# Building Machines that Discover Generalizable, Interpretable Knowledge

Kevin Ellis

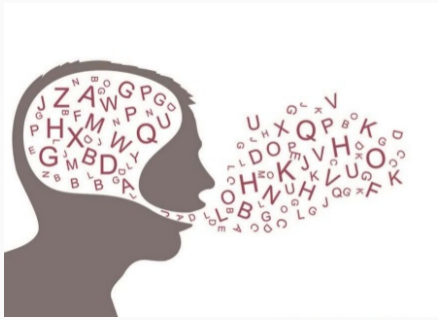
2020

MIT

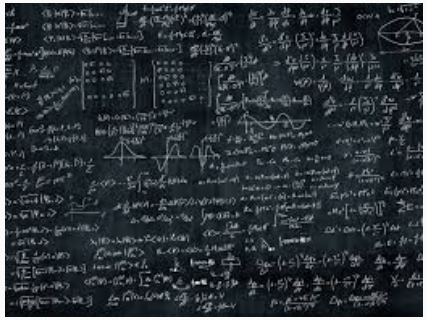
# What computational problems are solved by intelligence?

## an endless range of problems

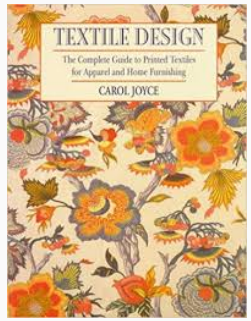
language



science

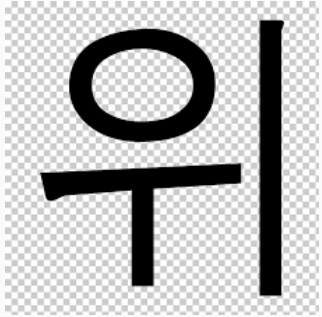


design




using new devices

writing new characters



coding

```
(MEMBER
 (LAMBDA (X L)
  (COND  ((NULL L) NIL)
         ((EQ X (FIRST L)) T)
         (T (MEMBER X (REST L))))))
```

Allen, Anatomy of Lisp, 1975

engineering

play

# What computational frameworks can contribute to this picture?

Three AI traditions

# What computational frameworks can contribute to this picture?

## Three AI traditions

### Symbolic


```
In[34]:= Solve[{(hw - hw^2) == Z}, h]
Out[34]= {}
```

```
In[33]:= Solve[{(hw-hw^2) ==Z},h]
```

Input interpretation:

solve | $h\,w - h\,w^2 = Z$ | for | $h$

Result:

$$h = \frac{Z}{w - w^2} \text{ and } w^2 \neq w$$

# What computational frameworks can contribute to this picture?

## Three AI traditions

### Symbolic


In[34]:= Solve[{(hw - hw^2) == Z}, h]

Out[34]= {}

In[33]:= Solve[{(hw-hw^2) ==Z},h]

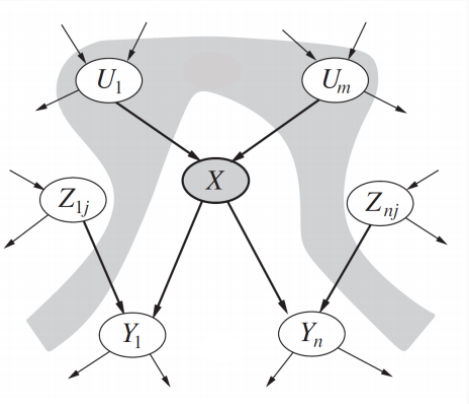
Input interpretation:

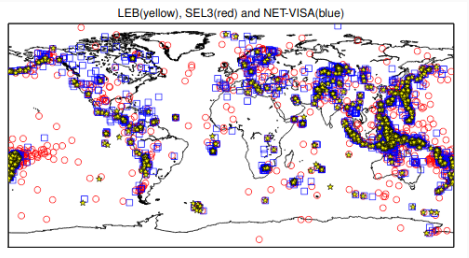
solve | $h\,w - h\,w^2 = Z$ | for | $h$

Result:

$$h = \frac{Z}{w - w^2} \quad \text{and} \quad w^2 \neq w$$

### Probabilistic



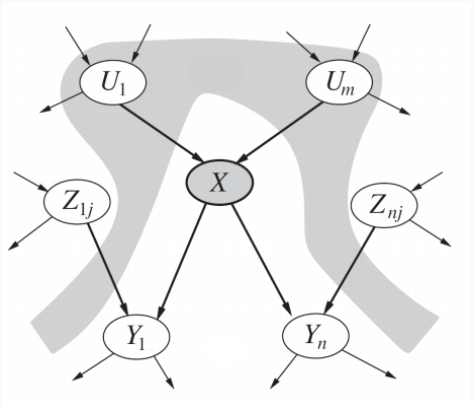
# What computational frameworks can contribute to this picture?

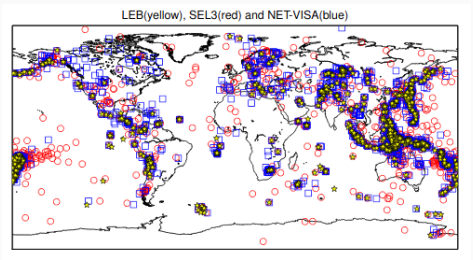
## Three AI traditions

### Symbolic

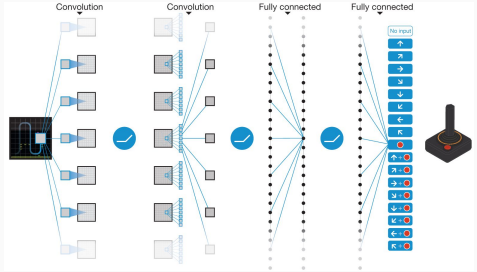

```
In[34]:= Solve[{(hw - hw^2) == Z}, h]
Out[34]= {}

In[33]:= Solve[{(hw-hw^2) ==Z},h]

Input interpretation:
solve | h w - h w^2 = Z | for | h

Result:
h = Z / (w - w^2)  and  w^2 ≠ w
```

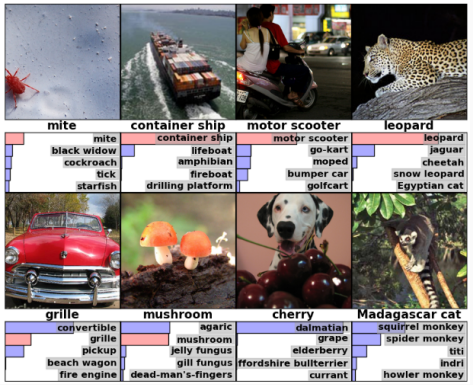
### Probabilistic




### Neural



# What computational frameworks can contribute to this picture?

Three AI traditions

## Symbolic


```
In[34]:= Solve[{(hw - hw^2)
Out[34]= {}
```

```
In[33]:= Solve[{(hw-hw^2) ==Z},h]
```

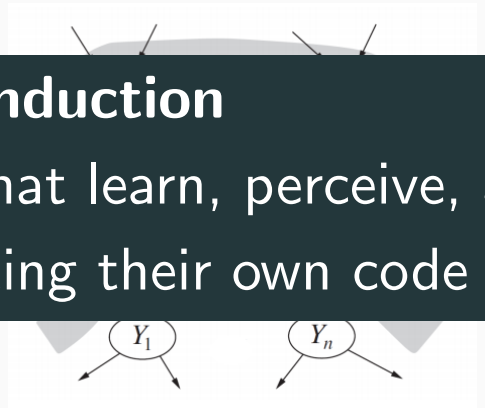
Input interpretation:

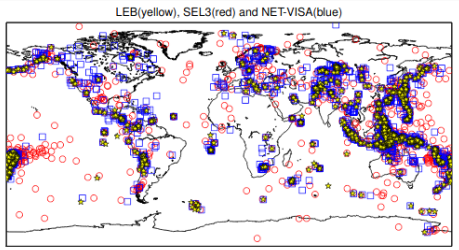
solve | $h\,w - h\,w^2 = Z$ | for | $h$

Result:

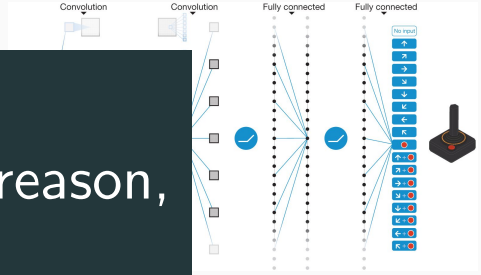
$$h = \frac{Z}{w - w^2} \quad \text{and} \quad w^2 \neq w$$

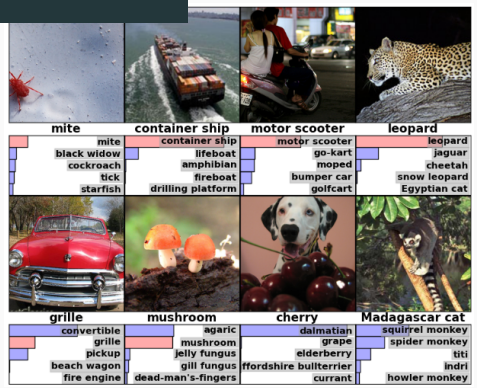
## Probabilistic




## Neural



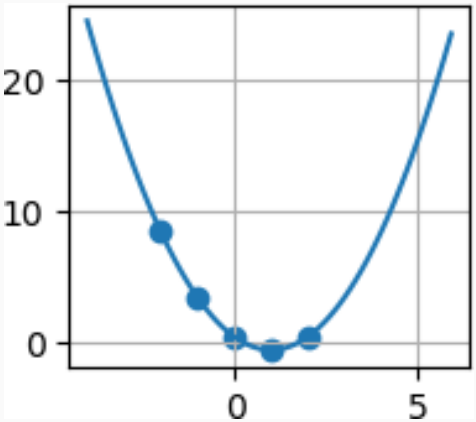




**Program induction**

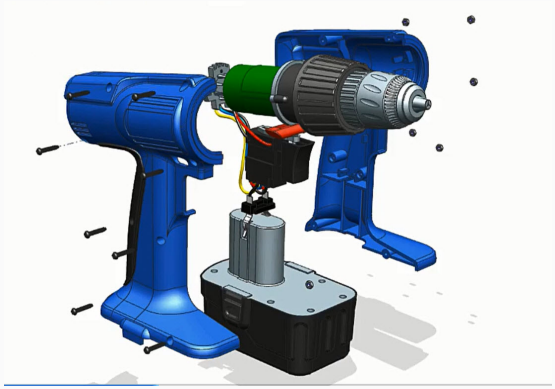
machines that learn, perceive, and reason, by writing their own code

# Why program induction?

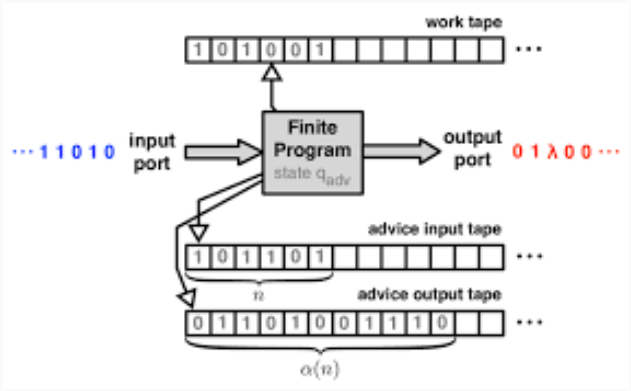
strong generalization
+data efficiency




interpretability



universal expressivity



# Why didn't this old idea work?

Program induction goes back to the 1956 Dartmouth Workshop that founded the field of AI

A PROPOSAL FOR THE

DARTMOUTH SUMMER RESEARCH PROJECT

ON ARTIFICIAL INTELLIGENCE

J. McCarthy, Dartmouth College
M. L. Minsky, Harvard University
N. Rochester, I. B. M. Corporation
C. E. Shannon, Bell Telephone Laboratories

# Why will it work this time?

better toolkits:

# Why will it work this time?

better toolkits:

- **probabilistic** methods for uncertainty and learning-to-learn

# Why will it work this time?

better toolkits:

- **probabilistic** methods for uncertainty and learning-to-learn
- **neural** methods for guiding combinatorial search

# Why will it work this time?

better toolkits:

- **probabilistic** methods for uncertainty and learning-to-learn
- **neural** methods for guiding combinatorial search
- **symbolic** methods, from the **programming languages** community
  - maturing **program synthesis** techniques
  - type systems, program analysis, constraint solving, ...

# Why will it work this time?

better toolkits:

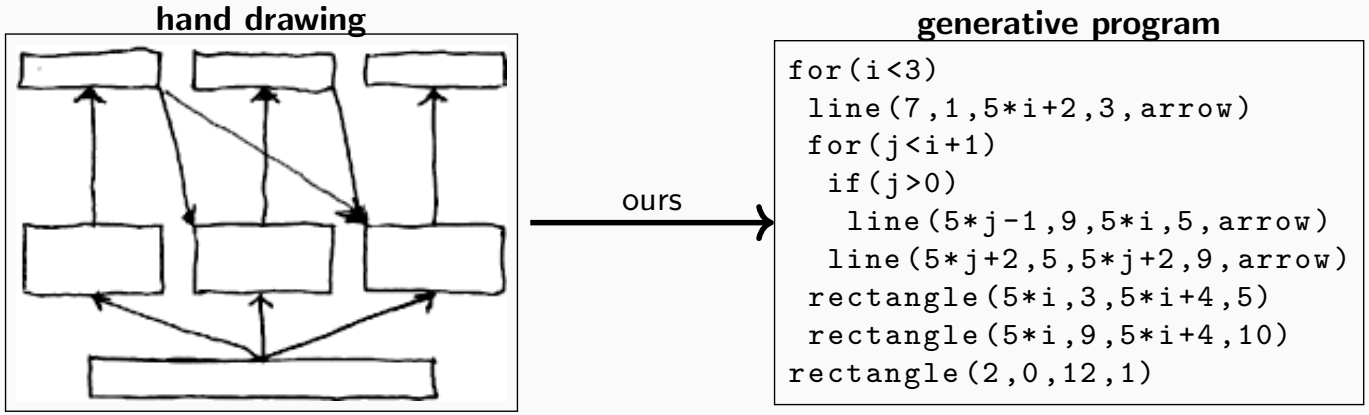
- **probabilistic** methods for uncertainty and learning-to-learn
- **neural** methods for guiding combinatorial search
- **symbolic** methods, from the **programming languages** community
  - maturing **program synthesis** techniques
  - type systems, program analysis, constraint solving, ...

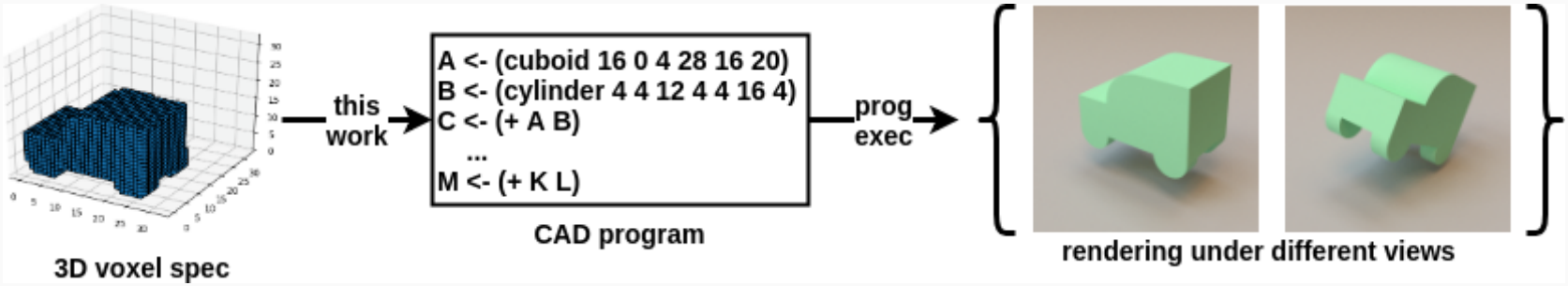
better problems:

- inverse CAD [Kulkarni et al. 2015]
- synthesizing human-understandable models [Evans et al. 2019]
- natural language→code [Liang et al. 2011; Zettlemoyer et al. 2007]
- programming by examples [Gulwani 2011], computer-aided-programming [Solar-Lezama 2008]

# Perception, Synthesizing models, Learning-to-Learn

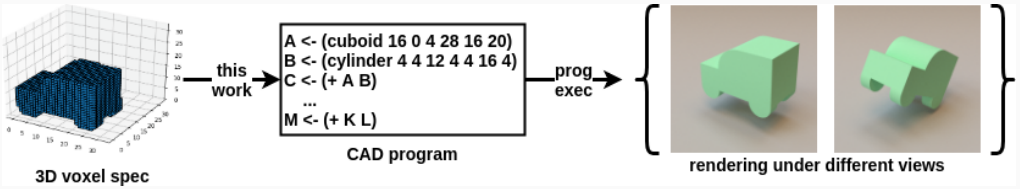
Theme: high-level visual understanding, pixels→programs



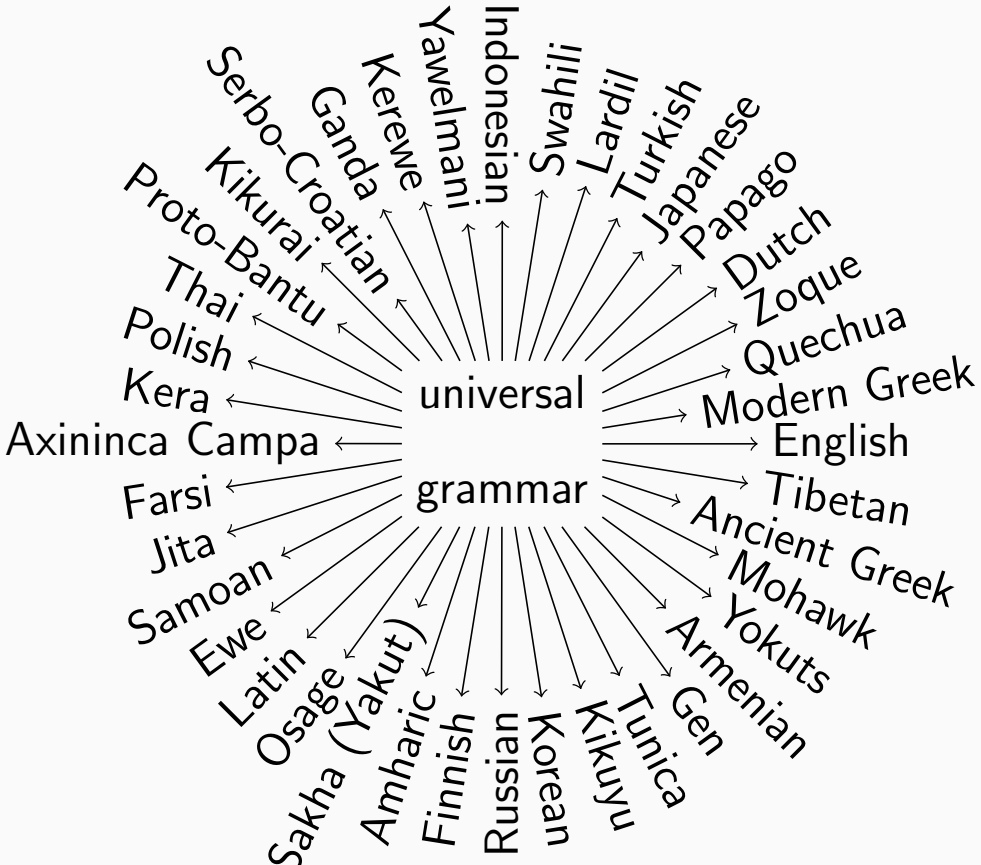



# Perception, Synthesizing models, Learning-to-Learn

Theme: high-level visual understanding, pixels→programs

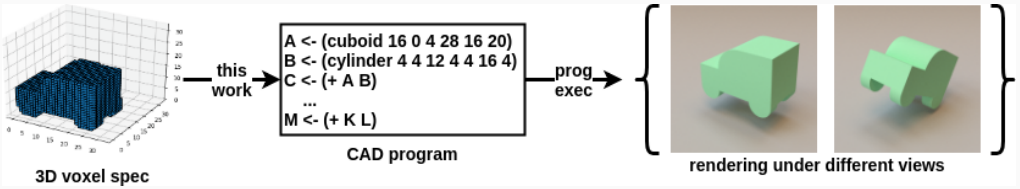



Theme: Synthesizing human-understandable models



# Perception, Synthesizing models, Learning-to-Learn

## Theme: high-level visual understanding, pixels→programs





## Theme #2: Synthesizing interpretable models

## Theme: Learning to synthesize programs

### List Processing

*Sum List*

```
[1 2 3] → 6
[4 6 8 1] → 17
```

*Double*

```
[1 2 3 4] → [2 4 6 8]
[6 5 1] → [12 10 2]
```

*Check Evens*

```
[0 2 3] → [T T F]
[2 4 9 6] → [T T F T]
```

### Text Editing

*Abbreviate*

```
Allen Newell → A.N.
Herb Simon → H.S.
```

*Drop Last Characters*

```
jabberwocky → jabberw
copycat → cop
```

*Extract*

```
see spot(run) → run
a (bee) see → bee
```

### Regexes

*Phone Numbers*

```
(555) 867-5309
(650) 555-2368
```

*Currency*

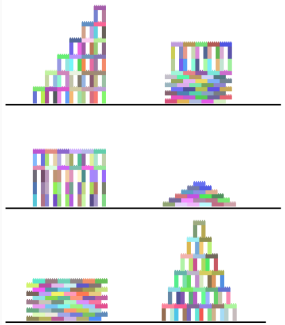
```
$100.25
$4.50
```

*Dates*

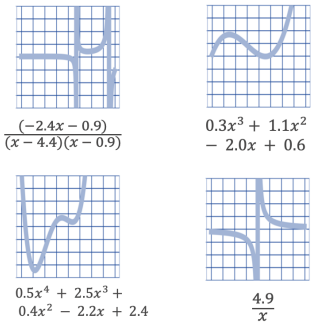
```
Y1775/0704
Y2000/0101
```

### LOGO Graphics

### Block Towers



### Symbolic Regression




### Recursive Programming

*Filter*

```
[■■■■■] → [■■■]
[■■■■■■] → [■■■■]
[■■■■■] → [■■■]
```

*Length*

```
[■■■■] → 4
[■■■■■■] → 6
[■■■] → 3
```

*Index List*

```
0, [■■■■■■] → ■
1, [■■■■■■] → ■
1, [■■■■■■■] → ■
```

*Every Other*

```
[■■■■■] → [■■■]
[■■■■■■] → [■■■■]
[■■■■■■■] → [■■■■]
```

### Physics

$$KE = \frac{1}{2}m|\vec{v}|^2$$

$$\vec{a} = \frac{1}{m}\sum_i \vec{F}_i$$

$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r_1} - \vec{r_2}|^2}\widehat{r_1 - r_2}$$

$$R_{total} = \left(\Sigma_i \frac{1}{R_i}\right)^{-1}$$

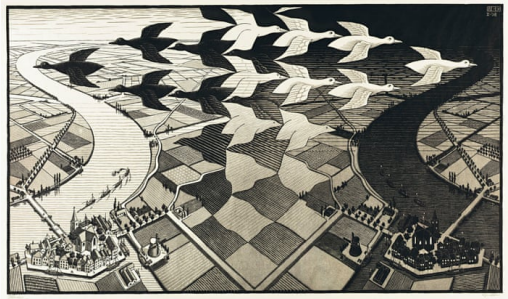
Program Induction and perception

learning to learn

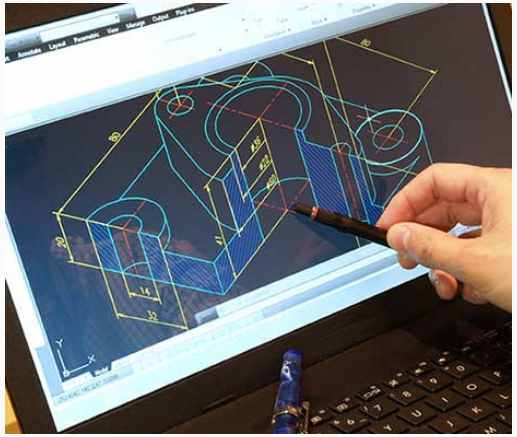
interpretable models

# High-level, abstract visual abilities

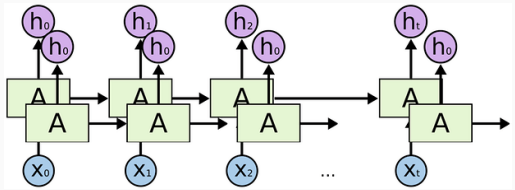
...in art

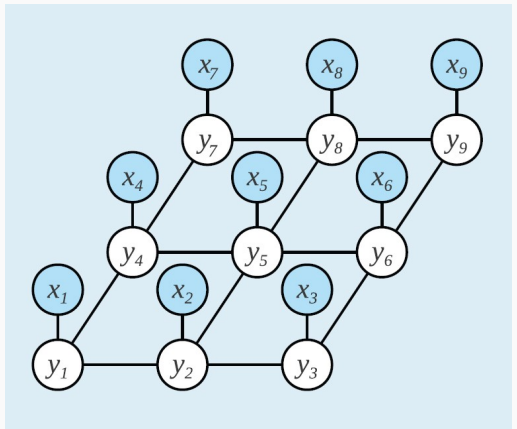


...in engineering



...in AI



# High-level, abstract visual abilities

...in art

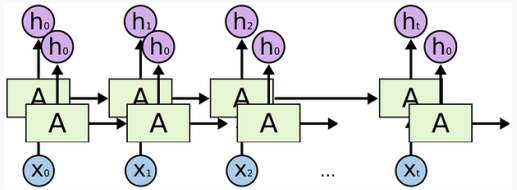
why?

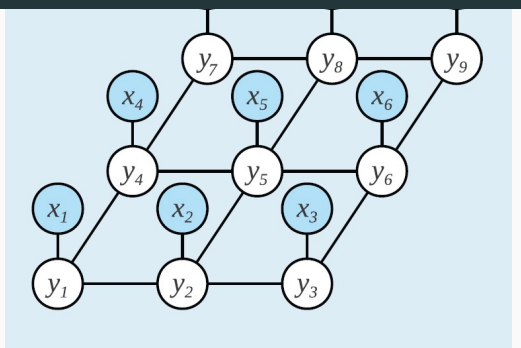
- impute missing objects, extrapolate percepts,
- learn visual concepts ('arch', 'spiral', 'Ising model'),
- assist graphic design, assist 3D modeling

how?

- machine learning +
- program synthesis techniques from programming languages community

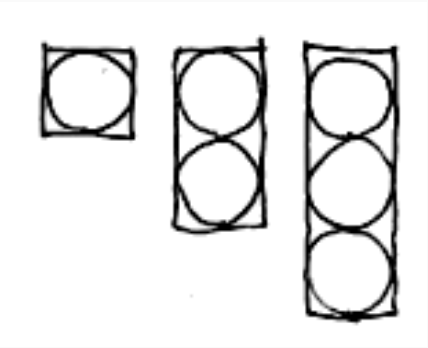
...in AI



# Learning to infer graphics programs from hand-drawn images

model infers program from drawing



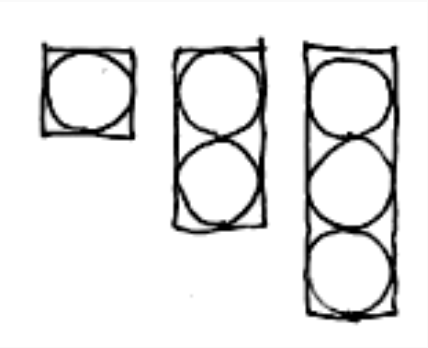
```
for (0 <= i < 3)
 rectangle(3*i,-2*i+4,
           3*i+2,6)
 for (0 <= j < i + 1)
  circle(3*i+1,-2*j+5)
```

Ellis, Ritchie, Solar-Lezama, Tenenbaum. NeurIPS 2018.

