

Program Induction: Bridging AI and program synthesis

Kevin Ellis

2020

MIT

What computational problems are solved by intelligence?

an endless range of problems

language



using new devices



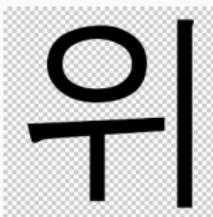
engineering



science



writing new characters



design



coding

```
(MEMBER  
(LAMBDA (X L)  
(COND ((NULL L) NIL)  
      ((EQ X (FIRST L)) T)  
      (T (MEMBER X (REST L)))))))
```

Allen, Anatomy of Lisp, 1975



play



What computational frameworks can contribute to this picture?

Three AI traditions

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Three AI traditions

Symbolic



In[34]:= **Solve**[{(h w - h w^2) == Z}, h]

Out[34]= {}



In[33]:= **Solve**[(h w - h w^2) == Z, h]

Input interpretation:

solve $h w - h w^2 = Z$ for h

Result:

$$h = \frac{Z}{w - w^2} \text{ and } w^2 \neq w$$

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Three AI traditions

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In[34]:= Solve[{(hw - hw^2) == z}, h]
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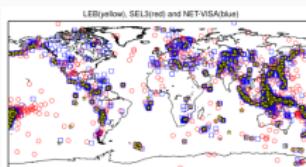
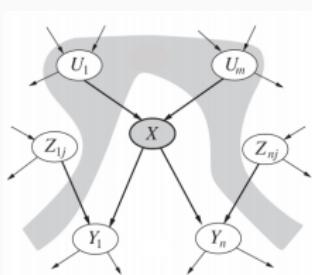
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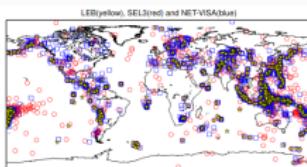
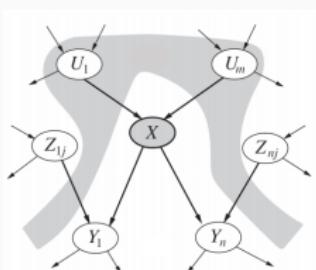
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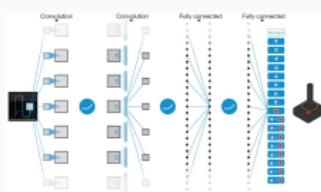
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Probabilistic



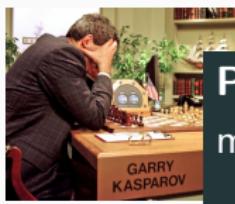
Neural



What computational frameworks can contribute to this picture?

Three AI traditions

Symbolic



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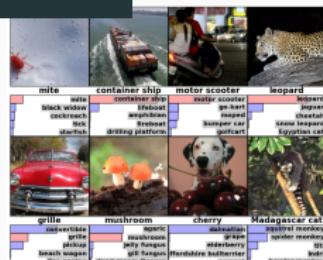
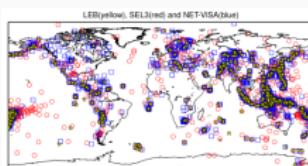
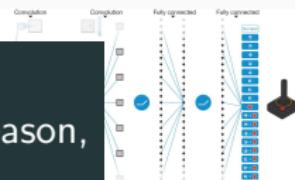
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Probabilistic



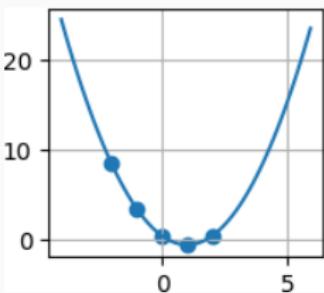
Program induction
machines that learn, perceive, and reason,
by writing their own code

Neural



Why program induction?

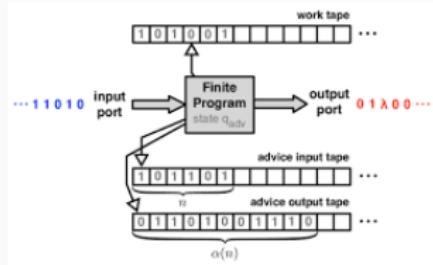
strong generalization
+ data efficiency



interpretability

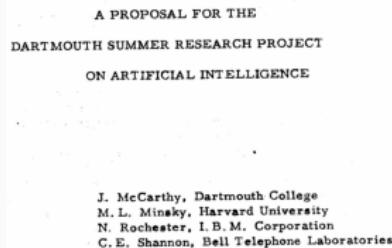


universal expressivity



Why didn't this old idea work?

Program induction goes back to 1956 Dartmouth Workshop that founded the field of AI



Why will it work this time?

better toolkits:

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better toolkits:

- **probabilistic** methods for uncertainty and learning-to-learn

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- **neural** methods for guiding combinatorial search

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better toolkits:

- **probabilistic** methods for uncertainty and learning-to-learn
- **neural** methods for guiding combinatorial search
- **symbolic** methods, from the **programming languages** community
 - maturing **program synthesis** techniques
 - type systems, program analysis, constraint solving, ...

Why will it work this time?

better toolkits:

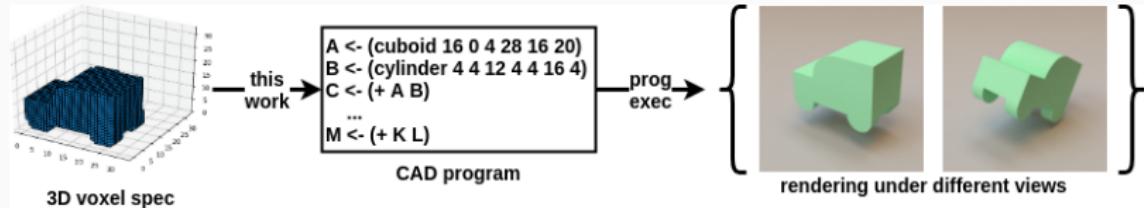
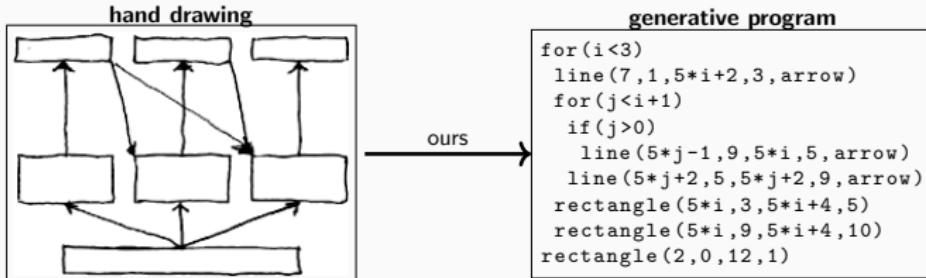
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- **neural** methods for guiding combinatorial search
- **symbolic** methods, from the **programming languages** community
 - maturing **program synthesis** techniques
 - type systems, program analysis, constraint solving, ...

better problems:

- inverse CAD [???
- probabilistic program synthesis [???
- synthesizing human-understandable models [Evans et al. 2019, ???]
- natural language→code [Liang et al. 2011; Zettlemoyer et al. 2007]
- programming by examples [Gulyani 2011]

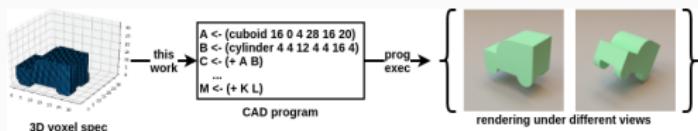
Perception, Synthesizing models, Learning-to-Learn

Theme #1: high-level scene understanding, pixels→programs

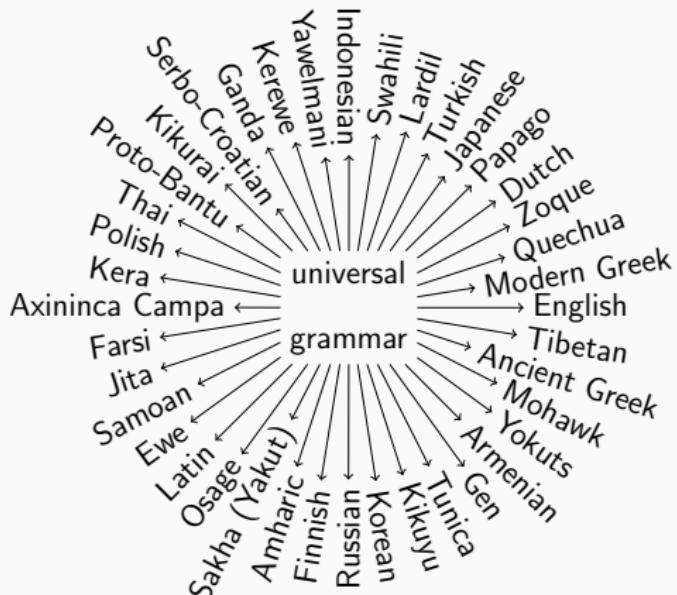


Perception, Synthesizing models, Learning-to-Learn

Theme #1: high-level scene understanding, pixels→programs

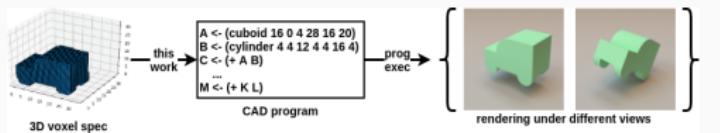


Theme #2: Synthesizing human-understandable models



Perception, Synthesizing models, Learning-to-Learn

Theme #1: high-level scene understanding, pixels→programs



Theme #2: Synthesizing interpretable models

Theme #3: Learning to synthesize programs

List Processing	Text Editing	Regexes	LOGO Graphics
Sum List [1 2 3] → 6 [4 6 8 1] → 17	Abbreviate Allen Newell → A.N. Herb Simon → H.S.	Phone Numbers (555) 867-5309 (650) 555-2368	⌚ ⚒ ⚓ ⚔
Double [1 2 3 4] → [2 4 6 8] [6 5 1] → [12 10 2]	Drop Last Characters jabberwocky → jabberw copycat → cop	Currency \$100.25 \$4.50	฿ ₧ ₩ ₪
Check Evens [0 2 3] → [T T F] [2 4 9 6] → [T T F T]	Extract see spot(run) → run a (bee) see → bee	Dates Y1775/0704 Y2000/0101	⌚ ⚒ ⚓ ⚔

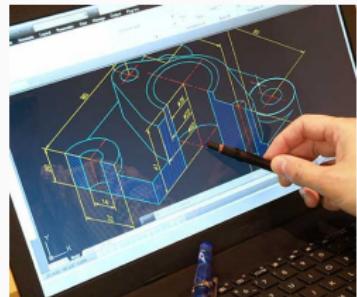
Block Towers	Symbolic Regression	Recursive Programming	Physics
	 $\frac{(-2.4x - 0.9)}{(x - 4.4)(x - 0.9)}$	 $0.3x^3 + 1.1x^2 - 2.0x + 0.6$	<i>Filter</i> [■■■■■] → [■] [■■■■■■] → [■■■■] [■■■■■■■] → [■■■]
	 $0.5x^4 + 2.5x^3 + 0.4x^2 - 2.2x + 2.4$	<i>Length</i> [■■■■■] → 4 [■■■■■■] → 6 [■■■■■■■] → 3	<i>Index List</i> 0, [■■■■■] → ■ 1, [■■■■■] → ■■ 1, [■■■■■■] → ■■■
	 $\frac{4.9}{x}$		$\vec{d} = \frac{1}{m} \sum_i \vec{F}_i$ $\vec{F} = \frac{q_1 q_2}{ \vec{r}_1 - \vec{r}_2 } \vec{r}_1 \vec{r}_2^{-1}$ $R_{total} = \left(\sum_i \frac{1}{R_i} \right)^{-1}$

High-level, abstract visual abilities

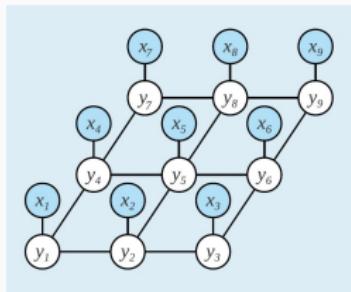
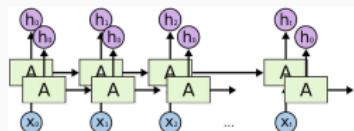
...in art



...in engineering



...in AI



High-level, abstract visual abilities

...in art



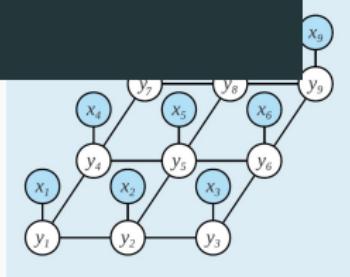
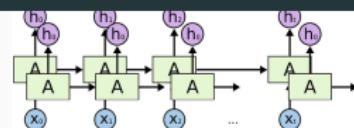
why?

impute missing objects, extrapolate percepts,
learn visual concepts ('arch', 'spiral', 'Ising model'),
assist graphic design, assist 3D modeling

how?

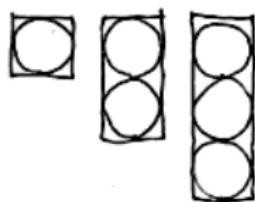
Bayesian inference of graphics program conditioned on image,
+program synthesis
+learning

...in AI



Learning to infer graphics programs from hand-drawn images

model infers program from drawing



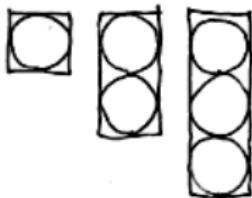
```
for (0 <= i < 3)
    rectangle(3*i, -2*i+4,
              3*i+2, 6)
    for (0 <= j < i + 1)
        circle(3*i+1, -2*j+5)
```

Ellis, Ritchie, Solar-Lezama, Tenenbaum. NeurIPS 2018.

