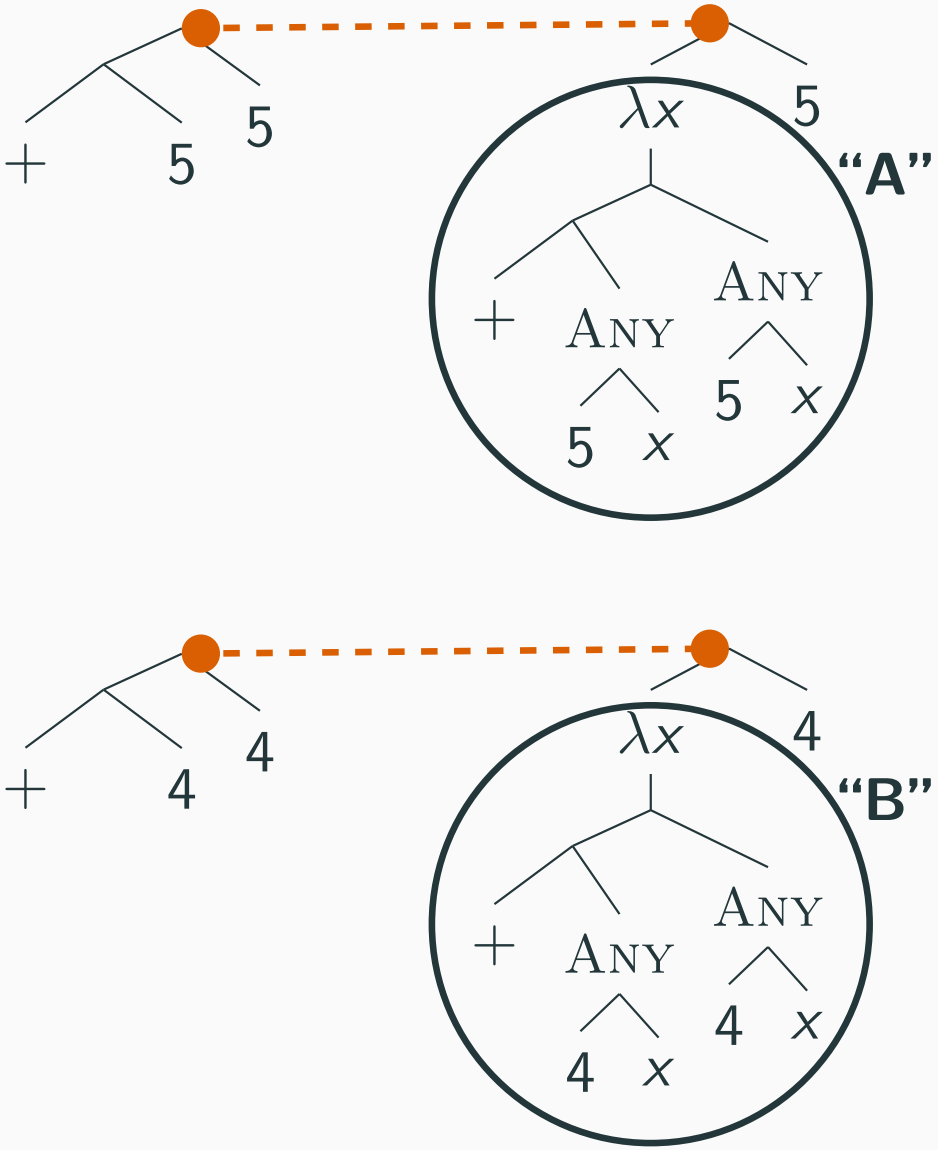


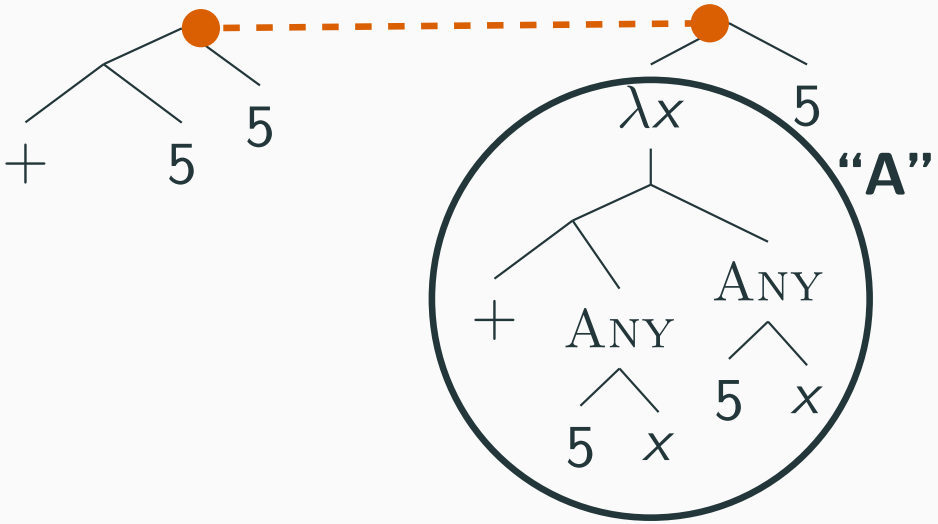


legend

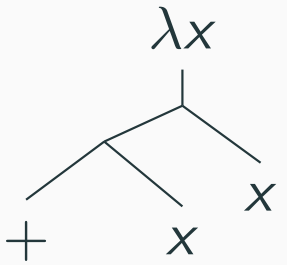
semantic equivalence

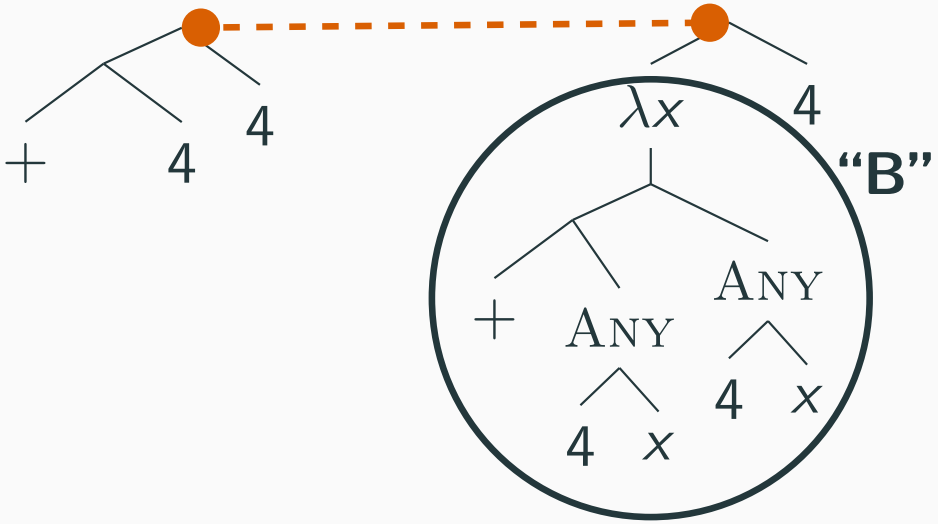
ANY nondeterministic choice


+
5
5
λx
5
“A”
+
ANY
ANY
5
x
5
x
+
4
4
λx
4
“B”
+
ANY
ANY
4
x
4
x
legend
semantic equivalence
ANY nondeterministic choice



A∩B is:

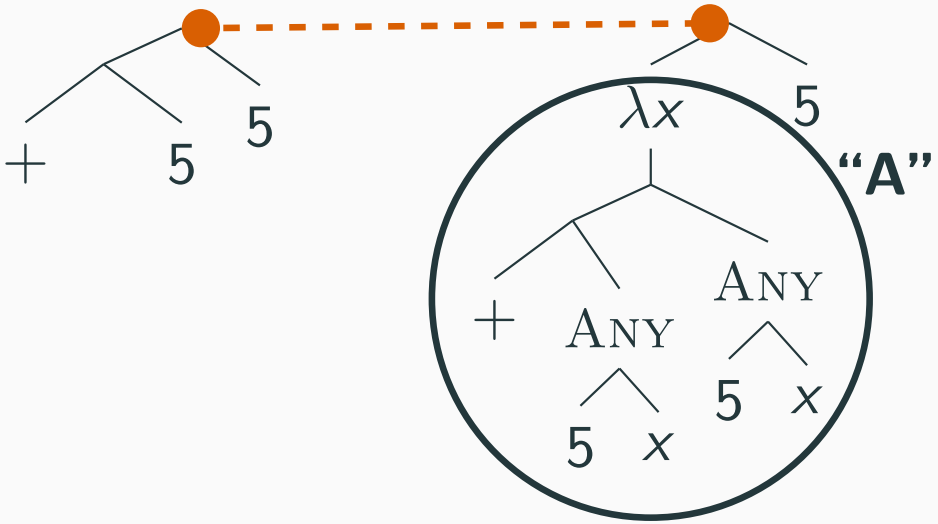




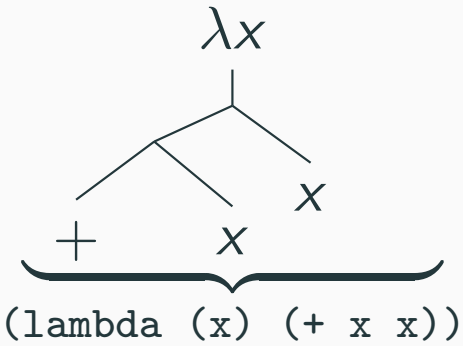
legend

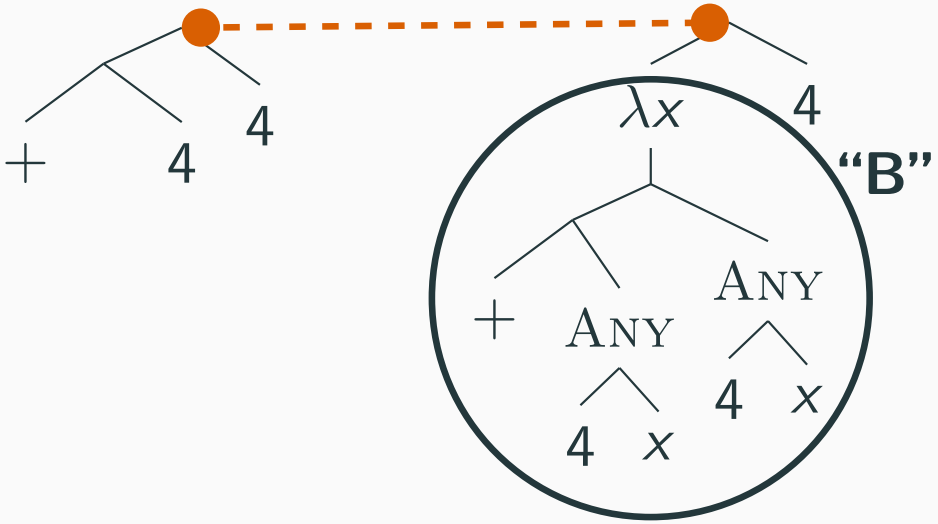
- - - - semantic equivalence

ANY nondeterministic choice



A∩B is:

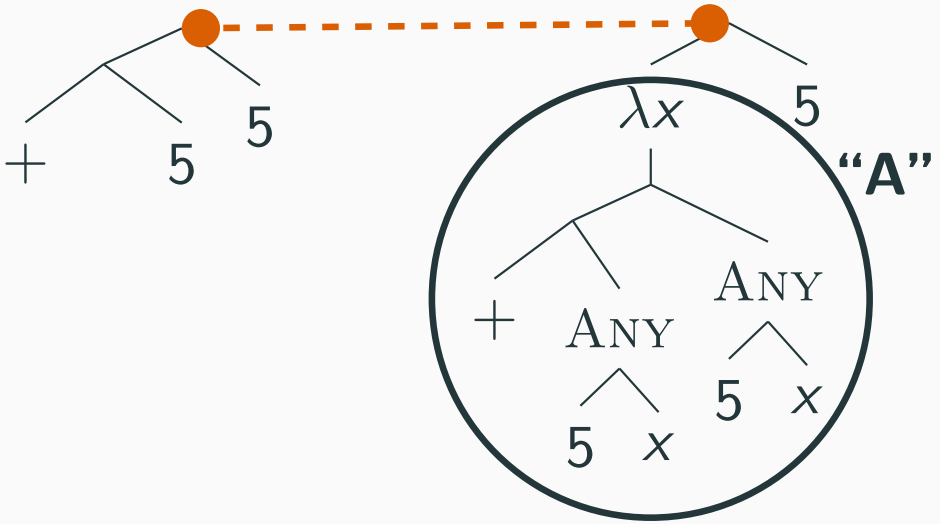




legend

- - - - semantic equivalence

ANY nondeterministic choice



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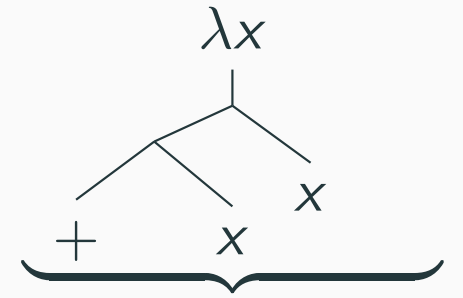
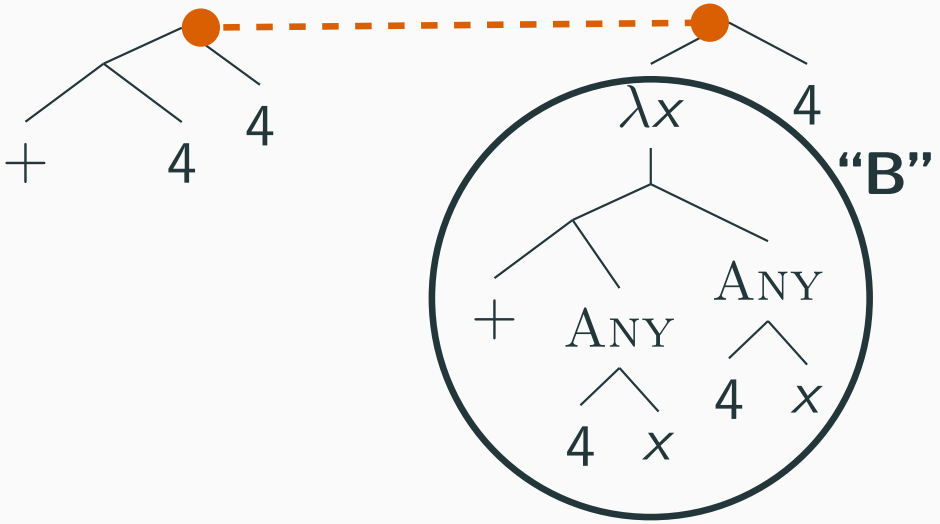


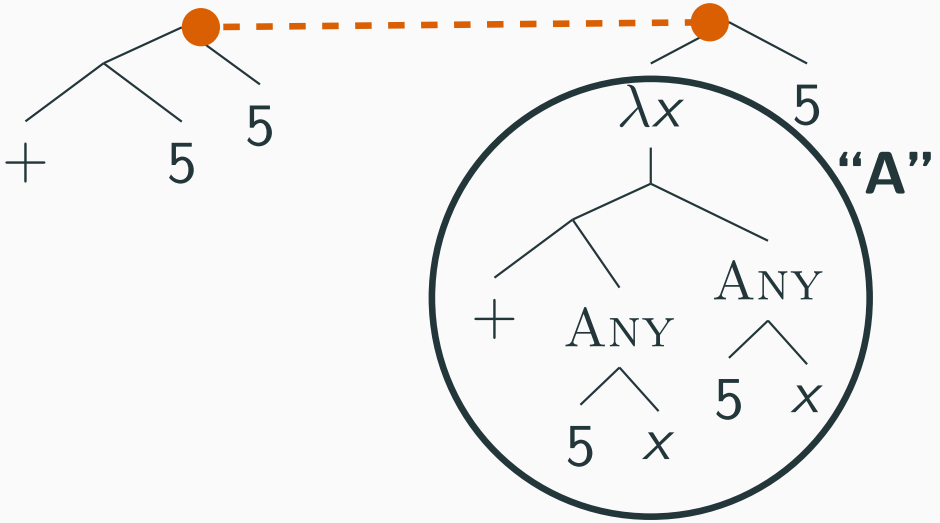
=double



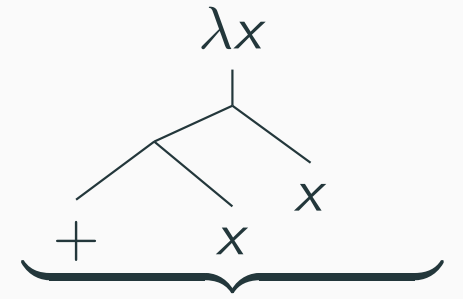
legend

- - - - semantic equivalence

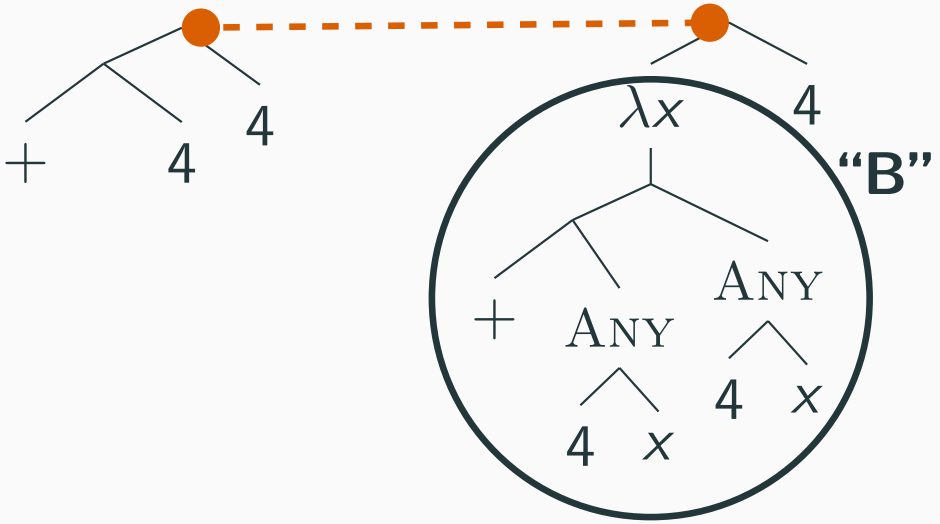
ANY nondeterministic choice



A∩B is:



=double



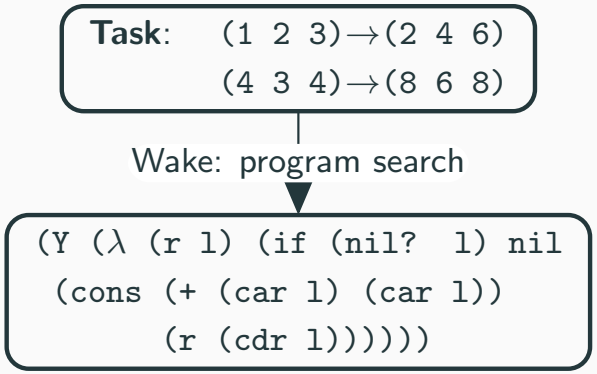
w/o double	w/ double
(+ 5 5)	(double 5)
(+ 4 4)	(double 4)
(+ 3 3)	(double 3)
	...