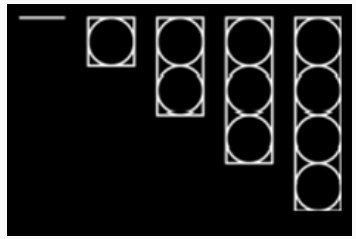
# Learning to infer graphics programs from hand-drawn images

model infers program from drawing

**zero-shot generalization / extrapolation**



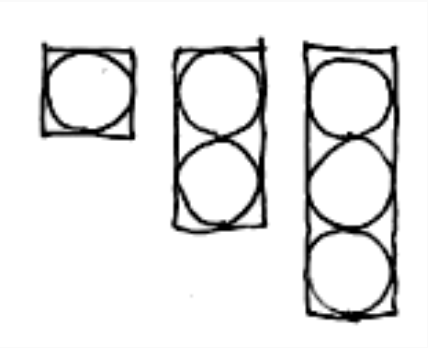
```
for (0 <= i < 3 + 1)
 rectangle(3*i,-2*i+4,
           3*i+2,6)
 for (0 <= j < i + 1)
  circle(3*i+1,-2*j+5)
```



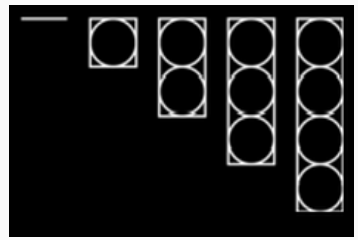
# Learning to infer graphics programs from hand-drawn images

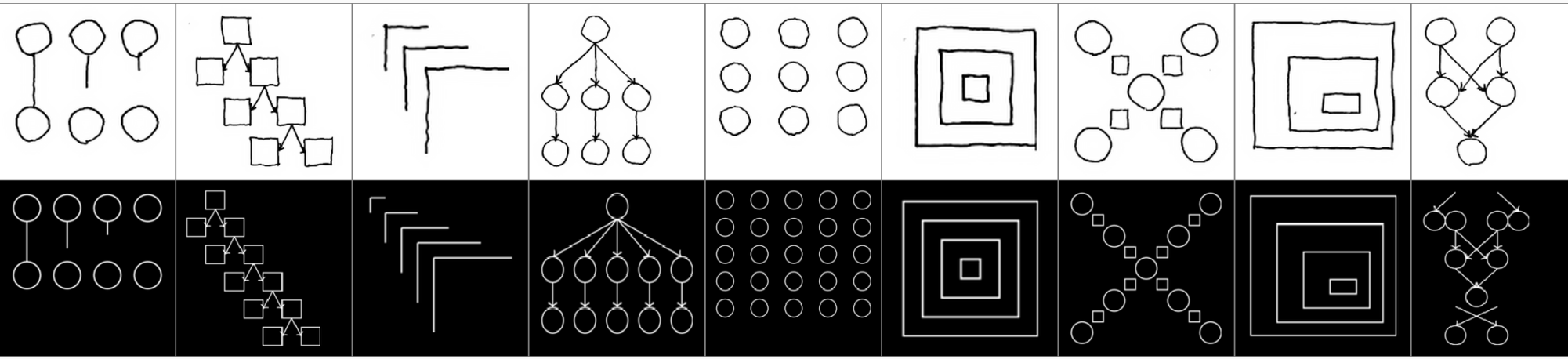
model infers program from drawing

**zero-shot generalization / extrapolation**

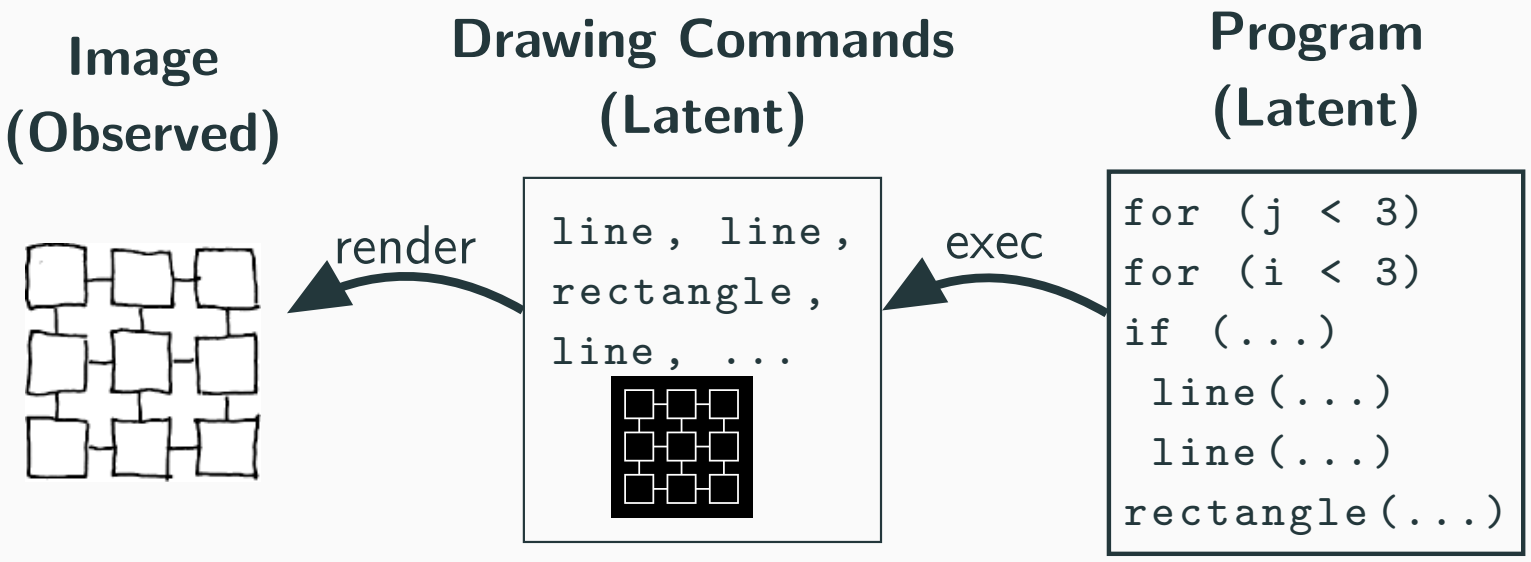


```
for (0 <= i < 3 + 1)
 rectangle(3*i,-2*i+4,
           3*i+2,6)
 for (0 <= j < i + 1)
  circle(3*i+1,-2*j+5)
```

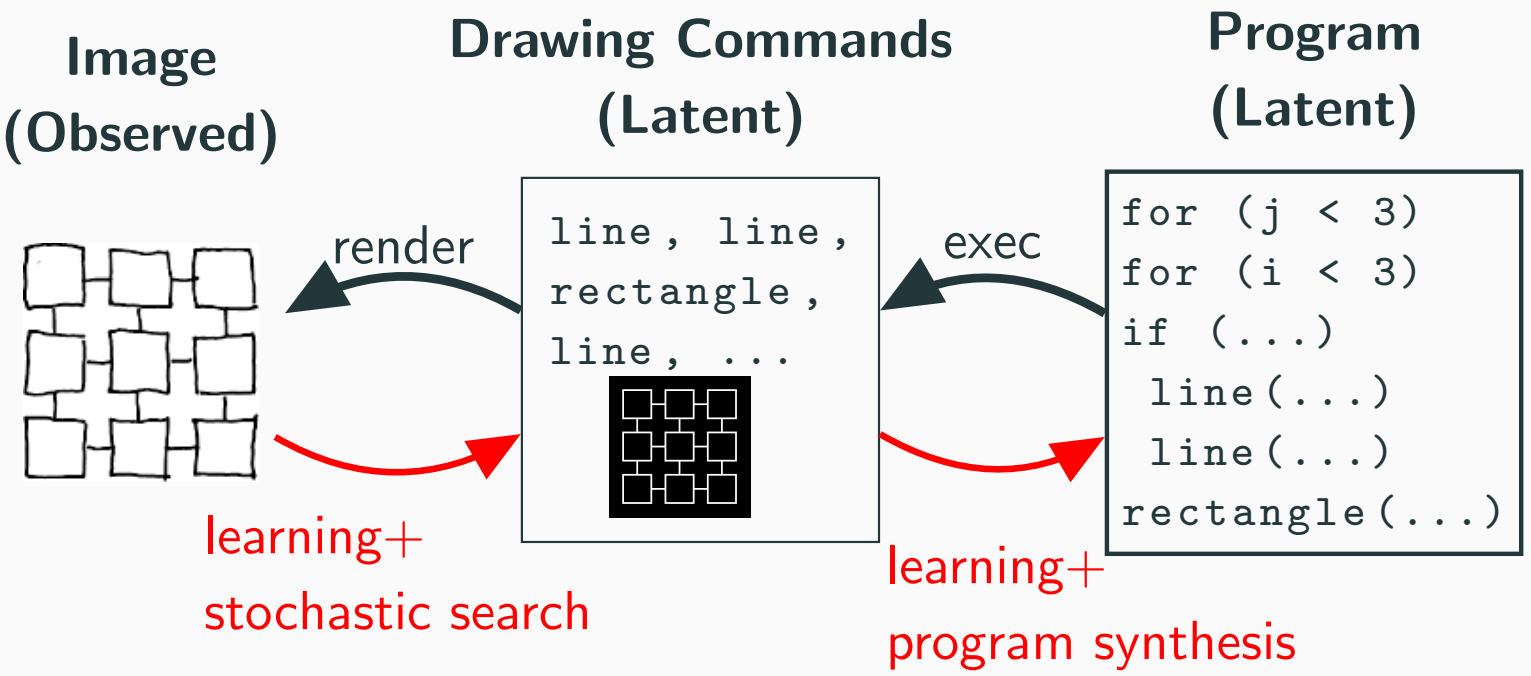




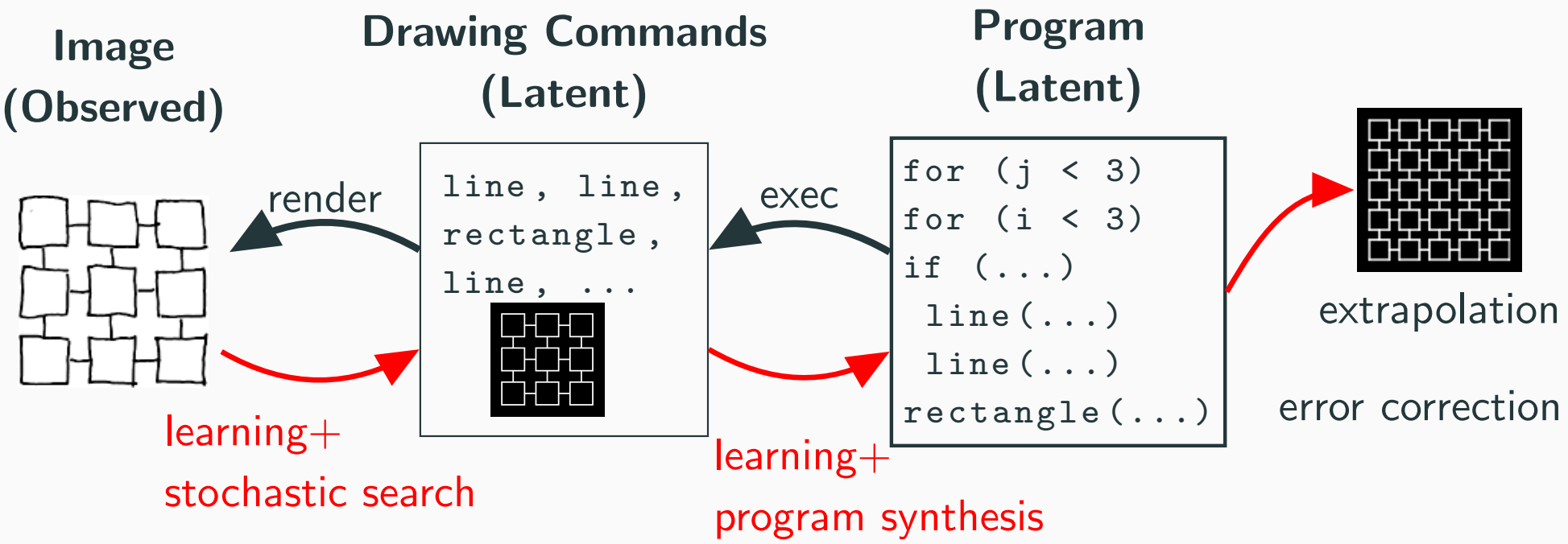
# How to infer graphics programs from hand-drawn images



# How to infer graphics programs from hand-drawn images

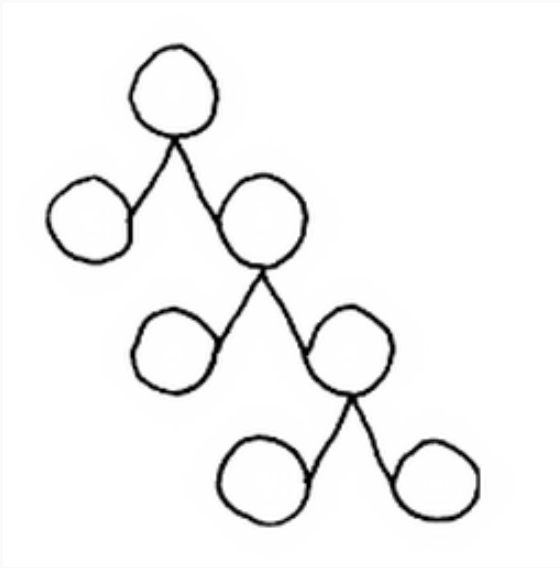


# How to infer graphics programs from hand-drawn images

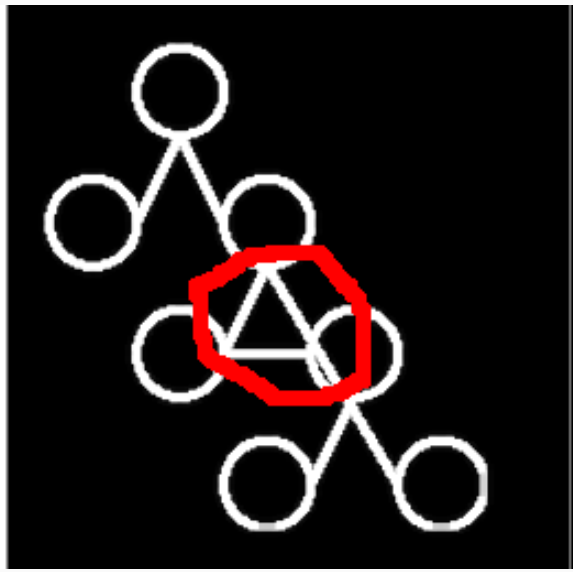


# Top-down influences on perception

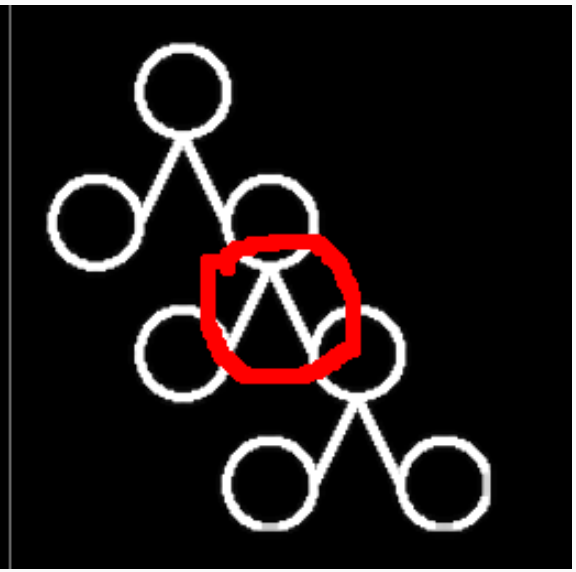
drawing



bottom-up neural net

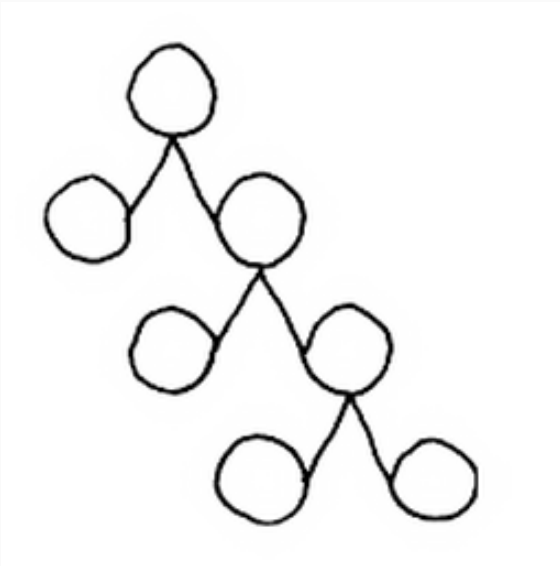


w/ top-down program bias

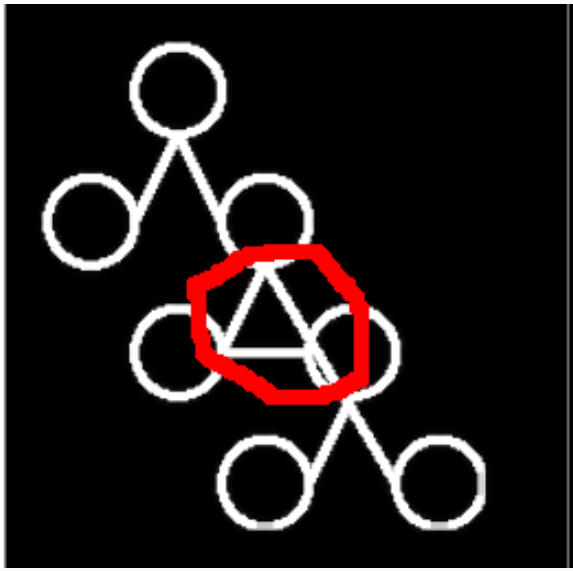


# Top-down influences on perception

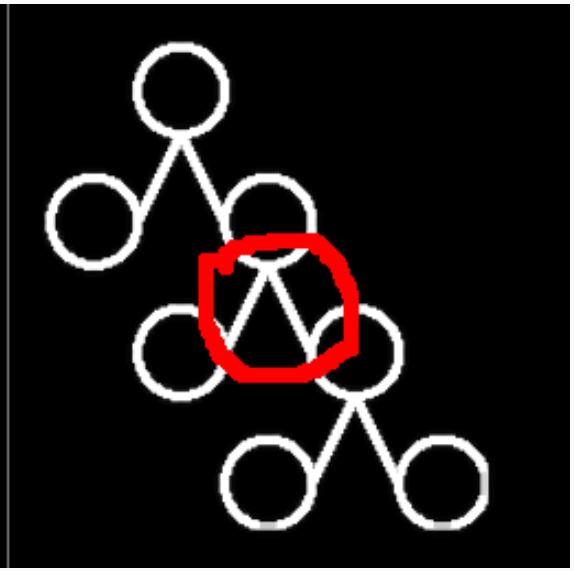
drawing

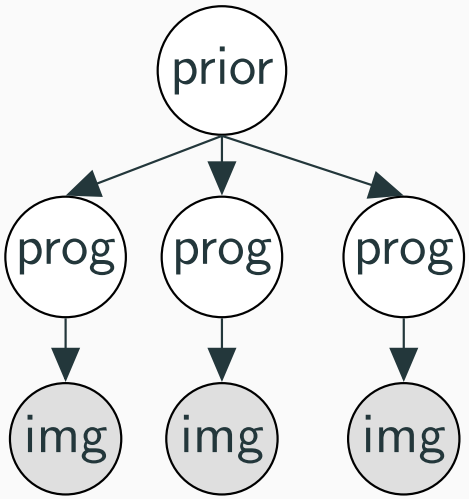


bottom-up neural net



w/ top-down program bias



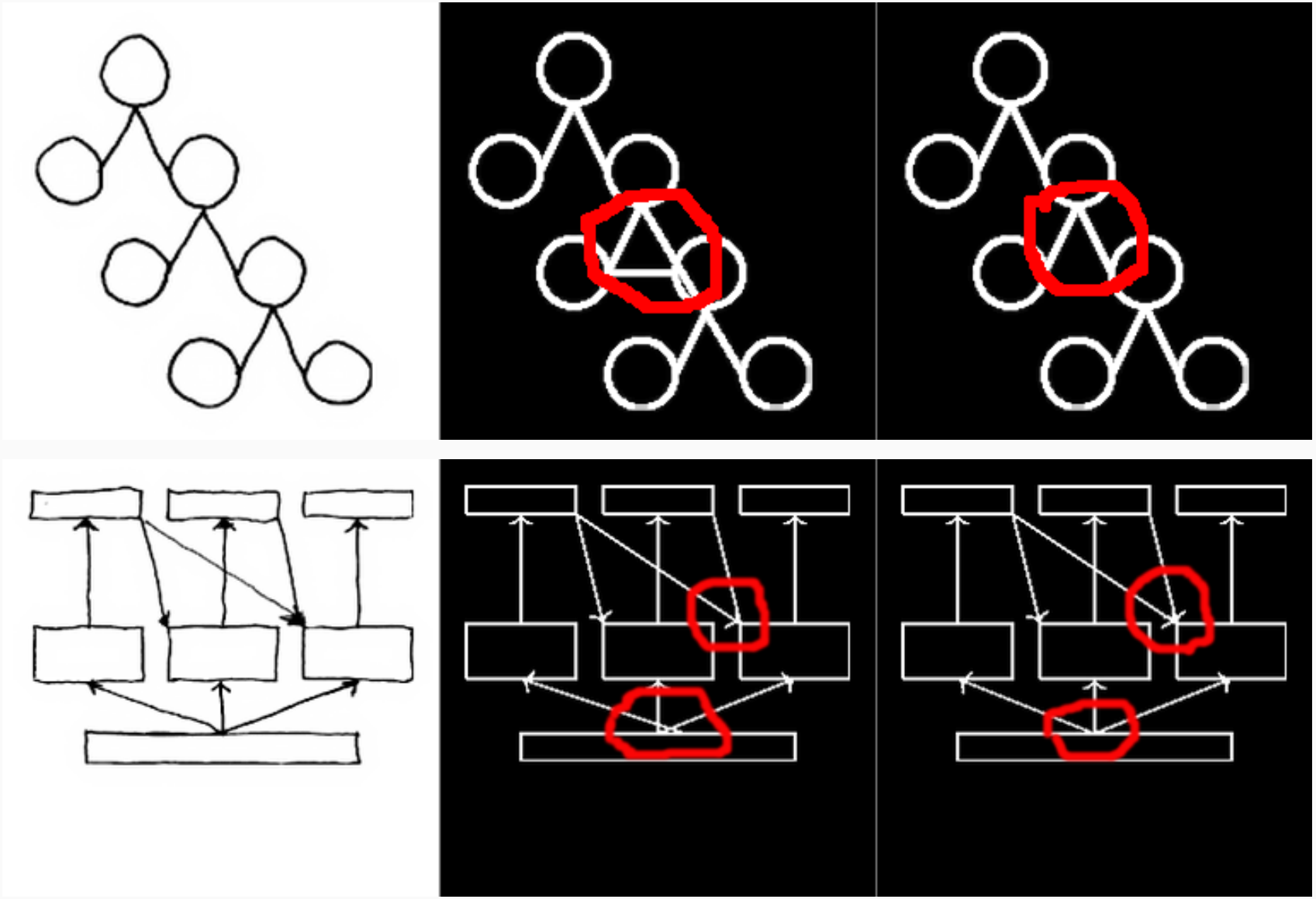



predicted program $=$

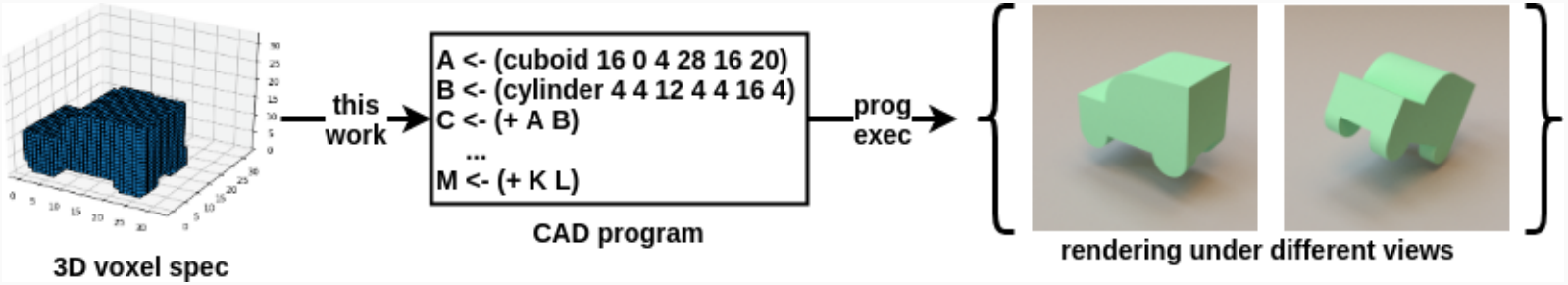
$\arg\max_{\text{progs}} \mathbb{P}\left[\text{img}|\text{prog}\right] \mathbb{P}\left[\text{prog}|\text{prior}\right]$

# Top-down influences on perception

drawing | bottom-up neural net | w/ top-down program bias

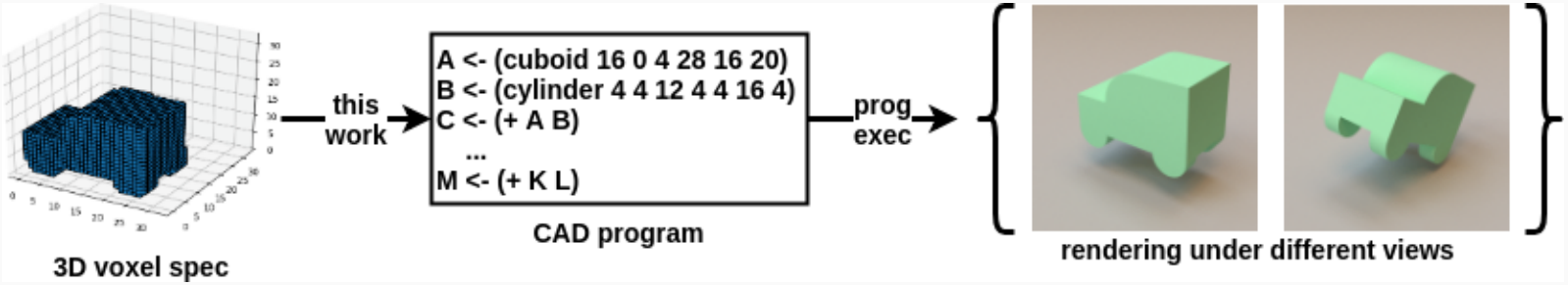


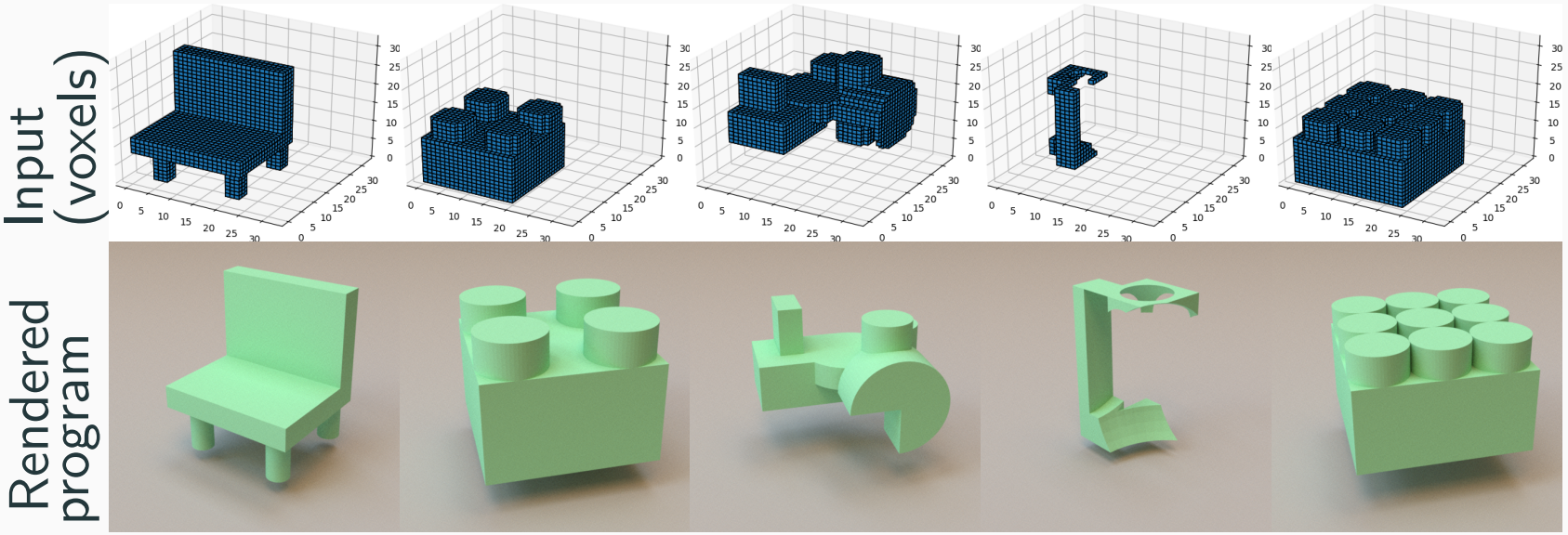
# 3D program induction




Ellis*, Nye*, Pu*, Sosa*, Tenenbaum, Solar-Lezama. NeurIPS 2019.
*equal contribution

# 3D program induction

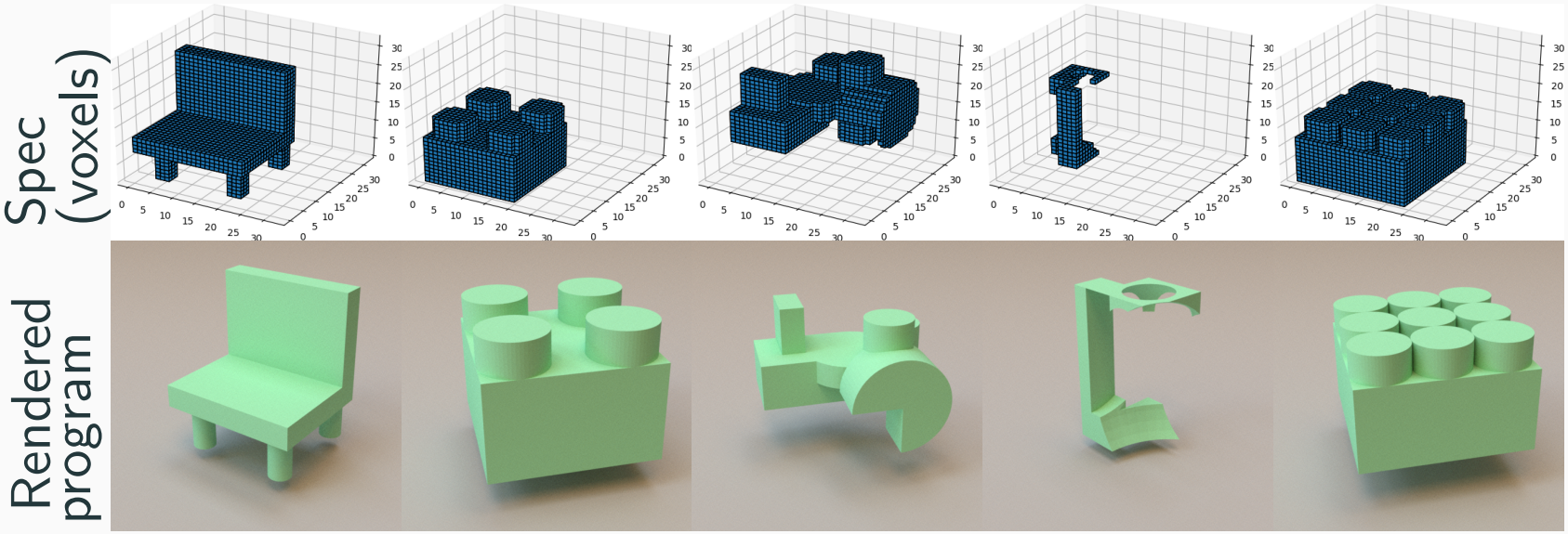



Ellis*, Nye*, Pu*, Sosa*, Tenenbaum, Solar-Lezama. NeurIPS 2019.

*equal contribution

# 3D program induction



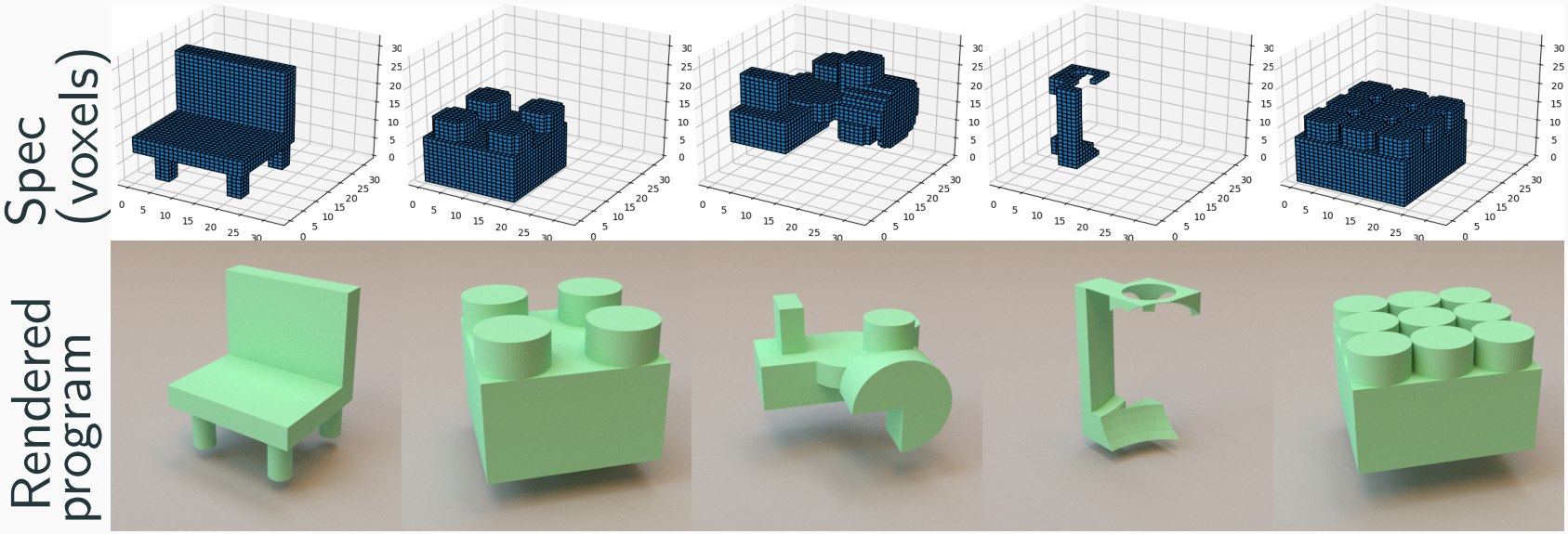

Challenge: combinatorics!

Branching factor: $>$ 1.3 million per line of code, $\approx$ 20 lines of code

Ellis*, Nye*, Pu*, Sosa*, Tenenbaum, Solar-Lezama. NeurIPS 2019.

*equal contribution

# 3D program induction




Challenge: combinatorics!

Branching factor: $>$ 1.3 million per line of code, $\approx$ 20 lines of code

Solution: stochastic **tree search** + learn **policy** that writes code + learn **value** function that assesses execution of program so far; analogous to AlphaGo

Ellis*, Nye*, Pu*, Sosa*, Tenenbaum, Solar-Lezama. NeurIPS 2019.

