DreamCoder: Bootstrapping Inductive Program Synthesis with Wake-Sleep Library Learning

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PLDI 2021

Inductive program synthesis

FlashFill (Gulwani 2012)

EXAMPLE 3 (Directory Name Extraction). *Consider the following example taken from an excel online help forum.*

Input v_1	*Output*
Company\Code\index.html	*Company\Code*
Company\Docs\Spec\specs.doc	*Company\Docs\Spec*

String Program:

$\texttt{SubStr}(v_1, \texttt{CPos}(0), \texttt{Pos}(\textit{SlashTok}, \epsilon, -1))$

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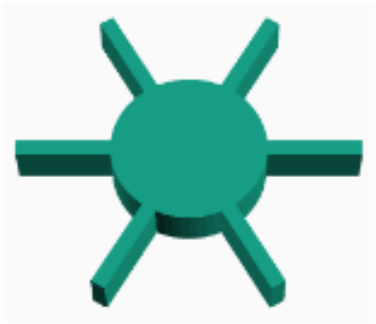
Szalinski (Nandi 2020)



(a) CAD model of ship's wheel

```
(Union
 (Cylinder [1, 5, 5])
 (Fold Union
  (Tabulate (i 6)
   (Rotate [0, 0, 60i]
    (Translate [1, −0.5, 0]
     (Cuboid [10, 1, 1])))))))
```

(b) Caddy program

Inductive program synthesis

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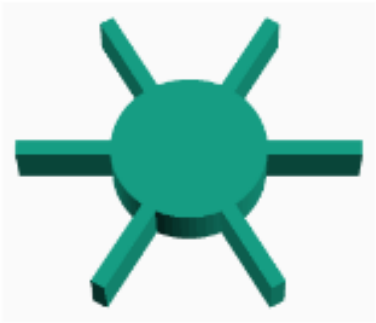
String expr P	:=	$\texttt{Switch}((\mathrm{b}_1, \mathrm{e}_1), \cdot\cdot, (\mathrm{b}_n, \mathrm{e}_n))$
Bool b	:=	$\mathrm{d}_1 \vee \cdot\cdot \vee \mathrm{d}_n$
Conjunct d	:=	$\pi_1 \wedge \cdot\cdot \wedge \pi_n$
Predicate π	:=	$\texttt{Match}(v_i, \mathrm{r}, k) \mid \neg\, \texttt{Match}(v_i, \mathrm{r}, k)$
Trace expr e	:=	$\texttt{Concatenate}(\mathrm{f}_1, \cdot\cdot, \mathrm{f}_n)$
Atomic expr f	:=	$\texttt{SubStr}(v_i, \mathrm{p}_1, \mathrm{p}_2)$
	|	$\texttt{ConstStr}(s)$
	|	$\texttt{Loop}(\lambda w : \mathrm{e})$
Position p	:=	$\texttt{CPos}(k) \mid \texttt{Pos}(\mathrm{r}_1, \mathrm{r}_2, \mathrm{c})$
Integer expr c	:=	$k \mid k_1 w + k_2$
Regular Expression r	:=	$\texttt{TokenSeq}(\mathrm{T}_1, \cdot\cdot, \mathrm{T}_m)$
Token T	:=	$C+ \mid [\neg C]+$
	|	SpecialToken

Szalinski (Nandi 2020)



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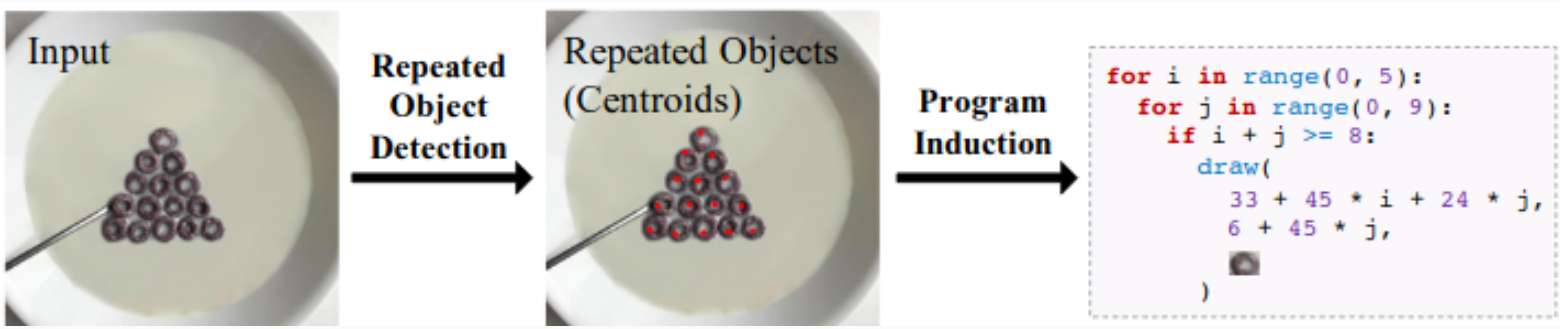
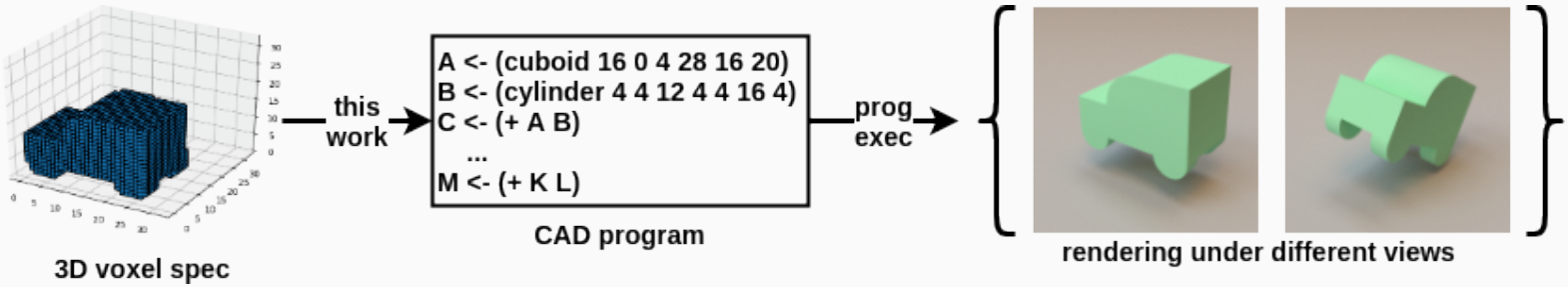
(b) Caddy program

op	::=	+ | - | × | /
num	::=	$\mathbb{R}$ | ⟨*var*⟩ | ⟨*num*⟩ ⟨*op*⟩ ⟨*num*⟩
vec2	::=	[⟨*num*⟩, ⟨*num*⟩]
vec3	::=	[⟨*num*⟩, ⟨*num*⟩, ⟨*num*⟩]
affine	::=	Translate | Rotate | Scale | **TranslateSpherical**
binop	::=	Union | Difference | Intersection
cad	::=	(Cuboid ⟨*vec3*⟩) | (Sphere ⟨*num*⟩)
	|	(Cylinder ⟨*vec2*⟩) | (HexPrism ⟨*vec2*⟩) | …
	|	(⟨*affine*⟩ ⟨*vec3*⟩ ⟨*cad*⟩)
	|	(⟨*binop*⟩ ⟨*cad*⟩ ⟨*cad*⟩)
	|	**(Fold** ⟨*binop*⟩ ⟨*cad-list*⟩**)**
cad-list	::=	(List ⟨*cad*⟩+)
	|	(Concat ⟨*cad-list*⟩+)
	|	(Tabulate (⟨*var*⟩ $\mathbb{Z}^+$)+ ⟨*cad*⟩)
	|	(Map2 ⟨*affine*⟩ ⟨*vec3-list*⟩ ⟨*cad-list*⟩)

Visual programs



Mao*, Zhang*, et al 2019



Ellis*, Nye*, Pu*, Sosa*, et al 2019

partial image x_{part}

```
for i = 1..3
  for j = 1..1
    draw(i*2, j*1, )
      ...
```

synthesized program P_{part}

Young et al 2019

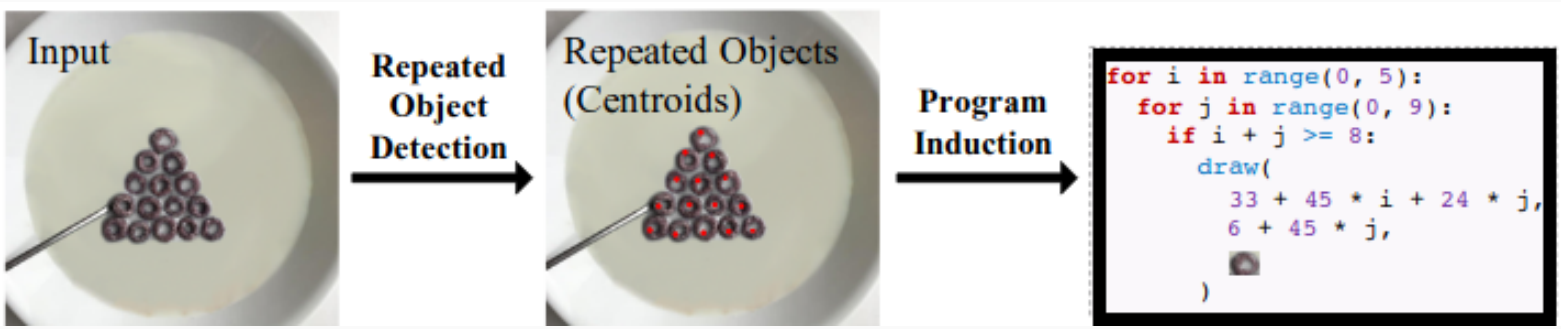
```
Draw("Top", "Circle", position, geometry)

for(i < 2, "translation", a)
  for(j < 2, "translation", b)
    Draw("Leg", "Cub", position + i*a + j*b,

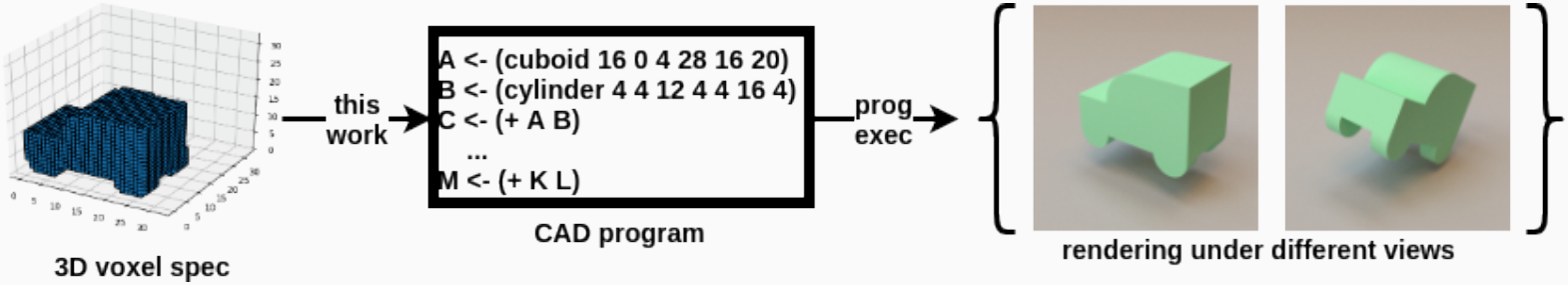
for(i < 2, "translation", c)
  Draw("Layer", "Rec", position + i*c, geomet
```

Tian et al 2019

Where does this language come from?



Mao*, Zhang*, et al 2019



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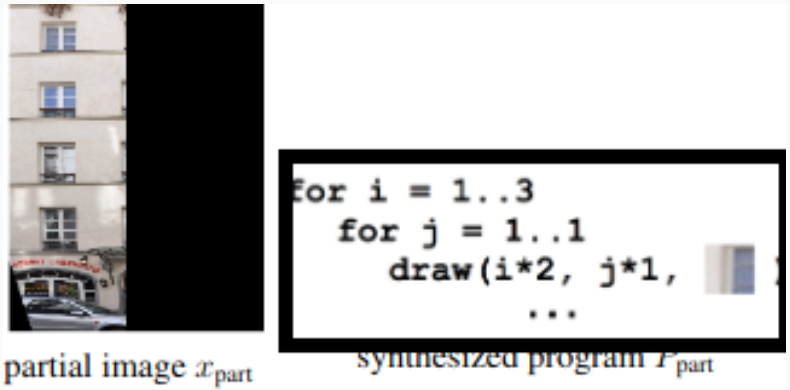

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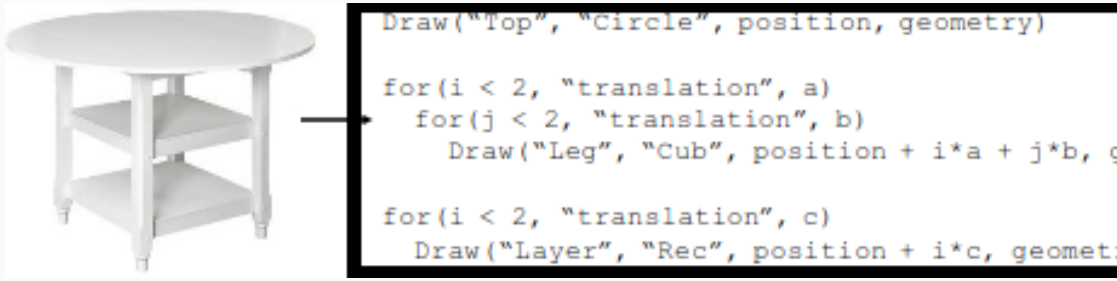
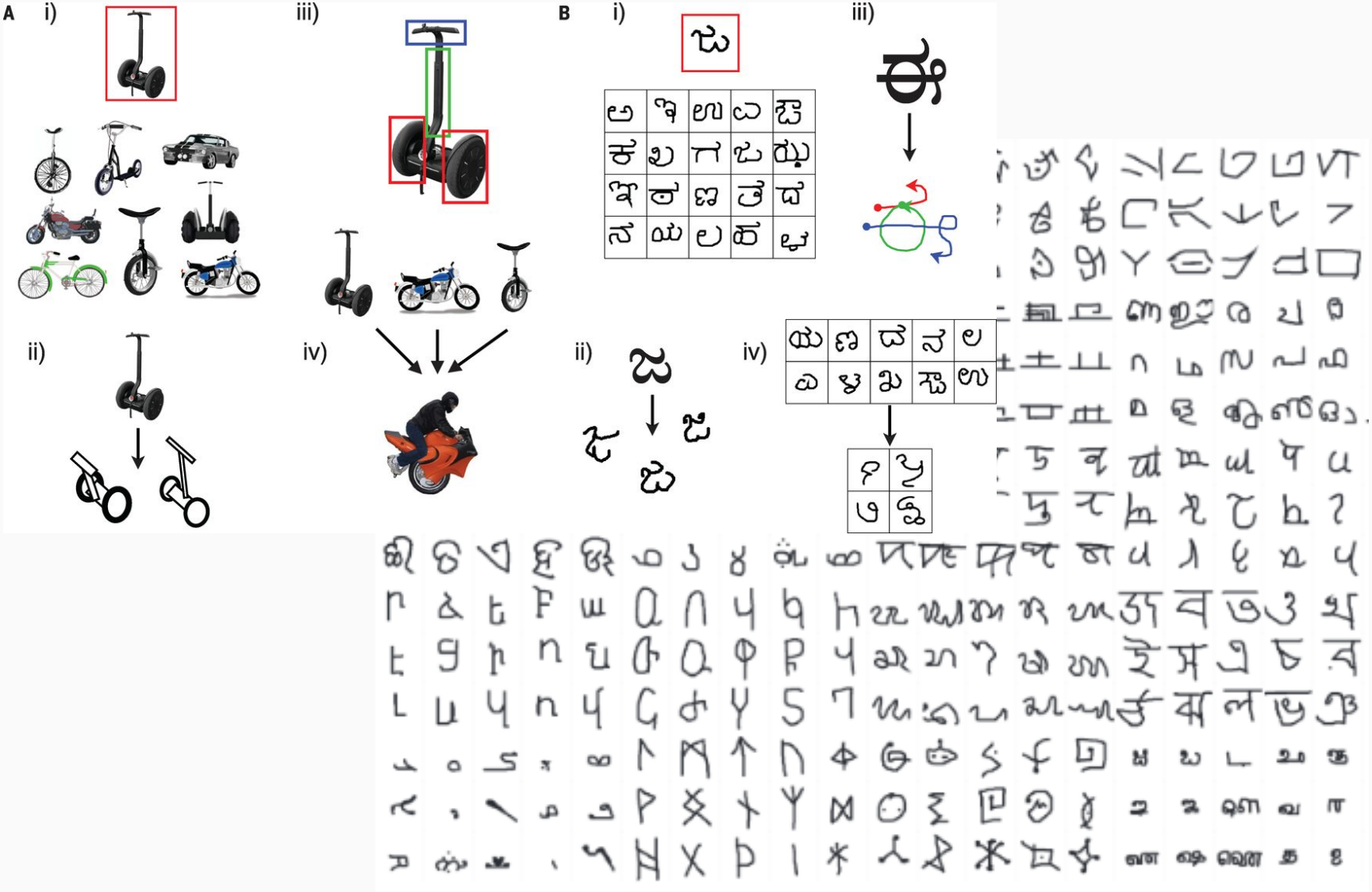


Tian et al 2019

Human-level Program induction



Lake et al 2015

DreamCoder and learning to learn

learning a library

learning to search

synergy between library+learned search

Learning to write code

Goal: acquire domain-specific knowledge needed to induce a class of programs

- Library of components
- Search strategy (synthesis algorithm)

Library learning

Initial Primitives

⋮

`map`

`fold`

`if`

`cons`

`>`

⋮

Sample Problem: `sort list`

[9 2 7 1] → [1 2 7 9]
[3 8 9 4 2] → [2 3 4 8 9]
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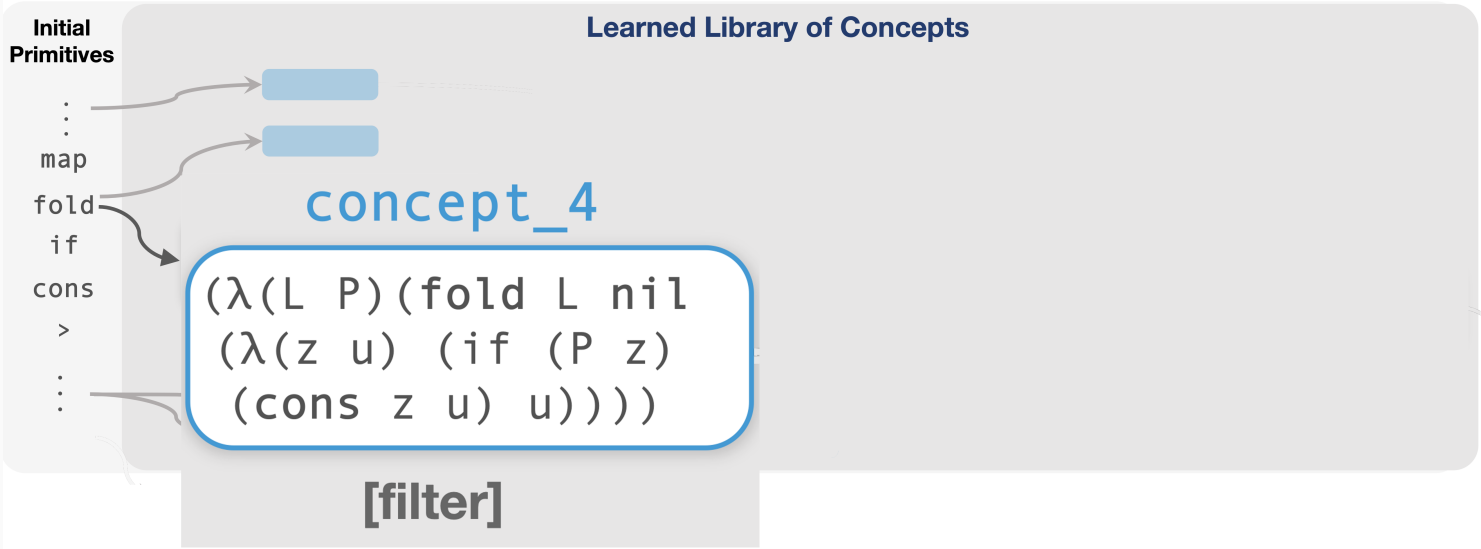

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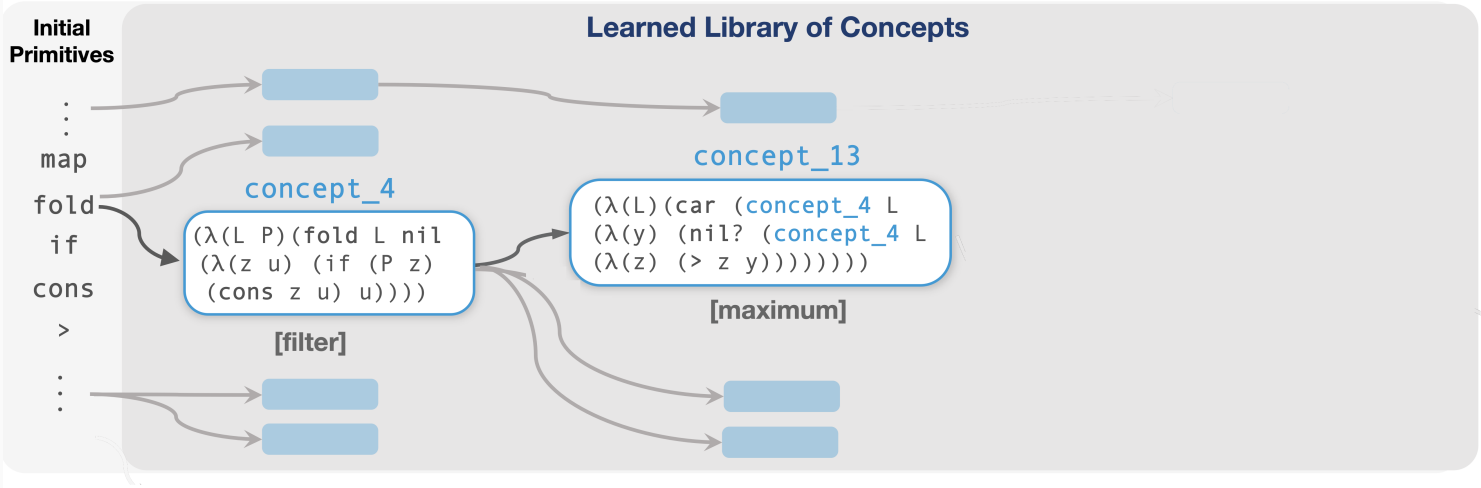

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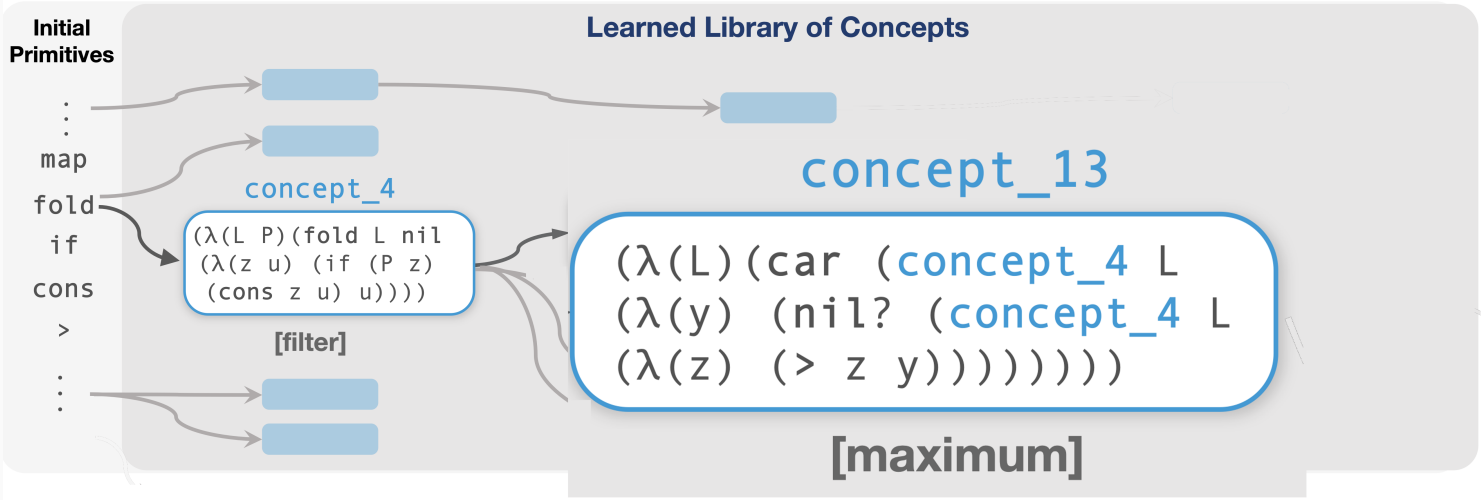

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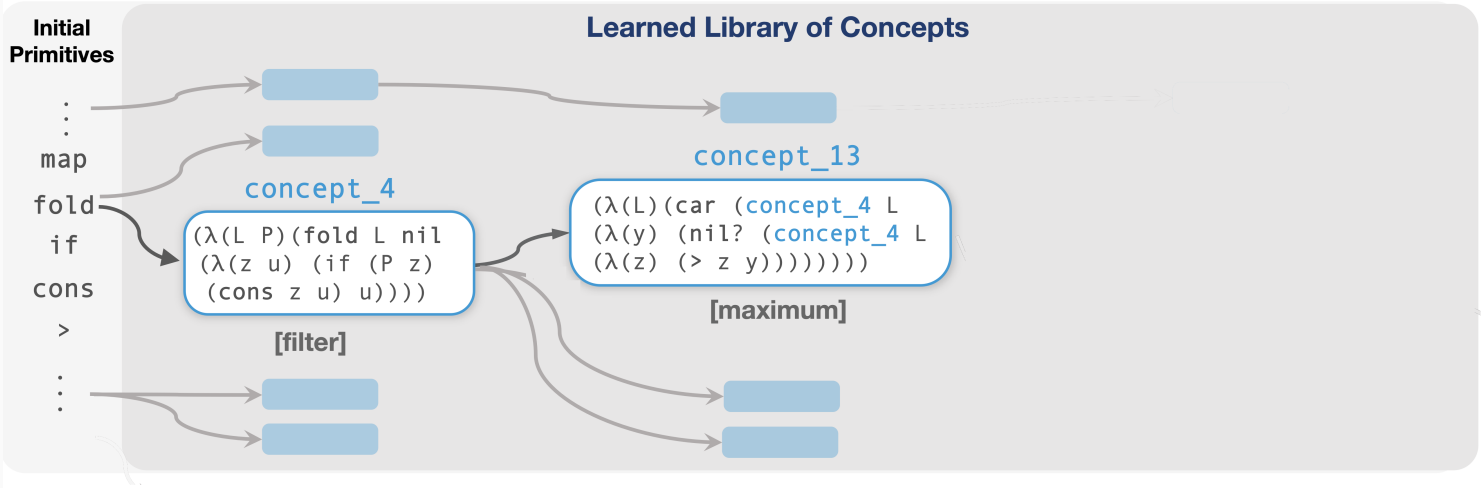

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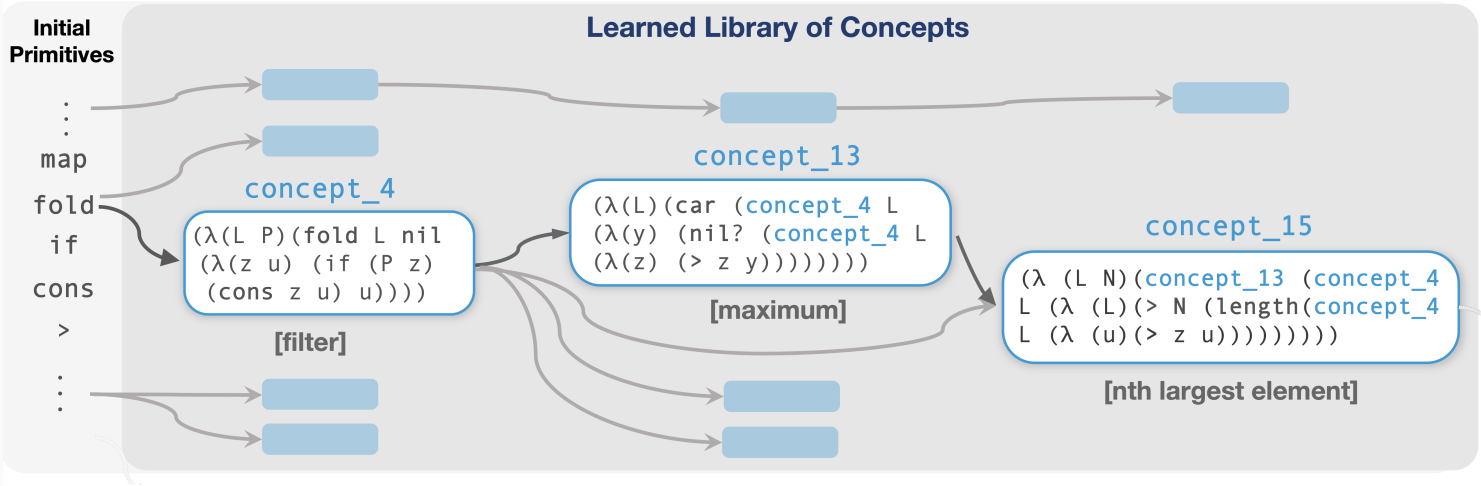

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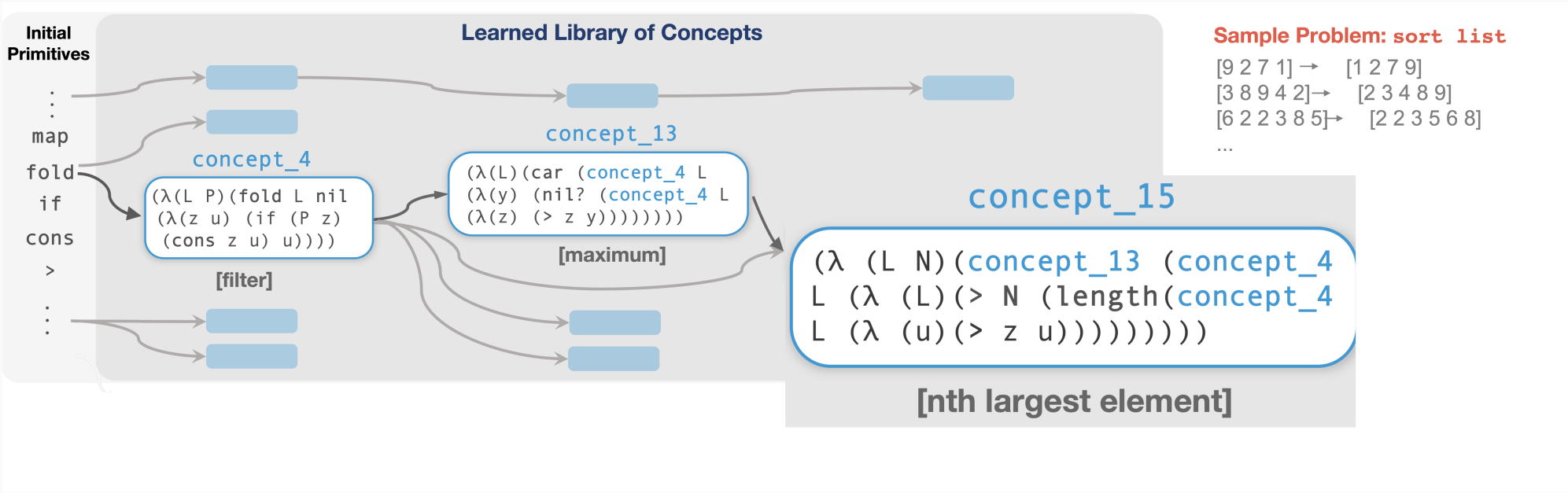

