Image Super-Resolution via Sparse Representation

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Outline

- Introduction
- Background on Super-Resolution
- Single image Super-Resolution via Sparse Representation
- Experimental Results
- Conclusion

Basic Idea of Super-resolution

Given multiple low-Resolution images

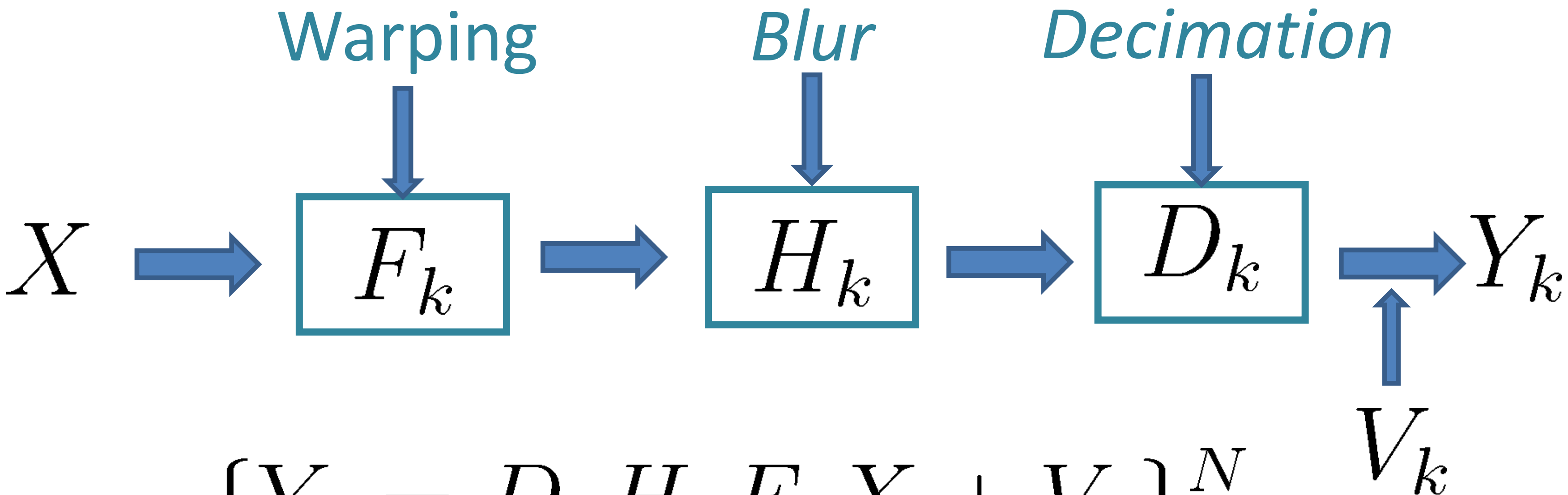
Reconstruct a high-resolution image

Algorithm

Applications

- Medical Imaging
- Satellite Imaging
- Pattern Recognition
- etc.

Model for Super-Resolution Problem



$$\{Y_k = D_k H_k F_k X + V_k\}_{k=1}^{N}$$

Note: in super-resolution problem, these are assumed to be known!

As a linear Inverse Problem

$$\{Y_k = D_k H_k F_k X + V_k\}_{k=1}^{N}$$

$$Y = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_N \end{bmatrix} = \begin{bmatrix} D_1 H_1 F_1 \\ D_2 H_2 F_2 \\ D_N H_N F_N \end{bmatrix} X + \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix} = HX + V$$

Classical Methods for Super-Resolution

- Frequency Domain Approach
- The Maximum-likelihood Method
- Projection onto Convex Sets Approach
- Bayesian Super-resolution Reconstruction
- Adaptive Filtering Approach
- etc.

Single Image Super-Resolution

Model: $z_l = DHy_h + v$

Algorithm

Given the low-resolution image z_l *, and the degradation process D, H, and the statistical property of v.*

We use patch-wise sparse representation for recovering the high-resolution image y_h

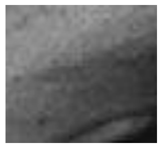
The Basic Idea

- We first interpolate the low-resolution image
- For the interpolated image, each patch has resolution enhancement process
- Average the image patched to get the final image

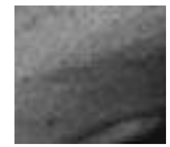
Question: How do we perform the enhancement for each patch? How to exploit the sparseness of each patch w.r.t a dictionary?

