

DreamCoder: Growing generalizable, interpretable knowledge with wake-sleep Bayesian program learning

Kevin Ellis

Collaborators: Catherine Wong, Maxwell Nye, Mathias Sablé-Meyer,
Lucas Morales, Armando Solar-Lezama, Joshua B. Tenenbaum

2020

colala group meeting

The premise of program induction

1. Represent knowledge as programs: as symbolic code

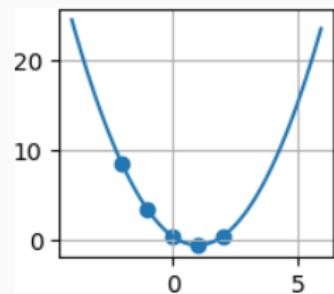
The premise of program induction

1. Represent knowledge as programs: as symbolic code
2. Learning=adding to that body of knowledge=
making new programs=program synthesis

Why program induction?

Why program induction?

strong generalization
+data efficiency

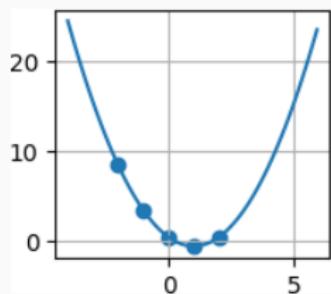


$$f(x) = (x-1)^{**2} - 0.5$$

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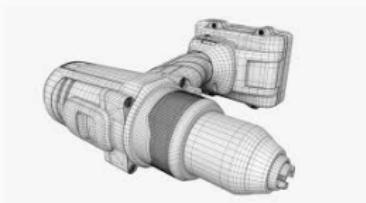
interpretability



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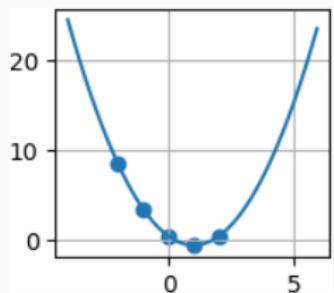


VS



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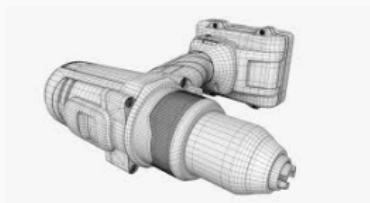


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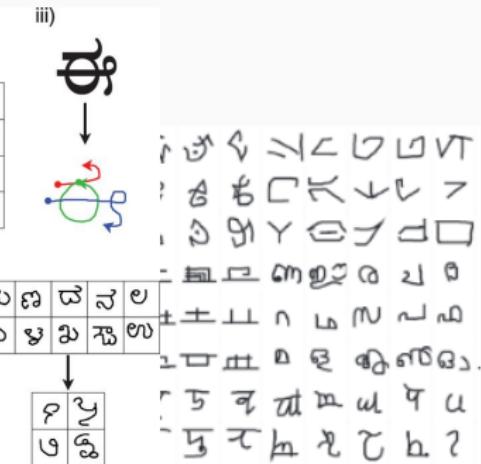
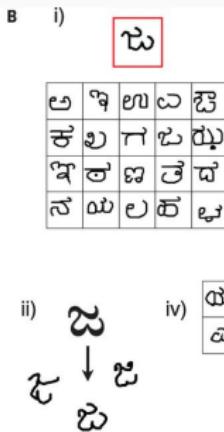
interpretability



universal expressivity

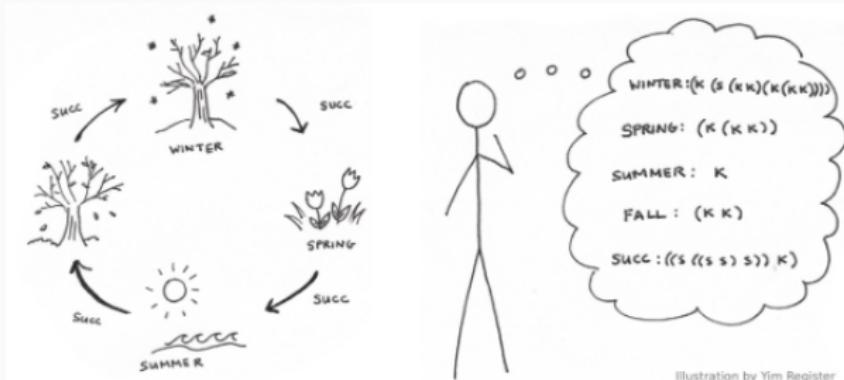


Human program induction



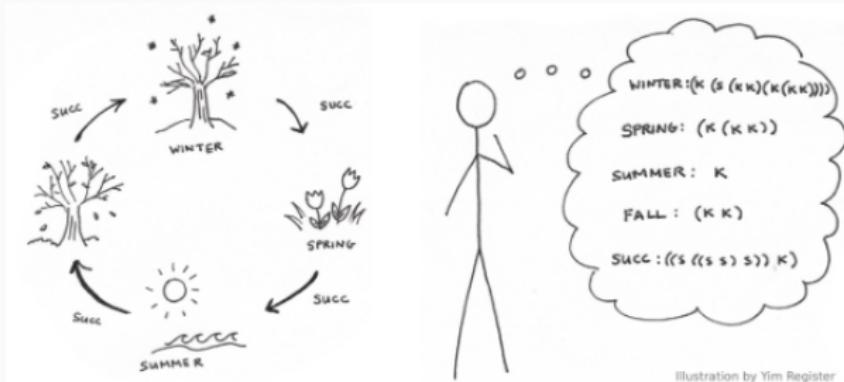
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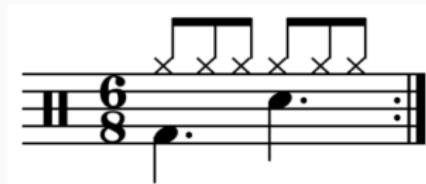


Piantadosi 2020: also kinship, subway routes, counting, ...

Human program induction



Piantadosi 2020: also kinship, subway routes, counting, ...



Evans 2019, "Apperception." Also visual occlusion, analogies, ...

Why didn't this old idea work?

Program induction goes back to the 1956 Dartmouth Workshop that founded the field of AI



A PROPOSAL FOR THE
DARTMOUTH SUMMER RESEARCH PROJECT
ON ARTIFICIAL INTELLIGENCE

J. McCarthy, Dartmouth College
M. L. Minsky, Harvard University
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John McCarthy, Ray Solomonoff

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main obstacle: combinatorial search is hard

Why try now?

better toolkits: neural+probabilistic+symbolic, and knowing how to combine them

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better compute+parallel algorithms

Program Induction and learning to learn

Learning to write code

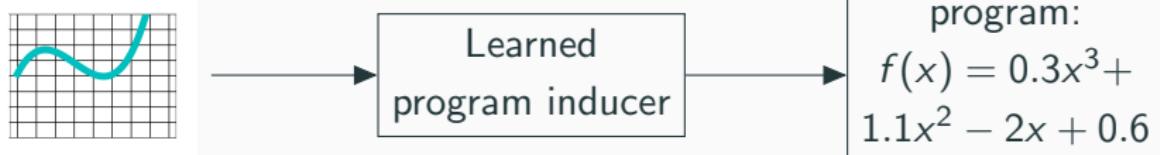
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- Library of concepts (declarative knowledge; domain specific language)
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Concepts: x^3 , $\alpha x + \beta$, etc

Inference strategy: neurosymbolic search for programs

Library learning

Initial Primitives

: 

map

fold 

if

cons

>

: 

Sample Problem: sort list

[9 2 7 1] → [1 2 7 9]

[3 8 9 4 2] → [2 3 4 8 9]

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Library learning

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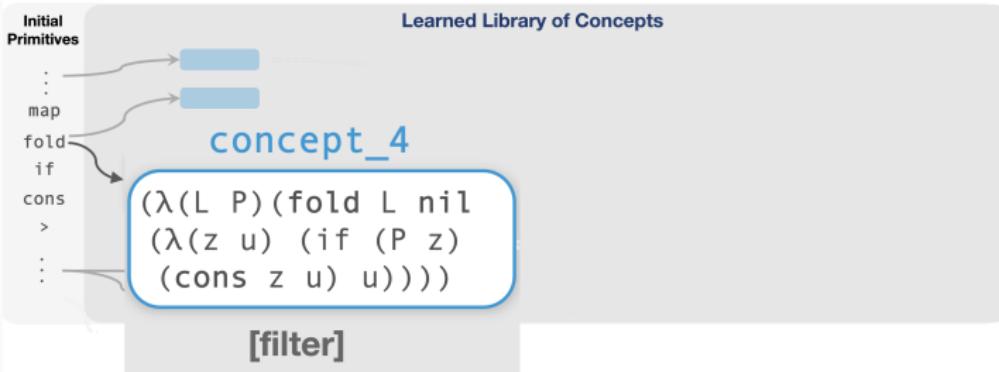
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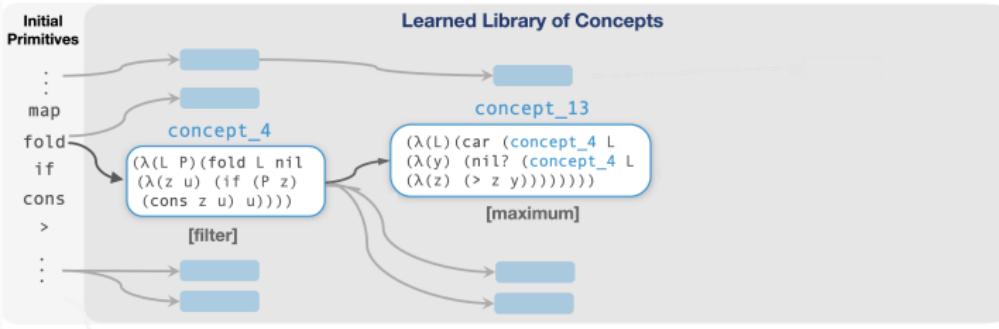
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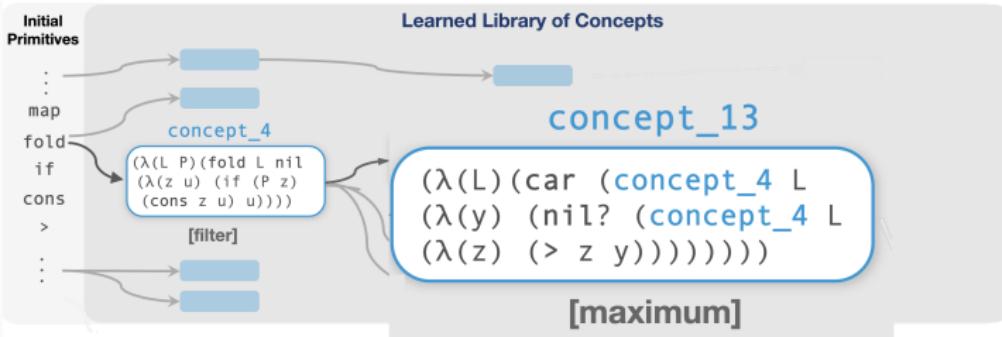
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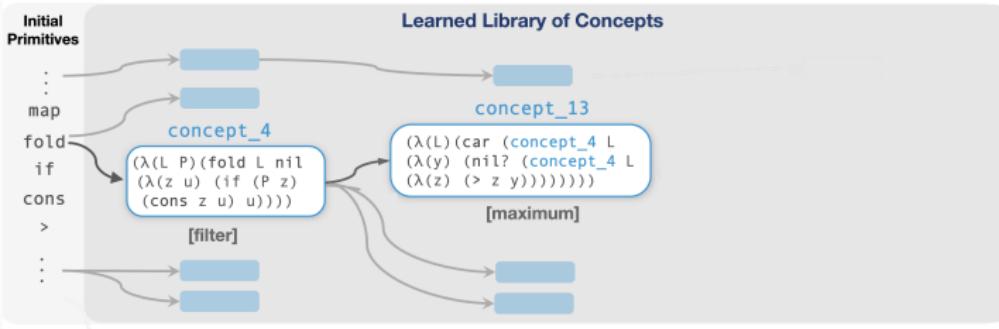
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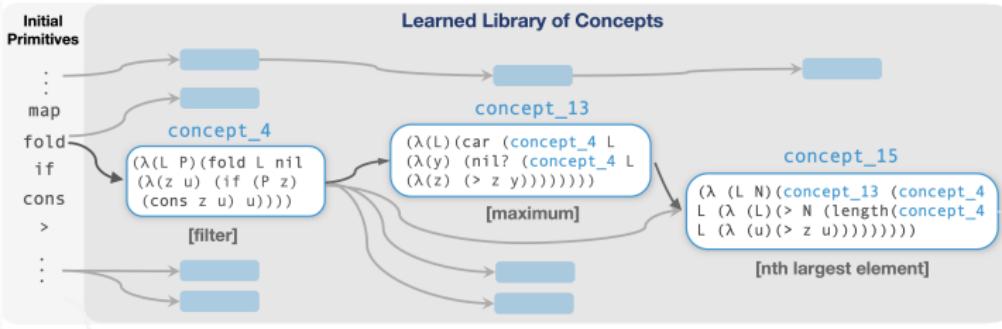
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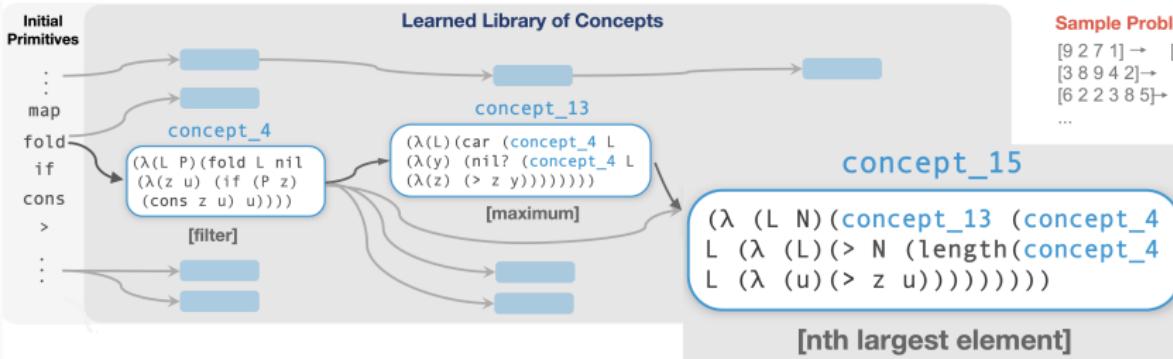
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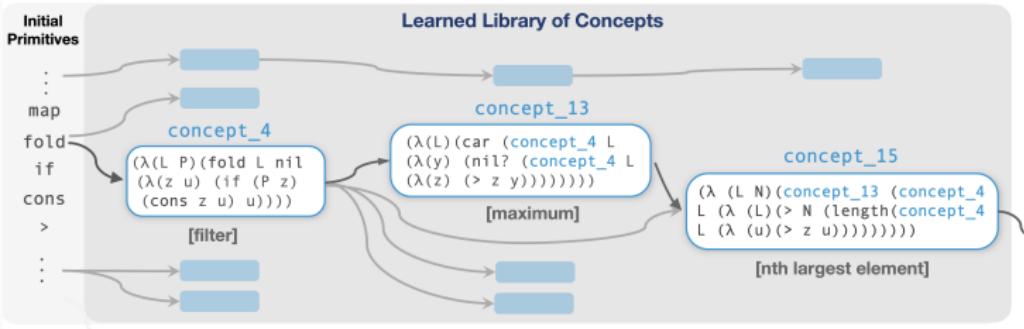
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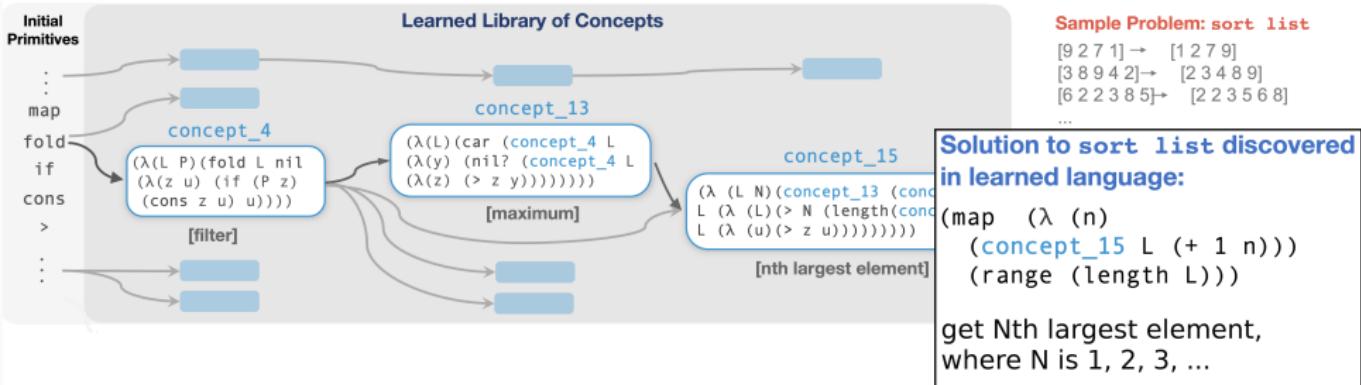
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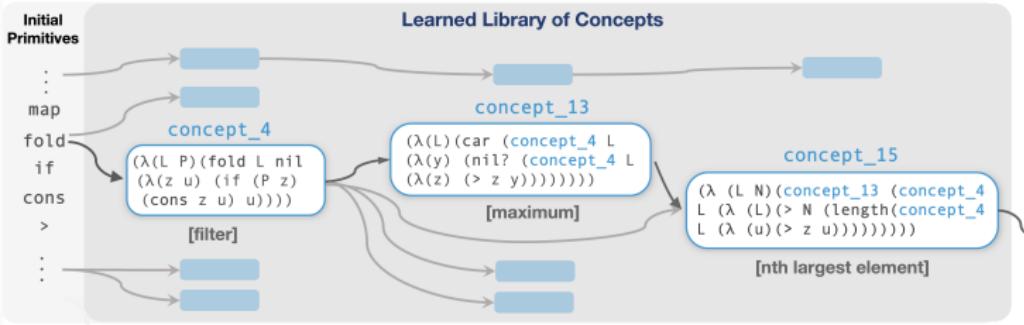
Solution to sort list discovered in learned language:

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(map (λ (n)
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  (range (length L)))
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Library learning



Library learning



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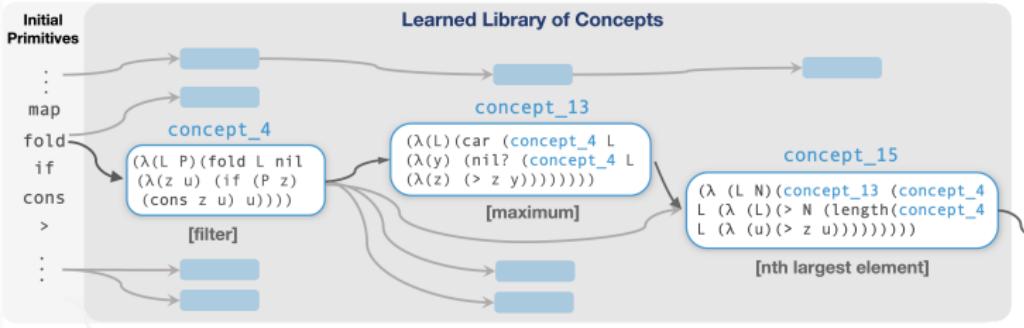
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get Nth largest element,
where N is 1, 2, 3, ...

