Building Machines that Discover Generalizable, Interpretable Knowledge

Kevin Ellis

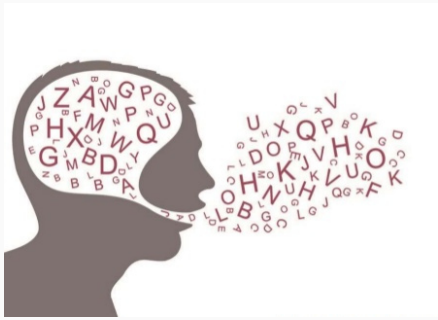
2020

MIT

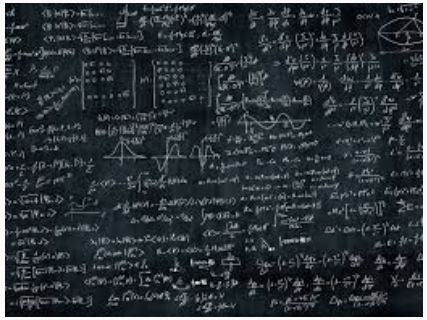
What computational problems are solved by intelligence?

an endless range of problems

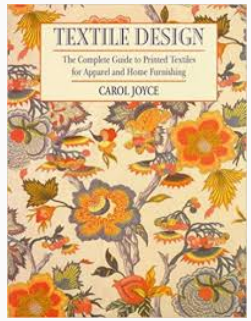
language



science

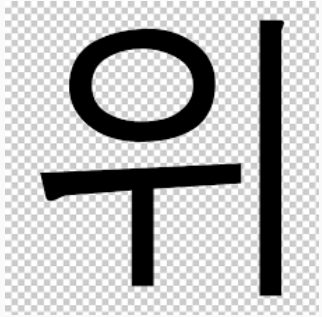


design



using new devices

writing new characters



coding

```
(MEMBER
 (LAMBDA (X L)
  (COND  ((NULL L) NIL)
         ((EQ X (FIRST L)) T)
         (T (MEMBER X (REST L))))))
```

Allen, Anatomy of Lisp, 1975

engineering

play

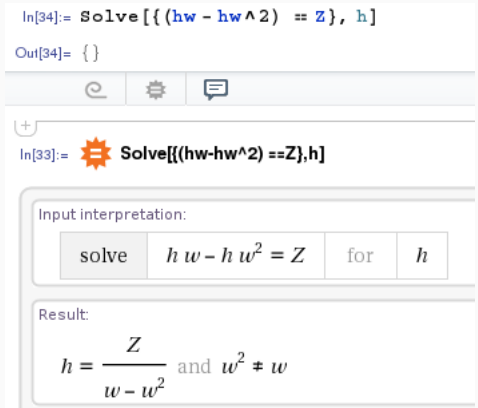
What computational frameworks can contribute to this picture?

Three AI traditions

What computational frameworks can contribute to this picture?

Three AI traditions

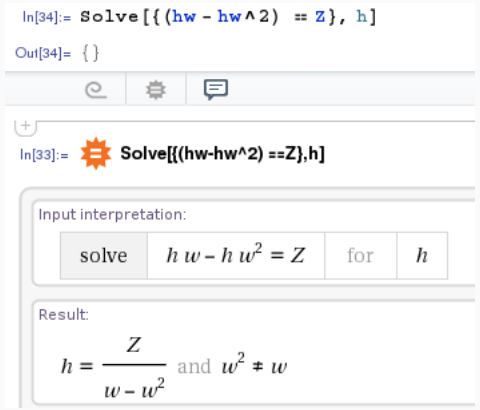
Symbolic



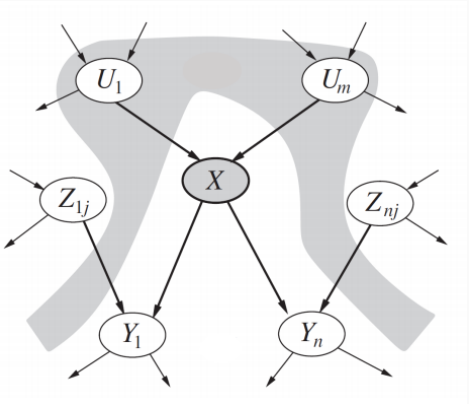
What computational frameworks can contribute to this picture?

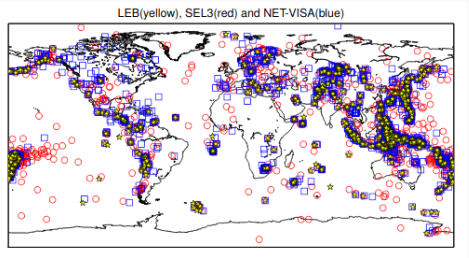
Three AI traditions

Symbolic



Probabilistic





What computational frameworks can contribute to this picture?

Three AI traditions

Symbolic


```
In[34]:= Solve[{(hw - hw^2) == Z}, h]
Out[34]= {}
```

```
In[33]:= Solve[{(hw-hw^2) ==Z},h]
```

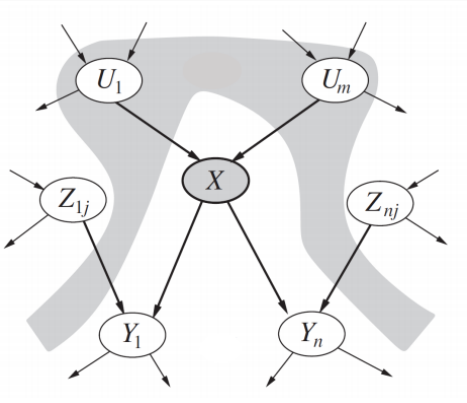
Input interpretation:

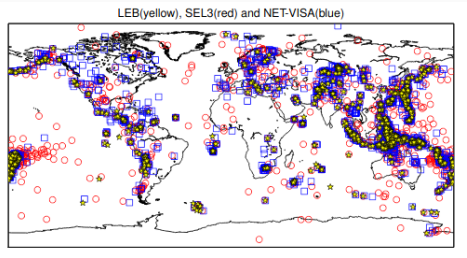
solve | $h\,w - h\,w^2 = Z$ | for | h

Result:

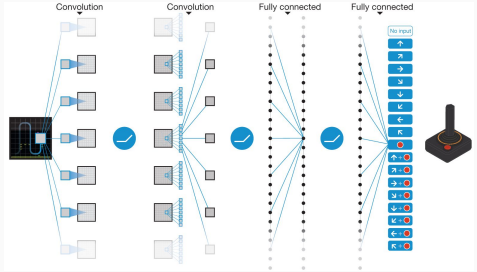
$$h = \frac{Z}{w - w^2} \quad \text{and} \quad w^2 \neq w$$

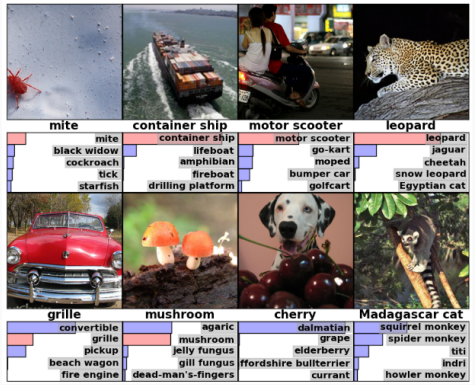
Probabilistic





Neural

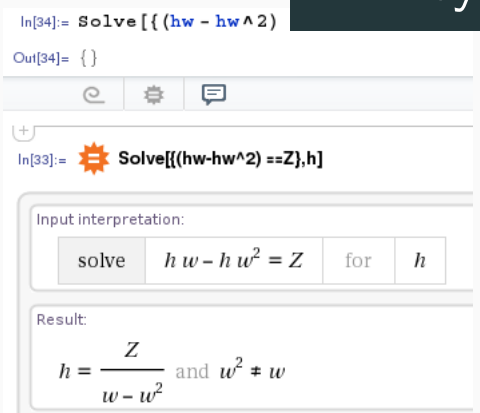




What computational frameworks can contribute to this picture?

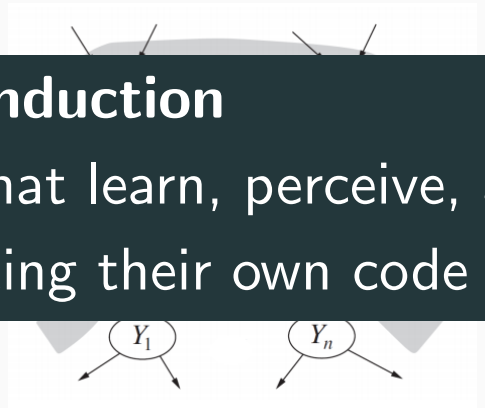
Three AI traditions

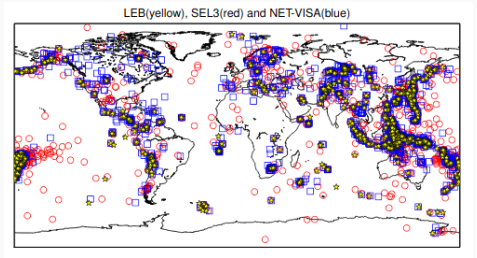
Symbolic | Probabilistic | Neural



Program induction

machines that learn, perceive, and reason, by writing their own code



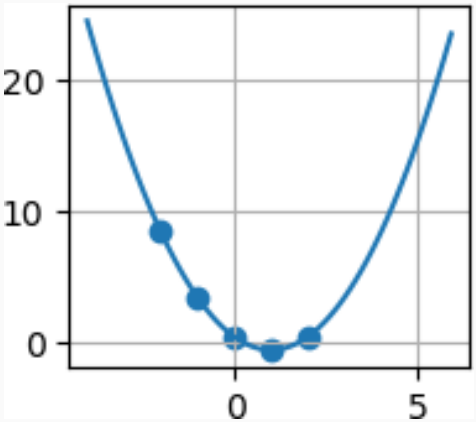


Why program induction?

Why program induction?

strong generalization

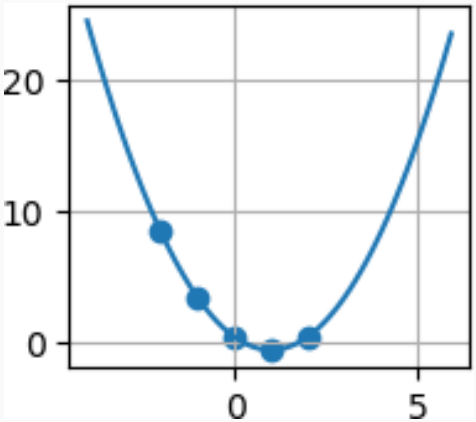
+data efficiency




```
f(x)=(x-1)**2 - 0.5
```

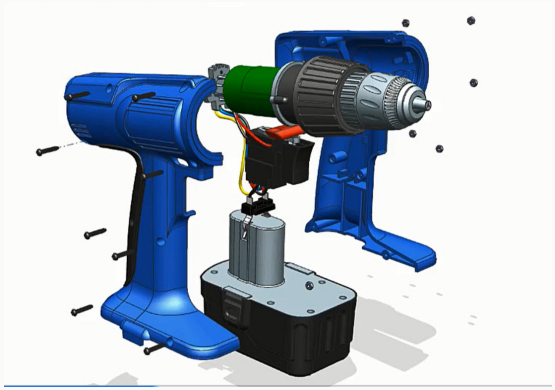
Why program induction?

strong generalization
+data efficiency

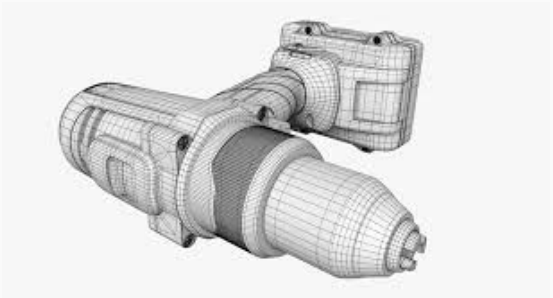



```
f(x)=(x-1)**2 - 0.5
```

interpretability

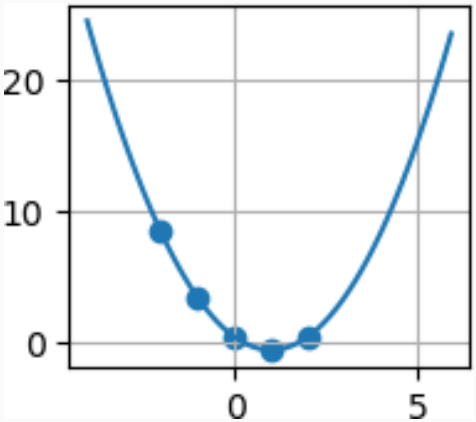


vs



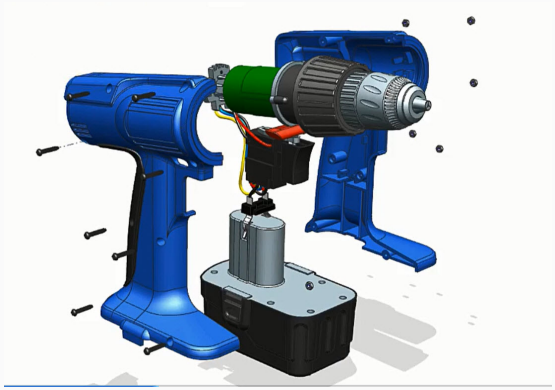
Why program induction?

strong generalization
+data efficiency

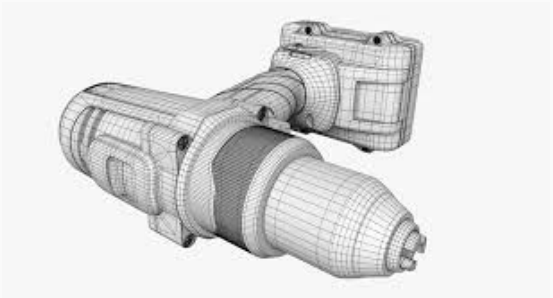



```
f(x)=(x-1)**2 - 0.5
```

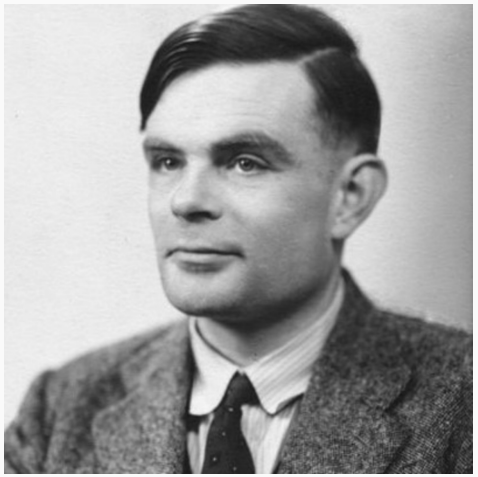
interpretability



vs

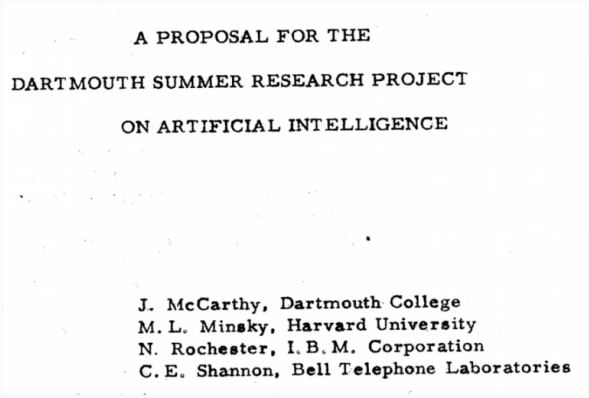


universal expressivity



Why didn't this old idea work?

Program induction goes back to the 1956 Dartmouth Workshop that founded the field of AI



A PROPOSAL FOR THE

DARTMOUTH SUMMER RESEARCH PROJECT

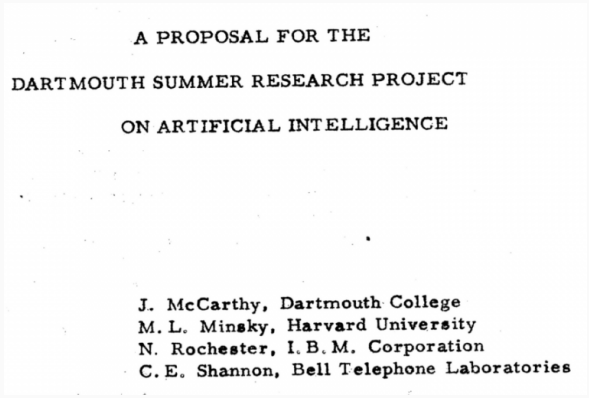
ON ARTIFICIAL INTELLIGENCE

J. McCarthy, Dartmouth College
M. L. Minsky, Harvard University
N. Rochester, I. B. M. Corporation
C. E. Shannon, Bell Telephone Laboratories

John McCarthy, Ray Solomonoff

Why didn't this old idea work?

Program induction goes back to the 1956 Dartmouth Workshop that founded the field of AI



A PROPOSAL FOR THE

DARTMOUTH SUMMER RESEARCH PROJECT

ON ARTIFICIAL INTELLIGENCE

J. McCarthy, Dartmouth College
M. L. Minsky, Harvard University
N. Rochester, I. B. M. Corporation
C. E. Shannon, Bell Telephone Laboratories

John McCarthy, Ray Solomonoff

main obstacle: combinatorial search is hard

Why try again?

better toolkits: neural+probabilistic+symbolic, and knowing how to combine them

Why try again?

better toolkits: neural+probabilistic+symbolic, and knowing how to combine them

maturing **program synthesis** techniques

Why try again?

better toolkits: neural+probabilistic+symbolic, and knowing how to combine them

maturing **program synthesis** techniques

better compute+parallel algorithms

A lesson from the AI winter

We need an on-ramp of practical, tractable problems:

semantic parsing [Liang et al. 2011; Zettlemoyer et al. 2007]

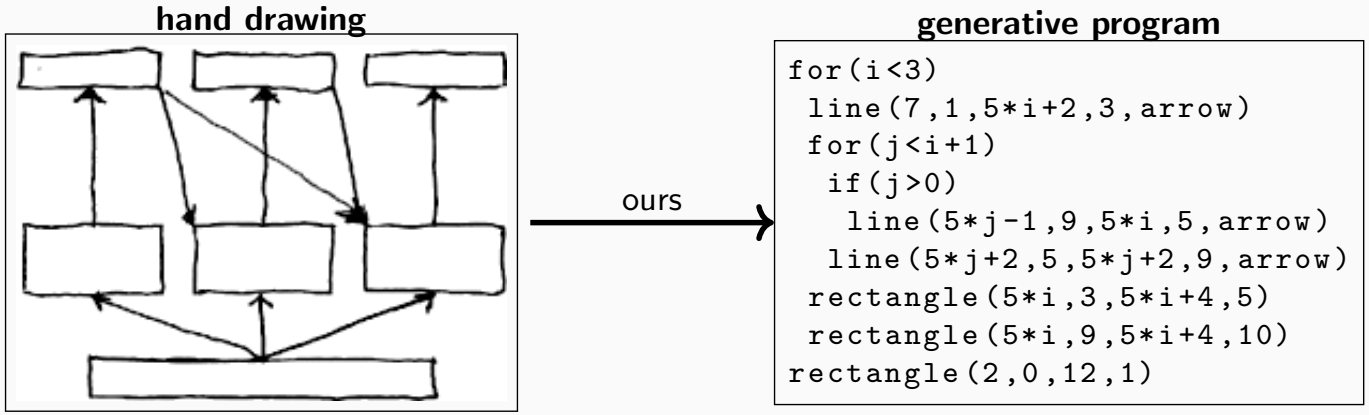
programming by examples [Gulwani 2011]

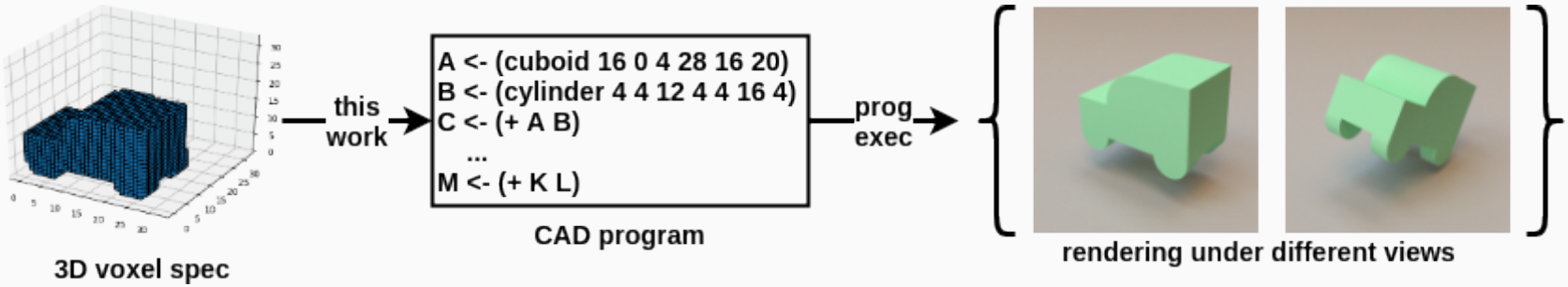
computer-aided-programming [Solar-Lezama 2008]

inverse procedural modeling [Kulkarni et al. 2015]

Perception, Synthesizing models, Learning-to-Learn

Theme #1: high-level visual understanding, pixels→programs

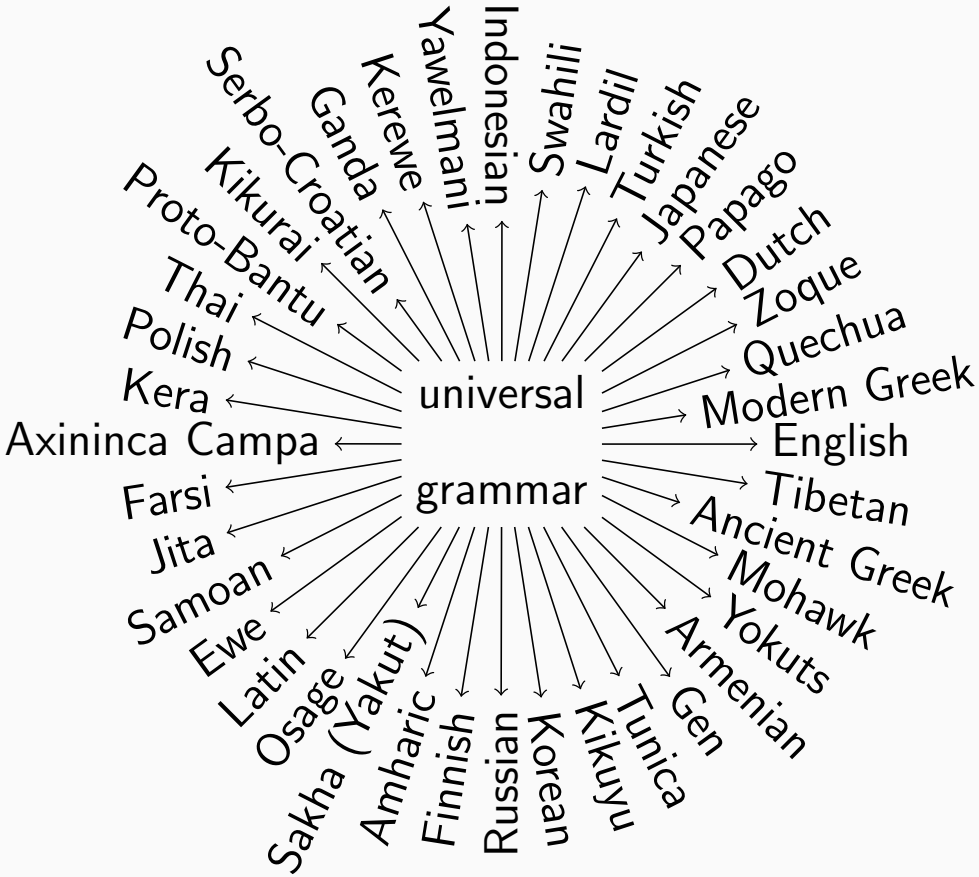




Perception, Synthesizing models, Learning-to-Learn

Theme #1: high-level visual understanding, pixels→programs

Theme #2: synthesizing human-understandable models



Perception, Synthesizing models, Learning-to-Learn

Theme #1: high-level visual understanding, pixels→programs

Theme #2: Synthesizing human-understandable models

Theme #3: learning to synthesize programs

List Processing

Sum List

```
[1 2 3] → 6
[4 6 8 1] → 17
```

Double

```
[1 2 3] → [2 4 6]
[4 5 1] → [8 10 2]
```

Text Editing

Abbreviate

```
Allen Newell → A.N.
Herb Simon → H.S.
```

Drop Last Three

```
shrdlu → shr
shakey → sha
```

Regexes

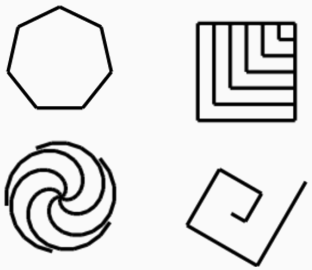
Phone numbers

```
(555) 867-5309
(650) 555-2368
```

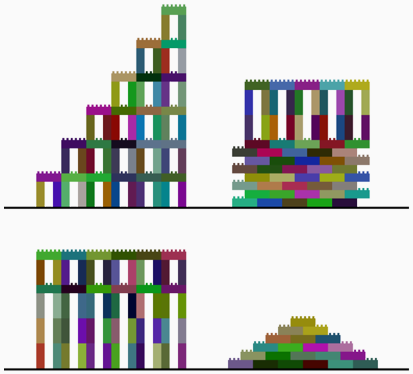
Currency

```
$100.25
$4.50
```

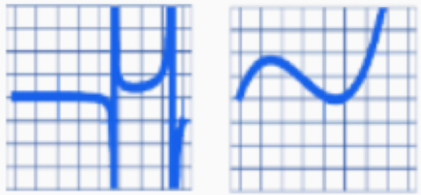
LOGO Graphics



Block Towers



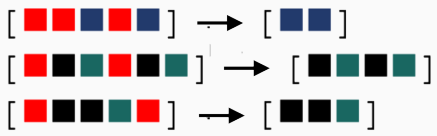
Symbolic Regression



$$y = f(x)$$

Recursive Programming

Filter Red



Physical Laws

$$\vec{a} = \frac{1}{m}\sum_i \vec{F}_i$$

$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r}|^2}\hat{r}$$

Program Induction and perception

model discovery

learning to learn

Vision is more than knowing what is where

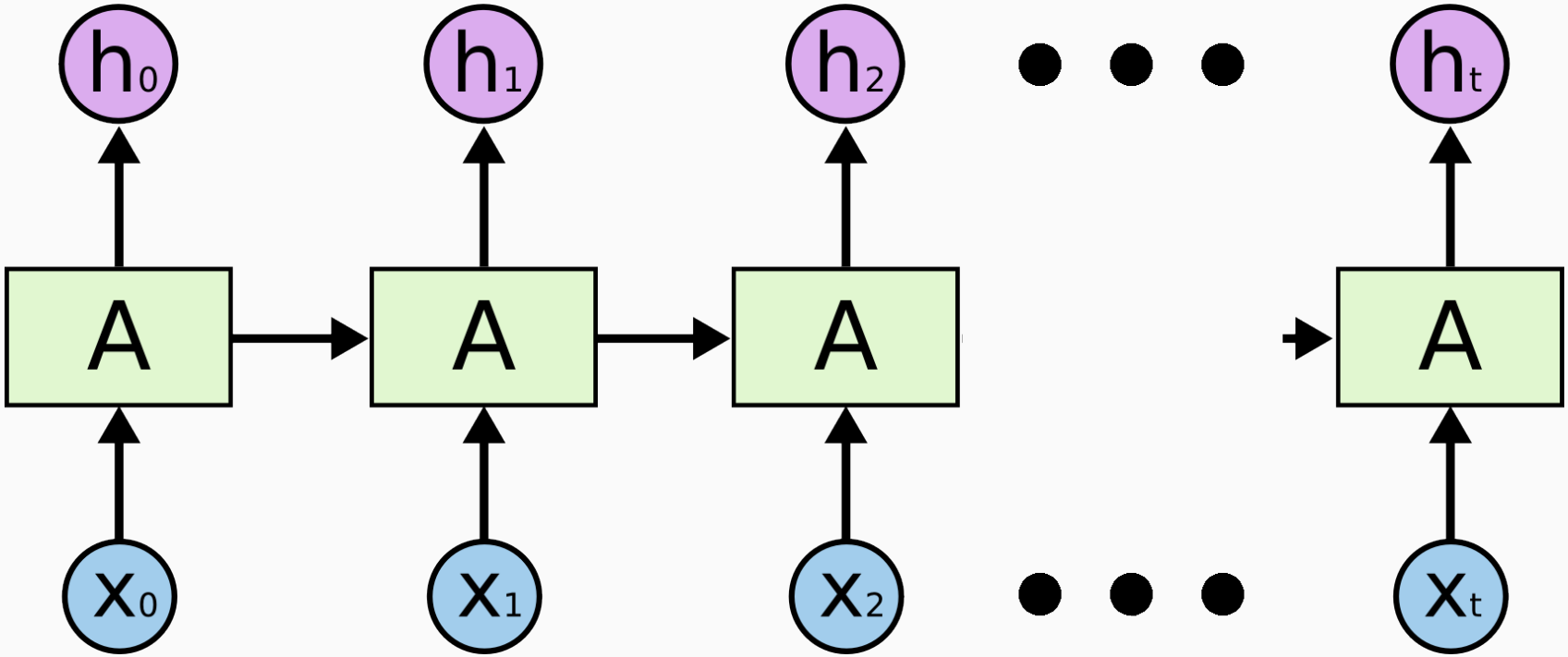
Can you visually extrapolate this aqueduct?

Vision is more than knowing what is where

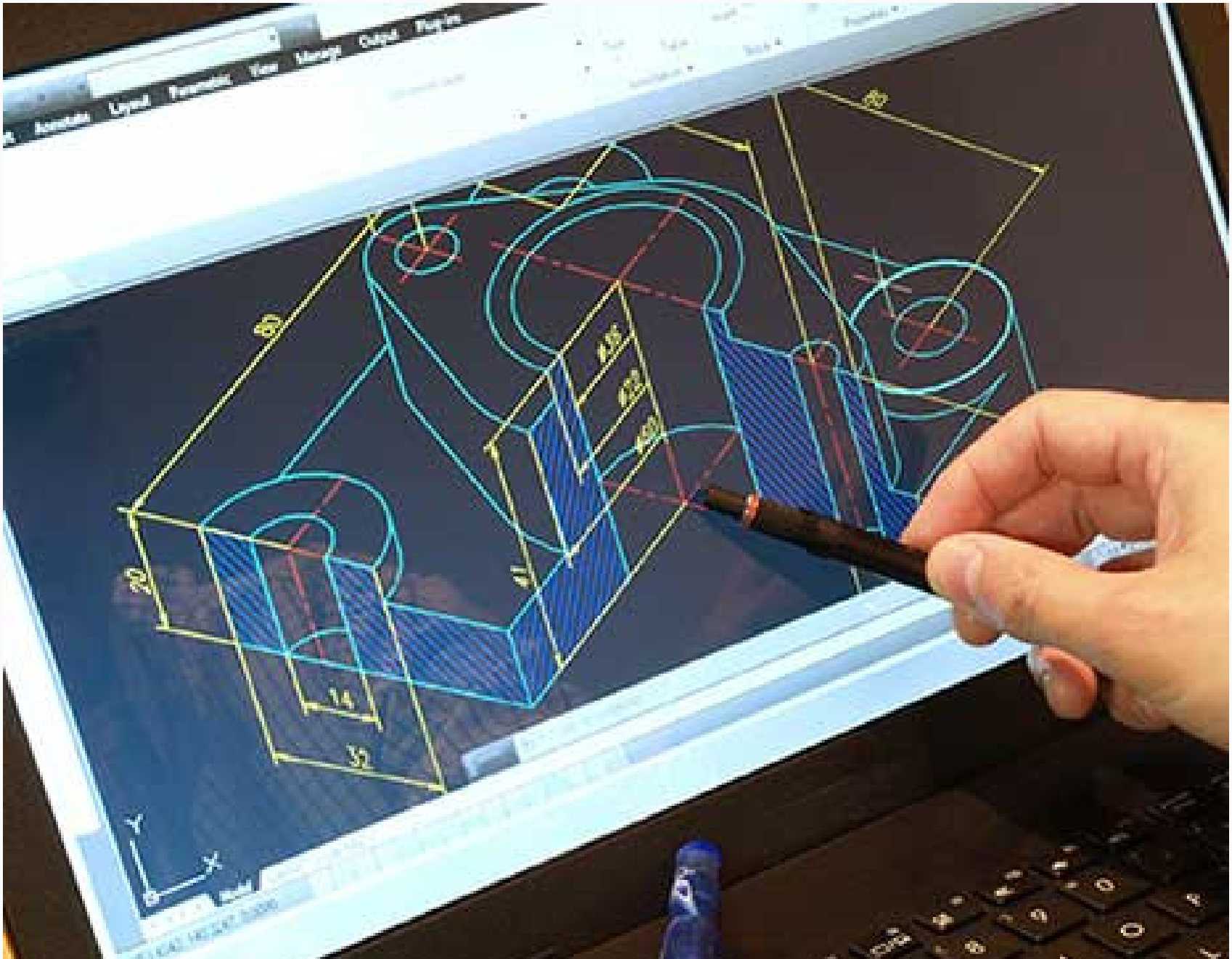
Can you visually extrapolate this aqueduct?

Vision is more than knowing what is where

Can you infer what goes in the ellipses?

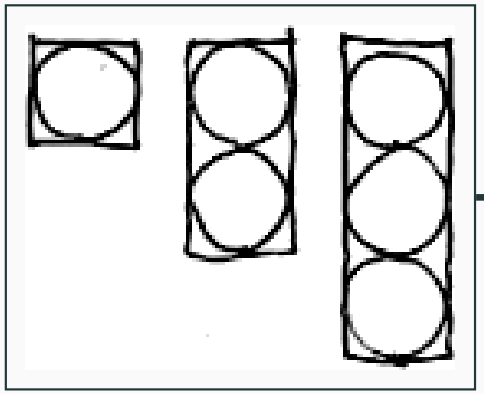


Vision is more than knowing what is where



Learning to infer graphics programs from hand-drawn images

hand drawing



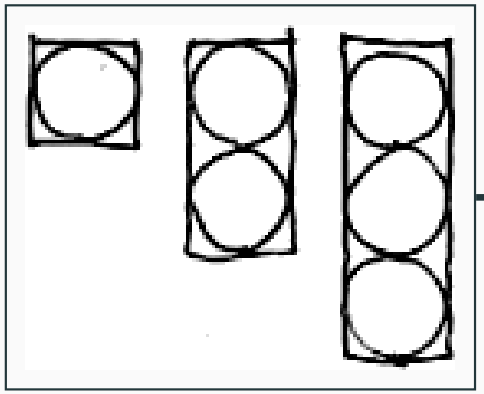
machine-generated program

```
for (0 <= i < 3)
 rectangle(3*i,-2*i+4,
           3*i+2,6)
 for (0 <= j < i + 1)
  circle(3*i+1,-2*j+5)
```

Ellis, Ritchie, Solar-Lezama, Tenenbaum. NeurIPS 2018.

Learning to infer graphics programs from hand-drawn images

hand drawing



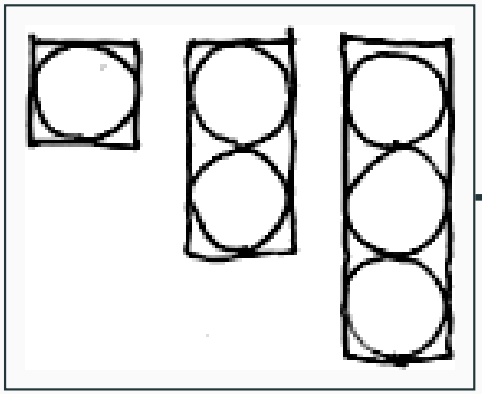
machine-generated program

```
for (0 <= i < 3 + 1)
 rectangle(3*i,-2*i+4,
           3*i+2,6)
 for (0 <= j < i + 1)
  circle(3*i+1,-2*j+5)
```

Ellis, Ritchie, Solar-Lezama, Tenenbaum. NeurIPS 2018.

Learning to infer graphics programs from hand-drawn images

hand drawing



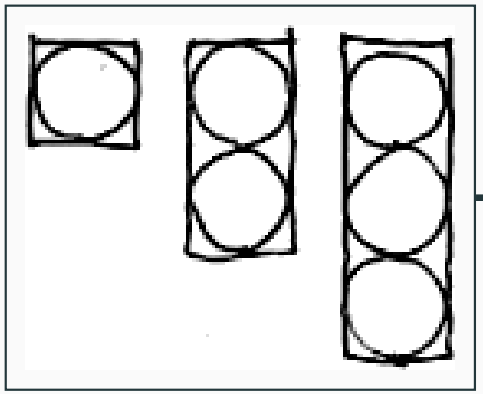
machine-generated program

```
for (0 <= i < 3 + 1)
 rectangle(3*i,-2*i+4,
           3*i+2,6)
 for (0 <= j < i + 1)
  circle(3*i+1,-2*j+5)
```

Ellis, Ritchie, Solar-Lezama, Tenenbaum. NeurIPS 2018.

Learning to infer graphics programs from hand-drawn images

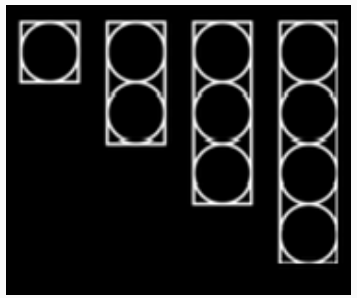
hand drawing



machine-generated program

```
for (0 <= i < 3 + 1)
 rectangle(3*i,-2*i+4,
           3*i+2,6)
 for (0 <= j < i + 1)
  circle(3*i+1,-2*j+5)
```

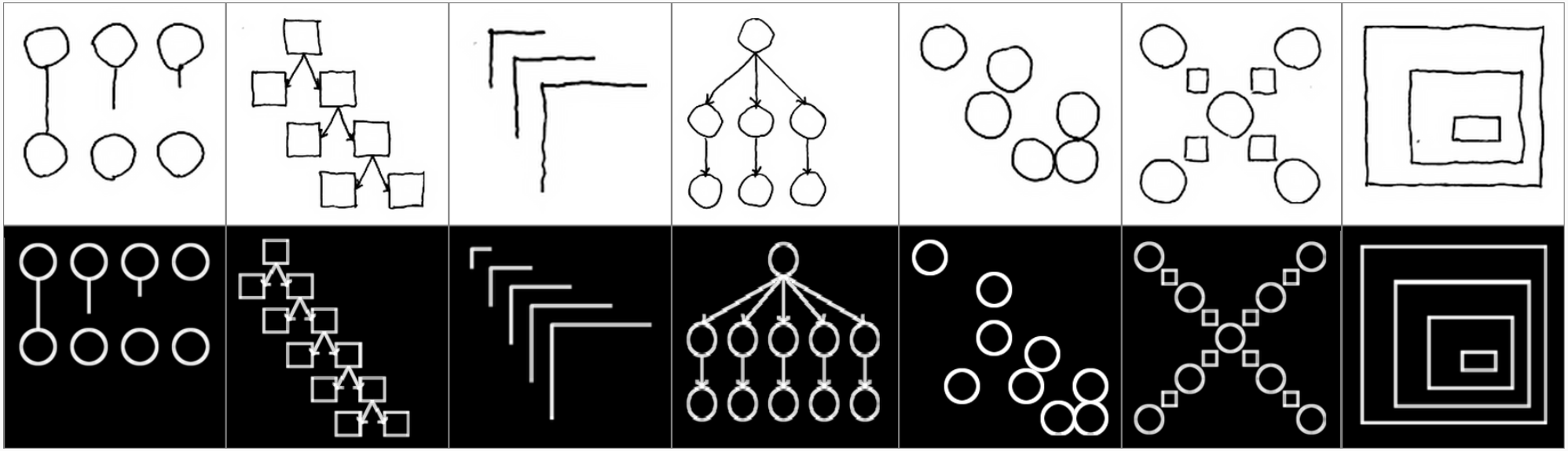
autogenerated extrapolation



one-shot generalization / extrapolation

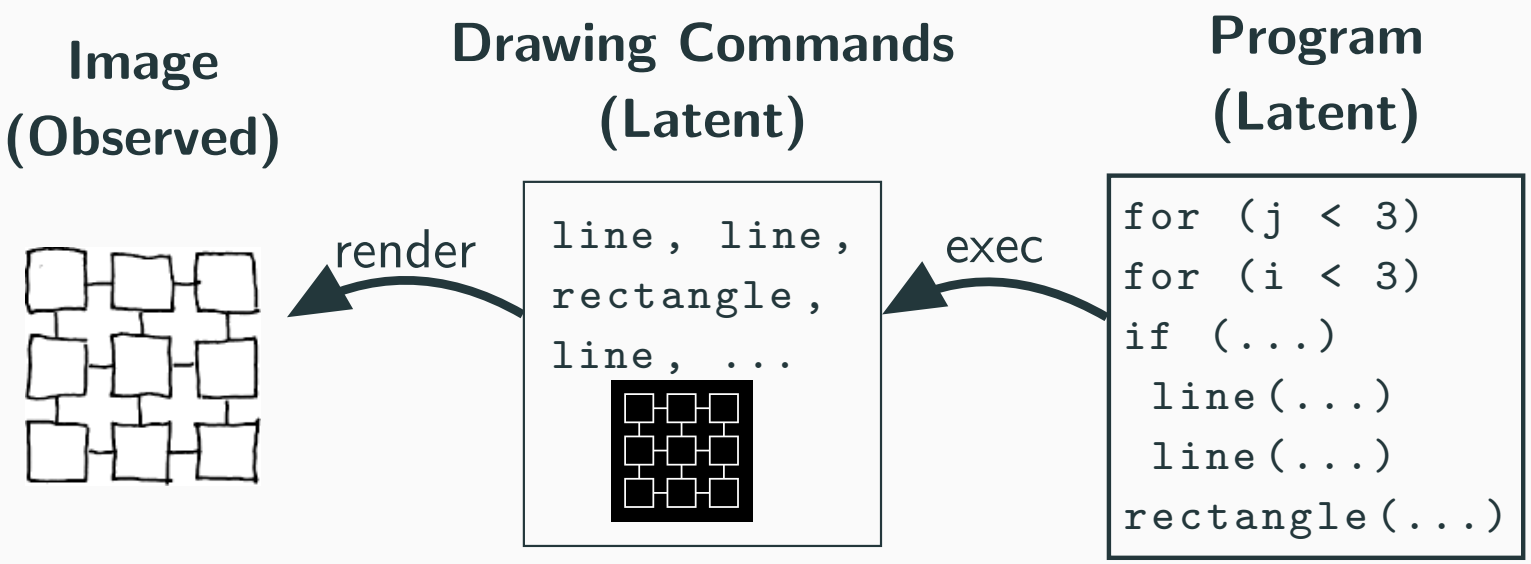
Ellis, Ritchie, Solar-Lezama, Tenenbaum. NeurIPS 2018.

Extrapolation from a single image



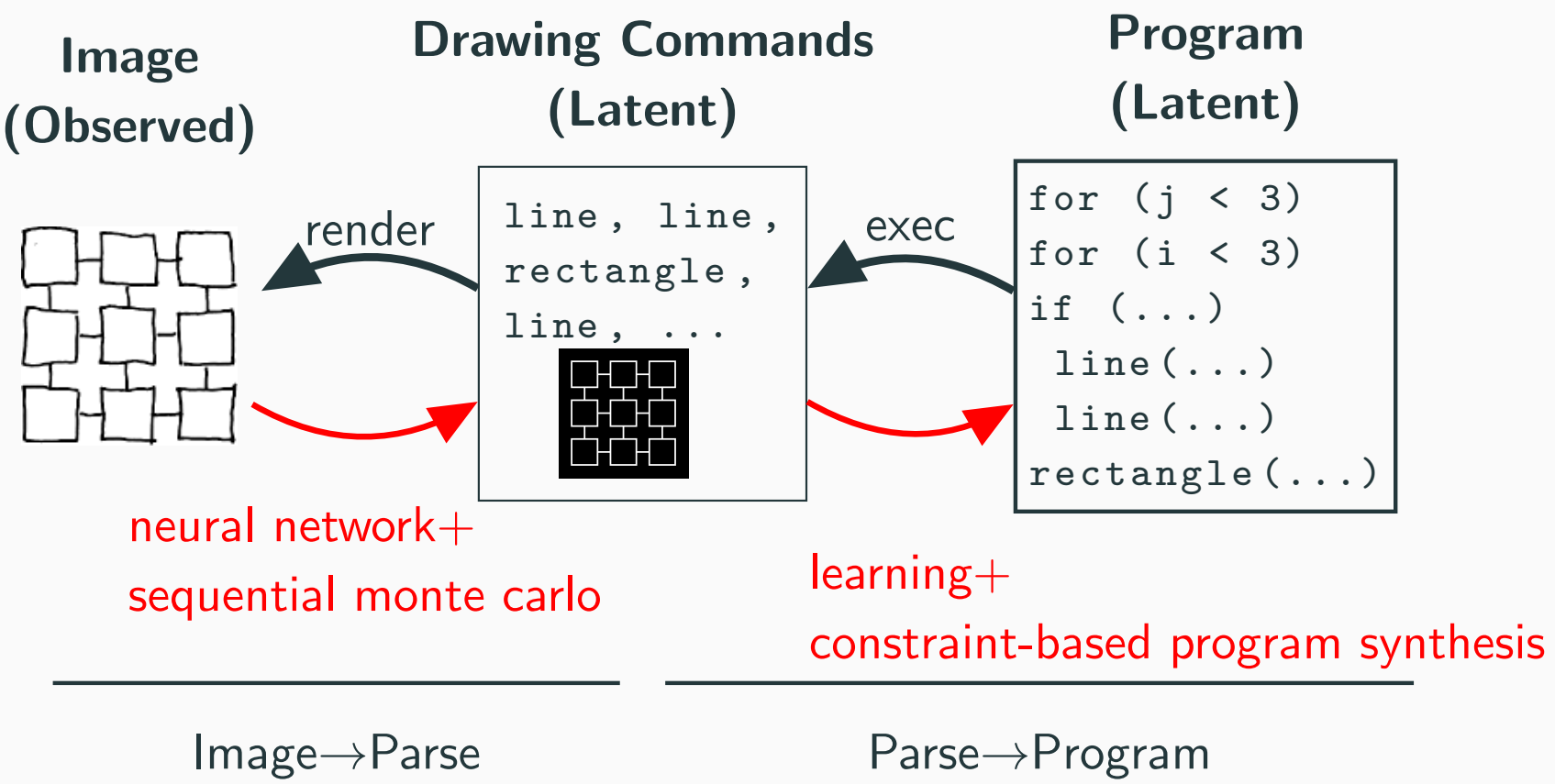
Ellis, Ritchie, Solar-Lezama, Tenenbaum. NeurIPS 2018.

How to infer graphics programs from hand-drawn images



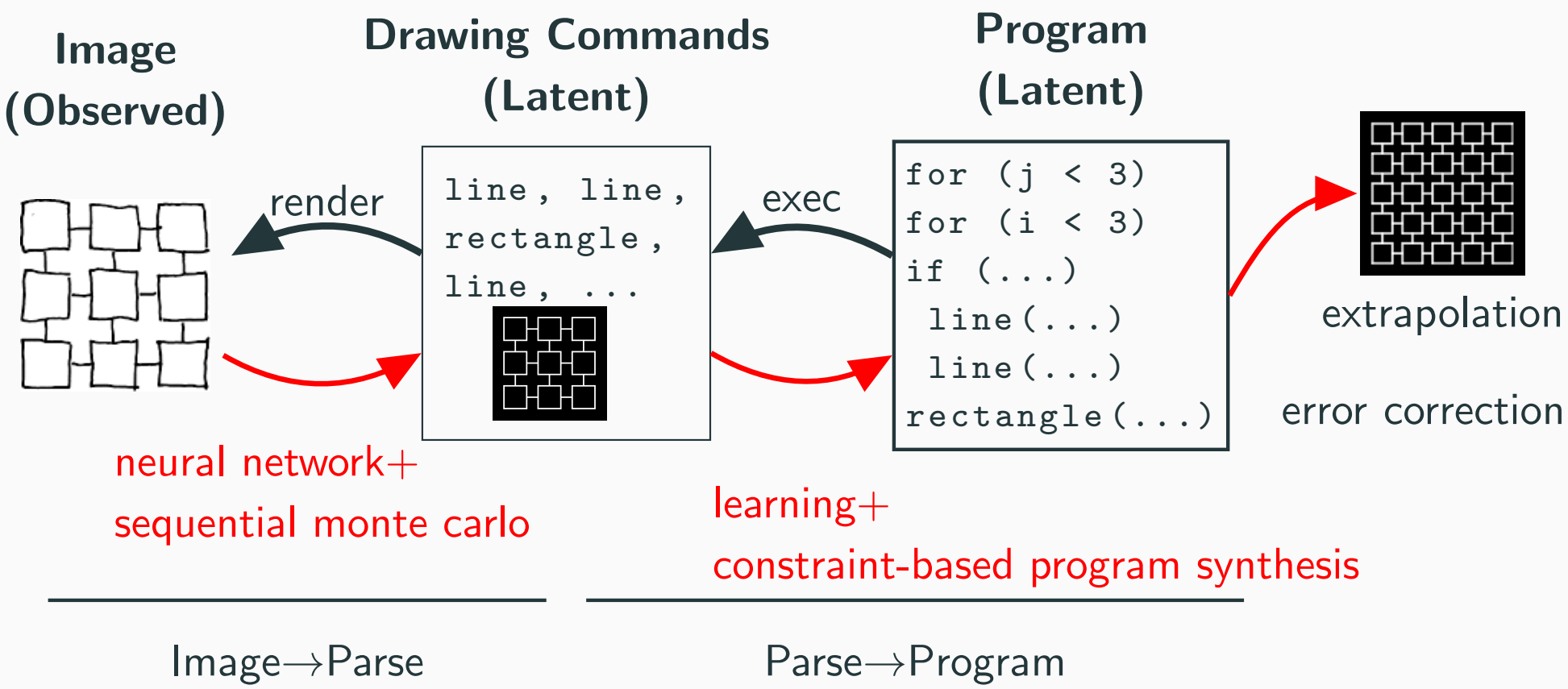
Ellis, Ritchie, Solar-Lezama, Tenenbaum. NeurIPS 2018.