*equal contribution

# Lessons

The bias from a programming language gives extrapolation, or strong generalization, sometimes even without meta-learning

Mix-and-match techniques to play to their strengths: neural nets for perception, symbols for reasoning, Bayesian methods for uncertainty

Program Induction and perception

learning to learn

interpretable models

# Learning to write code

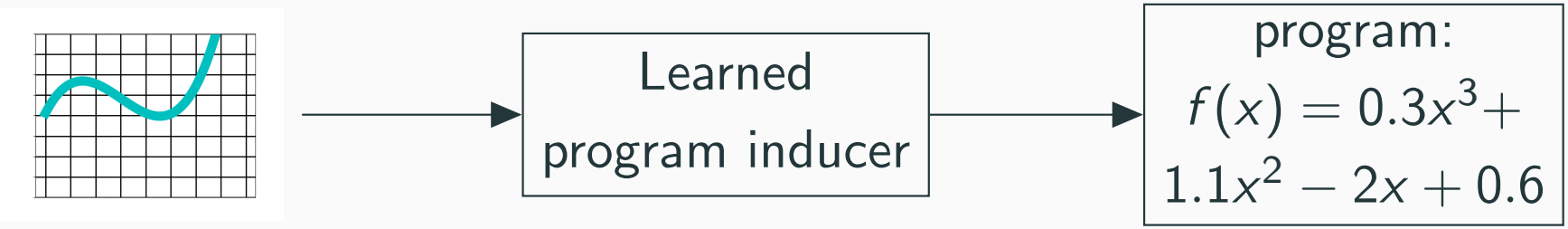
Goal: acquire domain-specific knowledge needed to induce a class of programs

- Library of concepts (declarative knowledge; domain specific language)
- Inference strategy (procedural knowledge; synthesis algorithm)

# Learning to write code

Goal: acquire domain-specific knowledge needed to induce a class of programs

- Library of concepts (declarative knowledge; domain specific language)
- Inference strategy (procedural knowledge; synthesis algorithm)



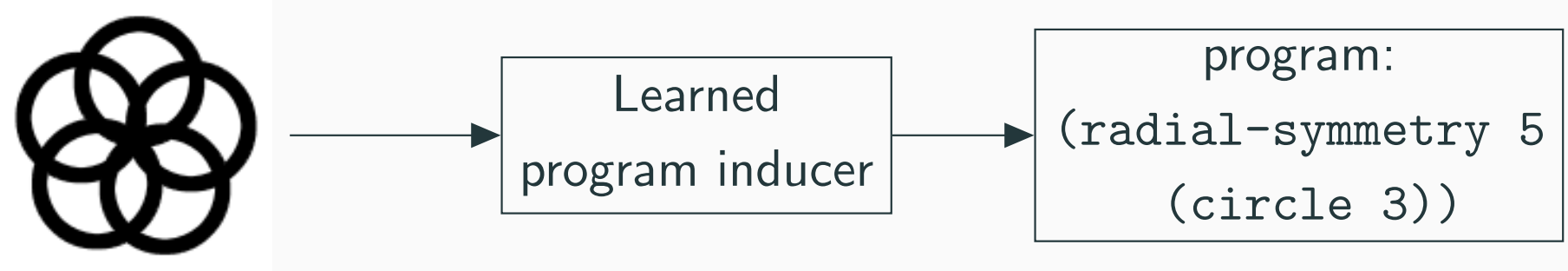

Concepts: $x^3$, $\alpha x + \beta$, etc

Inference strategy: neurosymbolic search for programs

# Learning to write code

Goal: acquire domain-specific knowledge needed to induce a class of programs

- Library of concepts (declarative knowledge; domain specific language)
- Inference strategy (procedural knowledge; synthesis algorithm)




Concepts: `circle`, `radial-symmetry`, etc

Inference strategy: neurosymbolic search for programs

# Library learning

# Library learning

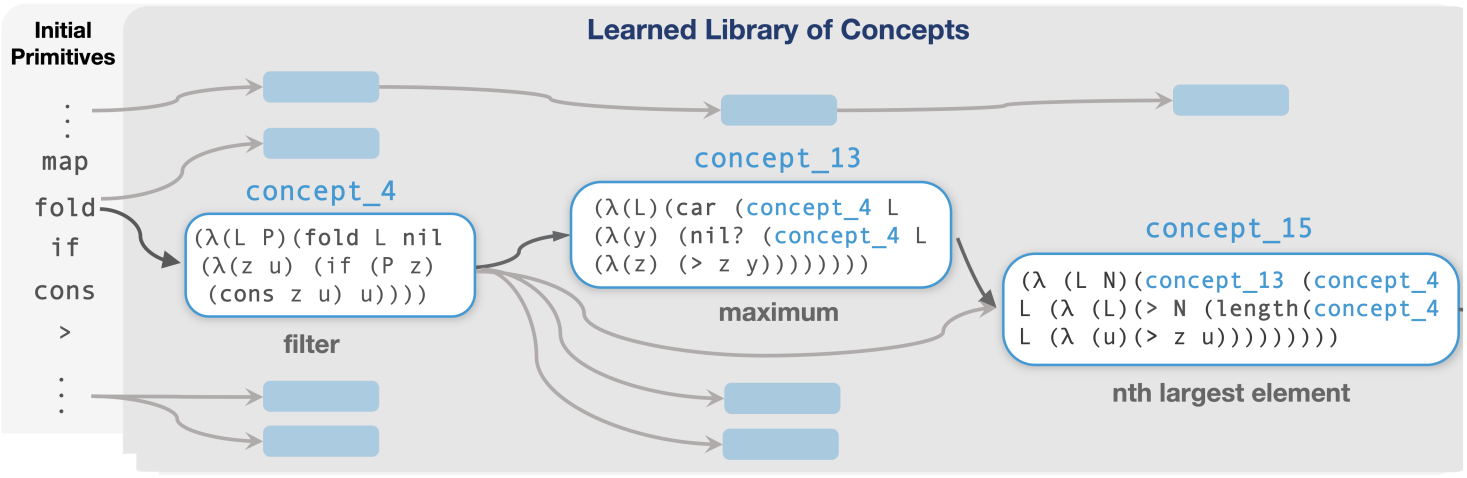
**Initial Primitives**

```
  ⋮
map
fold
 if
cons
 >
  ⋮
```

**Sample Problem:** `sort list`

```
[9 2 7 1]    ->   [1 2 7 9]
[3 8 9 4 2] ->   [2 3 4 8 9]
[6 2 2 3 5] ->   [2 32 3 5 6]
...
```

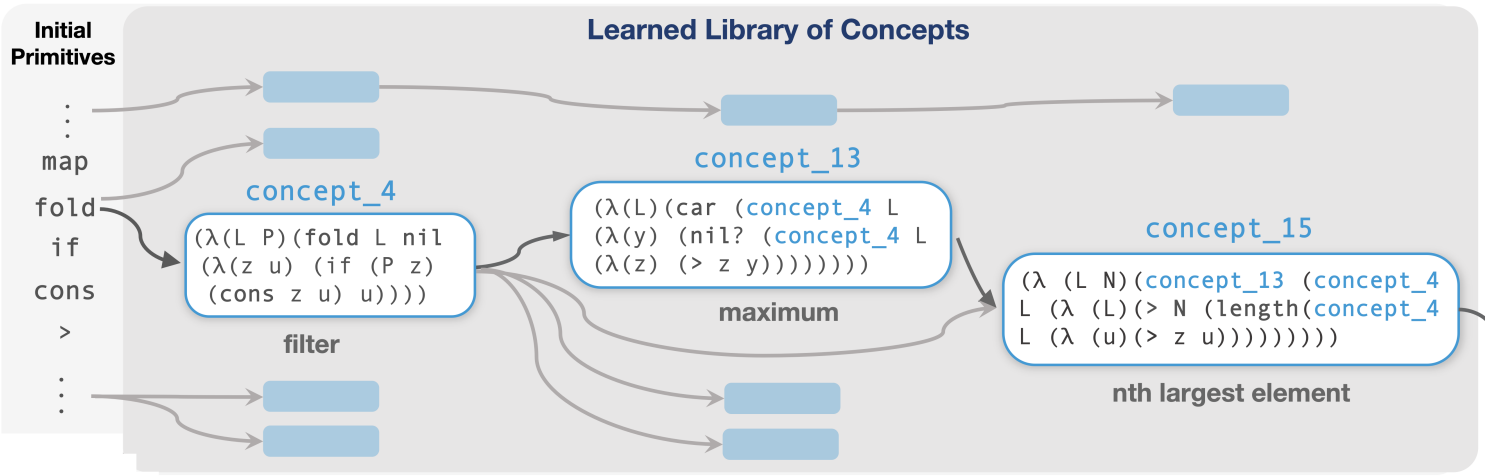
# Library learning




**Sample Problem: `sort list`**

[9 2 7 1] -> [1 2 7 9]
[3 8 9 4 2] -> [2 3 4 8 9]
[6 2 2 3 5] -> [2 32 3 5 6]
...

# Library learning



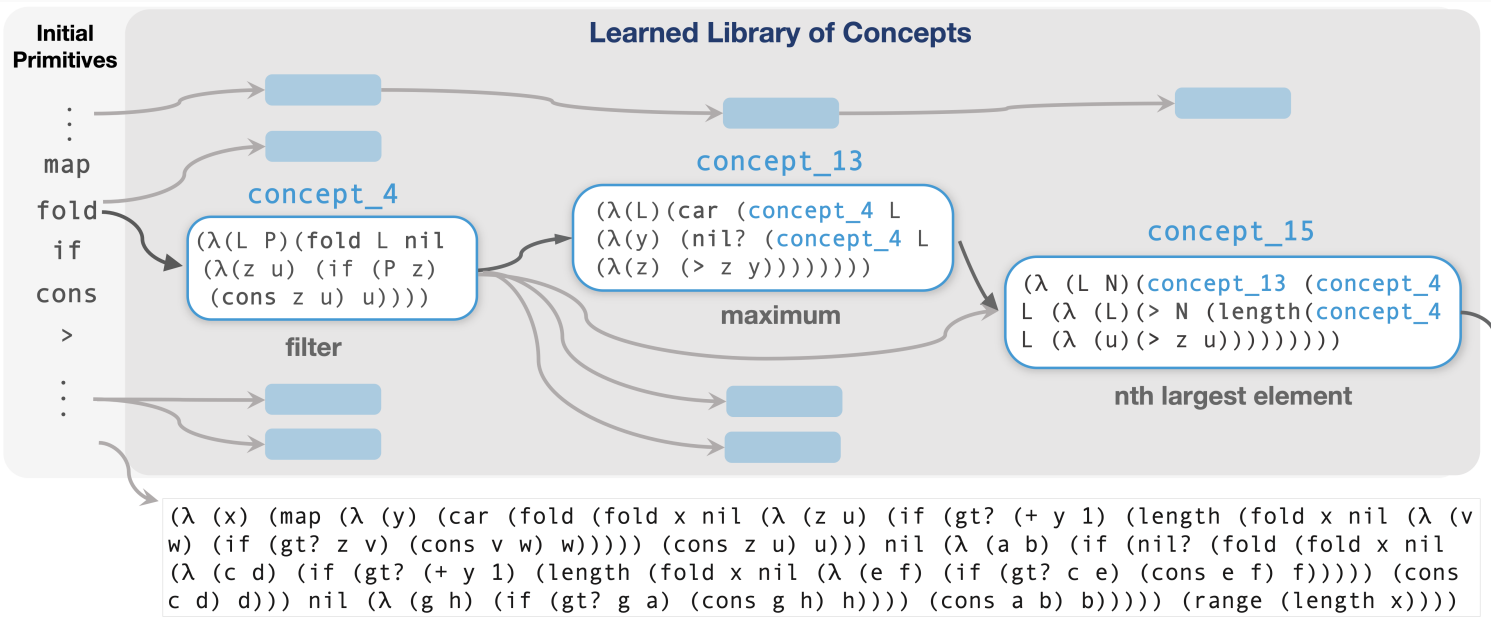

**Sample Problem: `sort list`**

```
[9 2 7 1]    ->  [1 2 7 9]
[3 8 9 4 2] ->  [2 3 4 8 9]
[6 2 2 3 5] ->  [2 32 3 5 6]
...
```

**Solution to `sort list` discovered in learned language:**

```
(map  (l (n)
  (concept_15 L (+ 1 n)))
  (range (length L)))
```

# Library learning




**Sample Problem: `sort list`**

```
[9 2 7 1]    ->  [1 2 7 9]
[3 8 9 4 2] ->  [2 3 4 8 9]
[6 2 2 3 5] ->  [2 32 3 5 6]
...
```

**Solution to `sort list` discovered in learned language:**

```
(map  (l (n)
  (concept_15 L (+ 1 n)))
  (range (length L)))
```

**Solution to `sort list` if expressed in initial primitives**

# DreamCoder

- **Wake:** Solve problems by writing programs
- **Sleep:** Improve library and neural recognition model:
  - **Abstraction sleep:** Improve library
  - **Dream sleep:** Improve neural inference model
- Combines ideas from Wake-Sleep & Exploration-Compression

# DreamCoder

- **Wake:** Solve problems by writing programs
- **Sleep:** Improve library and neural recognition model:
  - **Abstraction sleep:** Improve library
  - **Dream sleep:** Improve neural inference model
- Combines ideas from Wake-Sleep & Exploration-Compression

### List Processing

*Sum List*

```
[1 2 3] → 6
[4 6 8 1] → 17
```

*Double*

```
[1 2 3 4] → [2 4 6 8]
[6 5 1] → [12 10 2]
```

*Check Evens*

```
[0 2 3] → [T T F]
[2 4 9 6] → [T T F T]
```

### Text Editing

*Abbreviate*

```
Allen Newell → A.N.
Herb Simon → H.S.
```

*Drop Last Characters*

```
jabberwocky → jabberw
copycat → cop
```

*Extract*

```
see spot(run) → run
a (bee) see → bee
```

### Regexes

*Phone Numbers*

```
(555) 867-5309
(650) 555-2368
```

*Currency*

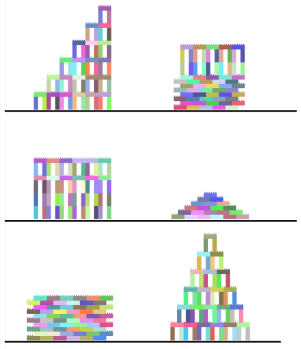
```
$100.25
$4.50
```

*Dates*

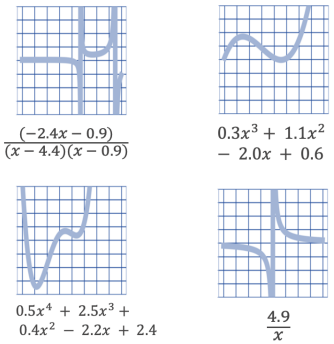
```
Y1775/0704
Y2000/0101
```

### LOGO Graphics

### Block Towers



### Symbolic Regression




### Recursive Programming

*Filter*

```
[■■■■■] → [■■]
[■■■■■■] → [■■■■]
[■■■■■] → [■■■]
```

*Index List*

```
0, [■■■■■■] → ■
1, [■■■■■■] → ■
1, [■■■■■■■] → ■
```

*Length*

```
[■■■■] → 4
[■■■■■■] → 6
[■■■] → 3
```

*Every Other*

```
[■■■■] → [■■]
[■■■■■] → [■■■]
[■■■■■■] → [■■■]
```

### Physics

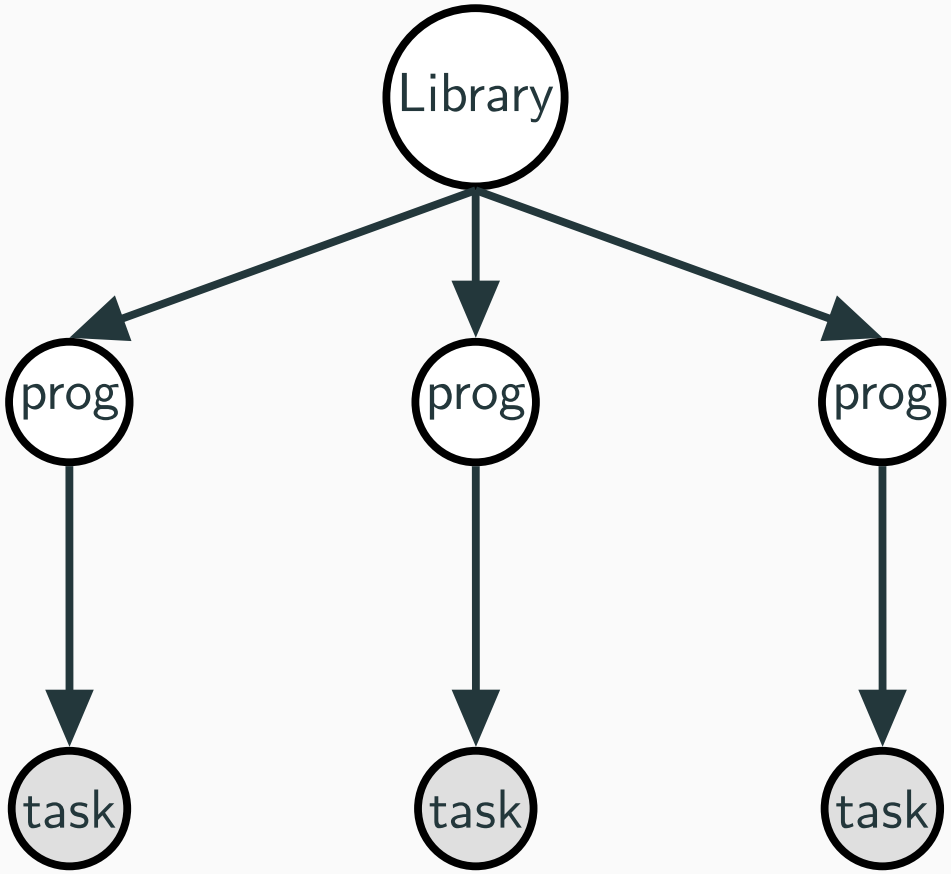
$$KE = \frac{1}{2}m|\vec{v}|^2$$

$$\vec{a} = \frac{1}{m}\sum_i \vec{F}_i$$

$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r_1} - \vec{r_2}|^2}\widehat{r_1 - r_2}$$

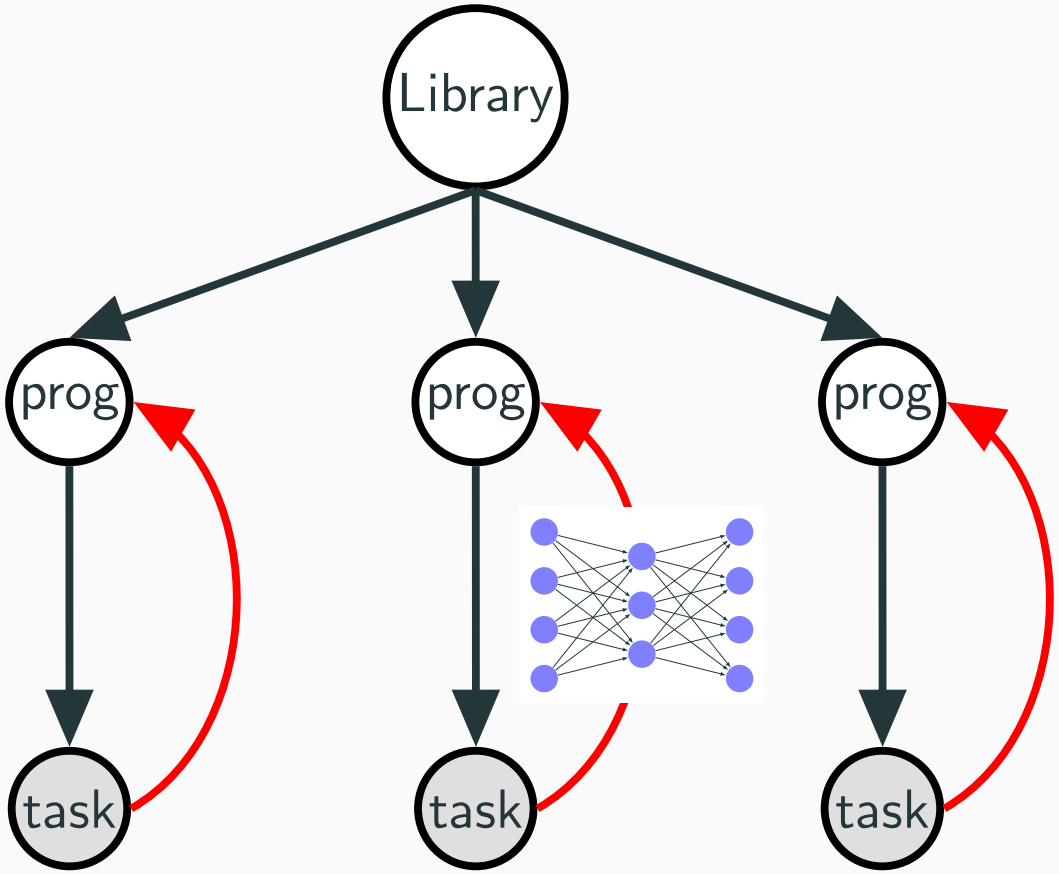
$$R_{total} = \left(\Sigma_i \frac{1}{R_i}\right)^{-1}$$

# Library learning as Bayesian inference




[Dechter et al., 2013] [Liang et al, 2010]; [Lake et al, 2015]

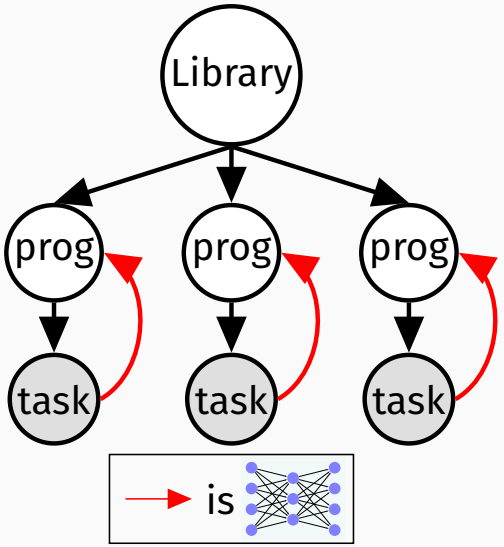
# Library learning as neurally-guided Bayesian inference




library learning via program analysis +

new neural inference network for program synthesis +

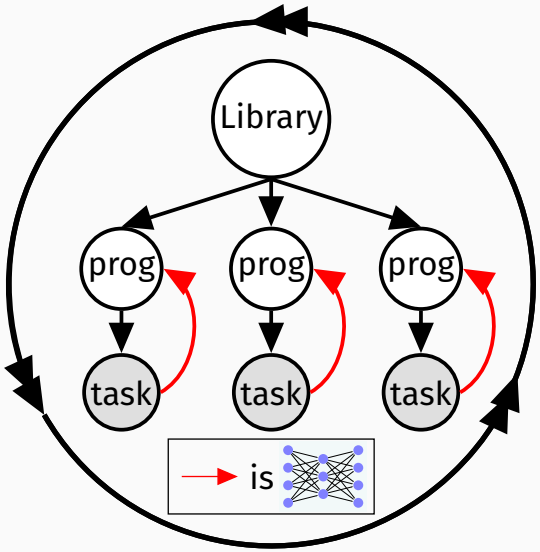
better program representation (Lisp+polymorphic types [Milner 1978])