Solution rewritten in initial primitives:

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induced sort program found in $\leq 10\text{min}$. Brute-force search
without learned library would take $\approx 10^{73}$ years

DreamCoder

- **Wake:** Solve problems by writing programs
- **Sleep:** Improve library and neural recognition model:
 - **Abstraction sleep:** Improve library
 - **Dream sleep:** Improve neural recognition model



cf. Helmholtz machine, wake/sleep neural network training algorithms

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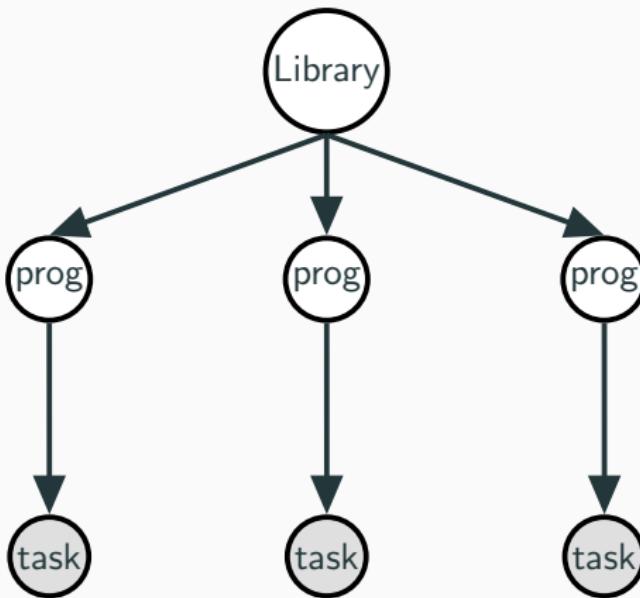
Memory consolidation during human sleep:

1. Slow-wave (~explicit, declarative)
2. Fast-wave REM (~implicit, procedural)



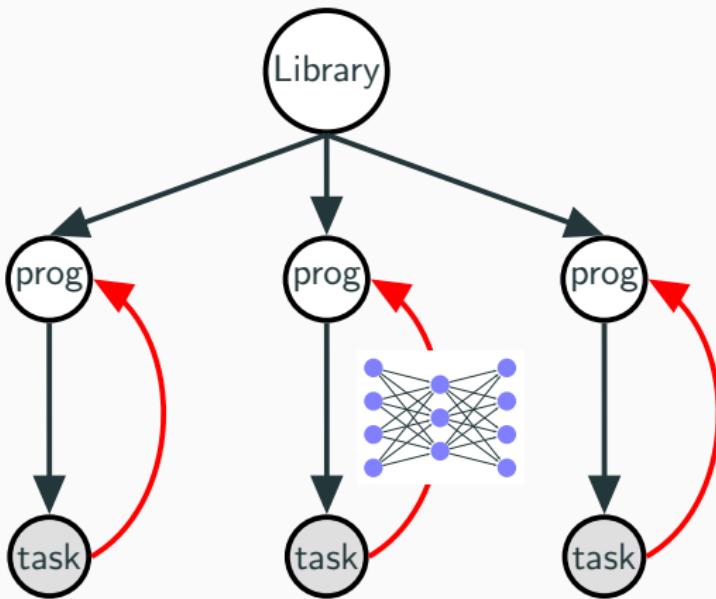
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Library learning as Bayesian inference

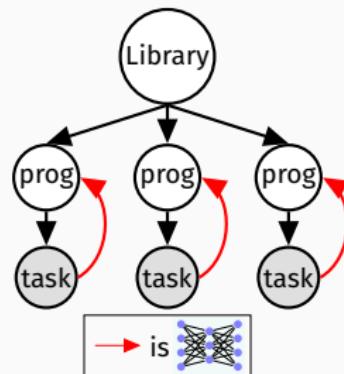


[Dechter et al, 2013] [Liang et al, 2010] [Lake et al, 2015]

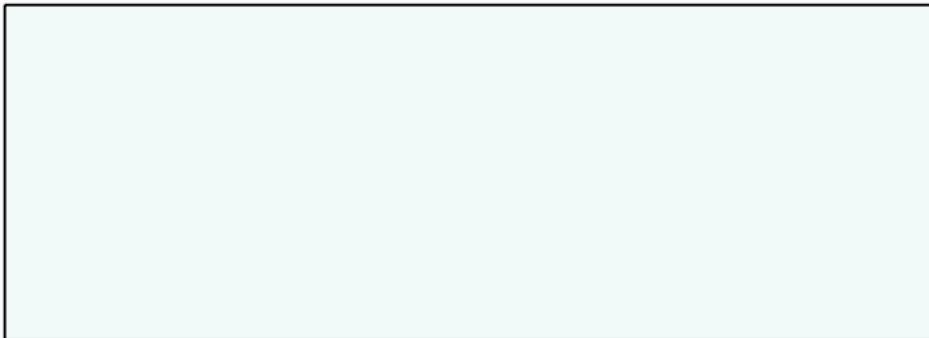
Library learning as neurally-guided Bayesian inference



library learning via program analysis +
new neural inference network for program synthesis +
better program representation (Lisp+polymorphic types [Milner 1978])



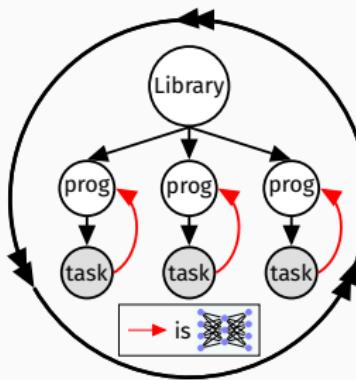
WAKE

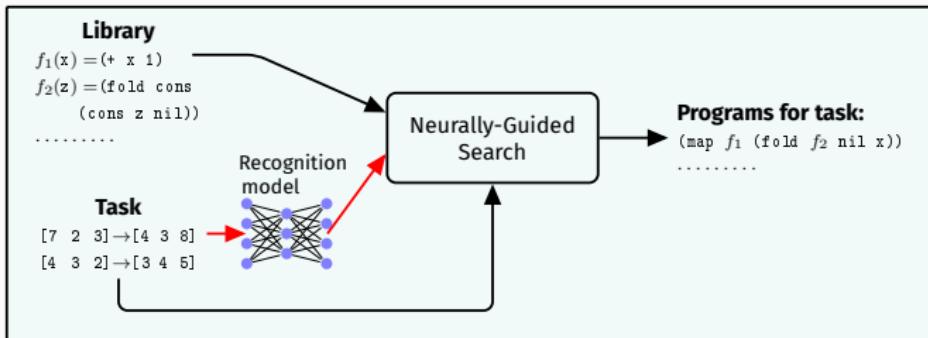
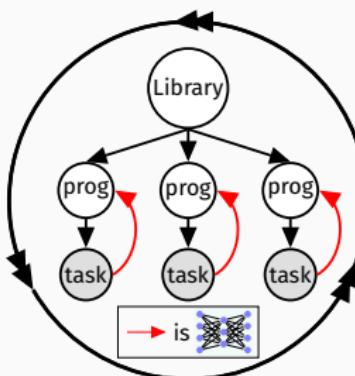


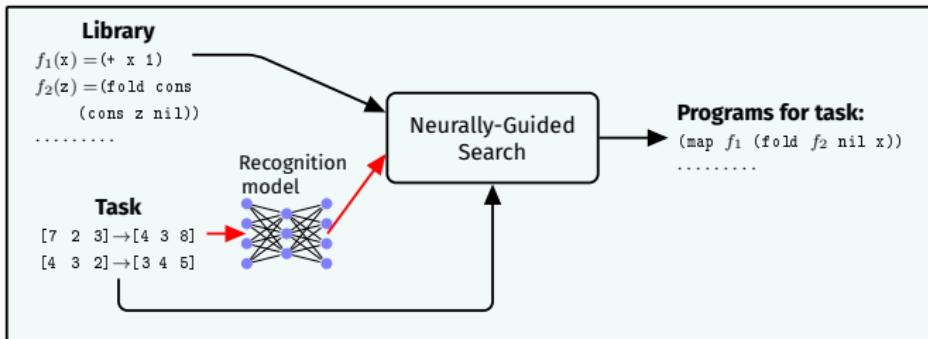
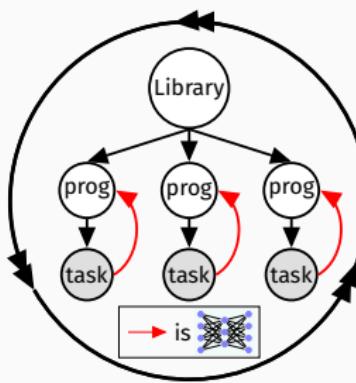
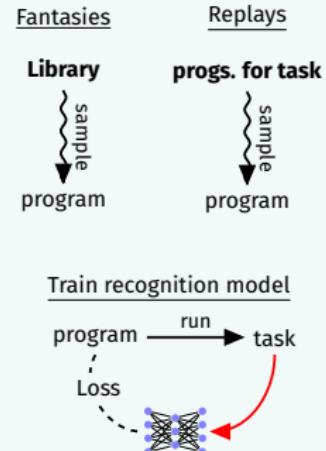
SLEEP: ABSTRACTION



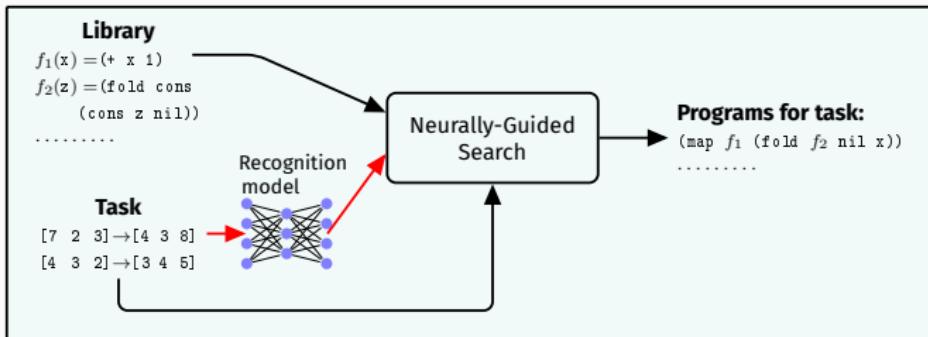
SLEEP: DREAMING



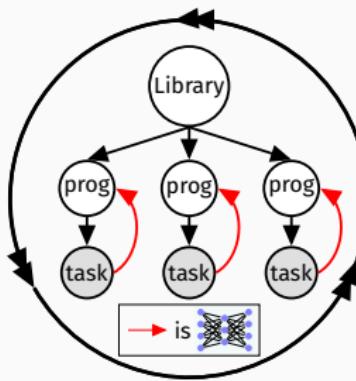
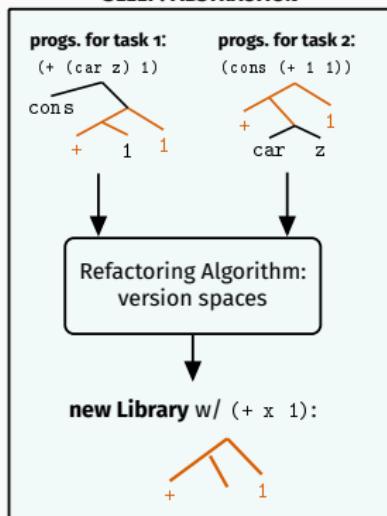
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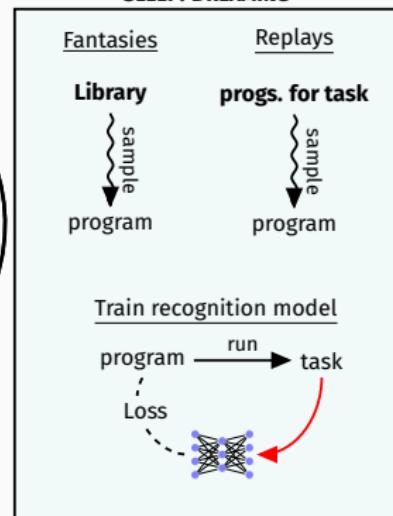
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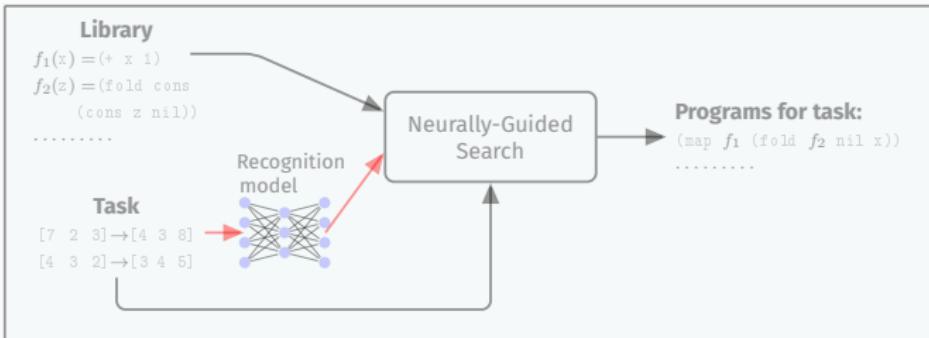
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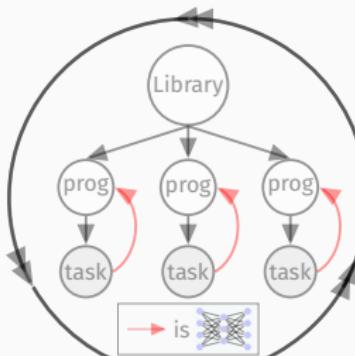
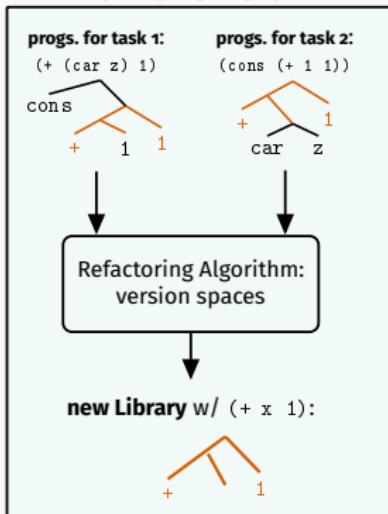
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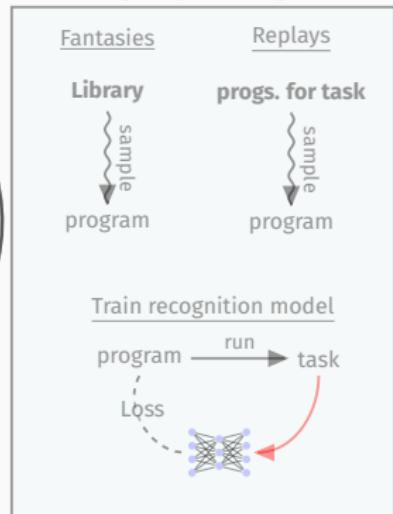
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Abstraction Sleep: Growing the library via refactoring

$$5 + 5$$

Abstraction Sleep: Growing the library via refactoring

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(+ 5 5)

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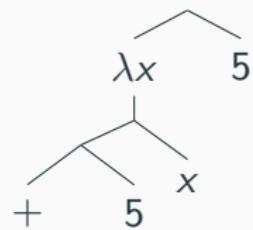
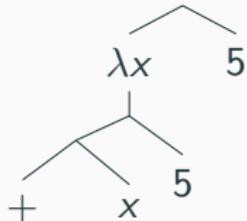
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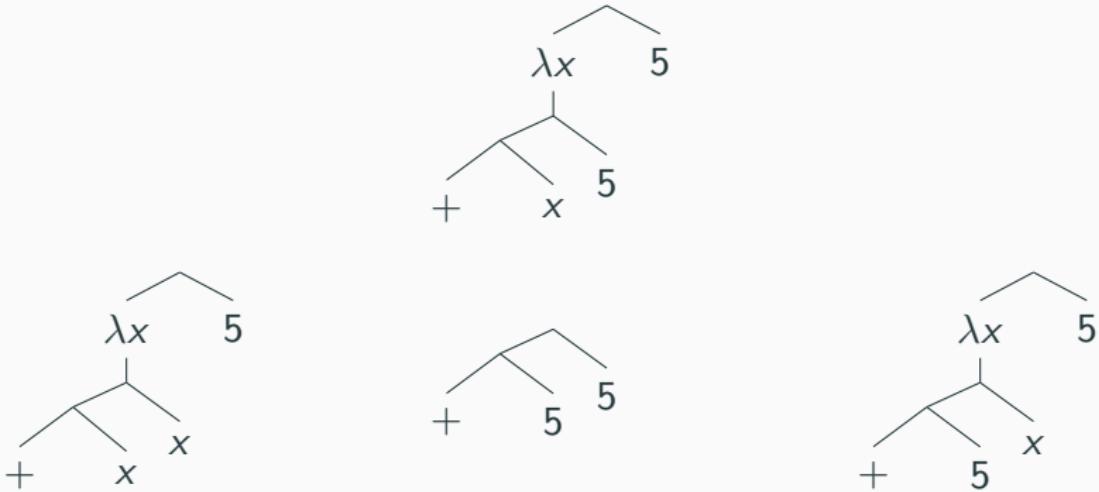
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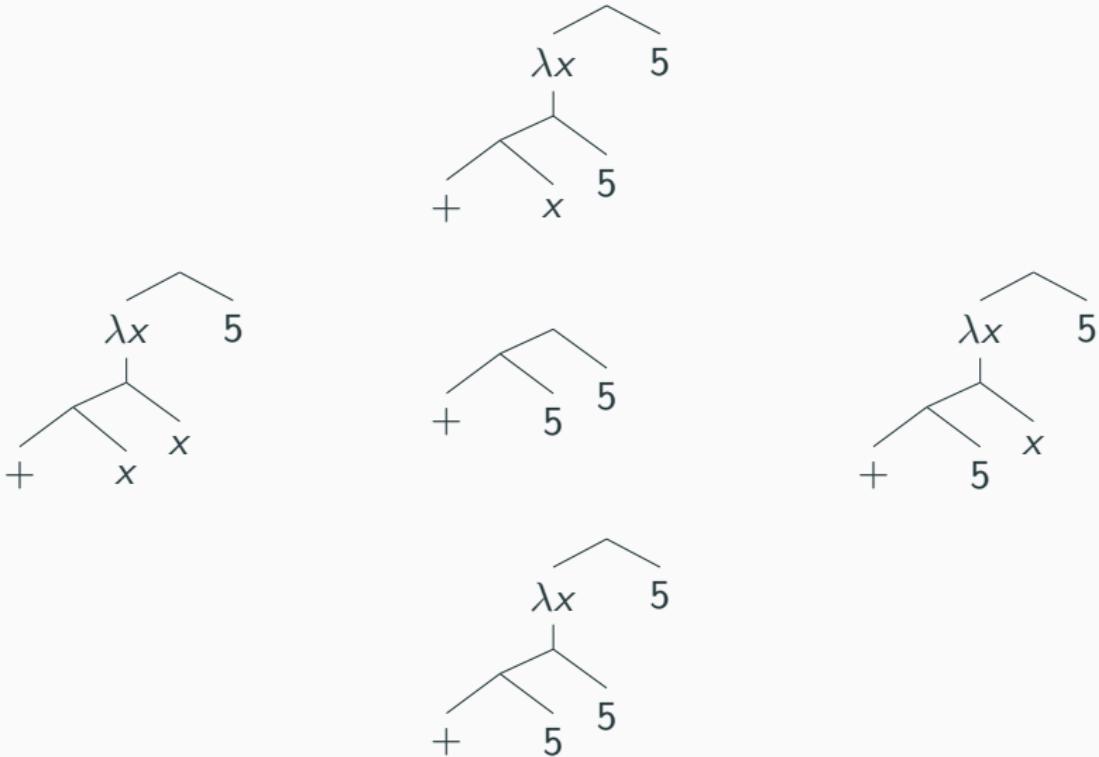
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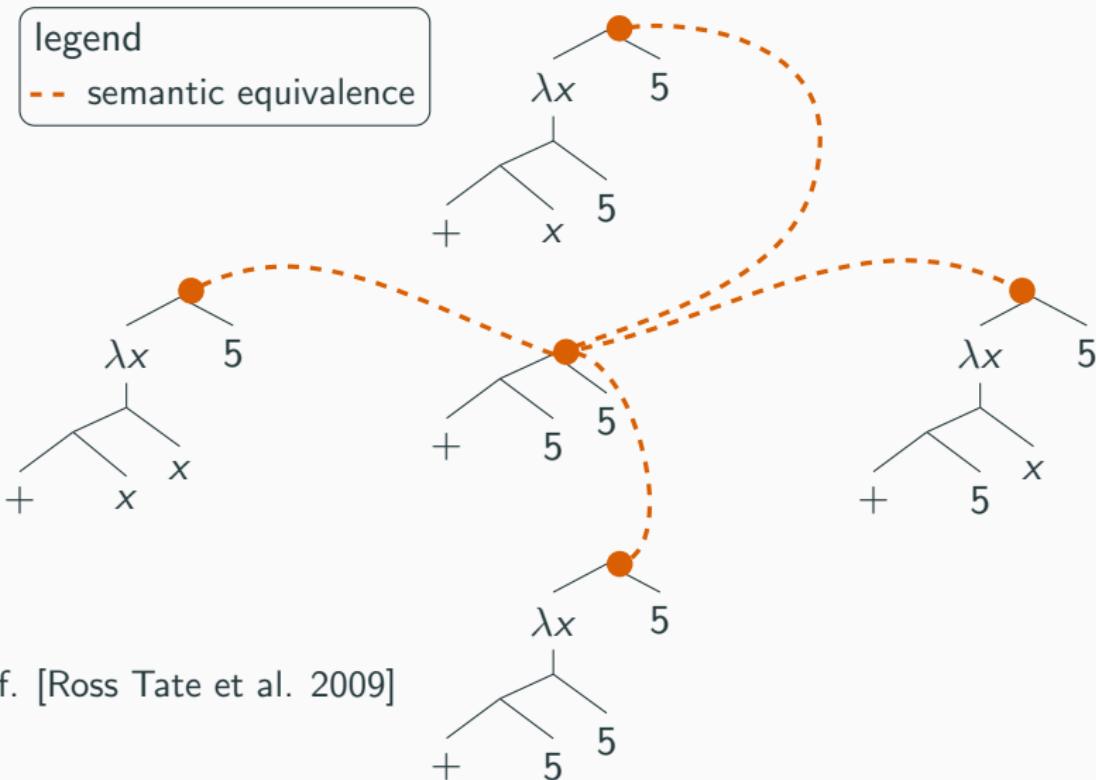
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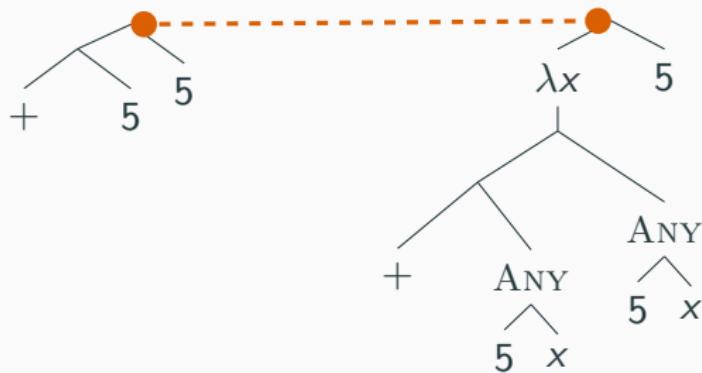


Abstraction Sleep: Growing the library via refactoring

legend

semantic equivalence

ANY nondeterministic choice



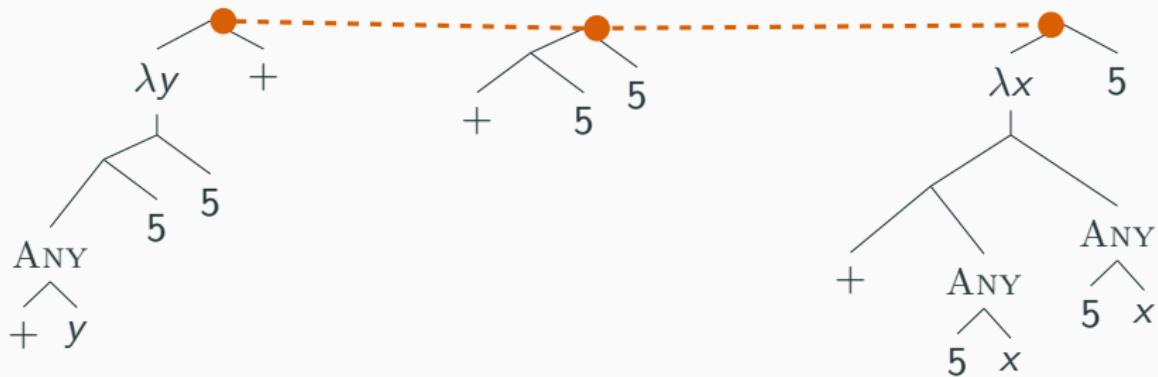
cf. [Gulwani 2012]