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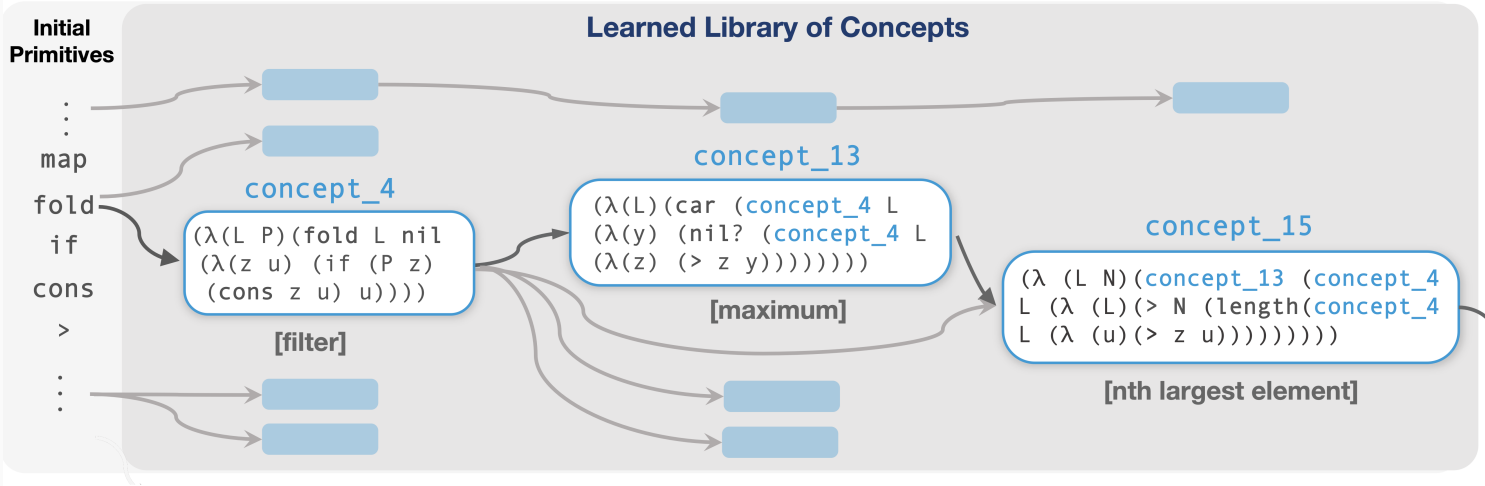

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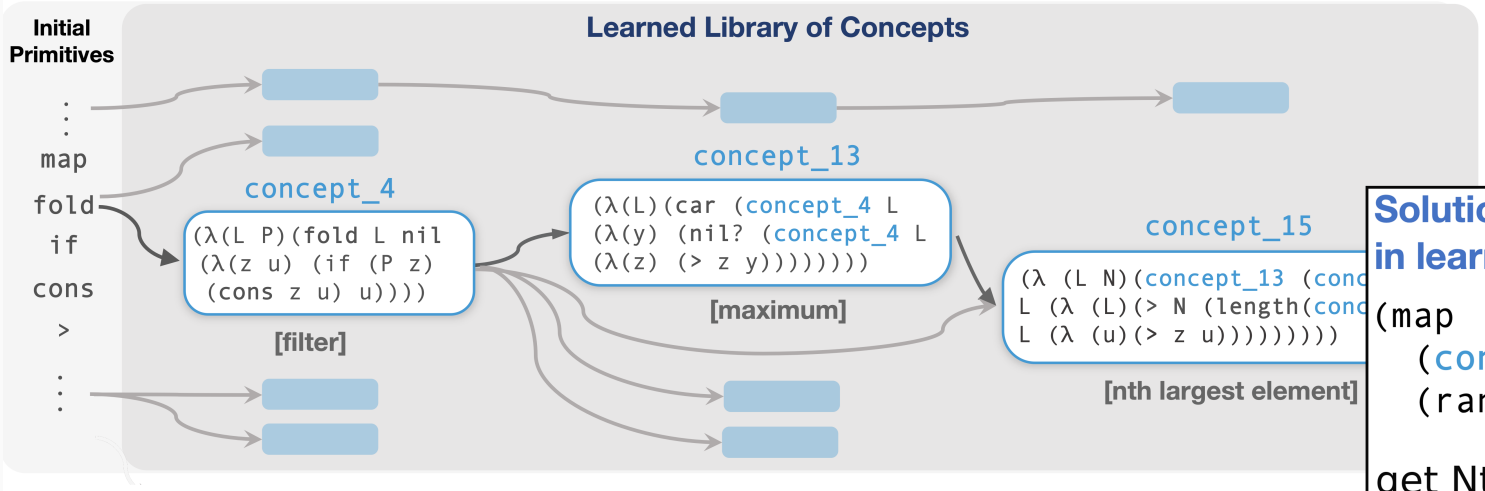
(map  (λ (n)
  (concept_15 L (+ 1 n)))
  (range (length L)))
```

Library learning



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get Nth largest element, where N is 1, 2, 3, ...

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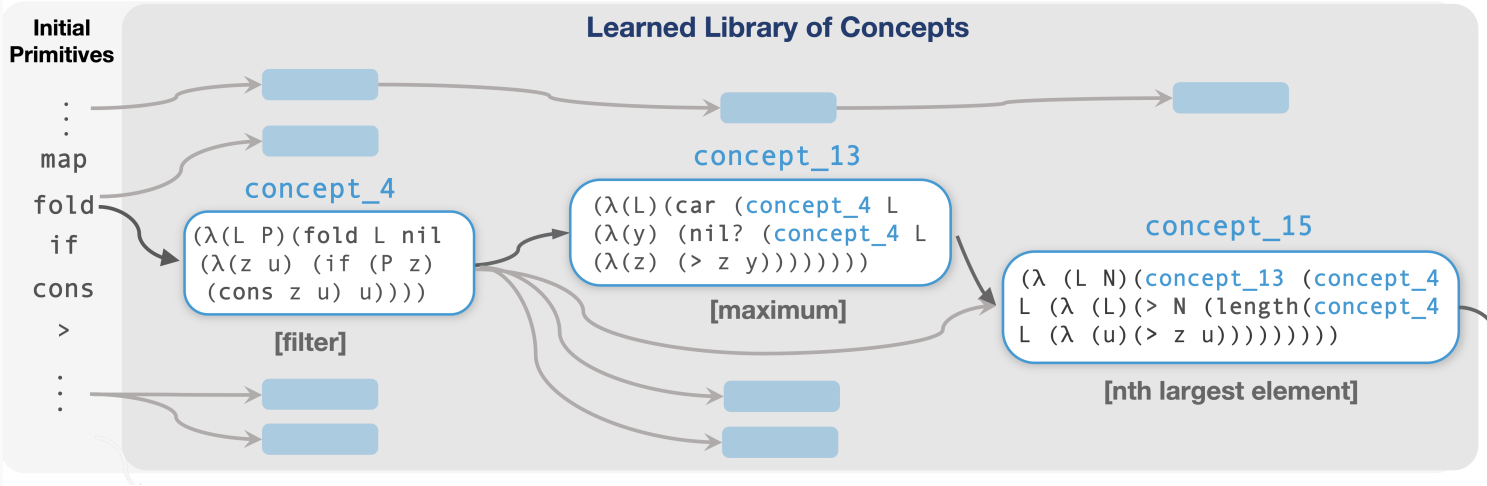

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Solution rewritten in initial primitives:

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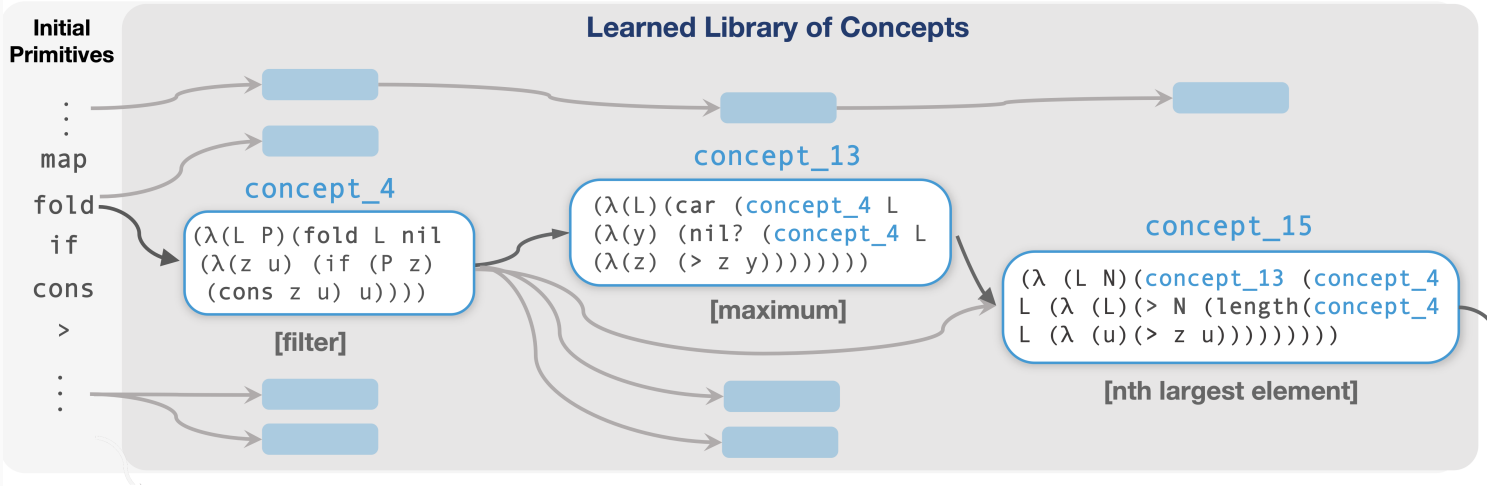
```

Library learning



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```

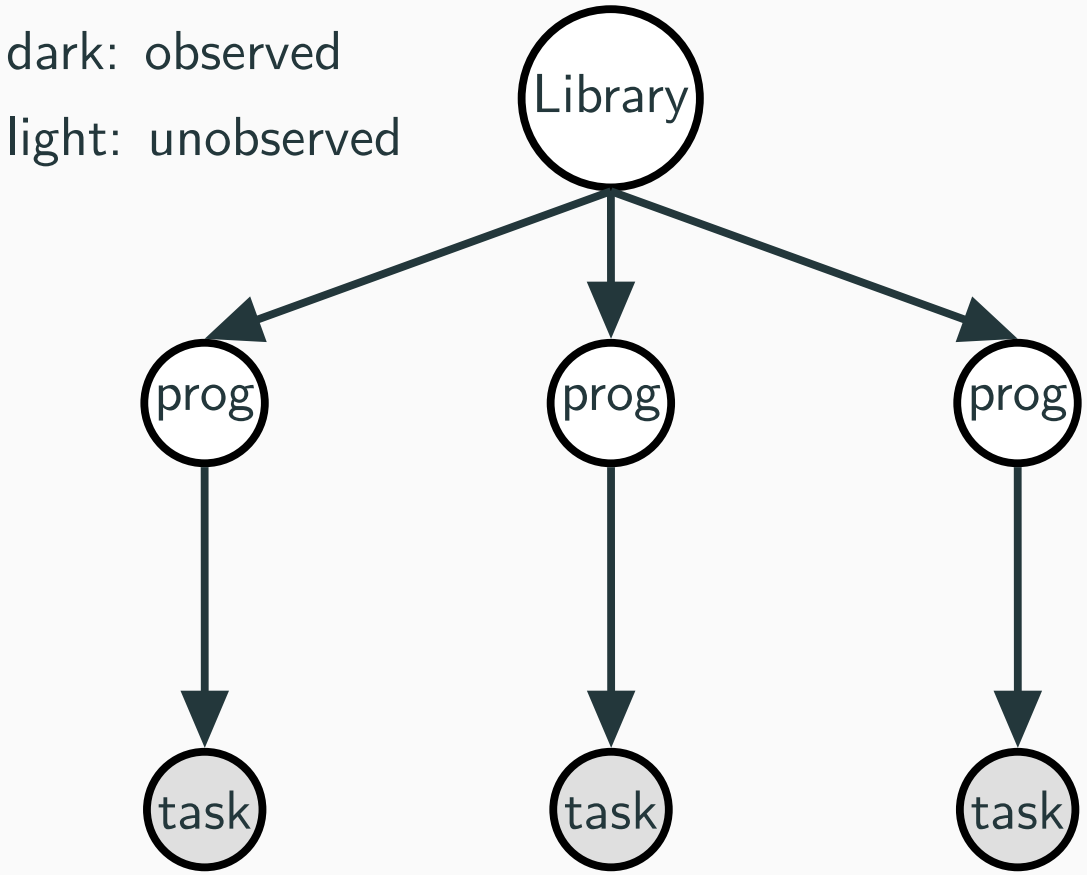
induced sort program found in $\leq$ 10min. Brute-force search without learned library would take $\approx 10^{73}$ years

DreamCoder

- **Wake:** Solve problems by writing programs
- **Sleep:** Improve library and neural recognition model:
  - **Abstraction sleep:** Improve library
  - **Dream sleep:** Improve neural recognition model

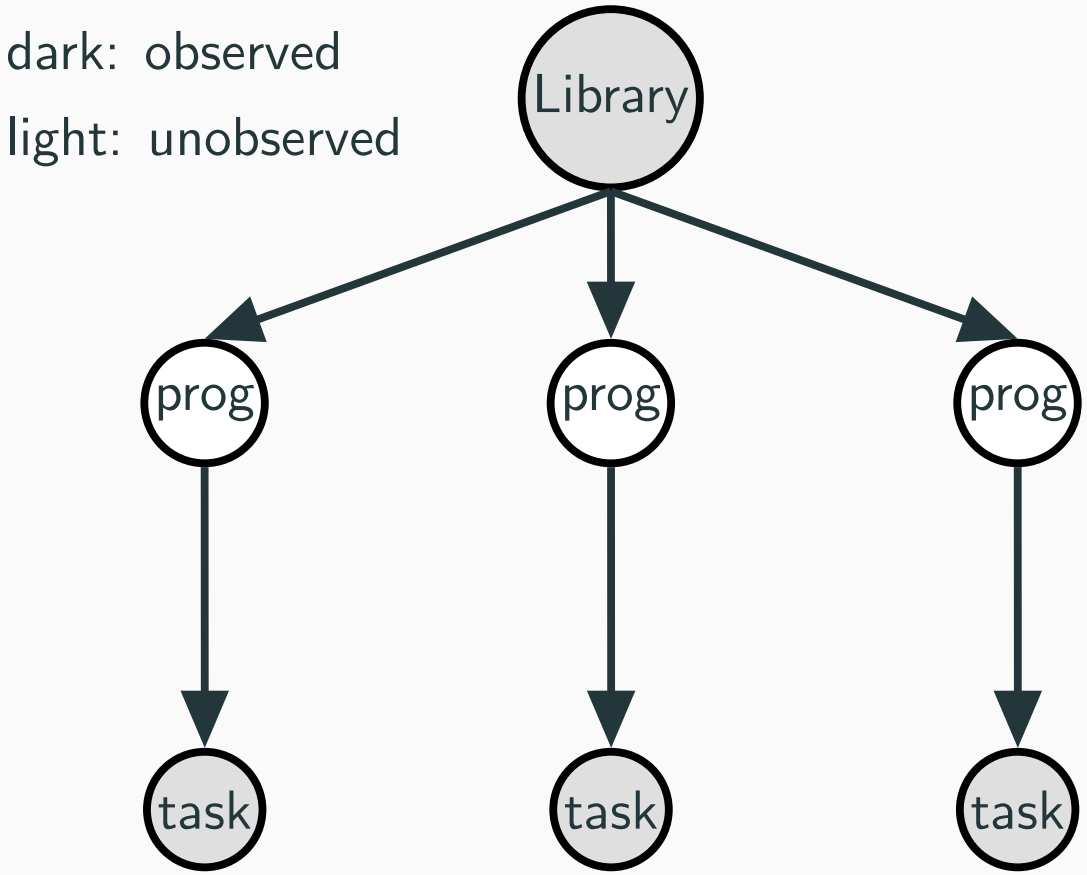
cf. Helmholtz machine, wake/sleep neural network training algorithms

Library learning as Bayesian inference



[Dechter et al, 2013] [Liang et al, 2010] [Lake et al, 2015]

Library learning as Bayesian inference



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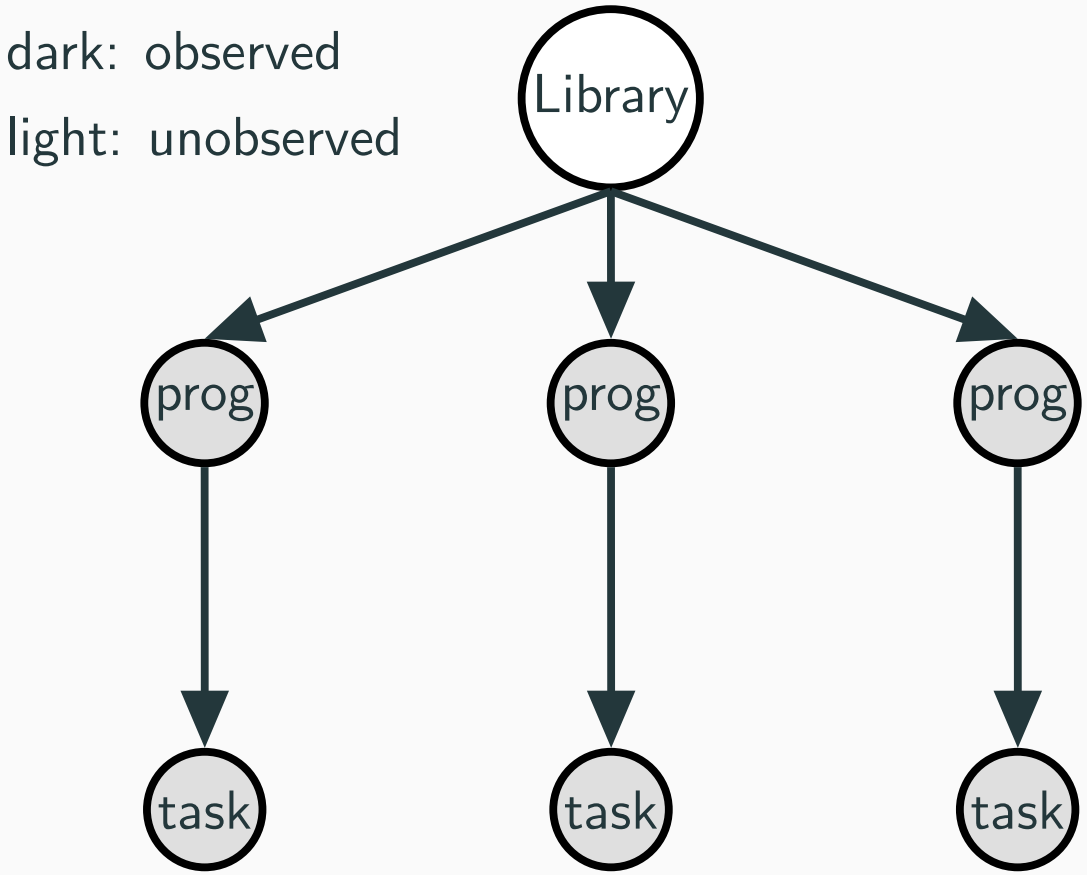

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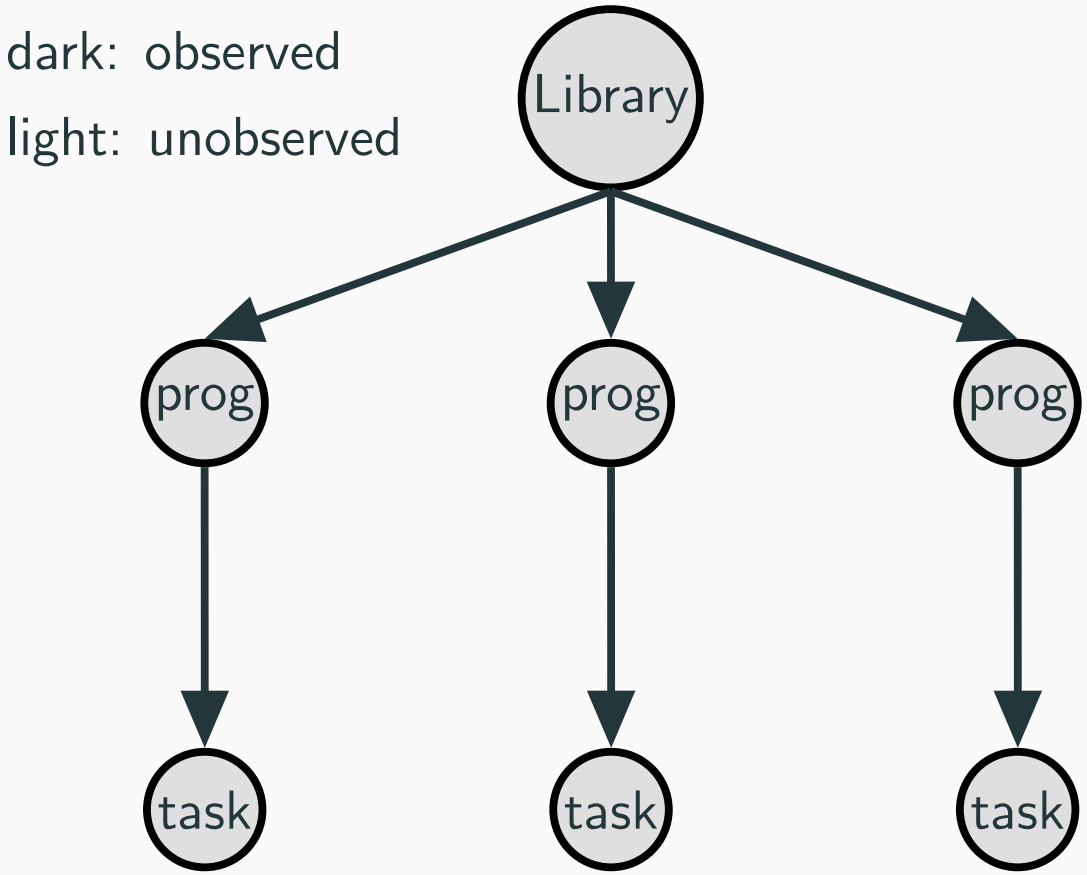

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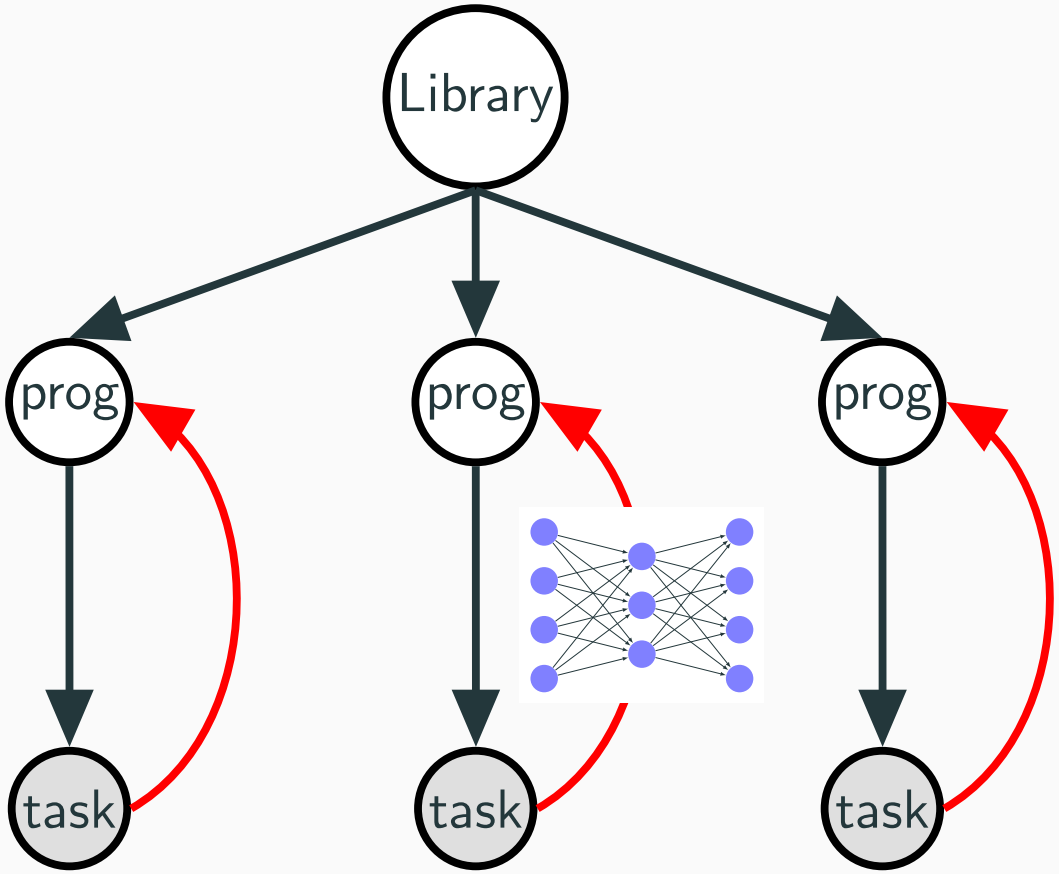
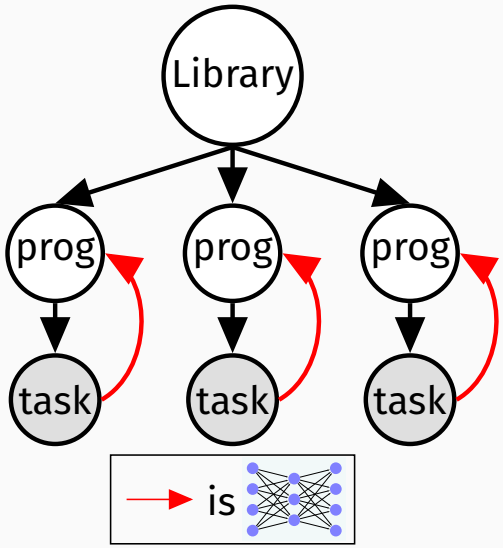
Library learning as neurally-guided Bayesian inference



library learning via program analysis +

new neural inference network for program synthesis +

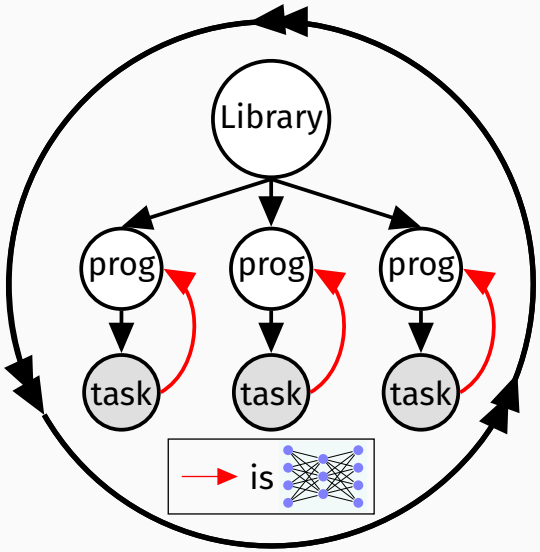
better program representation (Lisp+polymorphic types [Milner 1978])