How to infer graphics programs from hand-drawn images



Ellis, Ritchie, Solar-Lezama, Tenenbaum. NeurIPS 2018.

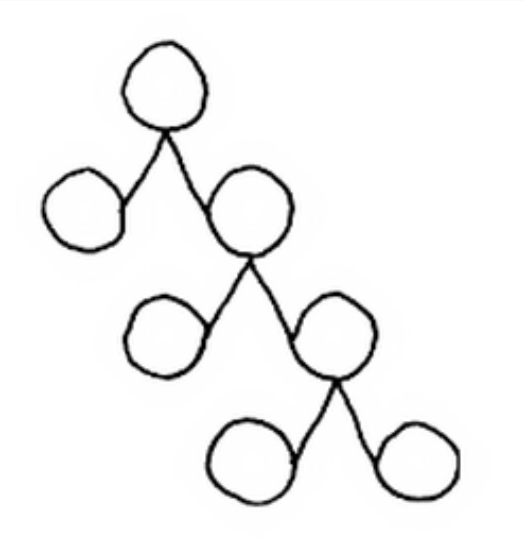
How to infer graphics programs from hand-drawn images



Ellis, Ritchie, Solar-Lezama, Tenenbaum. NeurIPS 2018.

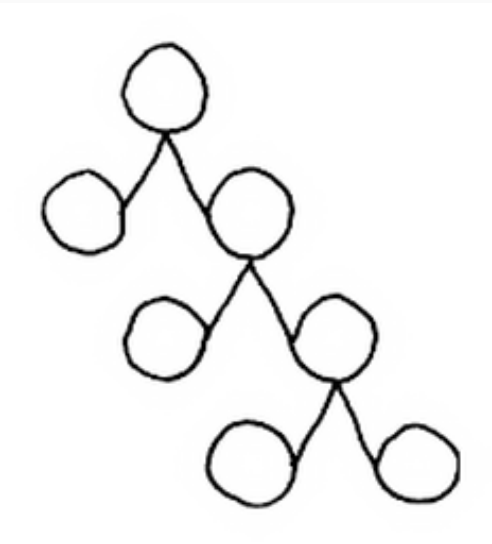
Top-down influences on perception

drawing

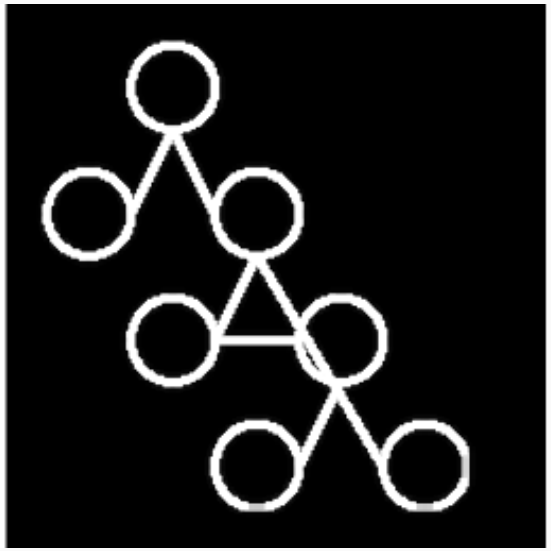


Top-down influences on perception

drawing

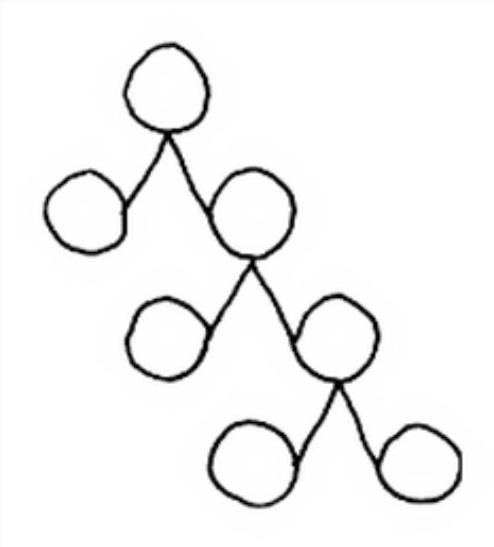


bottom-up neural net

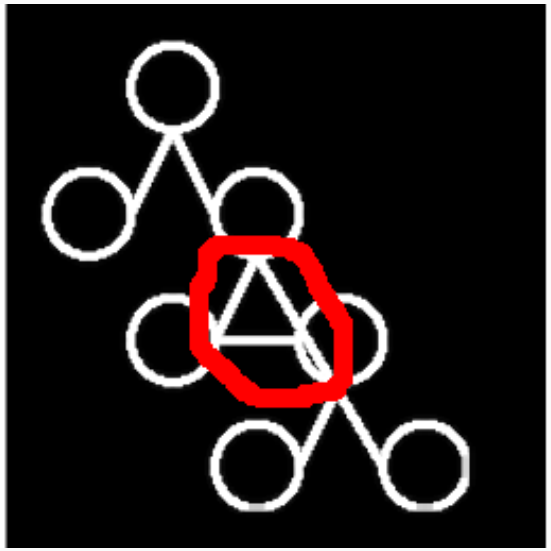


Top-down influences on perception

drawing

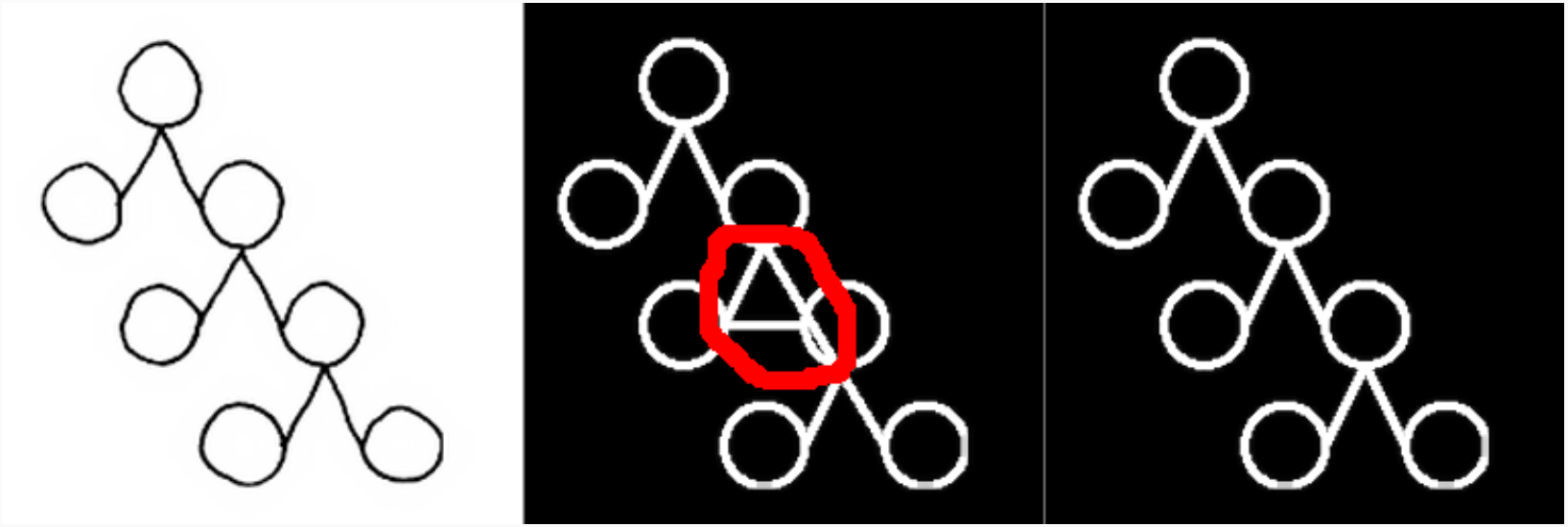


bottom-up neural net



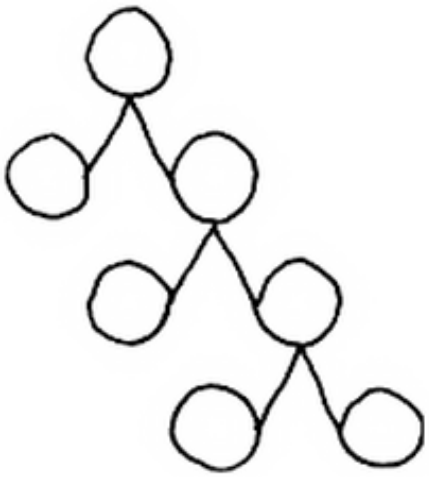
Top-down influences on perception

drawing | bottom-up neural net | w/ top-down program bias

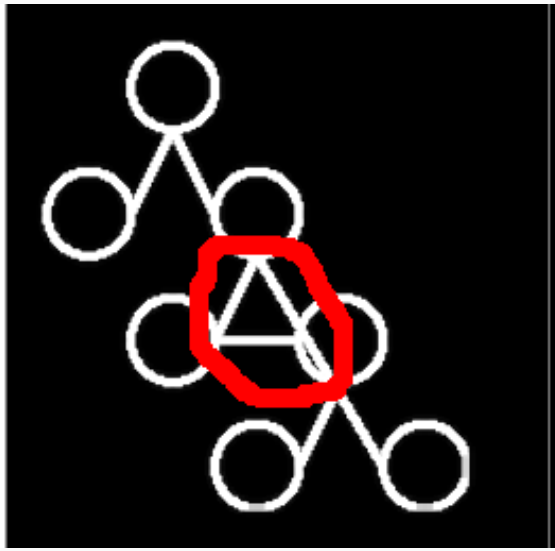


Top-down influences on perception

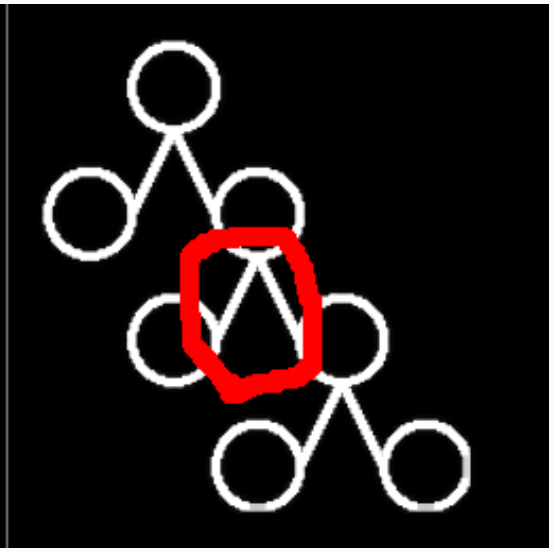
drawing



bottom-up neural net

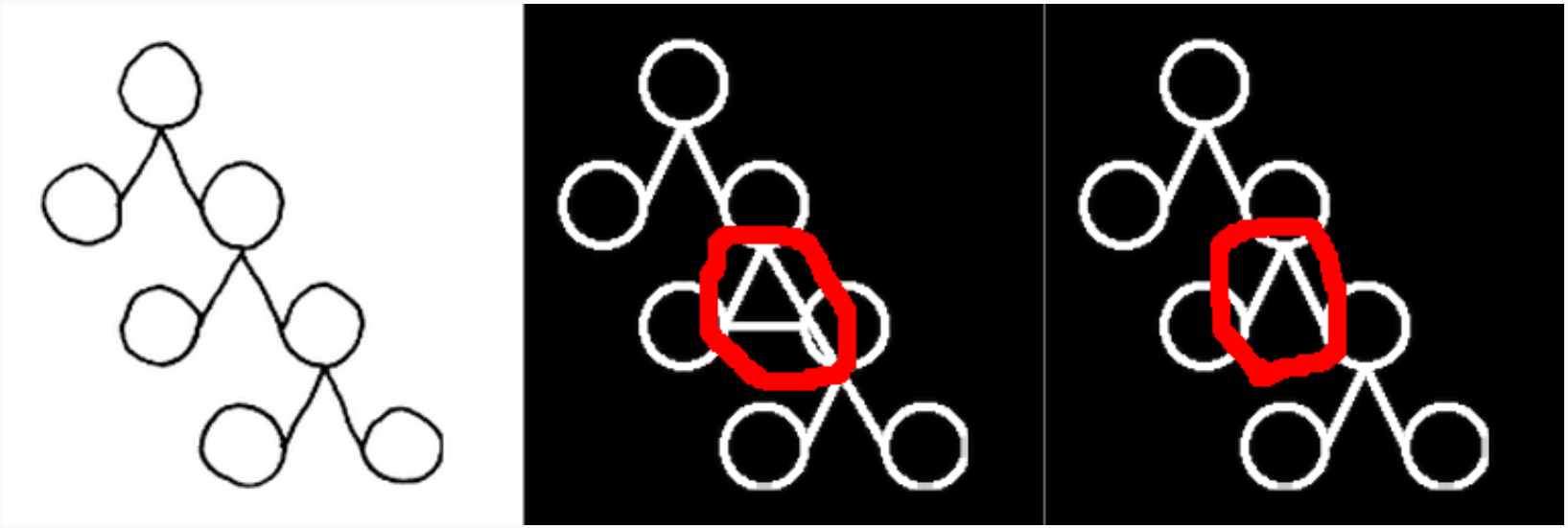


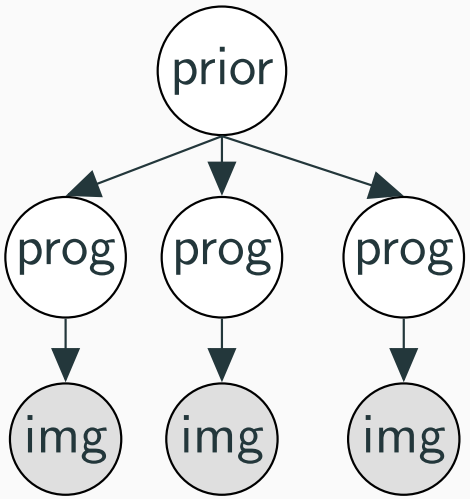
w/ top-down program bias



Top-down influences on perception

drawing | bottom-up neural net | w/ top-down program bias



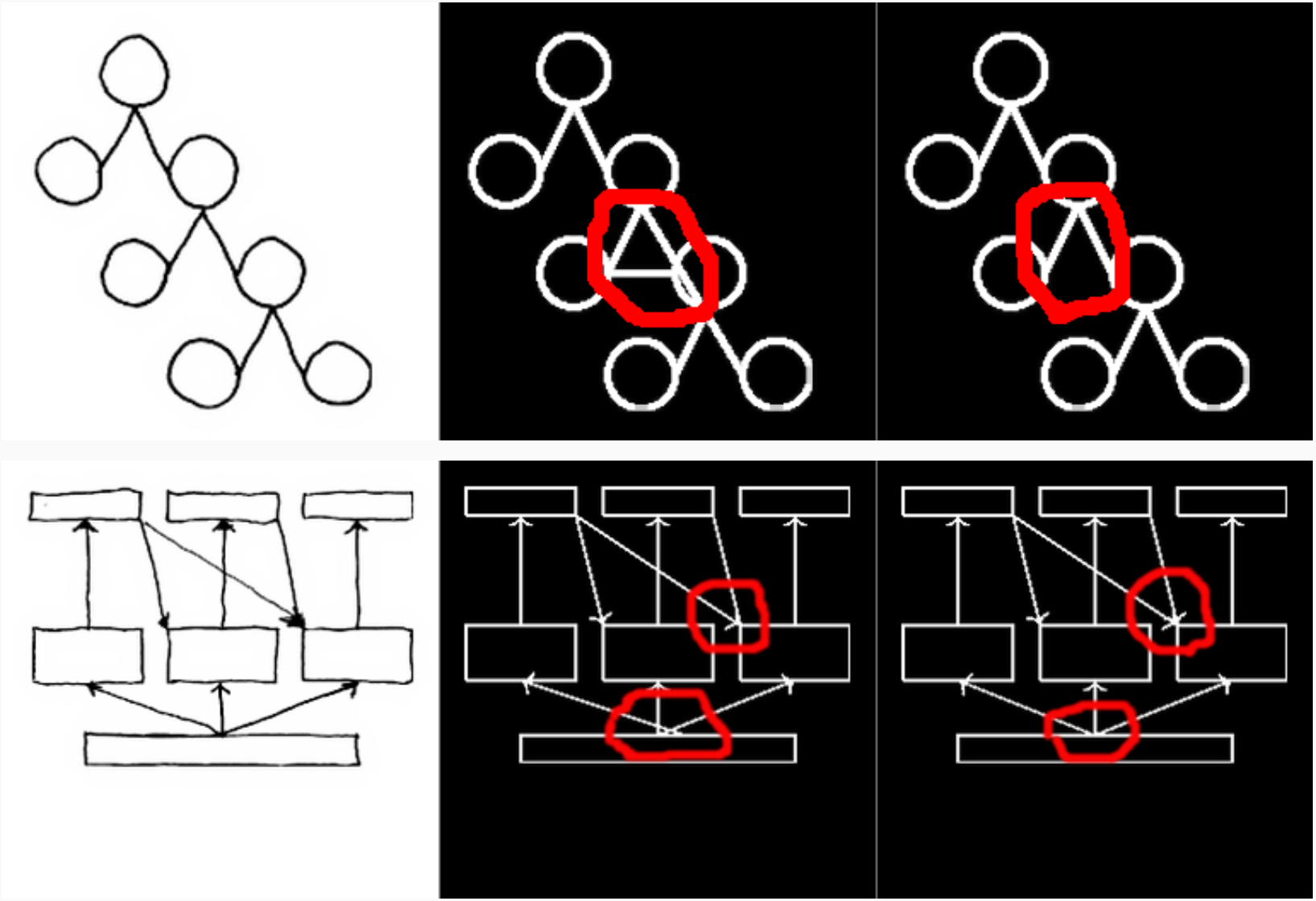


predicted program =

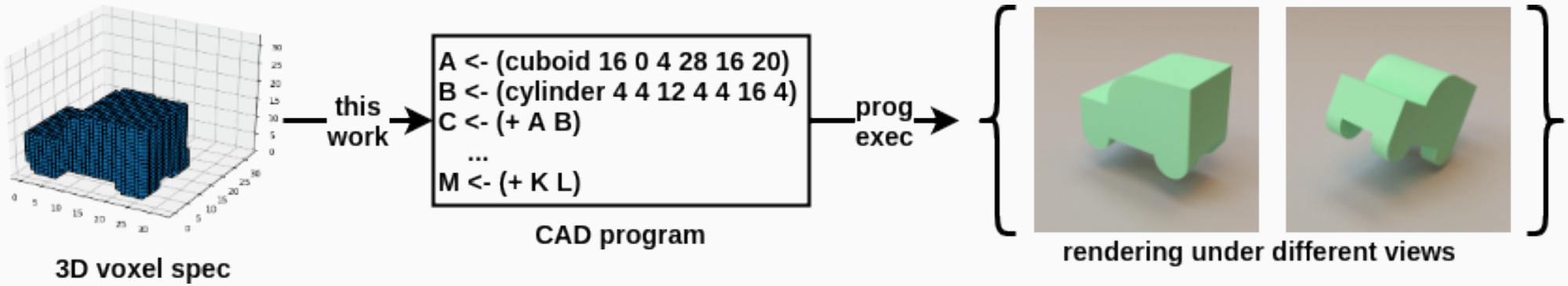
$\arg\max_{\text{progs}} \mathbb{P}[\text{img}|\text{prog}]\,\mathbb{P}[\text{prog}|\text{prior}]$

Top-down influences on perception

drawing | bottom-up neural net | w/ top-down program bias



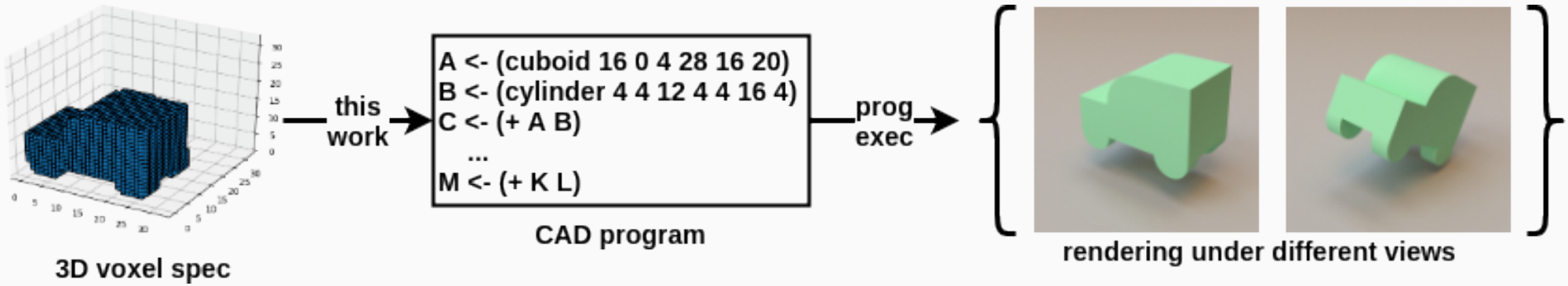
3D program induction



Ellis*, Nye*, Pu*, Sosa*, Tenenbaum, Solar-Lezama. NeurIPS 2019.

*equal contribution

3D program induction



Challenge: combinatorial search!

Branching factor: > 1.3 million per line of code, ≈ 20 lines of code

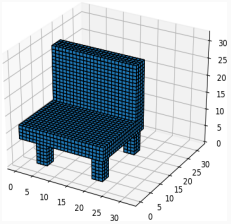
search space size: $(1.3 \text{ million})^{20} \approx 10^{122}$ programs

Ellis*, Nye*, Pu*, Sosa*, Tenenbaum, Solar-Lezama. NeurIPS 2019.

*equal contribution

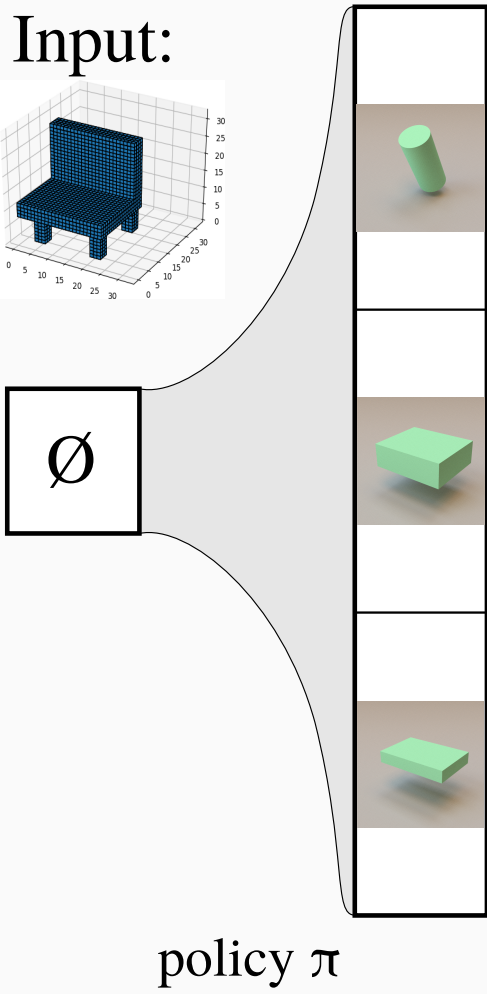
Solution: stochastic **tree search** + learn **policy** that writes code + learn **value** function that assesses execution of program so far; analogous to **AlphaGo** [Silver et al. 2016]

Input:

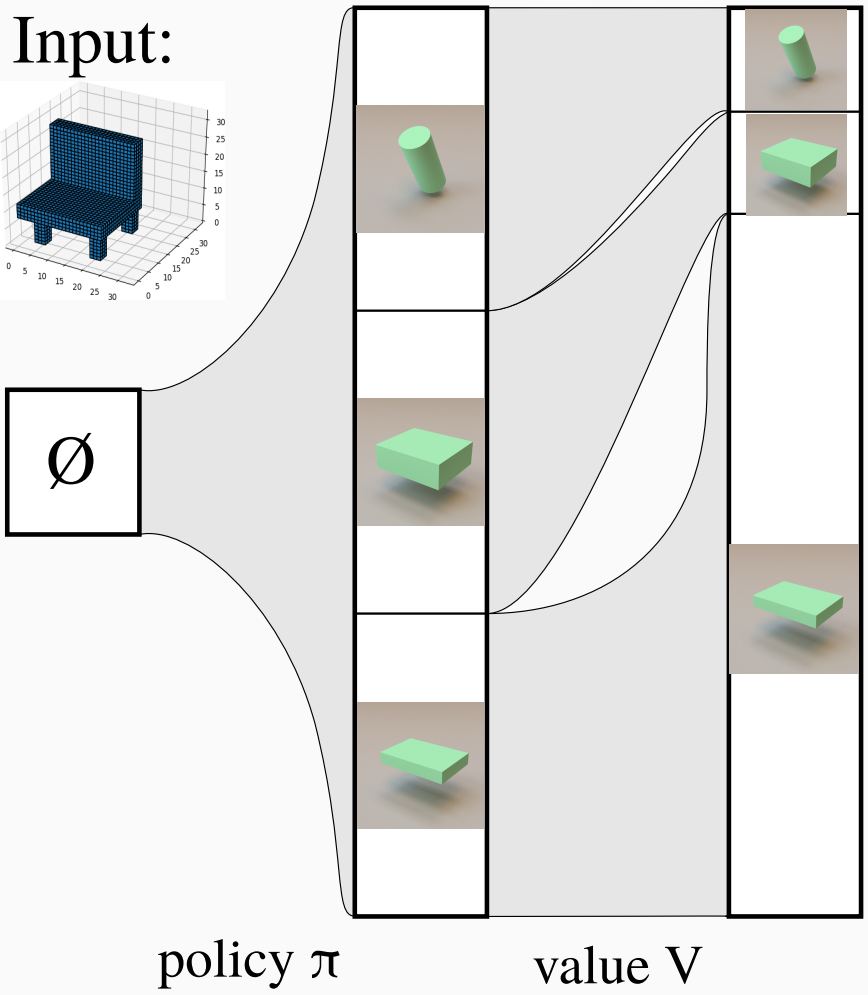


Ø

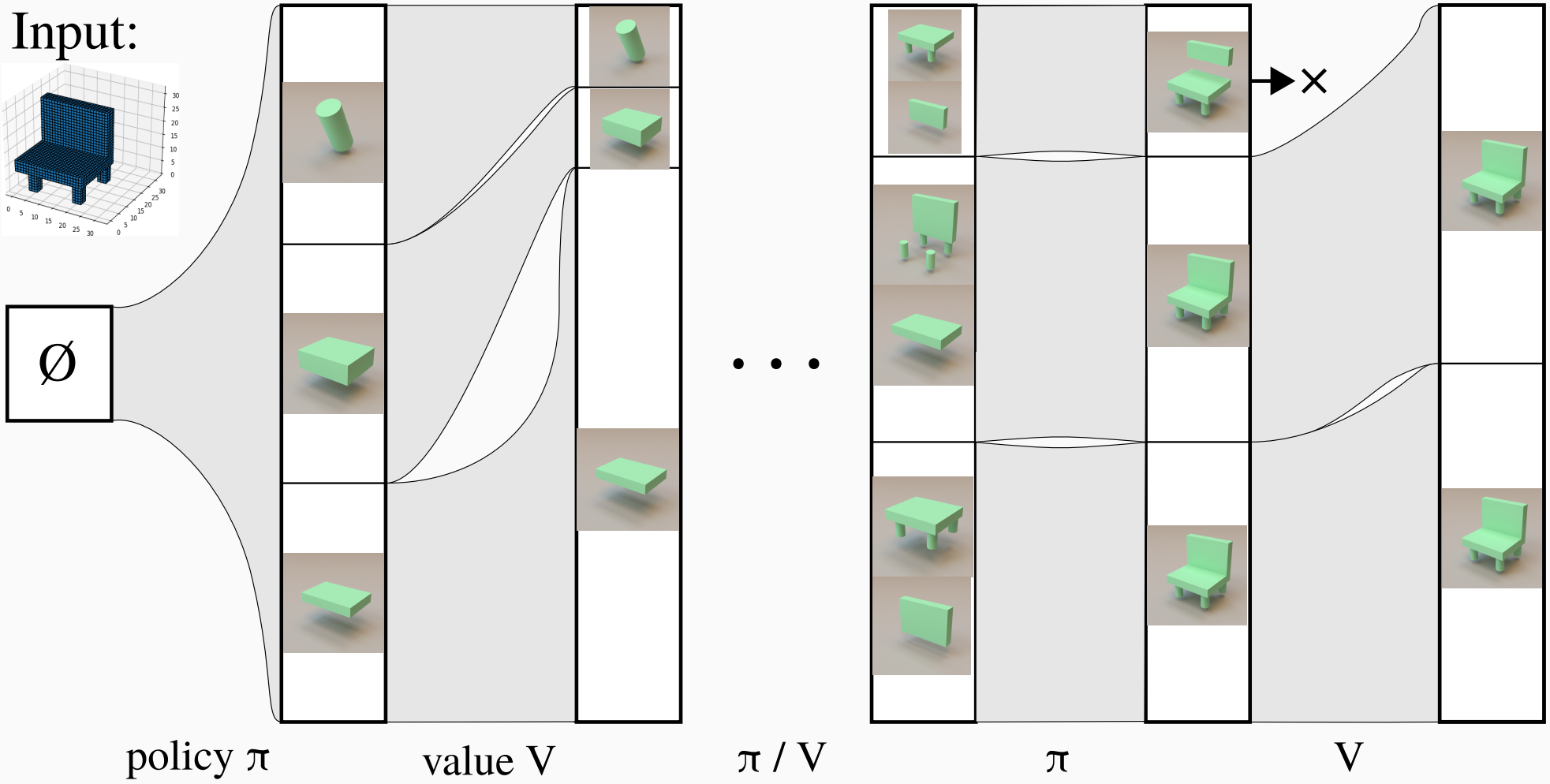
Solution: stochastic **tree search** + learn **policy** that writes code + learn **value** function that assesses execution of program so far; analogous to **AlphaGo** [Silver et al. 2016]



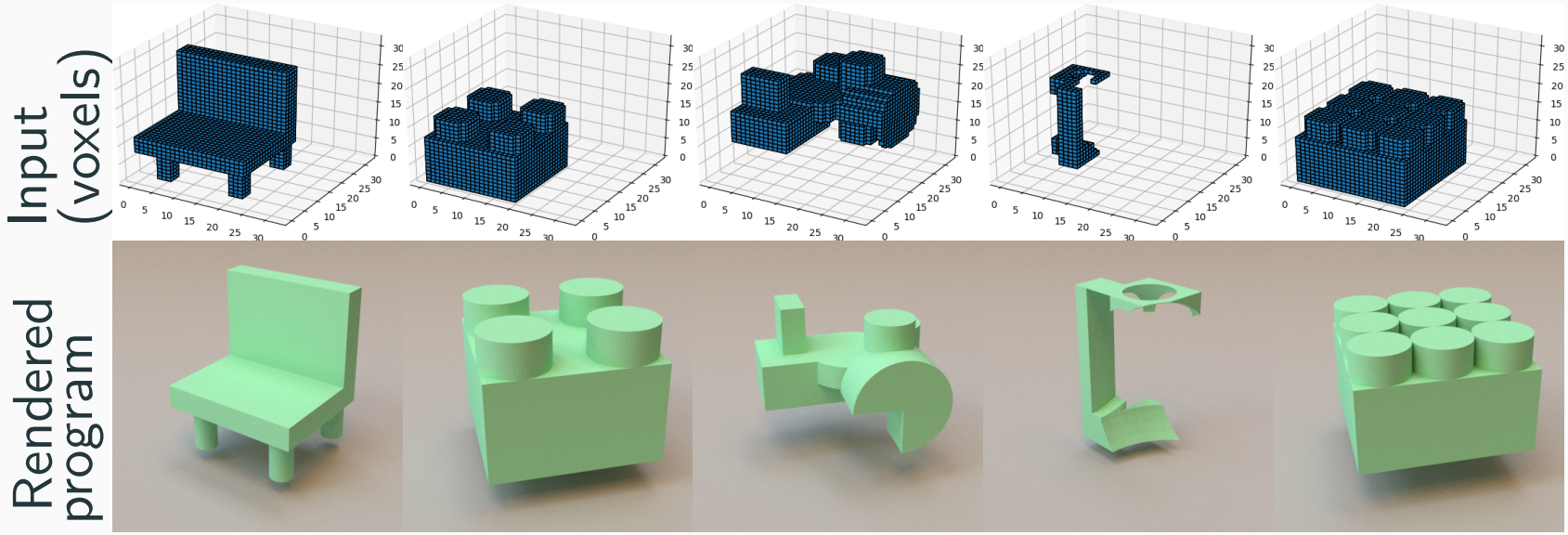
Solution: stochastic **tree search** + learn **policy** that writes code + learn **value** function that assesses execution of program so far; analogous to **AlphaGo** [Silver et al. 2016]



Solution: stochastic **tree search** + learn **policy** that writes code + learn **value** function that assesses execution of program so far; analogous to **AlphaGo** [Silver et al. 2016]



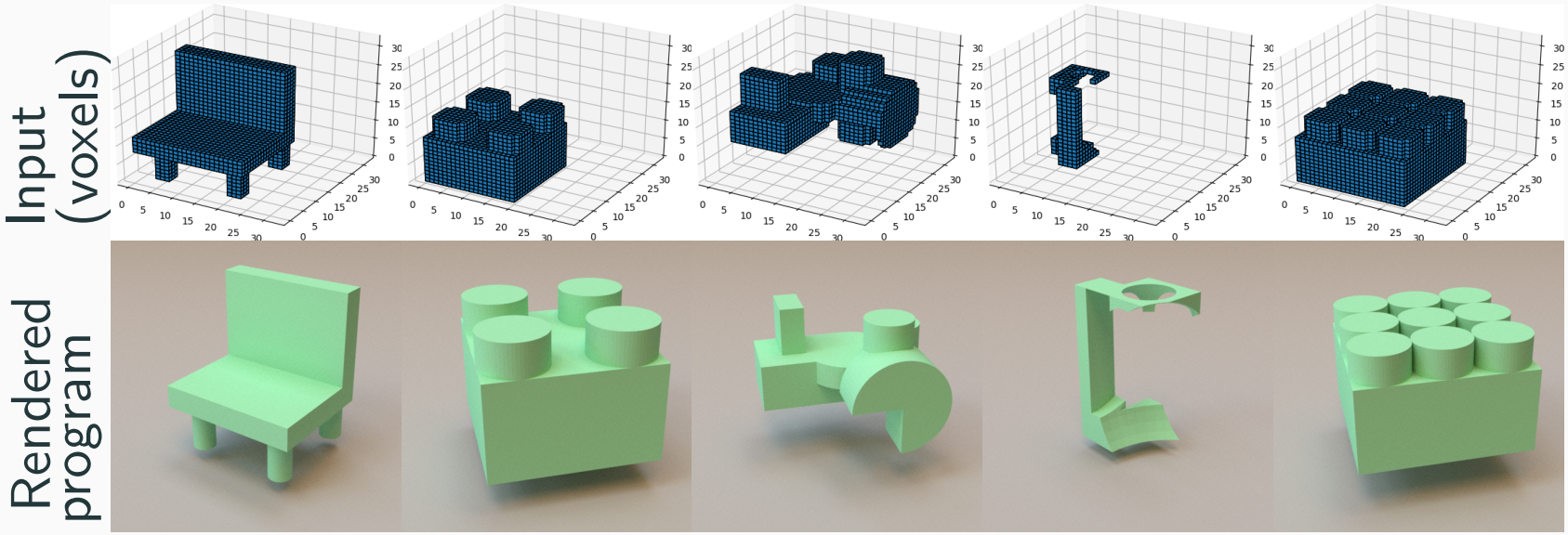
3D program induction



Ellis*, Nye*, Pu*, Sosa*, Tenenbaum, Solar-Lezama. NeurIPS 2019.

*equal contribution

3D program induction



same architecture learns to synthesize text editing programs (FlashFill, Gulwani 2012)

Ellis*, Nye*, Pu*, Sosa*, Tenenbaum, Solar-Lezama. NeurIPS 2019.
*equal contribution

Lessons

The inductive bias from a programming language gives extrapolation, or strong generalization

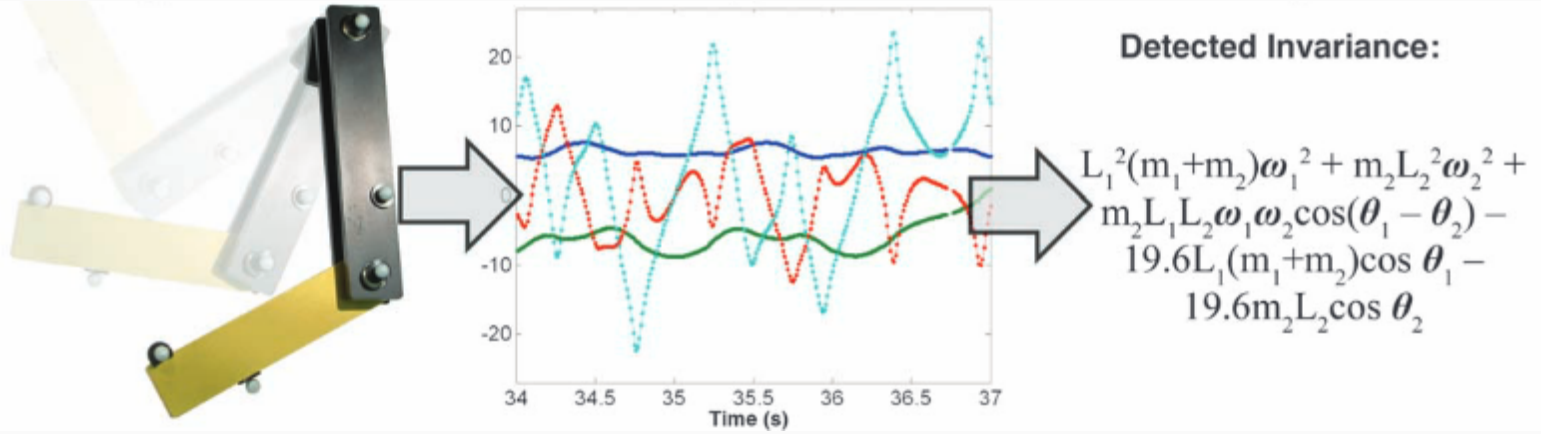
Combine the best of different techniques: neural nets for perception and pattern recognition, symbols for reasoning, Bayesian methods for uncertainty

Program Induction and perception

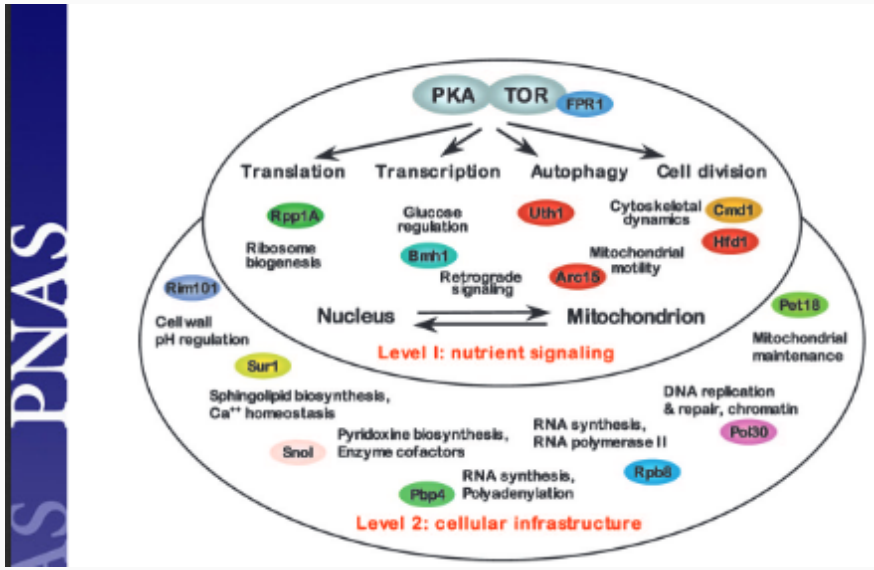
model discovery

learning to learn

Scientific discovery



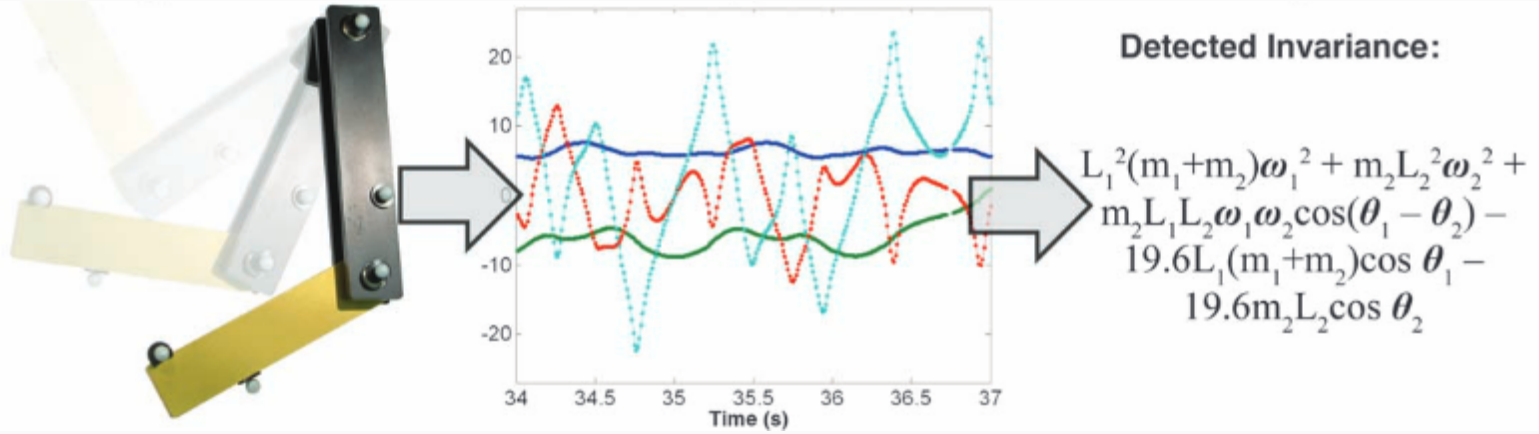
Schmidt & Lipson: "Distilling Free-Form Natural Laws from Experimental Data"



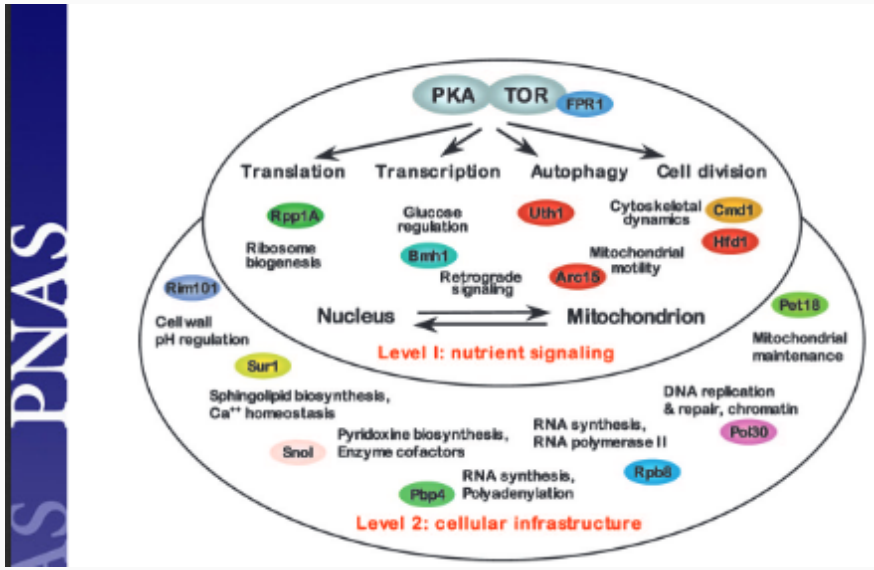
Lezon et al. 2006

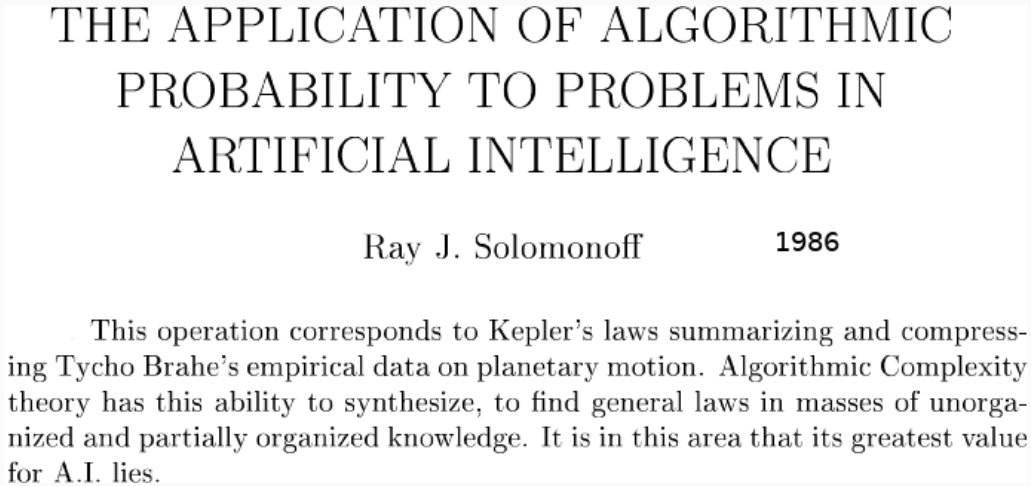
inferring genetic interaction networks

Scientific discovery



Schmidt & Lipson: "Distilling Free-Form Natural Laws from Experimental Data"





THE APPLICATION OF ALGORITHMIC PROBABILITY TO PROBLEMS IN ARTIFICIAL INTELLIGENCE

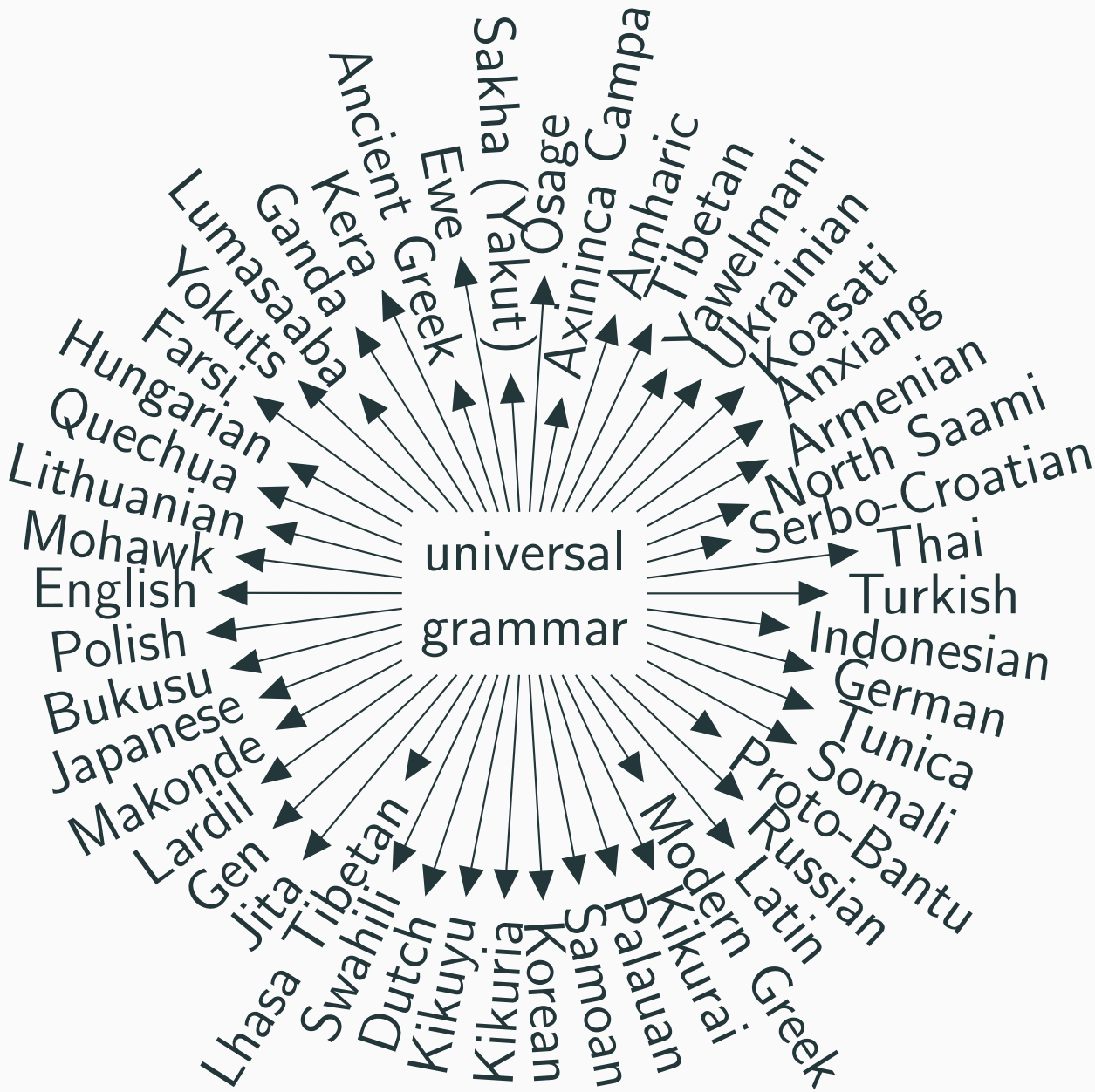
Ray J. Solomonoff 1986

This operation corresponds to Kepler's laws summarizing and compressing Tycho Brahe's empirical data on planetary motion. Algorithmic Complexity theory has this ability to synthesize, to find general laws in masses of unorganized and partially organized knowledge. It is in this area that its greatest value for A.I. lies.