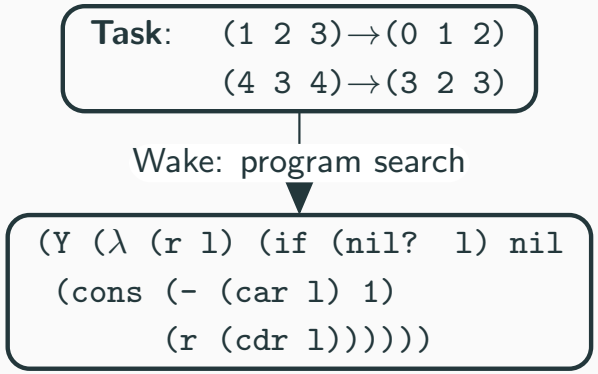
legend

- - - - semantic equivalence

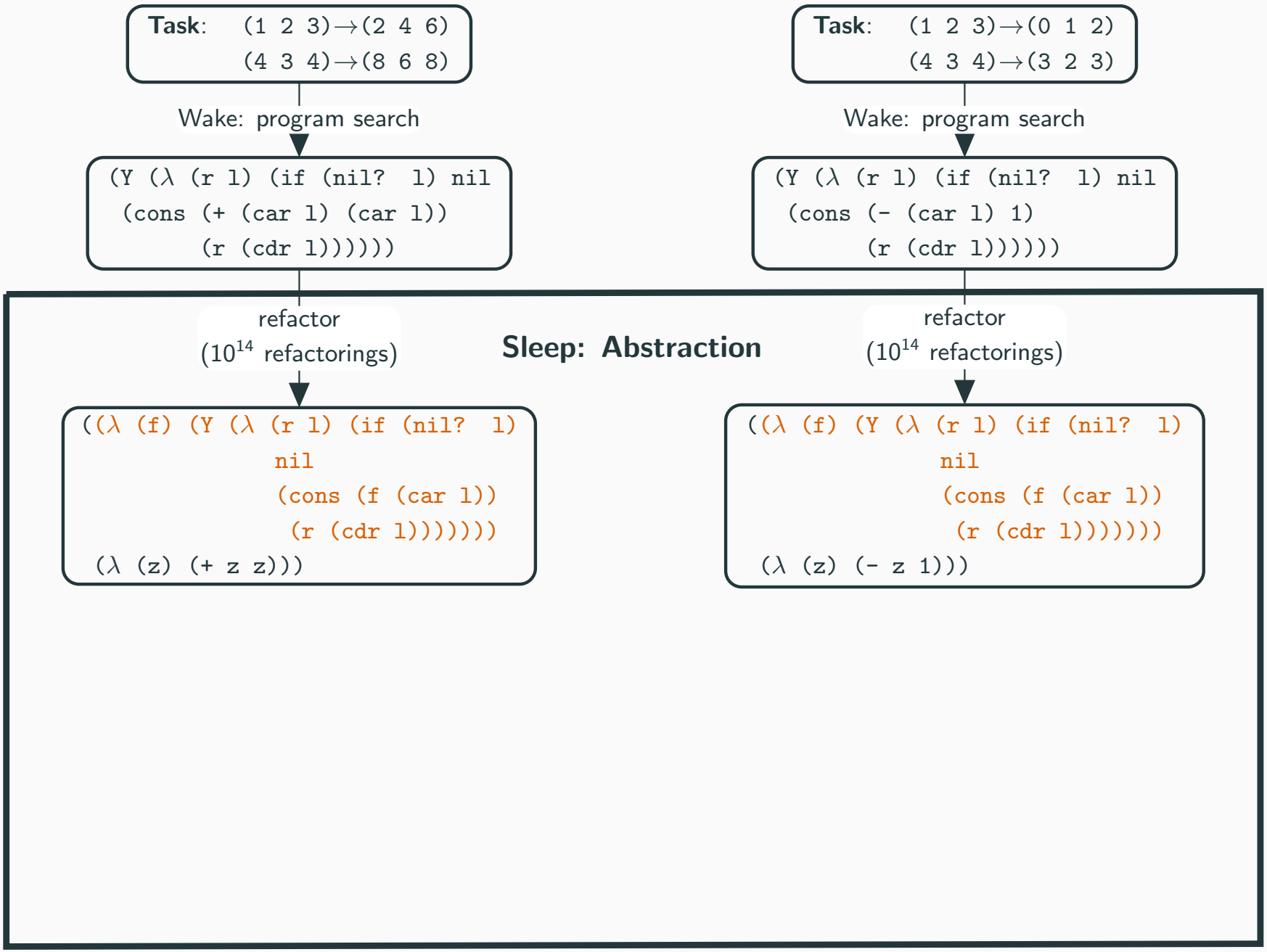
ANY nondeterministic choice

Abstraction Sleep: Growing the library via refactoring





Abstraction Sleep: Growing the library via refactoring



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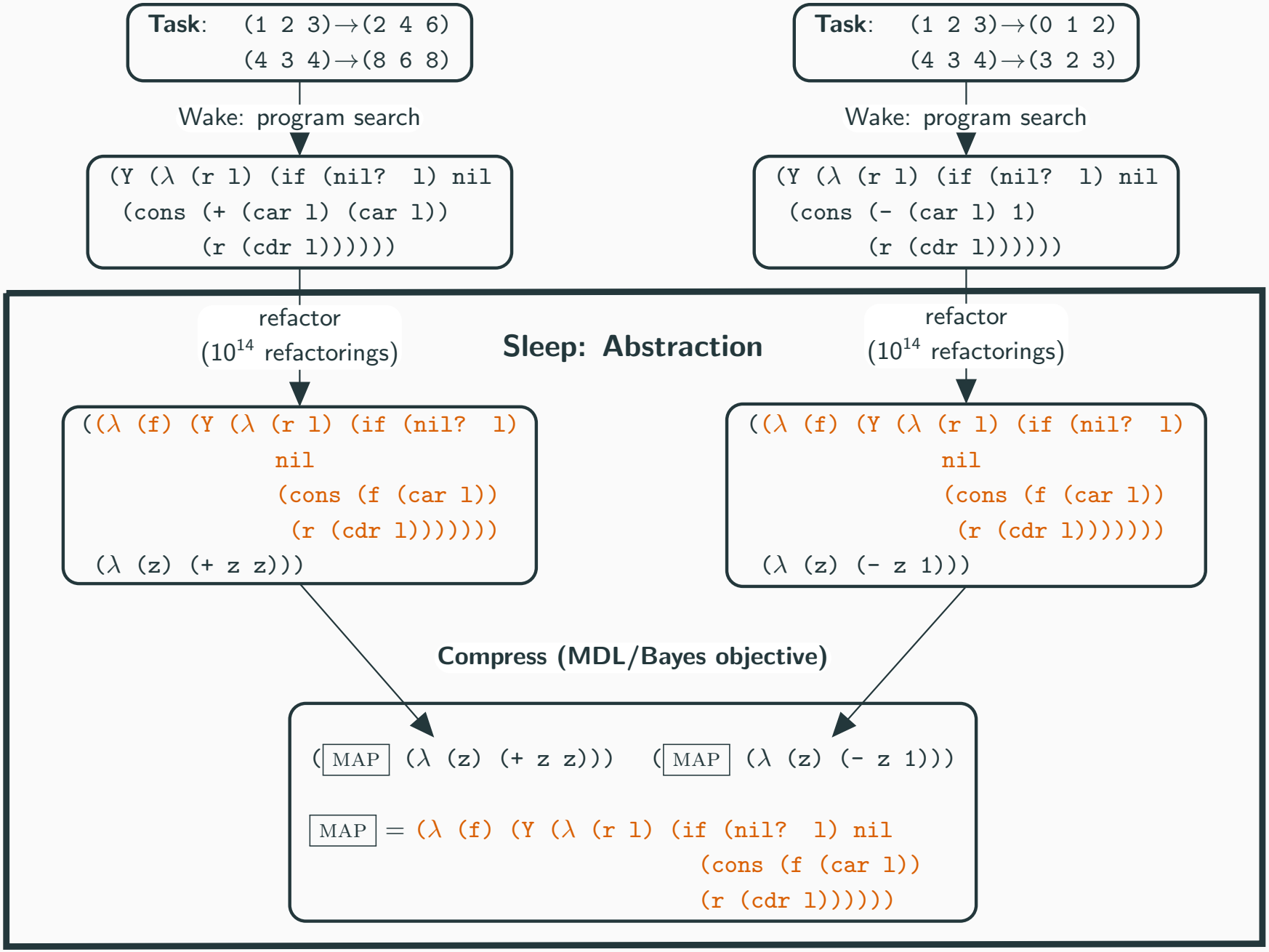


DreamCoder Domains

List Processing

Sum List

```
[1 2 3] → 6
[4 6 8 1] → 17
```

Double

```
[1 2 3 4] → [2 4 6 8]
[6 5 1] → [12 10 2]
```

Check Evens

```
[0 2 3] → [T T F]
[2 4 9 6] → [T T F T]
```

Text Editing

Abbreviate

```
Allen Newell → A.N.
Herb Simon → H.S.
```

Drop Last Characters

```
jabberwocky → jabberw
copycat → cop
```

Extract

```
see spot(run) → run
a (bee) see → bee
```

Regexes

Phone Numbers

```
(555) 867-5309
(650) 555-2368
```

Currency

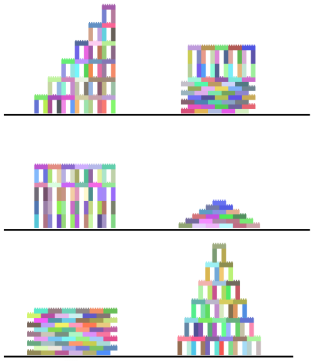
```
$100.25
$4.50
```

Dates

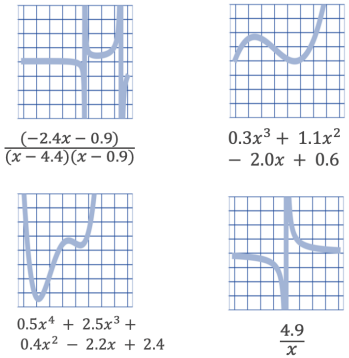
```
Y1775/0704
Y2000/0101
```

LOGO Graphics

Block Towers

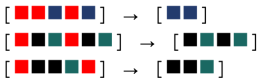


Symbolic Regression



Recursive Programming

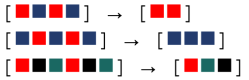
Filter



Index List

Length

Every Other



Physics

$$KE = \frac{1}{2}m|\vec{v}|^2$$

$$\vec{a} = \frac{1}{m}\sum_i \vec{F}_i$$

$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r_1} - \vec{r_2}|^2}\widehat{r_1 - r_2}$$

$$R_{total} = \left(\Sigma_i \frac{1}{R_i}\right)^{-1}$$

DreamCoder Domains

List Processing

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[4 6 8 1] → 17

Double

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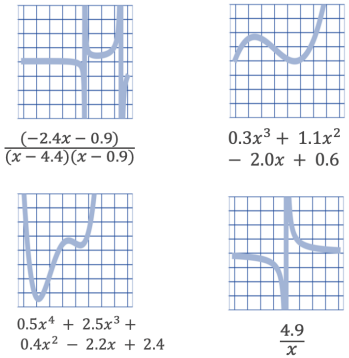
Block Towers

Symbolic Regression



Recursive Programming

Filter

[ ] → [ ]
[ ] → [ ]
[ ] → [ ]

Index List

0, [ ] →
1, [ ] →
1, [ ] →

Length

[ ] → 4
[ ] → 6
[ ] → 3

Every Other

[ ] → [ ]
[ ] → [ ]
[ ] → [ ]

Physics

$$KE = \frac{1}{2}m|\vec{v}|^2$$

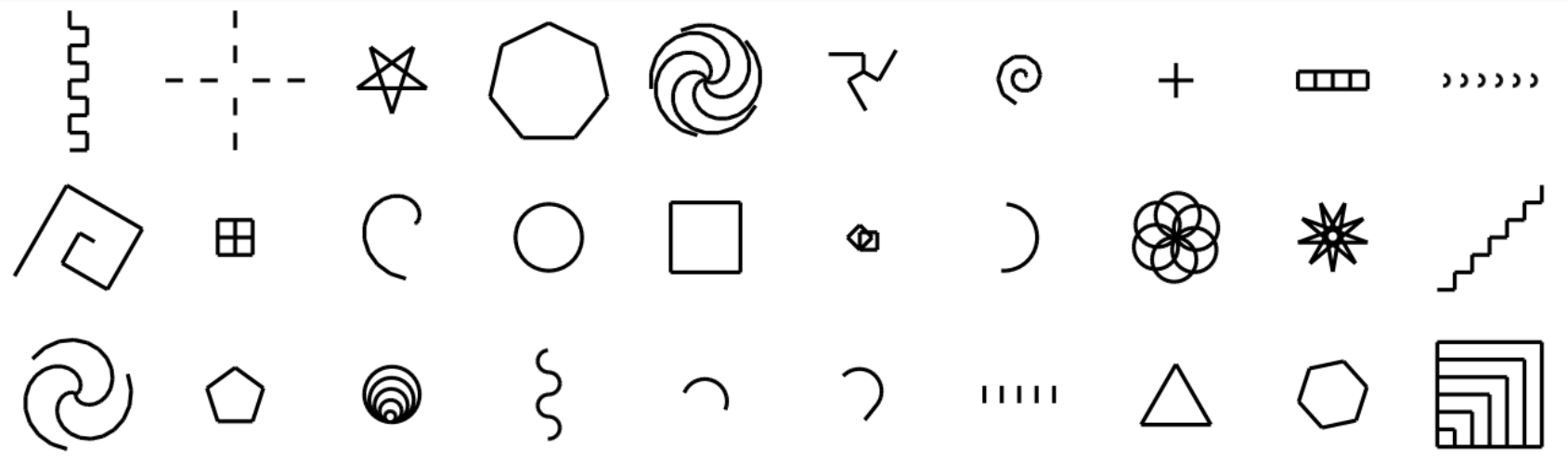
$$\vec{a} = \frac{1}{m}\sum_i \vec{F}_i$$

$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r_1} - \vec{r_2}|^2}\widehat{r_1 - r_2}$$

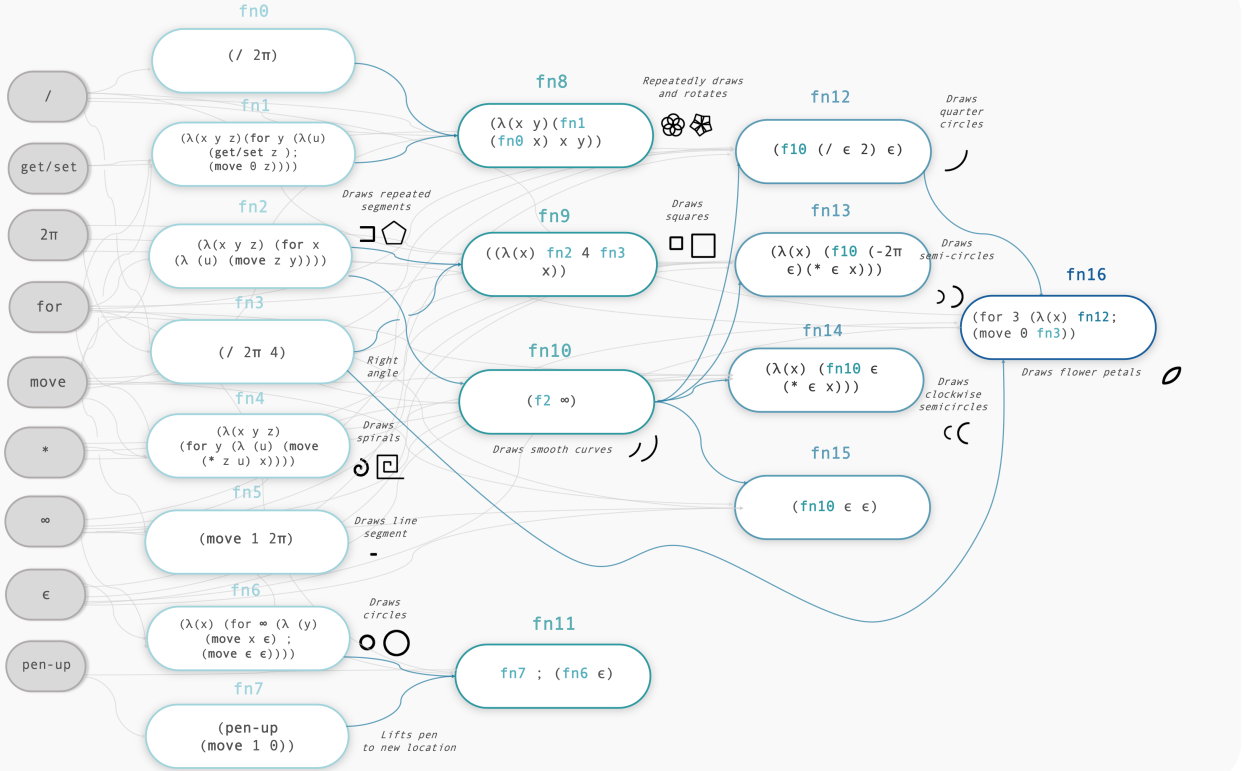
$$R_{total} = \left(\Sigma_i \frac{1}{R_i}\right)^{-1}$$

LOGO Graphics

30 out of 160 tasks



LOGO Graphics – learning interpretable library of concepts



```
(fn8 5 (fn4 (* ϵ 2) ∞ ϵ))
```

```
(for 2 (λ (x)
(fn14 2); (fn13 2)))
```

```
(for 7 (λ (x) (fn9 x)))
```

```
(fn10 (* (fn0 7) 3) 3)
```

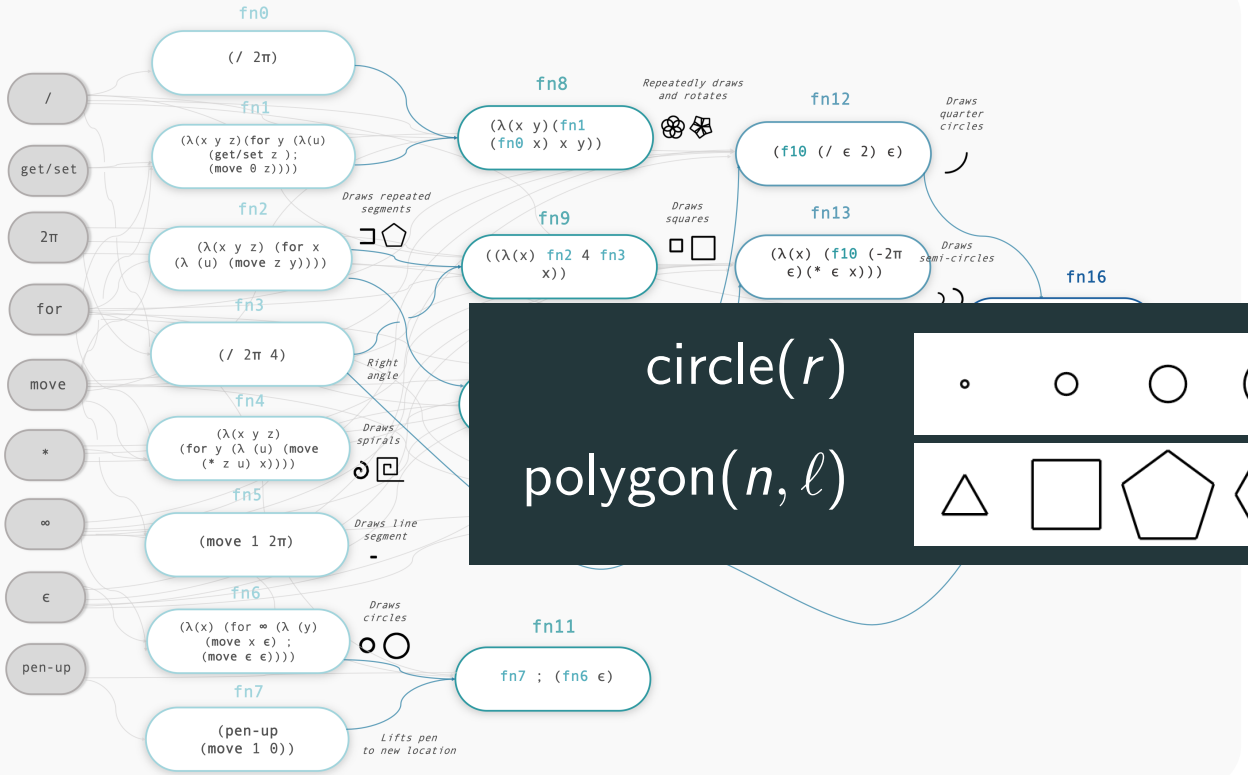
```
(fn8  6 (fn7 ; fn5 ;
fn7 ; fn5))
```

```
(fn8 5 fn16)
```

```
(move 0 (fn0 7)); fn5 ;
(fn13 4)
```

```
(fn4 fn3 9 1)
```