Library
prog
prog
prog
task
task
task
is

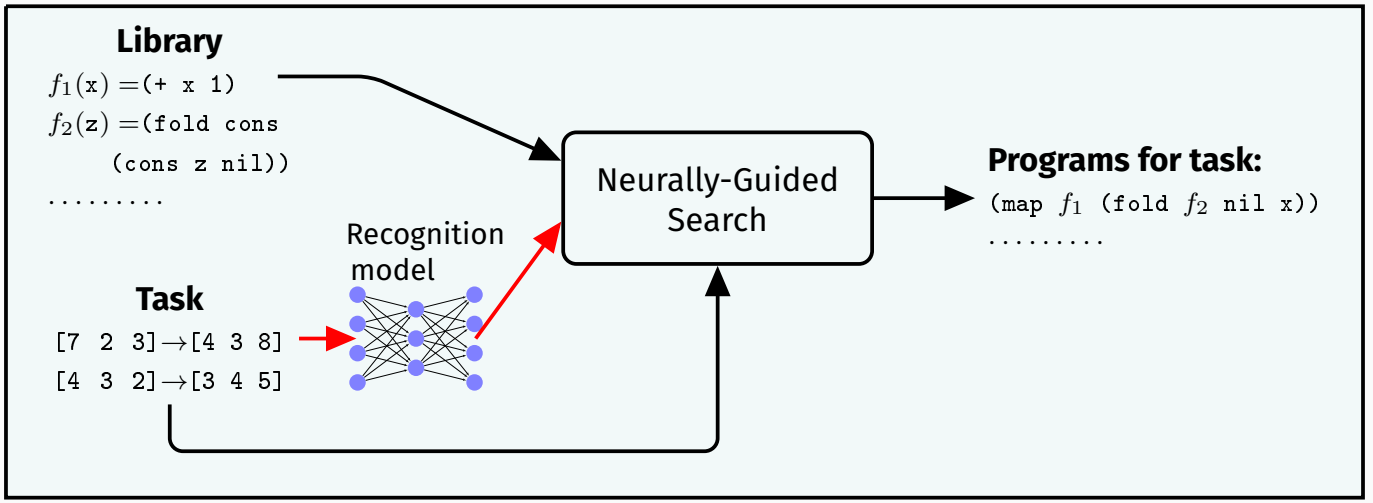
Wake

Sleep: Abstraction

Sleep: Dreaming

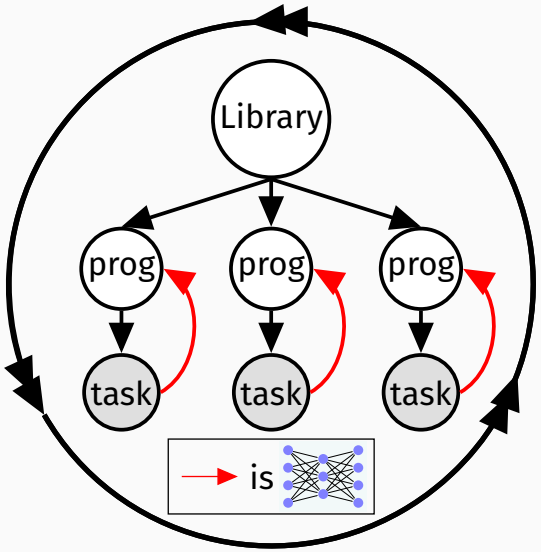


WAKE

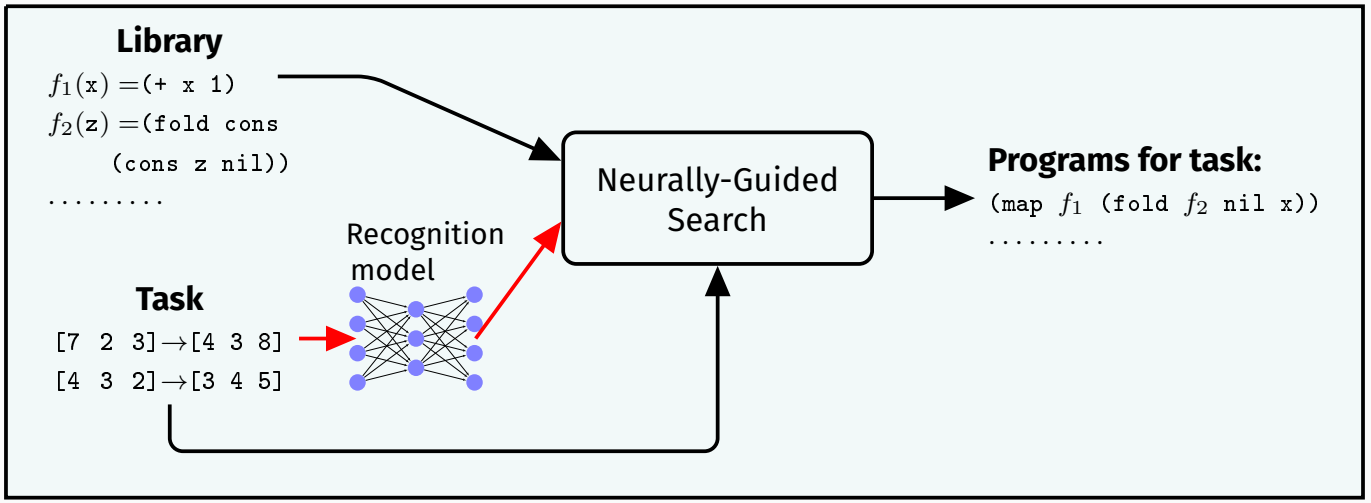


SLEEP: ABSTRACTION

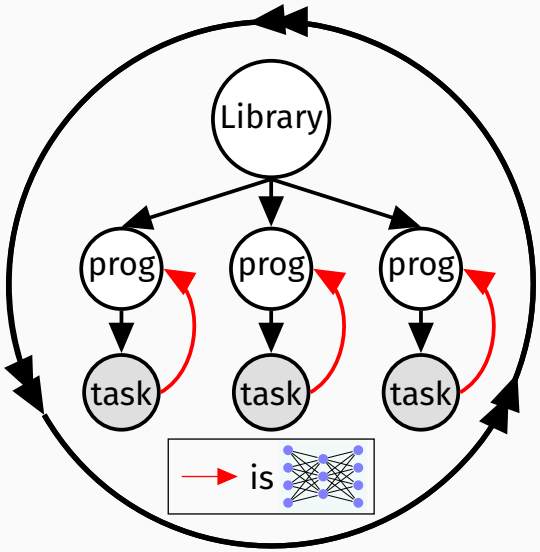
SLEEP: DREAMING



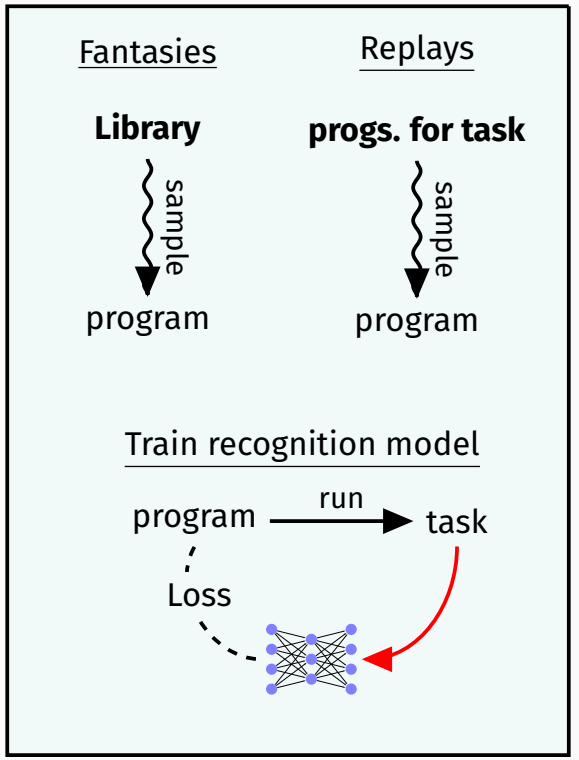
Wake

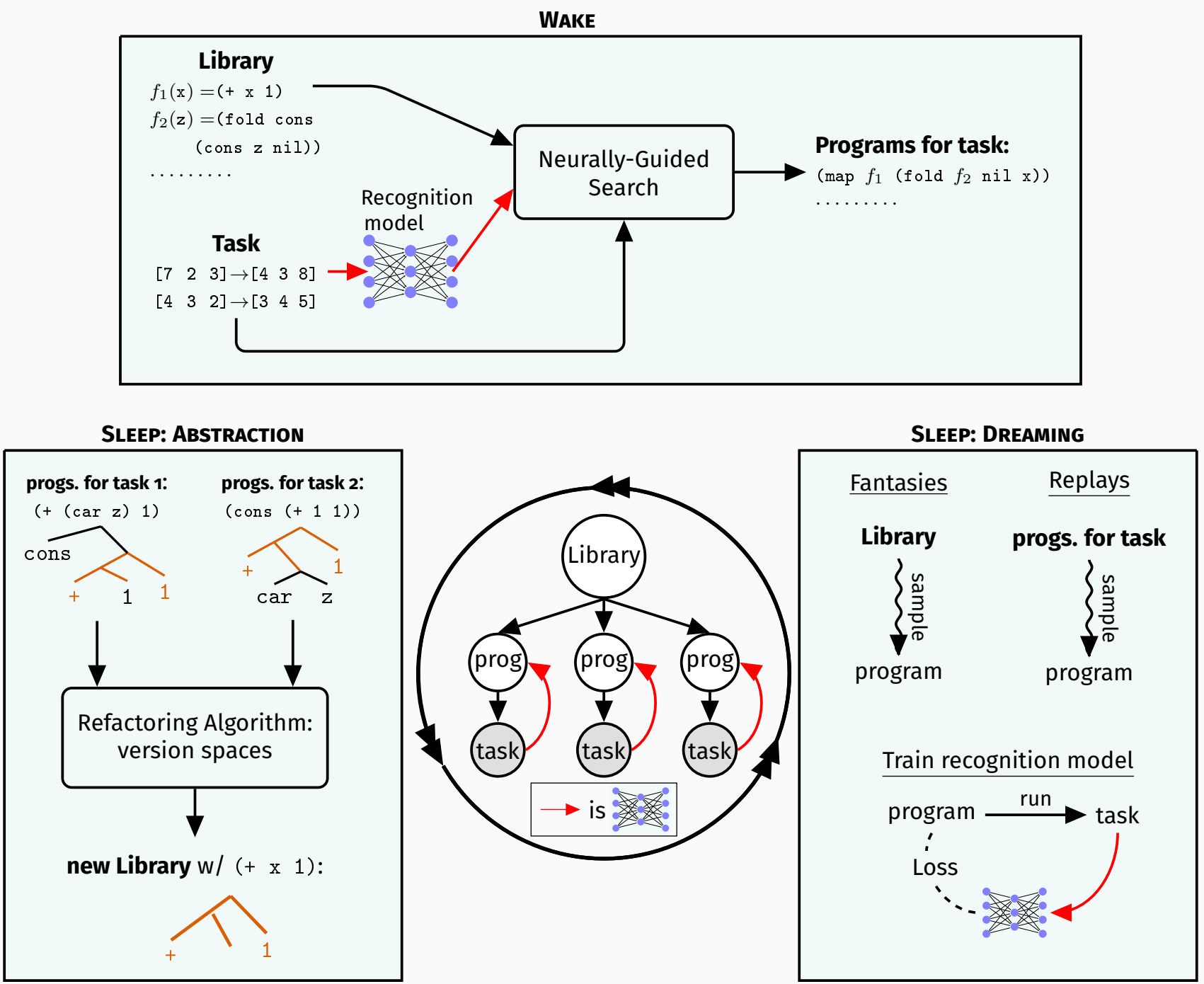


Sleep: Abstraction



Sleep: Dreaming




WAKE
Library
f_1(x) =(+ x 1)
f_2(z) =(fold cons
(cons z nil))
.....
Task
[7 2 3]→[4 3 8]
[4 3 2]→[3 4 5]
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Neurally-Guided Search
Programs for task:
(map f_1 (fold f_2 nil x))
.....
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progs. for task 1:
(+ (car z) 1)
cons
+
1
1
progs. for task 2:
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+
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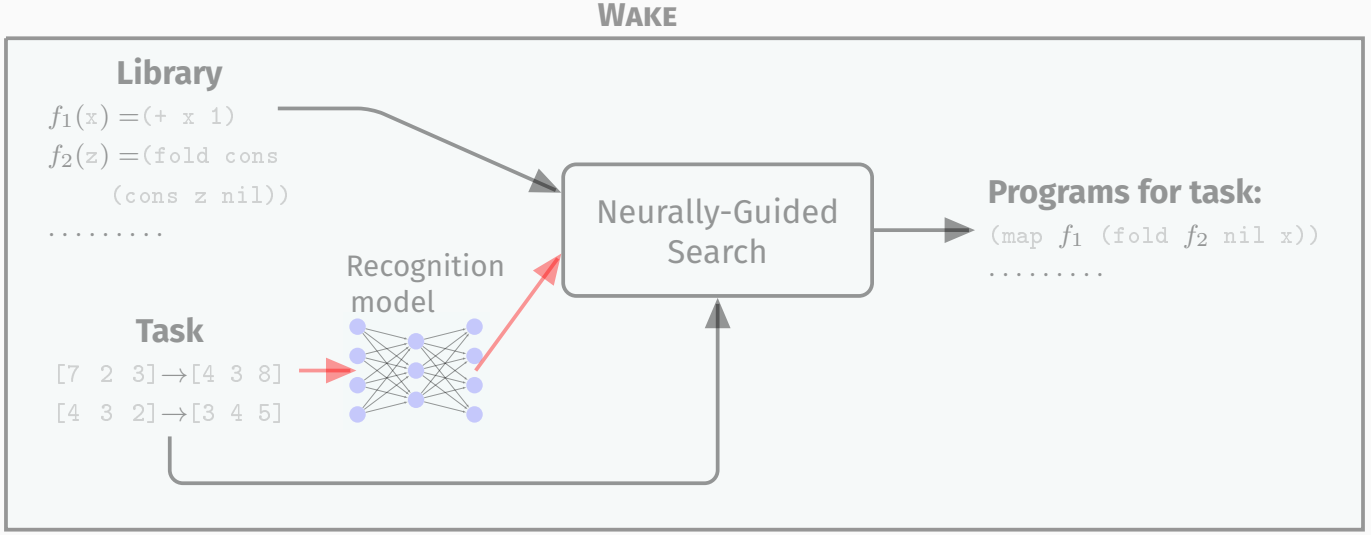
Refactoring Algorithm: version spaces
new Library w/ (+ x 1):
+
1
Library
prog
prog
prog
task
task
task
is
SLEEP: DREAMING
Fantasies
Replays
Library
progs. for task
sample
sample
program
program
Train recognition model
program
run
task
Loss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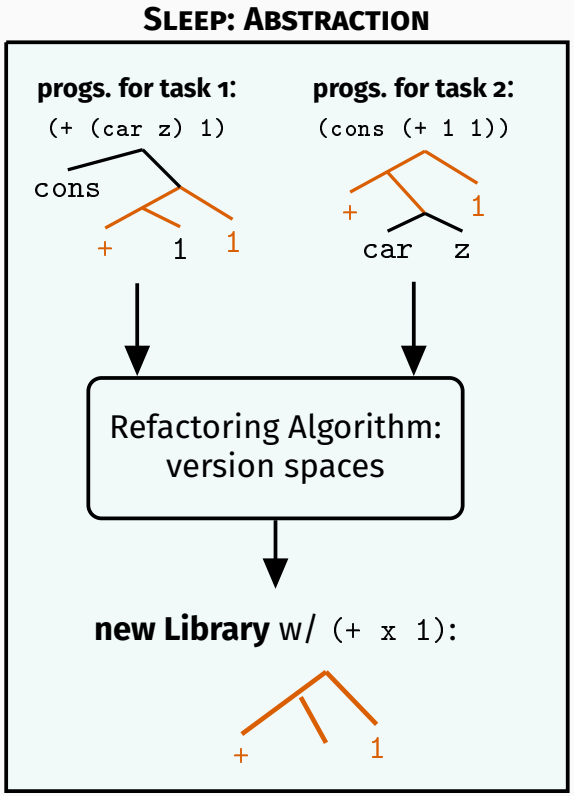
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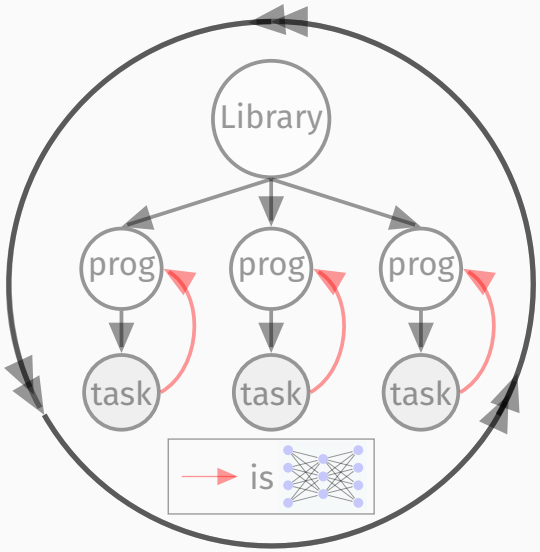
Library
prog
prog
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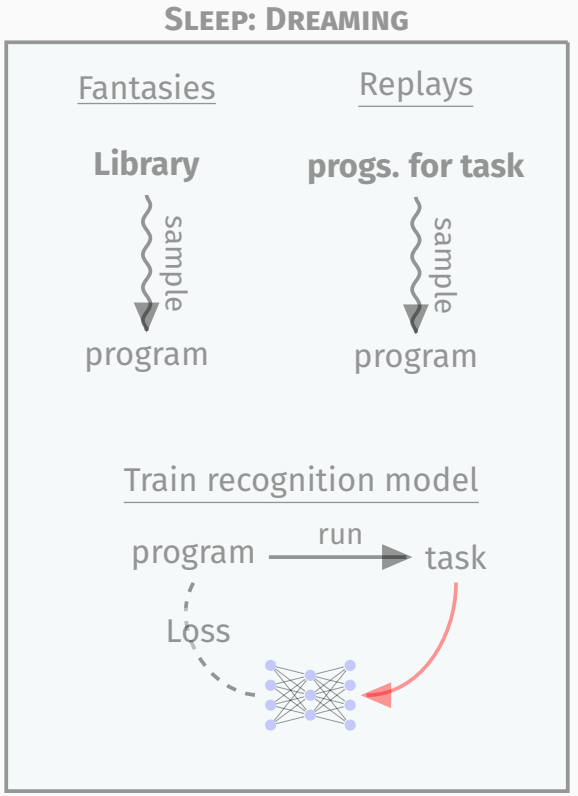
task
task
task
is


SLEEP: DREAMING
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Loss

DreamCoder and learning to learn

learning a library

learning to search

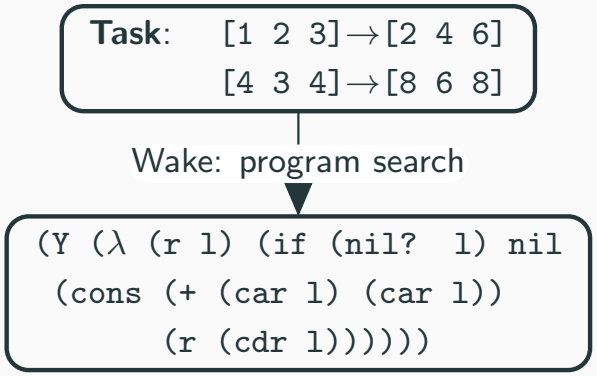
synergy between library+learned search

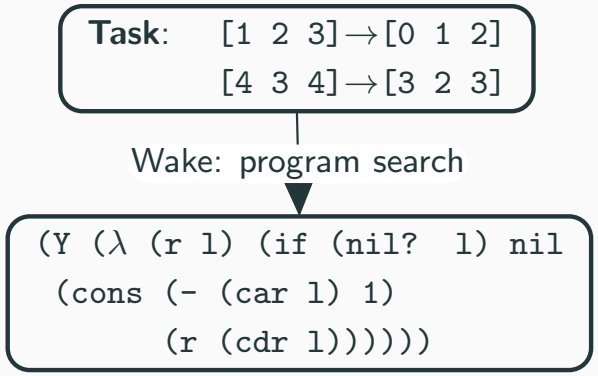
Abstraction Sleep: Growing the library via refactoring

Task: [1 2 3]→[2 4 6]
[4 3 4]→[8 6 8]

Task: [1 2 3]→[0 1 2]
[4 3 4]→[3 2 3]

Abstraction Sleep: Growing the library via refactoring





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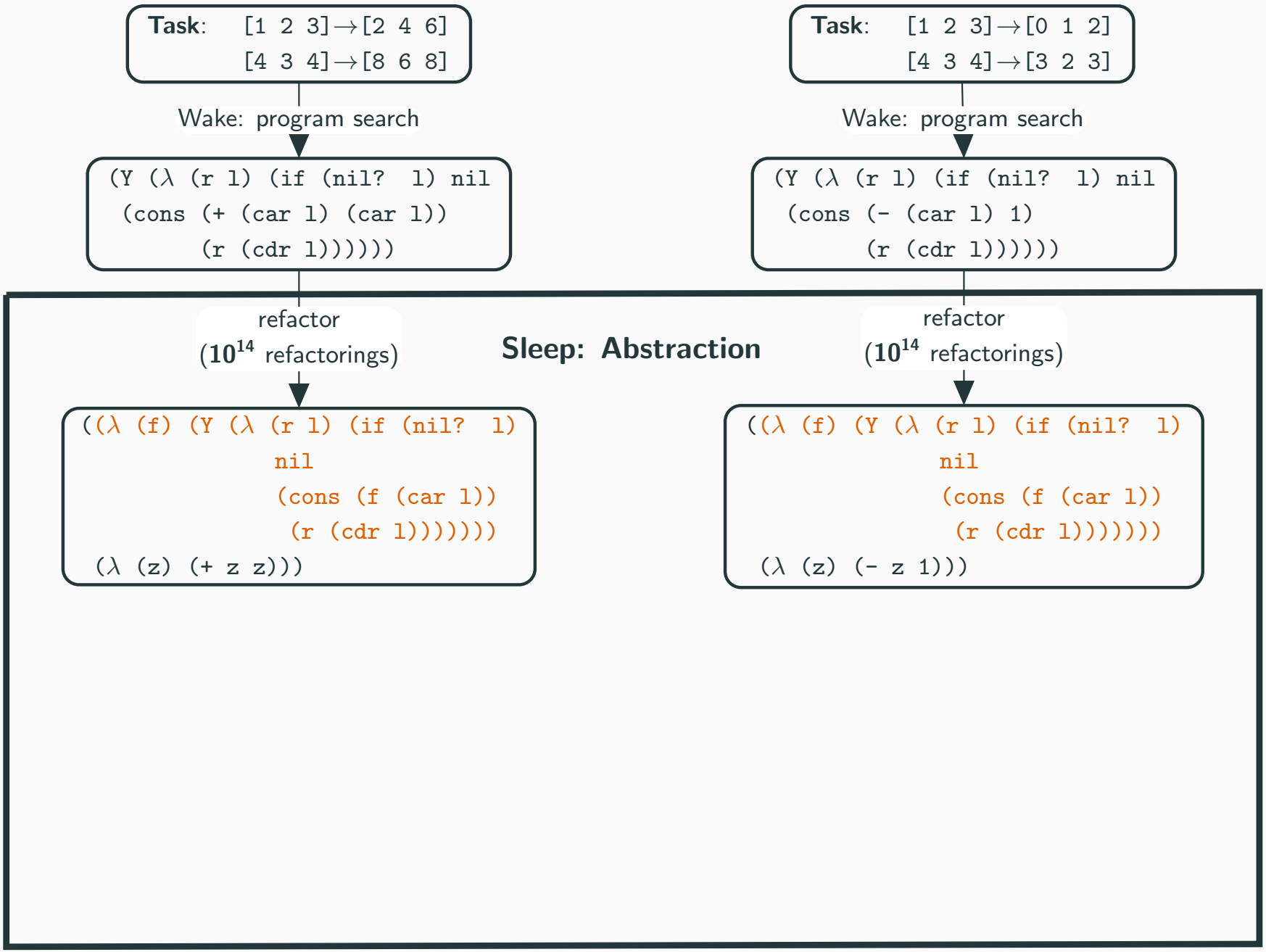

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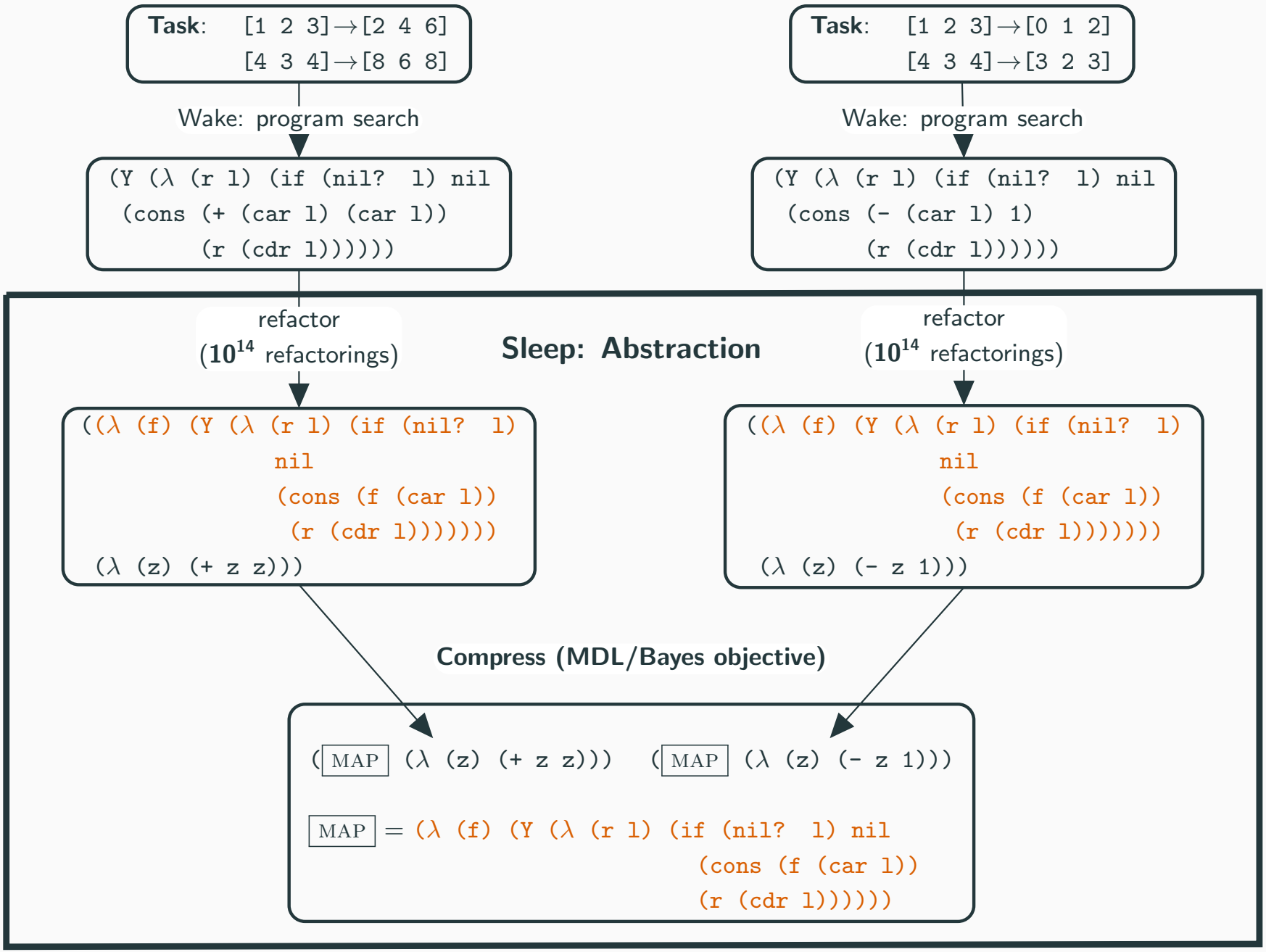

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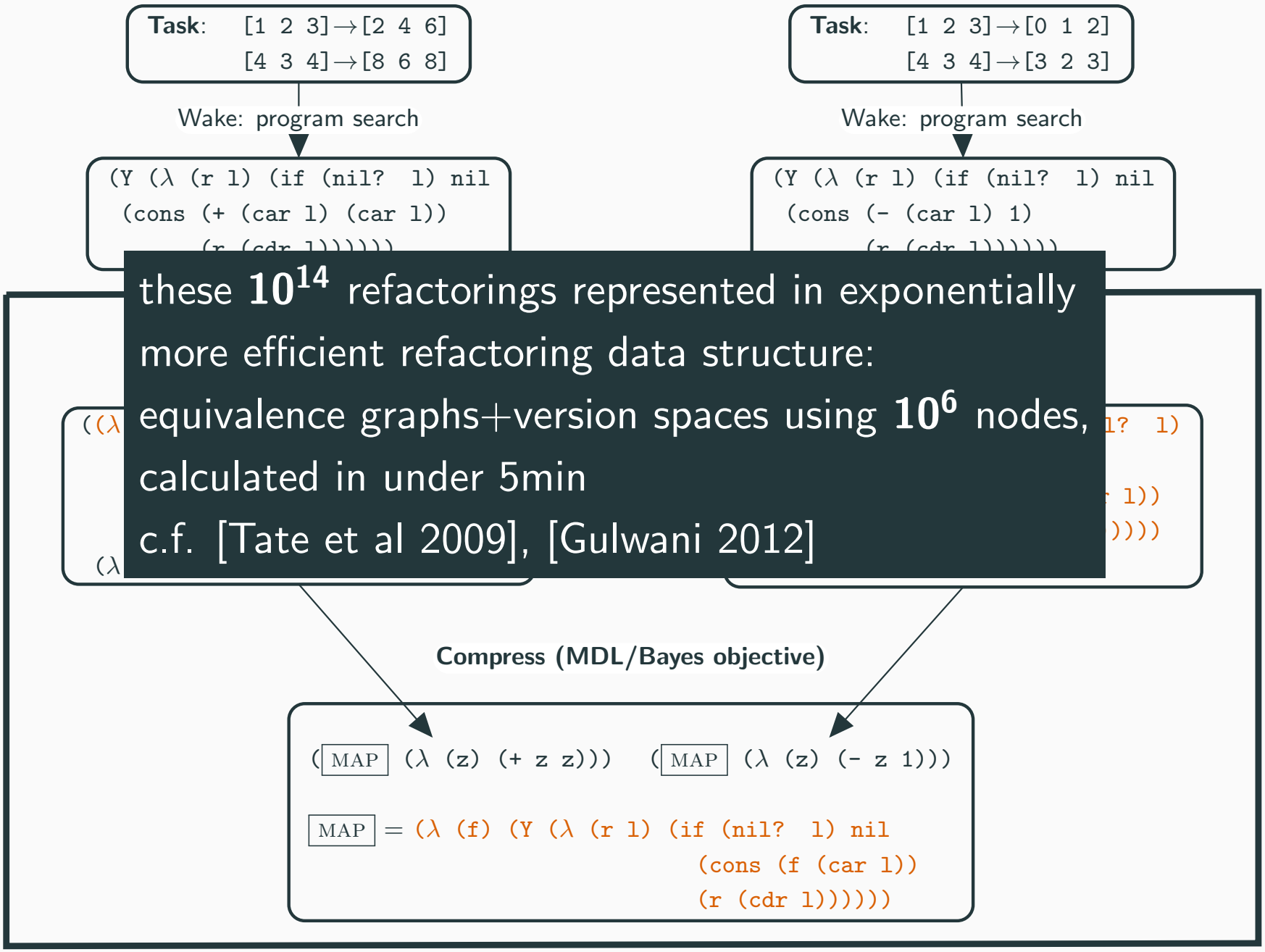

DreamCoder and learning to learn

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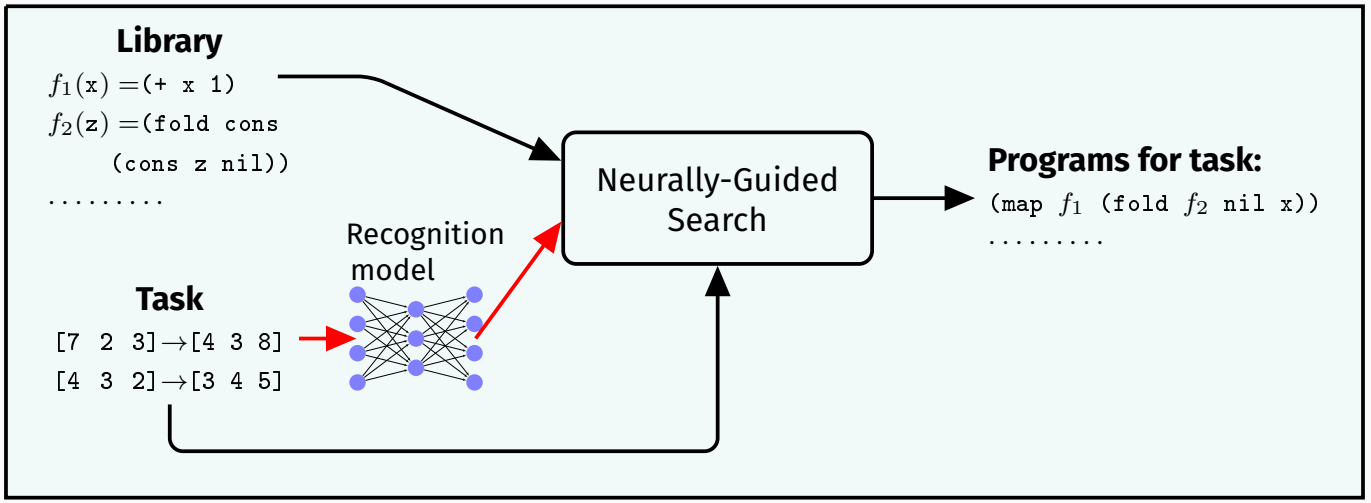
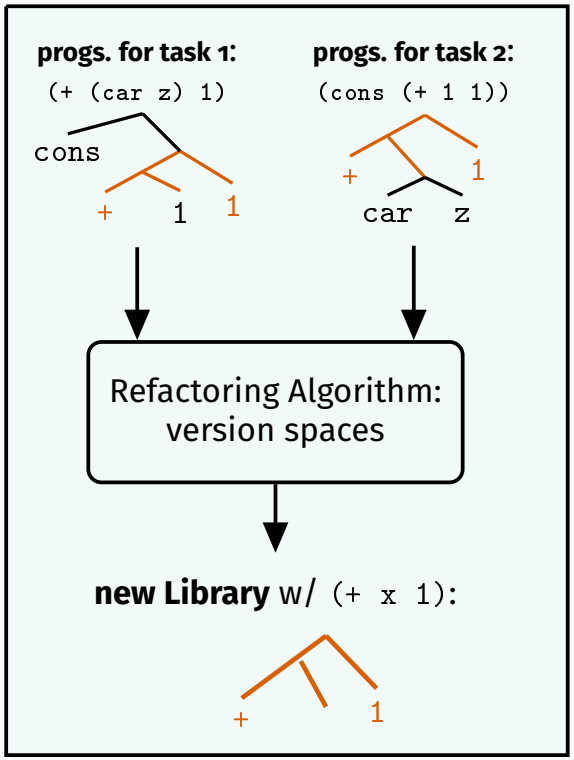
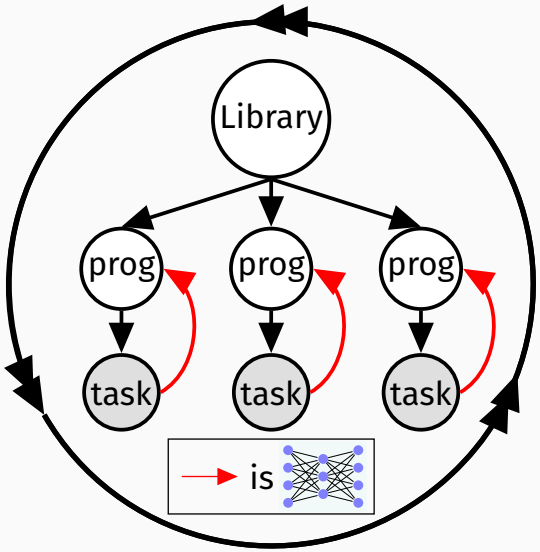
Wake