Lezon et al. 2006

inferring genetic interaction networks

Discovering human-understandable models of language



Few-shot language learning experiment

Mandarin:

	adjective	adverb
“slow”	man	manmandə
“small”	xiao	xiaoxiaodə
“fast”	kuai	???

Few-shot language learning experiment

Mandarin:

	adjective	adverb
“slow”	man	manmandə
“small”	xiao	xiaoxiaodə
“fast”	kuai	kuaikuaidə

Few-shot language learning experiment

Mandarin:

	adjective	adverb
“slow”	man	manmandə
“small”	xiao	xiaoxiaodə
“fast”	kuai	kuaikuaidə

stem+stem+də

Few-shot language learning experiment

Serbo-Croatian:

	masculine	feminine
“rich”	bogat	bogata
“mild”	blag	blaga
“green”	zelen	???

Few-shot language learning experiment

Serbo-Croatian:

	masculine	feminine
“rich”	bogat	bogata
“mild”	blag	blaga
“green”	zelen	zelena

Few-shot language learning experiment

Serbo-Croatian:

	masculine	feminine
“rich”	bogat	bogata
“mild”	blag	blaga
“green”	zelen	zelena

add “a” to stem to make feminine

Few-shot language learning experiment

Serbo-Croatian:

	masculine	feminine
“rich”	bogat	bogata
“mild”	blag	blaga
“green”	zelen	zelena
“clear”	???	yasna

add “a” to stem to make feminine

Few-shot language learning experiment

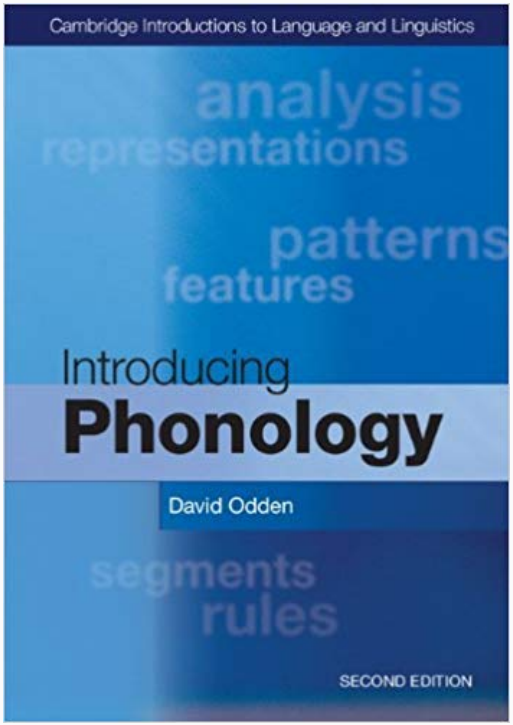
Serbo-Croatian:

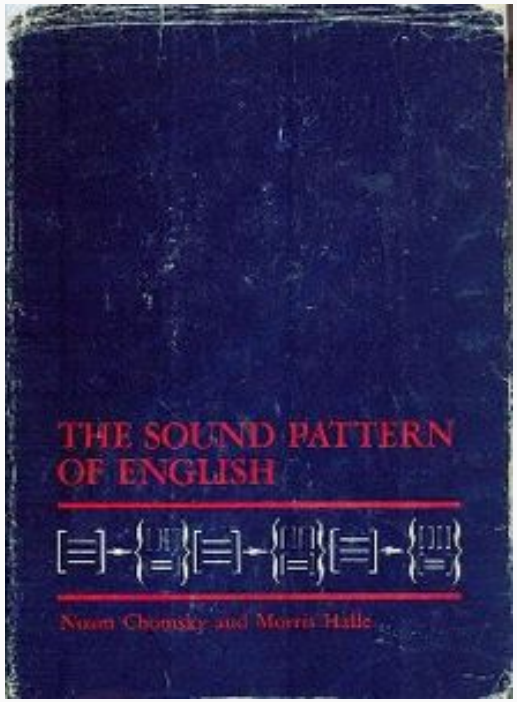
	masculine	feminine	stem (unobserved)
"rich"	bogat	bogata	bogat
"mild"	blag	blaga	blag
"green"	zelen	zelena	zelen
"clear"	**yasan**	yasna	yasn

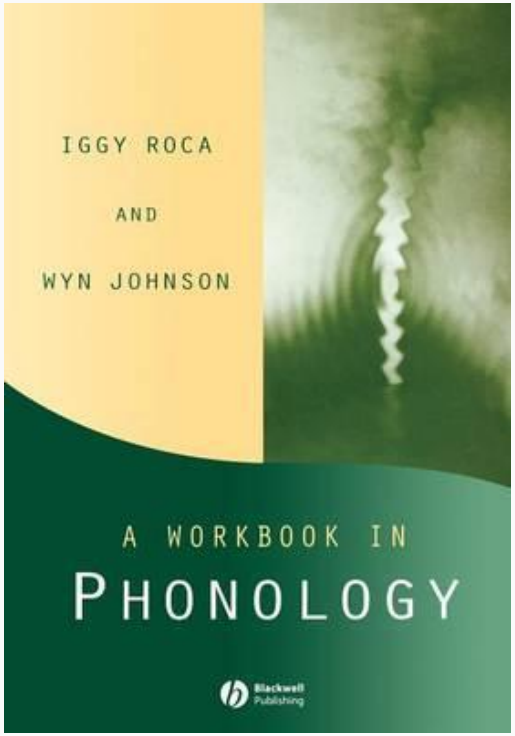
add "a" to stem to make feminine

insert "a" between two word-final consonants

$\varnothing \rightarrow$ a / C_C#


Cambridge Introductions to Language and Linguistics
analysis
representations
patterns
features
Introducing
Phonology
David Odden
segments
rules
SECOND EDITION


THE SOUND PATTERN
OF ENGLISH
Noam Chomsky and Morris Halle


IGGY ROCA
AND
WYN JOHNSON
A WORKBOOK IN
PHONOLOGY
Blackwell
Publishing

10 Sakha (Yakut)

Give a phonological analysis of the following case-marking paradigms of nouns in Sakha.

Noun	*Plural*	*Associative*	
aya	ayalar	ayalɨɨn	'father'
paarta	paartalar	paartalɨɨn	'school desk'
tɨa	tɨalar	tɨalɨɨn	'forest'
kinige	kinigeler	kinigeliin	'book'
ǰie	ǰieler	ǰieliin	'house'
iye	iyeler	iyeliin	'mother'
kini	kiniler	kiniliin	'3rd person'
bie	bieler	bieliin	'mare'
oyo	oyolor	oyoluun	'child'
χopto	χoptolor	χoptoluun	'gull'
börö	börölör	börölüün	'wolf'
tɨal	tɨallar	tɨalɨɨn	'wind'
ɨal	ɨallar	ɨalɨɨn	'neighbor'
kuul	kuullar	kuulluun	'sack'
at	attar	attɨɨn	'horse'
balɨk	balɨktar	balɨktɨɨn	'fish'
ɨskaap	ɨskaaptar	ɨskaaptɨɨn	'cabinet'
oyus	oyustar	oyustuun	'bull'
kus	kustar	kustuun	'duck'
tünnük	tünnükter	tünnüktüün	'window'
sep	septer	septiin	'tool'
et	etter	ettiin	'meat'
örüs	örüster	örüstüün	'river'
tiis	tiister	tiistiin	'tooth'
soroχ	soroχtor	soroχtuun	'some person'
oχ	oχtor	oχtuun	'arrow'
oloppos	oloppostor	oloppostuun	'chair'
ötöχ	ötöχtör	ötöχtüün	'abandoned farm'
ubay	ubaydar	ubaydɨɨn	'elder brother'
saray	saraydar	saraydɨɨn	'barn'
tɨy	tɨydar	tɨydɨɨn	'foal'
atɨɨr	atɨɨrdar	atɨɨrdɨɨn	'stallion'
oyuur	oyuurdar	oyuurduun	'forest'
üčügey	üčügeyder	üčügeydiin	'good person'
eǰiy	eǰiyder	eǰiydiin	'elder sister'
tomtor	tomtordor	tomtorduun	'knob'
moyotoy	moyotoydor	moyotoyduun	'chipmunk'
kötör	kötördör	kötördüün	'bird'
bölköy	bölköydör	bölköydüün	'islet'
χatɨŋ	χatɨŋnar	χatɨŋnɨɨn	'birch'
aan	aannar	aannɨɨn	'door'
tɨŋ	tɨŋner	tɨŋnɨɨn	'squirrel'
sordoŋ	sordoŋnor	sordoŋnuun	'pike'
olom	olomnor	olomnuun	'ford'
oron	oronnor	oronnuun	'bed'
bödöŋ	bödöŋnör	bödöŋnüün	'strong one'

Noun	*Partitive*	*Comparative*	*Ablative*	
aya	ayata	ayataayar	ayattan	'father'
paarta	paartata	paartataayar	paartattan	'school desk'
tɨa	tɨata	tɨataayar	tɨattan	'forest'
kinige	kinigete	kinigeteeyer	kinigetten	'book'
ǰie	ǰiete	ǰieteeyer	ǰietten	'house'
iye	iyete	iyeteeyer	iyetten	'mother'
kini	kinite	kiniteeyer	kinitten	'3rd person'
bie	biete	bieteeyer	bietten	'mare'
oyo	oyoto	oyotooyor	oyotton	'child'
χopto	χoptoto	χoptotooyor	χoptotton	'gull'
börö	börötö	börötööyör	böröttön	'wolf'
tɨal	tɨalla	tɨallaayar	tɨaltan	'wind'
ɨal	ɨalla	ɨallaayar	ɨaltan	'neighbor'
kuul	kuulla	kuullaayar	kuultan	'sack'
moχsoyol	moχsoyollo	moχsoyollooyor	moχsoyolton	'falcon'
at	atta	attaayar	attan	'horse'
balɨk	balɨkta	balɨktaayar	balɨktan	'fish'
ɨskaap	ɨskaapta	ɨskaaptaayar	ɨskaaptan	'cabinet'
oyus	oyusta	oyustaayar	oyustan	'bull'
kus	kusta	kustaayar	kustan	'duck'
tünnük	tünnükte	tünnükteeyer	tünnükten	'window'

Turkic Sakha (Yakut)

observed data

	Singular	Plural
Bed	oron	oronnor
Mare	bie	bieler
Cabinet	ɨskaap	ɨskaaptar

138 total examples

Ellis, Albright, Solar-Lezama, Tenenbaum, O'Donnell. 2020.

Turkic Sakha (Yakut)

grammar (unobserved)

SINGULAR→stem
PLURAL→stem+lar

observed data

	SINGULAR	PLURAL
BED	oron	oronnor
MARE	bie	bieler
CABINET	ɨskaap	ɨskaaptar

138 total examples

Ellis, Albright, Solar-Lezama, Tenenbaum, O'Donnell. 2020.

Turkic Sakha (Yakut)

grammar (unobserved)

SINGULAR→stem
PLURAL→stem+lar

r_1: l → d / [-lateral -tense]_
"l" becomes "d" next to "r", "t", but not "l"

r_2: C → [-voice] / [-voice]_
do not voice next to voiceless

r_3: V → [+rounded] / [+rounded] [-low]$_0$_
r_4: [+continuant -high] → [-rounded] / u C$_0$_
"harmonize" round vowels like "u", "o"

r_5: V → [-back -low] / [-back +vowel] [ ]$_0$_
"harmonize" vowels to be not at back of mouth

r_6: [-sonorant +voice] → [+nasal] / [+nasal]_
"nasalize" consonant next to a nasal, like "m"

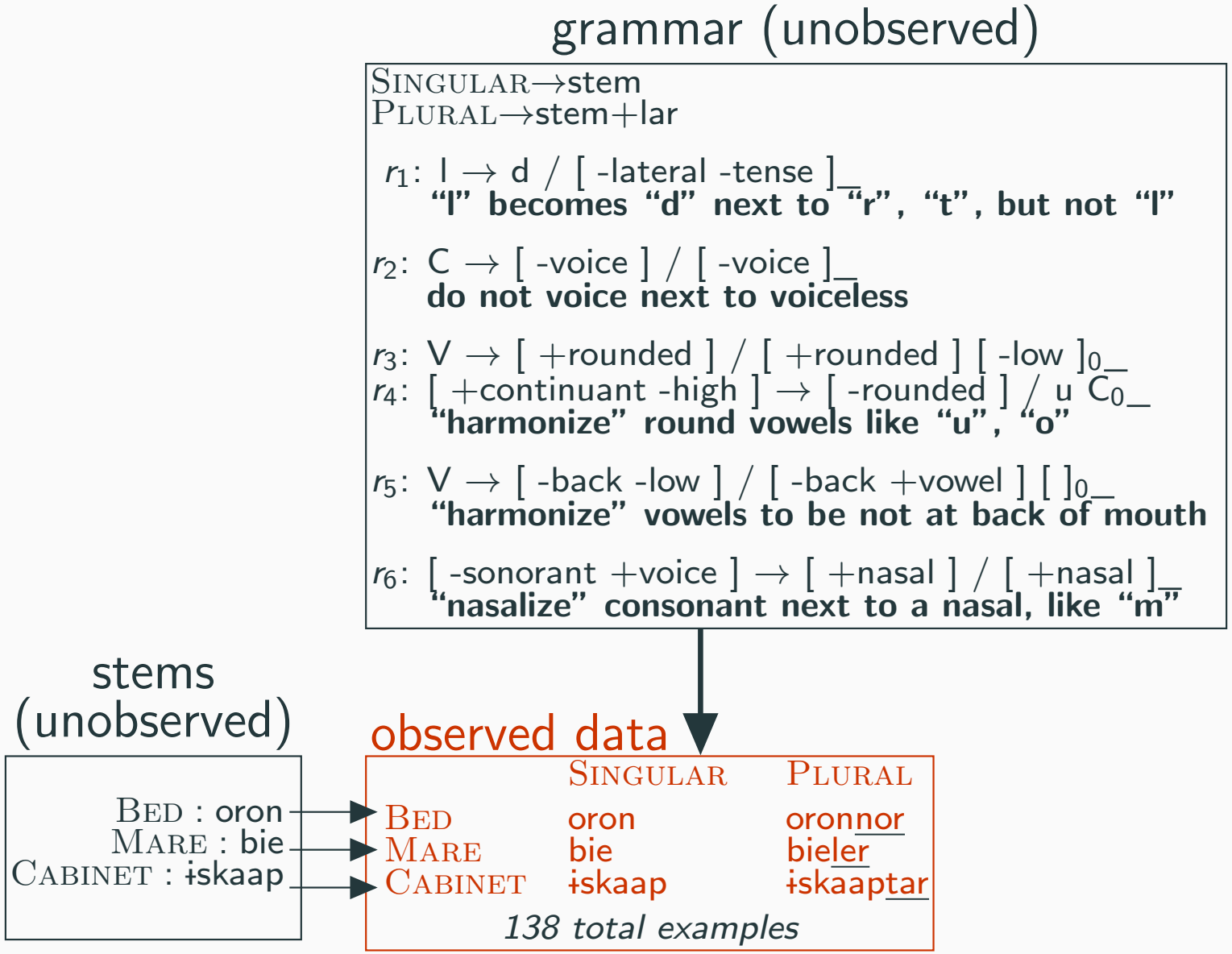
observed data

	SINGULAR	PLURAL
BED	oron	oronnor
MARE	bie	bieler
CABINET	ɨskaap	ɨskaaptar

138 total examples

Ellis, Albright, Solar-Lezama, Tenenbaum, O'Donnell. 2020.

Turkic Sakha (Yakut)



Ellis, Albright, Solar-Lezama, Tenenbaum, O'Donnell. 2020.

Turkic Sakha (Yakut)

grammar (unobserved)

SINGULAR→stem
PLURAL→stem+lar

r_1: l → d / [-lateral -tense]_
“l” becomes “d” next to “r”, “t”, but not “l”

r_2: C → [-voice] / [-voice]_
do not voice next to voiceless

r_3: V → [+rounded] / [+rounded] [-low]$_0$_
r_4: [+continuant -high] → [-rounded] / u C$_0$_
“harmonize” round vowels like “u”, “o”

r_5: V → [-back -low] / [-back +vowel] [ ]$_0$_
“harmonize” vowels to be not at back of mouth

r_6: [-sonorant +voice] → [+nasal] / [+nasal]_
“nasalize” consonant next to a nasal, like “m”

stems (unobserved)

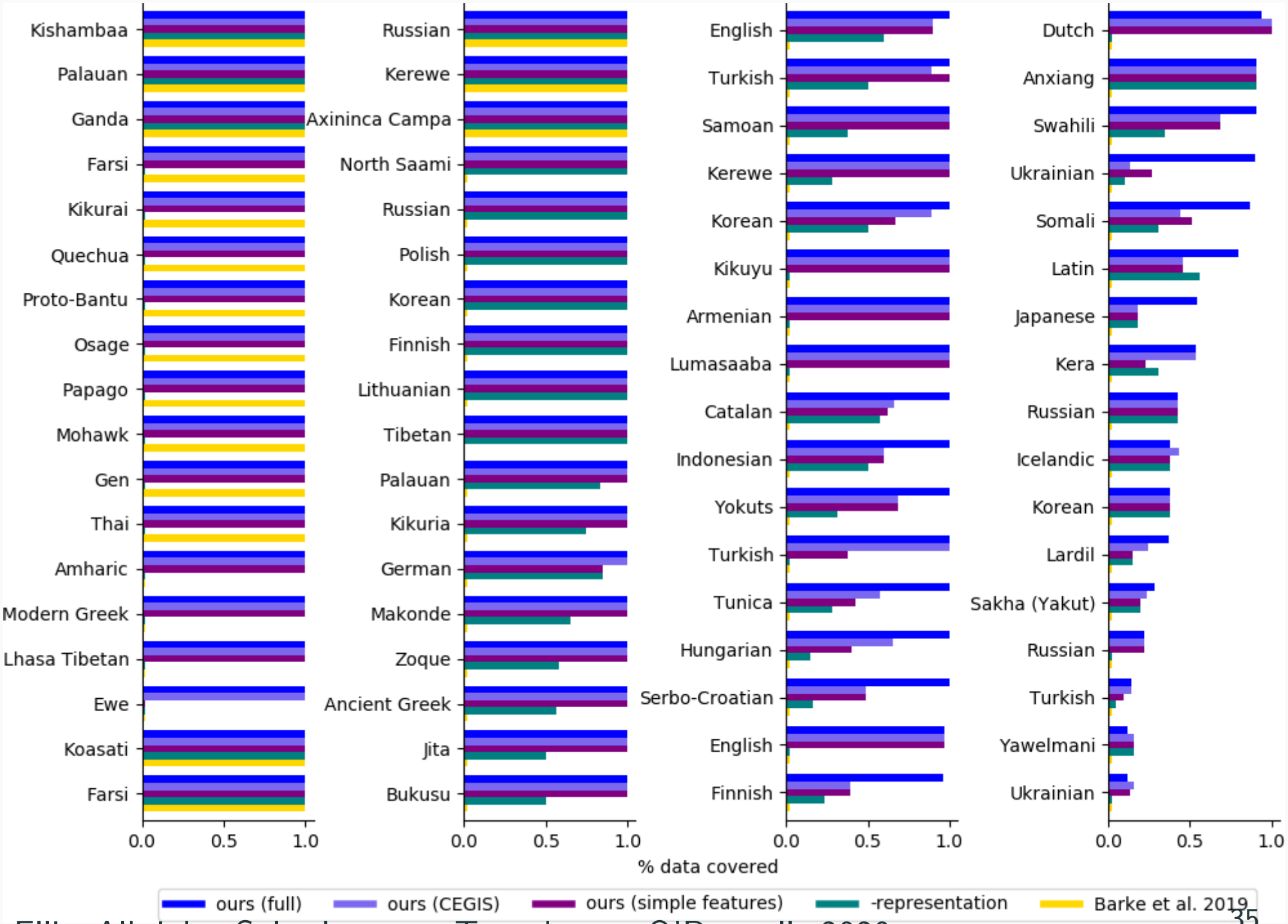
CABINET : ɨskaap
BED : oron
MARE : bie
RIVER : örus

observed data

CABINETS→ɨskaap+lar→ɨskaaplar$\xrightarrow{r_1}$ɨskaapdar$\xrightarrow{r_2}$ɨskaaptar
BEDS→oron+lar→oronlar$\xrightarrow{r_1}$orondar$\xrightarrow{r_3}$orondor$\xrightarrow{r_6}$oronnor
MARES→bie+lar→bielar$\xrightarrow{r_5}$bieler
RIVER (ASSOC)→örus+lɨɨn→öruslɨɨn$\xrightarrow{r_1}$örusdɨɨn$\xrightarrow{r_2}$
örustɨɨn$\xrightarrow{r_3}$örustuun$\xrightarrow{r_5}$[örüstüün]

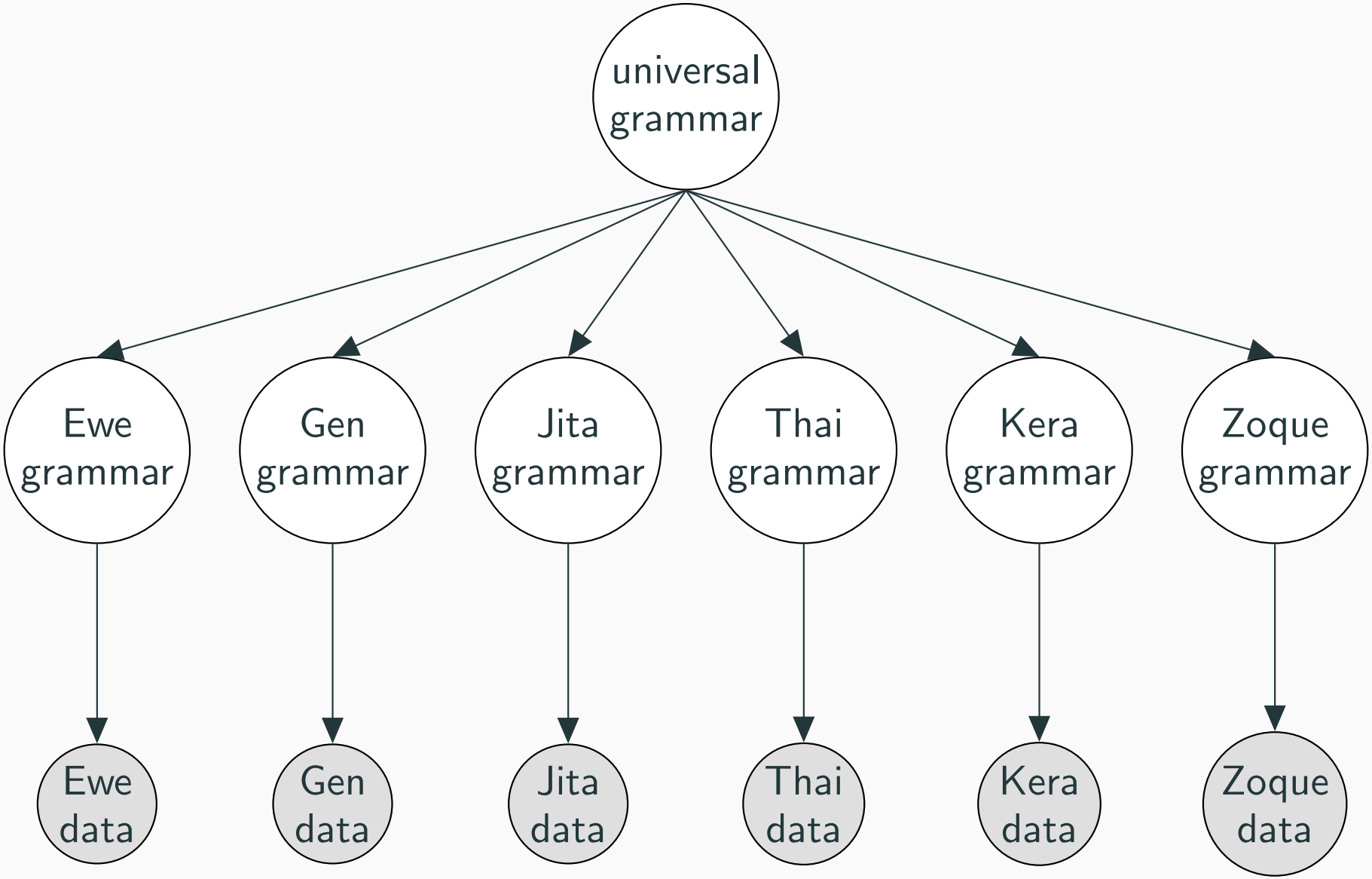
Kishambaa	Russian	English	Dutch
Palauan	Kerewe	Turkish	Anxiang
Ganda	Axininca Campa	Samoan	Swahili
Farsi	North Saami	Kerewe	Ukrainian
Kikurai	Russian	Korean	Somali
Quechua	Polish	Kikuyu	Latin
Proto-Bantu	Korean	Armenian	Japanese
Osage	Finnish	Lumasaaba	Kera
Papago	Lithuanian	Catalan	Russian
Mohawk	Tibetan	Indonesian	Icelandic
Gen	Palauan	Yokuts	Korean
Thai	Kikuria	Turkish	Lardil
Amharic	German	Tunica	kha (Yakut)
Modern Greek	Makonde	Hungarian	Russian
Lhasa Tibetan	Zoque	Serbo-Croatian	Turkish
Ewe	Ancient Greek	English	Yawelmani
Koasati	Jita	Finnish	Ukrainian
Farsi	Bukusu		

Ellis, Albright, Solar-Lezama, Tenenbaum, O'Donnell. 2020.



Ellis, Albright, Solar-Lezama, Tenenbaum, O'Donnell. 2020.

Distilling higher-level knowledge



Ellis, Albright, Solar-Lezama, Tenenbaum, O'Donnell. 2020.

Lessons

Higher-level knowledge matters ("universal grammar"). Get the basics of the representation correct

But *some* of this higher-level knowledge can be learned. You don't need millions of examples to learn it. But it's not a one-shot learning problem either

Program Induction and perception

model discovery

learning to learn

Learning to write code

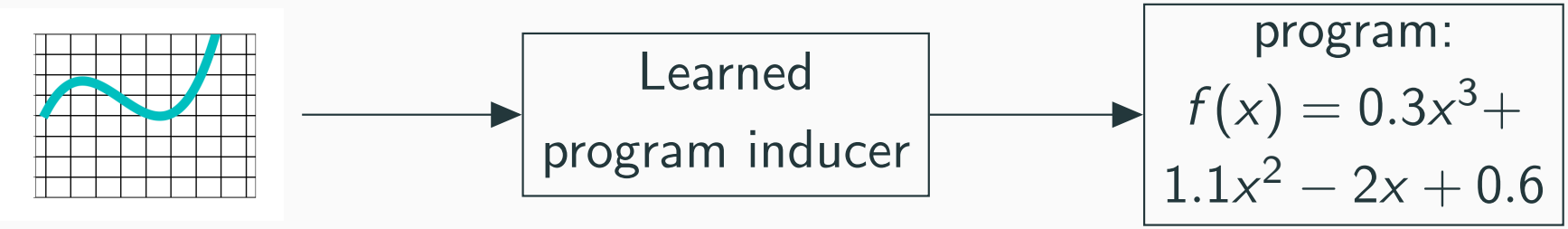
Goal: acquire domain-specific knowledge needed to induce a class of programs

- Library of concepts (declarative knowledge; domain specific language)
- Inference strategy (procedural knowledge; synthesis algorithm)

Learning to write code

Goal: acquire domain-specific knowledge needed to induce a class of programs

- Library of concepts (declarative knowledge; domain specific language)
- Inference strategy (procedural knowledge; synthesis algorithm)



Concepts: x^3, $\alpha x + \beta$, etc

Inference strategy: neurosymbolic search for programs

Library learning

Initial Primitives

```
map
fold
if
cons
>
```

Sample Problem: `sort list`

[9 2 7 1] → [1 2 7 9]
[3 8 9 4 2] → [2 3 4 8 9]
[6 2 2 3 8 5] → [2 2 3 5 6 8]
...

Ellis, Morales, Sable-Meyer, Solar-Lezama, Tenenbaum. NeurIPS 2018.

Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

Library learning

Initial Primitives

```
  :
 map
fold
 if
cons
 >
  :
```

Sample Problem: `sort list`

[9 2 7 1] → [1 2 7 9]
[3 8 9 4 2] → [2 3 4 8 9]
[6 2 2 3 8 5] → [2 2 3 5 6 8]
...

Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

Library learning

Initial Primitives

```
map
fold
if
cons
>
```

Sample Problem: `sort list`

[9 2 7 1] → [1 2 7 9]
[3 8 9 4 2] → [2 3 4 8 9]
[6 2 2 3 8 5] → [2 2 3 5 6 8]
...

Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

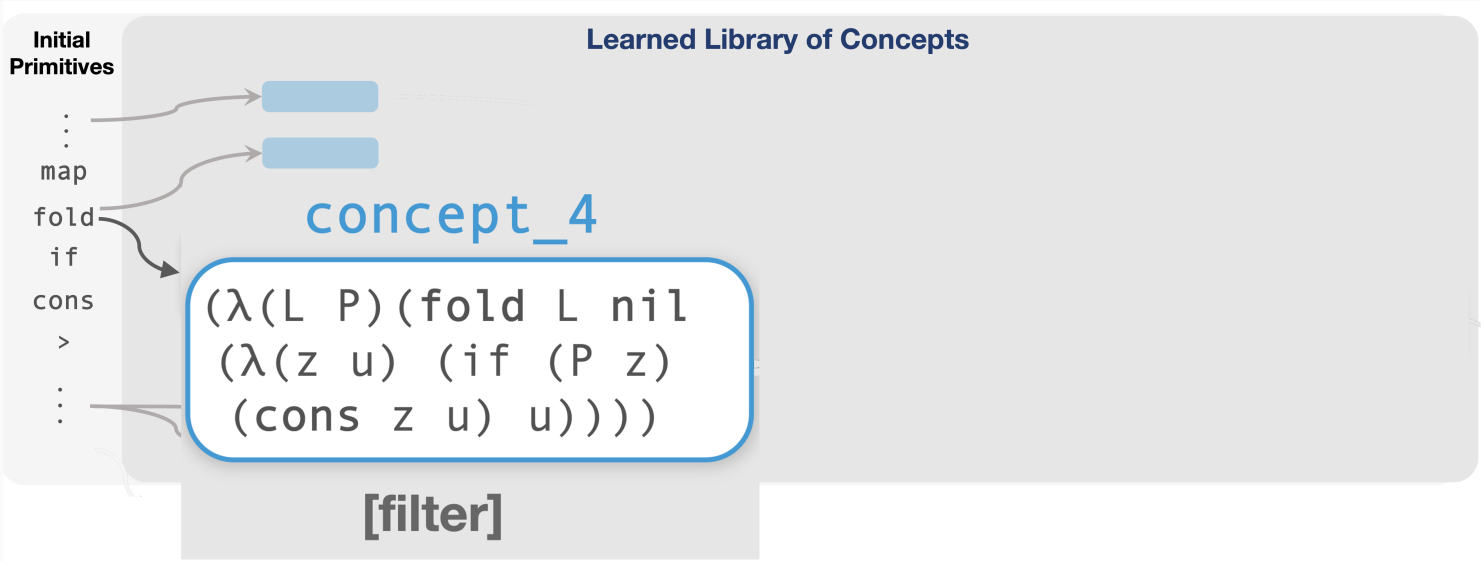
Library learning

Sample Problem: `sort list`

[9 2 7 1] → [1 2 7 9]
[3 8 9 4 2] → [2 3 4 8 9]
[6 2 2 3 8 5] → [2 2 3 5 6 8]
...

Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

Library learning



Sample Problem: `sort list`

[9 2 7 1] → [1 2 7 9]
[3 8 9 4 2] → [2 3 4 8 9]
[6 2 2 3 8 5] → [2 2 3 5 6 8]
...

Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

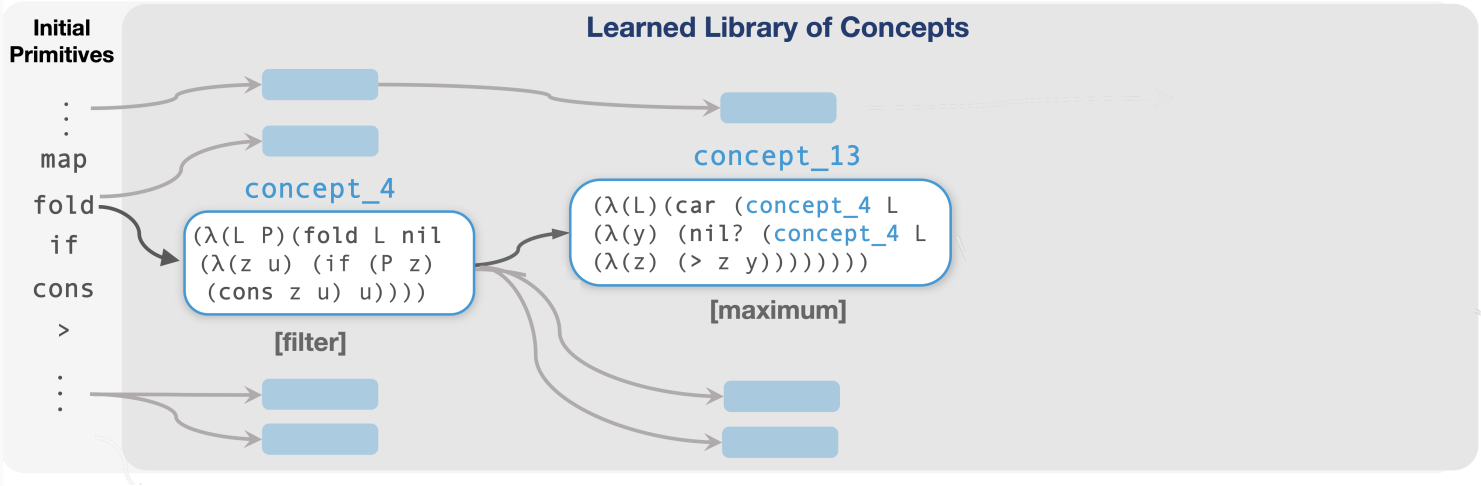
Library learning

Sample Problem: `sort list`

[9 2 7 1] → [1 2 7 9]
[3 8 9 4 2] → [2 3 4 8 9]
[6 2 2 3 8 5] → [2 2 3 5 6 8]
...

Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

Library learning

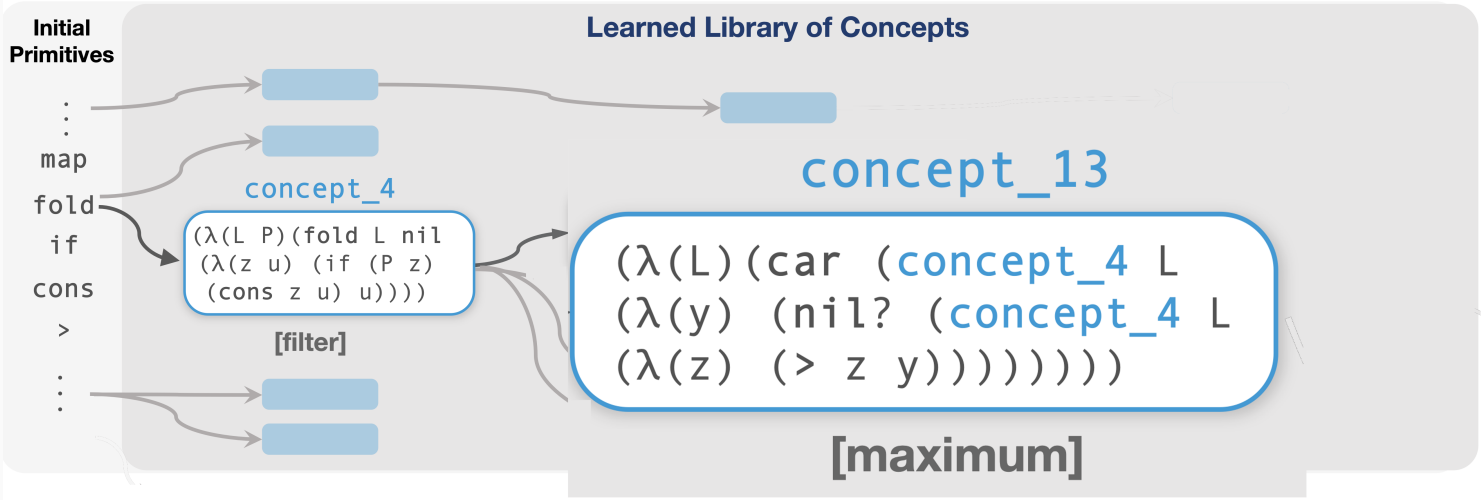


Sample Problem: `sort list`

[9 2 7 1] → [1 2 7 9]
[3 8 9 4 2] → [2 3 4 8 9]
[6 2 2 3 8 5] → [2 2 3 5 6 8]
...

Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

Library learning

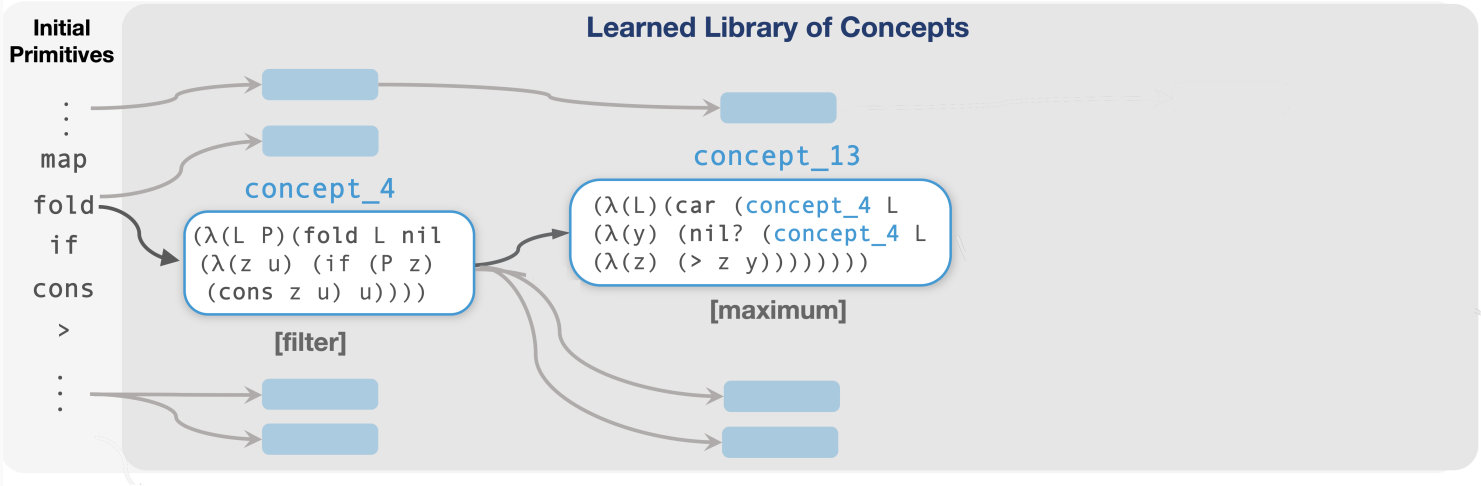


Sample Problem: `sort list`

[9 2 7 1] → [1 2 7 9]
[3 8 9 4 2] → [2 3 4 8 9]
[6 2 2 3 8 5] → [2 2 3 5 6 8]
...

Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

Library learning

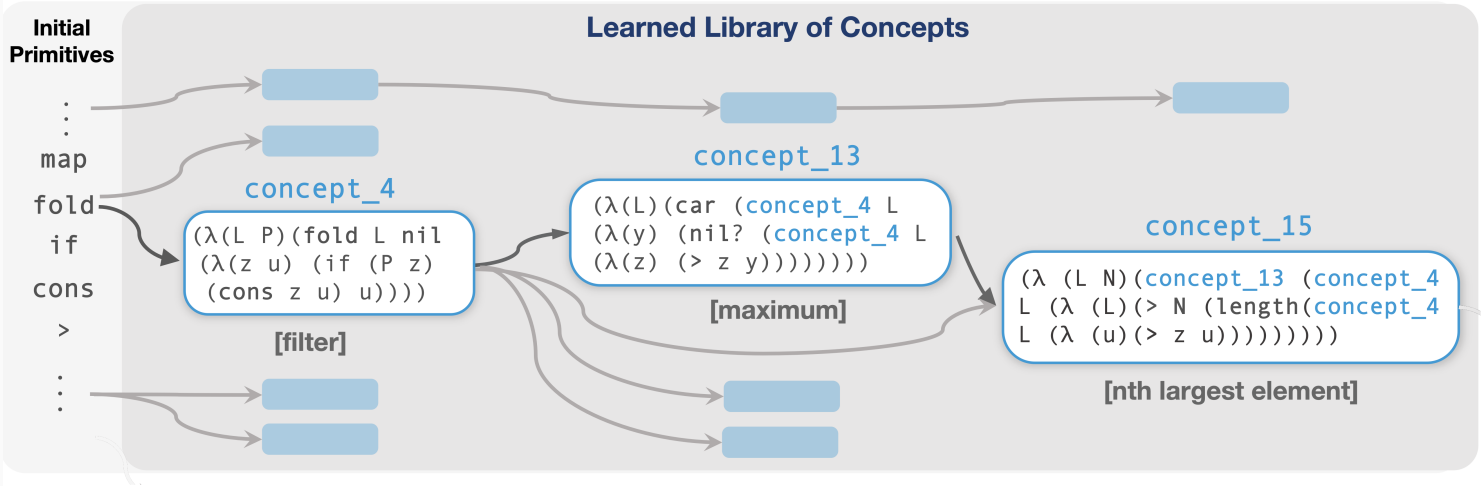


Sample Problem: `sort list`

[9 2 7 1] → [1 2 7 9]
[3 8 9 4 2] → [2 3 4 8 9]
[6 2 2 3 8 5] → [2 2 3 5 6 8]
...

Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

Library learning

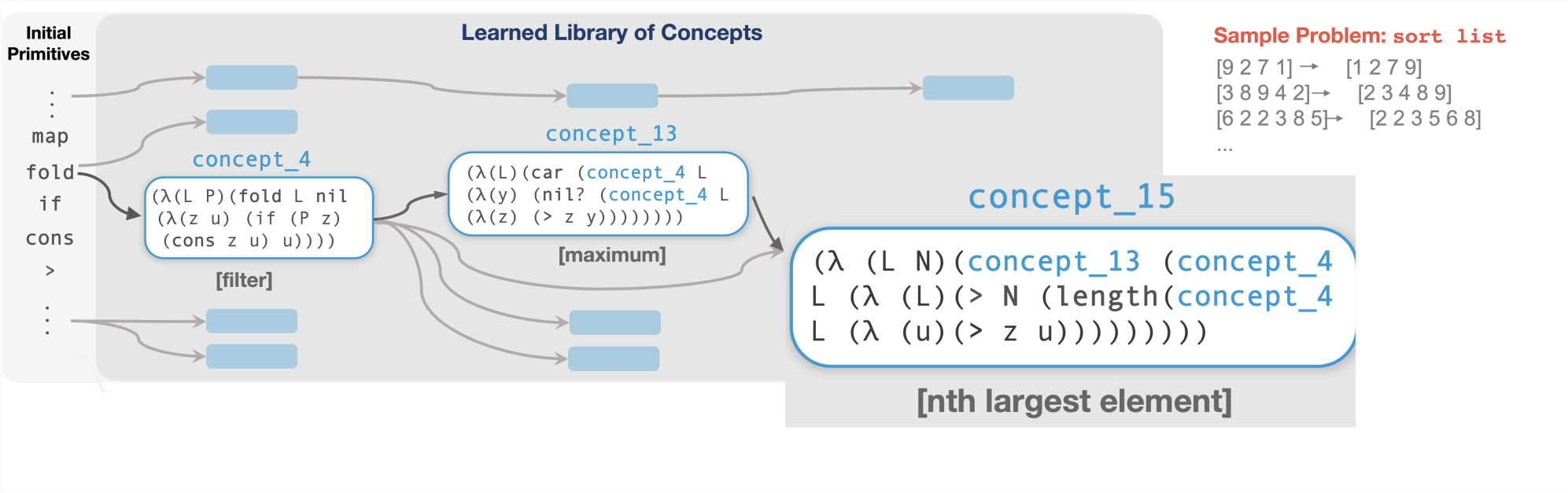


Sample Problem: `sort list`

[9 2 7 1] → [1 2 7 9]
[3 8 9 4 2] → [2 3 4 8 9]
[6 2 2 3 8 5] → [2 2 3 5 6 8]
...

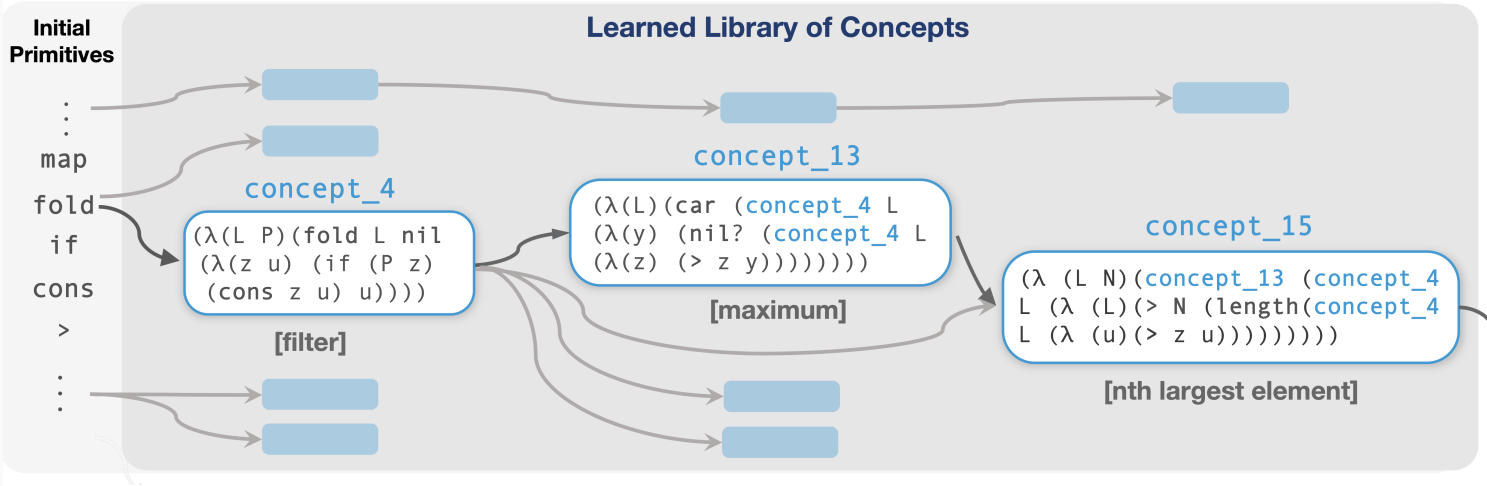
Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

Library learning



Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

Library learning



Sample Problem: `sort list`

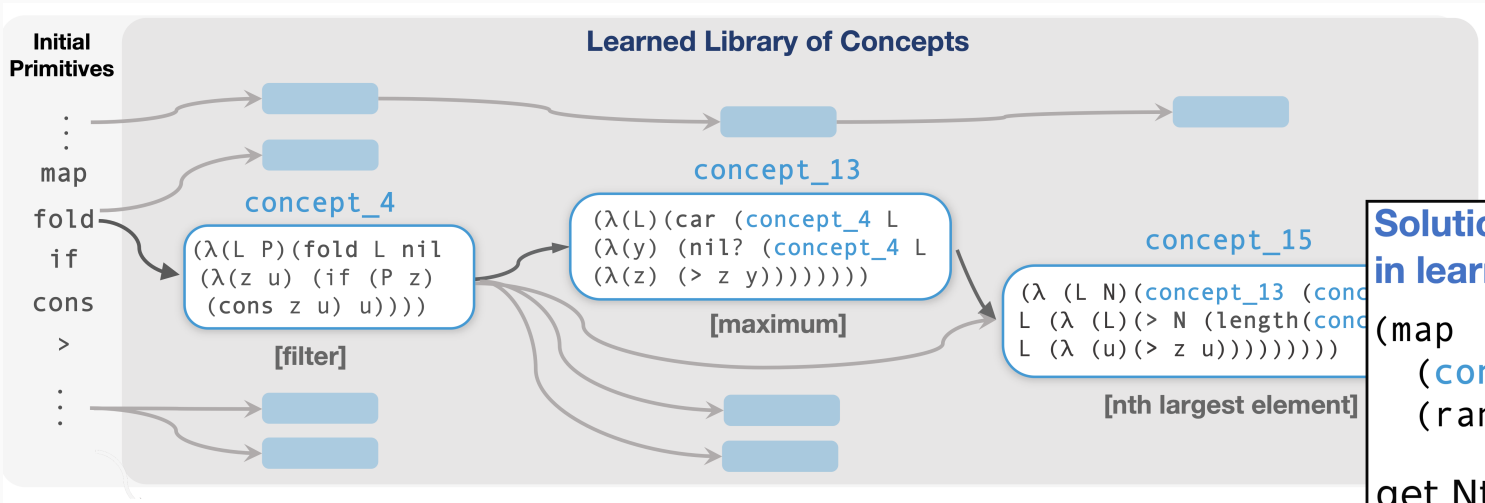
[9 2 7 1] → [1 2 7 9]
[3 8 9 4 2] → [2 3 4 8 9]
[6 2 2 3 8 5] → [2 2 3 5 6 8]
...

Solution to `sort list` discovered in learned language:

```
(map  (λ (n)
  (concept_15 L (+ 1 n)))
  (range (length L)))
```

Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

Library learning



Sample Problem: `sort list`

[9 2 7 1] → [1 2 7 9]
[3 8 9 4 2] → [2 3 4 8 9]
[6 2 2 3 8 5] → [2 2 3 5 6 8]
...