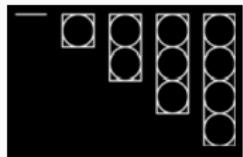
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zero-shot generalization / extrapolation



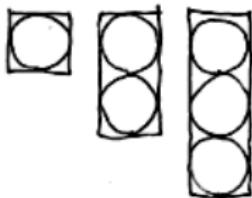
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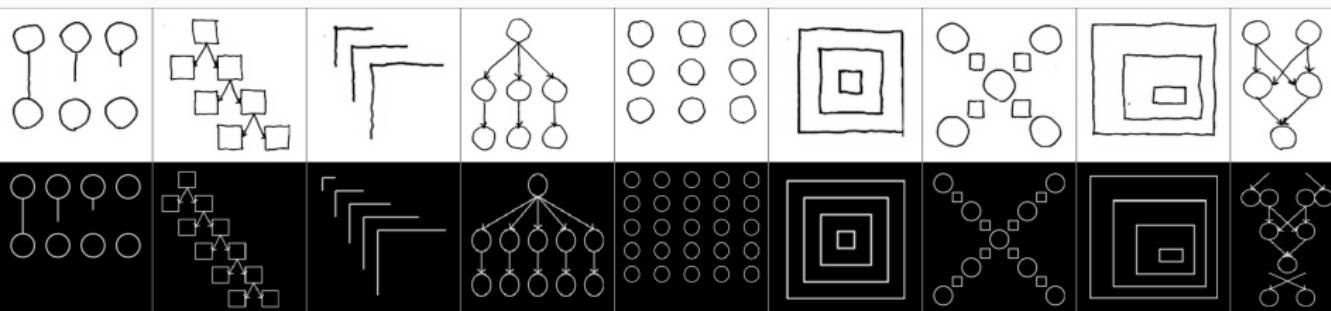
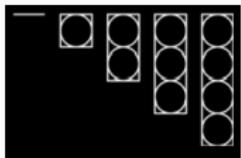
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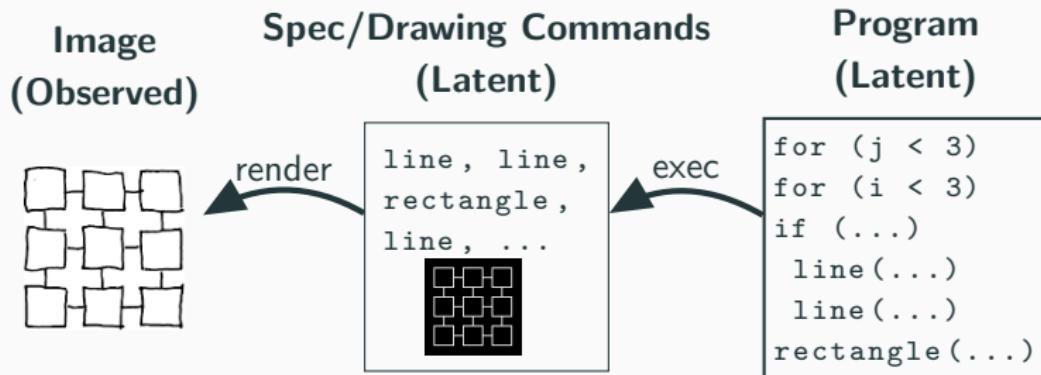
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How to infer graphics programs from hand-drawn images



How to infer graphics programs from hand-drawn images

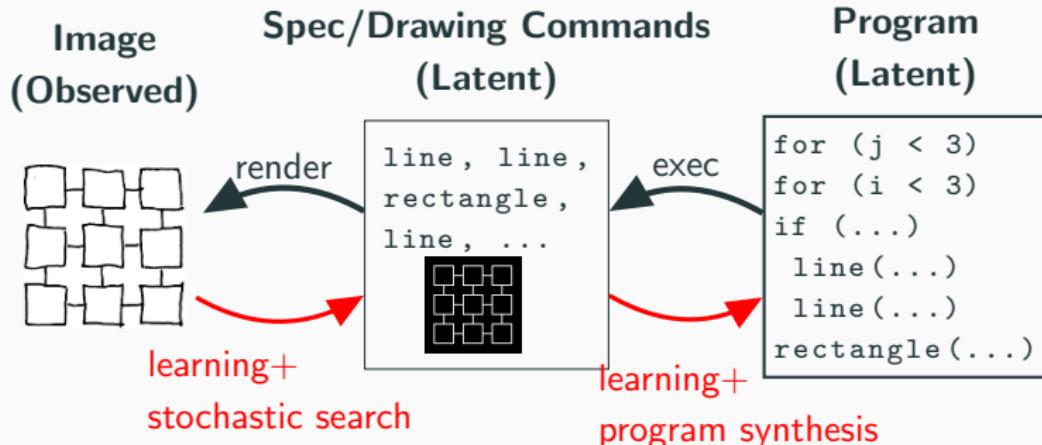
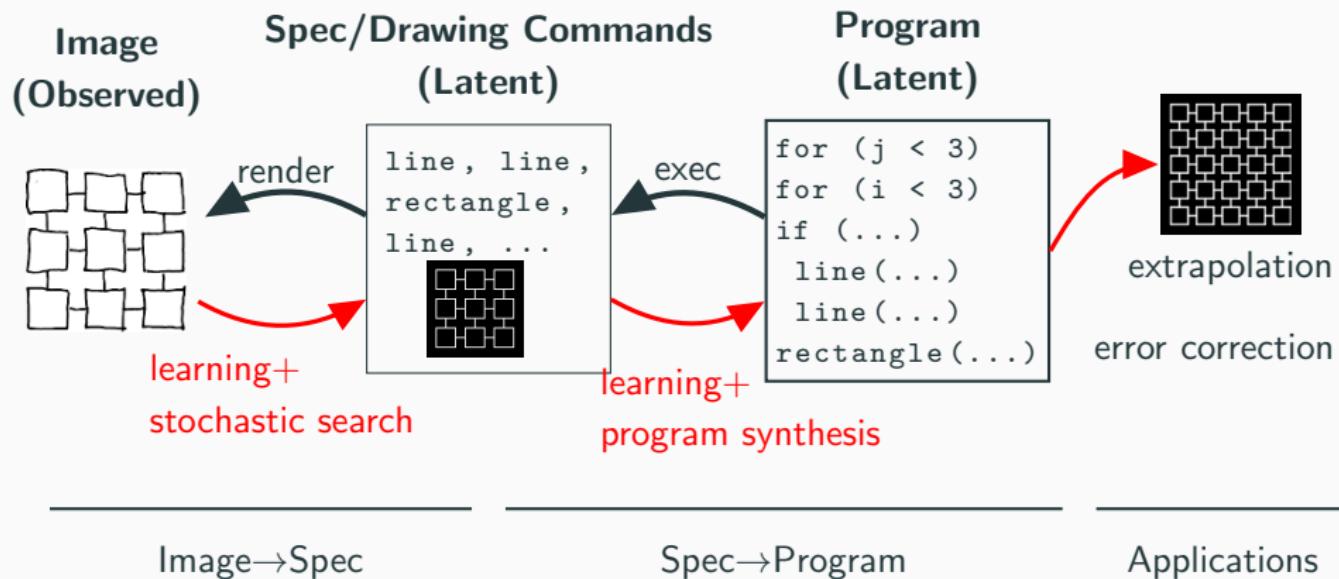


Image → Spec

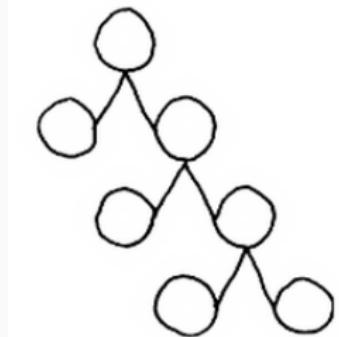
Spec → Program

How to infer graphics programs from hand-drawn images

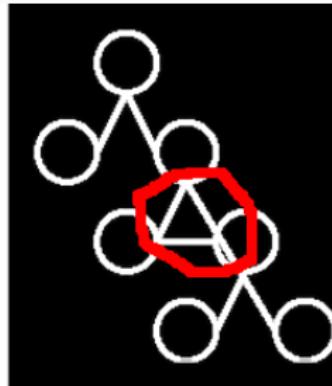


Top-down influences on perception

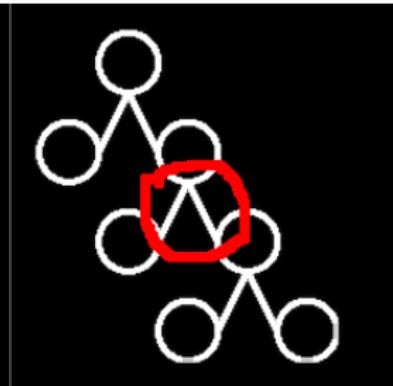
drawing



bottom-up neural net

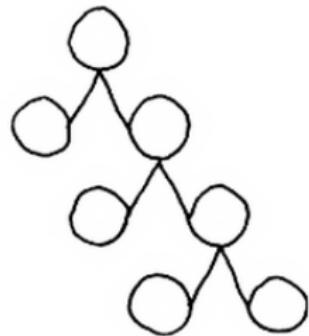


w/ top-down program bias

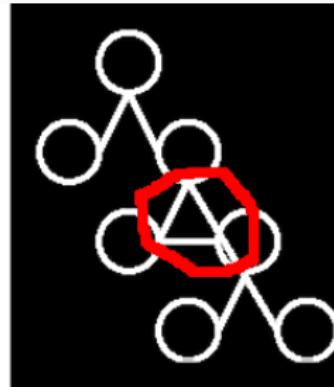


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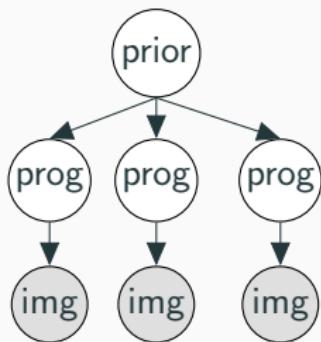
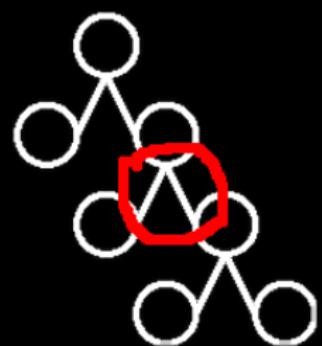
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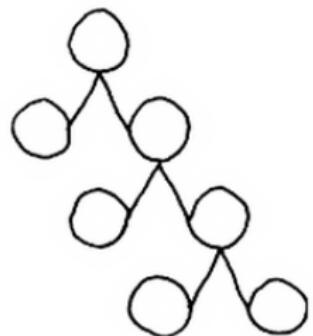
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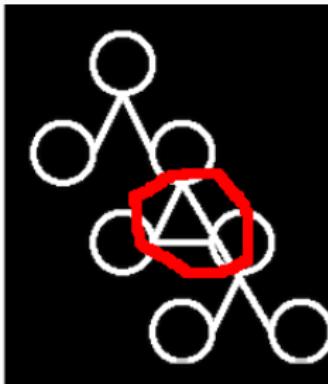
predicted program =
 $\arg \max_{\text{progs}} \mathbb{P} [\text{img} | \text{prog}] \mathbb{P} [\text{prog} | \text{prior}]$

Top-down influences on perception

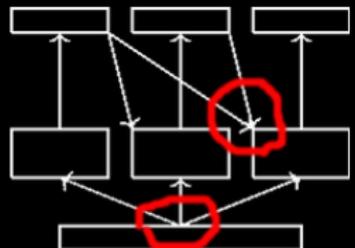
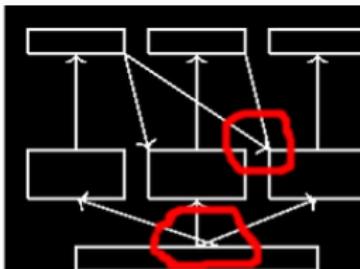
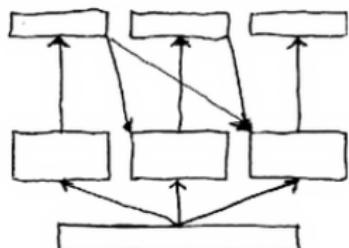
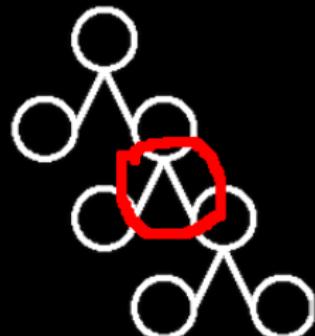
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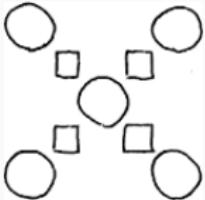
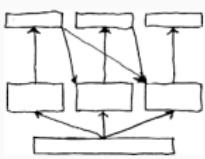
bottom-up neural net



w/ top-down program bias



Example programs

Drawing	Program
	<pre>for(i<3) line(i,-1*i+6, 2*i+2,-1*i+6) line(i,-2*i+4,i,-1*i+6)</pre>
	<pre>reflect(y=8) for(i<3) if(i>0) rectangle(3*i-1,2,3*i,3) circle(3*i+1,3*i+1)</pre>
	<pre>for(i<3) line(7,1,5*i+2,3,arrow) for(j<i+1) if(j>0) line(5*j-1,9,5*i,5,arrow) line(5*j+2,5,5*j+2,9,arrow) rectangle(5*i,3,5*i+4,5) rectangle(5*i,9,5*i+4,10) rectangle(2,0,12,1)</pre>

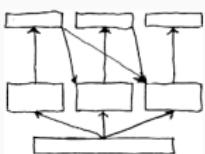
Example programs

Drawing	Program
	<pre>for(i<3) line(i,-1*i+6, 2*i+2,-1*i+6) line(i,-2*i+4,i,-1*i+6)</pre>

Learning played a role...

but much of this system is specific to 2-D graphics

Goal: a general algorithm for learning to synthesize programs

	<pre>for(i<3) line(7,1,5*i+2,3,arrow) for(j<i+1) if(j>0) line(5*j-1,9,5*i,5,arrow) line(5*j+2,5,5*j+2,9,arrow) rectangle(5*i,3,5*i+4,5) rectangle(5*i,9,5*i+4,10) rectangle(2,0,12,1)</pre>
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Synthesizing human-understandable models of language



Synthesizing human-understandable models of language

many languages, 70 diverse benchmarks

children and linguists can learn from sparse data

linguists can communicate their knowledge



Few-shot language learning experiment

Mandarin:

	adjective	adverb
“slow”	man	manmandə
“fast”	kuai	kuaikuaidə
“small”	xiao	???

Few-shot language learning experiment

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Few-shot language learning experiment

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stem+stem+də

Few-shot language learning experiment

Serbo-Croatian:

	masculine	feminine
“rich”	bogat	bogata
“mild”	blag	blaga
“green”	zelen	???

Few-shot language learning experiment

Serbo-Croatian:

	masculine	feminine
“rich”	bogat	bogata
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Few-shot language learning experiment

Serbo-Croatian:

	mASCULINE	fEMININE
“rich”	bogat	bogata
“mild”	blag	blaga
“green”	zelen	zelena

add “a” to stem to make feminine

Few-shot language learning experiment

Serbo-Croatian:

	masculine	feminine
“rich”	bogat	bogata
“mild”	blag	blaga
“green”	zelen	zelena
“clear”	???	yasna

add “a” to stem to make feminine

Few-shot language learning experiment

Serbo-Croatian:

	masculine	feminine	stem (unobserved)
“rich”	bogat	bogata	bogat
“mild”	blag	blaga	blag
“green”	zelen	zelena	zelen
“clear”	yasan	yasna	yasn

add “a” to stem to make feminine

insert “a” between two word-final consonants

$\emptyset \rightarrow a / C_C\#$

Diverse Linguistic Phenomena

Serbo-Croatian

grammar

MASC → stem, FEM → stem+a,
NEUT → stem+o, PL → stem+i

$r_1: [+vowel] \rightarrow [+stress] / [+stress] [-vowel]_0$
 $r_2: [+vowel] \rightarrow [-stress] / [-vowel]_0 [+stress]$
rules 1 & 2 shift stress to final vowel

$r_3: \emptyset \rightarrow a / [-vowel]_- [-vowel] \#$
rule 3 inserts “a” between word-final consonants

$r_4: [-sonorant] \rightarrow [-voice] / [-voice]$
rule 4 changes voicing of sound

$r_5: l \rightarrow o / -\#$
rule 5 changes word-final “l” to “o”

stems
(unobserved)

observed data
& unobserved derivation

- ⟨/sítn/, TINY⟩
- ⟨/blízk/, CLOSE⟩
- ⟨/túp/, BLUNT⟩
- ⟨/míl/, DEAR⟩

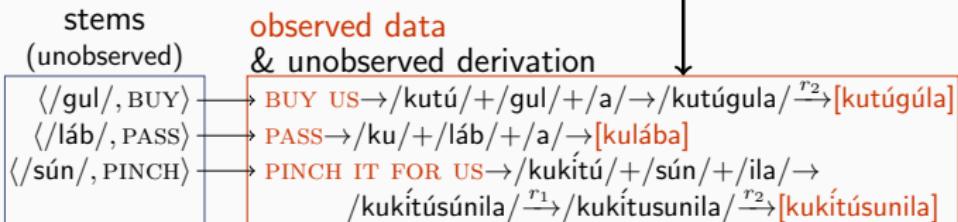
- TINY(MASC) → /sítn/ $\xrightarrow{r_3}$ [sítan]
- CLOSE(PL) → /blízk/+ /i/ → /blízki/ $\xrightarrow{r_1}$ /blízki/ $\xrightarrow{r_2}$ /blizkí/ $\xrightarrow{r_4}$ [bliskí]
- BLUNT(FEM) → /túp/+ /a/ → /túpa/ $\xrightarrow{r_1}$ /túpa/ $\xrightarrow{r_2}$ [tupá]
- DEAR(MASC) → /míl/ $\xrightarrow{r_5}$ [mío]

Languages with tones

Kerewe: Tanzanian Bantu

grammar

INF → /ku/+stem+/a/, V e.o. → /ku/+stem+/ana/,
V for → /ku/+stem+/ila/, V for e.o. → /ku/+stem+/ilana/
V us → /kutú/+stem+/a/, V it → /kuki/+stem+/a/,
V for us → /kutú/+stem+/ila/, V it for us → /kukítú/+stem+/ila/
 $r_1: [] \rightarrow [-\text{hiTone}] / [+ \text{hiTone}] [-\text{vowel}]_0$ _____
neutralize tone when right of tone
 $r_2: [+\text{vowel}] \rightarrow [+ \text{hiTone}] / [+ \text{hiTone}] [-\text{vowel}]_0 [-\text{vowel}]$ _____
spread tone rightward except to last vowel