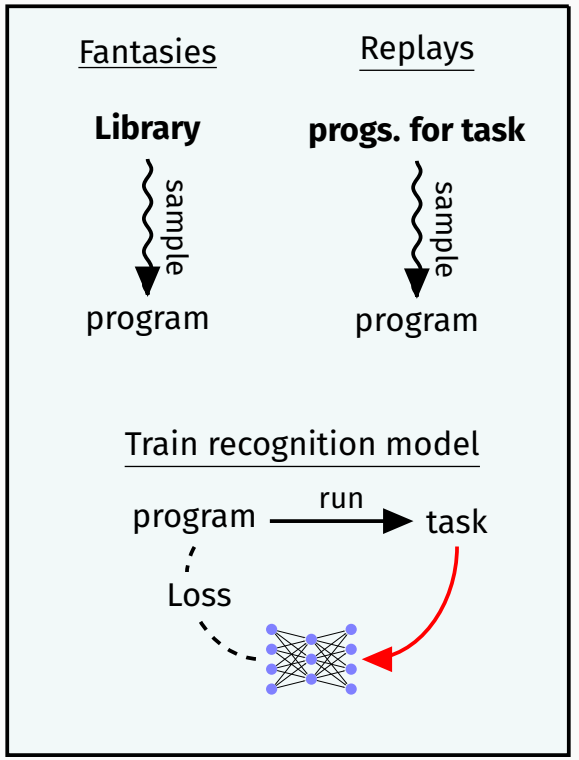


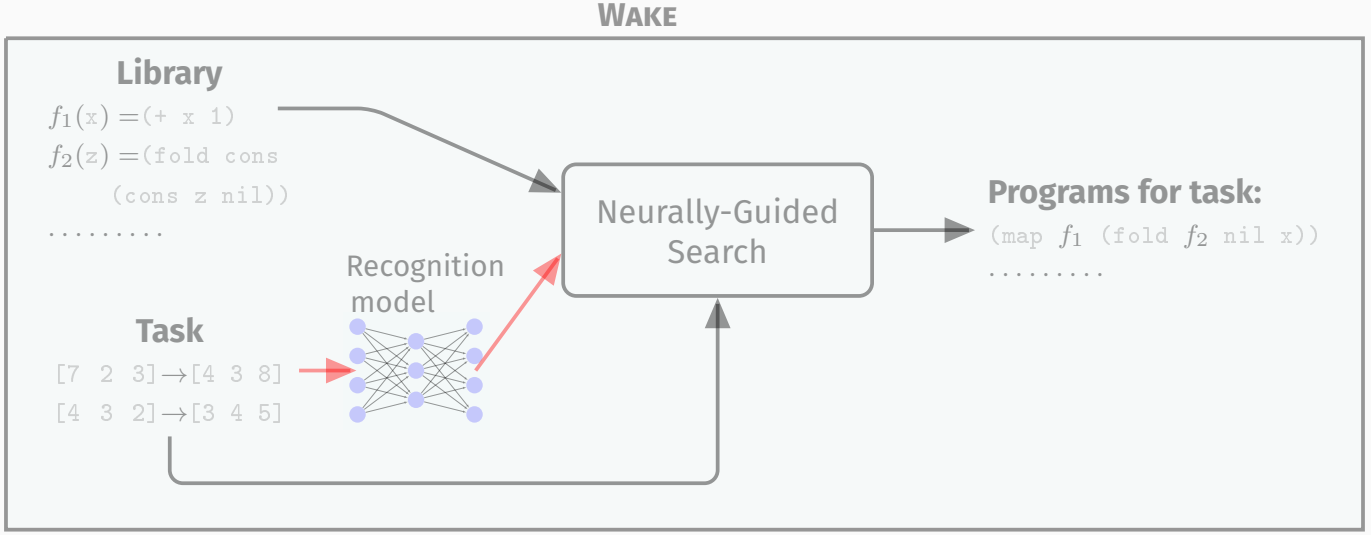
Sleep: Abstraction





Sleep: Dreaming

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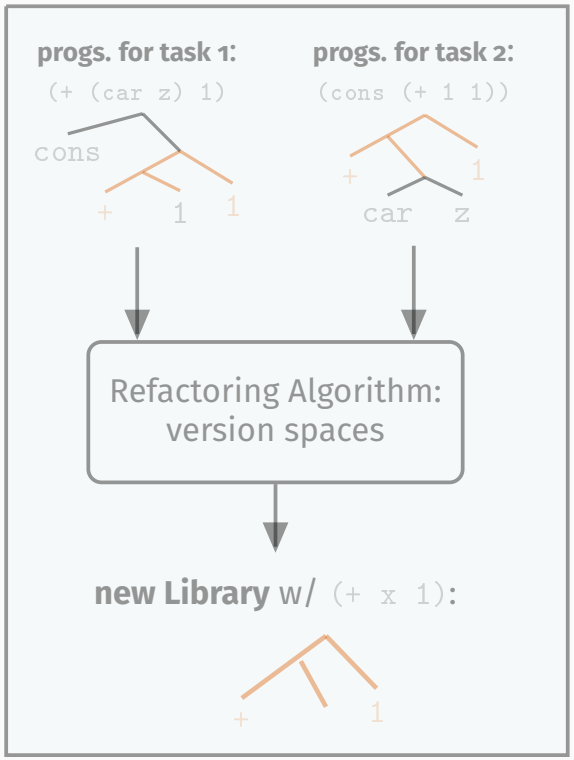
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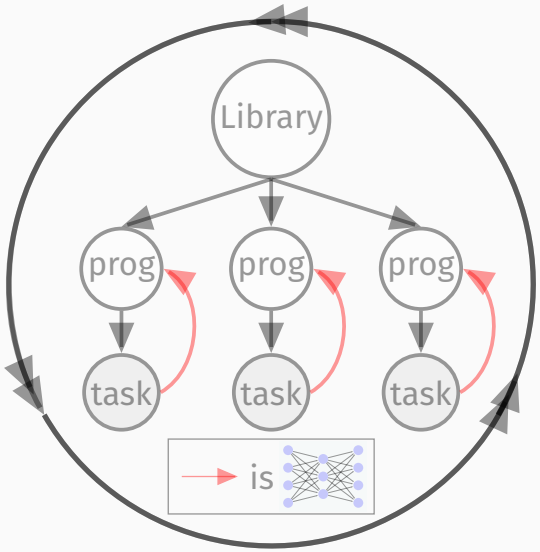
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task
task
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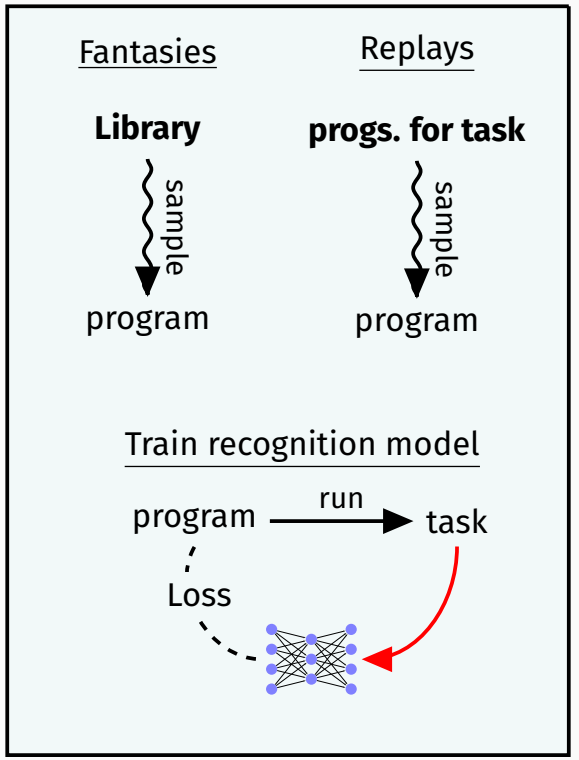
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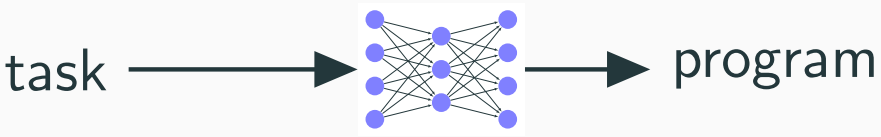
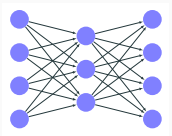
Neural recognition model guides search



Neural recognition model guides search

Neural recognition model guides search

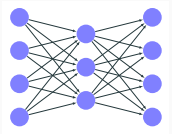


is a...

recurrent network (Devlin et al 2017)

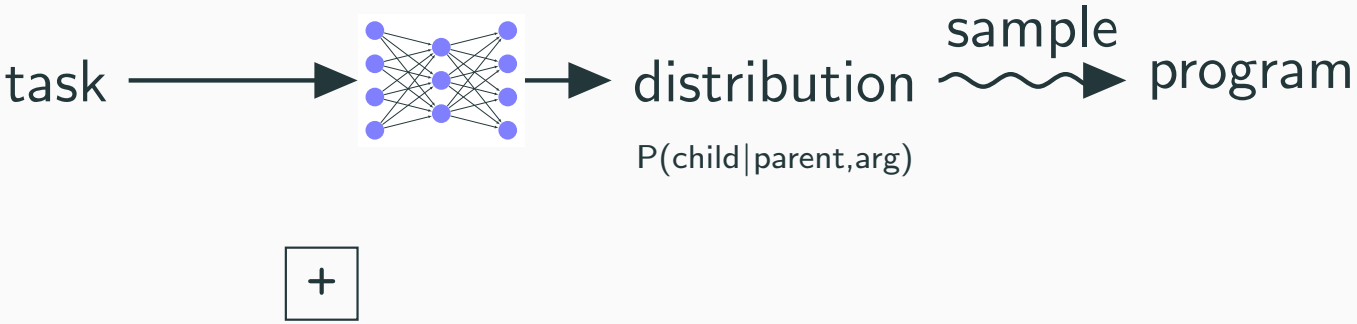
unigram model (Menon et al 2013; Balog et al 2016)

Neural recognition model guides search

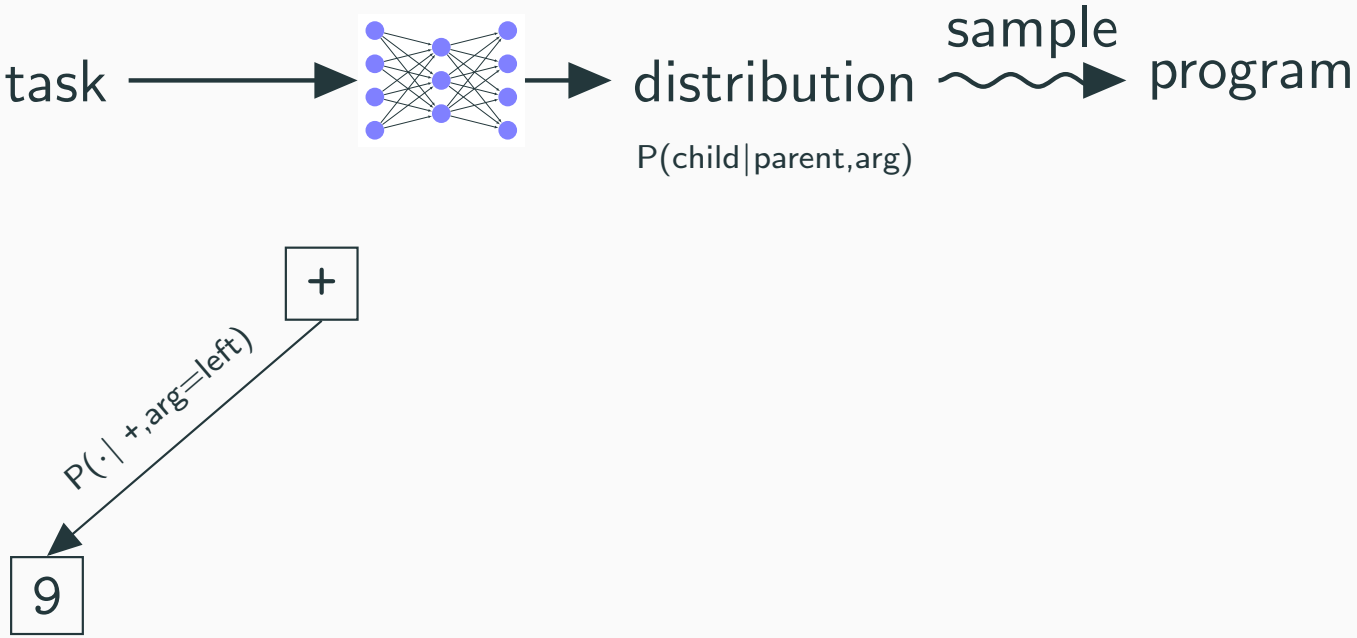


is a **"bigram" model over syntax trees**

Neural recognition model guides search



Neural recognition model guides search



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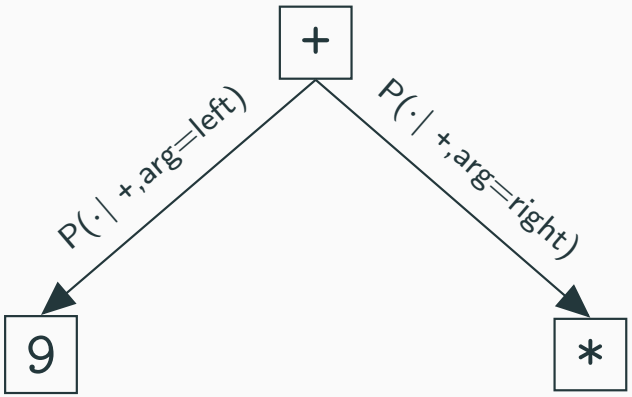

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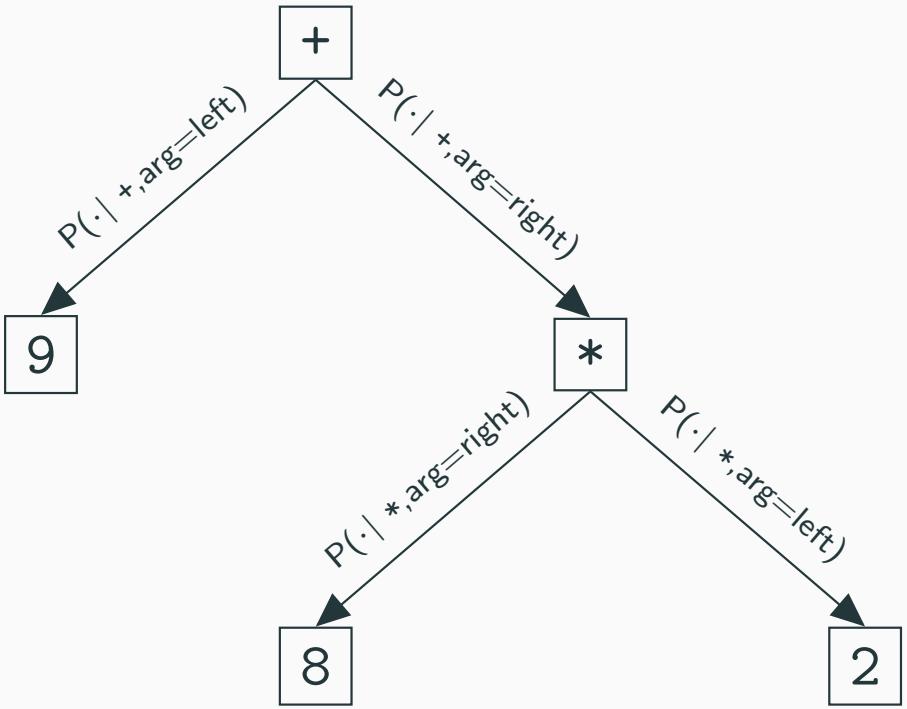

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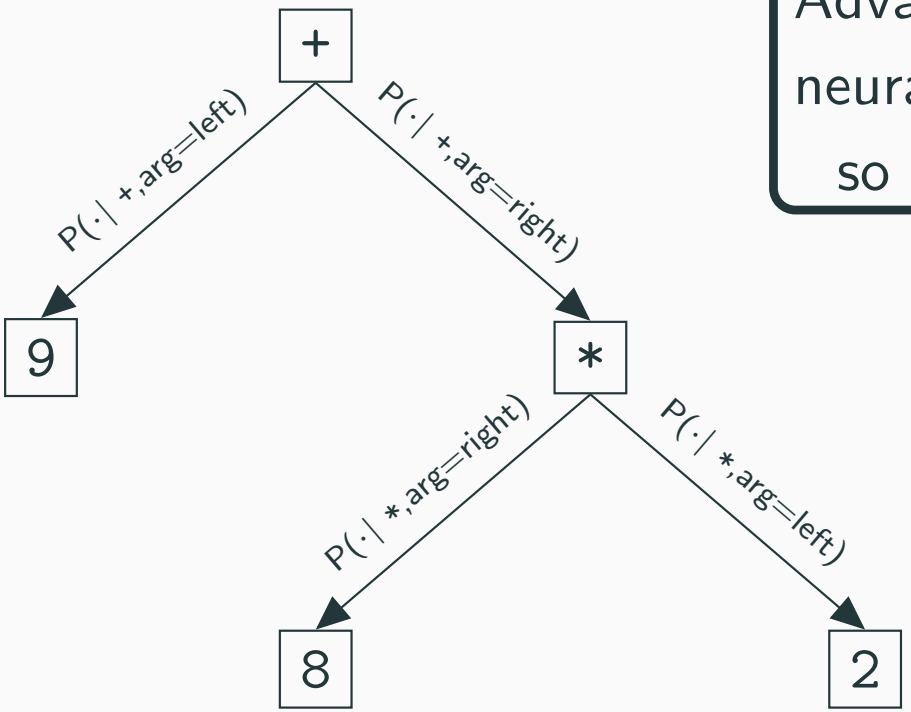
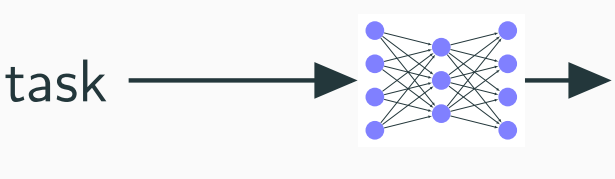


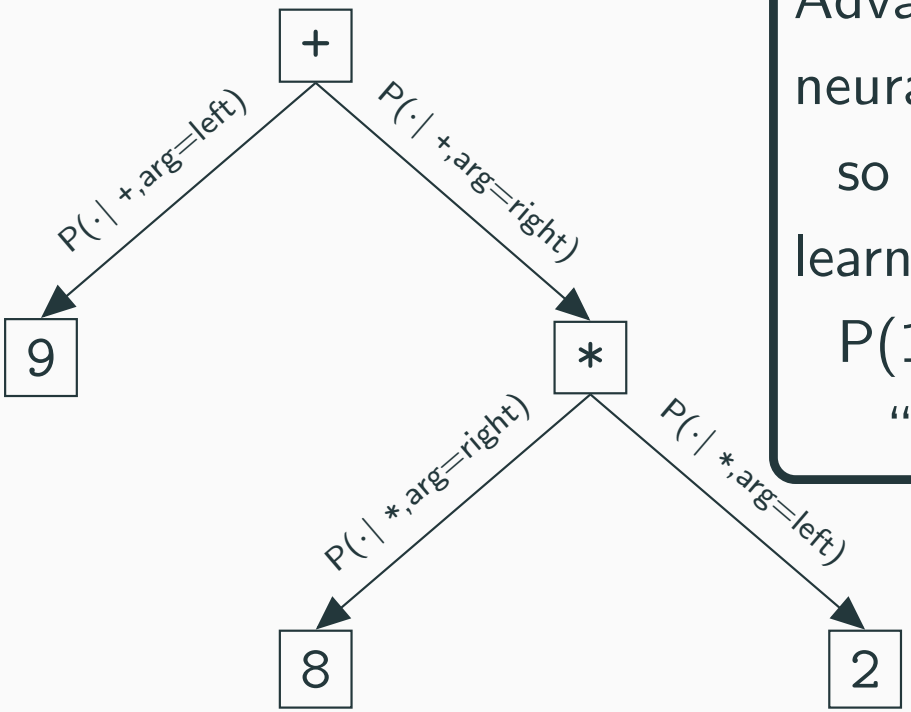
Advantages:

neural net runs once per task,

so CPU bottlenecks instead of GPU

Neural recognition model guides search





Advantages:

neural net runs once per task,

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learns to break syntactic symmetries:

$P(1|*,\text{arg}=\text{left})=0.0$

"do not multiply by one"

DreamCoder and learning to learn

learning a library

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DreamCoder Domains

List Processing

Sum List

```
[1 2 3] → 6
[4 6 8 1] → 17
```

Double

```
[1 2 3] → [2 4 6]
[4 5 1] → [8 10 2]
```

Text Editing

Abbreviate

```
Allen Newell → A.N.
Herb Simon → H.S.
```

Drop Last Three

```
shrdlu → shr
shakey → sha
```

Regexes

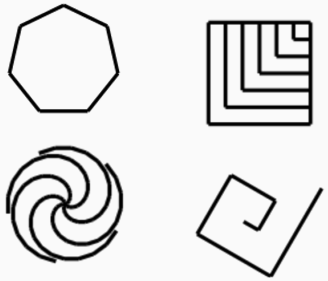
Phone numbers

```
(555) 867-5309
(650) 555-2368
```

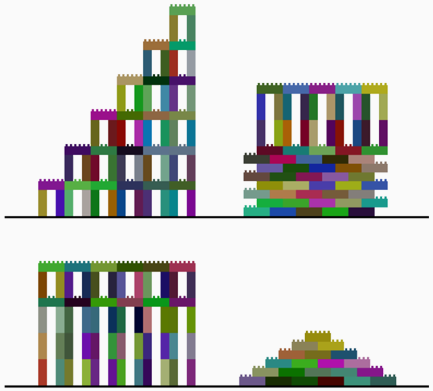
Currency

```
$100.25
$4.50
```

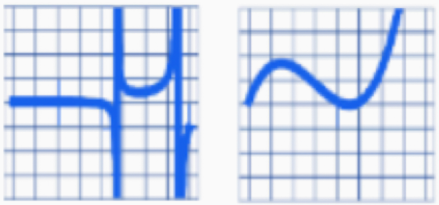
LOGO Graphics



Block Towers



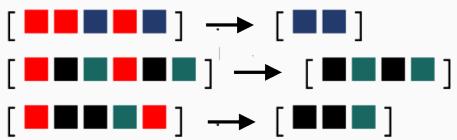
Symbolic Regression



$$y = f(x)$$

Recursive Programming

Filter Red



Physical Laws

$$\vec{a} = \frac{1}{m}\sum_i \vec{F}_i$$

$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r}|^2}\hat{r}$$

DreamCoder Domains

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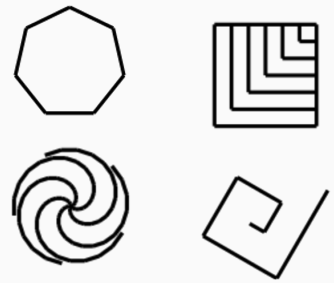

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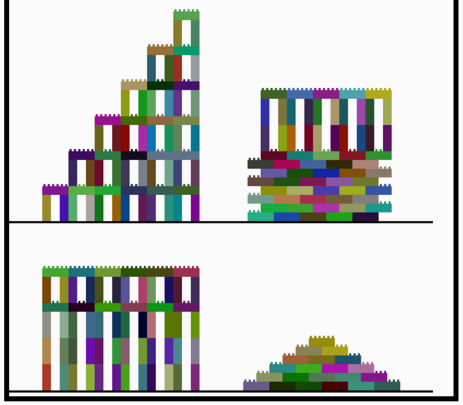

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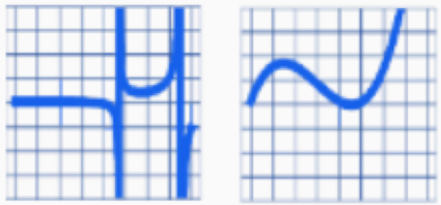

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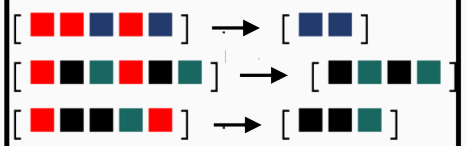
Recursive Programming

Filter Red



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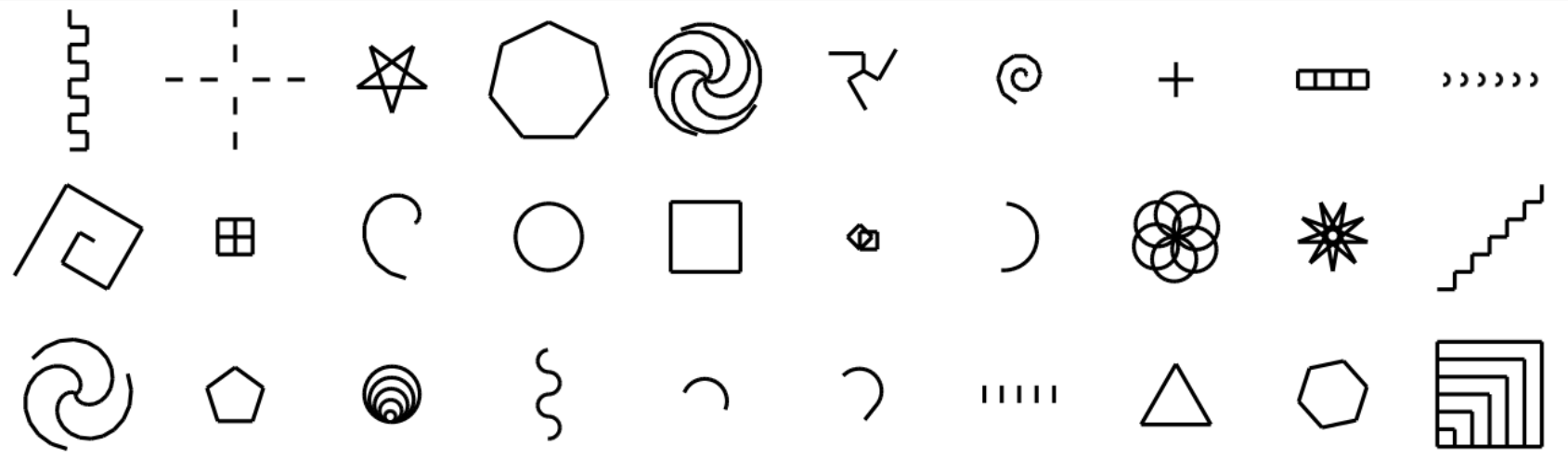
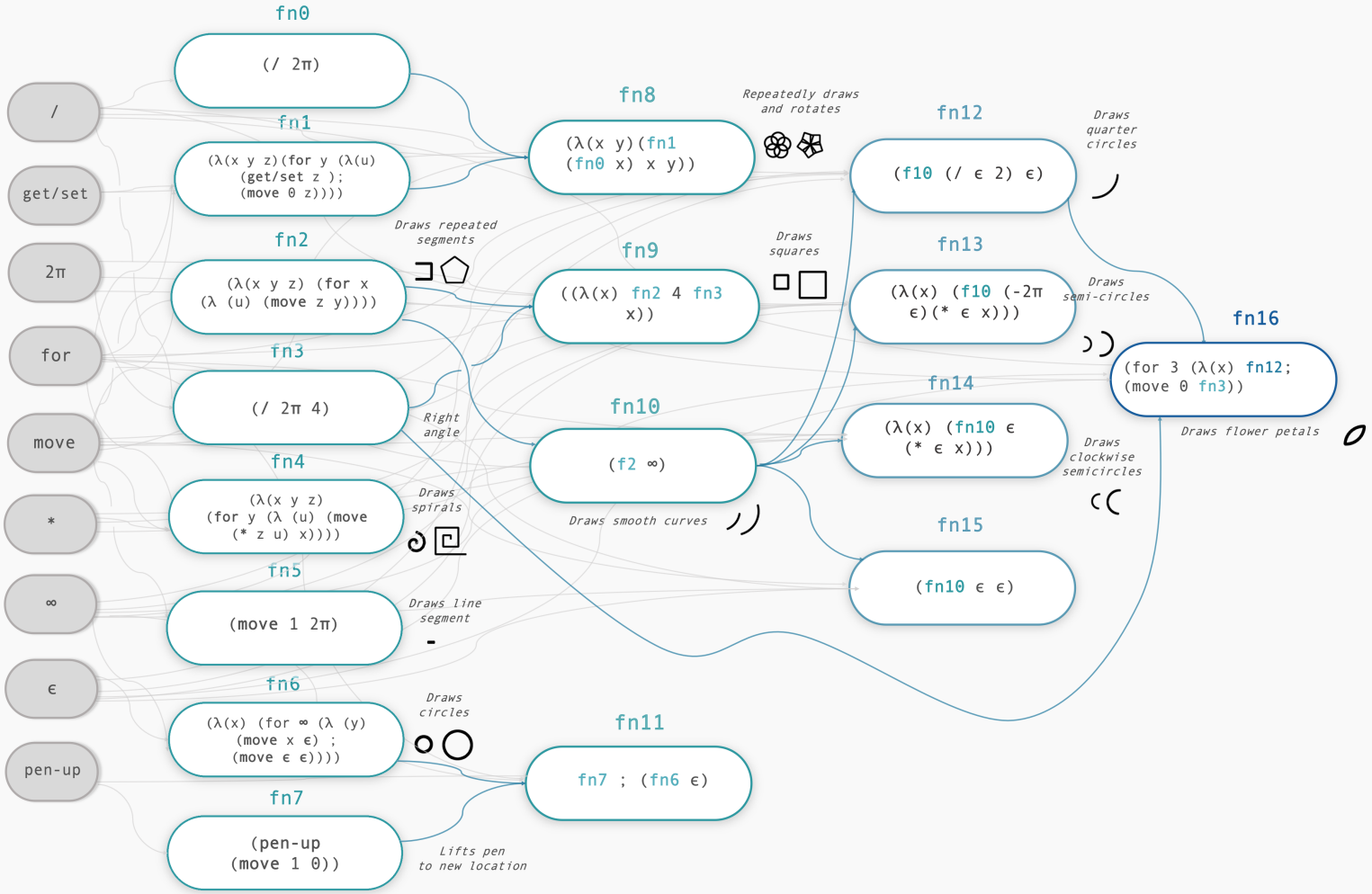
LOGO Turtle Graphics

30 out of 160 tasks

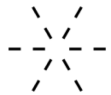


LOGO Turtle Graphics – learning an interpretable library



```
(fn8 5 (fn4 (* ϵ 2) ∞ ϵ))
```

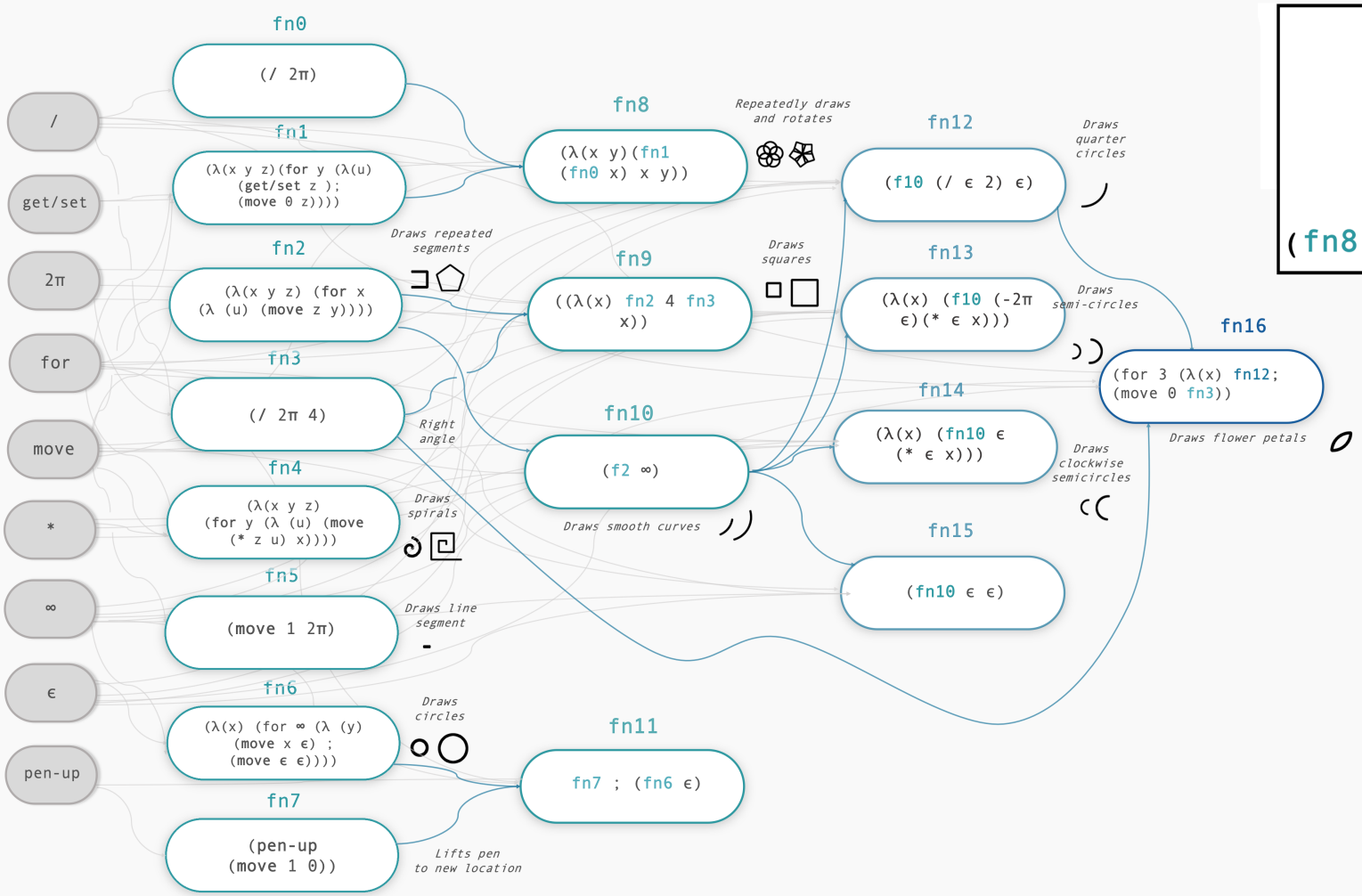
```
(for 7 (λ (x) (fn9 x)))
```

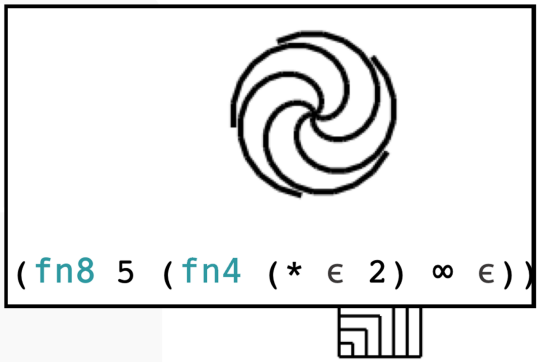


```
(fn8  6 (fn7 ; fn5 ;
fn7 ; fn5))
```

```
(move 0 (fn0 7)); fn5 ;
(fn13 4)
```

