

# DreamCoder: Growing domain-specific knowledge via wake-sleep program induction

---

Kevin Ellis

Collaborators: Catherine Wong, Maxwell Nye, Mathias Sablé-Meyer,  
Lucas Morales, Armando Solar-Lezama, Joshua B. Tenenbaum

2020

Conceptual Abstraction and Analogy in Natural and Artificial Intelligence

## The premise of program induction

1. Represent knowledge as programs: as symbolic code

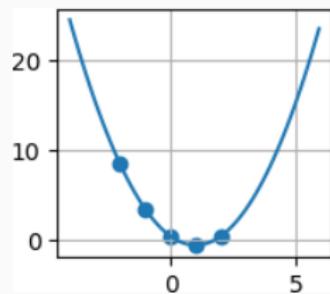
## The premise of program induction

1. Represent knowledge as programs: as symbolic code
2. Learning=adding to that body of knowledge=  
making new programs=program synthesis

# Why program induction?

# Why program induction?

strong generalization  
+data efficiency

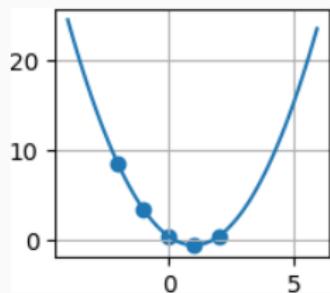


$$f(x) = (x-1)^{**2} - 0.5$$

# Why program induction?

strong generalization  
+data efficiency

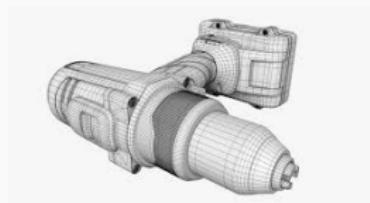
compositional reuse  
of abstractions



$$f(x) = (x-1)^2 - 0.5$$



VS

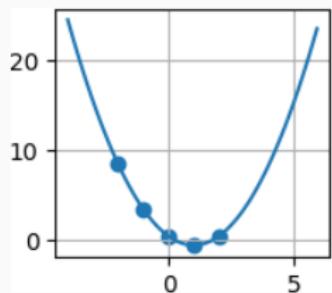


# Why program induction?

strong generalization  
+data efficiency

compositional reuse  
of abstractions

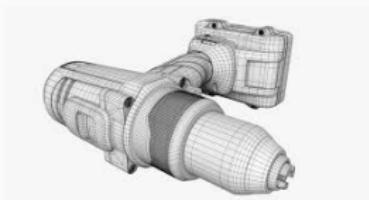
universal expressivity



$$f(x) = (x-1)^2 - 0.5$$



vs

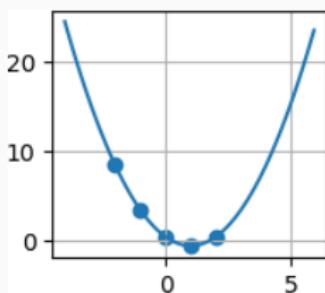


# Why program induction?

strong generalization  
+data efficiency

compositional reuse  
of abstractions

universal expressivity

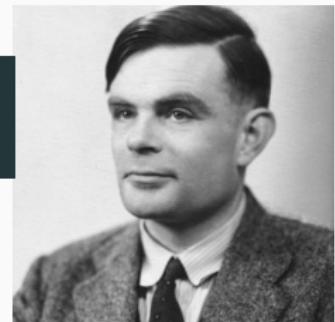
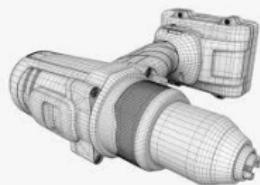


$$f(x) = (x-1)^2 - 0.5$$



Challenge: Combinatorial search,  
massive compute time

vs



## FlashFill (Gulwani 2012)

EXAMPLE 3 (Directory Name Extraction). Consider the following example taken from an excel online help forum.

Input $v_1$	Output
Company\Code\index.html	Company\Code\
Company\Docs\Spec\specs.doc	Company\Docs\Spec\

String Program:

$\text{SubStr}(v_1, \text{CPos}(0), \text{Pos}(\text{SlashTok}, \epsilon, -1))$

## FlashFill (Gulwani 2012)

EXAMPLE 3 (Directory Name Extraction). Consider the following example taken from an excel online help forum.

Input $v_1$	Output
Company\Code\index.html	Company\Code\
Company\Docs\Spec\specs.doc	Company\Docs\Spec\

String Program:

$\text{SubStr}(v_1, \text{CPos}(0), \text{Pos}(\text{SlashTok}, \epsilon, -1))$

## Szalinski (Nandi 2020)



(a) CAD model of ship's wheel

```
(Union  
  (Cylinder [1, 5, 5])  
  (Fold Union  
    (Tabulate (i 6)  
      (Rotate [0, 0, 60i]  
        (Translate [1, -0.5, 0]  
          (Cuboid [10, 1, 1]))))))
```

(b) Caddy program

# FlashFill (Gulwani 2012)

EXAMPLE 3 (Directory Name Extraction). Consider the following example taken from an excel online help forum.

Input $v_1$	Output
Company\Code\index.html	Company\Code\
Company\Docs\Spec\specs.doc	Company\Docs\Spec\

String Program:

$\text{SubStr}(v_1, \text{CPos}(0), \text{Pos}(\text{SlashTok}, \epsilon, -1))$

String expr $P$	$\text{Switch}((b_1, e_1), \dots, (b_n, e_n))$
Bool $b$	$d_1 \vee \dots \vee d_n$
Conjunct $d$	$\pi_1 \wedge \dots \wedge \pi_n$
Predicate $\pi$	$\text{Match}(v_i, r, k) \mid \neg \text{Match}(v_i, r, k)$
Trace expr $e$	$\text{Concatenate}(f_1, \dots, f_n)$
Atomic expr $f$	$\text{SubStr}(v_i, p_1, p_2)$   $\text{ConstStr}(s)$   $\text{Loop}(\lambda w : e)$
Position $p$	$\text{CPos}(k) \mid \text{Pos}(r_1, r_2, c)$
Integer expr $c$	$k \mid k_1 w + k_2$
Regular Expression $r$	$\text{TokenSeq}(T_1, \dots, T_m)$
Token $T$	$C + \mid [\neg C] + \mid \text{SpecialToken}$

# Szalinski (Nandi 2020)



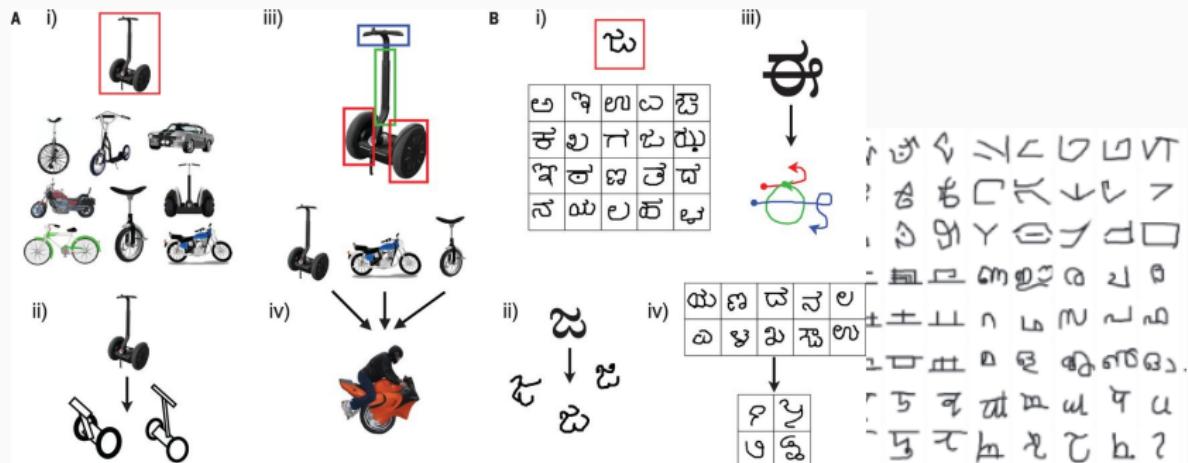
(a) CAD model of ship's wheel

```
(Union
  (Cylinder [1, 5, 5])
  (Fold Union
    (Tabulate (i 6)
      (Rotate [0, 0, 60i]
        (Translate [1, -0.5, 0]
          (Cuboid [10, 1, 1]))))))
```

(b) Caddy program

op	$\text{::= } + \mid - \mid \times \mid / \mid \text{num} \text{ ::= } \mathbb{R} \mid \langle \text{var} \rangle \mid \langle \text{num} \rangle \langle \text{op} \rangle \langle \text{num} \rangle$
vec2	$\text{::= } [\langle \text{num} \rangle, \langle \text{num} \rangle] \mid \text{vec3} \text{ ::= } [\langle \text{num} \rangle, \langle \text{num} \rangle, \langle \text{num} \rangle]$
affine	$\text{::= } \text{Translate} \mid \text{Rotate} \mid \text{Scale} \mid \text{TranslateSpherical}$
binop	$\text{::= } \text{Union} \mid \text{Difference} \mid \text{Intersection}$
cad	$\text{::= } (\text{Cuboid } \langle \text{vec3} \rangle) \mid (\text{Sphere } \langle \text{num} \rangle)$   $(\text{Cylinder } \langle \text{vec2} \rangle) \mid (\text{HexPrism } \langle \text{vec2} \rangle) \mid \dots$   $((\text{affine}) \langle \text{vec3} \rangle \langle \text{cad} \rangle)$   $((\text{binop}) \langle \text{cad} \rangle \langle \text{cad} \rangle)$   $(\text{Fold } \langle \text{binop} \rangle \langle \text{cad-list} \rangle)$
cad-list	$\text{::= } (\text{List } \langle \text{cad} \rangle^+)$   $(\text{Concat } \langle \text{cad-list} \rangle^+)$   $(\text{Tabulate } (\langle \text{var} \rangle \ Z^+)^+ \langle \text{cad} \rangle)$   $(\text{Map2 } \langle \text{affine} \rangle \langle \text{vec3-list} \rangle \langle \text{cad-list} \rangle)$
vec3-list	$\text{::= } (\text{List } \langle \text{vec3} \rangle^+)$   $(\text{Concat } \langle \text{vec3-list} \rangle^+)$   $(\text{Tabulate } (\langle \text{var} \rangle \ Z^+)^+ \langle \text{vec3} \rangle)$

# Program induction for learning and perception



ଶିତେହିତେମ୍ବାସ ପରିଦର୍ଶକ ଅପାଳୁଣ୍ଡନ୍  
 ରାତିଫିଲ୍ ପାଠୀକେ ନିଜାମୁନ୍ଦିବତ୍ତାର ଥ  
 କାନ୍ଦିପାଠୀକେ ପରିମାର୍ଗ କାହାରେ ଏହା କାହାରେ  
 ଲାପାଠୀକେ କାନ୍ଦିପାଠୀକେ କାନ୍ଦିପାଠୀକେ  
 ପାଠୀକେ କାନ୍ଦିପାଠୀକେ କାନ୍ଦିପାଠୀକେ  
 କାନ୍ଦିପାଠୀକେ କାନ୍ଦିପାଠୀକେ କାନ୍ଦିପାଠୀକେ  
 କାନ୍ଦିପାଠୀକେ କାନ୍ଦିପାଠୀକେ କାନ୍ଦିପାଠୀକେ  
 କାନ୍ଦିପାଠୀକେ କାନ୍ଦିପାଠୀକେ କାନ୍ଦିପାଠୀକେ

Program Induction and learning to learn  
learning a library  
synergy between library & neural search

# Learning to write code

Goal: acquire domain-specific knowledge needed to induce a class of programs

- Library of abstractions (domain specific language)
- Inference strategy (synthesis algorithm)

# Library learning

## Initial Primitives

: value  
:

map

fold func

if

cons

>

: value  
:

## Sample Problem: sort list

[9 2 7 1] → [1 2 7 9]

[3 8 9 4 2] → [2 3 4 8 9]

[6 2 2 3 8 5] → [2 2 3 5 6 8]

...

# Library learning

## Initial Primitives

:

⋮

map

fold

if

cons

>

⋮

⋮

## Sample Problem: sort list

[9 2 7 1] → [1 2 7 9]

[3 8 9 4 2] → [2 3 4 8 9]

[6 2 2 3 8 5] → [2 2 3 5 6 8]

⋮

# Library learning

## Initial Primitives

:

:

map

fold

if

cons

>

:

:

## Sample Problem: sort list

[9 2 7 1] → [1 2 7 9]

[3 8 9 4 2] → [2 3 4 8 9]

[6 2 2 3 8 5] → [2 2 3 5 6 8]

...

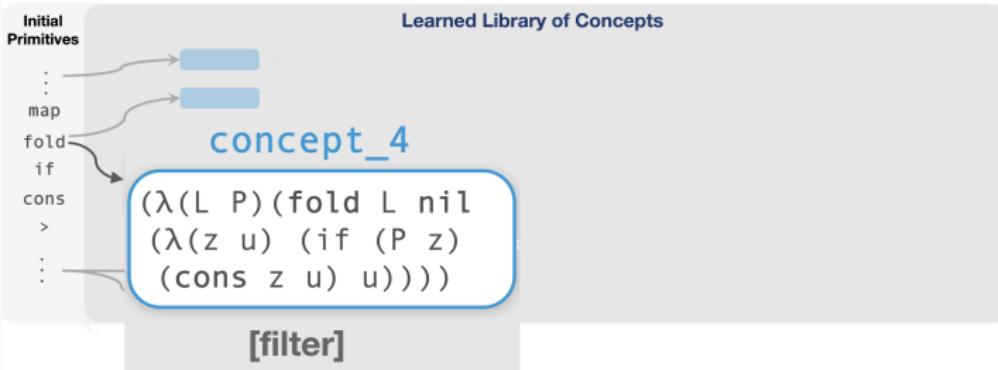
# Library learning



## Sample Problem: sort list

$[9 2 7 1] \rightarrow [1 2 7 9]$   
 $[3 8 9 4 2] \rightarrow [2 3 4 8 9]$   
 $[6 2 2 3 8 5] \rightarrow [2 2 3 5 6 8]$   
...

# Library learning



Sample Problem: sort list

$[9 2 7 1] \rightarrow [1 2 7 9]$   
 $[3 8 9 4 2] \rightarrow [2 3 4 8 9]$   
 $[6 2 2 3 8 5] \rightarrow [2 2 3 5 6 8]$   
...

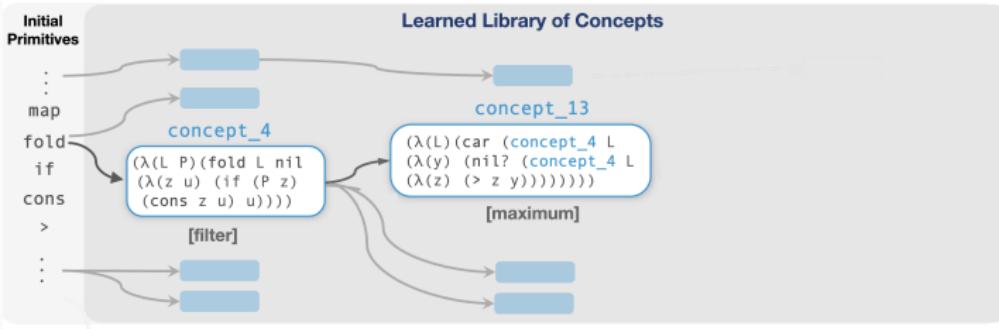
# Library learning



## Sample Problem: sort list

[9 2 7 1] → [1 2 7 9]  
[3 8 9 4 2] → [2 3 4 8 9]  
[6 2 2 3 8 5] → [2 2 3 5 6 8]  
...

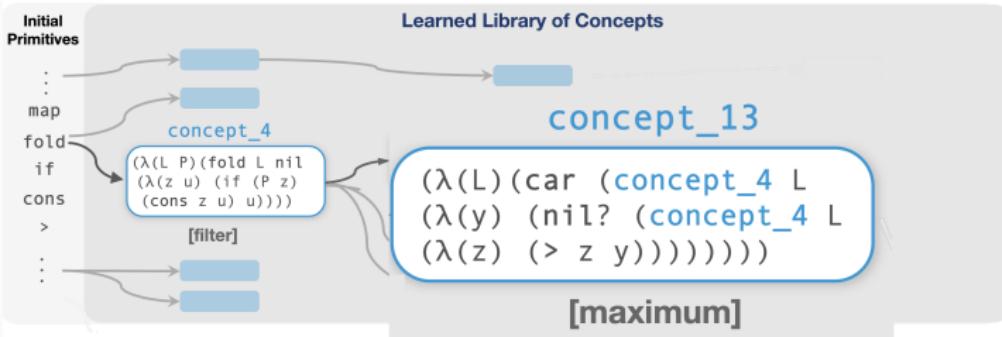
# Library learning



## Sample Problem: sort list

[9 2 7 1] → [1 2 7 9]  
[3 8 9 4 2] → [2 3 4 8 9]  
[6 2 2 3 8 5] → [2 2 3 5 6 8]  
...

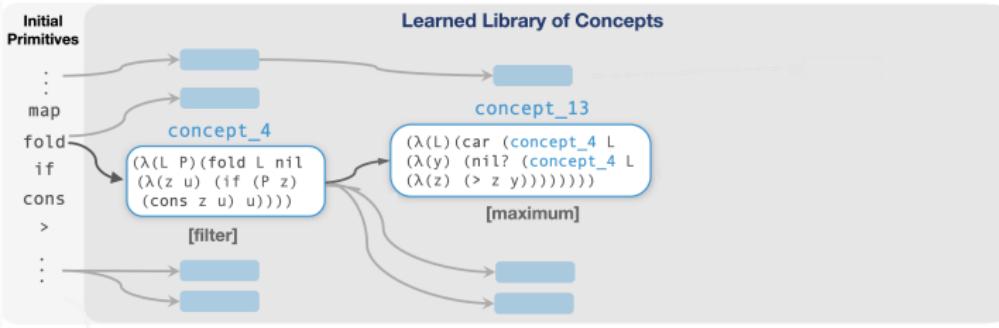
# Library learning



Sample Problem: sort list

[9 2 7 1] → [1 2 7 9]  
[3 8 9 4 2] → [2 3 4 8 9]  
[6 2 2 3 8 5] → [2 2 3 5 6 8]  
...

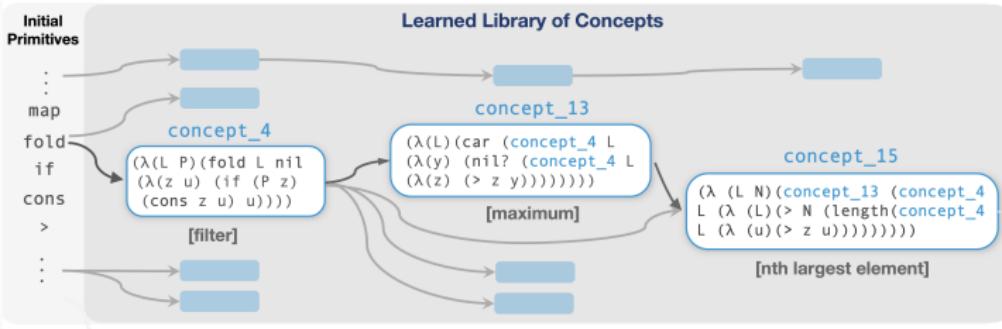
# Library learning



## Sample Problem: sort list

[9 2 7 1] → [1 2 7 9]  
[3 8 9 4 2] → [2 3 4 8 9]  
[6 2 2 3 8 5] → [2 2 3 5 6 8]  
...