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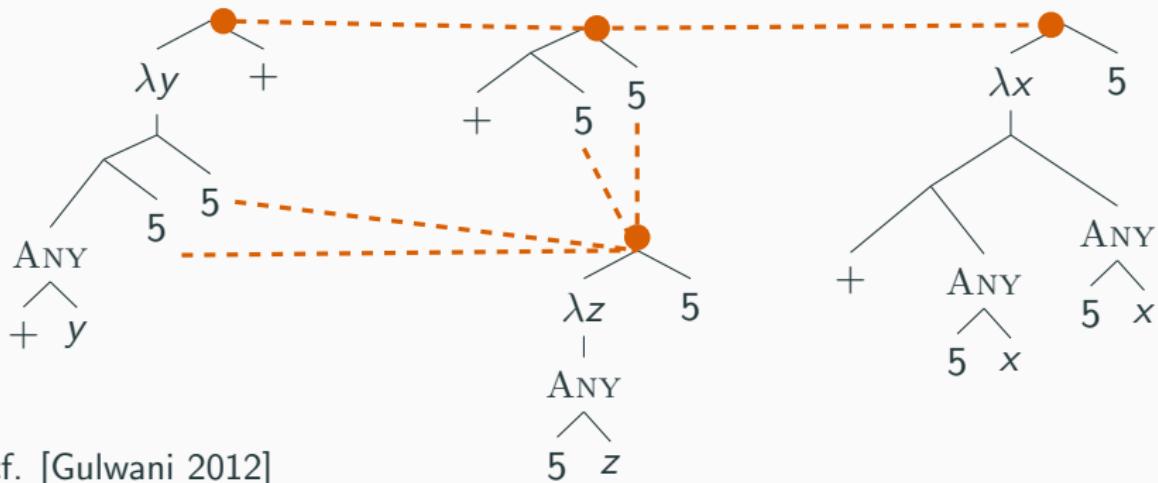


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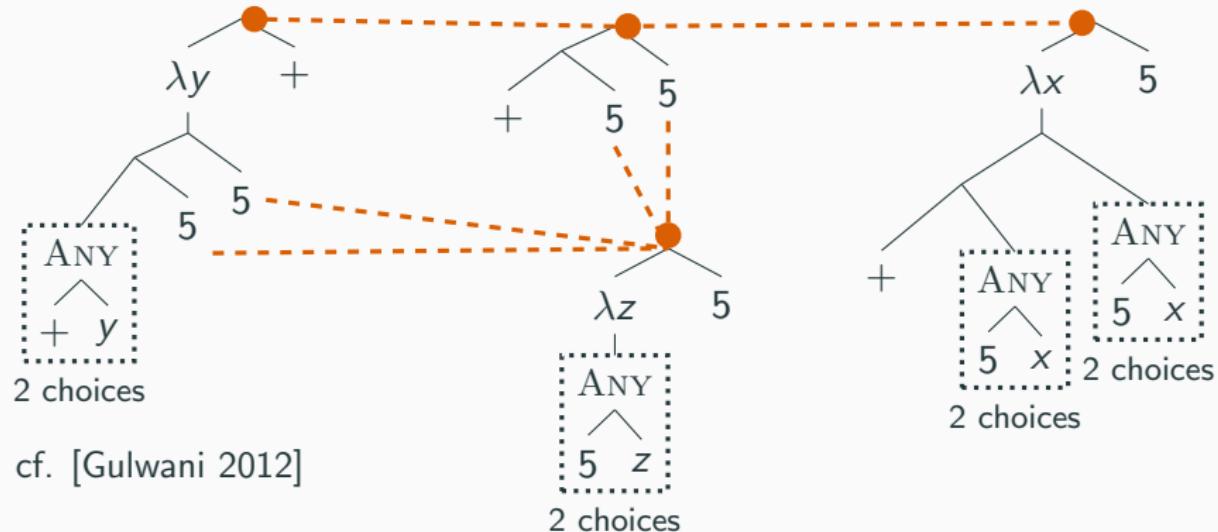
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Abstraction Sleep: Growing the library via refactoring

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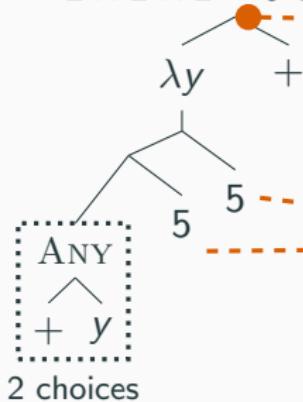
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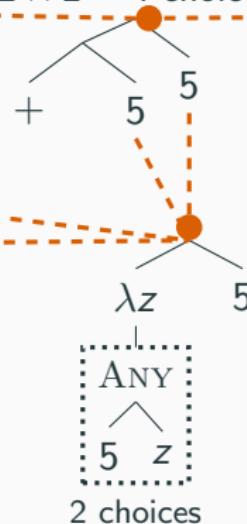
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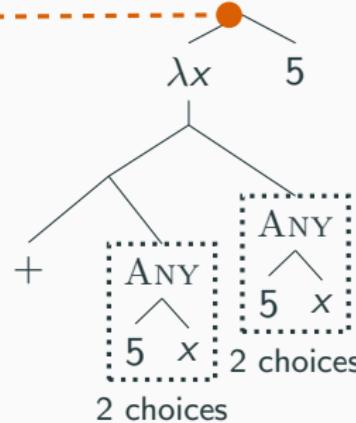
$2 \times 2 \times 2 = 8$ choices



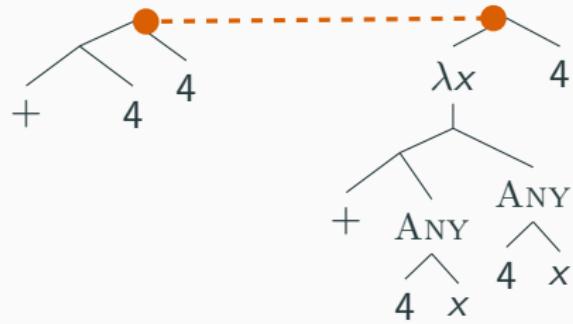
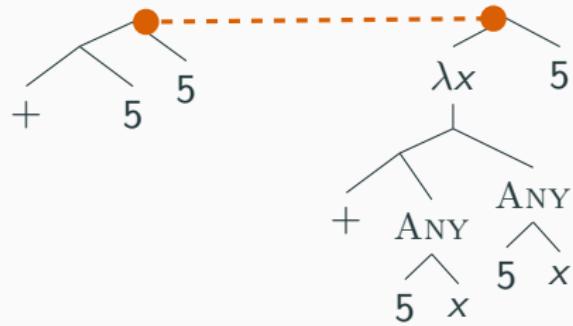
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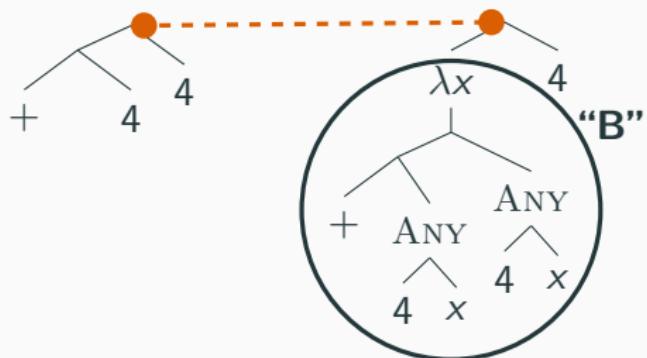
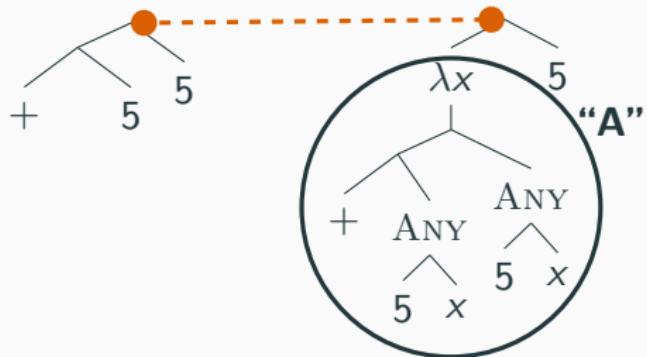
cf. [Gulwani 2012]



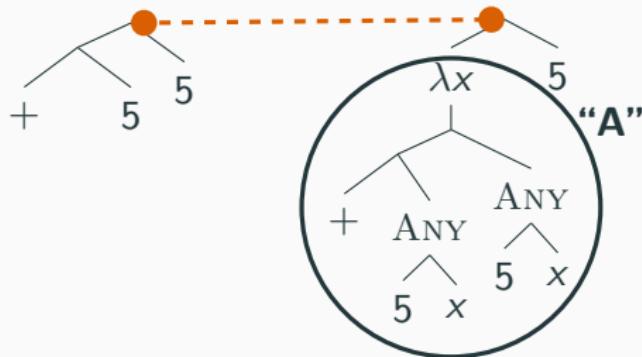
legend

— semantic equivalence

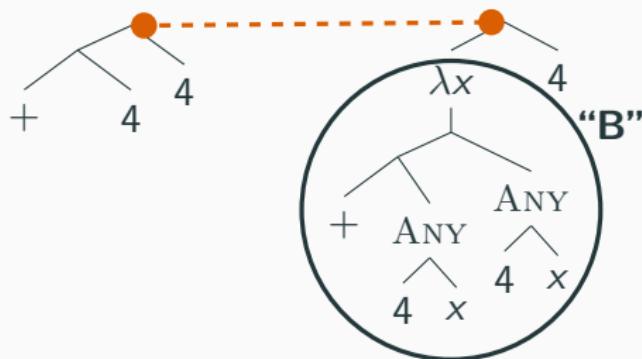
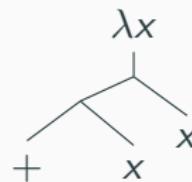
ANY nondeterministic choice



legend
 - - - semantic equivalence
 ANY nondeterministic choice



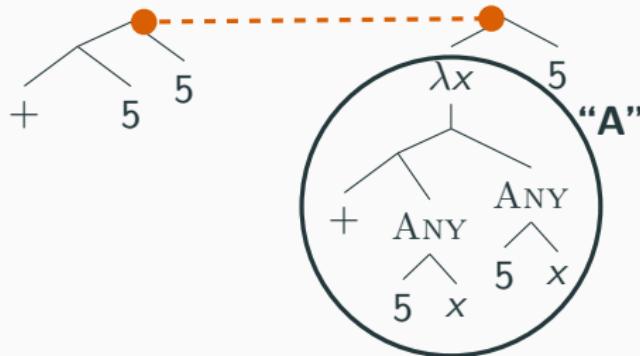
A intersect B:



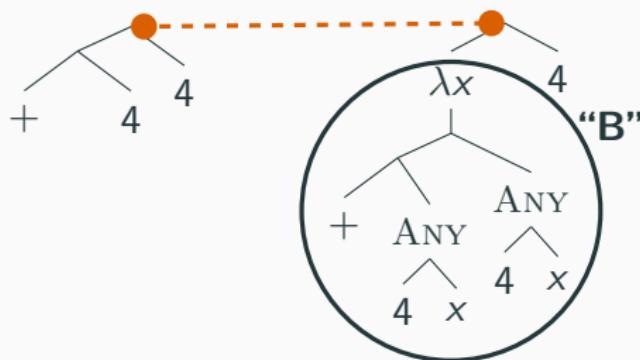
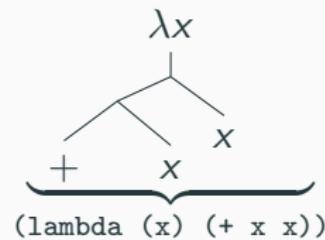
legend

— semantic equivalence

ANY nondeterministic choice



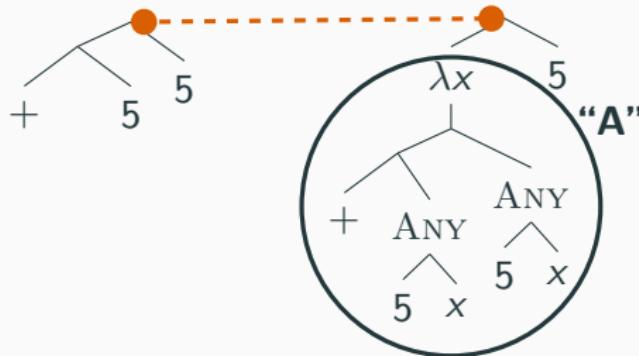
A intersect B:



legend

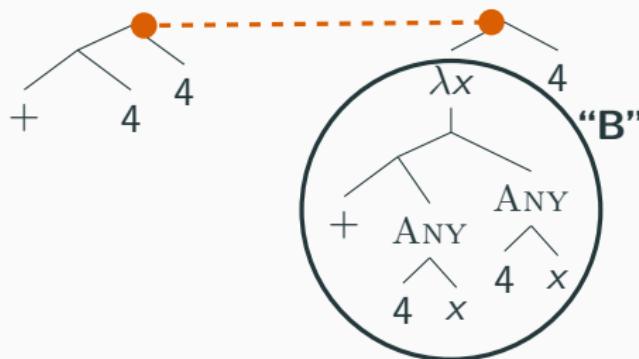
— semantic equivalence

ANY nondeterministic choice



A intersect B:

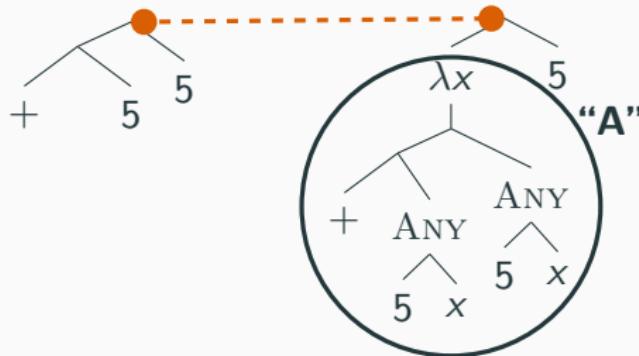
λx
 + x
 (lambda (x) (+ x x))
 = double



legend

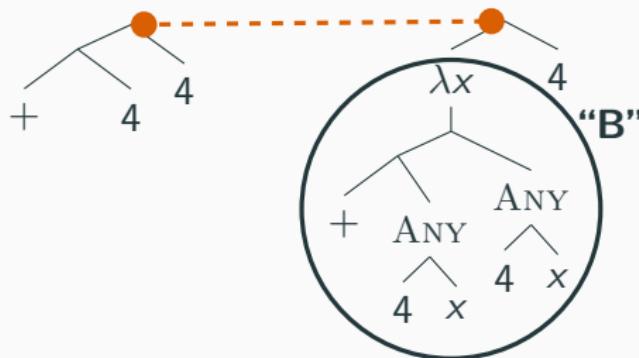
— dashed orange line semantic equivalence

ANY nondeterministic choice



A intersect B:

$$\begin{array}{c}
 \lambda x \\
 / \quad \backslash \\
 + \quad x \\
 \underbrace{\quad}_{(\text{lambda } (x) \ (\text{+ } x \ x))} \\
 = \text{double}
 \end{array}$$



w/o double	w/ double
$(+ \ 5 \ 5)$	$(\text{double } 5)$
$(+ \ 4 \ 4)$	$(\text{double } 4)$
$(+ \ 3 \ 3)$	$(\text{double } 3)$
...	

legend

— semantic equivalence
ANY nondeterministic choice

Abstraction Sleep: Growing the library via refactoring

Task: $[1\ 2\ 3] \rightarrow [2\ 4\ 6]$
 $[4\ 3\ 4] \rightarrow [8\ 6\ 8]$

Wake: program search

```
(Y (λ (r 1) (if (nil? 1) nil  
           (cons (+ (car 1) (car 1))  
                 (r (cdr 1)))))))
```

Task: $[1\ 2\ 3] \rightarrow [0\ 1\ 2]$
 $[4\ 3\ 4] \rightarrow [3\ 2\ 3]$

Wake: program search

```
(Y (λ (r 1) (if (nil? 1) nil  
           (cons (- (car 1) 1)  
                 (r (cdr 1)))))))
```

Abstraction Sleep: Growing the library via refactoring

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(Y (λ (r 1) (if (nil? 1) nil  
           (cons (- (car 1) 1)  
                  (r (cdr 1)))))))
```

refactor

$(10^{14}$ refactorings)

```
((λ (f) (Y (λ (r 1) (if (nil? 1)  
                           nil  
                           (cons (f (car 1))  
                                 (r (cdr 1)))))))  
  (λ (z) (+ z z)))
```

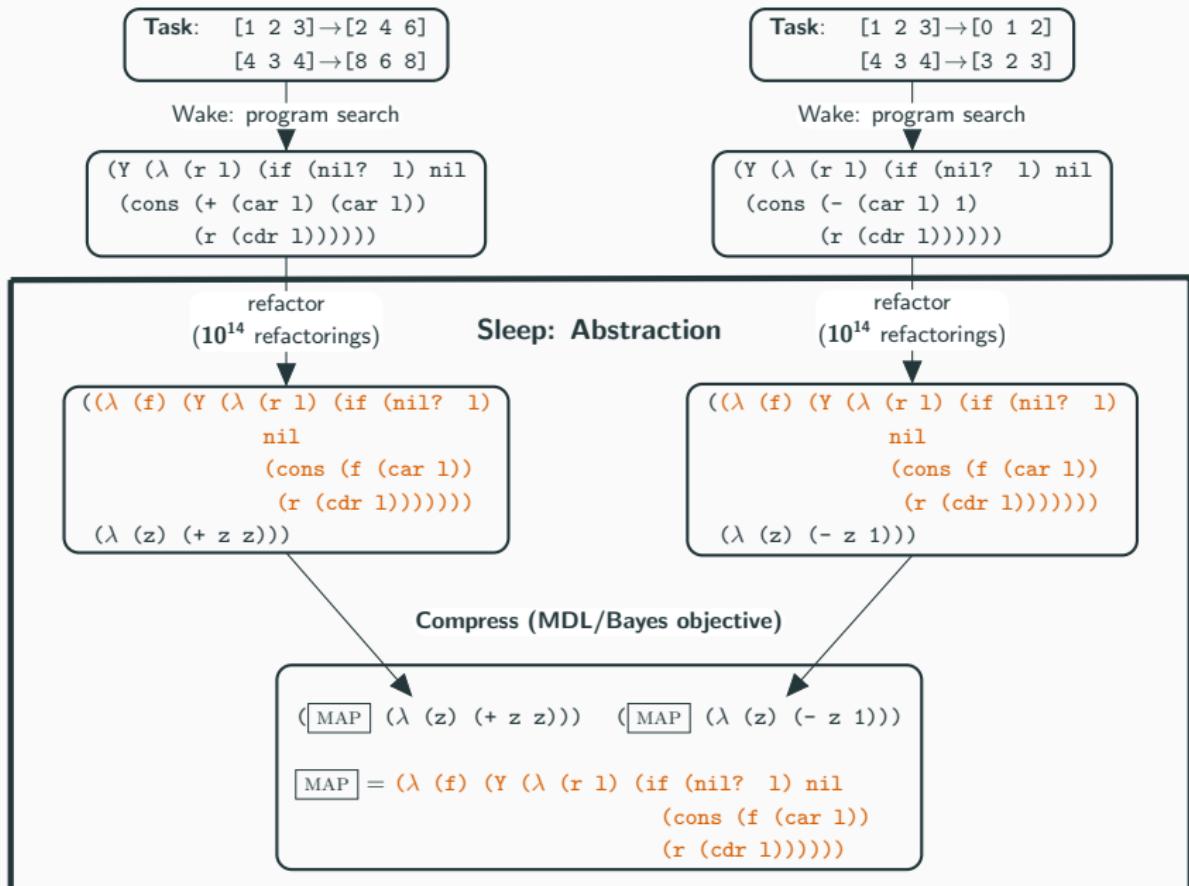
Sleep: Abstraction

refactor

$(10^{14}$ refactorings)

```
((λ (f) (Y (λ (r 1) (if (nil? 1)  
                           nil  
                           (cons (f (car 1))  
                                 (r (cdr 1)))))))  
  (λ (z) (- z 1)))
```

Abstraction Sleep: Growing the library via refactoring



Abstraction Sleep: Growing the library via refactoring

Task: $[1\ 2\ 3] \rightarrow [2\ 4\ 6]$
 $[4\ 3\ 4] \rightarrow [8\ 6\ 8]$

Wake: program search

```
(Y (λ (r 1) (if (nil? 1) nil  
           (cons (+ (car 1) (car 1))  
                  (r (cdr 1)))))))
```

Task: $[1\ 2\ 3] \rightarrow [0\ 1\ 2]$
 $[4\ 3\ 4] \rightarrow [3\ 2\ 3]$

Wake: program search