Vowel “harmony”

Turkic Sakha (Yakut)

grammar

NOUN→stem, PL→stem+/lar/,
ASSOCIATIVE→stem+/iin/

$r_1: l \rightarrow d / [-\text{lateral} \ -\text{tense}]$
“l” becomes “d” next to “r”, “t”, but not “l”

$r_2: C \rightarrow [-\text{voice}] / [-\text{voice}]$
do not voice next to voiceless

$r_3: V \rightarrow [+ \text{rounded}] / [+ \text{rounded}] [- \text{low}]_0$
 $r_4: [+ \text{continuant} \ -\text{high}] \rightarrow [- \text{rounded}] / u C_0$
“harmonize” round vowels like “u”, “o”

$r_5: V \rightarrow [-\text{back} \ -\text{low}] / [-\text{back} \ +\text{vowel}] []_0$
“harmonize” vowels to be not at back of mouth

$r_6: [-\text{sonorant} \ +\text{voice}] \rightarrow [+ \text{nasal}] / [+ \text{nasal}]$
“nasalize” consonant next to a nasal, like “n”, “m”

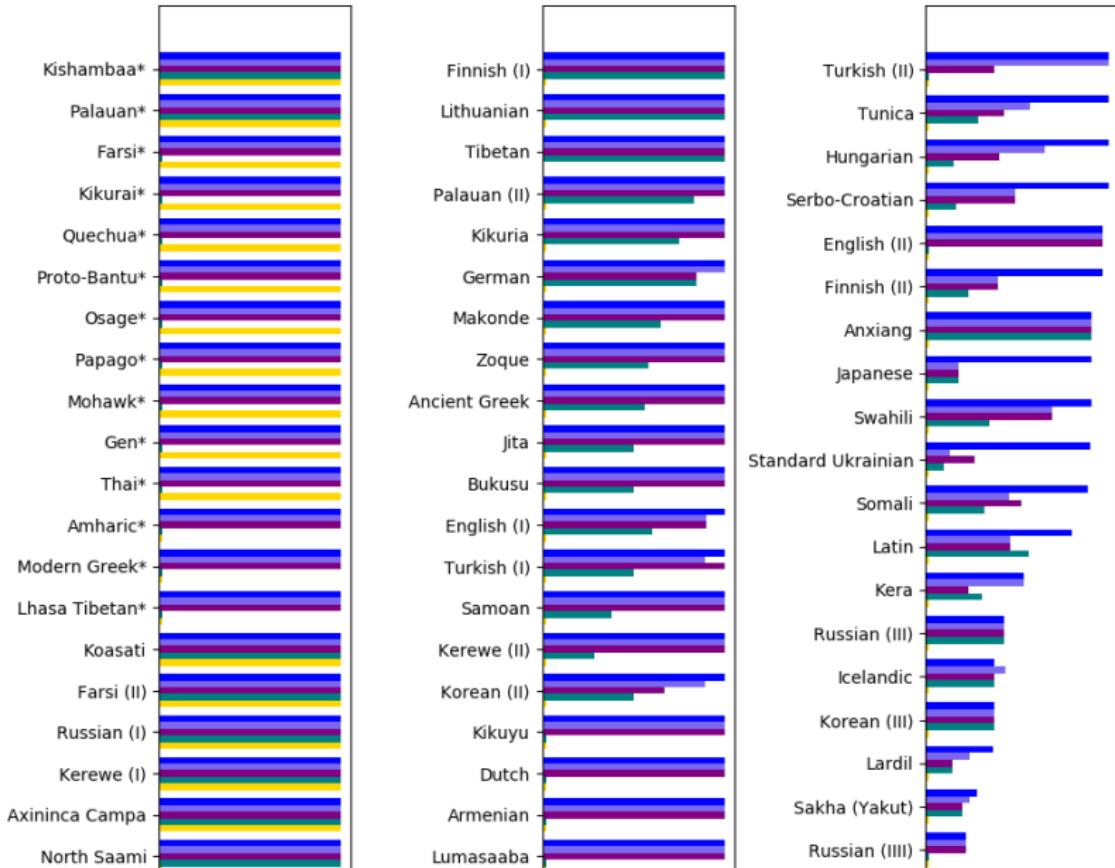
stems
(unobserved)

observed data
& unobserved derivation

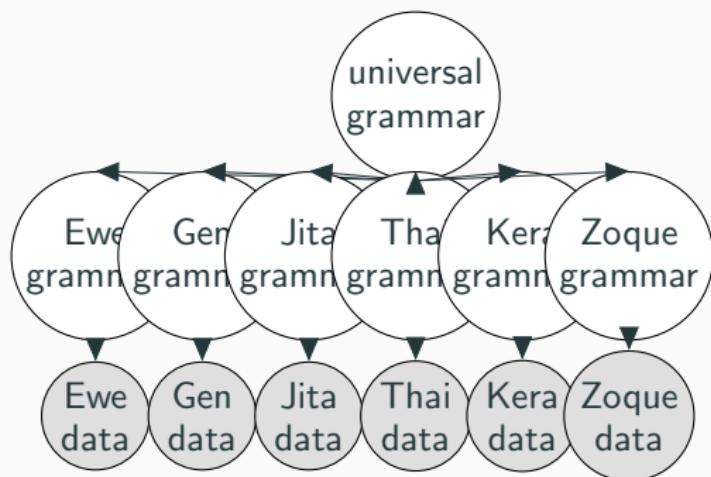
- ⟨/is̥kaap/, CABINET⟩
- ⟨/orɔn/, BED⟩
- ⟨/bie/, MARE⟩
- ⟨/örus/, RIVER⟩

CABINETS → /is̥kaap/+/lar/ → /is̥kaaplar/ $\xrightarrow{r_1}$ /is̥kaapdar/ $\xrightarrow{r_2}$ [is̥kaaptar]
BEDS → /orɔn/+/lar/ → /orɔnlar/ $\xrightarrow{r_1}$ /orondar/ $\xrightarrow{r_3}$ /orondor/ $\xrightarrow{r_6}$ [oronnor]
MARES → /bie/+/lar/ → /bielar/ $\xrightarrow{r_5}$ [bieler]
RIVER (ASSOC) → /örus/+/iin/ → /örusliin/ $\xrightarrow{r_1}$ /örusdiin/ $\xrightarrow{r_2}$
/örustiin/ $\xrightarrow{r_3}$ /örustuu/ $\xrightarrow{r_5}$ [örüstüün]

57 languages drawn from 70 datasets



Distilling higher-level knowledge



Distilling higher-level knowledge

Discovered
universal grammar
schema

w/o learned
universal grammar

w/ learned
universal grammar

consonant/vowel distinction

sounds \leftarrow [-vowel]

sounds \leftarrow [+vowel]

*a set of sounds is commonly
all consonants,
or all vowels*

Tibetan: [-nasal] $\rightarrow \emptyset / \#_$

[-vowel] $\rightarrow \emptyset / \# _ [-vowel]$

Growing domain-specific knowledge

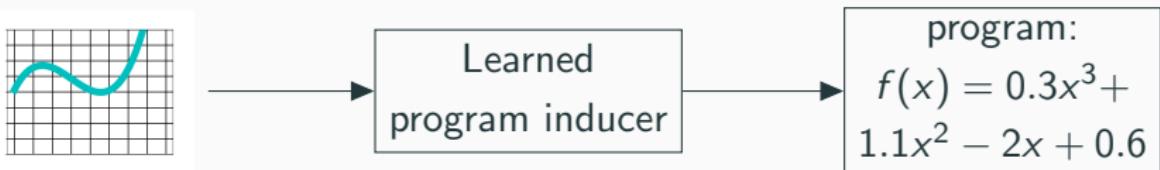
Goal: acquire domain-specific knowledge needed to induce a class of programs

- Library of concepts (declarative knowledge; domain specific language; generative model over programs)
- Inference strategy (procedural knowledge; synthesis algorithm)

Growing domain-specific knowledge

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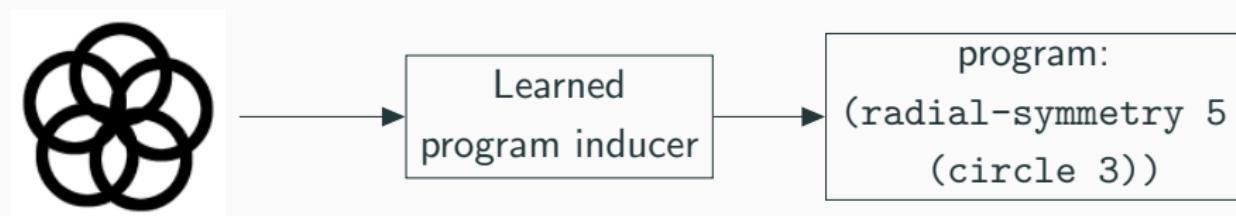
Concepts: x^3 , $\alpha x + \beta$, etc

Inference strategy: neurosymbolic search for programs

Growing domain-specific knowledge

Goal: acquire domain-specific knowledge needed to induce a class of programs

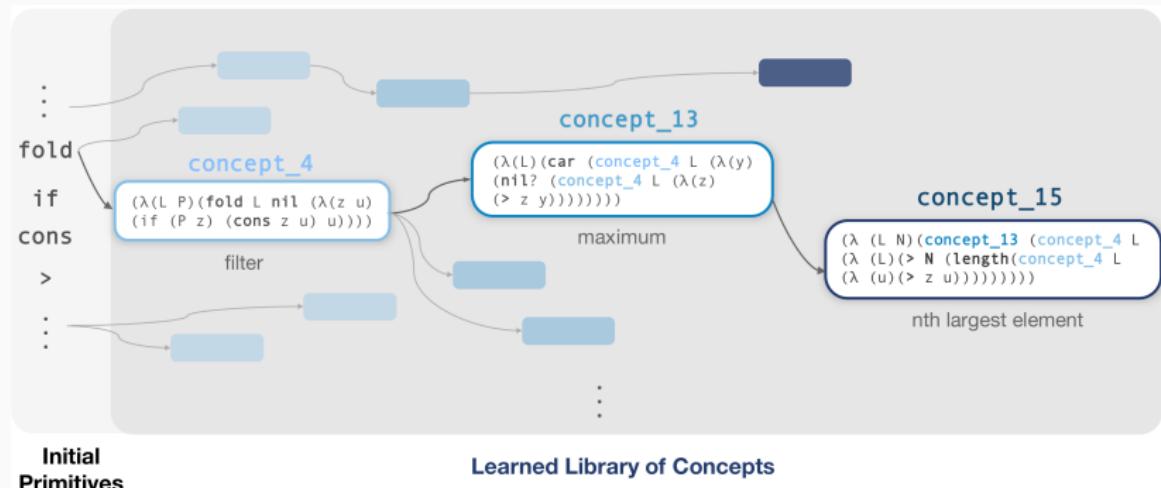
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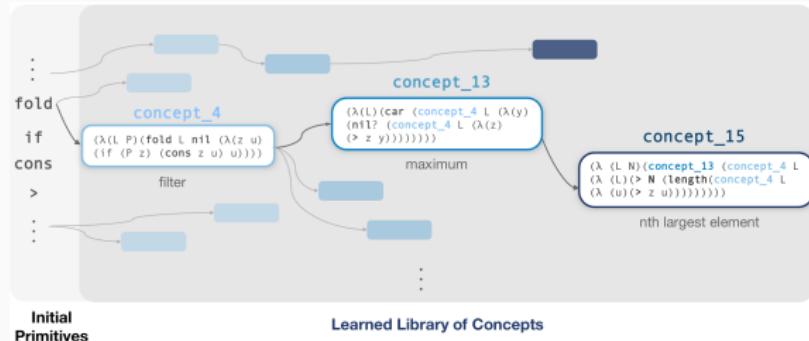
Concepts: circle, radial-symmetry, etc

Inference strategy: neurosymbolic search for programs

Library learning



Library learning



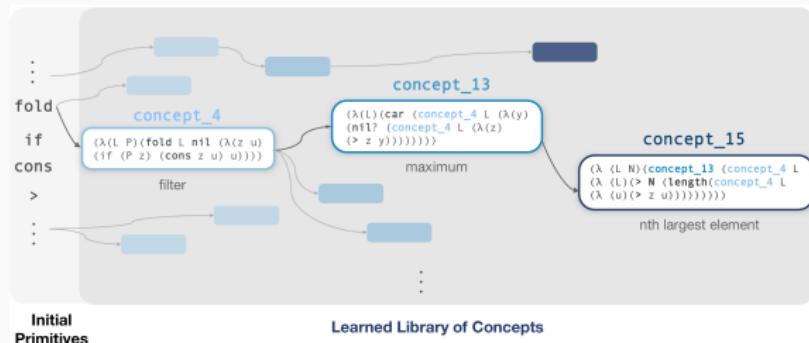
Problem: sort list

Solution:

```
(map (\n) (concept_15 L (+ 1 n))) (range (length L)))
```

get nth largest element where n = 1, 2, 3length of list

Library learning



Problem: sort list

Solution:

`(map (\lambda (n) (concept_15 L (+ 1 n))) (range (length L)))`
get nth largest element where n = 1, 2, 3 ... length of list

Solution in initial primitives:

```
(\lambda(x) (map (\lambda(y) (car (fold x nil (\lambda(z u) (if (gt? (+ y 1) (length (fold x nil (\lambda(v w) (if (gt? z v) (cons v w)))))) (cons z u) u))) nil (\lambda(a b) (if (nil? (fold (fold x nil (\lambda(c d) (if (gt? (+ y 1) (length (fold x nil (\lambda(e f) (if (gt? c e) (cons e f) f)))))) (cons c d) d))) nil (\lambda(g h) (if (gt? g a) (cons g h) h)))) (cons a b) b)))) (range (length x))))
```

Discovered Problem Solutions

DreamCoder

- **Wake:** Solve problems by writing programs
- **Sleep:** Improve DSL and neural recognition model:
 - **Abstraction sleep:** Improve library
 - **Dream sleep:** Improve neural inference model
- Combines ideas from Wake-Sleep & Exploration-Compression



DreamCoder

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List Processing

Sum List
 $[1 \ 2 \ 3] \rightarrow 6$
 $[4 \ 6 \ 8 \ 1] \rightarrow 17$

Double

$[1 \ 2 \ 3 \ 4] \rightarrow [2 \ 4 \ 6 \ 8]$
 $[6 \ 5 \ 1] \rightarrow [12 \ 10 \ 2]$

Check Evens

$[0 \ 2 \ 3] \rightarrow [T \ T \ F]$
 $[2 \ 4 \ 9 \ 6] \rightarrow [T \ T \ F \ T]$

Text Editing

Abbreviate
Allen Newell → A.N.
Herb Simon → H.S.