LOGO Turtle Graphics – learning an interpretable library





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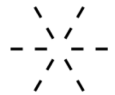
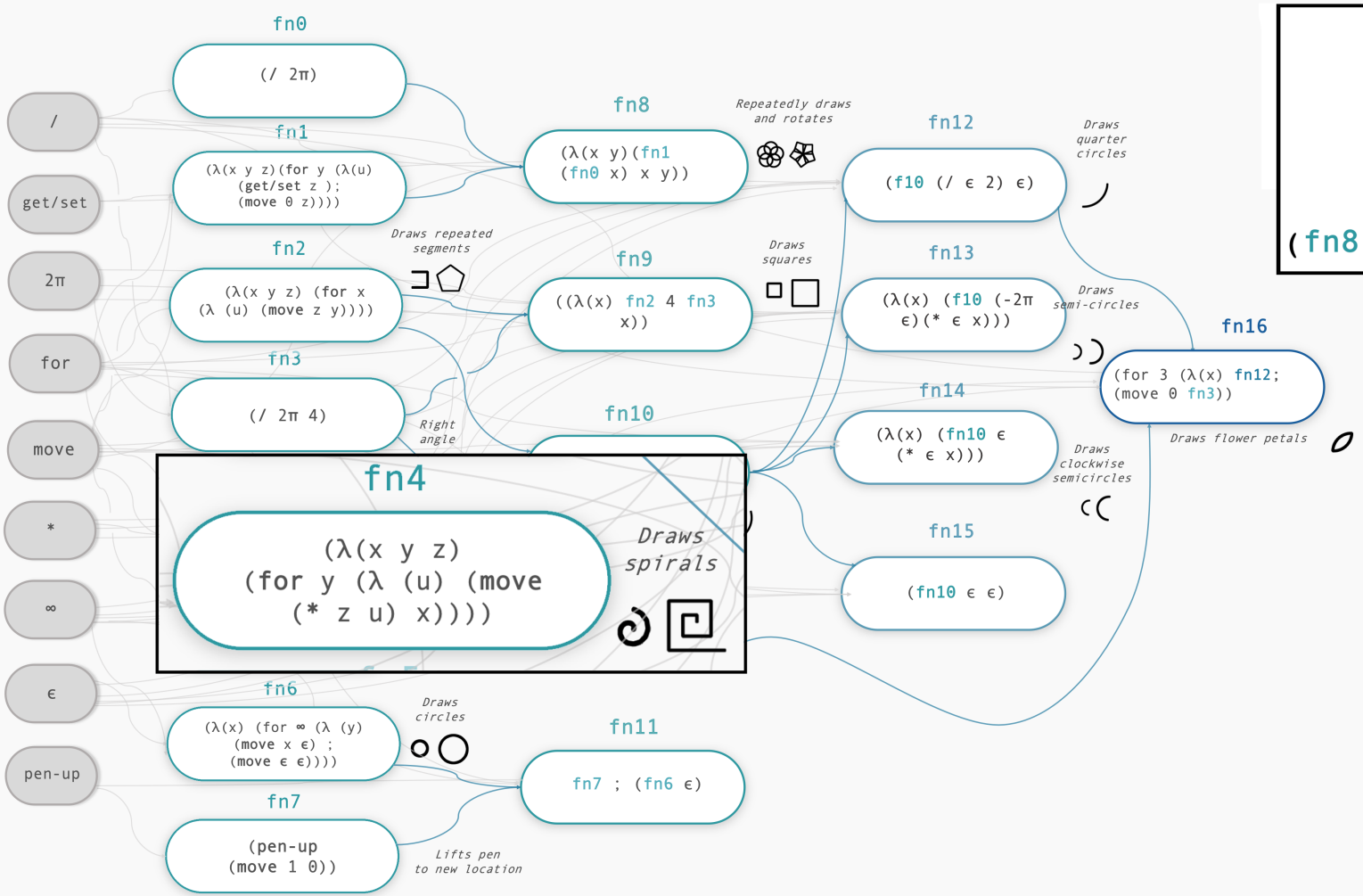
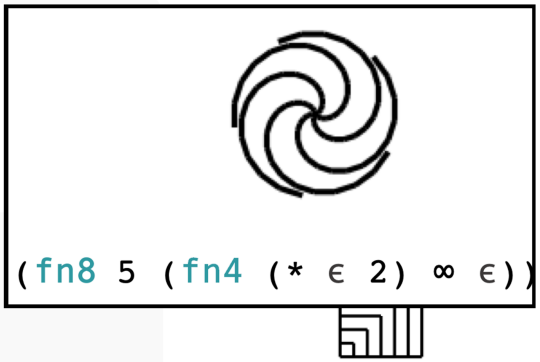


(fn8 6 (fn7 ; fn5 ; fn7 ; fn5))

(move 0 (fn0 7)); fn5 ; (fn13 4)

LOGO Turtle Graphics – learning an interpretable library



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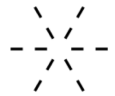

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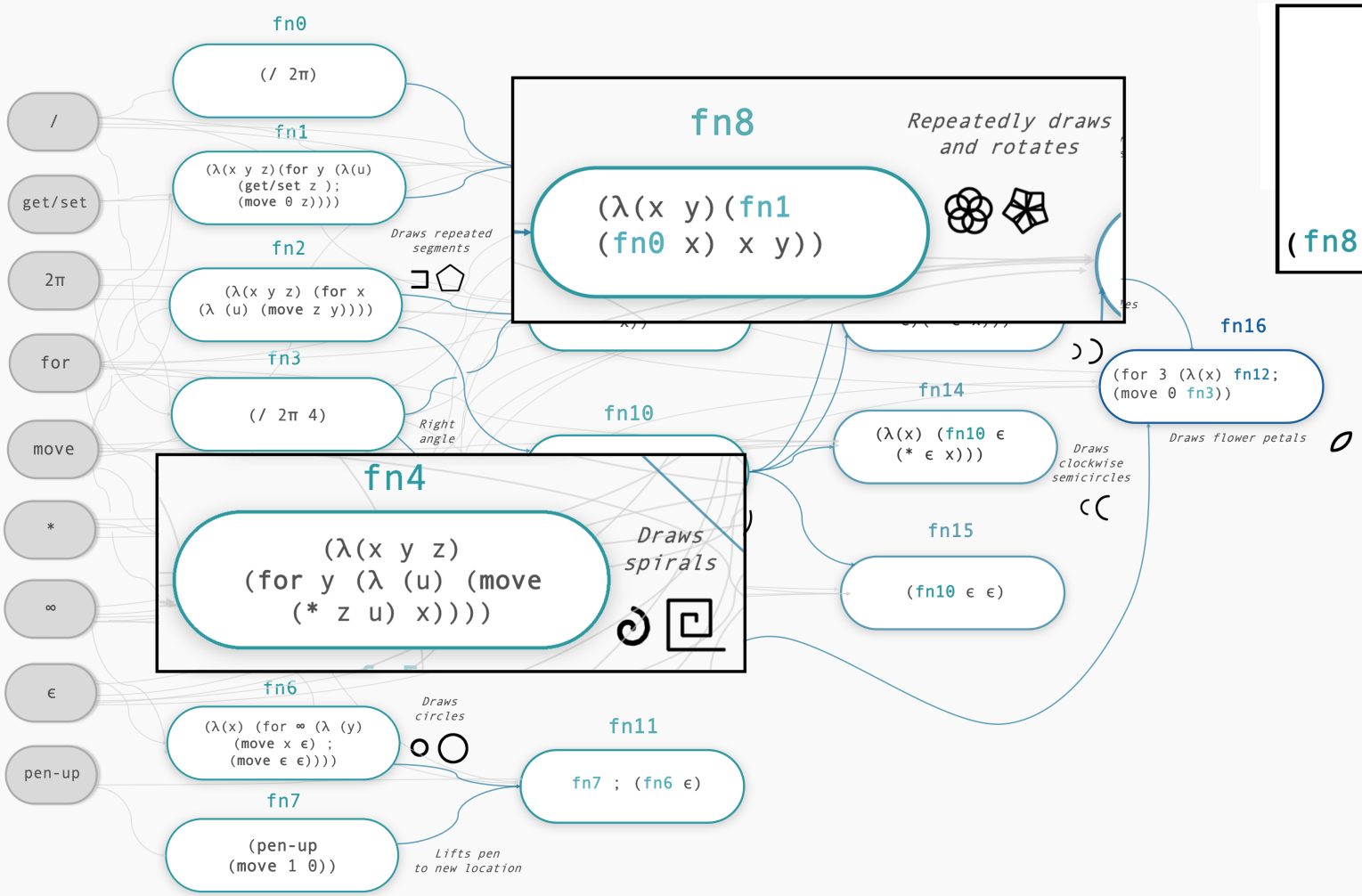
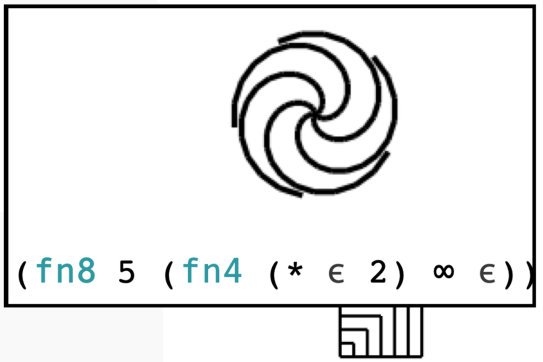
LOGO Turtle Graphics – learning an interpretable library





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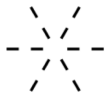

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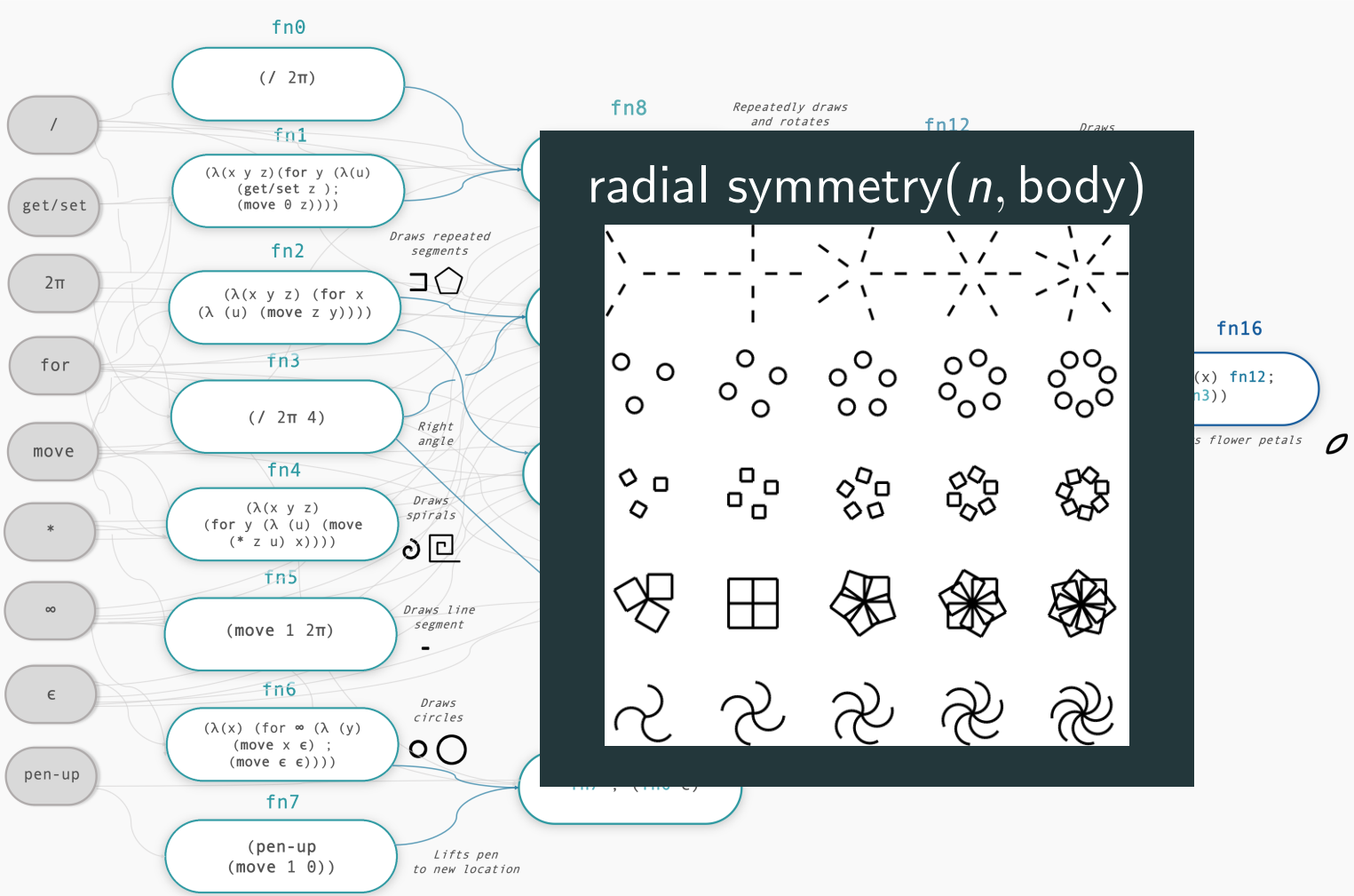
LOGO Turtle Graphics – learning an interpretable library



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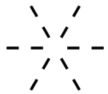

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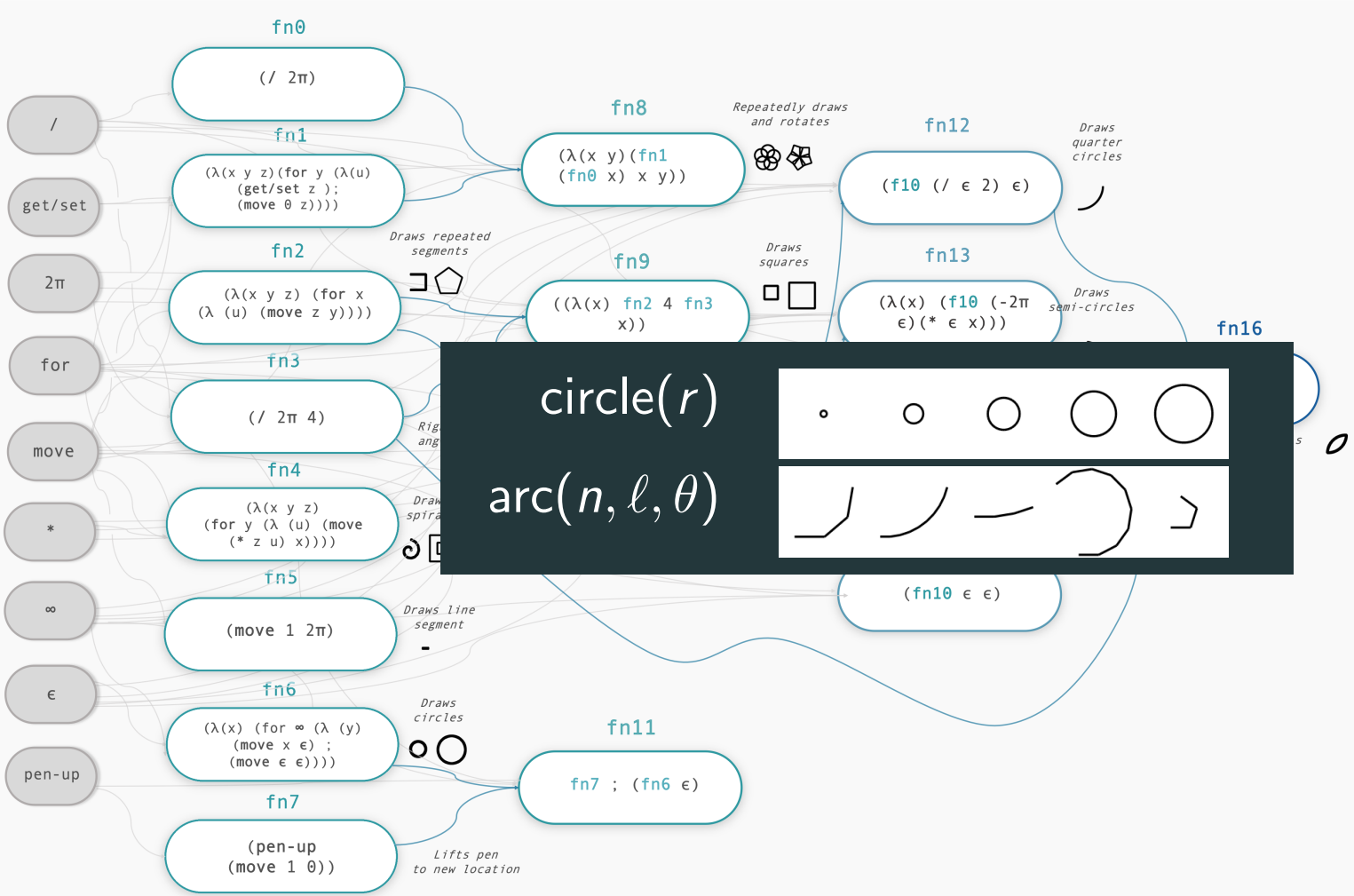
LOGO Turtle Graphics – learning an interpretable library



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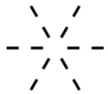

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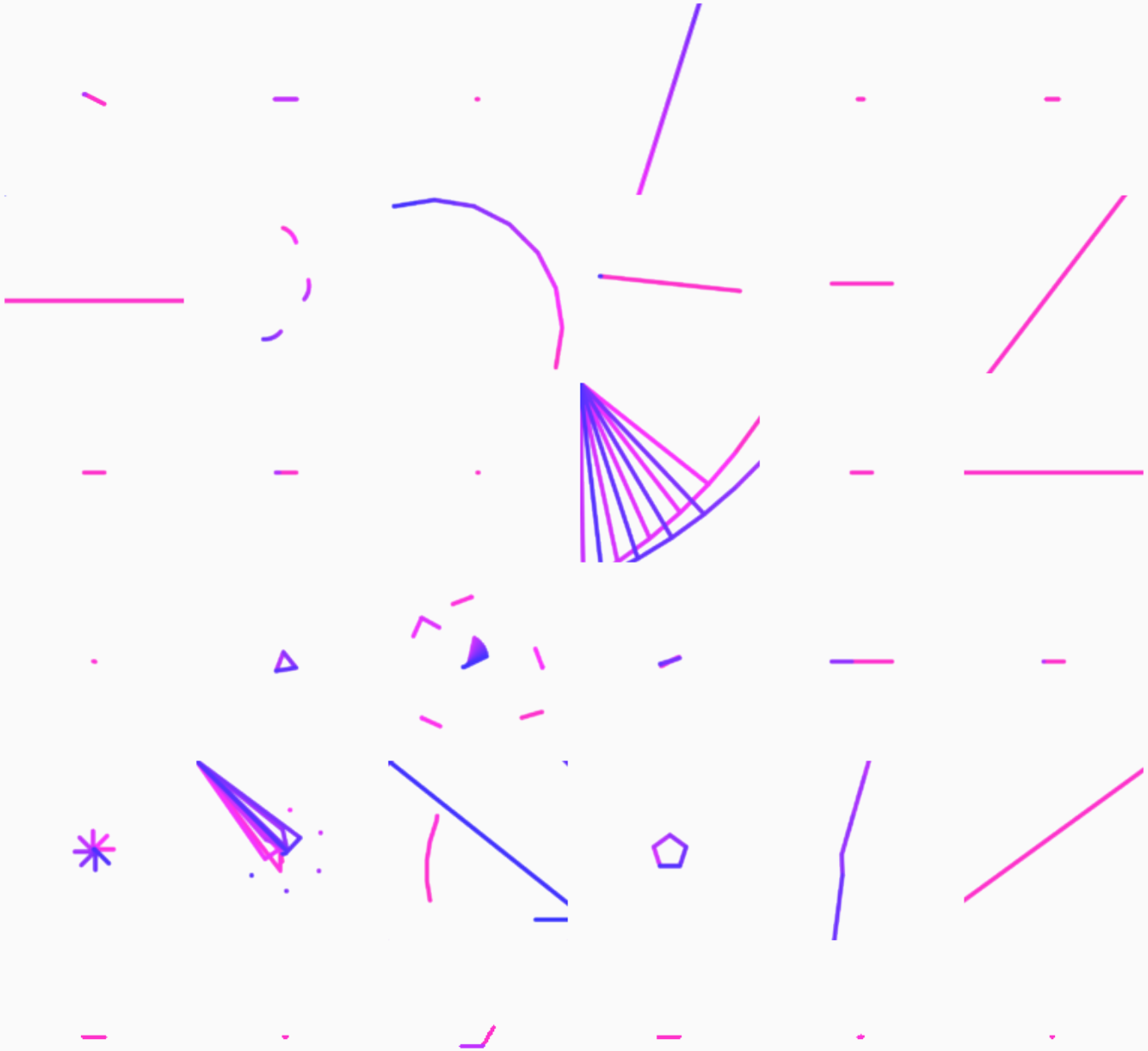
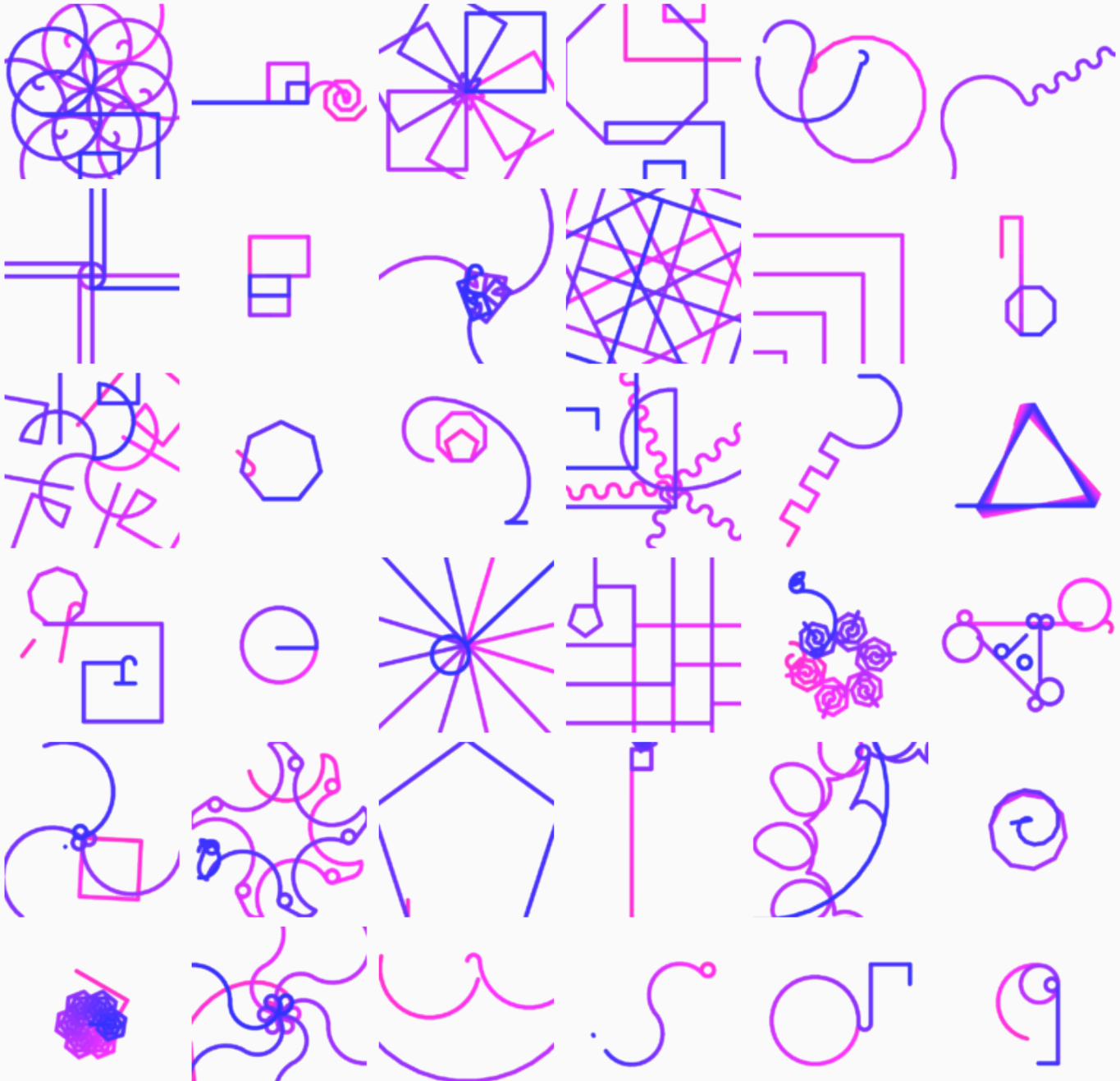
What does DreamCoder dream of? (before learning)

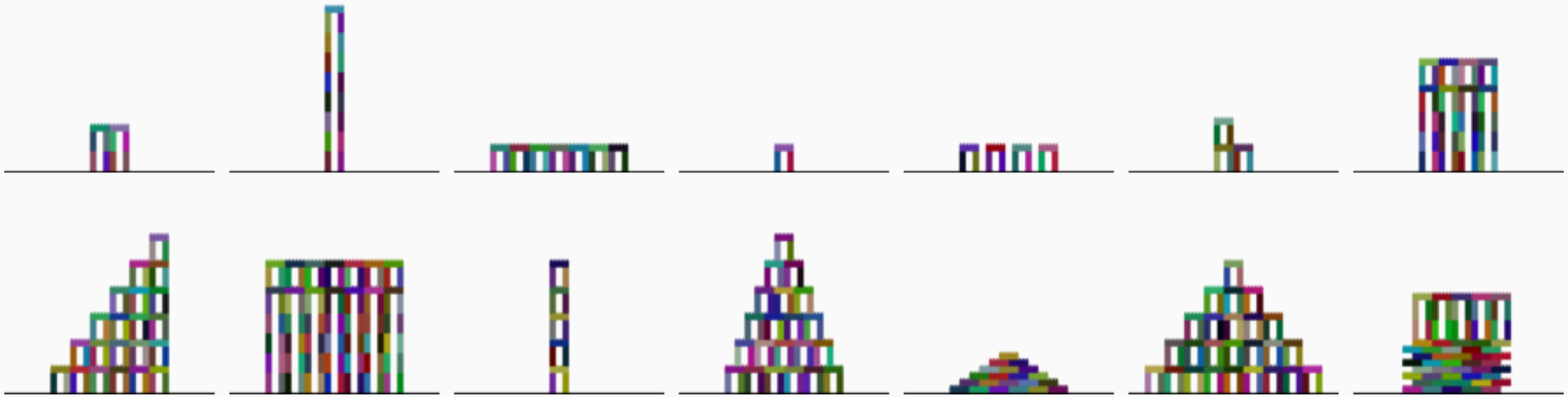


What does DreamCoder dream of? (after learning)



Planning to build towers

example tasks (112 total)



Planning to build towers

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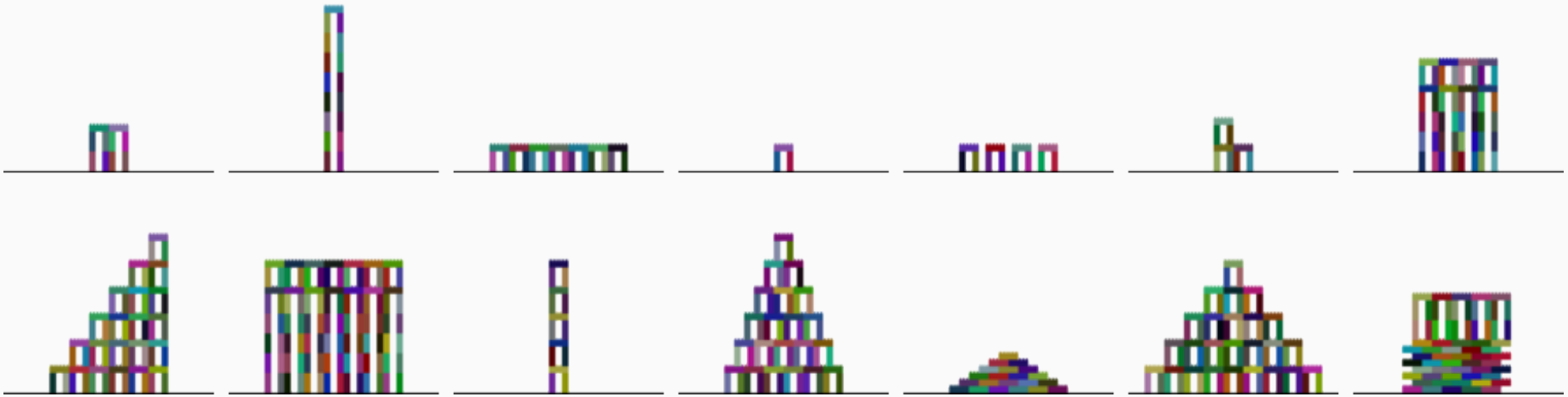
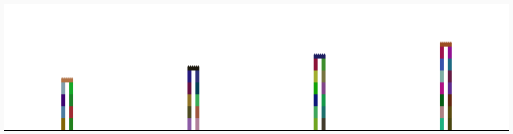
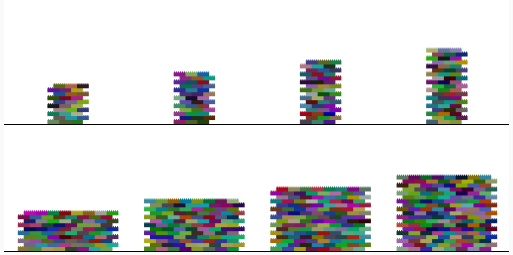


learned library routines ($\approx$ 20 total)

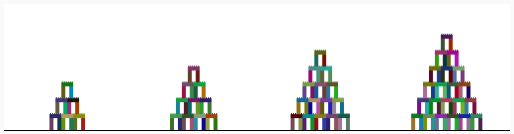
arch(h)



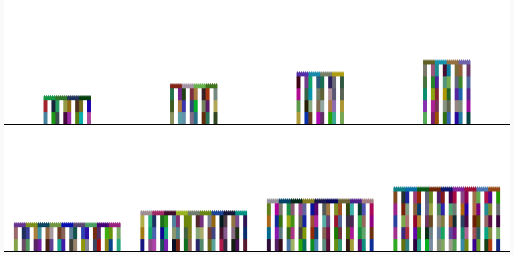
wall(w, h)