LOGO Graphics – learning interpretable library of concepts



circle(r)

polygon(n, ℓ)

(fn8 5 (fn4 (* ϵ 2) ∞ ϵ))

(for 2 (λ (x)
(fn14 2); (fn13 2)))

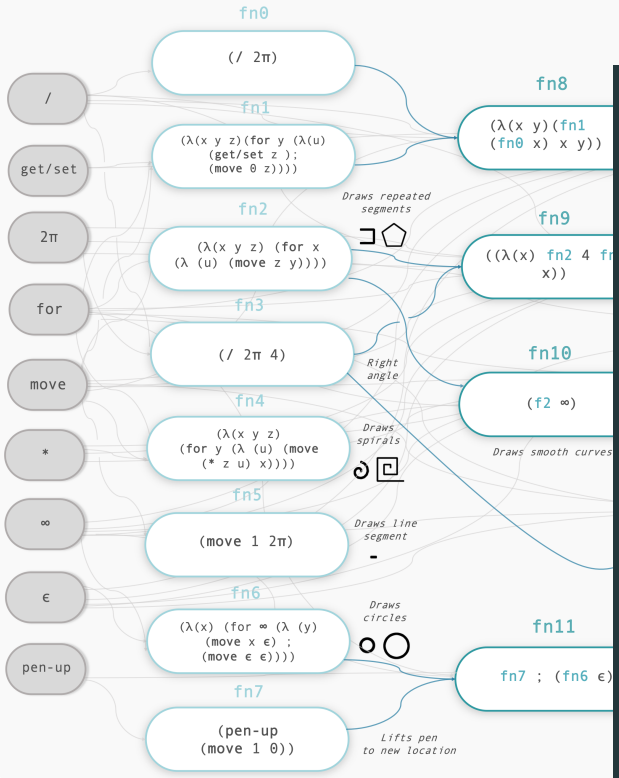
(fn10 (* (fn0 7) 3) 3)

(fn8 5 fn16)

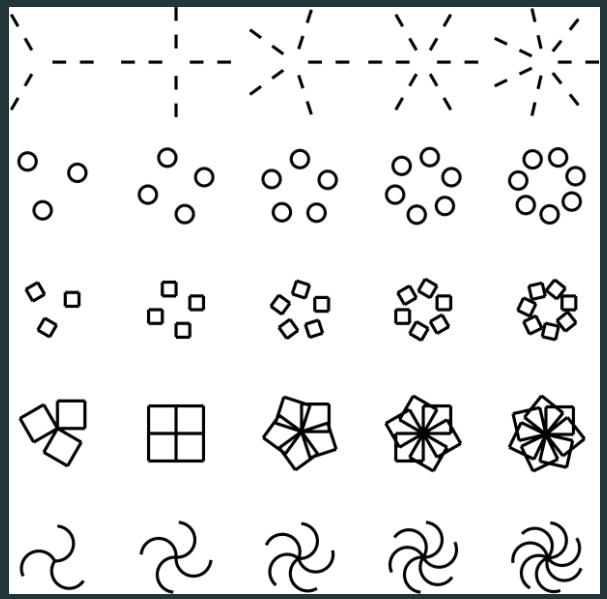
(move 0 (fn0 7)); fn5 ;
(fn13 4)

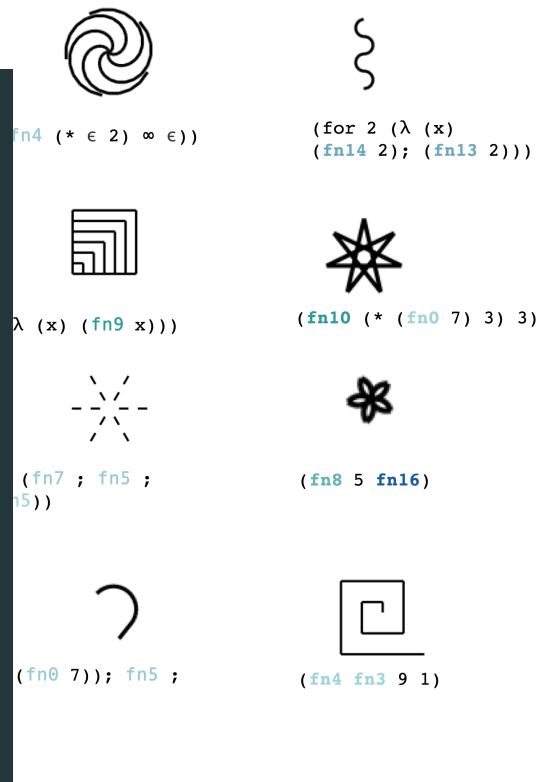
(fn4 fn3 9 1)

LOGO Graphics – learning interpretable library of concepts



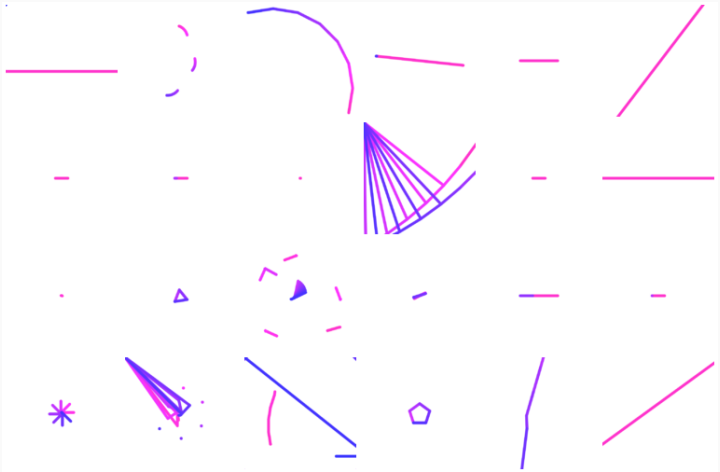
radial symmetry(n, body)



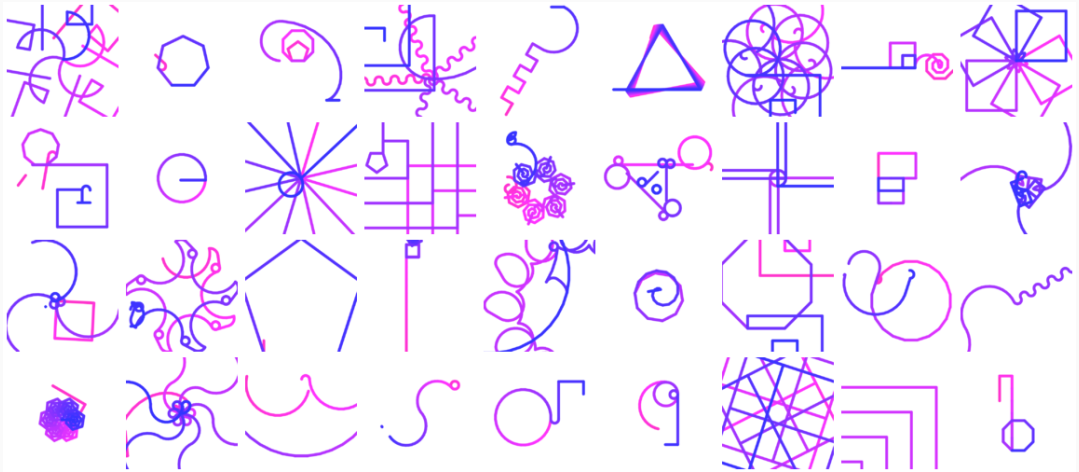


what does DreamCoder dream of?

before learning

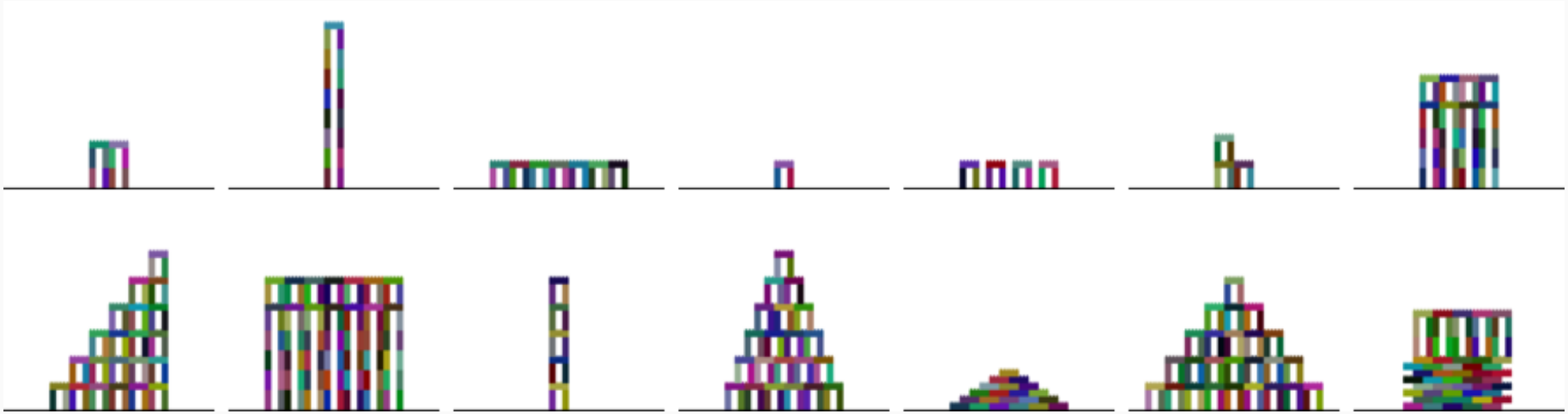


after learning



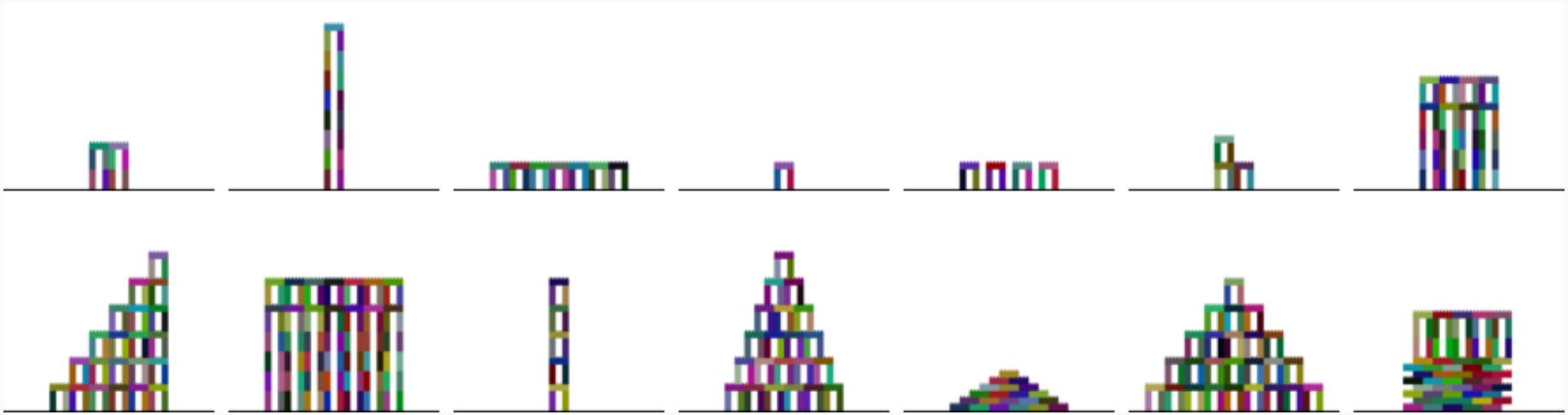
Planning to build towers

example tasks (112 total)



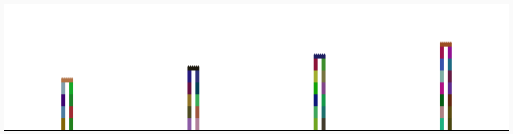
Planning to build towers

example tasks (112 total)

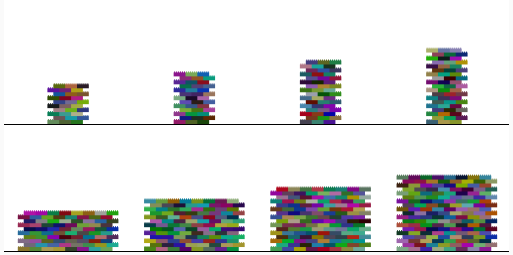


learned library routines ($\approx$ 20 total)

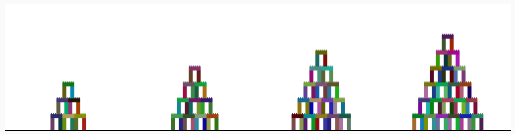
arch(h)



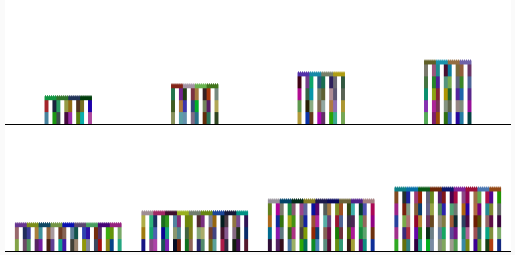
wall(w, h)



pyramid(h)

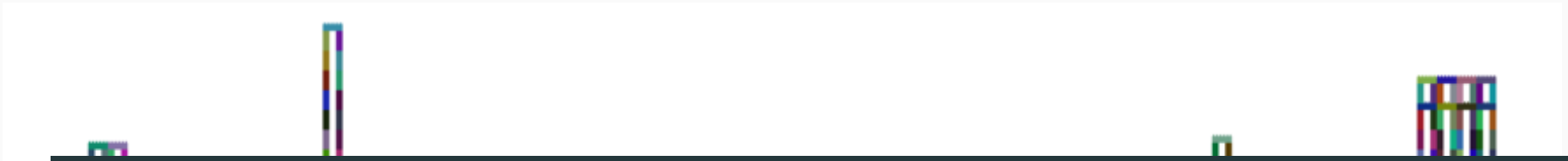


bridge(w, h)



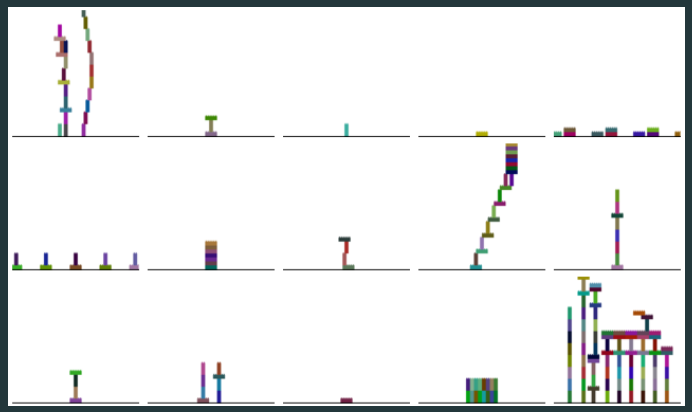
Planning to build towers

example tasks (112 total)

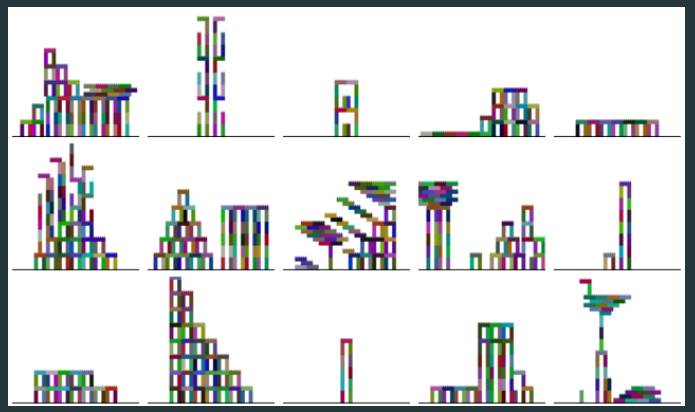


lea

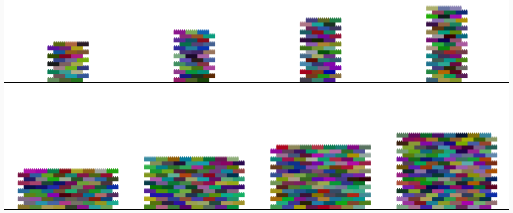
dreams before learning



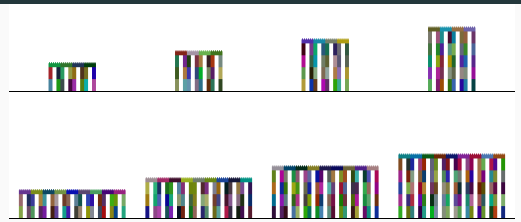
dreams after learning



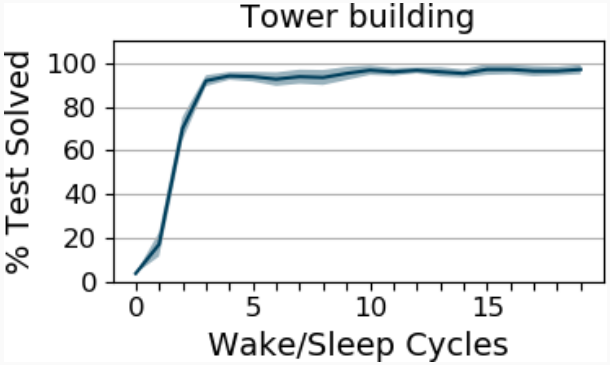
wall(w, h)



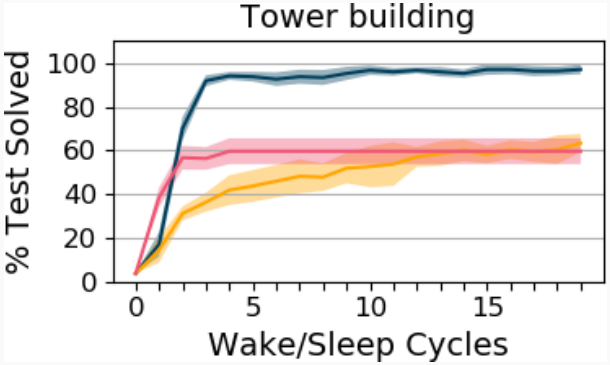
bridge(w, h)