Drop Last Characters

jabberwocky → jabberw
copycat → cop

Extract

see spot(run) → run
a (bee) see → bee

Regexes

Phone Numbers
(555) 867-5309
(650) 555-2368

Currency

\$100.25
\$4.50

Dates

Y1775/0704
Y2000/0101

LOGO Graphics



Physics

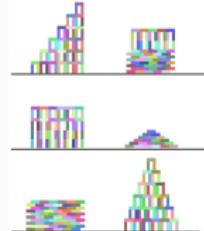
$$KE = \frac{1}{2}m|\vec{v}|^2$$

$$\bar{d} = \frac{1}{m} \sum_i \vec{F}_i$$

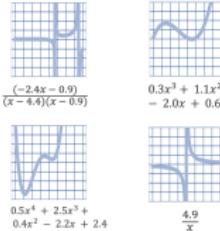
$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^2} \hat{\vec{r}}_1 \cdot \hat{\vec{r}}_2$$

$$R_{total} = \left(\sum_i \frac{1}{R_i} \right)^{-1}$$

Block Towers



Symbolic Regression

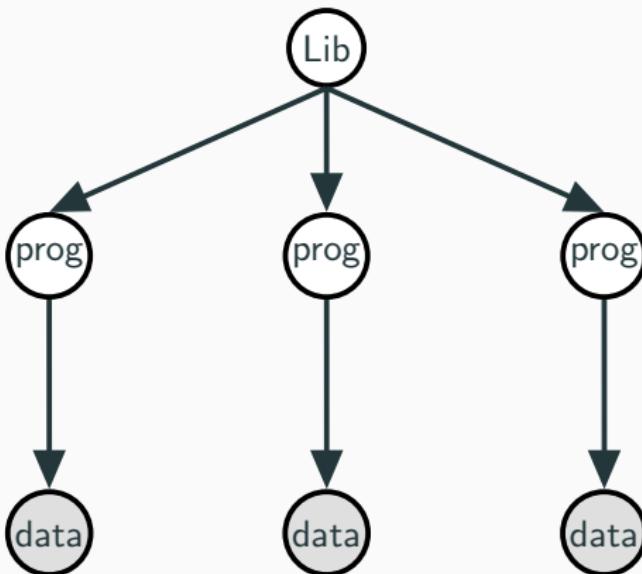


Recursive Programming

Filter	Index List
$[\text{■■■■■}] \rightarrow [\text{■■}]$	$0, [\text{■■■■■}] \rightarrow \text{■}$
$[\text{■■■■■■■■}] \rightarrow [\text{■■■■■■}]$	$1, [\text{■■■■■■■■}] \rightarrow \text{■}$
$[\text{■■■■■■■■■}] \rightarrow [\text{■■■■■■■}]$	$1, [\text{■■■■■■■■■}] \rightarrow \text{■}$

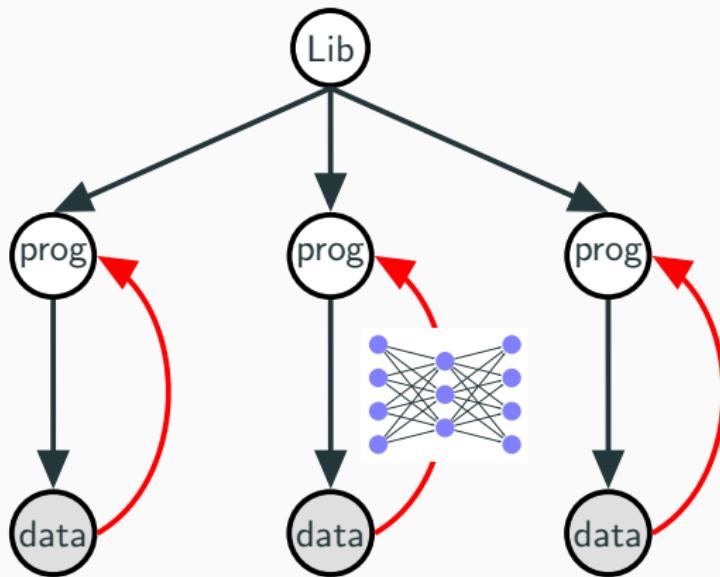
Length	Every Other
$[\text{■■■■■}] \rightarrow 4$	$[\text{■■■■■}] \rightarrow [\text{■■}]$
$[\text{■■■■■■■■}] \rightarrow 6$	$[\text{■■■■■■■■}] \rightarrow [\text{■■■■}]$
$[\text{■■■■■■■■■}] \rightarrow 3$	$[\text{■■■■■■■■■}] \rightarrow [\text{■■■}]$

Library learning as Bayesian inference

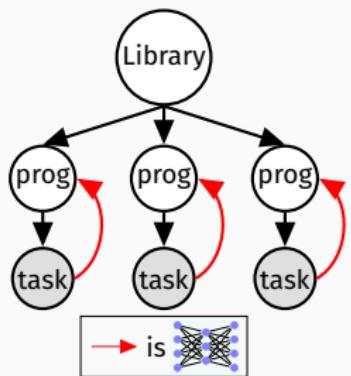


[Dechter et al., 2013] [Liang et al, 2010]; [Lake et al, 2015]

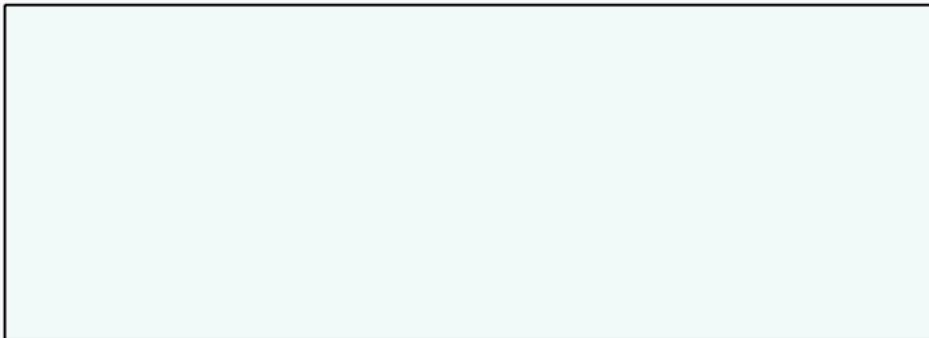
Library learning as amortized Bayesian inference



amortized inference +
better program representation (Lisp) +
library learning via program analysis +
new neural inference network for program synthesis

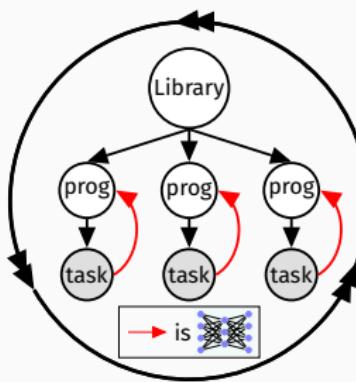


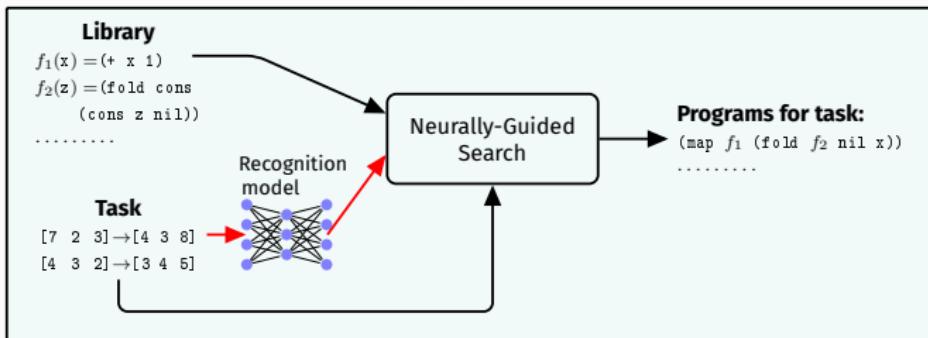
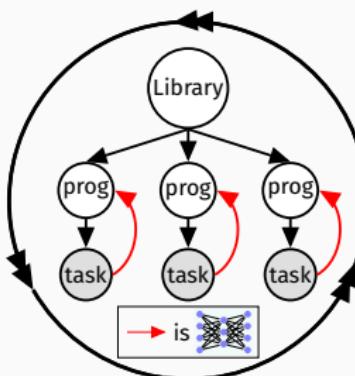
WAKE

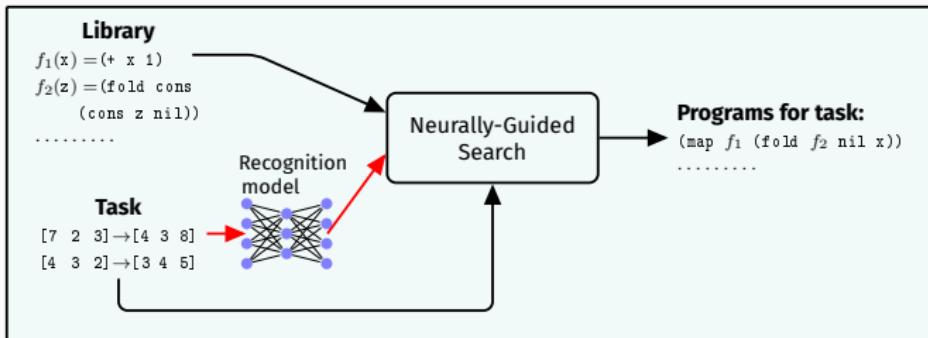
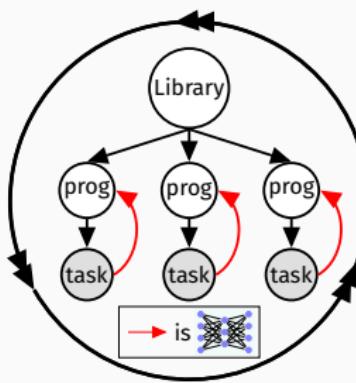
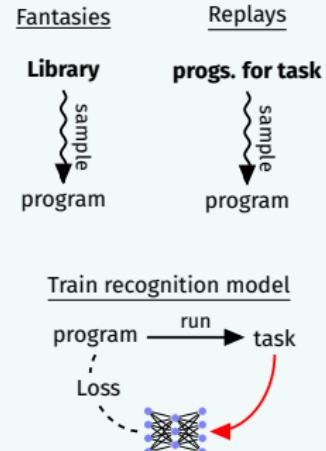


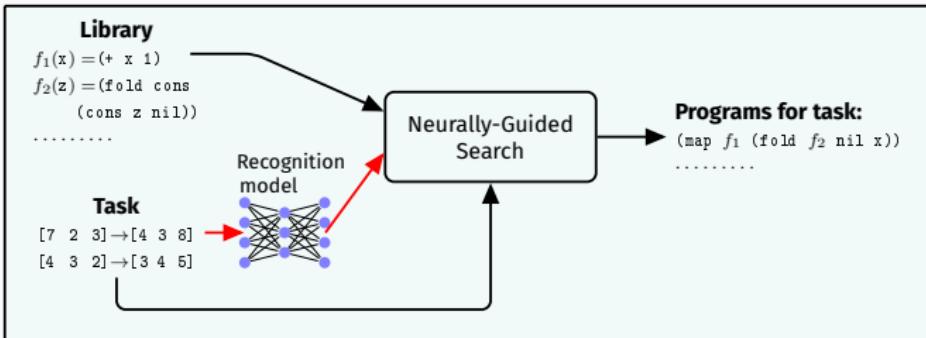
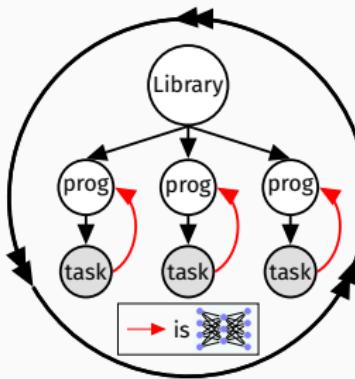
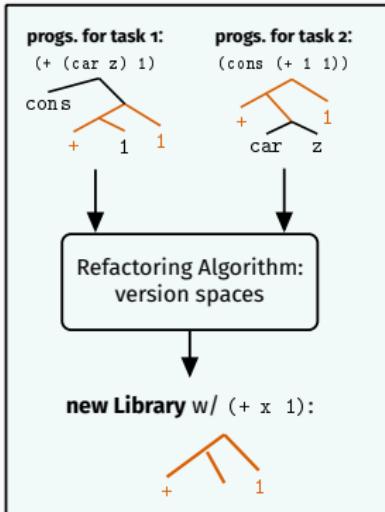
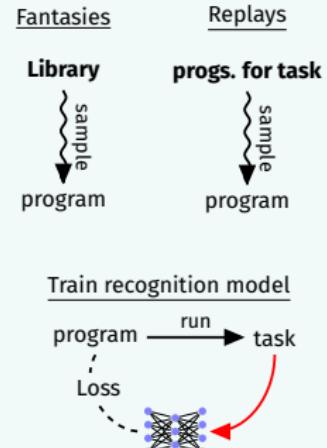
SLEEP: ABSTRACTION

SLEEP: DREAMING



WAKE**SLEEP: ABSTRACTION****SLEEP: DREAMING**

WAKE**SLEEP: ABSTRACTION****SLEEP: DREAMING**

WAKE**SLEEP: ABSTRACTION****SLEEP: DREAMING**

Abstraction Sleep: Growing the library via refactoring

$5 + 5$

Abstraction Sleep: Growing the library via refactoring

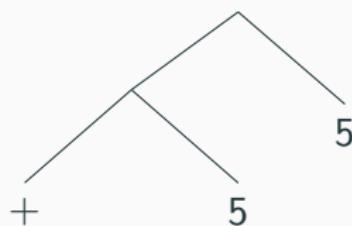
$5 + 5$

(+ 5 5)

Abstraction Sleep: Growing the library via refactoring

$5 + 5$

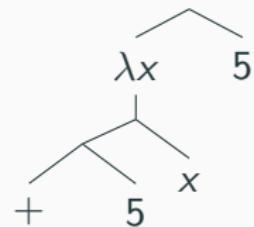
(+ 5 5)



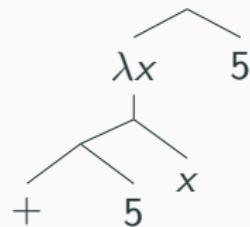
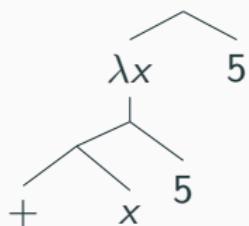
Abstraction Sleep: Growing the library via refactoring



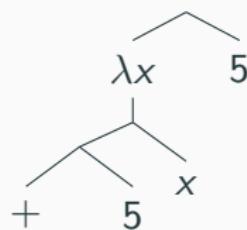
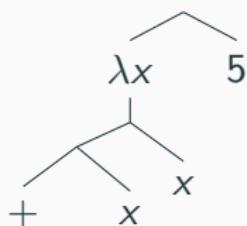
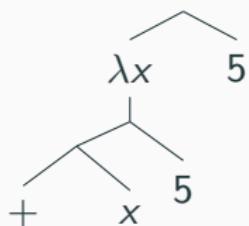
Abstraction Sleep: Growing the library via refactoring



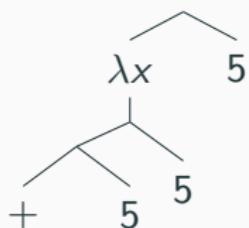
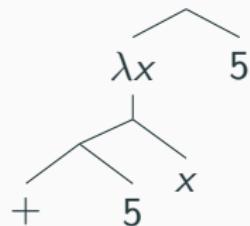
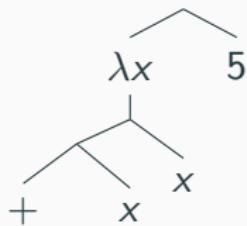
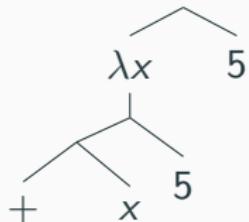
Abstraction Sleep: Growing the library via refactoring