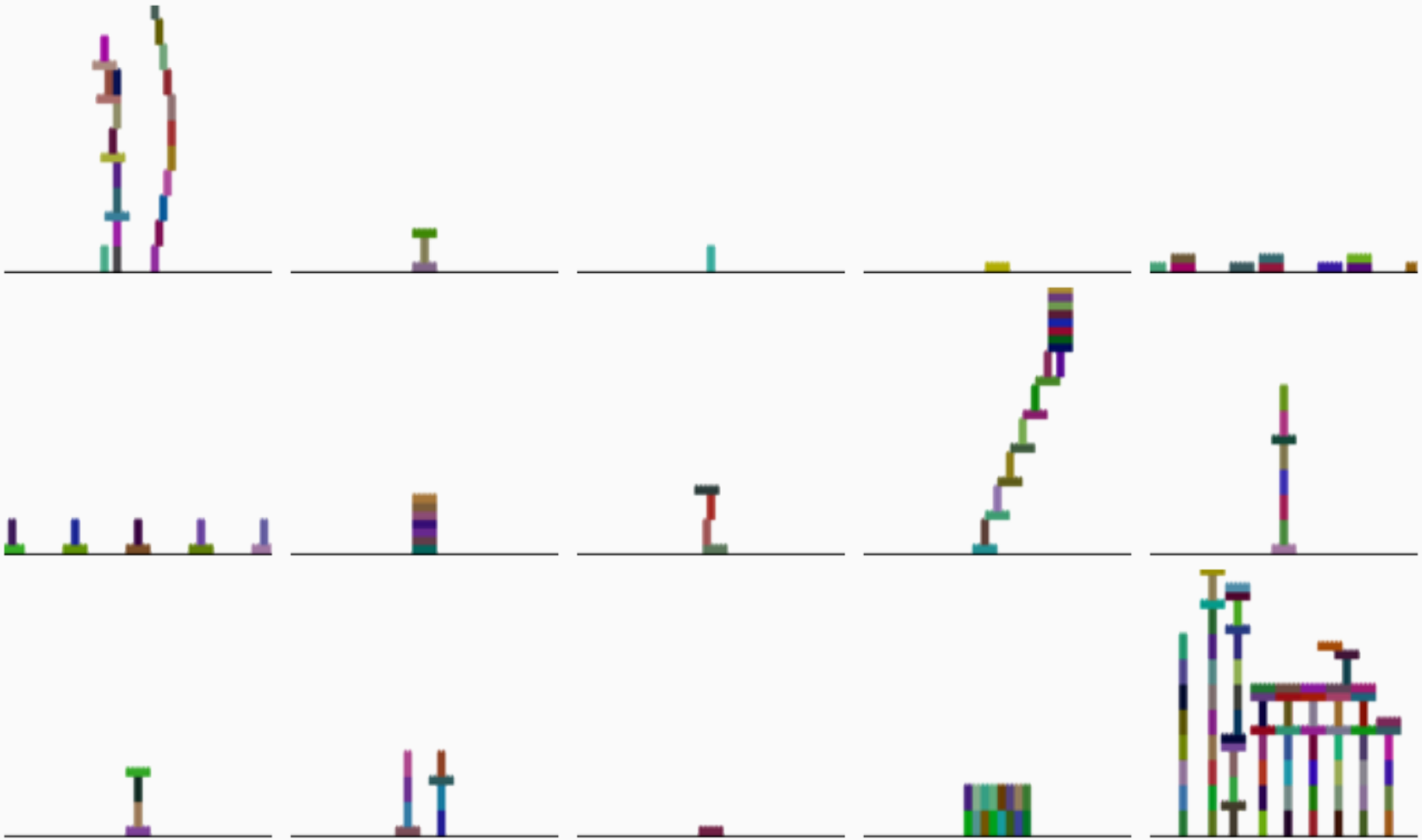
pyramid(h)



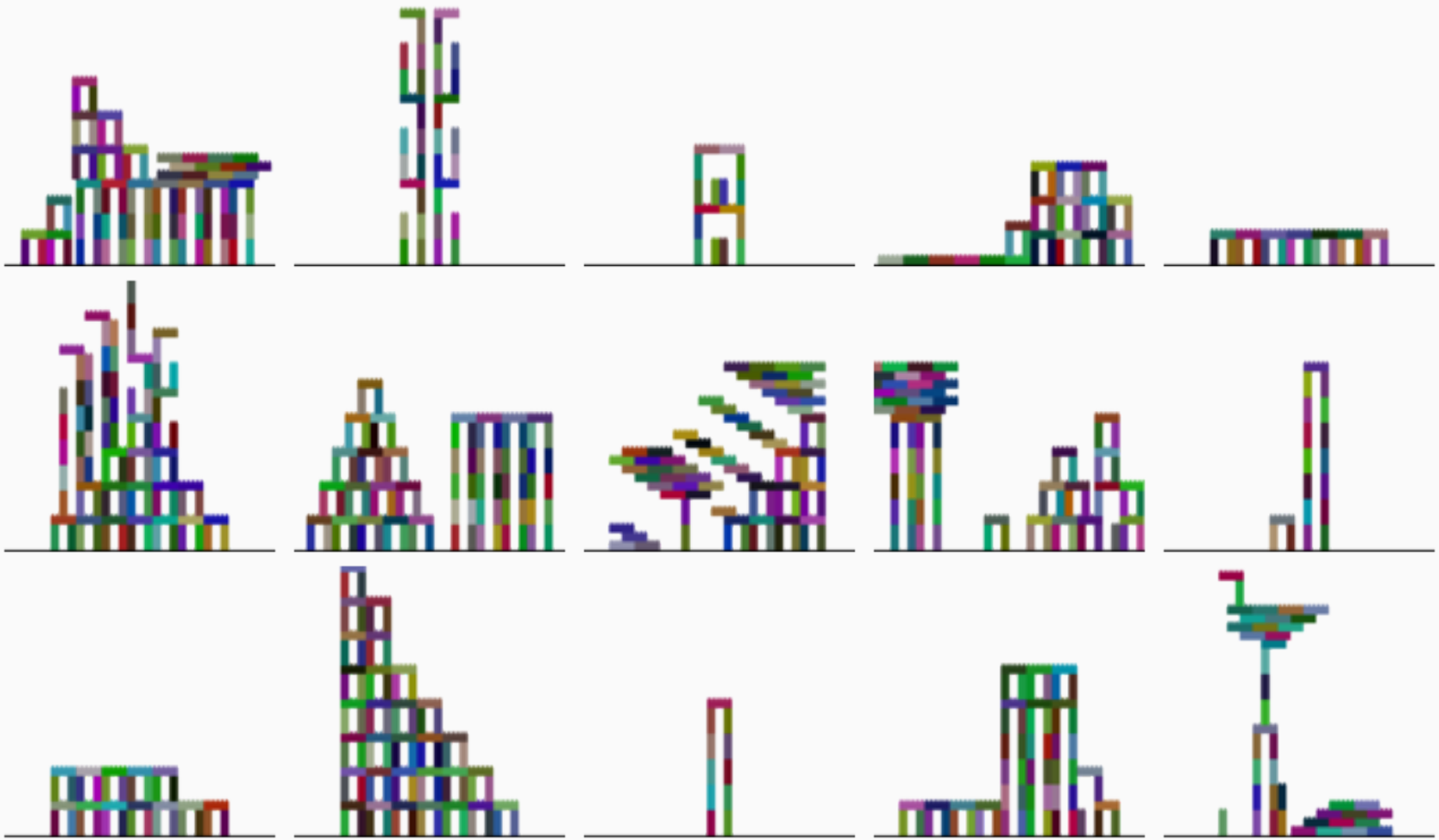
bridge(w, h)



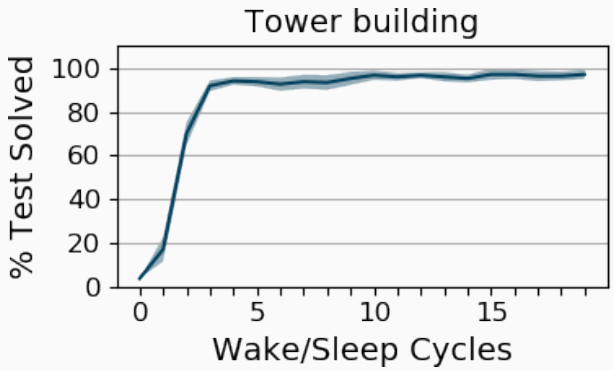
Dreams before learning



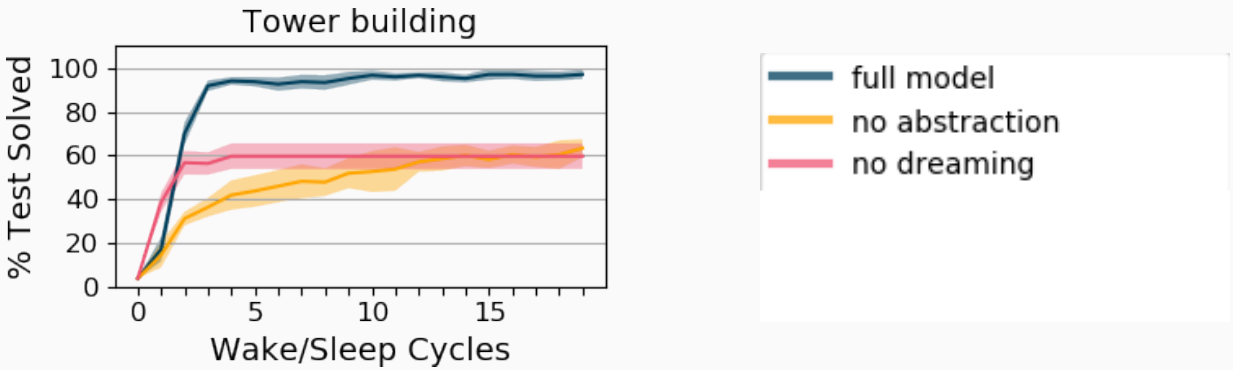
Dreams after learning



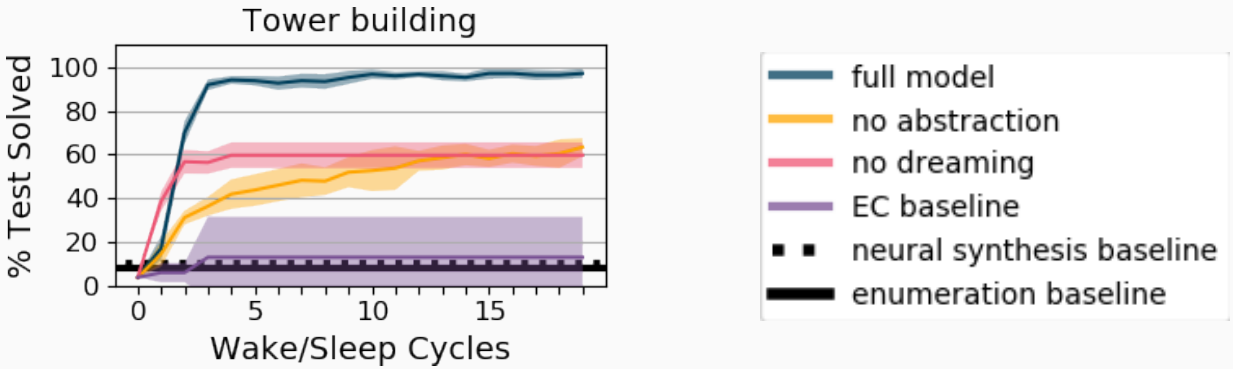
Learning dynamics



Learning dynamics

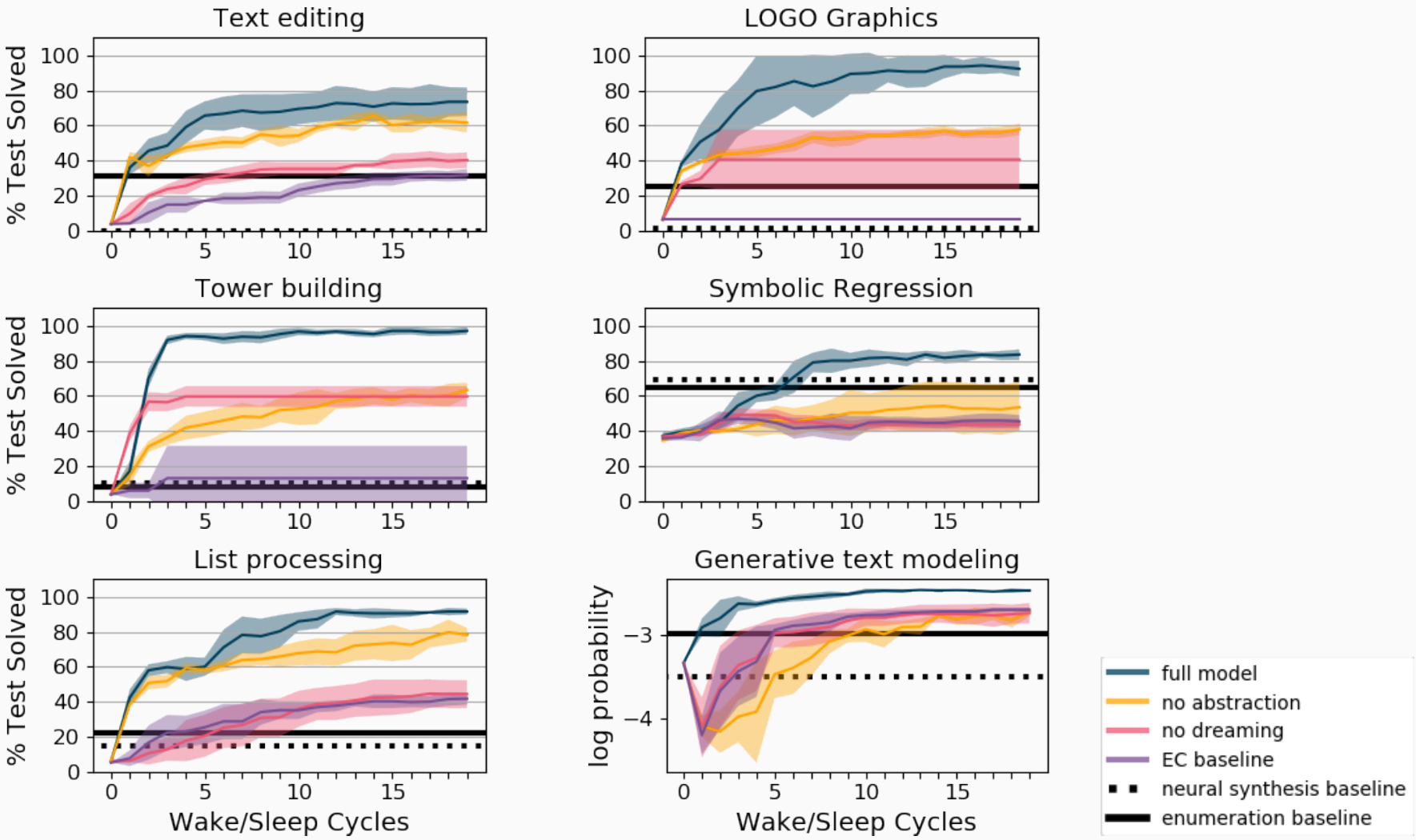


Learning dynamics



baselines: Exploration-Compression, EC [Dechter et al. 2013]
neural program synthesis, RobustFill [Devlin et al. 2017]
24 hours of brute-force enumeration

Learning dynamics



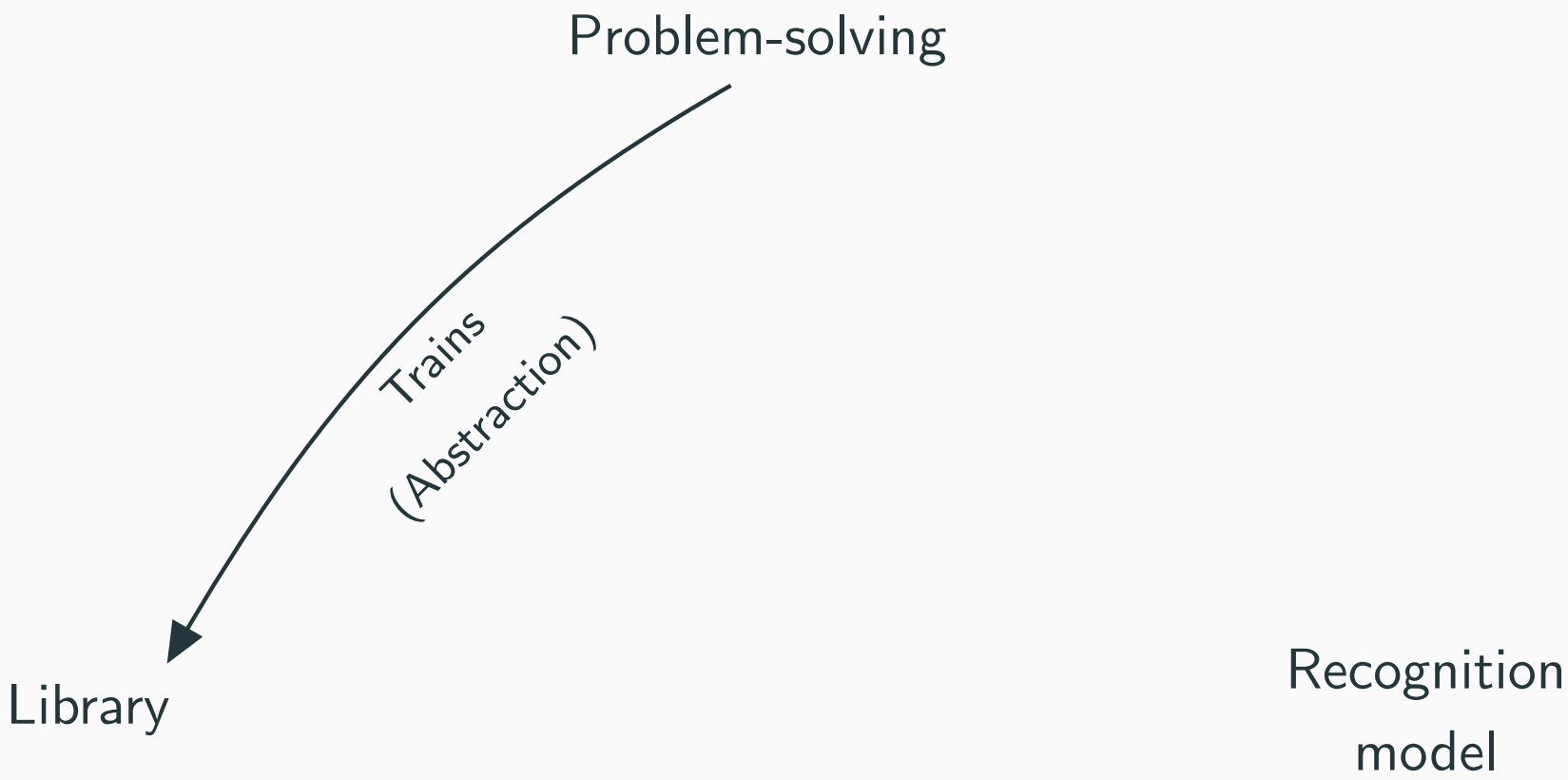
Synergy between recognition model and library learning

Problem-solving

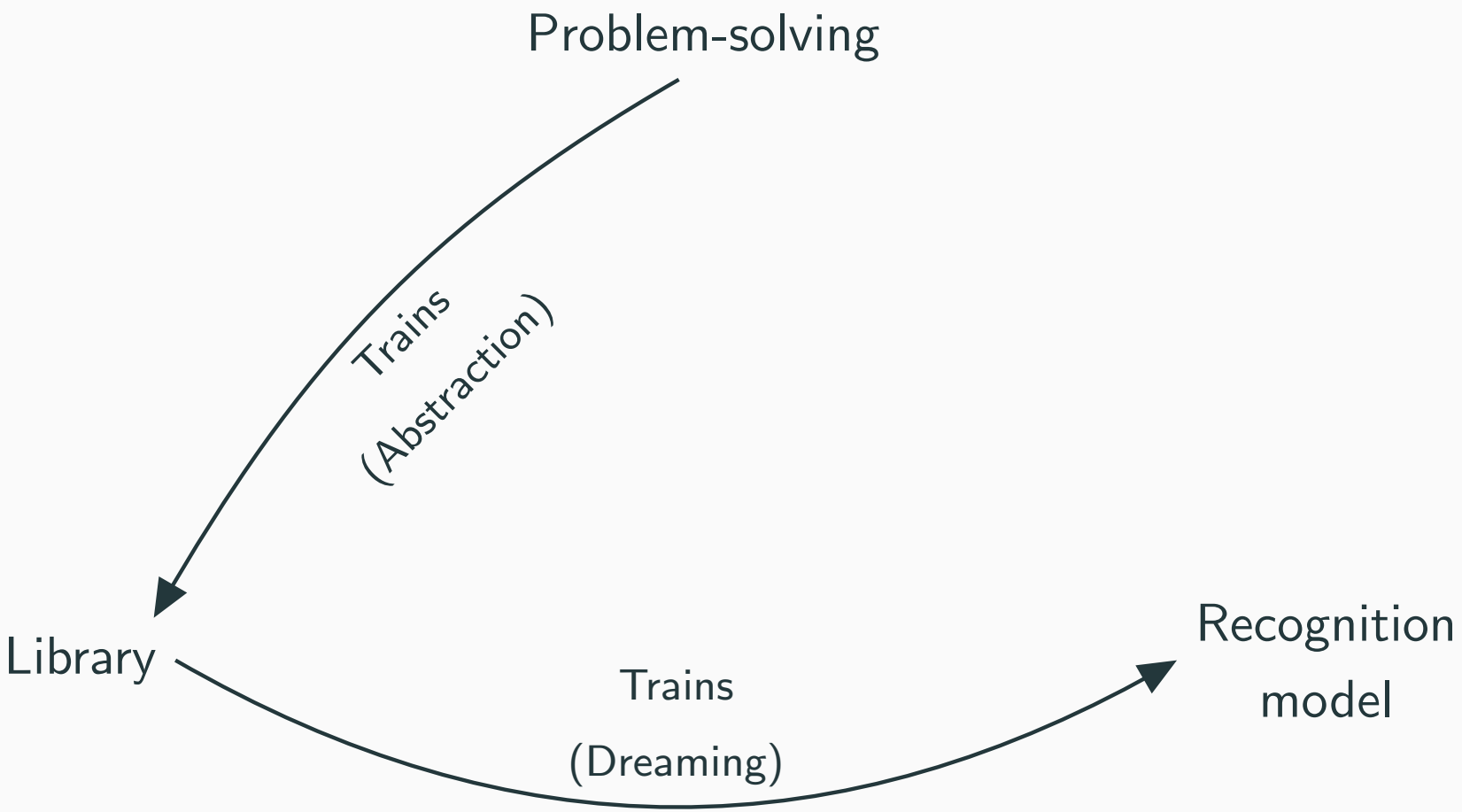
Library

Recognition model

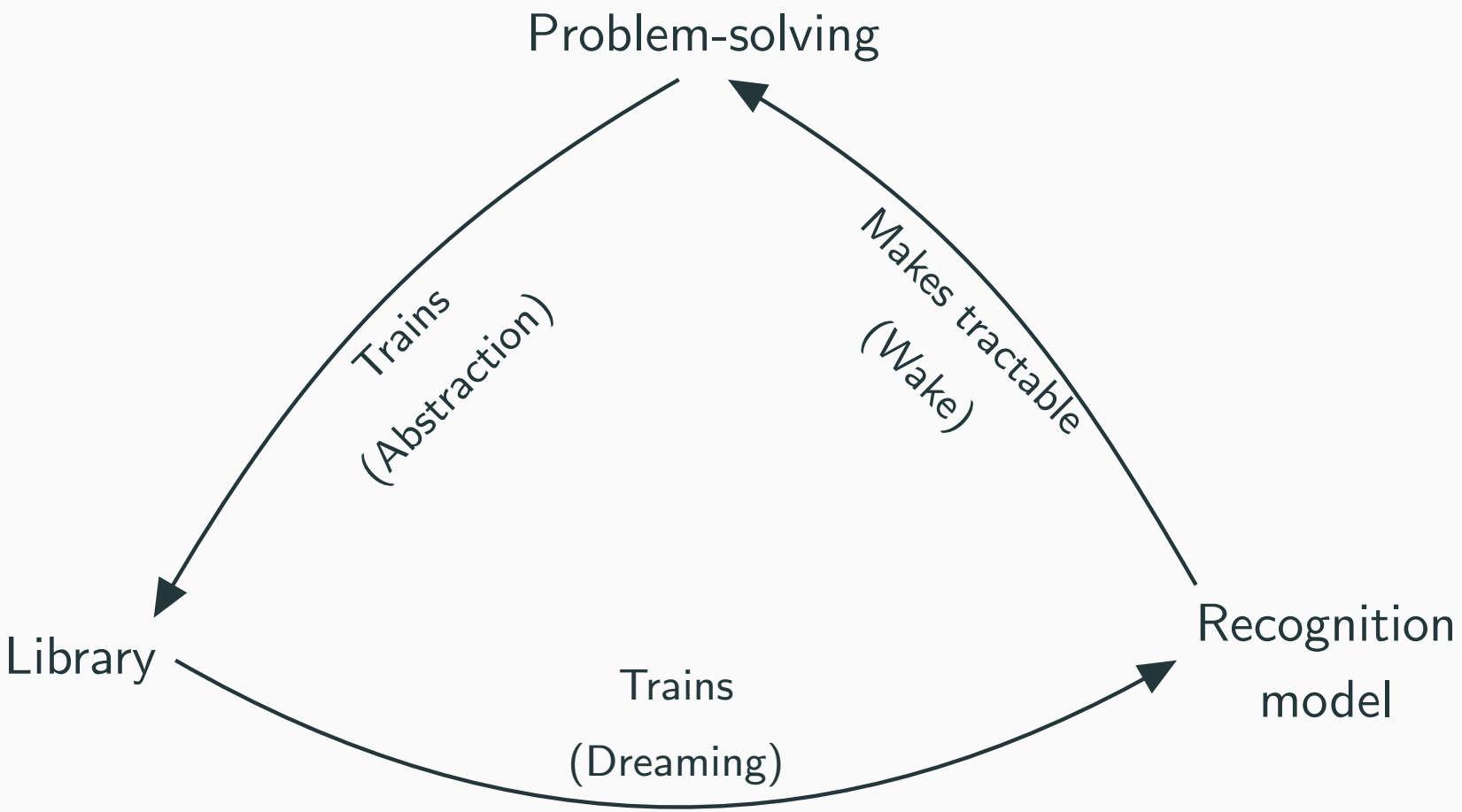
Synergy between recognition model and library learning



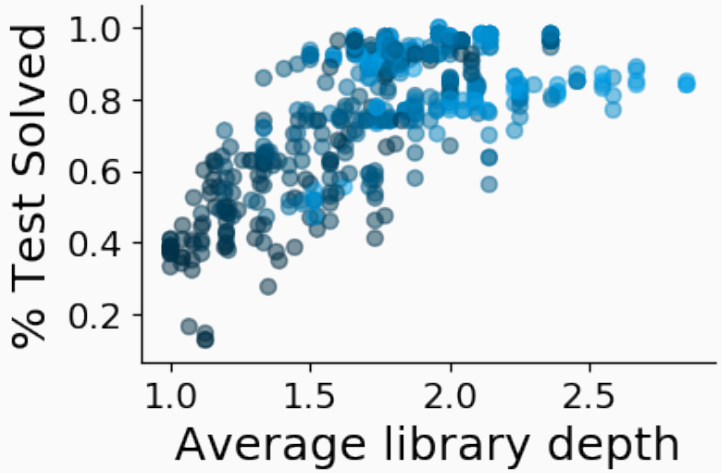
Synergy between recognition model and library learning



Synergy between recognition model and library learning



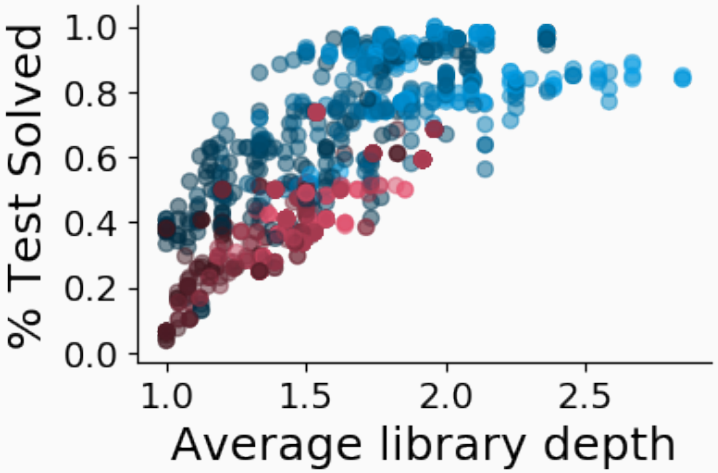
Evidence for dreaming bootstrapping better libraries

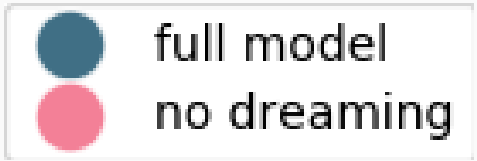


Darker: Early in learning

Brighter: Later in learning

Evidence for dreaming bootstrapping better libraries





Darker: Early in learning

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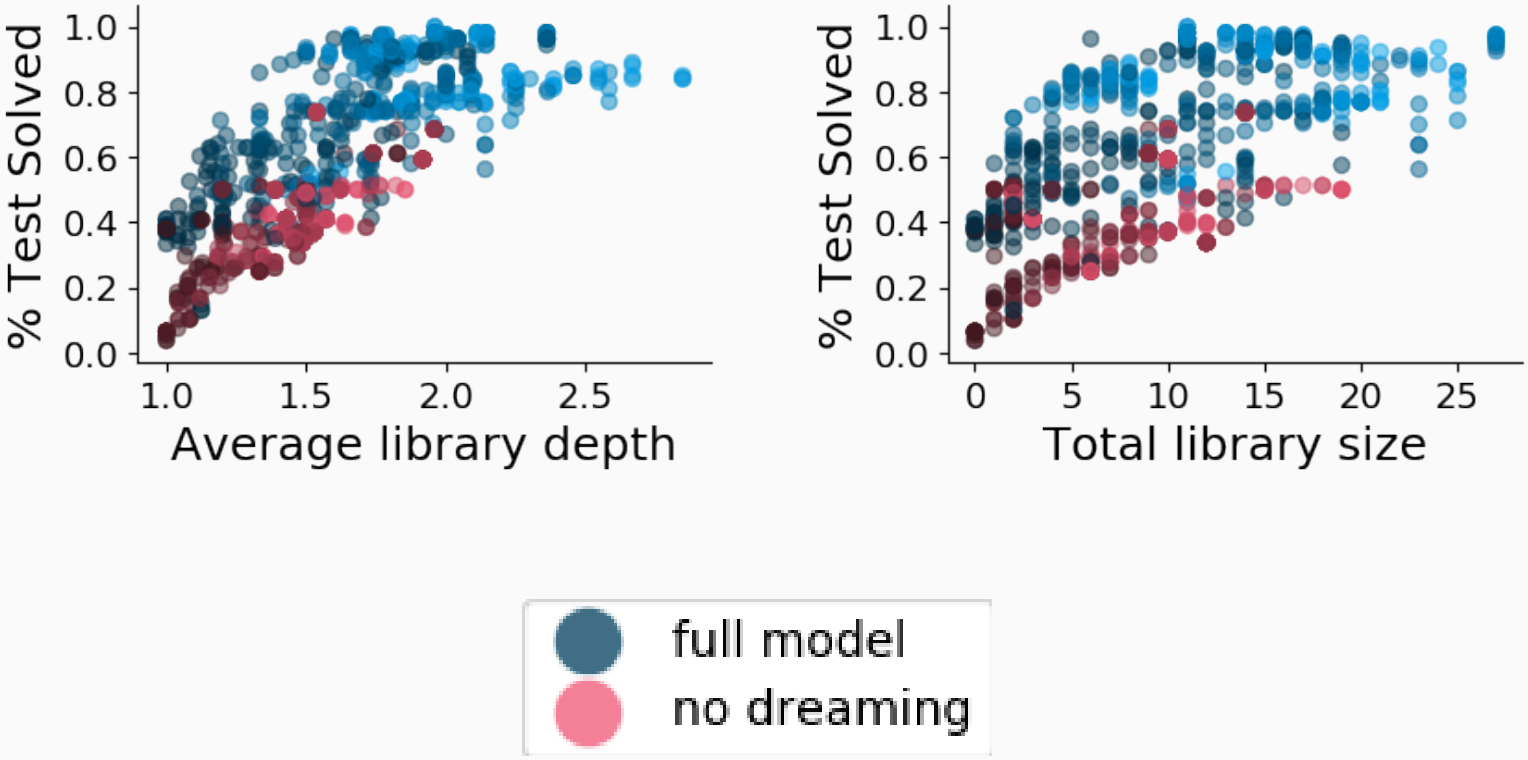
Evidence for dreaming bootstrapping better libraries



Darker: Early in learning

Brighter: Later in learning

From learning libraries, to learning languages

From learning libraries, to learning languages

modern functional programming $\rightarrow$ physics

From learning libraries, to learning languages

1950's Lisp $\rightarrow$ modern functional programming $\rightarrow$ physics

Physics Formula Sheet

Mechanics

$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$

$v = v_0 + at$

$v_x^2 - v_{x0}^2 = 2a(x - x_0)$

$\vec{a} = \frac{\Sigma \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$

$|\vec{F}_{friction}| \leq \mu |\vec{F}_{Normal}|$

$\vec{p} = m\vec{v}$

$\Delta \vec{p} = \vec{F}\Delta t$

$KE = \frac{1}{2}mv^2$

$\Delta PE = mg\Delta y$

$\Delta E = W = Fd\cos\theta$

$a_c = \frac{v^2}{r}$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$\omega = \omega_0 + \alpha t$

$T = \frac{2\pi}{\omega} = \frac{1}{f}$

$\nu = f\lambda$

$x = A\cos(2\pi f t)$

$\vec{\alpha} = \frac{\Sigma \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$