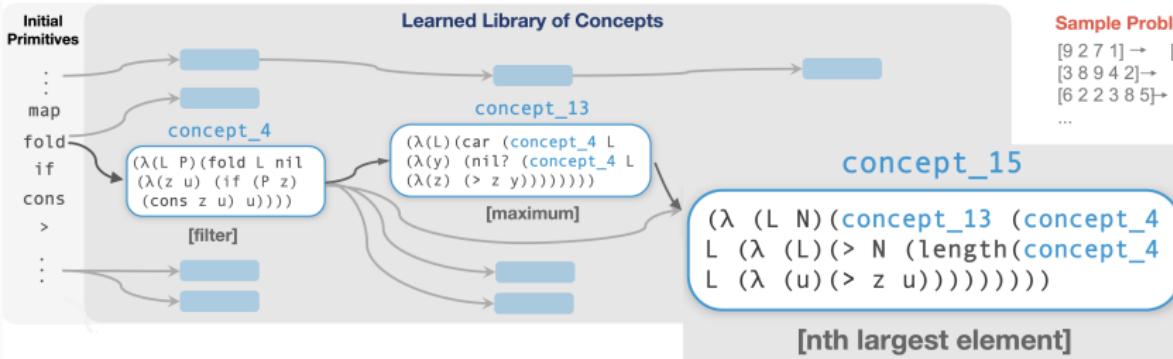
# Library learning



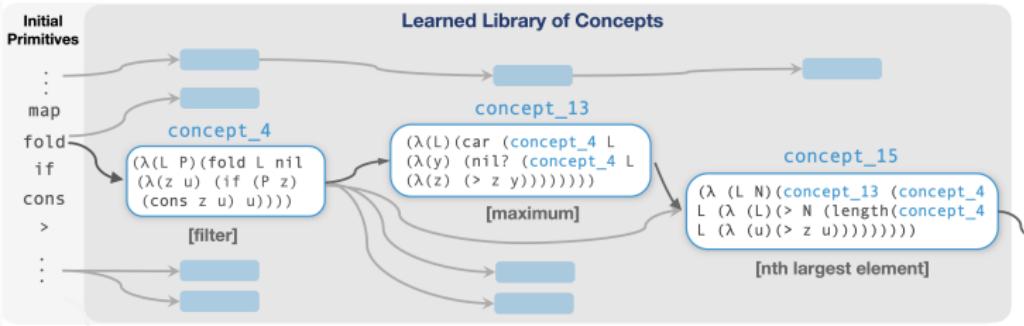
Sample Problem: sort list

[9 2 7 1] → [1 2 7 9]  
[3 8 9 4 2] → [2 3 4 8 9]  
[6 2 2 3 8 5] → [2 2 3 5 6 8]  
...

# Library learning



# Library learning



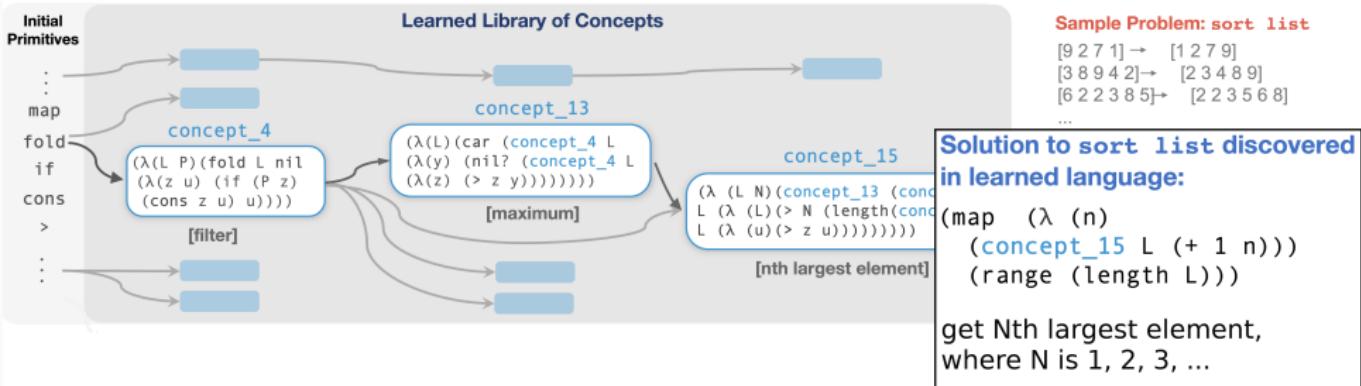
## Sample Problem: sort list

[9 2 7 1] → [1 2 7 9]  
[3 8 9 4 2] → [2 3 4 8 9]  
[6 2 2 3 8 5] → [2 2 3 5 6 8]  
...

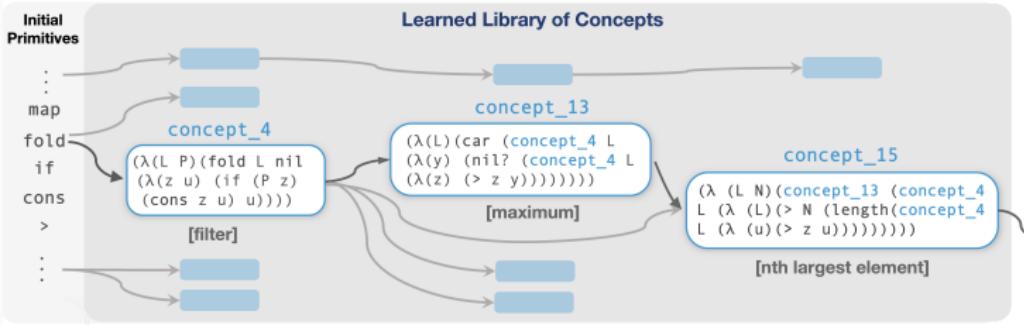
## Solution to sort list discovered in learned language:

```
(map (λ (n)
  (concept_15 L (+ 1 n)))
  (range (length L)))
```

# Library learning



# Library learning



## Sample Problem: sort list

[9 2 7 1] → [1 2 7 9]  
[3 8 9 4 2] → [2 3 4 8 9]  
[6 2 2 3 8 5] → [2 2 3 5 6 8]  
...

## Solution to sort list discovered in learned language:

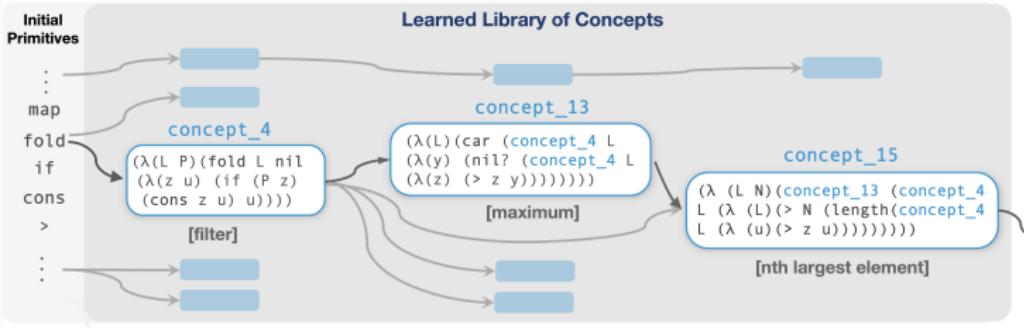
```
(map (λ (n)
  (concept_15 L (+ 1 n)))
  (range (length L)))
```

get Nth largest element,  
where N is 1, 2, 3, ...

## Solution rewritten in initial primitives:

```
(lambda (x) (map (lambda (y) (car (fold (fold x nil (lambda (z u) (if (gt? (+ y 1) (length
(fold x nil (lambda (v w) (if (gt? z v) (cons v w) w)))))) (cons z u) u))) nil (lambda (a b) (if
(nil? (fold (fold x nil (lambda (c d) (if (gt? (+ y 1) (length (fold x nil (lambda (e f) (if
(gt? c e) (cons e f) f)))))) (cons c d) d))) nil (lambda (g h) (if (gt? g a) (cons g h) h))) (cons a b) b)))) (range (length x))))
```

# Library learning



## Sample Problem: sort list

[9 2 7 1] → [1 2 7 9]  
[3 8 9 4 2] → [2 3 4 8 9]  
[6 2 2 3 8 5] → [2 2 3 5 6 8]  
...

## Solution to sort list discovered in learned language:

(map (lambda (n)  
  (concept\_15 L (+ 1 n)))  
  (range (length L)))

get Nth largest element,  
where N is 1, 2, 3, ...

## Solution rewritten in initial primitives:

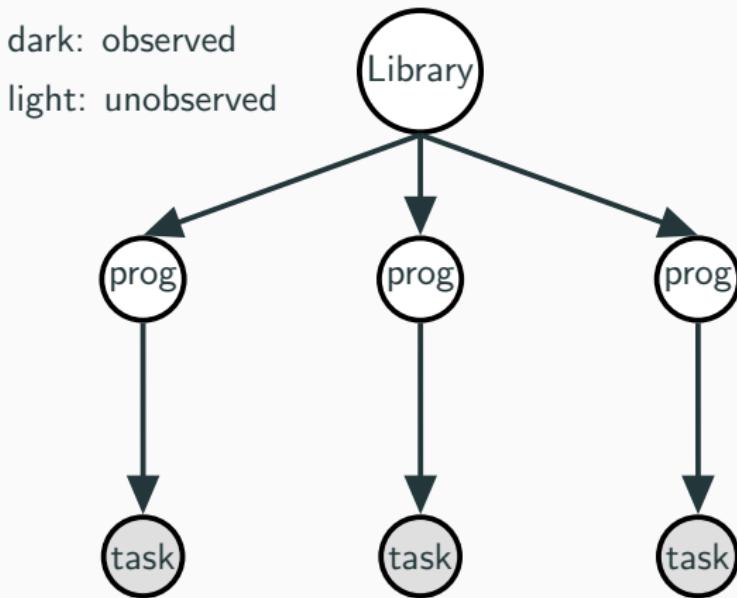
```
(lambda (x) (map (lambda (y) (car (fold (fold x nil (lambda (z u) (if (gt? (+ y 1) (length (fold x nil (lambda (v w) (if (gt? z v) (cons v w) w)))) (cons z u) u))) nil (lambda (a b) (if (nil? (fold (fold x nil (lambda (c d) (if (gt? (+ y 1) (length (fold x nil (lambda (e f) (if (gt? c e) (cons e f) f)))) (cons c d) d))) nil (lambda (g h) (if (gt? g a) (cons g h) h)))) (cons a b) b)))) (range (length x))))
```

induced sort program found in  $\leq 10\text{min}$ . Brute-force search without learned library would take  $\approx 10^{73}$  years

- **Wake:** Solve problems by writing programs
- **Sleep:** Improve library and neural recognition model:
  - **Abstraction sleep:** Improve library
  - **Dream sleep:** Improve neural recognition model

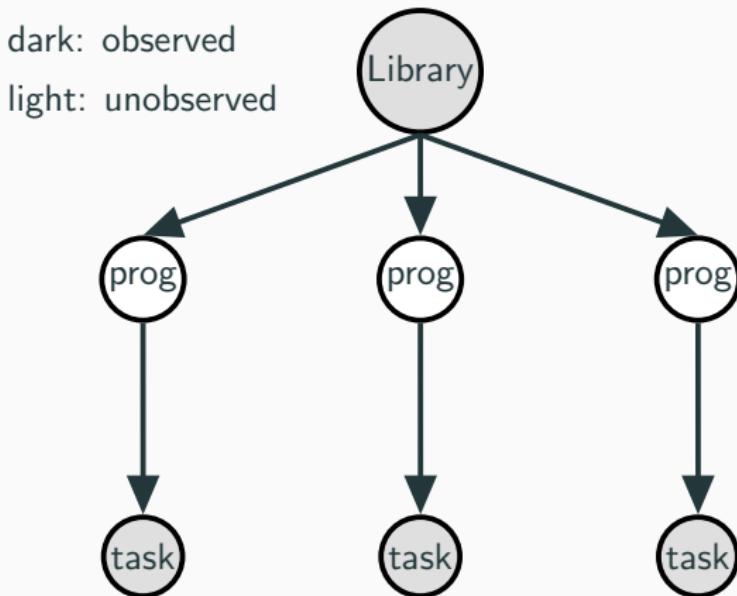
cf. Helmholtz machine, wake/sleep neural network training algorithms

# Library learning as Bayesian inference



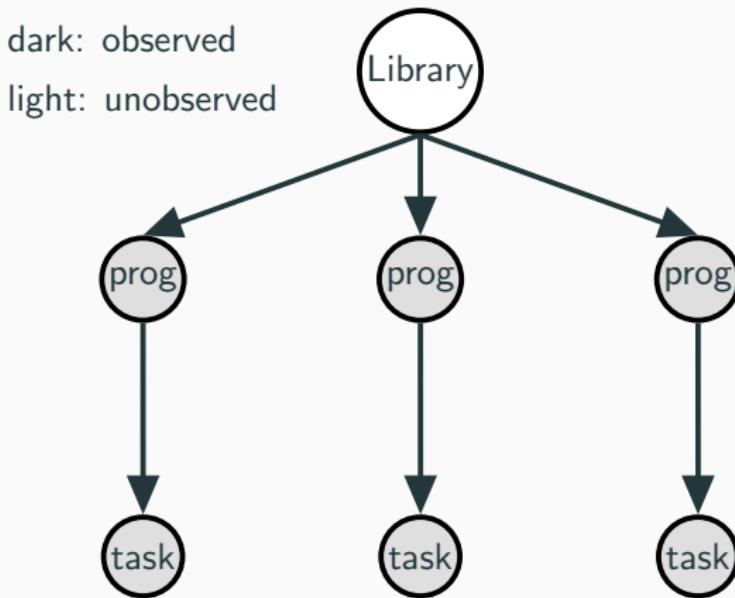
[Dechter et al, 2013] [Liang et al, 2010] [Lake et al, 2015]

# Library learning as Bayesian inference



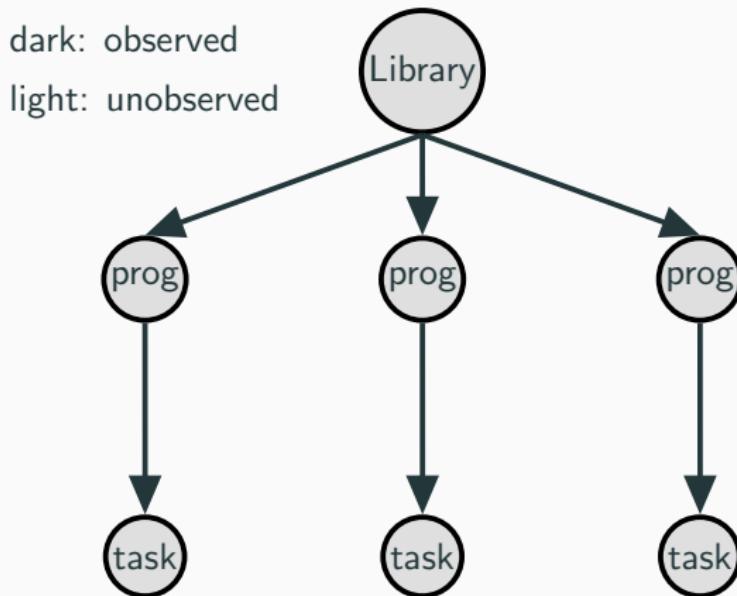
[Dechter et al, 2013] [Liang et al, 2010] [Lake et al, 2015]

# Library learning as Bayesian inference



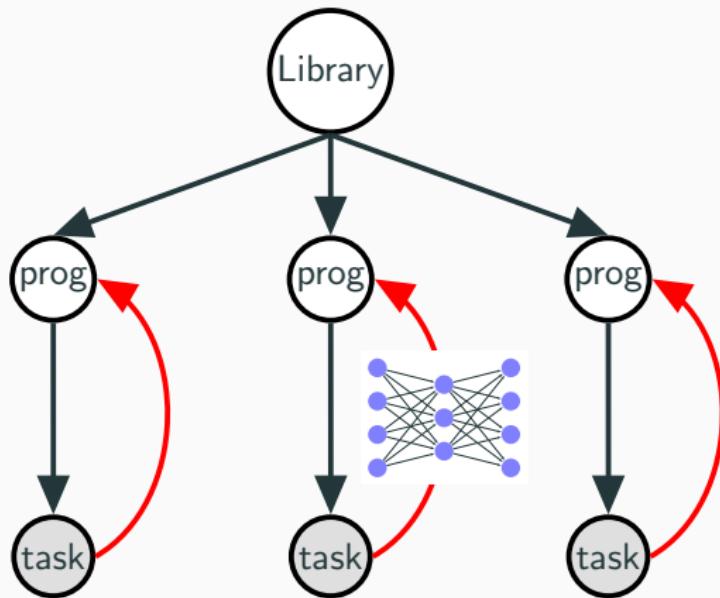
[Dechter et al, 2013] [Liang et al, 2010] [Lake et al, 2015]

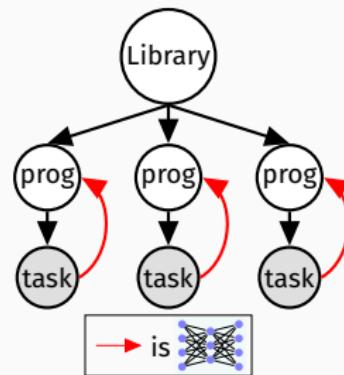
# Library learning as Bayesian inference



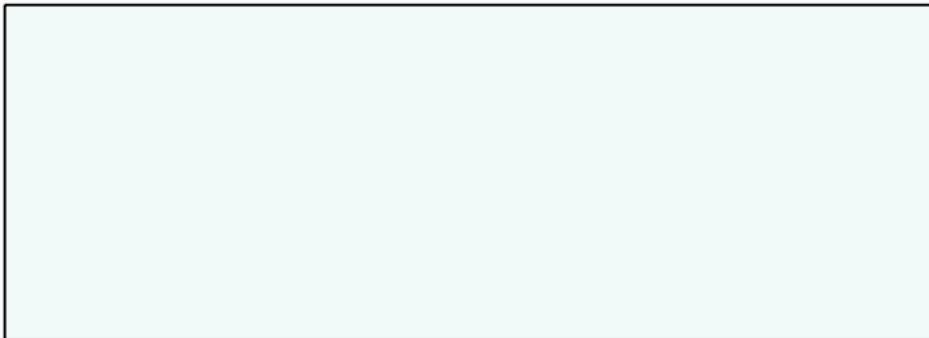
[Dechter et al, 2013] [Liang et al, 2010] [Lake et al, 2015]