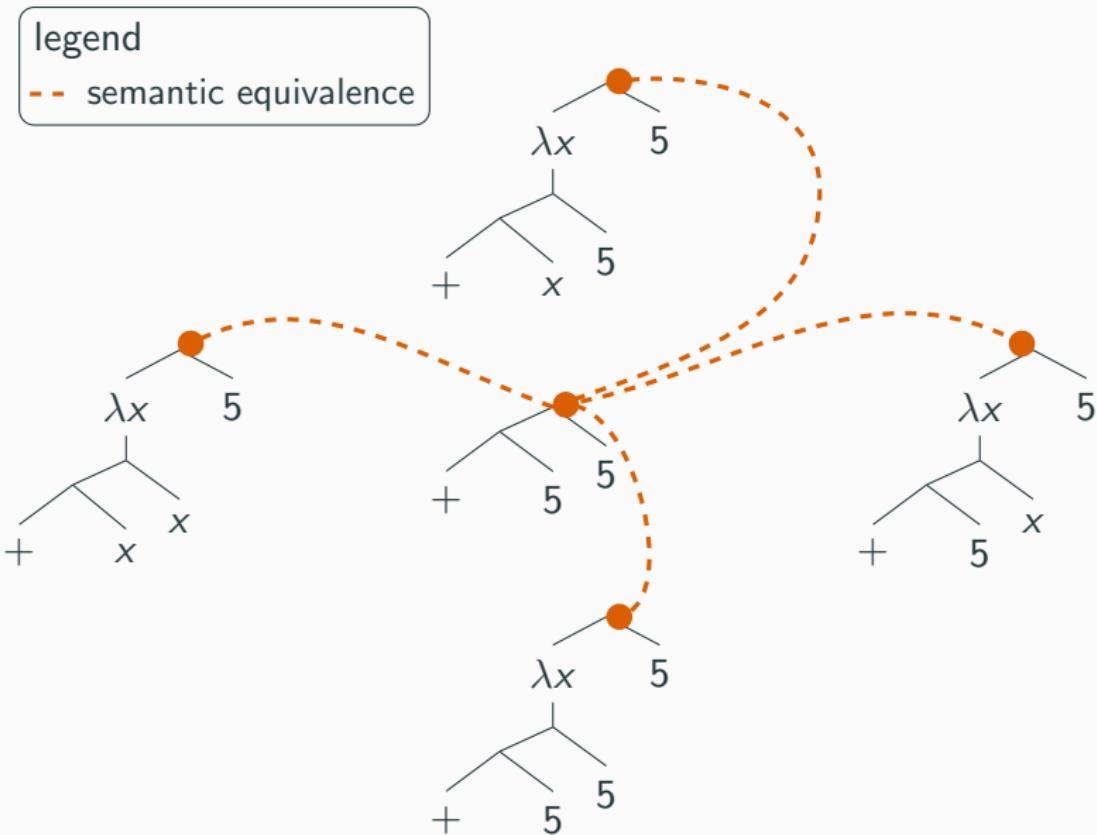
Abstraction Sleep: Growing the library via refactoring



Abstraction Sleep: Growing the library via refactoring



Abstraction Sleep: Growing the library via refactoring

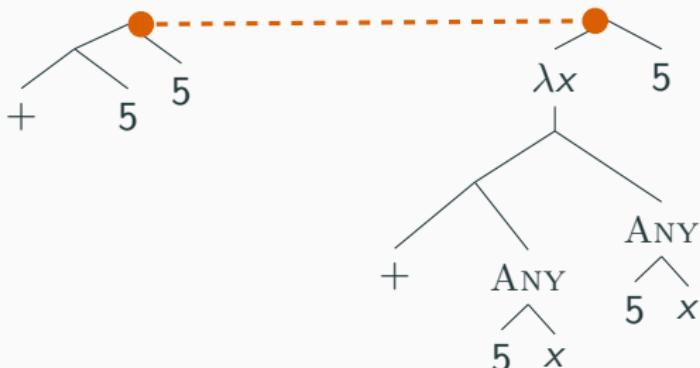


Abstraction Sleep: Growing the library via refactoring

legend

— semantic equivalence

ANY nondeterministic choice

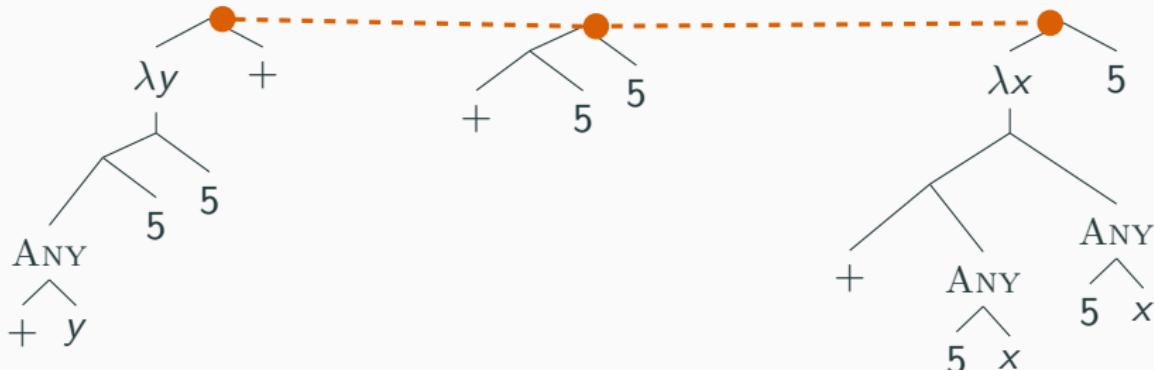


Abstraction Sleep: Growing the library via refactoring

legend

— semantic equivalence

ANY nondeterministic choice

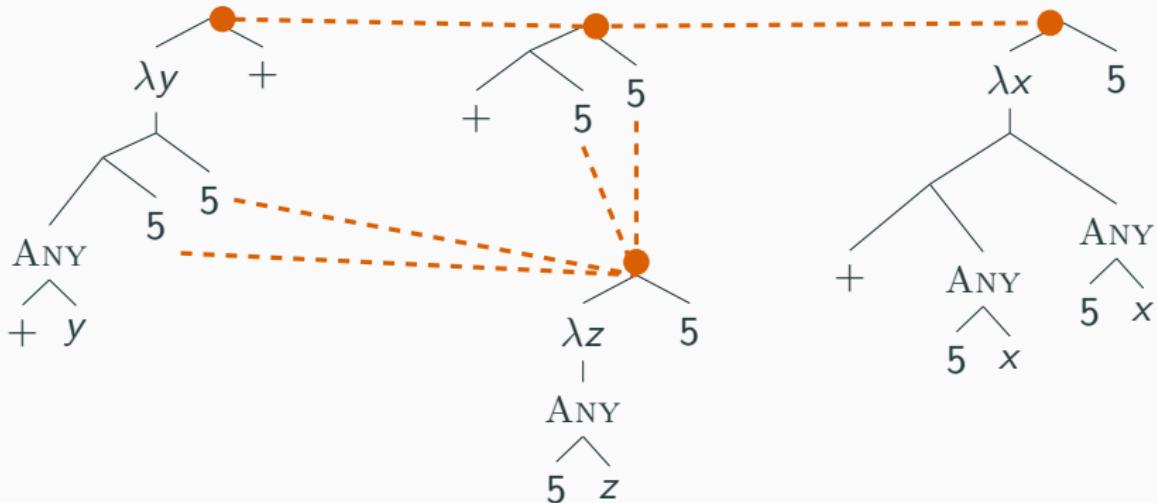


Abstraction Sleep: Growing the library via refactoring

legend

— semantic equivalence

ANY nondeterministic choice

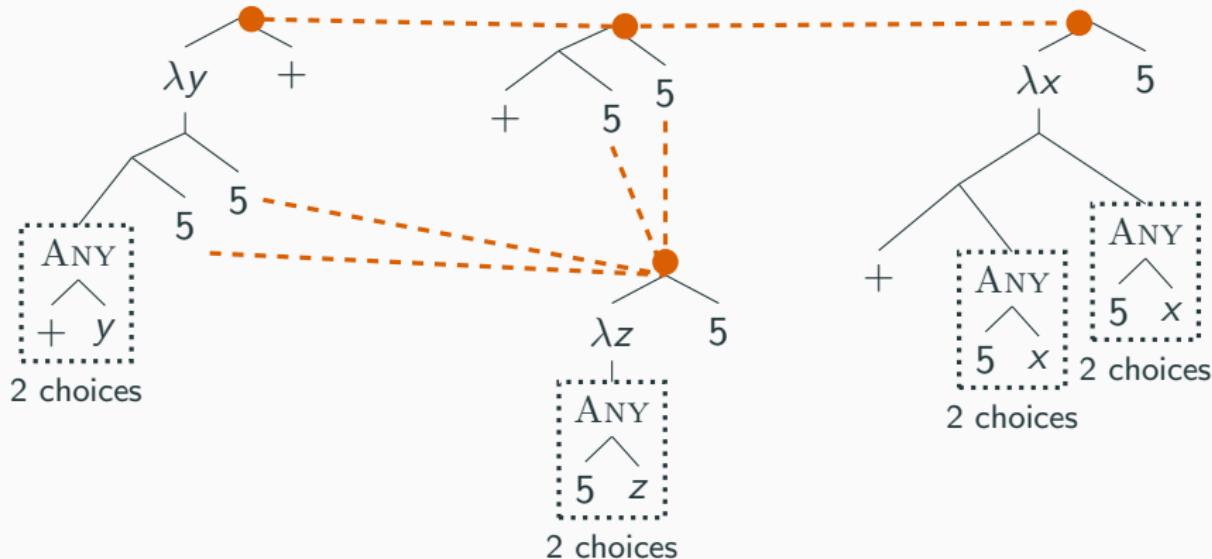


Abstraction Sleep: Growing the library via refactoring

legend

— semantic equivalence

ANY nondeterministic choice

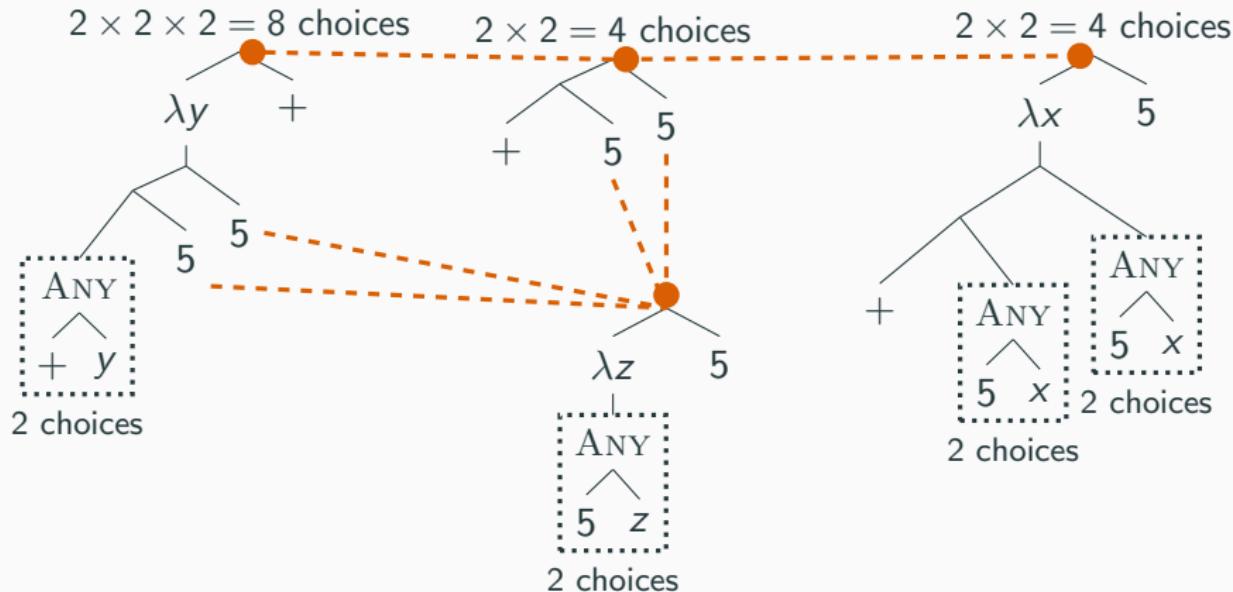


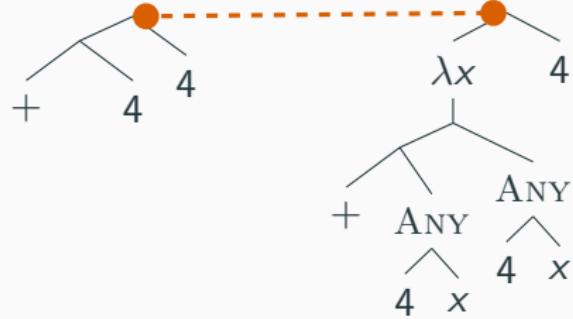
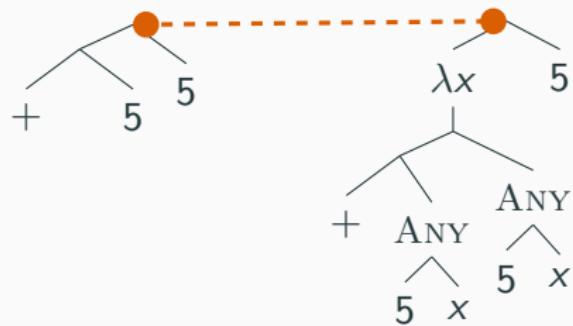
Abstraction Sleep: Growing the library via refactoring

legend

— semantic equivalence

ANY nondeterministic choice

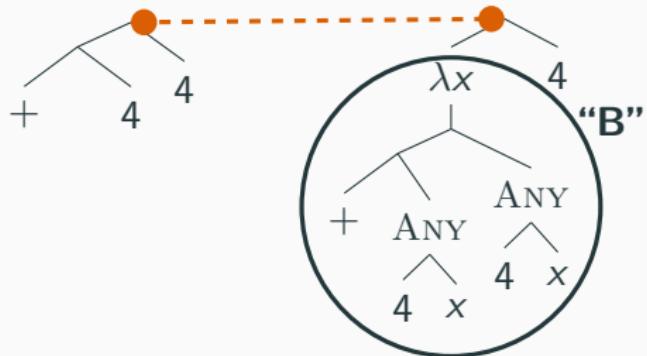
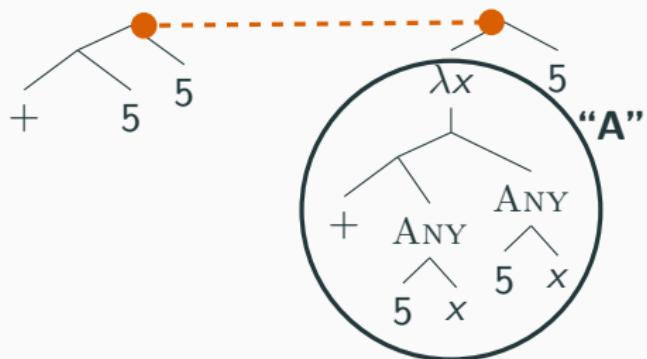




legend

— semantic equivalence

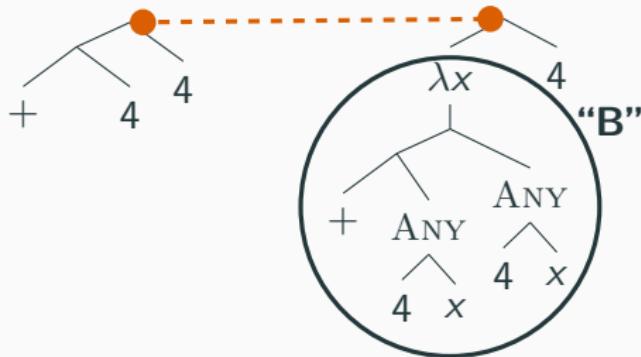
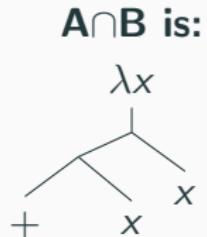
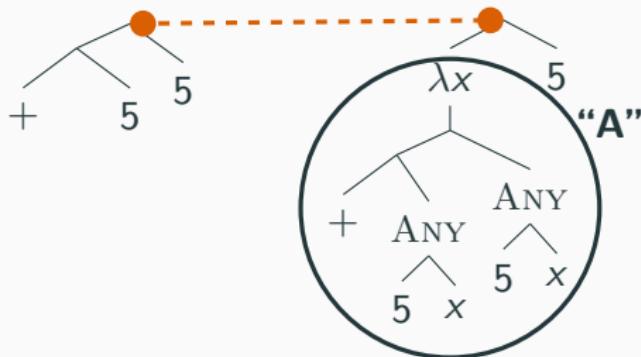
ANY nondeterministic choice



legend

— semantic equivalence

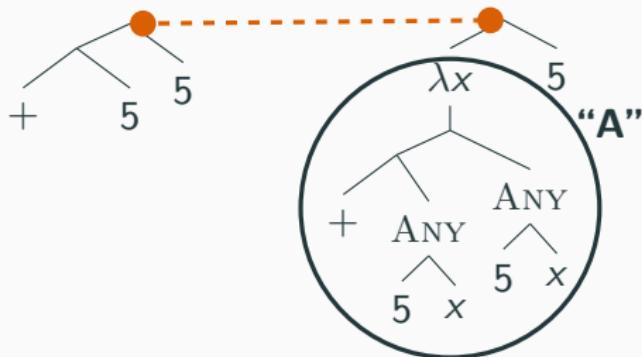
ANY nondeterministic choice



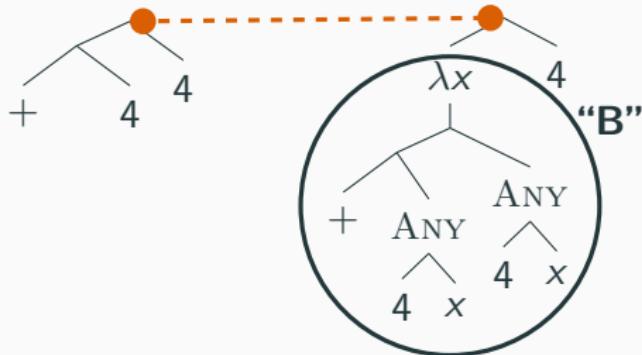
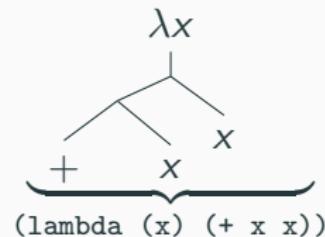
legend