Library
prog
prog
prog
task
task
task
is

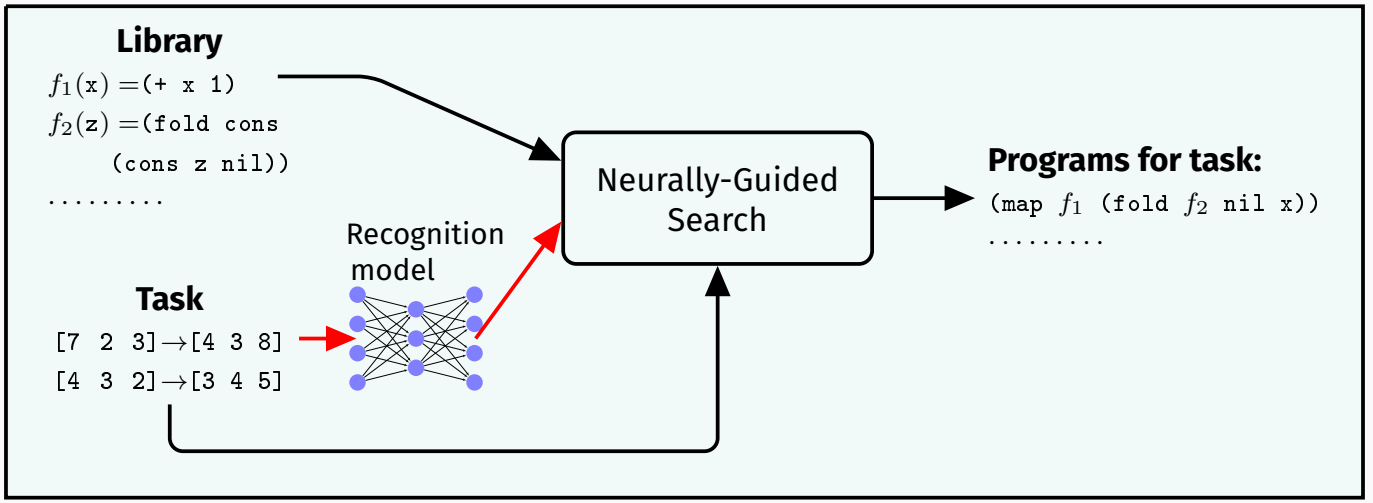
## Wake

## Sleep: Abstraction

## Sleep: Dreaming

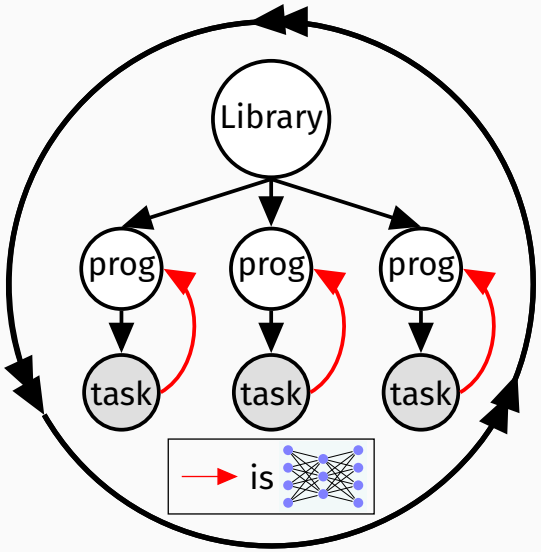


## WAKE

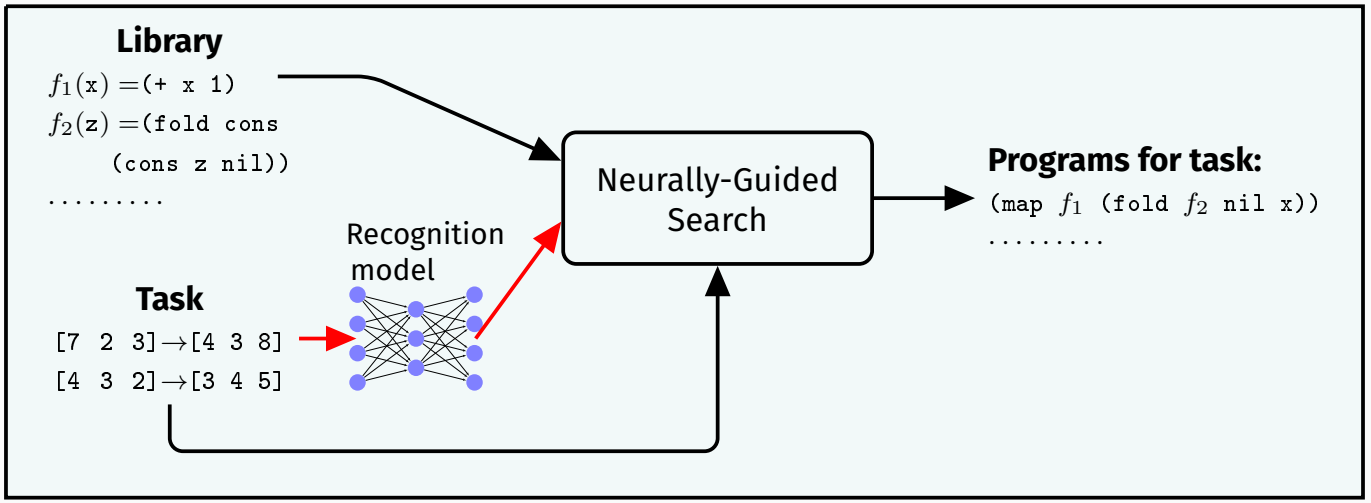



## SLEEP: ABSTRACTION

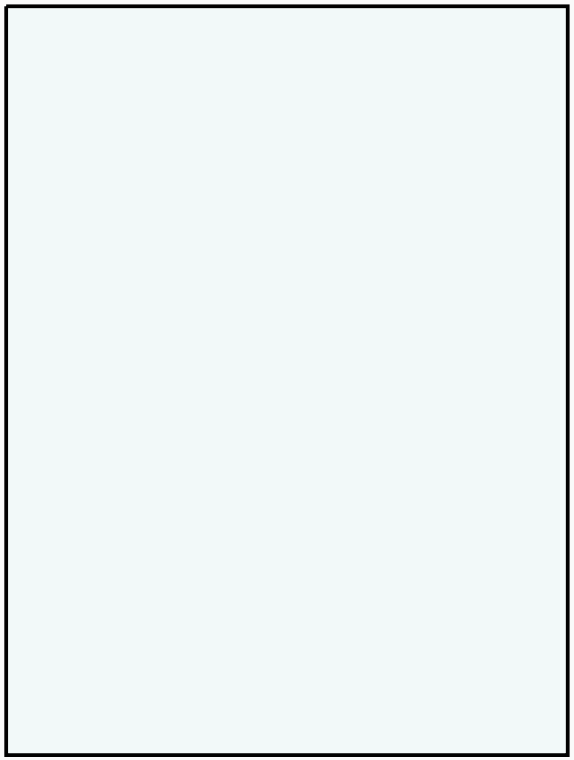
## SLEEP: DREAMING

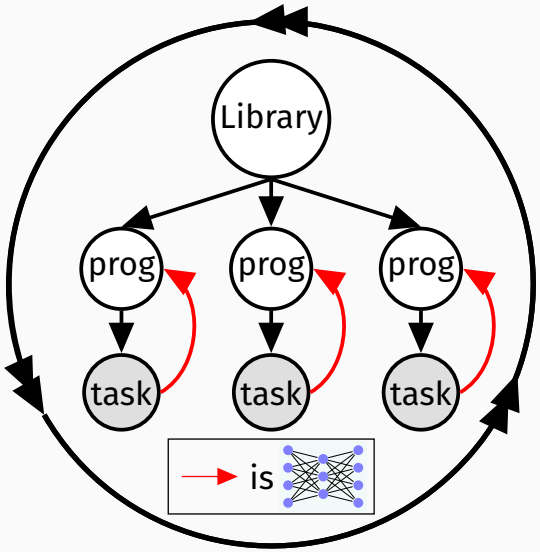


## Wake

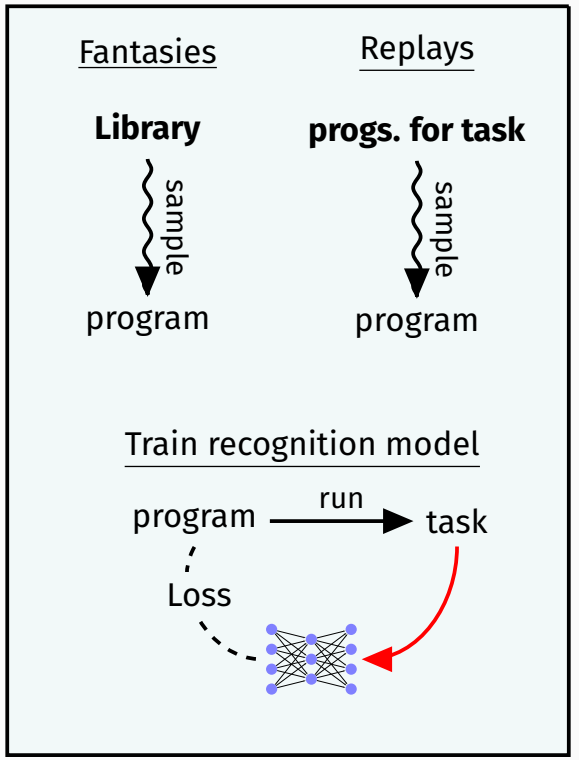


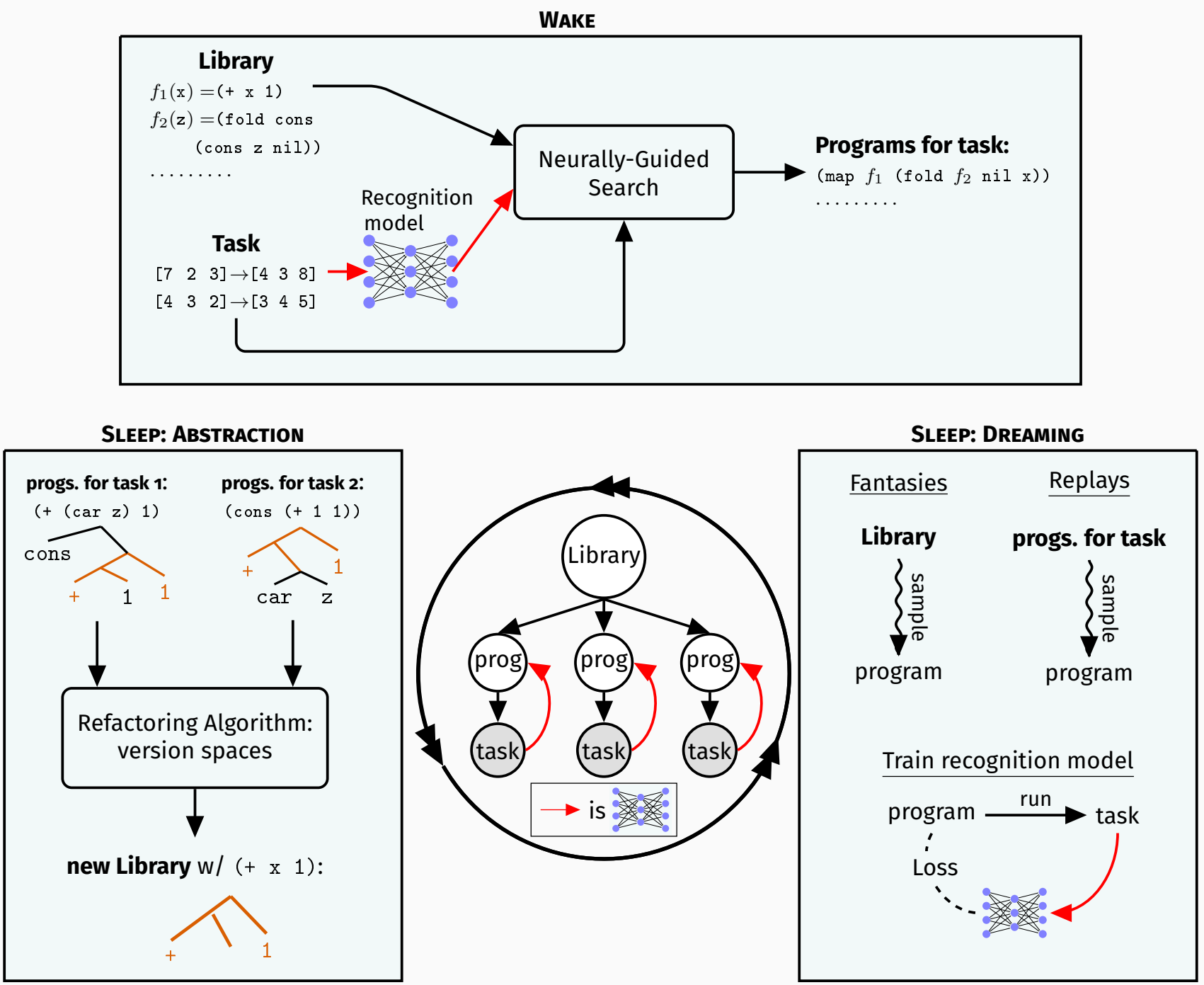

## Sleep: Abstraction






## Sleep: Dreaming




WAKE
Library
$f_1$(x) =(+ x 1)
$f_2$(z) =(fold cons
(cons z nil))
.....
Task
[7 2 3]→[4 3 8]
[4 3 2]→[3 4 5]
Recognition model
Neurally-Guided Search
Programs for task:
(map $f_1$ (fold $f_2$ nil x))
.....
SLEEP: ABSTRACTION
progs. for task 1:
(+ (car z) 1)
progs. for task 2:
(cons (+ 1 1))
cons
+
1
1
+
1
car
z
Refactoring Algorithm: version spaces
new Library w/ (+ x 1):
+
1
Library
prog
prog
prog
task
task
task
is
SLEEP: DREAMING
Fantasies
Replays
Library
progs. for task
sample
sample
program
program
Train recognition model
program
run
task
Loss

# Abstraction Sleep: Growing the library via refactoring

$5 + 5$

# Abstraction Sleep: Growing the library via refactoring

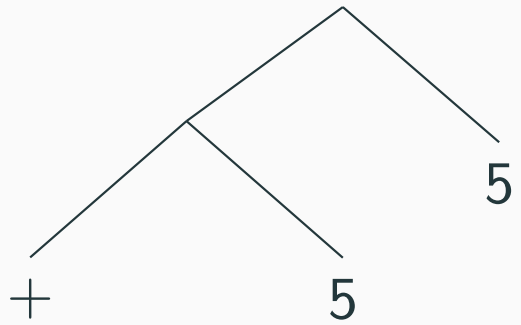
$5 + 5$

```
(+ 5 5)
```

# Abstraction Sleep: Growing the library via refactoring

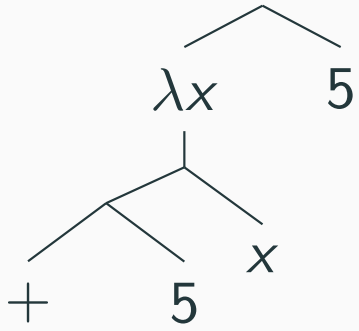
$5 + 5$

```
(+ 5 5)
```

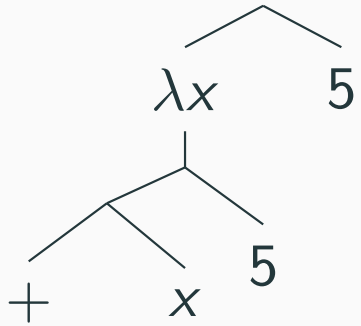


# Abstraction Sleep: Growing the library via refactoring

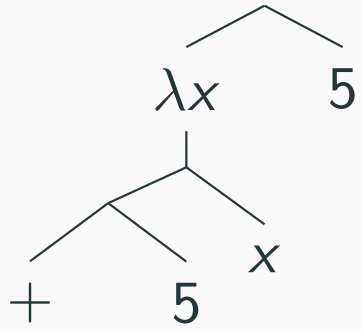
# Abstraction Sleep: Growing the library via refactoring

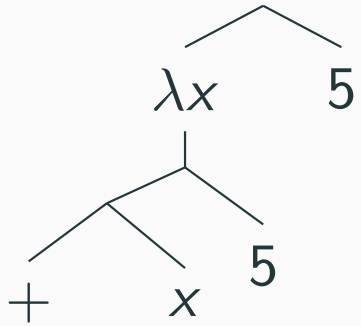
# Abstraction Sleep: Growing the library via refactoring

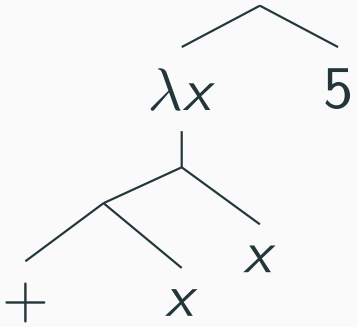




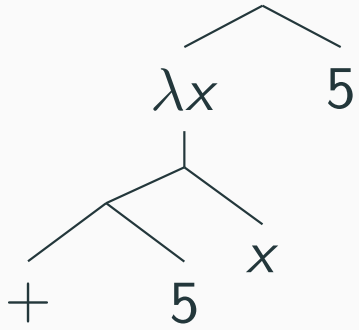


# Abstraction Sleep: Growing the library via refactoring

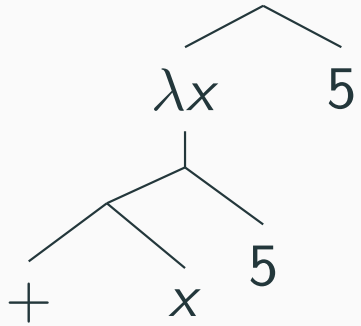


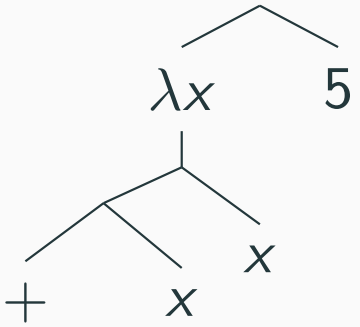





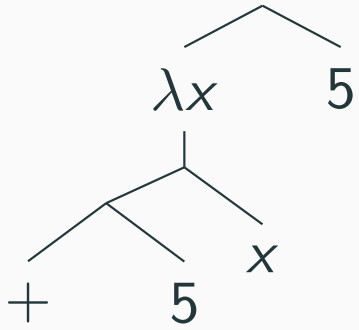


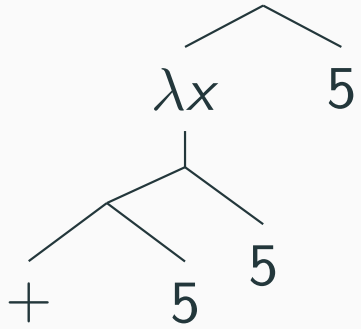
# Abstraction Sleep: Growing the library via refactoring



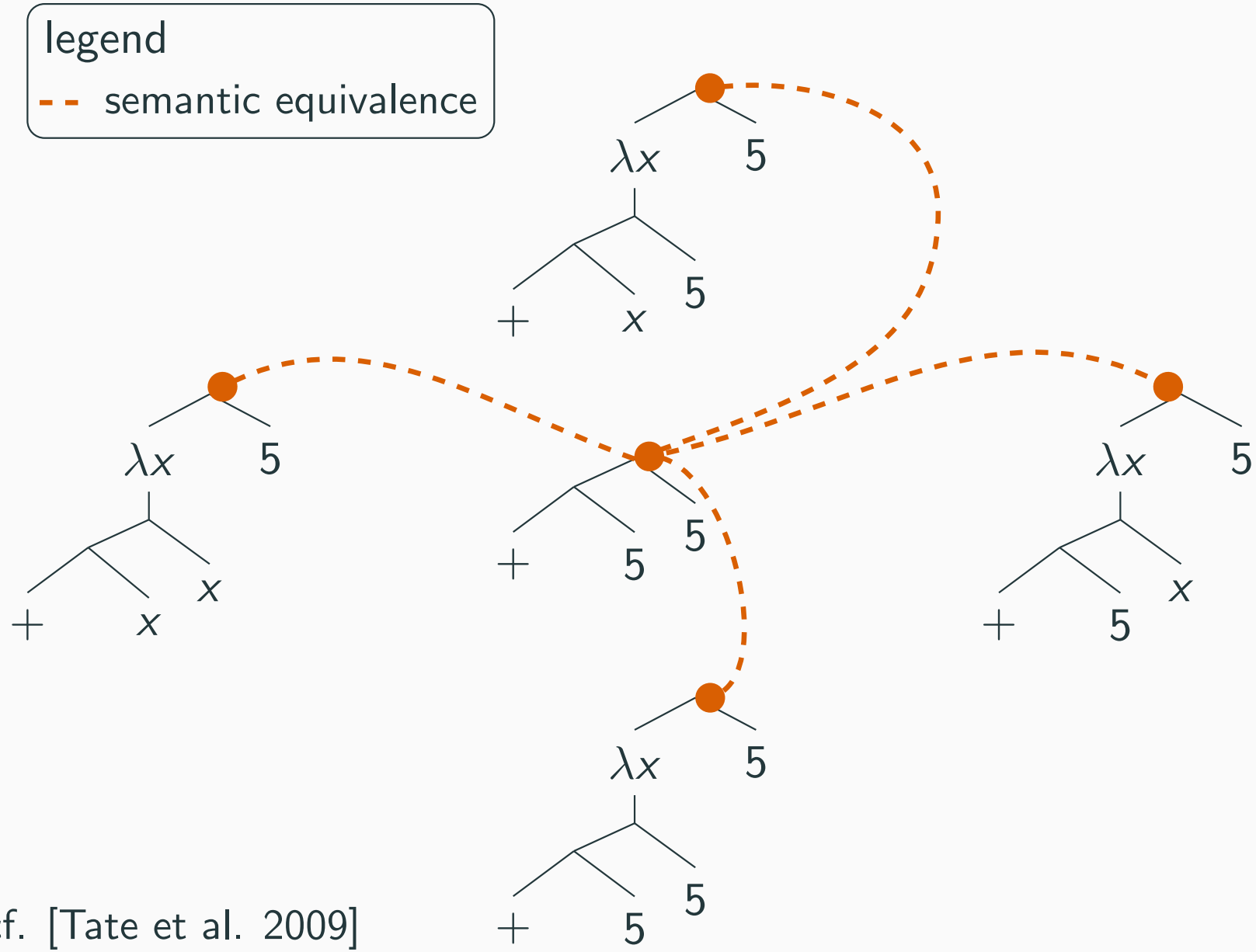







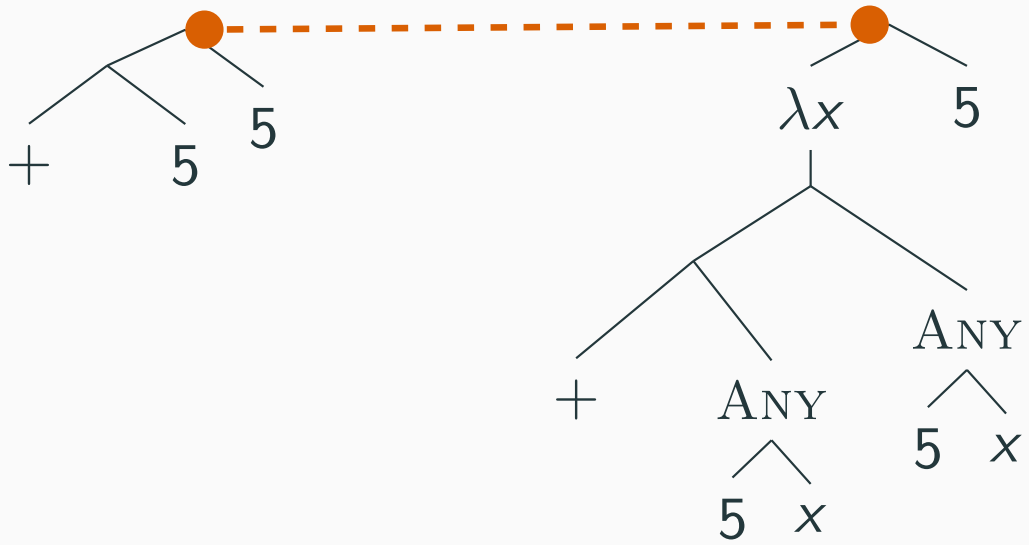
# Abstraction Sleep: Growing the library via refactoring




cf. [Tate et al. 2009]

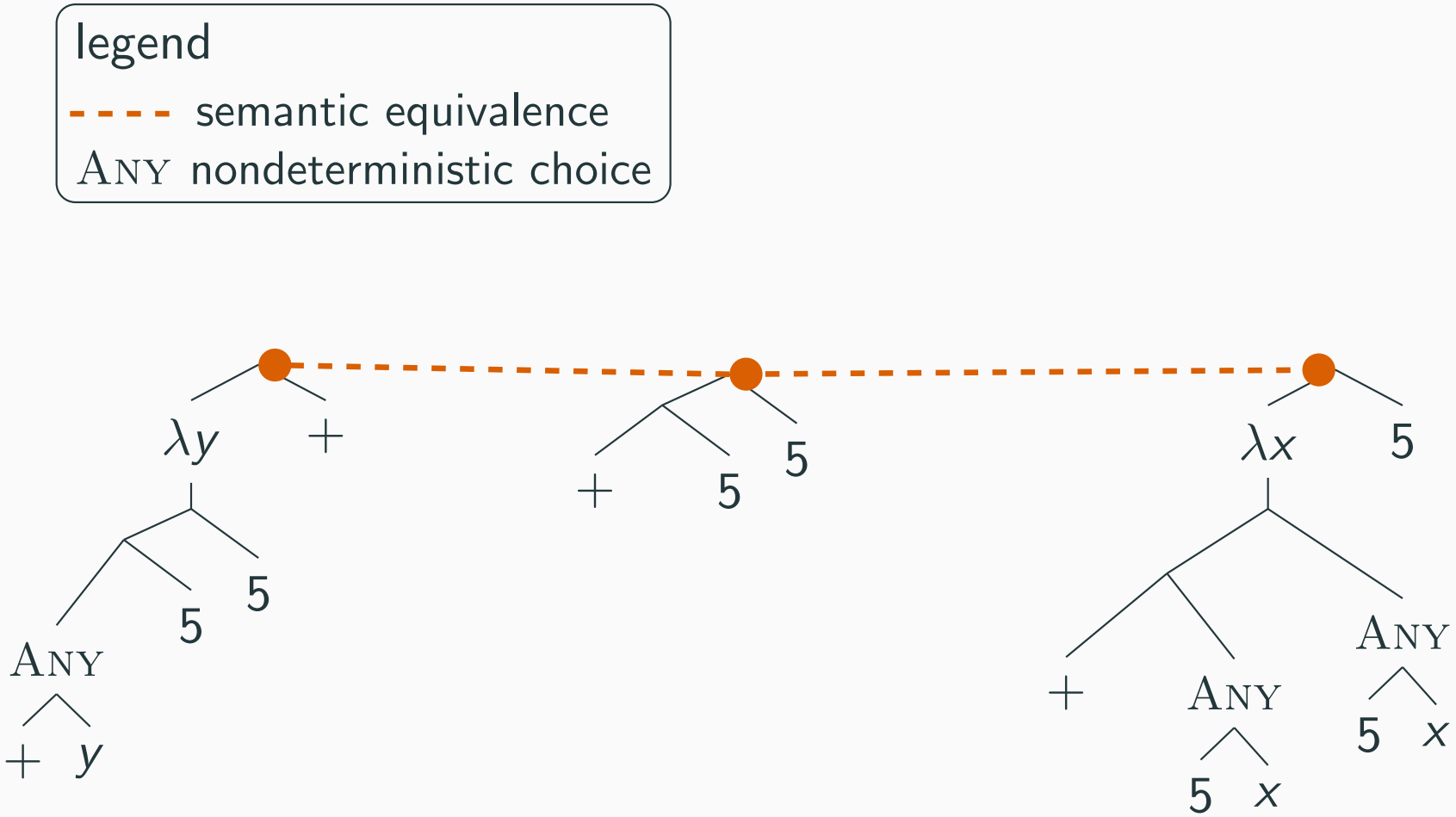
# Abstraction Sleep: Growing the library via refactoring

legend
- - - - semantic equivalence
ANY nondeterministic choice




cf. Tate et al. 2009
Gulwani 2012

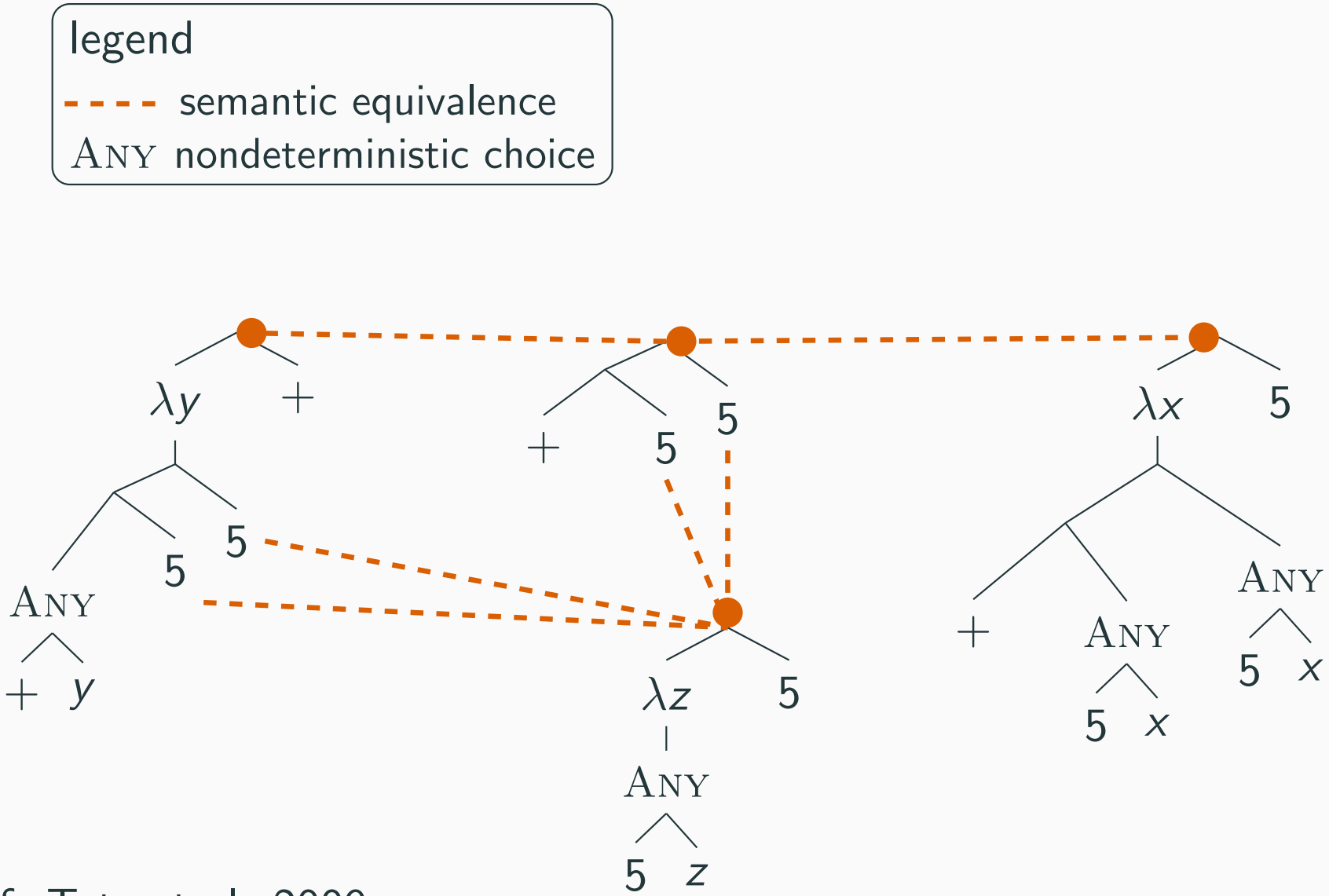
# Abstraction Sleep: Growing the library via refactoring




cf. Tate et al. 2009

Gulwani 2012

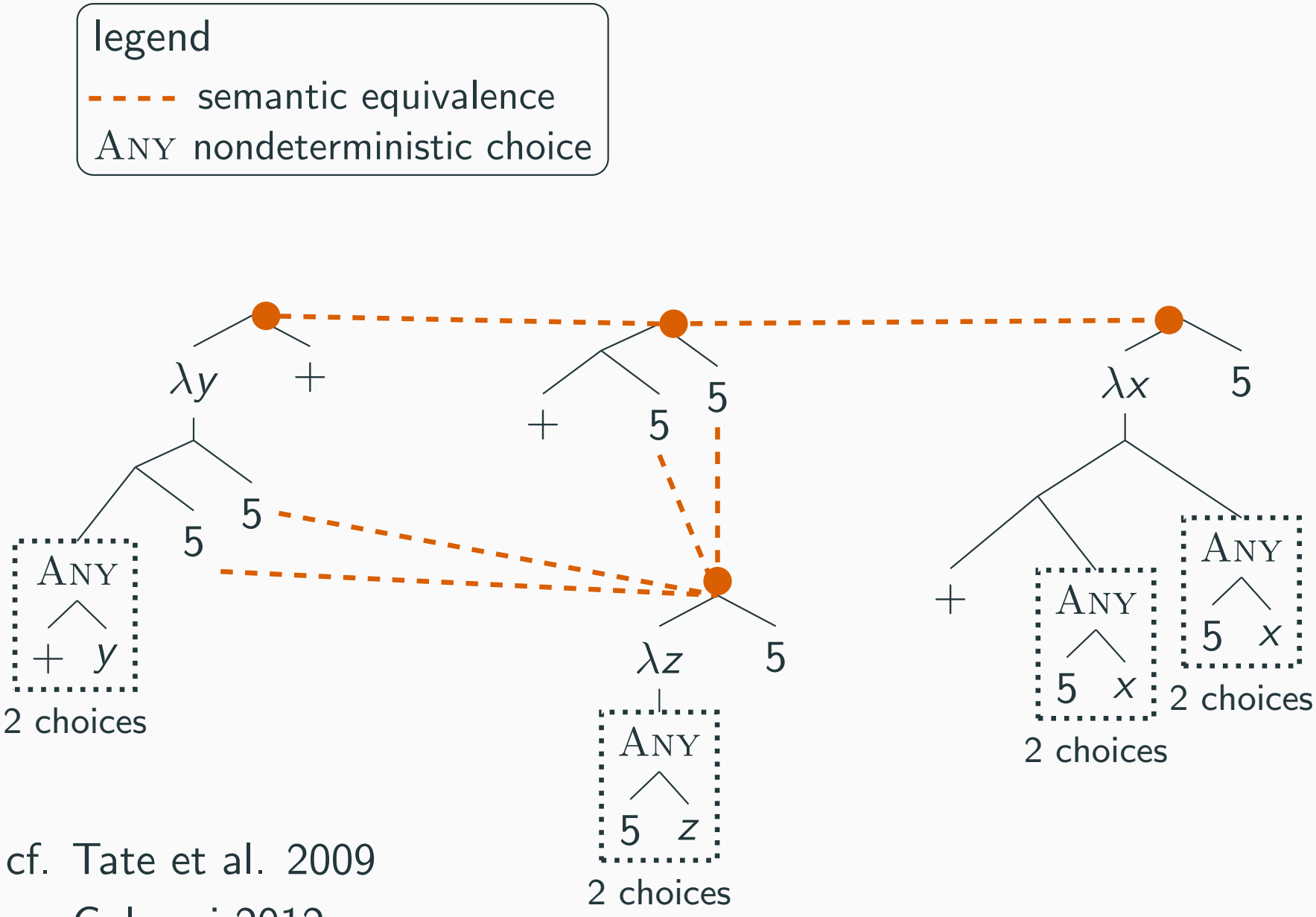
# Abstraction Sleep: Growing the library via refactoring




cf. Tate et al. 2009

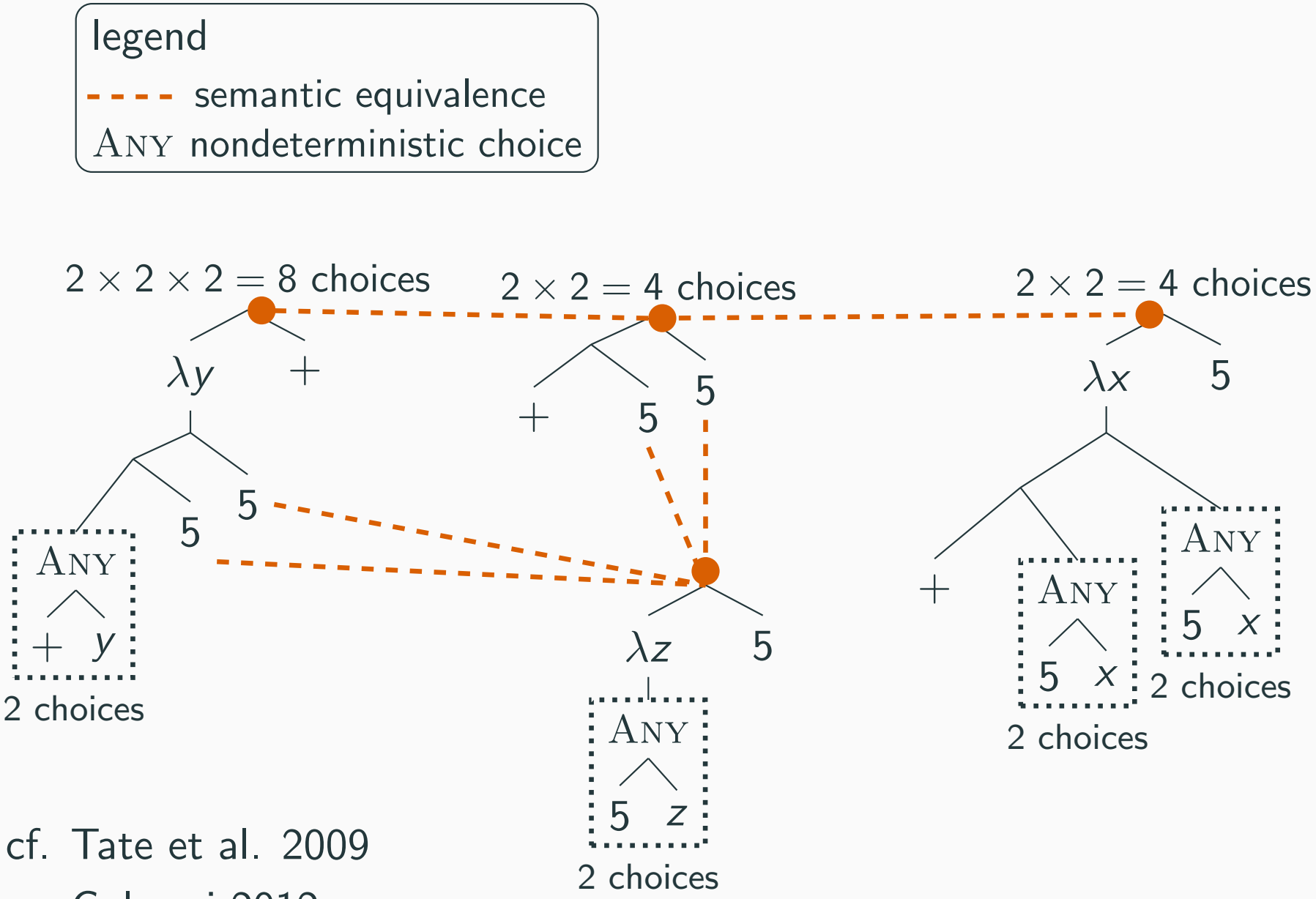
Gulwani 2012

# Abstraction Sleep: Growing the library via refactoring



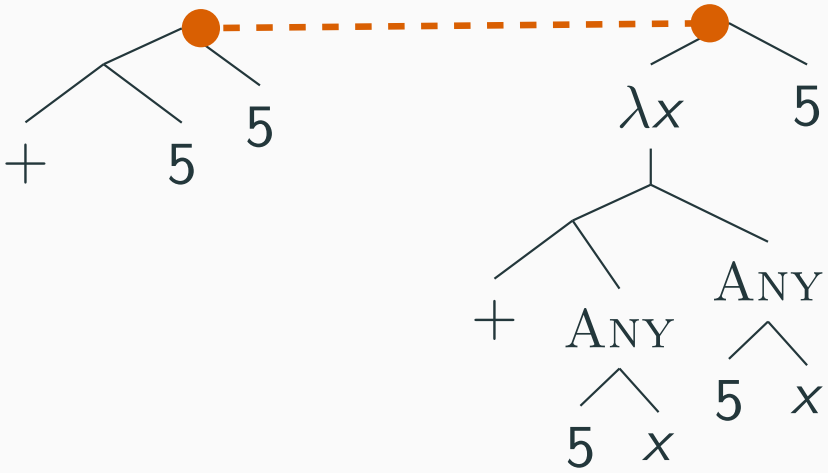

cf. Tate et al. 2009
Gulwani 2012

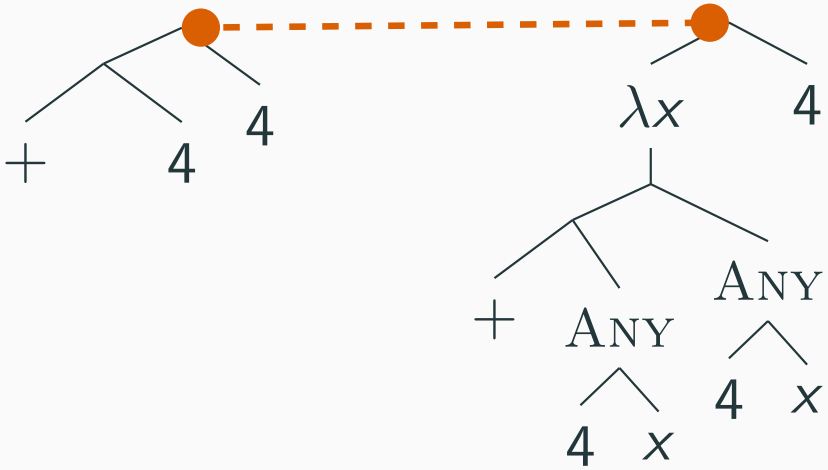
# Abstraction Sleep: Growing the library via refactoring