# Library learning as neurally-guided Bayesian inference



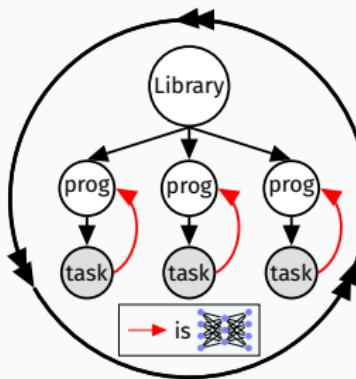


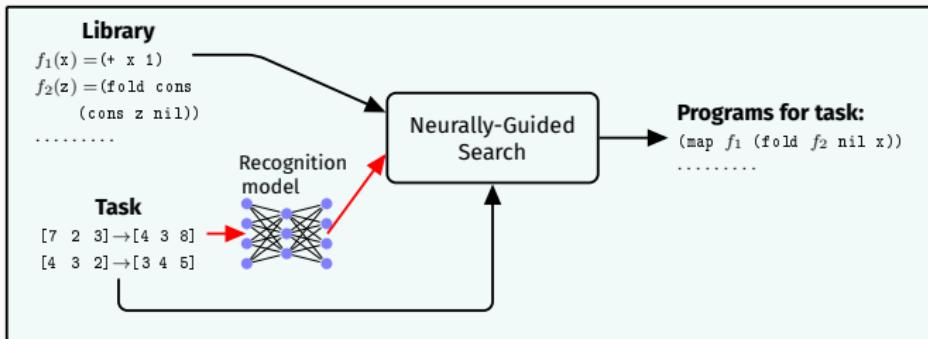
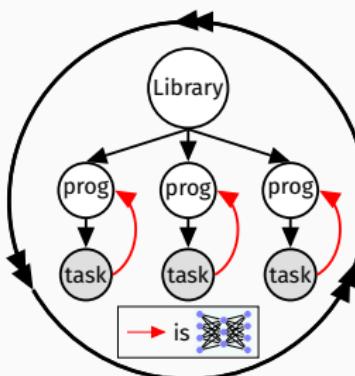
WAKE

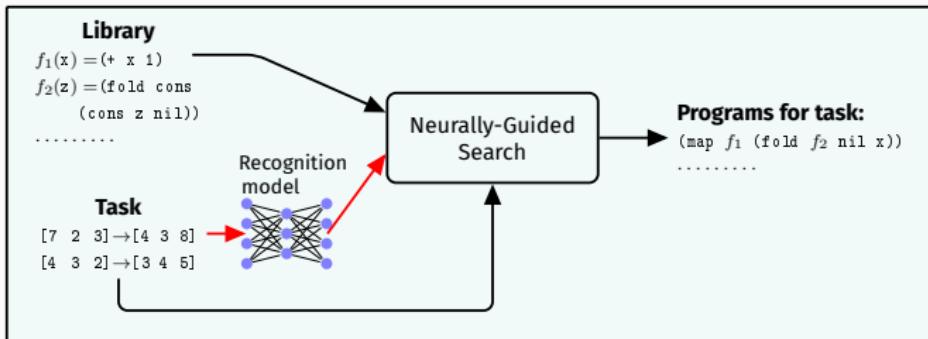
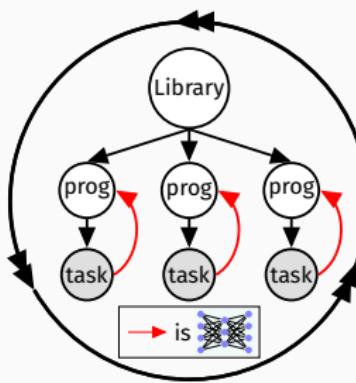
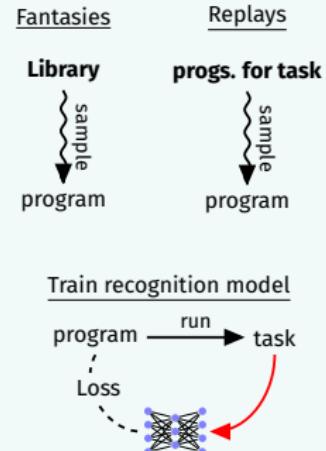


SLEEP: ABSTRACTION

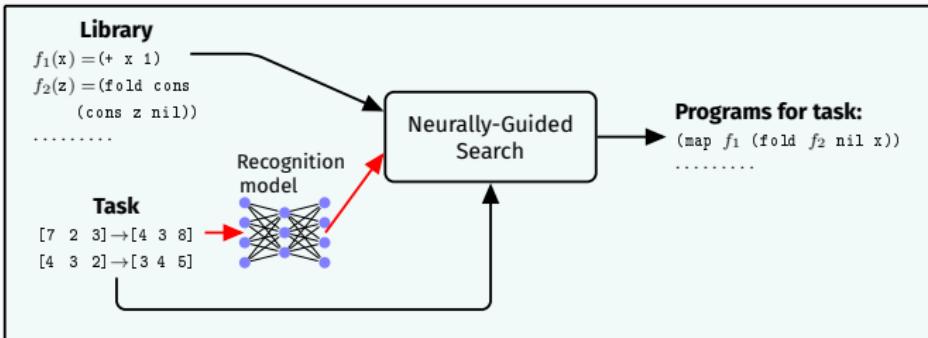
SLEEP: DREAMING



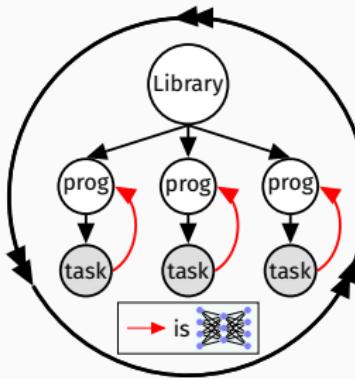
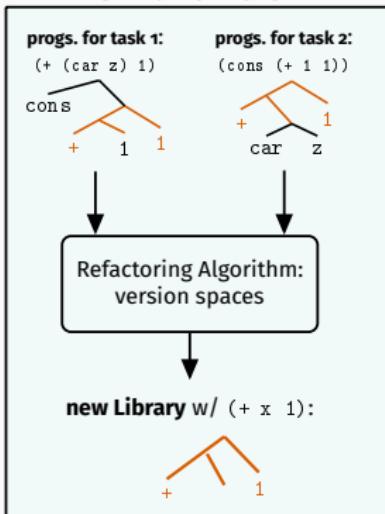
**WAKE****SLEEP: ABSTRACTION****SLEEP: DREAMING**

**WAKE****SLEEP: ABSTRACTION****SLEEP: DREAMING**

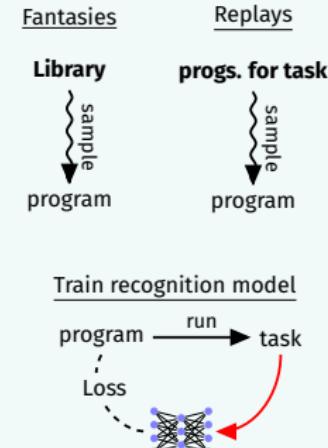
## WAKE



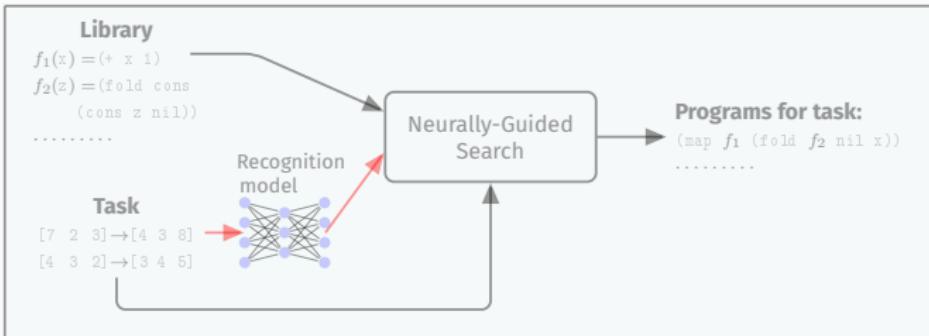
## SLEEP: ABSTRACTION



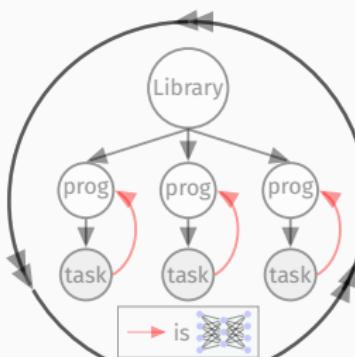
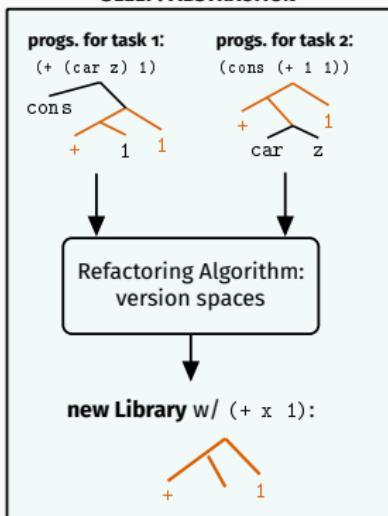
## SLEEP: DREAMING



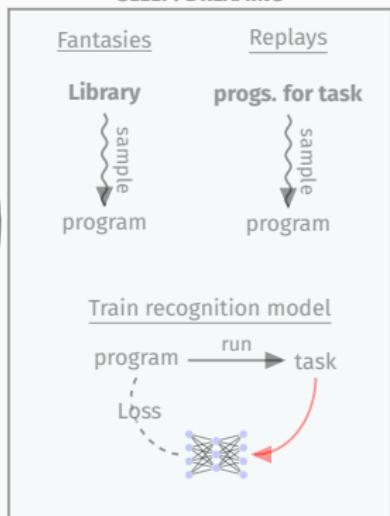
## WAKE



## SLEEP: ABSTRACTION



## SLEEP: DREAMING



# Abstraction as analogy — from François Chollet, yesterday

## Program-centric abstraction

- Graph of (usually discrete) operators where input nodes can take different values within a type
  - Example: function that sorts a list
- Abstract wrt input nodes values
- Obtained by merging specialized functions under a new abstract signature
  - This is a program analogy!

```
a = [4, 5, 2, 6]
result = 0
for i, e in enumerate(a):
    result += i * e
```

```
b = [6, 3, 4, 7]
result = 0
for i, e in enumerate(b):
    result += i * e
```

```
def process_item(x):
    result = 0
    for i, e in enumerate(x):
        result += i * e
    return result
```



# Abstraction Sleep: Growing the library via refactoring

**Task:**  $[1 \ 2 \ 3] \rightarrow [2 \ 4 \ 6]$   
 $[4 \ 3 \ 4] \rightarrow [8 \ 6 \ 8]$

**Task:**  $[1 \ 2 \ 3] \rightarrow [0 \ 1 \ 2]$   
 $[4 \ 3 \ 4] \rightarrow [3 \ 2 \ 3]$

# Abstraction Sleep: Growing the library via refactoring

Task:  $[1\ 2\ 3] \rightarrow [2\ 4\ 6]$   
 $[4\ 3\ 4] \rightarrow [8\ 6\ 8]$

Wake: program search

```
(Y (λ (r 1) (if (nil? 1) nil  
           (cons (+ (car 1) (car 1))  
                 (r (cdr 1)))))))
```

Task:  $[1\ 2\ 3] \rightarrow [0\ 1\ 2]$   
 $[4\ 3\ 4] \rightarrow [3\ 2\ 3]$

Wake: program search

```
(Y (λ (r 1) (if (nil? 1) nil  
           (cons (- (car 1) 1)  
                 (r (cdr 1)))))))
```

# Abstraction Sleep: Growing the library via refactoring

Task:  $[1\ 2\ 3] \rightarrow [2\ 4\ 6]$   
 $[4\ 3\ 4] \rightarrow [8\ 6\ 8]$

Wake: program search

```
(Y (λ (r 1) (if (nil? 1) nil  
           (cons (+ (car 1) (car 1))  
                  (r (cdr 1)))))))
```

Task:  $[1\ 2\ 3] \rightarrow [0\ 1\ 2]$   
 $[4\ 3\ 4] \rightarrow [3\ 2\ 3]$

Wake: program search

```
(Y (λ (r 1) (if (nil? 1) nil  
           (cons (- (car 1) 1)  
                  (r (cdr 1)))))))
```

refactor

$(10^{14}$  refactorings)

```
((λ (f) (Y (λ (r 1) (if (nil? 1)  
                           nil  
                           (cons (f (car 1))  
                                 (r (cdr 1)))))))  
  (λ (z) (+ z z)))
```

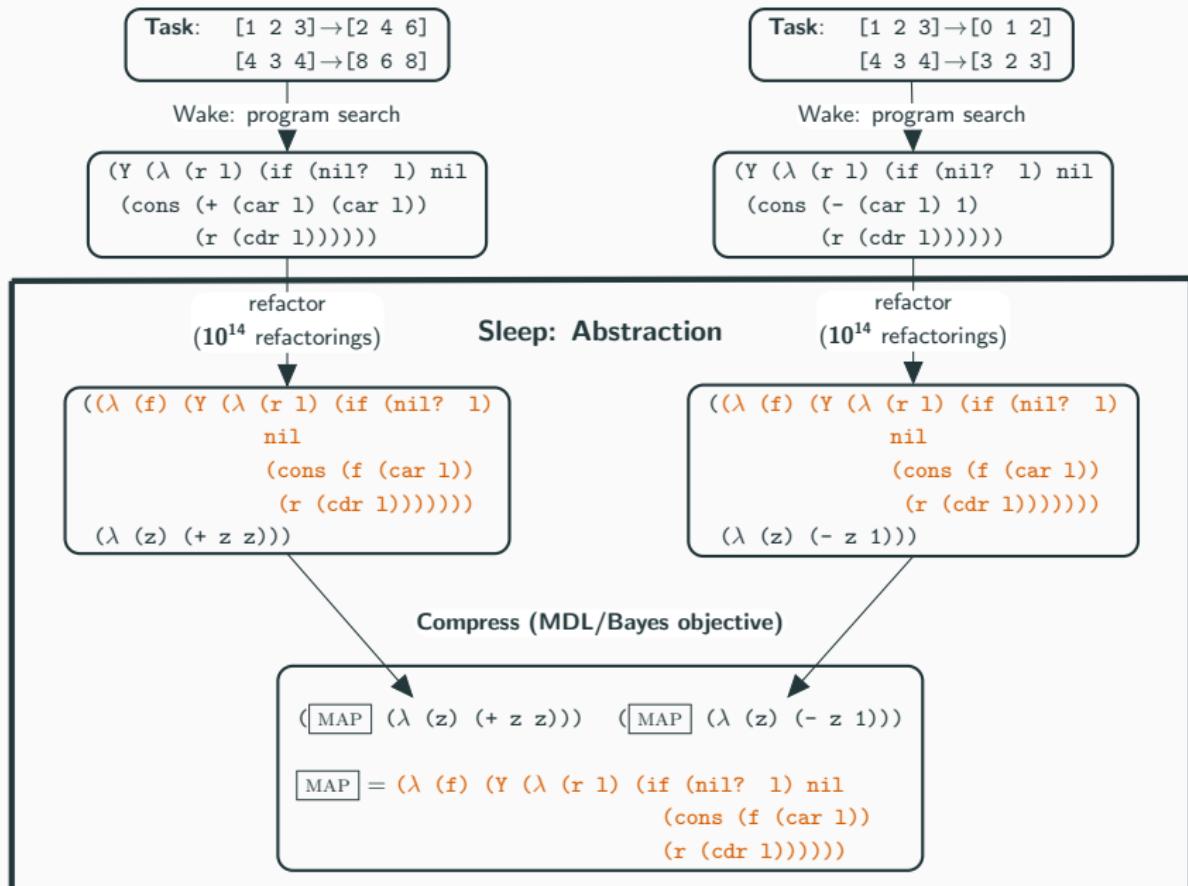
refactor

$(10^{14}$  refactorings)

```
((λ (f) (Y (λ (r 1) (if (nil? 1)  
                           nil  
                           (cons (f (car 1))  
                                 (r (cdr 1)))))))  
  (λ (z) (- z 1)))
```

## Sleep: Abstraction

# Abstraction Sleep: Growing the library via refactoring



# Abstraction Sleep: Growing the library via refactoring

Task:  $[1\ 2\ 3] \rightarrow [2\ 4\ 6]$   
 $[4\ 3\ 4] \rightarrow [8\ 6\ 8]$

Wake: program search

```
(Y (λ (r 1) (if (nil? 1) nil  
           (cons (+ (car 1) (car 1))  
                  (r (cdr 1)))))))
```

Task:  $[1\ 2\ 3] \rightarrow [0\ 1\ 2]$   
 $[4\ 3\ 4] \rightarrow [3\ 2\ 3]$

Wake: program search

```
(Y (λ (r 1) (if (nil? 1) nil  
           (cons (- (car 1) 1)  
                  (r (cdr 1)))))))
```

these  $10^{14}$  refactorings represented in exponentially more efficient refactoring data structure:

$(\lambda$  equivalence graphs+version spaces using  $10^6$  nodes,  
calculated in under 5min

c.f. [Tate et al 2009], [Gulwani 2012]

$\cdot 1?)$   
 $) \cdot 1))$   
 $) \cdot 1)))$

Compress (MDL/Bayes objective)

```
(MAP (λ (z) (+ z z))) (MAP (λ (z) (- z 1)))  
  
MAP = (λ (f) (Y (λ (r 1) (if (nil? 1) nil  
           (cons (f (car 1))  
                  (r (cdr 1)))))))
```

Program Induction and learning to learn  
learning a library  
synergy between library & neural search

# DreamCoder Domains

## List Processing

### Sum List

[1 2 3] → 6

[4 6 8 1] → 17

### Double

[1 2 3] → [2 4 6]

[4 5 1] → [8 10 2]

## Text Editing

### Abbreviate

Allen Newell → A.N.

Herb Simon → H.S.

### Drop Last Three

shrdlu → shr

shakey → sha

## Regexes

### Phone numbers

(555) 867-5309

(650) 555-2368

### Currency

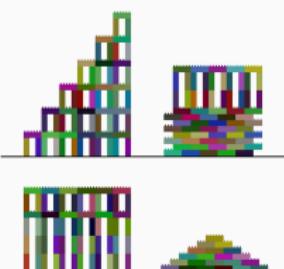
\$100.25

\$4.50

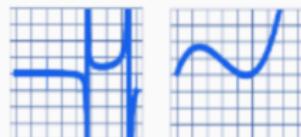
## LOGO Graphics



## Block Towers



## Symbolic Regression



$$y = f(x)$$

## Recursive Programming

### Filter Red

[■■■■■■■■] → [■■■■]

[■■■■■■■■■■] → [■■■■■■■■]

[■■■■■■■■■■] → [■■■■■■■■]

## Physical Laws

$$\vec{a} = \frac{1}{m} \sum_i \vec{F}_i$$

$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r}|^2} \hat{r}$$

# DreamCoder Domains

## List Processing

### Sum List

[1 2 3] → 6

[4 6 8 1] → 17

### Double

[1 2 3] → [2 4 6]

[4 5 1] → [8 10 2]

## Text Editing

### Abbreviate

Allen Newell → A.N.

Herb Simon → H.S.