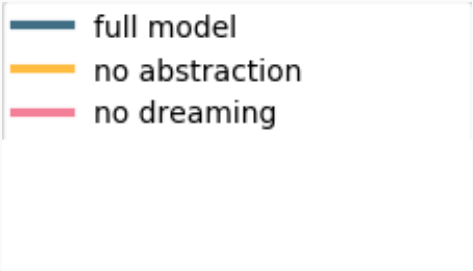


Learning dynamics

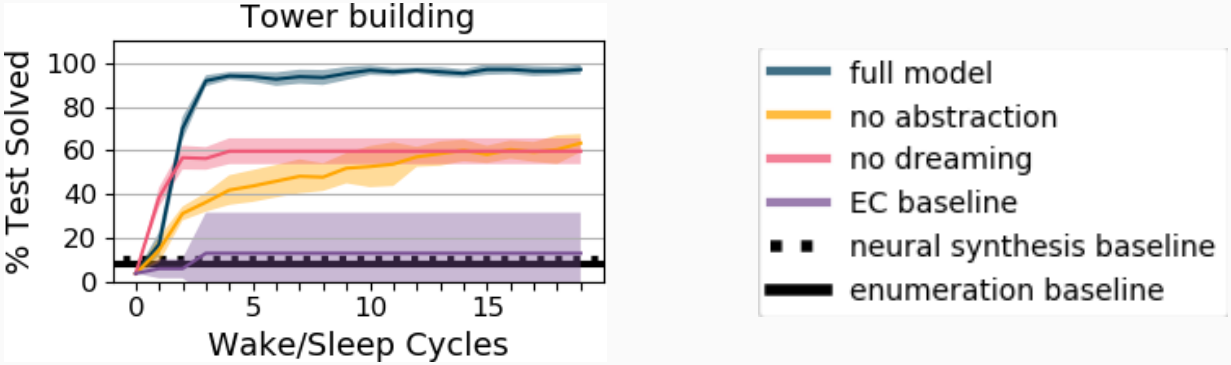


Learning dynamics



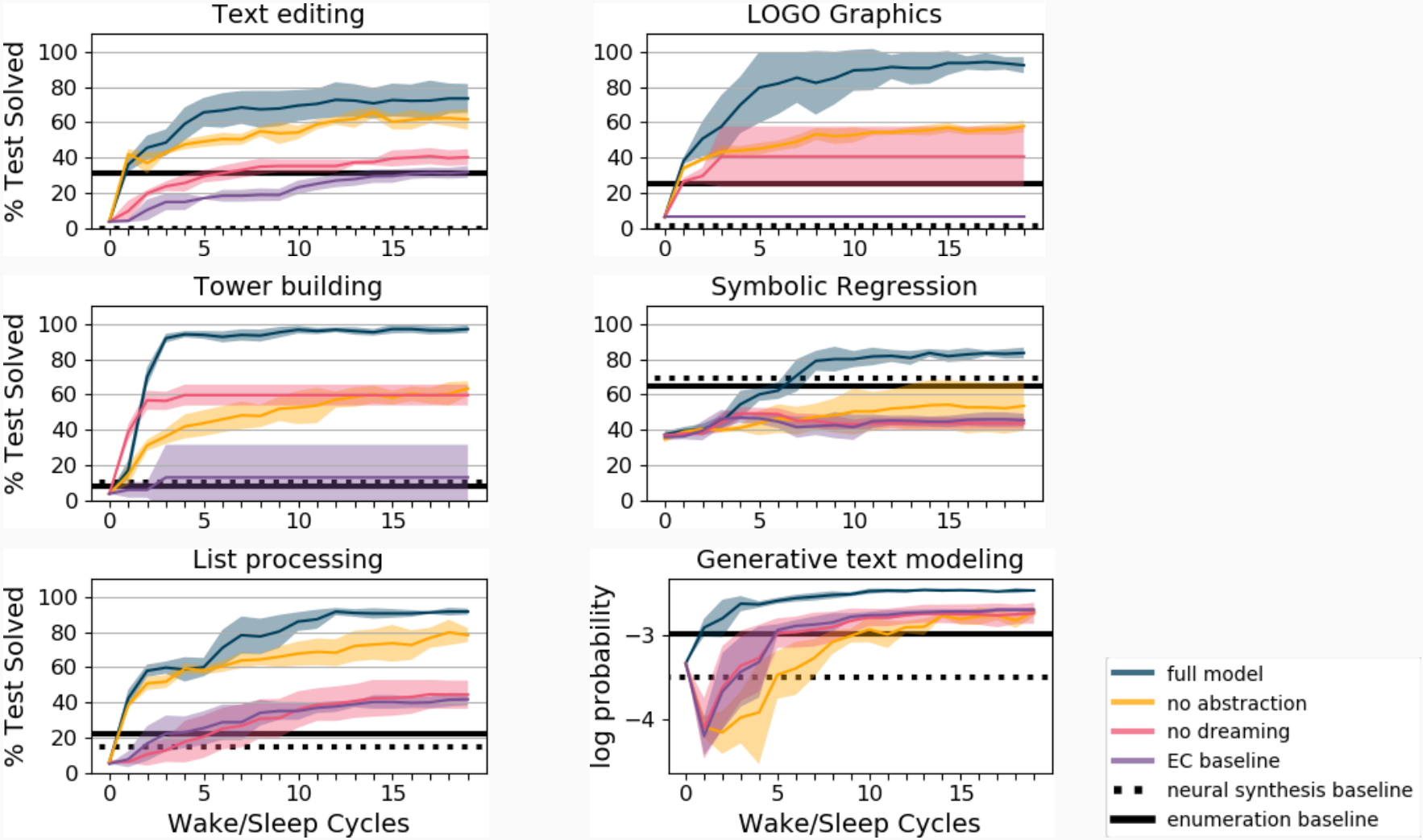


Learning dynamics

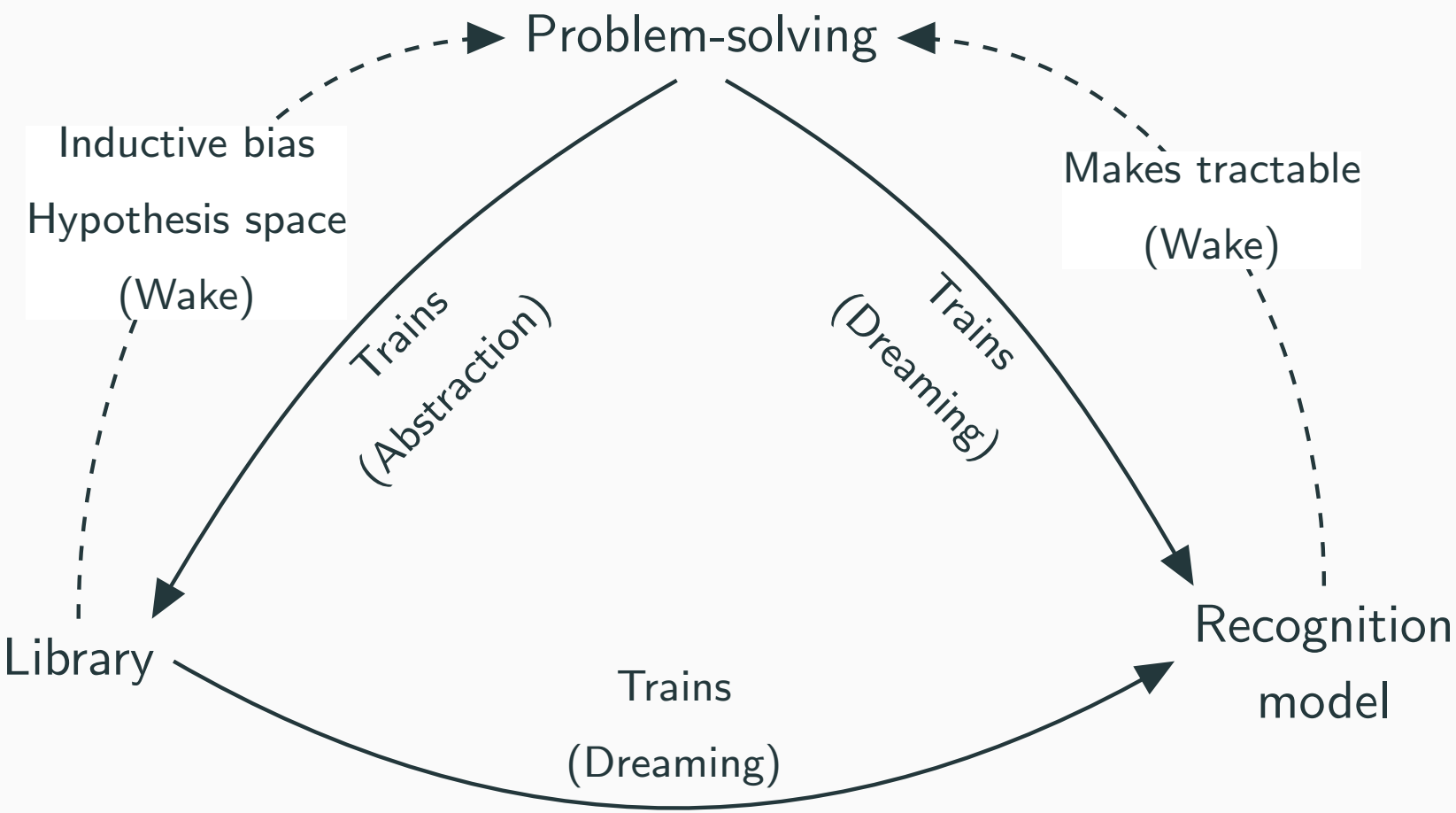


baselines: Exploration-Compression, EC (Dechter et al. 2013)
neural synthesis, RobustFill (Devlin et al. 2017)
24 hours of brute-force enumeration

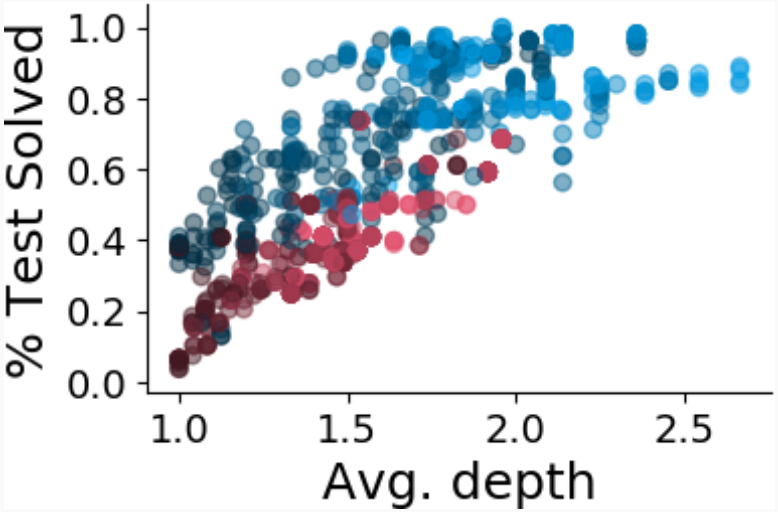
Learning dynamics

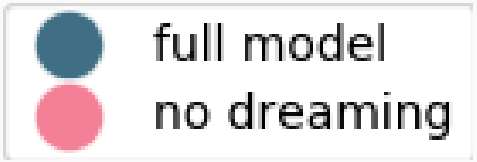


Synergy between dreaming and library learning



Evidence for dreaming bootstrapping better libraries

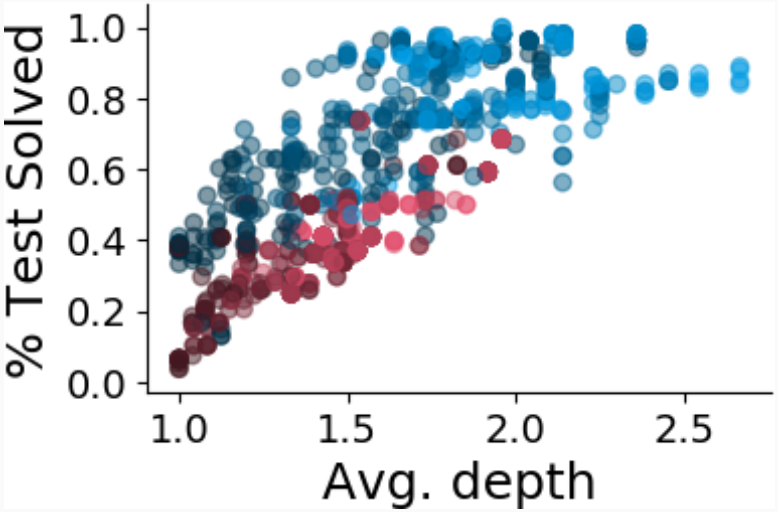


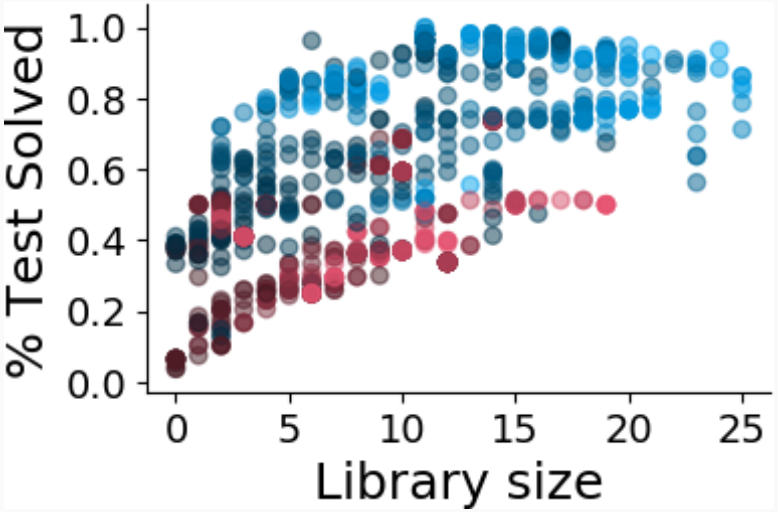


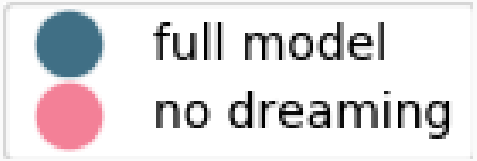
Darker: Early in learning

Lighter: Later in learning

Evidence for dreaming bootstrapping better libraries





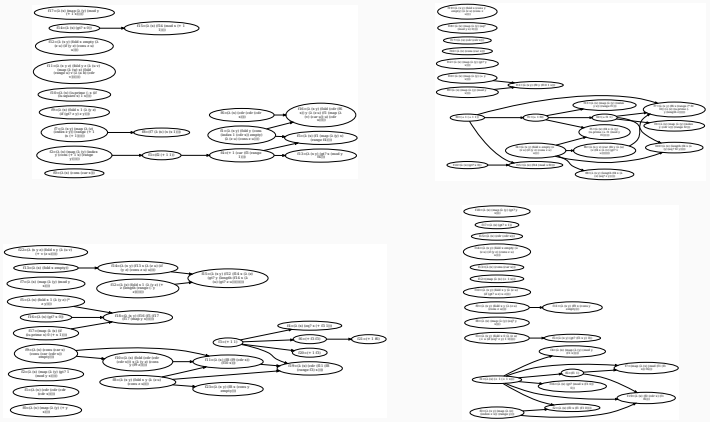


Darker: Early in learning

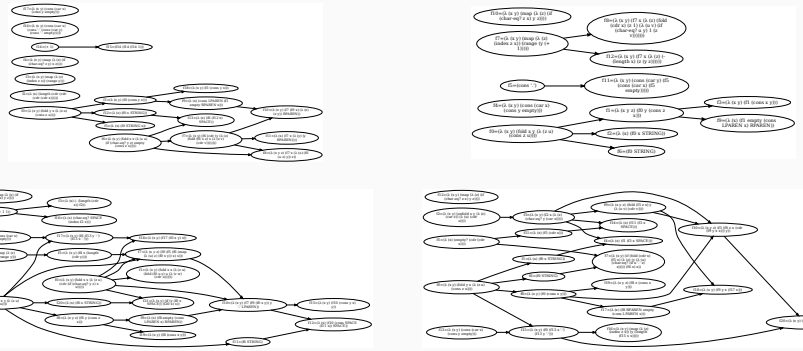
Lighter: Later in learning

Variability in learned library

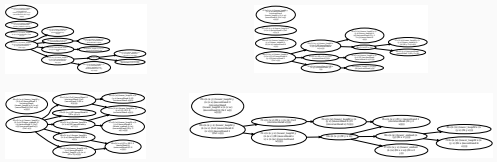
List processing



Text editing

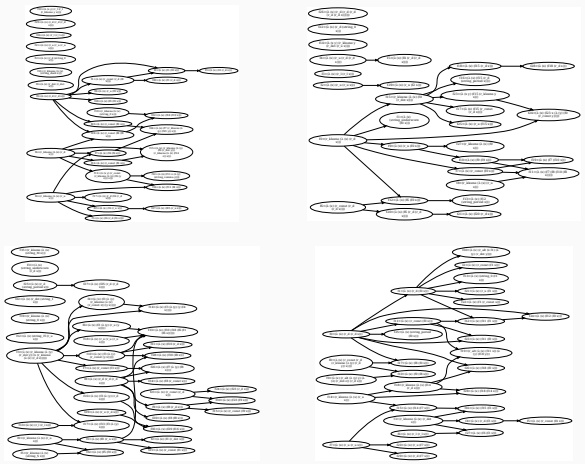


Tower building

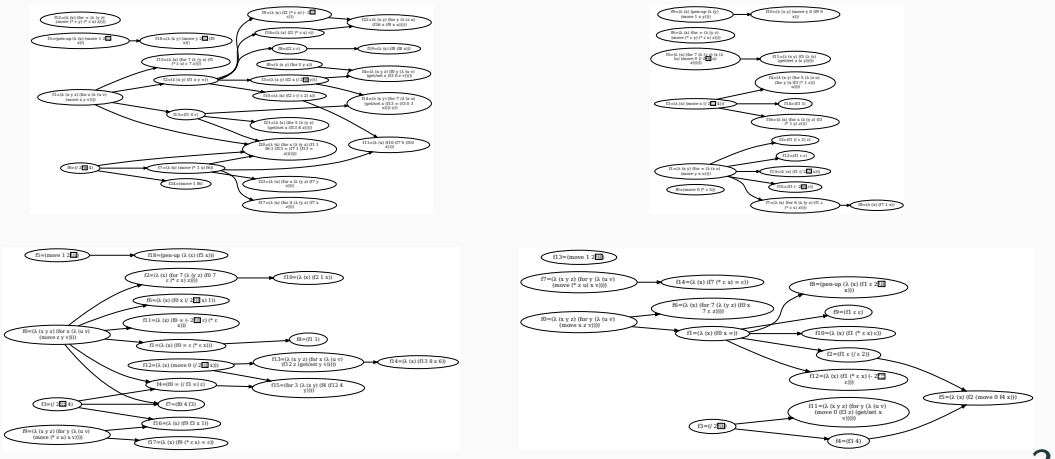


Symbolic regression

Generative regex



LOGO graphics



From learning libraries, to learning languages

From learning libraries, to learning languages

functional programming $\rightarrow$ physics

From learning libraries, to learning languages

1950's Lisp $\rightarrow$ modern functional programming $\rightarrow$ physics

Growing languages for vector algebra and physics

Initial Primitives

- map
- zip
- cons
- empty
- cdr
- power
- fold
- car
- +
- -
- *
- /
- 0
- 1
- π