cf. Tate et al. 2009

Gulwani 2012

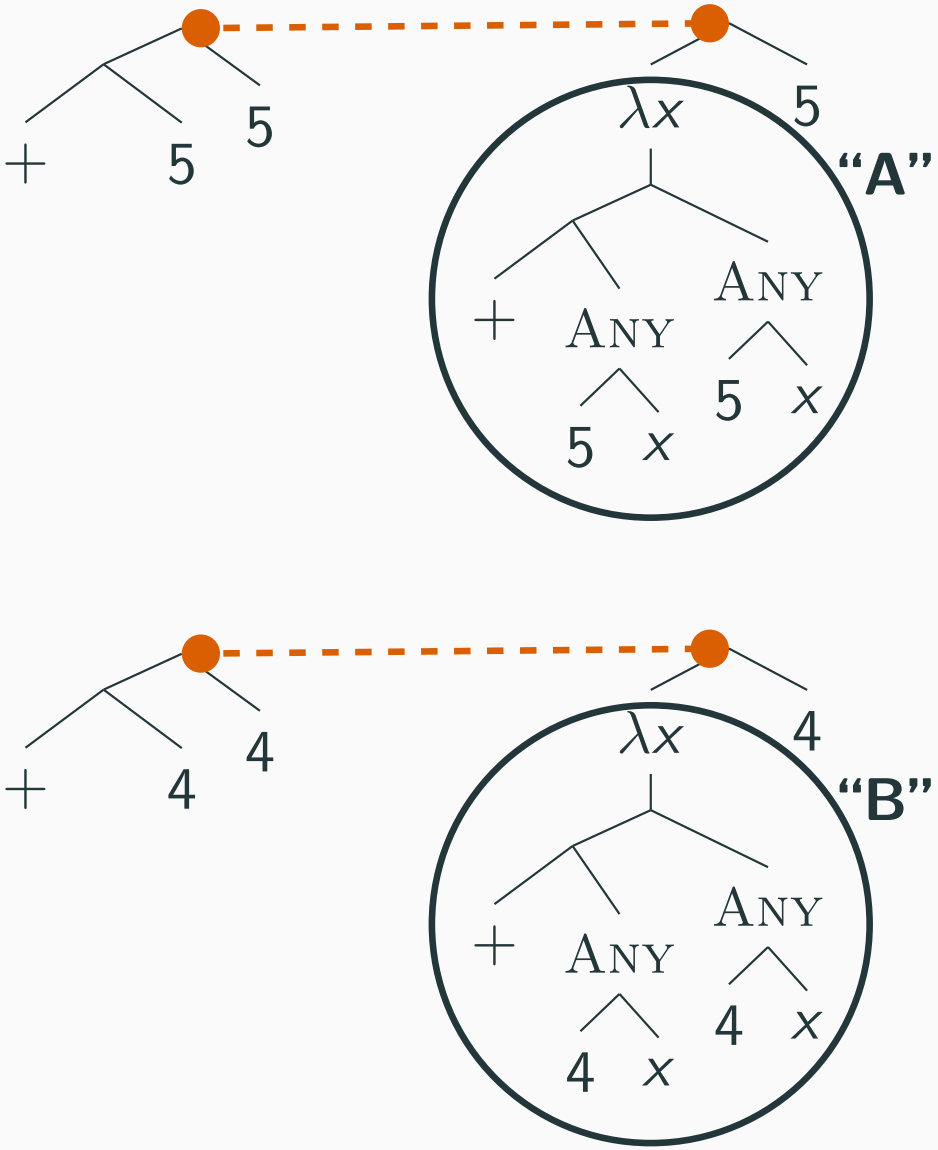


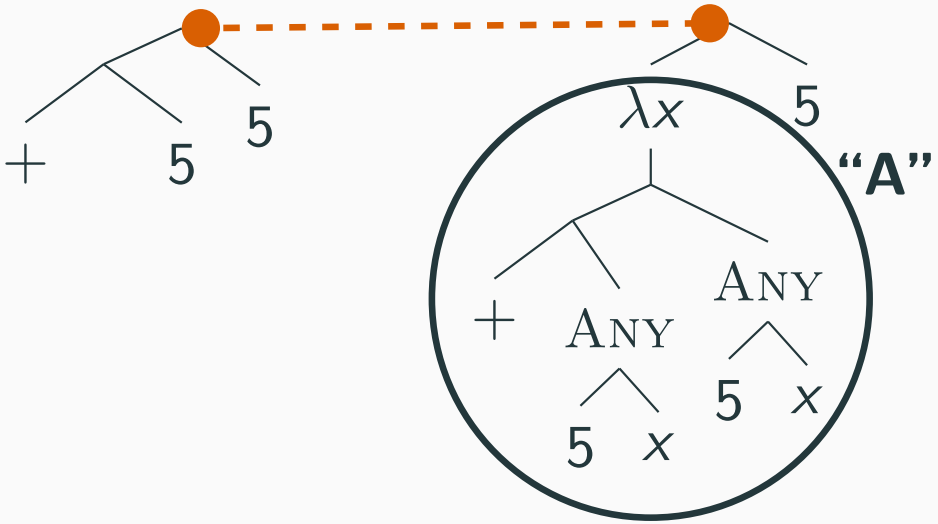




legend

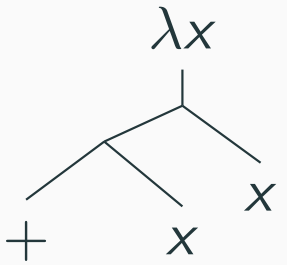
semantic equivalence

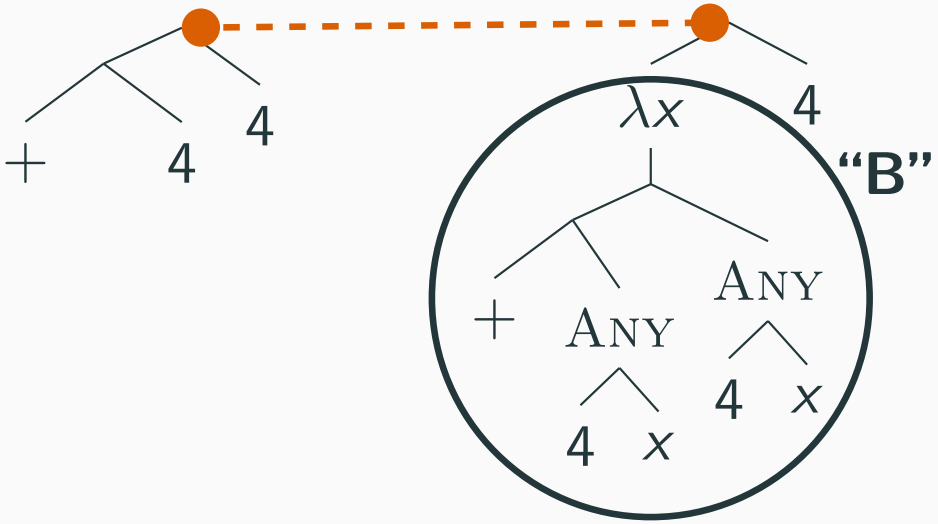
ANY nondeterministic choice


+
5
5
λx
5
“A”
+
ANY
ANY
5
x
5
x
+
4
4
λx
4
“B”
+
ANY
ANY
4
x
4
x
legend
semantic equivalence
ANY nondeterministic choice




**A∩B is:**

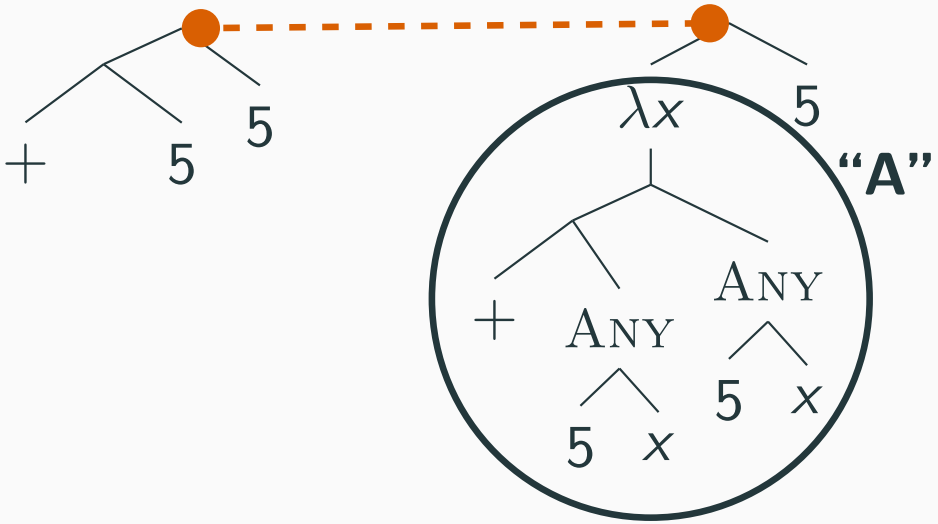


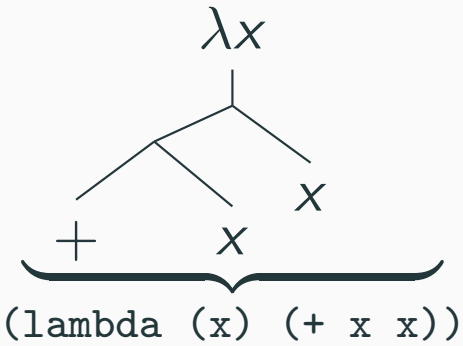

legend

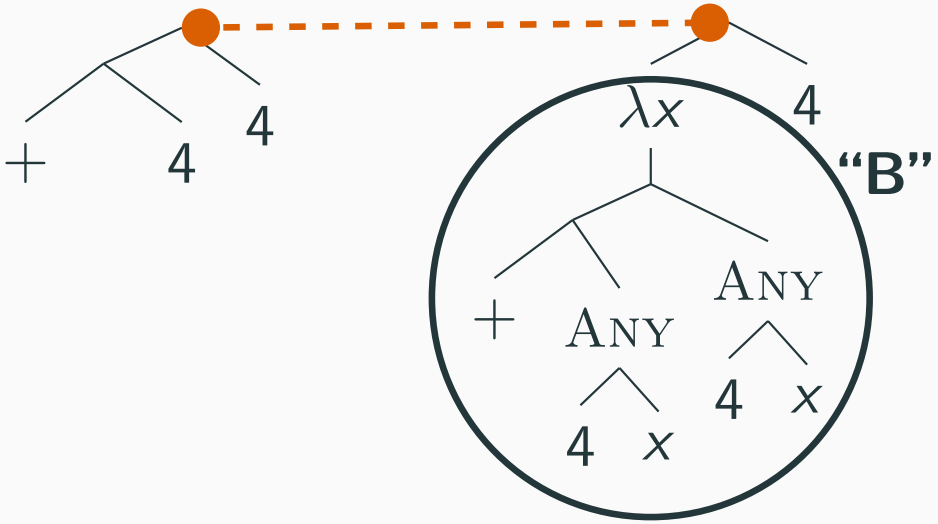
- - - - semantic equivalence

ANY nondeterministic choice




**A∩B is:**

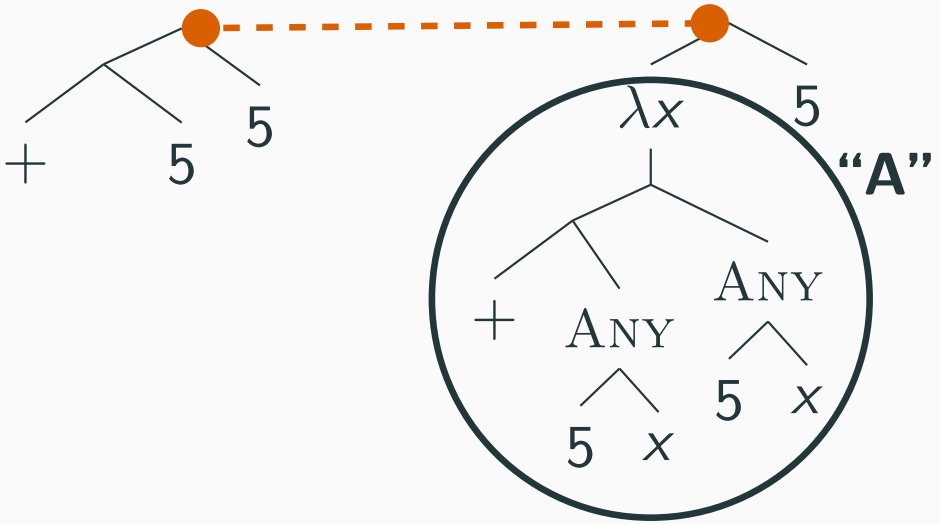


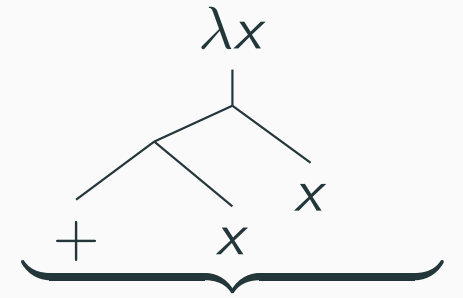

legend

- - - - semantic equivalence

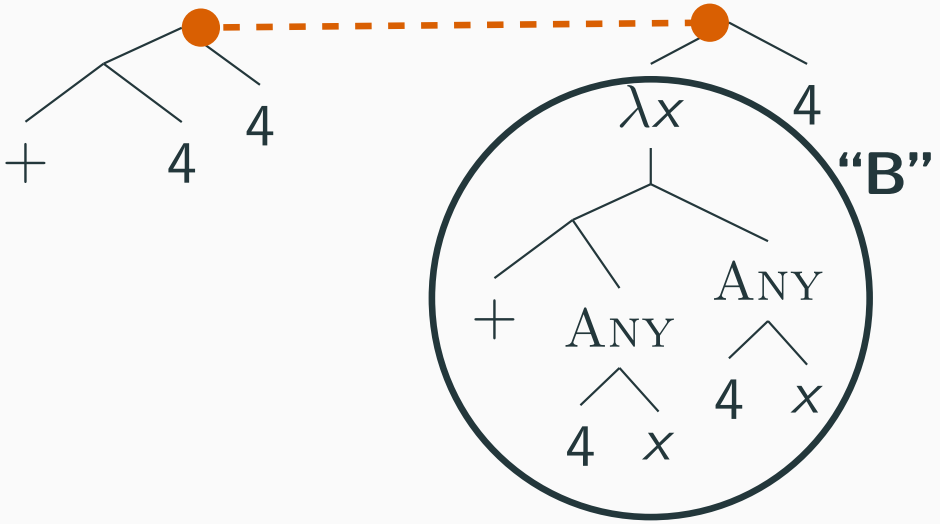
ANY nondeterministic choice




**A∩B is:**



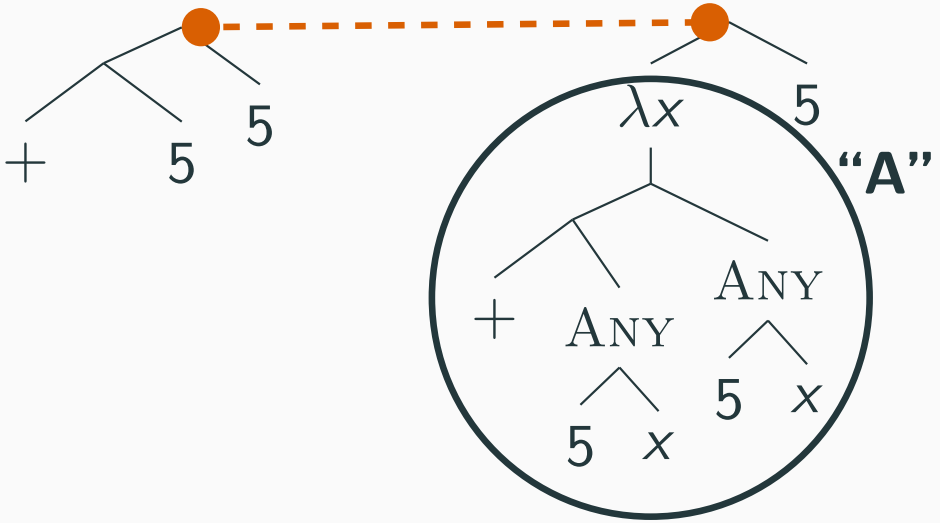

=double



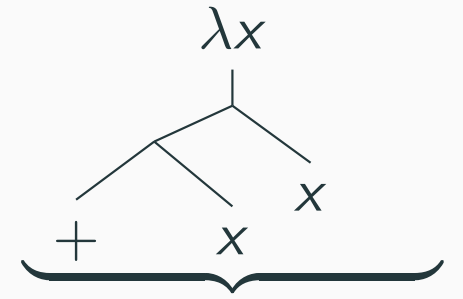

legend

- - - - semantic equivalence

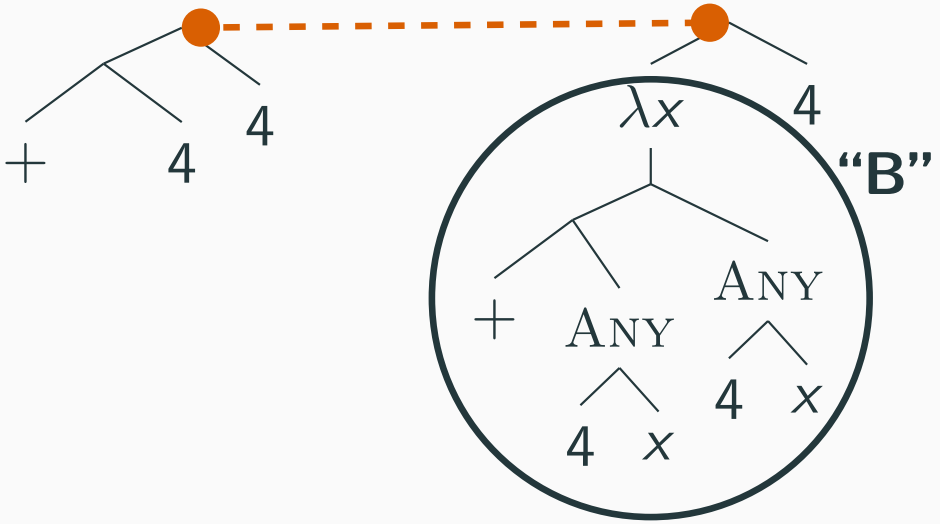
ANY nondeterministic choice




**A∩B is:**




=double



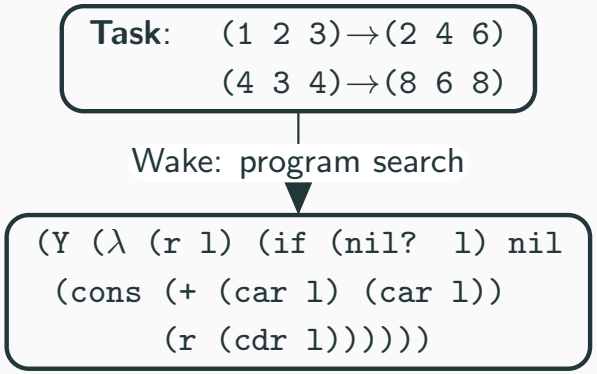

| w/o double | w/ double |
|---|---|
| (+ 5 5) | (double 5) |
| (+ 4 4) | (double 4) |
| (+ 3 3) | (double 3) |
| | ... |

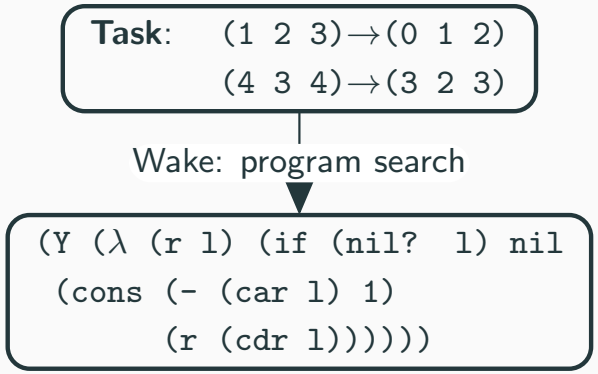
legend

- - - - semantic equivalence

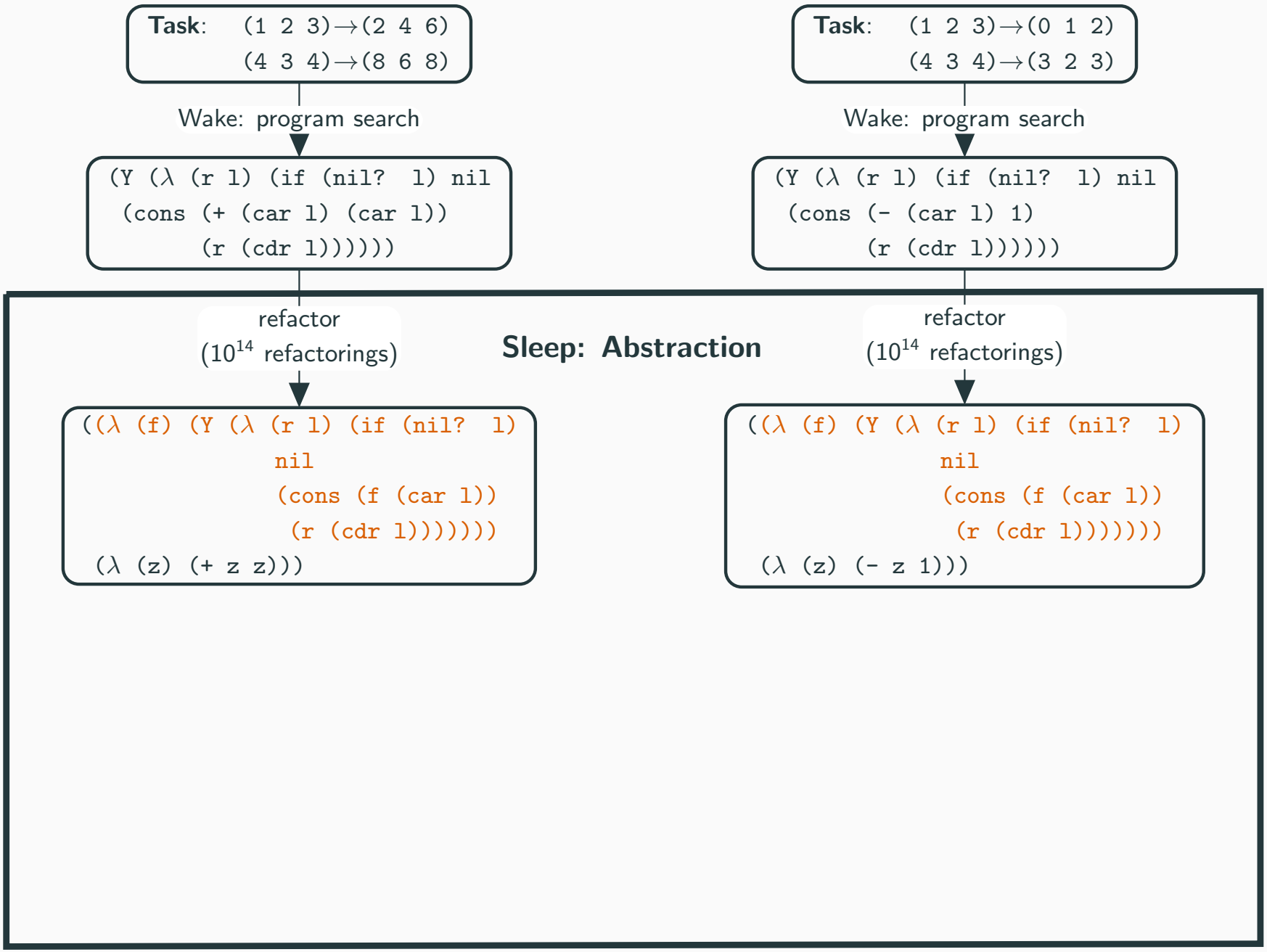
ANY nondeterministic choice

# Abstraction Sleep: Growing the library via refactoring

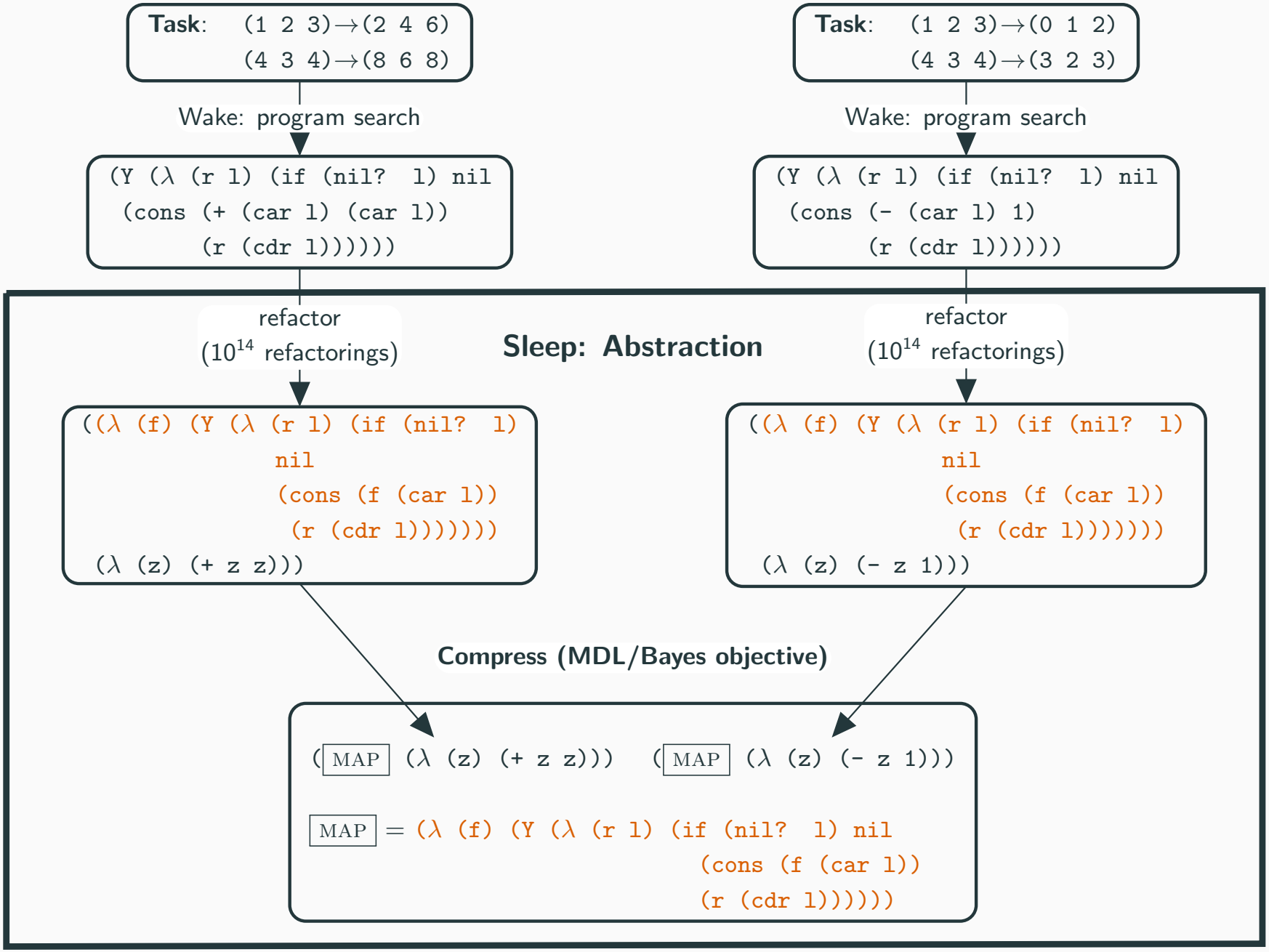


# Abstraction Sleep: Growing the library via refactoring



# Abstraction Sleep: Growing the library via refactoring



# DreamCoder Domains

## List Processing

*Sum List*

```
[1 2 3] → 6
[4 6 8 1] → 17
```

*Double*

```
[1 2 3 4] → [2 4 6 8]
[6 5 1] → [12 10 2]
```

*Check Evens*

```
[0 2 3] → [T T F]
[2 4 9 6] → [T T F T]
```

## Text Editing

*Abbreviate*

```
Allen Newell → A.N.
Herb Simon → H.S.
```

*Drop Last Characters*

```
jabberwocky → jabberw
copycat → cop
```

*Extract*

```
see spot(run) → run
a (bee) see → bee
```

## Regexes

*Phone Numbers*

```
(555) 867-5309
(650) 555-2368
```

*Currency*

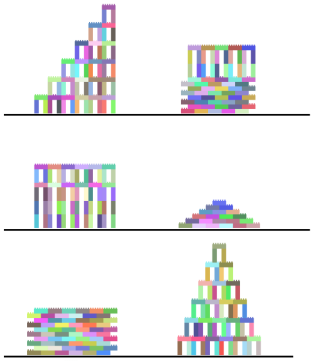
```
$100.25
$4.50
```

*Dates*

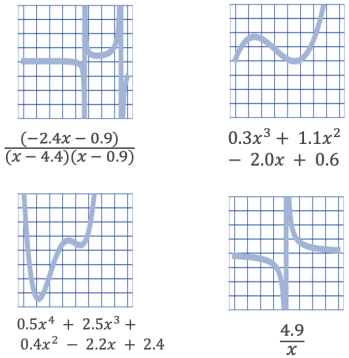
```
Y1775/0704
Y2000/0101
```

## LOGO Graphics

## Block Towers

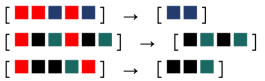


## Symbolic Regression




## Recursive Programming

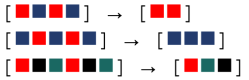
*Filter*



*Index List*

*Length*

*Every Other*



## Physics

$$KE = \frac{1}{2}m|\vec{v}|^2$$

$$\vec{a} = \frac{1}{m}\sum_i \vec{F}_i$$

$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r_1} - \vec{r_2}|^2}\widehat{r_1 - r_2}$$

$$R_{total} = \left(\Sigma_i \frac{1}{R_i}\right)^{-1}$$

# DreamCoder Domains

## List Processing

*Sum List*

[1 2 3] → 6
[4 6 8 1] → 17

*Double*

[1 2 3 4] → [2 4 6 8]
[6 5 1] → [12 10 2]

*Check Evens*

[0 2 3] → [T T F]
[2 4 9 6] → [T T F T]

## Text Editing

*Abbreviate*

Allen Newell → A.N.
Herb Simon → H.S.