### Drop Last Three

shrdlu → shr

shakey → sha

## Regexes

### Phone numbers

(555) 867-5309

(650) 555-2368

### Currency

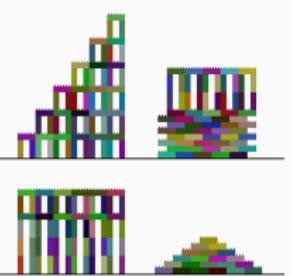
\$100.25

\$4.50

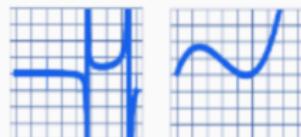
## LOGO Graphics



## Block Towers



## Symbolic Regression



$$y = f(x)$$

## Recursive Programming

### Filter Red

[■■■■■■■■] → [■■■■■■■■]

[■■■■■■■■■■] → [■■■■■■■■■■]

[■■■■■■■■■■■■] → [■■■■■■■■■■■■]

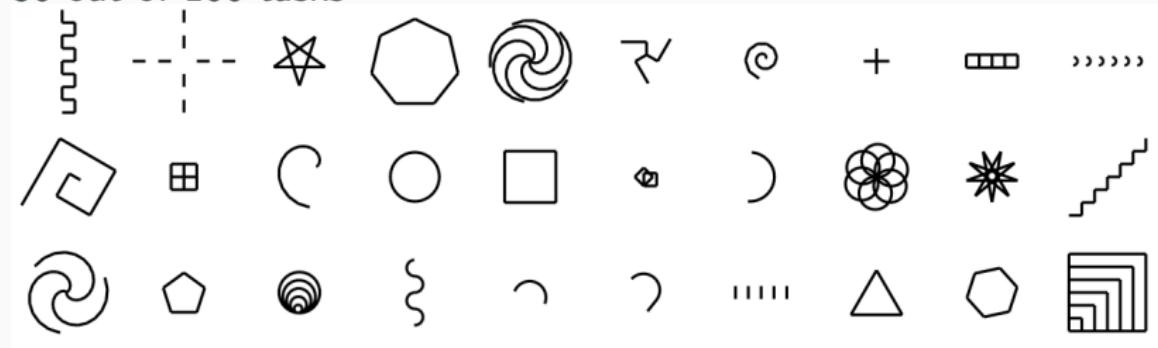
## Physical Laws

$$\vec{a} = \frac{1}{m} \sum_i \vec{F}_i$$

$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r}|^2} \hat{r}$$

# LOGO Turtle Graphics

30 out of 160 tasks



# LOGO Turtle Graphics

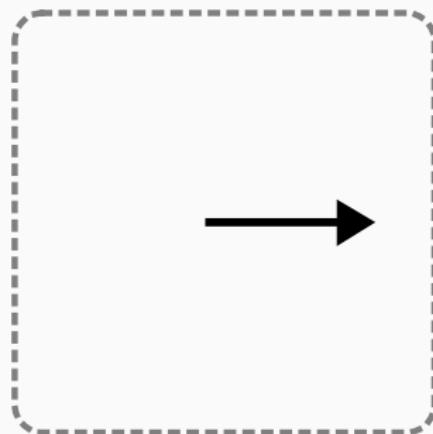
## DSL

OP ::= FW x | RT x | UP | DOWN | SET state

## Tasks

task : image

FW 1



# LOGO Turtle Graphics

## DSL

OP ::= FW x | RT x | UP | DOWN | SET state

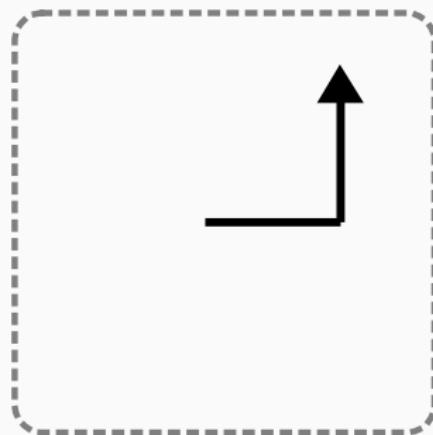
## Tasks

task : image

FW 1

RT  $\frac{\pi}{2}$

FW 1



# LOGO Turtle Graphics

## DSL

OP ::= FW x | RT x | UP | DOWN | SET state

## Tasks

task : image

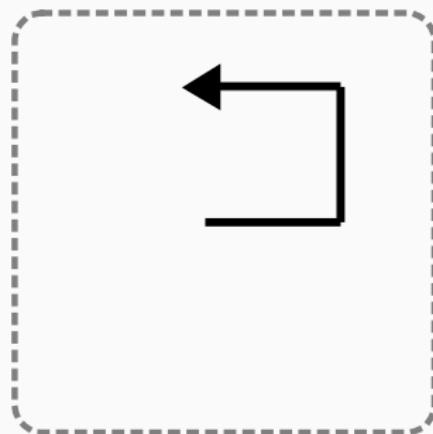
FW 1

RT  $\frac{\pi}{2}$

FW 1

RT  $\frac{\pi}{2}$

FW 1



# LOGO Turtle Graphics

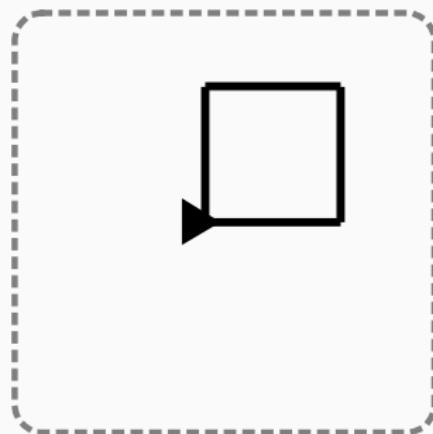
## DSL

OP ::= FW x | RT x | UP | DOWN | SET state

## Tasks

task : image

```
for i in range(4)
> FW 1
> RT π/2
```



# LOGO Turtle Graphics

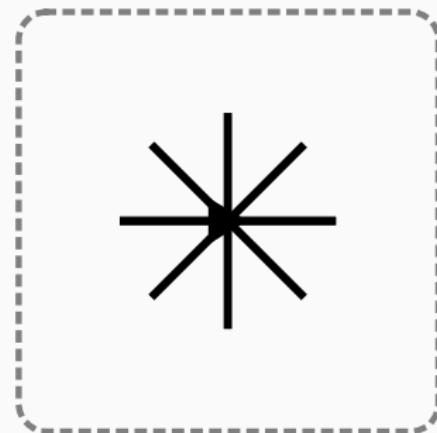
## DSL

OP ::= FW x | RT x | UP | DOWN | SET state

## Tasks

task : image

```
for i in range(8)
> FW 1
> SET origin
> RT  $\frac{2\pi}{8}$ 
```



# LOGO Turtle Graphics

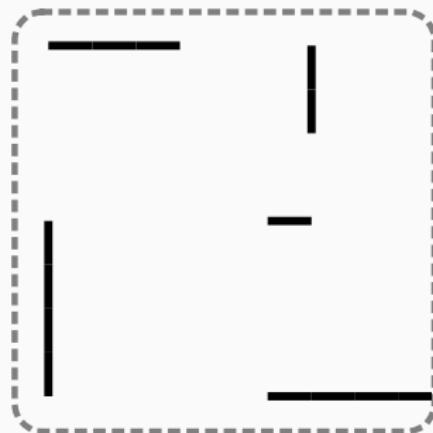
## DSL

OP ::= FW x | RT x | UP | DOWN | SET state

## Tasks

task : image

```
for i in range(8)
> PU
> FW  $\frac{i}{2}$ 
> PD
> FW  $\frac{i}{2}$ 
> RT  $\frac{\pi}{2}$ 
```



# LOGO Turtle Graphics

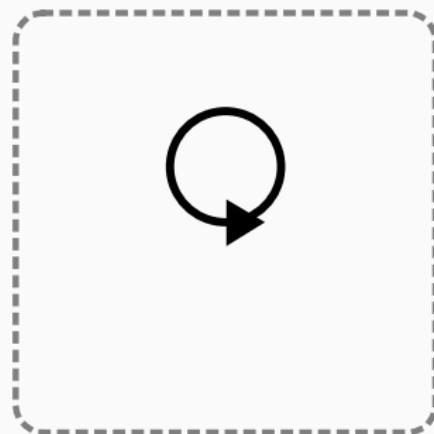
## DSL

OP ::= FW x | RT x | UP | DOWN | SET state

## Tasks

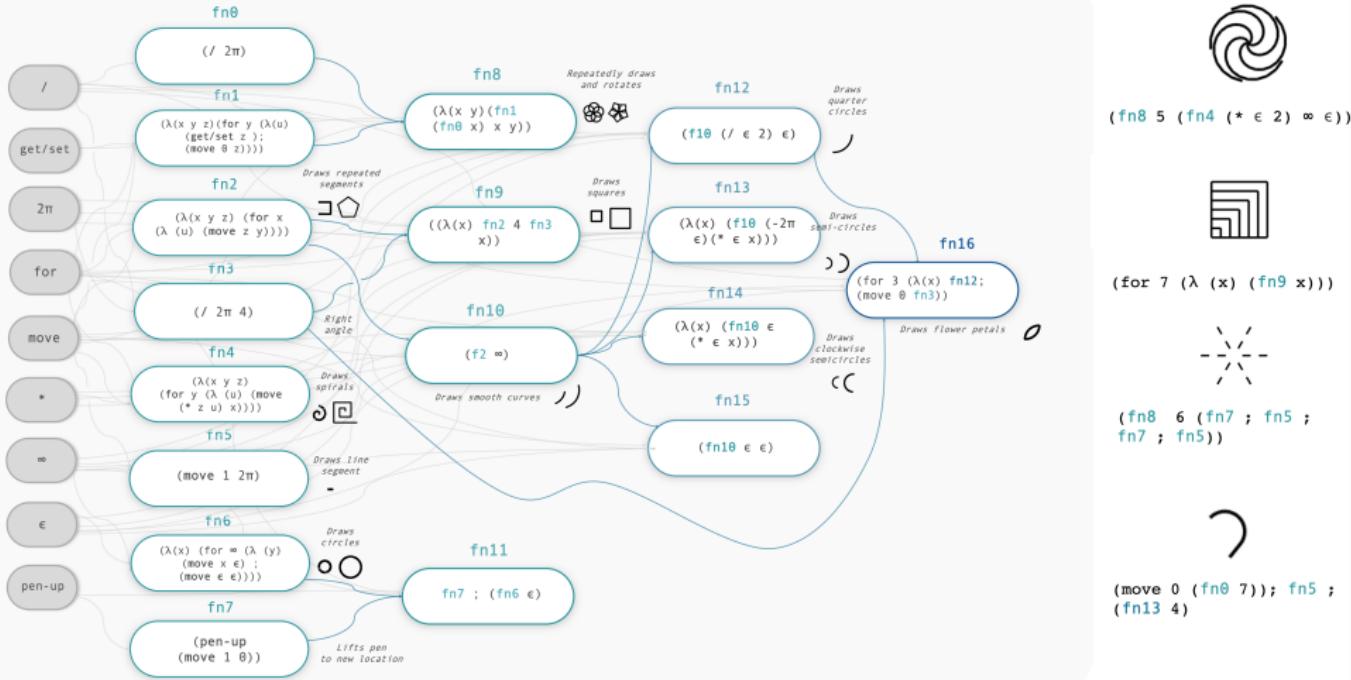
task : image

```
for i in range( $\infty$ )
> FW ε
> RT ε
```



NUM ::= 1 |  $\pi$  |  $\infty$  |  $\varepsilon$  | + | - | \* | /

# LOGO Turtle Graphics – learning an interpretable library



(fn8 5 (fn4 (\* ε 2) ∞))



(for 7 (λ (x) (fn9 x)))

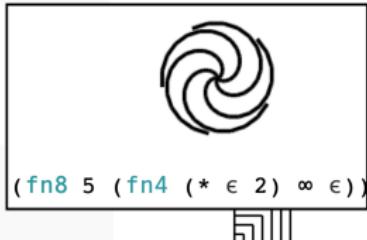
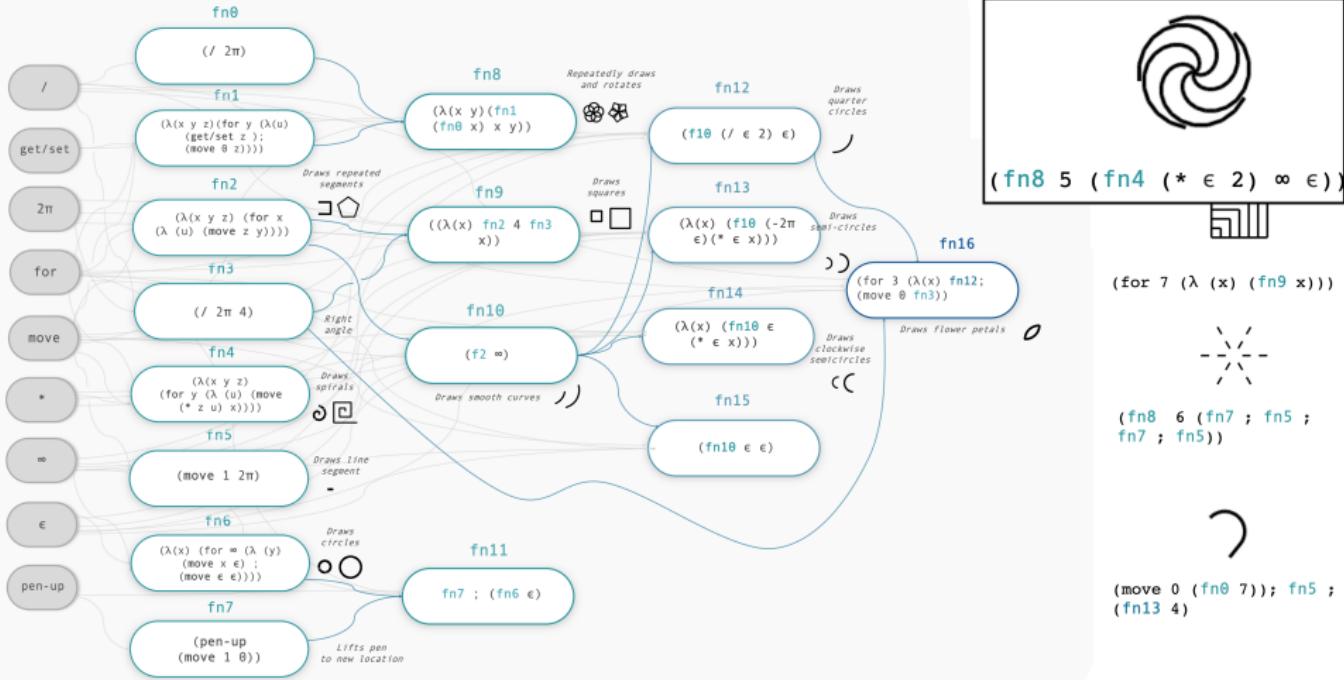


(fn8 6 (fn7 ; fn5 ; fn7 ; fn5))



(move 0 (fn0 7)); fn5 ;  
(fn13 4)

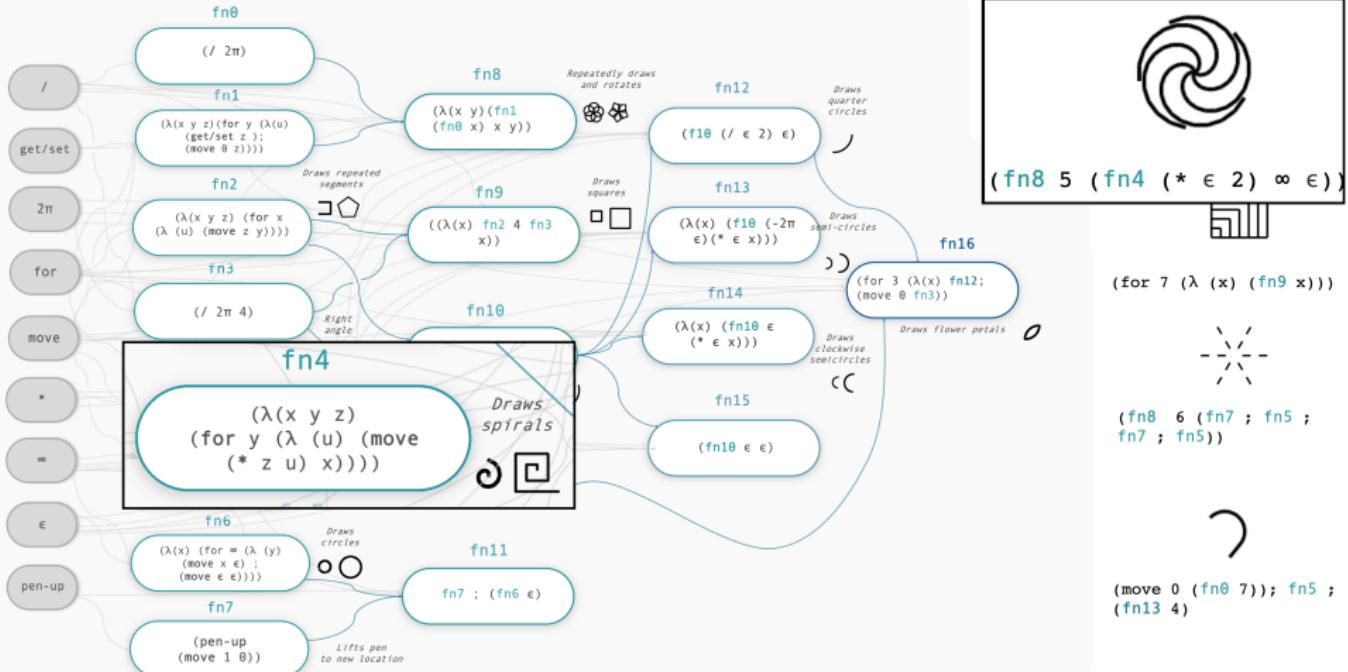
# LOGO Turtle Graphics – learning an interpretable library