```
(Y (λ (r 1) (if (nil? 1) nil  
           (cons (- (car 1) 1)  
                  (r (cdr 1)))))))
```

these 10^{14} refactorings are represented in DreamCoder's exponentially more efficient refactoring data structure using 10^6 nodes, calculated in under 5min

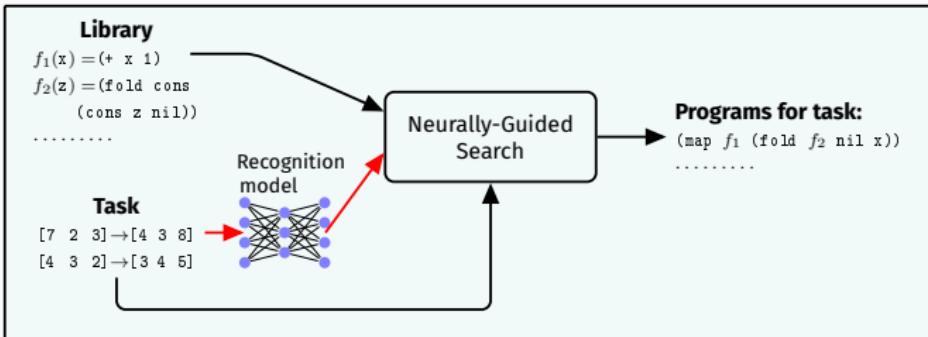
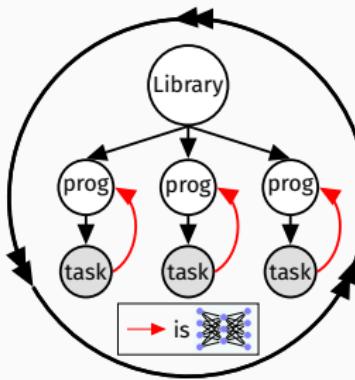
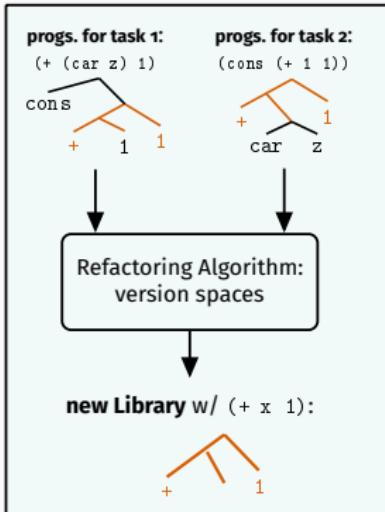
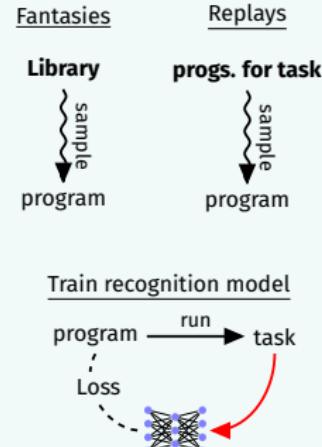
$(\lambda (z) (+ z z))$

$(\lambda (z) (- z 1))$

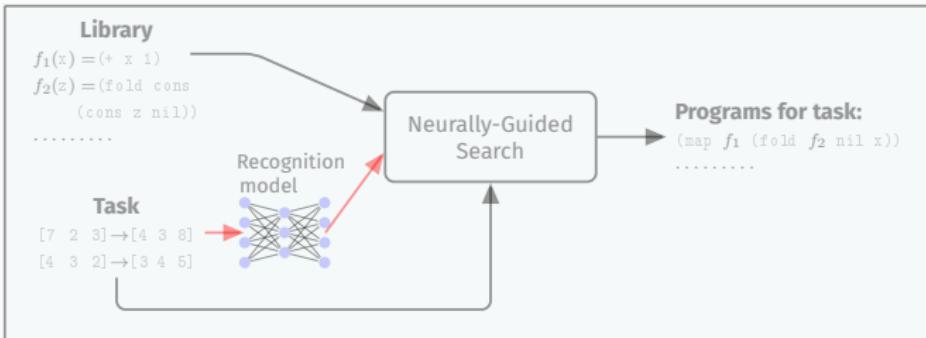
Compress (MDL/Bayes objective)

$(\boxed{\text{MAP}} (\lambda (z) (+ z z))) \quad (\boxed{\text{MAP}} (\lambda (z) (- z 1)))$

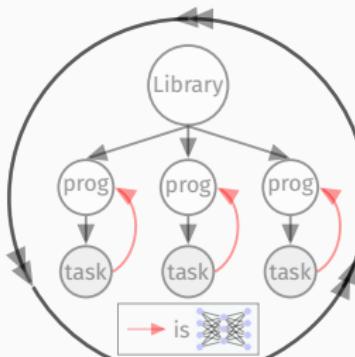
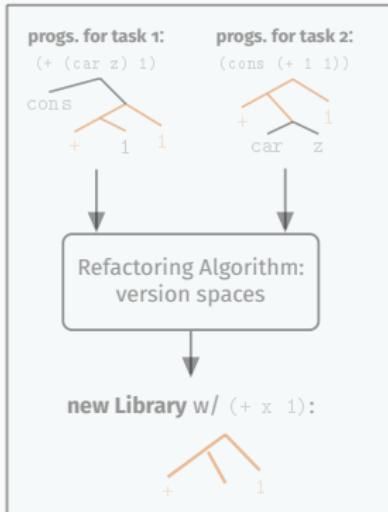
$\boxed{\text{MAP}} = (\lambda (f) (Y (\lambda (r 1) (if (nil? 1) nil
 (cons (f (car 1))
 (r (cdr 1)))))))$

WAKE**SLEEP: ABSTRACTION****SLEEP: DREAMING**

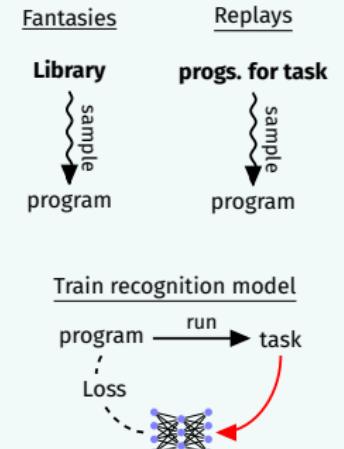
WAKE



SLEEP: ABSTRACTION



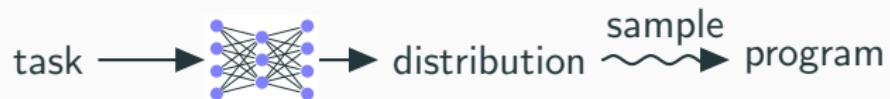
SLEEP: DREAMING



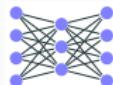
Neural recognition model guides search



Neural recognition model guides search



Neural recognition model guides search

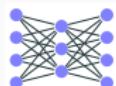


is a...

recurrent network (Devlin et al 2017)

unigram model (Menon et al 2013; Balog et al 2016)

Neural recognition model guides search

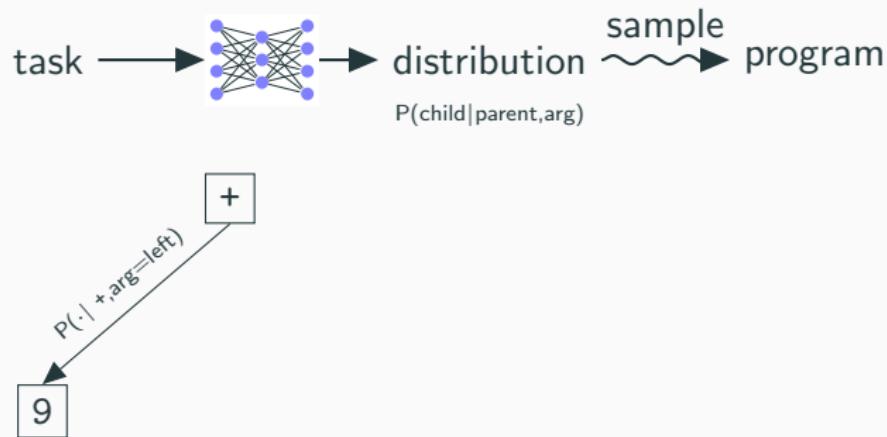


is a “**bigram**” model over syntax trees

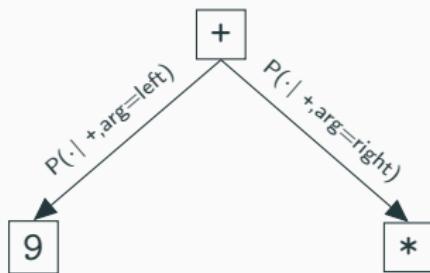
Neural recognition model guides search



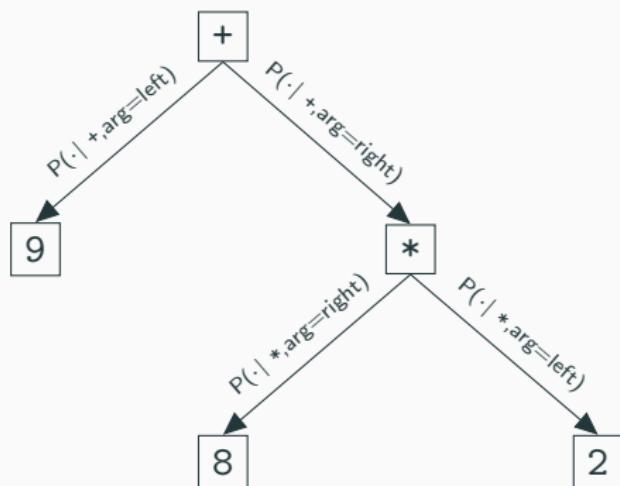
Neural recognition model guides search



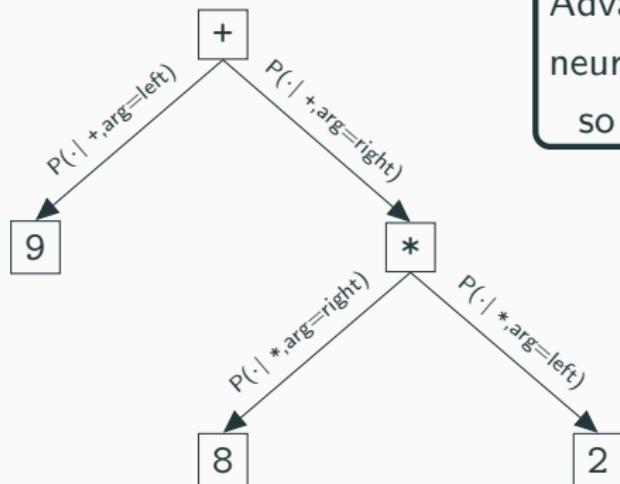
Neural recognition model guides search



Neural recognition model guides search

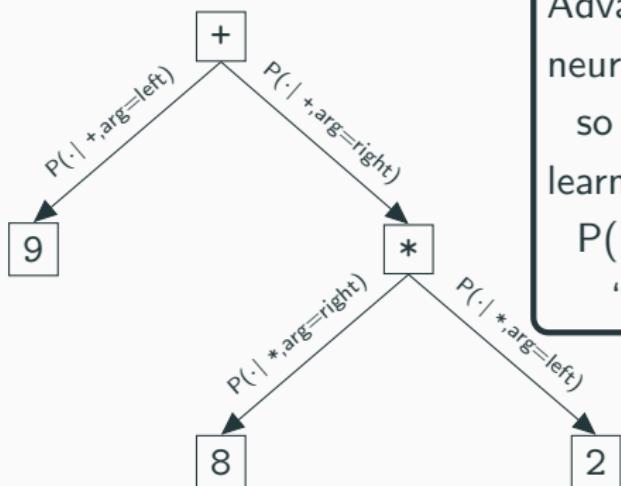


Neural recognition model guides search



Advantages:
neural net runs once per task,
so CPU bottlenecks instead of GPU

Neural recognition model guides search



Advantages:
neural net runs once per task,
so CPU bottlenecks instead of GPU
learns to break syntactic symmetries:
 $P(1|*,\text{arg}=left)=0.0$
“do not multiply by one”

DreamCoder Domains

List Processing

Sum List

[1 2 3] → 6

[4 6 8 1] → 17

Double

[1 2 3] → [2 4 6]

[4 5 1] → [8 10 2]

Text Editing

Abbreviate

Allen Newell → A.N.

Herb Simon → H.S.

Drop Last Three

shrdlu → shr

shakey → sha

Regexes

Phone numbers

(555) 867-5309

(650) 555-2368

Currency

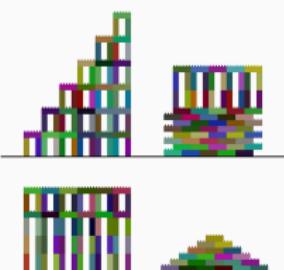
\$100.25

\$4.50

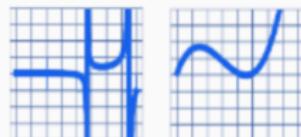
LOGO Graphics



Block Towers



Symbolic Regression



$$y = f(x)$$

Recursive Programming

Filter Red

[■■■■■■■■] → [■■■■]

[■■■■■■■■■■] → [■■■■■■■■]

[■■■■■■■■■■■■] → [■■■■■■■■■■]

Physical Laws

$$\vec{a} = \frac{1}{m} \sum_i \vec{F}_i$$

$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r}|^2} \hat{r}$$

DreamCoder Domains

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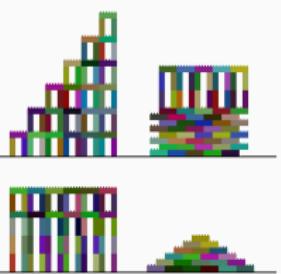
\$100.25

\$4.50

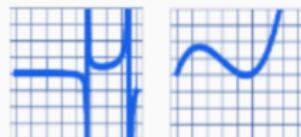
LOGO Graphics



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[■■■■■■■■■■■■] → [■■■■■■■■■■■■]

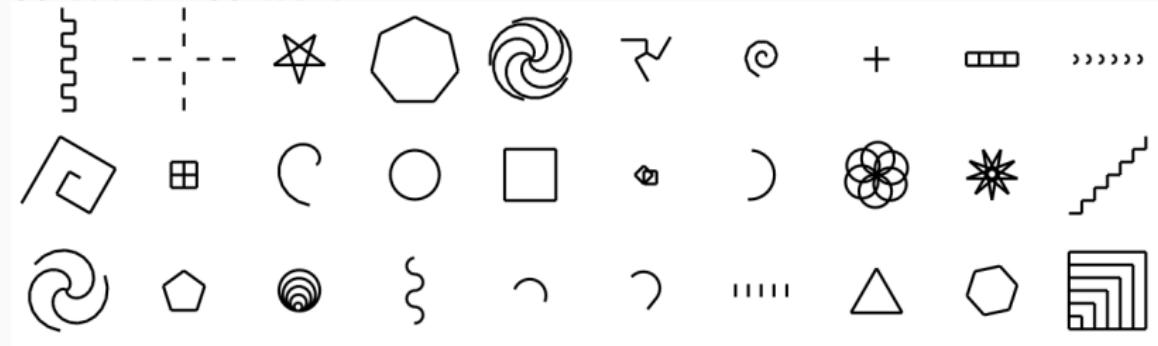
Physical Laws

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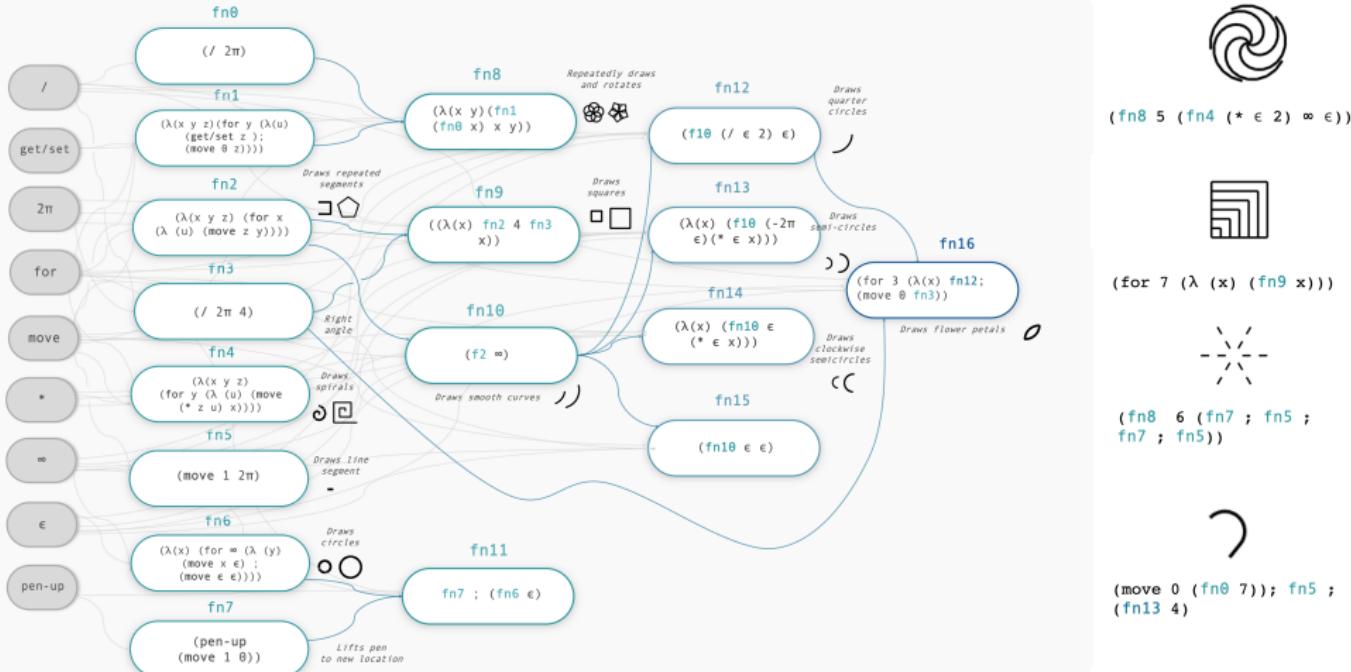
$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r}|^2} \hat{r}$$

LOGO Turtle Graphics

30 out of 160 tasks



LOGO Turtle Graphics – learning an interpretable library



(fn8 5 (fn4 (* ε 2) ∞))



(for 7 (λ (x) (fn9 x)))

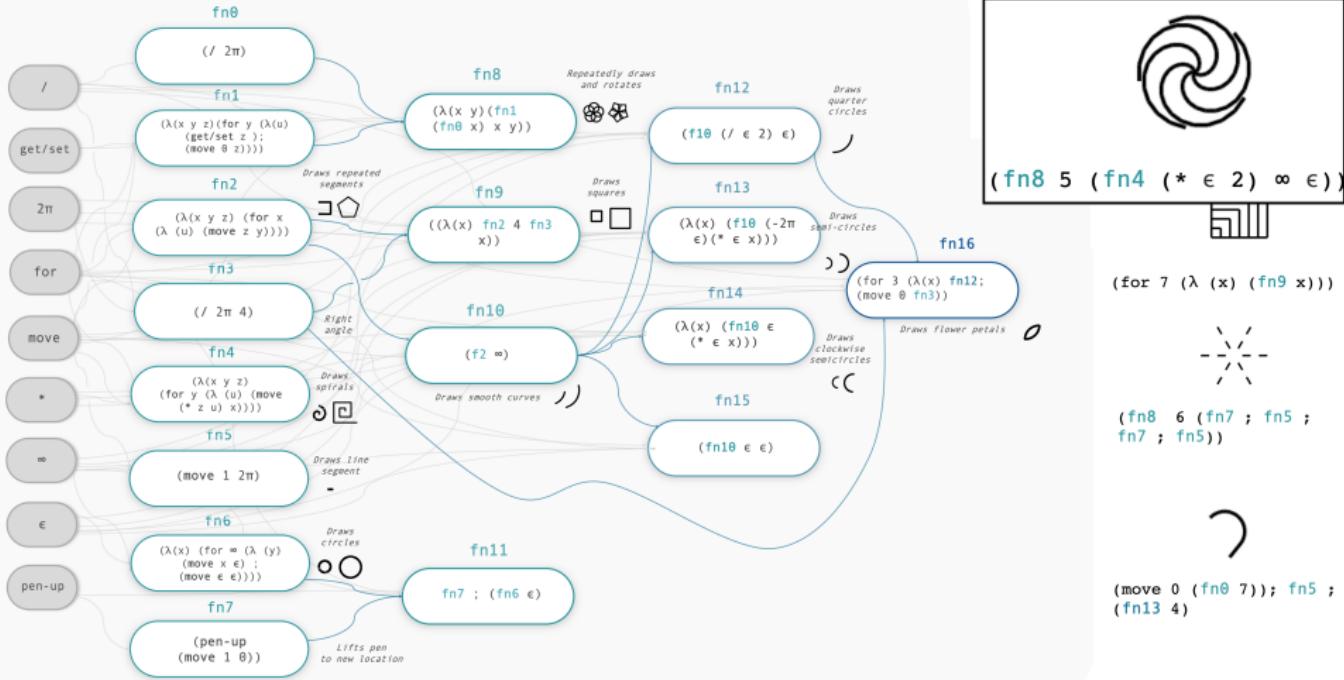


(fn8 6 (fn7 ; fn5 ; fn7 ; fn5))



(move 0 (fn0 7)); fn5 ;
(fn13 4)

LOGO Turtle Graphics – learning an interpretable library