— dashed orange line semantic equivalence

ANY nondeterministic choice



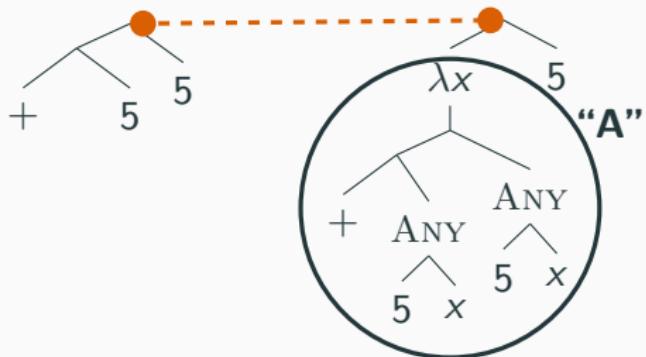
$A \cap B$ is:



legend

— semantic equivalence

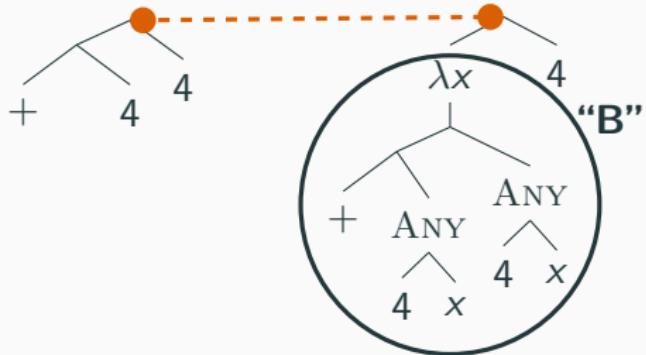
ANY nondeterministic choice



$A \cap B$ is:

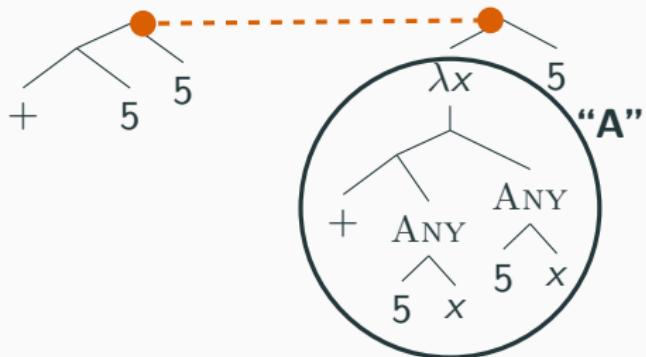
$(\lambda x \quad (+ x x))$

=double

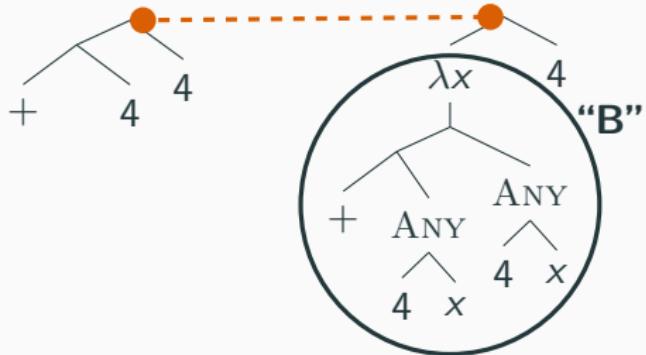
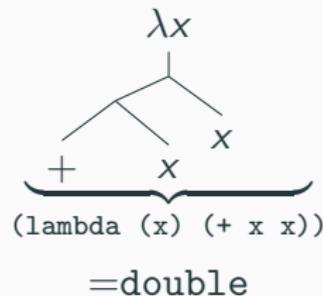


legend

- semantic equivalence
- ANY nondeterministic choice



$A \cap B$ is:



w/o double	w/ double
(+ 5 5)	(double 5)
(+ 4 4)	(double 4)
(+ 3 3)	(double 3)
...	

legend

— semantic equivalence
ANY nondeterministic choice

Abstraction Sleep: Growing the library via refactoring

Task: $(1\ 2\ 3) \rightarrow (2\ 4\ 6)$
 $(4\ 3\ 2) \rightarrow (8\ 6\ 8)$

Wake: program search

```
(Y (\lambda (r l) (if (nil? l) nil  
                      (cons (+ (car l) (car l))  
                            (r (cdr l)))))))
```

Task: $(1\ 2\ 3) \rightarrow (0\ 1\ 2)$
 $(4\ 3\ 2) \rightarrow (3\ 2\ 3)$

Wake: program search

```
(Y (\lambda (r l) (if (nil? l) nil  
                      (cons (- (car l) 1)  
                            (r (cdr l)))))))
```

Abstraction Sleep: Growing the library via refactoring

Task: $(1\ 2\ 3) \rightarrow (2\ 4\ 6)$
 $(4\ 3\ 2) \rightarrow (8\ 6\ 8)$

Wake: program search

```
(Y (λ (r 1) (if (nil? 1) nil  
           (cons (+ (car 1) (car 1))  
                  (r (cdr 1)))))))
```

Task: $(1\ 2\ 3) \rightarrow (0\ 1\ 2)$
 $(4\ 3\ 2) \rightarrow (3\ 2\ 3)$

Wake: program search

```
(Y (λ (r 1) (if (nil? 1) nil  
           (cons (- (car 1) 1)  
                  (r (cdr 1)))))))
```

refactor
 $(10^{14}$ refactorings)

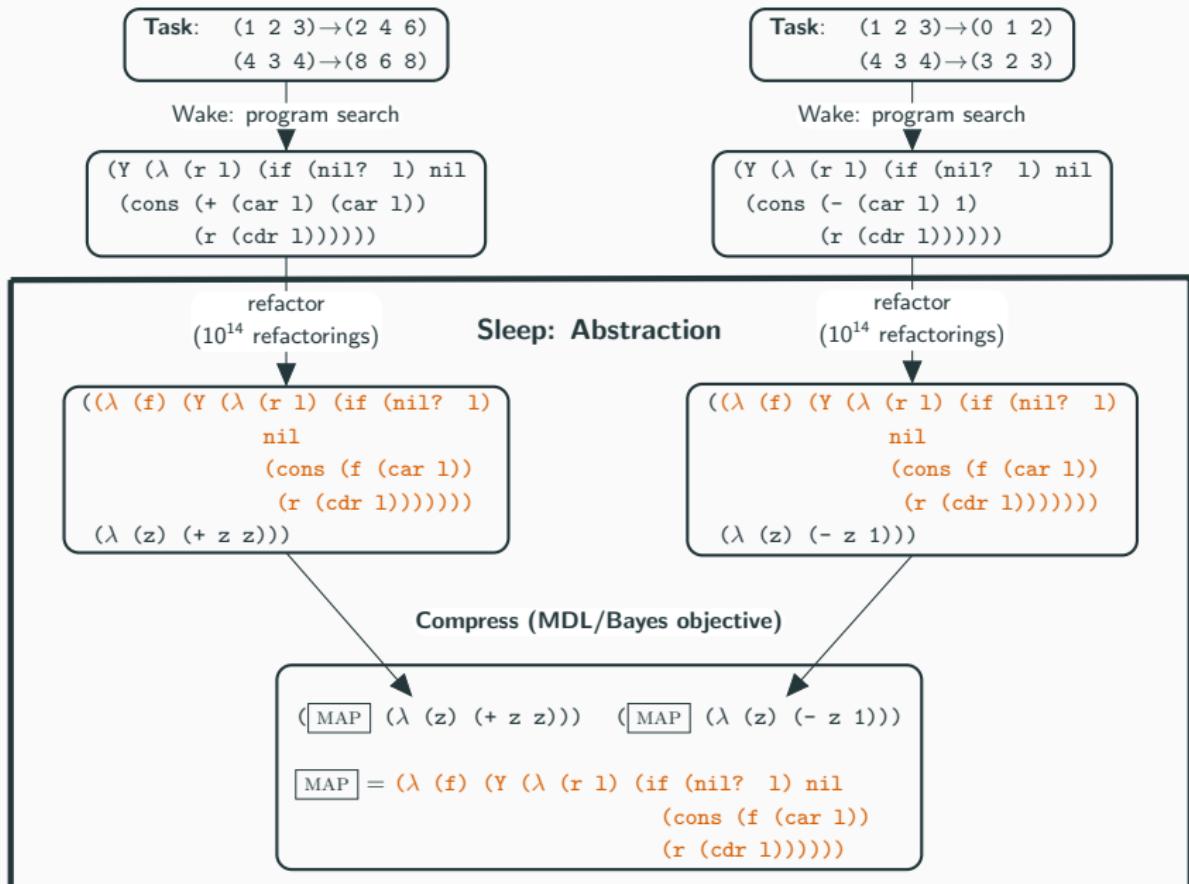
Sleep: Abstraction

refactor
 $(10^{14}$ refactorings)

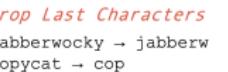
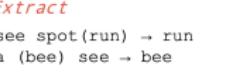
```
((λ (f) (Y (λ (r 1) (if (nil? 1)  
                           nil  
                           (cons (f (car 1))  
                                 (r (cdr 1)))))))  
  (λ (z) (+ z z)))
```

```
((λ (f) (Y (λ (r 1) (if (nil? 1)  
                           nil  
                           (cons (f (car 1))  
                                 (r (cdr 1)))))))  
  (λ (z) (- z 1)))
```

Abstraction Sleep: Growing the library via refactoring



DreamCoder Domains

List Processing	Text Editing	Regexes	LOGO Graphics
Sum List [1 2 3] → 6 [4 6 8 1] → 17	Abbreviate Allen Newell → A.N. Herb Simon → H.S.	Phone Numbers (555) 867-5309 (650) 555-2368	↶ ↷ ↸ ↹
Double [1 2 3 4] → [2 4 6 8] [6 5 1] → [12 10 2]	Drop Last Characters jabberwocky → jabberw copycat → cop	Currency \$100.25 \$4.50	✿ ⋯ ⋮ ⋆
Check Evens [0 2 3] → [T T F] [2 4 9 6] → [T T F T]	Extract see spot(run) → run a (bee) see → bee	Dates Y1775/0704 Y2000/0101	⊗ ⊖ ⊙ ⊚
Block Towers 	Symbolic Regression  $(-2.4x - 0.9)$ $(x - 4.4)(x - 0.9)$ $0.3x^3 + 1.1x^2 - 2.0x + 0.6$	Recursive Programming Filter [■■■■■] → [■■] [■■■■■■] → [■■■■] [■■■■■■■] → [■■■■■]	Physics $KE = \frac{1}{2} m \vec{v} ^2$ $\vec{a} = \frac{1}{m} \sum_i \vec{F}_i$
	 $0.5x^4 + 2.5x^3 + 0.4x^2 - 2.2x + 2.4$ $\frac{4.9}{x}$	Length [■■■■■] → 4 [■■■■■■] → 6 [■■■■] → 3	Index List 0, [■■■■■■■] → ■ 1, [■■■■■■■] → ■■ 1, [■■■■■■■■] → ■■■
		Every Other [■■■■■] → [■■] [■■■■■■] → [■■■■] [■■■■■■■] → [■■■■■]	$\vec{F} \propto \frac{q_1 q_2}{ \vec{r}'_1 - \vec{r}'_2 ^2} \hat{\vec{r}}_1 - \hat{\vec{r}}_2$ $R_{total} = \left(\sum_i \frac{1}{R_i} \right)^{-1}$

DreamCoder Domains

List Processing

Sum List

[1 2 3] → 6

[4 6 8 1] → 17

Double

[1 2 3 4] → [2 4 6 8]

[6 5 1] → [12 10 2]

Check Evens

[0 2 3] → [T T F]

[2 4 9 6] → [T T F T]

Text Editing

Abbreviate

Allen Newell → A.N.

Herb Simon → H.S.

Drop Last Characters

jabberwocky → jabberw

copycat → cop

Extract

see spot(run) → run

a (bee) see → bee

Regexes

Phone Numbers

(555) 867-5309

(650) 555-2368

Currency

\$100.25

\$4.50

Dates

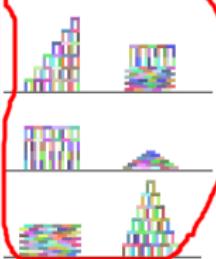
Y1775/0704

Y2000/0101

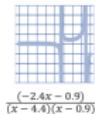
LOGO Graphics



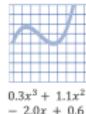
Block Towers



Symbolic Regression



$$0.5x^4 + 2.5x^3 + 0.4x^2 - 2.2x + 2.4$$



$$\frac{4.9}{x}$$

Recursive Programming

Filter

[■■■■] → [■■■]
[■■■■■] → [■■■■]
[■■■■] → [■■■]

Length

[■■■■] → 4
[■■■■■] → 6
[■■■] → 3

Index List

0, [■■■■■■] → ■
1, [■■■■■■] → ■■
1, [■■■■■■] → ■■■

Every Other

[■■■■■■] → [■■■]
[■■■■■■] → [■■■■]
[■■■■■■] → [■■■■■]

Physics

$$KE = \frac{1}{2} m |\vec{v}|^2$$

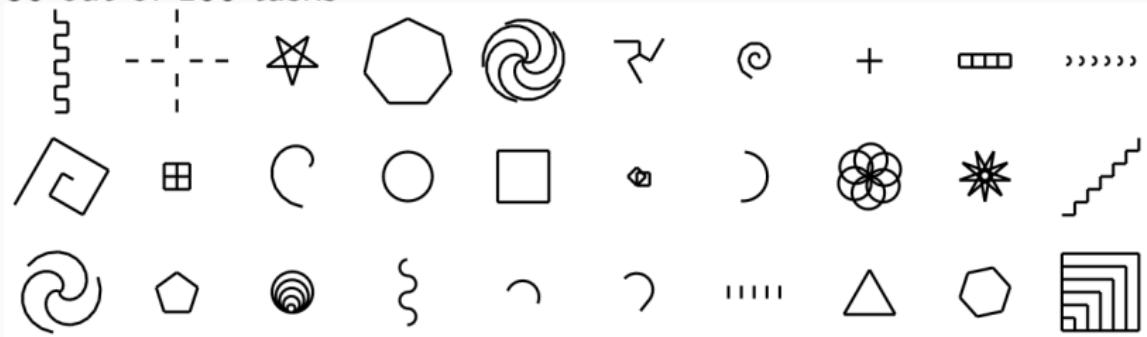
$$\vec{a} = \frac{1}{m} \sum_i \vec{F}_i$$

$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^2} \vec{r}_1 - \vec{r}_2$$