*Drop Last Characters*

jabberwocky → jabberw
copycat → cop

*Extract*

see spot(run) → run
a (bee) see → bee

## Regexes

*Phone Numbers*

(555) 867-5309
(650) 555-2368

*Currency*

$100.25
$4.50

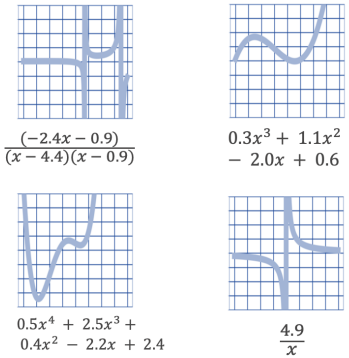
*Dates*

Y1775/0704
Y2000/0101

## LOGO Graphics

## Block Towers

## Symbolic Regression




## Recursive Programming

*Filter*

[ ] → [ ]
[ ] → [ ]
[ ] → [ ]

*Index List*

0, [ ] →
1, [ ] →
1, [ ] →

*Length*

[ ] → 4
[ ] → 6
[ ] → 3

*Every Other*

[ ] → [ ]
[ ] → [ ]
[ ] → [ ]

## Physics

$$KE = \frac{1}{2}m|\vec{v}|^2$$

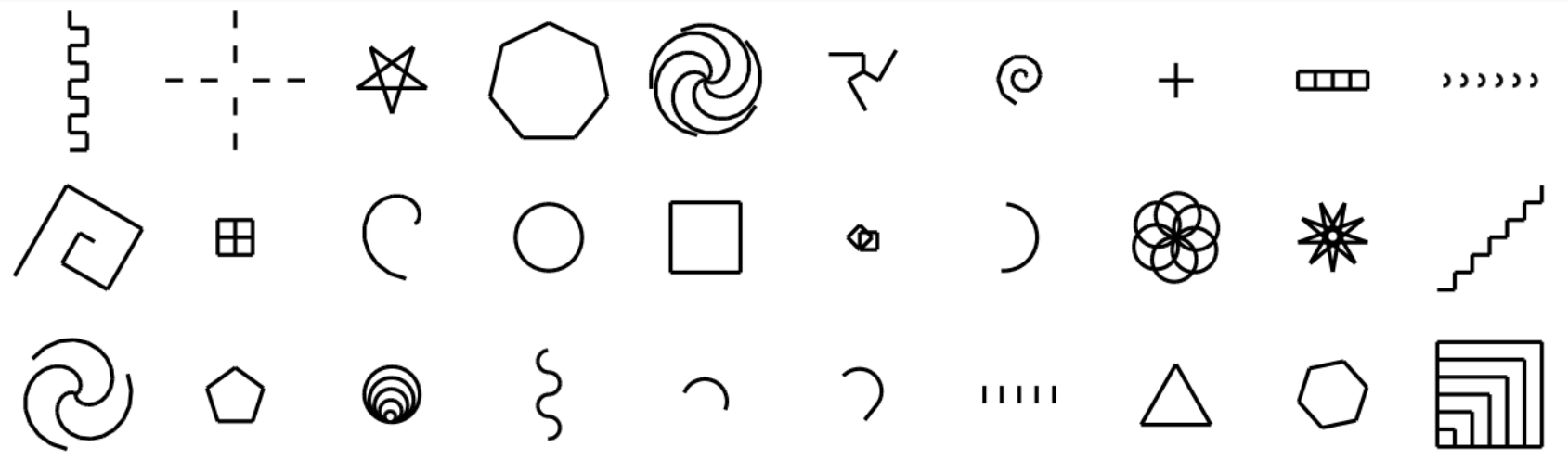
$$\vec{a} = \frac{1}{m}\sum_i \vec{F}_i$$

$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r_1} - \vec{r_2}|^2}\widehat{r_1 - r_2}$$

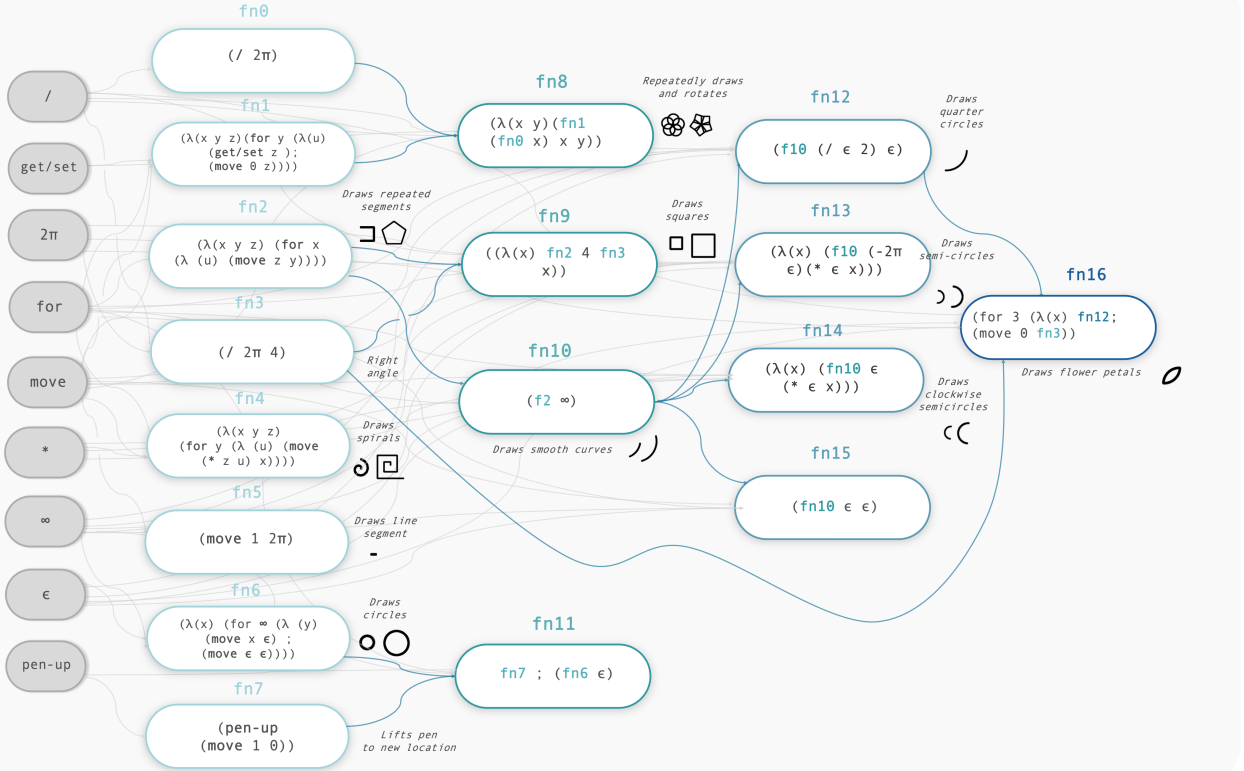
$$R_{total} = \left(\Sigma_i \frac{1}{R_i}\right)^{-1}$$

# LOGO Graphics

30 out of 160 tasks



# LOGO Graphics – learning interpretable library of concepts




```
(fn8 5 (fn4 (* ϵ 2) ∞ ϵ))
```

```
(for 2 (λ (x)
(fn14 2); (fn13 2)))
```

```
(for 7 (λ (x) (fn9 x)))
```

```
(fn10 (* (fn0 7) 3) 3)
```

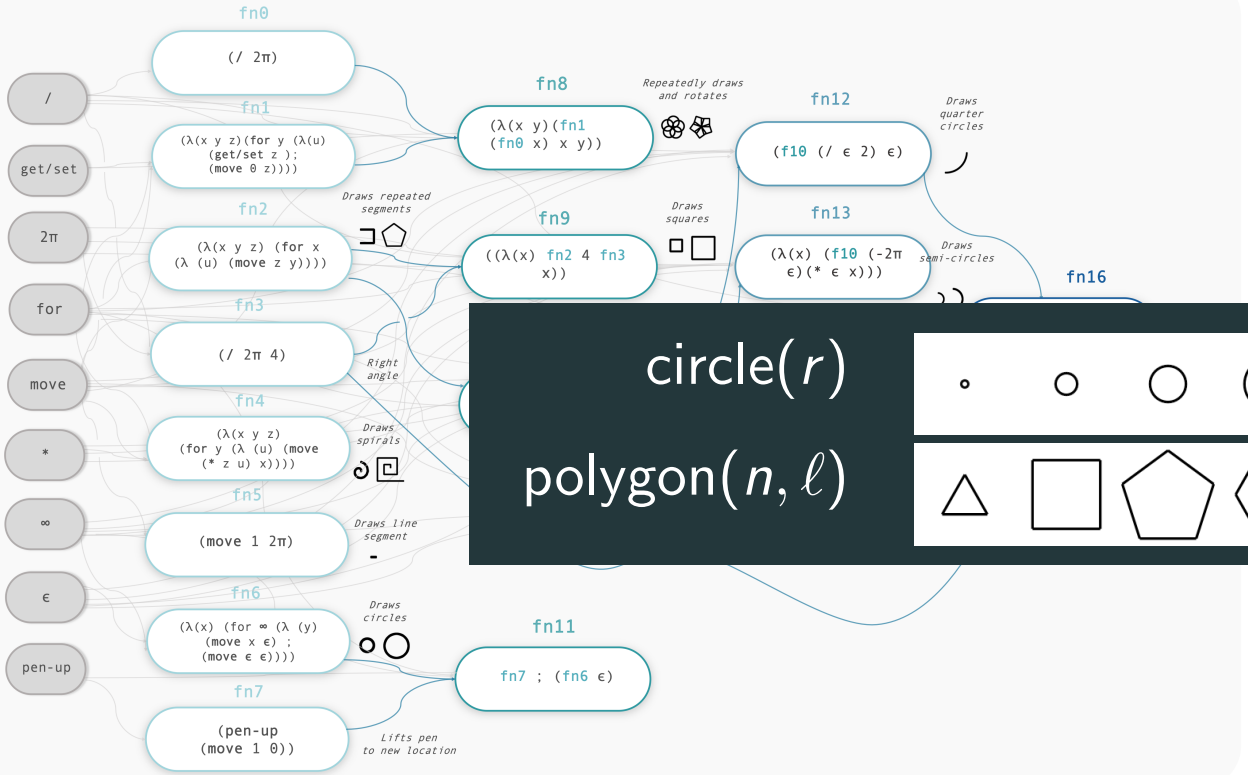
```
(fn8  6 (fn7 ; fn5 ;
fn7 ; fn5))
```

```
(fn8 5 fn16)
```

```
(move 0 (fn0 7)); fn5 ;
(fn13 4)
```

```
(fn4 fn3 9 1)
```

# LOGO Graphics – learning interpretable library of concepts




circle($r$)

polygon($n, \ell$)



(fn8 5 (fn4 (* ϵ 2) ∞ ϵ))

(for 2 (λ (x)
(fn14 2); (fn13 2)))

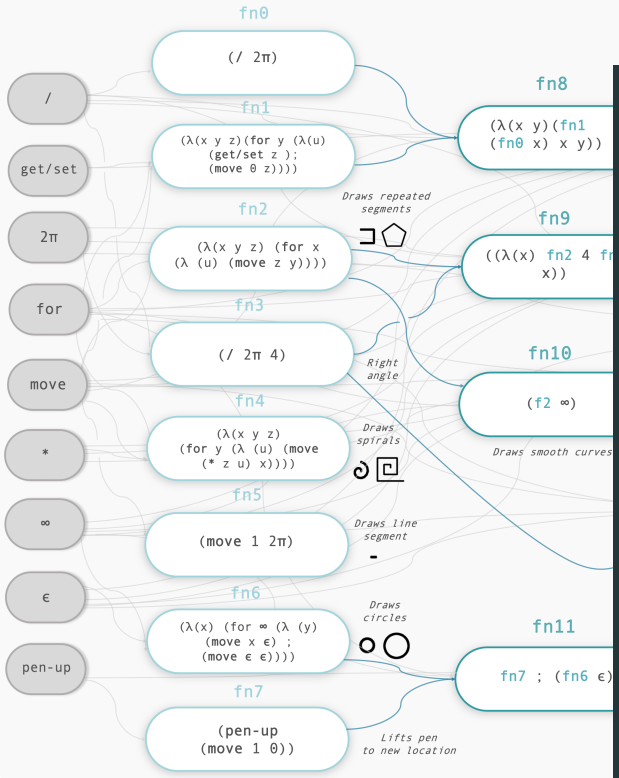
(fn10 (* (fn0 7) 3) 3)

(fn8 5 fn16)

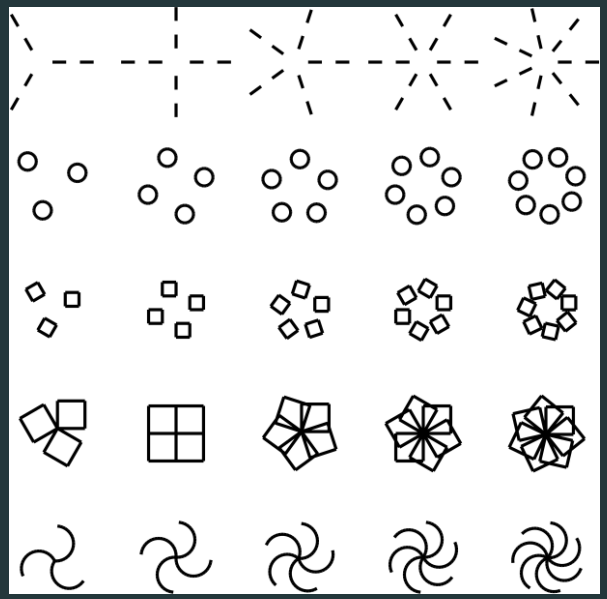
(move 0 (fn0 7)); fn5 ;
(fn13 4)

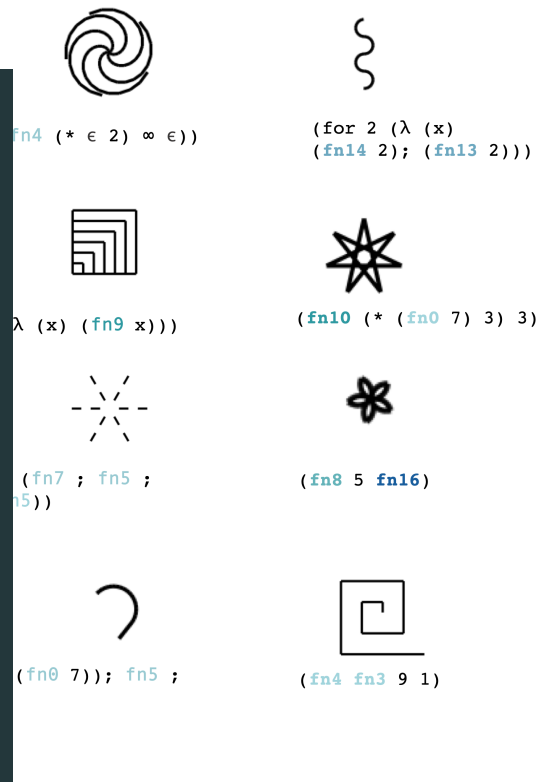
(fn4 fn3 9 1)

# LOGO Graphics – learning interpretable library of concepts



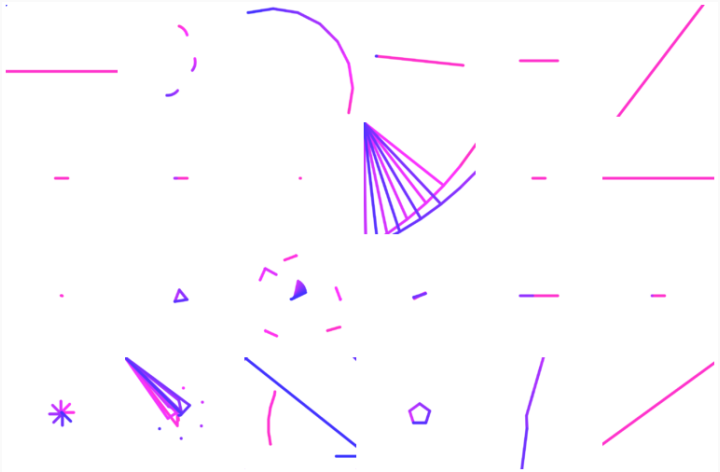

radial symmetry($n$, body)



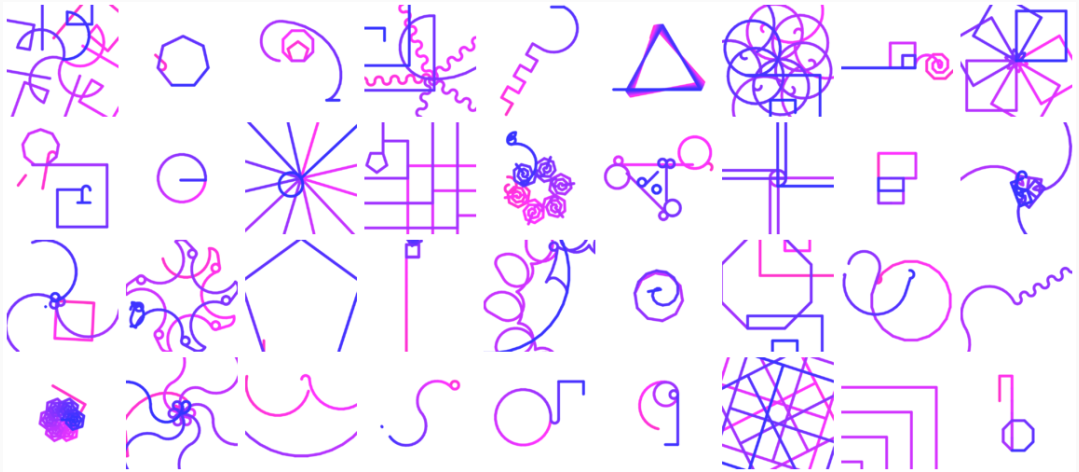


# what does DreamCoder dream of?

before learning

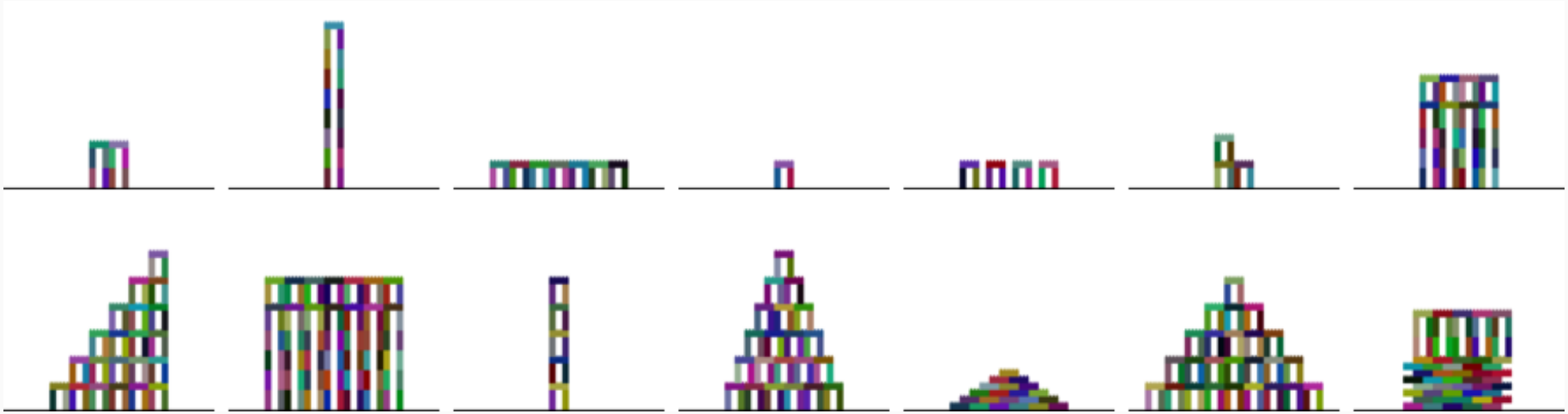


after learning



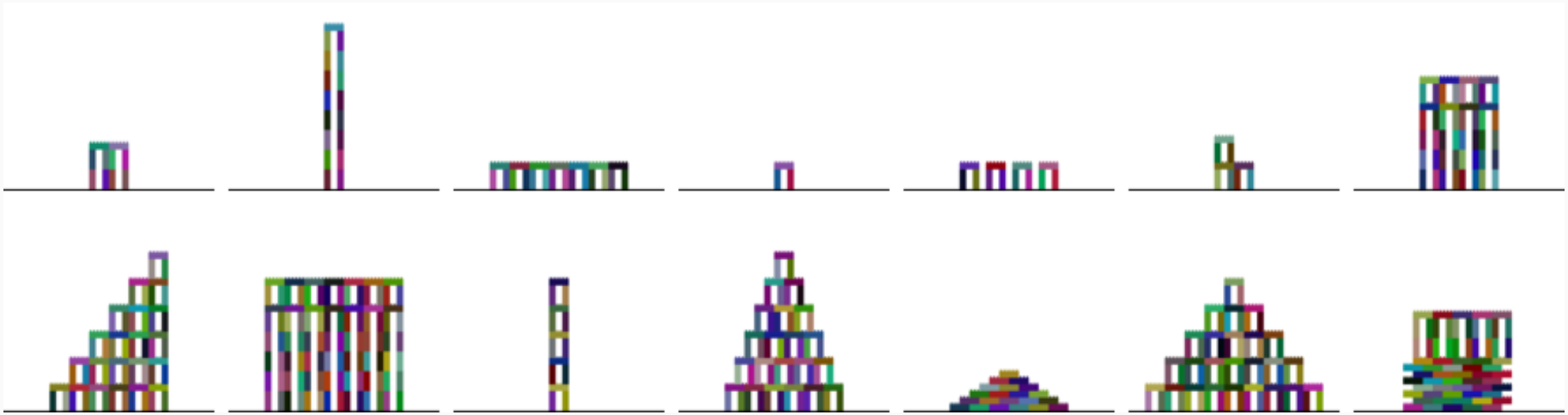
# Planning to build towers

example tasks (112 total)



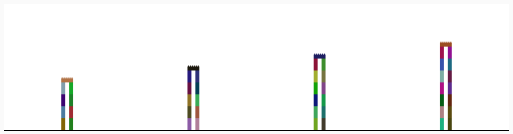
# Planning to build towers

example tasks (112 total)

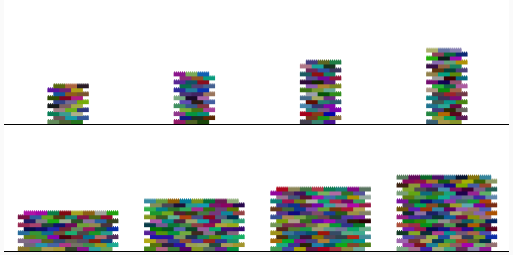


learned library routines ($\approx$ 20 total)

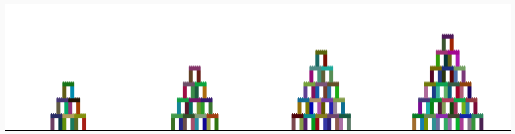
arch($h$)



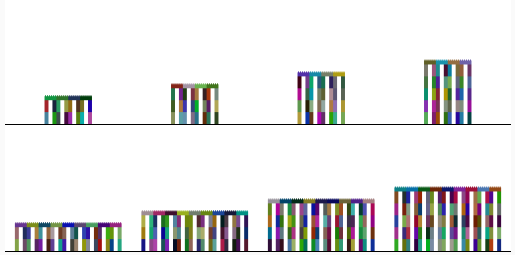
wall($w$, $h$)



pyramid($h$)

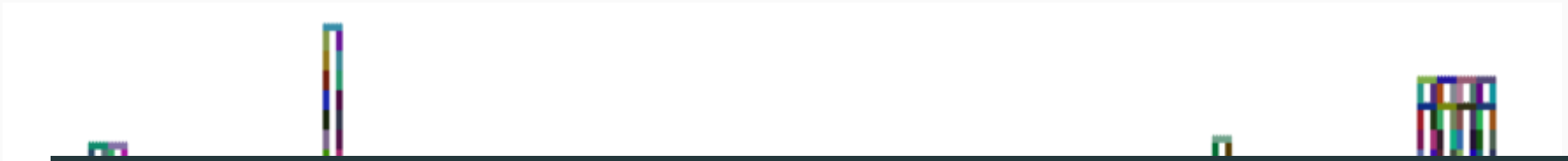


bridge($w$, $h$)



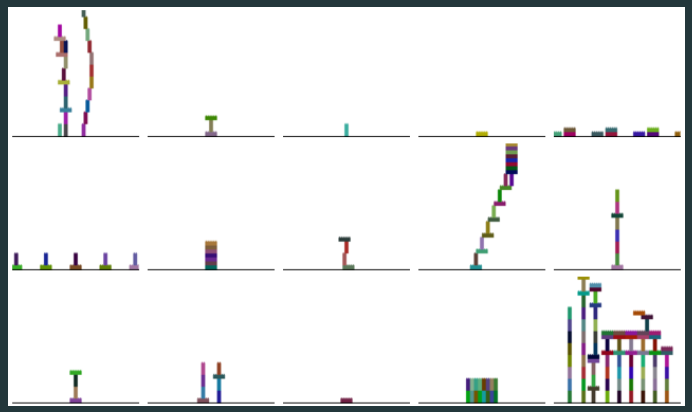
# Planning to build towers

example tasks (112 total)

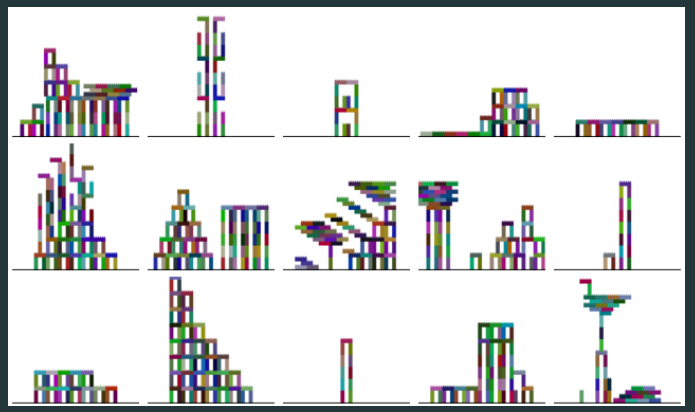


lea

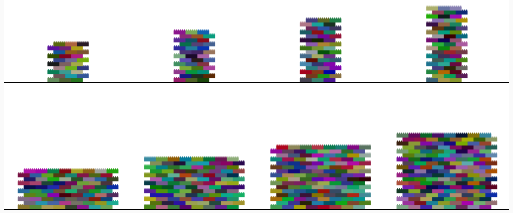
**dreams before learning**



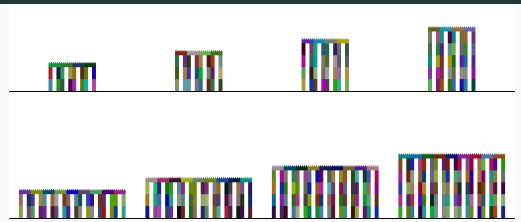
**dreams after learning**



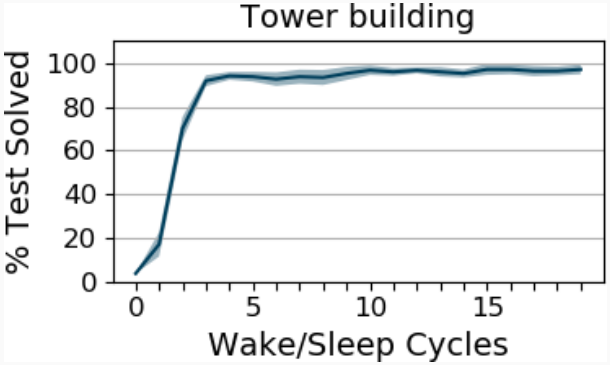
wall($w$, $h$)



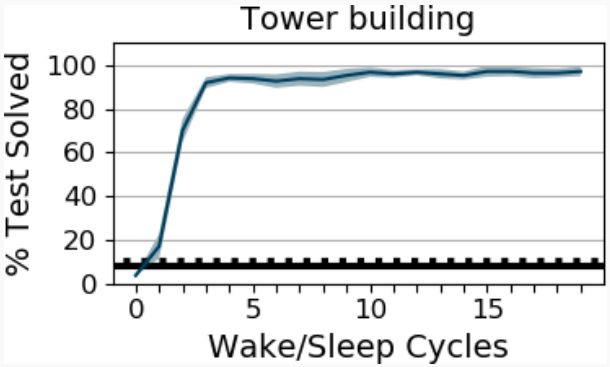
bridge($w$, $h$)