```
(fn8 5 (fn4 (* ε 2) ∞ ε))
```



```
(for 7 (λ (x) (fn9 x)))
```

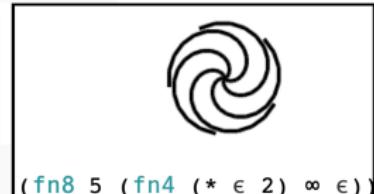
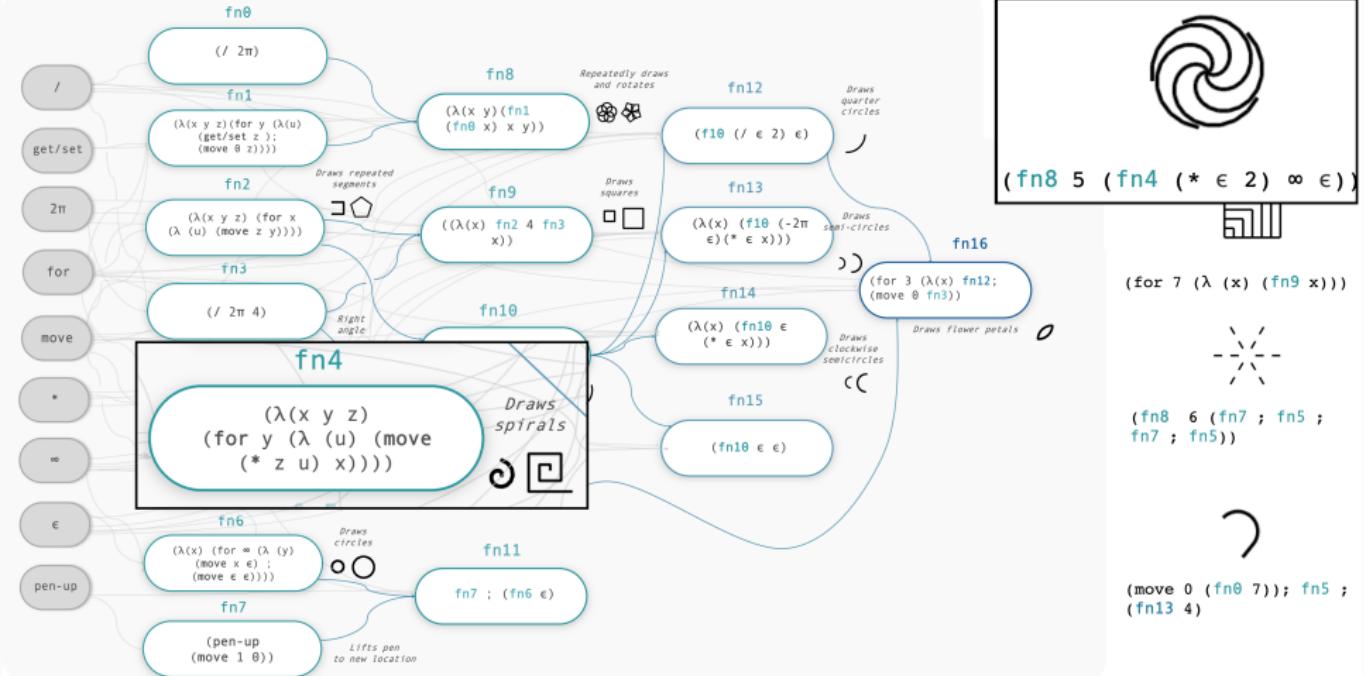


```
(fn8 6 (fn7 ; fn5 ; fn7 ; fn5))
```



```
(move 0 (fn0 7)); fn5 ; (fn13 4)
```

LOGO Turtle Graphics – learning an interpretable library



(fn8 5 (fn4 (* ∞ 2) ∞))



(for 7 ($\lambda(x)$ (fn9 x)))

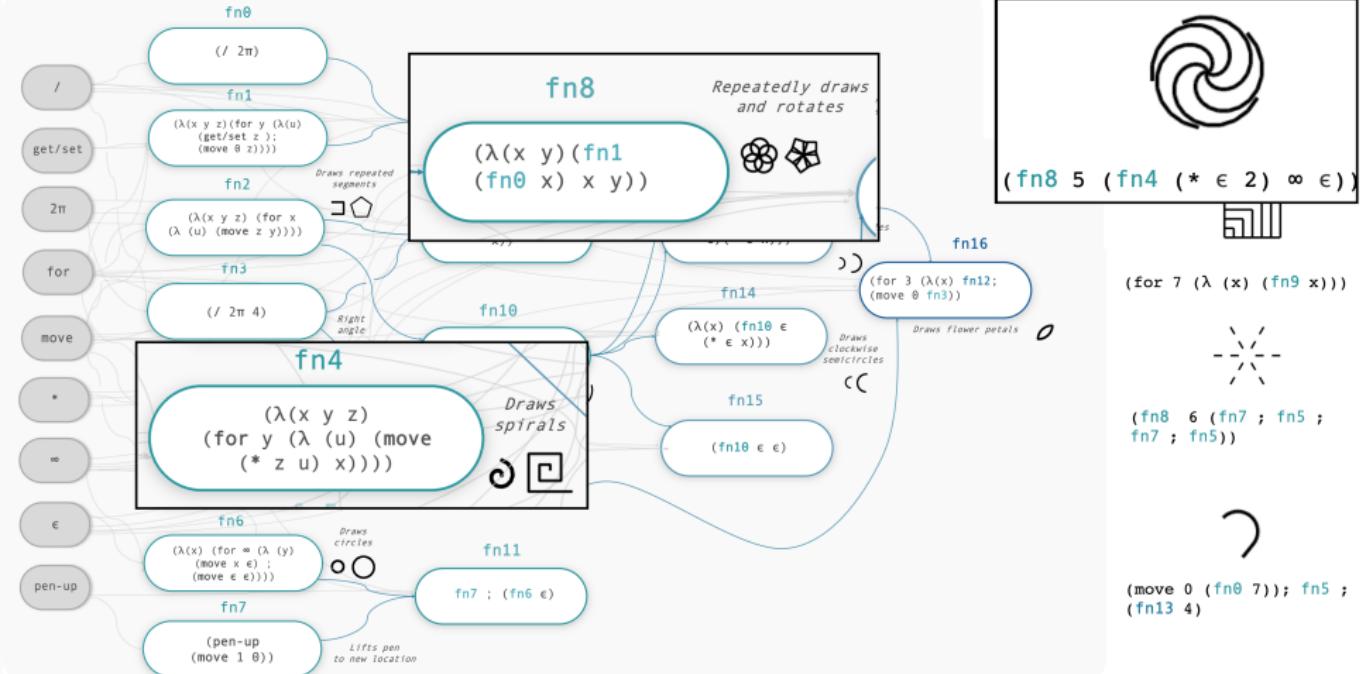


(fn8 6 (fn7 ; fn5 ; fn7 ; fn5))



(move 0 (fn0 7)); fn5 ; (fn13 4)

LOGO Turtle Graphics – learning an interpretable library



```
(fn8 5 (fn4 (* ∈ 2) ∞ ∈))
```



```
(for 7 (λ (x) (fn9 x)))
```

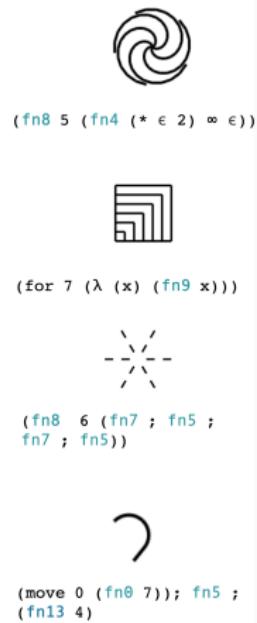
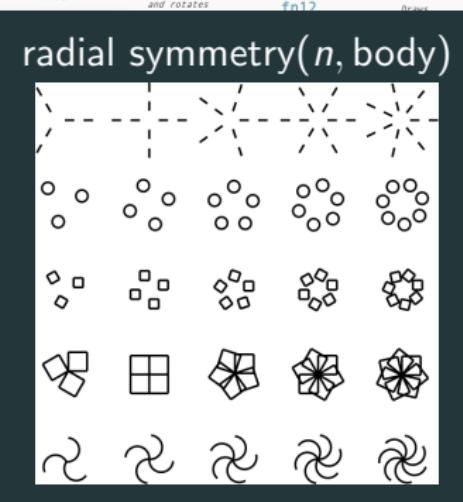
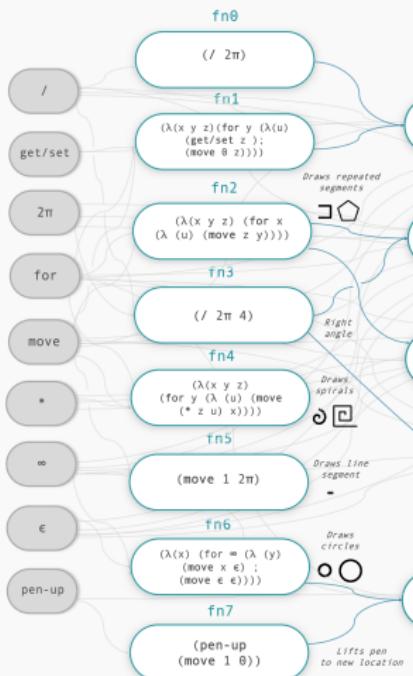


```
(fn8 6 (fn7 ; fn5 ; fn7 ; fn5))
```

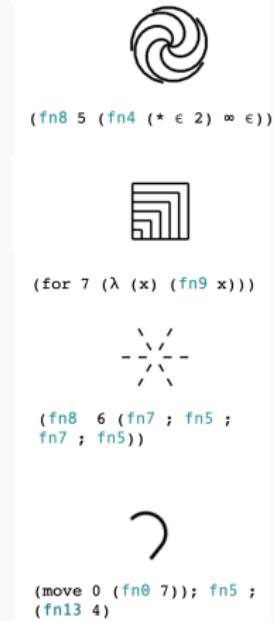
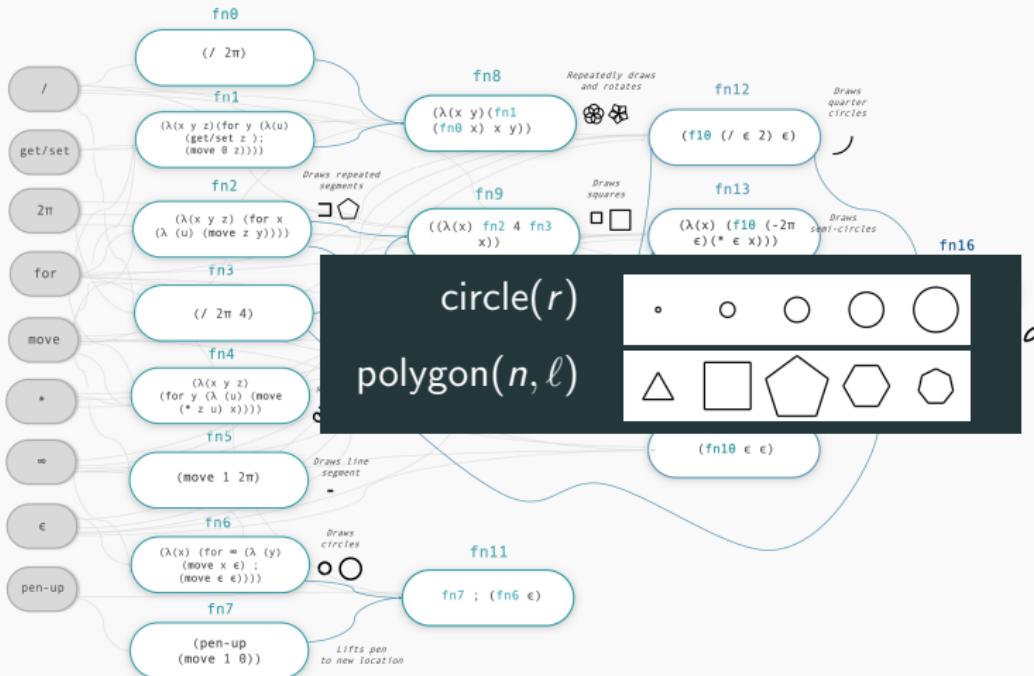