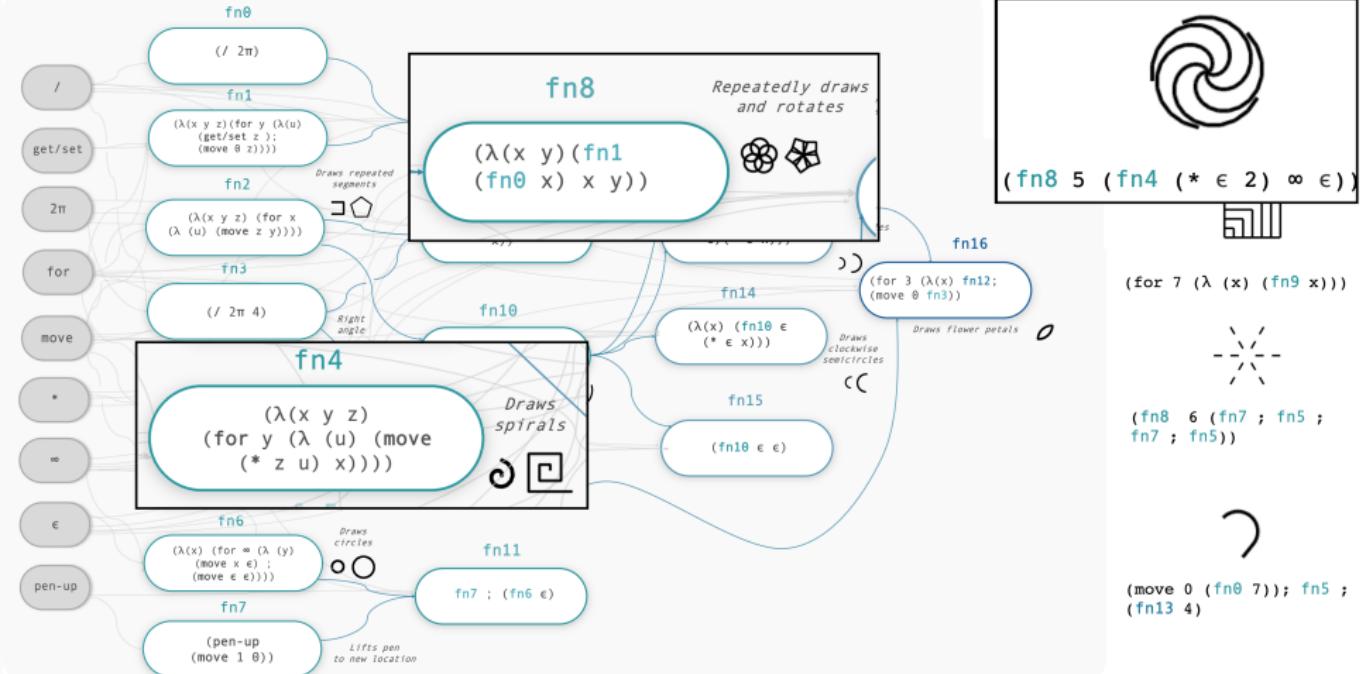
```
(fn8 5 (fn4 (* ε 2) ∞ ε))
(for 7 (λ (x) (fn9 x)))
(fn8 6 (fn7 ; fn5 ;
fn7 ; fn5))
```

```
(move 0 (fn0 7)); fn5 ;
(fn13 4)
```

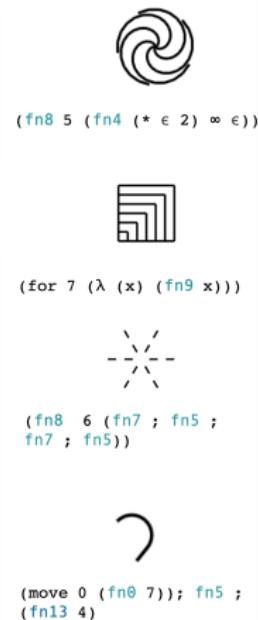
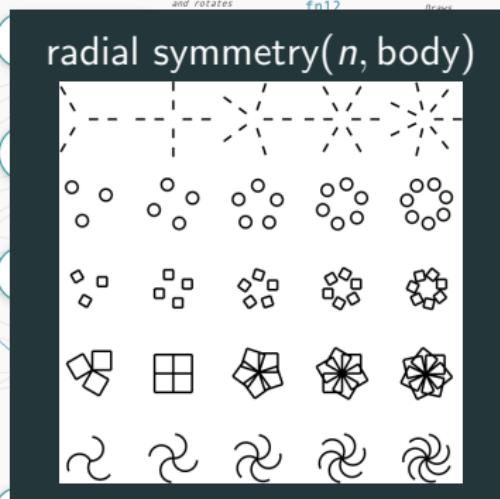
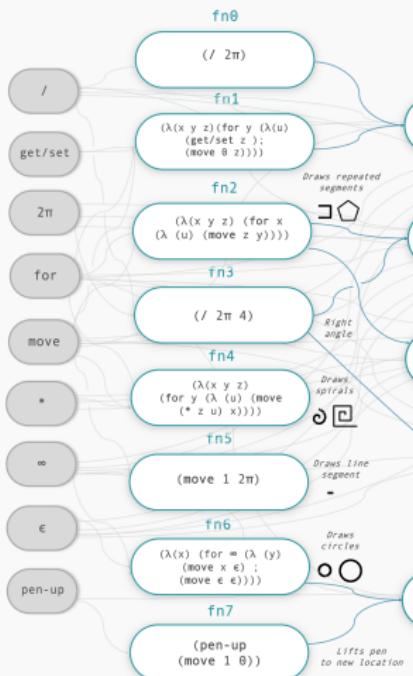
# LOGO Turtle Graphics – learning an interpretable library



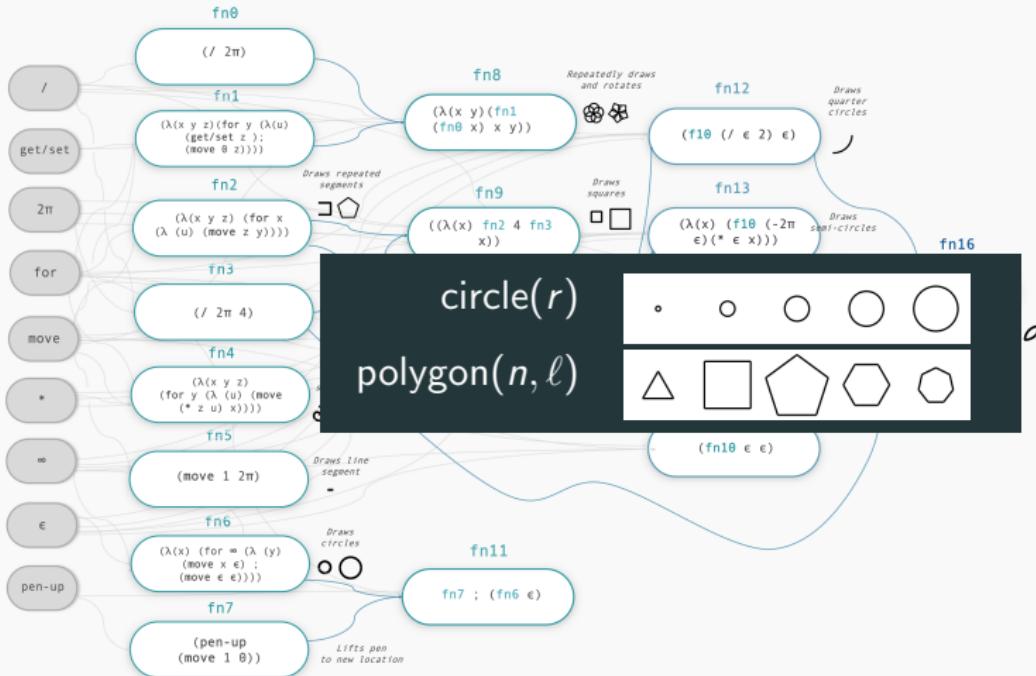
# LOGO Turtle Graphics – learning an interpretable library



# LOGO Turtle Graphics – learning an interpretable library



LOGO Turtle Graphics – learning an interpretable library



(fn8 5 (fn4 (\* \ 2) \ \ ))



```
(for 7 (λ (x) (fn9 x)))
```

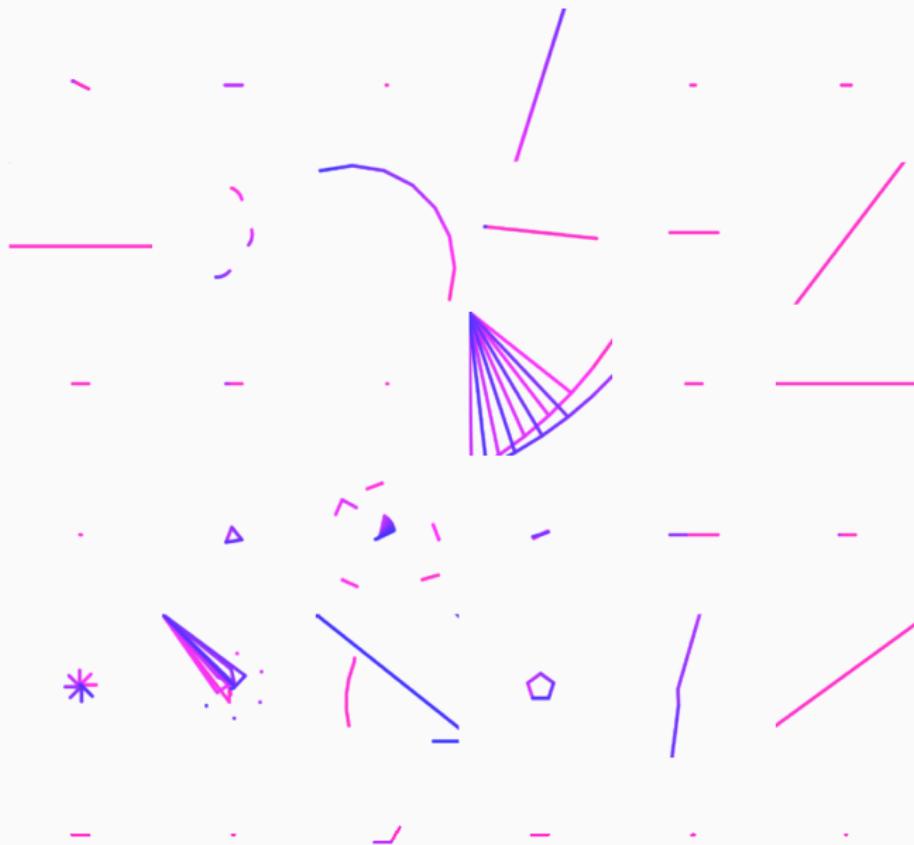


(fn8 6 (fn7 ; fn5 ;  
fn7 ; fn5))

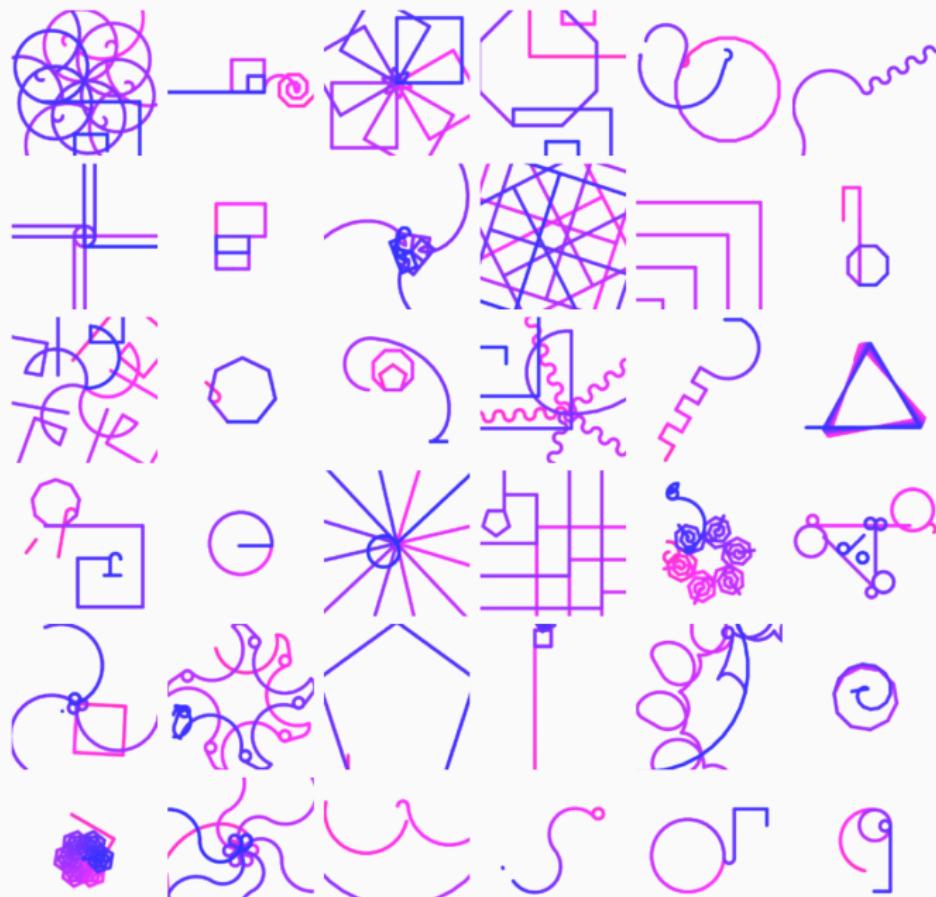


```
(move 0 (fn# 7)); fn5 ;  
(fn13 4)
```

# What does DreamCoder dream of? (before learning)

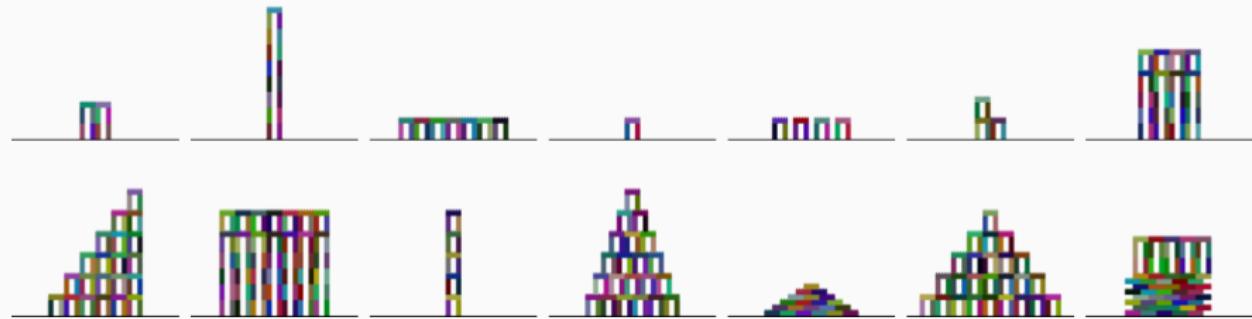


# What does DreamCoder dream of? (after learning)



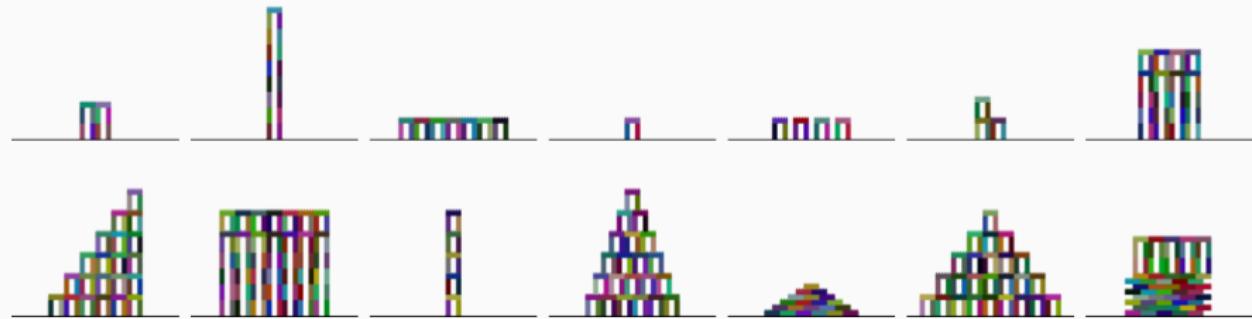
# Planning to build towers

example tasks (112 total)



# Planning to build towers

example tasks (112 total)



learned library routines ( $\approx 20$  total)