$\vec{\tau} = r \times F$

$L = I\,\omega$

$\Delta L = \tau \Delta t$

$|\vec{F}_{spring}| = k|\vec{x}|$

$PE_{spring} = \frac{1}{2}kx^2$

$T_{spring} = 2\pi\sqrt{\frac{m}{k}}$

$T_{pendulum} = 2\pi\sqrt{\frac{\ell}{g}}$

$|\vec{F}_{gravity}| = G\frac{m_1 m_2}{r^2}$

$|\vec{F}_{gravity}| = m\vec{g}$

$PE_{gravity} = -G\frac{m_1 m_2}{r}$

$\rho = \frac{m}{V}$

$KE = \frac{1}{2}I\,\omega^2$

Electricity

$|\vec{F}_E| = k\left|\frac{q_1 q_2}{r^2}\right|$

$\Delta V = IR$

$R = \frac{\rho \ell}{A}$

$I = \frac{\Delta q}{\Delta t}$

$P = I\Delta V$

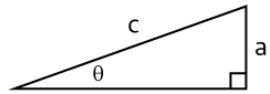
$R_{series} = R_1 + R_2 + \ldots + R_n$

$\frac{1}{R_{Parallel}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$

Geometry

Rectangle	$A = bh$	Rectangular Solid	$V = \ell wh$	Triangle	$A = \frac{1}{2}bh$
Circle	$A = \pi r^2$ $C = 2\pi r$	Cylinder	$V = \pi r^2 \ell$ $S = 2\pi r\ell + 2\pi r^2$	Sphere	$V = \frac{4}{3}\pi r^3$ $S = 4\pi r^2$

Trigonometry



$c^2 = a^2 + b^2$

$\sin\theta = \frac{a}{c}$ $\cos\theta = \frac{b}{c}$ $\tan\theta = \frac{a}{b}$

Variables

a = acceleration
A = amplitude
A = Area
b = base length
C = circumference
d = distance
E = energy
f = frequency
F = force
h = height
I = current
I = rotational inertia
KE = kinetic energy
k = spring constant
L = angular momentum
ℓ = length
m = mass
P = power
p = momentum
q = charge
r = radius
R = resistance
S = surface area
T = period
t = time
PE = potential energy
V = electric potential
V = volume
v = velocity
w = width
W = work
x = position
y = height
α = angular acceleration
λ = wavelength
μ = coefficient of friction

Growing languages for vector algebra and physics

Initial Primitives

- map
- zip
- cons
- empty
- cdr
- power
- fold
- car
- +
- -
- *
- /
- 0
- 1
- π

Physics Equations

Newton's Second Law

$$\vec{a} = \frac{1}{m}\sum_i \vec{F}_i$$

Parallel Resistors

$$R_{total} = \left(\Sigma_i \frac{1}{R_i}\right)^{-1}$$

Work

$$U = \vec{F} \cdot \vec{d}$$

Force in a Magnetic Field

$$|\vec{F}| = q|\vec{v} \times \vec{B}|$$

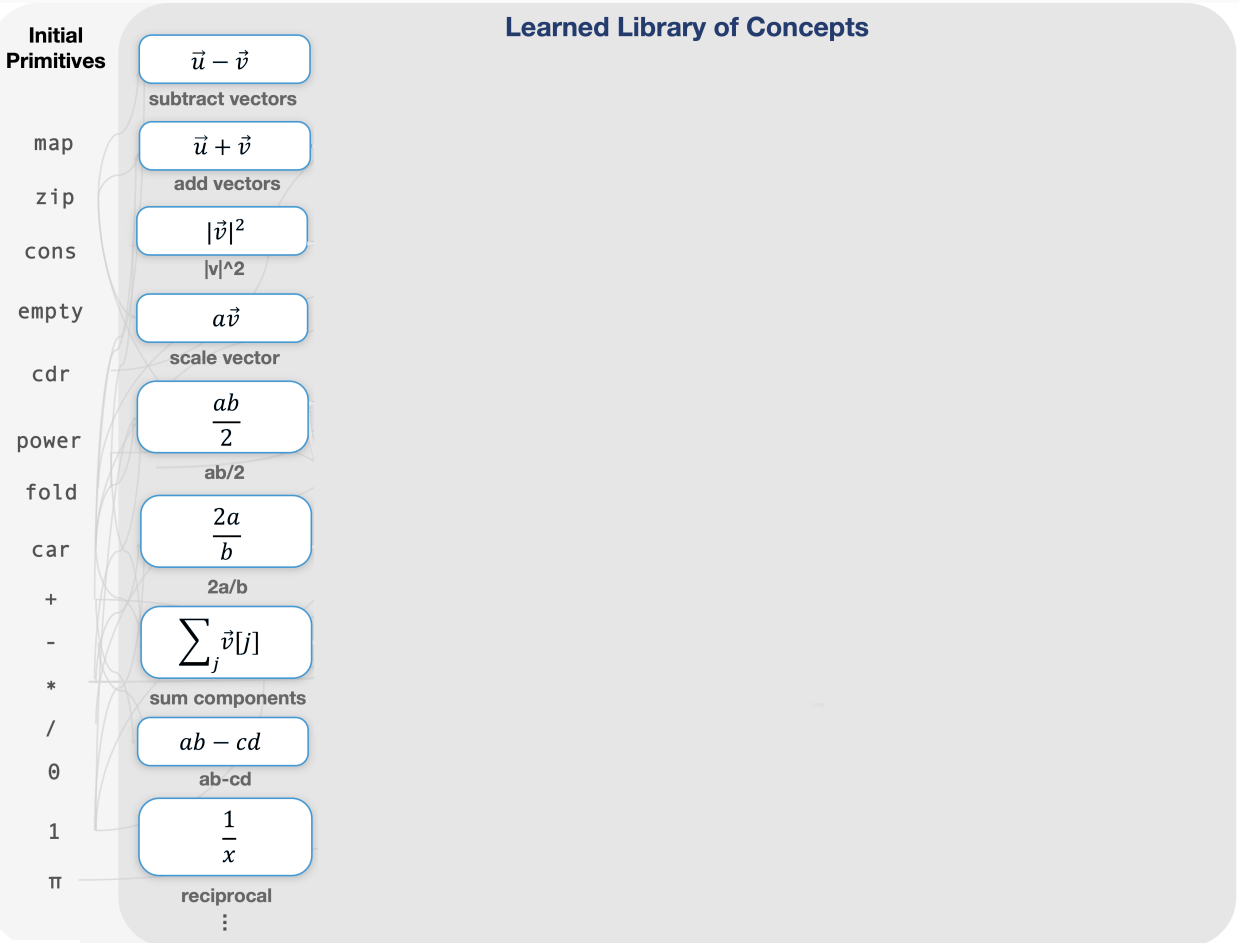
Kinetic Energy

$$KE = \frac{1}{2}m|\vec{v}|^2$$

Coulomb's Law

$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r_1} - \vec{r_2}|^2}\widehat{r_1 - r_2}$$

Growing languages for vector algebra and physics



Physics Equations

Newton's Second Law

$$\vec{a} = \frac{1}{m}\sum_i \vec{F}_i$$

Parallel Resistors

$$R_{total} = \left(\Sigma_i \frac{1}{R_i}\right)^{-1}$$

Work

$$U = \vec{F} \cdot \vec{d}$$

Force in a Magnetic Field

$$|\vec{F}| = q|\vec{v} \times \vec{B}|$$

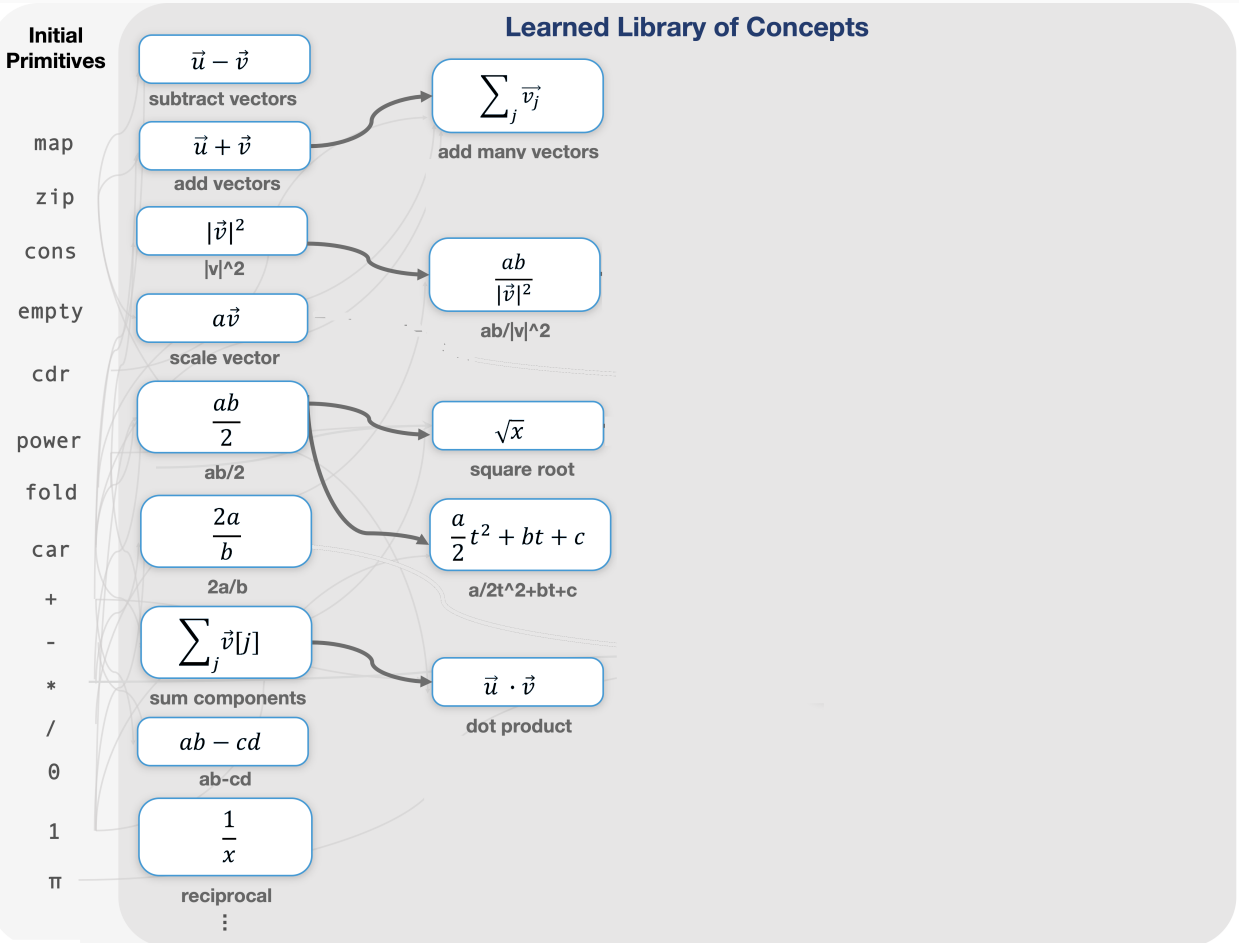
Kinetic Energy

$$KE = \frac{1}{2}m|\vec{v}|^2$$

Coulomb's Law

$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r_1} - \vec{r_2}|^2}\widehat{r_1 - r_2}$$

Growing languages for vector algebra and physics



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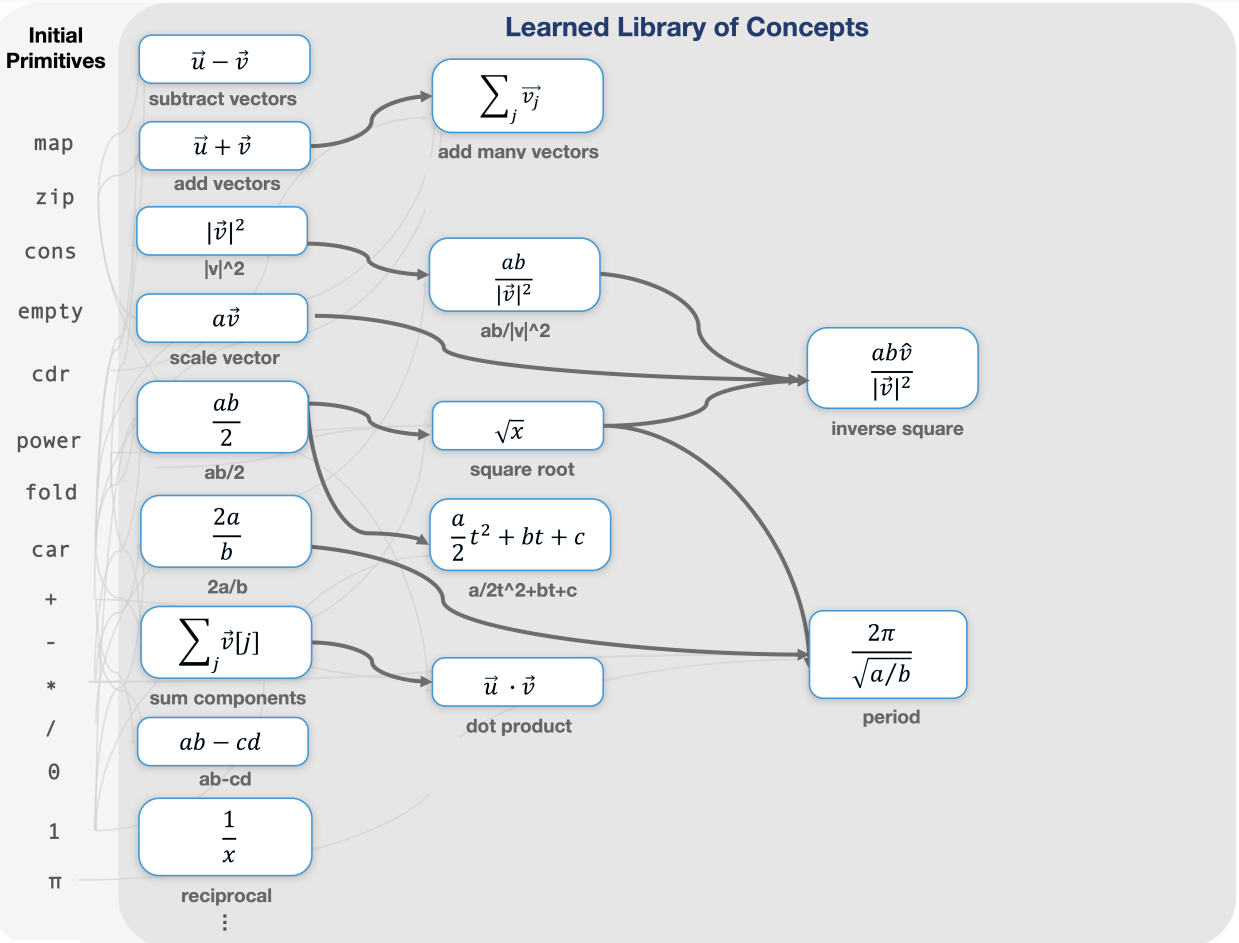

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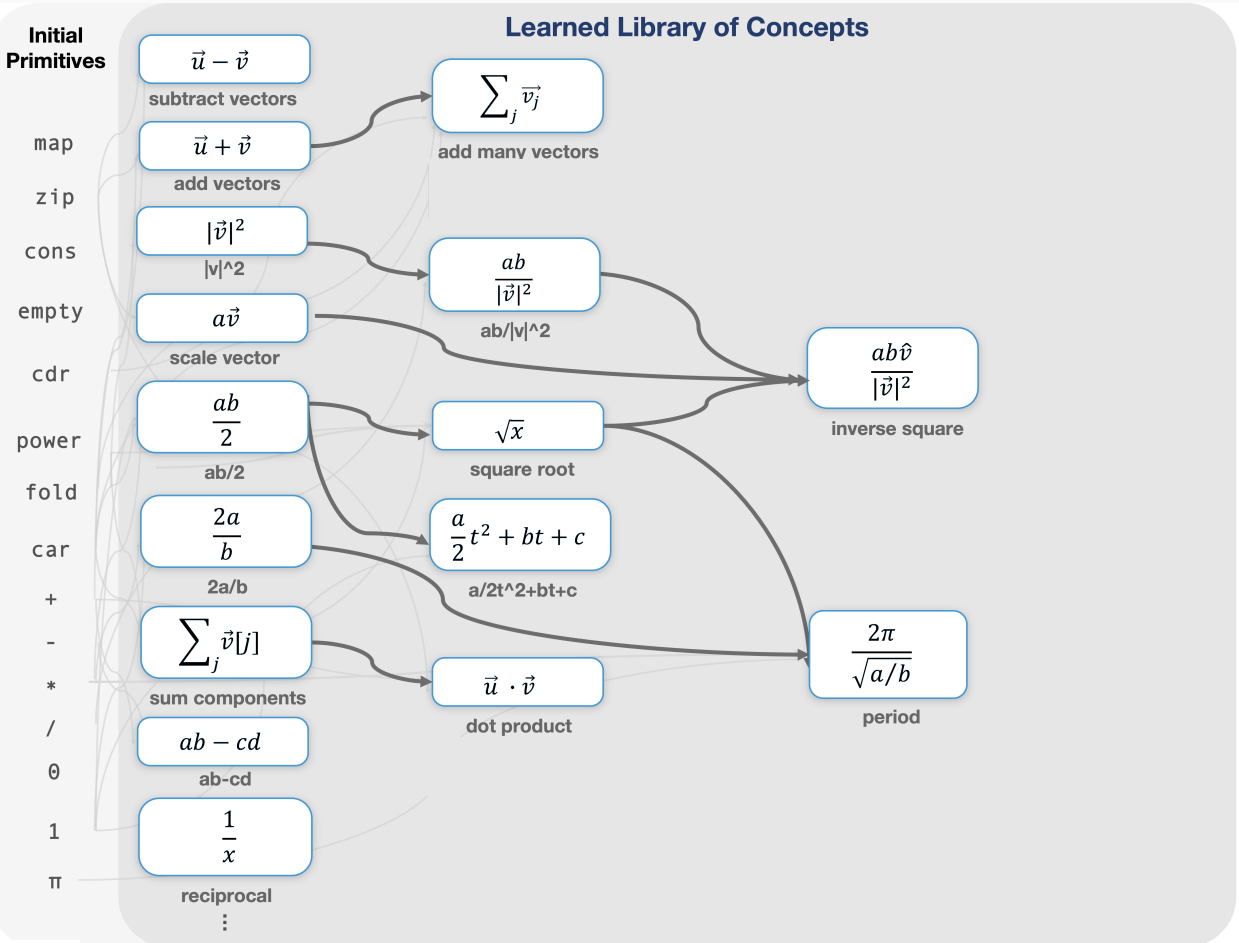

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```
(scale-vector(reciprocal m)
(add-many-vectors Fs))
```

Parallel Resistors

$$R_{total}=\left(\Sigma_i \frac{1}{R_i}\right)^{-1}$$

```
(reciprocal (sum-components
(map (λ(r) (reciprocal r))
Rs)))
```

Work

$$U=\vec{F}\cdot\vec{d}$$

```
(dot-product F d)
```

Force in a Magnetic Field

$$|\vec{F}|=q|\vec{v}\times\vec{B}|$$

```
(* q (ab-cd v_x b_y v_y b_x))
```

Kinetic Energy

$$KE=\frac{1}{2}m|\vec{v}|^2$$

```
(ab/2 m (|v|^2 v))
```

Coulomb's Law

$$\vec{F}\propto\frac{q_1 q_2}{|\vec{r_1}-\vec{r_2}|^2}\widehat{r_1-r_2}$$

```
(inverse-square q_1 q_2
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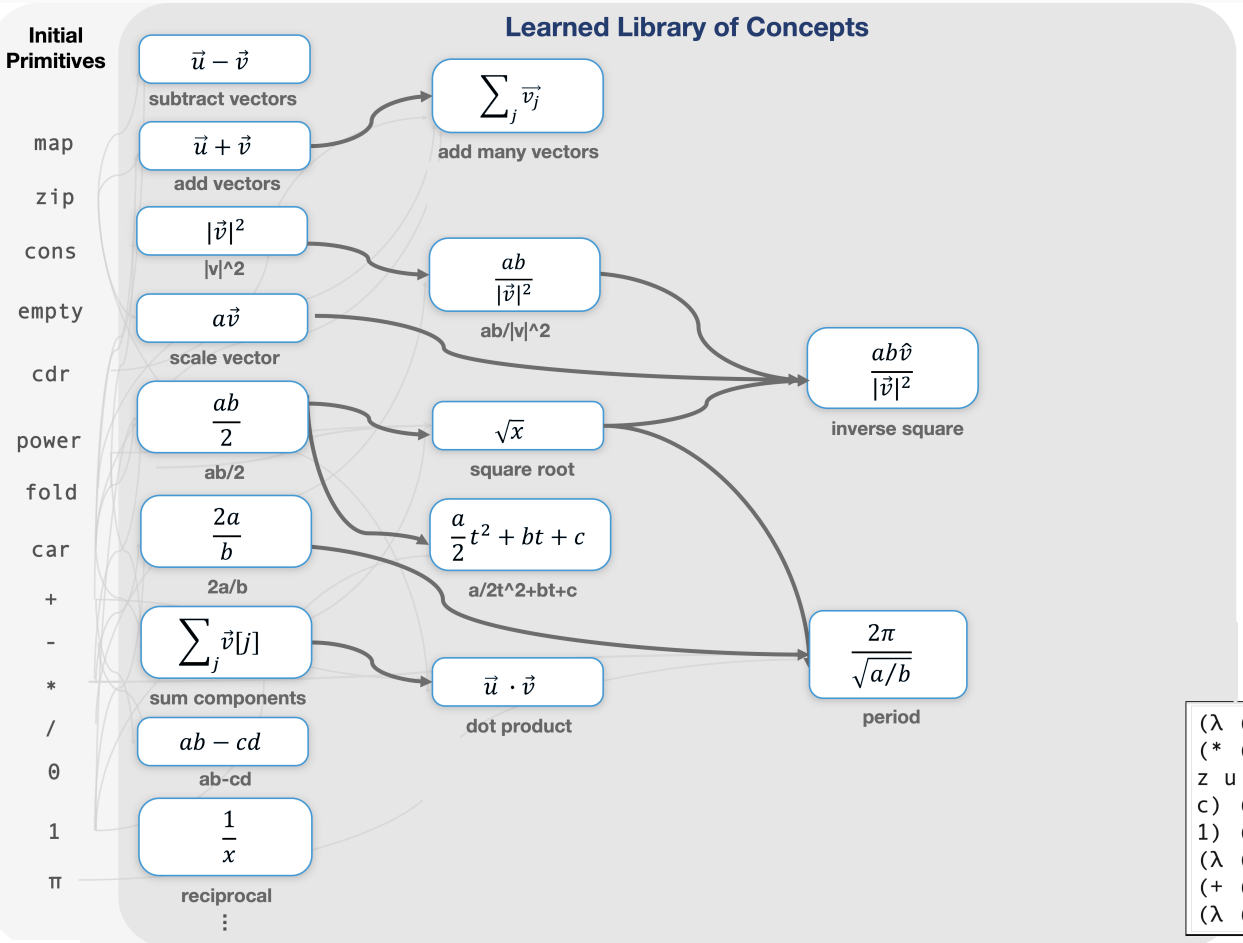
```

Growing languages for vector algebra and physics



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```
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```

```
(λ (x y z u) (map (λ (v) (* (/
(* (power (/ (* x x) (fold (zip
z u (λ (w a) (- w a))) 0 (λ (b
c) (+ (* b b)    c)))) (/ (* 1
1) (+ 1 1))) y) (fold (zip z u
(λ (d e)  (- d e))) 0 (λ (f g)
(+ (* f f) g)))) v)) (zip z u
(λ (h i) (- h i)))))
```

Solution to `Coulomb's Law` if expressed in initial primitives

Growing a language for recursive programming

Initial Primitives

- Y combinator
- cons
- car
- cdr
- nil
- if
- nil?
- +
- -
- 0
- 1
- =

Recursive Programming Algorithms

Stutter

[■■] → [■■■■]
[■■■] → [■■■■■■]

Take every other

[■■■■] → [■■]
[■■■■■■] → [■■■]

List lengths

[[■■■], [■]] → [3 1]
[[■■], [], [■]] → [2 0 1]

List differences

[1 8 2], [0 5 1] → [1 3 1]
[2 3 6], [1 2 4] → [1 1 2]

Growing a language for recursive programming

Recursive Programming Algorithms

Stutter

[■■] → [■■■■]
[■■■] → [■■■■■■]

```
(fold A (λ (u v) (cons
v (cons v u))) nil)
```

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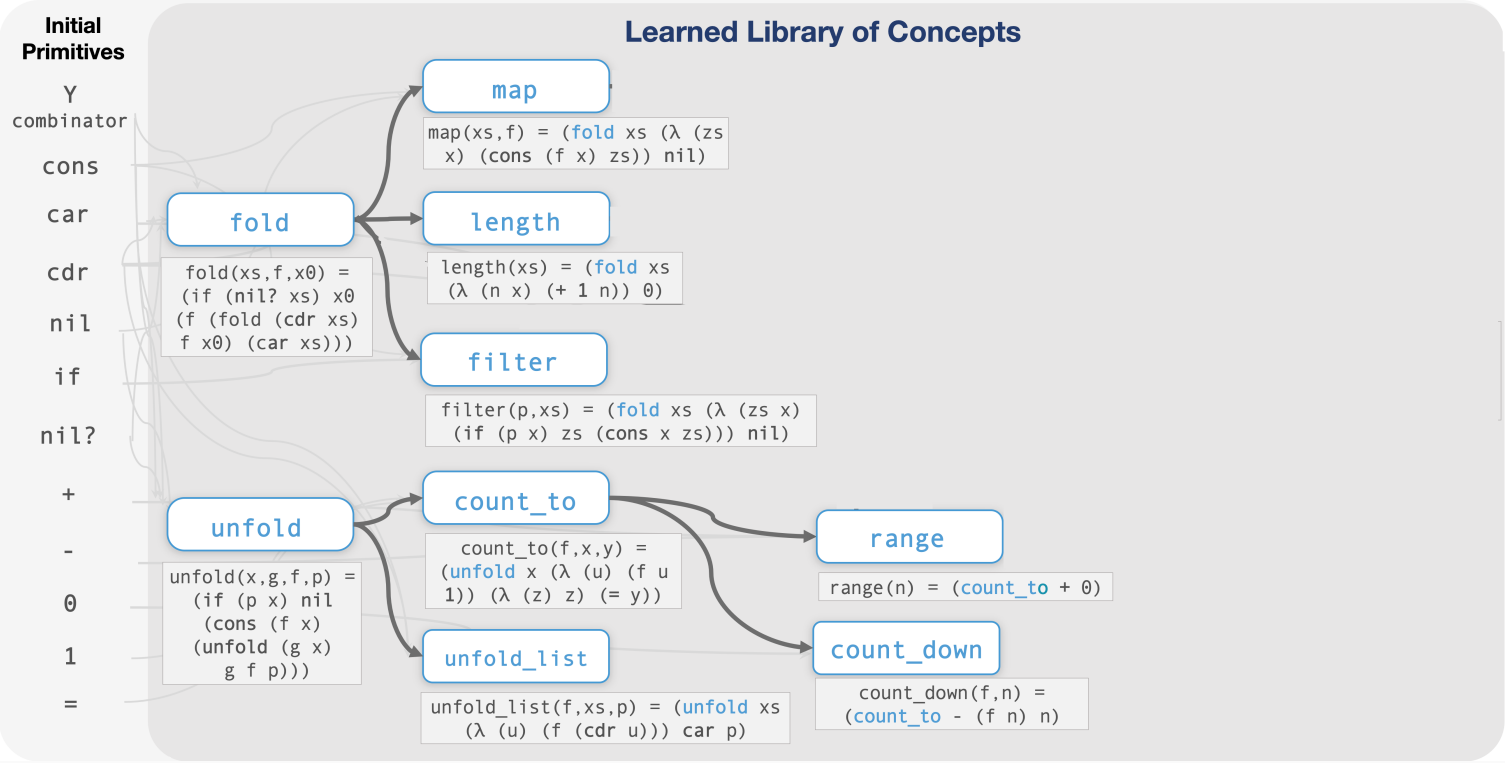

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```
(map A length)
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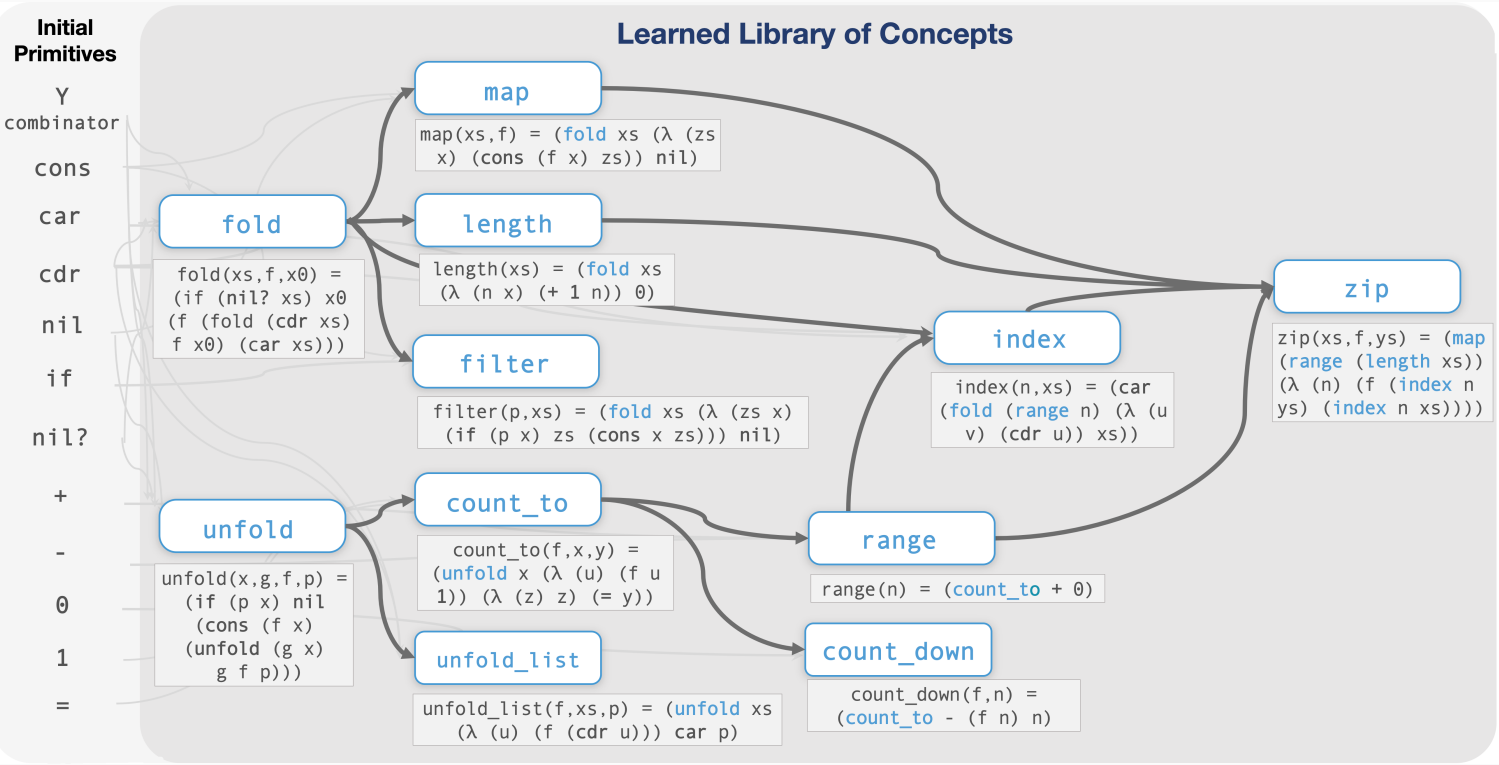

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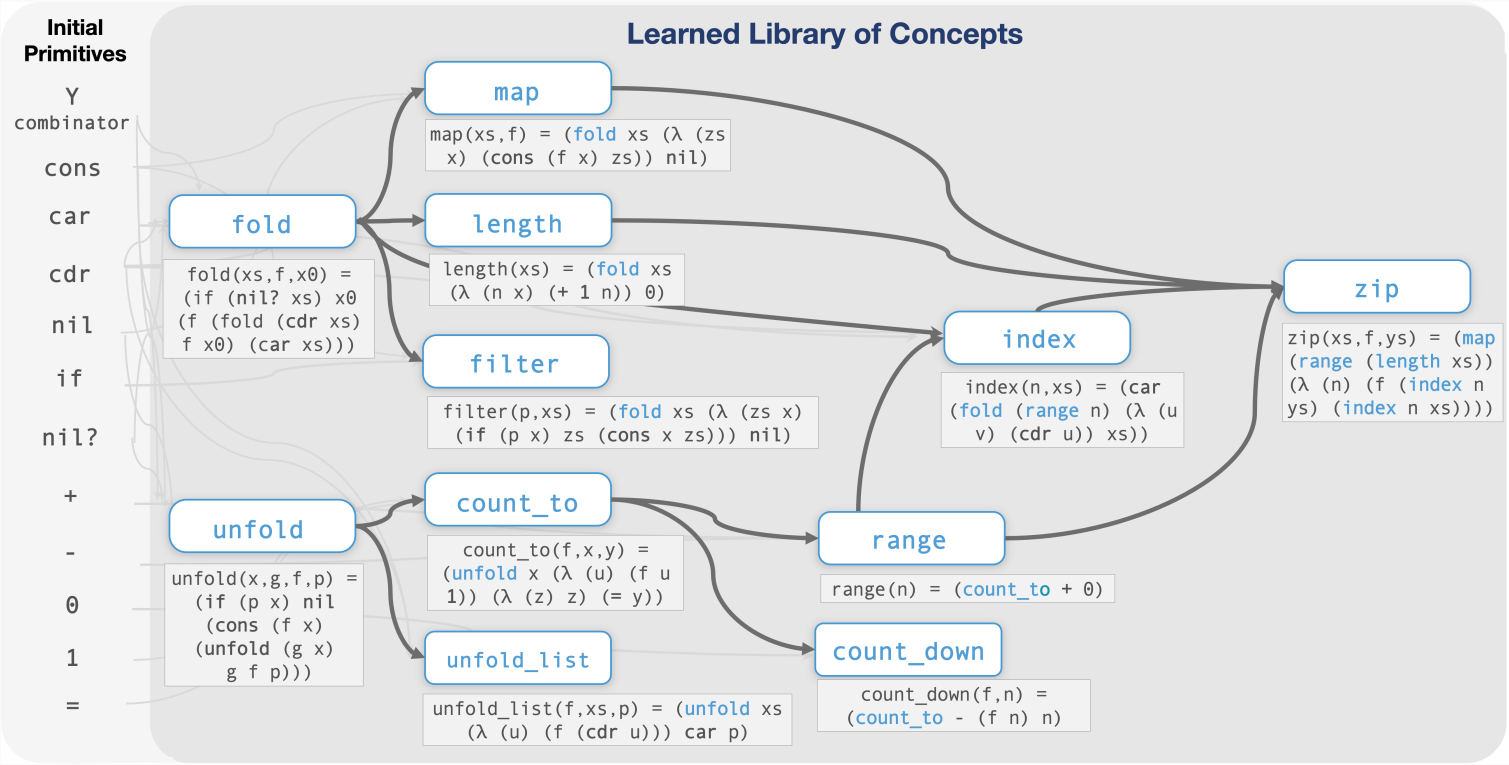
```
(zip A - B )
```

Growing a language for recursive programming



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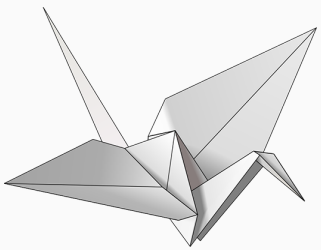
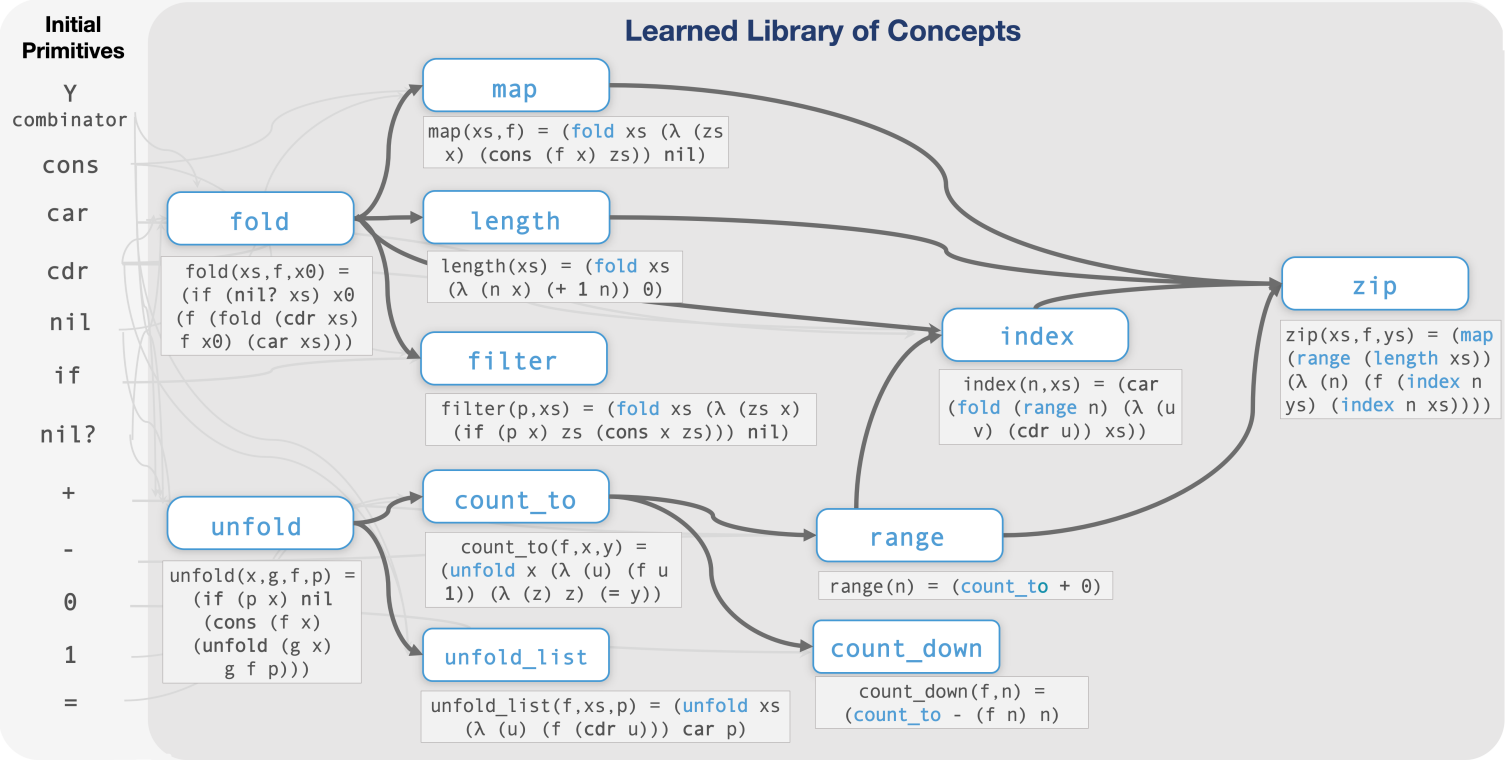


Origami Programming: Jeremy Gibbons, 2003

Growing a language for recursive programming



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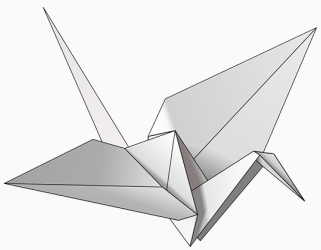
(zip A - B )
```