Physics Equations

Newton's Second Law

$$\vec{a} = \frac{1}{m}\sum_i \vec{F}_i$$

Parallel Resistors

$$R_{total} = \left(\Sigma_i \frac{1}{R_i}\right)^{-1}$$

Work

$$U = \vec{F} \cdot \vec{d}$$

Force in a Magnetic Field

$$|\vec{F}| = q|\vec{v} \times \vec{B}|$$

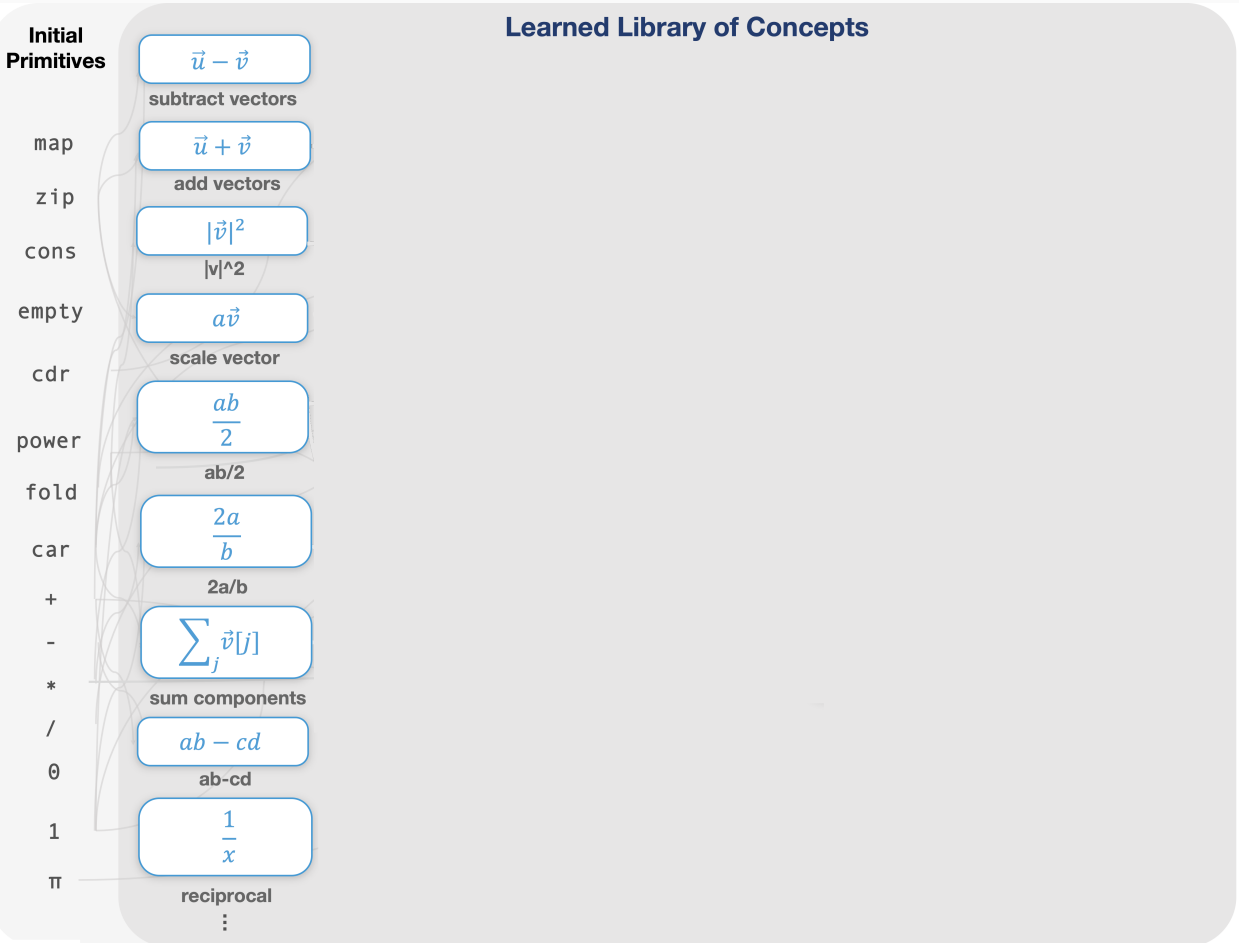
Kinetic Energy

$$KE = \frac{1}{2}m|\vec{v}|^2$$

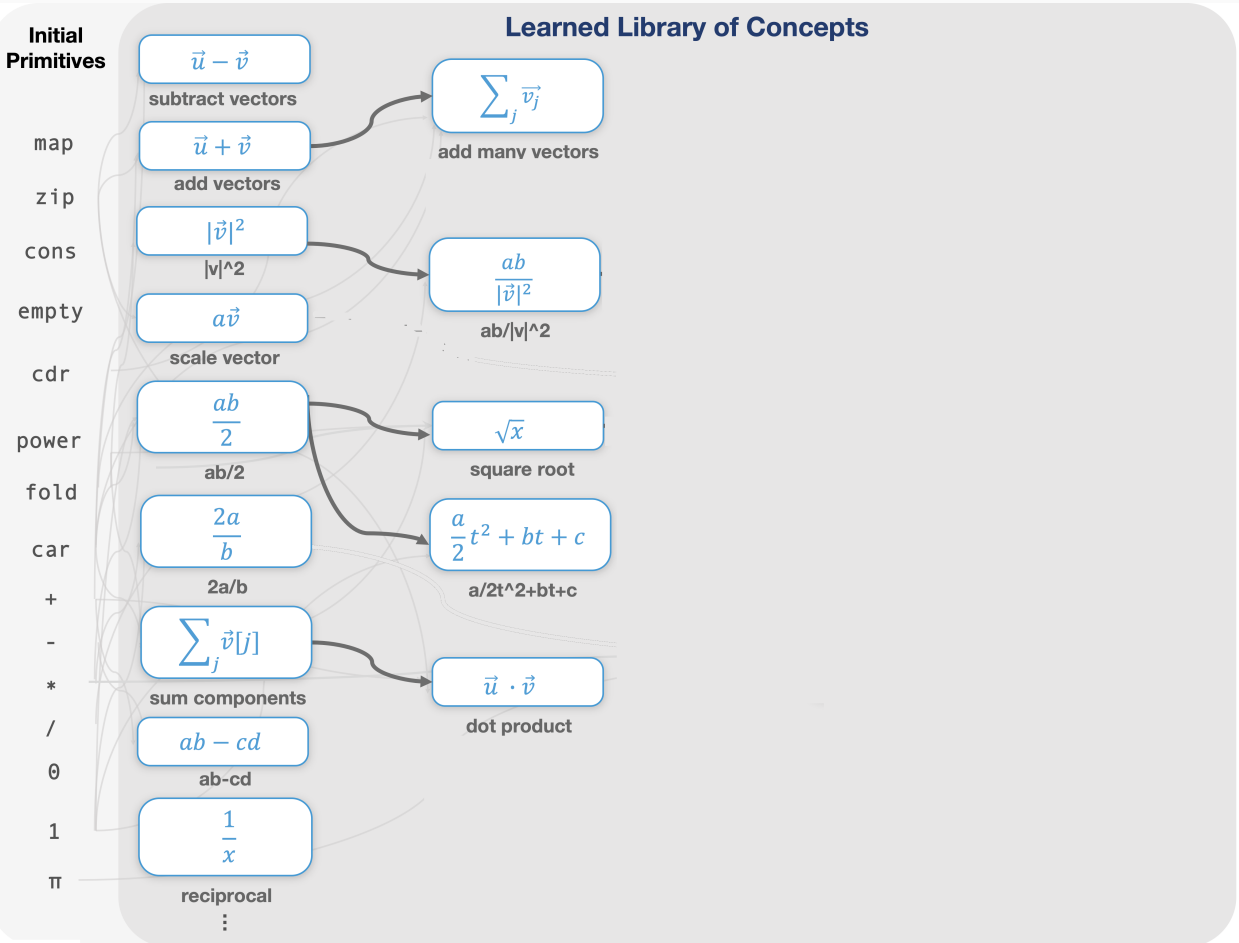
Coulomb's Law

$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r_1} - \vec{r_2}|^2}\widehat{r_1 - r_2}$$

Growing languages for vector algebra and physics



Growing languages for vector algebra and physics



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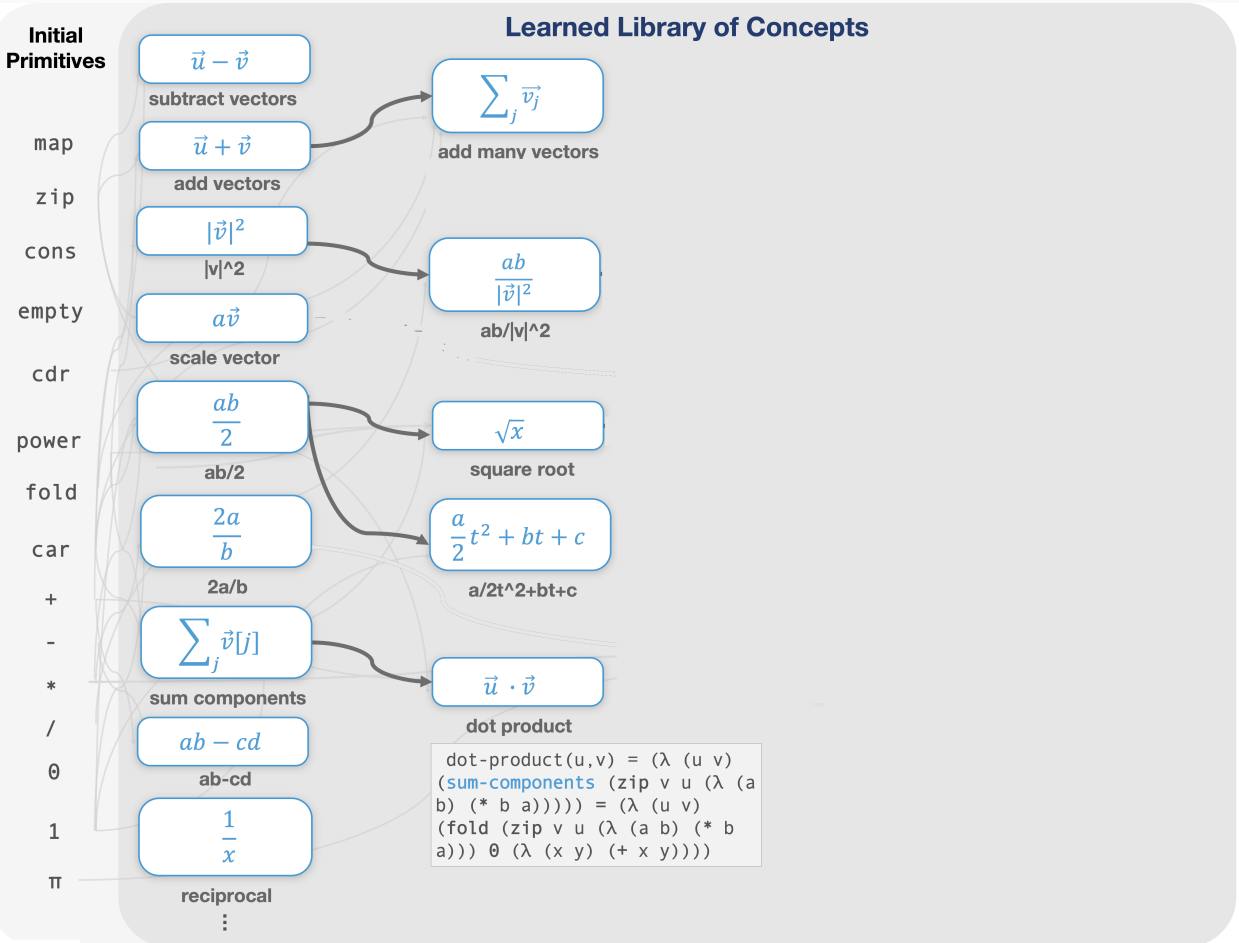

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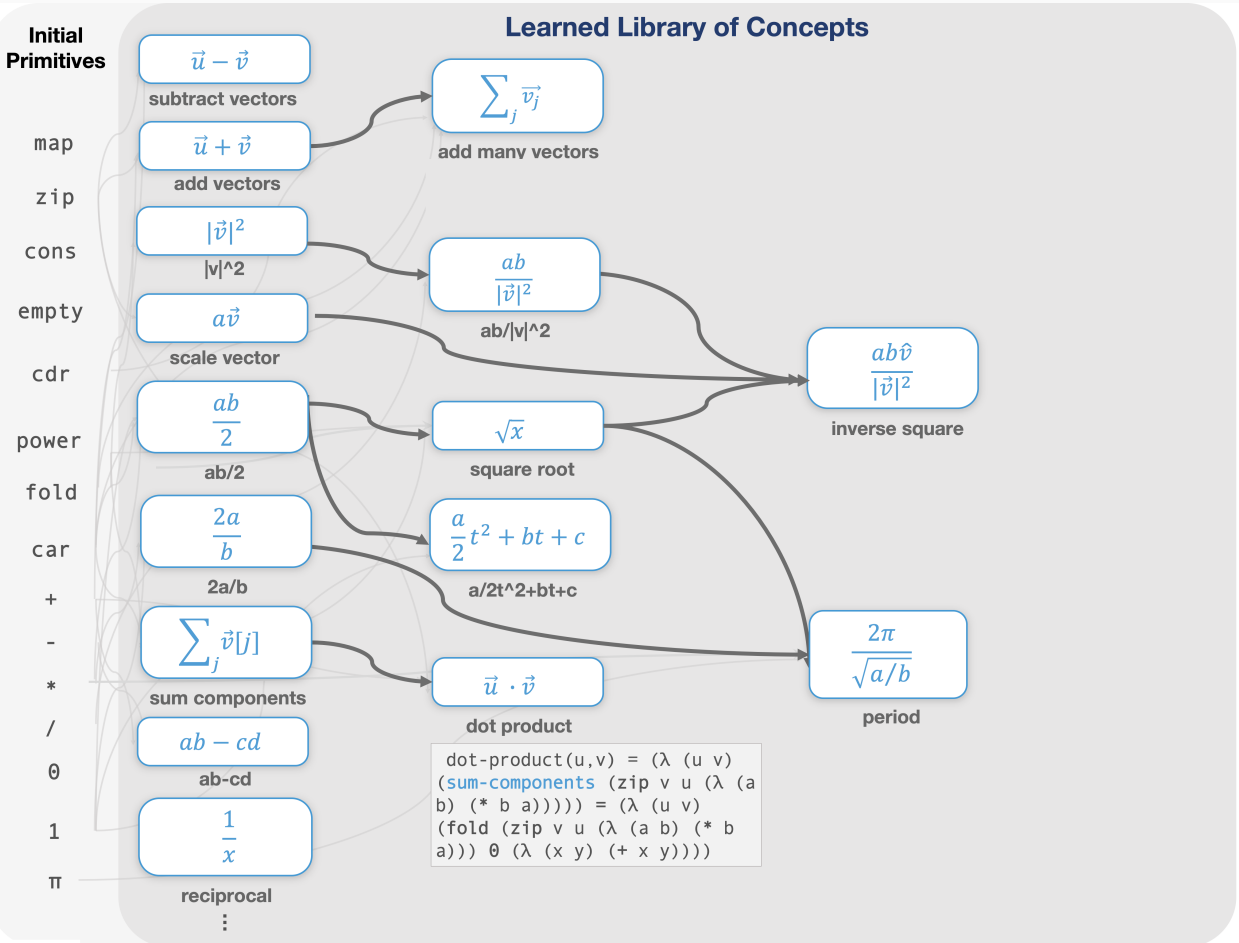

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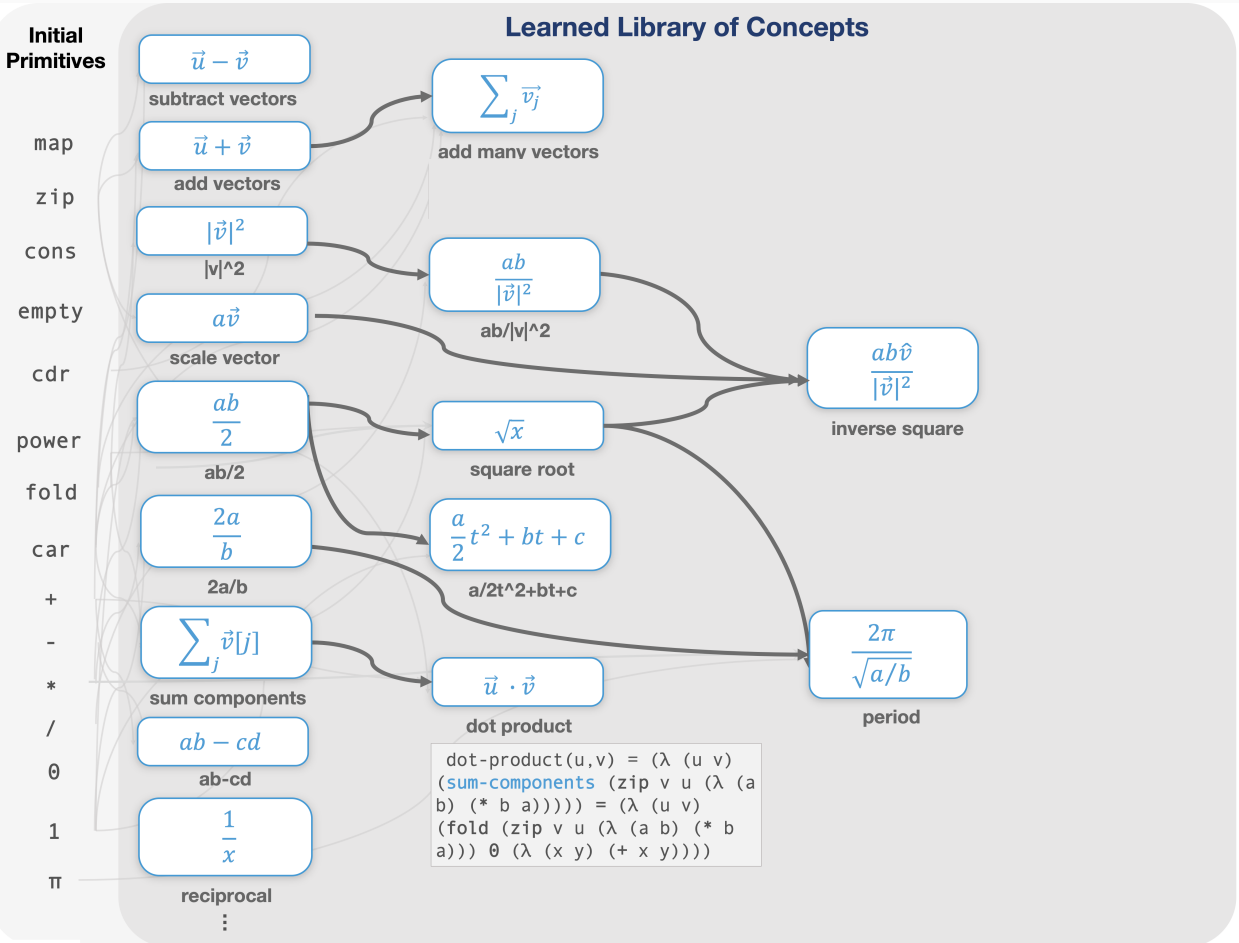

Physics Equations

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$$\vec{a}=\frac{1}{m}\sum_i \vec{F}_i$$

```
(scale-vector(reciprocal m)
(add-many-vectors Fs))
```

Parallel Resistors

$$R_{total}=\left(\Sigma_i \frac{1}{R_i}\right)^{-1}$$

```
(reciprocal (sum-components
(map (λ(r) (reciprocal r))
Rs)))
```