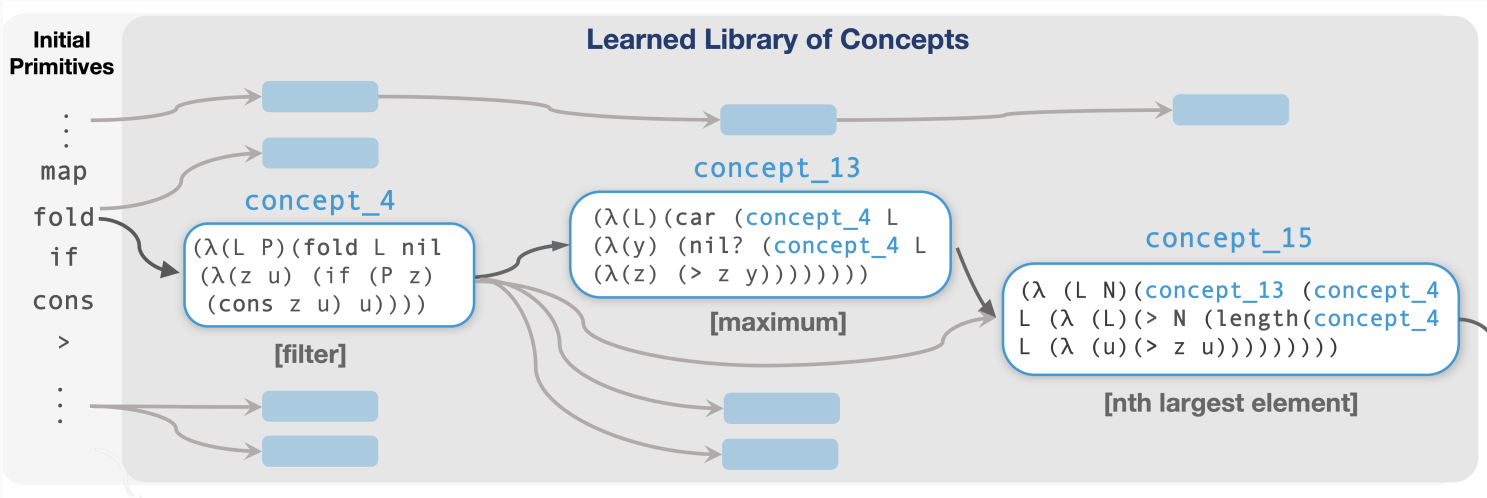
Solution to `sort list` discovered in learned language:

```
(map  (λ (n)
  (concept_15 L (+ 1 n)))
  (range (length L)))
```

get Nth largest element, where N is 1, 2, 3, ...

Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

Library learning



Sample Problem: `sort list`

[9 2 7 1] → [1 2 7 9]
[3 8 9 4 2] → [2 3 4 8 9]
[6 2 2 3 8 5] → [2 2 3 5 6 8]
...

Solution to `sort list` discovered in learned language:

```
(map  (λ (n)
  (concept_15 L (+ 1 n)))
  (range (length L)))
```

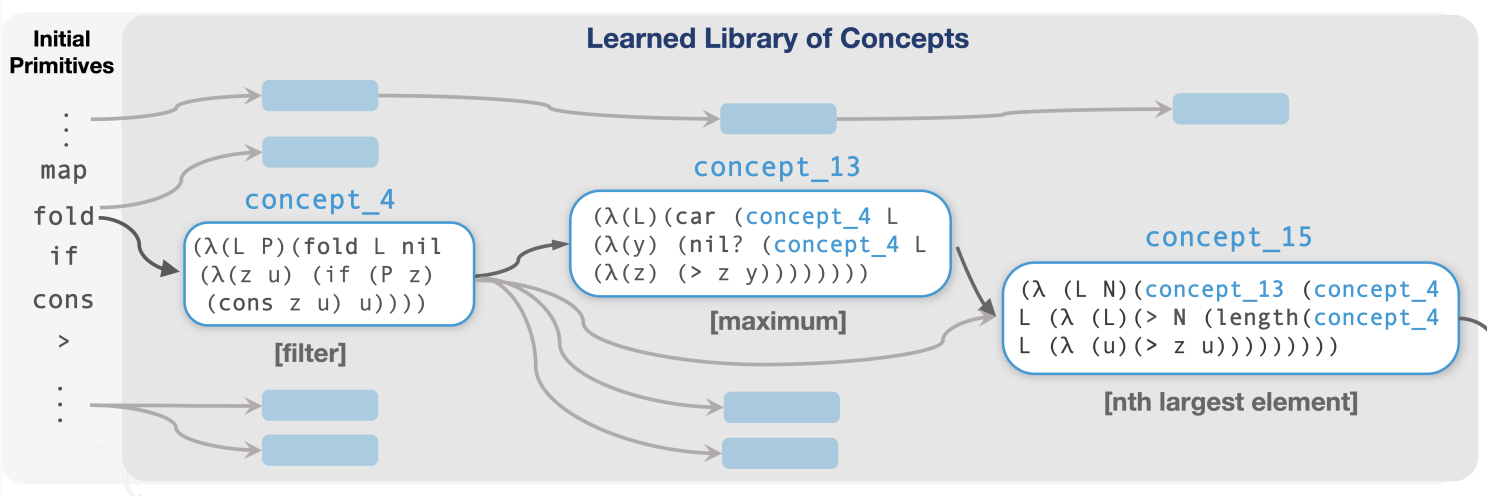
get Nth largest element, where N is 1, 2, 3, ...

Solution rewritten in initial primitives:

```
(lambda (x) (map (lambda (y) (car (fold (fold x nil (lambda (z u) (if (gt?  (+ y 1) (length
(fold x nil (lambda (v w) (if (gt?  z v) (cons v w) w))))) (cons z u) u))) nil (lambda (a b) (if
(nil?  (fold (fold x nil (lambda (c d) (if (gt?  (+ y 1) (length (fold x nil (lambda (e f) (if
(gt?  c e) (cons e f) f))))) (cons c d) d))) nil (lambda (g h) (if (gt?  g a) (cons g h) h))))
(cons a b) b)))))) (range (length x))))
```

Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

Library learning



Sample Problem: `sort list`

[9 2 7 1] → [1 2 7 9]
[3 8 9 4 2] → [2 3 4 8 9]
[6 2 2 3 8 5] → [2 2 3 5 6 8]
...

Solution to `sort list` discovered in learned language:

```
(map  (λ (n)
  (concept_15 L (+ 1 n)))
  (range (length L)))
```

get Nth largest element, where N is 1, 2, 3, ...

Solution rewritten in initial primitives:

```
(lambda (x) (map (lambda (y) (car (fold (fold x nil (lambda (z u) (if (gt?  (+ y 1) (length
(fold x nil (lambda (v w) (if (gt?  z v) (cons v w) w))))) (cons z u) u))) nil (lambda (a b) (if
(nil?  (fold (fold x nil (lambda (c d) (if (gt?  (+ y 1) (length (fold x nil (lambda (e f) (if
(gt?  c e) (cons e f) f))))) (cons c d) d))) nil (lambda (g h) (if (gt?  g a) (cons g h) h))))
(cons a b) b)))) (range (length x))))
```

induced sort program found in $\leq$ 10min. Brute-force search without learned library would take $\approx 10^{78}$ years

Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

DreamCoder

- **Wake:** Solve problems by writing programs
- **Sleep:** Improve library and neural recognition model:
  - **Abstraction sleep:** Improve library
  - **Dream sleep:** Improve neural recognition model

cf. Helmholtz machine, wake/sleep neural network training algorithms

DreamCoder

- **Wake:** Solve problems by writing programs
- **Sleep:** Improve library and neural recognition model:
  - **Abstraction sleep:** Improve library
  - **Dream sleep:** Improve neural recognition model

List Processing

Sum List

```
[1 2 3] → 6
[4 6 8 1] → 17
```

Double

```
[1 2 3] → [2 4 6]
[4 5 1] → [8 10 2]
```

Text Editing

Abbreviate

```
Allen Newell → A.N.
Herb Simon → H.S.
```

Drop Last Three

```
shrdlu → shr
shakey → sha
```

Regexes

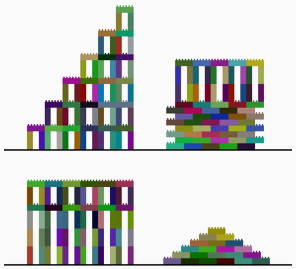
Phone numbers

```
(555) 867-5309
(650) 555-2368
```

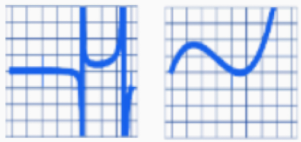
Currency

```
$100.25
$4.50
```

Block Towers



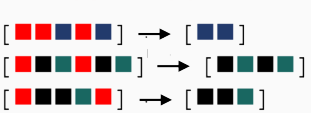
Symbolic Regression



$y = f(x)$

Recursive Programming

Filter Red

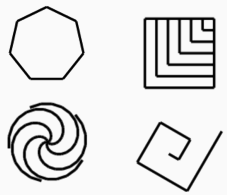


Physical Laws

$$\vec{a} = \frac{1}{m}\sum_i \vec{F}_i$$

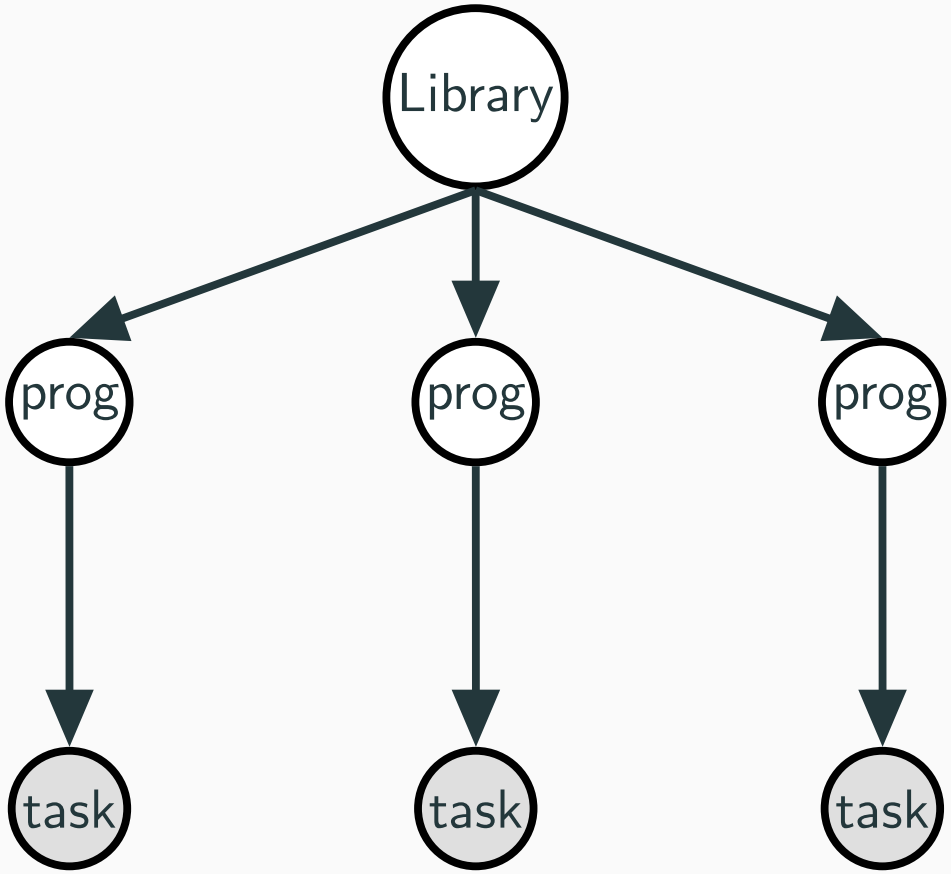
$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r}|^2}\hat{r}$$

LOGO Graphics



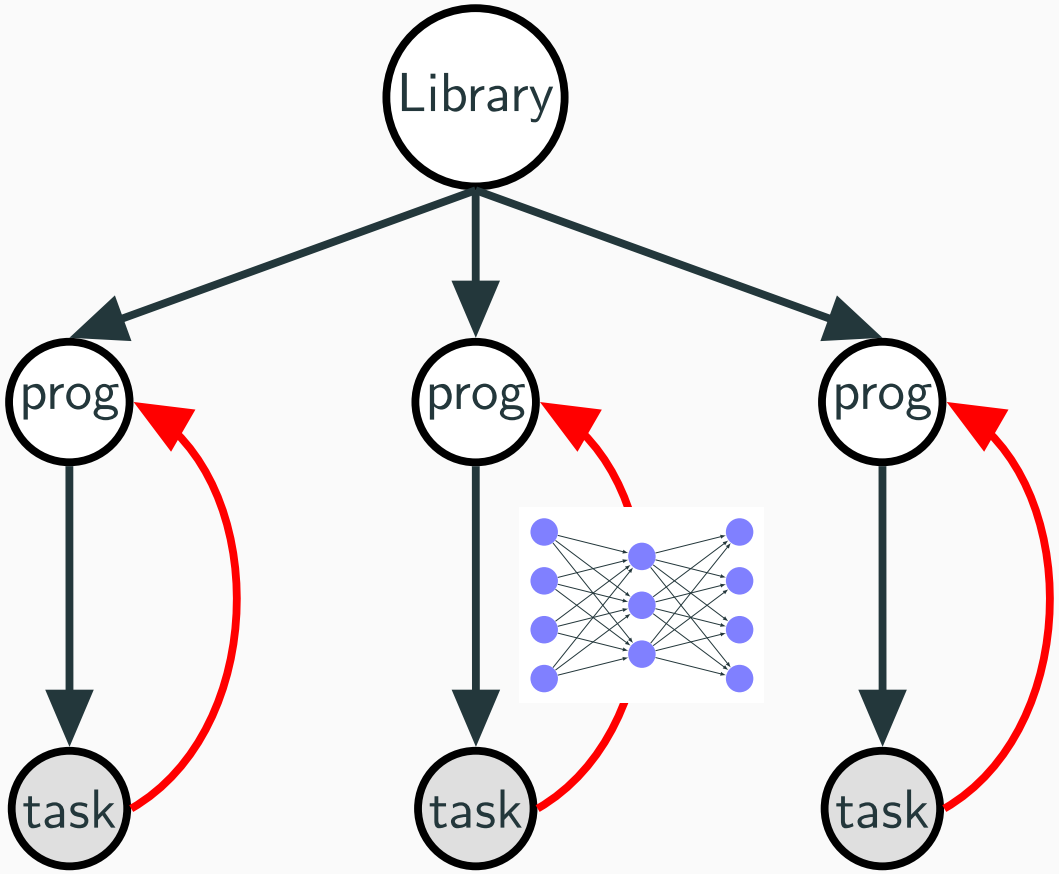
cf. Helmholtz machine, wake/sleep neural network training algorithms

Library learning as Bayesian inference



[Dechter et al., 2013] [Liang et al, 2010]; [Lake et al, 2015]

Library learning as neurally-guided Bayesian inference

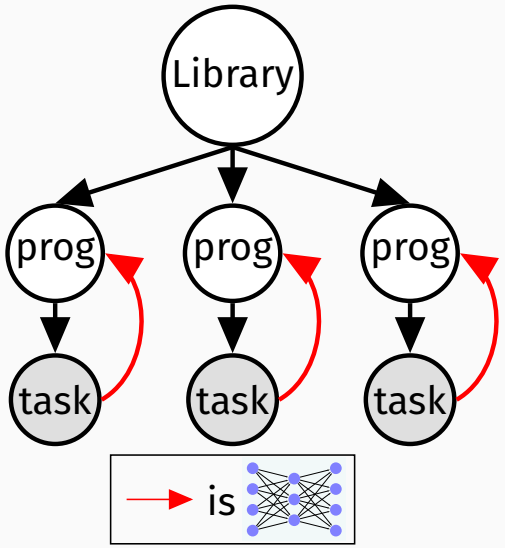


library learning via program analysis +

new neural inference network for program synthesis +

better program representation (Lisp+polymorphic types [Milner 1978])

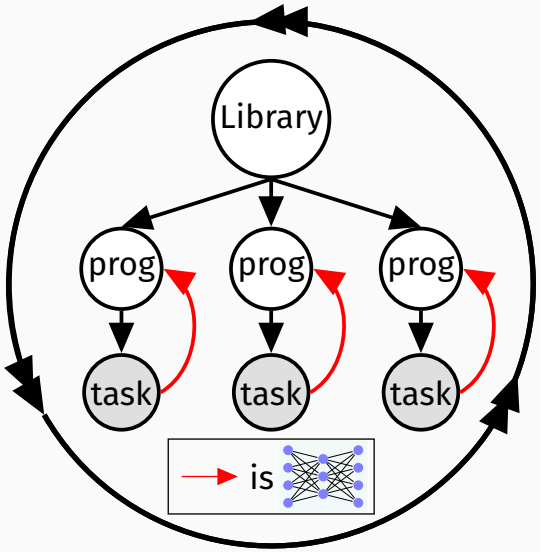
Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.


Library
prog
prog
prog
task
task
task
is

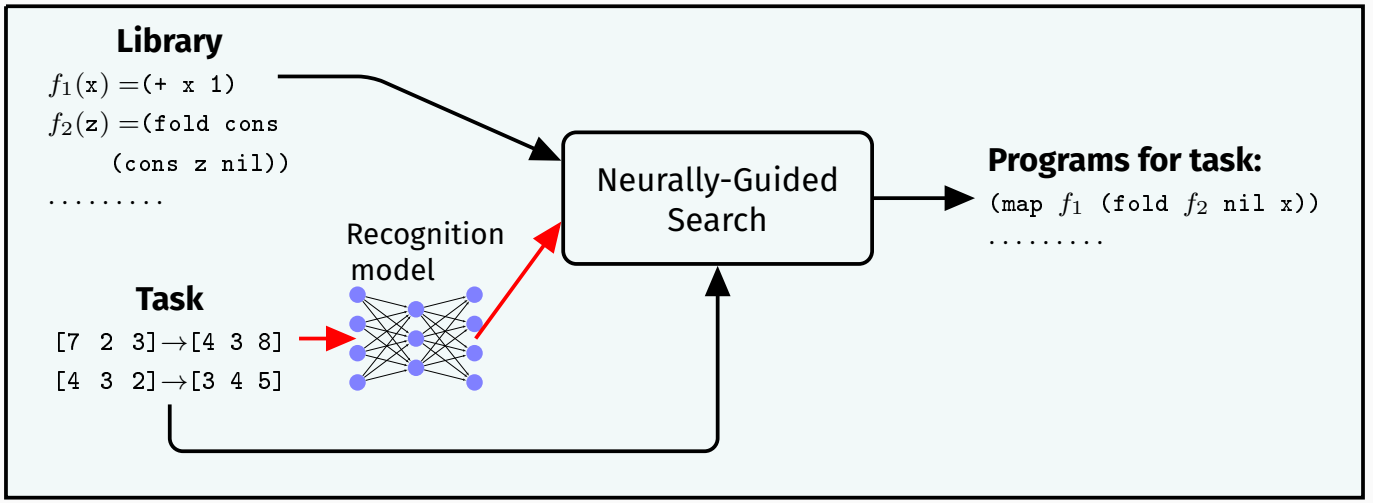
Wake

Sleep: Abstraction

Sleep: Dreaming

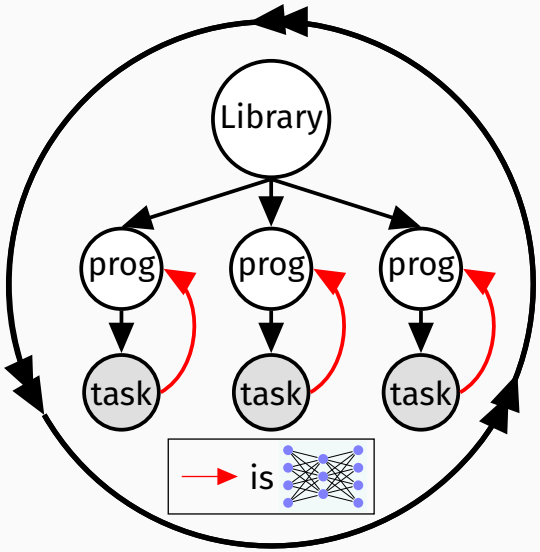


WAKE

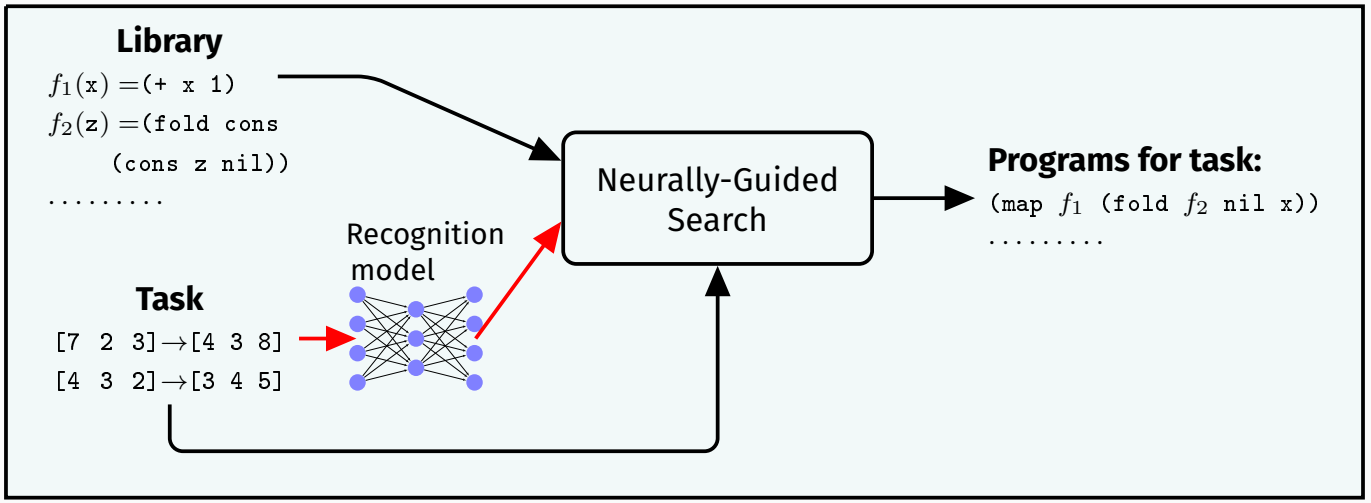


SLEEP: ABSTRACTION

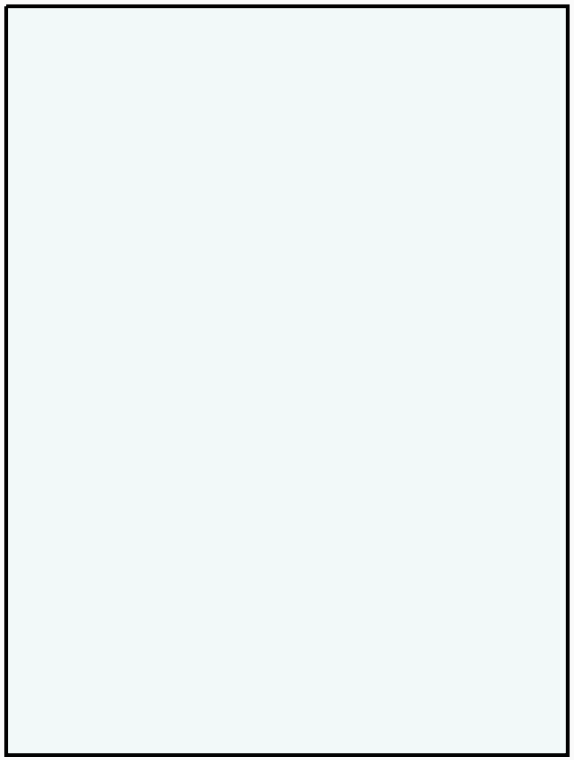
SLEEP: DREAMING

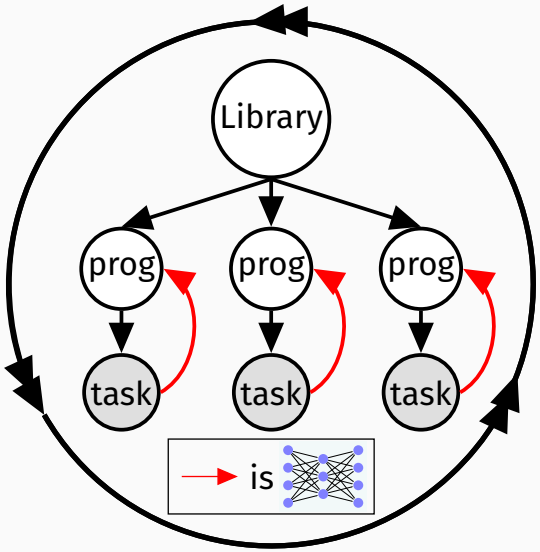


Wake

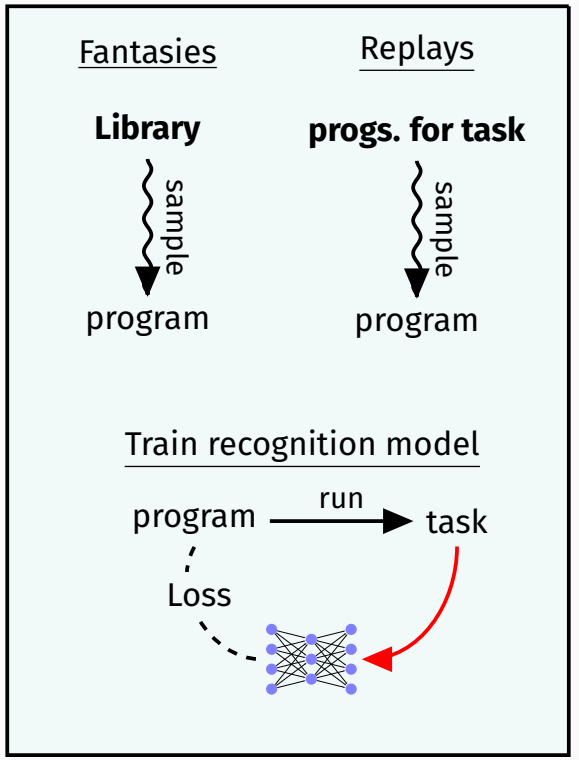


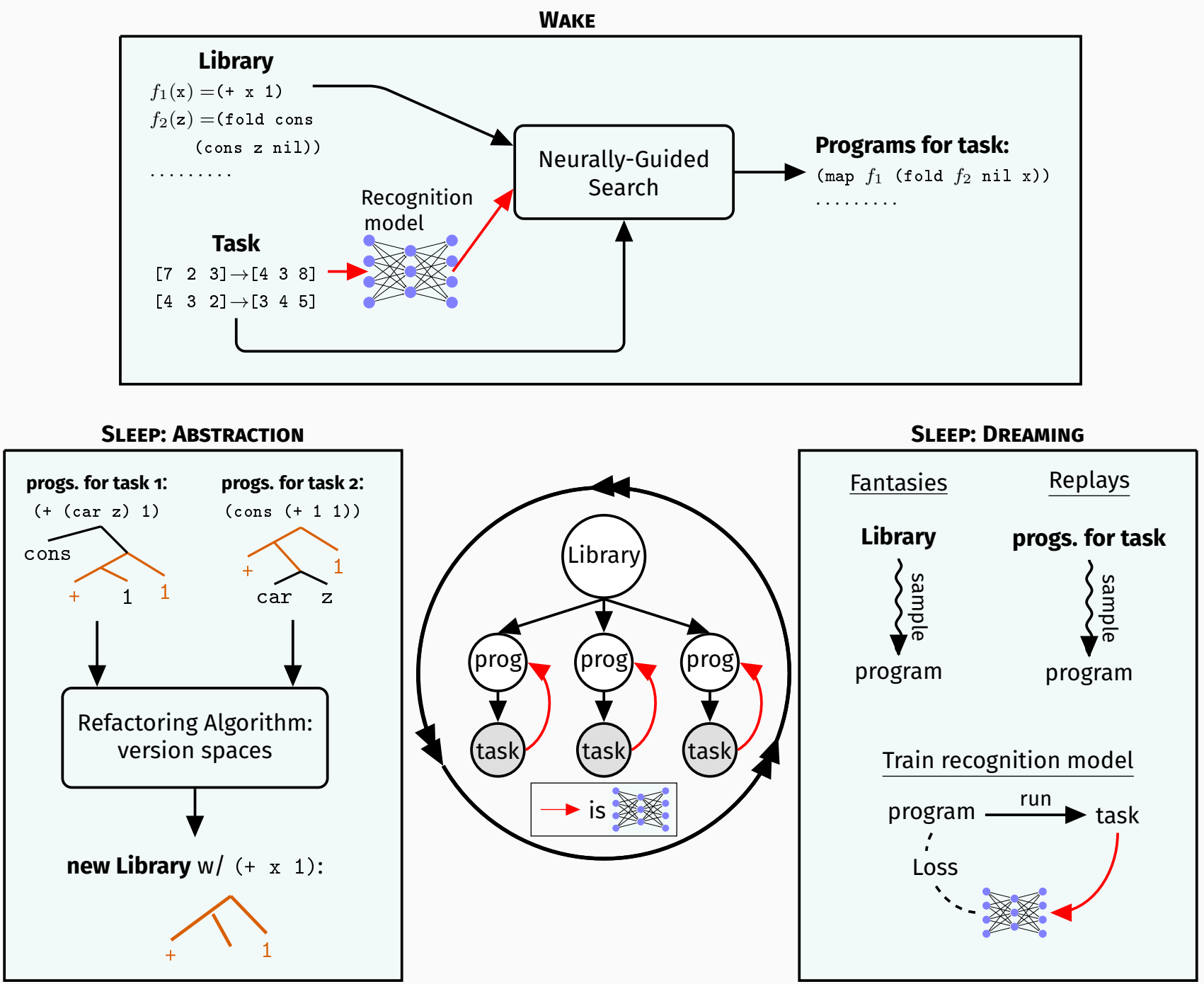
Sleep: Abstraction





Sleep: Dreaming




WAKE
Library
f_1(x) =(+ x 1)
f_2(z) =(fold cons
(cons z nil))
.....
Task
[7 2 3]→[4 3 8]
[4 3 2]→[3 4 5]
Recognition model
Neurally-Guided Search
Programs for task:
(map f_1 (fold f_2 nil x))
.....
SLEEP: ABSTRACTION
progs. for task 1:
(+ (car z) 1)
cons
+
1
1
progs. for task 2:
(cons (+ 1 1))
+
1
car
z
Refactoring Algorithm: version spaces
new Library w/ (+ x 1):
+
1
Library
prog
prog
prog
task
task
task
is
SLEEP: DREAMING
Fantasies
Replays
Library
progs. for task
sample
sample
program
program
Train recognition model
program
run
task
Loss

Abstraction Sleep: Growing the library via refactoring

$5 + 5$

Abstraction Sleep: Growing the library via refactoring

$5 + 5$

```
(+ 5 5)
```

Abstraction Sleep: Growing the library via refactoring

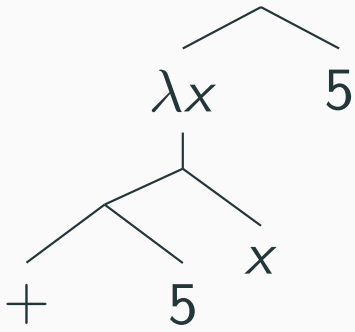
$5 + 5$

```
(+ 5 5)
```

Abstraction Sleep: Growing the library via refactoring

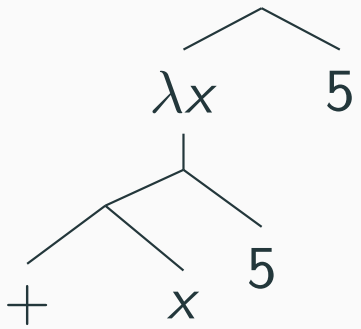
Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

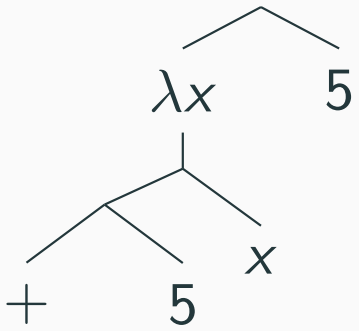
Abstraction Sleep: Growing the library via refactoring



Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

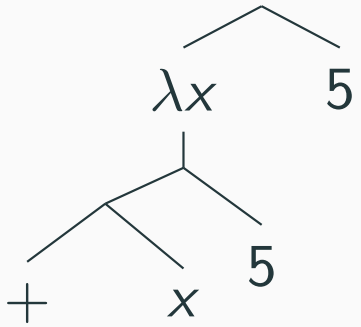
Abstraction Sleep: Growing the library via refactoring

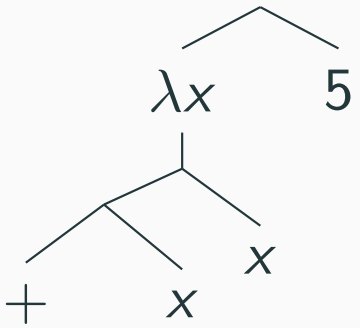


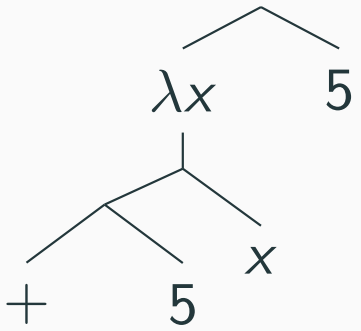


Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

Abstraction Sleep: Growing the library via refactoring

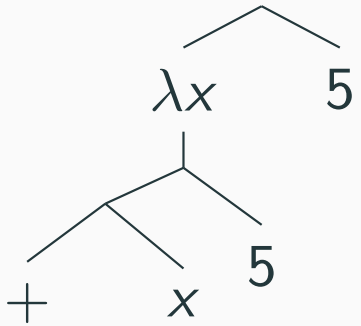


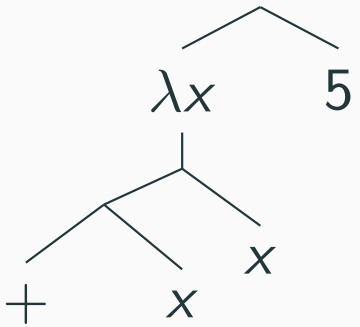


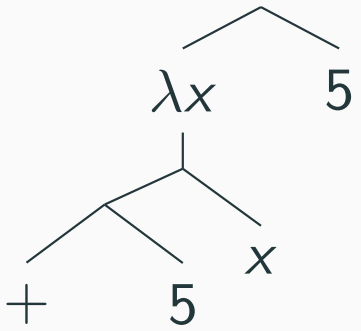


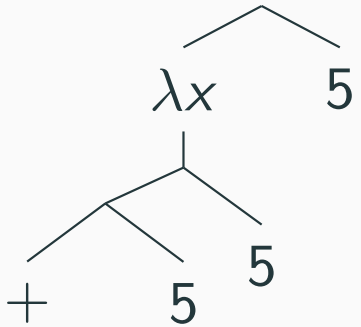
Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

Abstraction Sleep: Growing the library via refactoring



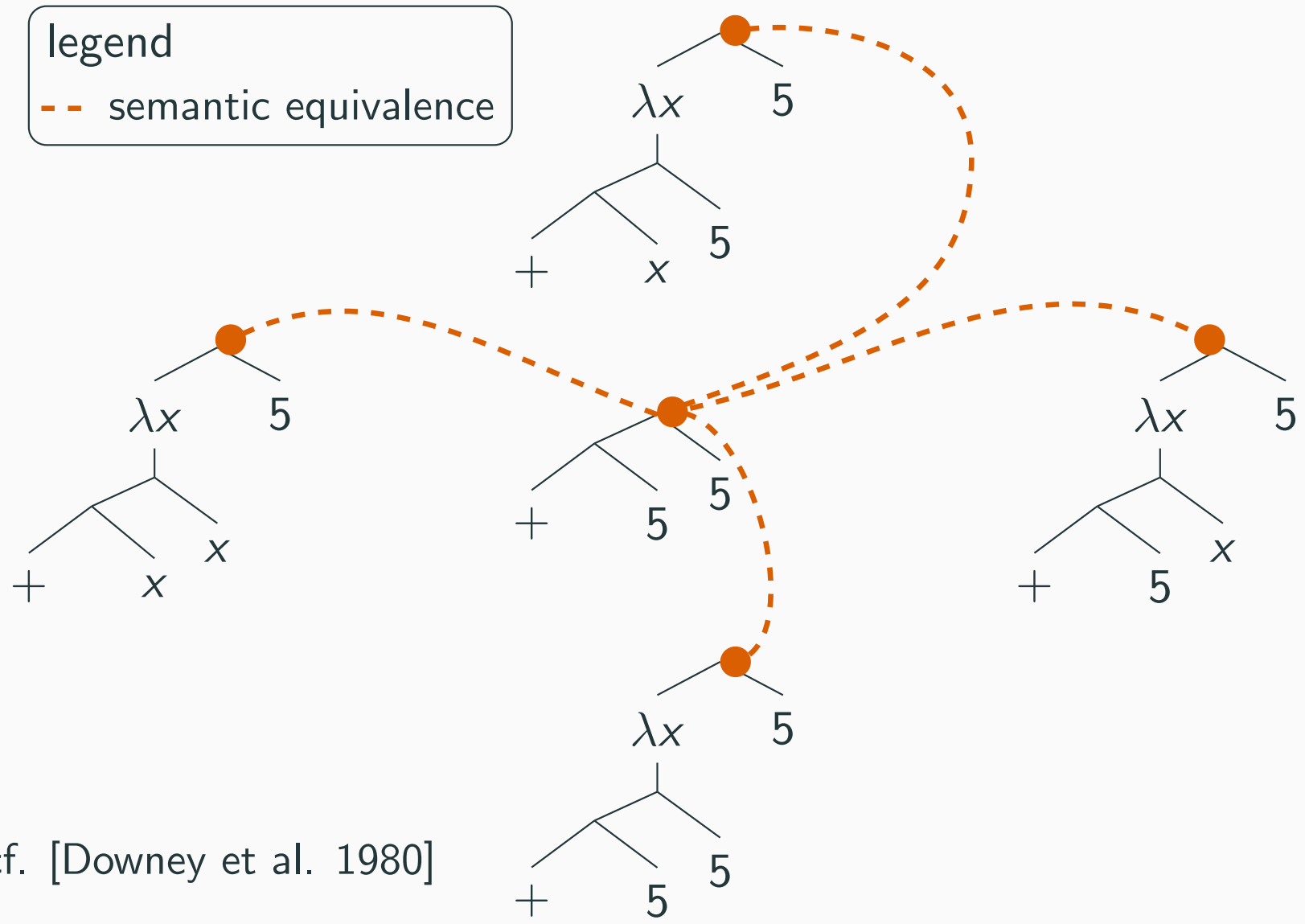






Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

Abstraction Sleep: Growing the library via refactoring

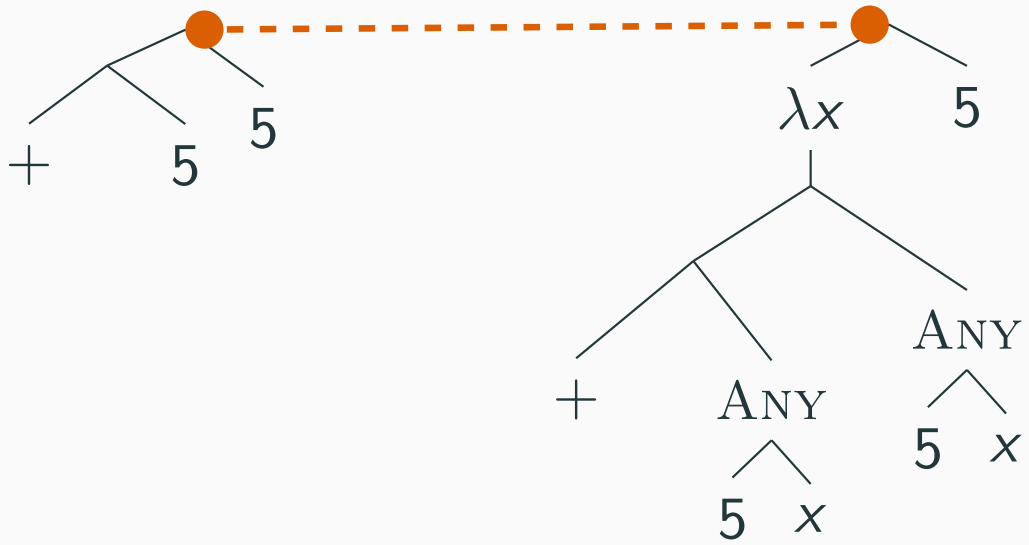


cf. [Downey et al. 1980]

Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

Abstraction Sleep: Growing the library via refactoring

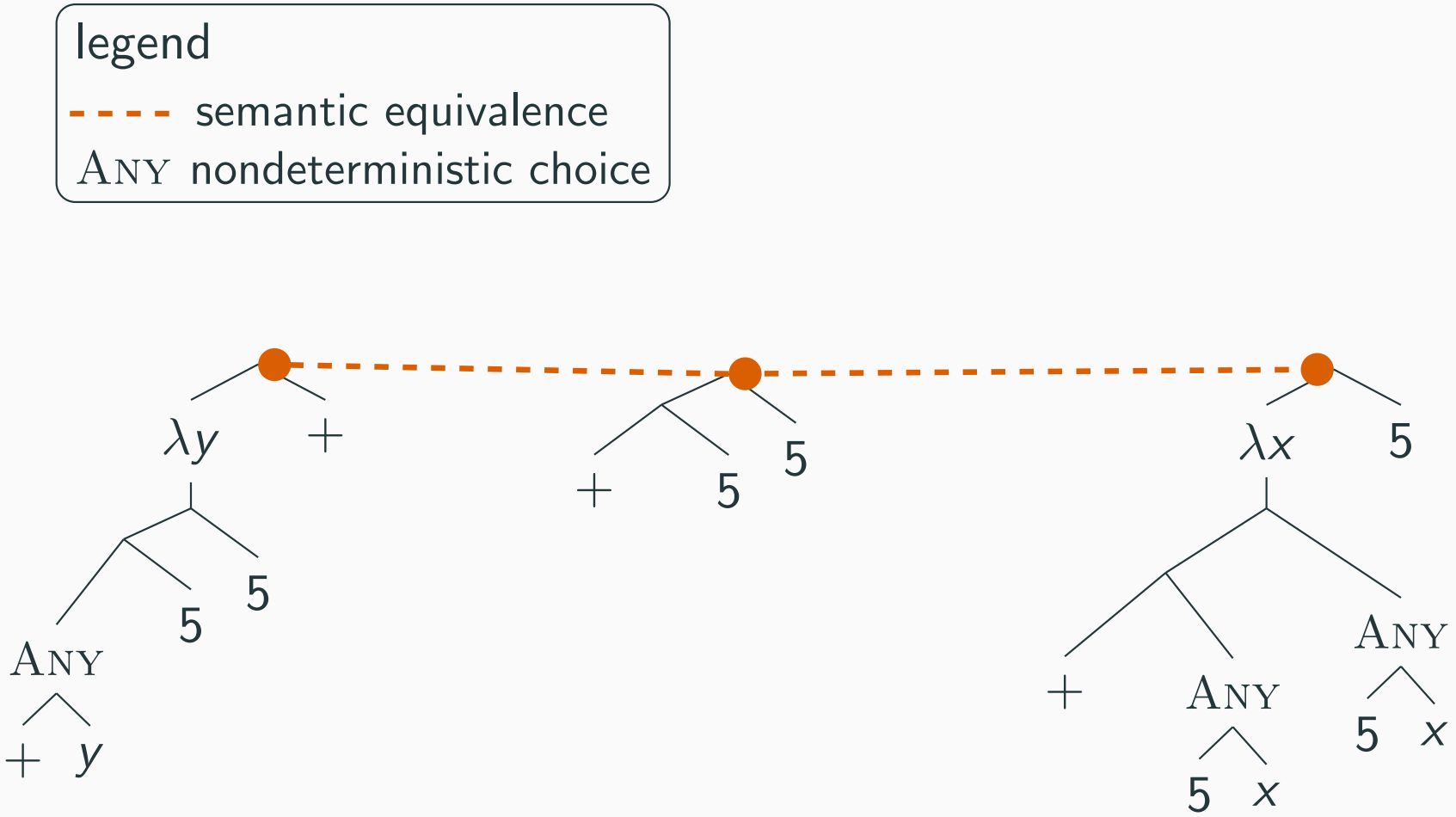
legend
- - - - semantic equivalence
ANY nondeterministic choice



cf. [Gulwani 2012]

Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

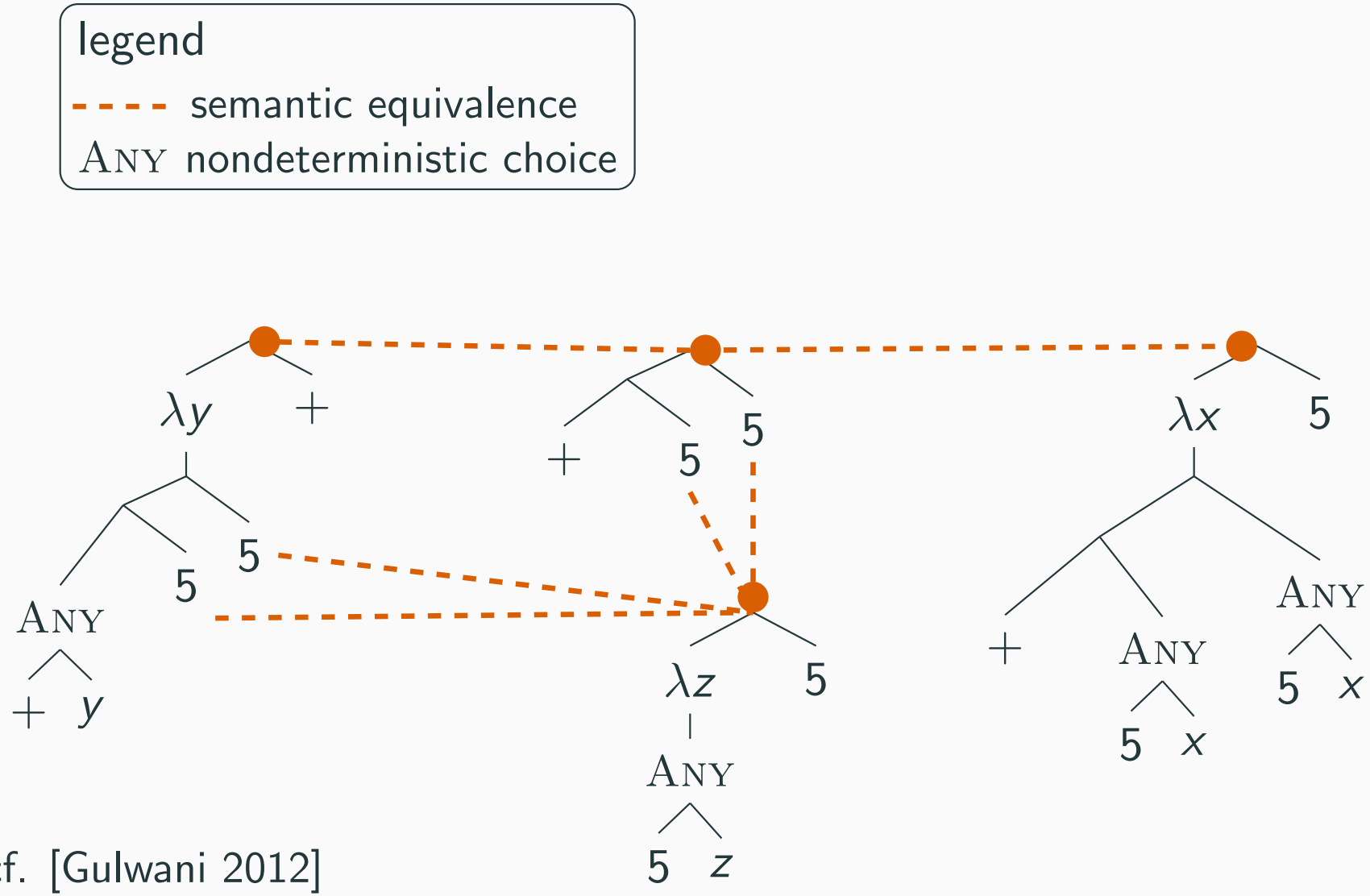
Abstraction Sleep: Growing the library via refactoring



cf. [Gulwani 2012]

Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

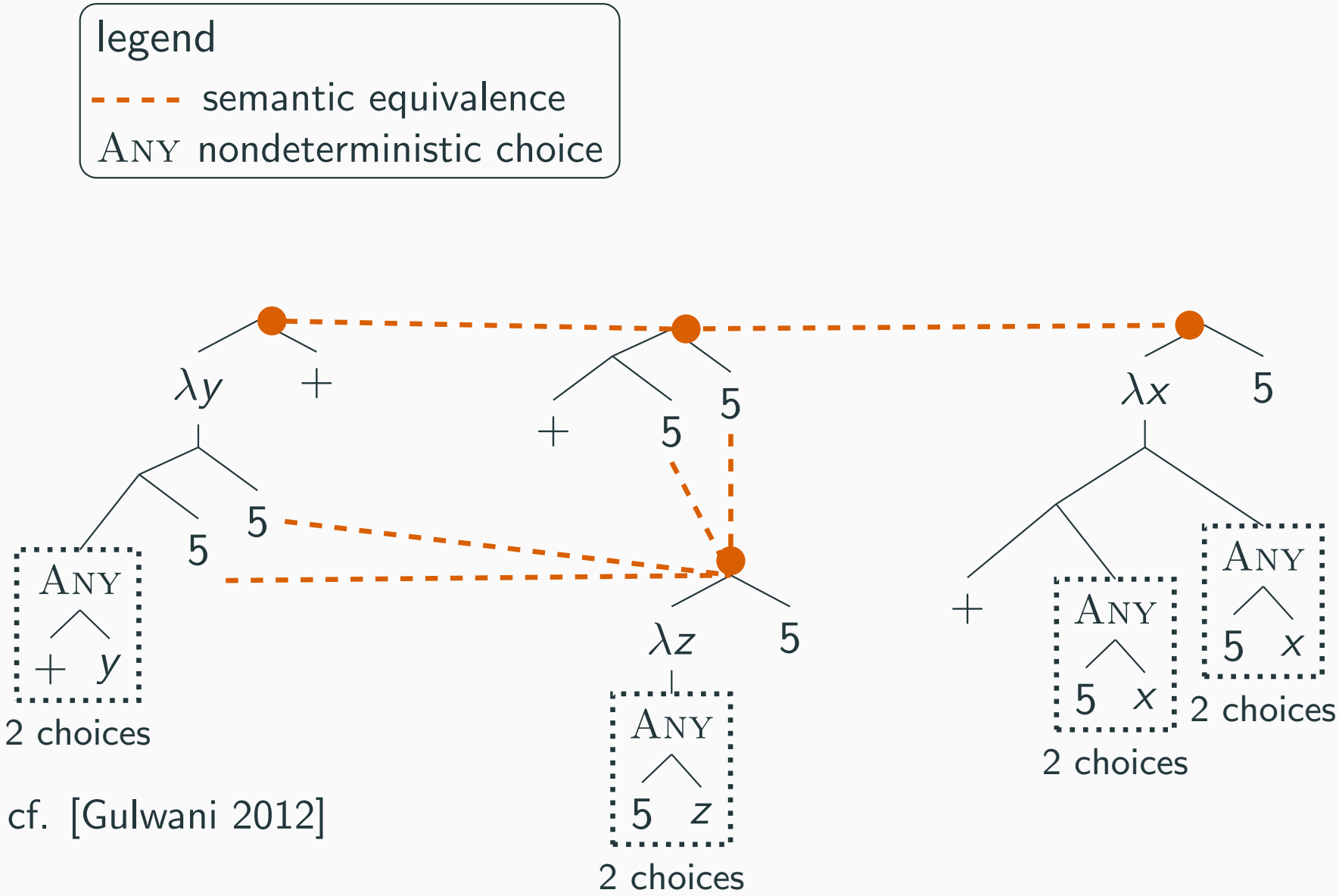
Abstraction Sleep: Growing the library via refactoring



cf. [Gulwani 2012]

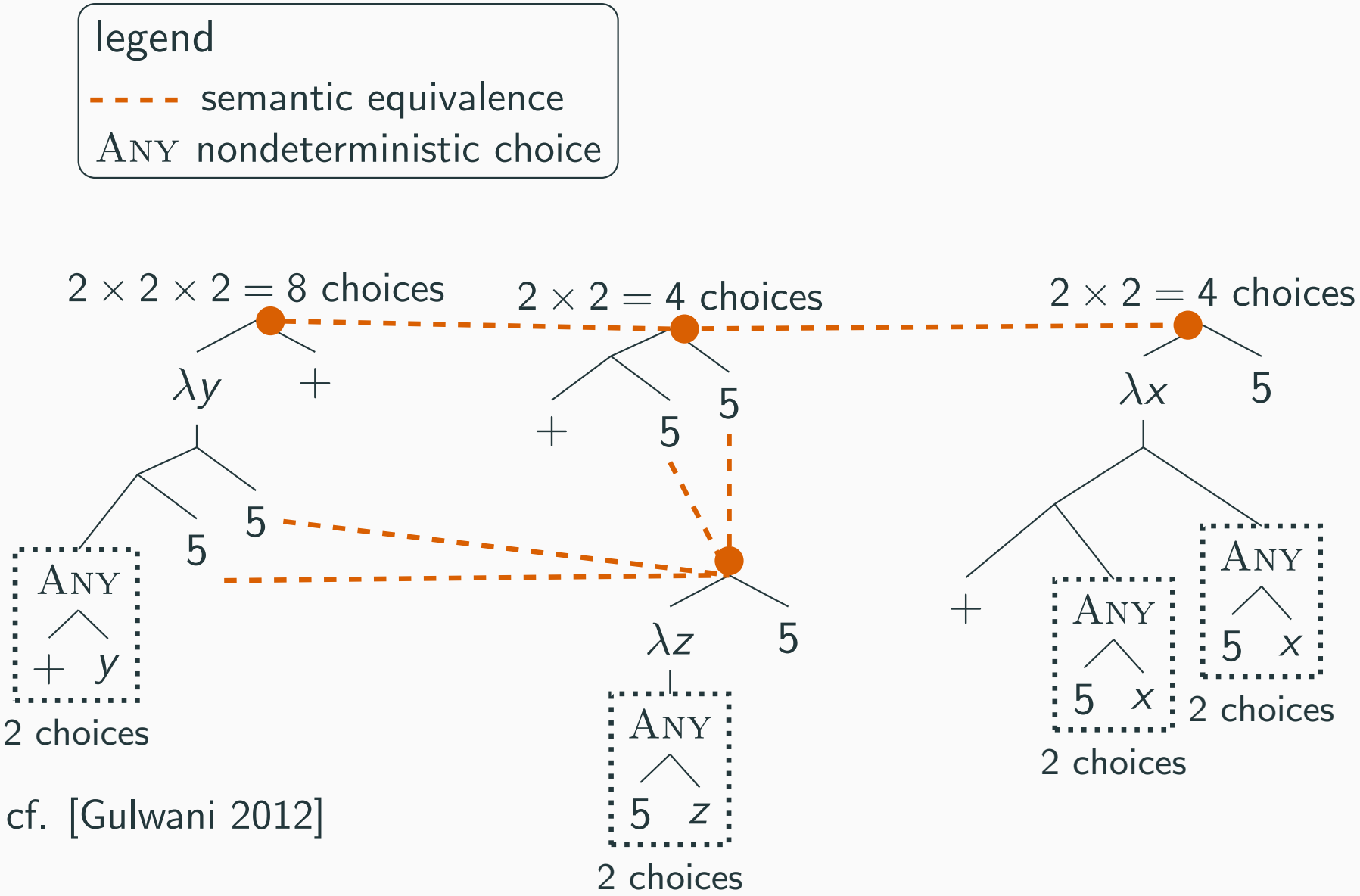
Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

Abstraction Sleep: Growing the library via refactoring



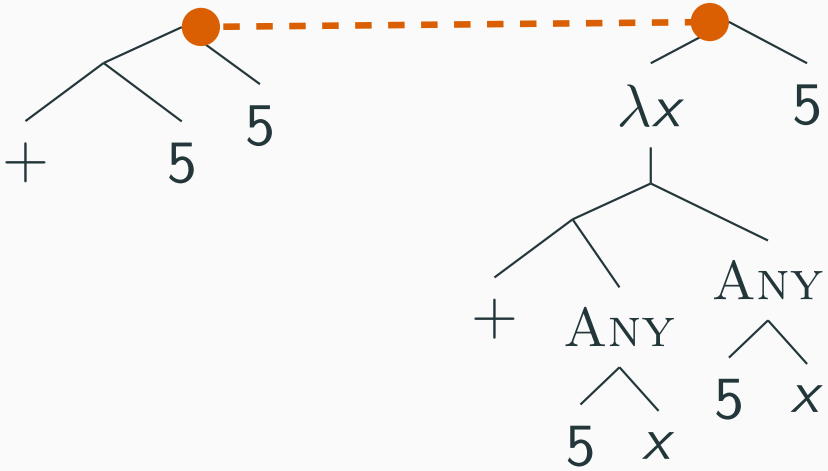
Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

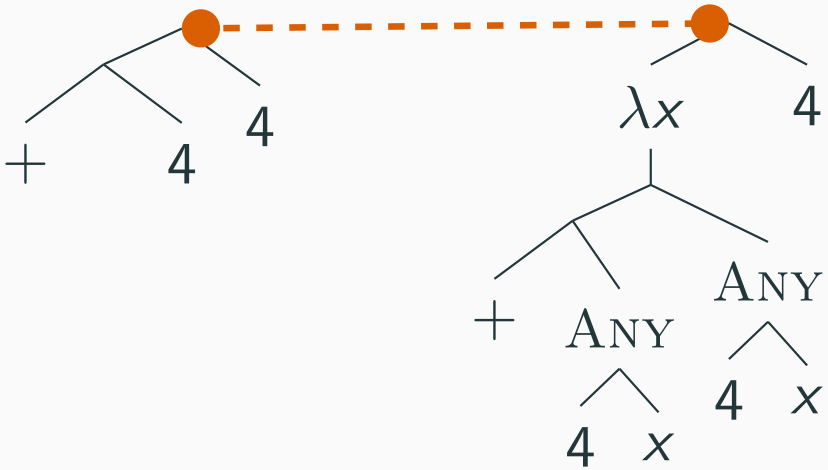
Abstraction Sleep: Growing the library via refactoring



cf. [Gulwani 2012]

Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.



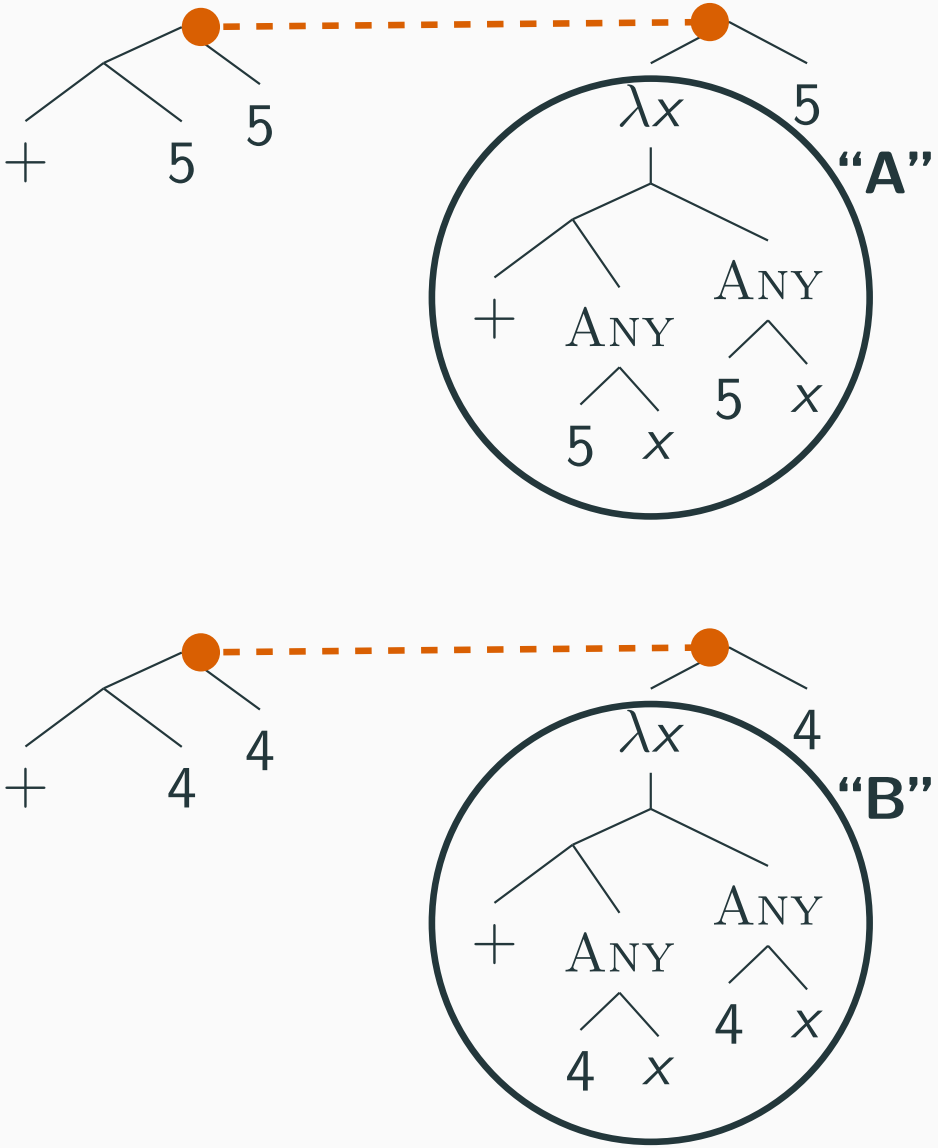


legend

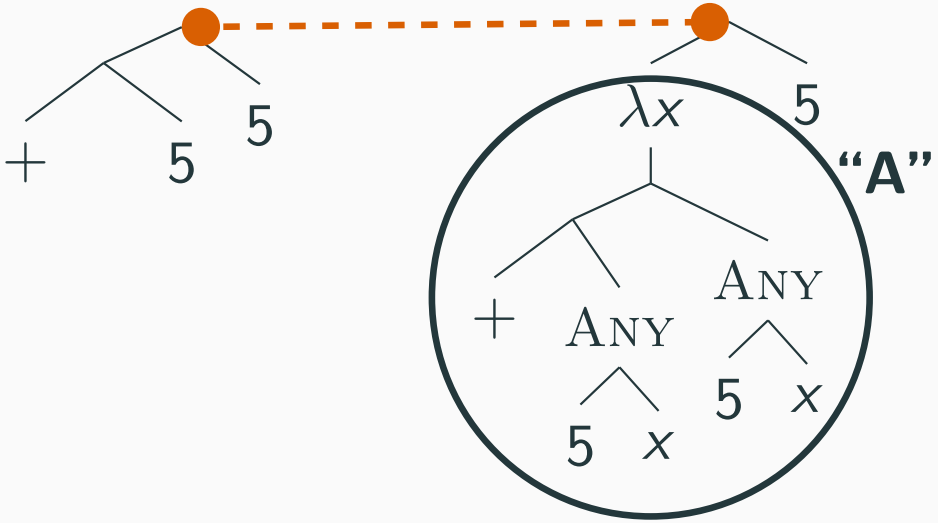
semantic equivalence

ANY nondeterministic choice

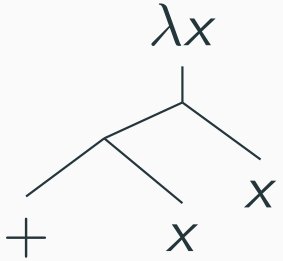
Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

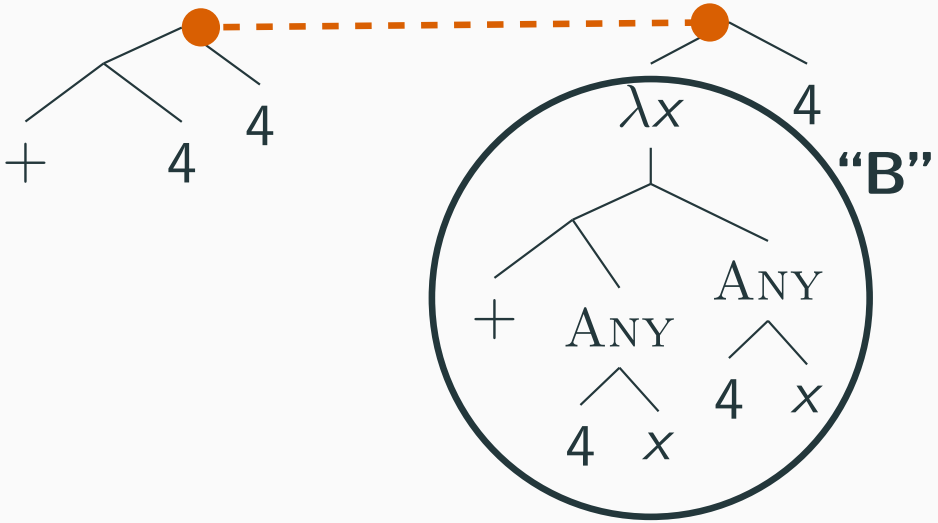


Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.



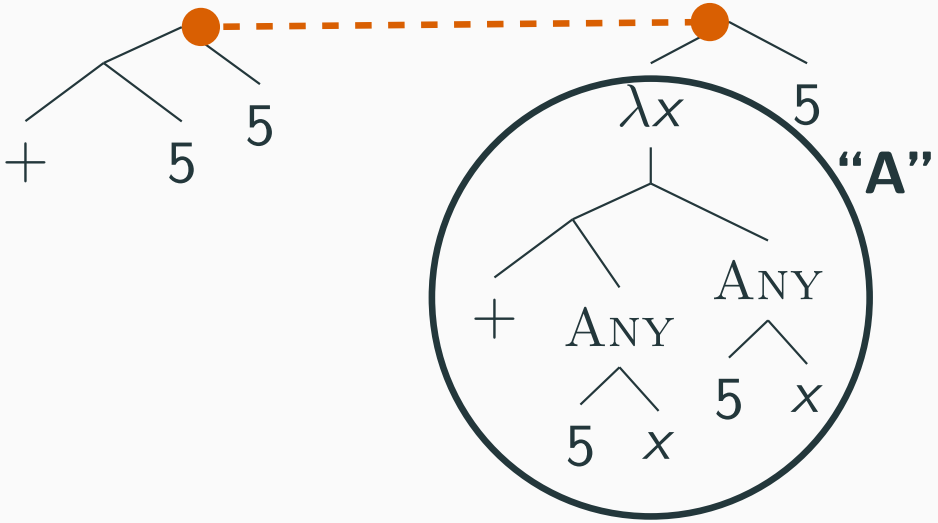
A intersect B:



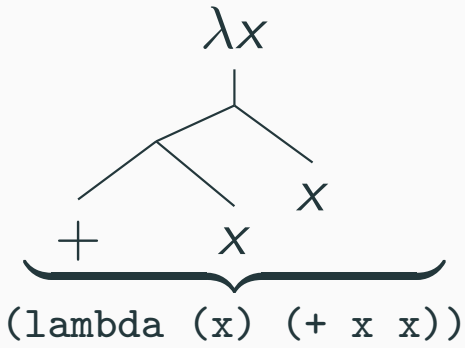


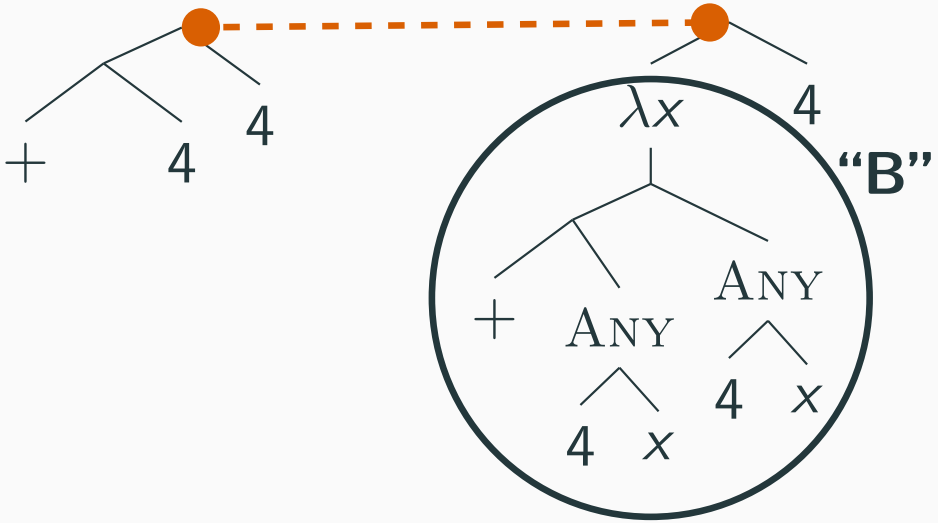
legend
- - - - semantic equivalence
ANY nondeterministic choice

Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.



A intersect B:



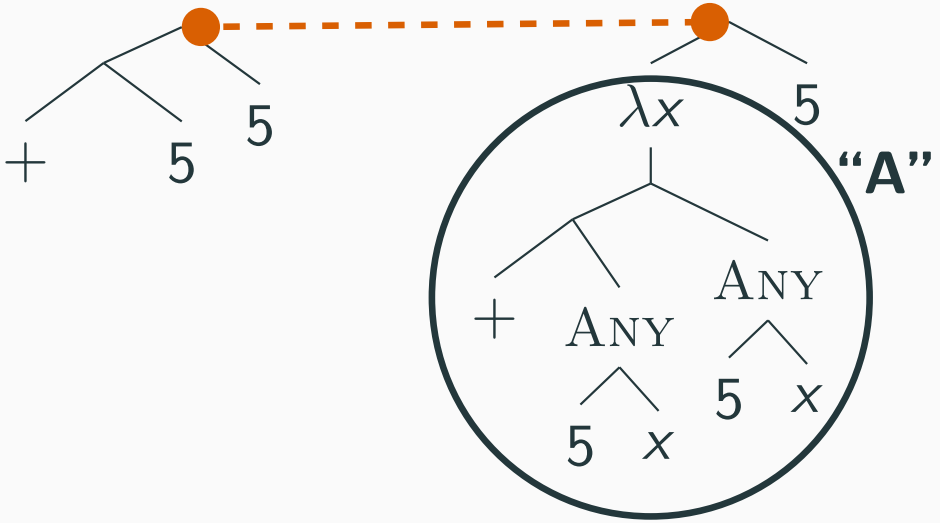


legend

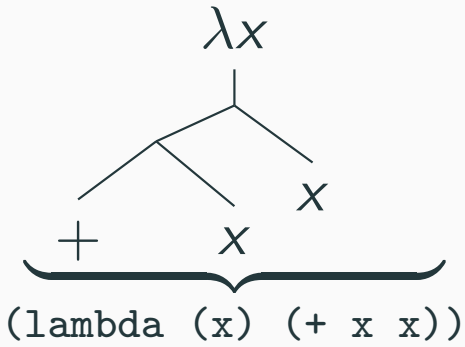
semantic equivalence

ANY nondeterministic choice

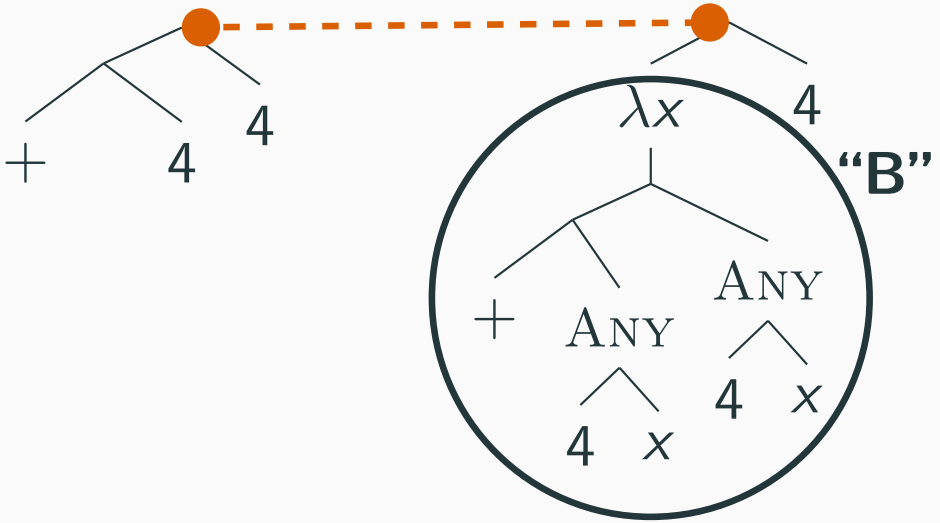
Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.



A intersect B:

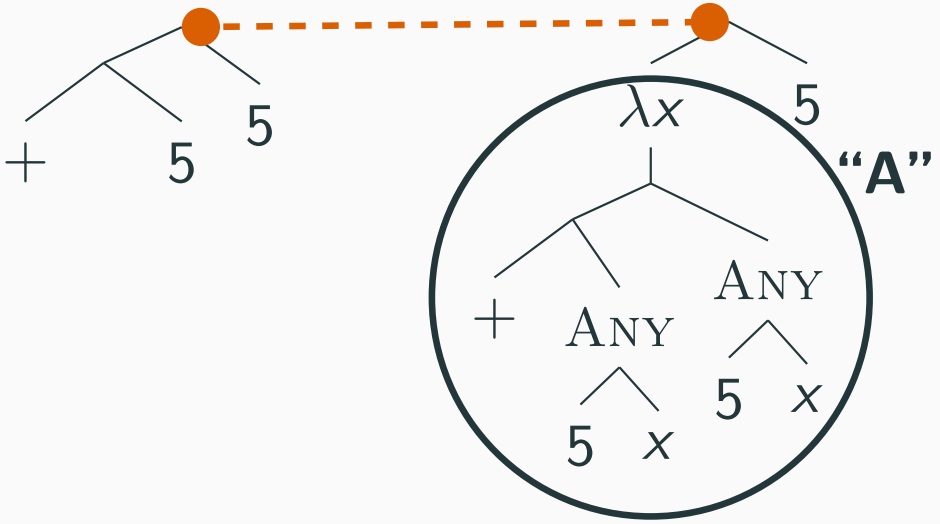


= `double`

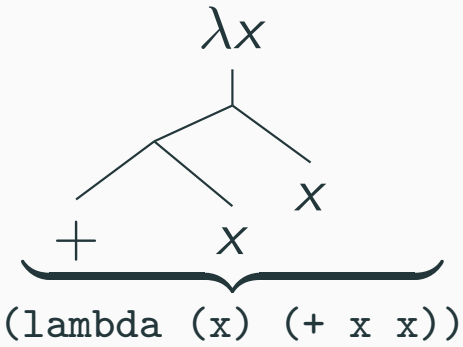


legend
- - - - semantic equivalence
ANY nondeterministic choice

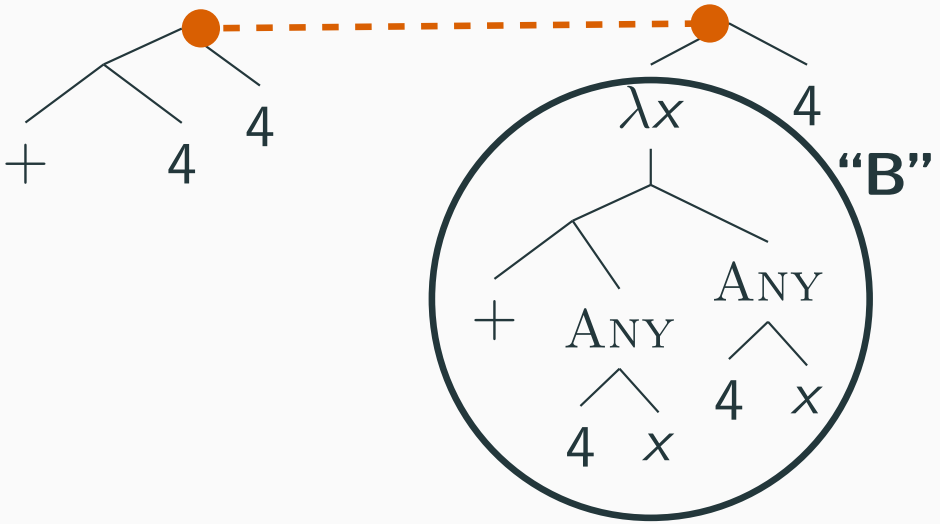
Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.



A intersect B:



= double

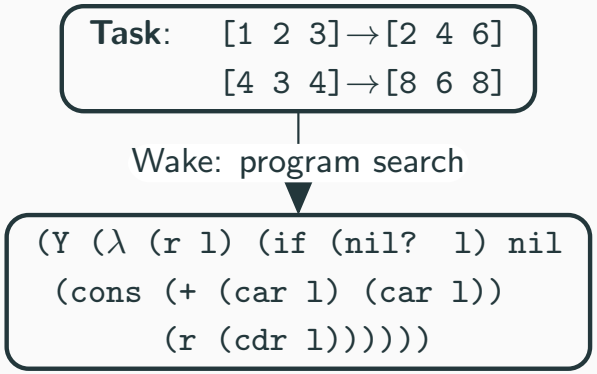


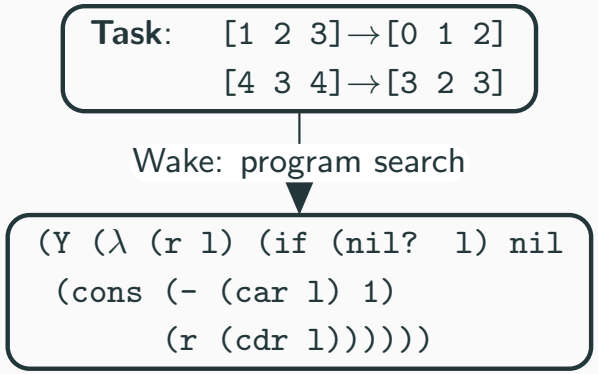
w/o double	w/ double
(+ 5 5)	(double 5)
(+ 4 4)	(double 4)
(+ 3 3)	(double 3)
...	

legend
- - - - semantic equivalence
ANY nondeterministic choice

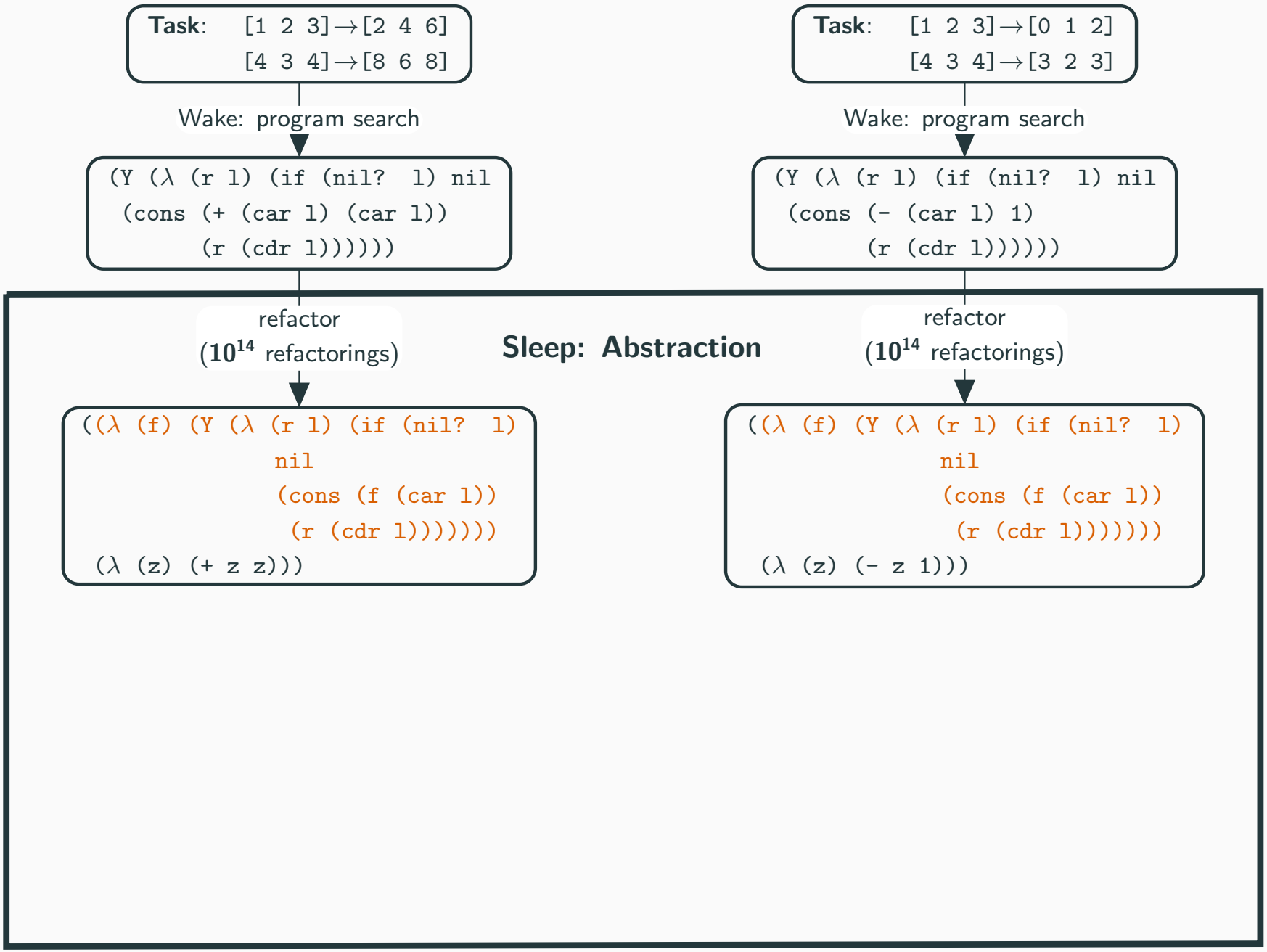
Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

Abstraction Sleep: Growing the library via refactoring

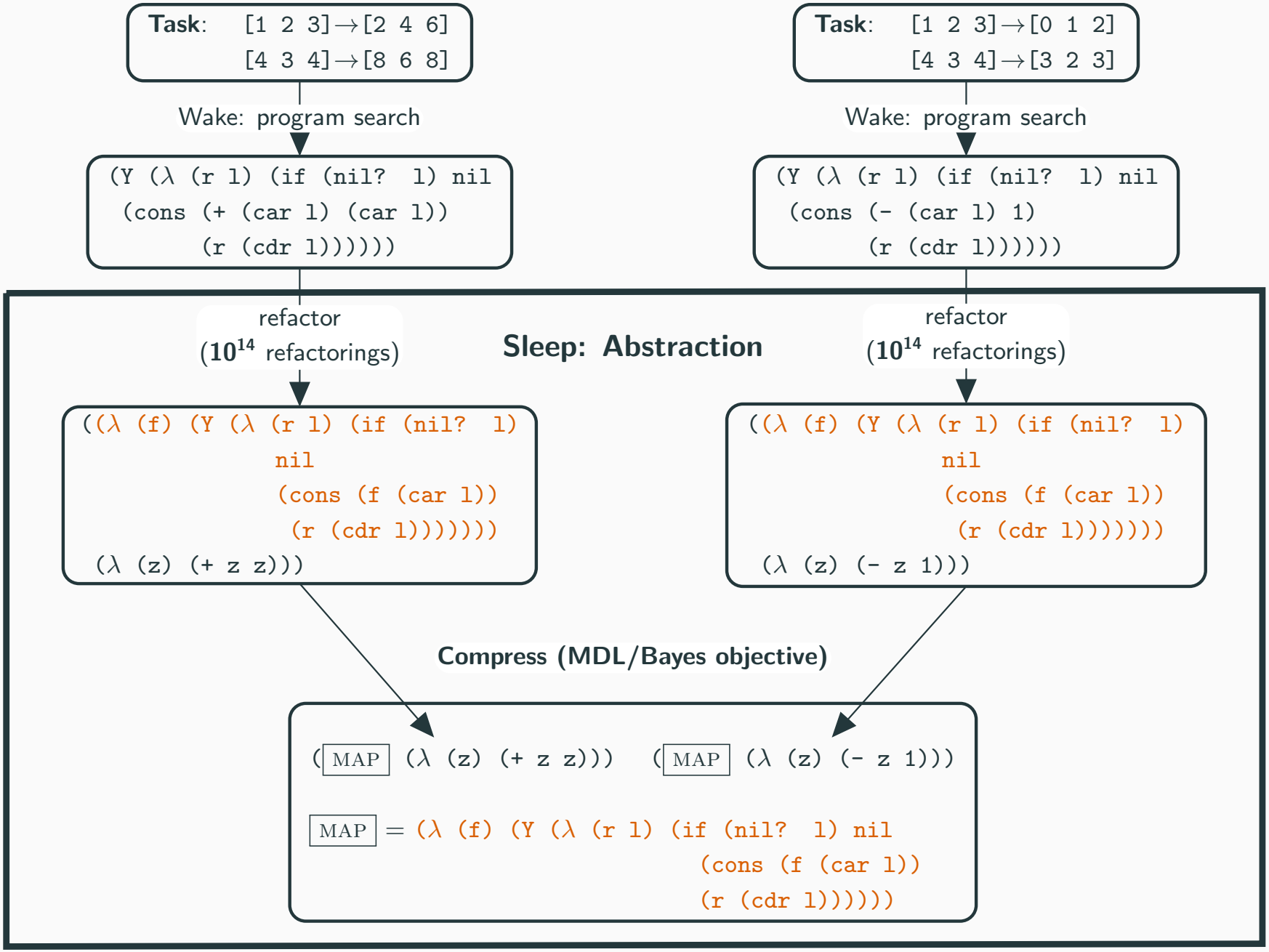




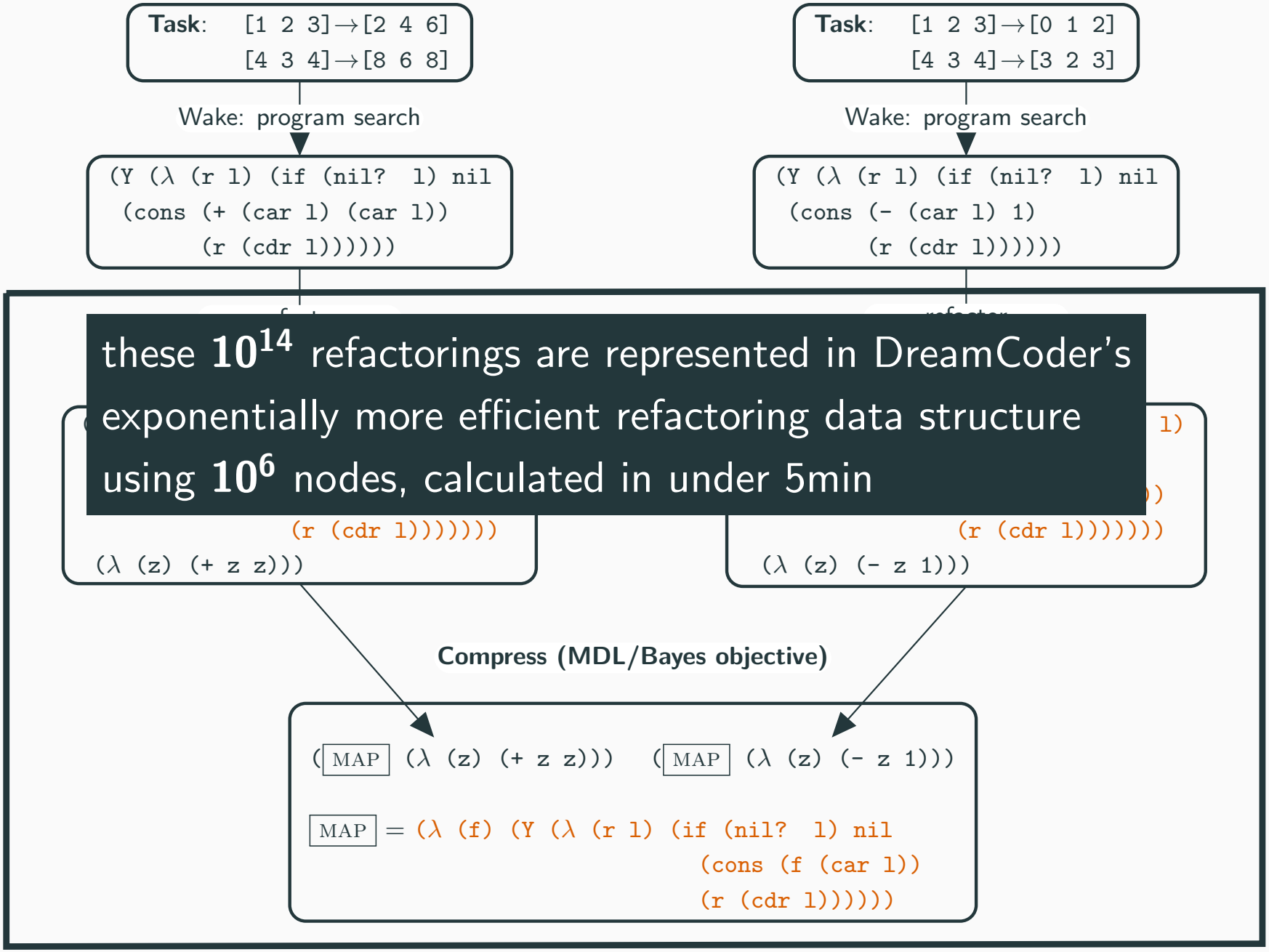
Abstraction Sleep: Growing the library via refactoring



Abstraction Sleep: Growing the library via refactoring



Abstraction Sleep: Growing the library via refactoring



DreamCoder Domains

List Processing

Sum List

```
[1 2 3] → 6
[4 6 8 1] → 17
```

Double

```
[1 2 3] → [2 4 6]
[4 5 1] → [8 10 2]
```

Text Editing

Abbreviate

```
Allen Newell → A.N.
Herb Simon → H.S.
```

Drop Last Three

```
shrdlu → shr
shakey → sha
```

Regexes

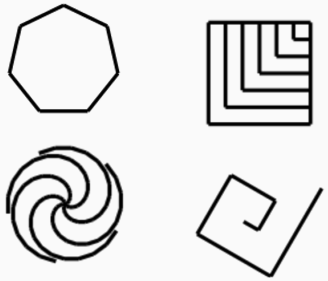
Phone numbers

```
(555) 867-5309
(650) 555-2368
```

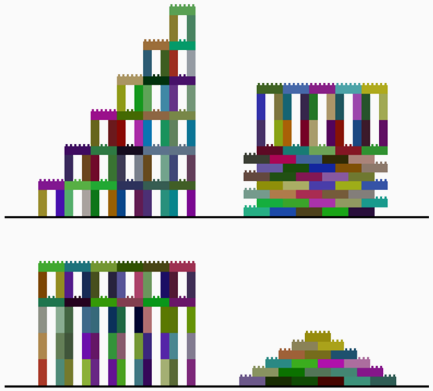
Currency

```
$100.25
$4.50
```

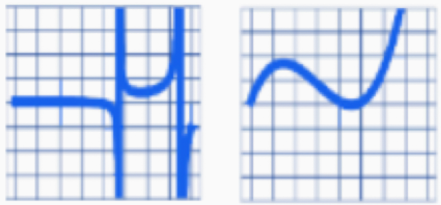
LOGO Graphics



Block Towers



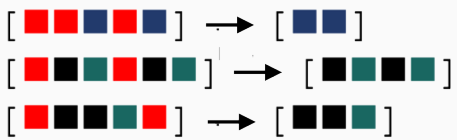
Symbolic Regression



$$y = f(x)$$

Recursive Programming

Filter Red



Physical Laws

$$\vec{a} = \frac{1}{m}\sum_i \vec{F}_i$$

$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r}|^2}\hat{r}$$

Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

DreamCoder Domains

List Processing

Sum List

```
[1 2 3] → 6
[4 6 8 1] → 17
```

Double

```
[1 2 3] → [2 4 6]
[4 5 1] → [8 10 2]
```

Text Editing

Abbreviate

```
Allen Newell → A.N.
Herb Simon → H.S.
```

Drop Last Three

```
shrdlu → shr
shakey → sha
```

Regexes

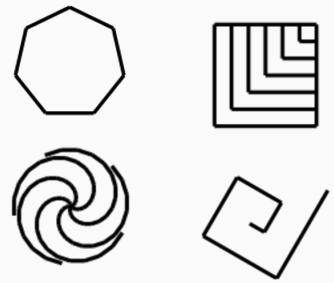
Phone numbers

```
(555) 867-5309
(650) 555-2368
```

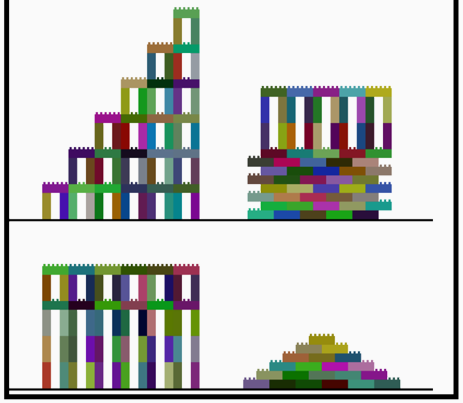
Currency

```
$100.25
$4.50
```

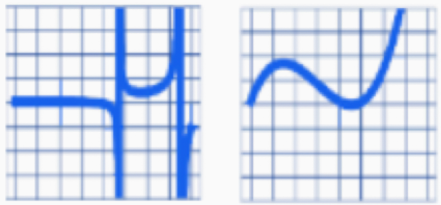
LOGO Graphics



Block Towers



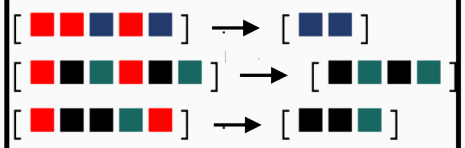
Symbolic Regression



$$y = f(x)$$

Recursive Programming

Filter Red



Physical Laws

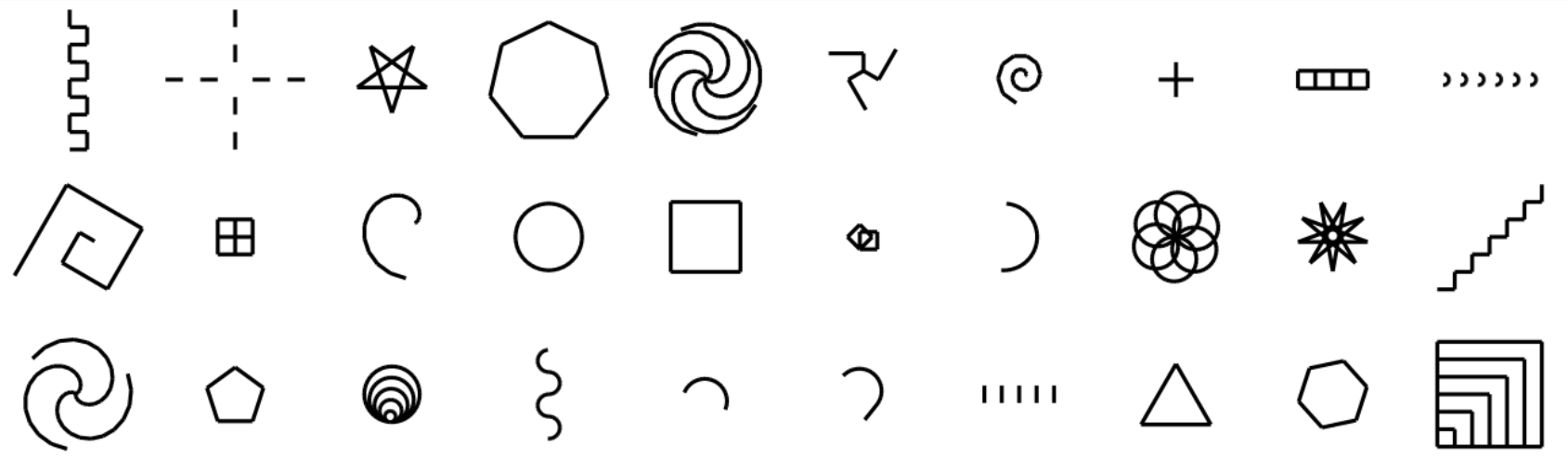
$$\vec{a} = \frac{1}{m}\sum_i \vec{F}_i$$

$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r}|^2}\hat{r}$$

Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

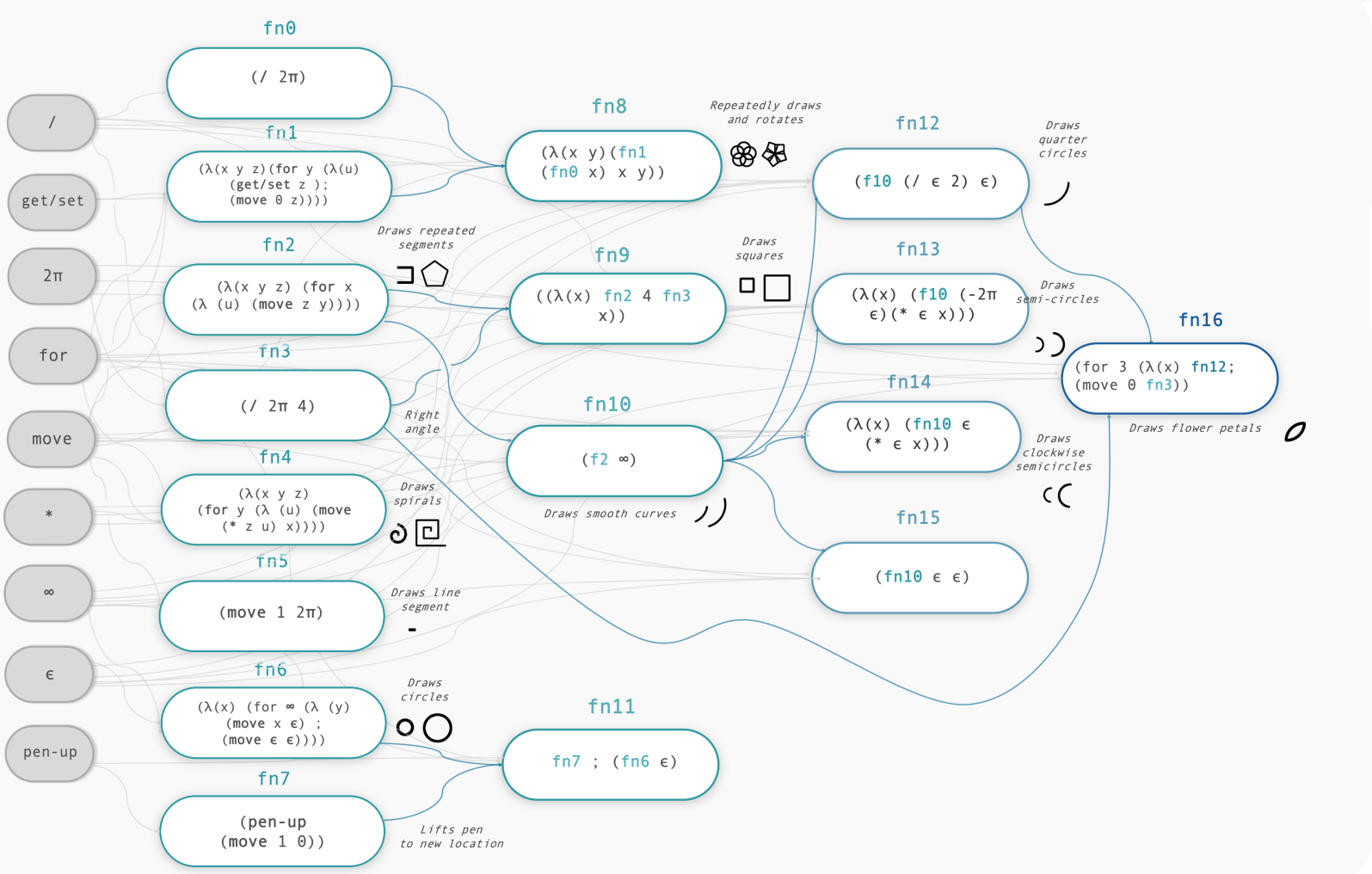
LOGO Turtle Graphics

30 out of 160 tasks

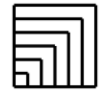


Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

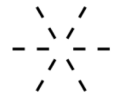
LOGO Turtle Graphics – learning an interpretable library