1 year of compute. 5 days on 64 CPUs.



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Lessons

Library learning interacts synergistically with neural synthesis: bootstrapping, more than sum of parts

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Symbols aren't necessarily interpretable. Grow the language based on experience to make it more powerful *and* more human understandable

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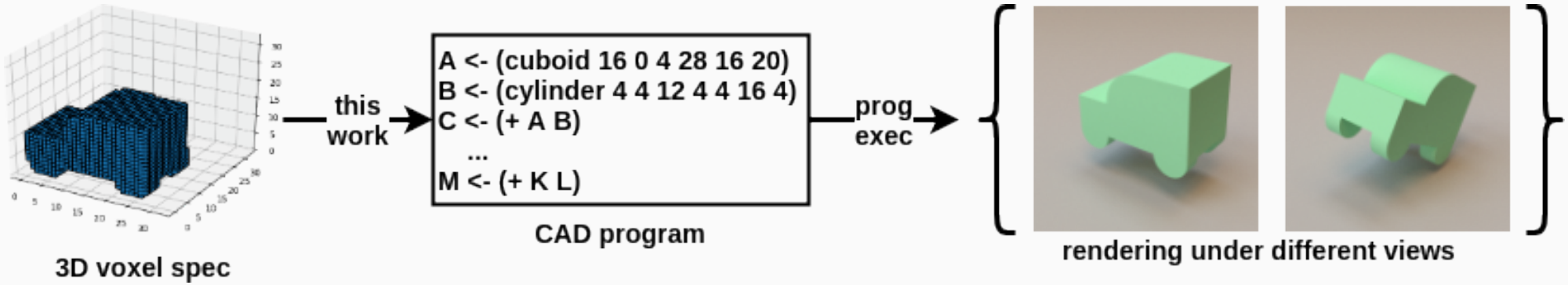
Library learning interacts synergistically with neural synthesis: bootstrapping, more than sum of parts

Symbols aren't necessarily interpretable. Grow the language based on experience to make it more powerful *and* more human understandable

Learning-from-scratch is possible in principle. Don't do it. But program induction makes it convenient to build in what we know how to build in, and then learn on top of that

the end.

3D program induction



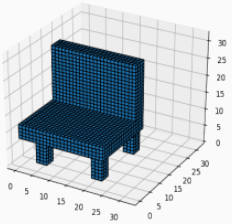
Challenge: combinatorial search!

Branching factor: > 1.3 million per line of code, ≈ 20 lines of code

search space size: $(1.3 \text{ million})^{20} \approx 10^{122}$ programs

Solution: stochastic **tree search** + learn **policy** that writes code + learn **value** function that assesses execution of program so far; analogous to **AlphaGo** [Silver et al. 2016]

Input:



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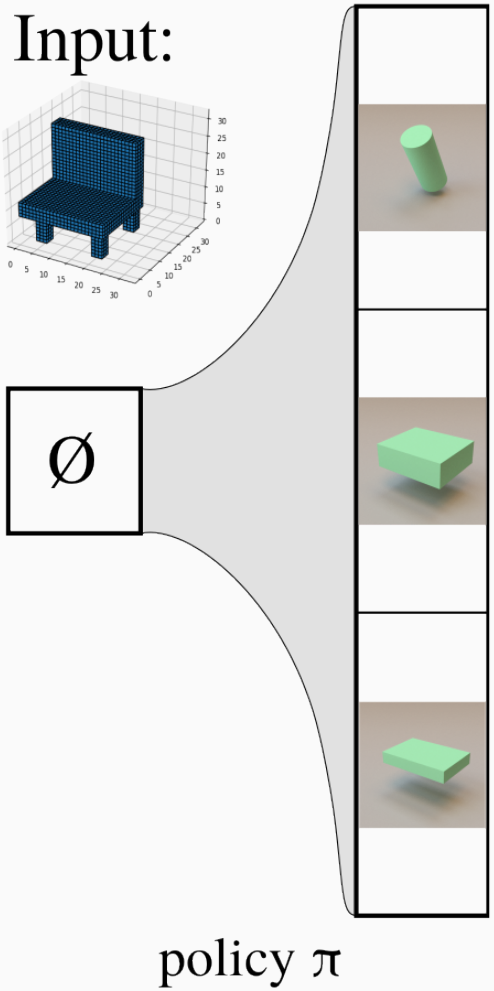

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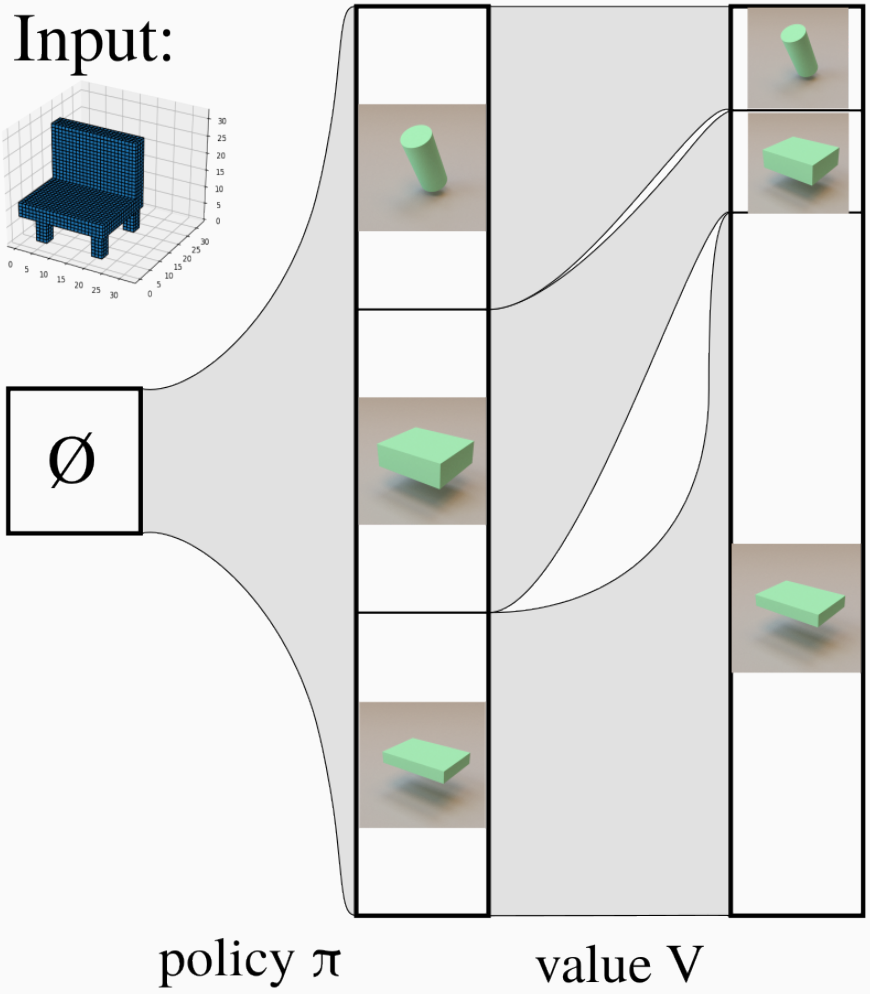

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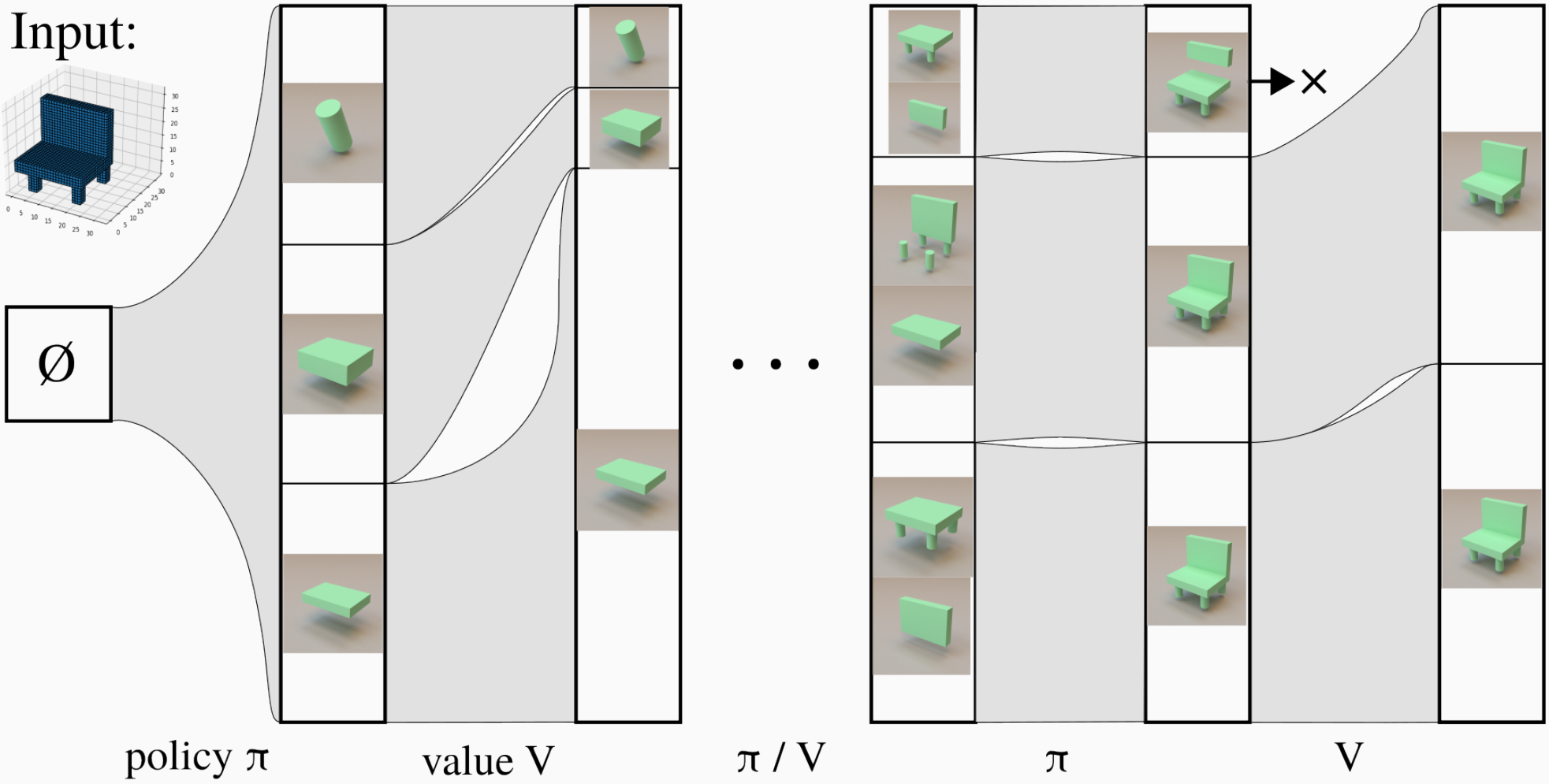
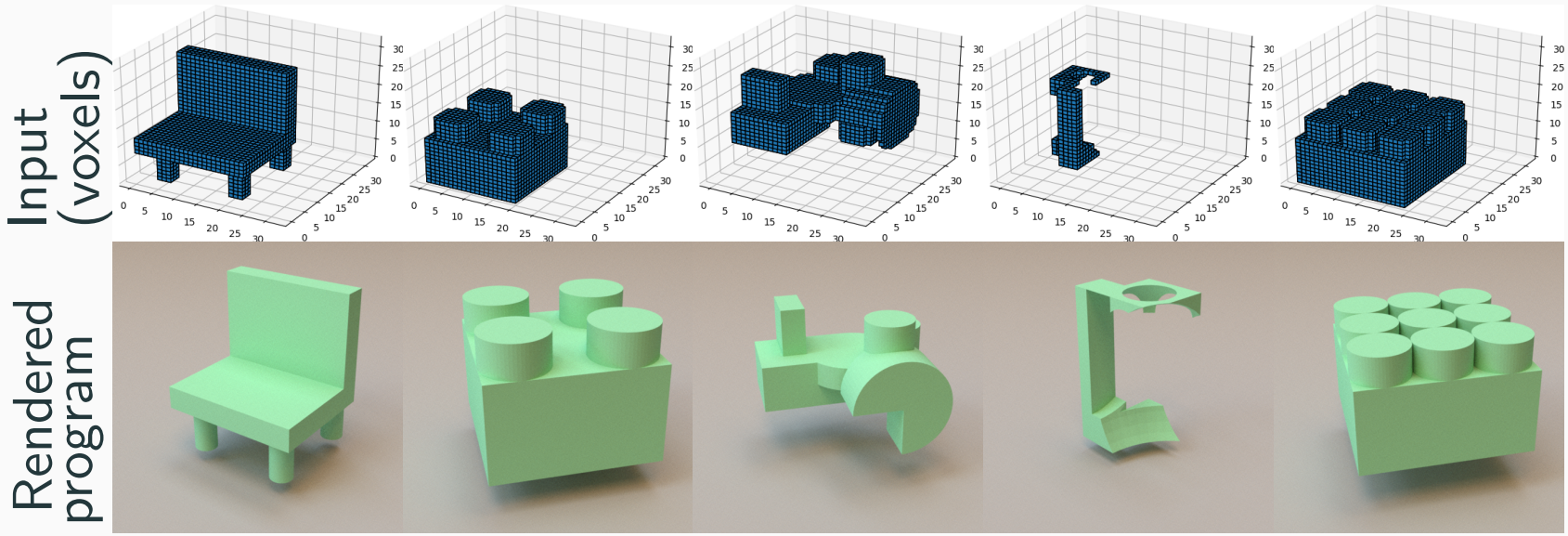


3D program induction



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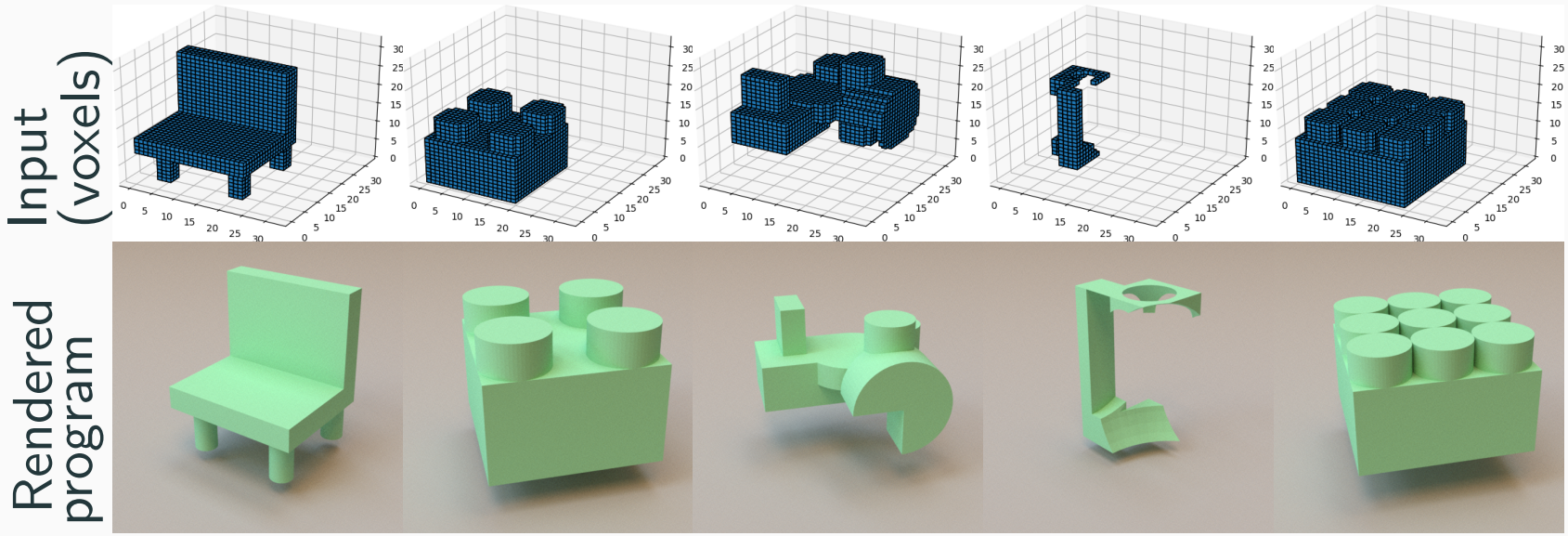
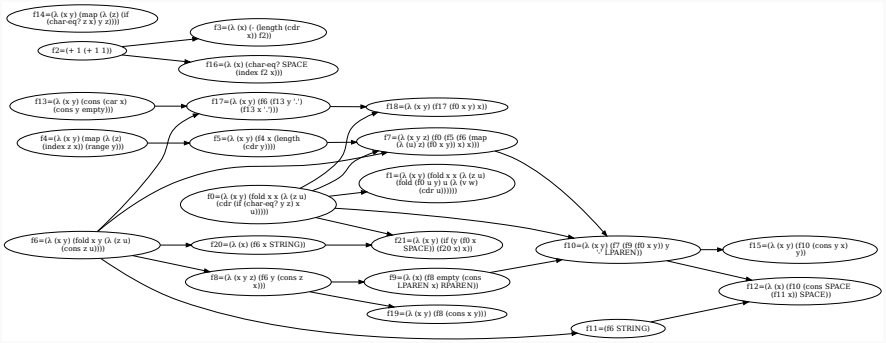
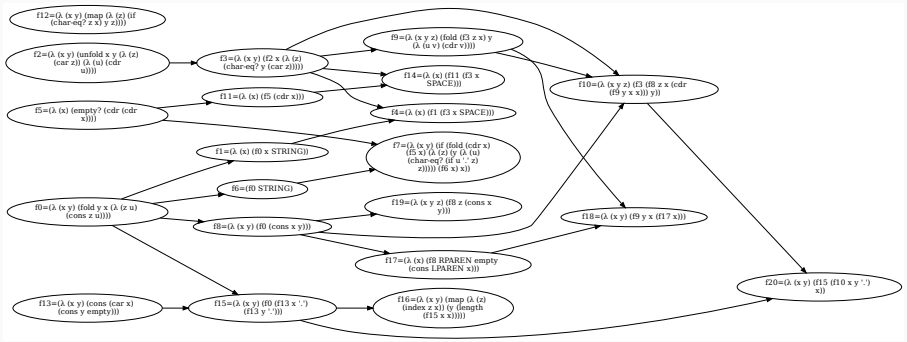


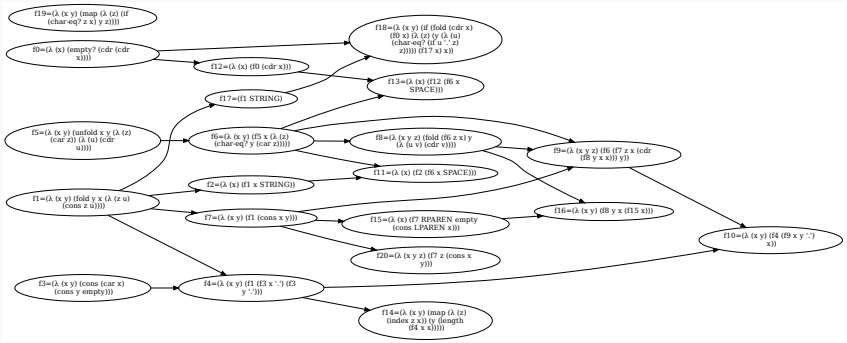
same architecture learns to synthesize text editing programs (FlashFill, Gulwani 2012)

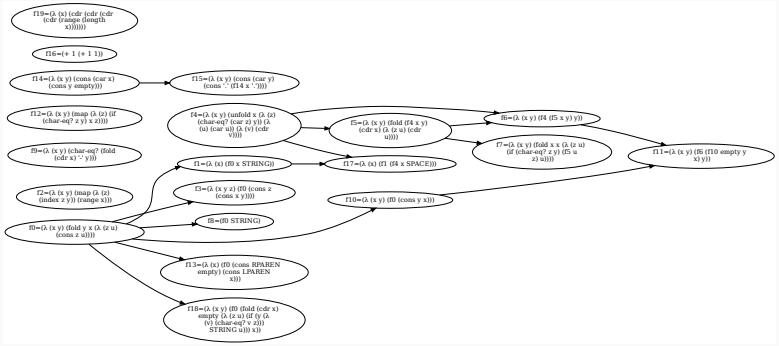
Library structure: Text Editing

DreamCoder learns libraries for FlashFill-style text editing [Gulwani 2012]



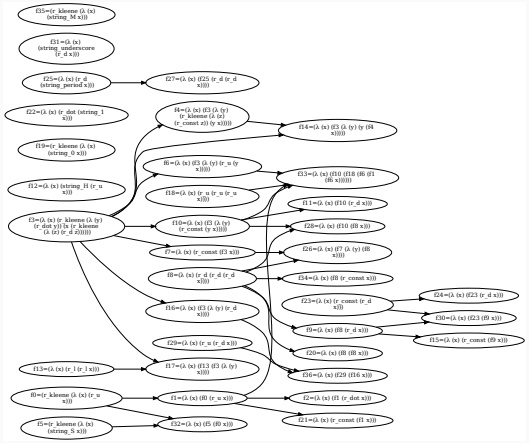


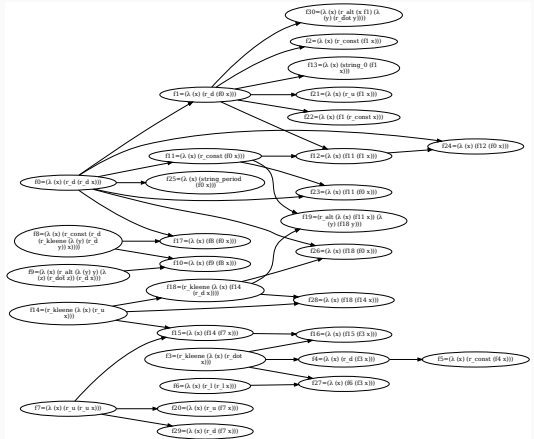




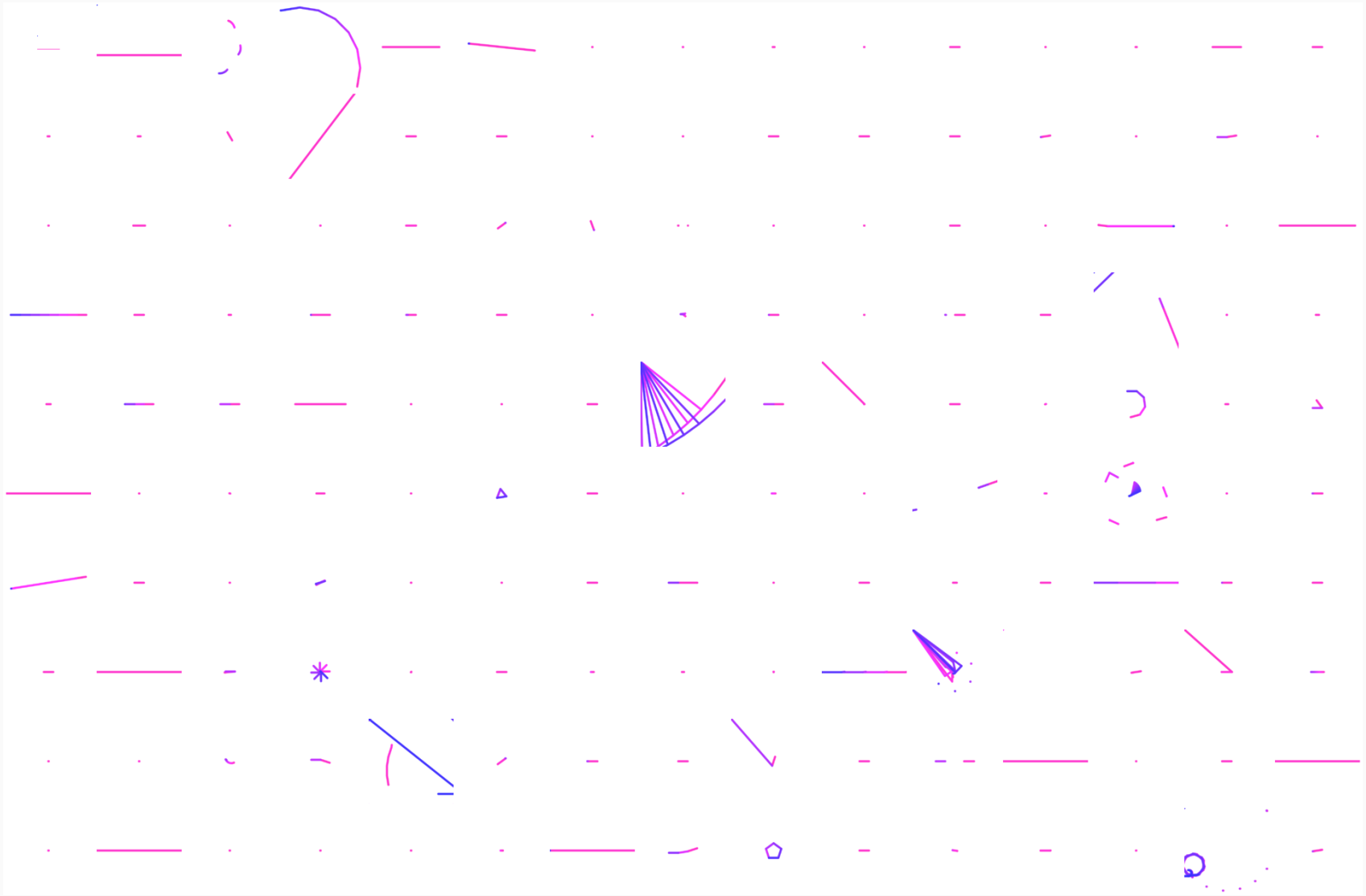
Library structure: Generating Text

Libraries for probabilistic generative models over text:
data from crawling web for CSV files





150 random dreams before learning



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