```
(move 0 (fn0 7)); fn5 ; (fn13 4)
```

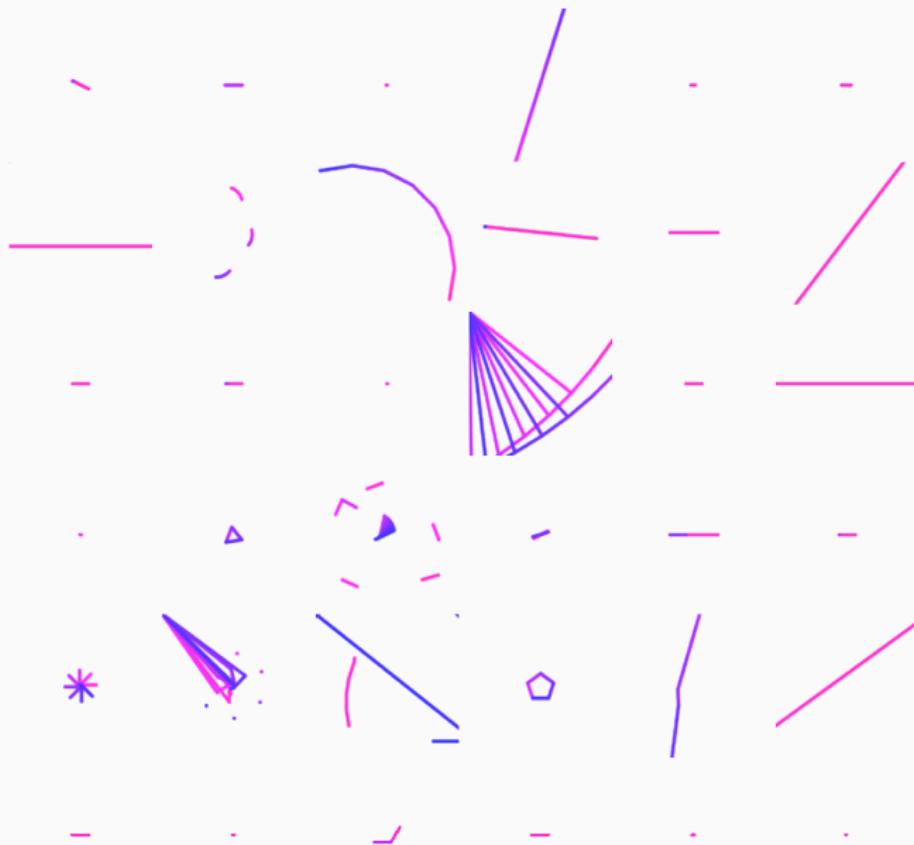
LOGO Turtle Graphics – learning an interpretable library



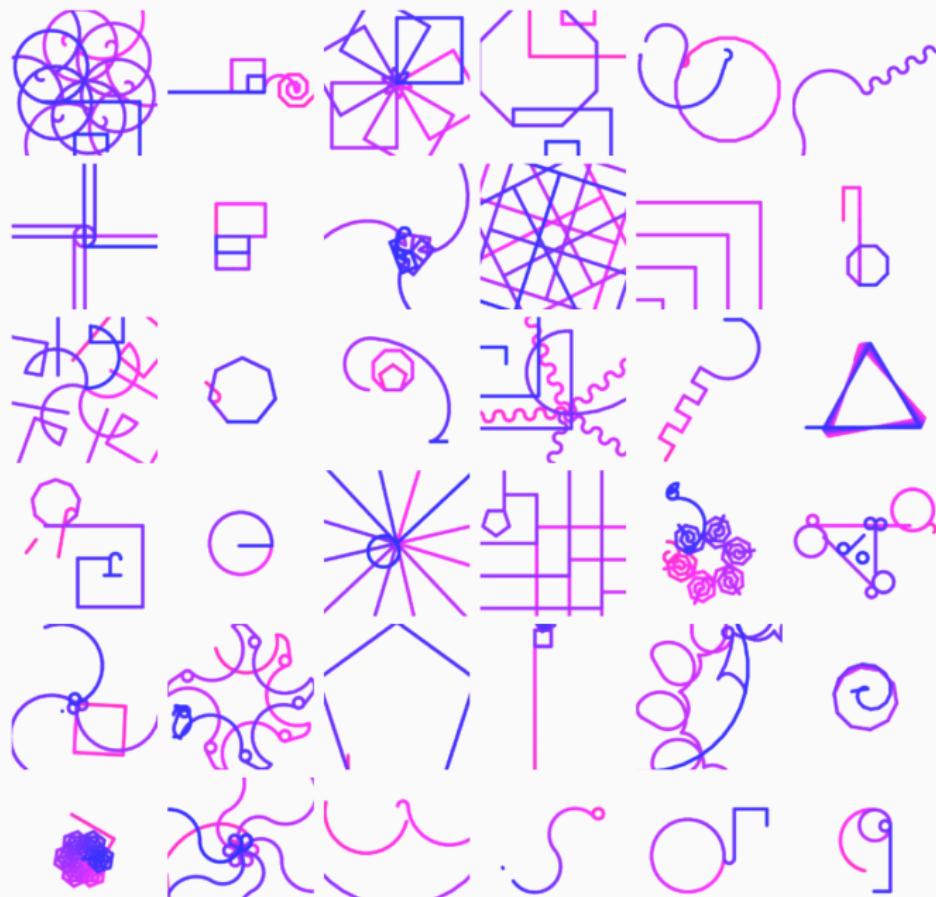
LOGO Turtle Graphics – learning an interpretable library



What does DreamCoder dream of? (before learning)

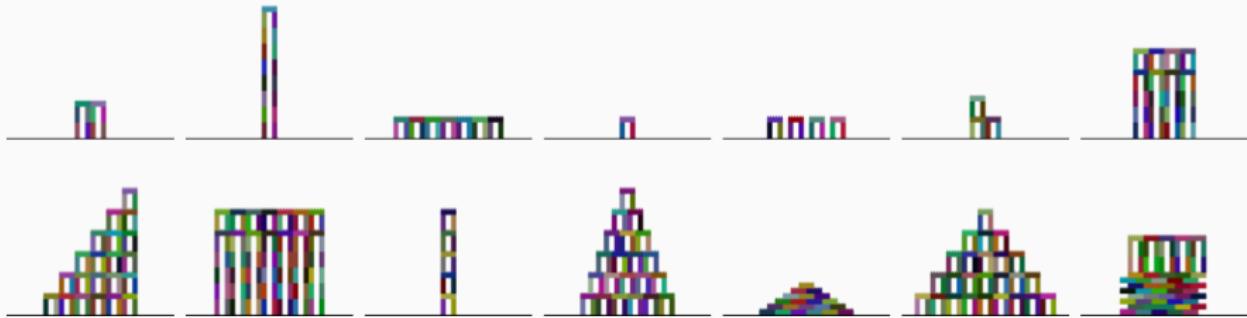


What does DreamCoder dream of? (after learning)



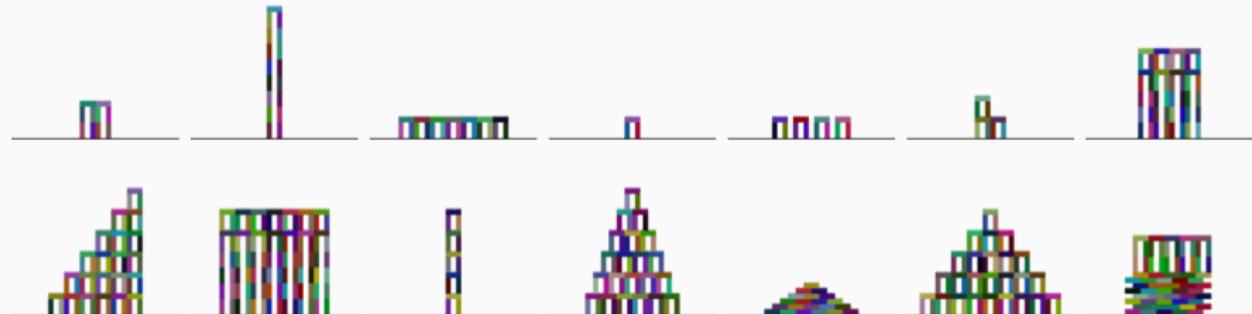
Planning to build towers

example tasks (112 total)

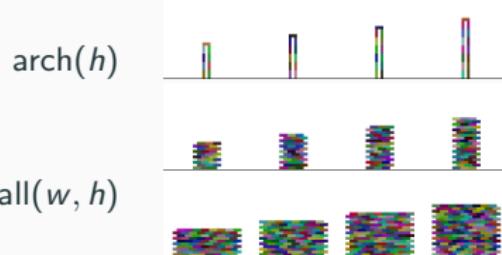


Planning to build towers

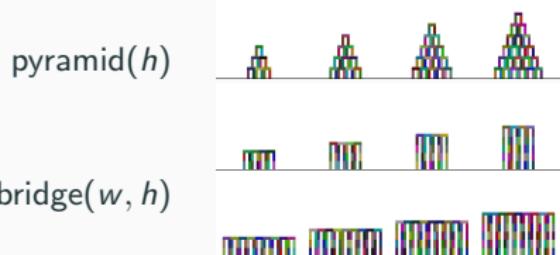
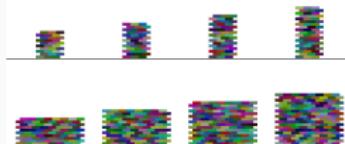
example tasks (112 total)



learned library routines (≈ 20 total)



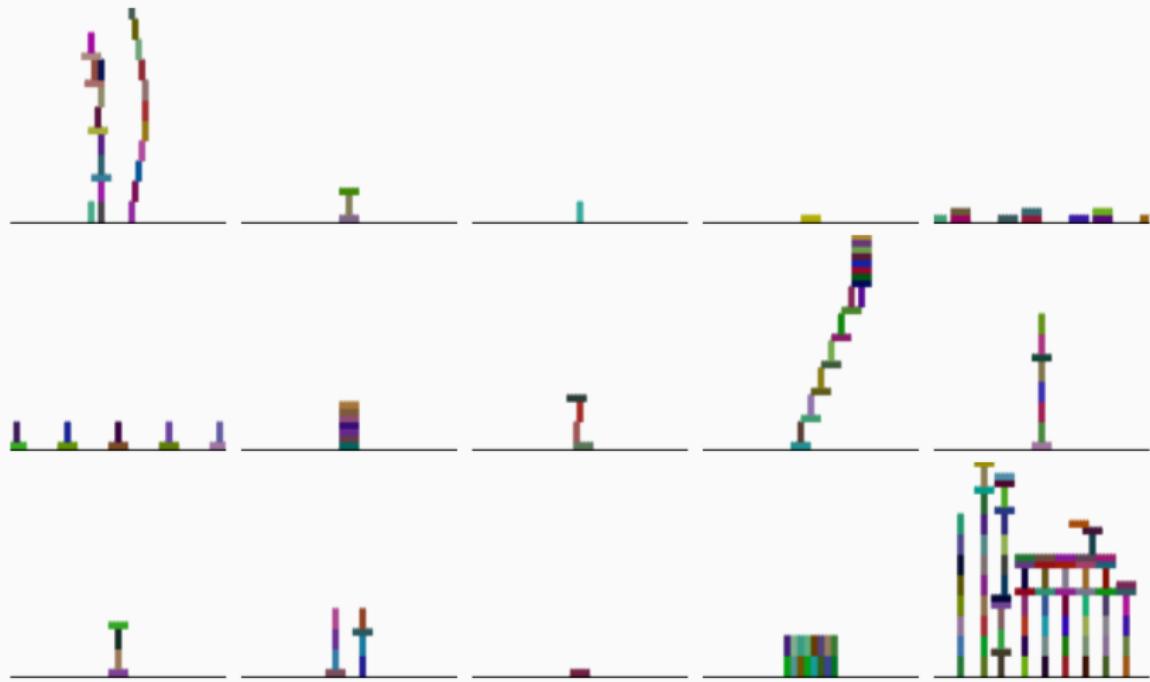
wall(w, h)



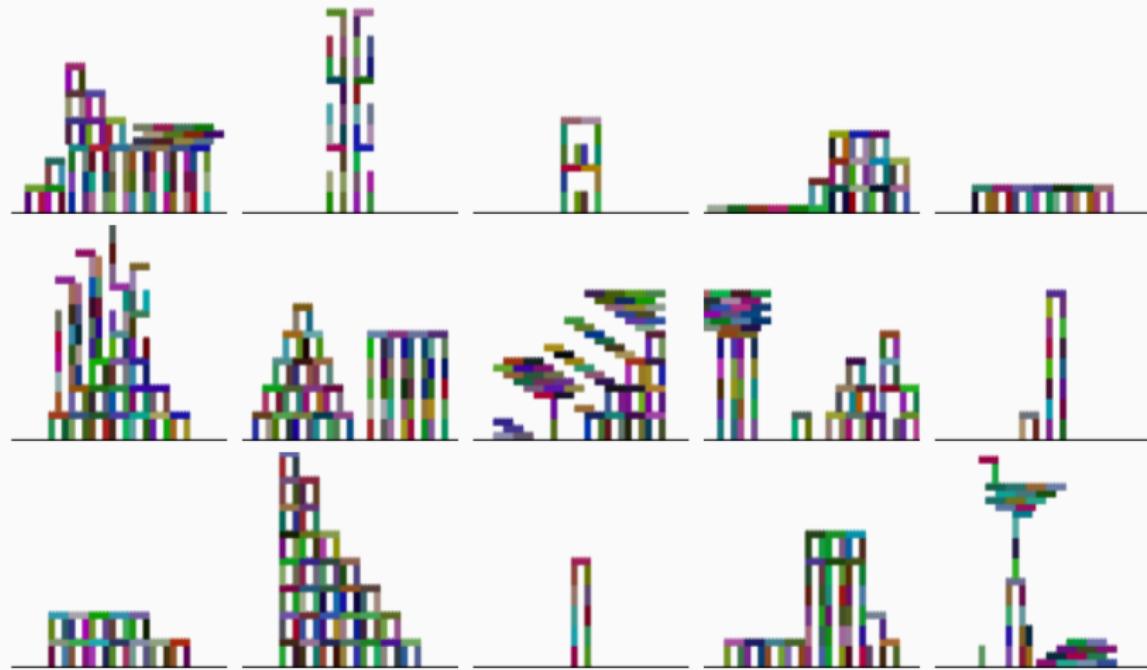
bridge(w, h)



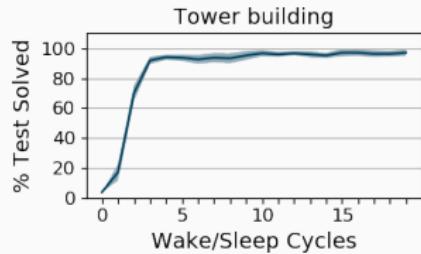
Dreams before learning



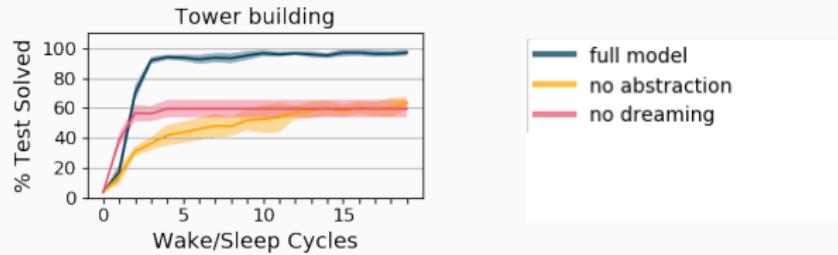
Dreams after learning



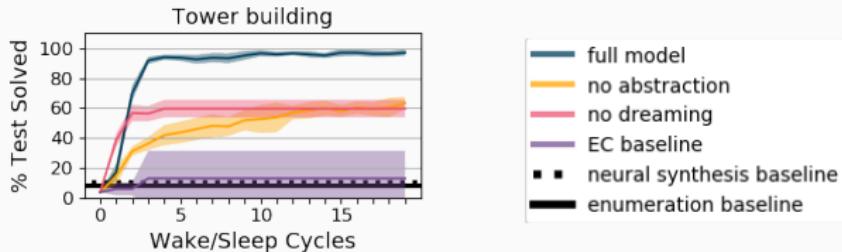
Learning dynamics



Learning dynamics

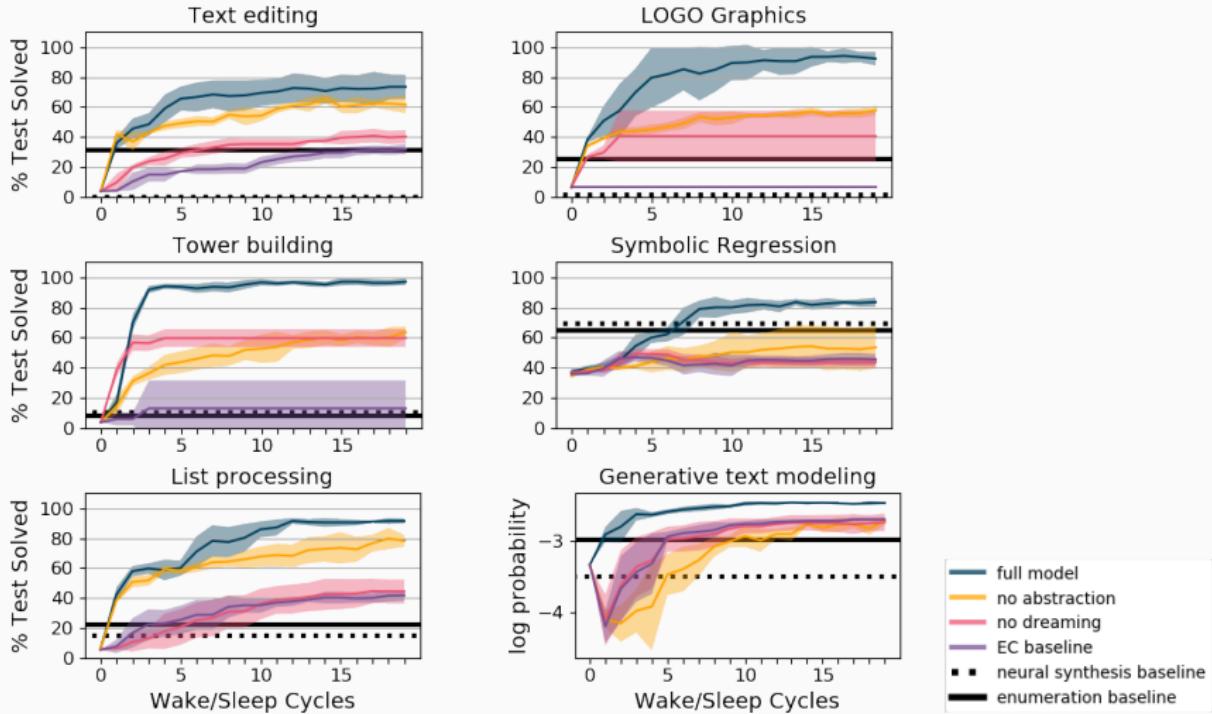


Learning dynamics



baselines: Exploration-Compression, EC [Dechter et al. 2013]
neural program synthesis, RobustFill [Devlin et al. 2017]
24 hours of brute-force enumeration

Learning dynamics



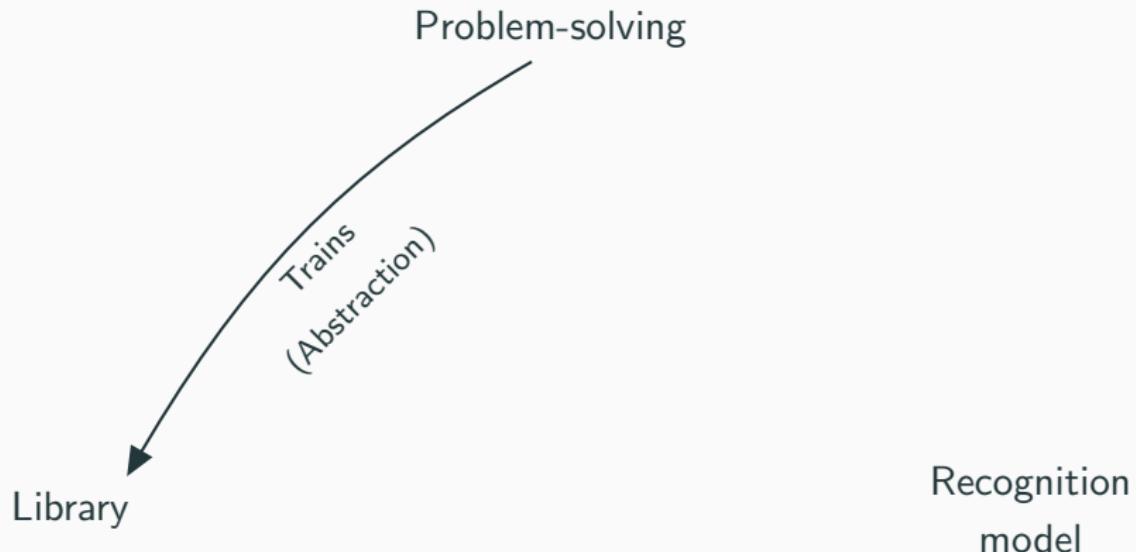
Synergy between dreaming and library learning

Problem-solving

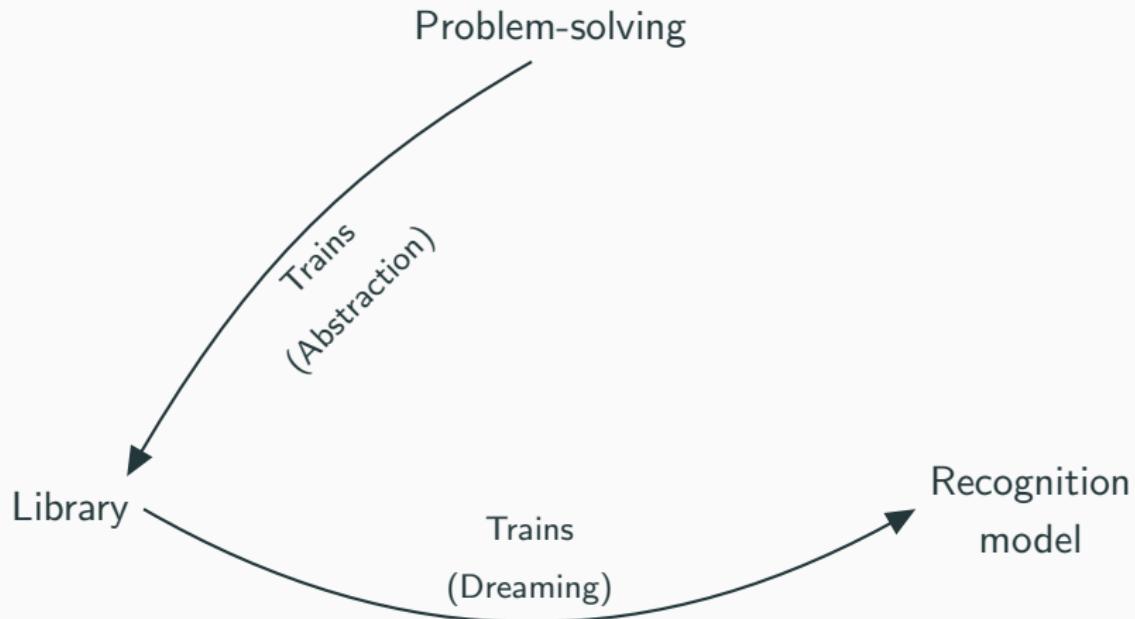
Library

Recognition
model

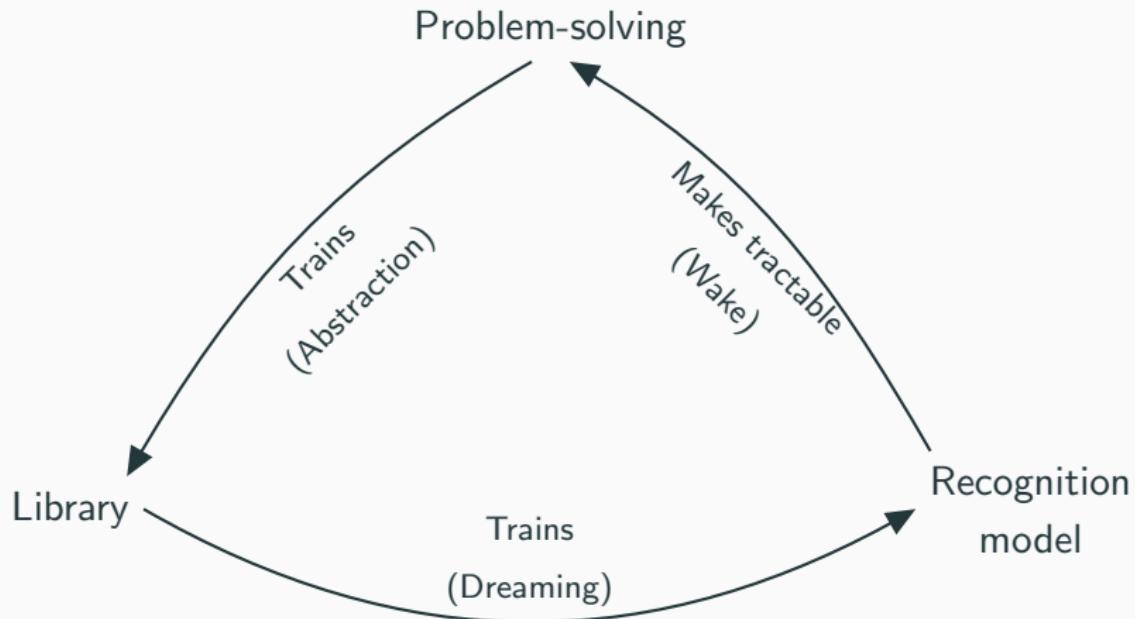
Synergy between dreaming and library learning



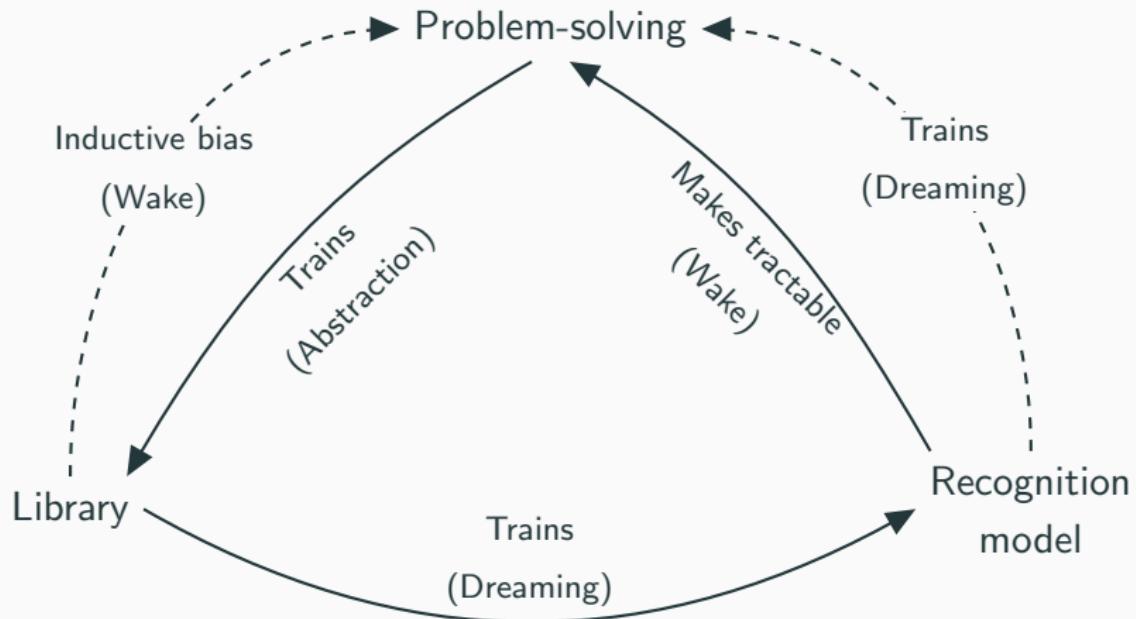
Synergy between dreaming and library learning



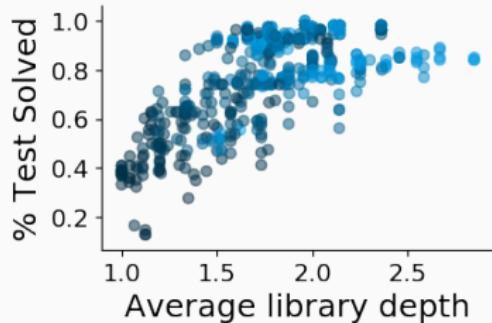
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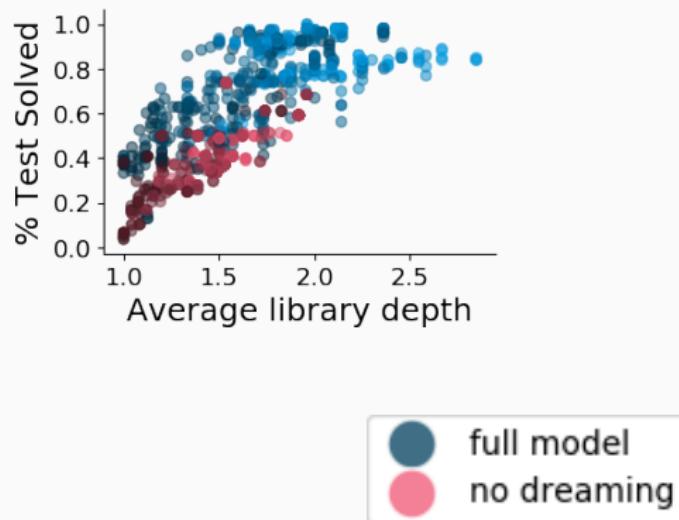
Evidence for dreaming bootstrapping better libraries



Darker: Early in learning

Brighter: Later in learning

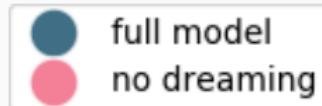
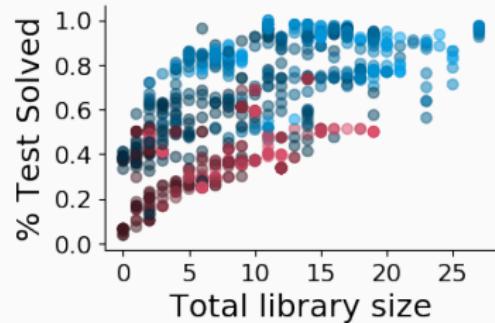
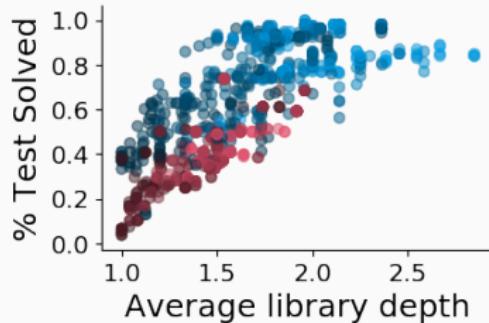
Evidence for dreaming bootstrapping better libraries



Darker: Early in learning

Brighter: Later in learning

Evidence for dreaming bootstrapping better libraries



Darker: Early in learning

Brighter: Later in learning

From learning libraries,
to learning languages

From learning libraries,
to learning languages

modern functional programming → physics

From learning libraries,
to learning languages

1950's Lisp → modern functional programming → physics

Physics Formula Sheet

Mechanics

$x = x_0 + v_{x0}t + \frac{1}{2}a_xt^2$	$a_t = \frac{v^2}{r}$	$ \vec{F}_{\text{spring}} = k \vec{x} $
$v = v_0 + at$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$	$P E_{\text{spring}} = \frac{1}{2}kx^2$
$v_s^2 - v_{s0}^2 = 2a(x - x_0)$	$\omega = \omega_0 + \alpha t$	$T_{\text{spring}} = 2\pi \sqrt{\frac{m}{k}}$
$\bar{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{\text{net}}}{m}$	$T = \frac{2\pi}{\omega} = \frac{1}{f}$	$T_{\text{pendulum}} = 2\pi \sqrt{\frac{L}{g}}$
$ \vec{F}_{\text{friction}} \leq \mu \vec{F}_{\text{Normal}} $	$v = f\lambda$	
$\bar{p} = m\bar{v}$	$x = A \cos(2\pi ft)$	$ \vec{F}_{\text{gravity}} = G \frac{m_1 m_2}{r^2}$
$\Delta \bar{p} = \vec{F} \Delta t$	$\bar{a} = \frac{\sum \vec{F}}{I} = \frac{\vec{F}_{\text{net}}}{I}$	$ \vec{F}_{\text{gravity}} = m\bar{g}$
$KE = \frac{1}{2}mv^2$	$\vec{r} = r \times \vec{F}$	$P E_{\text{gravity}} = -G \frac{m_1 m_2}{r}$
$\Delta PE = mg\Delta y$	$L = I\omega$	$p = \frac{m}{V}$
$\Delta E = W = Fd \cos\theta$	$\Delta L = \tau \Delta t$	$KE = \frac{1}{2}I\omega^2$

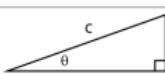
Electricity

$ \vec{F}_E = k \left \frac{q_1 q_2}{r^2} \right $	$\Delta V = IR$	$R = \frac{\rho \ell}{A}$
$I = \frac{\Delta q}{\Delta t}$		$P = I \Delta V$