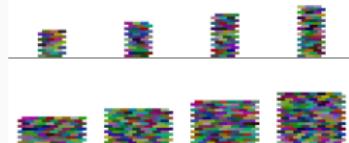
$\text{arch}(h)$



$\text{pyramid}(h)$



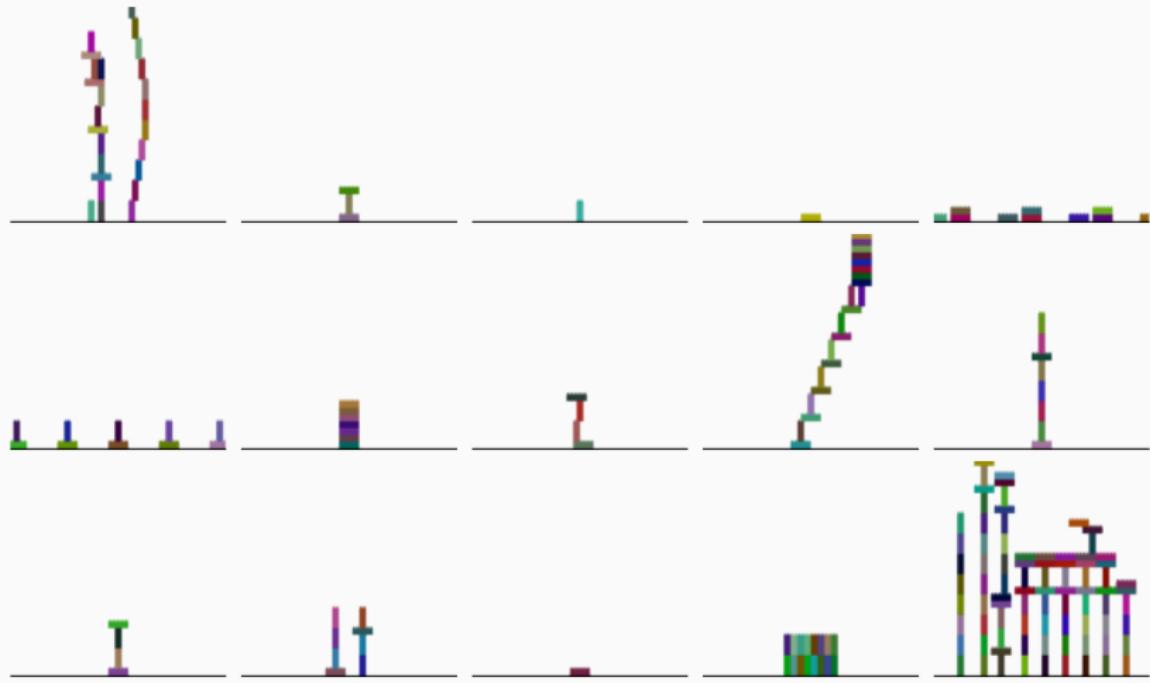
$\text{wall}(w, h)$



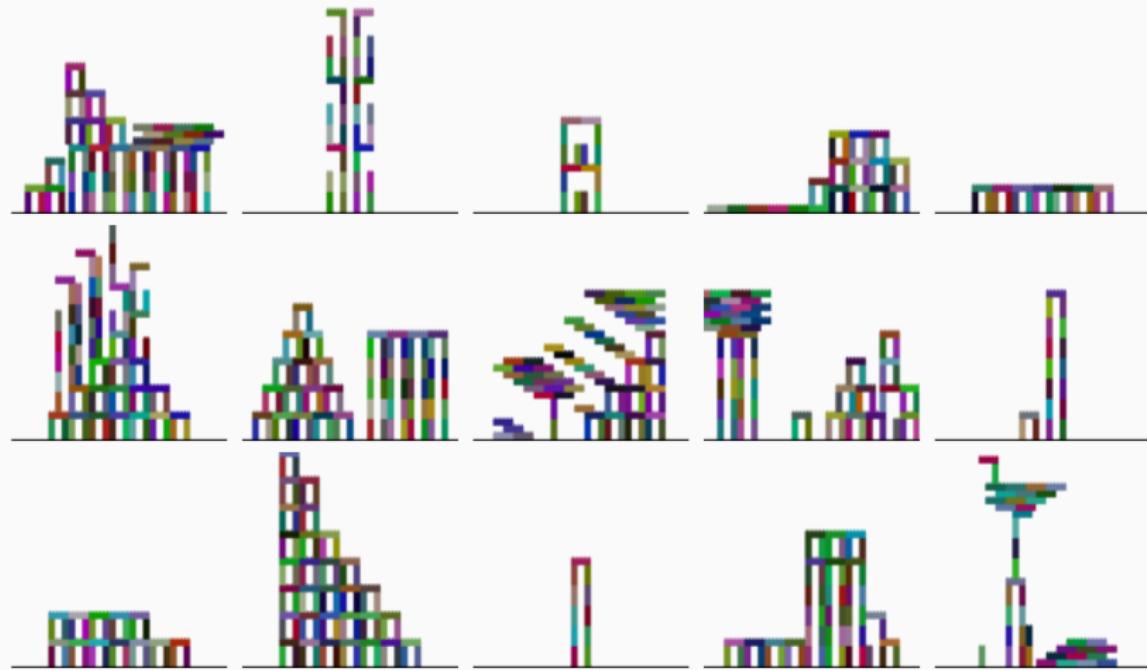
$\text{bridge}(w, h)$



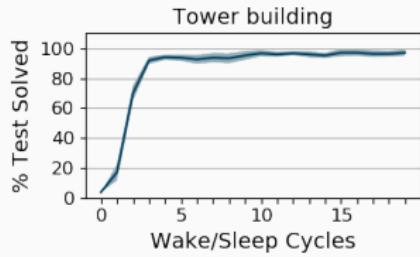
## Dreams before learning



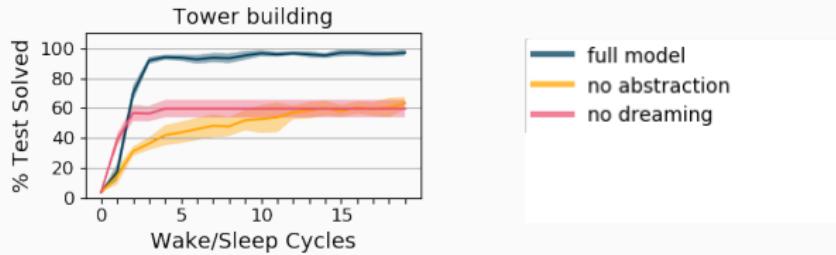
## Dreams after learning



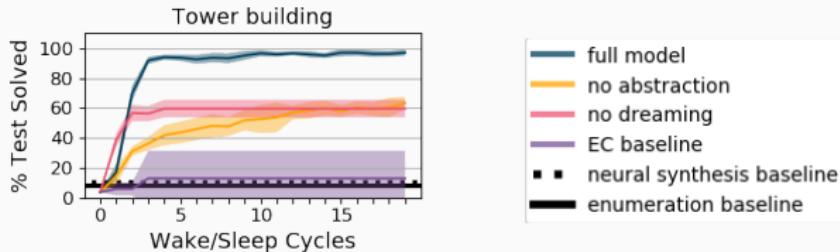
# Learning dynamics



# Learning dynamics

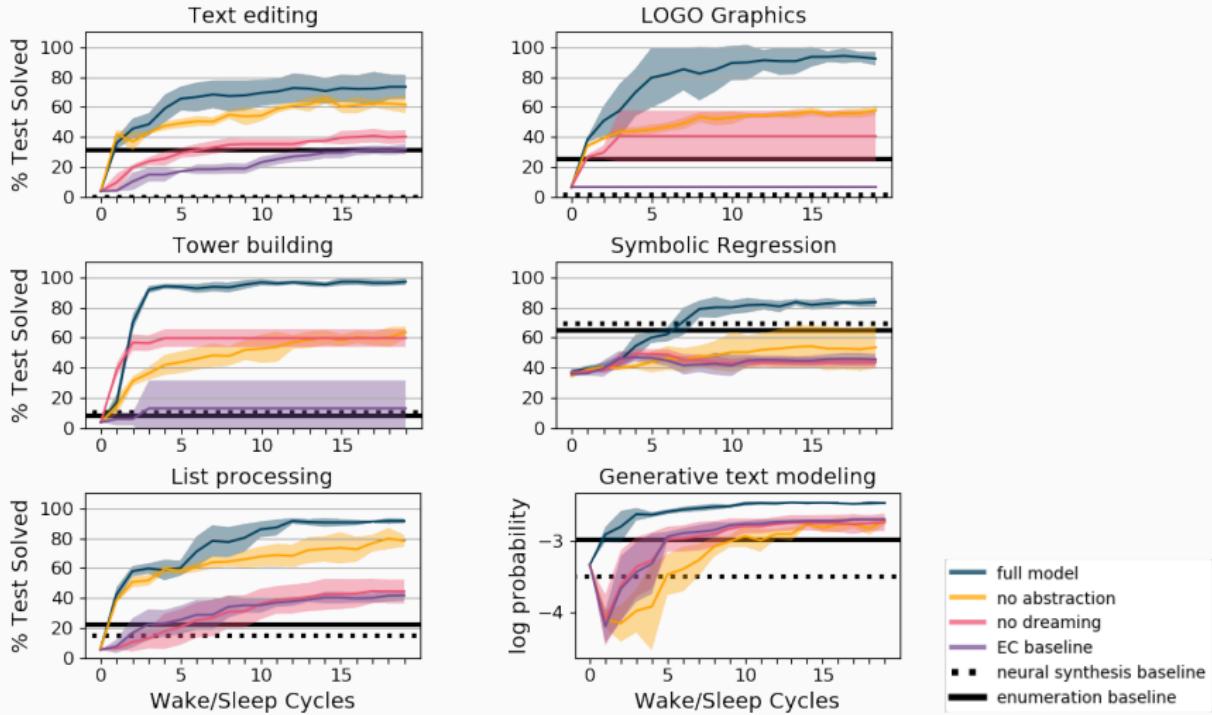


# Learning dynamics



baselines: Exploration-Compression, EC [Dechter et al. 2013]  
neural program synthesis, RobustFill [Devlin et al. 2017]  
24 hours of brute-force enumeration

# Learning dynamics



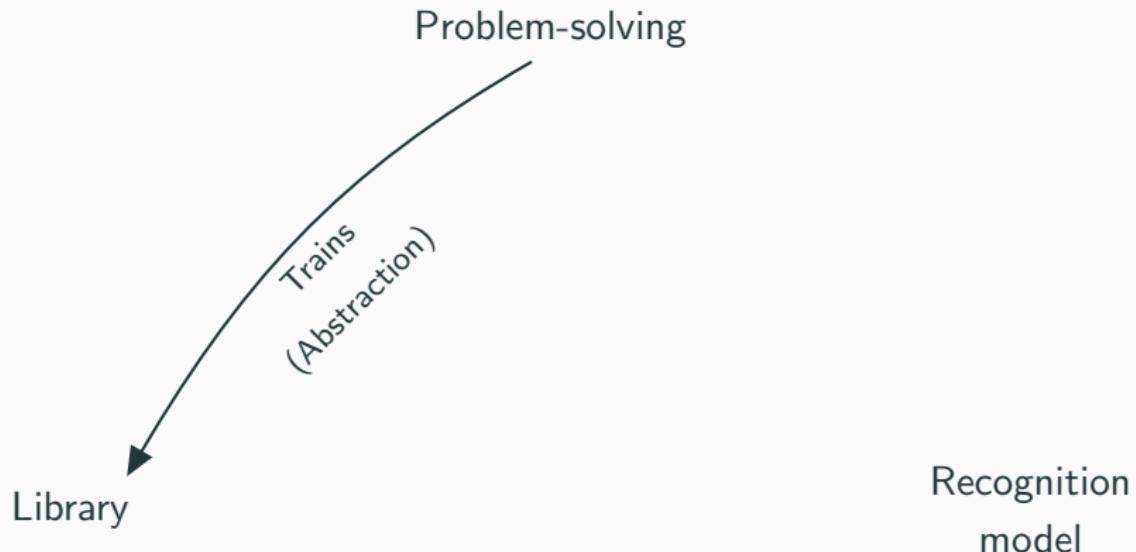
# Synergy between recognition model and library learning

Problem-solving

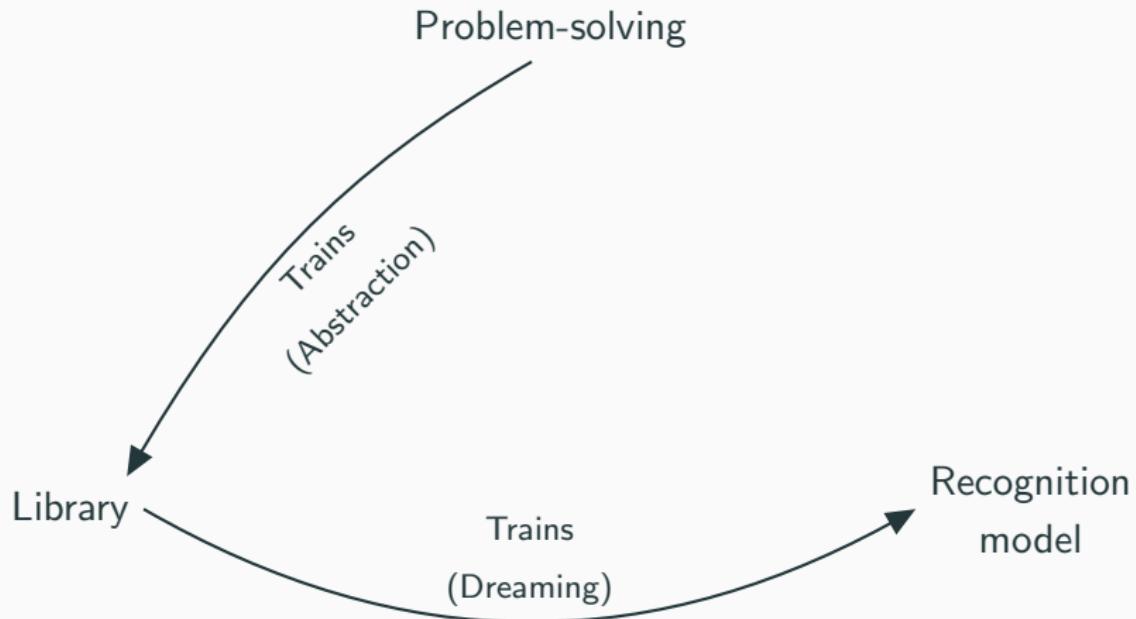
Library

Recognition  
model

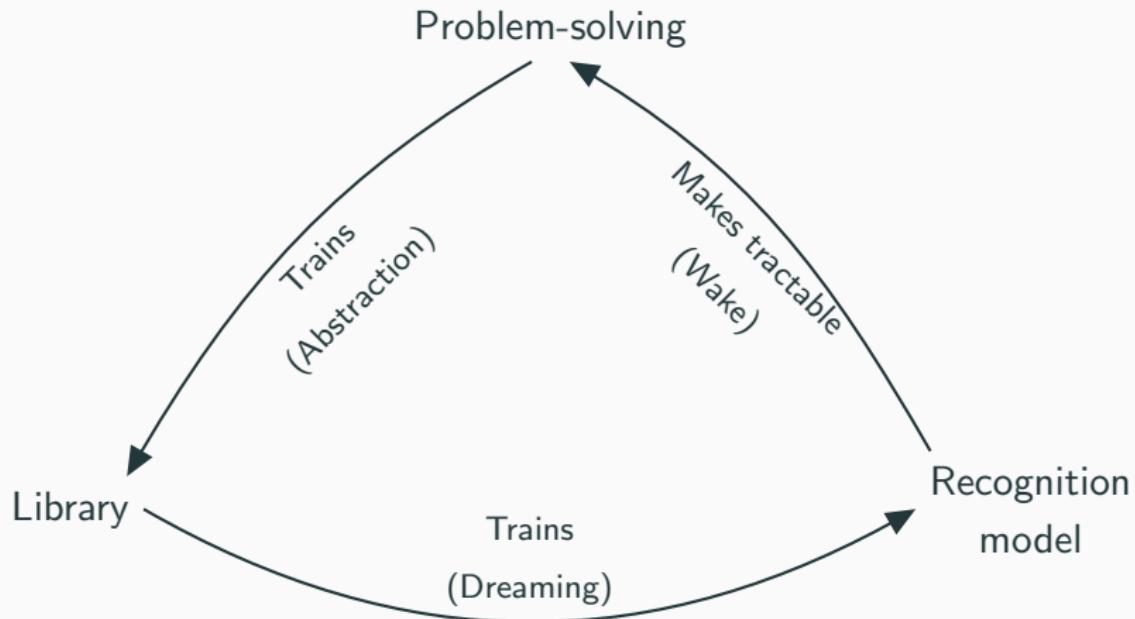
# Synergy between recognition model and library learning



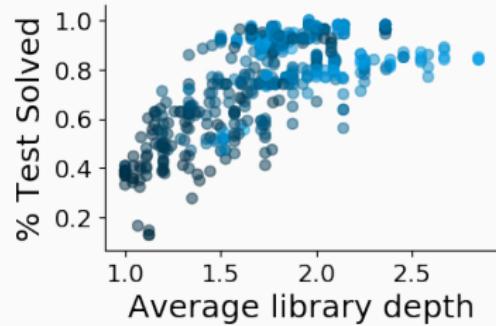
# Synergy between recognition model and library learning



# Synergy between recognition model and library learning



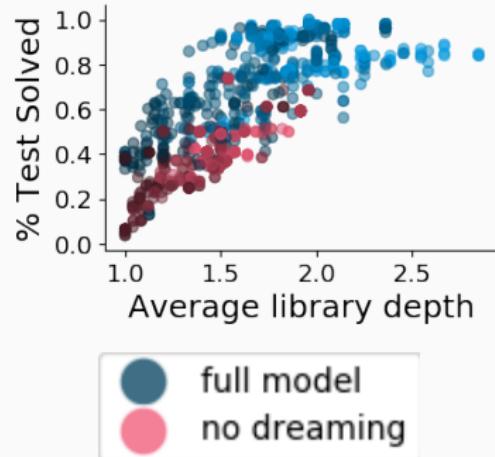
# Evidence for dreaming bootstrapping better libraries



Darker: Early in learning

Brighter: Later in learning

# Evidence for dreaming bootstrapping better libraries



Darker: Early in learning

Brighter: Later in learning

From learning libraries,  
to learning languages

From learning libraries,  
to learning languages

modern functional programming → physics

From learning libraries,  
to learning languages

1950's Lisp → modern functional programming → physics

# Physics Formula Sheet

## Mechanics

$x = x_0 + v_{x0}t + \frac{1}{2}a_xt^2$	$a_t = \frac{v^2}{r}$	$ \vec{F}_{\text{spring}}  = k  \vec{x} $
$v = v_0 + at$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$	$\text{PE}_{\text{spring}} = \frac{1}{2}kx^2$
$v_s^2 - v_{s0}^2 = 2a(x - x_0)$	$\omega = \omega_0 + \alpha t$	$T_{\text{spring}} = 2\pi \sqrt{\frac{m}{k}}$
$\bar{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{\text{net}}}{m}$	$T = \frac{2\pi}{\omega} = \frac{1}{f}$	$T_{\text{pendulum}} = 2\pi \sqrt{\frac{L}{g}}$
$ \vec{F}_{\text{friction}}  \leq \mu  \vec{F}_{\text{Normal}} $	$v = f\lambda$	
$\bar{p} = m\bar{v}$	$x = A\cos(2\pi ft)$	$ \vec{F}_{\text{gravity}}  = G \frac{m_1 m_2}{r^2}$
$\Delta \bar{p} = \vec{F} \Delta t$	$\bar{a} = \frac{\sum \vec{F}}{I} = \frac{\vec{F}_{\text{net}}}{I}$	$ \vec{F}_{\text{gravity}}  = m\bar{g}$
$KE = \frac{1}{2}mv^2$	$\vec{r} = r \times F$	$\text{PE}_{\text{gravity}} = -G \frac{m_1 m_2}{r}$
$\Delta PE = mg\Delta y$	$L = I\omega$	$p = \frac{m}{V}$
$\Delta E = W = Fd\cos\theta$	$\Delta L = \tau \Delta t$	$KE = \frac{1}{2}I\omega^2$

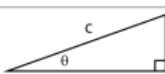
## Electricity

$ \vec{F}_E  = k \left  \frac{q_1 q_2}{r^2} \right $	$\Delta V = IR$	$R = \frac{\rho l}{A}$
$I = \frac{\Delta q}{\Delta t}$		$P = I\Delta V$
$R_{\text{series}} = R_1 + R_2 + \dots + R_n$	$\frac{1}{R_{\text{parallel}}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}$	

## Geometry

Rectangle	$A = bh$	Rectangular Solid	$V = lwh$	Triangle	$A = \frac{1}{2}bh$
Circle	$A = \pi r^2$	Cylinder	$V = \pi r^2 l$	Sphere	$V = \frac{4}{3}\pi r^3$
	$C = 2\pi r$		$S = 2\pi rl + 2\pi r^2$		$S = 4\pi r^2$

## Trigonometry



$$\begin{array}{ccccccc} c & & & & & & \\ & a & & & & & \\ & & & & & & \\ & & & & \theta & & \\ & & & & & & \\ & & & & \square & & \end{array}$$

$$c^2 = a^2 + b^2 \quad \sin\theta = \frac{a}{c} \quad \cos\theta = \frac{b}{c} \quad \tan\theta = \frac{a}{b}$$

## Variables

a = acceleration  
 A = amplitude  
 A = Area  
 b = base length  
 C = circumference  
 d = distance  
 E = energy  
 f = frequency  
 F = force  
 h = height  
 I = current  
 I = rotational inertia  
 KE = kinetic energy  
 k = spring constant  
 L = angular momentum  
 l = length  
 m = mass  
 P = power  
 p = momentum  
 q = charge  
 r = radius  
 R = resistance  
 S = surface area  
 T = period  
 t = time  
 PE = potential energy  
 V = electric potential  
 V = volume  
 v = velocity  
 w = width  
 W = work  
 x = position  
 y = height  
 $\alpha$  = angular acceleration  
 $\lambda$  = wavelength  
 $\mu$  = coefficient of friction