The Basic Idea

For each patch



Sparse Rep.



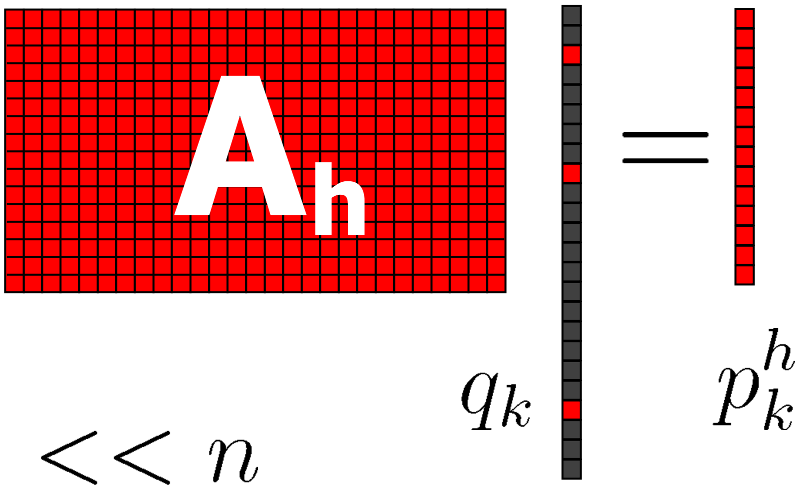
Do the sparse representation of the patches of low-resolution image under the learned dictionary A_l

Reconstruct the corresponding high-resolution patch by the the sparse representation coefficients and dictionary A_h

Same Representation Coefficients

Sparse Representation

p_k^h One arbitrary patch of high-resolution image



$$p_k^h = A_h q_k \quad and \quad ||q_k||_0 << n$$

$$||A_l q_k - p_k^l||_2 \leq \epsilon$$

Training: how to get two dictionaries?

Solution: by training

Steps:

1. We get a set of matching-patch pairs (training image)
2. Pre-process to get p_k^h and p_k^l
3. We can consider these are training samples
4. We construct the dictionaries based on training samples

Training: how to get A_l

- The training phase starts by collecting several images $\{y_h^j\}_j$
- Each of these images is blurred and down-scaled by a factor s to get $\{z_l^j\}_j$
- scaled $\{z_l^j\}_j$ up back to the original size, get $\{y_l^j\}_j$
- The process above can be formulated

$$y_l^j = Ly_h^j + v^j$$

Training: steps to get A_l

- Given $\{y_l^j\}_j$, we do filtering by K-high-pass filters in order to extract the feature in $\{y_l^j\}_j$
- From the result $f_k * y_l^j$, for one patch position, we have k patches, then concentrate together
- Use PCA to reduce size to get p_k^l
- By K-SVD/other dictionary learning method, we can get A_l provided p_k^l

Training: steps to get A_h

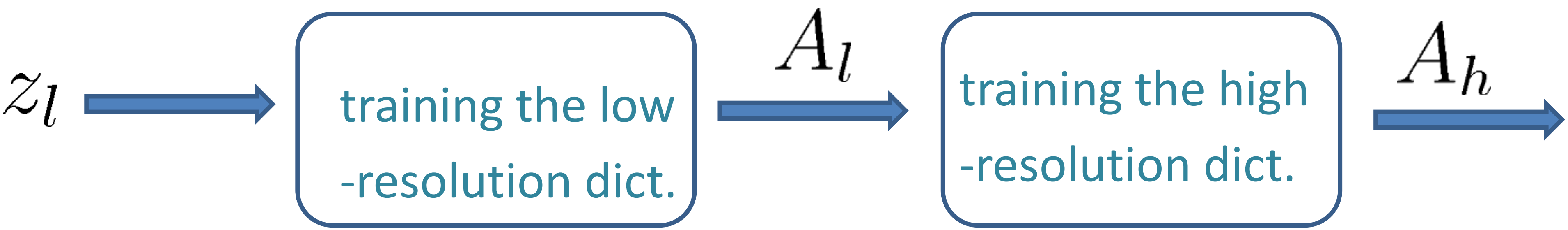
- For every j, compute $e_h^j = y_h^j - y_l^j$, since e_h^j contains edges and textures
- Extract patches from e_h^j, form p_k^h
- A_h can be obtained by following

$$A_h = \arg\min_{A_h} \sum_k ||p_k^h - A_h q_k||_2^2$$

- A_h has the closed form below

$$A_h = P_h Q^+ (QQ^T)^{-1}$$

Block Diagram for the Training



The trained low-resolution and high-resolution dictionaries are then used for high-resolution recovery of the low-resolution test images.