$R_{\text{series}} = R_1 + R_2 + \dots + R_n$	$\frac{1}{R_{\text{parallel}}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}$	

Geometry

Rectangle	$A = bh$	Rectangular Solid	$V = lwh$	Triangle	$A = \frac{1}{2}bh$
Circle	$A = \pi r^2$	Cylinder	$V = \pi r^2 l$	Sphere	$V = \frac{4}{3}\pi r^3$
	$C = 2\pi r$		$S = 2\pi rl + 2\pi r^2$		$S = 4\pi r^2$

Trigonometry



$$\begin{array}{ccccccc} c & & & & & & \\ & a & & & & & \\ & & & & & & \\ & & & & \theta & & \\ & & & & & & \\ & & & & \square & & \end{array}$$

$$c^2 = a^2 + b^2 \quad \sin\theta = \frac{a}{c} \quad \cos\theta = \frac{b}{c} \quad \tan\theta = \frac{a}{b}$$

Variables

a = acceleration
 A = amplitude
 A = Area
 b = base length
 C = circumference
 d = distance
 E = energy
 f = frequency
 F = force
 h = height
 I = current
 I = rotational inertia
 KE = kinetic energy
 k = spring constant
 L = angular momentum
 ℓ = length
 m = mass
 P = power
 p = momentum
 q = charge
 r = radius
 R = resistance
 S = surface area
 T = period
 t = time
 PE = potential energy
 V = electric potential
 V = volume
 v = velocity
 w = width
 W = work
 x = position
 y = height
 α = angular acceleration
 λ = wavelength
 μ = coefficient of friction

Growing languages for vector algebra and physics

Initial Primitives

map
zip

cons

empty

cdr

power

fold

car

+

-

*

/

0

1

π

Physics Equations

Newton's Second Law

$$\vec{a} = \frac{1}{m} \sum_l \vec{F}_l$$

Parallel Resistors

$$R_{total} = \left(\sum_i \frac{1}{R_i} \right)^{-1}$$

Work

$$U = \vec{F} \cdot \vec{d}$$

Force in a Magnetic Field

$$|\vec{F}| = q |\vec{v} \times \vec{B}|$$

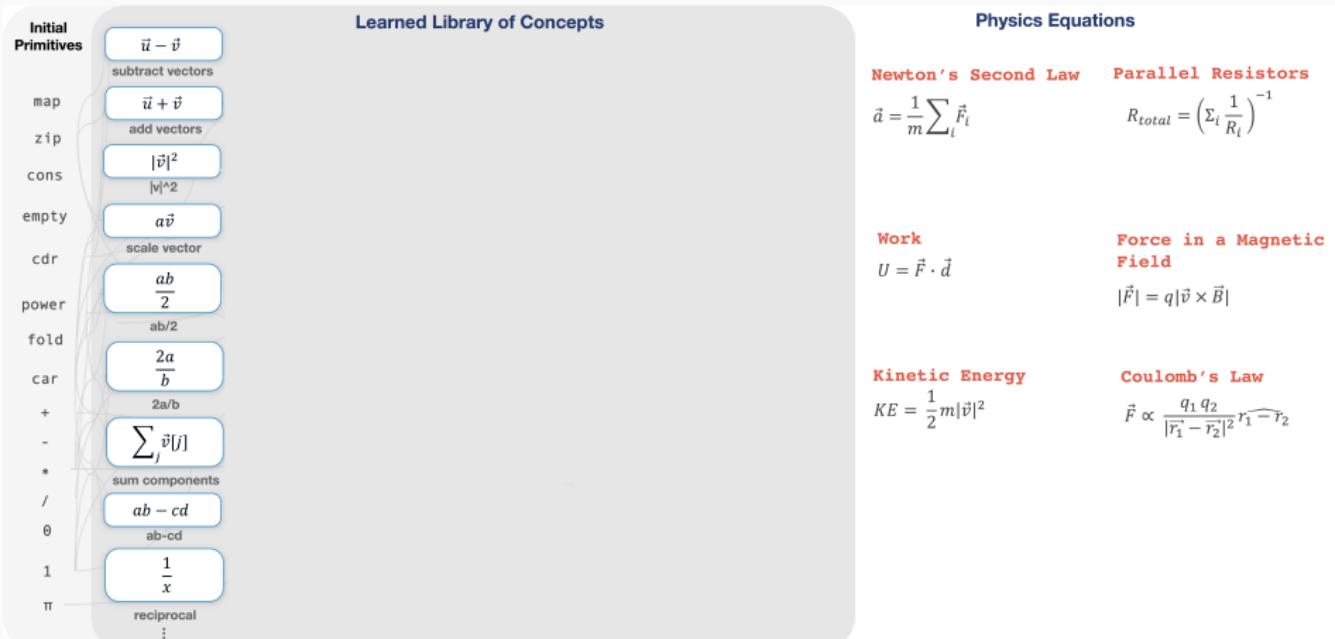
Kinetic Energy

$$KE = \frac{1}{2} m |\vec{v}|^2$$

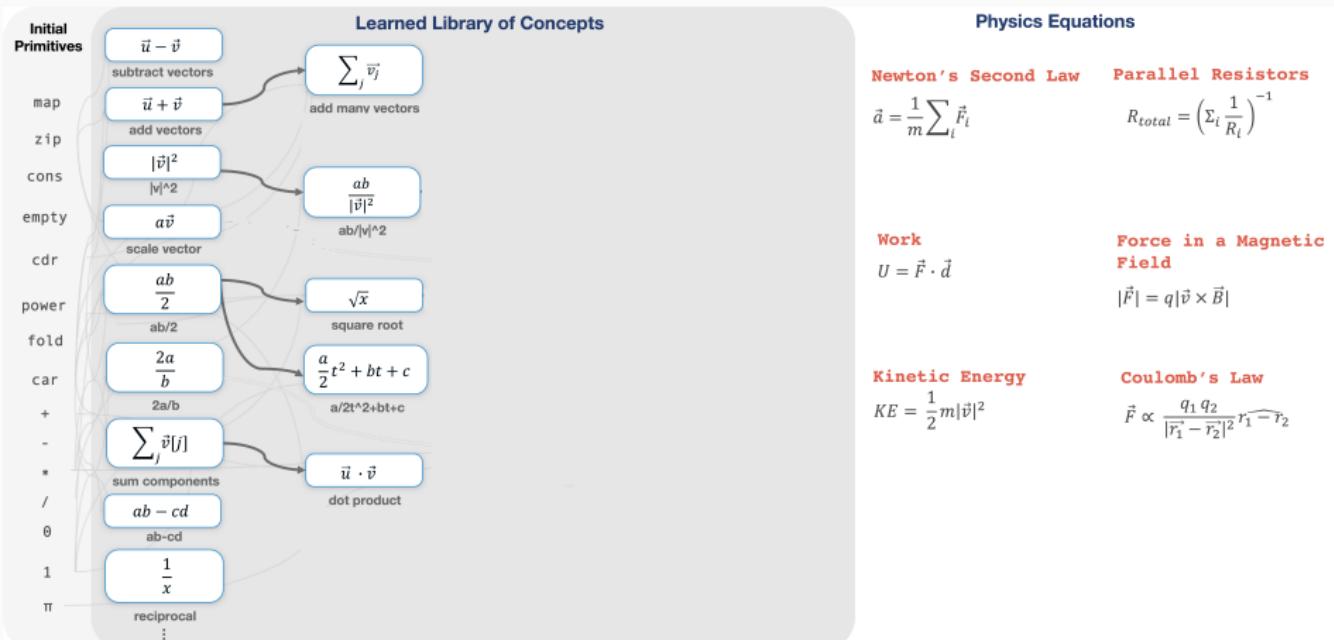
Coulomb's Law

$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^2} \hat{r}_1 - \hat{r}_2$$

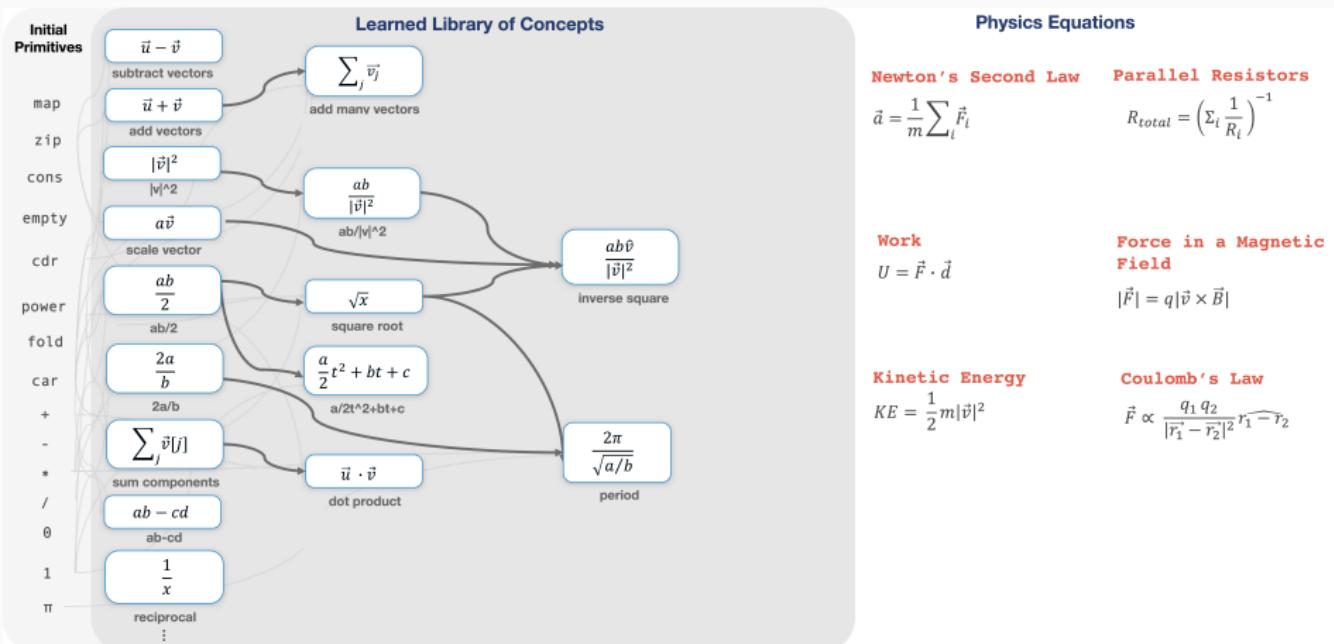
Growing languages for vector algebra and physics



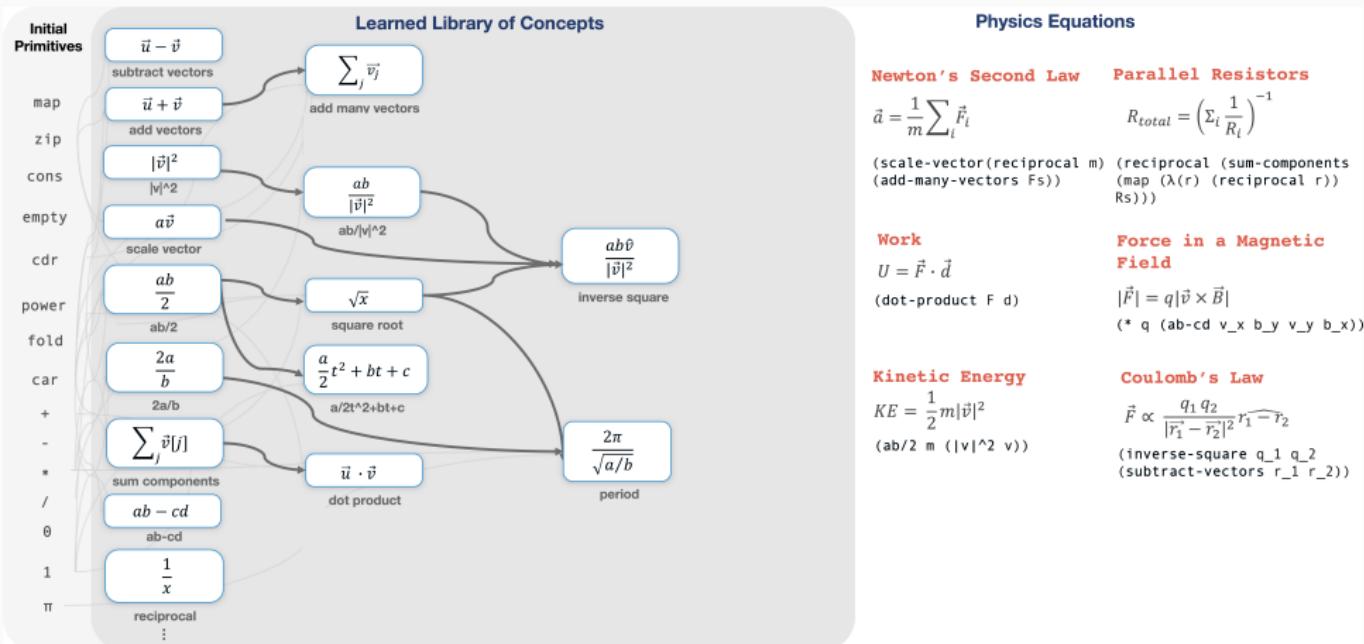
Growing languages for vector algebra and physics



Growing languages for vector algebra and physics



Growing languages for vector algebra and physics



Growing languages for vector algebra and physics

Initial
Primitives

$\vec{u} - \vec{v}$
subtract vectors

map

$\vec{u} + \vec{v}$
add vectors

zip

cons

empty

cdr

power

fold

car

+

-

*

/

0

1

π

Learned Library of Concepts

$$\sum_j \vec{v}_j$$

add many vectors

$$|\vec{v}|^2$$

$$|v|^2$$

$$ab$$

$$|v|^2$$

$$ab\vec{v}$$

scale vector

$$\frac{ab}{2}$$

$$ab/2$$

$$\sqrt{x}$$

square root

$$\frac{a}{2}t^2 + bt + c$$

$$a/2t^2+bt+c$$

$$\sum \vec{v}[j]$$

sum components

$$ab - cd$$

$$ab - cd$$

$$\frac{1}{x}$$

reciprocal

:

$$\frac{ab\hat{v}}{|v|^2}$$

inverse square

$$\frac{2\pi}{\sqrt{a/b}}$$

period

$$\vec{u} \cdot \vec{v}$$

dot product

Physics Equations

Newton's Second Law

$$\vec{a} = \frac{1}{m} \sum_l \vec{F}_l$$

Parallel Resistors

$$R_{total} = \left(\sum_i \frac{1}{R_i} \right)^{-1}$$

(scale-vector(reciprocal m) (reciprocal (sum-components (add-many-vectors Fs)))

(map (\lambda(r) (reciprocal r) Rs)))

Work

$$U = \vec{F} \cdot \vec{d}$$

(dot-product F d)

Force in a Magnetic Field

$$|\vec{F}| = q|\vec{v} \times \vec{B}|$$

(* q (ab-cd v_x b_y v_y b_x))

Kinetic Energy

$$KE = \frac{1}{2} m |\vec{v}|^2$$

(ab/2 m (|v|^2 v))

Coulomb's Law

$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^2} \widehat{\vec{r}_1 - \vec{r}_2}$$

(inverse-square q_1 q_2
(subtract-vectors r_1 r_2))

(\lambda (x y z u) (map (\lambda (v) (* (/ (* (power (/ (* x) (fold (zip z u (\lambda (w a) (- w a)))) \theta (\lambda (b c) (+ (* b b) c)))) (/ (* 1 1) (+ 1 1)))) y) (fold (zip z u (\lambda (d e) (- d e)))) \theta (\lambda (f g) (+ (* f f) g)))) v)) (zip z u (\lambda (h i) (- h i))))

Solution to Coulomb's Law if expressed in initial primitives

Growing a language for recursive programming