# Growing languages for vector algebra and physics

## Initial Primitives

map  
zip

cons

empty

cdr

power

fold

car

+

-

\*

/

0

1

$\pi$

## Physics Equations

### Newton's Second Law

$$\vec{a} = \frac{1}{m} \sum_l \vec{F}_l$$

### Parallel Resistors

$$R_{total} = \left( \sum_i \frac{1}{R_i} \right)^{-1}$$

### Work

$$U = \vec{F} \cdot \vec{d}$$

### Force in a Magnetic Field

$$|\vec{F}| = q |\vec{v} \times \vec{B}|$$

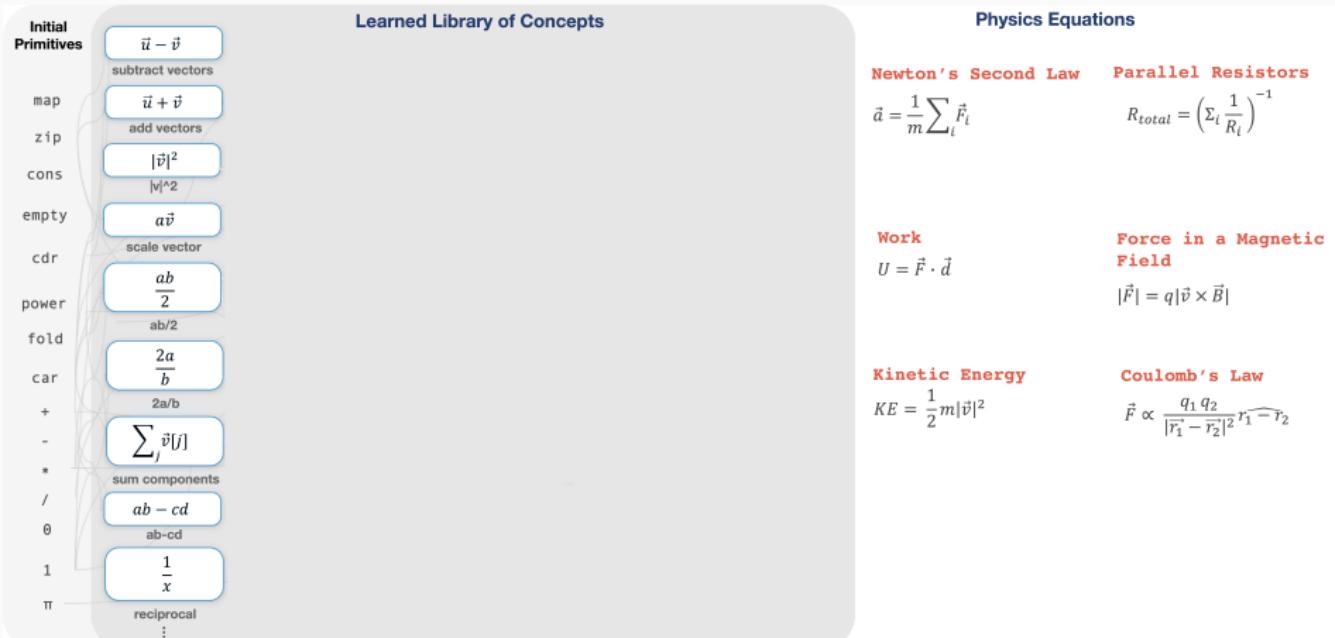
### Kinetic Energy

$$KE = \frac{1}{2} m |\vec{v}|^2$$

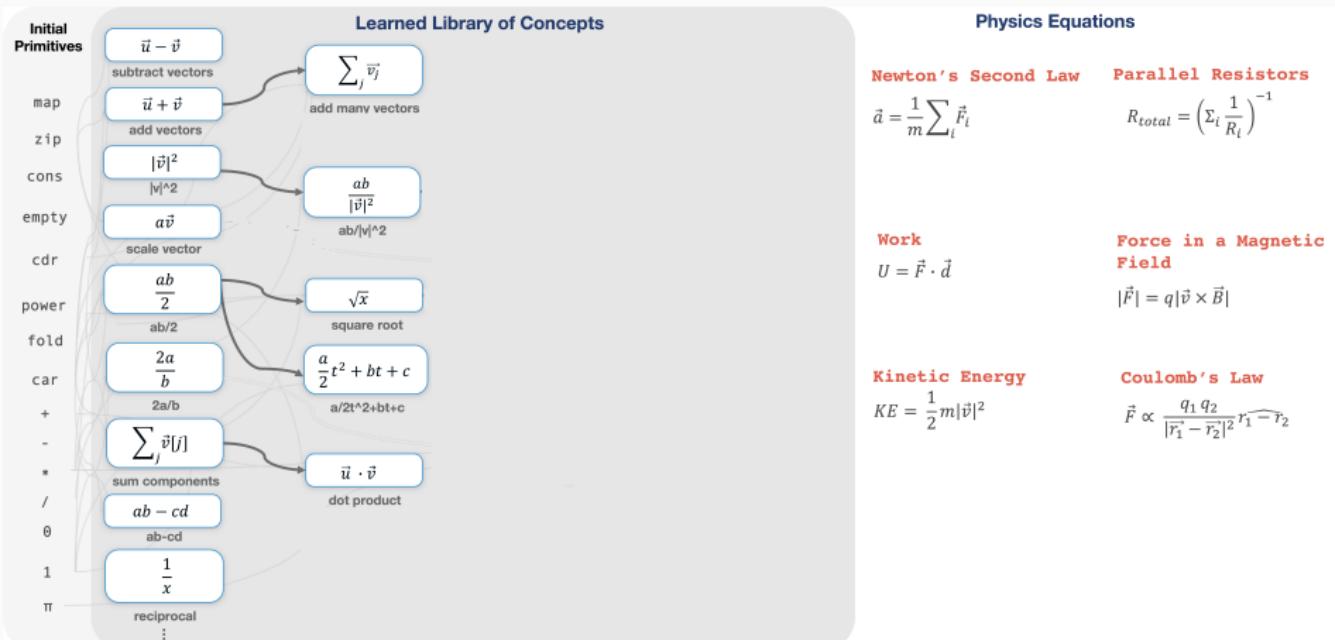
### Coulomb's Law

$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^2} \hat{r}_1 - \hat{r}_2$$

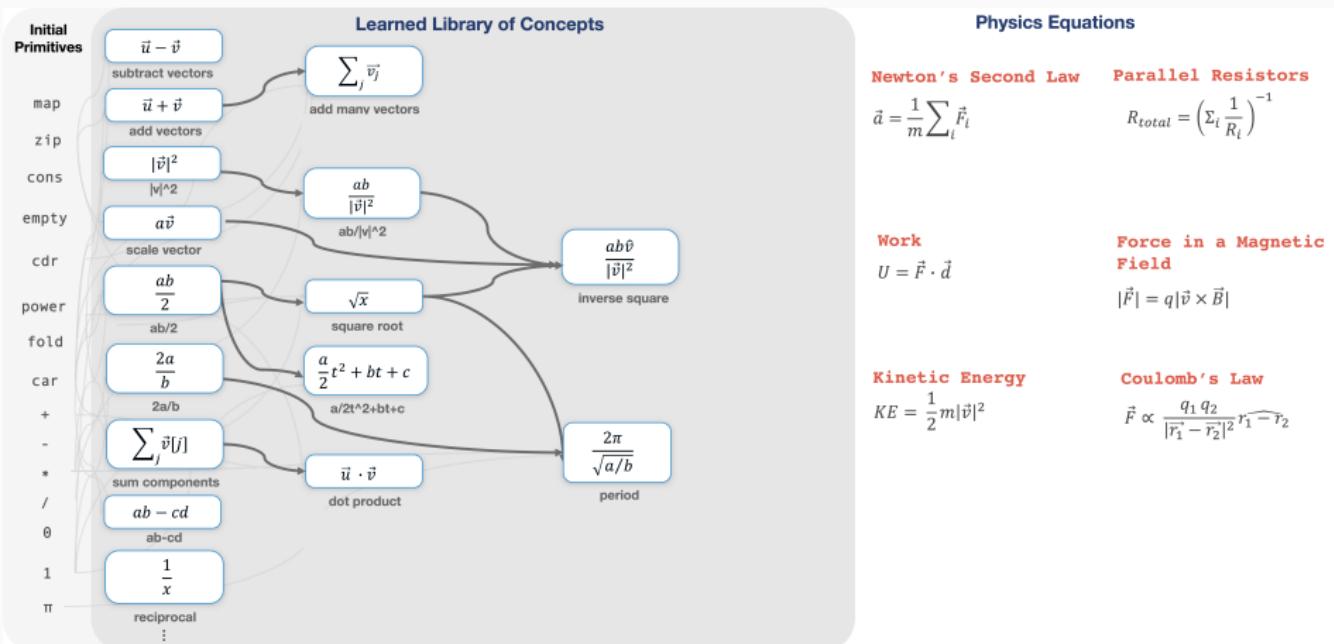
# Growing languages for vector algebra and physics



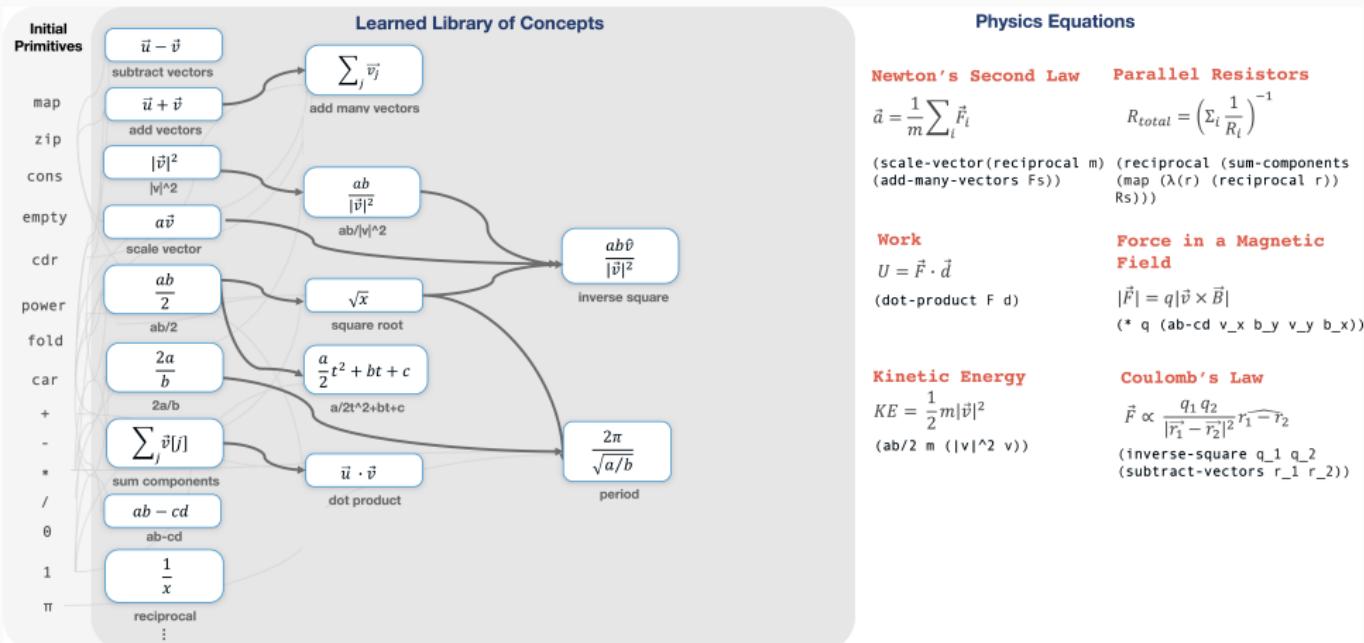
# Growing languages for vector algebra and physics



# Growing languages for vector algebra and physics



# Growing languages for vector algebra and physics



# Growing languages for vector algebra and physics

Initial  
Primitives

$\vec{u} - \vec{v}$   
subtract vectors

map

$\vec{u} + \vec{v}$   
add vectors

zip

cons

empty

cdr

power

fold

car

+

-

\*

/

0

1

$\pi$

Learned Library of Concepts

$$\sum_j \vec{v}_j$$

add many vectors

$$|\vec{v}|^2$$

$$|v|^2$$

$$ab$$

$$|v|^2$$

$$a\vec{v}$$

scale vector

$$\frac{ab}{2}$$

$$ab/2$$

$$\frac{2a}{b}$$

$$2a/b$$

$$\sum_j \vec{v}[j]$$

sum components

$$ab - cd$$

$$ab - cd$$

$$\frac{1}{x}$$

reciprocal

:

Physics Equations

Newton's Second Law

Parallel Resistors

$$\vec{a} = \frac{1}{m} \sum_l \vec{F}_l$$

$$R_{total} = \left( \sum_i \frac{1}{R_i} \right)^{-1}$$

(scale-vector(reciprocal m) (reciprocal (sum-components (add-many-vectors Fs)))  
(map (lambda(r) (reciprocal r)) Rs)))

Work

$$U = \vec{F} \cdot \vec{d}$$

(dot-product F d)

Force in a Magnetic Field

$$|\vec{F}| = q|\vec{v} \times \vec{B}|$$

(\* q (ab-cd v\_x b\_y v\_y b\_x))

Kinetic Energy

$$KE = \frac{1}{2} m |\vec{v}|^2$$

(ab/2 m (|v|^2 v))

Coulomb's Law

$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^2} \hat{r}_1 - \hat{r}_2$$

(inverse-square q\_1 q\_2  
(subtract-vectors r\_1 r\_2))

(lambda (x y z u) (map (lambda (v) (\* (/ (\* (power (/ (\* x x) (fold (zip z u (lambda (w a) (- w a)))) theta (lambda (b c) (+ (\* b b) c)))) (/ (\* 1 1) (+ 1 1)))) y) (fold (zip z u (lambda (d e) (- d e)))) theta (lambda (f g) (+ (\* f f) g)))) v)) (zip z u (lambda (h i) (- h i)))))

Solution to Coulomb's Law if expressed in initial primitives

# Growing a language for recursive programming

## Initial Primitives

Y  
combinator  
cons  
car  
cdr  
nil  
if  
nil?  
+  
-  
0  
1  
=

## Recursive Programming Algorithms

### Stutter

[ ] → [ ]  
[ ] → [ ]

### Take every other

[ ] → [ ]  
[ ] → [ ]

### List lengths

[ , []] → [3 1]  
[[ ], [], []] → [2 0 1]

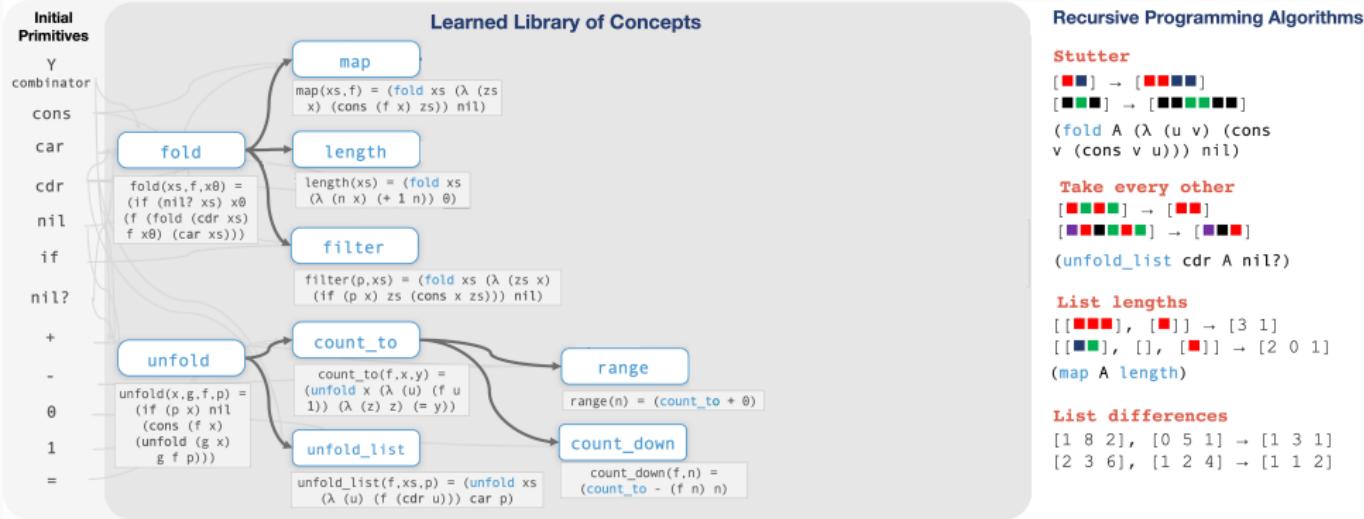
### List differences

[1 8 2], [0 5 1] → [1 3 1]  
[2 3 6], [1 2 4] → [1 1 2]

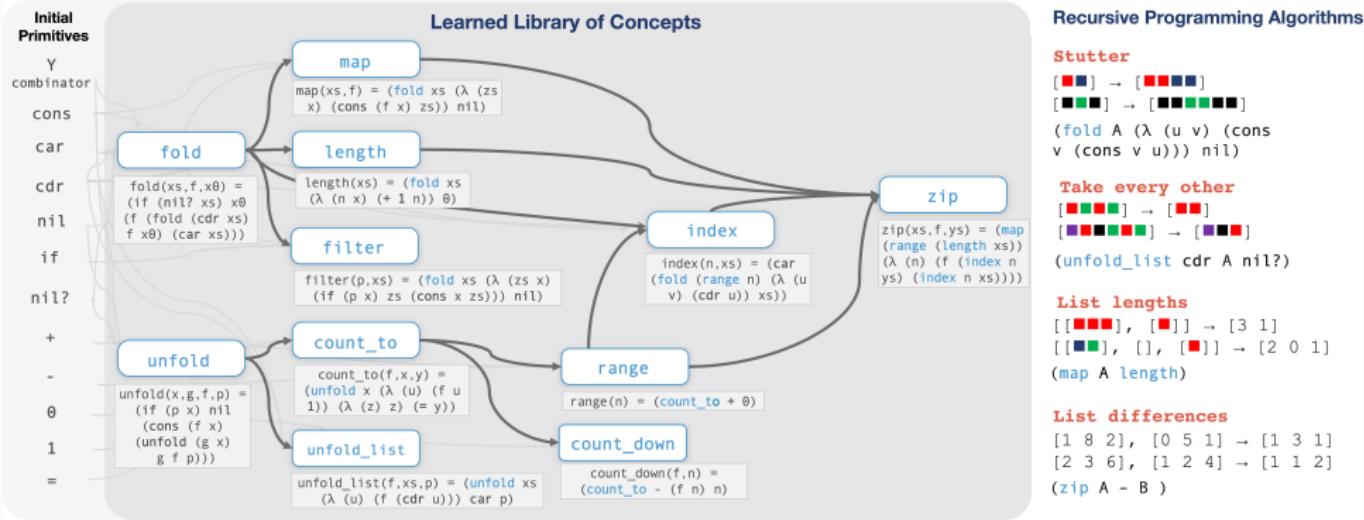
# Growing a language for recursive programming



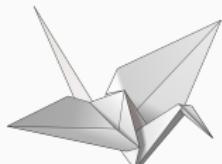
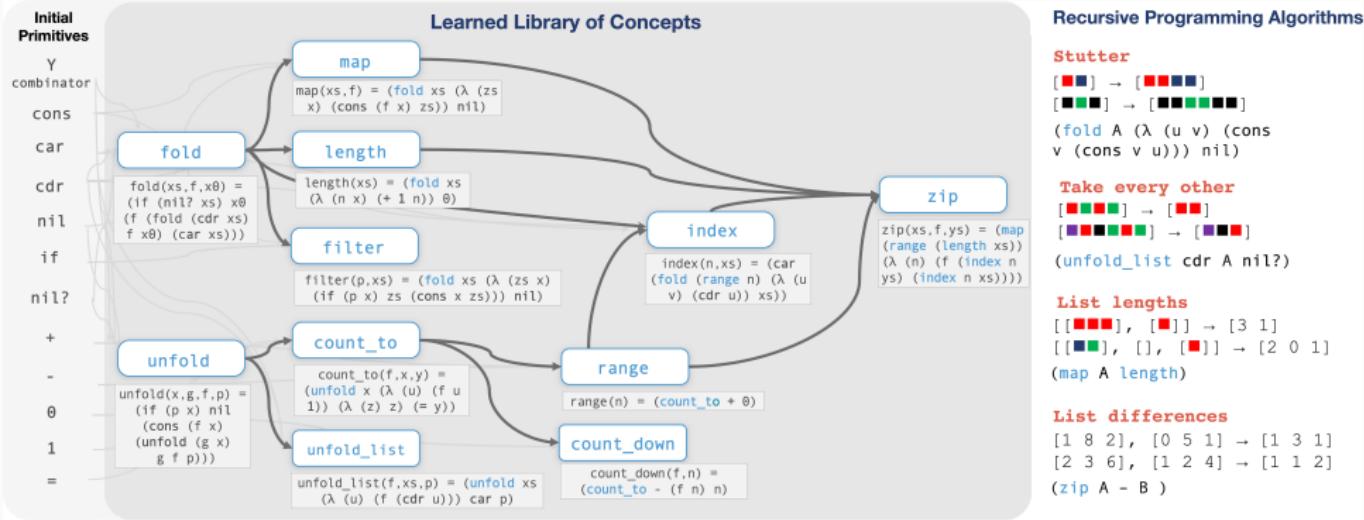
# Growing a language for recursive programming