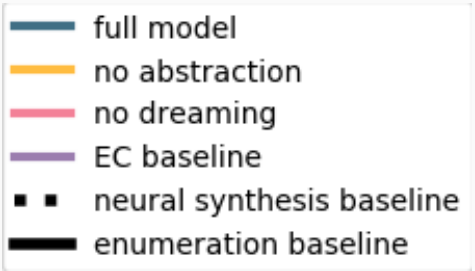


# Synergy between dreaming and library learning

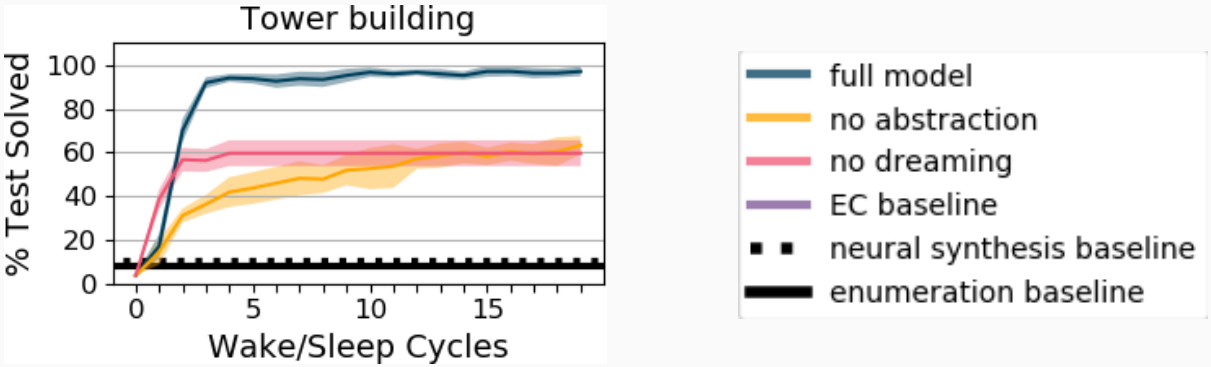


# Synergy between dreaming and library learning

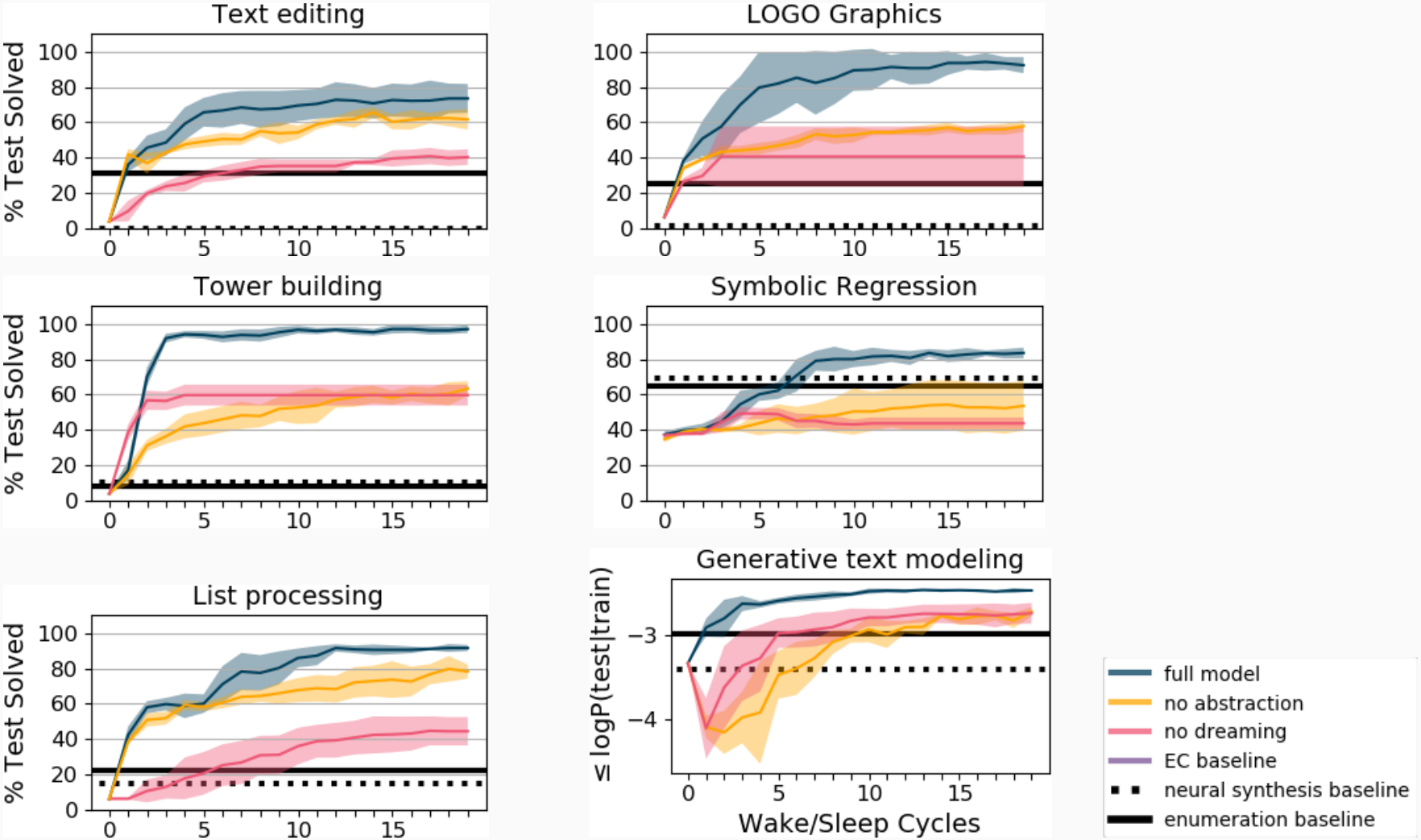


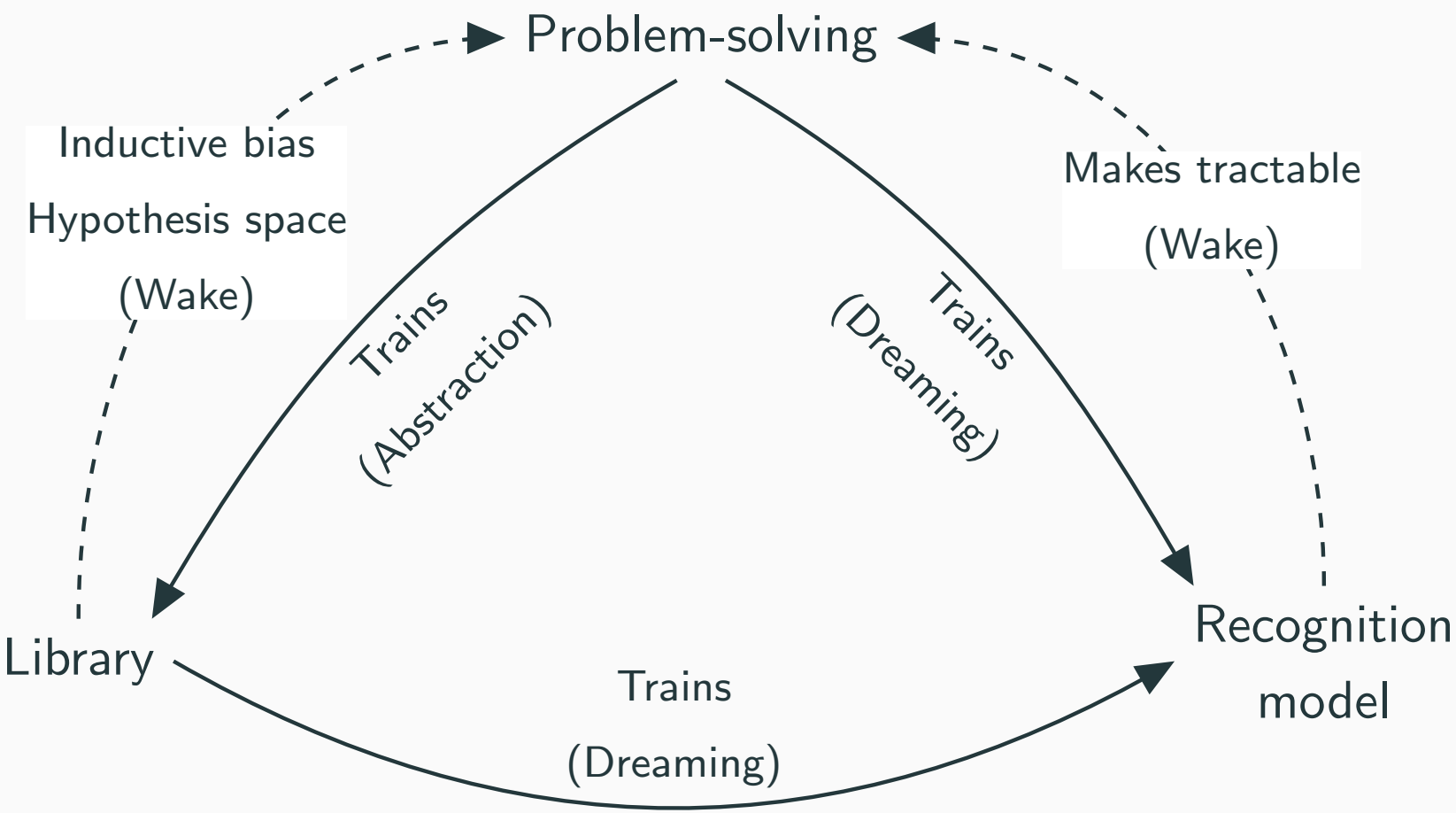
# Synergy between dreaming and library learning



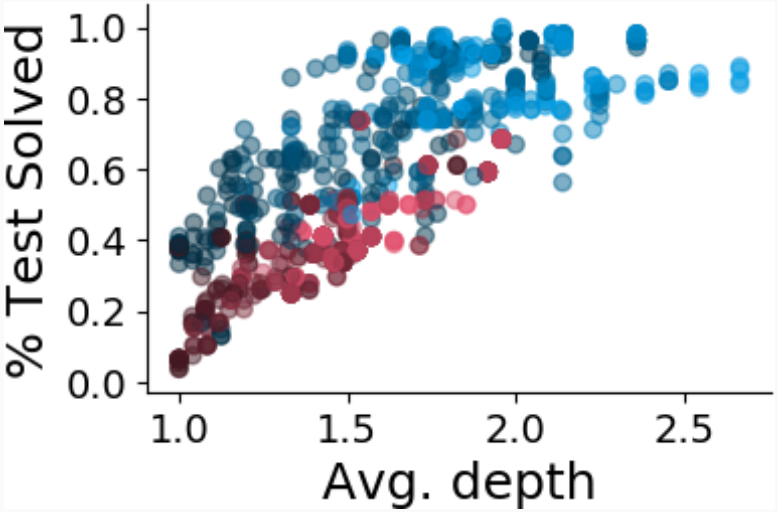
# Synergy between dreaming and library learning

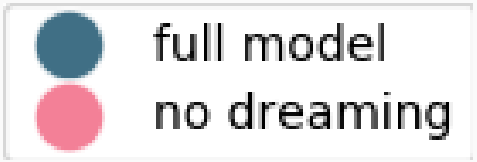


# synergy between dreaming and library learning



# Evidence for dreaming bootstrapping better libraries

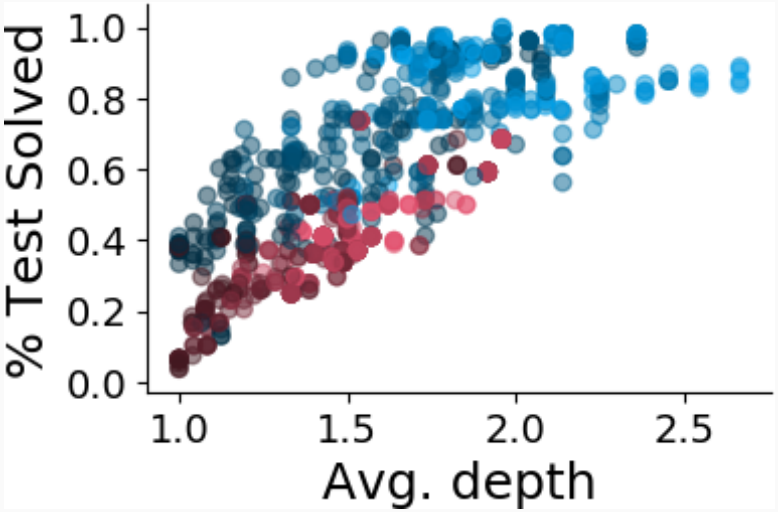


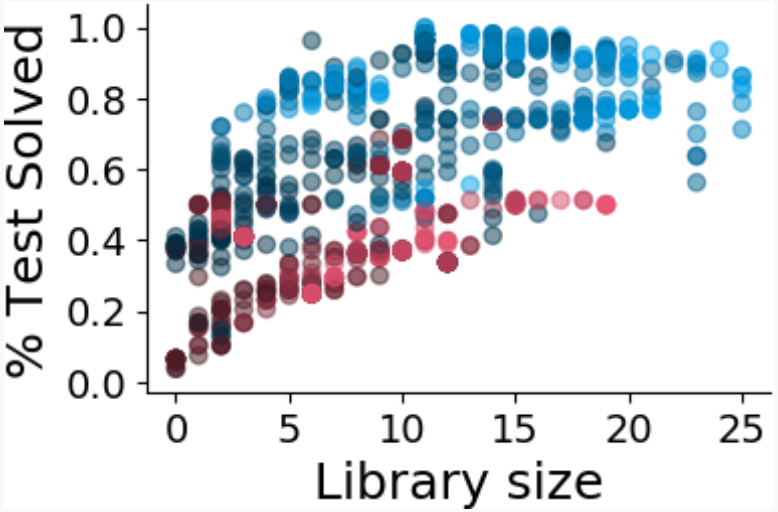



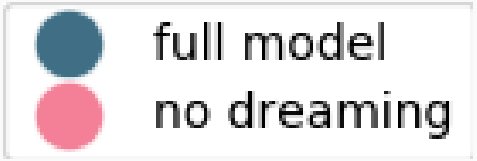

Darker: Early in learning

Lighter: Later in learning

# Evidence for dreaming bootstrapping better libraries



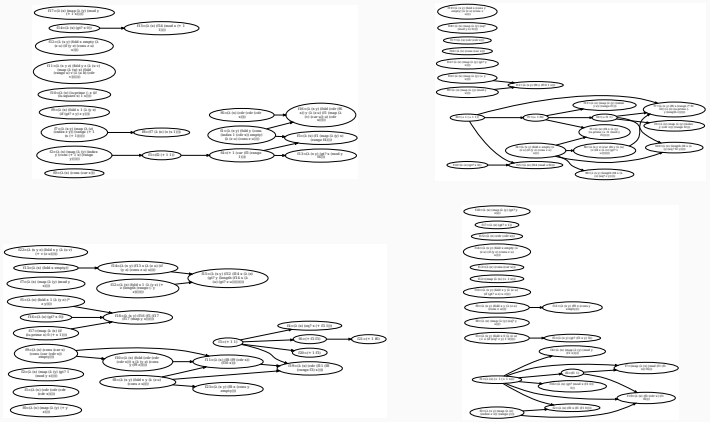




Darker: Early in learning

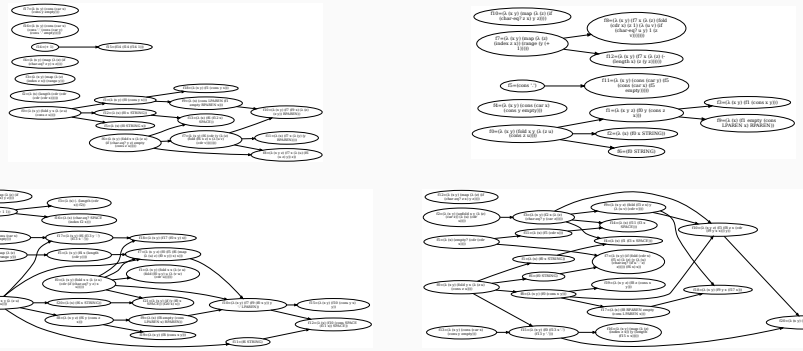
Lighter: Later in learning

# Variability in learned library

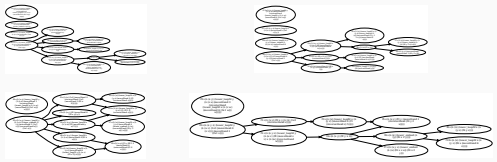
### List processing



### Text editing

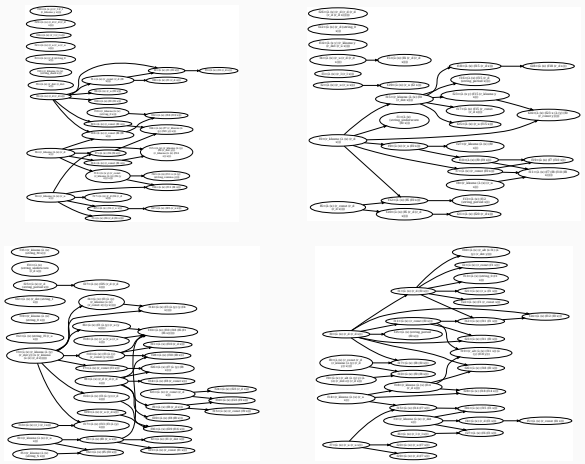


### Tower building

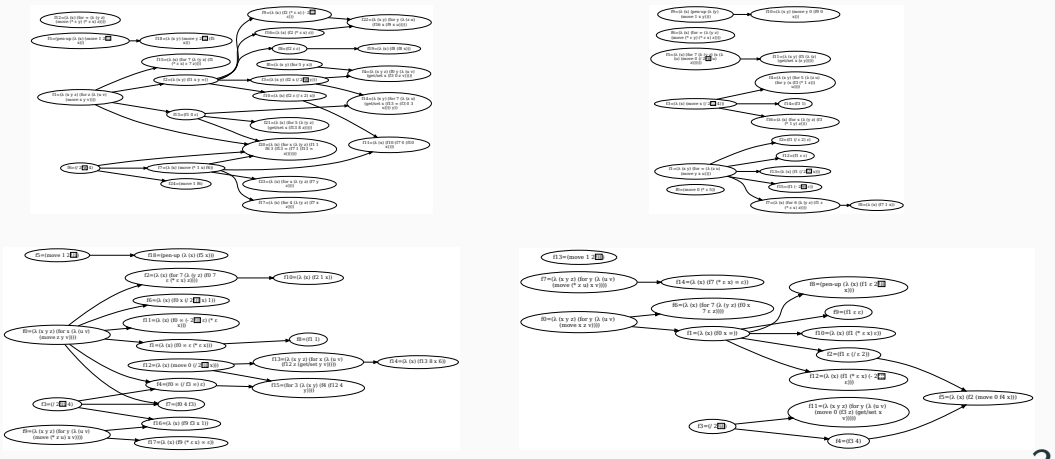


### Symbolic regression

### Generative regex



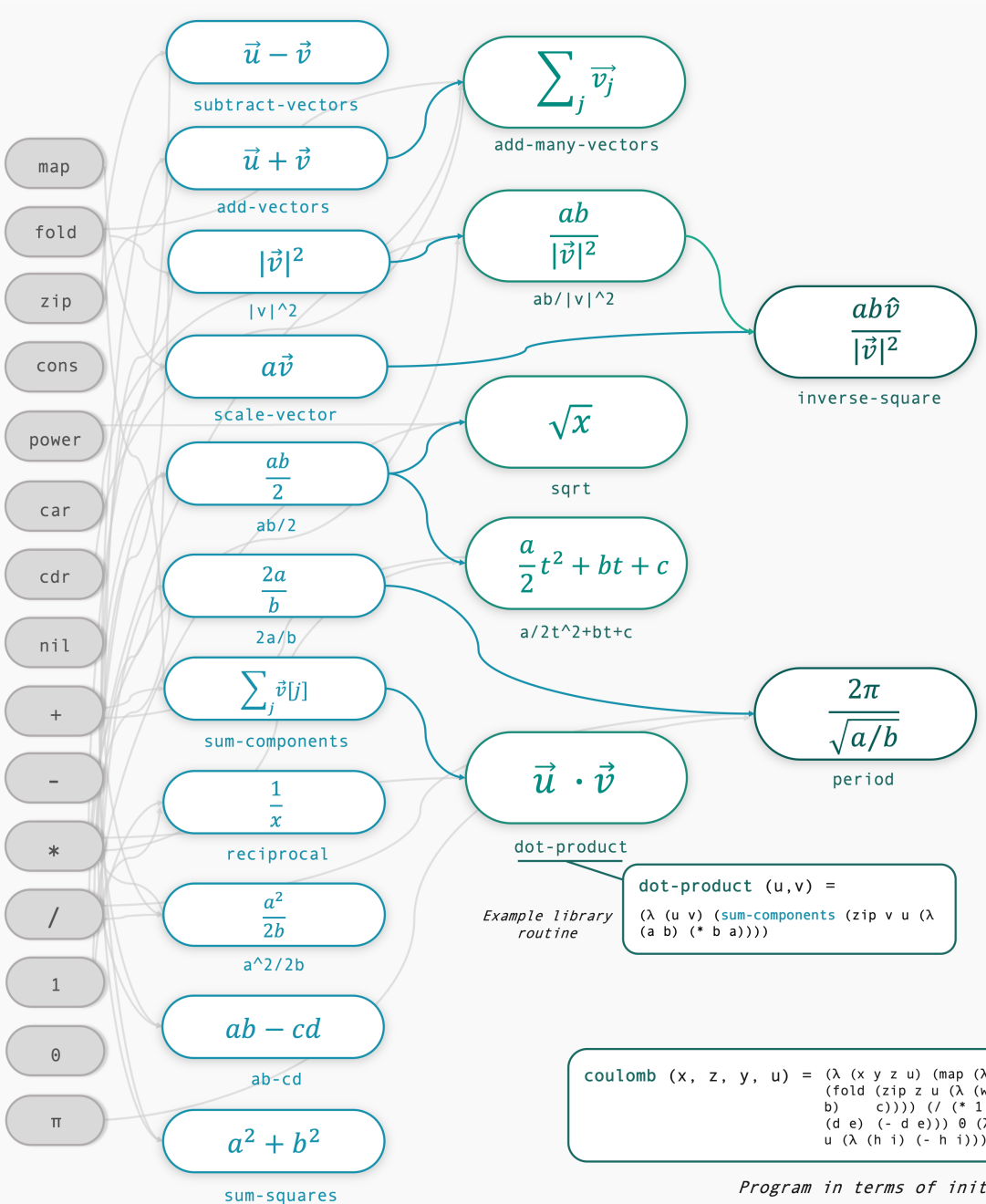
### LOGO graphics



# From learning libraries to learning languages

How far can we push library learning?

# Rediscovering vector algebra




*Electric Field to Charge Flux*

$$\vec{E}=\rho\vec{J}$$

```
(scale-vector p J)
```

*Newton's Second Law*

$$\vec{a}=\frac{1}{m}\sum_i \vec{F}_i$$

```
(scale-vector (reciprocal m)
(add-many-vectors Fs))
```

*Parallel Resistors*

$$R_{total}=\left(\Sigma_i \frac{1}{R_i}\right)^{-1}$$

```
(reciprocal (sum-components (map
(λ(r) (reciprocal r)) Rs)))
```

*Kinetic Energy*

$$KE=\frac{1}{2}m|\vec{v}|^2$$

```
(ab/2 m (|v|^2 v))
```

*Energy in a Capacitor*

$$U=\frac{1}{2}CV^2$$

```
(a^2/2b  v (reciprocal c))
```

*Ballistic Motion*

$$x(t)=\frac{a}{2}t^2+v_0t+x_0$$

```
(a/2t^2+bt+c c b a t)
```

*Force in a Magnetic Field*

$$|\vec{F}|=q|\vec{v}\times\vec{B}|$$

```
(* q (ab-cd v_x b_y v_y b_x))
```

*Work*

$$U=\vec{F}\cdot\vec{d}$$

```
(dot-product F d)
```

*Coulomb's Law*

$$\vec{F}\propto\frac{q_1 q_2}{|\vec{r_1}-\vec{r_2}|^2}\widehat{r_1-r_2}$$

```
(inverse-square q_1 q_2
(subtract-vectors r_1 r_2))
```

*Center of Mass*

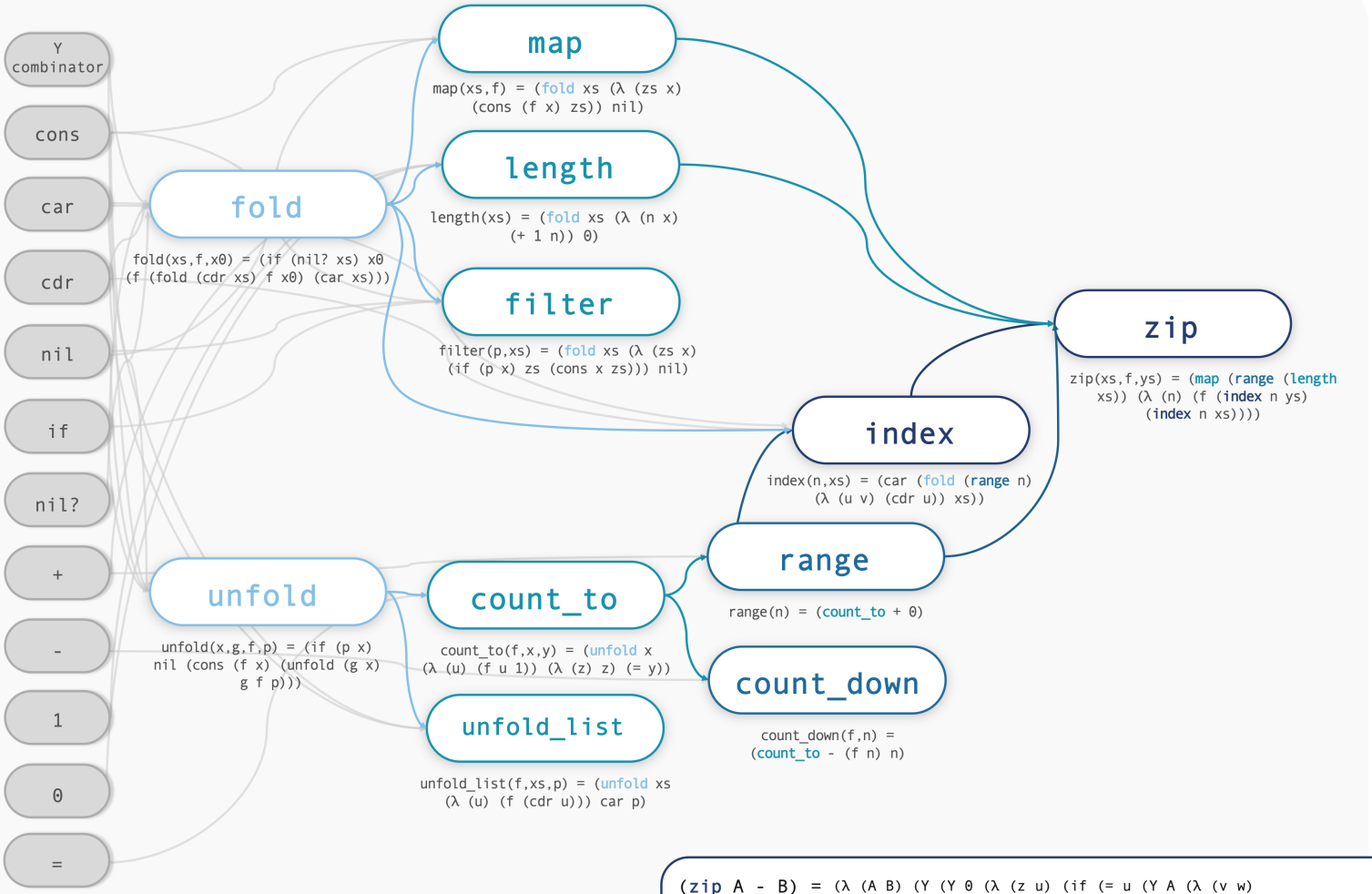
$$X_{CM}=\frac{\sum_i m_i x_i}{\sum_i m_i}$$

```
(/ (dot-product x m) (sum-
components m))
```

```
coulomb (x, z, y, u) = (λ (x y z u) (map (λ (v) (* (/ (* (power (/ (* x x)
(fold (zip z u (λ (w a) (- w a))) 0 (λ (b c) (+ (* b
b)    c)))) (/ (* 1 1) (+ 1 1))) y) (fold (zip z u (λ
(d e) (- d e))) 0 (λ (f g) (+ (* f f) g)))) v)) (zip z
u (λ (h i) (- h i)))))
```

*Program in terms of initial library*

# Rediscovering origami programming




*Stutter*

[■■] → [■■■■]
[■■■] → [■■■■■■]

```
(fold A (λ (u v) (cons
v (cons v u))) nil)
```

*Index List*

0, [■■■■■] → ■
1, [■■■■■] → ■

```
(index n A)
```

*Take Every Other*

[■■■■] → [■■]
[■■■■■■] → [■■■]

```
(unfold_list cdr A nil?)
```

*List Lengths*

[[■■■], [■]] → [3 1]
[[■■], [], [■]] → [2 0 1]

```
(map A length)
```

*Differences*

[1 8 2], [0 5 1] → [1 3 1]
[2 3 6], [1 2 4] → [1 1 2]

```
( zip A - B )
```

```
(zip A - B) = (λ (A B) (Y (Y 0 (λ (z u) (if (= u (Y A (λ (v w)
(if (nil? w) 0 (+ 1 (v (cdr w)))))))) nil (cons u (z (+ u 1))))))
(λ (a b) (if (nil? b) nil (cons (- (car (Y (Y 0 (λ (c d) (if (= d
(car b)) nil (cons d (c (+ d 1)))))))) (λ (e f) (if (nil? f) A (cdr
(e (cdr f)))))))) (car (Y (Y 0 (λ (g h) (if (= h (car b)) nil (cons
h (g (+ h 1)))))))) (λ (i j) (if (nil? j) B (cdr (i (cdr j))))))))
(a (cdr b)))))))
```

*Program in terms of initial library*

# Lessons

Symbols aren't necessarily interpretable. Flexibly grow the language based on experience to make it more powerful *and* more human understandable

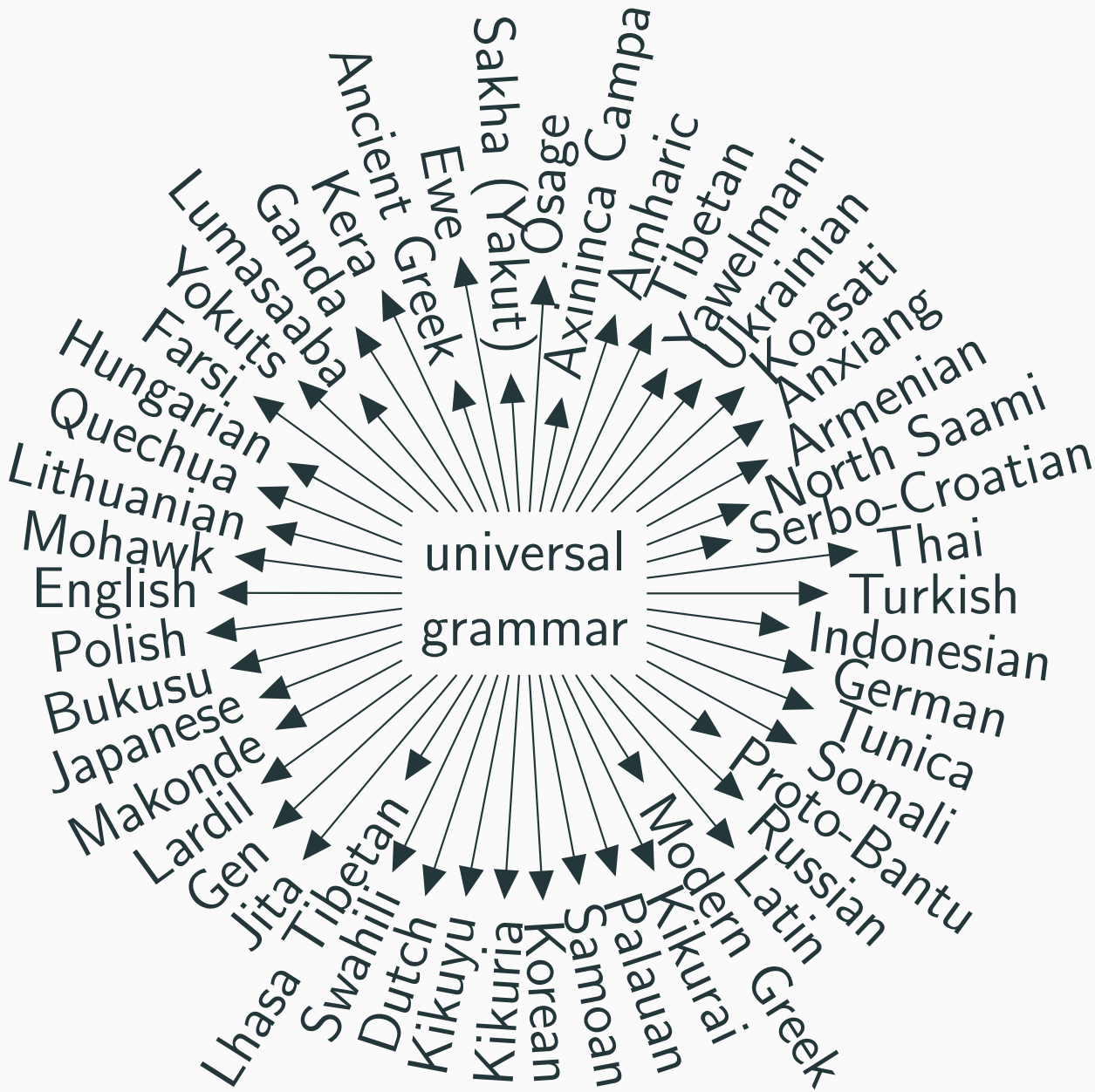
Learning-from-scratch is possible in principle. Don't do it. Choose wisely what to learn and what to build in