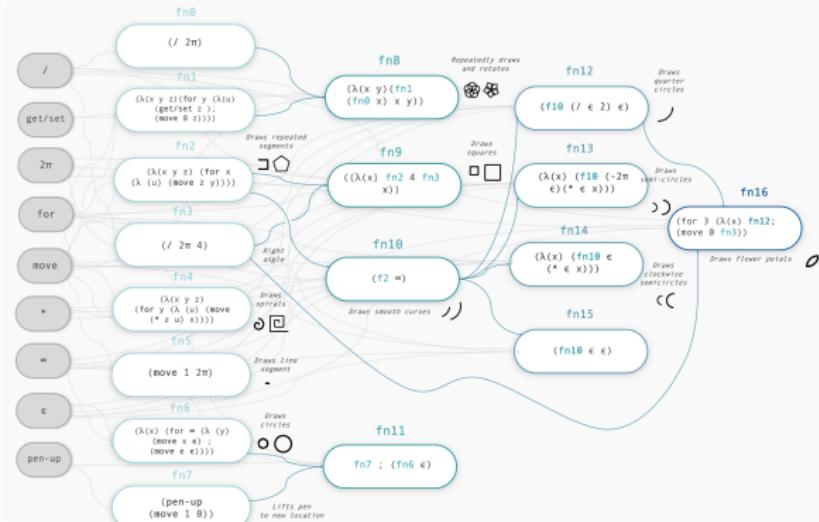
$$V_{total} = \left(\sum_i \frac{1}{R_i} \right)^{-1}$$

LOGO Graphics

30 out of 160 tasks



LOGO Graphics – learning interpretable library of concepts



fn0: 
 $\{\text{fn8 } 5 \text{ (fn4 } (* \epsilon 2) \text{ = } \epsilon)\}$

fn1: 
 $\{\text{fn14 } 2); (\text{fn13 } 2)\}$

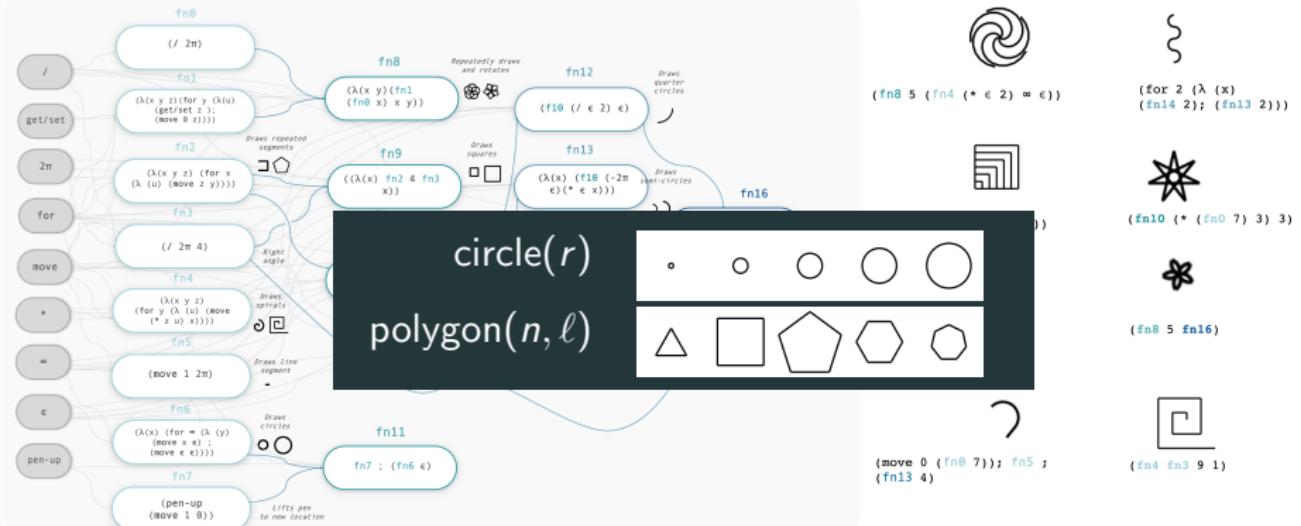
fn2: 
 $\{\text{fn10 } (\lambda (x) (\text{fn9 } x))\}$

fn3: 
 $\{\text{fn8 } 6 \text{ (fn7 ; fn5 ; fn7 ; fn5)}\}$

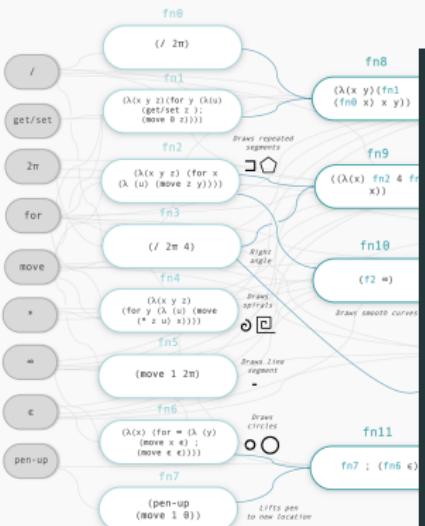
fn4: 
 $\{\text{fn8 } 5 \text{ fn16}\}$

fn5: 
 $\{\text{move } 0 \text{ (fn0 } 7\text{)}; \text{fn5 ; fn13 } 4\}$

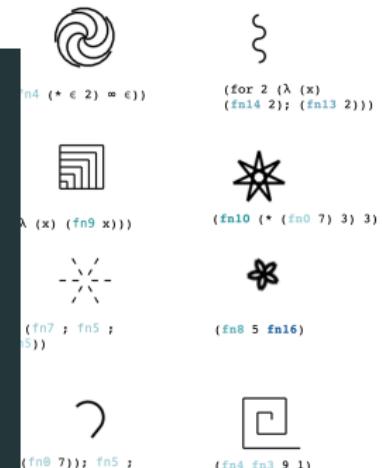
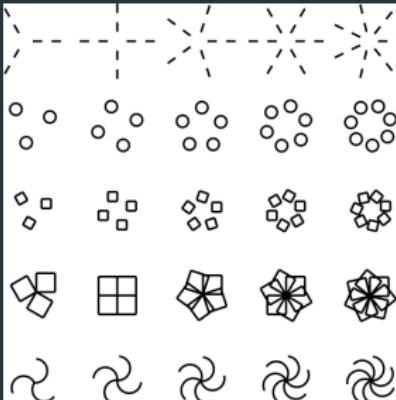
LOGO Graphics – learning interpretable library of concepts



LOGO Graphics – learning interpretable library of concepts

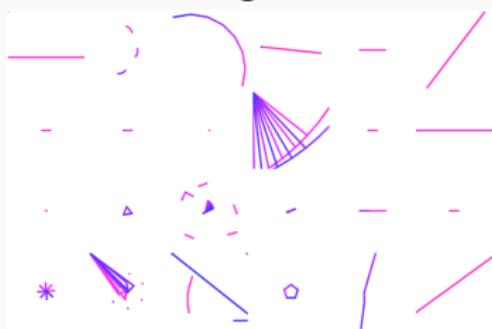


radial symmetry(n , body)

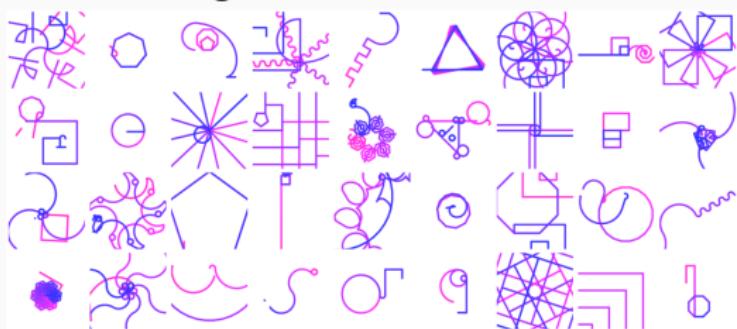


what does DreamCoder dream of?

before learning

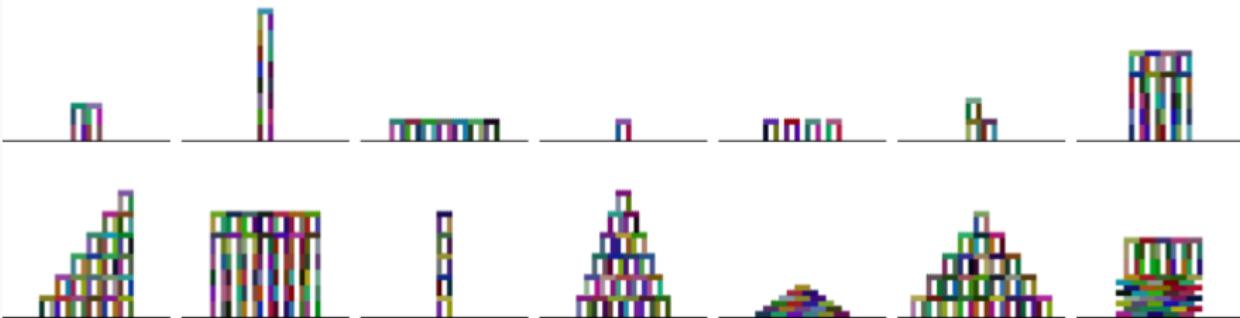


after learning



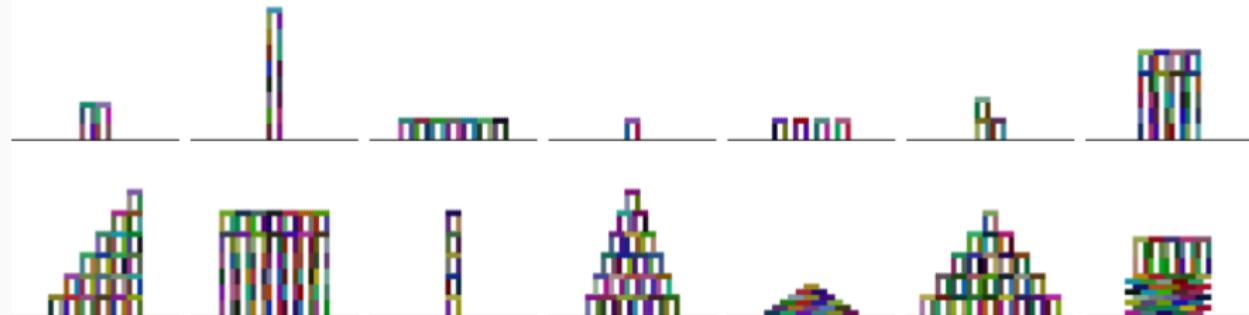
Planning to build towers

example tasks (112 total)

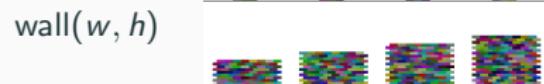
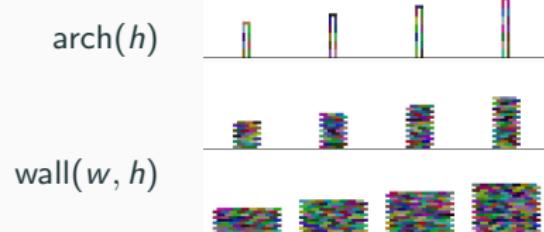


Planning to build towers

example tasks (112 total)

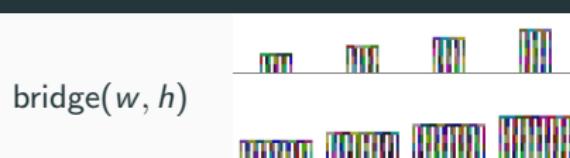
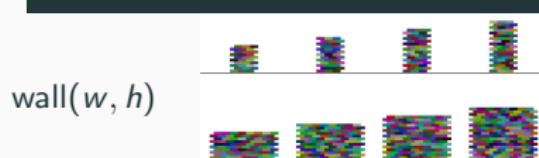
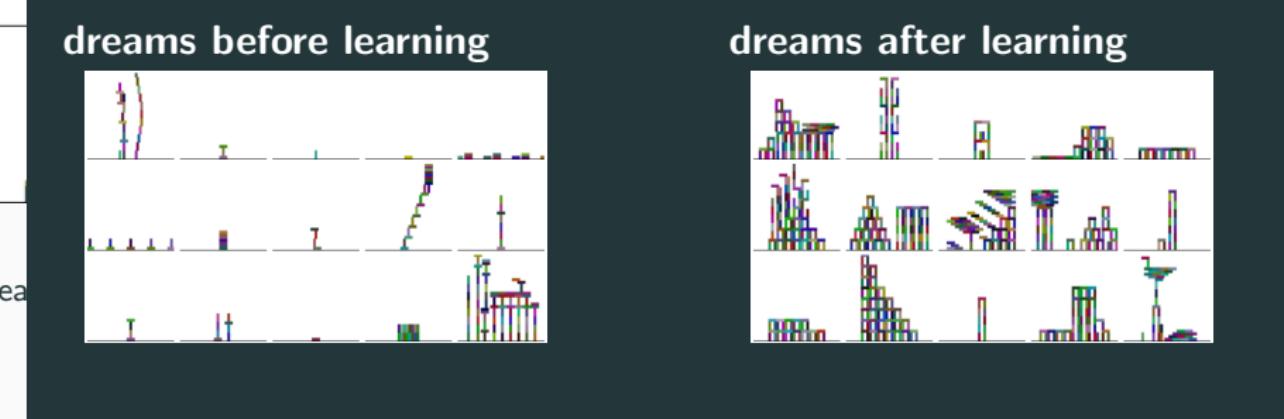


learned library routines (≈ 20 total)

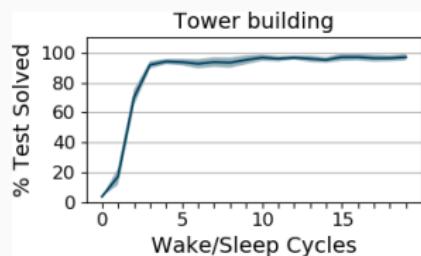


Planning to build towers

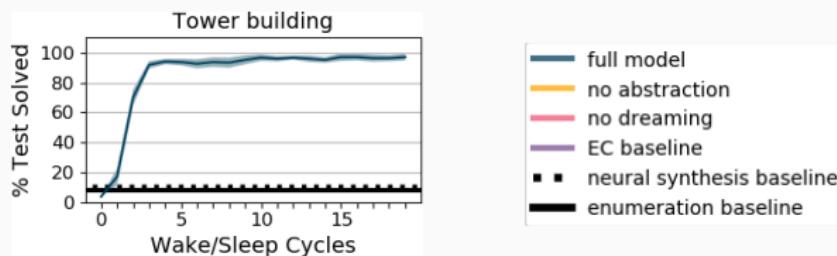
example tasks (112 total)



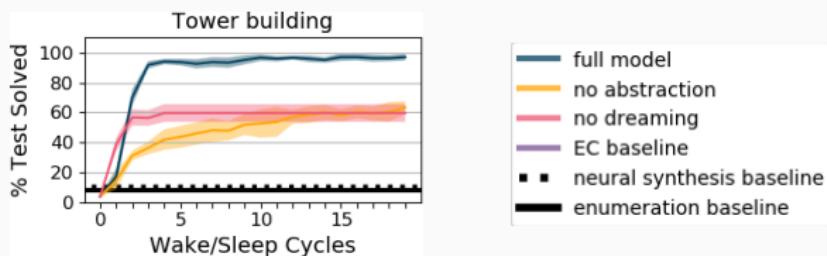
Synergy between dreaming and library learning