(fn8 5 (fn4 (* ϵ 2) ∞ ϵ))



(for 7 (λ (x) (fn9 x)))

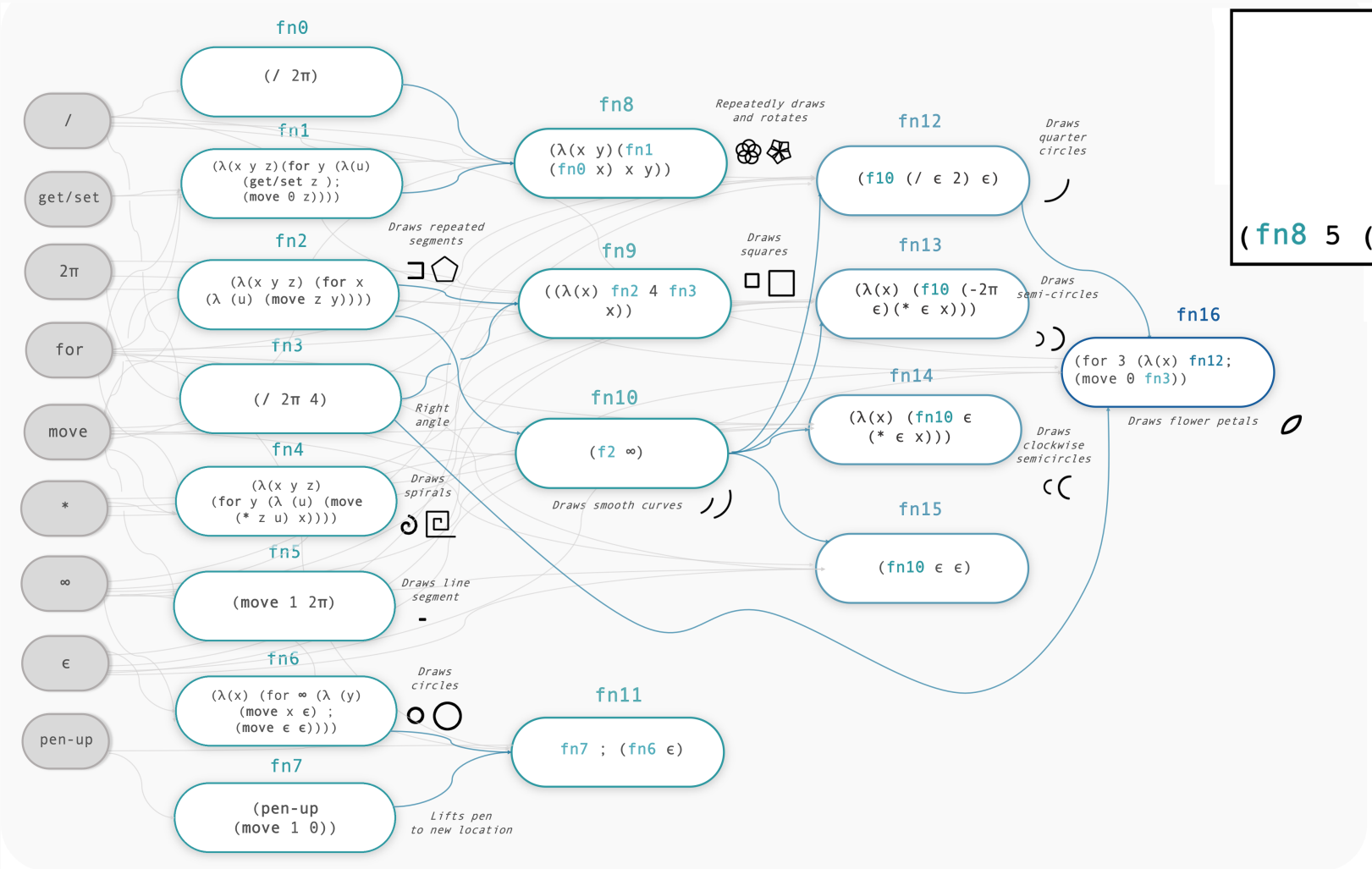


(fn8 6 (fn7 ; fn5 ; fn7 ; fn5))

(move 0 (fn0 7)); fn5 ; (fn13 4)

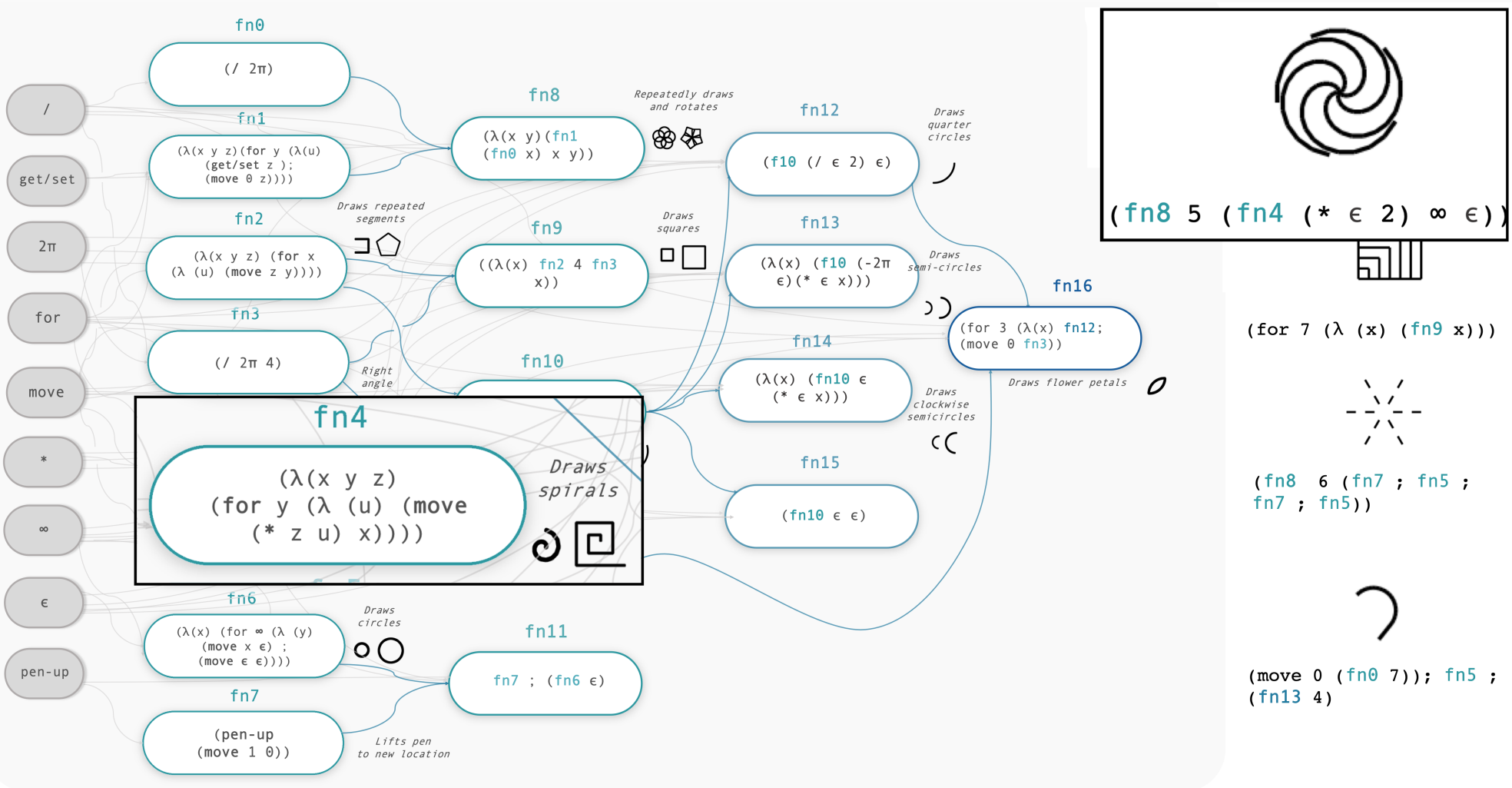
Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

LOGO Turtle Graphics – learning an interpretable library



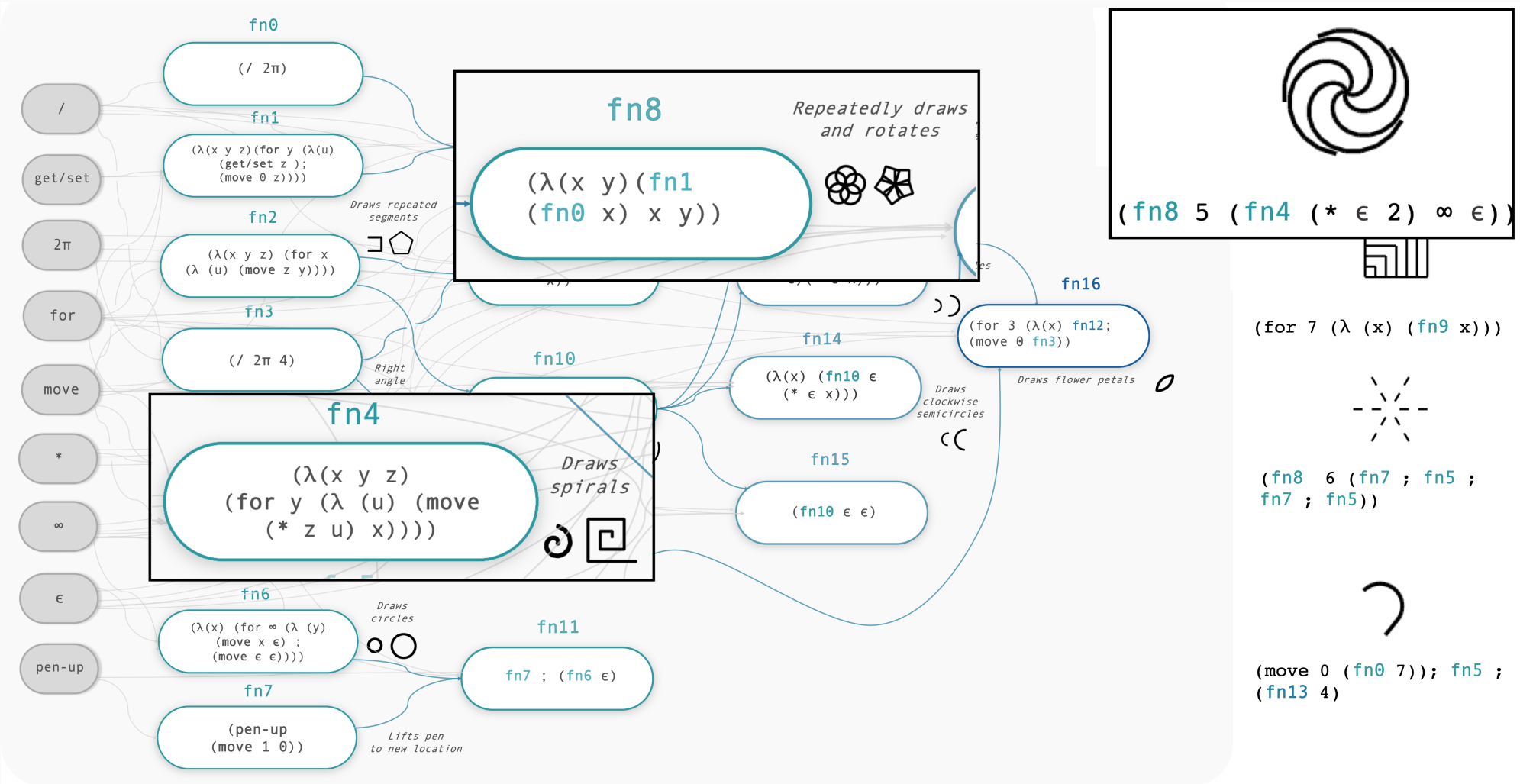
Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

LOGO Turtle Graphics – learning an interpretable library



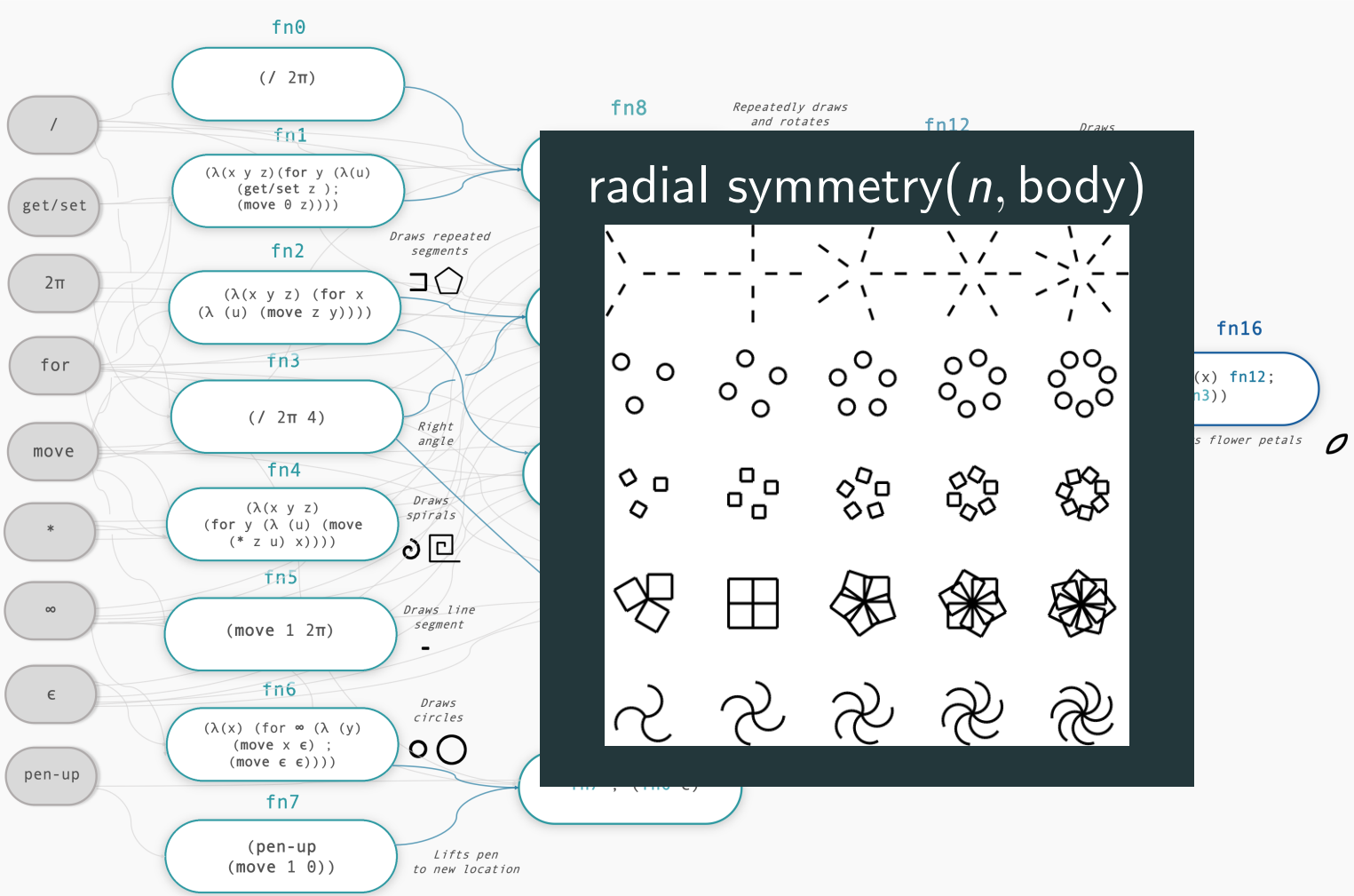
Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

LOGO Turtle Graphics – learning an interpretable library



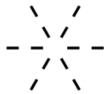
Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

LOGO Turtle Graphics – learning an interpretable library



(fn8 5 (fn4 (* ϵ 2) ∞ ϵ))

(for 7 (λ (x) (fn9 x)))

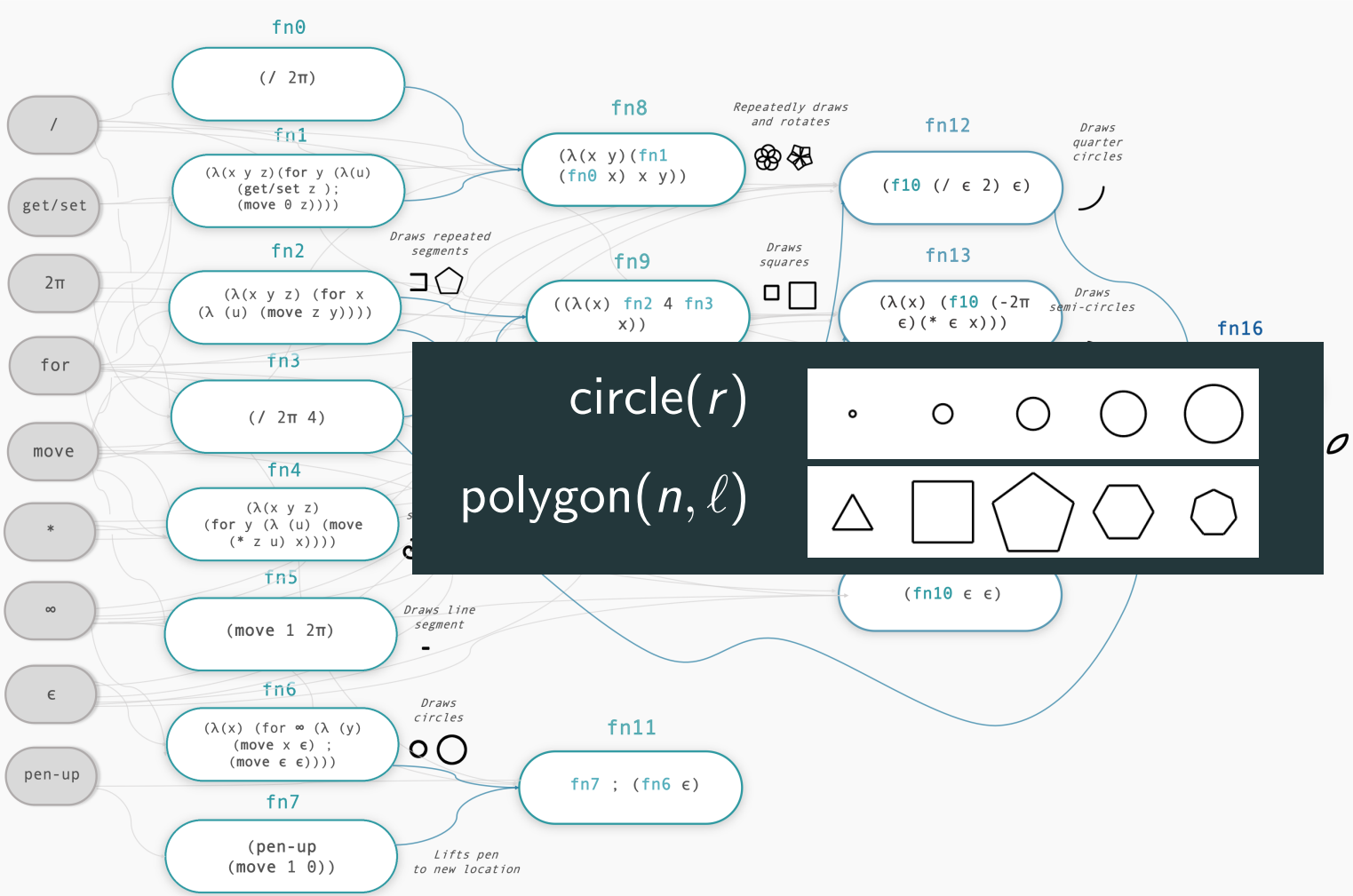


(fn8 6 (fn7 ; fn5 ; fn7 ; fn5))

(move 0 (fn0 7)); fn5 ; (fn13 4)

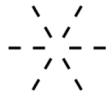
Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

LOGO Turtle Graphics – learning an interpretable library



```
(fn8 5 (fn4 (* ϵ 2) ∞ ϵ))
```

```
(for 7 (λ (x) (fn9 x)))
```

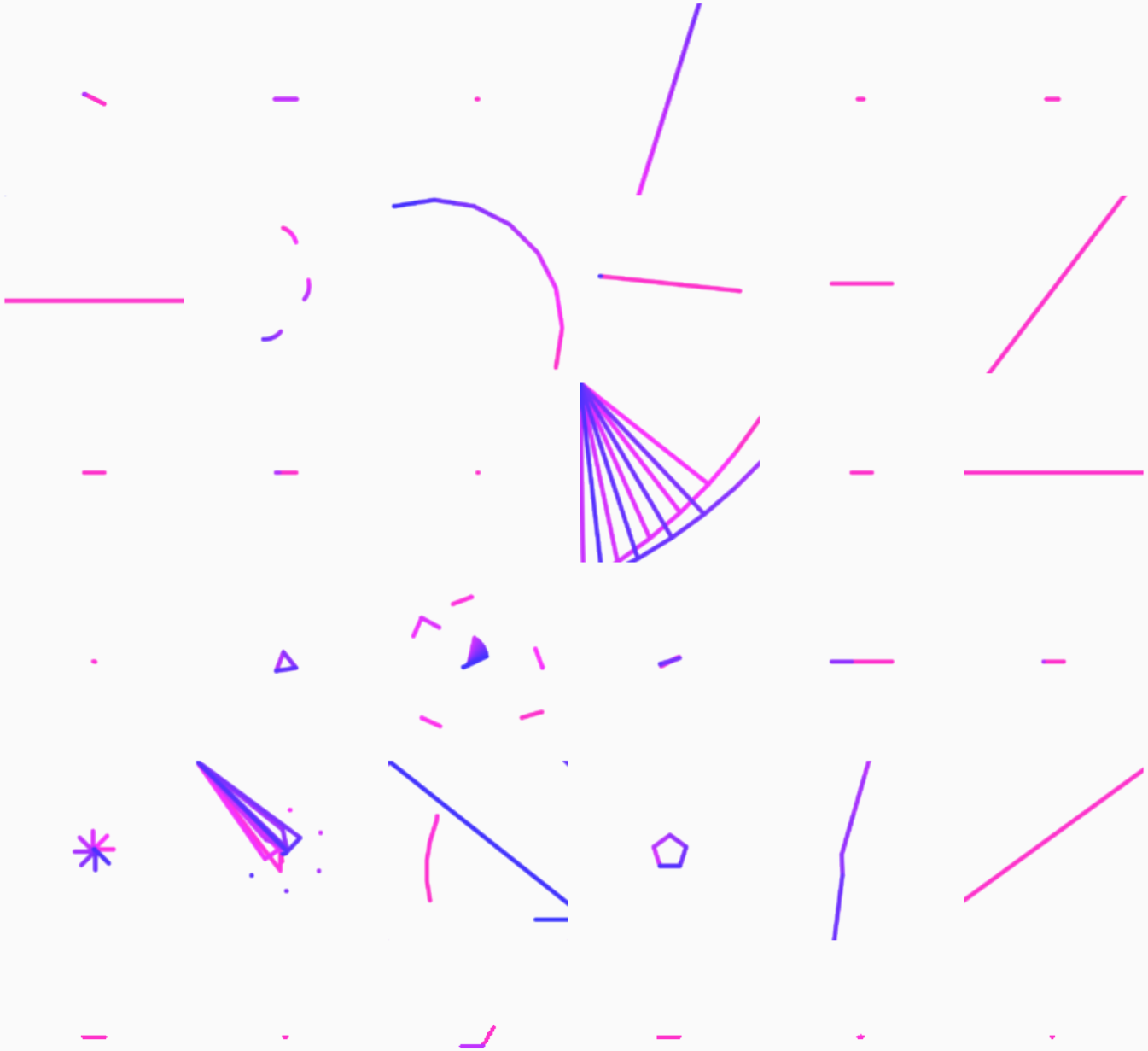


```
(fn8  6 (fn7 ; fn5 ;
fn7 ; fn5))
```

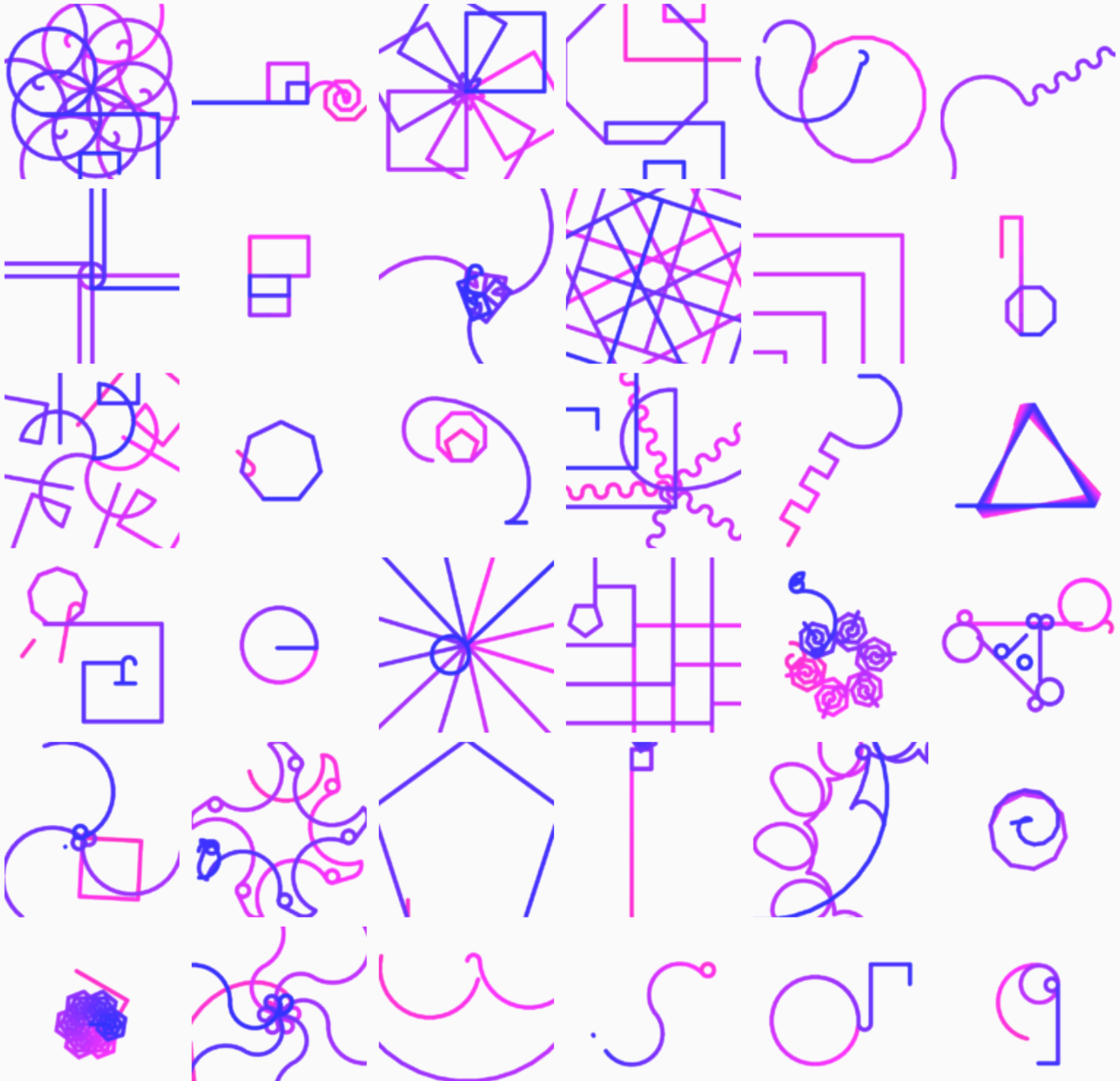
```
(move 0 (fn0 7)); fn5 ;
(fn13 4)
```

Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

What does DreamCoder dream of? (before learning)

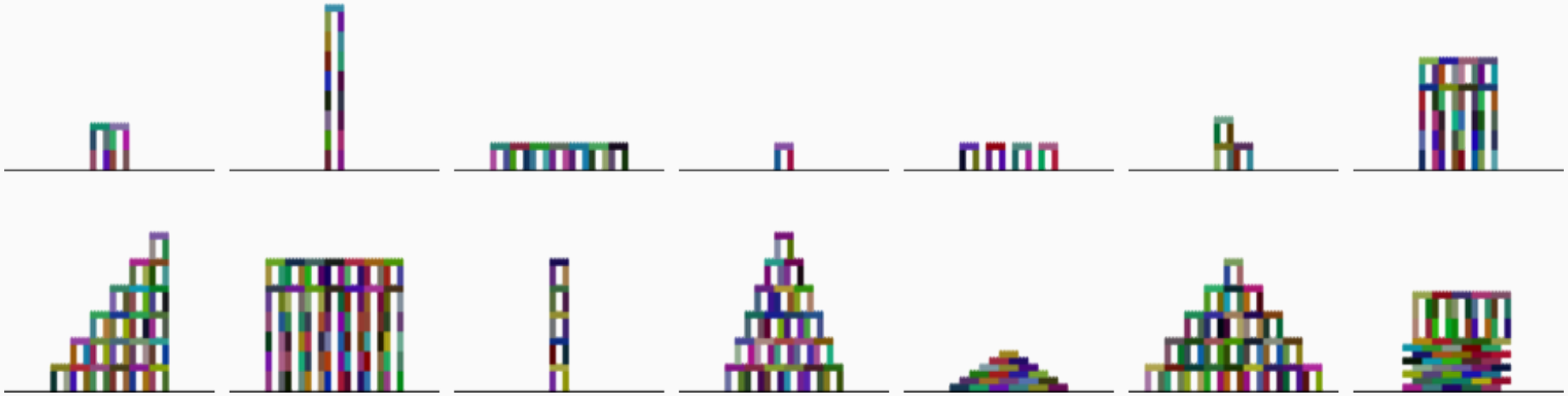


What does DreamCoder dream of? (after learning)



Planning to build towers

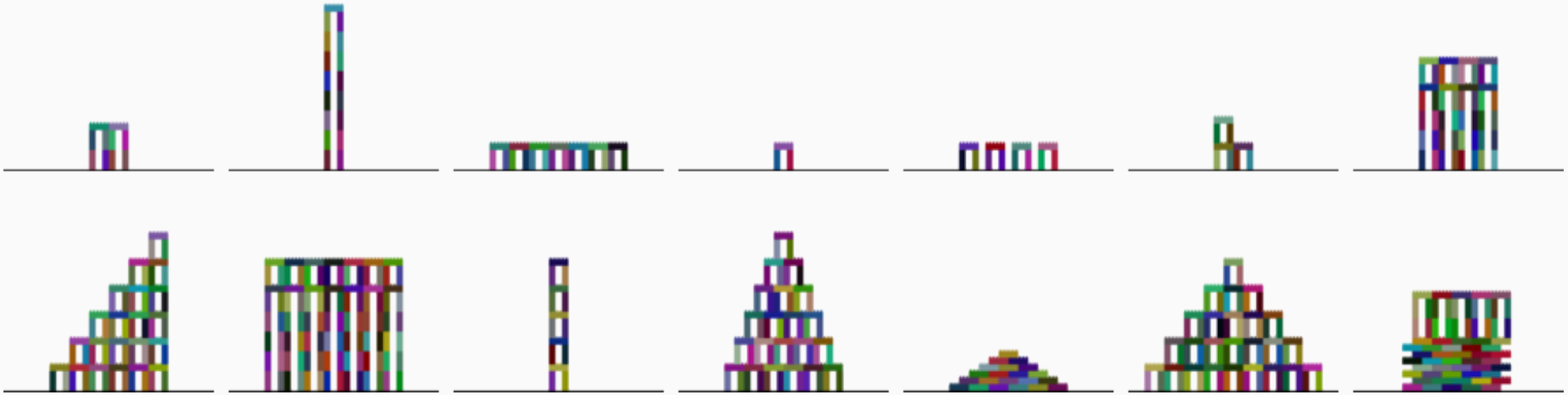
example tasks (112 total)



Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

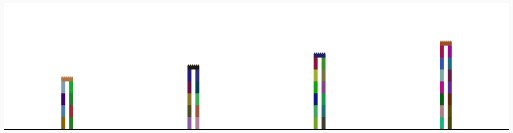
Planning to build towers

example tasks (112 total)

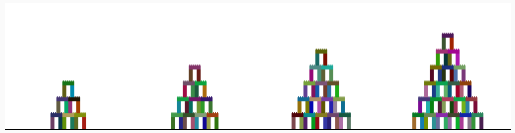


learned library routines ($\approx$ 20 total)

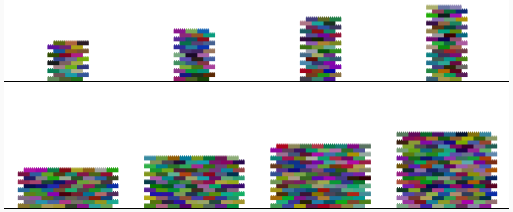
arch(h)



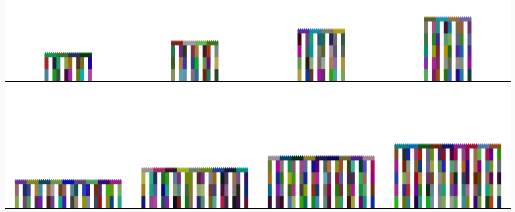
pyramid(h)



wall(w, h)

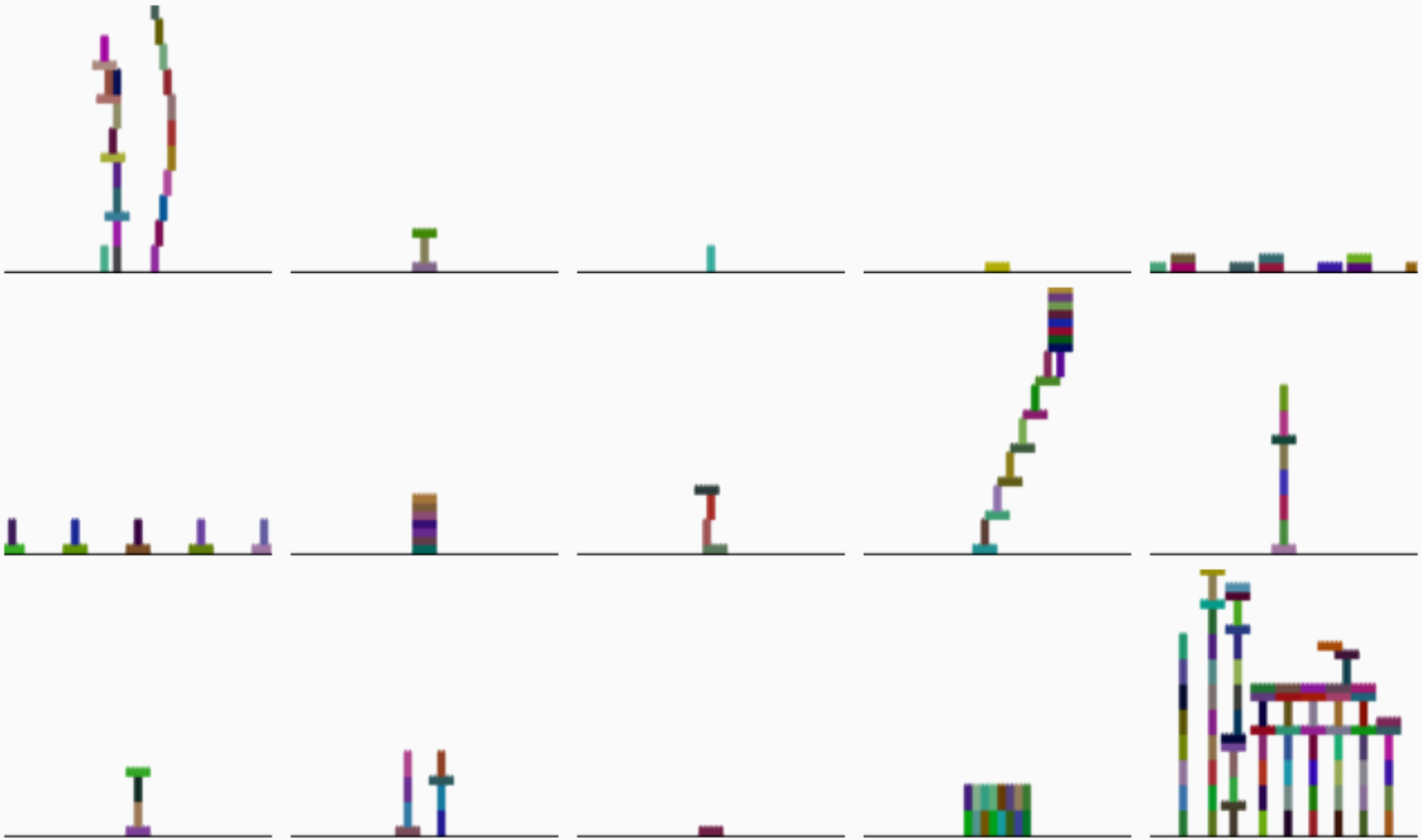


bridge(w, h)

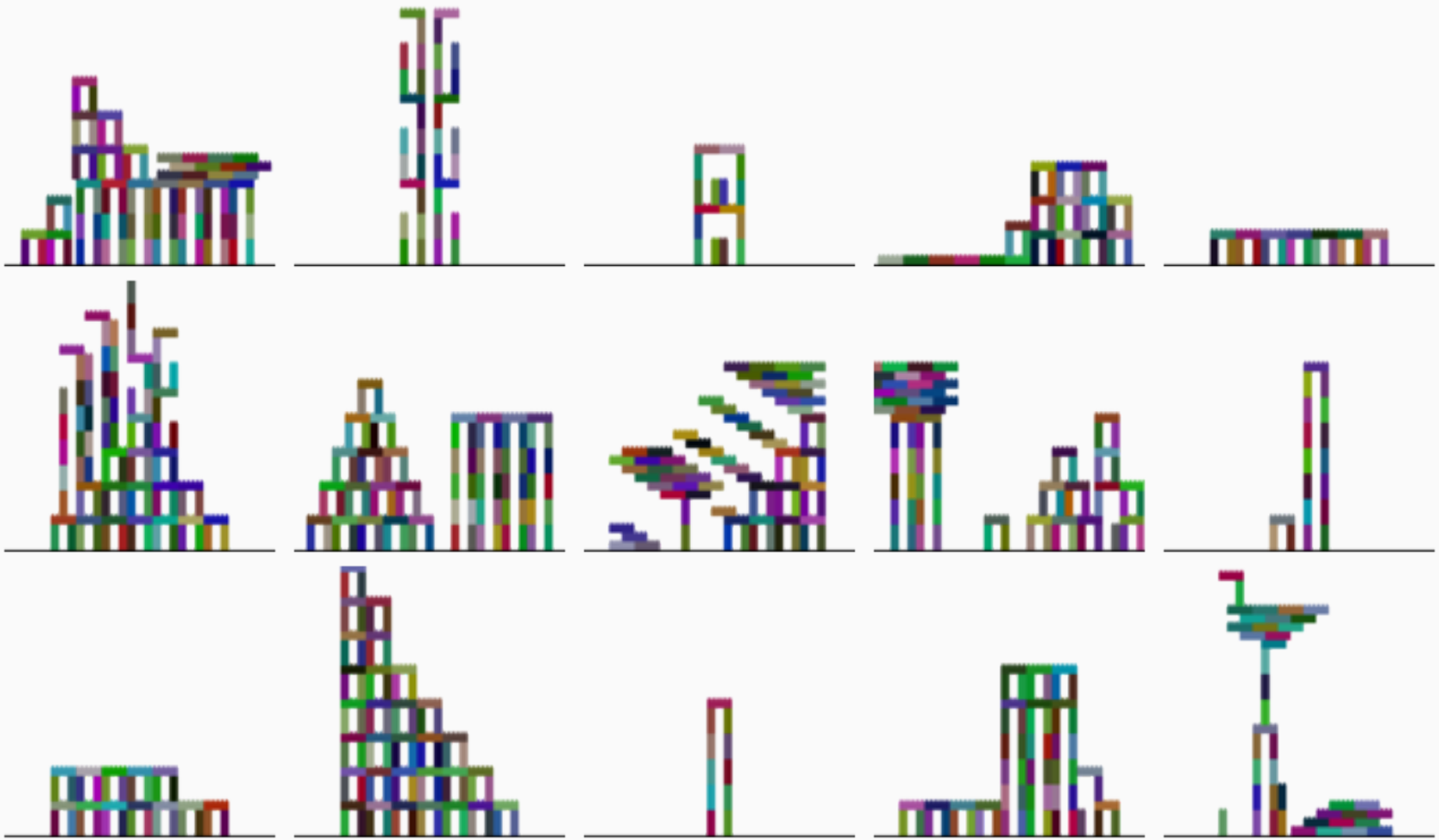


Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

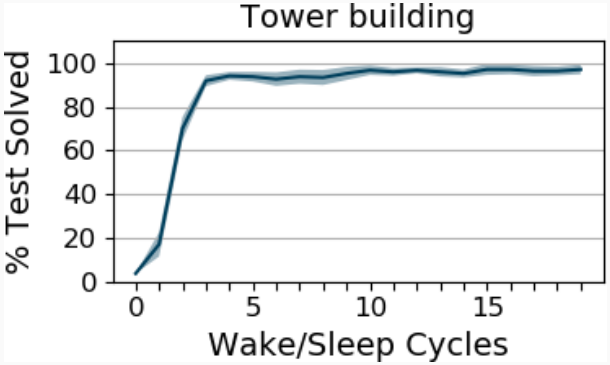
Dreams before learning



Dreams after learning

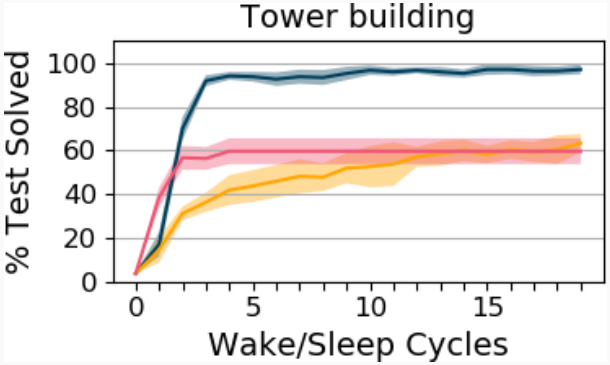


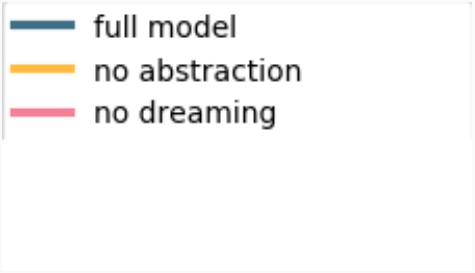
Learning dynamics



Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

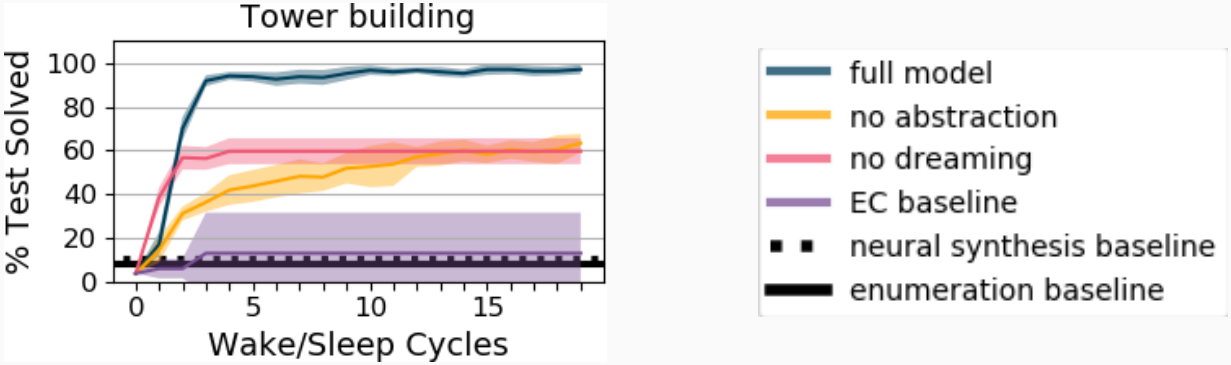
Learning dynamics





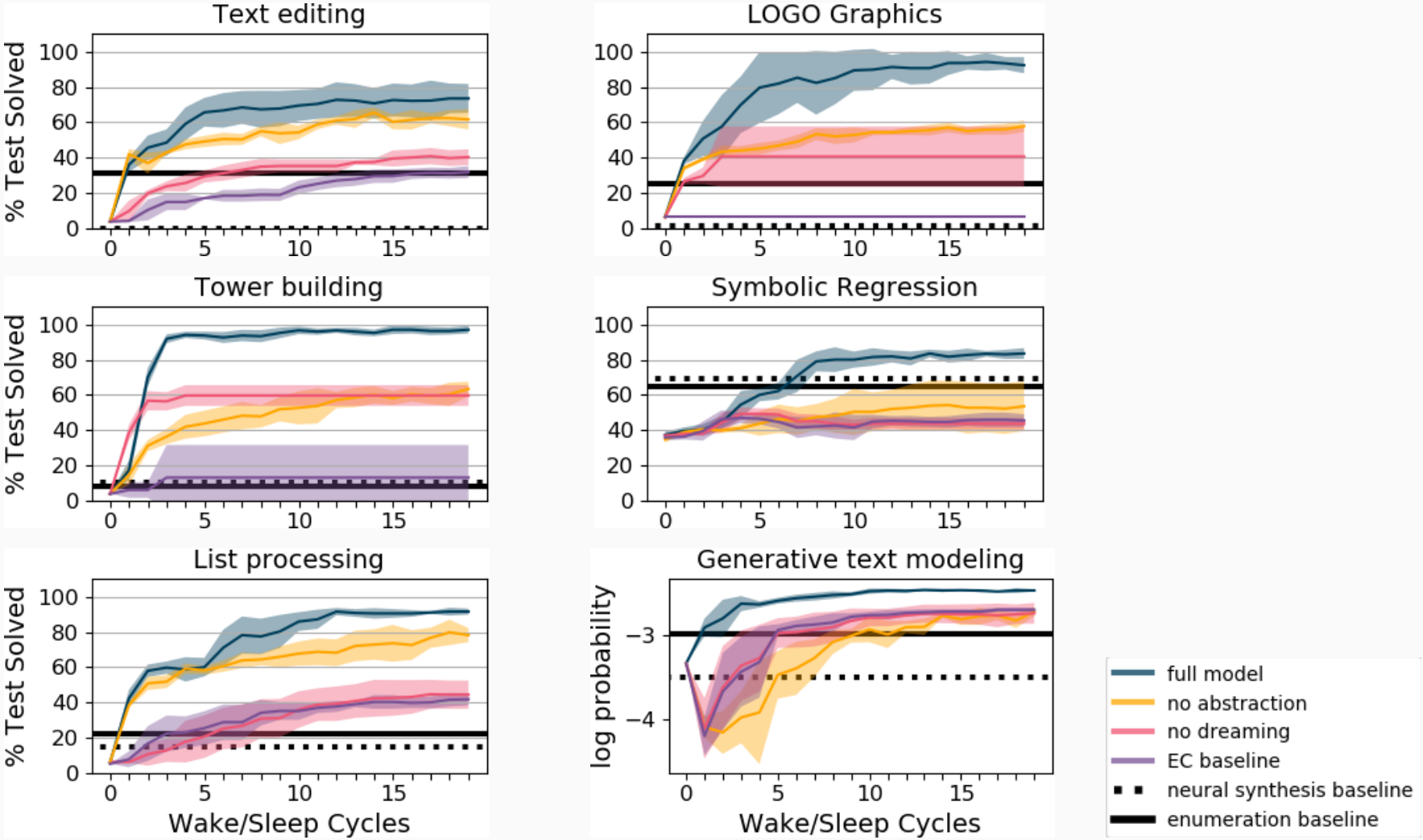
Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

Learning dynamics



baselines: Exploration-Compression, EC [Dechter et al. 2013]
neural program synthesis, RobustFill [Devlin et al. 2017]
24 hours of brute-force enumeration