Program Induction and perception

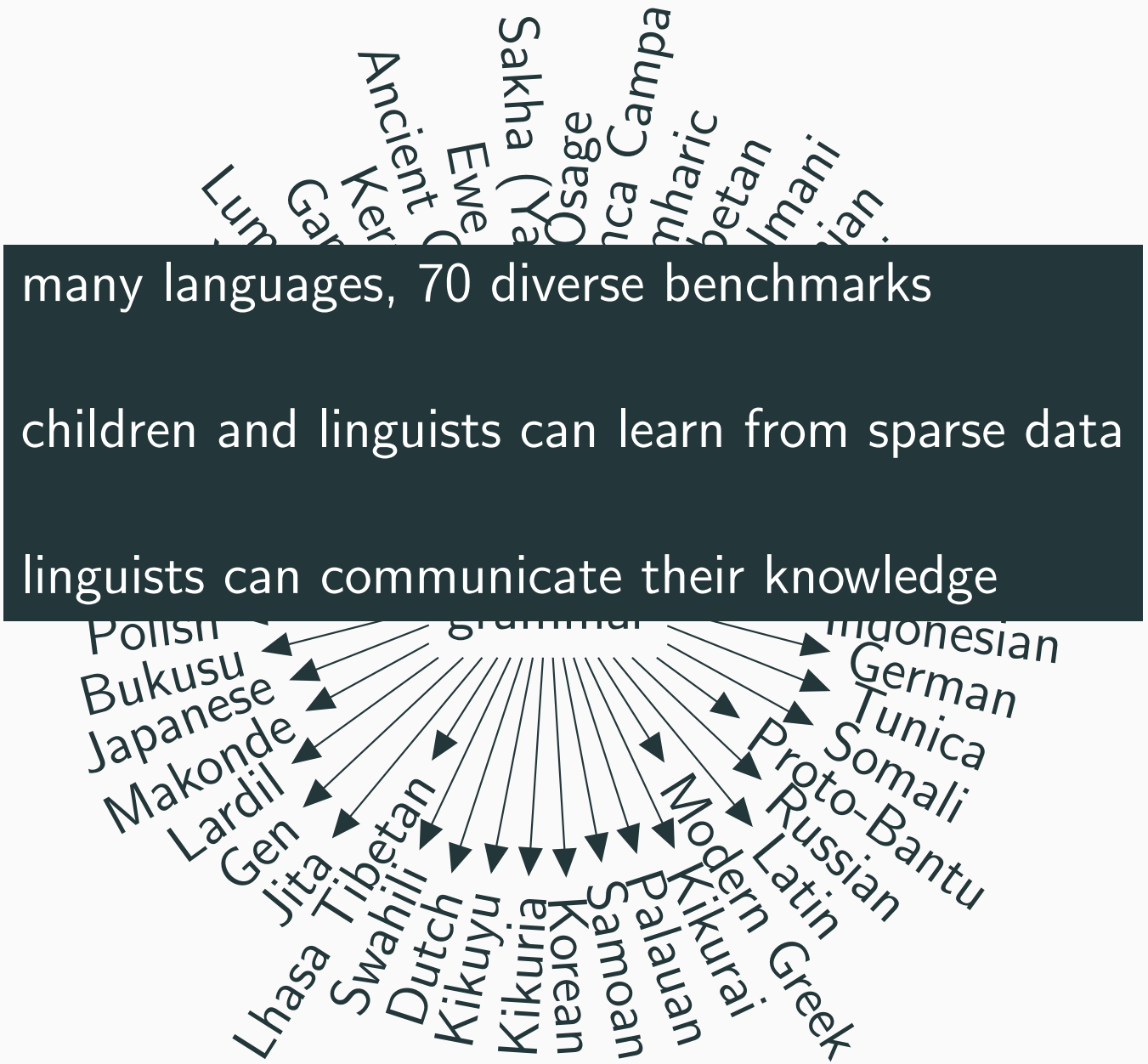
learning to learn

interpretable models

# Synthesizing human-understandable models of language



# Synthesizing human-understandable models of language



# Few-shot language learning experiment

Mandarin:

| | adjective | adverb |
|---|---|---|
| “slow” | man | manmandə |
| “fast” | kuai | kuaikuaidə |
| “small” | xiao | ??? |

# Few-shot language learning experiment

Mandarin:

| | adjective | adverb |
|---|---|---|
| “slow” | man | manmandə |
| “fast” | kuai | kuaikuaidə |
| “small” | xiao | xiaoxiaodə |

# Few-shot language learning experiment

Mandarin:

| | adjective | adverb |
|---|---|---|
| “slow” | man | manmandə |
| “fast” | kuai | kuaikuaidə |
| “small” | xiao | xiaoxiaodə |

stem+stem+də

# Few-shot language learning experiment

Serbo-Croatian:

| | masculine | feminine |
|---|---|---|
| “rich” | bogat | bogata |
| “mild” | blag | blaga |
| “green” | zelen | ??? |

# Few-shot language learning experiment

Serbo-Croatian:

| | masculine | feminine |
|---|---|---|
| “rich” | bogat | bogata |
| “mild” | blag | blaga |
| “green” | zelen | zelena |

# Few-shot language learning experiment

Serbo-Croatian:

| | masculine | feminine |
|---|---|---|
| “rich” | bogat | bogata |
| “mild” | blag | blaga |
| “green” | zelen | zelena |

*add “a” to stem to make feminine*

# Few-shot language learning experiment

Serbo-Croatian:

| | masculine | feminine |
|---|---|---|
| “rich” | bogat | bogata |
| “mild” | blag | blaga |
| “green” | zelen | zelena |
| “clear” | ??? | yasna |

*add “a” to stem to make feminine*

# Few-shot language learning experiment

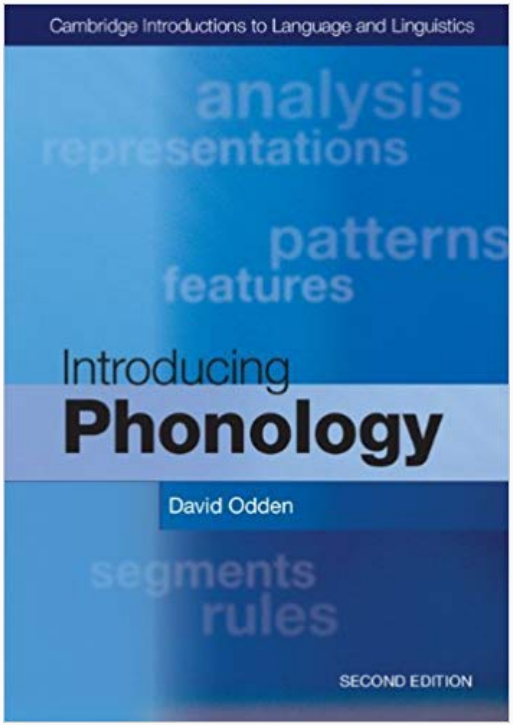
Serbo-Croatian:

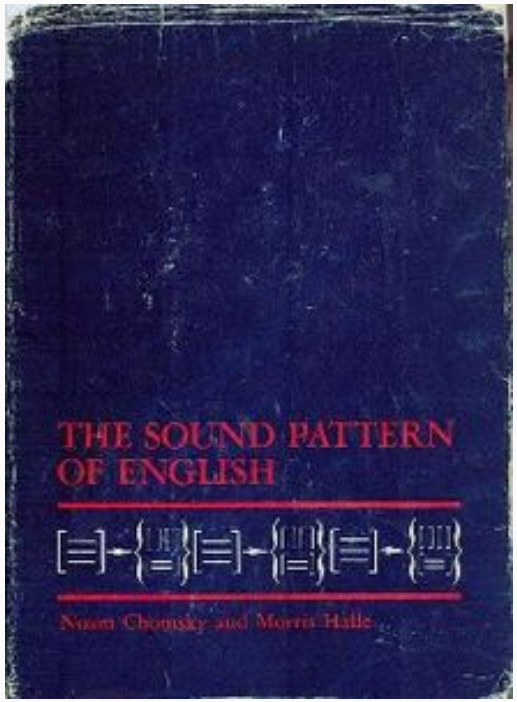
| | masculine | feminine | stem (unobserved) |
|---|---|---|---|
| "rich" | bogat | bogata | bogat |
| "mild" | blag | blaga | blag |
| "green" | zelen | zelena | zelen |
| "clear" | **yasan** | yasna | yasn |

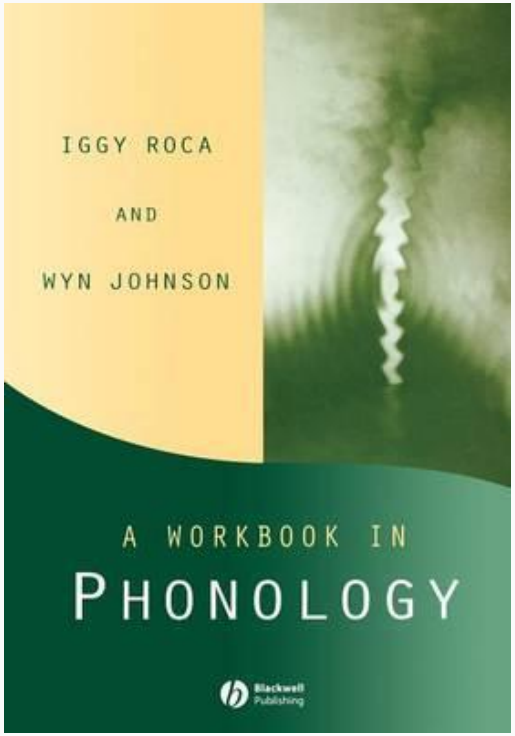
*add "a" to stem to make feminine*

*insert "a" between two word-final consonants*

$\varnothing \rightarrow$ a / C_C#


Cambridge Introductions to Language and Linguistics
analysis
representations
patterns
features
Introducing
Phonology
David Odden
segments
rules
SECOND EDITION


THE SOUND PATTERN
OF ENGLISH
Noam Chomsky and Morris Halle


IGGY ROCA
AND
WYN JOHNSON
A WORKBOOK IN
PHONOLOGY
Blackwell
Publishing

## 10 Sakha (Yakut)

Give a phonological analysis of the following case-marking paradigms of nouns in Sakha.

| *Noun* | *Plural* | *Associative* | |
|---|---|---|---|
| aya | ayalar | ayalɨɨn | 'father' |
| paarta | paartalar | paartalɨɨn | 'school desk' |
| tɨa | tɨalar | tɨalɨɨn | 'forest' |
| kinige | kinigeler | kinigeliin | 'book' |
| ǰie | ǰieler | ǰieliin | 'house' |
| iye | iyeler | iyeliin | 'mother' |
| kini | kiniler | kiniliin | '3rd person' |
| bie | bieler | bieliin | 'mare' |
| oyo | oyolor | oyoluun | 'child' |
| χopto | χoptolor | χoptoluun | 'gull' |
| börö | börölör | börölüün | 'wolf' |
| tɨal | tɨallar | tɨalɨɨn | 'wind' |
| ɨal | ɨallar | ɨalɨɨn | 'neighbor' |
| kuul | kuullar | kuulluun | 'sack' |
| at | attar | attɨɨn | 'horse' |
| balɨk | balɨktar | balɨktɨɨn | 'fish' |
| ɨskaap | ɨskaaptar | ɨskaaptɨɨn | 'cabinet' |
| oyus | oyustar | oyustuun | 'bull' |
| kus | kustar | kustuun | 'duck' |
| tünnük | tünnükter | tünnüktüün | 'window' |
| sep | septer | septiin | 'tool' |
| et | etter | ettiin | 'meat' |
| örüs | örüster | örüstüün | 'river' |
| tiis | tiister | tiistiin | 'tooth' |
| soroχ | soroχtor | soroχtuun | 'some person' |
| oχ | oχtor | oχtuun | 'arrow' |
| oloppos | oloppostor | oloppostuun | 'chair' |
| ötöχ | ötöχtör | ötöχtüün | 'abandoned farm' |
| ubay | ubaydar | ubaydɨɨn | 'elder brother' |
| saray | saraydar | saraydɨɨn | 'barn' |
| tɨy | tɨydar | tɨydɨɨn | 'foal' |
| atɨɨr | atɨɨrdar | atɨɨrdɨɨn | 'stallion' |
| oyuur | oyuurdar | oyuurduun | 'forest' |
| üčügey | üčügeyder | üčügeydiin | 'good person' |
| eǰiy | eǰiyder | eǰiydiin | 'elder sister' |
| tomtor | tomtordor | tomtorduun | 'knob' |
| moyotoy | moyotoydor | moyotoyduun | 'chipmunk' |
| kötör | kötördör | kötördüün | 'bird' |
| bölköy | bölköydör | bölköydüün | 'islet' |
| χatɨŋ | χatɨŋnar | χatɨŋnɨɨn | 'birch' |
| aan | aannar | aannɨɨn | 'door' |
| tɨŋ | tɨŋner | tɨŋnɨɨn | 'squirrel' |
| sordoŋ | sordoŋnor | sordoŋnuun | 'pike' |
| olom | olomnor | olomnuun | 'ford' |
| oron | oronnor | oronnuun | 'bed' |
| bödöŋ | bödöŋnör | bödöŋnüün | 'strong one' |

| *Noun* | *Partitive* | *Comparative* | *Ablative* | |
|---|---|---|---|---|
| aya | ayata | ayataayar | ayattan | 'father' |
| paarta | paartata | paartataayar | paartattan | 'school desk' |
| tɨa | tɨata | tɨataayar | tɨattan | 'forest' |
| kinige | kinigete | kinigeteeyer | kinigetten | 'book' |
| ǰie | ǰiete | ǰieteeyer | ǰietten | 'house' |
| iye | iyete | iyeteeyer | iyetten | 'mother' |
| kini | kinite | kiniteeyer | kinitten | '3rd person' |
| bie | biete | bieteeyer | bietten | 'mare' |
| oyo | oyoto | oyotooyor | oyotton | 'child' |
| χopto | χoptoto | χoptotooyor | χoptotton | 'gull' |
| börö | börötö | börötööyör | böröttön | 'wolf' |
| tɨal | tɨalla | tɨallaayar | tɨaltan | 'wind' |
| ɨal | ɨalla | ɨallaayar | ɨaltan | 'neighbor' |
| kuul | kuulla | kuullaayar | kuultan | 'sack' |
| moχsoyol | moχsoyollo | moχsoyollooyor | moχsoyolton | 'falcon' |
| at | atta | attaayar | attan | 'horse' |
| balɨk | balɨkta | balɨktaayar | balɨktan | 'fish' |
| ɨskaap | ɨskaapta | ɨskaaptaayar | ɨskaaptan | 'cabinet' |
| oyus | oyusta | oyustaayar | oyustan | 'bull' |
| kus | kusta | kustaayar | kustan | 'duck' |
| tünnük | tünnükte | tünnükteeyer | tünnükten | 'window' |

# Turkic Sakha (Yakut)

observed data

| | | | |
|---|---|---|---|
| CABINET (SINGULAR): | ɨskaap | CABINET (PLURAL): | ɨskaaptar |
| BED (SINGULAR): | oron | BED (PLURAL): | oronnor |
| MARE (SINGULAR): | bie | MARE (PLURAL): | bieler |

*138 total examples*

# Turkic Sakha (Yakut)

grammar (unobserved)

SINGULAR→stem
PLURAL→stem+lar

observed data

| | | | |
|---|---|---|---|
| CABINET (SINGULAR): | ɨskaap | CABINET (PLURAL): | ɨskaaptar |
| BED (SINGULAR): | oron | BED (PLURAL): | oronnor |
| MARE (SINGULAR): | bie | MARE (PLURAL): | bieler |

*138 total examples*

# Turkic Sakha (Yakut)

## grammar (unobserved)

SINGULAR→stem
PLURAL→stem+lar

$r_1$: l → d / [ -lateral -tense ]_
**"l" becomes "d" next to "r", "t", but not "l"**

$r_2$: C → [ -voice ] / [ -voice ]_
**do not voice next to voiceless**

$r_3$: V → [ +rounded ] / [ +rounded ] [ -low ]$_0$_
$r_4$: [ +continuant -high ] → [ -rounded ] / u C$_0$_
**"harmonize" round vowels like "u", "o"**

$r_5$: V → [ -back -low ] / [ -back +vowel ] [ ]$_0$_
**"harmonize" vowels to be not at back of mouth**

$r_6$: [ -sonorant +voice ] → [ +nasal ] / [ +nasal ]_
**"nasalize" consonant next to a nasal, like "m"**

## observed data

| | | | |
|---|---|---|---|
| CABINET (SINGULAR): | ɨskaap | CABINET (PLURAL): | ɨskaaptar |
| BED (SINGULAR): | oron | BED (PLURAL): | oronnor |
| MARE (SINGULAR): | bie | MARE (PLURAL): | bieler |

*138 total examples*

# Turkic Sakha (Yakut)

## grammar (unobserved)

SINGULAR→stem
PLURAL→stem+lar

$r_1$: l → d / [ -lateral -tense ]_
**"l" becomes "d" next to "r", "t", but not "l"**

$r_2$: C → [ -voice ] / [ -voice ]_
**do not voice next to voiceless**

$r_3$: V → [ +rounded ] / [ +rounded ] [ -low ]$_0$_
$r_4$: [ +continuant -high ] → [ -rounded ] / u C$_0$_
**"harmonize" round vowels like "u", "o"**

$r_5$: V → [ -back -low ] / [ -back +vowel ] [ ]$_0$_
**"harmonize" vowels to be not at back of mouth**

$r_6$: [ -sonorant +voice ] → [ +nasal ] / [ +nasal ]_
**"nasalize" consonant next to a nasal, like "m"**

## stems (unobserved)

CABINET : ɨskaap
BED : oron
MARE : bie

## observed data

| | | | |
|---|---|---|---|
| CABINET (SINGULAR): | ɨskaap | CABINET (PLURAL): | ɨskaaptar |
| BED (SINGULAR): | oron | BED (PLURAL): | oronnor |
| MARE (SINGULAR): | bie | MARE (PLURAL): | bieler |

*138 total examples*

# Turkic Sakha (Yakut)

## grammar (unobserved)

SINGULAR→stem
PLURAL→stem+lar

$r_1$: l → d / [ -lateral -tense ]_
**"l" becomes "d" next to "r", "t", but not "l"**

$r_2$: C → [ -voice ] / [ -voice ]_
**do not voice next to voiceless**

$r_3$: V → [ +rounded ] / [ +rounded ] [ -low ]$_0$_
$r_4$: [ +continuant -high ] → [ -rounded ] / u C$_0$_
**"harmonize" round vowels like "u", "o"**

$r_5$: V → [ -back -low ] / [ -back +vowel ] [ ]$_0$_
**"harmonize" vowels to be not at back of mouth**

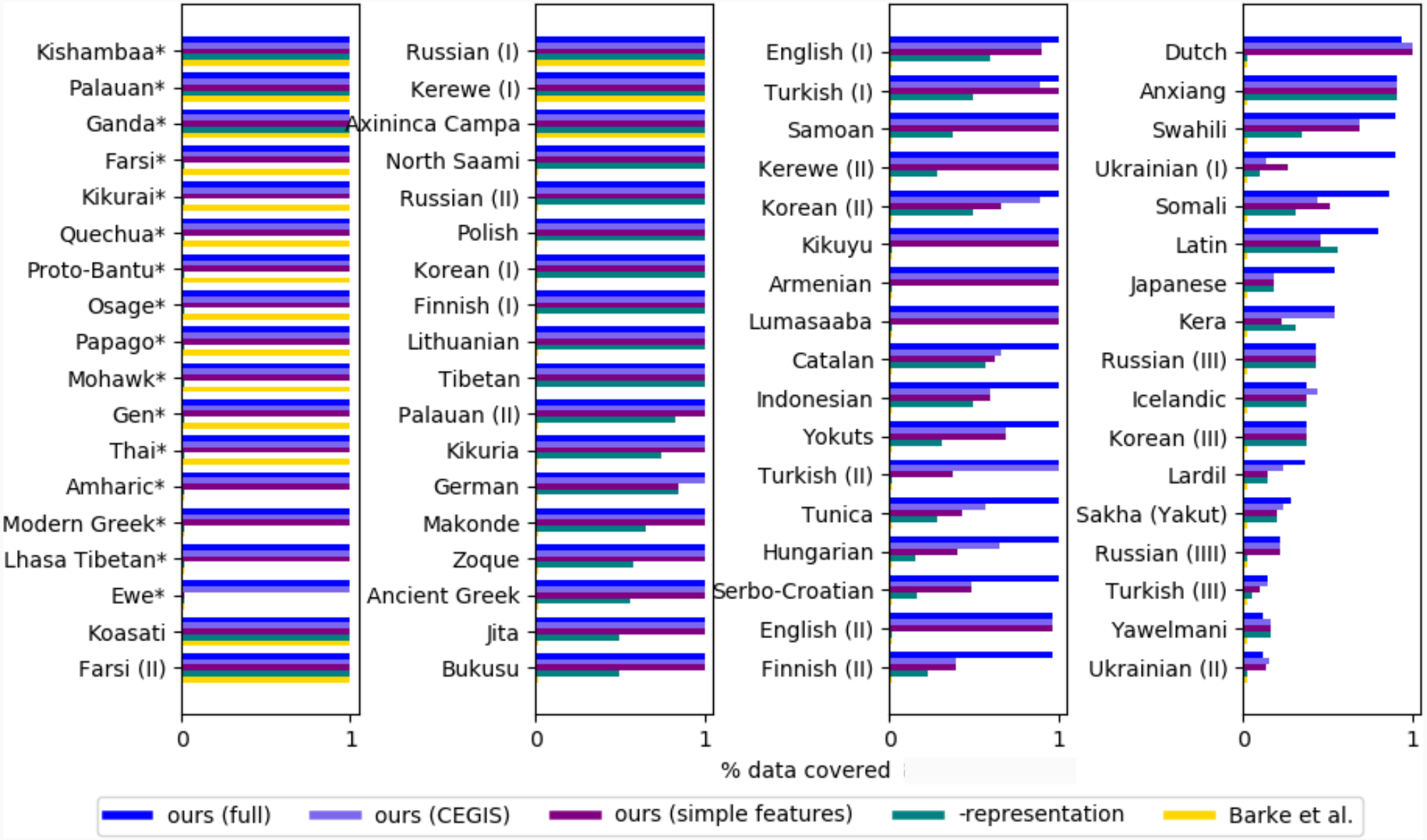
$r_6$: [ -sonorant +voice ] → [ +nasal ] / [ +nasal ]_
**"nasalize" consonant next to a nasal, like "m"**

## stems (unobserved)

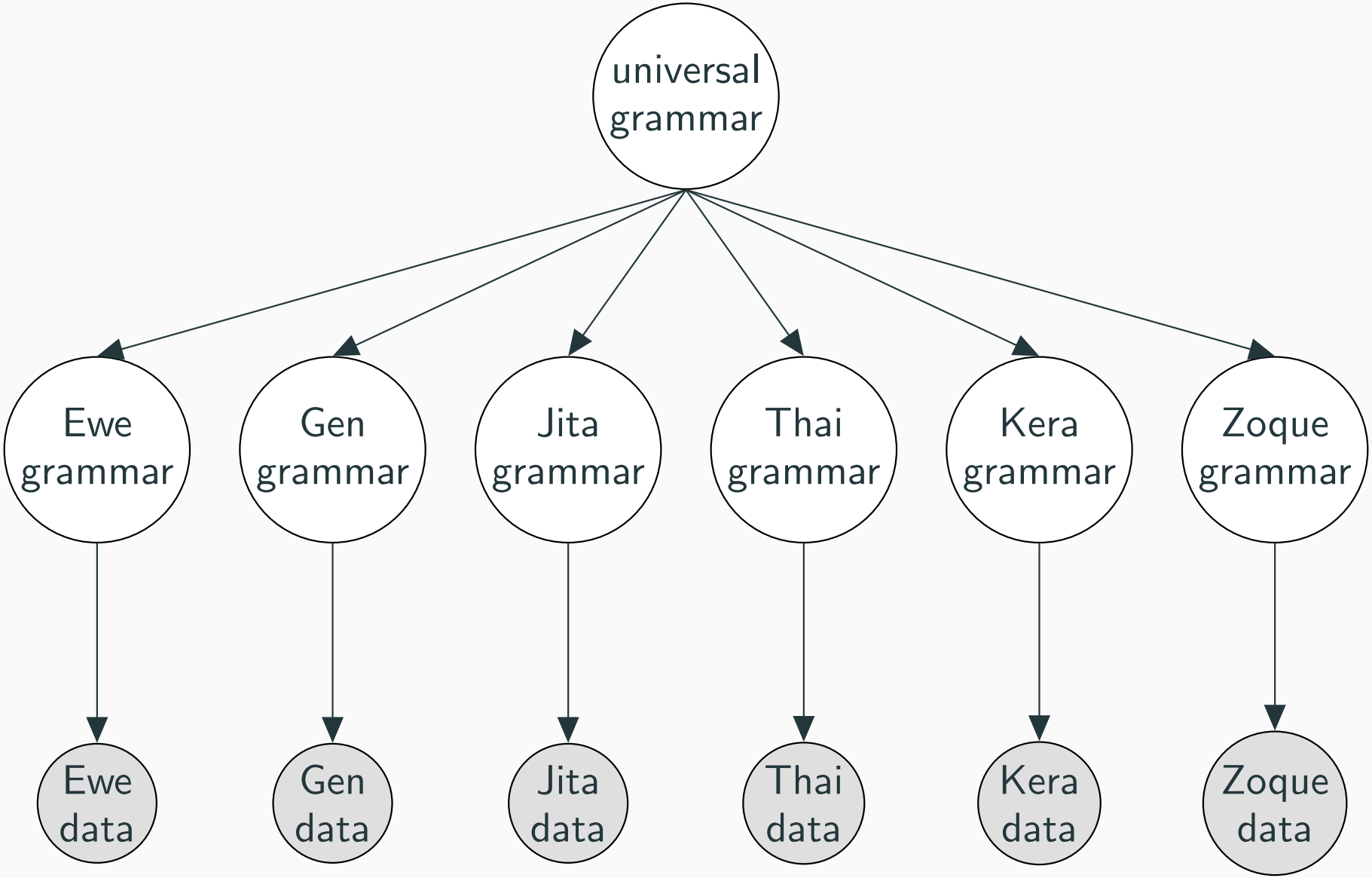
CABINET : ɨskaap
BED : oron
MARE : bie
RIVER : örus

## observed data

CABINETS→ɨskaap+lar→ɨskaaplar$\xrightarrow{r_1}$ɨskaapdar$\xrightarrow{r_2}$ɨskaaptar
BEDS→oron+lar→oronlar$\xrightarrow{r_1}$orondar$\xrightarrow{r_3}$orondor$\xrightarrow{r_6}$oronnor
MARES→bie+lar→bielar$\xrightarrow{r_5}$bieler
RIVER (ASSOC)→örus+lɨɨn→öruslɨɨn$\xrightarrow{r_1}$örusdɨɨn$\xrightarrow{r_2}$
örustɨɨn$\xrightarrow{r_3}$örustuun$\xrightarrow{r_5}$[örüstüün]


Kishambaa*
Palauan*
Ganda*
Farsi*
Kikurai*
Quechua*
Proto-Bantu*
Osage*
Papago*
Mohawk*
Gen*
Thai*
Amharic*
Modern Greek*
Lhasa Tibetan*
Ewe*
Koasati
Farsi (II)
Russian (I)
Kerewe (I)
Axininca Campa
North Saami
Russian (II)
Polish
Korean (I)
Finnish (I)
Lithuanian
Tibetan
Palauan (II)
Kikuria
German
Makonde
Zoque
Ancient Greek
Jita
Bukusu
English (I)
Turkish (I)
Samoan
Kerewe (II)
Korean (II)
Kikuyu
Armenian
Lumasaaba
Catalan
Indonesian
Yokuts
Turkish (II)
Tunica
Hungarian
Serbo-Croatian
English (II)
Finnish (II)
Dutch
Anxiang
Swahili
Ukrainian (I)
Somali
Latin
Japanese
Kera
Russian (III)
Icelandic
Korean (III)
Lardil
Sakha (Yakut)
Russian (IIII)
Turkish (III)
Yawelmani
Ukrainian (II)
0
1
% data covered
ours (full)
ours (CEGIS)
ours (simple features)
-representation
Barke et al.

# Distilling higher-level knowledge



# Lessons

Higher-level knowledge matters ("universal grammar"). Get the basic computational substrate correct

But *some* of this higher-level knowledge can be learned. You don't need millions of examples to learn it. But it's not a one-shot learning problem either

Program Induction and perception

learning to learn

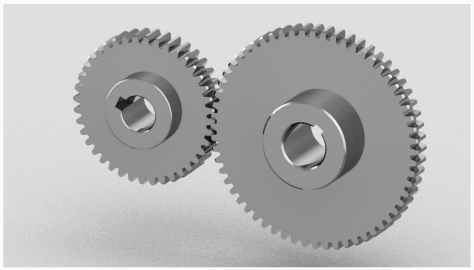
interpretable models

the future

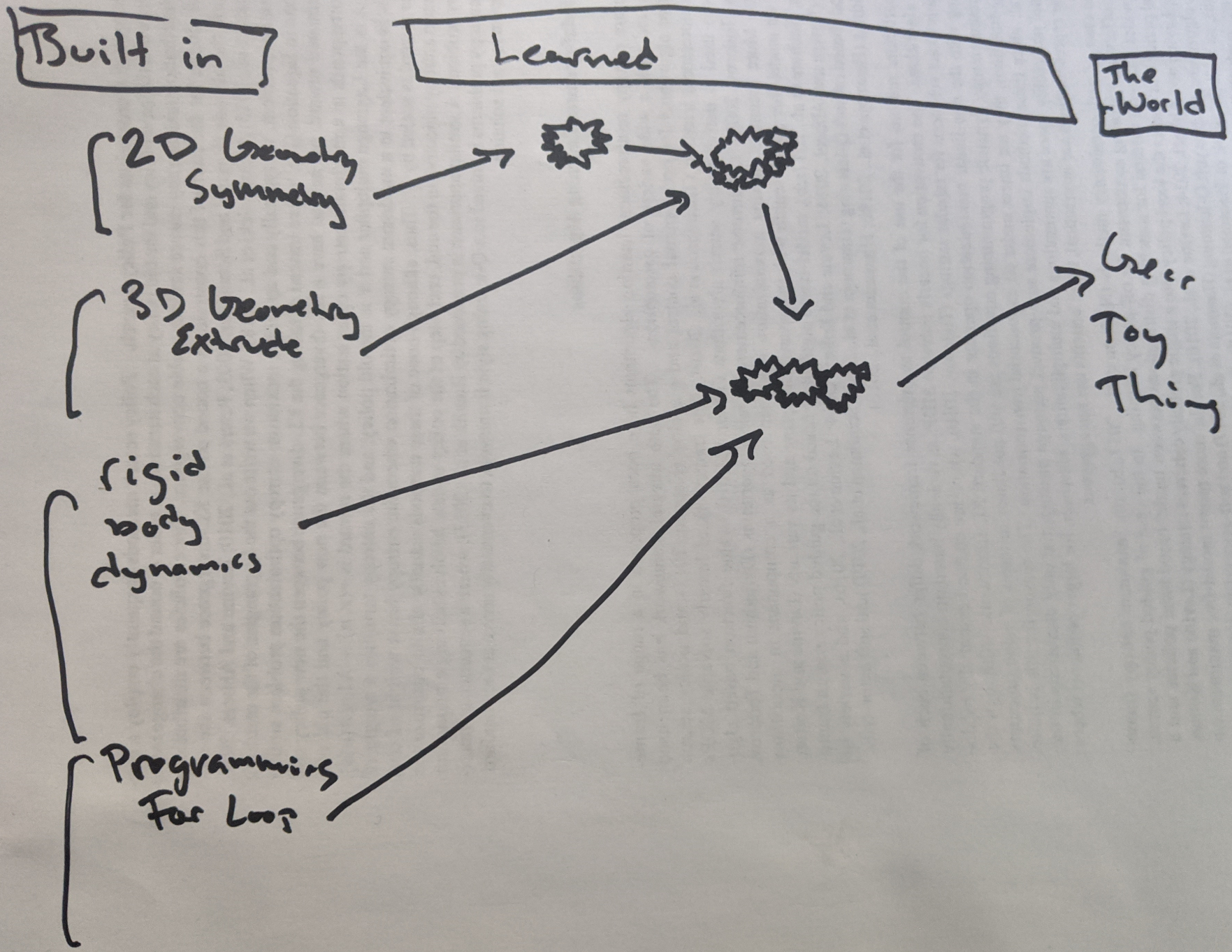
# Models of the physical world

hinge

gear



doorknob


Built in
Learned
The World
2D Geometry Symmetry
3D Geometry Extrude
rigid body dynamics
Programming For Loop
Creature Toy Thing

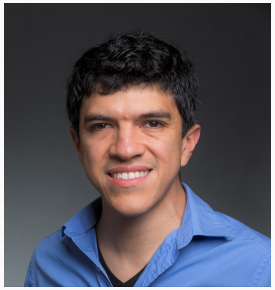
# Collaborators

Tim O'Donnell

Josh Tenenbaum

Adam Albright

Armando Solar-Lezama



Max Nye

Cathy Wong

Yewen Pu

Dan Ritchie

Mathias Sable-Meyer

Lucas Morales

**thank you**