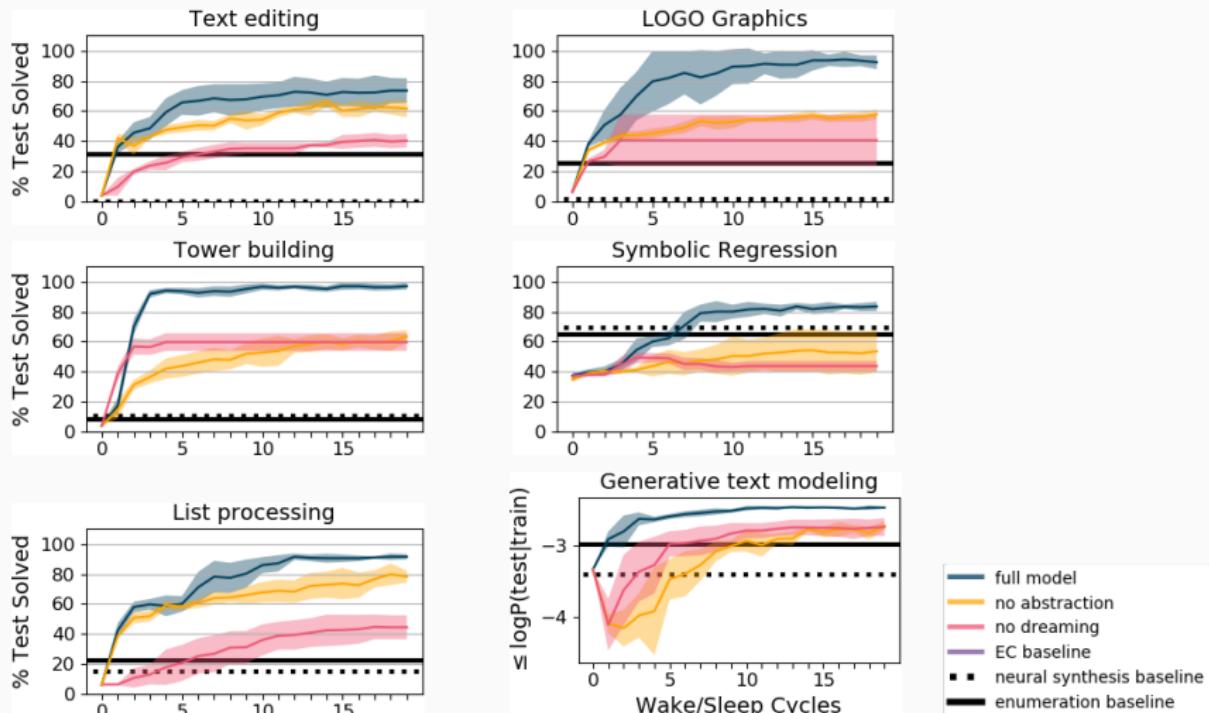
Synergy between dreaming and library learning



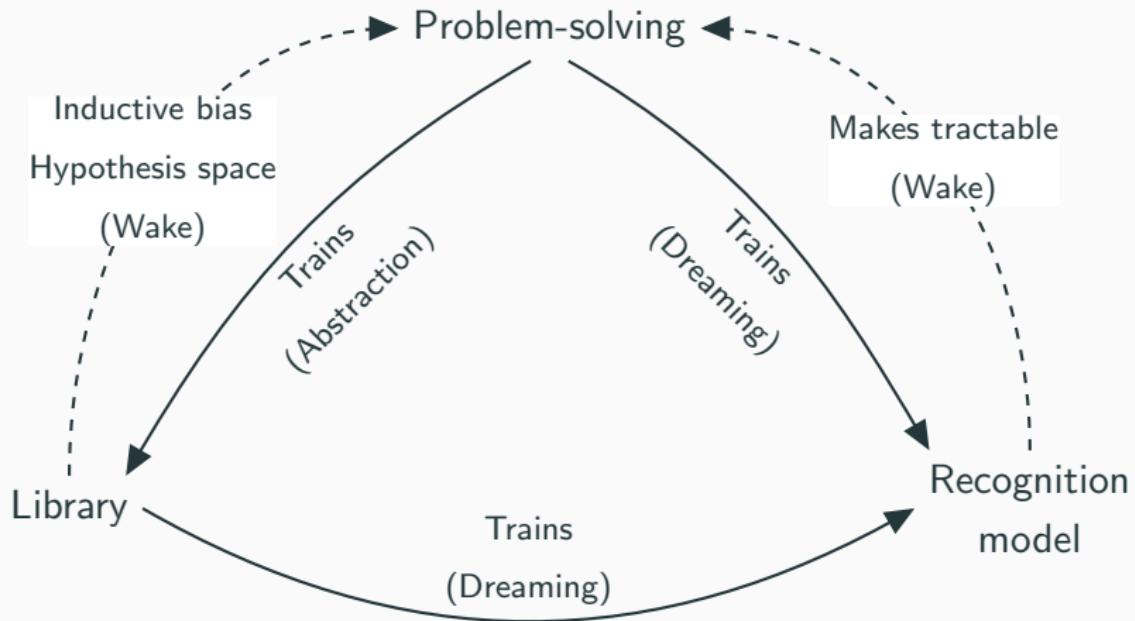
Synergy between dreaming and library learning



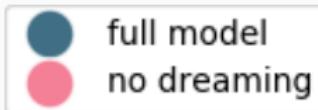
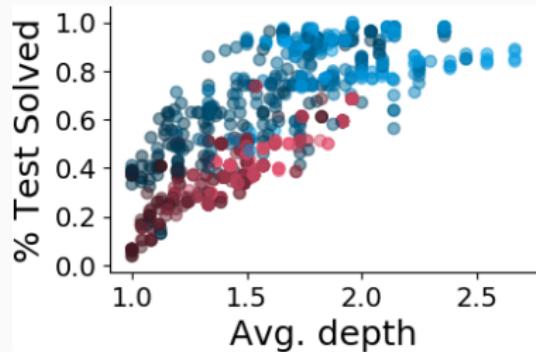
Synergy between dreaming and library learning



synergy between dreaming and library learning



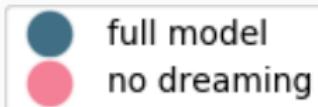
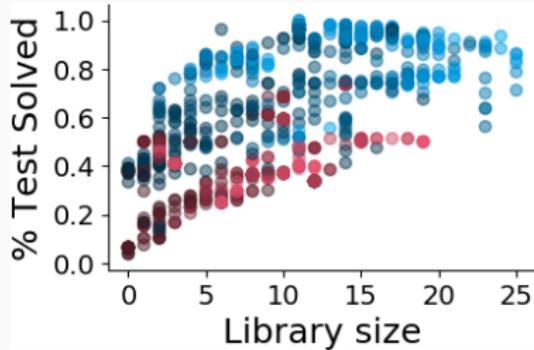
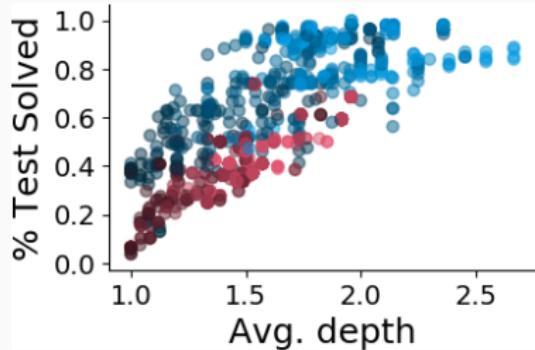
Evidence for dreaming bootstrapping better libraries



Darker: Early in learning

Lighter: Later in learning

Evidence for dreaming bootstrapping better libraries

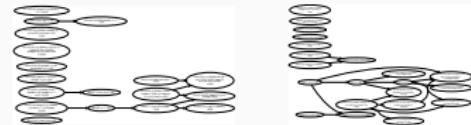


Darker: Early in learning

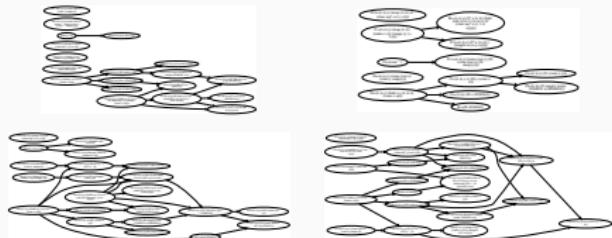
Lighter: Later in learning

Variability in learned library

List processing



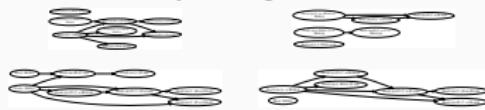
Text editing



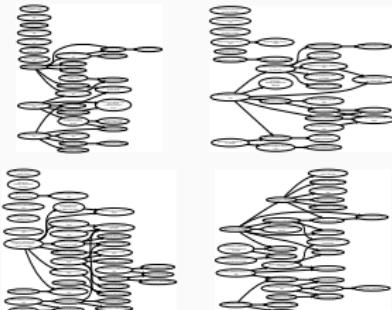
Tower building



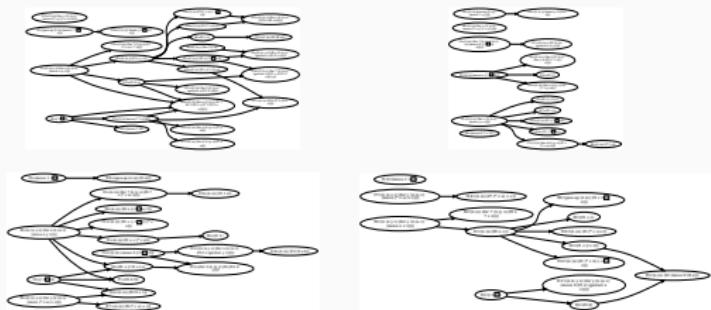
Symbolic regression



Generative regex



LOGO graphics



From learning libraries to learning languages

these experiments study how DreamCoder grows from a “beginner” state to “expert”:

- “beginner:” basic domain-specific procedures, only easiest problems have short solutions
- “expert:” learned library allowing hardest problems to have short meaningful solutions

From learning libraries to learning languages

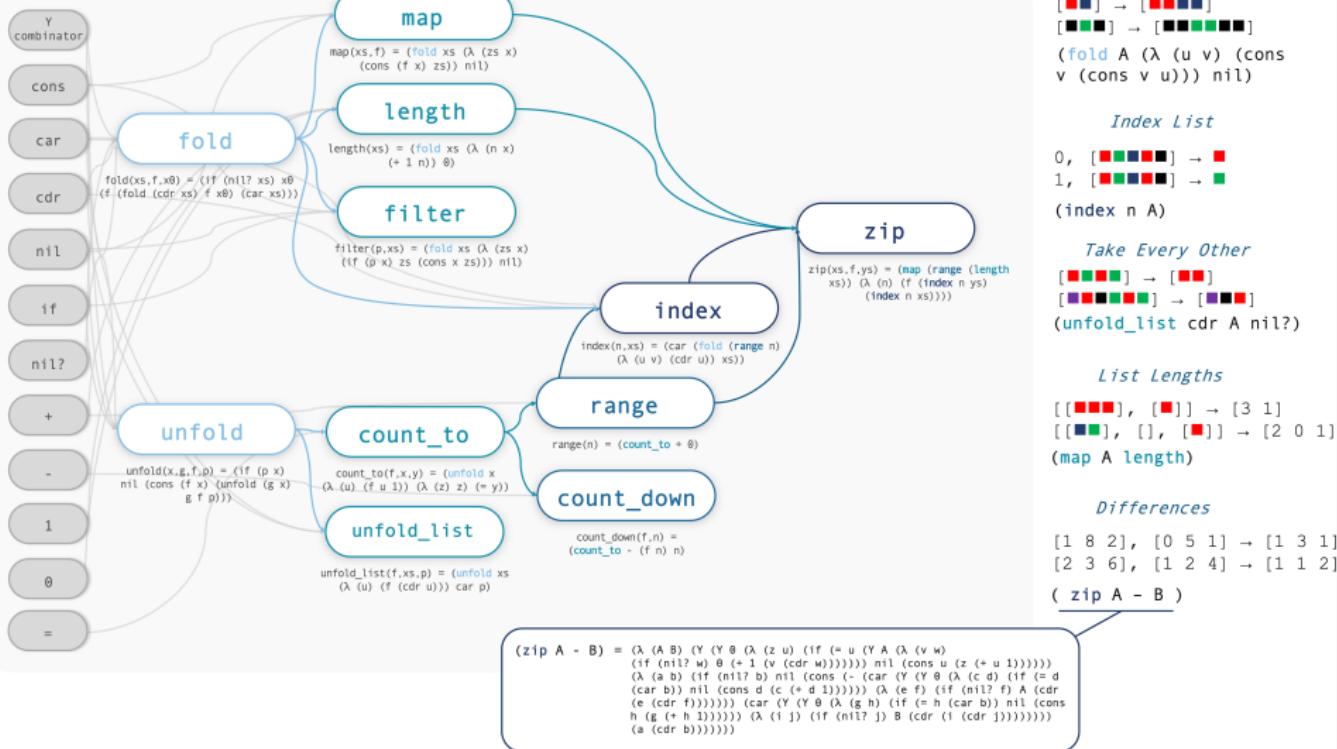
these experiments study how DreamCoder grows from a “beginner” state to “expert”:

- “beginner:” basic domain-specific procedures, only easiest problems have short solutions
- “expert:” learned library allowing hardest problems to have short meaningful solutions

go beyond: start with generic arithmetic & control flow, learn fundamentals of domain

- Physics from recursive higher-order functions
- Recursive higher-order language from 1959 proto-Lisp & Y-combinator

Rediscovering origami programming



Program in terms of initial library

From learning libraries to using interpreters

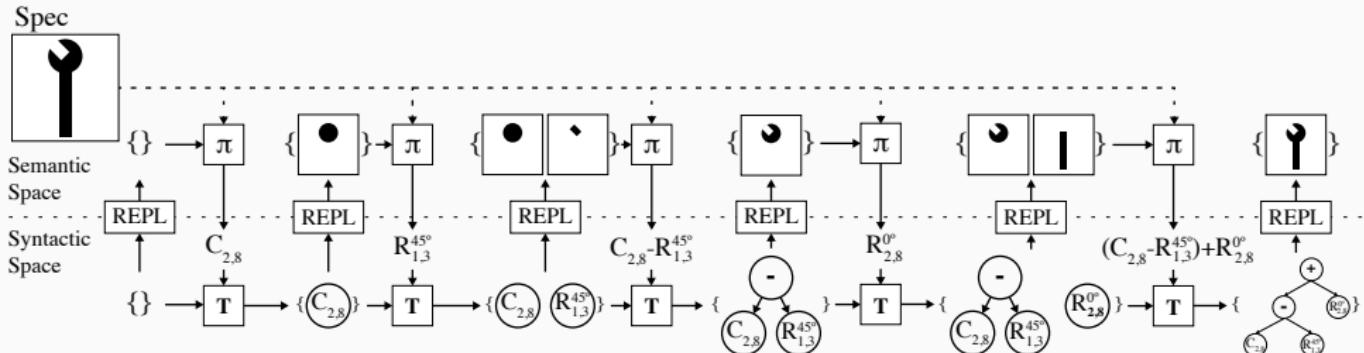
DreamCoder: library building as inspiration

Software engineers: build libraries, use interpreters, version control, debuggers, ...

Ellis*, Nye*, Pu*, Sosa*, Tenenbaum, Solar-Lezama. NeurIPS 2019.

*equal contribution

Synthesis with a REPL



REPL: Bridges syntax and semantics

π : policy, writes code conditioned on REPL state

T : Markov Decision Process transition function

Spec: Image to draw

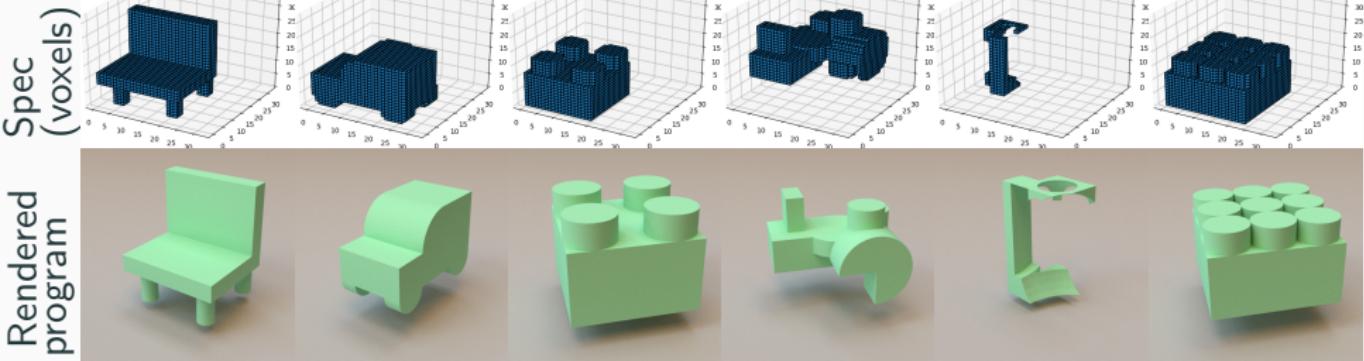
Ellis*, Nye*, Pu*, Sosa*, Tenenbaum, Solar-Lezama. NeurIPS 2019.

*equal contribution

Scaling to long programs

Branching factor: > 1.3 million per line of code

Successfully synthesizes >20-line programs in seconds (>100 tokens)



Programmatic models of the physical world

hinge



gear



doorknob