Learning dynamics



Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

Synergy between dreaming and library learning

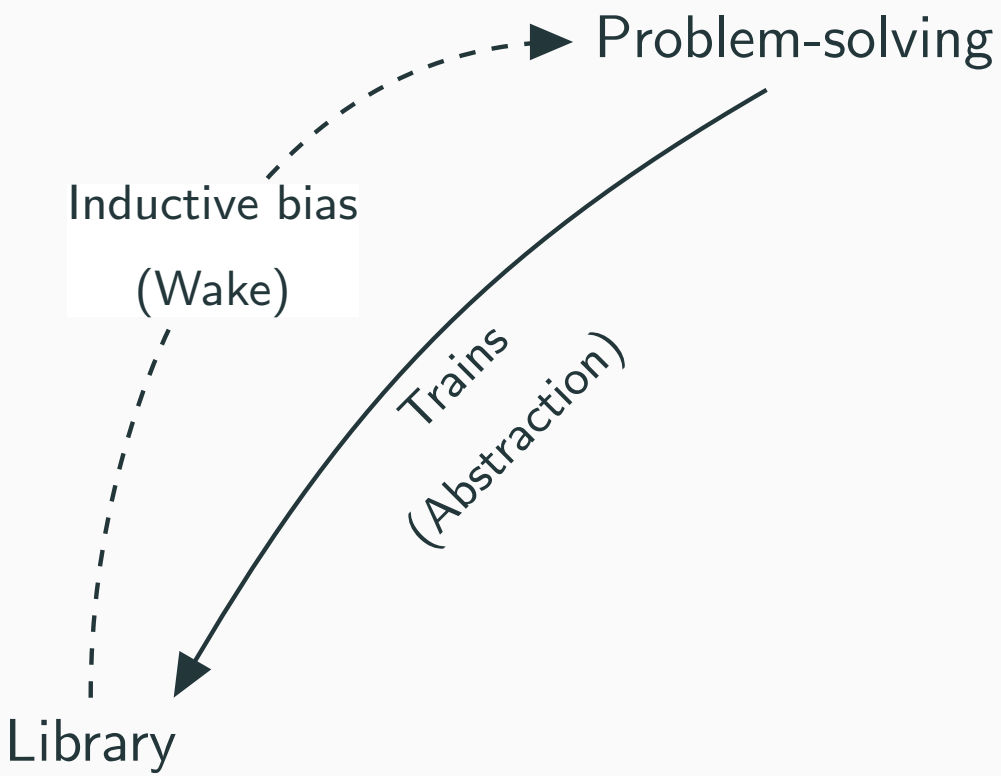
Problem-solving

Library

Recognition model

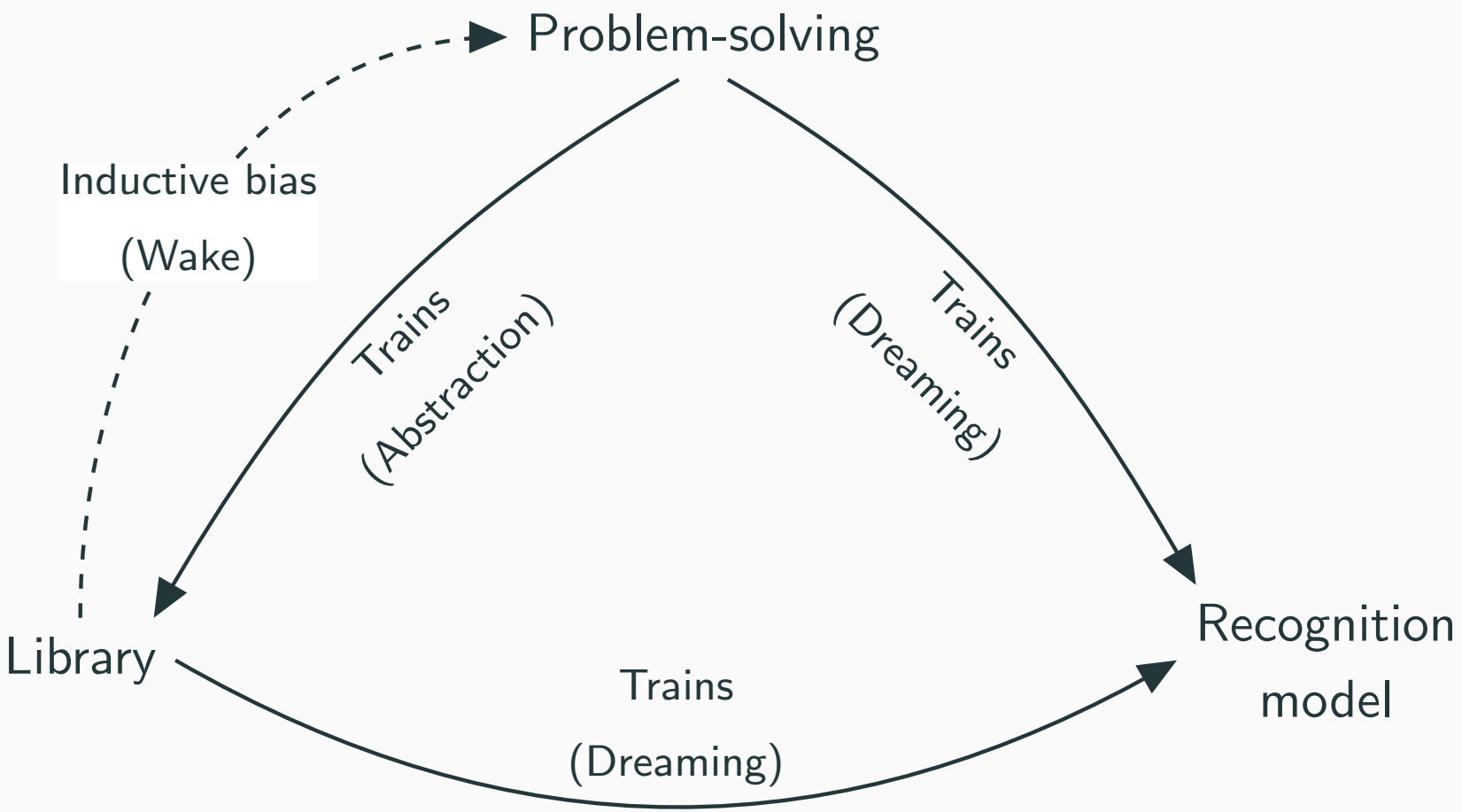
Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

Synergy between dreaming and library learning



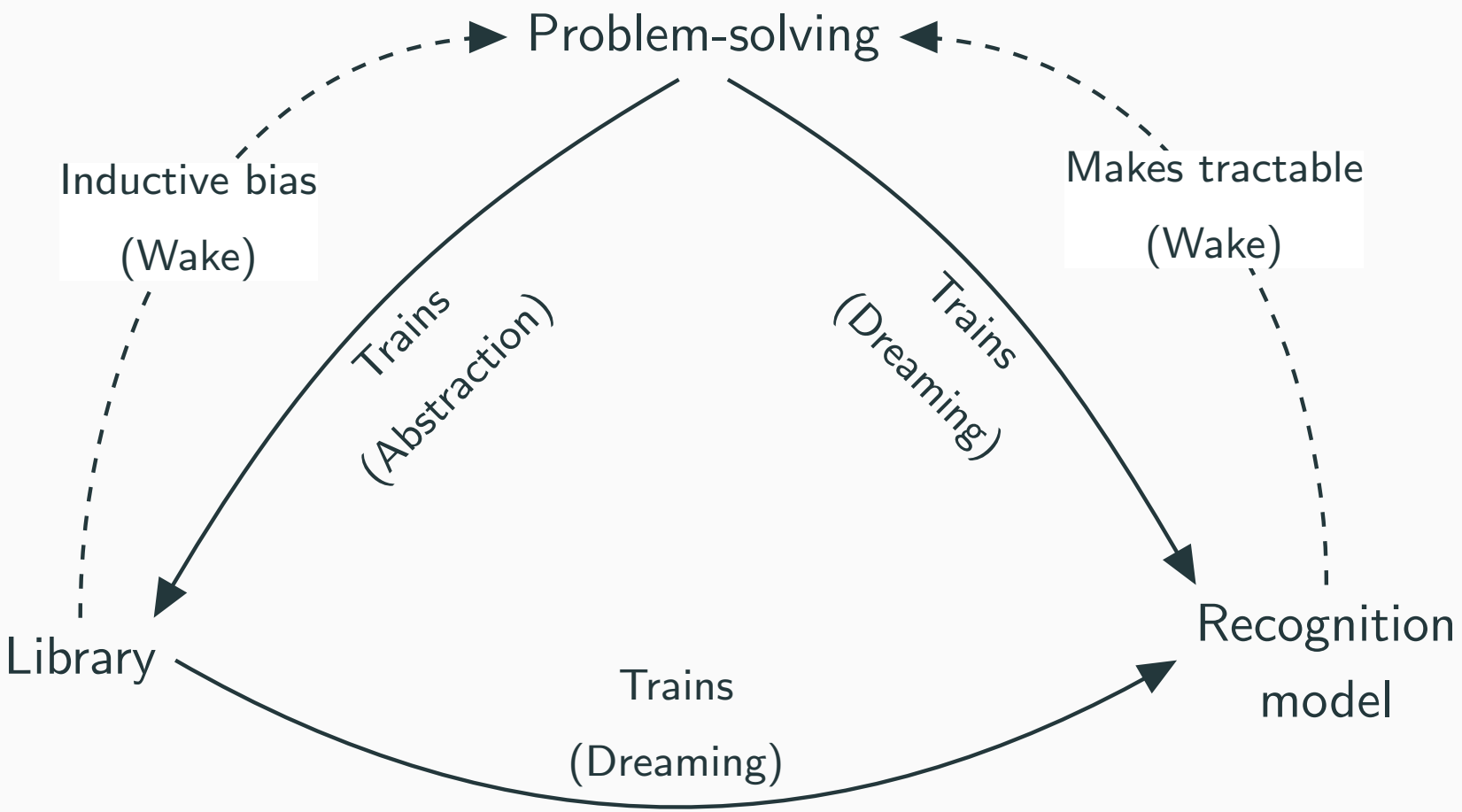
Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

Synergy between dreaming and library learning



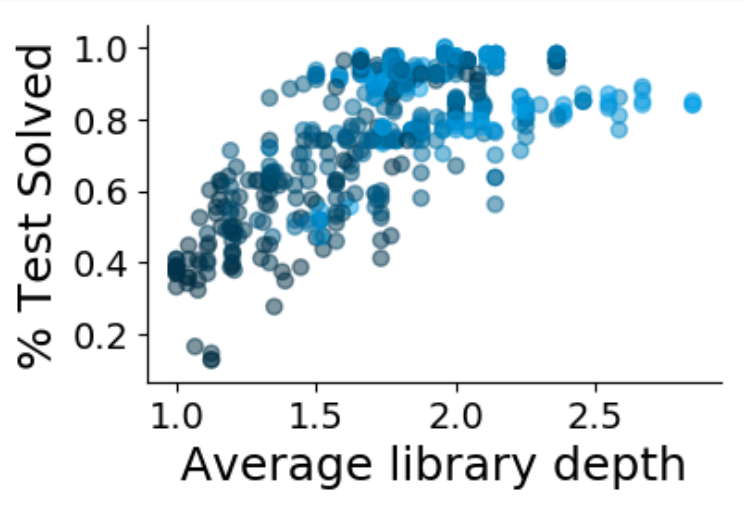
Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

Synergy between dreaming and library learning



Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

Evidence for dreaming bootstrapping better libraries

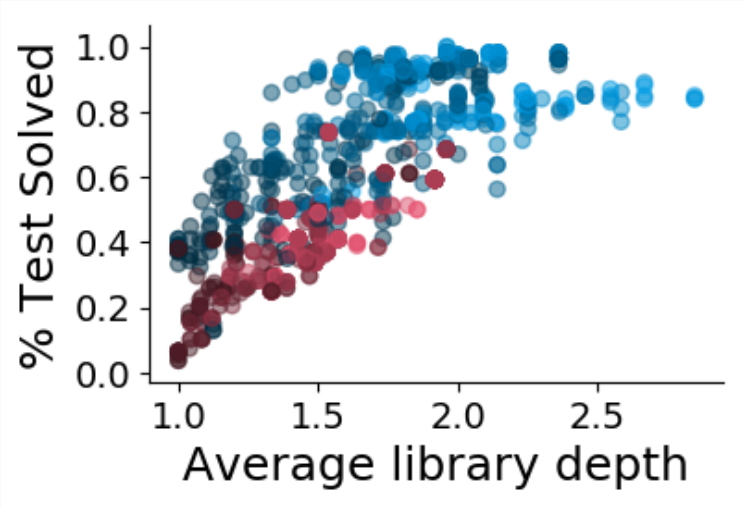


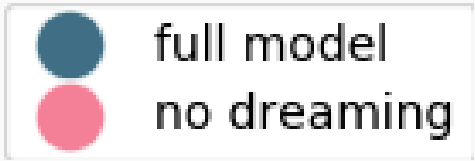
Darker: Early in learning

Brighter: Later in learning

Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

Evidence for dreaming bootstrapping better libraries



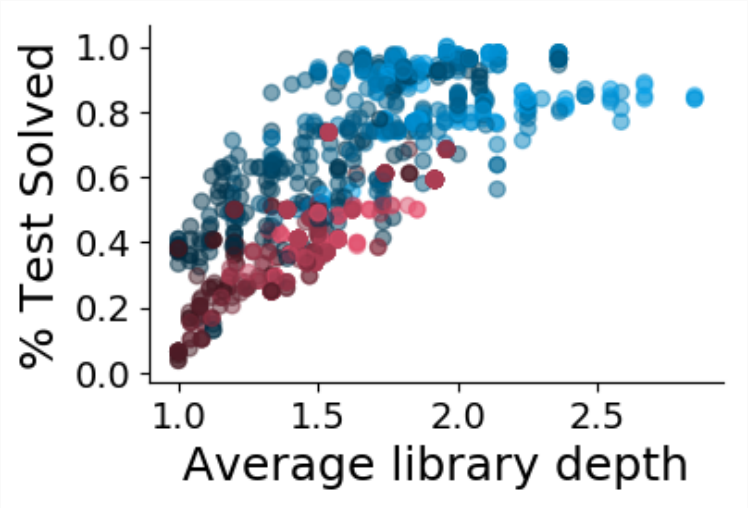


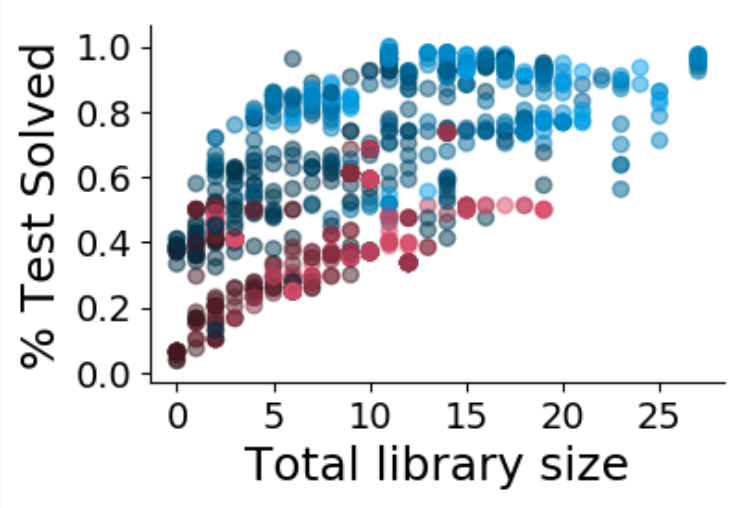
Darker: Early in learning

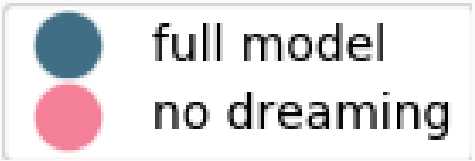
Brighter: Later in learning

Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

Evidence for dreaming bootstrapping better libraries







Darker: Early in learning

Brighter: Later in learning

Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

From learning libraries, to learning languages

From learning libraries, to learning languages

modern functional programming $\rightarrow$ physics

From learning libraries, to learning languages

1950's Lisp $\rightarrow$ modern functional programming $\rightarrow$ physics

Physics Formula Sheet

Mechanics

$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$

$v = v_0 + at$

$v_x^2 - v_{x0}^2 = 2a(x - x_0)$

$\vec{a} = \frac{\Sigma \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$

$|\vec{F}_{friction}| \leq \mu |\vec{F}_{Normal}|$

$\vec{p} = m\vec{v}$

$\Delta \vec{p} = \vec{F}\Delta t$

$KE = \frac{1}{2}mv^2$

$\Delta PE = mg\Delta y$

$\Delta E = W = Fd\cos\theta$

$a_c = \frac{v^2}{r}$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$\omega = \omega_0 + \alpha t$

$T = \frac{2\pi}{\omega} = \frac{1}{f}$

$\nu = f\lambda$

$x = A\cos(2\pi f t)$

$\vec{\alpha} = \frac{\Sigma \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$

$\vec{\tau} = r \times F$

$L = I\,\omega$

$\Delta L = \tau \Delta t$

$|\vec{F}_{spring}| = k|\vec{x}|$

$PE_{spring} = \frac{1}{2}kx^2$

$T_{spring} = 2\pi\sqrt{\frac{m}{k}}$

$T_{pendulum} = 2\pi\sqrt{\frac{\ell}{g}}$

$|\vec{F}_{gravity}| = G\frac{m_1 m_2}{r^2}$

$|\vec{F}_{gravity}| = m\vec{g}$

$PE_{gravity} = -G\frac{m_1 m_2}{r}$

$\rho = \frac{m}{V}$

$KE = \frac{1}{2}I\,\omega^2$

Electricity

$|\vec{F}_E| = k\left|\frac{q_1 q_2}{r^2}\right|$

$\Delta V = IR$

$R = \frac{\rho \ell}{A}$

$I = \frac{\Delta q}{\Delta t}$

$P = I\Delta V$

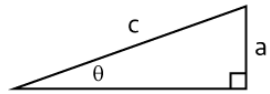
$R_{series} = R_1 + R_2 + \ldots + R_n$

$\frac{1}{R_{Parallel}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$

Geometry

Rectangle	$A = bh$	Rectangular Solid	$V = \ell wh$	Triangle	$A = \frac{1}{2}bh$
Circle	$A = \pi r^2$ $C = 2\pi r$	Cylinder	$V = \pi r^2 \ell$ $S = 2\pi r\ell + 2\pi r^2$	Sphere	$V = \frac{4}{3}\pi r^3$ $S = 4\pi r^2$

Trigonometry



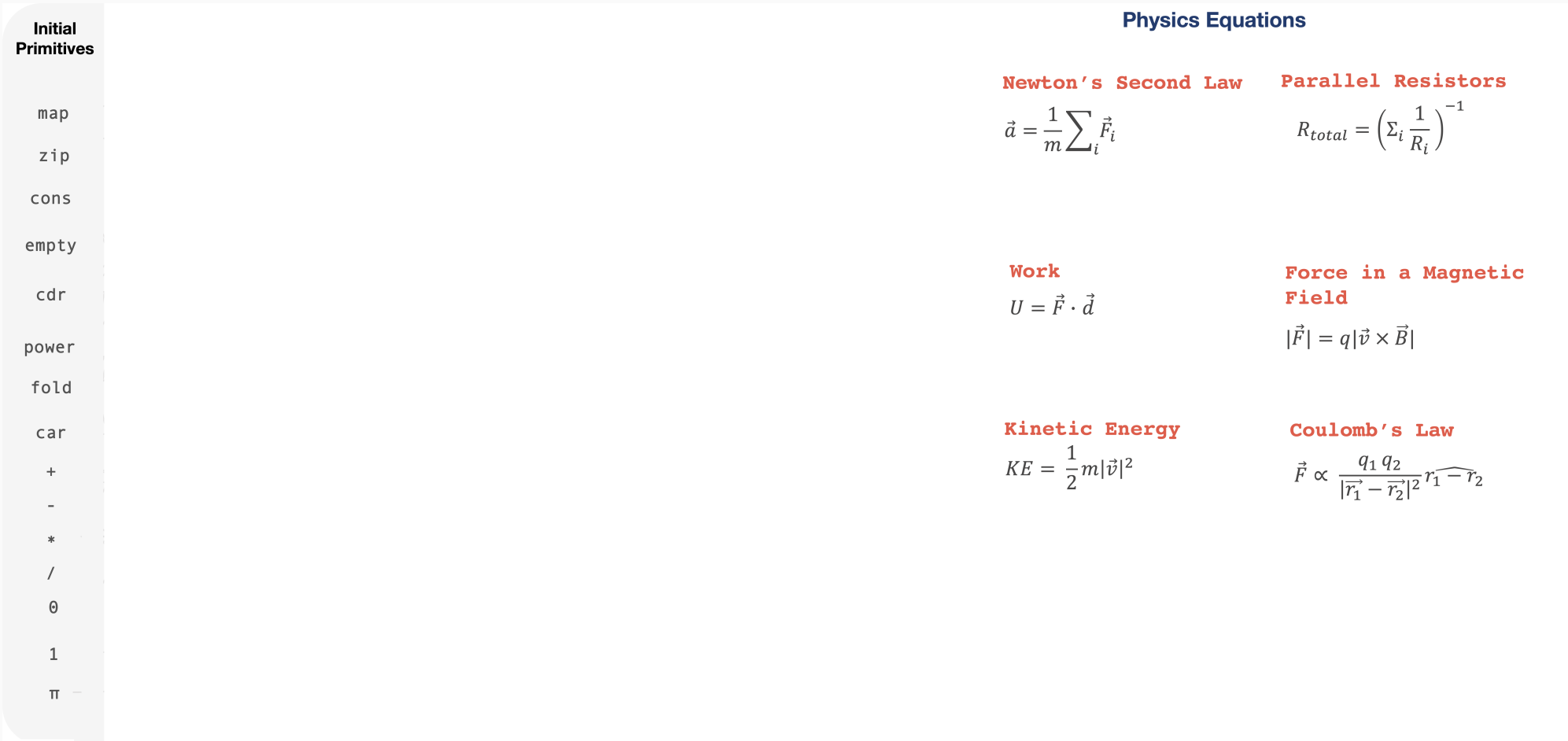
$c^2 = a^2 + b^2$

$\sin\theta = \frac{a}{c}$ $\cos\theta = \frac{b}{c}$ $\tan\theta = \frac{a}{b}$

Variables

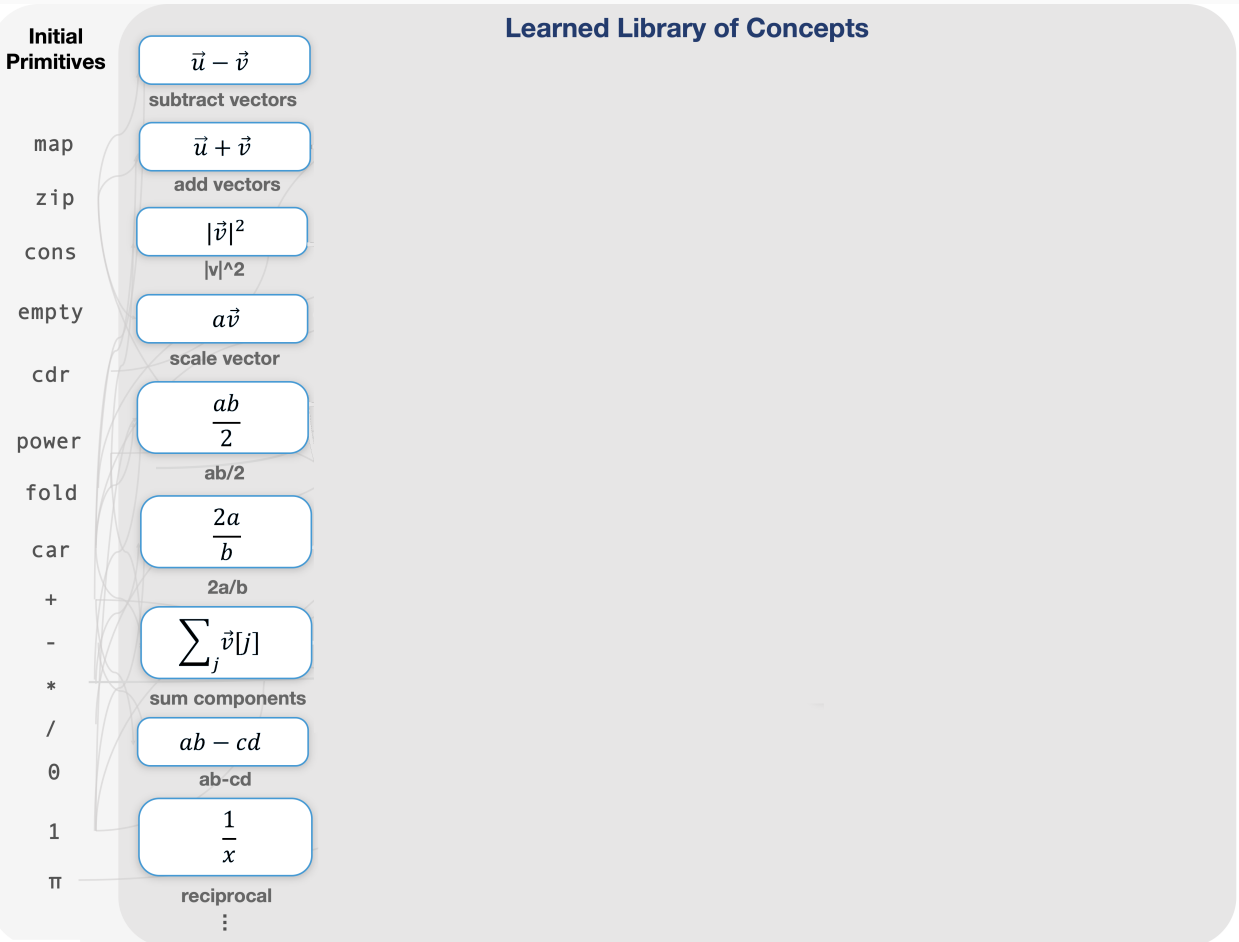
a = acceleration
A = amplitude
A = Area
b = base length
C = circumference
d = distance
E = energy
f = frequency
F = force
h = height
I = current
I = rotational inertia
KE = kinetic energy
k = spring constant
L = angular momentum
ℓ = length
m = mass
P = power
p = momentum
q = charge
r = radius
R = resistance
S = surface area
T = period
t = time
PE = potential energy
V = electric potential
V = volume
v = velocity
w = width
W = work
x = position
y = height
α = angular acceleration
λ = wavelength
μ = coefficient of friction

Growing languages for vector algebra and physics



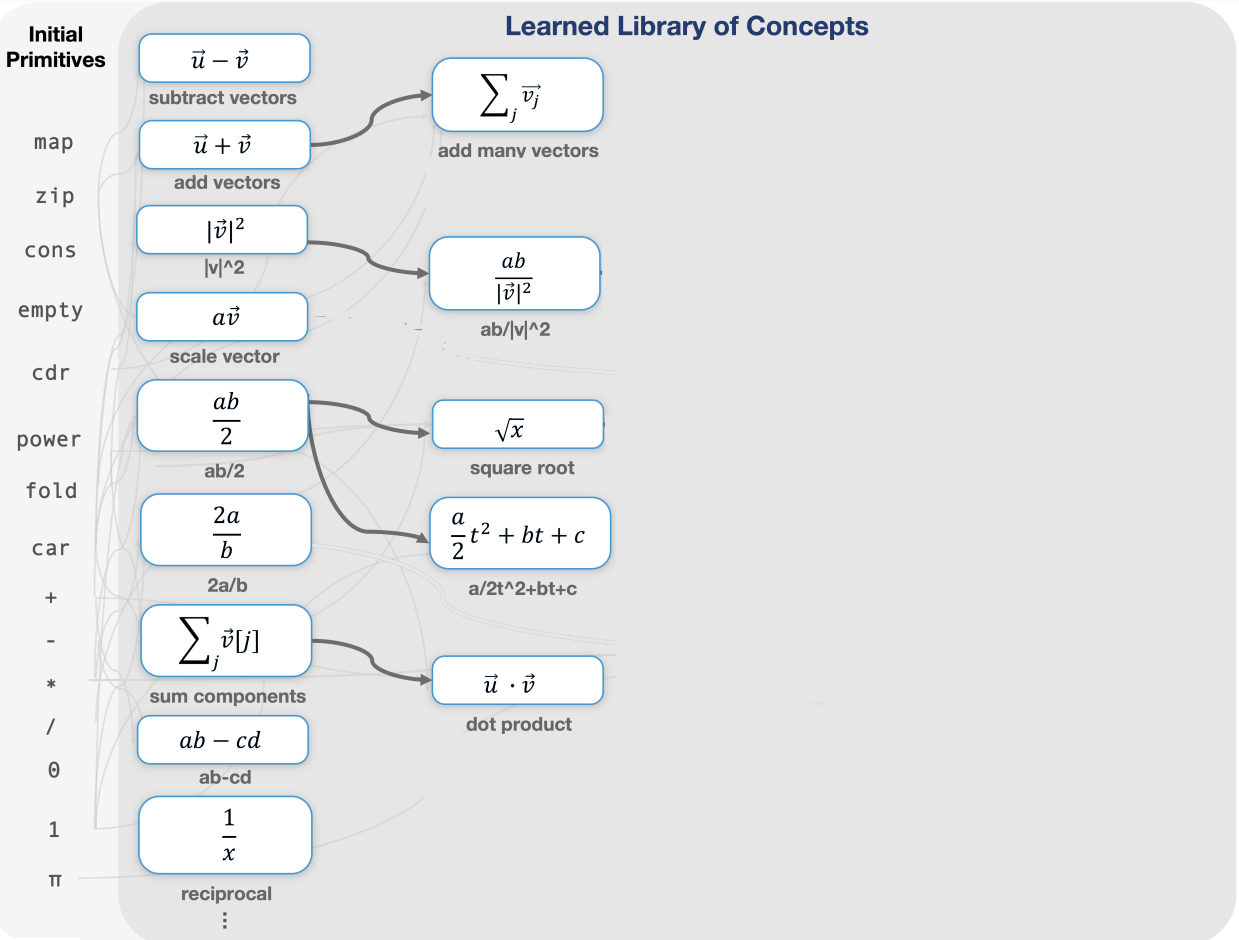
Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

Growing languages for vector algebra and physics



Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

Growing languages for vector algebra and physics



Physics Equations

Newton's Second Law

$$\vec{a} = \frac{1}{m}\sum_i \vec{F}_i$$

Parallel Resistors

$$R_{total} = \left(\Sigma_i \frac{1}{R_i}\right)^{-1}$$

Work

$$U = \vec{F} \cdot \vec{d}$$

Force in a Magnetic Field

$$|\vec{F}| = q|\vec{v} \times \vec{B}|$$

Kinetic Energy

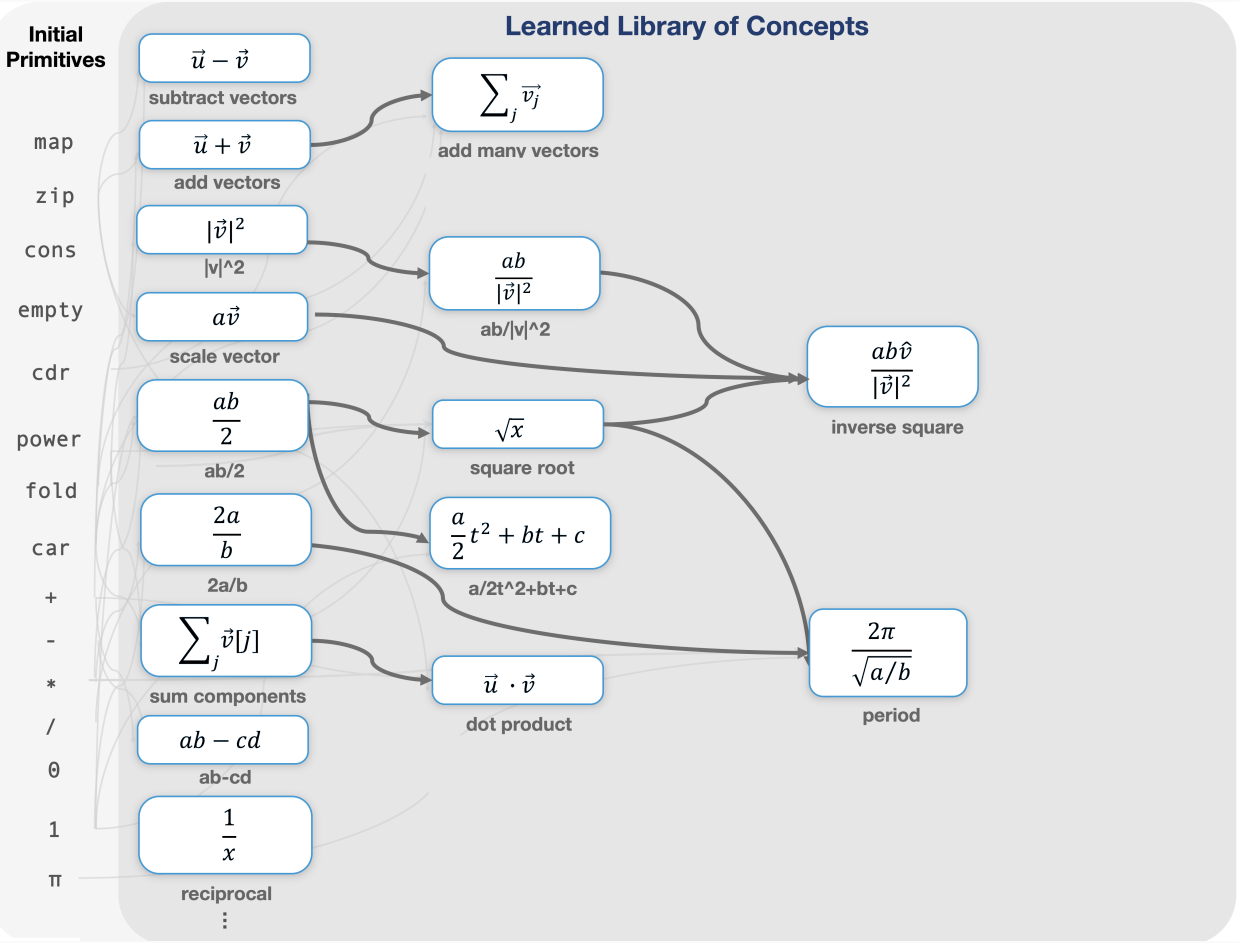
$$KE = \frac{1}{2}m|\vec{v}|^2$$

Coulomb's Law

$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r_1} - \vec{r_2}|^2}\widehat{r_1 - r_2}$$

Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

Growing languages for vector algebra and physics



Physics Equations

Newton's Second Law

$$\vec{a}=\frac{1}{m}\sum_i \vec{F}_i$$

Parallel Resistors

$$R_{total}=\left(\Sigma_i \frac{1}{R_i}\right)^{-1}$$

Work

$$U=\vec{F}\cdot\vec{d}$$

Force in a Magnetic Field

$$|\vec{F}|=q|\vec{v}\times\vec{B}|$$

Kinetic Energy

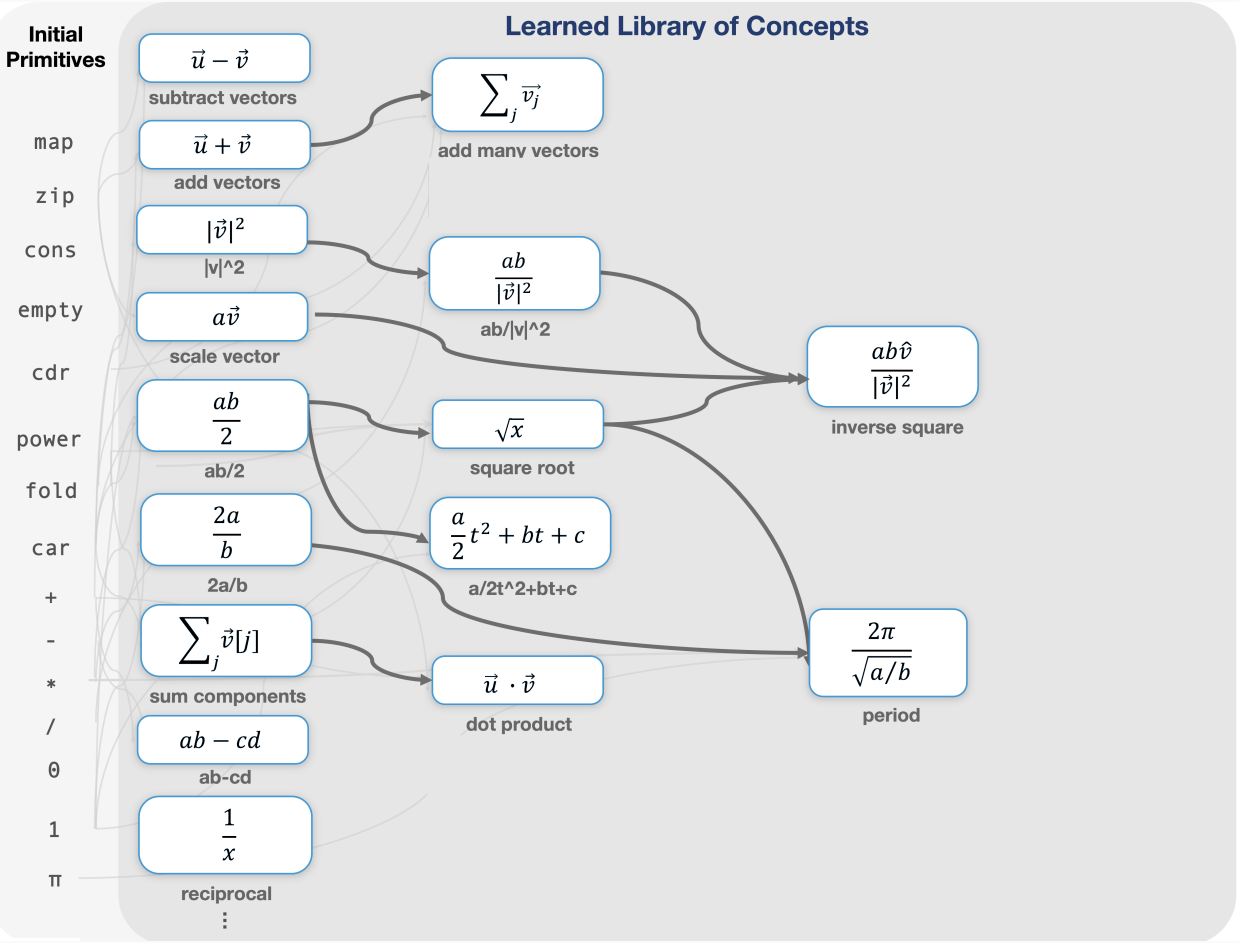
$$KE=\frac{1}{2}m|\vec{v}|^2$$

Coulomb's Law

$$\vec{F}\propto\frac{q_1 q_2}{|\vec{r_1}-\vec{r_2}|^2}\widehat{r_1-r_2}$$

Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

Growing languages for vector algebra and physics



Physics Equations

Newton's Second Law

$$\vec{a}=\frac{1}{m}\sum_i \vec{F}_i$$

```
(scale-vector(reciprocal m)
(add-many-vectors Fs))
```

Parallel Resistors

$$R_{total}=\left(\Sigma_i \frac{1}{R_i}\right)^{-1}$$

```
(reciprocal (sum-components
(map (λ(r) (reciprocal r))
Rs)))
```

Work

$$U=\vec{F}\cdot\vec{d}$$

```
(dot-product F d)
```

Force in a Magnetic Field

$$|\vec{F}|=q|\vec{v}\times\vec{B}|$$

```
(* q (ab-cd v_x b_y v_y b_x))
```

Kinetic Energy

$$KE=\frac{1}{2}m|\vec{v}|^2$$

```
(ab/2 m (|v|^2 v))
```

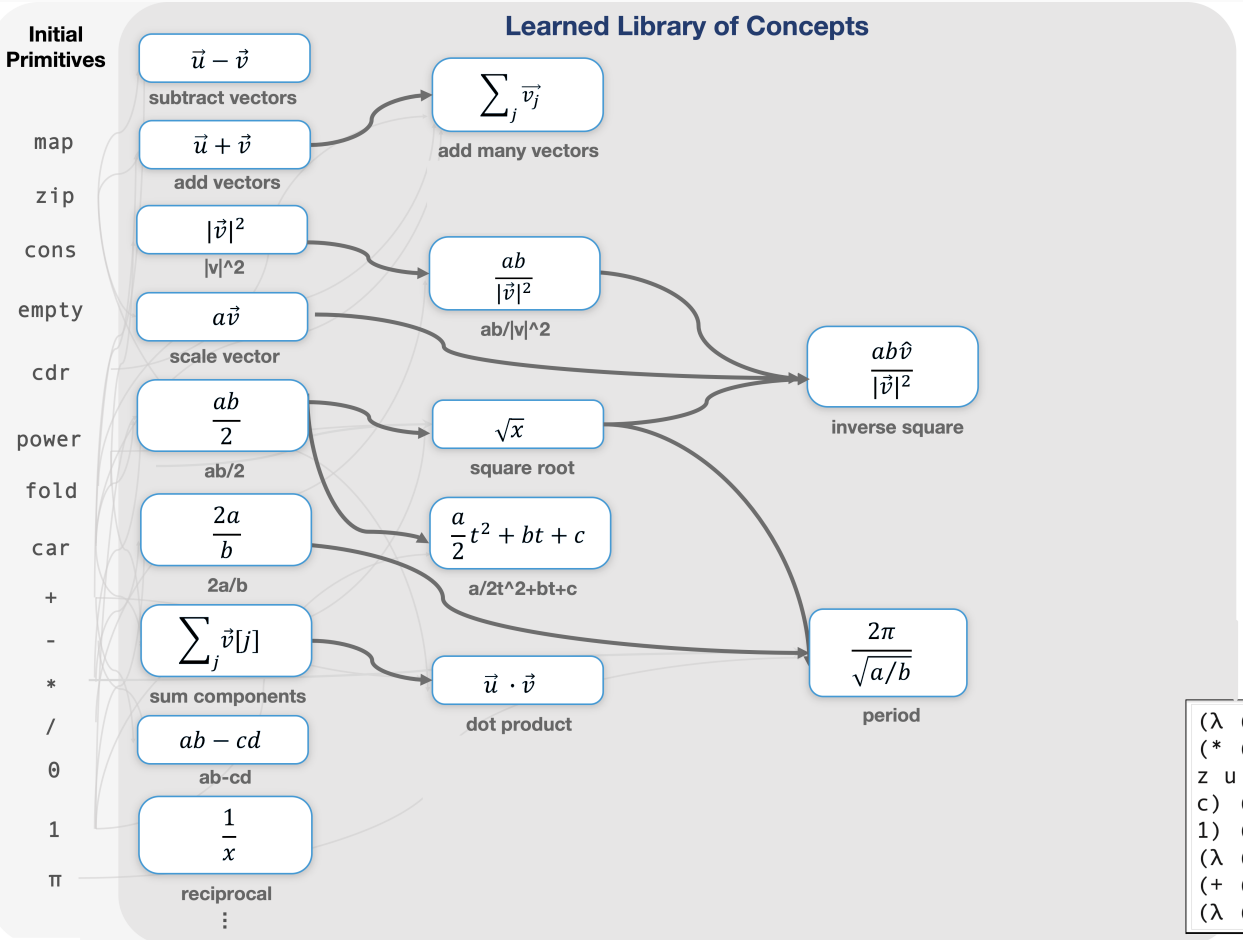
Coulomb's Law

$$\vec{F}\propto\frac{q_1 q_2}{|\vec{r_1}-\vec{r_2}|^2}\widehat{r_1-r_2}$$

```
(inverse-square q_1 q_2
(subtract-vectors r_1 r_2))
```

Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

Growing languages for vector algebra and physics



Physics Equations

Newton's Second Law

$$\vec{a} = \frac{1}{m}\sum_i \vec{F}_i$$

```
(scale-vector(reciprocal m)
(add-many-vectors Fs))
```

Parallel Resistors

$$R_{total} = \left(\Sigma_i \frac{1}{R_i}\right)^{-1}$$

```
(reciprocal (sum-components
(map (λ(r) (reciprocal r))
Rs)))
```

Work

$$U = \vec{F} \cdot \vec{d}$$

```
(dot-product F d)
```

Force in a Magnetic Field

$$|\vec{F}| = q|\vec{v} \times \vec{B}|$$

```
(* q (ab-cd v_x b_y v_y b_x))
```

Kinetic Energy

$$KE = \frac{1}{2}m|\vec{v}|^2$$

```
(ab/2 m (|v|^2 v))
```

Coulomb's Law

$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r_1} - \vec{r_2}|^2}\widehat{r_1 - r_2}$$

```
(inverse-square q_1 q_2
(subtract-vectors r_1 r_2))
```

```
(λ (x y z u) (map (λ (v) (* (/
(* (power (/ (* x x) (fold (zip
z u (λ (w a) (- w a))) 0 (λ (b
c) (+ (* b b)    c)))) (/ (* 1
1) (+ 1 1))) y) (fold (zip z u
(λ (d e)  (- d e))) 0 (λ (f g)
(+ (* f f) g)))) v)) (zip z u
(λ (h i) (- h i)))))
```

Solution to `Coulomb's Law` if expressed in initial primitives

Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

Growing a language for recursive programming

Initial Primitives

- Y combinator
- cons
- car
- cdr
- nil
- if
- nil?
- +
- -
- 0
- 1
- =

Recursive Programming Algorithms

Stutter

[■■] → [■■■■]
[■■■] → [■■■■■■]

Take every other

[■■■■] → [■■]
[■■■■■■] → [■■■]

List lengths

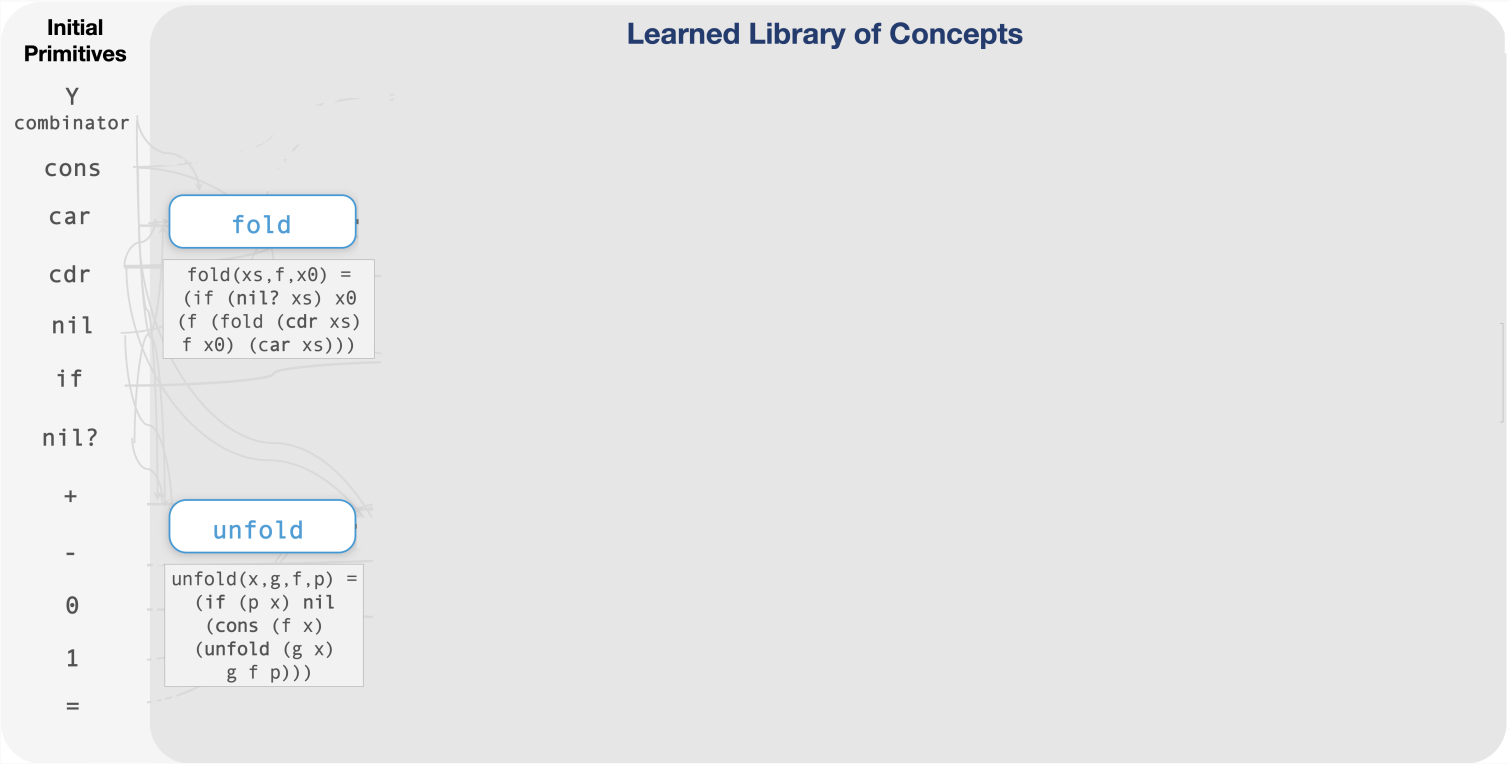
[[■■■], [■]] → [3 1]
[[■■], [], [■]] → [2 0 1]

List differences

[1 8 2], [0 5 1] → [1 3 1]
[2 3 6], [1 2 4] → [1 1 2]