Initial Primitives

Y
combinator
cons
car
cdr
nil
if
nil?
+
-
0
1
=

Recursive Programming Algorithms

Stutter

[] → []
[] → []

Take every other

[] → []
[] → []

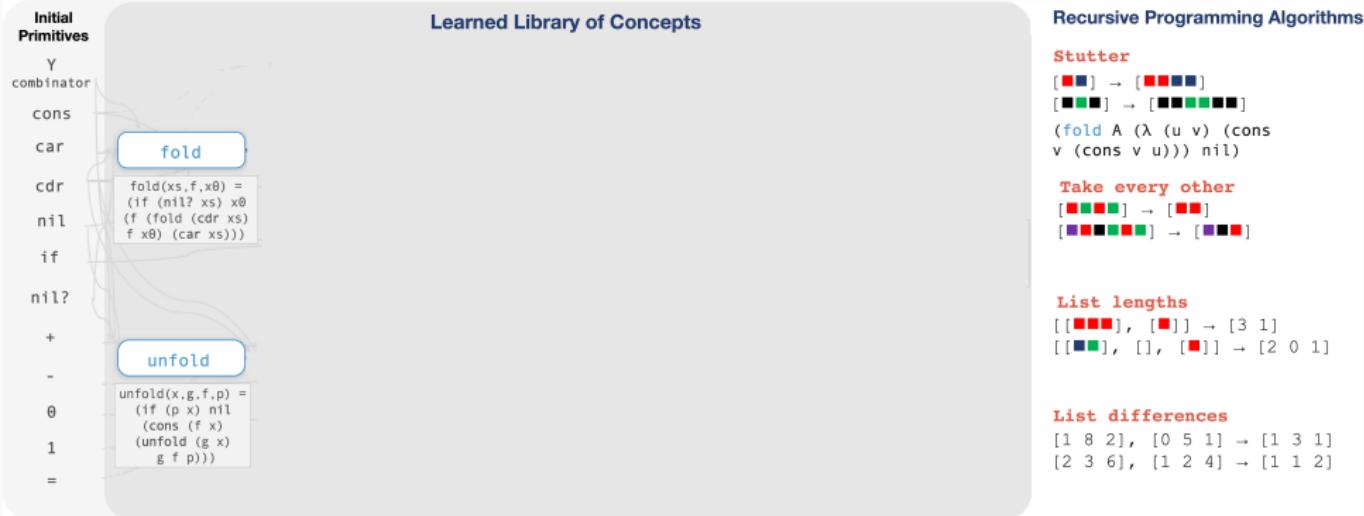
List lengths

[, []] → [3 1]
[[], [], []] → [2 0 1]

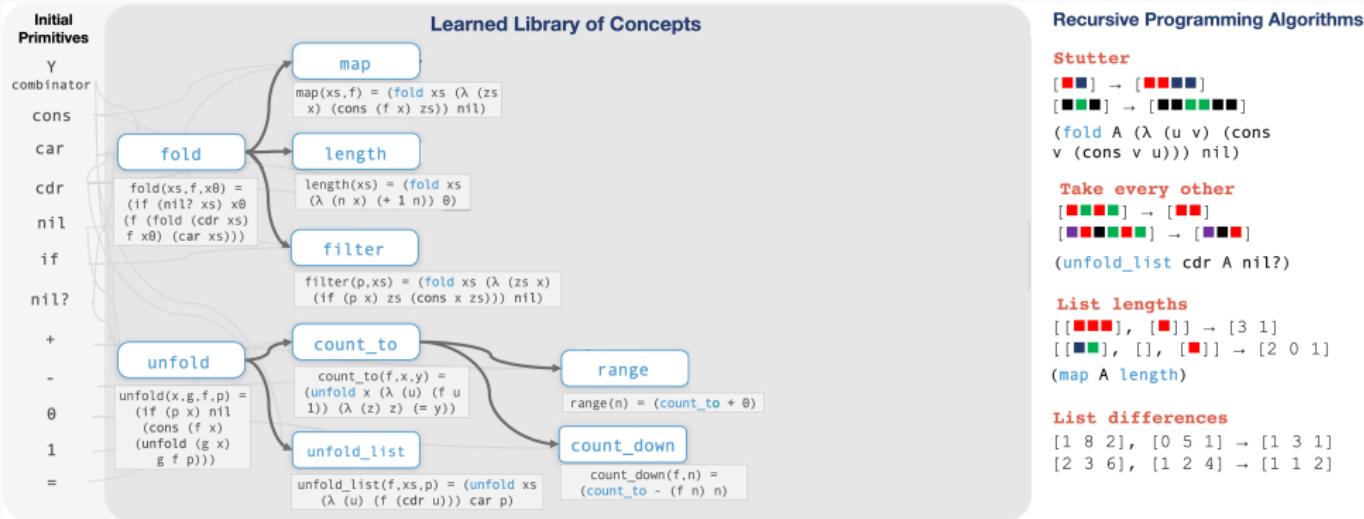
List differences

[1 8 2], [0 5 1] → [1 3 1]
[2 3 6], [1 2 4] → [1 1 2]

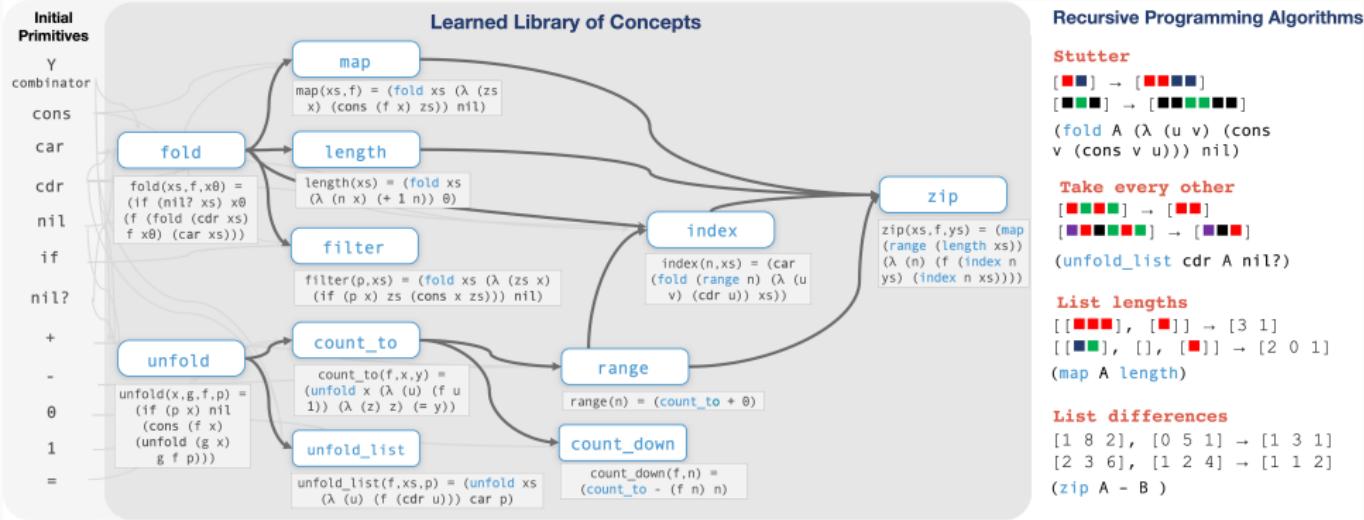
Growing a language for recursive programming



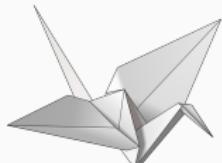
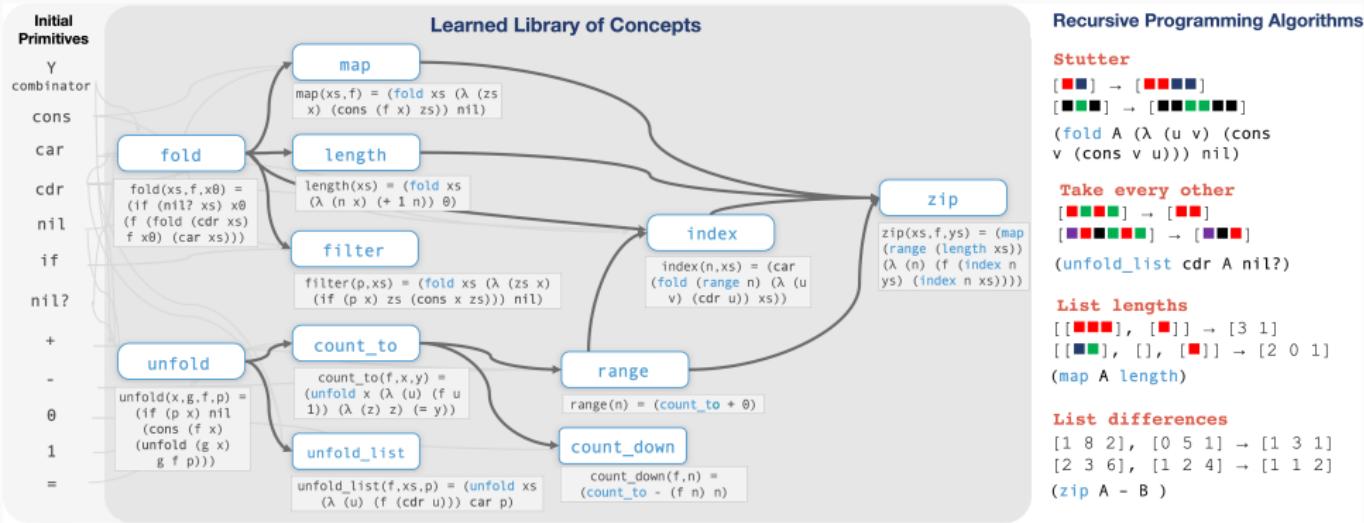
Growing a language for recursive programming



Growing a language for recursive programming

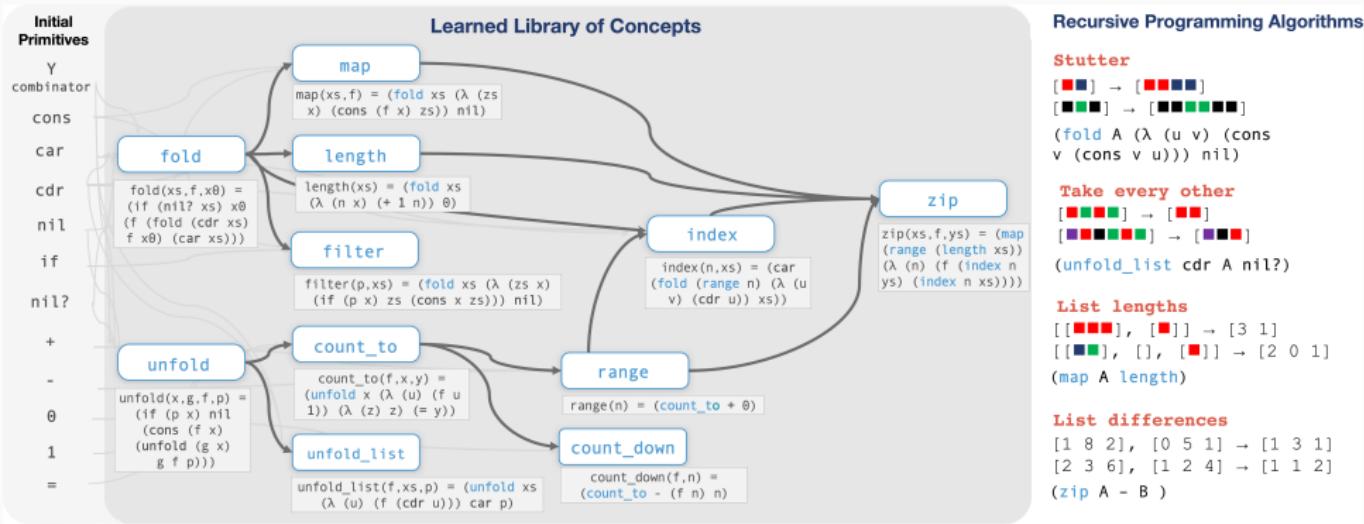


Growing a language for recursive programming



Origami Programming: Jeremy Gibbons, 2003

Growing a language for recursive programming



1 year of compute. 5 days on 64 CPUs.



Origami Programming: Jeremy Gibbons, 2003

Lessons

Symbols aren't necessarily interpretable. Flexibly grow the language based on experience to make it more powerful *and* more human understandable

Learning-from-scratch is possible in principle. Don't do it. But program induction makes it convenient to build in what we know how to build in, and then learn and adapt on top of that

Program Induction and learning to learn the future

Some unresolved questions

Not everything is crisp and symbolic. How do we learn neurosymbolic hybrid programs?

Some unresolved questions

Not everything is crisp and symbolic. How do we learn neurosymbolic hybrid programs?

Search is still hard

Some unresolved questions

Not everything is crisp and symbolic. How do we learn neurosymbolic hybrid programs?

Search is still hard

Is this a good model for human learning and thinking?

What we want for the future of machine learning

Strong generalization

What we want for the future of machine learning

Strong generalization

Bootstrapping, learning-to-learn, representation learning

What we want for the future of machine learning

Strong generalization

Bootstrapping, learning-to-learn, representation learning

Discovering knowledge that humans can understand and build on

Collaborators

Josh
Tenenbaum



Armando
Solar-Lezama



Max Nye



Cathy Wong



Mathias Sable-Meyer

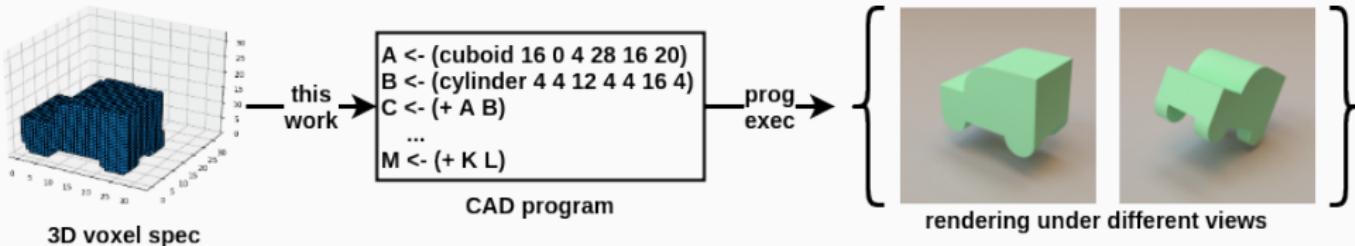


Lucas Morales



thank
you

3D program induction

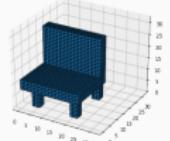


Challenge: combinatorial search!

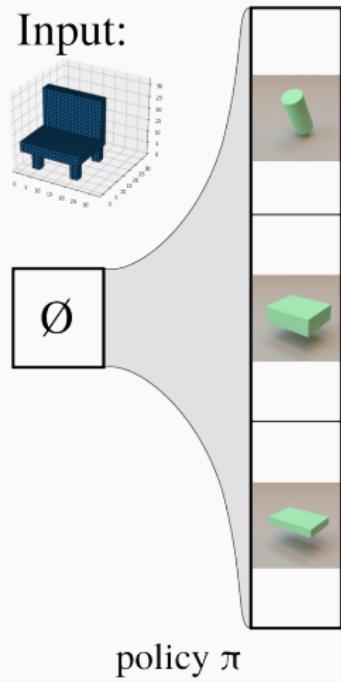
Branching factor: > 1.3 million per line of code, ≈ 20 lines of code
search space size: $(1.3 \text{ million})^{20} \approx 10^{122}$ programs

Solution: stochastic **tree search** + learn **policy** that writes code
+ learn **value** function that assesses execution of program so far;
analogous to **AlphaGo** [Silver et al. 2016]

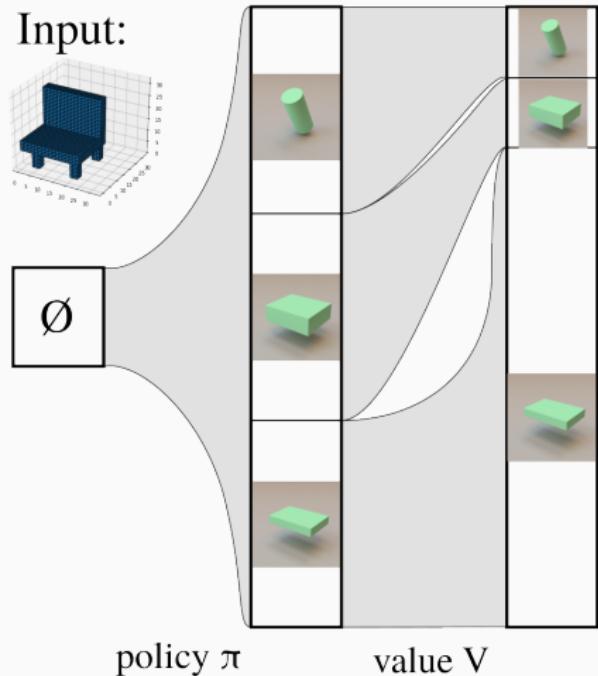
Input:



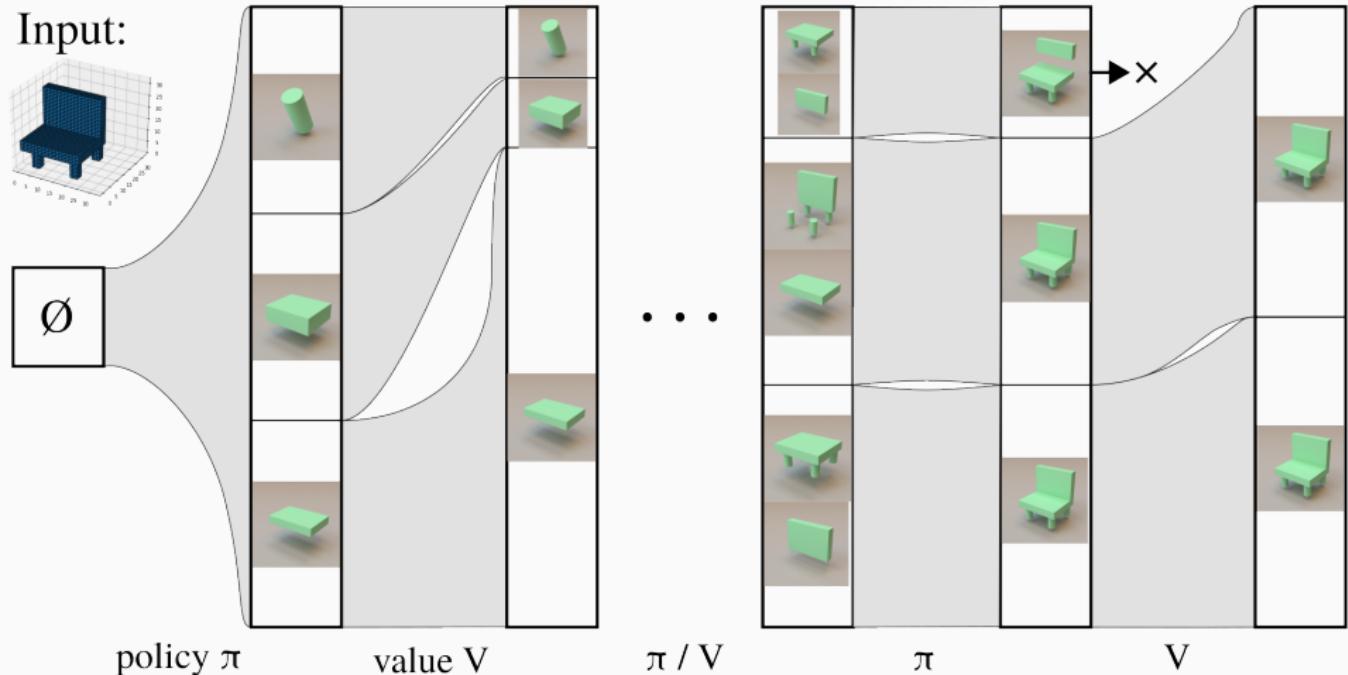
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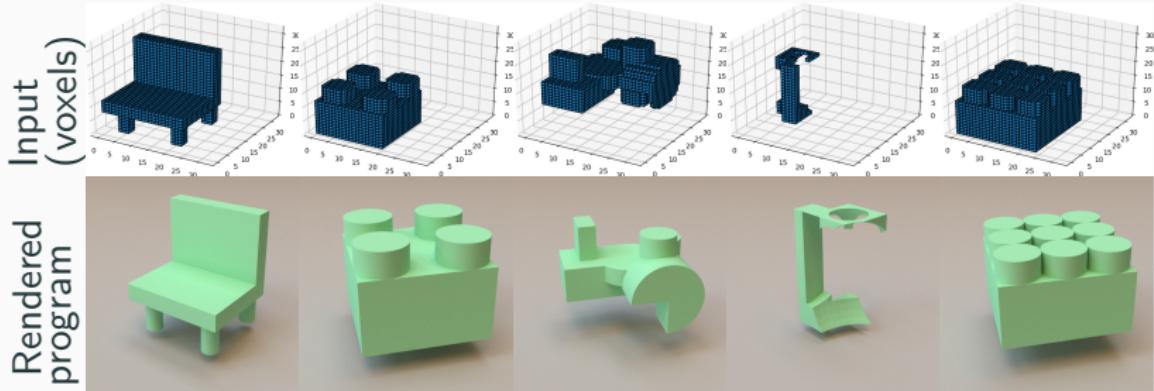
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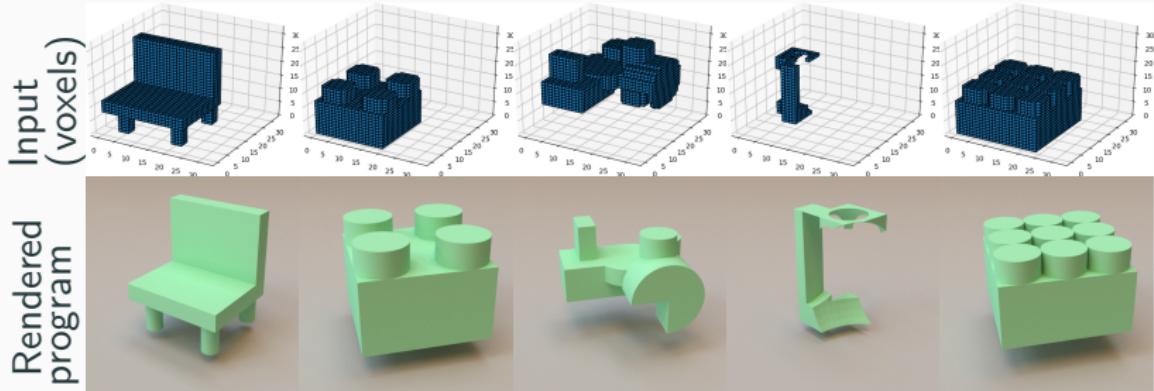
3D program induction



Ellis*, Nye*, Pu*, Sosa*, Tenenbaum, Solar-Lezama. NeurIPS 2019.

*equal contribution

3D program induction



same architecture learns to synthesize text editing programs
(FlashFill, Gulwani 2012)

Ellis*, Nye*, Pu*, Sosa*, Tenenbaum, Solar-Lezama. NeurIPS 2019.

*equal contribution

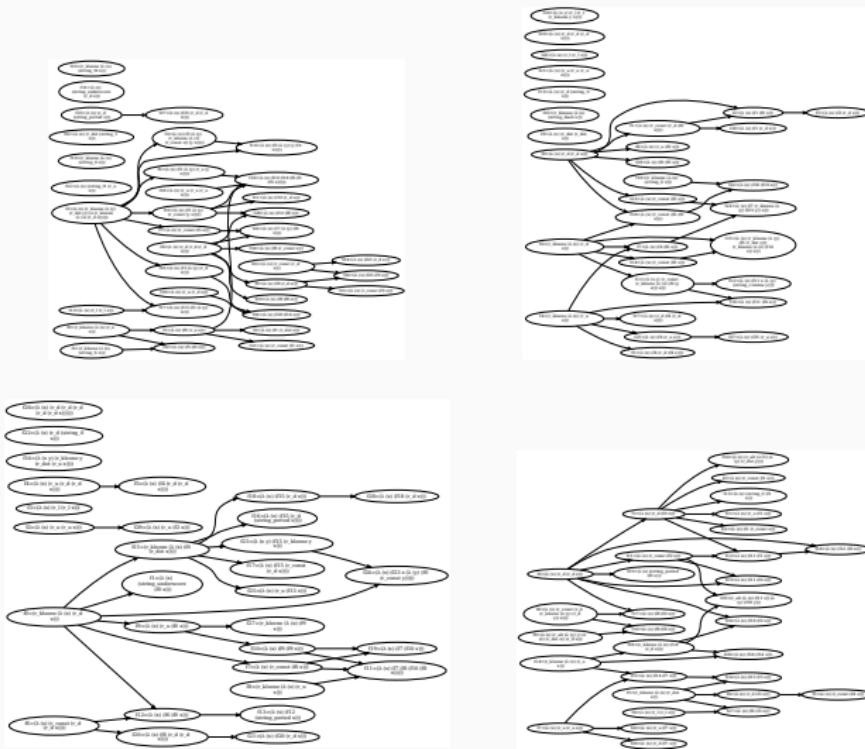
Library structure: Text Editing

DreamCoder learns libraries for FlashFill-style text editing [Gulwani 2012]

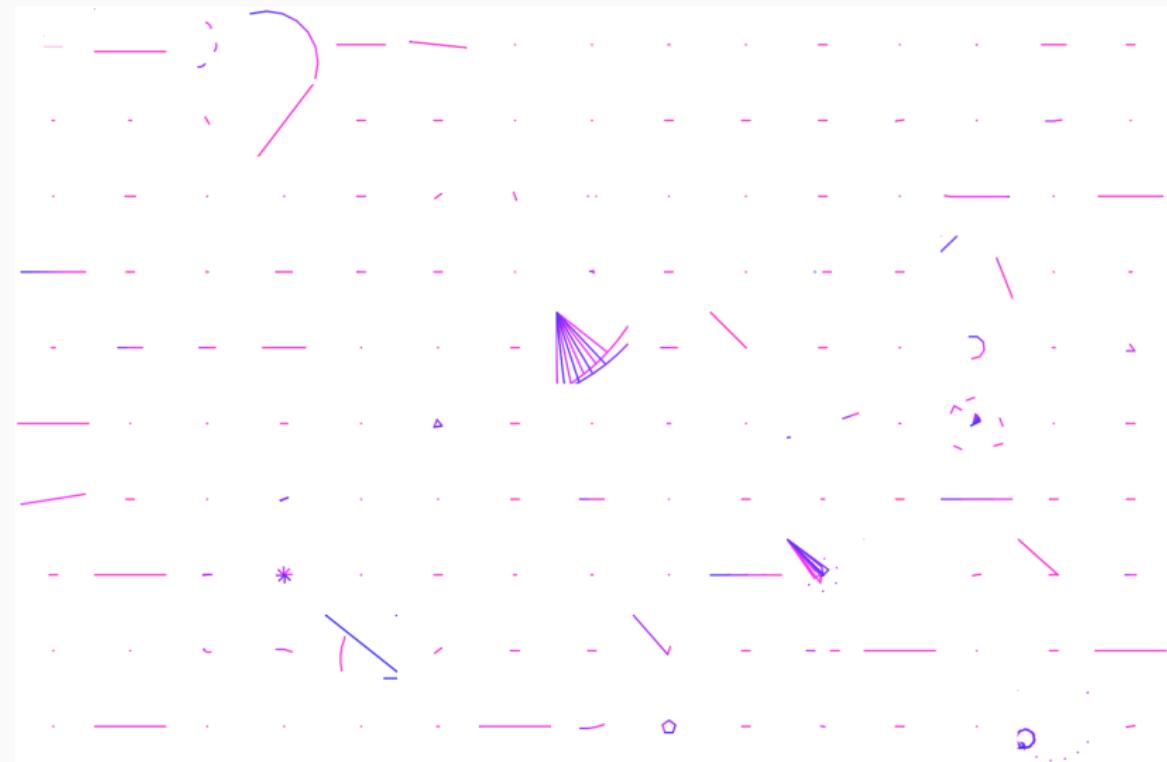


Library structure: Generating Text

Libraries for probabilistic generative models over text:
data from crawling web for CSV files



150 random dreams before learning



150 random dreams after learning