Testing steps for super-resolution

- Assume the test image z_l generated from a high resolution image y_h
- y_h undergoing the same blur and decimation process as used in the training process
- Scale z_l to the same size of y_h, denoted by y_l
- Extract the feature by K high-pass filter: $f_k * y_l$
- As the same procedure to produce $\{p_k^l\}_k$

Sparse Coding and Reconstruction

- With $\{p_k^l\}_k$, use OMP to find the sparse representation coefficients $\{q_k\}_k$, namely,

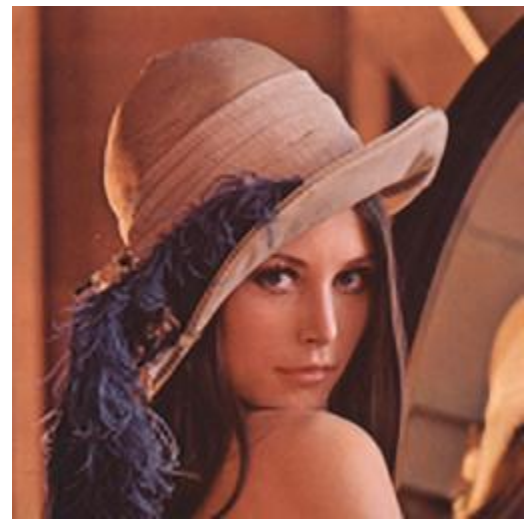
$$p_k^l = A_l q_k$$

- By $\{q_k\}_k$, compute the high-resolution image patch $\hat{p}_k^h = A_h q_k$ corresponding to p_k^l
- Emerge $\{p_k^l\}_k$ together in their locations, and average the overlapping regions
- Output: $\hat{y}_h$

Experimental Results

Bicubic Interpolation

Sparse Recovery



RMSE=5.8453

RMSE=4.5154

Experimental Results

Input image

Experimental Results

Bicubic interpolation

Experimental Results

Our method

Text Super-Resolution

Training data

This book is about *convex optimization*, a special class of mathematical optimiza
tion problems, which includes least-squares and linear programming problems. It
is well known that least-squares and linear programming problems have a fairly
complete theory, arise in a variety of applications, and can be solved numerically
very efficiently. The basic point of this book is that the same can be said for the
larger class of convex optimization problems.

While the mathematics of convex optimization has been studied for about a
century, several related recent developments have stimulated new interest in the
topic. The first is the recognition that interior-point methods, developed in the
1980s to solve linear programming problems, can be used to solve convex optimiza-
tion problems as well. These new methods allow us to solve certain new classes
of convex optimization problems, such as semidefinite programs and second-order
cone programs, almost as easily as linear programs.

The second development is the discovery that convex optimization problems
(beyond least-squares and linear programs) are more prevalent in practice than
was previously thought. Since 1990 many applications have been discovered in
areas such as automatic control systems, estimation and signal processing, com-
munications and networks, electronic circuit design, data analysis and modeling
statistics, and finance. Convex optimization has also found wide application in com-
binatorial optimization and global optimization, where it is used to find bounds on
the optimal value, as well as approximate solutions. We believe that many other
applications of convex optimization are still waiting to be discovered.

There are great advantages to recognizing or formulating a problem as a convex
optimization problem. The most basic advantage is that the problem can then be
solved, very reliably and efficiently, using interior-point methods or other special
methods for convex optimization. These solution methods are reliable enough to be
embedded in a computer-aided design or analysis tool, or even a real-time reactive
or automatic control system. There are also theoretical or conceptual advantages
of formulating a problem as a convex optimization problem. The associated dual

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Conclusion

- Image super-resolution is an important issue
- We assume the degradation is known
- Sparse representation seems a good prior for super-resolution problem
- Performance verifies this point
- Other efficient algorithms are desired!

References

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