Work

$$U=\vec{F}\cdot\vec{d}$$

```
(dot-product F d)
```

Force in a Magnetic Field

$$|\vec{F}|=q|\vec{v}\times\vec{B}|$$

```
(* q (ab-cd v_x b_y v_y b_x))
```

Kinetic Energy

$$KE=\frac{1}{2}m|\vec{v}|^2$$

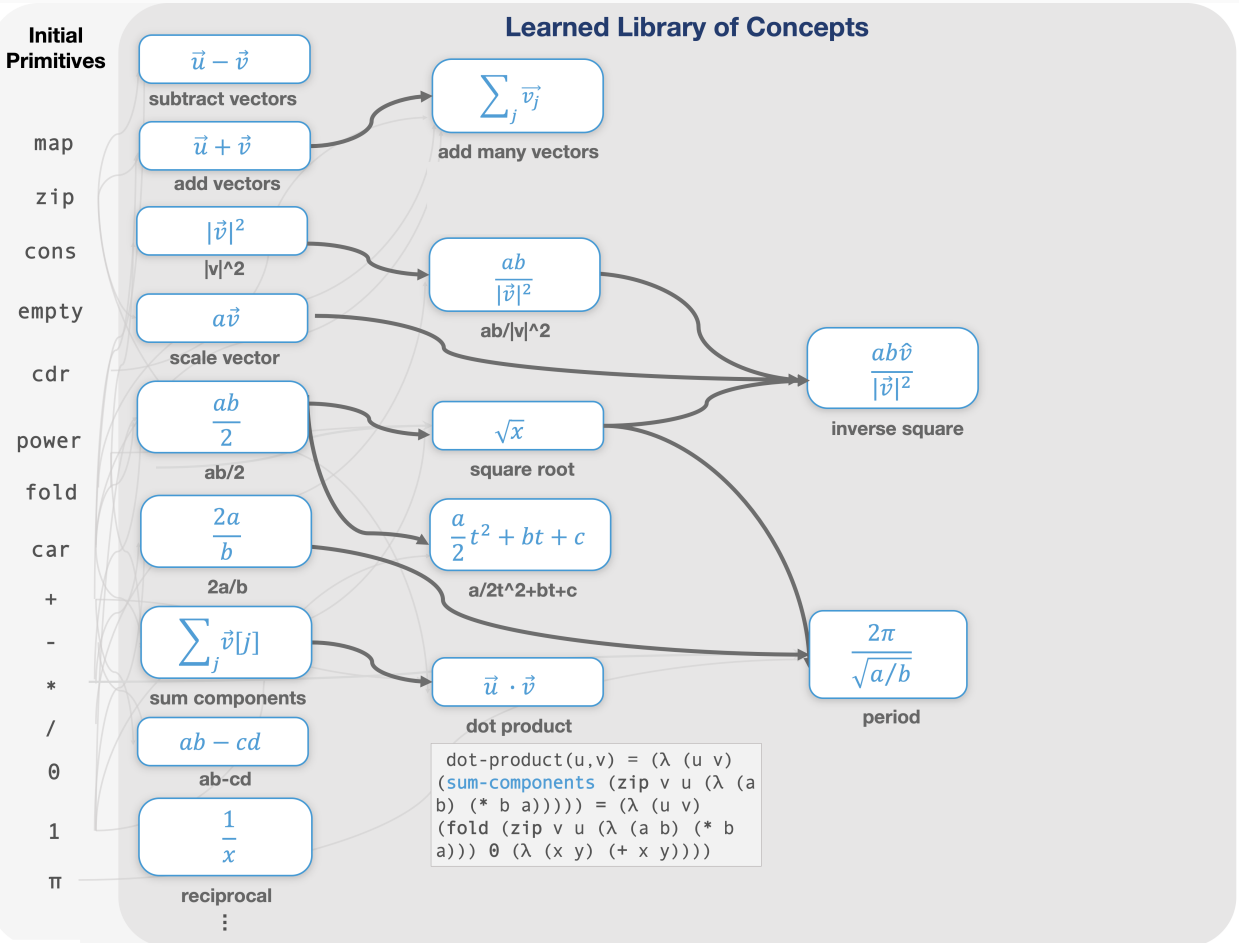
```
(ab/2 m (|v|^2 v))
```

Coulomb's Law

$$\vec{F}\propto\frac{q_1 q_2}{|\vec{r_1}-\vec{r_2}|^2}\widehat{r_1-r_2}$$

```
(inverse-square q_1 q_2
(subtract-vectors r_1 r_2))
```

Growing languages for vector algebra and physics



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```
(inverse-square q_1 q_2
(subtract-vectors r_1 r_2))
```

```
(λ (x y z u) (map (λ (v) (* (/
(* (power (/ (* x x) (fold (zip
z u (λ (w a) (- w a))) 0 (λ (b
c) (+ (* b b)    c)))) (/ (* 1
1) (+ 1 1))) y) (fold (zip z u
(λ (d e)  (- d e))) 0 (λ (f g)
(+ (* f f) g)))) v)) (zip z u
(λ (h i) (- h i)))))
```

Solution to `Coulomb's Law` if expressed in initial primitives

Rediscovering origami programming

Initial Primitives

- Y combinator
- cons
- car
- cdr
- nil
- if
- nil?
- +
- -
- 0
- 1
- =

Recursive Programming Algorithms

Stutter

[■■] → [■■■■]
[■■■] → [■■■■■■]

```
(fold A (λ (u v) (cons
v (cons v u))) nil)
```

Take every other

[■■■■] → [■■]
[■■■■■■] → [■■■]

```
(unfold_list cdr A nil?)
```

List lengths

[[■■■], [■]] → [3 1]
[[■■], [], [■]] → [2 0 1]

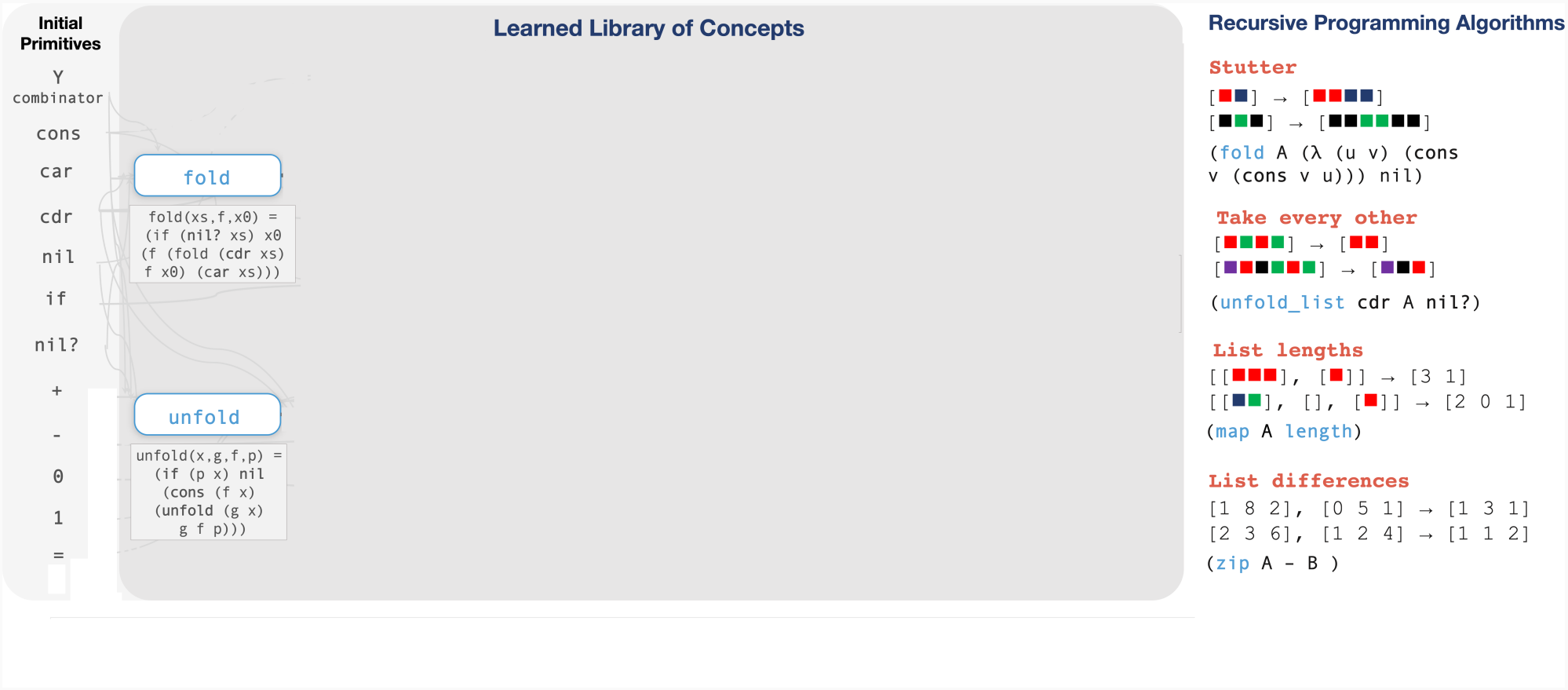
```
(map A length)
```

List differences

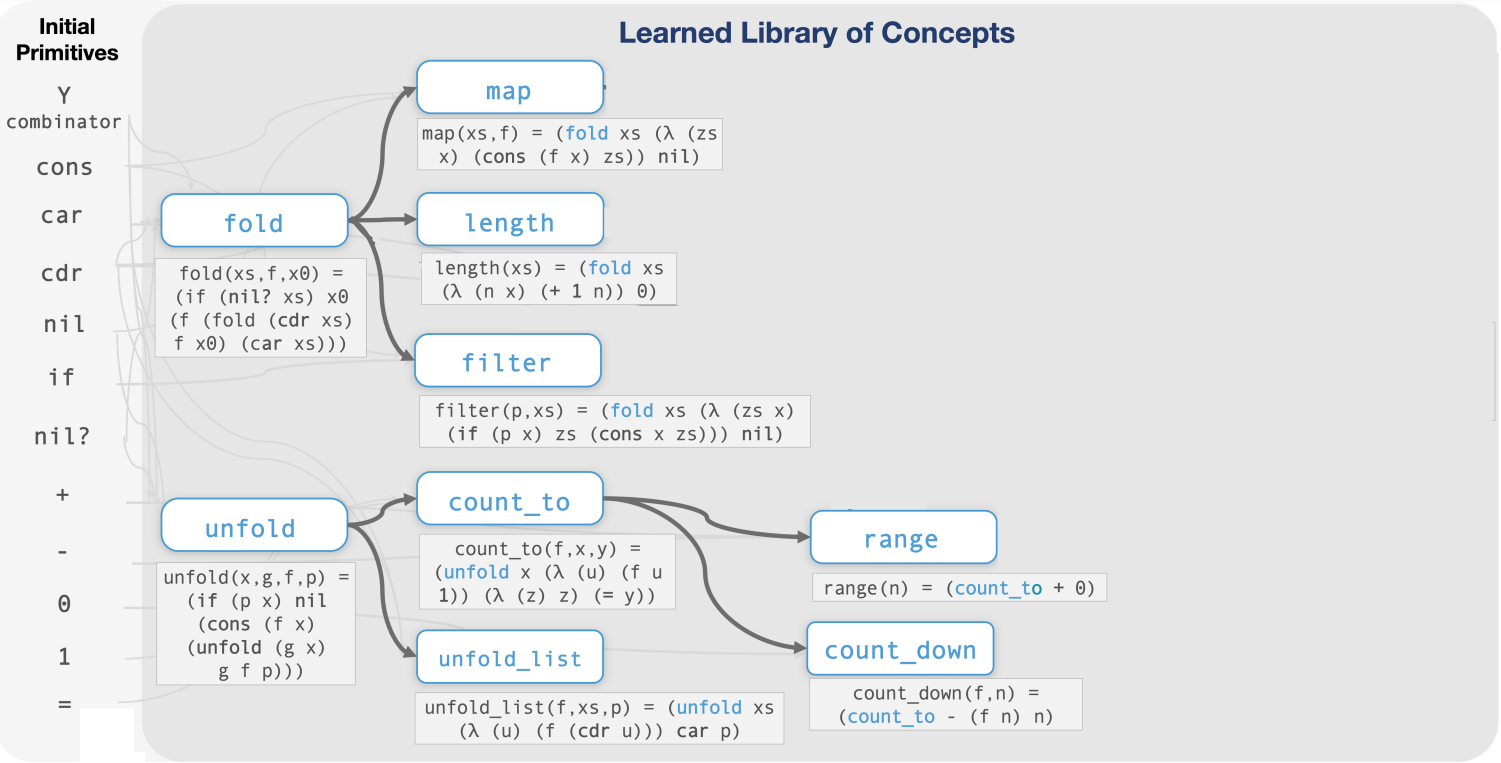
[1 8 2], [0 5 1] → [1 3 1]
[2 3 6], [1 2 4] → [1 1 2]

```
(zip A - B )
```

Rediscovering origami programming



Rediscovering origami programming



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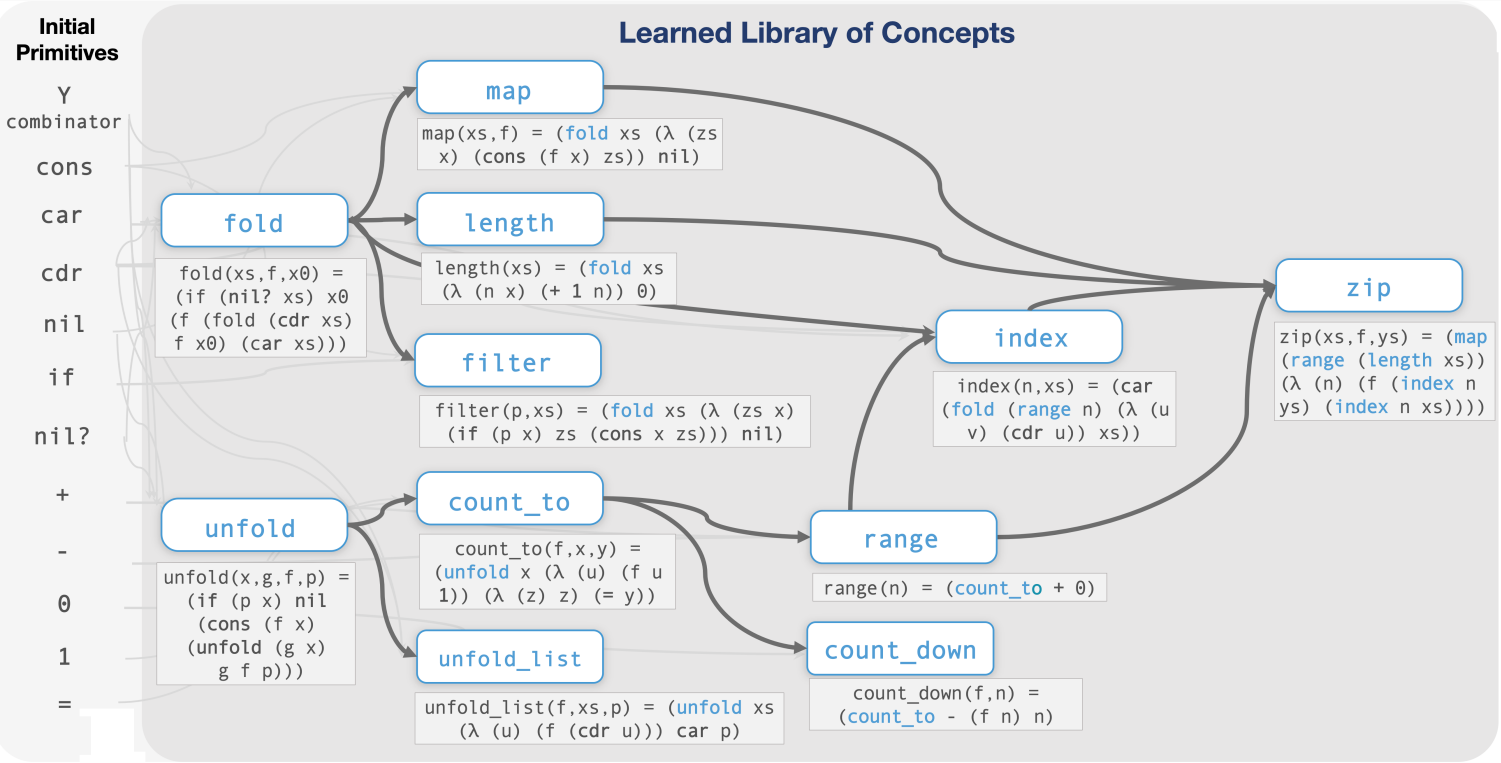

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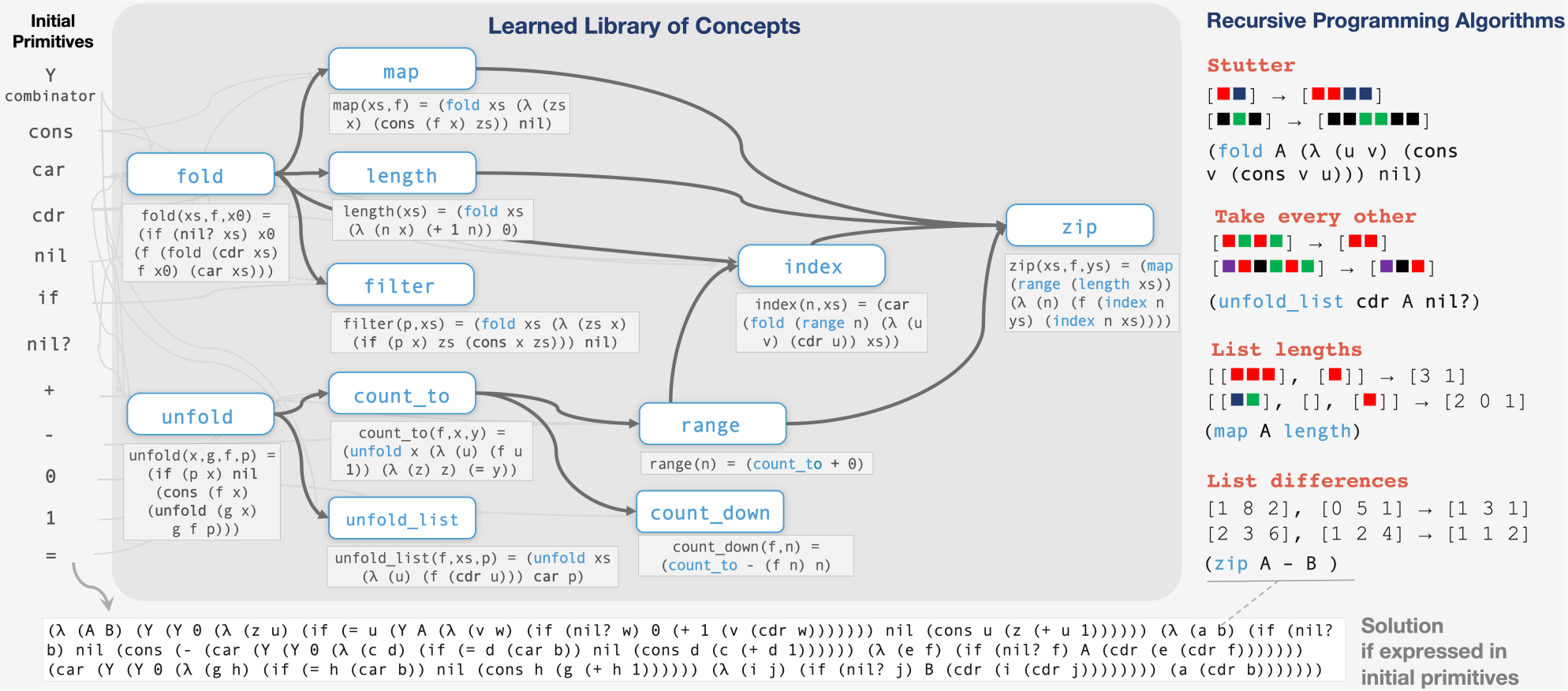