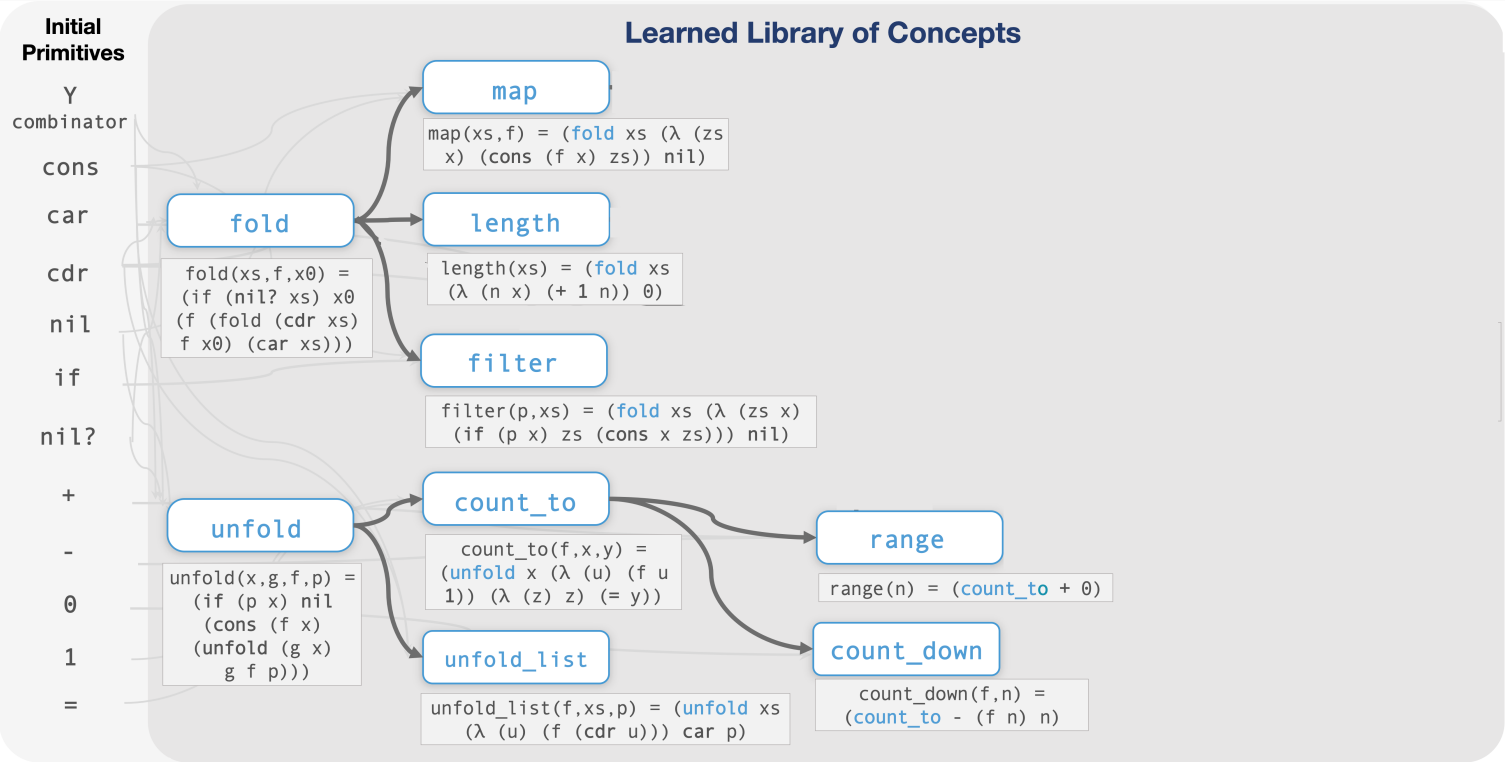
Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

Growing a language for recursive programming



Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

Growing a language for recursive programming



Recursive Programming Algorithms

Stutter

[■■] → [■■■■]
[■■■] → [■■■■■■]

```
(fold A (λ (u v) (cons
v (cons v u))) nil)
```

Take every other

[■■■■] → [■■]
[■■■■■■] → [■■■]

```
(unfold_list cdr A nil?)
```

List lengths

[[■■■], [■]] → [3 1]
[[■■], [], [■]] → [2 0 1]

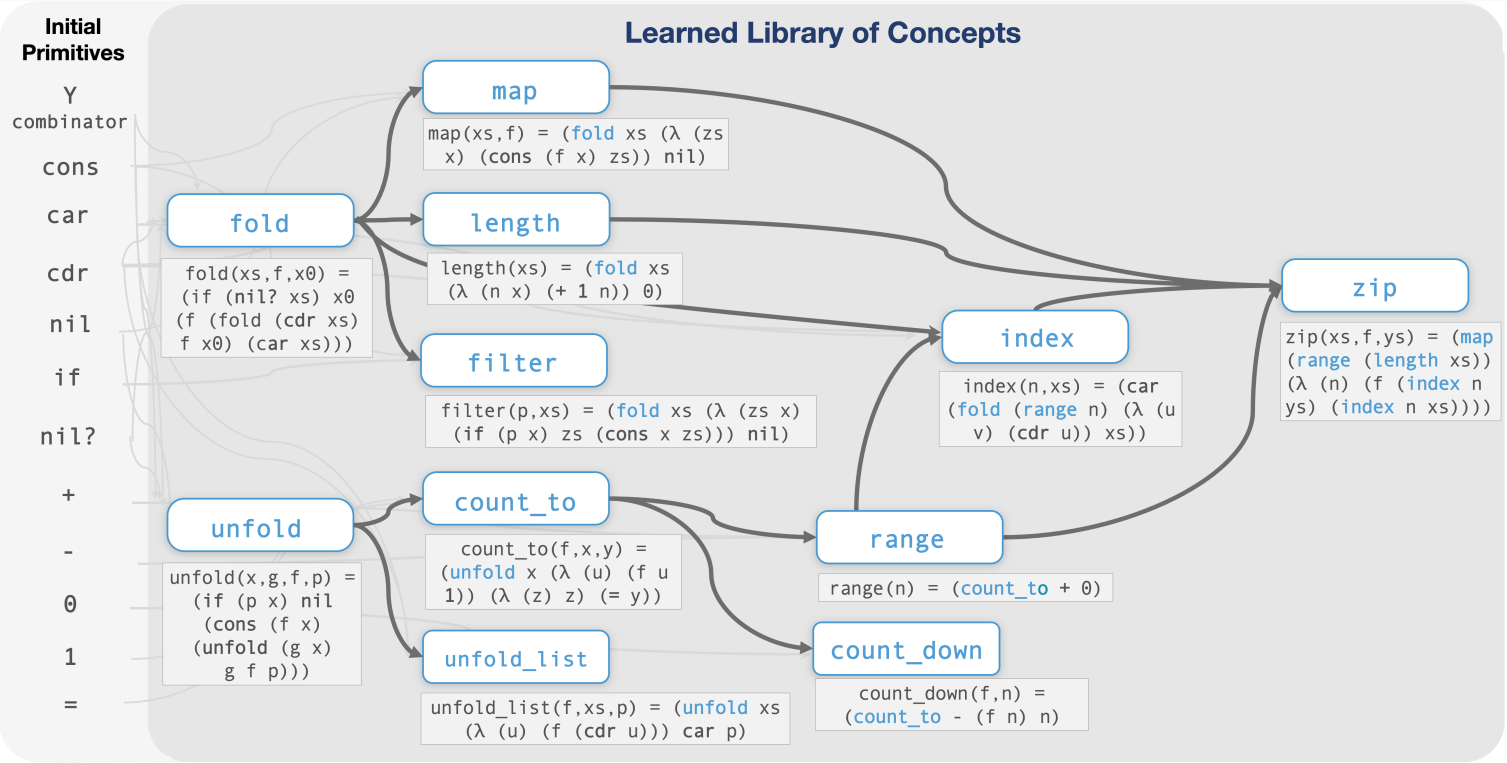
```
(map A length)
```

List differences

[1 8 2], [0 5 1] → [1 3 1]
[2 3 6], [1 2 4] → [1 1 2]

Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

Growing a language for recursive programming



Recursive Programming Algorithms

Stutter

[■■] → [■■■■]
[■■■] → [■■■■■■]

```
(fold A (λ (u v) (cons
v (cons v u))) nil)
```

Take every other

[■■■■] → [■■]
[■■■■■■] → [■■■]

```
(unfold_list cdr A nil?)
```

List lengths

[[■■■], [■]] → [3 1]
[[■■], [], [■]] → [2 0 1]

```
(map A length)
```

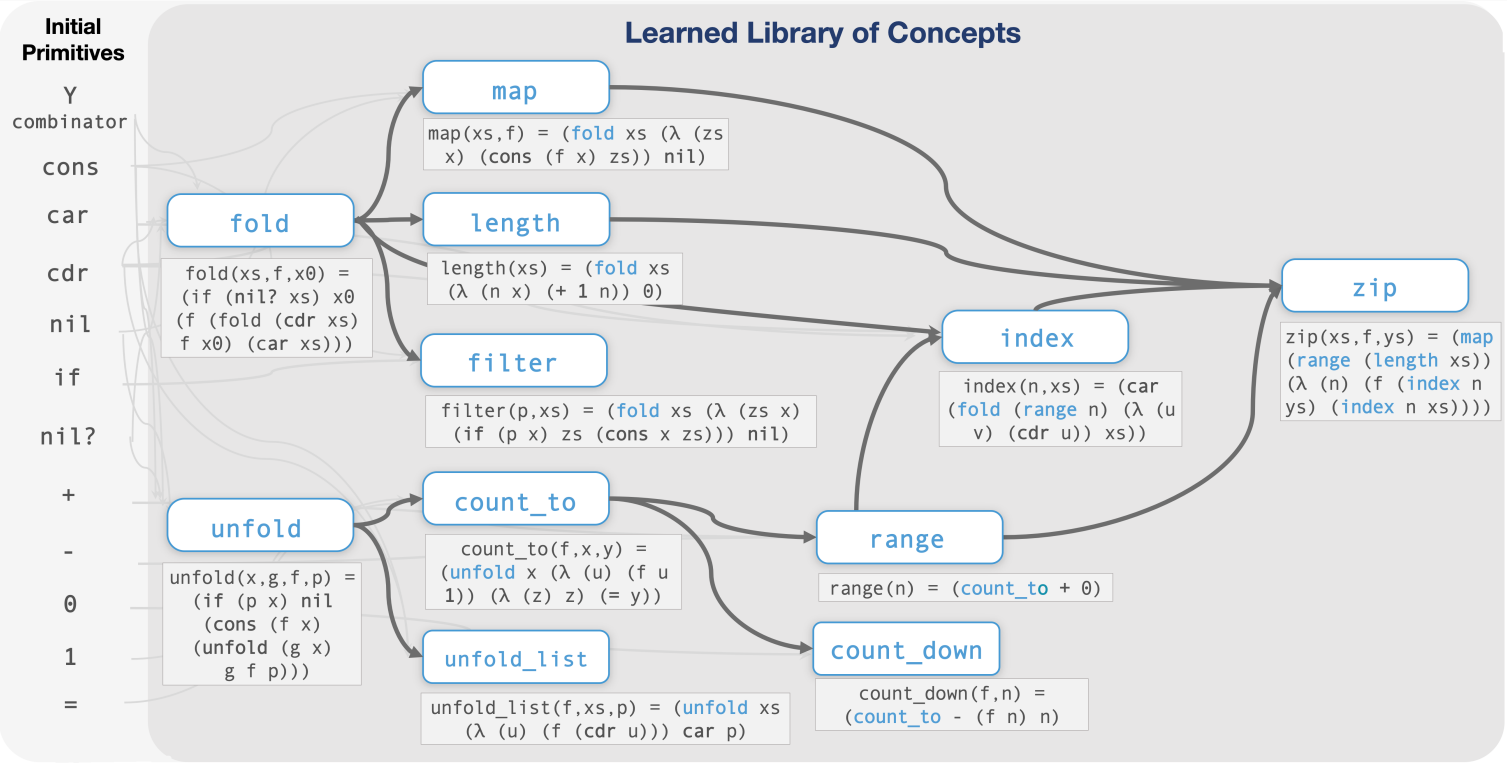
List differences

[1 8 2], [0 5 1] → [1 3 1]
[2 3 6], [1 2 4] → [1 1 2]

```
(zip A - B )
```

Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

Growing a language for recursive programming



Recursive Programming Algorithms

Stutter

[■■] → [■■■■]
[■■■] → [■■■■■■]

```
(fold A (λ (u v) (cons
v (cons v u))) nil)
```

Take every other

[■■■■] → [■■]
[■■■■■■] → [■■■]

```
(unfold_list cdr A nil?)
```

List lengths

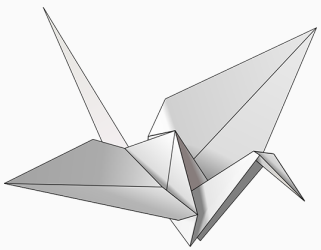
[[■■■], [■]] → [3 1]
[[■■], [], [■]] → [2 0 1]

```
(map A length)
```

List differences

[1 8 2], [0 5 1] → [1 3 1]
[2 3 6], [1 2 4] → [1 1 2]

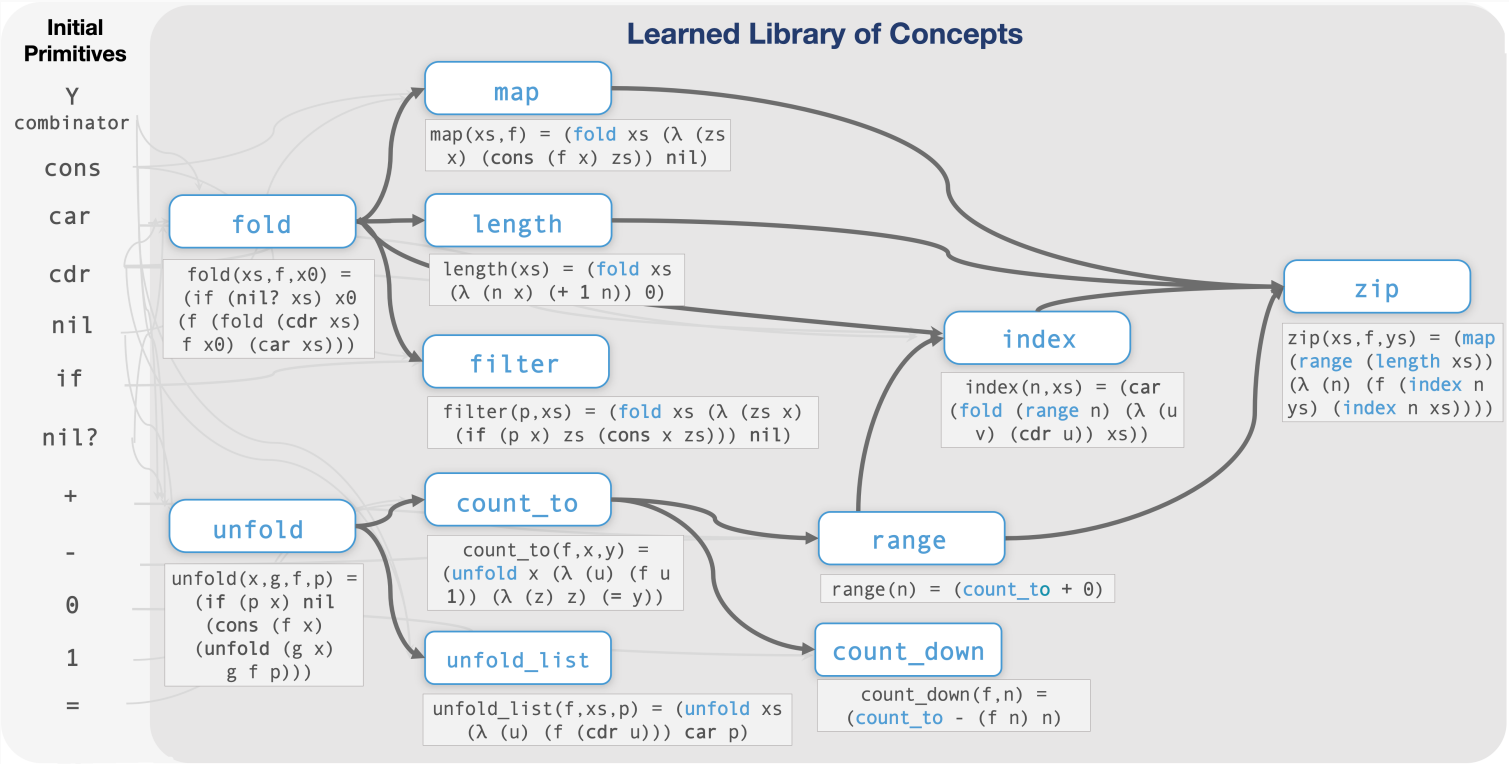
```
(zip A - B )
```



Origami Programming: Jeremy Gibbons, 2003

Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

Growing a language for recursive programming



Recursive Programming Algorithms

Stutter

[■■] → [■■■■]
[■■■] → [■■■■■■]

```
(fold A (λ (u v) (cons
v (cons v u))) nil)
```

Take every other

[■■■■] → [■■]
[■■■■■■] → [■■■]

```
(unfold_list cdr A nil?)
```

List lengths

[[■■■], [■]] → [3 1]
[[■■], [], [■]] → [2 0 1]

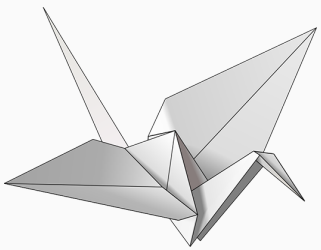
```
(map A length)
```

List differences

[1 8 2], [0 5 1] → [1 3 1]
[2 3 6], [1 2 4] → [1 1 2]

```
(zip A - B )
```

1 year of compute. 5 days on 64 CPUs.



Origami Programming: Jeremy Gibbons, 2003

Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

Lessons

Symbols aren't necessarily interpretable. Flexibly grow the language based on experience to make it more powerful *and* more human understandable

Learning-from-scratch is possible in principle. Don't do it. But program induction makes it convenient to build in what we know how to build in, and then learn and adapt on top of that

Program Induction and perception

learning to learn

model discovery

the future

What we've got now:

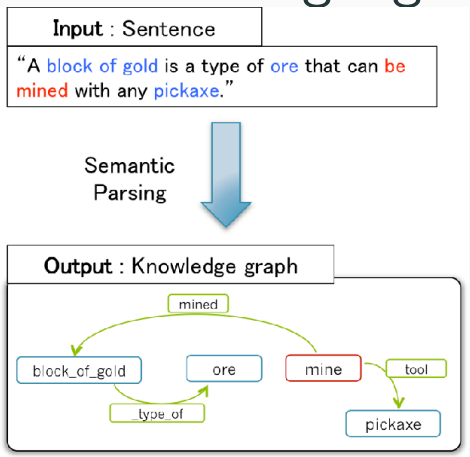
a toolkit for program induction,

addressing combinatorial program search via learning, integrating symbolic, probabilistic, and neural techniques

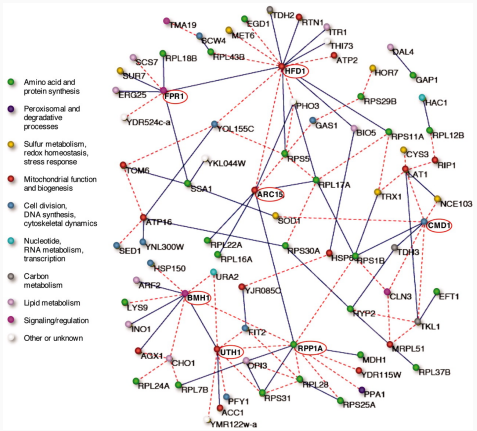
Where will this toolkit take us?

What's in reach

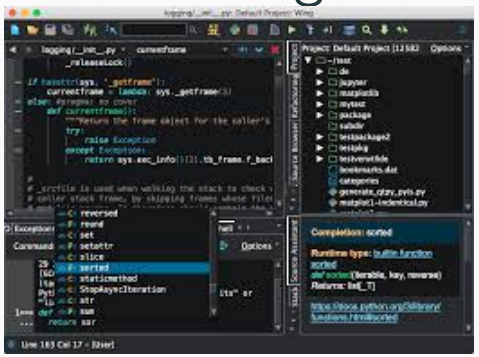
Library learning+ Natural language



Computer-Aided Science

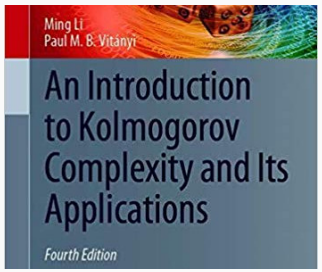


Synthesis for software engineers

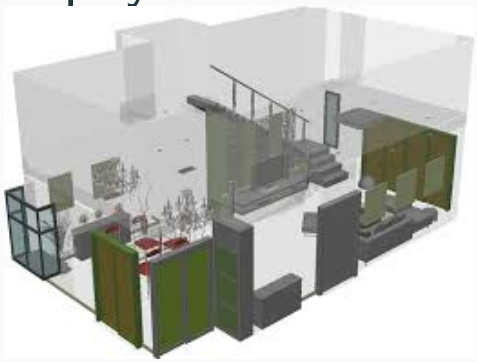


Theory for program induction

$$P_M(x) = \sum_{i=1}^{\infty} 2^{-|s_i(x)|}$$

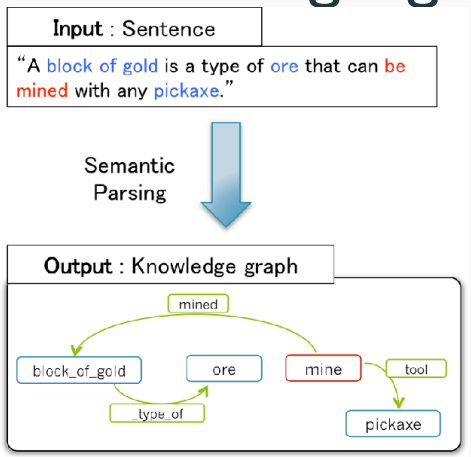


Modeling the physical world

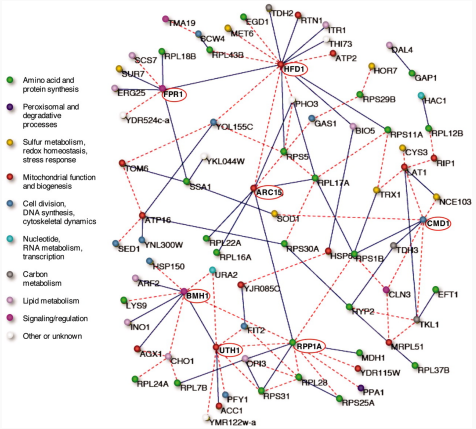


What's in reach

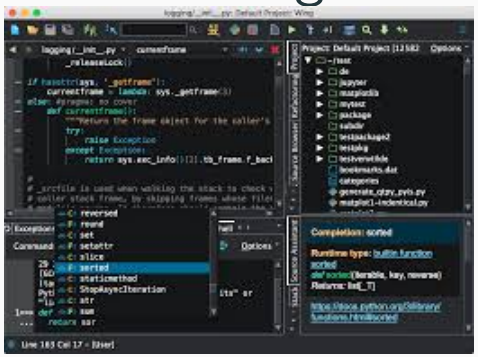
Library learning+ Natural language



Computer-Aided Science

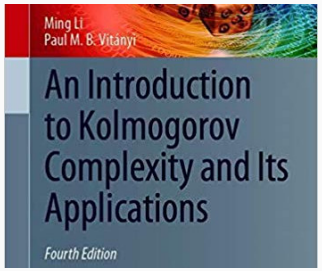


Synthesis for software engineers

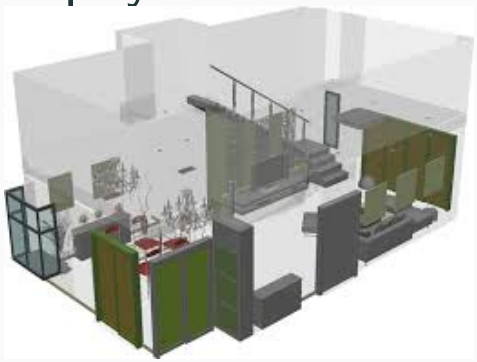


Theory for program induction

$$P_M(x) = \sum_{i=1}^{\infty} 2^{-|s_i(x)|}$$

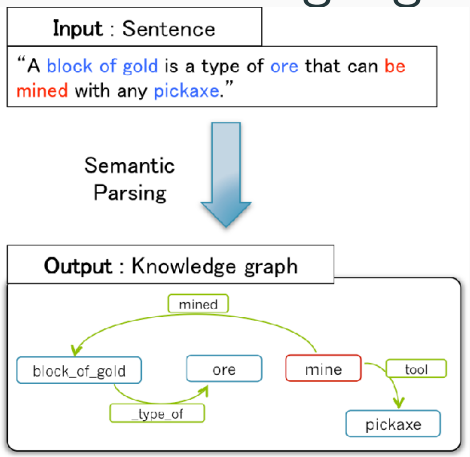


Modeling the physical world

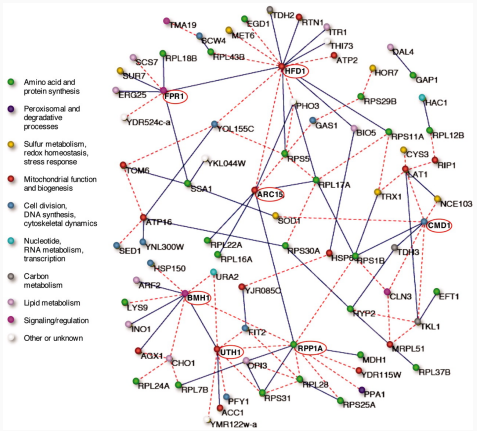


What's in reach

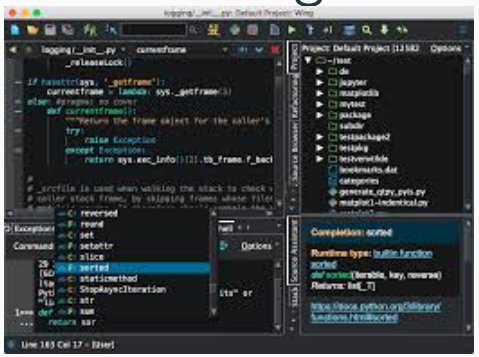
Library learning+ Natural language



Computer-Aided Science

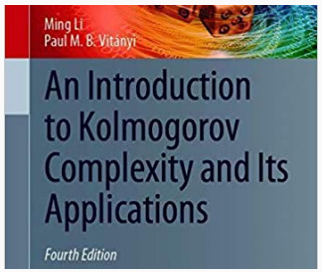


Synthesis for software engineers

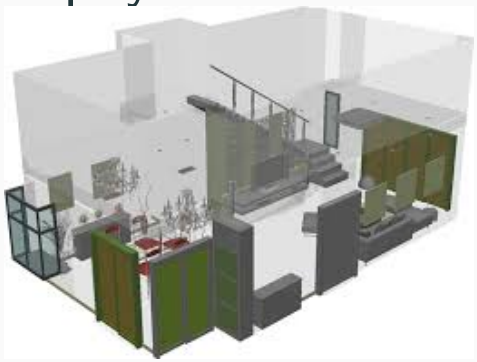


Theory for program induction

$$P_M(x) = \sum_{i=1}^{\infty} 2^{-|s_i(x)|}$$

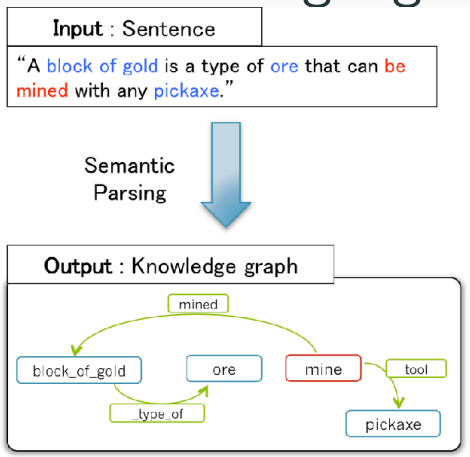


Modeling the physical world

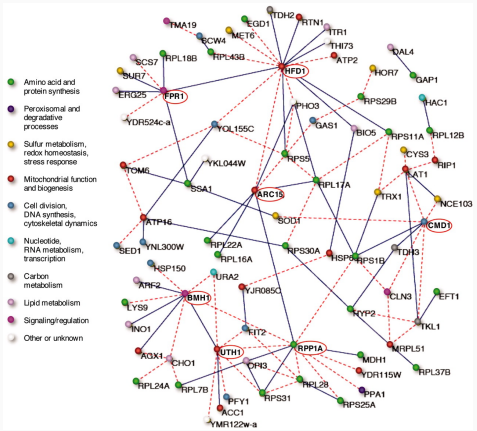


What's in reach

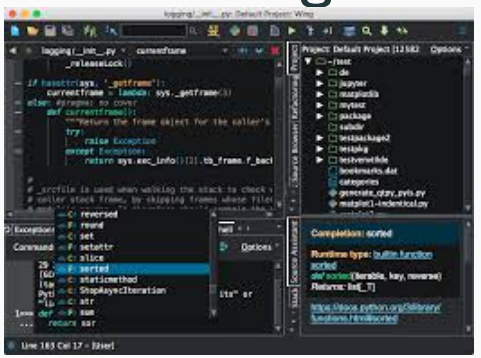
Library learning+ Natural language



Computer-Aided Science

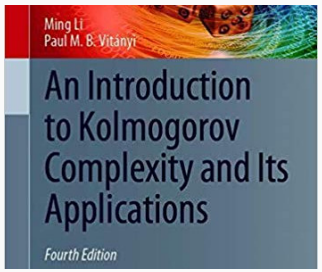


Synthesis for software engineers

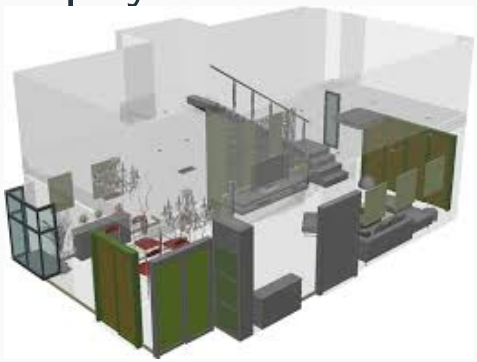


Theory for program induction

$$P_M(x) = \sum_{i=1}^{\infty} 2^{-|s_i(x)|}$$

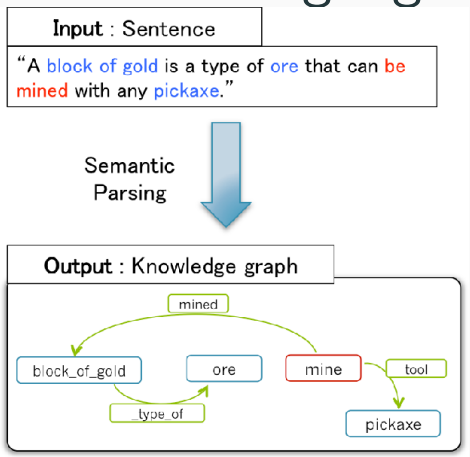


Modeling the physical world

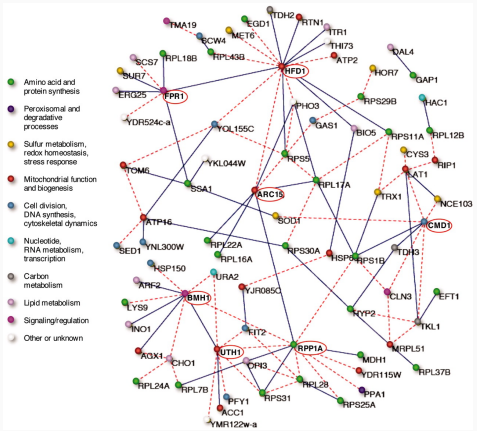


What's in reach

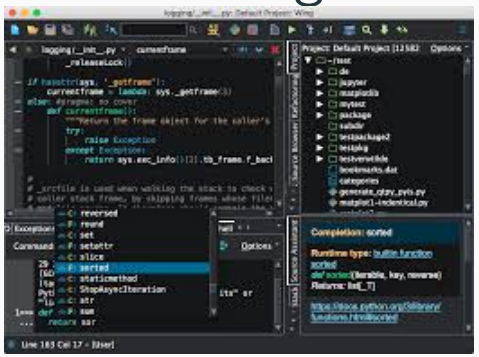
Library learning+ Natural language



Computer-Aided Science

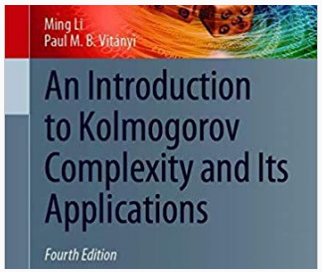


Synthesis for software engineers

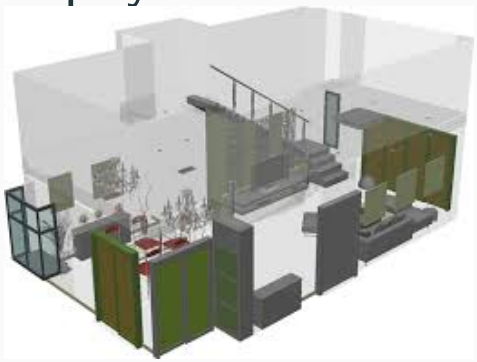


Theory for program induction

$$P_M(x) = \sum_{i=1}^{\infty} 2^{-|s_i(x)|}$$

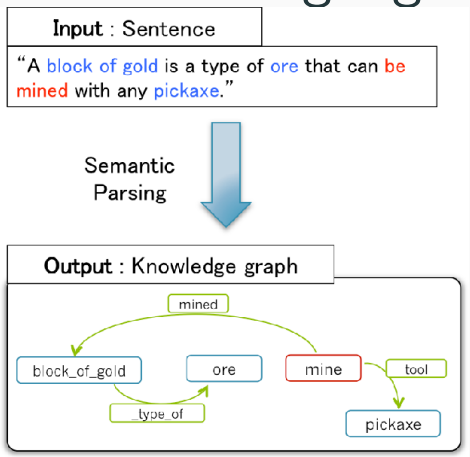


Modeling the physical world

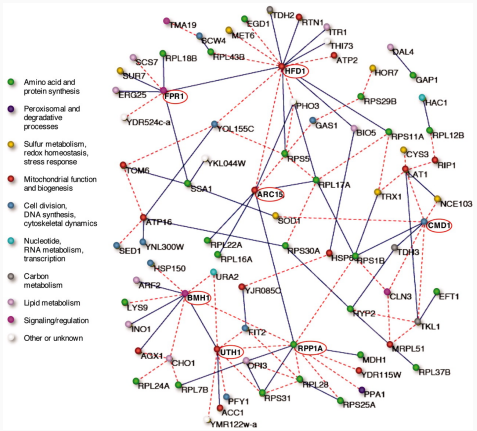


What's in reach

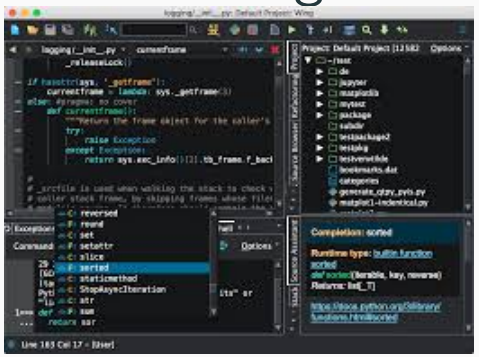
Library learning+ Natural language



Computer-Aided Science

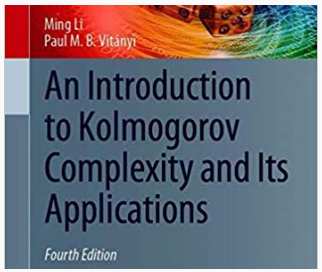


Synthesis for software engineers



Theory for program induction

$$P_M(x) = \sum_{i=1}^{\infty} 2^{-|s_i(x)|}$$

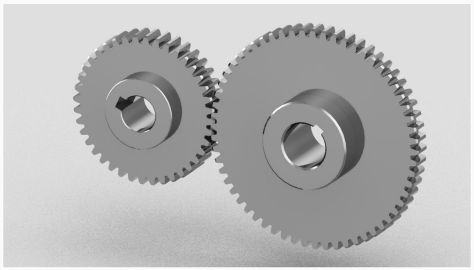


Modeling the physical world

Within reach: Modeling the physical world

hinge

gear



doorknob

Within reach: Modeling the physical world

Built in | Learned Library | The world

2D geometry

symmetry

...

3D geometry

extrude

...

Rigid body

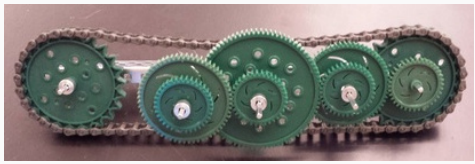
dynamics

...

Programming

for loop

...



Within reach: Modeling the physical world

Built in | Learned Library | The world

2D geometry

symmetry

...

3D geometry

extrude

...

Rigid body

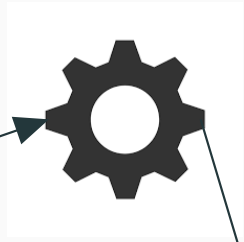
dynamics

...

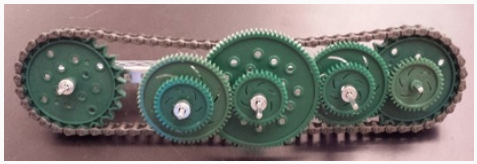
Programming

for loop

...



gear(n, ℓ, r)



Within reach: Modeling the physical world

Built in | Learned Library | The world

2D geometry

symmetry

...

3D geometry

extrude

...

Rigid body

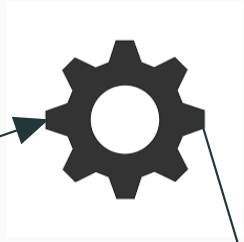
dynamics

...

Programming

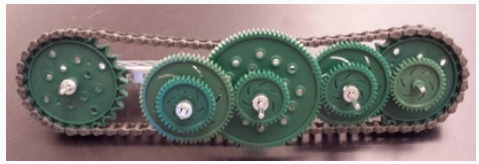
for loop

...



gear(n,ℓ,r)

geartrain(K,$\vec{n}$,$\vec{\ell}$,$\vec{r}$)



Within reach: Modeling the physical world

Built in

Learned Library

The world

2D geometry

symmetry

...

3D geometry

extrude

...

Rigid body

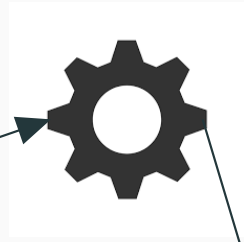
dynamics

...

Programming

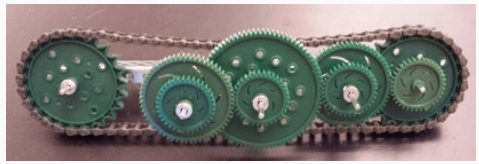
for loop

...



gear(n,ℓ,r)

geartrain(K,$\vec{n}$,$\vec{\ell}$,$\vec{r}$)



What's far off, but worthwhile?

What kinds of future machines could learn all this?

using new devices

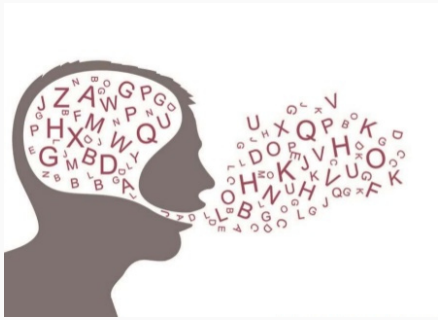
play

coding

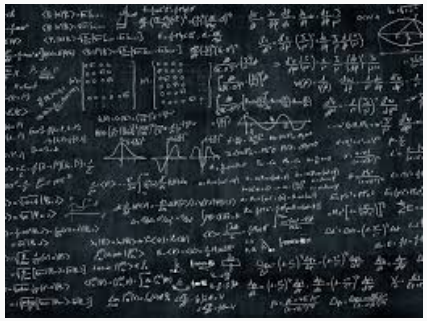
```
(MEMBER
 (LAMBDA (X L)
  (COND  ((NULL L) NIL)
         ((EQ X (FIRST L)) T)
         (T (MEMBER X (REST L))))))
```

Allen, Anatomy of Lisp, 1975

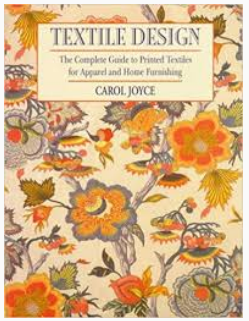
language



science



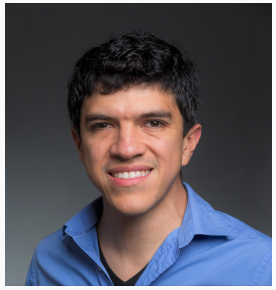
design



Collaborators

Josh Tenenbaum

Armando Solar-Lezama



Tim O'Donnell

Adam Albright

Max Nye

Cathy Wong

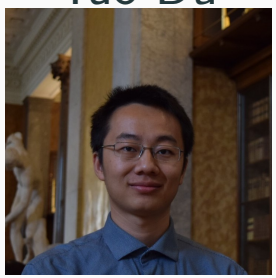
Yewen Pu

Dan Ritchie

Mathias Sable-Meyer

Lucas Morales

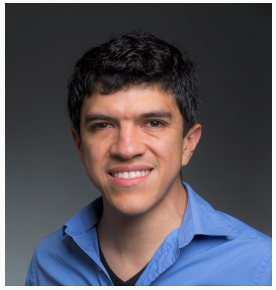
Tao Du



Collaborators

Josh Tenenbaum

Armando Solar-Lezama



Tim O'Donnell

Adam Albright

Max Nye

Cathy Wong

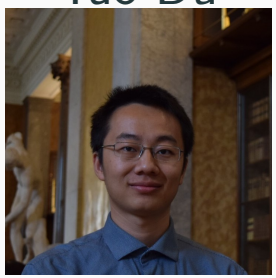
Yewen Pu

Dan Ritchie

Mathias Sable-Meyer

Lucas Morales

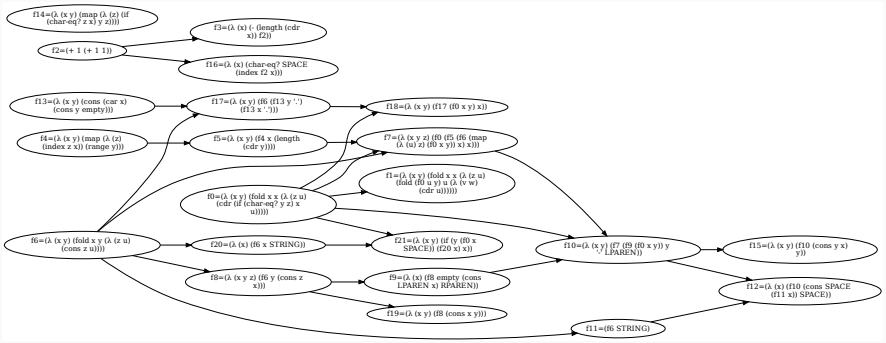
Tao Du

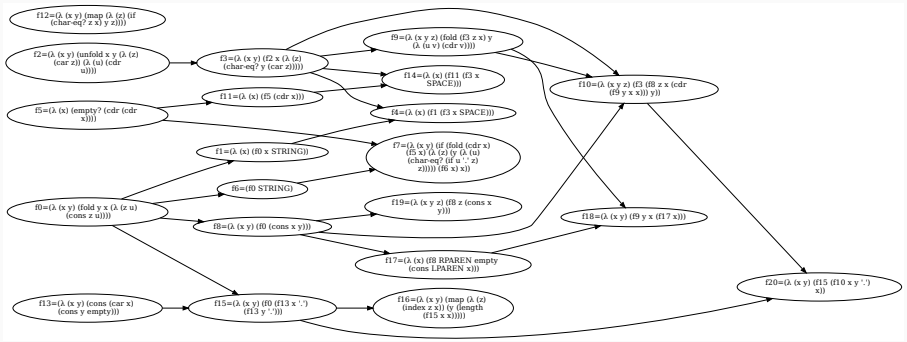


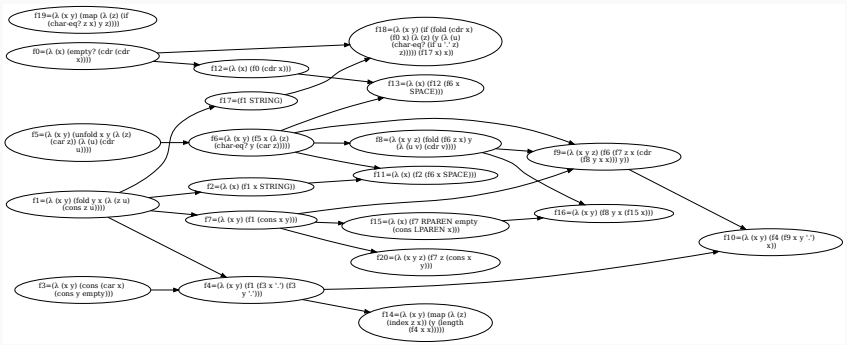
thank you

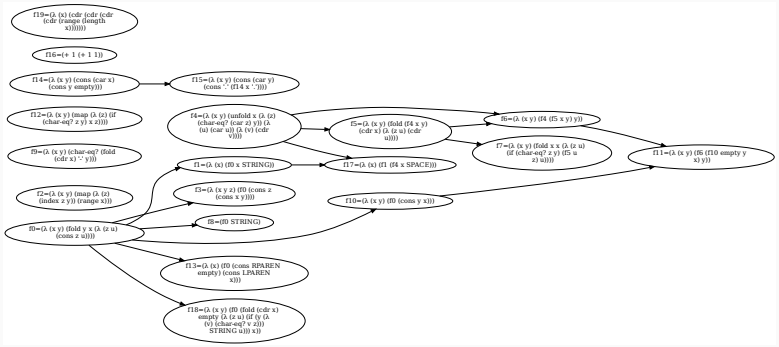
Library structure: Text Editing

DreamCoder learns libraries for FlashFill-style text editing [Gulwani 2012]





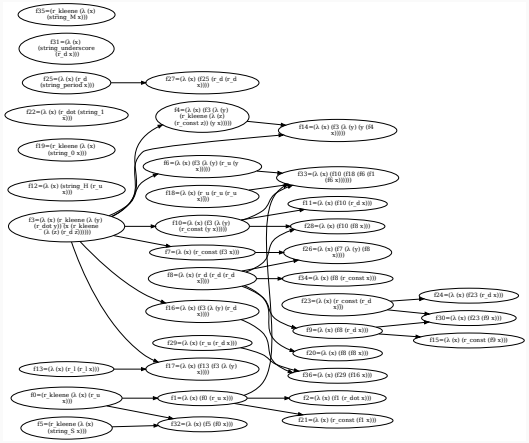


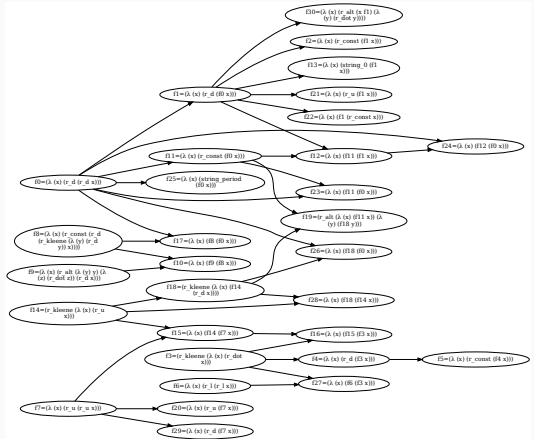


Ellis, Wong, Nye, ..., Solar-Lezama, Tenenbaum. 2020.

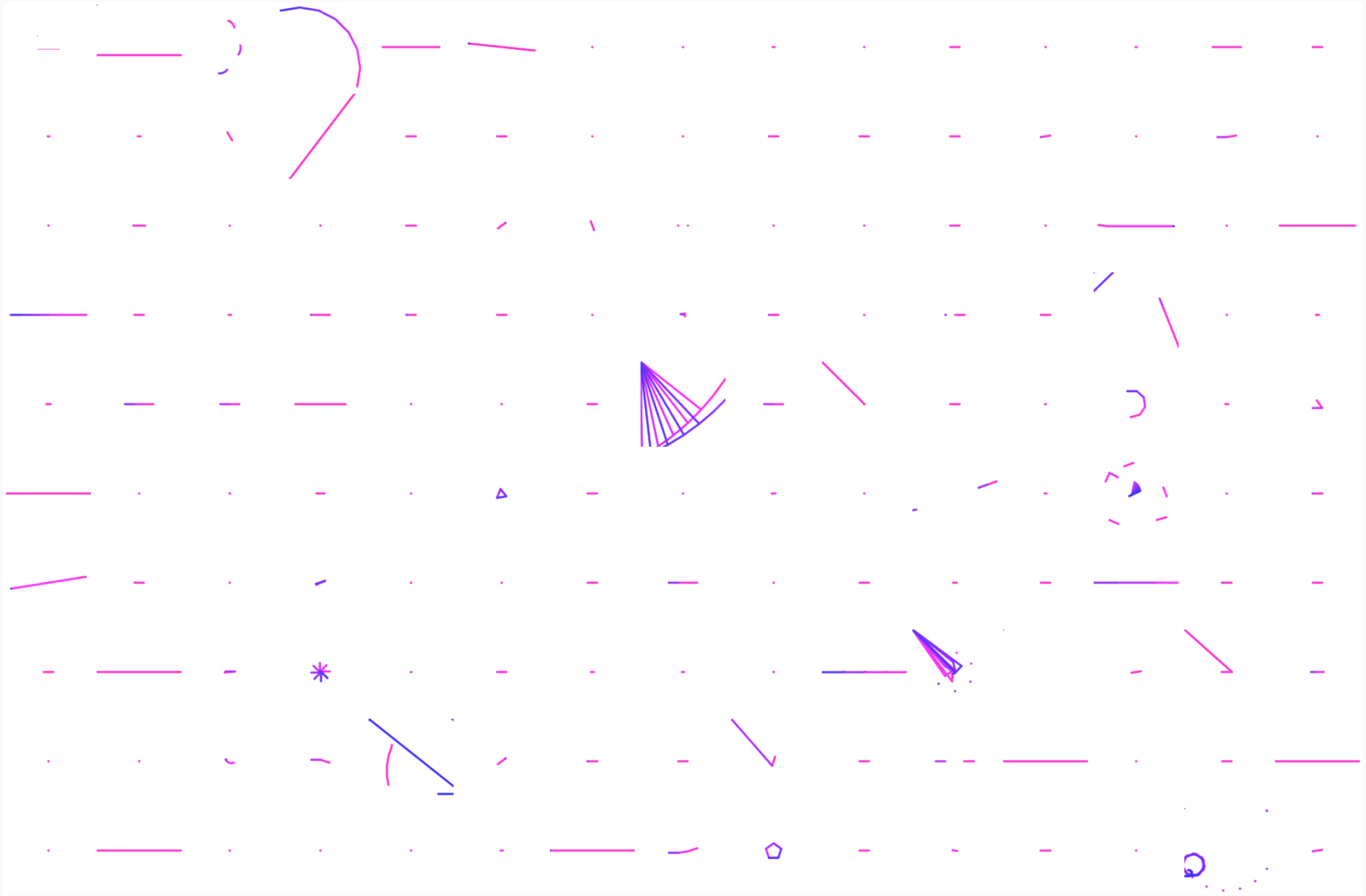
Library structure: Generating Text

Libraries for probabilistic generative models over text:
data from crawling web for CSV files





150 random dreams before learning



150 random dreams after learning