# Growing a language for recursive programming

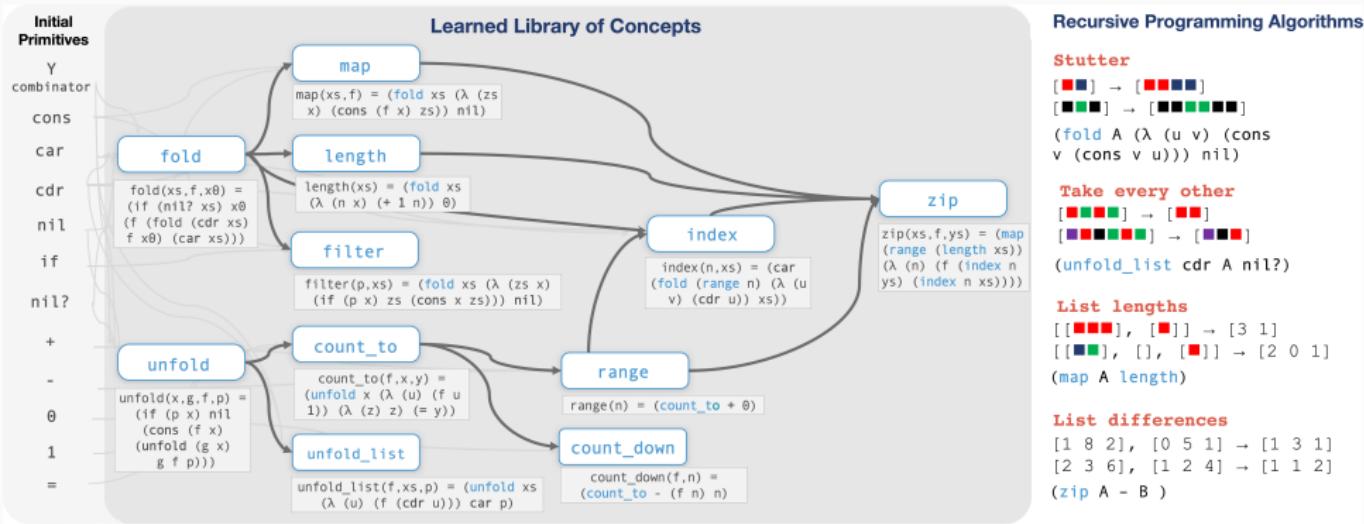


# Growing a language for recursive programming

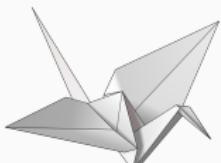


Origami Programming: Jeremy Gibbons, 2003

# Growing a language for recursive programming



1 year of compute. 5 days on 64 CPUs.



Origami Programming: Jeremy Gibbons, 2003

## Lessons

---

Growing a library of abstractions interacts synergistically with learning how to use those abstractions

## Lessons

Growing a library of abstractions interacts synergistically with learning how to use those abstractions

Bayesian, symbolic learning algorithms mean you don't need massive data sets to learn libraries of abstractions

## Lessons

Growing a library of abstractions interacts synergistically with learning how to use those abstractions

Bayesian, symbolic learning algorithms mean you don't need massive data sets to learn libraries of abstractions

Some abstractions can be bootstrapped without language—but surely natural language supervision would help...?

# Collaborators

Josh  
Tenenbaum



Armando  
Solar-Lezama



Cathy Wong



Max Nye



Mathias Sable-Meyer

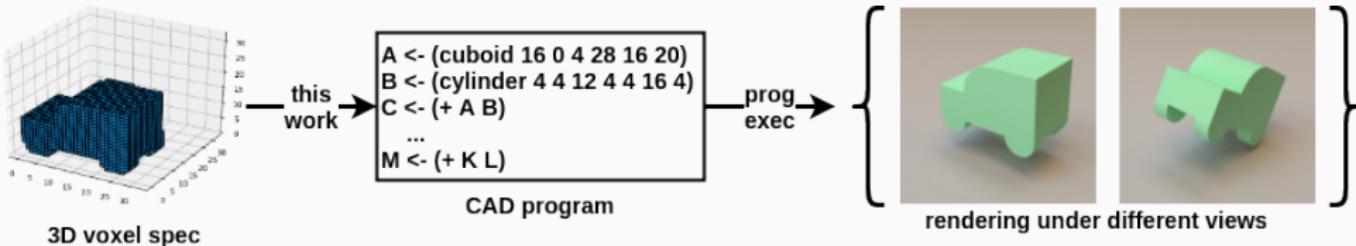


Lucas Morales



thank  
you

# 3D program induction

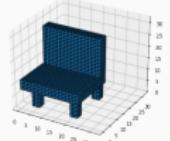


Challenge: combinatorial search!

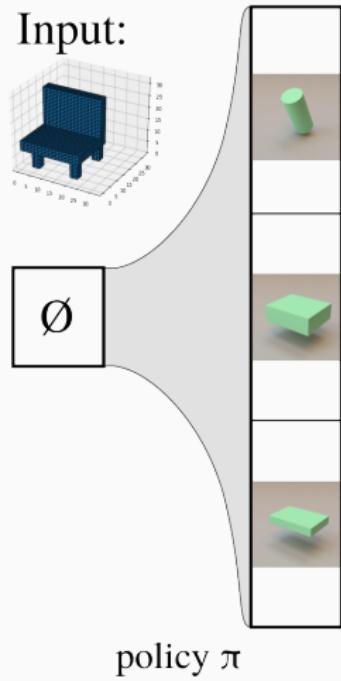
Branching factor:  $> 1.3$  million per line of code,  $\approx 20$  lines of code  
search space size:  $(1.3 \text{ million})^{20} \approx 10^{122}$  programs

Solution: stochastic **tree search** + learn **policy** that writes code  
+ learn **value** function that assesses execution of program so far;  
analogous to **AlphaGo** [Silver et al. 2016]

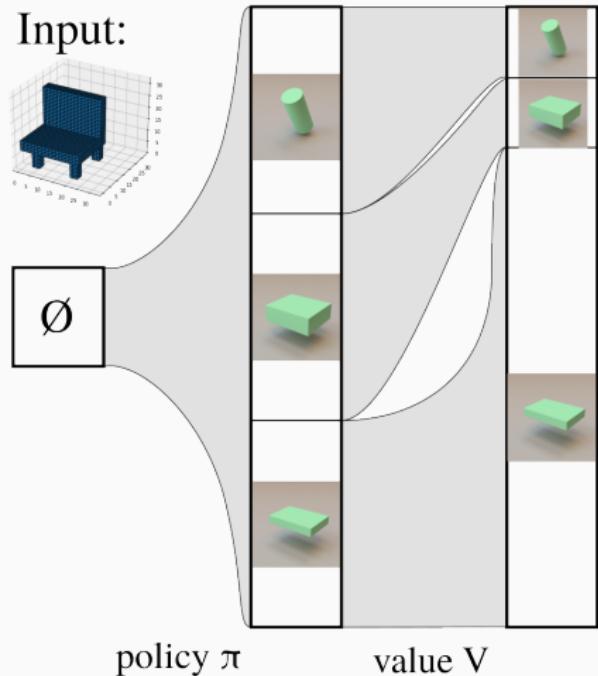
Input:



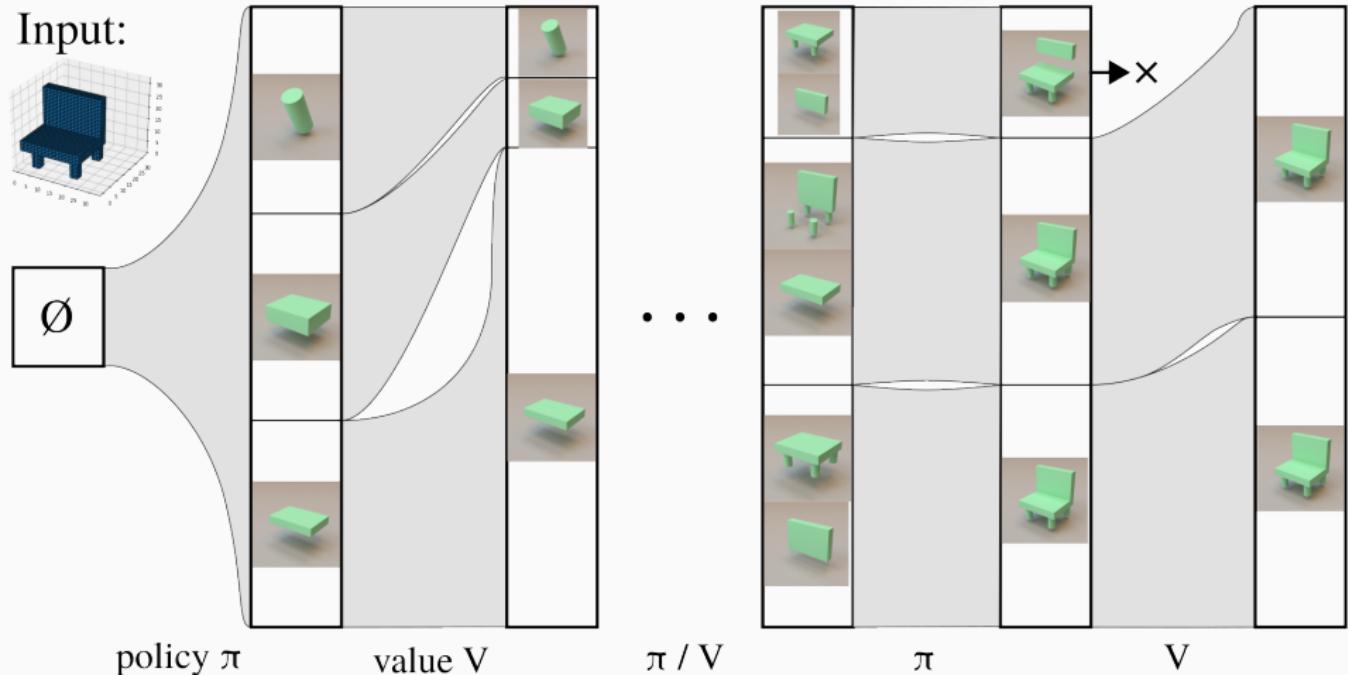
Solution: stochastic **tree search** + learn **policy** that writes code  
+ learn **value** function that assesses execution of program so far;  
analogous to **AlphaGo** [Silver et al. 2016]



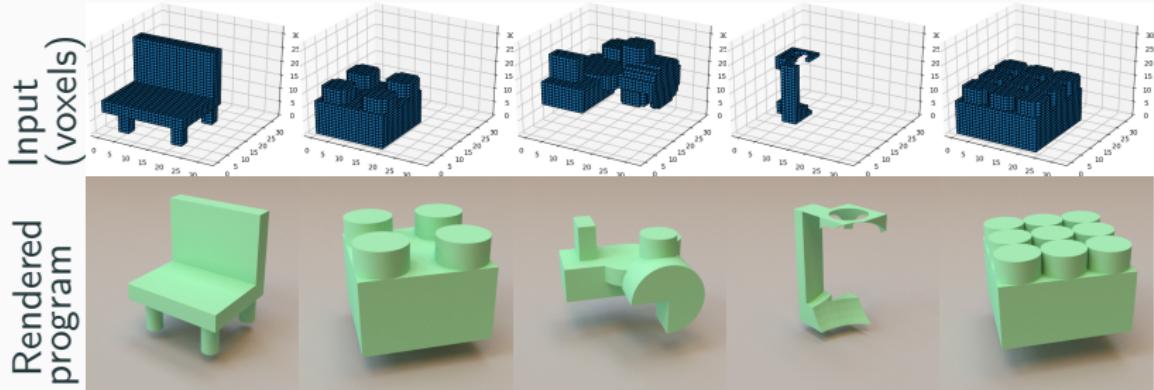
Solution: stochastic **tree search** + learn **policy** that writes code  
+ learn **value** function that assesses execution of program so far;  
analogous to **AlphaGo** [Silver et al. 2016]



Solution: stochastic **tree search** + learn **policy** that writes code  
+ learn **value** function that assesses execution of program so far;  
analogous to **AlphaGo** [Silver et al. 2016]



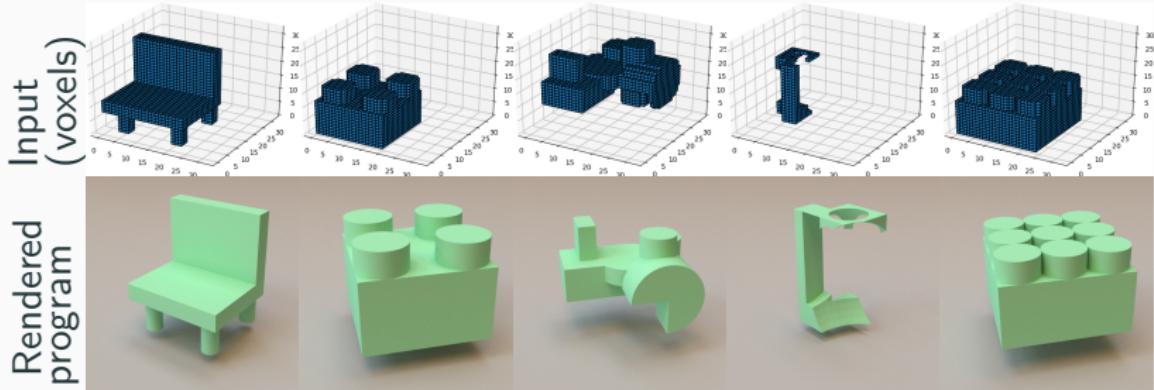
# 3D program induction



Ellis\*, Nye\*, Pu\*, Sosa\*, Tenenbaum, Solar-Lezama. NeurIPS 2019.

\*equal contribution

# 3D program induction



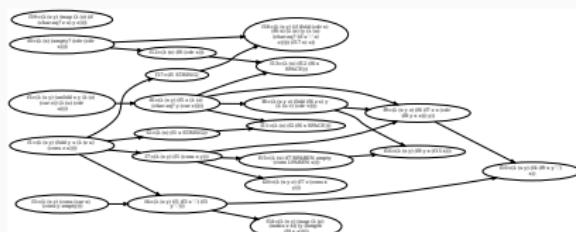
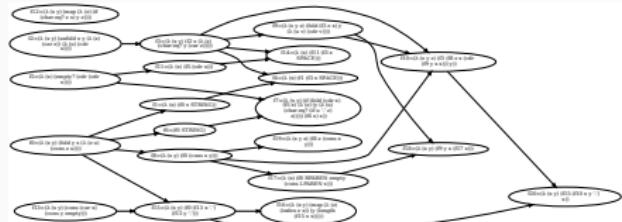
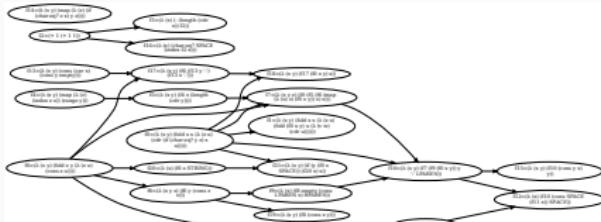
same architecture learns to synthesize text editing programs  
(FlashFill, Gulwani 2012)

Ellis\*, Nye\*, Pu\*, Sosa\*, Tenenbaum, Solar-Lezama. NeurIPS 2019.

\*equal contribution

# Library structure: Text Editing

DreamCoder learns libraries for FlashFill-style text editing [Gulwani 2012]

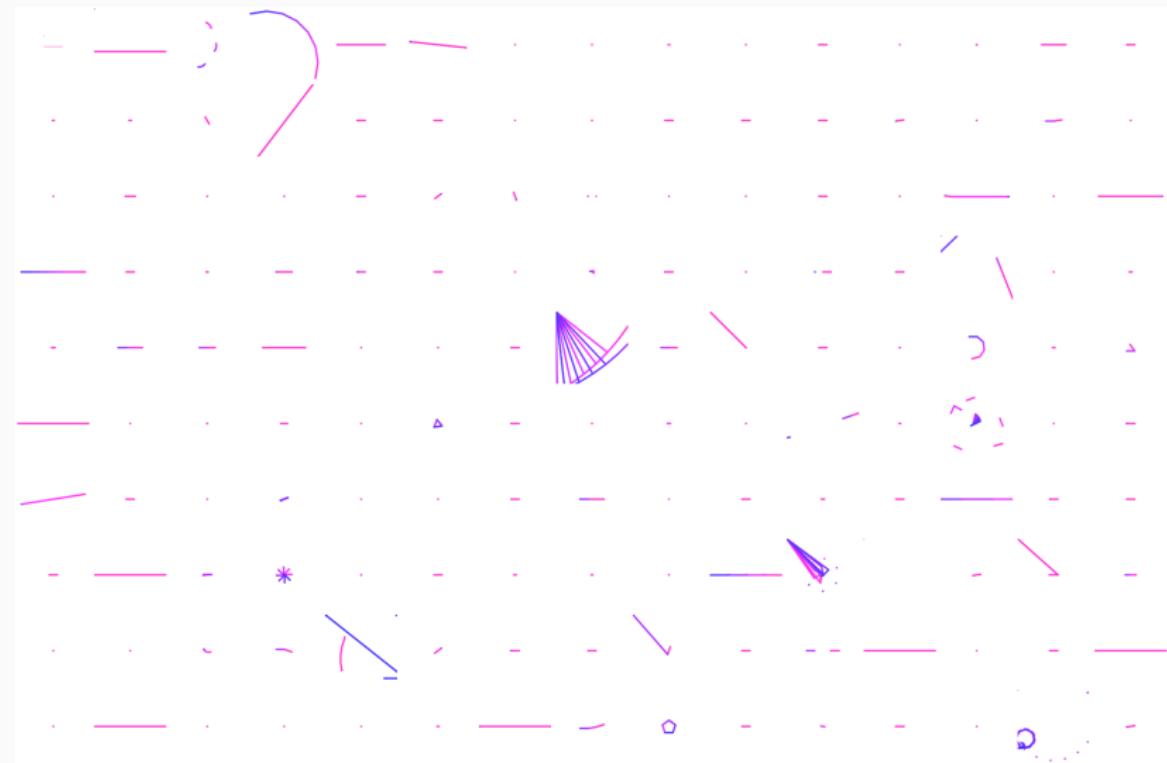


# Library structure: Generating Text

## Libraries for probabilistic generative models over text: data from crawling web for CSV files



# 150 random dreams before learning



# 150 random dreams after learning