```
(λ (A B) (Y (Y 0 (λ (z u) (if (= u (Y A (λ (v w) (if (nil? w) 0 (+ 1 (v (cdr w)))))))) nil (cons u (z (+ u 1)))))) (λ (a b) (if (nil?
b) nil (cons (- (car (Y (Y 0 (λ (c d) (if (= d (car b)) nil (cons d (c (+ d 1)))))) (λ (e f) (if (nil? f) A (cdr (e (cdr f)))))))
(car (Y (Y 0 (λ (g h) (if (= h (car b)) nil (cons h (g (+ h 1)))))) (λ (i j) (if (nil? j) B (cdr (i (cdr j)))))))) (a (cdr b)))))))
```

Solution if expressed in initial primitives

Lessons

Symbols aren't necessarily interpretable. Flexibly grow the language based on experience to make it more powerful *and* more human understandable

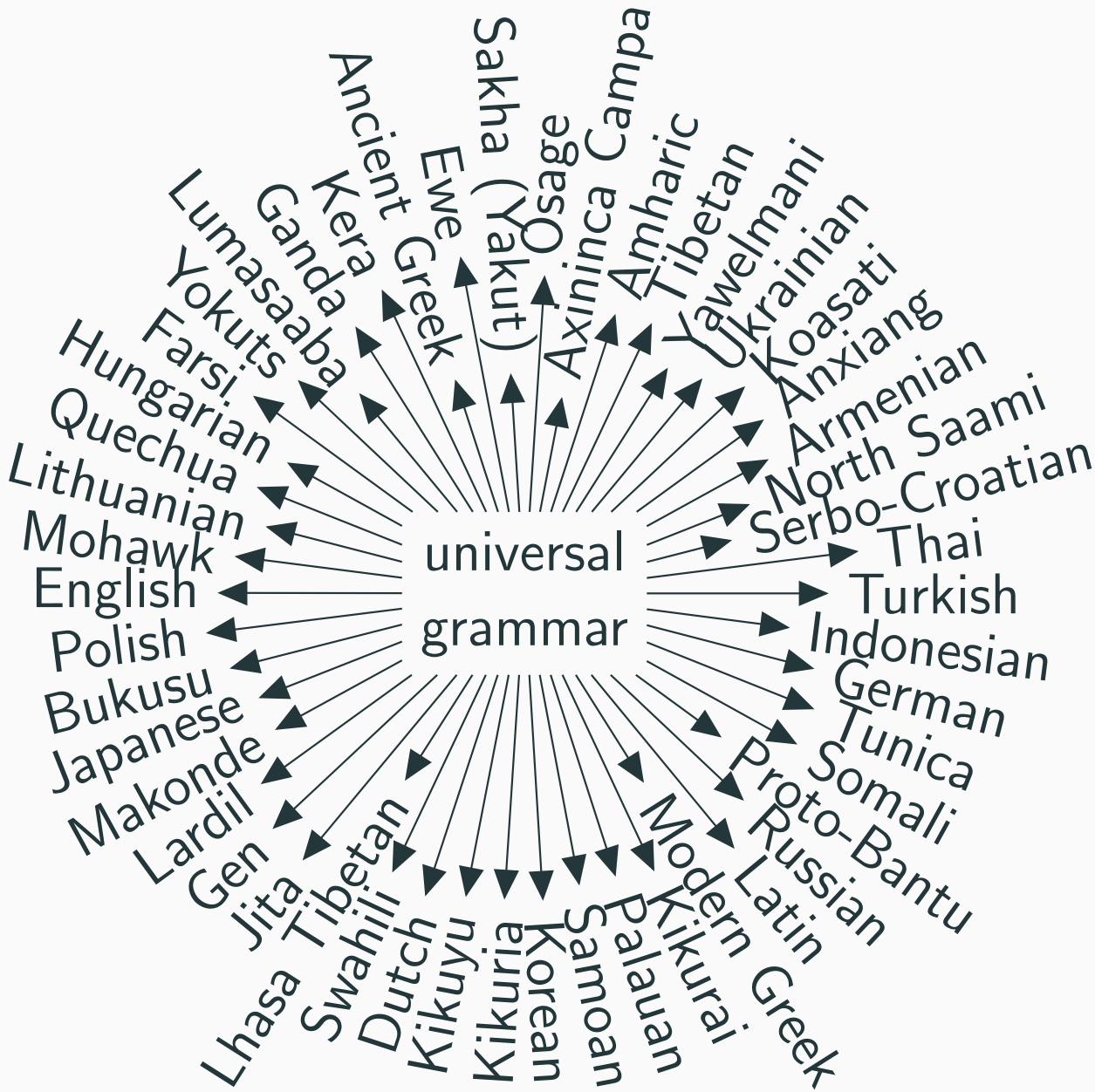
Learning-from-scratch is possible in principle. Don't do it. Choose wisely what to learn and what to build in

Program Induction and perception

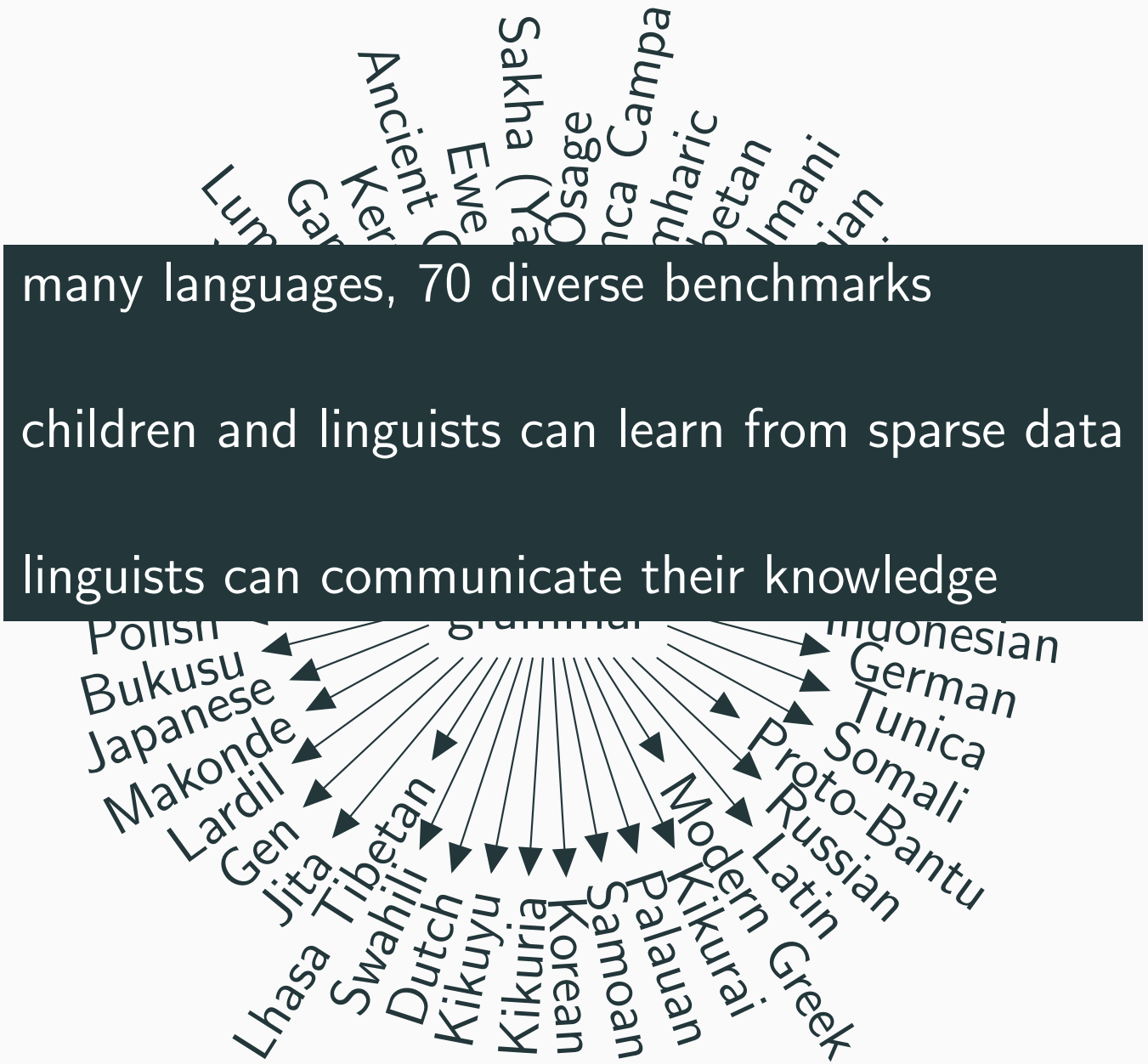
learning to learn

discovering models

Synthesizing human-understandable models of language



Synthesizing human-understandable models of language



Few-shot language learning experiment

Mandarin:

	adjective	adverb
“slow”	man	manmandə
“fast”	kuai	kuaikuaidə
“small”	xiao	???

Few-shot language learning experiment

Mandarin:

	adjective	adverb
“slow”	man	manmandə
“fast”	kuai	kuaikuaidə
“small”	xiao	xiaoxiaodə

Few-shot language learning experiment

Mandarin:

	adjective	adverb
“slow”	man	manmandə
“fast”	kuai	kuaikuaidə
“small”	xiao	xiaoxiaodə

stem+stem+də

Few-shot language learning experiment

Serbo-Croatian:

	masculine	feminine
“rich”	bogat	bogata
“mild”	blag	blaga
“green”	zelen	???

Few-shot language learning experiment

Serbo-Croatian:

	masculine	feminine
“rich”	bogat	bogata
“mild”	blag	blaga
“green”	zelen	zelena

Few-shot language learning experiment

Serbo-Croatian:

	masculine	feminine
“rich”	bogat	bogata
“mild”	blag	blaga
“green”	zelen	zelena

add “a” to stem to make feminine

Few-shot language learning experiment

Serbo-Croatian:

	masculine	feminine
“rich”	bogat	bogata
“mild”	blag	blaga
“green”	zelen	zelena
“clear”	???	yasna

add “a” to stem to make feminine

Few-shot language learning experiment

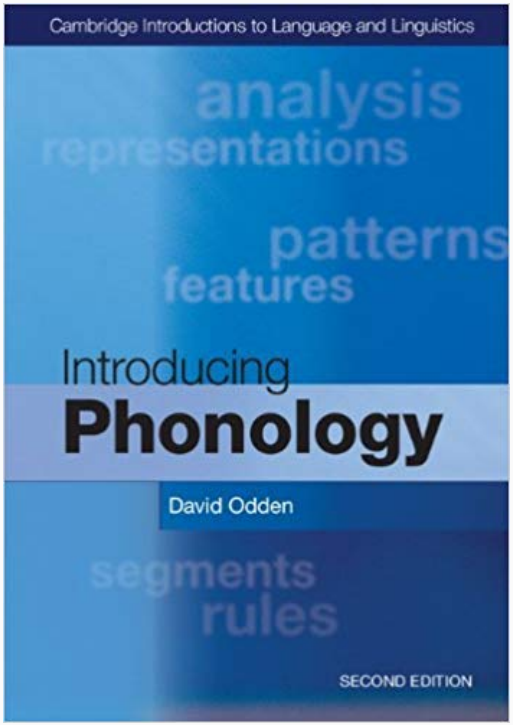
Serbo-Croatian:

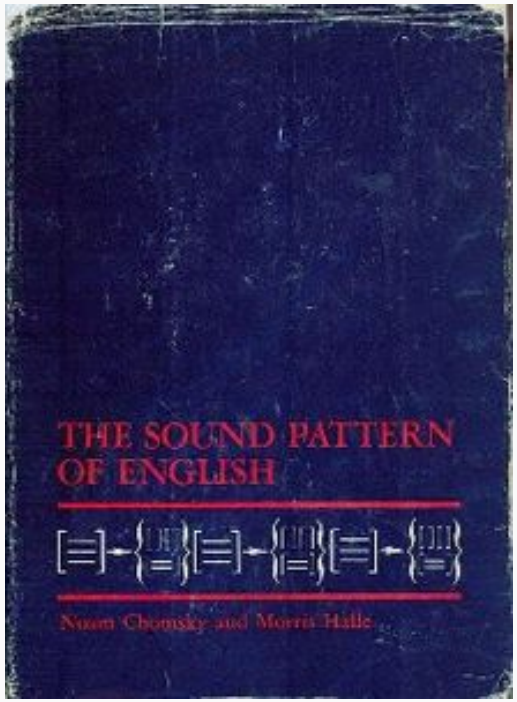
	masculine	feminine	stem (unobserved)
"rich"	bogat	bogata	bogat
"mild"	blag	blaga	blag
"green"	zelen	zelena	zelen
"clear"	**yasan**	yasna	yasn

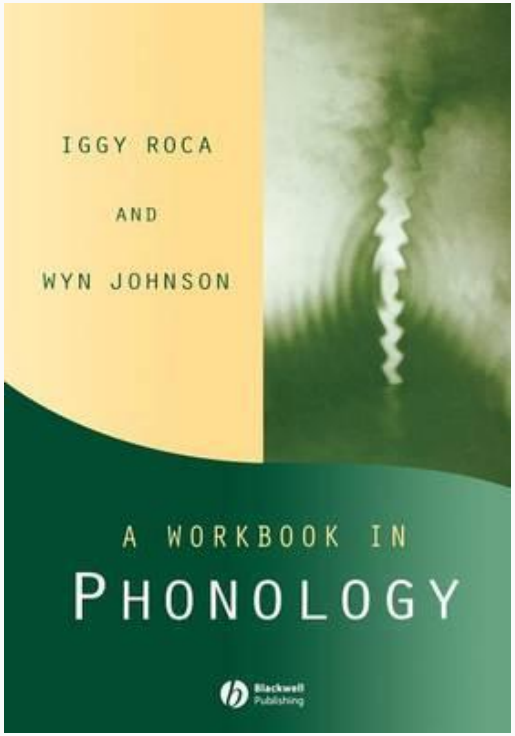
add "a" to stem to make feminine

insert "a" between two word-final consonants

$\varnothing \rightarrow$ a / C_C#


Cambridge Introductions to Language and Linguistics
analysis
representations
patterns
features
Introducing
Phonology
David Odden
segments
rules
SECOND EDITION


THE SOUND PATTERN
OF ENGLISH
Noam Chomsky and Morris Halle


IGGY ROCA
AND
WYN JOHNSON
A WORKBOOK IN
PHONOLOGY
Blackwell
Publishing

10 Sakha (Yakut)

Give a phonological analysis of the following case-marking paradigms of nouns in Sakha.

Noun	*Plural*	*Associative*	
aya	ayalar	ayalɨɨn	'father'
paarta	paartalar	paartalɨɨn	'school desk'
tɨa	tɨalar	tɨalɨɨn	'forest'
kinige	kinigeler	kinigeliin	'book'
ǰie	ǰieler	ǰieliin	'house'
iye	iyeler	iyeliin	'mother'
kini	kiniler	kiniliin	'3rd person'
bie	bieler	bieliin	'mare'
oyo	oyolor	oyoluun	'child'
χopto	χoptolor	χoptoluun	'gull'
börö	börölör	börölüün	'wolf'
tɨal	tɨallar	tɨalɨɨn	'wind'
ɨal	ɨallar	ɨalɨɨn	'neighbor'
kuul	kuullar	kuulluun	'sack'
at	attar	attɨɨn	'horse'
balɨk	balɨktar	balɨktɨɨn	'fish'
ɨskaap	ɨskaaptar	ɨskaaptɨɨn	'cabinet'
oyus	oyustar	oyustuun	'bull'
kus	kustar	kustuun	'duck'
tünnük	tünnükter	tünnüktüün	'window'
sep	septer	septiin	'tool'
et	etter	ettiin	'meat'
örüs	örüster	örüstüün	'river'
tiis	tiister	tiistiin	'tooth'
soroχ	soroχtor	soroχtuun	'some person'
oχ	oχtor	oχtuun	'arrow'
oloppos	oloppostor	oloppostuun	'chair'
ötöχ	ötöχtör	ötöχtüün	'abandoned farm'
ubay	ubaydar	ubaydɨɨn	'elder brother'
saray	saraydar	saraydɨɨn	'barn'
tɨy	tɨydar	tɨydɨɨn	'foal'
atɨɨr	atɨɨrdar	atɨɨrdɨɨn	'stallion'
oyuur	oyuurdar	oyuurduun	'forest'
üčügey	üčügeyder	üčügeydiin	'good person'
eǰiy	eǰiyder	eǰiydiin	'elder sister'
tomtor	tomtordor	tomtorduun	'knob'
moyotoy	moyotoydor	moyotoyduun	'chipmunk'
kötör	kötördör	kötördüün	'bird'
bölköy	bölköydör	bölköydüün	'islet'
χatɨŋ	χatɨŋnar	χatɨŋnɨɨn	'birch'
aan	aannar	aannɨɨn	'door'
tɨŋ	tɨŋner	tɨŋnɨɨn	'squirrel'
sordoŋ	sordoŋnor	sordoŋnuun	'pike'
olom	olomnor	olomnuun	'ford'
oron	oronnor	oronnuun	'bed'
bödöŋ	bödöŋnör	bödöŋnüün	'strong one'

Noun	*Partitive*	*Comparative*	*Ablative*	
aya	ayata	ayataayar	ayattan	'father'
paarta	paartata	paartataayar	paartattan	'school desk'
tɨa	tɨata	tɨataayar	tɨattan	'forest'
kinige	kinigete	kinigeteeyer	kinigetten	'book'
ǰie	ǰiete	ǰieteeyer	ǰietten	'house'
iye	iyete	iyeteeyer	iyetten	'mother'
kini	kinite	kiniteeyer	kinitten	'3rd person'
bie	biete	bieteeyer	bietten	'mare'
oyo	oyoto	oyotooyor	oyotton	'child'
χopto	χoptoto	χoptotooyor	χoptotton	'gull'
börö	börötö	börötööyör	böröttön	'wolf'
tɨal	tɨalla	tɨallaayar	tɨaltan	'wind'
ɨal	ɨalla	ɨallaayar	ɨaltan	'neighbor'
kuul	kuulla	kuullaayar	kuultan	'sack'
moχsoyol	moχsoyollo	moχsoyollooyor	moχsoyolton	'falcon'
at	atta	attaayar	attan	'horse'
balɨk	balɨkta	balɨktaayar	balɨktan	'fish'
ɨskaap	ɨskaapta	ɨskaaptaayar	ɨskaaptan	'cabinet'
oyus	oyusta	oyustaayar	oyustan	'bull'
kus	kusta	kustaayar	kustan	'duck'
tünnük	tünnükte	tünnükteeyer	tünnükten	'window'

Turkic Sakha (Yakut)

observed data

	Singular	Plural
Bed	oron	oron<u>nor</u>
Mare	bie	bie<u>ler</u>
Cabinet	ɨskaap	ɨskaaptar<u>tar</u>

138 total examples

Turkic Sakha (Yakut)

grammar (unobserved)

SINGULAR→stem
PLURAL→stem+lar

observed data

	SINGULAR	PLURAL
BED	oron	oronnor
MARE	bie	bieler
CABINET	ɨskaap	ɨskaaptartar

138 total examples

Turkic Sakha (Yakut)

grammar (unobserved)

SINGULAR→stem
PLURAL→stem+lar

r_1: l → d / [-lateral -tense]_
"l" becomes "d" next to "r", "t", but not "l"

r_2: C → [-voice] / [-voice]_
do not voice next to voiceless

r_3: V → [+rounded] / [+rounded] [-low]$_0$_
r_4: [+continuant -high] → [-rounded] / u C$_0$_
"harmonize" round vowels like "u", "o"

r_5: V → [-back -low] / [-back +vowel] [ ]$_0$_
"harmonize" vowels to be not at back of mouth

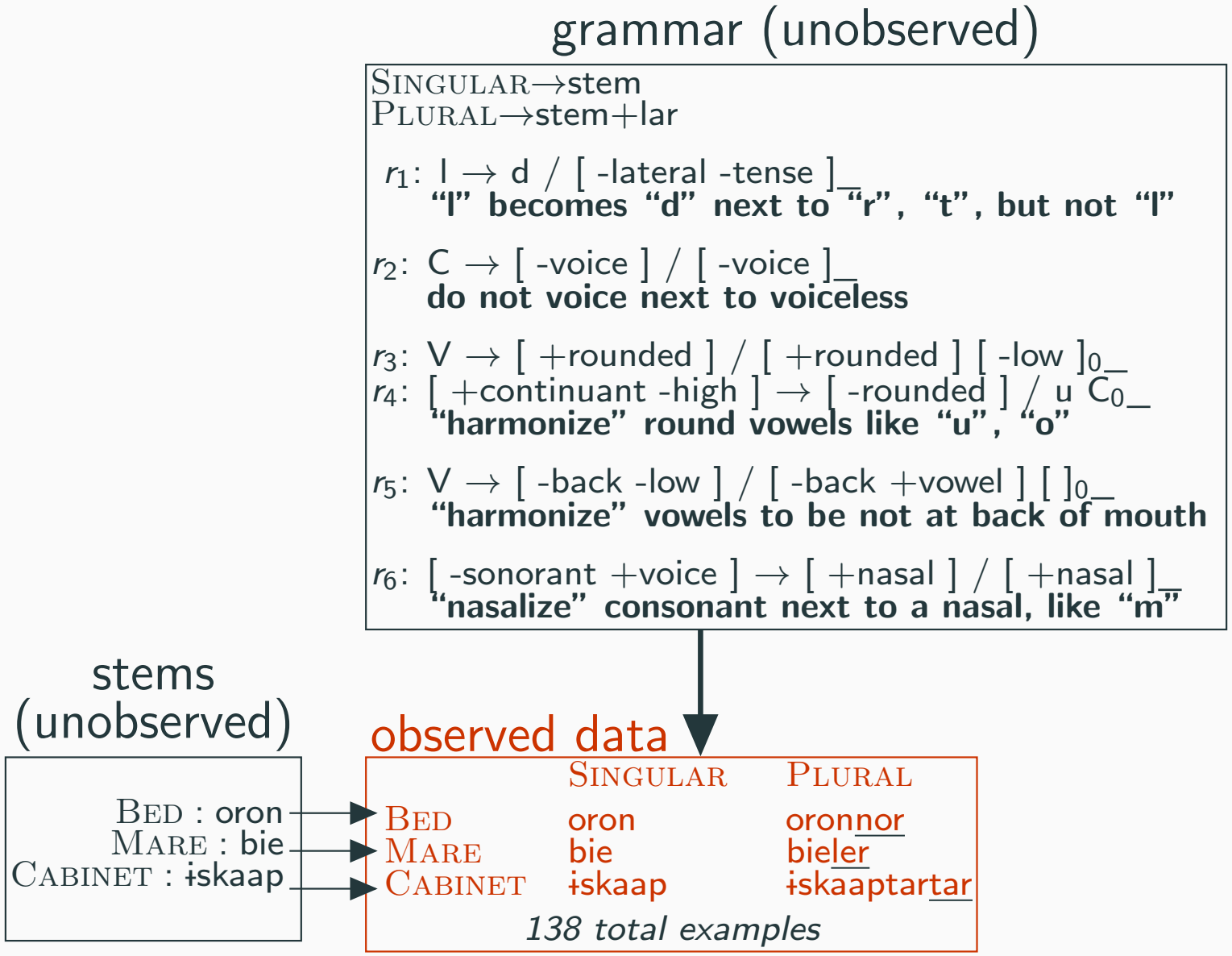
r_6: [-sonorant +voice] → [+nasal] / [+nasal]_
"nasalize" consonant next to a nasal, like "m"

observed data

	SINGULAR	PLURAL
BED	oron	oronnor
MARE	bie	bieler
CABINET	ɨskaap	ɨskaaptartar

138 total examples

Turkic Sakha (Yakut)



Turkic Sakha (Yakut)

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r_4: [+continuant -high] → [-rounded] / u C$_0$_
"harmonize" round vowels like "u", "o"

r_5: V → [-back -low] / [-back +vowel] [ ]$_0$_
"harmonize" vowels to be not at back of mouth

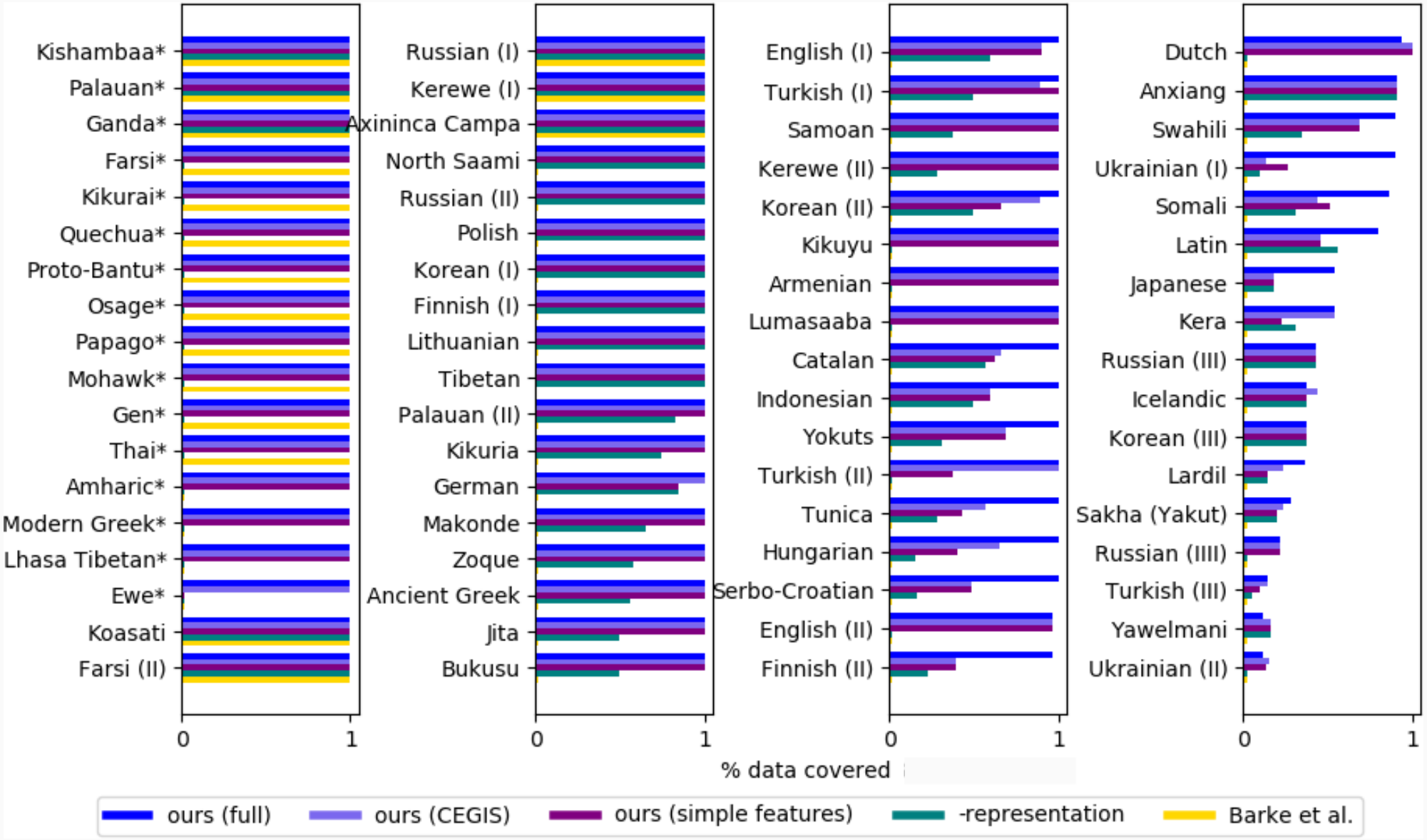
r_6: [-sonorant +voice] → [+nasal] / [+nasal]_
"nasalize" consonant next to a nasal, like "m"

stems (unobserved)

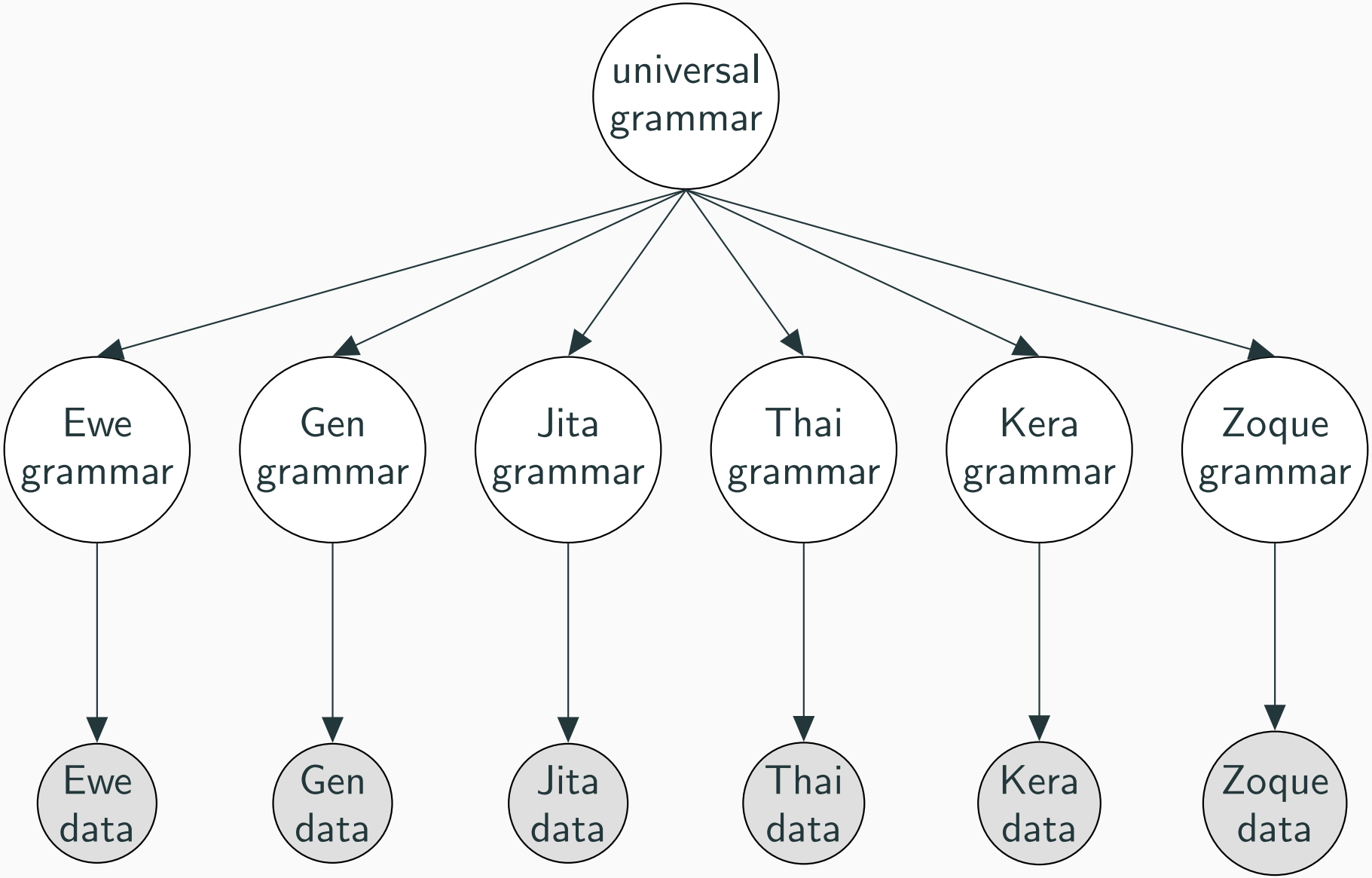
CABINET : ɨskaap
BED : oron
MARE : bie
RIVER : örus

observed data

CABINETS→ɨskaap+lar→ɨskaaplar$\xrightarrow{r_1}$ɨskaapdar$\xrightarrow{r_2}$ɨskaaptar
BEDS→oron+lar→oronlar$\xrightarrow{r_1}$orondar$\xrightarrow{r_3}$orondor$\xrightarrow{r_6}$oronnor
MARES→bie+lar→bielar$\xrightarrow{r_5}$bieler
RIVER (ASSOC)→örus+lɨɨn→öruslɨɨn$\xrightarrow{r_1}$örusdɨɨn$\xrightarrow{r_2}$
örustɨɨn$\xrightarrow{r_3}$örustuun$\xrightarrow{r_5}$[örüstüün]


Kishambaa*
Palauan*
Ganda*
Farsi*
Kikurai*
Quechua*
Proto-Bantu*
Osage*
Papago*
Mohawk*
Gen*
Thai*
Amharic*
Modern Greek*
Lhasa Tibetan*
Ewe*
Koasati
Farsi (II)
Russian (I)
Kerewe (I)
Axininca Campa
North Saami
Russian (II)
Polish
Korean (I)
Finnish (I)
Lithuanian
Tibetan
Palauan (II)
Kikuria
German
Makonde
Zoque
Ancient Greek
Jita
Bukusu
English (I)
Turkish (I)
Samoan
Kerewe (II)
Korean (II)
Kikuyu
Armenian
Lumasaaba
Catalan
Indonesian
Yokuts
Turkish (II)
Tunica
Hungarian
Serbo-Croatian
English (II)
Finnish (II)
Dutch
Anxiang
Swahili
Ukrainian (I)
Somali
Latin
Japanese
Kera
Russian (III)
Icelandic
Korean (III)
Lardil
Sakha (Yakut)
Russian (IIII)
Turkish (III)
Yawelmani
Ukrainian (II)
0
1
% data covered
ours (full)
ours (CEGIS)
ours (simple features)
-representation
Barke et al.

Distilling higher-level knowledge



Lessons

Higher-level knowledge matters ("universal grammar"). Get the basic computational substrate correct

But *some* of this higher-level knowledge can be learned. You don't need millions of examples to learn it. But it's not a one-shot learning problem either

Program Induction and perception

learning to learn

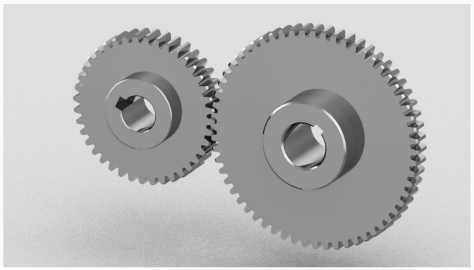
model discovery

the future

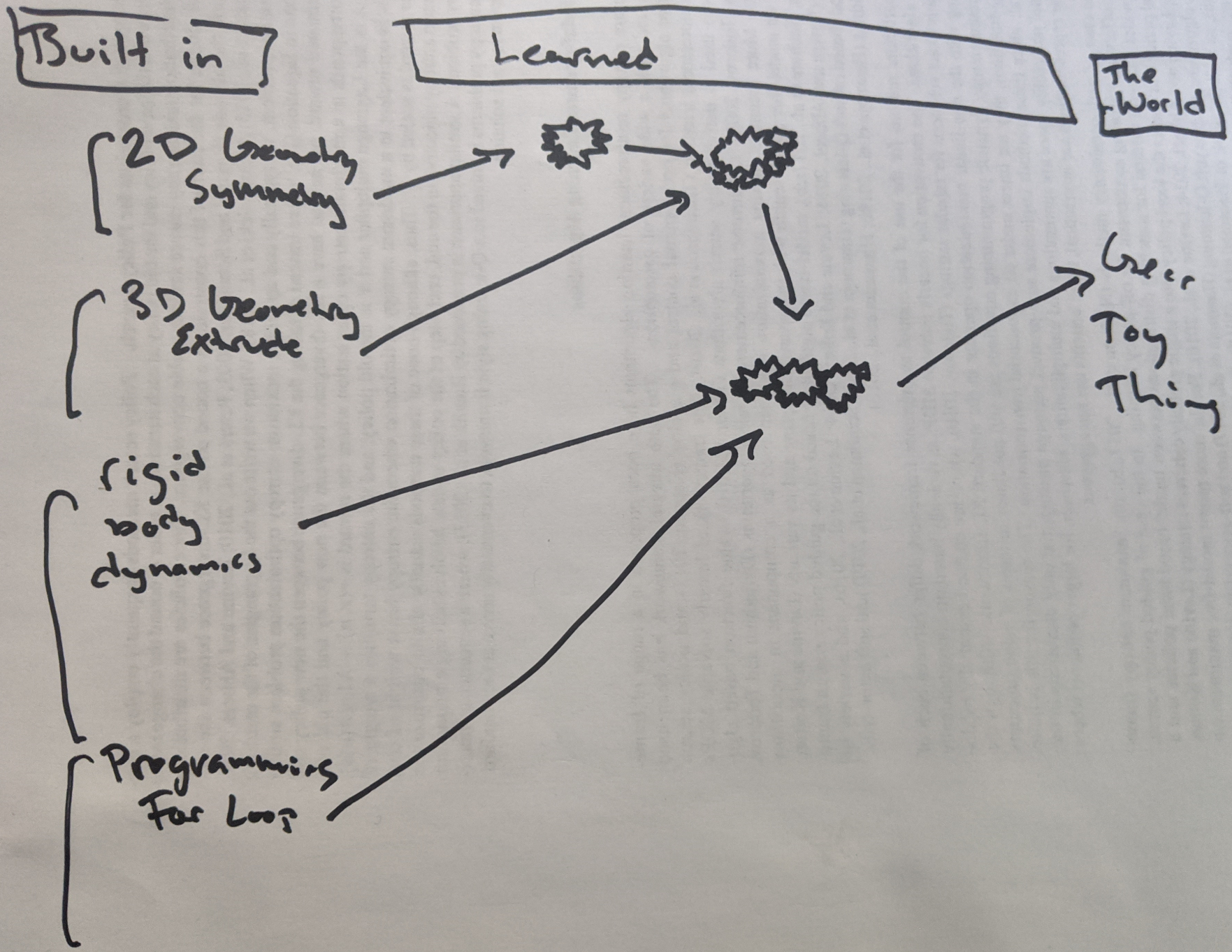
Models of the physical world

hinge

gear



doorknob


Built in
Learned
The World
2D Geometry Symmetry
3D Geometry Extrude
rigid body dynamics
Programming For Loop
Creature Toy Thing

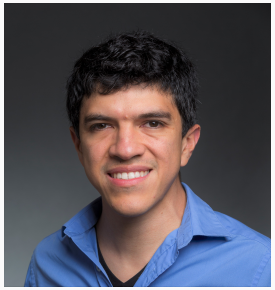
Collaborators

Tim O'Donnell

Josh Tenenbaum

Adam Albright

Armando Solar-Lezama



Max Nye

Cathy Wong

Yewen Pu

Dan Ritchie

Mathias Sable-Meyer

Lucas Morales

thank you